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Paper No. 1955

TAPERED STRUCTURAL MEMBERS: AN ANALYTICAL TREATMENT

BY WALTER H. WEISKOPF[1], AND JOHN W. PICKWORTH[1], MEMBERS AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. FRED L. PLUMMER, LEROY W. CLARK, E. G. PAULET, J. CHARLES RATHBUN, HALVARD W. BIRKELAND, C. W. DUNHAM, FANG-YIN TSAI, A. A. EREMIN, AUSTIN H. REEVES, A. W. FISCHER, L. LEGENS, AND WALTER H. WEISKOPF AND JOHN W. PICKWORTH.

SYNOPSIS

The object of this paper is to present a method of analyzing structures composed of members which are not uniform in cross-section throughout their lengths. The application to some of the classical methods of analysis is indicated, and as applied to the method of moment distribution a complete development is given. Formulas for variously shaped members under various loadings are given for design use.

INTRODUCTION

The tapered member, one in which the cross-section varies from point to point along the length, is the natural and, in every respect, the correct design for a beam with restrained ends. In fact, unless the members are tapered the full advantage of continuous beams and rigidly framed structures cannot be realized. With the marked present-day trend toward the use of such structures the need increases for a satisfactory mathematical treatment of this subject.

The term, tapered member, as used herein is not confined to members in which the upper or lower edge has a curvature or haunch. Broadly stated a tapered member is one in which the moment of inertia is not constant

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NOTE.—Published in October, 1935, *Proceedings*.

[1] Cons. Engr. (Weiskopf & Pickworth), New York, N. Y.

throughout the length. In this sense a plate girder, in which the flanges are straight, but vary in area as certain parts, such as cover-plates, are added is a tapered member.

When one seeks to apply the usual theory of structures to a tapered beam difficulties appear immediately. It becomes necessary at the beginning to perform some operation, usually an integration, upon an expression of the form, $\frac{M\,dx}{E\,I}$, $\frac{M^2\,dx}{E\,I}$, or $\frac{M\,x\,dx}{E\,I}$. For a uniform member this presents no unusual difficulties since I and E are constant. In a tapered member, however, the moment of inertia, I, is usually a complicated function of x, subject to irregularities such as occur at haunches, or involving sharp discontinuities where parts of the flange begin. Although the moment of inertia, I, may be a simple function of x, it is likely to be one which will make the integration of $\frac{M\,dx}{E\,I}$ difficult or impossible.

The method explained herein for overcoming these difficulties introduces, for the actual I of the member, a function of x, the nature of which has the properties of: (1) Closely approximating the actual I-curve; and (2) being of such a form that the expression, $\frac{M\,dx}{E\,I}$, can be integrated easily. This function,[2] which is termed the substitute I-curve, is:

$$I_x = \frac{I_0}{1 + A\left(\frac{x}{l}\right)^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which I_x is the moment of inertia at any point; I_0 is the moment of inertia at the end where $x = 0$; and l is the length of the member. Constants A and n are determined so as to make the substitute I-curve fit closely to the actual curve.

FITTING THE *I*-CURVE

Fig. 1 represents a member above which are plotted the actual, and a substitute, I-curve. Obviously, the substitute I-curve (Equation (1)) agrees with the actual curve where $x = 0$, both having the value, I_0, for this point. The constant A, can be determined to make the two curves coincide at the other end of the member, $x = l$. Substituting I_l for I_x, and l for x in Equation (1) and solving for A:

$$A = \frac{I_0 - I_l}{I_l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Constant A then depends solely upon the moments of inertia at the ends of the member. From Equation (2) it is evident that the greater the taper, the greater the difference between the extreme I-values, and, therefore, the greater the value of A. For this reason, A is termed the "taper modulus" of the member. For a member of uniform cross-section the taper modulus becomes zero.

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[2] Similar formulas have been suggested by George E. Large, Assoc. M. Am. Soc. C. E., and Clyde T. Morris, M. Am. Soc. C. E., *Bulletin No. 66*. Ohio State Univ.,Columbus. Ohio, and by Max Ritter. *Schweitzerische Bauzeitung*, Vol. LIII, No. 18.

Since the taper modulus, A, depends solely upon the end values of the I-curve, the constant, n, can only affect the path between these two values, or the shape of the I-curve between the ends. Therefore, n is termed the "shape exponent".

The effect of varying the shape exponent, n, is illustrated in Fig. 2. For the purpose of this illustration, I_o is chosen as 10, and I_l as 1. Therefore, by Equation (2) $A = 9$; and, by Equation (1):

$$I_x = \frac{10}{1 + 9\left(\frac{x}{l}\right)^n} \quad \dots\dots\dots\dots\dots\dots\dots (3)$$

For each value of n the substitute I-curve takes a different path between the ends of the member. As n approaches infinity, the substitute I-curve approaches that of a uniform member of moment of inertia, I_o, while as n approaches zero the curve approaches that of a uniform member, of moment of inertia, I_l. Theoretically, the shape exponent, n, can have any value between zero and plus infinity.

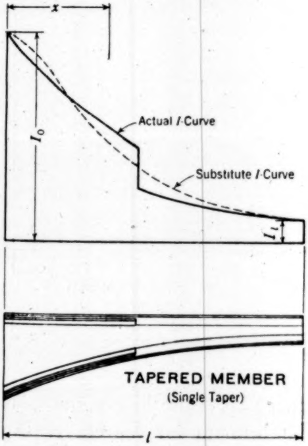


FIG. 1.

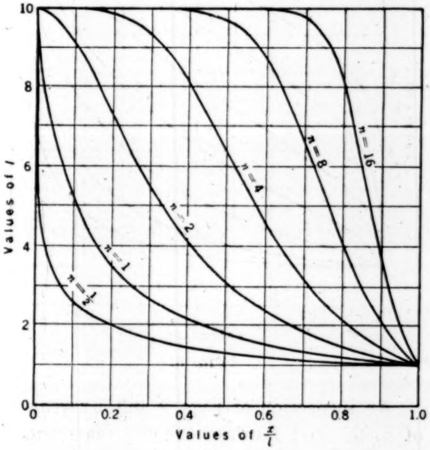


FIG. 2.

Although in Fig. 2, I_o is chosen greater than I_l, the method is equally applicable when I_l is greater than I_o. In Fig. 3 the curves are plotted for $I_o = 1$, and $I_l = 10$. In this example, A is negative and, by Equation (2), is equal to -0.9. By Equation (1),

$$I_x = \frac{1}{1 - 0.9\left(\frac{x}{l}\right)^n} \quad \dots\dots\dots\dots\dots\dots\dots (4)$$

As before, n can have any positive value. When it becomes infinity the curve again represents a uniform beam, of moment of inertia, I_0; and when n becomes zero the curve represents a uniform beam, of moment of inertia, I_l.

By properly choosing a value for n, the substitute I-curve can be made to approximate closely the actual I-curve. As an aid in selecting n, a value for it can be determined which will make the substitute I-curve pass through any intermediate point. Suppose it is desired that the substitute I-curve shall have a value, I_e, where $x = x_e$. Substitute I_e for I_x, and x_e for x in Equation (1), and solve for n; then:

$$n = \frac{\log \dfrac{I_0 - I_e}{A\, I_e}}{\log \dfrac{x_e}{l}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

By means of Equations (2) and (5) values of A and n can be determined, which will make the substitute I-curve coincide with the actual I-curve at the ends of the member and at one intermediate point.

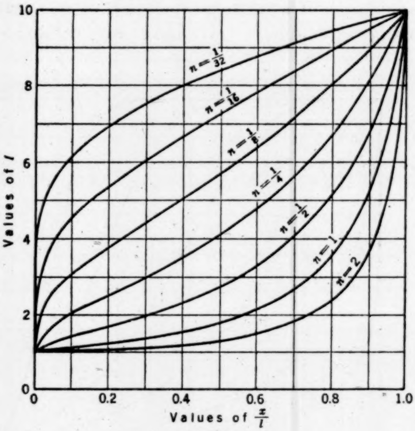


FIG. 3.

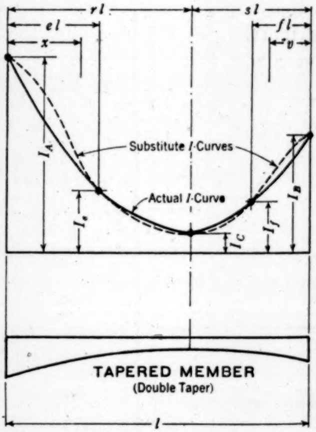


FIG. 4.

An inspection of Figs. 2 and 3 reveals the fact that regardless of the value of n, the end I-values are the maximum and minimum for the curve. No intermediate value of I can be greater than the larger end value, or less than the smaller. Since members in continuous frames very frequently have I-values at the center less than that of either end, this might appear at first to be a severe limitation to the method. The difficulty, however, can be overcome easily by treating such a member in sections. Thus, let Fig. 4 represent a member with end I-values, I_A and I_B, that are large, and with a minimum I_C, that occurs at some point a distance $r\,l$ from the left end. Such a member which can be defined as one of double taper, is divided into a left and a right section. A substitute *I*-curve can be fitted to the left section of the member by substituting $r\,l$ for l in Equation (1), which becomes,

$$I_x = \frac{I_A}{1 + A\,\dfrac{x^n}{r^n\,l^n}} \quad\text{.......................(6)}$$

For this section of the member, Equation (2) becomes,

$$A = \frac{I_A - I_C}{I_C} \quad\text{.......................(7)}$$

The substitute *I*-curve can also be made to pass through another point in the section of the member, (I_e), by means of Equation (5) which becomes,

$$n = \frac{\log \dfrac{I_A - I_e}{A\,I_e}}{\log \dfrac{e}{r}} \quad\text{.......................(8)}$$

For the right section of the member take the origin of co-ordinates at the other end, using v instead of x as the ordinate. If B is the taper modulus, and m, the shape exponent for this section, the equations of the substitute *I*-curve become,

$$I_v = \frac{I_B}{1 + B\,\dfrac{v^m}{s^m\,l^m}} \quad\text{.......................(9)}$$

$$B = \frac{I_B - I_C}{I_C} \quad\text{.......................(10)}$$

and,

$$m = \frac{\log \dfrac{I_B - I_f}{B\,I_f}}{\log \dfrac{f}{s}} \quad\text{.......................(11)}$$

It is possible, of course, to divide a member into three, four, or more sections, and fit a substitute *I*-curve to each. Practically, however, it seems that dividing into two sections gives sufficient accuracy for most engineering purposes. Ordinarily, it is not necessary to obtain an extremely close approximation. The taper modulus can be obtained exactly, and then a variation in the shape exponent produces a surprisingly small change in the resulting stress distribution.

APPLICATIONS

Many of the commonly used methods of analyzing structures composed of uniform members can be applied to structures composed of tapered members by using substitute *I*-curves. Some of these methods are: (1) Finding the elastic curve of a member; (2) finding the strain energy of a member and, from the strain energy, the deflection; (3) the method of least work; (4) the slope deflection method; and (5) the Cross method of distributing fixed-end moments.

(*1*).—*Finding the Elastic Curve of a Member.*—By means of a double integration of the fundamental expression, $\frac{d^2 y}{dx^2} = \frac{M}{EI}$, the deflection, y, is obtained as a function of x.

(*2*).—*Finding the Strain Energy of a Member and from This Energy, the Deflection.*—This problem consists of integrating between the proper limits the expression for the strain energy, W:

$$W = \int \frac{M^2\,dx}{2\,EI} \qquad (12)$$

(*3*).—*The Method of Least Work.*—To find a statically unknown force, H, this method consists in equating $\frac{\partial W}{\partial H}$ to zero and solving the resulting equation which is in the form:

$$\frac{\partial W}{\partial H} = \int \frac{M}{EI}\,\frac{\partial M}{\partial H}\,dx = 0 \qquad (13)$$

(*4*).—*The Slope Deflection Method.*—The fundamental equations of this method are derived by integrating the $\frac{M}{EI}$-diagrams; thus:

$$\theta_A - \theta_B = \int_B^A \frac{M}{EI}\,dx \qquad (14)$$

and,

$$y_A - \theta_A\,l = \int_B^A \frac{M\,x\,dx}{EI} \qquad (15)$$

in which θ_A and θ_B are the changes in slope of the points, A and B, and y_A is the deflection of one end of the member with respect to the other end.

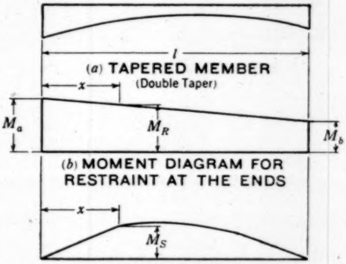
(*5*).—*The Cross Method of Distributing Fixed-End Moments.*—This method requires first finding the fixed-end moments, the stiffness, and the carry-over factors. By means of expressions of the form, $\int \frac{Mx\,dx}{EI}$ and $\int \frac{x^2\,dx}{EI}$, these moments and factors can be found for tapered members. A full development of these terms is given subsequently.

INTEGRATING THE $\frac{M}{EI}$-CURVE

All the foregoing equations are variations and recurrences of the familiar expressions, $\int \frac{M\,dx}{EI}$, $\int \frac{M^2\,dx}{EI}$, $\int \frac{M\,x\,dx}{EI}$, etc. They can all be integrated according to the following general procedure.

The bending moment at any point in a member can always be expressed as the algebraic sum of the moment due to restraint at the ends, and that due to the loads acting on the beam as if the ends were simply supported. These moment diagrams are shown in Fig 5. If M_R is the moment at any point due to end restraint, and M_s is the simple beam moment:

$$M = M_R + M_S \text{..........} (16)$$



(a) TAPERED MEMBER (Double Taper)

(b) MOMENT DIAGRAM FOR RESTRAINT AT THE ENDS

(c) SIMPLE BEAM MOMENT DIAGRAM

FIG. 5.

Since the moment diagram for end restraint is a straight line,

$$M_R = M_a \frac{(l-x)}{l} + M_b \frac{x}{l} = M_a + (M_b - M_a) \frac{x}{l} \text{ ......} (17)$$

in which M_a and M_b are the end moments.

In treating the moment, M_S, the member is divided into sections between the concentrated loads. It has already been shown how the member can be divided into sections for a discontinuity in the I-curve. Each one of these can again be divided for a discontinuity in the M_s-diagram, so that sections are obtained over which both the I-curve, and the M_S-diagram are continuous functions of x. The integral of the entire member will then be the sum of the integrals of the separate sections. Subdividing the member into sections, although it increases the work, does not add to the difficulties encountered in any other respect.

For a complicated system of loading it is often easier to plot the influence line of the quantity desired for a single concentrated load at any point on the member, and then to sum up the effects of simultaneous loads. If there are no loads other than concentrations the M_s-diagram will consist of a series of straight lines, any one of which can be expressed by an equation of the form:

$$M_S = a_0 + a_1 x \text{........................} (18)$$

If parts of the beam carry uniform loads, sections of the M_S-diagram could be expressed in the form:

$$M_S = a_0 + a_1 x + a_2 x^2 \text{.....................} (19)$$

For a uniformly varying load, there would be:

$$M_S = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \text{..................} (20)$$

When more complicated loadings occur they can be treated by the influence-line method. Therefore, it is plain that the M_S-diagram can always be divided into sections any one of which can be expressed by a formula not more complicated than Equation (20).

Furthermore, for any section of the member, from Equations (16), (17), and (20),

$$M = M_a + (M_b - M_a)\,\frac{x}{l} + a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots\ldots (21)$$

Using the substitute I-curve:

$$I_x = \frac{I_A}{1 + \dfrac{A\,x^n}{r^n\,l^n}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

it follows that,

$$\frac{M}{E\,I} = \frac{1}{E\,I_A}\left(1 + \frac{A\,x^n}{r^n\,l^n}\right)\left[M_a + (M_b - M_a)\,\frac{x}{l} + a_0 + a_1 x + a_2 x^2 + a_3 x^3\right]. (23)$$

Then,

$$\int \frac{M\,dx}{E\,I} = \int \frac{1}{E\,I_A}\left[M_a + (M_b - M_a)\,\frac{x}{l} + a_0 + a_1 x + a_2 x^2 + a_3 x^3\right] dx$$

$$+ \int \frac{A}{E\,I_A\,r^n l^n}\left[M_a x^n + (M_b - M_a)\,\frac{x^{n+1}}{l} + a_0 x^n + a_1 x^{n+1} + a_2 x^{n+2} + a_3 x^{n+3}\right] dx. (24)$$

In Equation (24) M_a and M_b are sometimes known (as is the case when a deflection is sought) and sometimes the unknowns which are to be found. In either case they do not vary with values of x, and are constants as far as the integration of Equation (24) is concerned. Since E, I_A, r, n, l, and the a-terms are constants, Equation (24) can be integrated term by term by the simplest processes of integral calculus.

For integrals of the form, $\int \frac{M\,x\,dx}{E\,I}$, all the terms of Equation (24) would contain values of x one power higher. For the form, $\int \frac{M^2\,dx}{E\,I}$, the terms would have still higher powers of x, but the integrations could be performed in the same general manner.

APPLICATION TO THE CROSS METHOD OF DISTRIBUTING FIXED-END MOMENTS

The method of moment distributions[3] introduced by Hardy Cross, M. Am. Soc. C. E., is so useful that its adaptation to tapered members will be developed in full. This will not only afford an illustration of the methods of integrating the various $\frac{M}{E\,I}$ - functions, but will also furnish a set of formulas which will enable a designer to use the Cross method without integrating each time. Formulas will be derived that will give the fixed-end moments for various conditions of loading, and the stiffness and carry-over factors, for variously shaped members. Once values for these quantities have been found by means of the formulas, the designer can proceed to distribute the fixed-end moments according to the method of Professor Cross in the same manner as for uniform members.

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[3] "Analysis of Continuous Frames by Distributing Fixed-End Moments", by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

First the expressions for beams in general will be derived. From Equations (16) and (17), a formula can be obtained for the moment at any point in a beam; thus,

$$M = M_a \frac{(l - x)}{l} + M_b \frac{x}{l} + M_S \qquad (25)$$

In Equation (25), M_a and M_b are any restraining moments at the ends. The values of these moments for fixed ends are those which would make the strain energy of the member a minimum. The strain energy is:

$$W = \int_0^l \frac{M^2 \, dx}{2 \, E \, I_x} \qquad (26)$$

Then,

$$\frac{\partial W}{\partial M_A} = \int_0^l \frac{M}{E \, I_x} \, \frac{\partial M}{\partial M_A} \, dx = 0 \qquad (27)$$

with a similar equation for $\frac{\partial W}{\partial M_B}$. For fixed-end moments, Equation (25) becomes:

$$M = M_A \left(\frac{l - x}{l} \right) + M_B \frac{x}{l} + M_S \qquad (28)$$

Then,

$$\frac{\partial M}{\partial M_A} = \frac{l - x}{l} \qquad (29)$$

and,

$$\frac{\partial M}{\partial M_B} = \frac{x}{l} \qquad (30)$$

Substituting Equations (28) (29), and (30) in Equation (27) and its companion formula for $\frac{\partial W}{\partial M_B}$, and then simplifying and solving for M_A and M_B:

$$M_A = -l \, \frac{\displaystyle\int_0^l \frac{M_S (l - x) \, dx}{I_x} \int_0^l \frac{x^2 \, dx}{I_x} - \int_0^l \frac{M_S \, x \, dx}{I_x} \int_0^l \frac{x (l - x) \, dx}{I_x}}{\displaystyle\int_0^l \frac{x^2 \, dx}{I_x} \int_0^l \frac{(l - x)^2 \, dx}{I_x} - \left[\int_0^l \frac{x (l - x) \, dx}{I_x} \right]^2} \qquad (31)$$

and,

$$M_B = -l \, \frac{\displaystyle\int_0^l \frac{M_S \, x \, dx}{I_x} \int_0^l \frac{(l - x)^2 \, dx}{I_x} - \int_0^l \frac{M_S (l - x) \, dx}{I_x} \int_0^l \frac{x (l - x) \, dx}{I_x}}{\displaystyle\int_0^l \frac{x^2 \, dx}{I_x} \int_0^l \frac{(l - x)^2 \, dx}{I_x} - \left[\int_0^l \frac{x (l - x) \, dx}{I_x} \right]^2} \qquad (32)$$

Denote the integrals in Equations (31) and (32) as follows:

$$\int_0^l \frac{x(l-x)\,dx}{I_x} = \frac{l^3 F_1}{I_A} \qquad \text{(33)}$$

$$\int_0^l \frac{x^2\,dx}{I_x} = \frac{l^3 F_2}{I_A} \qquad \text{(34)}$$

and,

$$\int_0^l \frac{(l-x)^2\,dx}{I_x} = \frac{l^3 F_3}{I_A} \qquad \text{(35)}$$

If P is a concentrated load located at any point on the A-section of a member of double taper,

$$\int_0^l \frac{M_S\, x\,dx}{I_x} = \frac{P\, l^3 F_4}{I_A} \qquad \text{(36)}$$

and,

$$\int_0^l \frac{M_S\,(l-x)\,dx}{I_x} = \frac{P\, l^3 F_5}{I_A} \qquad \text{(37)}$$

Then,

$$M_A = -P\,l\,\frac{F_5 F_2 - F_4 F_1}{F_2 F_3 - F_1^2} \qquad \text{(38)}$$

and,

$$M_B = -P\,l\,\frac{F_4 F_3 - F_5 F_1}{F_2 F_3 - F_1^2} \qquad \text{(39)}$$

If the load, P, is on the B-section of a member of double taper,

$$\int_0^l \frac{M_S\, x\,dx}{I_x} = \frac{P\, l^3 F_6}{I_A} \qquad \text{(40)}$$

and,

$$\int_0^l \frac{M_S\,(l-x)\,dx}{I_x} = \frac{P\, l^3 F_7}{I_A} \qquad \text{(41)}$$

Then,

$$M_A = -P\,l\,\frac{F_7 F_2 - F_6 F_1}{F_2 F_3 - F_1^2} \qquad \text{(42)}$$

and,

$$M_B = -P\,l\,\frac{F_6 F_3 - F_7 F_1}{F_2 F_3 - F_1^2} \qquad \text{(43)}$$

If a member supports a uniformly distributed load, w, per unit length:

$$\int_0^l \frac{M_S\, x\,dx}{I_x} = \frac{w\, l^4 F_8}{I_A} \qquad \text{(44)}$$

and,

$$\int_0^l \frac{M_S\,(l-x)\,dx}{I_x} = \frac{w\, l^4 F_9}{I_A} \qquad \text{(45)}$$

Then,

$$M_A = -w\,l^2\,\frac{F_9 F_2 - F_8 F_1}{F_2 F_3 - F_1^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots(46)$$

and,

$$M_B = -w\,l^2\,\frac{F_8 F_3 - F_9 F_1}{F_2 F_3 - F_1^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots(47)$$

CARRY-OVER FACTOR

According to Professor Cross the carry-over factor is defined[4] as follows:

> "If one end of a member which is on unyielding supports at both ends is rotated while the other end is held fixed the ratio of the moment at the fixed end to the moment producing rotation at the rotating end is herein called the 'carry-over factor'."

Since there are no loads on the member except the end moments, Equation (25) becomes,

$$M = M_a\left(\frac{l-x}{l}\right) + M_b\,\frac{x}{l} \quad\ldots\ldots\ldots\ldots\ldots\ldots(48)$$

If End B is fixed, M_b in Equation (48) becomes M_B. If C_{AB} is the carry-over factor from End A to End B, the foregoing definition may be expressed algebraically, as follows:

$$C_{AB} = \frac{M_B}{M_a} \quad\ldots\ldots\ldots\ldots\ldots\ldots(49)$$

The strain energy for the member (see Equation (26)) is:

$$W = M_a^2 \int_0^l \frac{(l-x)^2\,dx}{2\,l^2\,E\,I_x} + M_a\,M_B \int_0^l \frac{x\,(l-x)\,dx}{l^2\,E\,I_x} + M_B^2 \int_0^l \frac{x^2\,dx}{2\,l^2\,E\,I_x} \quad\ldots(50)$$

For fixation at End B:

$$\frac{\partial W}{\partial M_B} = M_a \int_0^l \frac{x\,(l-x)\,dx}{l^2\,E\,I_x} + M_B \int_0^l \frac{x^2\,dx}{l^2\,E\,I_x} = 0 \quad\ldots\ldots(51)$$

Therefore,

$$C_{AB} = \frac{M_B}{M_a} = -\frac{\displaystyle\int_0^l \frac{x\,(l-x)\,dx}{I_x}}{\displaystyle\int_0^l \frac{x^2\,dx}{I_x}} = -\frac{F_1}{F_2} \quad\ldots\ldots\ldots(52)$$

Similarly, for the other end,

$$C_{BA} = -\frac{F_1}{F_3} \quad\ldots\ldots\ldots\ldots\ldots\ldots(53)$$

STIFFNESS

Professor Cross defines[4] "stiffness" of a member as follows:

> "'Stiffness', as herein used, is the moment at one end of a member (which is on unyielding supports at both ends) necessary to produce unit rotation of that end when the other end is fixed."

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[4] "Analysis of Continuous Frames by Distributing Fixed-End Moments", by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 2.

According to this definition the stiffness of a member would be $\frac{M_a}{\theta_A}$. For uniform members Professor Cross does not use this value, however, but uses instead $\frac{I}{l}$, which is proportional to it, the relation between the two values being,

$$\frac{I}{l} = \frac{M_a}{4\,E\,\theta_A} \qquad \text{(54)}$$

It is immaterial, of course, whether one uses $\frac{I}{l}$, or $\frac{M_a}{\theta_A}\left(\text{which is } \frac{4\,E\,I}{l}\right)$ as long as one uses the same form for all the members of the structure. Since for uniform members, $\frac{I}{l}$ is a most convenient form, and is in common use, and since uniform and tapered members may be combined in the same structure, the stiffness of a member as used herein is in a form that corresponds to $\frac{I}{l}$, for uniform members. Therefore, the stiffness, S_A, of End A may be expressed by the formula:

$$S_A = \frac{M_a}{4\,E\,\theta_A} \qquad \text{(55)}$$

The slope deflection, θ_A, can be found by equating the external to the internal work; thus:

$$\frac{M_a\,\theta_A}{2} = \int_0^l \frac{M^2\,dx}{2\,E\,I_x} = M_a^2 \int_0^l \frac{(l-x)^2\,dx}{2\,l^2\,E\,I_x} + M_a\,M_B \int_0^l \frac{x\,(l-x)\,dx}{l^2\,E\,I_x} + M_B^2 \int_0^l \frac{x^2\,dx}{2\,l^2\,E\,I_x} \qquad \text{(56)}$$

$$\theta_A = \frac{M_a \int_0^l \frac{(l-x)^2\,dx}{I_x} + 2\,M_B \int_0^l \frac{x\,(l-x)\,dx}{I_x} + \frac{M_B^2}{M_a} \int_0^l \frac{x^2\,dx}{I_x}}{E\,l^2} \qquad \text{(57)}$$

and,

$$\theta_A = \frac{\left(M_a\,F_3 + 2\,M_B\,F_1 + \frac{M_B^2}{M_a}\,F_2\right) l}{E\,I_A} \qquad \text{(58)}$$

From Equation (52),

$$\frac{M_B}{M_a} = -\,\frac{F_1}{F_2} \qquad \text{(59)}$$

Then,

$$\theta_A = \frac{(M_a\,F_3 + M_B\,F_1)\,l}{E\,I_A} \qquad \text{(60)}$$

and (see Equation (55)):

$$S_A = \frac{M_a\,I_A\,E}{4\,E\,(M_a\,F_3 + M_B\,F_1)\,l} = \frac{F_2\,I_A}{4\,(F_2\,F_3 - F_1^2)\,l} \qquad \text{(61)}$$

Similarly, for the other end,

$$S_B = \frac{F_3 I_A}{4(F_2 F_3 - F_1^2)\,l} \quad \text{....................(62)}$$

Equations (38), (39), (42), (43), (46), (47), (52), (53), (61), and (62) are general expressions for the fixed-end moments, carry-over factors, and stiffness for members of any kind in terms of the integrals expressed by the symbols, F_1, F_2, etc. For uniform members these F-terms can be evaluated directly. For tapered members they can be found by performing the integrations indicated, using substitute I-curves.

A summary of the formulas used in this method giving values for the F-terms is given subsequently in Cases 1 to 5. In each example, the quantities, F_1, F_2, and F_3, are functions of the properties of the member and are independent of the loading. The stiffness and carry-over factors, which also depend on properties of a member only, are given in terms of F_1, F_2, and F_3. The expression, $F_2 F_3 - F_1^2$, occurs repeatedly as the denominator in formulas for stiffness and for values of the fixed-end moments. The F-terms with subscripts greater than 3 are functions of the loading as well as of the properties of the member.

EVALUATION OF F-TERMS

Case 1.—Unsymmetrical Member of Double Taper.—This is the most general case (see Fig. 6). First, the values of A, B, n, m, C_{AB}, C_{BA}, S_A, and S_B, are determined by Equations (7), (8), (10), (11), (52), (53), (61), and (62), respectively, in which:

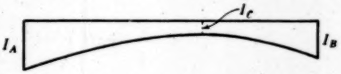

FIG. 6.—CASE 1, UNSYMMETRICAL MEMBER (DOUBLE TAPER.)

$$F_1 = r^2\left[\frac{1}{2} - \frac{r}{3} + A\left(\frac{1}{n+2} - \frac{r}{n+3}\right)\right]$$
$$+ s^2\,\frac{I_A}{I_B}\left[\frac{1}{2} - \frac{s}{3} + B\left(\frac{1}{m+2} - \frac{s}{m+3}\right)\right] \quad \text{.........(63)}$$

$$F_2 = r^3\left[\frac{1}{3} + \frac{A}{n+3}\right] + s\,\frac{I_A}{I_B}\left[1 - s + \frac{s^2}{3}\right.$$
$$\left. + B\left(\frac{1}{m+1} - \frac{2s}{m+2} + \frac{s^2}{m+3}\right)\right] \quad \text{..............(64)}$$

and,

$$F_3 = r\left[1 - r + \frac{r^2}{3} + A\left(\frac{1}{n+1} - \frac{2r}{n+2} + \frac{r^2}{n+3}\right)\right]$$
$$+ s^3\,\frac{I_A}{I_B}\left[\frac{1}{3} + \frac{B}{m+3}\right] \quad \text{....................(65)}$$

When a load, P, is placed on the left section, a distance, $k\,l$, from the origin, ($k \leqq r$), the moments are determined by Equations (38) and (39), in which:

$$F_4 = k\,F_1 - \frac{k^3}{6} - A\,\frac{k^{n+3}}{r^n\,(n+2)\,(n+3)} \qquad \text{(66)}$$

and,

$$F_5 = k\,F_3 - \frac{3\,k^2 - k^3}{6} - A\,\frac{k^{n+2}(n+3-kn-k)}{r^n\,(n+1)\,(n+2)\,(n+3)} \qquad \text{(67)}$$

When the load is on the right section, ($k \geqq r$), the moments are determined by Equations (42) and (43), in which,

$$F_6 = (1-k)\,F_2 - \frac{I_A\,(2 - 3\,k + k^3)}{6\,I_B} - B\,\frac{I_A\,(1-k)^{m+2}\,(2 + k\,m + k)}{I_B\,s^m\,(m+1)\,(m+2)\,(m+3)} \qquad \text{(68)}$$

and,

$$F_7 = (1-k)\,F_1 - \frac{I_A\,(1-k)^3}{6\,I_B} - B\,\frac{I_A\,(1-k)^{m+3}}{I_B\,s^m\,(m+2)\,(m+3)} \qquad \text{(69)}$$

In Equations (66) to (69) the values of F_1, F_2, and F_3 are those of Equations (63), (64), and (65), respectively.

The moments for a uniform load over this entire beam are determined by Equations (46) and (47), in which:

$$F_8 = \frac{r^2}{2}\left[\frac{1}{3} - \frac{r}{4} + A\left(\frac{1}{n+3} - \frac{r}{n+4}\right)\right] + \frac{s^2}{2}\,\frac{I_A}{I_B}\left[\frac{1}{2} - \frac{2\,s}{3} + \frac{s^2}{4} + B\left(\frac{1}{m+2} - \frac{2\,s}{m+3} + \frac{s^2}{m+4}\right)\right] \qquad \text{(70)}$$

and,

$$F_9 = \frac{r^2}{2}\left[\frac{1}{2} - \frac{2\,r}{3} + \frac{r^2}{4} + A\left(\frac{1}{n+2} - \frac{2\,r}{n+3} + \frac{r^2}{n+4}\right)\right] + \frac{s^3\,I_A}{2\,I_B}\left[\frac{1}{3} - \frac{s}{4} + B\left(\frac{1}{m+3} - \frac{s}{m+4}\right)\right] \qquad \text{(71)}$$

Case 2.—Symmetrical Member of Double Taper.—The formulas applying to the symmetrical member of double taper (see Fig. 7) are obtained by making the two ends alike in the formulas of Case 1; thus: $I_A = I_B$; $r = s = 0.5$; and $A = B$.

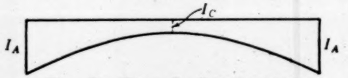

FIG. 7.—CASE 2, SYMMETRICAL MEMBER (DOUBLE TAPER.)

As before, the values of A, n, $C_{AB} = C_{BA}$, and $S_A = S_B$ are determined by Equations (7), (8), (52), and (61), respectively, in which,

$$F_1 = \frac{1}{6} + A\,\frac{n+4}{4\,(n+2)\,(n+3)} \qquad \text{(72)}$$

$$F_2 = \frac{1}{3} + A\,\frac{(n+1)\,(n+4) + 4}{4\,(n+1)\,(n+2)\,(n+3)} \qquad \text{(73)}$$

and,

$$F_3 = F_2 \qquad \text{(74)}$$

In applying Equation (61) to this case it is to be noted that since $F_3 = F_2$ in a symmetrical member, the denominator for the stiffness and fixed-end moments becomes $F_2^2 - F_1^2$.

When the load is on the left section, ($k \leqq \frac{1}{2}$), the moments are determined by Equations (38) and (39) in which,

$$F_4 = k F_1 - \frac{k^3}{6} - A \frac{2^n k^{n+3}}{(n+2)(n+3)} \quad \ldots\ldots\ldots\ldots\ldots (75)$$

and,

$$F_5 = k F_2 - \frac{3k^2 - k^3}{6} - A \frac{2^n k^{n+2}(n + 3 - kn - k)}{(n+1)(n+2)(n+3)} \quad \ldots\ldots (76)$$

When the load is on the right section, ($k \geqq \frac{1}{2}$), the moments are determined by Equations (42) and (43), in which:

$$F_6 = (1-k) F_2 - \frac{2 - 3k + k^3}{6} - A \frac{2^n (1-k)^{n+2}(2 + kn + k)}{(n+1)(n+2)(n+3)} \quad \ldots (77)$$

and,

$$F_7 = (1-k) F_1 - \frac{(1-k)^3}{6} - A \frac{2^n (1-k)^{n+3}}{(n+2)(n+3)} \quad \ldots\ldots\ldots (78)$$

In Equations (75) to (78) the values of F_1 and F_2 are those of Equations (72), and (73), respectively.

When the loading is symmetrical a further simplification occurs in the fixed-end moments due to the fact that $F_4 = F_5$. Thus, for a concentrated load at the center of a symmetrical beam, Equation (38) becomes,

$$M_A = M_B = - P l \frac{F_{10}}{F_2 + F_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

in which,

$$F_{10} = \frac{1}{16} + \frac{A}{8(n+2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (80)$$

Similarly, for a uniformly distributed load, $F_9 = F_8$, Equation (46) becomes,

$$M_A = M_B = - w l^2 \frac{F_8}{F_2 + F_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (81)$$

in which,

$$F_8 = \frac{1}{24} + A \frac{n + 4}{16(n+2)(n+3)} \quad \ldots\ldots\ldots\ldots\ldots (82)$$

Case 3.—Unsymmetrical Member of Single Taper.—For this case (see Fig. 8), $r = 1$ and $s = 0$. Making the necessary substitutions in Equations (7), (8), (52), (53), (61), and (62), the values of A, n, C_{AB}, C_{BA}, S_A and S_B, are determined as in Case 1, with:

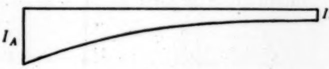

FIG. 8.—CASE 3, TAPERED MEMBER (SINGLE TAPER.)

$$F_1 = \frac{1}{6} + \frac{A}{(n+2)(n+3)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

$$F_2 = \frac{1}{3} + \frac{A}{n+3} \quad \text{.......................(84)}$$

and,

$$F_3 = \frac{1}{3} + \frac{2A}{(n+1)(n+2)(n+3)} \quad \text{..............(85)}$$

The moments with a concentrated load at any distance, $k\,l$, from the left end are determined by Equations (38) and (39), with:

$$F_4 = k F_1 - \frac{k^3}{6} - A \frac{k^{n+3}}{(n+2)(n+3)} \quad \text{.............(86)}$$

and,

$$F_5 = k F_2 - \frac{3k^2 - k^3}{6} - A \frac{k^{n+2}(n+3-kn-k)}{(n+1)(n+2)(n+3)} \quad \text{......(87)}$$

In Equations (86) and (87) the values of F_1, F_2, and F_3 are those of Equations (83), (84), and (85), respectively.

The moments for a uniform load over the entire beam, are determined by Equations (46) and (47), in which:

$$F_8 = \frac{1}{24} + \frac{A}{2(n+3)(n+4)} \quad \text{.................(88)}$$

and,

$$F_9 = \frac{1}{24} + \frac{A}{(n+2)(n+3)(n+4)} \quad \text{..............(89)}$$

Since members of this kind are sometimes required to withstand earth pressure, formulas are necessary for triangular loading. For example, when the member is loaded as shown in Fig. 9(a), Equation (31) becomes,

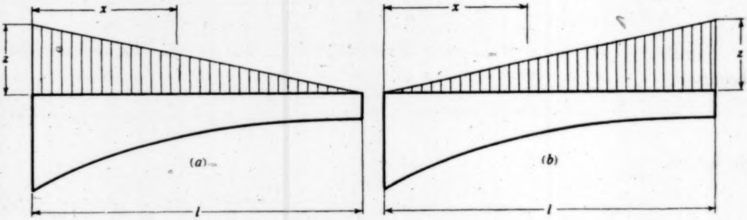

Fig. 9.—Triangular Loadings.

$$M_A = -z\,l^2 \frac{F_{12} F_2 - F_{11} F_1}{F_2 F_3 - F_1^2} \quad \text{..................(90)}$$

and Equation (32) becomes,

$$M_B = -z\,l^2 \frac{F_{11} F_3 - F_{12} F_1}{F_2 F_3 - F_1^2} \quad \text{..................(91)}$$

in which,

$$F_{11} = \frac{7}{360} + \frac{A\,(n+7)}{6\,(n+3)\,(n+4)\,(n+5)} \qquad \text{(92)}$$

and,

$$F_{12} = \frac{1}{45} + \frac{A\,(n+8)}{3\,(n+2)\,(n+3)\,(n+4)\,(n+5)} \qquad \text{(93)}$$

When the member is loaded as shown in Fig. 9(*b*), Equations (31) and (32) become,

$$M_A = -z\,l^2\,\frac{F_{14}\,F_2 - F_{13}\,F_1}{F_2\,F_3 - F_1^2} \qquad \text{(94)}$$

and,

$$M_B = -z\,l^2\,\frac{F_{13}\,F_3 - F_{14}\,F_1}{F_2\,F_3 - F_1^2} \qquad \text{(95)}$$

in which,

$$F_{13} = \frac{1}{45} + \frac{A}{3\,(n+3)\,(n+5)} \qquad \text{(96)}$$

and,

$$F_{14} = \frac{7}{360} + \frac{A\,(2\,n+7)}{3\,(n+2)\,(n+3)\,(n+4)\,(n+5)} \qquad \text{(97)}$$

By adding the moments due to uniform and triangular loadings, moments for a trapezoidal loading diagram can be obtained.

Case 4.—"Stepped" Member, without Taper.—The stepped member (see Fig. 10) finds application in the columns of mill buildings, for example. In this case, A and B are equal to zero and C_{AB}, C_{BA}, S_A, and S_B are determined by Equations (52), (53), (61), and (62), in which:

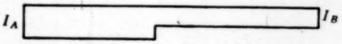

FIG. 10.—CASE 4, STEPPED MEMBER.

$$F_1 = r^2\left(\frac{1}{2} - \frac{r}{3}\right) + s^2\,\frac{I_A}{I_B}\left(\frac{1}{2} - \frac{s}{3}\right) \qquad \text{(98)}$$

$$F_2 = \frac{r^3}{3} + \frac{s\,I_A}{I_B}\left(1 - s + \frac{s^2}{3}\right) \qquad \text{(99)}$$

and,

$$F_3 = r\left(1 - r + \frac{r^2}{3}\right) + \frac{s^3\,I_A}{3\,I_B} \qquad \text{(100)}$$

When the concentrated load is to the left of the step the moments are determined by Equations (38) and (39), substituting values of F_4 and F_5 from Equations (66) and (67), in which the taper modulus, A, is equal to zero.

When the concentrated load is to the right of the step, the moments are determined by Equations (42) and (43), except that the taper modulus, B, in Equations (68) and (69) is zero.

The moments for a uniform load over the entire beam are determined by Equations (46) and (47) in which:

$$F_8 = \frac{r^3}{2}\left(\frac{1}{3} - \frac{r}{4}\right) + \frac{s^2 I_A}{2 I_B}\left(\frac{1}{2} - \frac{2s}{3} + \frac{s^2}{4}\right) \text{........(101)}$$

and,

$$F_9 = \frac{r^2}{2}\left(\frac{1}{2} - \frac{2r}{3} + \frac{r^2}{4}\right) + \frac{s^3 I_A}{2 I_B}\left(\frac{1}{3} - \frac{s}{4}\right) \text{.........(102)}$$

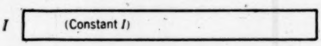

FIG. 11.—CASE 5, UNIFORM MEMBER.

Case 5.—Member with Uniform Depth and Cross-Section.—For the simplest type of member, with a uniform depth and cross-section (see Fig. 11), in which $r = 1$ and $s = 0$, the formulas are derived directly from the equations of Case 4; that is, $F_1 = \frac{1}{6}$; $F_2 = F_3 = \frac{1}{3}$; $C_{AB} = C_{BA} = -\frac{1}{2}$; and $S_A = S_B = \frac{I}{l}$. With a concentrated load, the moments are determined by Equations (38) and (39), in which:

$$F_4 = \frac{k - k^3}{6} \text{........................(103)}$$

and,

$$F_6 = \frac{2k - 3k^2 + k^3}{6} \text{....................(104)}$$

or simply:

$$M_A = -P\,l\,k\,(1 - k)^2 \text{..................(105)}$$

and,

$$M_B = -P\,l\,k^2\,(1 - k) \text{..................(106)}$$

When the concentrated load is at the center of the beam, $F_{10} = \frac{1}{16}$ and Equation (79) becomes:

$$M_A = M_B = -P\,l\,\frac{F_{10}}{F_2 + F_1} = -\frac{P\,l}{8} \text{...........(107)}$$

For a uniformly distributed load, $F_8 = F_9 = \frac{1}{24}$, and Equation (81) becomes:

$$M_A = M_B = -w\,l^2\,\frac{F_8}{F_2 + F_1} = -\frac{w\,l^2}{12} \text{..........(108)}$$

For a triangular loading, such as that in Fig. 9(*a*), $M_A = -\frac{z\,l^2}{20}$; and $M_B = -\frac{z\,l^2}{30}$. The moments may also be determined by Equations (90) and (91), recalling that $F_2 = F_3$; $F_{11} = \frac{7}{360}$; and $F_{12} = \frac{1}{45}$. Thus, by breaking down from Case 1 to Case 5 the familiar values for uniform beams are obtained.

VARIATION OF THE CROSS METHOD

Professor Cross gives one variation in his method well worth mention. When one end of a member is free to rotate, no moment need be carried over to it, but a modified stiffness should be used for the other end. This modified stiffness for tapered members is found as follows: Assuming End *B* as free to rotate, make M_B zero in Equation (58); thus,

$$\theta_A = \frac{M_a F_3 l}{E I_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (109)$$

Then, Equation (55) becomes:

$$S'_A = \frac{M_a}{4 E \theta_A} = \frac{I_A}{4 l F_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (110)$$

Similarly, if End *A* is hinged:

$$S'_B = \frac{I_A}{4 l F_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (111)$$

WORKING LINES

In applying the Cross method to framed structures composed of tapered members it is important to fix the working lines correctly. It has been found that where the curvature of a member is not too great, the working line can be taken as a straight line near the neutral axis of the minimum section. The reason for this location becomes obvious when one considers that the work terms are maximum where the moment of inertia is small, and that the working line should be located to be correct for the region contributing the greatest amount to the internal work. As an illustration Fig. 12 shows the working line for a part of a framed structure. Such lines are to be used only for the determination of the statically unknown quantities. In determining the maximum fiber stresses, the statically unknown factors having been found, the working lines, of course, should be the neutral surfaces of the members.

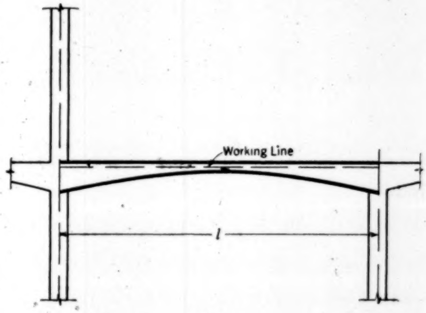

FIG. 12.

APPROXIMATE SOLUTION

Since structures composed of tapered members are usually statically indeterminate the procedure is first to assume a trial design, analyze the stresses, modify the design, re-analyze the stresses, etc. It is very convenient in work of this nature to have an approximate solution that can be applied quickly.

Values of 2 for the shape exponent, n, have been noted frequently where the taper modulus is positive. If this value is used the work is considerably shortened, often with little sacrifice of accuracy. It has also been found in members of double taper that the I-curve is quite flat for a considerable distance near the central part of the member. There would then be small loss in accuracy by making r and s equal to one-half.

As an illustration of this procedure formulas for the F-terms of Cases 1 to 3, with the aforementioned approximations, are as follows:

In Case 1 (Equations (63) to (71)):

$$F_1 = \frac{1}{12} + \frac{3A}{80} + \frac{I_A}{I_B}\left[\frac{1}{12} + \frac{3B}{80}\right] \tag{112}$$

$$F_2 = \frac{1}{24} + \frac{A}{40} + \frac{I_A}{I_B}\left[\frac{7}{24} + \frac{B}{15}\right] \tag{113}$$

$$F_3 = \frac{7}{24} + \frac{A}{15} + \frac{I_A}{I_B}\left[\frac{1}{24} + \frac{B}{40}\right] \tag{114}$$

$$F_4 = k\,F_1 - \frac{k^3}{6} - \frac{A\,k^5}{5} \tag{115}$$

$$F_5 = k\,F_3 - \frac{3\,k^2 - k^3}{6} - A\,\frac{k^4\,(5 - 3\,k)}{15} \tag{116}$$

$$F_6 = (1 - k)\,F_2 - \frac{I_A\,(2 - 3\,k + k^3)}{6\,I_B} - B\,\frac{I_A\,(1 - k)^4\,(2 + 3\,k)}{15\,I_B} \tag{117}$$

$$F_7 = (1 - k)\,F_1 - \frac{I_A\,(1 - k)^3}{6\,I_B} - B\,\frac{I_A\,(1 - k)^5}{5\,I_B} \tag{118}$$

$$F_8 = \frac{5}{384} + \frac{7A}{960} + \frac{I_A}{I_B}\left[\frac{11}{384} + \frac{11B}{960}\right] \tag{119}$$

and,

$$F_9 = \frac{11}{384} + \frac{11A}{960} + \frac{I_A}{I_B}\left[\frac{5}{384} + \frac{7B}{960}\right] \tag{120}$$

In Case 2 (Equations (72) to (82)):

$$F_1 = \frac{1}{6} + \frac{3A}{40} \tag{121}$$

$$F_2 = \frac{1}{3} + \frac{11A}{120} = F_3 \tag{122}$$

$$F_4 = k\,F_1 - \frac{k^3}{6} - \frac{A\,k^5}{5} \tag{123}$$

$$F_5 = k\,F_2 - \frac{3\,k^2 - k^3}{6} - A\,\frac{k^4\,(5 - 3\,k)}{15} \tag{124}$$

$$F_6 = (1 - k)\,F_3 - \frac{2 - 3\,k + k^3}{6} - A\,\frac{(1 - k)^4\,(2 + 3\,k)}{15} \; \ldots\ldots (125)$$

$$F_7 = (1 - k)\,F_1 - \frac{(1 - k)^3}{6} - A\,\frac{(1 - k)^5}{5} \; \ldots\ldots (126)$$

$$F_{10} = \frac{1}{16} + \frac{A}{32} \; \ldots\ldots (127)$$

and,

$$F_8 = \frac{1}{24} + \frac{3\,A}{160} = F_9 \; \ldots\ldots (128)$$

In Case 3 (Equations (83) to (97)):

$$F_1 = \frac{1}{6} + \frac{A}{20} \; \ldots\ldots (129)$$

$$F_2 = \frac{1}{3} + \frac{A}{5} \; \ldots\ldots (130)$$

$$F_3 = \frac{1}{3} + \frac{A}{30} \; \ldots\ldots (131)$$

$$F_4 = k\,F_1 - \frac{k^3}{6} - A\,\frac{k^5}{20} \; \ldots\ldots (132)$$

$$F_5 = k\,F_3 - \frac{3\,k^2 - k^3}{6} - A\,\frac{k^4\,(5 - 3\,k)}{60} \; \ldots\ldots (133)$$

$$F_8 = \frac{1}{24} + \frac{A}{60} \; \ldots\ldots (134)$$

$$F_9 = \frac{1}{24} + \frac{A}{120} \; \ldots\ldots (135)$$

$$F_{11} = \frac{7}{360} + \frac{A}{140} \; \ldots\ldots (136)$$

$$F_{12} = \frac{1}{45} + \frac{A}{252} \; \ldots\ldots (137)$$

$$F_{13} = \frac{1}{45} + \frac{A}{105} \; \ldots\ldots (138)$$

and,

$$F_{14} = \frac{7}{360} + \frac{11\,A}{2520} \; \ldots\ldots (139)$$

CONCLUSION

It is intended that this analysis be taken broadly as suggesting a general method of treating tapered members. Although the material presented gives the application to certain types of analysis and to some special kinds of members, it can be elaborated and extended to apply to many other kinds of analysis and other forms of members. It is hoped that the method of substitute I-curves will thus aid in the development of classes of structures which have been hampered in the past by mathematical difficulties.

APPENDIX

NOTATION

a_0, a_1, a_2, etc. = constants in the equation of a simple-beam moment diagram;
e = proportion of ordinate to span;
f = proportion of ordinate to span;
k = proportion of the ordinate of a concentrated load to the span;
l = length, or span, of a structural member;
m = a shape exponent;
n = a shape exponent;
r = proportion of length of section of a member to span;
s = proportion of length of section of a member to span;
v = a horizontal distance measured from the right end of a member;
w = load per unit distance;
x = horizontal distance measured from the left end of a member;
y = deflection; y_A = deflection of one end of a member from its original position;
z = maximum intensity of a uniformly varying load;
A = taper modulus;
B = taper modulus;
C_{AB} = carry-over factor from End A to End B, etc.;
E = modulus of elasticity;
F_1, F_2, F_3, etc. = constants in the Cross method of distributing fixed-end moments;
H = an unknown force in the method of least work;
I = rectangular moment of inertia; I_x = moment of inertia at any point; I_0, I_A, etc. = moment of inertia where $x = 0$, at Point A, etc.;
M = bending moment; M_R = bending moment at any point due to end restraint; M_S = moment at any point in a member, assuming the ends to be simply supported; M_a = restraining moment at End A, etc.; M_A = fixed-end moment at End A, etc.;
P = concentrated load;
S_A = stiffness of a member in the Cross method at End A, etc.;
S'_A = modified stiffness at End A of a member when End B is free to rotate, etc.;
W = work, or energy; strain energy;
θ = slope deflection; θ_A = slope deflection at Point A, etc.

DISCUSSION

Fred L. Plummer,[5] M. Am. Soc. C. E. (by letter).—In an effective manner the authors illustrate a valuable, although not new, method of expressing mathematically the approximate variation in moment of inertia of structural members of variable cross-section. The form of the empirical equation is such that the integrations required for the solution of most problems involving deflections or redundant forces in structures consisting of members subjected to bending moments, can be performed very easily. In 1925, the writer developed a modified form of the slope deflection method which could be used for members of variable cross-section (printed in blue-print form only). Formulas corresponding to Equation (31) to Equation (47) were developed. In applying these formulas to specific types of members the true variation in moment of inertia was used in many cases, even though the resulting integrations were quite difficult. In a few cases a variation identical with that proposed by the authors was used.

Unfortunately, many engineers in design offices are unwilling or unable to carry through the very simple integrations which are required when the method suggested by the authors is followed. Formulas or tabulated coefficients must be made available before such engineers, many of whom have had little or no training in the analysis of continuous structures, can be expected to use such structures and to design them easily and competently. These men will welcome the formulas developed by the authors. Extensive tables of coefficients have been prepared and published by L. T. Evans[6], Assoc. M. Am. Soc. C. E., E. B. Russell[7], T. F. Hickerson[8], M. Am. Soc. C. E., and others. The mere availability of tables and formulas giving coefficients for tapered members, however, will not produce well designed structures. Such aids are only tools to help eliminate some of the analysis routine. The values depend upon definite assumptions[9] which must be recognized and given consideration when the designing engineer attempts to predict how a given structure will act and what stresses will probably exist when such action takes place.

A number of tables containing constants for use in designing tapered beams[10] has been produced by A. Strassner. These values were revised by Walter Ruppel, Assoc. M. Am. Soc. C. E., and printed in a form suitable for use in connection with the conjugate and fixed-point methods of analysis.[11] The same values are equally important in connection with other methods since they represent some of the general integrals set up by the authors. Six sets of values are given by the tables for various types of members and kinds of loads. The tables cover several forms of members and types of loading. Equations (52), (53), (61), (62), (110), and (111) of the paper may be extended, as follows, respectively:

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[5] Cons. Engr.; Assoc. Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

[6] "Handbook of Rigid Frame Analysis", 1934, Edwards Bros., Inc.

[7] "Analysis of Continuous Frames", by Ellison and Russell, 1934.

[8] "Structural Frameworks", 1934, Univ. of North Carolina Press.

[9] "Continuous Frames of Reinforced Concrete", 1932, John Wiley & Sons, Inc.

[10] "Neuere Methoden", Vol. 1, Second Edition, Berlin, 1921.

[11] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 167 *et seq.*

$$C_{AB} = -\frac{F_1}{F_2} = \frac{-v}{1-v} \quad \dots\dots\dots\dots\dots\dots (140)$$

$$C_{BA} = -\frac{F_1}{F_3} = \frac{-u}{1-u} \quad \dots\dots\dots\dots\dots\dots (141)$$

$$S_A = \frac{I_A}{4\,L}\,\frac{F_2}{(F_2\,F_3 - F^2_1)} = \frac{I}{4\,L}\,\frac{1-v}{p\,(1-u-v)} \dots\dots\dots (142)$$

$$S_B = \frac{I_A}{4\,L}\,\frac{F_3}{(F_2\,F_3 - F^2_1)} = \frac{I}{4\,L}\,\frac{1-u}{q\,(1-u-v)} \dots\dots (143)$$

$$S'_A = \frac{I_A}{4\,L\,F_3} = \frac{I}{4\,L\,p\,(1-u)} \quad \dots\dots\dots\dots\dots (144)$$

and,

$$S'_B = \frac{I_A}{4\,L\,F_2} = \frac{I}{4\,L\,q\,(1-v)} \quad \dots\dots\dots\dots\dots (145)$$

In Equations (140) to (145), I is the minimum value for the members and is not necessarily equal to I_A, the moment of inertia at the left end of the member. Furthermore (differing from the notation of the paper),

$$u = \frac{\dfrac{1}{L}\displaystyle\int_0^L \frac{(L-x)\,x\,dx}{Z}}{\displaystyle\int_0^L \frac{(L-x)\,dx}{Z}}$$

and,

$$v = \frac{\dfrac{1}{L}\displaystyle\int_0^L \frac{(L-x)\,x\,dx}{Z}}{\displaystyle\int_0^L \frac{x\,dx}{Z}}$$

in which Z is the ratio of the moment of inertia of the cross-section to the least value of the moment of inertia for the member.

The writer has reproduced and demonstrated the use of these same constants in connection with the slope deflection, moment distribution, and other methods.[13] While these tables cover a wide range of conditions, the original tables by Strassner are even more extensive and include all forms that are commonly used in engineering structures.

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[13] "Statically Indeterminate Structures", 1934, Edwards Bros., Inc.

Any engineer who attempts to develop additional tables, however, will find that the method suggested by the authors will very materially facilitate his work and, at the same time, result in no appreciable loss of accuracy.

LeRoy W. Clark,[13] M. Am. Soc. C. E. (by letter).—The method of analyzing tapered members presented by the authors is interesting and ingenious, but the writer doubts whether it will be generally adopted. To test its usefulness he assigned to a group of graduate students some problems using haunched beams, and found that except in a few cases they could obtain correct results in less time by using the column analogy[14], while the substitute *I*-curve gave results which were very unreliable.

For beams with moderate haunches the results agree very well with correct values. For the beam, as shown in Fig. 13(*a*), using the authors' suggested approximate value of 2 for the shape modulus, *n* gives results not exceeding 8% in error. With *n* based on *I* at a distance of 7.5 ft from End *A*, the errors do not exceed $2\frac{1}{2}$ per cent.

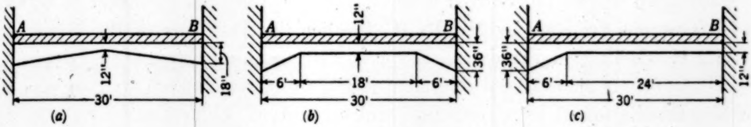

FIG. 13.—EXAMPLES OF TAPERED MEMBERS.

For beams with deep short haunches the method is not so satisfactory. Computations were made for the beam shown in Fig. 13(*b*). The shape exponent was taken first as 2, and then calculated for the moments of inertia at distances of 1, 2, 3, 4, and 5 ft from End *A*. The percentage of error in each case is shown in Table 1. If *n* is computed for the moment of inertia at a distance of 6 ft, the results are the same as those for a prismatic beam of uniform depth of 1 ft.

TABLE 1.—PERCENTAGE OF ERROR, WEISKOPF-PICKWORTH ANALYSIS

Values of *n*	Bending moment, *M*.	Stiffness factor, *S*.	Carry-over factor, *C*.
2	4.8	146	9.3
Values of *n* corresponding to the moment of inertia at:			
$x = 1$ ft	2.4	85	4.9
$x = 2$ ft	2.6	90	5.3
$x = 3$ ft	2.2	81	4.5
$x = 4$ ft	0.7	55	1.9
$x = 5$ ft	—3.5	7	—5.3

In the case of an unsymmetrical beam of the dimensions shown in Fig. 13(*c*), an attempt to solve the problem by using one section for the entire beam and a value of $n = 2$, gave absurd results, as might have been expected.

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[13] Prof. of Mechanics, Rensselaer Polytechnic Inst.; Cons. Engr., Troy, N. Y.
[14] *Bulletin 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

With this solution, using a value of n based on the moment of inertia at $x = 3$ ft, the stiffness factors at Ends A and B were 110% and 25% in error. respectively. Using two I-curves reduced the maximum error to about 4%, but the labor involved is at least as great as in the column analogy. It is evident that the greatest errors always occur in the stiffness factor, S.

In practically all the problems solved the results were considerably in error, and the errors varied widely, depending upon the point through which the substitute I-curve is made to pass. This is a matter of judgment, and a designer with limited experience would have no criterion by which to judge the correctness of his work, whereas he could have absolute confidence in the results of the column analogy. Although an experienced designer could doubtless use the substitute I-curve satisfactorily, it is doubtful whether he would save much time in so doing, and in all probability he would have constructed curves from which the desired results could be read easily. Such a series of curves was prepared by Mr. R. H. Trathen, in "A Study of the Effect of Beam Haunching on Fixed End Moment, Stiffness, and Carry-Over Factor."[15]

The authors should be complimented for an extremely well written, and a clear presentation of an interesting solution.

E. G. PAULET,[16] ASSOC. M. AM. SOC. C. E. (by letter).—The formula for the variation of moment of inertia introduced by Professor Max Ritter and presented by Walter Ruppel, Assoc. M. Am. Soc. C. E.[17], is directly reducible to Equation (1) suggested by the authors. When the shape exponent is made equal to 1, a special case of Professor Ritter's formula results.[18] The writer has used the formula for this special case in the analysis of tapered members encountered in structures, such as swing and bascule girder bridges, and rigid-frame bridges.

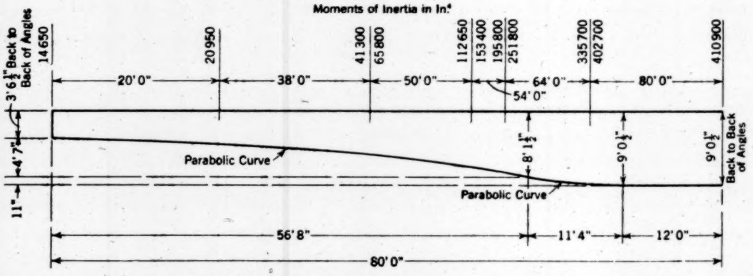

FIG. 14.—CHARACTERISTICS OF ONE ARM OF A SWING SPAN.

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[15] Thesis presented to Rensselaer Polytechnic Inst. in partial fulfillment of the requirements for the degree of Master of Civil Engineering.

[16] Bridge Design Engr., State Highway Comm., Baton Rouge, La.

[17] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 159; also, "Analysis of Continuous Frames", by E. B. Russell, Second Edition, p. 23, San Francisco, Calif., 1934.

[18] *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio.

In Fig. 14 is shown one arm of an actual girder swing span with equal arms, the bottom chord of which is on a tangent and two reverse curves, the ratio of the curves being one-fifth, although one-fourth is also employed. Using the authors' notation and letting Fig. 15 represent the tapered member with its elastic behavior similar to that of the swing span arm, the ordinates of the influence line for the center reaction are given by the following equations:

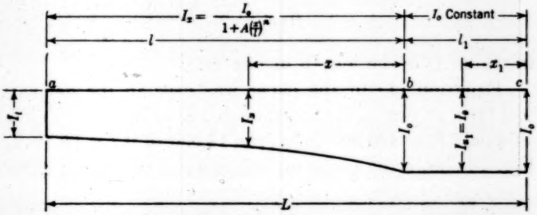

FIG. 15.—TAPERED MEMBER USED FOR ANALYSIS OF SWING SPAN.

$$y_{(x=0\ \mathrm{to}\ l)} = \frac{3}{\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \left\{ \frac{L^2 l}{2} + l^3 \left[-\frac{1}{6} + A\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right] + \frac{x\,(l^2 - L^2)}{2} + x^2 \left[-\frac{l}{2} + \frac{x}{6} - \frac{A\,x^n}{l^{n-1}}\left(\frac{1}{n+1} - \frac{1}{n+2}\right) - \frac{A\,x^{n+1}}{l^n}\left(-\frac{1}{n+2} + \frac{1}{n+3}\right)\right]\right\} \quad ..(146)$$

and,

$$y_{(x_1=0\ \mathrm{to}\ l_1)} = 1.0 - \frac{x_1^2\,(3\,L - x_1)}{2\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \quad ..(147)$$

From Equations (146) and (147), the positive center reaction due to a uniform load, $w = 1.0$ per unit length, covering both arms, is:

$$R_{\mathrm{center}} = \frac{6\,l^3}{\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]} \left\{ \frac{L^3}{4} + l^2 \left[-\frac{1}{24} + A\,l^3 \left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} - \frac{\left(\frac{1}{n+1} - \frac{1}{n+2}\right)}{n+3} - \frac{\left(-\frac{1}{n+2} + \frac{1}{n+3}\right)}{n+4}\right)\right]\right\} + \left[2 - \frac{l_1^2\,(4\,L - l_1)}{4\left[L^3 + 3\,A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right]}\right] l_1 \quad .....(148)$$

and the negative end reaction due to a uniform load, $w = 1.0$ per unit length, covering one arm, is:

$$R_{\text{end}} = -\frac{\text{Equation (148)} - L}{4} \qquad \text{(149)}$$

in which L is the length of one arm.

Equation (1) of the paper was used in the derivation of Equations (146) to (149)[19]. For a shape exponent, $n = 1.0$, the following design values were adopted: $I_0 = 403\,000$ in.^{4}; $I_l = 14\,650$ in.^{4}; $l = 60$ ft; $l_1 = 20$ ft; and $A = 26.5$. Fig. 16 shows the paths of the actual I-curve and substitute I-curve. A value of $l = 64$ ft for the length of the tapered member would be preferable, provided a suitable shape exponent, n, were determined. However, since Equations (146) to (149) had first been derived with a value of $n = 1.0$, and the substitute I-curve with such a value would rise quickly as shown in Fig. 16, a length, $l = 60$ ft, for the tapered member was chosen as a rational value.

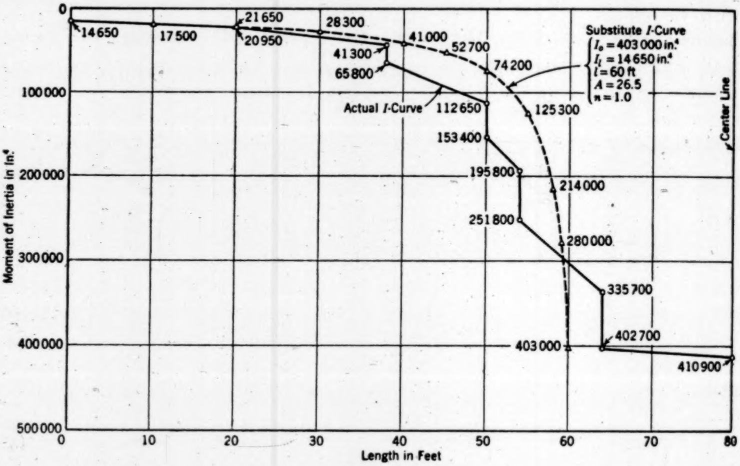

FIG. 16.

After substitution of the design values in Equations (148) and (149), the center reaction is found to be $1.471\,w\,L$, and the negative end reaction, $-\,0.1178\,w\,L$. By the moment-area method, the center reaction is $1.472\,w\,L$, and the negative end reaction is $-\,0.1179\,w\,L$. Therefore, when a shape exponent, $n = 1.0$, is used, and judgment is exercised in the selection of a rational value for l, similar cases may be thusly approached and results obtained satisfactory for all practical purposes. However, since the shape exponent, n, appears in Equations (146) to (149), its rational value could well be substituted in these equations. The most probable value of n, obtained by the least square method, is 1.64, and the arithmetic mean value is 1.55 ($l = 64$ ft being used). Obviously, such values for n can be obtained only when the sections at various points along the tapered member are already known. Consequently, a value of n that is suitable for the majority of cases would be desirable. The authors mention a value of 2 for n. If $n = 2$, then $l = 64$ ft, $l_1 = 16$ ft, $L = 80$ ft, and $A = 26.5$, are substituted in Equations (148) and (149), the center reaction is found to be 1.451 $w\,L$ and the negative end reaction is $-\,0.1129\ w\,L$. These values of reactions compare very favorably with those found by the moment-area method, and a shape exponent, $n = 2$, would be a satisfactory value for this class of structures.

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[19] The complete discussion, with supporting derivation, has been filed for reference in Engineering Societies Library, New York, N. Y.

A comparison of the values for the center reaction and the negative end reaction obtained by considering I to be variable, with those obtained when I is considered constant, shows that the center and end reactions of the former case are, respectively, 18% and 88% greater than in the latter case. For uplift, the consideration of the tapered member analysis is important. This analysis yields final design moments which are greater than those obtained by the analysis when I is considered constant, the excess varying from 9% at the quarter-point from the end to 4% at the center support.

The deflection at each end of the tapered arms of a swing span with equal arms due to a concentrated load, P, applied at each end is:

$$y = \frac{P}{E\,I_0}\left[\frac{L^3}{3} + A\,l^3\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right] \ldots\ldots (150)$$

Let $P = 15\,000$ lb; $E = 30\,000\,000$ lb per sq in.; $I_0 = 403\,000$ in.^{4}; $L = 80$ ft; $l = 64$ ft; and $A = 26.5$. Substituting these values in Equation (150), the end deflection, y, is found to be 1.03 in. when $n = 1.64$, 1.09 in. when $n = 1.55$, and 1.62 in. when $n = 1.0$. By the moment-area method, $y = 1.11$ in.

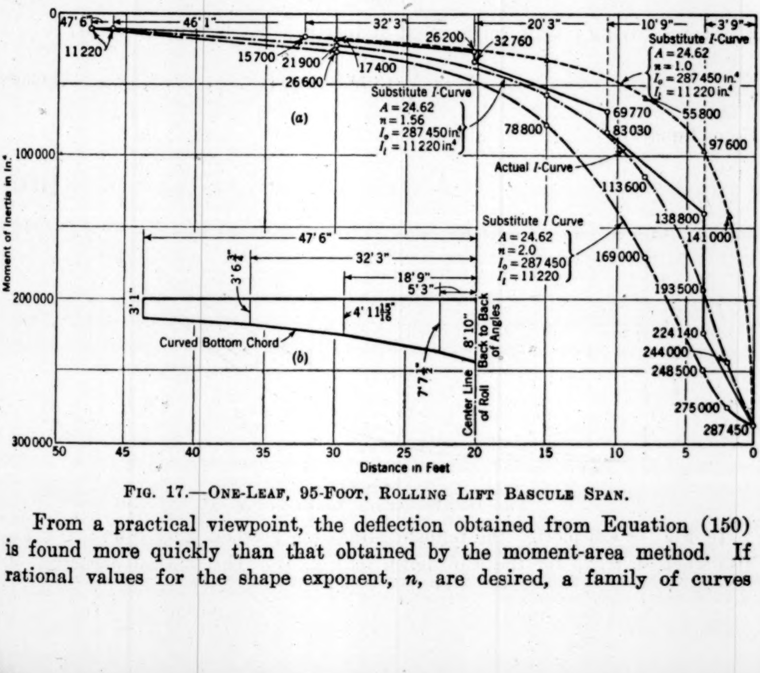

FIG. 17.—ONE-LEAF, 95-FOOT, ROLLING LIFT BASCULE SPAN.

From a practical viewpoint, the deflection obtained from Equation (150) is found more quickly than that obtained by the moment-area method. If rational values for the shape exponent, n, are desired, a family of curves similar to Fig. 2 may be prepared and used in conjunction with a diagram of the actual I-curve of the member drawn to the same scale as the n-curves. By properly placing the n-curves over the actual I-curve, as shown in Fig. 1, a satisfactory value for the shape exponent, n, may be chosen.

In Fig. 17 are given the general dimensions and moments of inertia at various sections of one leaf of a double-leaf, rolling, lift, bascule girder bridge. The taper modulus, A, is 24.62. The most probable value of the shape exponent, n, is 1.56, and the arithmetic mean value is 1.42. The graph of the three substitute I-curves having shape exponents, 1.0, 1.56, and 2.0, shows the relative departure of these curves from the actual I-curve.

The shear, V, carried by the center lock of a double-leaf bascule bridge, the main girders of which are tapered and loaded with a concentrated load, P, distant $k\,l$ from the center line of trunnion or roll (the leaves being assumed cantilevered from the trunnion or roll, and $I_x = \dfrac{I_0}{1 + A\left(\dfrac{x}{l}\right)^n}$) is:

$$V = P\left[\frac{k^2}{2} - \frac{k^3}{6} + A\,k^{n+3}\left(-\frac{1}{n+2} + \frac{1}{n+3}\right) + A\,k^{n+2}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right] \div 2\left[\frac{1}{3} + A\left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}\right)\right] \quad\ldots\ldots\ldots(151)$$

and adopting a shape exponent of $n = 2.0$,

$$V = P\left[\frac{2\,(15\,k^2 - 5\,k^3) + 5\,A\,k^4 - 3\,A\,k^5}{40 + 4\,A}\right] \quad\ldots\ldots\ldots\ldots(152)$$

For leaves having a uniform moment of inertia (that is, with $A = 0$),

$$V = \frac{P}{4}\,(3\,k^2 - k^3) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(153)$$

From Equation (152), the limit of V as A approaches infinity is:

$$\frac{P}{4}\,(5\,k^4 - 3\,k^5) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(154)$$

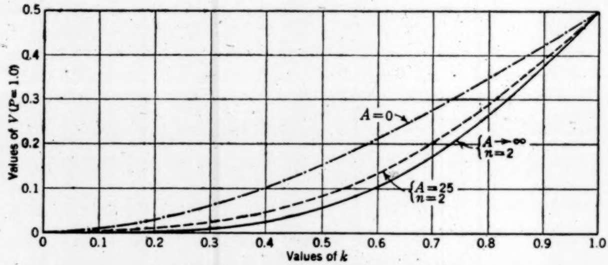

FIG. 18.—INFLUENCE LINES FOR V.

In Fig. 18 are plotted the influence lines for V when $A = 0$ and $A \to \infty$, also when $A = 25$ for the leaf shown in Fig. 17. The marked departure of the curves for $A = 0$ and $A \rightarrow \infty$ near the center of the leaf where it is a maximum at $k = 0.624$, indicates whether the leaf should be analyzed as a member of constant moment of inertia or as a tapered member. The choice is a matter of individual preference. Considering the uncertainty of end-fixity of the leaves at the trunnion, or the center line of the roll, and the value of the positive taper modulus, A, as being in general above 20, the writer has adopted the following equation for the influence line for V,

$$V = \frac{P k^2}{8} (3 - k + 5 k^2 - 3 k^3) \text{ ....} (155)$$

The influence line given by Equation (155) is for $A = 10$, and plots midway between the curves for $A = 0$ and $A \rightarrow \infty$.

The shear, V, carried by the center lock, due to a uniform load, w, per unit length, extending throughout the length, l, of one leaf is: For a tapered member:

$$V = \frac{w\,l \left[\frac{1}{4} + A \left(\frac{1}{n+1} - \frac{3}{n+2} + \frac{3}{n+3} - \frac{1}{n+4} \right) \right]}{4 \left[\frac{1}{3} + A \left(\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \right) \right]} \text{ ...} (156)$$

and for a member of uniform moment of inertia,

$$V = \frac{3\,w\,l}{16} \text{ ....} (157)$$

From Equation (157), when $n = 2$ and $A = 10$,

$$V = \frac{5\,w\,l}{32} \text{ ....} (158)$$

Furthermore, when $n = 2$ and $A \rightarrow \infty$, the limit of,

$$V = \frac{w\,l}{8} \text{ ....} (159)$$

It may be seen from Equations (157), (158), and (159), that as the value of A increases, the value of V rapidly converges toward $\frac{w\,l}{8}$. For a concentrated load, P, on one leaf, a rapid convergence also exists.

With the aid of Equation (1), of the paper, an analysis of various types of structures having tapered members will show directly the relative behavior of their component parts. The derivation of formulas for sought values, while not difficult, may be lengthy and tedious at times; however, for the simpler types of structures which are of frequent application, an analytical treatment should certainly receive consideration. The authors[20] have already contributed much information in this direction.

In his analysis of symmetrical parabolic arches, F. Guerrini[21] uses a function for the substitute *I*-curve similar to the authors' Equation (4), and which is:

$$I_x \cos \theta_x = \frac{I_0}{1 - 0.5 \left(\frac{x}{l} \right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (160)$$

in which θ_x is the angle that the tangent to the parabolic axis of the arch makes with the horizontal at any point distant x from the crown, and l is the half-span length, I_0 and I_x being, respectively, the moments of inertia of the arch at the crown and at any point distant x from the crown.

Professor J. Rieger[22] has analyzed members of variable moment of inertia involving straight haunches. His analysis is exact but usually leads into complicated expressions unless certain approximations are adopted. He has also considered such approximations and has compared the results with those obtained by exact analysis. Tapered members with straight haunches may be analyzed with Equation (1) when a suitable shape exponent, n, is used. For positive taper modulus, $A = 4$ to $A = 10$, a value of 2 for n would be satisfactory for all practical purposes. For straight haunches, with positive taper modulus, $A = 0$ to $A = 20$, the curve of arithmetic mean values for n is approximately given by, $n = \sqrt{\frac{A + 0.14}{3.5}} + 0.8$, and the curve of most probable values for n is approximately given by, $n = 0.63 \sqrt{\frac{A + 0.14}{3.5}} + 0.874$. The variable error occurring when these approximate equations are used does not exceed 6 per cent.

The method of substitute *I*-curves is convenient for the direct analysis of structures having tapered members. When more than three statically indeterminate quantities are sought and, therefore, a direct solution becomes involved if not impracticable, recourse to a method such as the slope-deflection method or the Cross method of distributing fixed-end moments, is convenient. With the authors' method of substitute *I*-curves all the necessary coefficients entering into the slope-deflection and Cross methods may be computed and put in tabular[23] or graphical[18] form.

Although Equation (1) is Professor Ritter's general formula re-stated in a different form, its convenient presentation, its interpretation, and its applications, undoubtedly demonstrate the convenient adaptability and far-reaching possibilities of the method of substitute *I*-curves.

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[20] "Symmetrical Rigid Frames", and "Unsymmetrical Rigid Frames", by W. H. Weiskopf and J. W. Pickworth, Am. Inst. of Steel Construction, N. Y., 1934.

[21] "Calcul des Ponts avec Arc Parabolique et Tablier Supérieur", by F. Guerrini. *Le Constructeur de Ciment Armé*, Paris, July, 1935, p. 137.

[22] "Calcul des Constructions Hyperstatiques", by J. Rieger, Vol. II, 1931, p. 446, Paris, France.

[23] "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, 1932, p. 289.

J. CHARLES RATHBUN,[24] M. AM. SOC. C. E. (by letter).—As a study of one variation of the beam theory, this paper is of undoubted value. The authors have developed, rather completely, the theory as applied to a beam with moments of inertia that vary according to a given mathematical formula that contains three arbitrary constants. As the paper is well written, it should help very much in clarifying the beam theory.

From an engineering standpoint the writer feels that the value of this work depends upon its superiority over other methods both as to ease of application and of understanding. Whether the method can stand this test is open to question. Using this paper as a guide the analysis of a few beams would involve considerable study on the part of the engineer. Some labor could be saved in the authors' methods if one were to develop a series of tables or diagrams covering the various changes in the formula for taper as the assumed constants change. This would only be economical if the engineer were required to deal with a very large number of tapered beams to which this formula applies. The writer feels that such an addition should not be made as the merit of the paper lies in its theoretical rather than its practical interest.

The most obvious variation of the method used by the authors is to assume that the $\frac{1}{I}$-curve varies as a parabola, or that,

$$\frac{1}{I} = p_2 x^2 + p_1 x + p_0 \ldots\ldots\ldots\ldots\ldots\ldots (161)$$

in which p_2, p_1, and p_0 are constants so chosen that the curve will pass through predetermined points. By assuming more terms, the curve of Equation (161) may be made to pass through four or more points. Loads are usually concentrated, uniform, or, in a few cases, uniformly varying. These loads lead to moment curves with equations of the form:

$$M = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \ldots\ldots\ldots\ldots\ldots (162)$$

in which, again, a_3, a_2, a_1, and a_0 are constants. It follows at once that the $\frac{M}{I}$-curve is of this same form. Equation (162) is a form that is readily integrable as the only integral form used is $\int x^n \, dx = \frac{x^{n+1}}{n+1}$. This statement is true also of the expression in Equations (12), (13), (14), and (15), of the paper, and the expressions used in the Cross method. These parabolic curves can be made to fit the $\frac{M}{I}$-curve more closely than those of the authors as the irregularities of the curves (as in Fig. 1) need not be ironed out. If Equation (161) is substituted in Equation (52), the carry-over factor, C_{AB}, is obtained.

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[24] Associate Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

As examples of the method of using Equations (161) and (162), consider the five cases cited by the authors, E being considered equal to unity.

Case 1.—Assume Equation (161) as the formula of the $\frac{1}{I}$-curve, and define i_A as $\frac{1}{I}$ at the point, A. When $x = 0$, $\frac{1}{I_A} = i_A$; when $x = l$, $\frac{1}{I_B} = i_B$; and, when $x = \frac{l}{2}$, $\frac{1}{I_C} = i_C$, the origin being taken at the left end. Consequently, $i_A = p_0$; $i_B = p_2 l^2 + p_1 l + p_0$; and, $i_C = \frac{p_2 l^2}{4} + p_1 \frac{l}{2} + p_0$. From these equations it follows that, $p_0 = i_A$; $p_1 = \frac{4 i_C - i_B - 3 i_A}{l}$; and $p_2 = \frac{2 (i_A - i_B - 2 i_C)}{l^2}$.

The $\frac{M}{I}$-curve becomes, for a uniform load: $\frac{M}{I} = \left(\frac{w l x - w x^2}{2}\right)(p_2 x^2 + p_1 x + p_0)$; and, by double integration:

$$y = \frac{w l}{2}\left(p_0 \frac{x^3}{6} + p_1 \frac{x^4}{12} + p_2 \frac{x^5}{20}\right) - \frac{w}{2}\left(p_0 \frac{x^4}{12} + p_1 \frac{x^5}{20} + p_2 \frac{x^6}{30}\right) + C x \quad \text{..(163)}$$

The constant, C, is obtained by placing $y = 0$ and $x = l$.

Case 2.—In this example, i_A may equal i_B as suggested by the authors, where $p_0 = i_A$; $p_1 = \frac{4 i_C - 4 i_A}{l}$; and $p_2 = \frac{4 i_A - 4 i_C}{l^2} = \frac{-p_1}{l}$. On the other hand, one may shift the origin to the center taking advantage of the symmetry of the equations. Then the $\frac{M}{I}$-curve becomes $\left(\frac{w l^2}{8} - \frac{w x^2}{2}\right)(i_C + p x^2)$; and, by double integration:

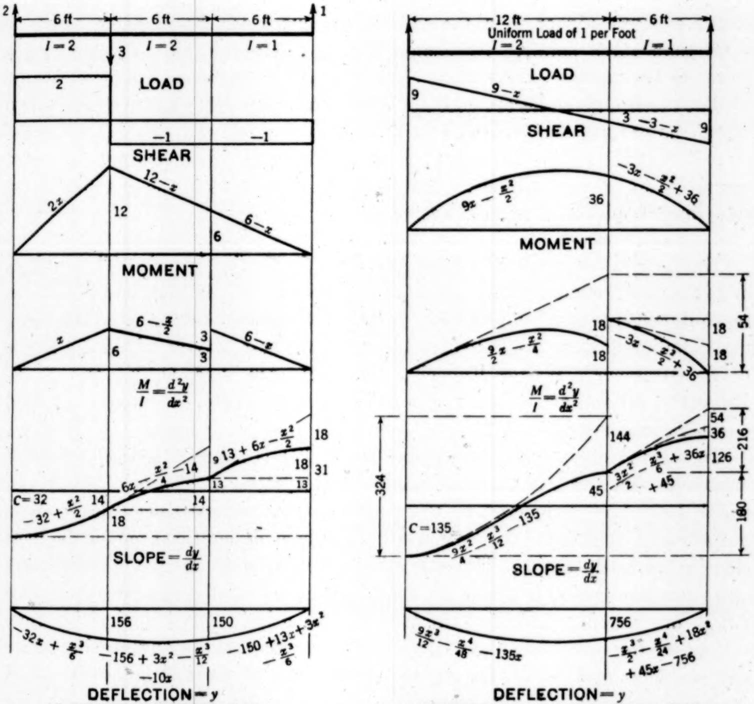
$$y = \frac{w l^2}{16} i_C x^2 + \frac{w (i_A - 2 i_C)}{24} x^4 - \frac{w (i_A - i_C)}{15 l^2} x^6 + C x \quad \text{.....(164)}$$

and the constant can be obtained by letting $x = \frac{l}{2}$ and $y = 0$.

Case 3.—This is the same as Case 2, or Case 1, depending upon the shape of the curve—that is, depending on whether or not it has zero slope at Point C.

Case 4.—In this case the authors make the approximation of substituting a continuous curve for a discontinuous one, as shown in Fig. 1. The I-curve is not continuous. In Fig. 19, the writer has developed numerical examples for both a concentrated and a uniform load in order to show that the conventional method presents no difficulties; nor does it involve excessive labor. For example, referring to Fig. 19(a), $18\ C = \frac{18 \times 6}{3} + 18 \times 6 + \frac{36 \times 6}{2} - \frac{9 \times 6}{3} + 45 \times 6 + \frac{36 \times 6}{2} - \frac{18 \times 6}{3}$; and $C = 32$. Furthermore, at the third-points, the ordinates to the deflection curve are: $\frac{6^3}{6} - 32 \times 6 = - 156$; and $- 156 + 3 \times 6^2 - \frac{6^3}{6} - 14 \times 6 = - 150$.

Referring to Fig. 19(*b*), $18\ C = \dfrac{324 \times 12}{3} - \dfrac{144 \times 12}{4} + 180 \times 6 + \dfrac{216 \times 6}{2} - \dfrac{54 \times 6}{3} - \dfrac{36 \times 6}{4} = 2\,430.$ $C = 135.$ Furthermore, at the third-point indicated, the ordinate to the deflection curve is: $\dfrac{9 \times 12^3}{12} - \dfrac{3 \times 12^4}{144} - 135 \times 12 = -756.$


2
6 ft
6 ft
6 ft
1
$I = 2$
3
$I = 2$
$I = 1$
2
LOAD
-1
-1
SHEAR
$2x$
$12 - x$
12
6
$6 - x$
MOMENT
x
$6 - \frac{x}{2}$
6
3
3
$6 - x$
$\frac{M}{I} = \frac{d^2y}{dx^2}$
$13 + 6x - \frac{x^2}{2}$
18
9
18
31
$6x - \frac{x^2}{4} - 14$
13
13
$C = 32$
$-32 + \frac{x^2}{2}$
14
14
18
SLOPE $= \frac{dy}{dx}$
156
150
$-32x + \frac{x^3}{6}$
$-156 + 3x^2 - \frac{x^3}{12} - 10x$
$-150 + 13x + 3x^2 - \frac{x^3}{6}$
DEFLECTION $= y$
12 ft
6 ft
Uniform Load of 1 per Foot
$I = 2$
$I = 1$
LOAD
9
$9 - x$
3
$-3 - x$
9
SHEAR
$9x - \frac{x^2}{2}$
36
$-3x - \frac{x^2}{2} + 36$
MOMENT
18
18
$\frac{9}{2}x - \frac{x^2}{4}$
18
18
$-3x - \frac{x^2}{2} + 36$
54
$\frac{M}{I} = \frac{d^2y}{dx^2}$
144
54
36
$\frac{3x^2}{2} - \frac{x^3}{6} + 36x + 45$
126
216
45
324
180
$C = 135$
$\frac{9x^2}{4} - \frac{x^3}{12} - 135$
SLOPE $= \frac{dy}{dx}$
756
$\frac{9x^3}{12} - \frac{x^4}{48} - 135x$
$-\frac{x^3}{2} - \frac{x^4}{24} + 18x^2 + 45x - 756$
DEFLECTION $= y$

(*a*) CONCENTRATED LOAD (*b*) UNIFORMLY DISTRIBUTED LOAD

FIG. 19.—ALTERNATE SOLUTION, CASE 4.

This method has the advantage that it is theoretically exact. All the computations are given here or in the illustration. With more complicated numbers the use of the slide-rule will reduce the work to approximately that of the examples.

The equations are written on the curves for those who prefer the multiple-integration method and the areas are divided for those who prefer to integrate by computing areas. With the curves the origin is taken at the left of the section for which the curve is written. When this system is followed the constant of integration is obtained by placing x equal to the section length in the section to the left of the one being integrated. For the constant of the slope curve in the extreme left section, *C*, the area under the entire curve, considering the temporary axes as at the beginning of the left end of the curve, is computed. This area is divided by the length of the beam.

Case 5.—This problem is only the simple beam theory as given in texts on the subject.

It is the writer's opinion that the substitution of a parabolic curve for the reciprocal inertia curve is simpler to understand, is as readily applied, and is more elastic than the method given by the authors. By using a parabola of a power higher than the second, the curve can be made to check at more than three points. By changing the equation where sudden changes occur in the inertia curve, these changes can be considered mathematically.

The method given herein does not involve any ideas that are not familiar to those who are engaged in work on the beam theory. If the writer has demonstrated that it is superior to that given by the authors he has not detracted greatly from a very excellent mathematical treatise on the comparatively complicated subject of tapered beams.

HALVARD W. BIRKELAND,[25] JUN. AM. SOC. C. E. (by letter).—The authors have written a commendable paper on a subject that has increased in importance during the last ten to fifteen years. Two references are given for equations similar to those presented in the paper; to these may be added the names of A. Strassner[26], Walter Ruppel[27], Assoc. M. Am. Soc. C. E., and E. B. Russell[28]. There are also several other references for tapered structural members in general.[29]

In their "Conclusion", the authors state: "It is hoped that the method of substitute *I*-curves will thus aid in the development of classes of structures which have been hampered in the past by mathematical difficulties." It is quite true, and pathetic, that some engineers have difficulties even in the most elementary mathematics. However, the writer believes, from his experiences and from those of others, that the use of haunched members has been retarded not so much by mathematical difficulties as by a lack of geometrical interpretations of the beam constants. It is fortunate that many engineering terms and problems may be reduced to their elements, and there is no advantage in ignoring the physical significance of those elements. This is not only the writer's observation. The same opinion was expressed editorially[30] in 1935, although it was written in connection with a rather general article on structural analysis:

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[25] Junior Engr., U. S. Bureau of Reclamation, Denver, Colo.

[26] "Neuere Methoden", by A. Strassner, Band I. Third Edition, Berlin, 1925.

[27] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points", by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E.; Discussion by Walter Ruppel, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

[28] "Analysis of Continuous Frames by the Method of Restraining Stiffnesses", by E. B. Russell, Second Edition, San Francisco, Calif., 1934.

[29] "Continuous Frames of Reinforced Concrete", by Hardy Cross, and N. D. Morgan, Members, Am. Soc. C. E., N. Y., 1932; "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, Fourth Edition, N. Y., 1932; "Concrete, Plain and Reinforced", by F. W. Taylor, S. E. Thompson, and E. Smulski, Members, Am. Soc. C. E., Vol. 2, Fourth Edition, N. Y., 1928; and, "The Modified Slope Deflection Equations", by L. T. Evans, Jun. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 28, 1932.

[30] *Engineering News-Record*, October 17, 1935, p. 551.

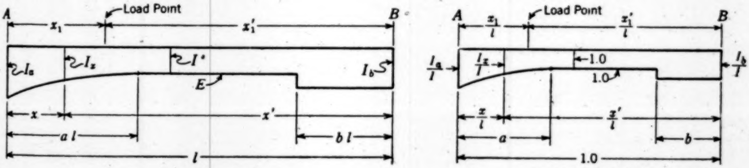
> "Just as every complex operation is made up of simple elements, so all phases of engineering consist of elementary facts and relations. Much of the engineer's skill lies in his grasp of these elements and his ability to recognize them in the particular problem with which he may be dealing. Nowhere is this truer than in the field of structures. Here the engineer's success at all times depends on how intimately he understands elementary physical facts and behavior."

With the preceding in mind, the physical meanings of the F-term will be presented. In this discussion the following fundamental F-terms will be considered: F_1, F_2, F_3, F_4, and F_5. An inspection of Equations (36), (37), (40), and (41) shows that $F_6 \equiv F_4$ and $F_7 \equiv F_5$. so far as any general discussion is concerned, and all other F-terms may be obtained by the proper integrations of F_4 and F_5. To illustrate the last statement, Equation (88) may be obtained from Equation (86) thus:

$$F_8 = \int_{k=0}^{k=1.0} F_4\, dk \ldots\ldots\ldots\ldots\ldots\ldots (165)$$

Before giving the geometrical interpretations, it is necessary to introduce a fictitious beam, the "unit beam", so called for reasons which will be apparent later.

Definitions of the Actual Beam and the Unit Beam.—An actual beam is a beam or a member as it is in a structure, with all its actual dimensions, regardless of end restraints and loads. An example of an actual beam is shown in Fig. 20(a). Each actual beam has a unit beam which corresponds to it.



(a) ACTUAL BEAM (b) UNIT BEAM

FIG. 20.

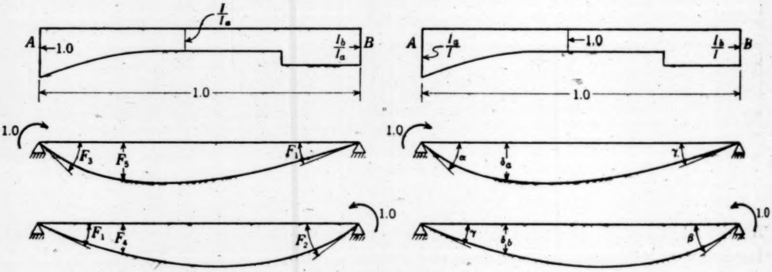
All dimensions of this unit beam are proportional to those of the actual beam: Longitudinal dimensions are reduced by the factor, l, usually the span length; the modulus of elasticity at every point is reduced by the factor, E, usually the modulus of elasticity of the material; and the moment of inertia at every point is reduced by the factor, I, usually the moment of inertia at the minimum section of the beam. An example of a unit beam is shown in Fig. 20(b). The beam in Fig. 20 is chosen as an illustration because it embodies dissymmetry, and constant section as well as gradual and abrupt changes in section, and will signify a beam with any arbitrary variation in moment of inertia.

The unit beam was defined by deriving it from the actual beam. Imagine the procedure reversed, and it is seen that one unit beam may be the source of any number of actual beams. Another unit beam may be the source of another group of actual beams. Unit beams, therefore, may be said to be a criterion of actual beams.

Credit for originating the unit beam concept should go to Mr. Ruppel, who states:[31] "When values for special loadings or beams are determined, it is suggested that they be computed for, or reduced to, coefficients for a unit load on the span of unity with I and E equal to unity."

Geometrical Interpretations of F-Terms.—Consider a simple-span unit beam in which the reduction factors are the span length, the modulus of elasticity of the material, and the moment of inertia at End A, as shown in Fig. 21(a). Then F_3 = the rotation at A due to a unit moment at A; F_2 = the rotation at B due to a unit moment at B; F_1 = the rotation at B due to a unit moment at A, or the rotation at A due to a unit moment at B; F_5 = the rotation at A due to a unit load at the load point, or the deflection at the load point due to a unit moment at A; and F_4 = the rotation at B due to a unit load at the load point, or the deflection at the load point due to a unit moment at B. The double definitions of F_1, F_5, and F_4 are due to Maxwell's law of reciprocal displacements.

The foregoing study of the physical meanings of the F-terms suggests that better symbols might have been chosen. The writer, therefore, proposes the following substitutions: α for F_3; β for F_2; γ for F_1; δ_a for F_5; and δ_b for F_4. The writer also proposes that the reduction factor for moment of inertia be I_c, the minimum moment of inertia of the member. This latter suggestion will make many of the authors' equations more symmetrical, and will simplify the relations between their paper and most of the material that has been published before. The relations between the authors' and the writer's symbols are illustrated in Fig. 21. The mathematical relations are:



(a) BY THE AUTHOR (b) BY THE WRITER

FIG. 21.—SIMPLE-SPAN UNIT BEAMS, WITH DEFLECTION CURVES AND BEAM CONSTANT.

$$\frac{F_3}{\alpha} = \frac{F_2}{\beta} = \frac{F_1}{\gamma} = \frac{F_5}{\delta_a} = \frac{F_4}{\delta_b} = \frac{I_A}{I_c} \quad \ldots\ldots\ldots\ldots\ldots (166)$$

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[31] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 161.

TABLE 2.—CONVERSION FORMULAS FOR BEAM CONSTANTS (ARRANGED VERTICALLY).
(See Footnotes 2, 26, 27, 28, and 29 for complete reference.)

Authors	F_2	F_3	F_1	F_5	F_4
Writer's simple span unit beam..	$\frac{IA}{I}\alpha$	$\frac{IA}{I}\beta$	$\frac{IA}{I}\gamma$	$\frac{IA}{I}\delta_a$	$\frac{IA}{I}\delta_b$
Strassner..........	$\frac{J_a}{J}\frac{\alpha_a}{3}$	$\frac{J_a}{J}\frac{\alpha_b}{3}$	$\frac{J_a}{J}\frac{\beta}{6}$	$\frac{J_a}{J}\frac{\beta S_a}{6}$	$\frac{J_a}{J}\frac{\beta S_b}{6}$
Ruppel...........	$\frac{I'}{I}(1-u)\,p$	$\frac{I'}{I}(1-v)\,q$	$\frac{I'}{I}u\,p=\frac{I'}{I}v\,q$	$\frac{I'}{I}p\,s$	$\frac{I'}{I}q\,t$
Russell...........	$\frac{I_f'}{I}\frac{1}{4K_f(1-C_n C_f)}$	$\frac{I_f'}{I}\frac{1}{4K_n(1-C_n C_f)}$	$\frac{I_f'}{I}\frac{C_f}{4K_f(1-C_n C_f)}=\frac{I_f'}{I}\frac{C_n}{4K_n(1-C_n C_f)}$	$\frac{I_f'}{I}\frac{F_f+C_f F_n}{4K_f(1-C_n C_f)}$	$\frac{I_f'}{I}\frac{F_n+C_n F_f}{4K_n(1-C_n C_f)}$
Cross and Morgan.	$\frac{IA}{I}\frac{1}{S_a(1-r_a r_b)}$	$\frac{IA}{I}\frac{1}{S_b(1-r_a r_b)}$	$\frac{IA}{I}\frac{r_b}{S_a(1-r_a r_b)}=\frac{IA}{I}\frac{r_a}{S_b(1-r_a r_b)}$	$\frac{IA}{I}\frac{M_a+r_b M_b}{S_a(1-r_a r_b)}$	$\frac{IA}{I}\frac{M_b+r_a M_a}{S_b(1-r_a r_b)}$
Turneaure and Maurer*	$\frac{I_1}{I_2}\frac{B}{A B^2+A C^2}$	$\frac{I_1}{I_2}\frac{B}{A B^2+A C^2}$	$\frac{I_1}{I_2}\frac{C}{A B^2+A C^2}$	$\frac{I_1}{I_2}\frac{B C D_1+C^2 D_2}{A B^2+A C^2}$	$\frac{I_1}{I_2}\frac{B C D_2+C^2 D_1}{A B^2+A C^2}$
Taylor, Thompson and Smulski*...	$\frac{I_1}{I}\frac{\alpha}{3}$	$\frac{I_1}{I}\frac{\alpha}{3}$	$\frac{I_1}{I}\frac{\beta}{6}$	$\frac{I_1}{I}\frac{\beta C_{v2}}{6}$	$\frac{I_1}{I}\frac{\beta C_{v1}}{6}$
Evans............	$\frac{IA}{I_c}\frac{C_3}{C_1 C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_1}{C_1 C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_2}{C_1 C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_3 CAB+C_2 CBA}{C_1 C_3-C_2^2}$	$\frac{IA}{I_c}\frac{C_1 CBA+C_2 CAB}{C_1 C_3-C_2^2}$
Large and Morris†	$\frac{1}{4S_a(1-r_a r_b)}$	$\frac{1}{4S_b(1-r_a r_b)}$	$\frac{r_b}{4S_a(1-r_a r_b)}=\frac{r_a}{4S_b(1-r_a r_b)}$	(No influence lines for fixed-end moments.)	

* Only for symmetrical beams.

† Symbols similar to those of Cross and Morgan have been assigned by the writer.

There are several books, booklets, and articles which have tabulations of beam constants for variously haunched beams, and these constants are usually directly applicable only to the particular method with which they are presented. Because one set of tables may cover loads or haunches not included in other tables, it may be useful to have conversion formulas so that any one set of tables may be used for all methods. The writer has used such formulas since 1933 and has made them more complete from time to time. An extract is presented in Table 2, with the F-terms of the authors and the simple-span unit beam of the writer included. The table is arranged so that all terms in one column are equal. If the equations are solved for the constants used by the many other writers on the subject, the complete set of conversion formulas may be obtained.

Concerning the substitute I-curves in general, the writer advances the suggestion that there is little need for such substitutions, but what is needed is more accurate and more complete tables (not curves) of beam constants, similar to those presented by Strassner, Ruppel, and Russell.

C. W. Dunham,[32] M. Am. Soc. C. E. (by letter).—The method of analysis set forth in this paper should be useful as a means of checking design calculations for certain special structures after the sections and stresses have been determined by other simple and approximate methods. However, there is one point or feature which is touched upon almost too lightly; namely, the "working lines."

One of the most common uses of the tapered section is in rigid frames and the location of the working lines may cause considerable variation in the results of the analysis. It may be advantageous for the authors to give more emphasis to this matter and to include in their method the necessary data to serve as a guide of sufficient accuracy so that the possible error which may be introduced, due to incorrect working lines, shall not exceed that which would result from the use of more simple methods of analysis.

In one rather extreme case of rigid frames, the stresses resulting from the use of a working line through the center of gravity of the center section varied more than 10% from those computed by using a working line through the center of gravity of the haunch. Of course, this error works in opposite directions as far as the stresses at the crown and haunch are concerned, but, in any event, it is too large to neglect.

Although the rigid frame is only one of the types of structures in which the tapered member is used, it is an important type. Such frames often have a definite camber or curve in the top chord. This adds a thrust in the frame, because of the tendency toward a partial arch action which is not included in the formulas given in the paper, but which should not be omitted in the calculations. Furthermore, assuming a portal-like member supported by columns in a building, as suggested by Fig. 12, the location of the working line in the tapered section may greatly affect the moments in the columns themselves.

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[32] Asst. Engr., Design Div., Port of New York Authority, New York, N. Y.

FANG-YIN TSAI,[33] ASSOC. M. AM. SOC. C. E. (by letter).—The outstanding feature of the method described in this paper is the substitution of an approximate *I*-curve for the actual one so that the various constants can be expressed by formulas derived by direct integration. Several sets of such formulas, most of which are rather lengthy and involve fractional exponents, are presented for the computation of the various constants involved in the moment distribution method proposed by Hardy Cross, M. Am. Soc. C. E. To those who prefer to solve the problem by formulas, this paper may be of value.

In the aforementioned formulas, the authors have introduced the various constants, F_1, F_2, F_3, etc., without assigning to them any physical meaning. Elsewhere[34], the writer has stated that,

> "* * * to analyze, by any method, a continuous structure with a moment of inertia varying in any manner, five independent constants or coefficients must be known (three beam coefficients and two load coefficients) for every span of the structure considered as a simply supported beam; and these five coefficients may be expressed in various ways to suit any particular method of analysis."

It may be of interest to compare the authors' *F*-constants with those of the other systems, and, before doing so, a brief explanation of the latter with regard to their characteristics and adaptability to certain methods of analysis will be necessary. Among the various prevailing systems of expressing these beam and load constants, the following are the most outstanding: (1) The method of angle changes; (2) the method of moment areas; (3) application of Ruppel's constants; (4) application of Strassner's constants; and (5) application of constants in the Cross method.

(1).—Angle Changes.—The system of expressing the constants in terms of angle changes was first introduced apparently, by German authors and was used by Ernst Suter[35] and A. Strassner[36] in developing their methods of analysis. Considering the beam, *LR*, in Fig. 22, as simply supported and subjected to any loading, the following angle changes occur: $\alpha^0 =$ angle change due to the given loading (see Fig. 22(*b*)); $\alpha =$ angle change due to a moment of unity applied at the same end as α^0 (see Fig. 22(*c*) and Fig. 22(*d*)); and $\beta =$ angle change at either end of a beam due to a moment of unity applied at the other end (see Fig. 22(*c*) and Fig. 22(*d*)). The subscripts, *L* and *R*, denote left end and right end, respectively.

(2).—Moment Areas.—Again, considering Beam *LR* (Fig. 22) as simply supported and subjected to any loading[34]: $A_0 =$ area of the $\frac{M}{I}$-diagram due to loading; $g =$ abscissa ratio of the centroid of A_0 from End L; $A =$ area of the $\frac{M}{I}$-diagram due to a moment of unity applied at the designated end (the

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[33] Prof. of Structural Eng., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

[34] *Transactions,* Am. Soc. C. E., Vol. 101 (1936), p. 130.

[35] "Die Methode der Festpunkte", Julius Springer, Berlin, 1932.

[36] "Neuere Methoden zur Statik der Rahmenträgwerke und der Elastischen Bogenträger", Vol. 1, Wilhelm Ernst & Sohn, Berlin, 1925.

end indicated by the subscripts, L or R); and u and v = abscissa ratios, of the centroid of A_L and A_R from Ends L and R, respectively.

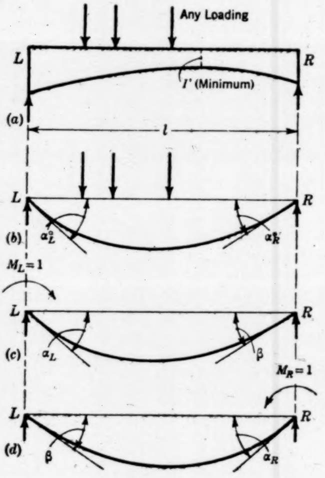
Any Loading
L
R
I' (Minimum)
(a)
l
L
R
(b)
α_L^o
α_R^o
$M_L=1$
L
R
(c)
α_L
β
$M_R=1$
L
R
(d)
β
α_R

FIG. 22.

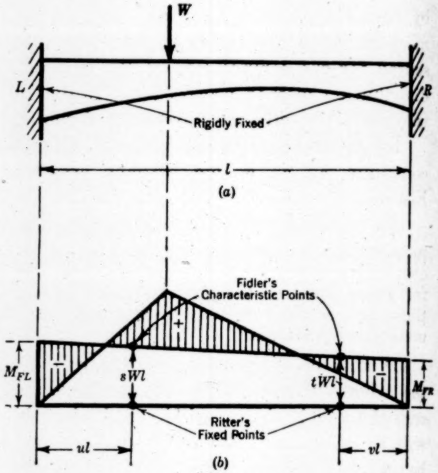
W
L
R
Rigidly Fixed
l
(a)
Fidler's Characteristic Points
+
−
−
M_{FL}
sWl
tWl
M_{FR}
Ritter's Fixed Points
ul
vl
(b)

FIG. 23.

Of the six constants, only five are independent, since it can be shown easily that, according to Maxwell's principle of reciprocal deflections (angular), the following relation is always valid:

$$A_L u = A_R v \qquad (167)$$

(3).—Constants in Ruppel's Tables[37].—The tables introduced by Walter Ruppel, Assoc. M. Am. Soc. C. E., were converted and also extended from Strassner's tables[36] for use in the graphical method of fixed points or conjugate points developed by T. C. Fidler[38], A. Ostenfeld[39], and L. H. Nishkian and D. B. Steinman[40], Members, Am. Soc. C. E. In these tables are found the beam constants or coefficients, p, q, u, and v, and the load coefficients, s and t. The constants, u and v, are identical with those of the moment areas previously mentioned. The physical meaning of the constants, u, v, s, and t, is shown in Fig. 23. Again, only five of the six constants are independent, since the following relationship is always valid:

$$p\,u = q\,v \qquad (168)$$

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[37] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

[38] *Minutes of Proceedings*, Inst. C. E., London, Vol. 74 (1883), p. 196.

[39] "Teknisk Statik" (in Danish), Vol. II, Copenhagen, 1925, pp. 83–142.

[40] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 1–48.

(4).—Constants in Strassner's Tables[38].—The beam constants[41], ϕ_{aL}, ϕ_{aR}, and ϕ_{β}, and the load constants, ϕ_s and ϕ_t, are presented in Strassner's tables. An important feature of this system is that, for a uniform load over the full length of a symmetrical double-tapered member of any shape, the load constants, ϕ_s and ϕ_t, are always equal to unity. Hence, for the given case, no tables are needed for those two constants.

(5).—Constants in Cross' Method of Moment Distribution.—In applying the method of moment distribution developed by Professor Cross[42], the following constants for every span of a continuous structure must be first determined: C_{LR} = the factor required to carry over the moment at End L, to End R. (C_{RL} = the factor required to carry over the moment at End R, to End L. The sign for C_{LR} and C_{RL} will be considered here as positive); S_L = the stiffness factor when a moment is applied at End L with End R fixed. (S_R refers to the moment applied at End R with End L fixed. This stiffness factor is in accordance with Professor Cross' definition[43]. That given by the authors' in Equation (61) or Equation (62) is equal to $\frac{S}{4E}$, which corresponds to $\frac{I}{l}$ for beams of constant moment of inertia. Similarly, the modified stiffness factor given by the authors' in Equation (110) or Equation (111) is equal to $\frac{S'}{4E}$, which corresponds to $\frac{3I}{4l}$ for beams of constant moment of inertia); and M_{FL} = the fixed-end moment at End L, due to loading, with both ends fixed. Again, only five of the six constants are independent since the following relation is always valid:

$$C_{LR}\,S_L = C_{RL}\,S_R \qquad (169)$$

In addition, the following modified constants are worthy of note: S'_L = the stiffness factor when the moment is applied at End L with the other end simply supported; and M'_{FL} = the fixed-end moment at End L due to the given loading, with the other end simply supported. The following relations can be found easily:

$$S'_L = S_L\,(1 - C_{LR}\,C_{RL}) \qquad (170)$$

$$S'_R = S_R\,(1 - C_{LR}\,C_{RL}) \qquad (171)$$

$$M'_{FL} = M_{FL} + C_{RL}\,M_{FR} \qquad (172)$$

$$M'_{FR} = M_{FR} + C_{LR}\,M_{FL} \qquad (173)$$

and,

$$C_{LR}\,S'_L = C_{RL}\,S'_R \qquad (174)$$

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[41] There are no such notations in Strassner's tables ("Neuere Methoden zur Statik der Rahmenträgwerke und der Elastischen Bogenträger", Vol. 1, pp. 101–112), and they are introduced by the writer, indicating that they are the functions of a_L, a_R, β, s, and t. Similar tables are also found in "Die Methode der Festpunkte", by Ernst Suter, pp. 413–419.

[42] *Transactions*, Am. Soc. C. E., Vol. 96 (1982), p. 1.

[43] *Loc. cit.*, p. 2.

TABLE 3.—CONVERSION OF BEAM AND LOAD CONSTANTS IN THE VARIOUS SYSTEMS*

Units	Angle changes	Moment areas	Ruppel	Strassner	Cross
$\frac{\text{Radians}}{\text{Kip-feet}}$	α_L	$\frac{A_L}{E}(1-u)$	$\frac{pl}{EI'}(1-u)$	$\frac{l}{3EI'}\phi\alpha_L$	$\frac{1}{S_L(1-C_{LR}C_{RL})}$
	α_R	$\frac{A_R}{E}(1-v)$	$\frac{ql}{EI'}(1-v)$	$\frac{l}{3EI'}\phi\alpha_R$	$\frac{1}{S_R(1-C_{LR}C_{RL})}$
	β	$\frac{A_L u}{E}$ or $\frac{A_R v}{E}$	$\frac{pul}{EI'}$ or $\frac{qvl}{EI'}$	$\frac{l}{6EI'}\phi_\beta$	$\frac{C_{RL}}{S_L(1-C_{LR}C_{RL})}$ or $\frac{C_{LR}}{S_R(1-C_{LR}C_{RL})}$
Radians	α°_L	$\frac{A_0}{E}(1-g)$	$\frac{Wl^2}{EI'}sp$	$\frac{Wl^2}{K6EI'}\phi_\beta\phi_s$	$\frac{M_{FL}+C_{RL}M_{FR}}{S_L(1-C_{LR}C_{RL})}$
	α°_R	$\frac{A_0}{E}g$	$\frac{Wl^2}{EI'}tq$	$\frac{Wl^2}{K6EI'}\phi_\beta\phi_t$	$\frac{M_{FR}+C_{LR}M_{FL}}{S_R(1-C_{LR}C_{RL})}$
$\frac{\text{Ratio}}{\text{Cubic feet}}$	$E(\alpha_L+\beta)$	A_L NOTE.— $A_L u = A_R v$	$\frac{pl}{I'}$	$\frac{l}{6I'}(2\phi_{\alpha_L}+\phi_\beta)$	$\frac{E(1+C_{RL})}{S_L(1-C_{LR}C_{RL})}$
	$E(\alpha_R+\beta)$	A_R	$\frac{ql}{I'}$	$\frac{l}{6I'}(2\phi_{\alpha_R}+\phi_\beta)$	$\frac{E(1+C_{LR})}{S_R(1-C_{LR}C_{RL})}$
$\frac{\text{Kips}}{\text{Sq. ft.}}$	$E(\alpha^\circ_L+\alpha^\circ_R)$	A_0	$\frac{Wl^2}{I'}(sp+tq)$	$\frac{Wl^2}{K6I'}\phi_\beta(\phi_s+\phi_t)$	$E[M_{FL}S_R(1+C_{RL}) + M_{FR}S_L(1+C_{LR}) \div S_L S_R(1-C_{LR}C_{RL})$
Ratio	$\frac{\beta}{\alpha_L+\beta}$	u	u	$\frac{\phi_\beta}{2\phi_{\alpha_L}+\phi_\beta}$	$\frac{C_{RL}}{1+C_{RL}}$
	$\frac{\beta}{\alpha_R+\beta}$	v	v	$\frac{\phi_\beta}{2\phi_{\alpha_R}+\phi_\beta}$	$\frac{C_{RL}}{1+C_{LR}}$
	$\frac{\alpha^\circ_R}{\alpha^\circ_L+\alpha^\circ_R}$	g	$\frac{tg}{sp+tq}$	$\frac{\phi_t}{\phi_s+\phi_t}$	$\cfrac{1}{1+\cfrac{S_R(M_{FL}+C_{RL}M_{FR})}{S_L(M_{FR}+C_{LR}M_{FL})}}$
Ratio	$\frac{EI'}{l}(\alpha_L+\beta)$	$\frac{A_L I'}{l}$	p NOTE.— $pu = qv$	$\frac{2\phi_{\alpha_L}+\phi_\beta}{6}$	$\frac{EI'}{l}\,\frac{(1+C_{RL})}{S_L(1-C_{LR}C_{RL})}$
	$\frac{EI'}{l}(\alpha_R+\beta)$	$\frac{A_R I'}{l}$	q	$\frac{2\phi_{\alpha_R}+\phi_\beta}{6}$	$\frac{EI'}{l}\,\frac{(1+C_{LR})}{S_R(1-C_{LR}C_{RL})}$
	$\frac{\alpha^\circ_L}{Wl(\alpha_L+\beta)}$	$\frac{A_0(1-g)}{A_L Wl}$	s	$\frac{\phi_\beta\phi_s}{K(2\phi_{\alpha_L}+\phi_\beta)}$	$\frac{M_{FL}+C_{RL}M_{FR}}{Wl(1+C_{RL})}$
	$\frac{\alpha^\circ_R}{Wl(\alpha_R+\beta)}$	$\frac{A_0 g}{A_R Wl}$	t	$\frac{\phi_\beta\phi_t}{K(2\phi_{\alpha_R}+\phi_\beta)}$	$\frac{M_{FR}+C_{LR}M_{FR}}{Wl(1+C_{LR})}$
Ratio	$\frac{3EI'}{l}\alpha_L$	$\frac{3I'A_L(1-u)}{l}$	$3p(1-u)$	ϕ_{α_L}	$\frac{3EI'}{lS_L(1-C_{LR}C_{RL})}$
	$\frac{3EI'}{l}\alpha_R$	$\frac{3I'A_R(1-v)}{l}$	$3q(1-v)$	ϕ_{α_R}	$\frac{3EI'}{lS_R(1-C_{LR}C_{RL})}$
	$\frac{6EI'}{l}\beta$	$\frac{6I'A_L u}{l}$ or $\frac{6I'A_R v}{l}$	$6pu = 6qv$	ϕ_β	$\frac{6EI'C_{RL}}{lS_L(1-C_{LR}C_{RL})}$ or $\frac{6EI'C_{LR}}{lS_R(1-C_{LR}C_{RL})}$
	$\frac{K\alpha^\circ_L}{Wl\beta}$	$\frac{KA_0(1-g)}{A_L uWl}$ or $\frac{KA_0(1-g)}{A_R vWl}$	$\frac{Ks}{u}$	ϕ_s	$\frac{K}{Wl}\times\frac{M_{FL}+C_{RL}M_{FR}}{C_{RL}}$
	$\frac{K\alpha^\circ_R}{Wl\beta}$	$\frac{KA_0 g}{A_L uWl}$ or $\frac{KA_0 g}{A_R vWl}$	$\frac{Kt}{v}$	ϕ_t	$\frac{K}{Wl}\times\frac{M_{FR}+C_{LR}M_{FL}}{C_{LR}}$
Ratio	$\frac{\beta}{\alpha_R}$	$\frac{v}{1-v}$	$\frac{v}{1-v}$	$\frac{\phi_\beta}{2\phi_{\alpha_R}}$	C_{LR} NOTE.— $C_{LR}S_L = C_{RL}S_R$ $S'_L = S_L(1 - C_{LR}C_{RL})$ $S'_R = S_R(1 - C_{LR}C_{RL})$ $M'_{FL} = M_{FL} + C_{RL}M_{FR}$ $M'_{FR} = M_{FR} + C_{LR}M_{FL}$ $C_{LR}S'_L = C_{RL}S'_R$
	$\frac{\beta}{\alpha_L}$	$\frac{u}{1-u}$	$\frac{u}{1-u}$	$\frac{\phi_\beta}{2\phi_{\alpha_L}}$	C_{RL}
Kip-feet	$\frac{\alpha_R}{\alpha_L\alpha_R-\beta^2}$	$\frac{E(1-v)}{A_L(1-u-v)}$	$\frac{EI'(1-v)}{pl(1-u-v)}$	$\frac{3EI'\phi_{\alpha_R}}{l(\phi_{\alpha_L}\phi_{\alpha_R}-\frac{1}{2}\phi^2_\beta)}$	S_L
	$\frac{\alpha_L}{\alpha_L\alpha_R-\beta^2}$	$\frac{E(1-u)}{A_R(1-u-v)}$	$\frac{EI'(1-u)}{ql(1-u-v)}$	$\frac{3EI'\phi_{\alpha_L}}{l(\phi_{\alpha_L}\phi_{\alpha_R}-\frac{1}{2}\phi^2_\beta)}$	S_R
	$\frac{\alpha_R\alpha^\circ_L-\beta\alpha^\circ_R}{\alpha_L\alpha_R-\beta^2}$	$\frac{A_0[(1-g)-v]}{A_L(1-u-v)}$	$\frac{Wl[s-(sv+tu)]}{(1-u-v)}$	$\frac{Wl}{2K}\times\frac{\phi_\beta(\phi_{\alpha_R}\phi_s-\frac{1}{2}\phi_\beta\phi_t)}{(\phi_{\alpha_L}\phi_{\alpha_R}-\frac{1}{2}\phi^2_\beta)}$	M_{FL}
	$\frac{\alpha_L\alpha^\circ_R-\beta\alpha^\circ_L}{\alpha_L\alpha_R-\beta^2}$	$\frac{A_0[g-u]}{A_R(1-u-v)}$	$\frac{Wl[t-(sv+tu)]}{(1-u-v)}$	$\frac{Wl}{2K}\times\frac{\phi_\beta(\phi_{\alpha_L}\phi_t-\frac{1}{2}\phi_\beta\phi_s)}{(\phi_{\alpha_L}\phi_{\alpha_R}-\frac{1}{2}\phi^2_\beta)}$	M_{FR}

* Signs for the various quantities in the table are disregarded; 1 kip = 1 000 lb.

Comments.—The relation between all the constants of the aforementioned five systems are presented in Table 3[44], in which all the quantities on the same horizontal line are equal to one another and the additional notation is defined as follows: I' = minimum moment of inertia of the tapered member, in feet^4; E = modulus of elasticity, in kips per square foot; l = span length, in feet; W = total load on the span, in kips (in applying Strassner's and Ruppel's tables, the load constants for each load must be computed separately and used with the proper summation if there are several loads on the span); and K = a factor to be used in connection with Strassner's tables only. $K = 1$ for concentrated load and $K = 4$ for uniform load over the full span length. By means of this conversion table (Table 3), the constants of any other system may be computed easily when those of any one system are known.

Of the five systems of expressing the constants, each has its own especial adaptability to a particular method of analysis. That the angle changes and Strassner's constants are especially adapted to Suter's and Strassner's methods and that Ruppel's constants are especially computed for the graphical method of fixed or conjugate points have already been indicated. Elsewhere[45] the writer has used the angle changes in developing the general form of the three-moment equation, resulting in an exceedingly simple formula as follows (see Fig. 24):

$$\beta_1 M_A + (\alpha_{R1} + \alpha_{L2}) M_B + \beta_2 M_C = - \alpha^\circ_{R1} - \alpha^\circ_{L2} \ldots\ldots (175)$$

The same constants have also been used by the writer in developing the generalized graphical analysis of restrained and continuous beams[46] and the slope-deflection equations[47], as follows:

$$M_{LR} = \frac{1}{\alpha_L \alpha_R - \beta^2} \left[\alpha_R \theta_L + \beta\theta_R - (\alpha_R + \beta) \frac{d}{l} \mp (\alpha_R \alpha^\circ_L - \beta\alpha^\circ_R)\right] . (176)$$

and,

$$M_{RL} = \frac{1}{\alpha_L \alpha_R - \beta^2} \left[\alpha_L \theta_R + \beta\theta_L - (\alpha_L + \beta) \frac{d}{l} \pm (\alpha_L \alpha^\circ_R - \beta\alpha^\circ_L)\right] . (177)$$

The constants of moment areas have been used by A. W. Earl[48], M. Am. Soc. C. E., in writing the slope-deflection equations; by Thomas F. Hickerson[49], M. Am. Soc. C. E., in developing his method of analysis; and by Ralph W.

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[44] "'Cross' Constants for Members with Varying Moment of Inertia", by Fang-Yin Tsai and Tseng-I Yang, *Journal*, Tsing Hua Civ. Eng. Soc., Peiping, China, No. 2, July, 1931 (in English).

[45] "Theorem of Three-Moment in General Form", by Fang-Yin Tsai, The Science Repts., National Tsing Hua Univ., Peiping, China, Series A, Vol. II, pp. 19–36, April, 1933 (in English).

[46] "Generalized Graphical Analysis of Restrained and Continuous Beams", by Fang-Yin Tsai, *Journal*, Tsing Hua Civ. Eng. Soc., Peiping, China, No. 3, May, 1932 (in English).

[47] "Slope-Deflection Equations for the Analysis of Rigid Frames with Varying Moment of Inertia", by Fang-Yin Tsai, The Science Repts., National Tsing Hua Univ., Peiping, China, Series A, Vol. II, pp. 75–81, July, 1933. Similar equations have also been obtained by L. T. Evans, Assoc. M. Am. Soc. C. E., *Journal*, Am. Concrete Inst., October, 1931, p. 109; see, also, "Handbook of Rigid Frame Analysis", by L. T. Evans, Edwards Brothers, Inc., Ann Arbor, Mich., 1934.

[48] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 114.

[49] "Structural Frameworks", by T. F. Hickerson, The Univ. of North Carolina Press, Chapel Hill, N. C., 1934.

Stewart[50], M. Am. Soc. C. E., in his paper entitled "Analysis of Continuous Structures by Traversing the Elastic Curves." Besides their imperative use in the method of moment distribution, Professor Cross' constants have been used by E. B. Russell[51] in his method of restraining stiffness, and may be also used in writing the slope-deflection equations, as follows:

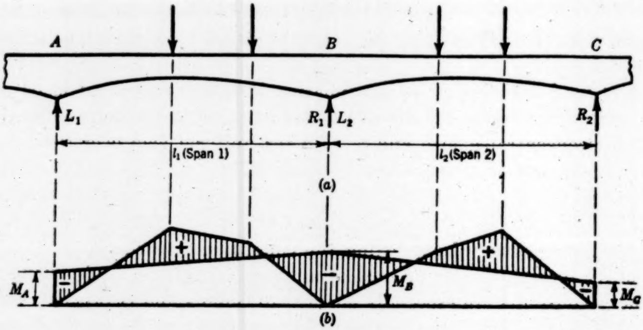

FIG. 24.

$$M_{LR} = S_L \left[\theta_L + C_{LR}\,\theta_R - (1 + C_{LR})\,\frac{d}{l}\right] \mp M_{FL} \ldots\ldots (178)$$

and,

$$M_{RL} = S_R \left[\theta_R + C_{RL}\,\theta_L - (1 + C_{RL})\,\frac{d}{l}\right] \pm M_{FR} \ldots\ldots (179)$$

The comparison of the authors' F-constants with those of the five systems previously mentioned are presented in Table 4, in which the value of A is computed by the authors' Equation (7). This factor, $A + 1$, is introduced because, for reference of computation, the writer and others have all used the minimum moment of inertia, I', or I_C, whereas the authors have used the maximum moment of inertia, I_A, which is equal to $I_C\ (A + 1)$, or $I'\ (A + 1)$. Tables 3 and 4 not only facilitate the application of the authors' formulas to any method of analysis, but also furnish a comprehensive view of the relation between all the constants of the various systems.

Of the aforementioned systems, that of angle changes is perhaps the most fundamental and elegant inasmuch as such changes not only possess a definite physical meaning, but they also represent, directly, the predominant deformations of continuous structures, on the basis of which most of the methods of analysis are developed. Moreover, their use in certain methods, gives exceed-

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[50] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

[51] "Analysis of Continuous Frames by the Method of Restraining Stiffness", by E. B. Russell, San Francisco, Second Edition 1934. Tables for the constants, $\frac{S_L}{4\,E\,I'}$, $\frac{S_R}{4\,E\,I'}$, C_{LR}, C_{RL}, $\frac{M_{FL}}{W\,l}$, and $\frac{M_{FR}}{W\,l}$, computed from Ruppel's tables may also be found in this book, pp. 24–42.

TABLE 4.—RELATIONS OF THE AUTHORS' *F*-CONSTANTS WITH THOSE OF THE OTHER SYSTEMS

(Units are All Ratios)

The authors' constants	Angle change	Moment area	Ruppel	Strassner	Cross
F_1	$\frac{EI'(A+1)}{l}\beta$	$\frac{I'(A+1)}{l}A_R v$ or $\frac{I'(A+1)}{l}A_L u$	$qv(A+1)$ or $pu(A+1)$	$\frac{A+1}{6}\phi_\beta$	$\frac{EI'(A+1)}{l}\frac{C_{RL}}{S_L(1-C_{LR}C_{RL})}$ or $\frac{EI'(A+1)}{l}\frac{C_{LR}}{S_R(1-C_{LR}C_{RL})}$
F_2	$\frac{EI'(A+1)}{l}\alpha_R$	$\frac{I'(A+1)}{l}A_R(1-v)$	$q(1-v)(A+1)$	$\frac{A+1}{3}\phi\alpha_R$	$\frac{EI'(A+1)}{l}\frac{1}{S_R(1-C_{LR}C_{RL})}$
F_3	$\frac{EI'(A+1)}{l}\alpha_L$	$\frac{I'(A+1)}{l}A_L(1-u)$	$p(1-u)(A+1)$	$\frac{A+1}{3}\phi\alpha_L$	$\frac{EI'(A+1)}{l}\frac{1}{S_L(1-C_{LR}C_{RL})}$
$F_4, F_6, F_8, F_{11}, F_{13}$	$\frac{EI'(A+1)}{Wl^2}\alpha^{\circ}_R$	$\frac{I'(A+1)}{Wl^2}A_0 g$	$tq(A+1)$	$\frac{A+1}{K6}\phi_\beta\phi_t$	$\frac{EI'(A+1)}{Wl^2}\frac{M_{FR}+C_{LR}M_{FL}}{S_R(1-C_{LR}C_{RL})}$
$F_5, F_7, F_9, F_{12}, F_{14}$	$\frac{EI'(A+1)}{Wl^2}\alpha^{\circ}_L$	$\frac{I'(A+1)}{Wl^2}A_0(1-g)$	$sp(A+1)$	$\frac{A+1}{K6}\phi_\beta\phi_s$	$\frac{EI'(A+1)}{Wl^2}\frac{M_{FL}+C_{RL}M_{FR}}{S_L(1-C_{LR}C_{RL})}$

ingly simple results, as in the case of the theorem of three moments presented by Equation (175). From Table 4, it is seen that the authors' F-constants are similar and proportional to those angle changes, and the former may be considered as the coefficients for the latter. Angle changes are, perhaps, most easily computed by the method of moment areas, applied either graphically, as shown by Ernst Suter[52], or by numerical summation, as shown by G. E. Large[53], Assoc. M. Am. Soc. C. E.

The authors' method is inherently approximate, since it requires the replacement of the actual I-curve by a substitute I-curve, and the latter can rarely fit the former exactly, especially when the actual I-curve is very irregular, or has some abrupt changes, as illustrated by the authors' Fig. 1. Although close approximation is not objectionable in the analysis of continuous structures (particularly in the case of reinforced concrete, in view of the many other uncertain and even untrue assumptions involved), the application of the authors' formulas for computing the F-constants requires much more labor than the moment-area method. It is regrettable that they have not given a numerical example to indicate how approximate the results of their method are and how much labor their computations require. For this purpose, the writer has solved the numerical example given by Professor Large[53] (Mr. Tsun-Kuai Liu has computed the various constants). Using the results of Professor Large's computations, the following constants of the moment-area method are derived (see Fig. 25): $A_0 = 89.15 \times 3 = 267.45$ kip-ft^{-2}; $A_L = 9.12 \times \dfrac{3}{10} = 2.736$ ft^{-3}; $v = 1 - \dfrac{19.62}{30} = 0.346$; $g = \dfrac{18.03}{30} = 0.601$; and $u = \dfrac{13.92}{30} = 0.464$. (The reason for using the multiplier, 3, in the computation for A_L is explained by Professor Large. The denominator, 10, is necessary since Professor Large used an end moment of 10 instead of unity, which is the value used by the writer in defining A_L and A_R). By Equation (167), $A_R = \dfrac{2.736 \times 0.464}{0.346} = 3.669$ ft^{-3}; or, using Professor Large's results, $A_R = 12.27 \times \dfrac{3}{10} = 3.681$ ft^{-3}. Both values of A_R are in close agreement. However, in order to check all the results exactly, the value, 3.669, will be used for A_R in all the computations. For the purpose of illustrating and comparing all the beam and load constants of the various systems, they have been computed for the same example in accordance with Table 3 and are arranged for comparison in Table 5. All the results in this table are cross-checked by computing them with 6-place logarithm tables to a precision of at least four significant figures, which, of course, is rarely necessary in ordinary practice.

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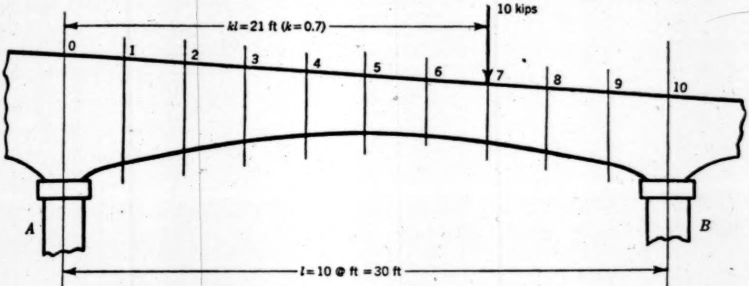
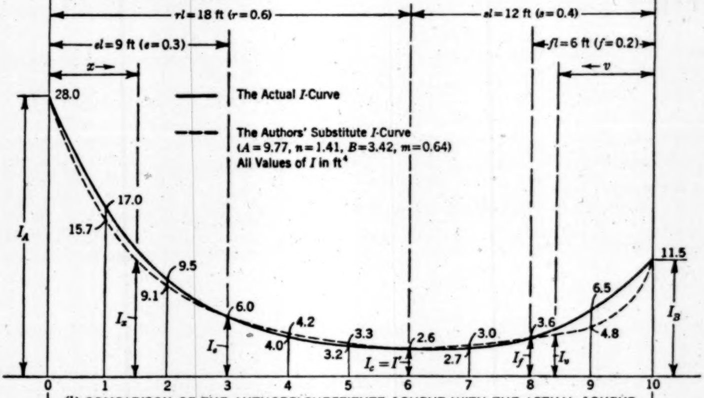
[52] "Die Methode der Festpunkte", pp. 77–80.

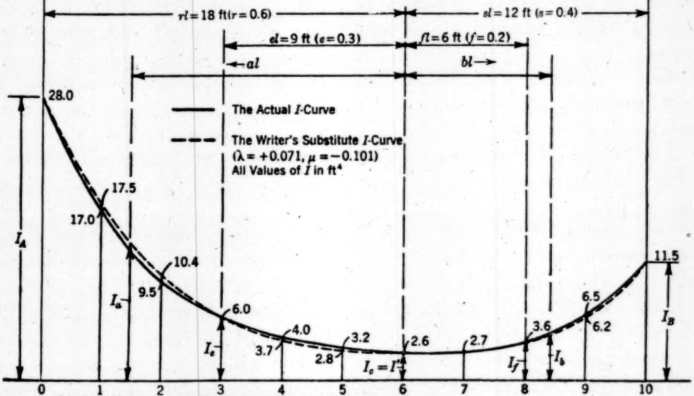
[53] *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio, November, 1932, p. 14. A similar but slightly different treatment by the same author is also found in *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 102.


10 kips
kl = 21 ft (k = 0.7)
0
1
2
3
4
5
6
7
8
9
10
A
B
l = 10 @ ft = 30 ft

(a) BEAM AND LOADING



(b) COMPARISON OF THE AUTHORS' SUBSTITUTE I-CURVE WITH THE ACTUAL I-CURVE



(c) COMPARISON OF THE WRITER'S SUBSTITUTE I-CURVE WITH THE ACTUAL I-CURVE

FIG. 25.

Using the authors' method and referring to Fig. 25(b): By Equation (7),

$$A = \frac{28 - 2.6}{2.6} = 9.77\text{; by Equation (8), } n = \frac{\log \dfrac{28 - 6}{9.77 \times 6}}{\log \dfrac{0.3}{0.6}} = 1.41\text{; by}$$

Equation (10), $B = \dfrac{11.5 \times 2.6}{2.6} = 3.42$; and, by Equation (11),

$$m = \frac{\log \dfrac{11.5 - 3.6}{3.42 \times 3.6}}{\log \dfrac{0.2}{0.4}} = 0.64.$$

TABLE 5.—THE VALUES OF ALL CONSTANTS OF THE VARIOUS SYSTEMS FOR THE BEAM SHOWN IN FIG. 25(a)

Angle changes	Moment areas	Ruppel	Strassner	Cross
$\alpha_L = \dfrac{1.470}{E}\ \dfrac{\text{radians}}{\text{kip-ft}}$	$A_L = 2.736\ \text{ft}^{-3}$	$p = 0.2371$	$\phi_{\alpha_L} = 0.3812$	$C_{LR} = 0.5291$ $C_{RL} = 0.8657$
$\alpha_R = \dfrac{2.400}{E}\ \dfrac{\text{radians}}{\text{kip-ft}}$	$A_R = 3.669\ \text{ft}^{-3}$	$q = 0.3180$	$\phi_{\alpha_R} = 0.6237$	$S_L = 1.2581\ E$ kip-ft $S_R = 0.7689\ E$ kip-ft
	$u = 0.464$	$u = 0.464$	$\phi_\beta = 0.6604$	$M_{FL} = 27.26$ kip-ft $M_{FR} = 52.55$ kip-ft
$\beta = \dfrac{1.270}{E}\ \dfrac{\text{radians}}{\text{kip-ft}}$	$v = 0.346$	$v = 0.346$	$\phi_s = 0.2801$	Check: $C_{LR}S_L = C_{RL}S_R = 0.665\ E$
$\alpha^\circ_L = \dfrac{106.71}{E}$ radians	$A_0 = 267.45\ \text{kip-ft}^{-2}$	$s = 0.1300$	$\phi_t = 0.4219$	Modified:
	$g = 0.601$	$t = 0.1457$		$S'_L = 0.6819\ E$ kip-ft $S'_R = 0.4167\ E$ kip-ft
$\alpha^\circ_R = \dfrac{160.74}{E}$ radians	Check:	Check:		$M'_{FL} = 72.75$ kip-ft $M'_{FR} = 66.97$ kip-ft
	$A_L u = A_R v = 1.270$	$pu = qv = 0.110$		Check: $C_{LR}S'_L = C_{RL}S'_R = 0.361E$

With the foregoing four constants known, the authors' F-constants and Professor Cross' constants are computed by their "exact" and approximate formulas. The same constants are also computed from Professor Large's results in accordance with Tables 3 and 4. All the values are arranged in Table 6, in which the percentages of the errors in the values of the authors' formulas in comparison with those by the moment-area method are also shown. Since the results of Professor Large's computations are sufficiently accurate (he has checked them, at least partly, by models), the authors' "exact" formulas yield errors as great as 12% and their approximate formulas yield errors as great as 50% even for this quite ordinary case. As far as the writer's experience is concerned, the application of the authors' lengthy formulas involving fractional exponents requires considerably more time and is also more susceptible to errors than the moment-area method applied either graphically or by numerical summation. In view of the foregoing comparison, it is very doubtful whether the authors' method will replace the moment-area method and find favor among practising engineers. In fact, since the design of continuous structures usually requires a long cut-and-try process (as stated by the authors under the heading "Approximate Solution"), it seems to the writer that there is absolutely no reason why, ordinarily, a designer should not design his tapered members in accordance with those prevailing types (straight, parabolic, and sharply curved tapers), for which many sets of very comprehensive tables and charts are now available, in order to save the enormous amount of labor which would be required if he chooses to do otherwise.

TABLE 6.—COMPARISON OF THE VALUES OF CONSTANTS COMPUTED BY MOMENT AREAS AND BY THE AUTHORS' METHOD
($A = 9.77$; $B = 3.42$; $n = 1.41$; and $m = 0.64$.)

Description	Moment-area method	THE AUTHORS' METHOD: "Exact"	Equation No.	Percentage error	Approximate	Equation No.	Percentage error
The Author's Constants:							
F_1	1.185	1.162	(63)	− 1.2	0.965	(112)	−18.6
F_2	2.240	2.355	(64)	+ 5.1	1.552	(113)	−30.7
F_3	1.369	1.358	(65)	− 0.8	1.253	(114)	− 8.3
F_6	0.500	0.483	(68)	− 3.4	0.348	(117)	−30.4
F_7	0.332	0.318	(69)	− 4.2	0.275	(118)	−17.2
The Cross' Constants:							
C_{LR}, or C_{AB}	0.5291	0.4935	(52)	− 6.7	0.6219	(52)	+17.5
C_{RL}, or C_{BA}	0.8657	0.8554	(53)	− 1.2	0.7702	(53)	−11.0
$\frac{S_L}{4E}$, or S_A, in feet^3	0.3145	0.2972	(61)	− 5.5	0.3576	(61)	+13.7
$\frac{S_R}{4E}$, or S_B, in feet^3	0.1922	0.1715	(62)	−10.8	0.2888	(62)	+50.2
M_{FL}, or M_A, in kip-feet	27.26	30.49	(42)	+11.8	26.80	(42)	− 1.7
M_{FR}, or M_B, in kip-feet	52.55	46.53	(43)	−11.5	50.56	(43)	− 1.9

From Fig. 25(*b*) it is seen that the authors' substitute *I*-curve falls short quite considerably in comparison with the actual *I*-curve at Section 9. Since a change in the *I*-value near the end of a member usually has a greater effect on the various functions than a change near the center, the large errors in the fixed-end moments computed by the authors' "exact" formulas may be attributed to this cause. The errors might possibly diminish somewhat if the substitute *I*-curve for the left half of the beam had been made to pass through the actual *I*-curve at Section 9 (in which case, $f = 0.1$; $I_f = 6.5$; and $m = 1.075$) instead of Section 8. However, if this is done, the substitute *I*-values to the right of Section 9 would be much too large, and the fitting of the substitute *I*-curve would also require a few trials. For this reason, the writer has tried to find some other equation for the substitute *I*-curve which will fit the actual *I*-curve better than that of the authors', and obtained the following results (see Fig. 25(*c*)):

$$I_a = I_c + (I_A - I_c)\left[\lambda\left(\frac{a}{r}\right)^2 + (1-\lambda)\left(\frac{a}{r}\right)^3\right] \quad \ldots\ldots(180)$$

$$\lambda = \frac{\dfrac{I_e - I_C}{I_A - I_C}\left(\dfrac{r}{e}\right)^3 - 1}{\dfrac{r}{e} - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(181)$$

$$I_b = I_C + (I_B - I_C)\left[\mu\left(\frac{b}{s}\right)^2 + (1-\mu)\left(\frac{b}{s}\right)^3\right] \quad\ldots\ldots(182)$$

and,

$$\mu = \frac{\dfrac{I_f - I_C}{I_B - I_C}\left(\dfrac{s}{f}\right)^3 - 1}{\dfrac{s}{f} - 1} \quad\ldots\ldots\ldots\ldots\ldots\ldots(183)$$

Equations (180) to (183) correspond, respectively, to the author's Equations (6), (8), (9), and (11), with the difference that the variable abscissa ratios, a and b, are measured from the ordinate of I_C to the left and right, respectively, and that the values of λ and μ may be either positive or negative within certain limits. Applying Equations (181) and (183) to the foregoing example:

$$\lambda = \frac{\dfrac{6-2.6}{28-2.6}\left(\dfrac{0.6}{0.3}\right)^3 - 1}{\dfrac{0.6}{0.3} - 1} = +\ 0.071$$

and,

$$\mu = \frac{\dfrac{3.6-2.6}{11.5-2.6}\left(\dfrac{0.4}{0.2}\right)^3 - 1}{\dfrac{0.4}{0.2} - 1} = -\ 0.101.$$

The writer's substitute I-curve, together with the actual I-curve, is plotted in Fig. 25(c), from which it is seen that the fit is much better than that shown in Fig. 25(b), particularly at the left half where the curves almost coincide. The writer hopes that the authors will investigate the possibility of using the foregoing equations in their method.

The position of the working line shown in the authors' Fig. 12 is very reasonable and is in agreement with that used by German writers[35, 36]. However, a study should also be made of the condition at the intersections of beams and columns, which affects greatly their I-values and, consequently, the moments[54]. As indicated in Fig. 12, the authors have assumed the condition of Fig. 26(b) for considering the I-values of beams, which is also in agreement with German writers[35, 36]; but the condition shown in Fig. 26(c) is also very reasonable and may represent the actual condition more or less, particularly when the width of columns is considerable. For considering the I-values of columns, the conditions shown in Fig. 26(d) and Fig. 26(e) have usually been assumed, in which case the following angle changes may be easily derived when the cross-section of the columns is constant:

$$a_R = \frac{(h')^3}{3\,E\,I\,h^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(184)$$

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[54] "Analysis of Continuous Frames by the Method of Restraining Stiffness", by E. B. Russell, San Francisco, Second Edition, 1904, p. 45.

$$\alpha_L = \frac{h^3 - (h'')^3}{3\,E\,I\,h^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (185)$$

and,

$$\beta = \frac{(h')^2\,(h + 2\,h'')}{3\,E\,I\,h^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (186)$$

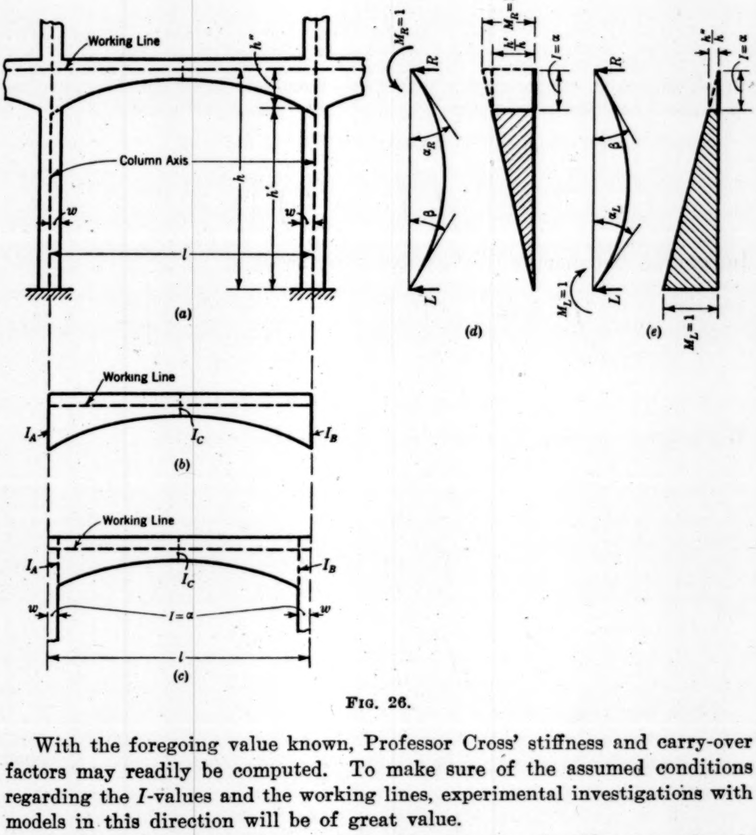
Working Line
h''
Column Axis
h
h'
w
w
l
(a)
Working Line
I_A
I_C
I_B
(b)
Working Line
I_A
I_C
I_B
w
$I=\alpha$
w
l
(c)
$M_R=1$
R
α_R
β
L
$\frac{h'}{h}$
$M_R=1$
$I=\alpha$
(d)
$M_L=1$
R
β
α_L
L
$\frac{h''}{h}$
$I=\alpha$
$M_L=1$
(e)

FIG. 26.

With the foregoing value known, Professor Cross' stiffness and carry-over factors may readily be computed. To make sure of the assumed conditions regarding the I-values and the working lines, experimental investigations with models in this direction will be of great value.

Its title indicates that the paper should cover a much wider scope than it actually does, inasmuch as the term, "structural members", as commonly understood, includes such members under the action of flexure, compression, tension, or torsion, or any combination of them. Actually, the paper treats such members under the action of flexure only, and, therefore, it would be more appropriate and also more precise if the word, "structural", in the title were changed to read "flexural."

A. A. EREMIN,[55] ASSOC. M. AM. SOC. C. E. (by letter).—An interesting method of computing the elastic coefficients of tapered members is presented in this paper. With the equations developed by the authors, a complete analytical analysis of perfectly rigid frames can be made.

The computations of the coefficients, *F*, may be simplified, as follows: A general integral expression of the elastic coefficients of tapered members is:

$$R = \int_0^l \frac{x^p\,dx}{I_x} \qquad \text{.......................(187)}$$

In which p is an exponent of x that varies from zero to 5. In the most common cases (see Equation (24)), $p = 5$.

Substitute Equation (1) in Equation (187) as follows:

$$R = \frac{1}{I_0} \int_0^l \left[1 + A\left(\frac{x}{l}\right)^n\right] x^p\,dx \qquad \text{.............(188)}$$

Integrating and simplifying, Equation (188) becomes:

$$\begin{aligned}\int_0^l \frac{x^p\,dx}{I_x} &= \frac{1}{I_0}\left[\frac{x^{p+1}}{p+1} + \frac{A}{l^n}\,\frac{x^{n+p+1}}{(n+p+1)}\right]\\ &= \frac{l^{p+1}}{I_0}\left[\frac{1}{p+1} + \frac{A}{(n+p+1)}\right] \qquad \text{..............(189)}\end{aligned}$$

The integrals entering Equation (189) may be expressed as:

$$\int_0^l \frac{dx}{I_x} = \frac{l\,R_0}{I_0} \qquad \text{......................(190)}$$

$$\int_0^l \frac{x\,dx}{I_x} = \frac{l^2\,R_1}{I_0} \qquad \text{.....................(191)}$$

and,

$$\int_0^l \frac{x^2\,dx}{I_x} = \frac{l^3\,R_2}{I_0} \qquad \text{.....................(192)}$$

From Equation (189), the values of R are:

$$R_0 = 1 + \frac{A}{n+1} \qquad \text{......................(193)}$$

$$R_1 = \frac{1}{2} + \frac{A}{n+2} \qquad \text{.....................(194)}$$

and,

$$R_2 = \frac{1}{3} + \frac{A}{n+3} \qquad \text{....................(195)}$$

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[55] Assoc. Bridge Designing Engr., Div. of Hignways, State Dept. of Public Works, Sacramento, Calif.

Therefore, from Equations (33), (34), (35), (190), (191), and (192):

$$\int_0^l \frac{x(l-x)\,dx}{I_x} = \frac{l^3}{I_A}(R_1 - R_2) \quad \text{..............(196)}$$

$$\int_0^l \frac{x^2\,dx}{I_x} = \frac{l^3}{I_A} R_2 \quad \text{......................(197)}$$

and,

$$\int_0^l \frac{(l-x)^2\,dx}{I_x} = \frac{l^3}{I_A}(R_0 - 2R_1 + R_2) \quad \text{............(198)}$$

Equation (189) is especially convenient in computing the loading coefficients, with bending moments in which the power of x is the highest, as in Equation (24).

Computation of the coefficients for tapered members may be further simplified by means of Strassner's tables[56], in which the elastic properties of members with variable sections are expressed by formulas for moment of inertia similar to those in this paper.

Assume that $N = \frac{I_0}{I_e}$; and that $t = \frac{x}{l}$. Then Equation (1) for the moment of inertia of a tapered member becomes the same as that introduced by Professor Strassner[56],

$$I_x = \frac{I_0}{1-(1-N)\,t^n} \quad \text{...................(199)}$$

The formulas developed by Messrs. Weiskopf and Pickworth suggest the construction of various tables and curves to simplify the computation of stresses in statically indeterminate frames. The authors have made a valuable contribution to the profession.

Austin H. Reeves,[57] Assoc. M. Am. Soc. C. E. (by letter).—The great worth of this paper is proved more conclusively with each increment of study. Having designed many types of structural frameworks containing members of variable moment of inertia by both conjugate points (fixed points) and the method of fixed-end moment distribution, it was with a critical (if not, antagonistic) state of mind that the writer approached the paper. This condition was caused by a casual glance disclosing plenty of calculus.

However, a start was made by checking about twenty of the equations against several designs which the writer knew to be correct. The accuracy of the results of the analytical treatment was thus established. Nevertheless, a further and much more exhaustive study was made, which produced an ever-increasing respect for the authors' treatment of the subject. For example, the writer solved the problem shown in Fig. 27, a beam with a lateral width of 12 in. Within 5 min it was determined that n should be less than 1.3.

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[56] "Der Durchlaufende Rahmen", by A. Strassner.

[57] Newark, N. J.

Within an additional 5 min it was decided that an accurate enough value of n would be 1.1; that is, since $e\ l = 4.9$ ft; $I_e = 4\,411$ in.^{4}; $I_A = 46\,656$ in.^{4}; and $I_C = 1\,728$ in.^{4}, A is found by Equation (7) to be 26. Then, it is a matter of simple substitution in Equation (8) to determine that $n = 1.1$.

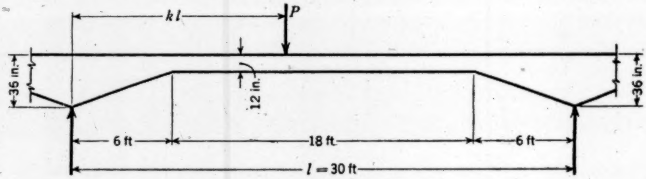

FIG. 27.

It is impossible to have more than one correct value for n. A correct substitute I-curve cannot run through any arbitrarily selected point because it must satisfy the following two conditions[58]:

(a) The area of the actual I-curve must be maintained in selecting a substitute curve; and,

(b) The distance from the left support to the center of gravity of this area must be the same for the substitute I-curve as for the actual curve because it controls the size of the carry-over factor, whereas the area controls the rigidity and is also a factor in determining the amount of the fixed-end moments.

It is incorrect to run a substitute I-curve, successively, through a number of points selected arbitrarily by measurements from the left support. Of course, such procedure would give to n a different value for each point selected.

The paper is so excellent that the writer hesitates to make suggestions. However, emphasis should be laid upon the fact that $e\ l$ and I_e should be selected correctly first (using Conditions (a) and (b) as criteria), and then the correct value of n can be computed by Equation (8) of the paper.

In conclusion, the writer wishes to express his admiration for the thorough manner in which so complicated a subject has been presented. The paper should be used extensively in the future as its full import becomes appreciated.

A. W. FISCHER,[59] ESQ. (by letter).—A very thorough mathematical treatment for solving the values of M_A, C_{AB}, and S_A, is presented in this paper. The results are reliable when the substitute I-curve practically coincides with the actual I-curve. By solving for the shape exponent, n, at several points along the tapered member a value can be selected such that the substitute I-curve will fit the actual I-curve very well. Members of uniform depth, with straight-line haunches at either end, or at both ends, should be divided into sections so as to fit the substitute I-curve more closely to the actual I-curve. The taper modulus, A, can be determined readily, but the accurate determination of n is another problem. Since an average can be selected, however, so that the substitute I-curve will give reliable results, it seems that the authors have added a very important item in the theory pertaining to this subject.

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[58] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 90.

[59] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

For the solution of general tapered members used in ordinary practice it seems that the values of the constants, M_A, C_{AB}, and S_A, can be taken directly from charts[60] in just a fraction of the time required to calculate the constants using the authors' method; and especially is this true for straight haunched beams in which the haunch does not extend to the center of the span.

As an example, consider the symmetrical rigid-frame concrete bridge[61], shown in Fig. 28. If a width of 12 ft is used, $I = d^3$, in feet. Substituting in Equation (7): $A = \dfrac{76.77 - 5.359}{5.359} = 13.33$ = taper modulus for the horizontal member.

At a point 12 ft from the left support for the horizontal member the depth = 2.65 ft, from which $I_e = 18.61$, and substituting in Equation (8), $n = \log \dfrac{76.77 - 18.61}{13.33 \times 18.61} \div \log \dfrac{12}{30} = 1.583$ = the shape exponent for the horizontal member. In solving the value of n at other points, it seems that the value, 1.583, is somewhat too great, and a value of 1.55 is selected so that the substitute I-curve will fit quite well with the actual I-curve for the horizontal member.

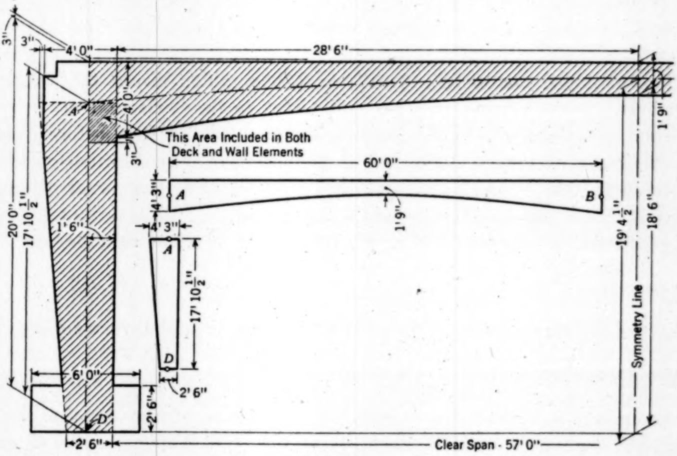

FIG. 28.

Case 2 will apply for the horizontal member and, from Equation (72), $F_1 = 1.3117$; from Equation (73), with $n = 1.55$, $F_2 = 1.8023$; and, from Equation (74), $F_3 = F_2$.

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[60] See "Design of Continuous Frames Having Variable Moments of Inertia", *Civil Engineering*, October, 1932, pp. 647–648.

[61] "Analysis of Rigid Frame Concrete Bridges", Portland Cement Assoc., Third Edition, 1935.

For a uniformly distributed load of $w = 90$ lb per lin ft, on a span of $l = 60$ ft, with $n = 1.55$, Equation (82) yields $F_6 = 0.32797$, and, from Equation (81), $M_A = M_B = -34\,120$ ft-lb.

For a concentrated load of 2 500 lb at the center of the span, Equation (80) gives $F_{10} = 0.5319$, and, from Equation (79), with $P = 2\,500$ lb, $M_A = M_B = -25\,620$ ft-lb. Therefore, the total, live load, fixed-end moment $= -34\,120 - 25\,620 = -59\,740$ ft-lb. From Equation (52), $C_{AB} = -\dfrac{F_1}{F_2} = -0.7278$; and, from Equation (61), with $I_A = 76.77$, $S_A = 0.3775$.

For the vertical member, from Equation (7), $A = 3.912$. At a point 6 ft from the top of the member the thickness $= 3.66$ ft, and $I_6 = 49.03$. Substituting in Equation (8), with $e = 6$ and $r = 17.88$, $n = 1.771$. In solving for the value of n at other points, it seems that 1.771 is slightly too small; therefore, a value of 1.8 will be used. The vertical member comes under Case 3, and, from Equation (85), $F_2 = 0.4865$.

Variation of the Cross Method When One End Is Hinged.—From Equation (110), $S'_A = 2.206$. As the horizontal member is symmetrical and $M_A = M_B$, a modified S_A can be used[62] so that one distribution is all that is required. Calling the new stiffness factor, S''_A, it is equal to $S_A \times (1 + C_{AB}) = 0.1028$.

The combined factors, $S'_A + S''_A = 2.3088$, which means that 95.55% is distributed to the vertical member and 4.45% to the horizontal member. As the total, live load, fixed-end moment, $M_A = -59\,740$ ft-lb before distribution, the actual moment at the end of Member AB after distribution will be, $-59\,740\ (1.0000 - 0.0455) = -57\,080$ ft-lb, which is the corner moment at Point A when the deck is straight. On account of the horizontal member being curved, as shown in Fig. 28, the corner moment will be less. The reduction factor[63] will be $\dfrac{17.88 + 0.75}{19.38}$, and this value times $-57\,080 = -54\,870$ ft-lb. As the corner moment is $-54\,870$, the horizontal thrust acting at the hinged support will be $\dfrac{54\,870}{17.88} = 3\,069$ lb.

Assuming, now, that the horizontal component of the reaction for the foregoing loading is 3 069 lb, then, by statics, the moment at the crown is,

$$
\begin{array}{lcl}
+\ 90 \times 30 \times 0.5 \times 30 & = & +\ 40\,500 \\
+\ 0.5 \times 2\,500 \times 30 & = & +\ 37\,500 \\
-\ 3\,069 \times 19.375 & = & -\ 59\,450 \\
\hline
\text{Total}\ldots\ldots\ldots\ldots\ldots & = & +\ 18\,550 \text{ ft-lb}
\end{array}
$$

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[62] "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., N. Y., John Wiley & Sons, 1932, Fig. 88, p. 119.

[63] "Analysis of Rigid Frame Concrete Bridges", Portland Cement Assoc., Third Edition, 1935, p. 17.

Using a method advanced by the Portland Cement Association[64], the total positive live load moment at the crown, assuming a simply supported deck, is found to be 78 000 ft-lb. Then the difference between this moment and the negative corner moment created by the same loading is 78 000 − 57 080 = + 20 920 ft-lb, which is the moment at the crown of the frame with a straight deck. The reduction factor[63] will be, $\frac{17.88 + 0.5 \times 1.50}{17.88 + 1.50} = \frac{18.63}{19.38}$, and this value times 20 920 = 20 120 ft-lb.

The foregoing example was also solved by the writer using the "method of elastic weights", and the corner moments by that method were found to be − 55 100 ft-lb, and the crown moments, + 18 310 ft-lb. As the results by the authors' method check very closely with the method of elastic weights it shows that the former is sufficiently accurate for practical purposes.

The authors' method for the analysis of tapered members will give the desired results; but for the analysis of such a structure as the rigid-frame bridge, hinged at the supports, there are other analytical methods that are shorter, and if a systematic arrangement of tables is used the writer is of the opinion that the method of elastic weights will give the desired results in the shortest time.

By means of the method of elastic weights, the depth of the frame can vary along its length in any proportion, the axis can be of any shape, the end-posts need not be vertical, and, if the division points are taken sufficiently close, the results will agree very well with any of the analytical methods.

The one great disadvantage of the authors' method for the analysis of the rigid-frame bridge is that it is based on a straight-line axis for the horizontal member, and as practically all horizontal members have a curved axis a correction must be made by introducing a reduction factor which is only approximate. Another disadvantage is that, when the load is not symmetrical, there will be side-sway, and a correction must be made for that also.

Even if the method is not generally adopted, the authors should be commended for introducing a mathematical treatment which, some day, may be further elaborated, and eventually an average value of *n* can be selected more readily, so that the substituted *I*-curve will give as close results as the actual *I*-curve.

L. LEGENS,[66] ESQ. (by letter).—A method of analyzing structures, composed of members of non-uniform cross-section, is discussed in this paper, and the authors have indicated its application to some classical methods of analysis. The hope is expressed in conclusion that the resulting method of substitute *I*-curves may aid in the development of classes of structures which have been hampered in the past by mathematical difficulties of design. The use of this method, in general, is most advantageous and becomes necessary for the calculation of many basically and indeterminate systems.

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[64] "Analysis of Rigid Frame Concrete Bridges", Portland Cement Assoc., Third Edition, 1935, p. 24.

[66] Engr., Alsace-Lorraine R. R., Strasbourg, France.

For the arch fixed at the two ends, in the past, it was generally agreed, for the purpose of computation, that the reduced moment of inertia, $I' = I \cos \phi$, should be constant and equal to the moment of inertia, I_c, at the crown (see Fig. 29). Müller-Breslau[96] was one of the first to state that this assumption was not admissible, and to suggest that the arch should be computed as a tapered beam.

Assuming a parabolic arch barrel and selecting a Cartesian system of co-ordinates with the origin in O (Fig. 29), he determined the three statically indeterminate reactions, X_a, X_b, and X_c, for a concentrated load, P_m, with the general equations of elasticity.

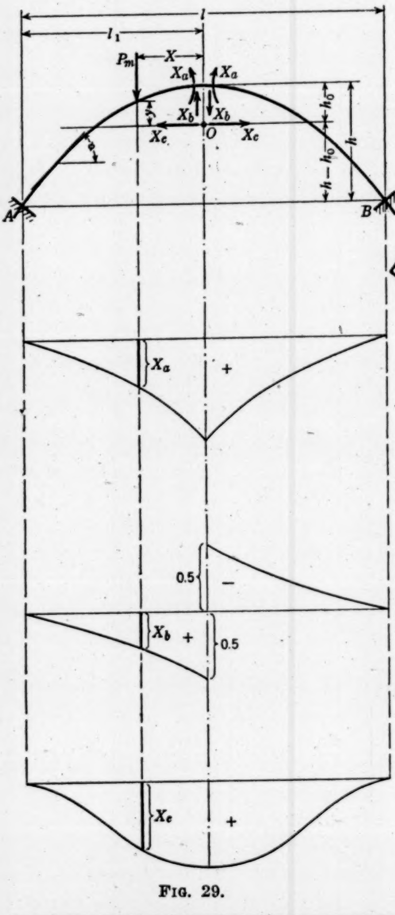

FIG. 29.

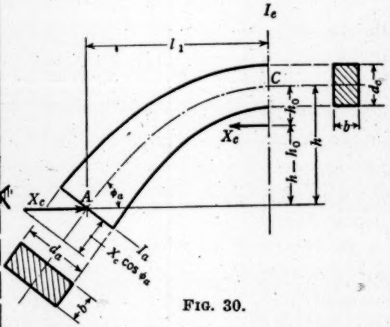

FIG. 30.

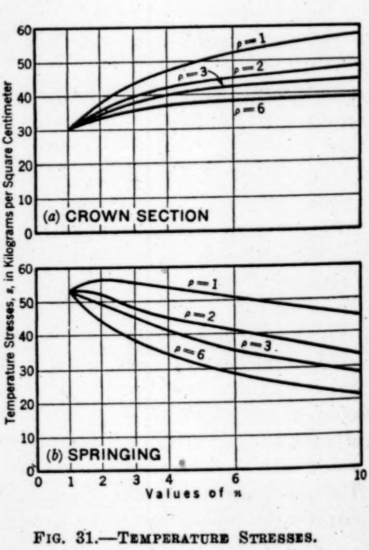

FIG. 31.—TEMPERATURE STRESSES.

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[96] "Die Graphische Statik der Baukonstruktionen", von Müller-Breslau, Band II, 2 Abteilung, Seite 556.

Expressing the reduced moment of inertia, I'_n, of the end cross-sections by:

$$I'_n = \frac{1}{\alpha} I_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (200)$$

he chose for the function of the I'-curve, the formula,

$$\frac{I_c}{I'} = 1 - (1 - \alpha) \left(\frac{x}{l_1}\right)^n \ldots\ldots\ldots\ldots\ldots\ldots (201)$$

which is similar to Equation (1) of the paper, except that the center of ordinates for Equation (201) is at the mid-points, whereas in the paper (Equation (1)) it is at the ends. For $\alpha = 1$, Equation (201) will reduce to $I' = I \cos \phi = I_c$, an expression which was recognized by early mathematicians; and, for the distance, h_0 (see Fig. 30), Müller-Breslau derives the expression:

$$h_0 = \left(\frac{h\,(\rho + 1)}{3\,(\rho + \alpha)}\right) \left(\frac{\rho + 3\,\alpha}{\rho + 3}\right) \ldots\ldots\ldots\ldots\ldots (202)$$

in which values of ρ, corresponding to n, are shown in Fig. 31.

Fig. 29 presents the influence lines for the reactions, X_a, X_b, and X_c. These values have been calculated by Müller-Breslau for various ratios, $\frac{x}{l_1}$, as shown in Table 7. The difference between the values resulting from the Müller-Breslau "shape exponents", ρ =1 and $\rho = 2$, is not very great; but there is considerable variation between the different values, X_a, X_b, and X_c, for $\alpha = 0.25$ or for $\alpha = 1.00$. For practical purposes, $\alpha = 0.25$ will be a good average value and it is not permissible to assume that $\alpha = 1$ (that is, to neglect the super-modulus).

TABLE 7.—COMPUTATION OF REACTIONS FOR VARIOUS POSITIONS OF LOAD

$\frac{x}{l_1}$	VALUES OF X_a			VALUES OF X_b			VALUES OF X_c		
	$\alpha = 0.25$		$\alpha = 1.00$	$\alpha = 0.25$		$\alpha = 1.00$	$\alpha = 0.25$		$\alpha = 1.00$
	$\rho = 1$	$\rho = 2$		$\rho = 1$	$\rho = 2$		$\rho = 1$	$\rho = 2$	
0.0	6.0000	6.2500	7.5000	0.5000	0.5000	0.5000	0.9973	0.9950	0.9290
0.2	3.4560	3.6480	4.8000	0.3328	0.3331	0.3520	0.8941	0.8981	0.8562
0.4	1.7280	1.8180	2.7000	0.1882	0.1872	0.2160	0.6371	0.6443	0.6555
0.6	0.6720	0.6880	1.2000	0.0814	0.0789	0.1040	0.3315	0.3321	0.3805
0.8	0.1440	0.1380	0.3000	0.0191	0.0174	0.0280	0.0904	0.0864	0.1204
h_0*	......	......	......	......	......	......	3.50	3.67	5.00

* Values of h_0, in meters.

It is well known that for flat arches $\left(\frac{h}{l} \leq \frac{1}{10}\right)$, the influence of temperature is very considerable and in every case is more important than that of loads.

Referring to the general equations of elasticity, let δ = deformation; then: $\delta_{at} = 0$; $\delta_{bt} = 0$; $\delta_{ct} = 2\,\epsilon\, t\, l_1 = \epsilon\, t\, l = \Delta\, l$; $X_a = \dfrac{\delta_{at}}{\delta_{aa}} = 0$; $X_b = \dfrac{\delta_{bt}}{\delta_{bb}} = 0$; $X_c = \dfrac{\delta_{ct}}{\delta_{cc}}$; and,

$$X_c = \frac{\Delta\, l\, E\, I_c}{h\, l\left(\dfrac{1}{5}\, h\, \dfrac{\rho + 5\,\alpha}{\rho + 5} - \dfrac{1}{3}\, h_0\, \dfrac{\rho + 3\,\alpha}{\rho + 3}\right)} \qquad (203)$$

The writer prefers to use the reciprocal values introduced by Müller-Breslau. Therefore, according to Equations (200) and (201), $n = \dfrac{1}{\alpha}$; $I'_n = n\, I_c$; and,

$$\frac{I_c}{I'} = 1 - \frac{n-1}{n}\left(\frac{x}{l_1}\right)^{\rho} \qquad (204)$$

Equation (203) becomes:

$$X_c = \frac{E\, I_c}{h^2}\,\frac{\Delta\, l}{l}\,\frac{1}{\dfrac{1}{5}\,\dfrac{n\,\rho + 5}{n\,(\rho + 5)} - \dfrac{1}{9}\,\dfrac{n\,(\rho + 1)}{n\,\rho + 1}\left[\dfrac{n\,\rho + 3}{n\,(\rho + 3)}\right]^2} \qquad (205)$$

Referring to Fig. 30 the unit stress, s, is obtained for the crown section, C, by the equation:

$$s = \frac{X_c}{A_c} \pm \frac{X_c\, h_0}{I_c}\,\frac{d_c}{2} \qquad (206)$$

in which A_c and I_c are the area and the moment of inertia and are equal, respectively, to: $A_c = b\, d_c$; and $I_c = \dfrac{b\, d^3_c}{12}$. When the signs, ± or ∓, appear, the positive refers to the intrados and the negative refers to the extrados. Finally, according to Equations (202), (205), and (206), for h_0, X_c, A_c, and I_c:

$$s = \frac{E\,\dfrac{\Delta\, l}{l}\left(\dfrac{d_c}{h}\right)^2 \dfrac{1}{12}\left[1 \pm \dfrac{2\, h}{d_c}\,\dfrac{\rho + 1}{\rho + 3}\,\dfrac{n\,\rho + 3}{n\,\rho + 1}\right]}{\dfrac{1}{5}\,\dfrac{n\,\rho + 5}{n\,(\rho + 5)} - \dfrac{1}{9}\,\dfrac{n\,(\rho + 1)}{n\,\rho + 1}\left[\dfrac{n\,\rho + 3}{n\,(\rho + 3)}\right]^2} \qquad (207)$$

For Sections A at the springing (see Fig. 30), the unit stress, s, will be:

$$s = \frac{X_c \cos\phi_a}{A_a} \mp \frac{X_c\,(h - h_0)}{I_n}\,\frac{d_a}{2} \qquad (208)$$

in which $I_n = \dfrac{b\, d^3_a}{12}$; and, $A_a = b\, d_a$. Furthermore,

$$I'_a = I_a \cos\phi_a = n\, I_c = \frac{n\, b\, d^3_c}{12} \qquad (209)$$

$$\frac{I_a}{I_c} = \frac{n}{\cos \phi_a} \frac{d^3_a}{d^3_c} \quad \ldots\ldots (210)$$

$$\cos \phi_a = \frac{1}{\sqrt{1 + \left(\frac{4\,h}{l}\right)^2}} \quad \ldots\ldots (211)$$

and,

$$h - h_0 = \frac{2}{3}\, h\, \frac{\rho\, n\,(\rho + 4) + 3}{\rho\,[\,n\,(\rho + 3) + 1\,] + 3} \quad \ldots\ldots (212)$$

from which Equation (208) becomes:

$$s = \frac{X_c \cos \phi_a}{b\, d_a} \left[1 \mp \frac{h - h_0}{b\, d^3_a} \times \frac{12\, b\, d_a}{\cos \phi_a} \times \frac{d_a}{2}\right]$$

$$= \frac{X_c \cos \phi_a}{b\, d_a} \left[1 \mp \frac{6\,(h - h_0)}{d_a \cos \phi_a}\right] \quad \ldots\ldots (213)$$

and,

$$s = \frac{E\, \frac{\Delta\, l}{l}\, \frac{\cos \phi_a}{12} \left(\frac{d_c}{h}\right)^2 \left[1 \mp \frac{4\,h}{d_a}\, \frac{1}{\cos \phi_a}\, \frac{\rho\, n\,(\rho + 4) + 3}{\rho\,[\,n\,(\rho + 3) + 1\,] + 3}\right]}{\frac{1}{5}\, \frac{n\,\rho + 5}{n\,(\rho + 5)} - \frac{1}{9}\, \frac{n\,(\rho + 1)}{n\,\rho + 1} \left[\frac{n\,\rho + 3}{n\,(\rho + 3)}\right]^2} \quad .(214)$$

in which,

$$\frac{d_a}{d_c} = \frac{\sqrt[3]{n}}{\sqrt[3]{\cos \phi_a}} \quad \ldots\ldots (215)$$

and,

$$d_a \cos \phi_a = d_c \sqrt[3]{n}\, \sqrt[3]{\cos^2 \phi_a} \quad \ldots\ldots (216)$$

It follows that:

$$s = \frac{E\frac{\Delta l}{l}\, \frac{\cos \phi_a}{12} \left(\frac{d_c}{h}\right)^2 \frac{\sqrt[3]{\cos \phi_a}}{\sqrt[3]{n}} \left[1 \mp \frac{4\,h}{d_c}\, \frac{1}{\sqrt[3]{n}}\, \frac{1}{\sqrt[3]{\cos^2 \phi_a}}\, \frac{\rho\, n\,(\rho + 4) + 3}{\rho\,[n\,(\rho + 3) + 1] + 3}\right]}{\frac{1}{5}\, \frac{n\,\rho + 5}{n\,(\rho + 5)} - \frac{1}{9}\, \frac{n\,(\rho + 1)}{n\,(\rho + 1)} \left[\frac{n\,\rho + 3}{n\,(\rho + 3)}\right]^2} \quad \ldots\ldots (217)$$

For flat arches one may assume that $\sqrt[3]{\cos \phi_a}$ varies as 1 and $\sqrt[3]{\cos^2 \phi_a}$ varies as 1, in which case, Equation (217) becomes:

$$s \propto \frac{E\frac{\Delta l}{l} \left(\frac{d_c}{h}\right)^2 \frac{1}{12\, \sqrt[3]{n}}\, \frac{1}{\sqrt{1 + 4\left(\frac{h}{l}\right)^2}} \left[1 \mp \frac{4\,h}{d_c}\, \frac{1}{\sqrt[3]{n}}\, \frac{\rho\, n\,(\rho + 4) + 3}{\rho\,[n\,(\rho + 3) + 1] + 3}\right]}{\frac{1}{5}\, \frac{n\,\rho + 5}{n\,(\rho + 5)} - \frac{1}{9}\, \frac{n\,(\rho + 1)}{n\,\rho + 1} \left[\frac{n\,\rho + 3}{n\,(\rho + 3)}\right]^2}$$

Finally, for the crown section:

$$s = E\,\frac{\Delta l}{l}\left(\frac{d_c}{h}\right)\frac{1}{6}\,h_3(\rho, n)\left[\frac{1}{2}\left(\frac{d_c}{h}\right) \mp h_4(\rho, n)\right] \ldots\ldots\ (218)$$

and, for the section at the springing:

$$s \backsim E\,\frac{\Delta l}{l}\left(\frac{d_c}{h}\right)\frac{1}{3}\,\frac{1}{\sqrt{1+\left(\frac{4h}{l}\right)^2}}\,h_1(\rho, n)\left[\frac{1}{4}\left(\frac{d_c}{h}\right) \pm h_2(\rho, n)\right] \ldots\ (219)$$

in which:

$$h_1(\rho, n) = \frac{1}{\frac{1}{5}\,\frac{n\,\rho+5}{n\,(\rho+5)} - \frac{1}{9}\,\frac{n\,(\rho+1)}{n\,\rho+1}\left[\frac{n\,\rho+3}{n\,(\rho+3)}\right]^2}\,\frac{1}{\sqrt[3]{n}} \ldots (220)$$

$$h_2(\rho, n) = \frac{\rho\,n\,(\rho+4)+3}{\rho\,[n\,(\rho+3)+1]+3}\,\frac{1}{\sqrt[3]{n}} \ldots\ldots\ldots\ldots (221)$$

$$\begin{aligned} h_3(\rho, n) &= \frac{1}{\frac{1}{5}\,\frac{n\,\rho+5}{n\,(\rho+5)} - \frac{1}{9}\,\frac{n\,(\rho+1)}{n\,\rho+1}\left[\frac{n\,\rho+3}{n\,(\rho+3)}\right]^2} \\ &= \sqrt[3]{n}\,h_1(\rho, n) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (222) \end{aligned}$$

and,

$$h_4(\rho, n) = \frac{\rho+1}{\rho+3}\,\frac{n\,\rho+3}{n\,\rho+1} \ldots\ldots\ldots\ldots\ldots\ldots (223)$$

Values of h are listed in Table 8. The maximum stresses are found for the intrados at the crown and for the extrados at both springing lines.

TABLE 8.—VALUES OF $h\ (\rho, n)$ IN EQUATIONS (220) TO (223), INCLUSIVE

Values of n	FOR VALUES OF ρ EQUAL TO:				FOR VALUES OF ρ EQUAL TO:			
	1	2	3	6	1	2	3	6
	(a) VALUES OF h_1 (EQUATION (220))				(c) VALUES OF h_3 (EQUATION (222))			
1	11.25	11.25	11.25	11.25	11.25	11.25	11.25	11.25
2	13.50	12.55	12.01	11.11	17.01	15.81	15.14	14.00
3	14.68	13.00	12.08	10.65	21.18	18.75	17.42	15.36
4	15.38	13.11	11.92	10.19	24.41	20.82	18.93	16.18
6	16.03	12.95	11.45	9.41	29.13	23.53	20.80	17.11
10	16.20	12.27	10.52	8.33	34.90	26.43	22.66	17.95
	(b) VALUES OF h_2 (EQUATION (221))				(d) VALUES OF h_4 (EQUATION (223))			
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	0.86	0.86	0.85	0.83	0.83	0.84	0.86	0.90
3	0.78	0.77	0.77	0.74	0.75	0.77	0.80	0.86
4	0.72	0.71	0.70	0.68	0.70	0.73	0.77	0.84
6	0.65	0.64	0.62	0.61	0.64	0.69	0.74	0.82
10	0.56	0.54	0.53	0.51	0.59	0.66	0.71	0.80

The writer has computed these stresses for a concrete arch bridge, 74 m (242.8 ft) long and 6.75 m (22.1 ft) high for super-moduli of $n = 1, 2, 3, 4, 6$, and 10, and the shape exponents, $\rho = 1, 2, 3$, and 6. They are shown in Fig. 31. With increasing values of n and ρ the stresses decrease at the springing, increase at the crown, and are almost equal at both sections for $n = 5$ and $\rho = 2$ which are good average values as determined for a considerable number of actual bridges.

For practical purposes it may be sufficient to compute the influence of temperature by the old formulas (assuming $n = 1$), the actual stresses being always less than those calculated.

In conclusion, the writer wishes to express his appreciation of the clear method described in this paper. It permits a rapid analysis of structures with variable moments of inertia. Although Müller-Breslau anticipated the method proposed by the authors, he did not apply it in such a general manner. Of course, the equations presented by the writer could be easily transformed to make them agree with the equation for the moment of inertia of tapered members presented by Messrs. Weiskopf and Pickworth.

WALTER H. WEISKOPF,[67] AND JOHN W. PICKWORTH,[67] MEMBERS, AM. SOC. C. E. (by letter).—The broad approach to the subject adopted by discussers of this paper is gratifying. It is a strong human trait to be distrustful of new methods and engineers are undoubtedly unfamiliar with the general application of substitute I-curves. The many suggestions and examples of use, as well as the fund of data that has been so generously contributed, are greatly appreciated, constituting as they do a tribute to the progressive spirit of the profession.

The frank statement that this method employs a substitute I-curve might lead some students of the subject to feel that for this reason the analysis is inherently inaccurate and to be distrusted. It should be realized that for all tapered members, except a few very simple forms, there is no exact treatment. Whether the designer realizes it or not, all methods are approximations or, in a sense, use substitute I-curves.

In the commonly used methods the integrations are performed by dividing the member into sections along its length. The value of $\frac{1}{I}$, or $\frac{M}{I}$, at the mid-point of each of these sections is assumed to apply to its entire length. In reality, this substitutes a series of steps, not only for the I-curves, but for the moment diagram as well. The latter inaccuracy is one which the writers' method does not introduce. The accuracy of the common method depends on the number of sections or steps employed. Usually, these sections are from 3 to 6 ft long, and the results are sufficiently accurate for ordinary engineering purposes, but such approximations are no more trustworthy than the use of a smooth curve which follows the general course of the I-diagram.

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[67] Cons. Engr. (Weiskopf & Pickworth), New York, N. Y.

Messrs. Large and Morris[53] have improved on the "stepped" substitute I-curve by introducing a series of inclined straight lines or chords. The area under the curve is thus divided into a series of trapezoids instead of rectangles. This method still involves a substitute I-curve, although an excellent one.

The accuracy of the method of substitute I-curves depends on the skill employed in fitting the curve. As Professor Clark discovered in turning his students loose on it, the method is not a fool-proof one that can be used safely by any beginner, but should be applied intelligently.

Mr. Reeves mentions two conditions that should be satisfied by the substitute curve: The area and the center of gravity of the area under the I-curve should be maintained. An additional thought should be borne in mind, namely, that where an exact fit is impossible it is more important to have a close fit at parts of the member where the moment of inertia is small rather than where it is large. This follows from the fact that the contribution of the work terms is large where the value of I is small, and *vice versa.*

Professor Plummer gives the relations between the terms F_1, F_2, and F_3 and the constants used in the conjugate and fixed-point methods of analysis. The following relations can be added to those which he gives:

$$u = \frac{F_1}{F_1 + F_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (224)$$

and,

$$v = \frac{F_1}{F_1 + F_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (225)$$

Professor Clark finds results obtained by means of the substitute I-curves satisfactory where he used the formulas for two-section members on a member which has two sections (Fig. 13(a)). In Fig. 13(b), however, he tried the formulas for two-section members on what is very obviously one with three distinct parts, a long uniform center section with sharp haunches at the two ends. It would have been more in the spirit of the writers' work to treat the center section as uniform and fit a substitute I-curve to each of the haunches. This can be done either by integrating with numerical values for an individual member, or by integrating to derive formulas. For such a member the formulas are no more complicated than those of Case 1 and are much more accurate.

Even if it is stretching the mathematics considerably to apply two-section formulas to a three-section member, the results are quite accurate if judgment is exercised. As mentioned previously, it is more important to have the two I-curves fit where the member is small than where it is large. This explains why Professor Clark obtained best results using a shape exponent which made the curves coincide where $x = 5$, and is obvious to one skilled in the method. Trying the other values ($x = 1$, $x = 2$, $x = 3$, etc.) was misleading. On the other hand Mr. Reeves treating the same member (Fig. 27) immediately selected a value of 4.9 for $e\,l$.

Professor Clark tried to test the speed of the method of substitute I-curves by having some of his students work the same problem, by using it, and by using the column analogy. The column analogy is a general method of analyzing statically indeterminate structures. The substitute I-curve affords a means of integrating the functions of x divided by I, which can be used in any method of analysis, the column analogy as well as any other. The two are not comparable. What Professor Clark means, of course, is that he compared the speed in obtaining results using formulas derived by the least work theory and substitute I-curves for the integrations, with the column analogy and a step-by-step summation for the integrations. Such a comparison in any case is not very clear, but considering that the students were presumably familiar with the step-by-step summation and totally unfamiliar with the method of substitute I-curves, it is of no value.

Mr. Paulet has contributed several illuminating examples of the use of the method. His thorough grasp of the fundamentals enables him to make variations in the application which add to the flexibility. For instance, his arbitrary assumption of the length in order to maintain a simple shape exponent is an ingenious hint that should prove useful to the skillful designer. Another excellent suggestion by Mr. Paulet to those who desire a visual guide in the selection of the shape exponent is to prepare a family of curves similar to Figs. 2 and 3 for ready use.

Professor Rathbun suggests another substitute curve, based on the assumption that the reciprocal of I varies as a parabola, and he further suggests that by increasing the power of x and the number of terms, the I-curves can be made to coincide at three, four, or more points. This is a possibility, of course, and there may be designers who prefer this curve. A disadvantage, however, is that the constants which Professor Rathbun designates p_1, p_2, etc., are expressed by formulas and have lost all obvious relationship to the form of the member. The constants, A (taper modulus) and n (shape exponent), in the writers' curve (Equation (1)), have a clear and obvious significance.

Professor Rathbun is incorrect in stating that the writers make the approximation of substituting a continuous curve for the two sections of the "stepped" member, Case 4. This case is treated by making the two parts of the member uniform, but not the same ($A = 0$, $B = 0$, I_A does not equal I_B). Since the terms, A, B, n, and m disappear, the formulas foı Case 4 are not approximate as Professor Rathbun states, but are exact.

Mr. Birkeland stresses the importance of an understanding of the physical significance of the mathematical terms employed by the engineer. He presents a geometrical interpretation of the F-terms by means of angular changes in the unit beam due to various unit moments and loads, which is undoubtedly clarifying to those accustomed to study the behavior of structures in terms of slope changes and deflections.

Correctly, he points out that all F-terms beyond F_5 can be obtained from F_4 and F_5, but such formulas as his Equation (165), should be used with care. In members of more than one section the integrations must be performed separately for each section.

Mr. Birkeland suggests using I_C instead of I_A as a reduction factor to obtain the F-terms. This is entirely a matter of personal preference. The writers intended to present a general method which an individual engineer can use to best advantage by adaptation to his own particular taste. If a designer prefers using I_C to I_A, and has sufficient mastery of the method to make this modification throughout, there can be no objections.

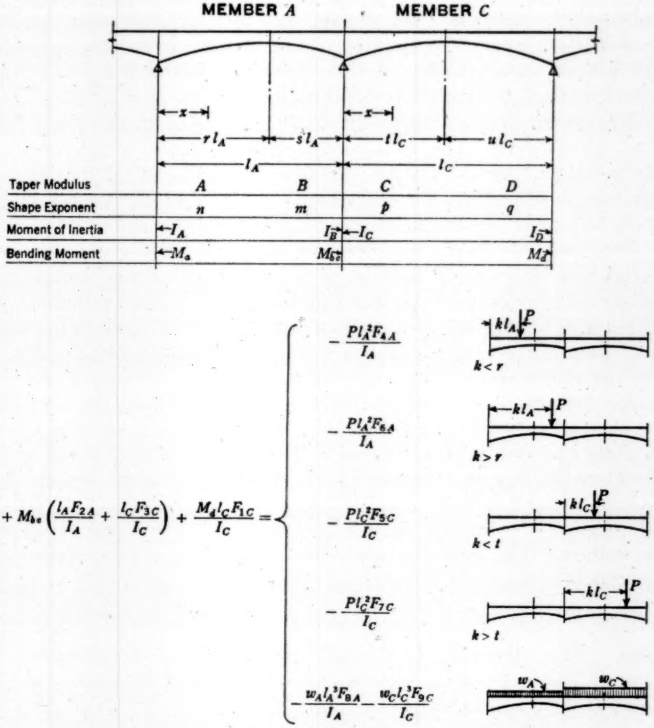
Mr. Dunham very properly emphasizes the importance of fixing the working lines correctly. The suggestions embodied in Fig. 12, and in Professor Tsai's Fig. 26 should be followed. Another method, brought out by Mr. Fischer in connection with the curved girder of a rigid frame, is to use a working line through the center of gravity of the ends of the member and later apply a correction.

Professor Tsai presents an extensive discussion of the various methods of analyzing structures composed of tapered members, and very complete tables showing the equivalents of the constants of the various systems. These are valuable records compiled in a scholarly manner. He is to be complimented on his presentation of the generalized equation of three moments. The method so often used in the past for want of a better one, assuming that a tapered continuous beam is uniform for the purpose of computing the reactions, is frequently grossly inaccurate. There has been a real need for this equation, but to the best of the writers' knowledge it has never appeared in any text. The writers had, independently, derived a generalized three-moment equation for tapered members using F-terms (see Fig. 32).

Professor Tsai tried the method of substitute I-curves on the member shown in Fig. 25[63] for a concentrated load at Point $k = 0.7$ and found that the resulting fixed-end moments differed about 12% from those obtained by step-by-step summation. A better test for accuracy is the area under the influence line, or what amounts to the same thing, the fixed-end moment for uniform load. This will more nearly approach the actual loading on the member.

For a uniform load the difference between the step-by-step value and that obtained using Professor Tsai's substitute I-curve is 6 per cent. Professor Tsai's shape exponent ($m = 0.64$), however, is poor. He seems to have sensed this for he states that better values might have been obtained using $f = 0.1$. Using a value of $m = 1$ the difference for the concentrated load is 5% instead of 12%, and for the uniform load, 2% instead of 6 per cent. If he had continued his comparative studies in order to develop influence lines by both methods, he would have appreciated the savings in time and labor which accrue once F_1, F_2, and F_3 are found.

Apparently, Professor Tsai does not fully appreciate the inaccuracy of the step-by-step summation. For M_A on this same member he obtains 27.26 ft-kips, whereas Professor Large obtained 27.95, a difference of $2\frac{1}{2}\%$; yet Professor Tsai uses his step-by-step summation as a standard of comparison for the substitute I-curve. The writers obtained a value of $M_A = 28$ ft-kips using $m = 1$. Professor Tsai refers to the writers' "exact" formulas. These formulas are not exact, and in no place are they so designated. An exception to this generality is Case 4, the "stepped" member (and, of course, Case 5), the formulas for which are exact.


MEMBER A
MEMBER C
x
x
rl_A
sl_A
tl_C
ul_C
l_A
l_C
Taper Modulus
A
B
C
D
Shape Exponent
n
m
p
q
Moment of Inertia
I_A
I_B
I_C
I_D
Bending Moment
M_a
M_{bc}
M_d
kl_A
P
$k<r$
kl_A
P
$k>r$
kl_C
P
$k<t$
kl_C
P
$k>t$
w_A
w_C

$$\frac{M_a l_A F_{1A}}{I_A} + M_{bc}\left(\frac{l_A F_{2A}}{I_A} + \frac{l_C F_{3C}}{I_C}\right) + \frac{M_d l_C F_{1C}}{I_C} = \begin{cases} -\dfrac{Pl_A^2F_{4A}}{I_A} \\[2ex] -\dfrac{Pl_A^2F_{6A}}{I_A} \\[2ex] -\dfrac{Pl_C^2F_{5C}}{I_C} \\[2ex] -\dfrac{Pl_C^2F_{7C}}{I_C} \\[2ex] -\dfrac{w_A l_A^3 F_{8A}}{I_A} - \dfrac{w_C l_C^3 F_{9C}}{I_C} \end{cases}$$

FIG. 32.—THREE-MOMENT EQUATION FOR TAPERED MEMBERS

Professor Tsai suggests another substitute I-curve in the form, $I_x = a + bx^2 + cx^3$; but in order to integrate functions of x divided by I, the value of I must be expressed in some reciprocal relation to x. The writers do not understand how Professor Tsai's curve makes it possible to perform these integrations; nor does he furnish any light in this direction.

Professor Tsai mentions the condition at the intersections of members. When members are shallow this presents no difficulties, but when they are deep and the working points are well within the adjacent members, the point is well worth considering. The most rational treatment of this condition is to assign a value of infinity to the moment of inertia for the length between the edge of an adjacent member and the working point. Thus, the columns in Fig. 26(a) would have an infinite moment of inertia for h'', the upper part of the length, h. This can easily be taken care of in the writers' formulas by considering the column as a two-section "stepped" member (Case 4) with I_B equal to infinity. The resulting formulas, of course, are quite simple.

Mr. Eremin gives a simple method of computing the F-terms which is convenient for members of one section. It is not applicable, however, to multi-section members. Mr. Fischer presents an interesting application of the substitute I-curves to the rigid frame bridge. Although the writers differ with him in the selection of working lines they have found by various checks, as he has, that the results are accurate.

Mr. Legens presents Müller-Breslau's analysis of an arch, which utilizes a substitute I-curve. This is a good example of a use of the method beyond any mentioned by the writers. It should be emphasized that their purpose was to furnish, not a list of formulas but a method, general in its scope. which has many applications beyond the few mentioned.

For those who have repetitive cases to handle, tables are most useful and as has been pointed out by more than one discusser, the method will be an aid to those computing and checking such tables. The writers do not agree that design should be restricted in any way by the scope of existing tables. It is desirable to have available a method (or rather, methods) for analyzing any and all shapes.

Mr. Birkeland and Professor Tsai refer to the importance of understanding the physical meaning of the F-terms. They interpret these constants in terms of angle changes. Three discussers, Professor Plummer, Mr. Birkeland, and Professor Tsai present conversion tables which show the equivalents of the F-terms to the constants of other methods. The writers agree that the physical meaning of such constants is important to a thorough understanding of the subject. The following analytical interpretation of the meaning of the F-terms is preferred by the writers.

The F-terms are ratios, the integrals being divided by $\frac{l^3}{I_A}$ to obtain them. This simplifies numerical computation. For theoretical purposes, however, it is better to denote each integral as it stands by a symbol. Using G then instead of F:

$$G_1 = \frac{l^3\,F_1}{I_A} = \int \frac{x\,(l-x)\,dx}{I_x} \quad\ldots\ldots\ldots\ldots\ldots \quad (226)$$

$$G_2 = \frac{l^3\,F_2}{I_A} = \int \frac{x^2\,dx}{I_x} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (227)$$

and,

$$G_3 = \frac{l^3\,F_3}{I_A} = \int \frac{(l-x)^2\,dx}{I_x} \quad\ldots\ldots\ldots\ldots\ldots \quad (228)$$

It is apparent from Equations (226), (227), and (228), that G_2 is the moment of inertia of the reciprocal I-diagram about an axis at the left end of the member, and G_3 is the moment of inertia of the reciprocal I-diagram about an axis at the right end; G_1 is a fourth dimensional quantity for which there seems to be no name in engineering nomenclature. It is the summation of the product of each elementary strip of the area under the reciprocal I-curve times, first, its distance from the left end, and then its distance from the right end. It is like a product of inertia, except that the two axes are parallel instead of at right angles to each other. It will, therefore, be termed the "parallel product of inertia."

Other properties of the reciprocal I-diagram can be expressed in terms of the constants, G_1, G_2, and G_3, thus:

$$A_0 = \int \frac{dx}{I_x} = \frac{2\,G_1 + G_2 + G_3}{l^2} \quad \text{.............. (229)}$$

$$\int \frac{x\,dx}{I_x} = \frac{G_1 + G_2}{l} \quad \text{.................... (230)}$$

$$\int \frac{(l - x)\,dx}{I_x} = \frac{G_1 + G_3}{l} \quad \text{................ (231)}$$

$$x_0 = l\,\frac{G_1 + G_2}{2\,G_1 + G_2 + G_3} \quad \text{.................... (232)}$$

and,

$$v_0 = l\,\frac{G_1 + G_3}{2\,G_1 + G_2 + G_3} = l - x_0 \quad \text{.............. (233)}$$

Equation (229) is the area under the reciprocal I-curve; Equations (230) and (231) are the statical moments of the area under the reciprocal I-curve about the left end and the right end, respectively; and x_0 and v_0, respectively (Equations (232) and (233)), are the distances to the center of gravity from the left and right ends of the member.

The moment of inertia of the reciprocal I-curve about its center of gravity axis is found to be:

$$G_0 = \frac{G_2\,G_3 - G_1^2}{2\,G_1 + G_2 + G_3} \quad \text{.................. (234)}$$

It is interesting to note the appearance of the expression, $G_2 G_3 - G_1^2$, analogous to $F_2 F_3 - F_1^2$ which appeared in the denominator of the equations for fixed-end moments and for stiffness.

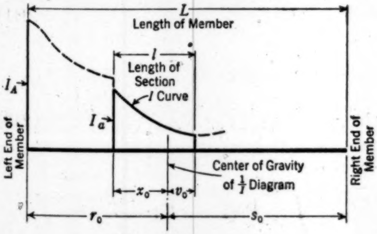


FIG. 33

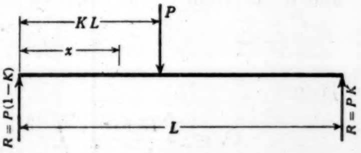


FIG. 34

Since G_2 and G_3 are moments of inertia, their values can be transferred to different axes as is done with ordinary moments of inertia. This suggests that in a complicated member composed of several sections the values of G_2 and G_3 can be summed up as the area of each section, the location of its center of gravity, and its G_0 are known.

Fig. 33 represents the I-curve for a section of a member. In general, small letters will be used to represent properties of the section about axes of the section, and capital letters to represent properties of the member about its axes. The values of g_1, g_2, and g_3 for this section about its own ends can be found by the formulas of Case 3, the section of single taper, multiplying the F-terms by $\frac{l^3}{I_a}$. Then, from Equations (229) to (234),

$$a_0 = \frac{2\,g_1 + g_2 + g_3}{l^2} \quad \text{.................... (235)}$$

$$x_0 = l\,\frac{g_1 + g_2}{2\,g_1 + g_2 + g_3} \quad \text{.................... (236)}$$

and,

$$g_0 = \frac{g_2\,g_3 - g_1^2}{2\,g_1 + g_2 + g_3} \quad \text{.................... (237)}$$

Knowing x_0, values of r_0 and s_0 can be found. Then for the entire member,

$$G_2 = \Sigma\,(g_0 + a_0\,r_0^2) \text{.................... (238)}$$

and,

$$G_3 = \Sigma\,(g_0 + a_0\,s_0^2) \text{.................... (239)}$$

An expression analogous to these for G_1 is:

$$G_1 = \Sigma\,(-\,g_0 + a_0\,r_0\,s_0) \text{.................... (240)}$$

Thus the properties of a member can be found simply no matter into how many sections it is divided. For a concentrated load, P (see Fig. 34), the terms, G_4 and G_5, analogous to F_4 and F_5, can similarly be found. To the left of the load the simple beam moment is:

$$M_s = P\,(1 - K)\,x \text{.................... (241)}$$

and to the right of the load the simple beam moment is:

$$M_s = PK\,(L - x) \text{.................... (242)}$$

Then,

$$G_4 = \frac{P\,L^3\,F_4}{I_A} = \int_0^L \frac{M_s\,x\,dx}{I_x} = \int_0^{KL} P\,(1 - K)\,\frac{x^2\,dx}{I_x}$$

$$+ \int_{KL}^{L} P\;K\,x\,(L - x)\,\frac{dx}{I_x} \quad \text{.................. (243)}$$

By Equations (238), (239), and (240):

$$G_4 = P\,(1 - K) \sum_{0}^{KL} (g_0 + a_0\, r_0^2) + P\,K \sum_{KL}^{L} (-\,g_0 + a_0\, r_0\, s_0) \; \ldots (244)$$

In a similar manner,

$$G_5 = \frac{P\,L^3\,F_5}{I_A} = P\,(1 - K) \sum_{0}^{KL} (-\,g_0 + a_0\, r_0\, s_0)$$

$$+ \; P\,K \sum_{KL}^{L} (g_0 + a_0\, s_0^2) \; \ldots\ldots\ldots\ldots\ldots\ldots (245)$$

Thus, all the elastic properties can be obtained for a member of any number of sections (see Table 9).

TABLE 9.—PROPERTIES OF RECIPROCAL I-DIAGRAM FOR A SECTION OF A MEMBER

INTEGRAL	DEFINITION	FORMULA
$\int \frac{dx}{I_x}$	AREA under $\frac{1}{I}$-curve	$\frac{2\,g_1 + g_2 + g_3}{l^2}$
$\int \frac{x\,dx}{I_x}$	STATICAL MOMENT about left end of section	$\frac{g_1 + g_2}{l}$
$\int \frac{(l - x)\,dx}{I_x}$	STATICAL MOMENT about right end of section	$\frac{g_1 + g_3}{l}$
$\int \frac{x\,(l - x)\,dx}{I_x}$	PARALLEL PRODUCT OF INERTIA	$g_1 = \frac{l^3\,F_1}{I_a}$
$\int \frac{x^2\,dx}{I_x}$	MOMENT OF INERTIA about left end of section	$g_2 = \frac{l^3\,F_2}{I_a}$
$\int \frac{(l - x)^2\,dx}{I_x}$	MOMENT OF INERTIA about right end of section	$g_3 = \frac{l^3\,F_3}{I_a}$
	Distance from left end of section to center of gravity	$l\,\frac{g_1 + g_2}{2\,g_1 + g_2 + g_3} = x_0$
	Distance from right end of section to center of gravity	$l\,\frac{g_1 + g_3}{2\,g_1 + g_2 + g_3} = v_0$
	MOMENT OF INERTIA about axis through center of gravity	$\frac{g_2\,g_3 - g_1^2}{2\,g_1 + g_2 + g_3} = g_0$

This theory of combining sections of a member has been found particularly convenient where a part of the member is uniform, since for this section the g-terms are very simple. From the formulas given in Case 5, the uniform member, there is obtained:

$$g_1 = \frac{l^3}{6\,I_a} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (246)$$

$$g_2 = g_3 = \frac{l^3}{3\,I_a} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (247)$$

and,

$$g_0 = \frac{l^3}{12\,I_a} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (248)$$

Equations (247) and (248) are obviously the moments of inertia of a rectangle of length, l, and width, $\frac{1}{I_a}$, about axes at its edge and center of gravity.

In the member given by Professor Clark shown in Fig. 13(*b*) (a uniform center section with a straight haunch at each end) this method was found more convenient than the three section formulas. It was also found possible to integrate the section with the straight haunch using the actual *I*-curve, $I_x = (a + bx)^3$, to find g_1, g_2, and g_3. The foregoing relations thus make an exact solution possible for a member of any number of sections where the moment of inertia varies as the cube of the depth and the edges of each section are straight.

It has even been found possible to perform the integrations and thus obtain an exact solution where the contours of portions of a member are conic sections and I varies as the cube of the depth, $I_x = (a + bx + cx^2)^3$, but this solution is too long for use in design work.

It is apparent that all the relations of the properties of the reciprocal *I*-curve given in Equations (226) to (248) are mathematical theorems that do not depend on, and are not limited by, substitute *I*-curves. In the original paper it was brought out that the substitute *I*-curve furnishes a way of integrating the functions of x divided by I which can be used in any method of analysis. Conversely, it is possible to use these theorems without employing substitute *I*-curves. Thus, if one chose, one could find the values of g_1, g_2, and g_3 of the sections of a member by the step-by-step summation or by the Large-Morris approximation, and then, by means of the relations of Equations (235) to (245), find all the elastic properties of the member.

It has been found that many different kinds of members can be treated more conveniently by summing the properties of individual sections than by endeavoring to consider the member as a whole in the beginning. Examples of such members are:

Example (*a*).—A member framing into two deep members of such proportions that an infinite moment of inertia should be used at both ends. There would be three or more sections to such a member. The numerical work, however, is greatly reduced by the fact that for the end sections, in which I equals infinity, the *g*-terms are zero.

Example (*b*).—Steel members in which cover-plates discontinue at frequent intervals. In some cases it is accurate enough to use a smooth substitute *I*-curve for the entire length. In others, any number of sections can be employed so that the results can be obtained with any desired degree of accuracy. This is an example of the flexibility of the general method.

The writers recognize fully the thought and careful work which has gone into the preparation of the many discussions of this paper. They wish to thank all the contributors most sincerely, and express appreciation of the manner in which they have amplified a subject that was treated, with some difficulty, within the confines of limited space.

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INFLUENCE OF DIVERSION ON THE MISSISSIPPI AND ATCHAFALAYA RIVERS

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WITH DISCUSSION BY MESSRS. LEO M. ODOM, E. W. LANE, AND E. F. SALISBURY.

SYNOPSIS

The effect of an outlet, operative at all stages, from the Mississippi River into the Atchafalaya River, is discussed in this paper. This continuous diversion decreases the volume of the Mississippi and increases that of the Atchafalaya. The result of this diversion is set forth and provides a means of proving or of disproving some of the present hydraulic theories and opinions.

For present purposes, a like volume of water (bank-full stage) through the various years, and the passage of this like volume of water past the gauge station as registered by the gauge height are analyzed at points on both rivers.

On the Mississippi River, gauge stations are analyzed where they are not affected, as well as where they are affected, by continuous diversion, in order to obtain comparative data and to place before the mind of the reader some of the ordinary causes that influence the variation in gauge height for the same volume of discharge. This analysis shows: (1) That at points not affected by diversion operative at all stages, the same volume of water is now passing at the same elevation as in earlier years; (2) that the Mississippi River has conformed itself to the hydraulic theory and increased its slope as the volume has been diminished by diversion; and, likewise (3), the Atchafalaya River has conformed itself to the hydraulic theory and flattened its slope as its volume has been increased through diversion.

NOTE.—Published in November, 1935, *Proceedings*.

[1] Chf. Engr., L. & A. R. R., Minden, La.

INTRODUCTION

At only one point in the entire reach of the Mississippi River is its water diverted at practically all stages, namely, at Red River Landing (Angola, La.), about 190 miles up stream from New Orleans, where it is diverted into the Atchafalaya River. The result of this diversion is clearly reflected in the available data and should prove or disprove present theories and opinions.

In his study of sediment-bearing streams, Guglielmini, an Italian engineer of the Seventeenth Century, advanced the hydraulic theory[2] that: (1) The greater the quantity of water a stream carries, the less will be its fall; (2) the greater the force of a stream, the less will be the slope of its bed; and (3), the slope of the bottom in rivers will diminish in the same proportion in which the body of water is increased, and *vice versa.*

Commenting on this theory, Humphreys and Abbot state[2]:

> "* * *. These rules have their explanation in the facts, that the beds of rivers, of the character above mentioned, are capable of resisting, unchanged, only a certain velocity of current; and, on the other hand, that the sedimentary matter, contained in the river-water requires a certain degree of velocity to keep it in suspension. From the counteracting tendencies of the above two causes, a mean becomes established at which the current ceases to deposit its sediment, and the bottom ceases to be abraded; in other words, the bottom becomes permanent. But if, from any cause, such as throwing off a portion of the water through a waste-weir, the velocity of the current is diminished, it is no longer able to maintain its sediment in suspension, but will continue to deposit in its bed until, through the elevation of the bed, its velocity again becomes, what it was before it was disturbed, sufficient to maintain its sediment in permanent suspension."

While he was a member of the Mississippi River Commission, the late James B. Eads, F. Am. Soc. C. E., clearly and ably expounded this same theory. In a letter to the Mississippi River Commission[3] dated April 12, 1882, he stated emphatically that, in flood time, the current cannot be checked in the slightest degree without causing a deposition of some part of the sediment. In this connection he named three controlling factors that are involved in every problem presented by the characteristic phenomenon of the Mississippi River: (1) The force of the current; (2) the frictional resistance of the river bed; and (3) the intimate relation between the quantity of sediment carried in the water, and the velocity of the current. If the current is increased or decreased from any cause (such as the friction between the water and the bed of the stream) the quantity of sediment carried in suspension is likewise increased or decreased. If the volume is diminished, the ratio of friction to volume will be increased; and, conversely, if the volume is increased the ratio of frictional resistance will be decreased.

Diagrammatically, Mr. Eads demonstrated the rapidity with which frictional resistance increases if the sedimentary river is divided into two or more channels. Unless the two new channels have steeper slopes it is impossible for the water to flow as fast as it would in the original, single channel. Quoting from the letter:

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[2] "Report on the Physics and Hydraulics of the Mississippi River", by A. A. Humphreys and H. L. Abbot, 1861 (reprinted 1876), *Professional Papers No. 4*, Corps of Topographical Engrs., U. S. A., p. 415.

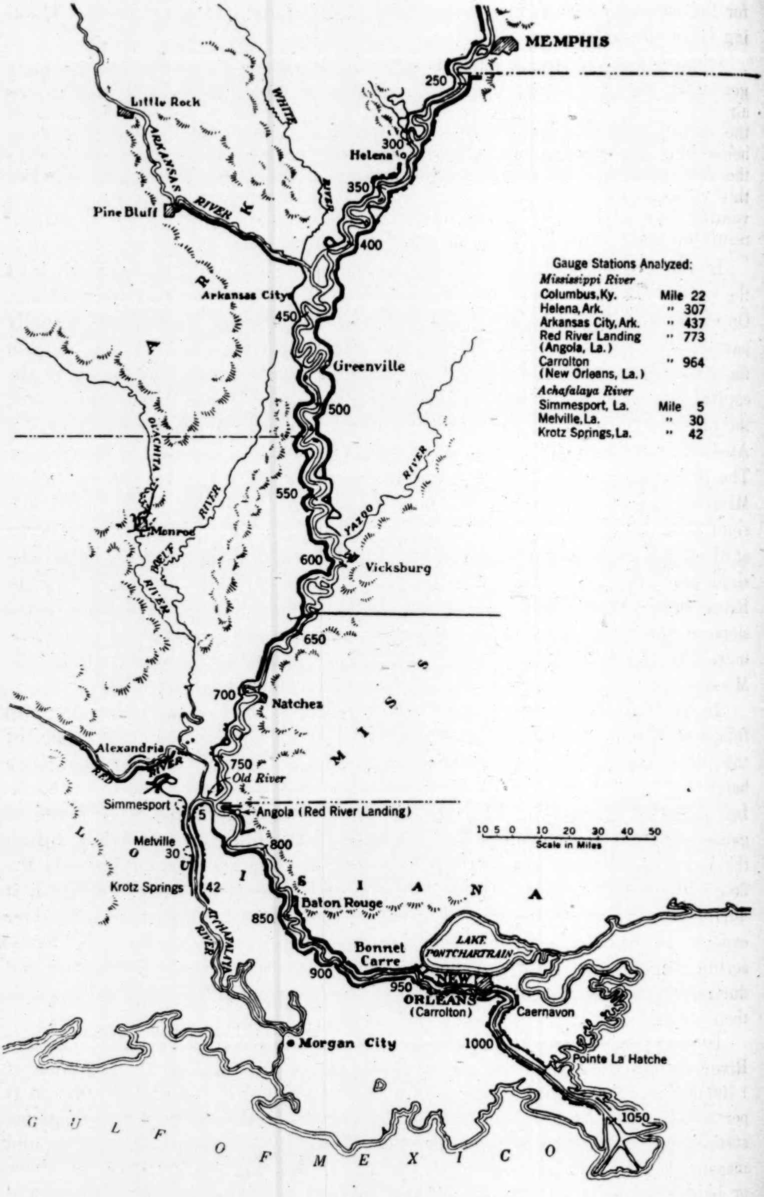
[3] Annual Rept. of the Chf. of Engrs., U. S. Army, to the Secretary of War, for the Year, 1882, Pt. III, Appendix RR, p. 2769 *et seq.*

> "There are no truths more capable of complete demonstration or more generally recognized by hydraulic engineers than these and the effects already treated by the Atchafalaya bear ample testimony to the soundness of the deductions thus advanced. For instance, the volume in the main river below the Atchafalaya outlet is decreased by the volume carried off through the Atchafalaya. Therefore, in proportion as the lower Mississippi has lost this volume the slope of the river or its flood line should be steepened. The volume has been steadily increased in the Atchafalaya and as a natural result of such increase its slope has diminished."

If the foregoing theory is correct, the action of the Mississippi River at the point of almost constant diversion is a logical place to test its accuracy. Opposite Red River Landing the Red and Atchafalaya Rivers are virtually parallel to the Mississippi River (see Fig. 1), with a connection between the two called "Old River." This junction has been ably compared to the capital letter, H. The right-hand side of the letter is the Mississippi River; the other side above the cross-bar, the Red River, and below the cross-bar, the Atchafalaya; the cross-bar connection, seven miles in length, is Old River. The direction of flow in Old River is dependent on stages prevailing on the Mississippi and Red Rivers, the flow being most frequently from the Mississippi River through Old River into the Atchafalaya. This condition can occur at all stages of the Mississippi. The volume of water diverted from the Mississippi into the Atchafalaya decreases the volume of water in the Mississippi River below Old River. Therefore, if the hydraulic theory is correct, the slope of the Mississippi River below Old River should be increased. Furthermore, as the volume in the Atchafalaya is increased by waters from the Mississippi, the slope of the Atchafalaya River should be diminished.

In testing the correctness of this hydraulic theory, it is necessary to follow a like volume of water through the various years and the passage of this like volume of water by the gauge stations as registered by the gauge heights. In selecting this like volume of water it should be close to the bank-full stage of the river. This selection will give a desirable wide range of gauge readings for consideration not only in the different years, but during the same year. Furthermore (and this is very important), it represents the flow at bank-full stage, the form of the river which all the forces acting on it during the year are striving to create. The usual mistake made by river experts is to confine their discussions to flood stages, ignoring the forces acting during low water. A flood acts but a comparatively short time and during the remainder of the 365 days the river modifies the results which then occur.

It is apparent from the discharge observations recorded by the Mississippi River Commission that the above requirements are fulfilled by a volume of 1 100 000 to 1 200 000 cu ft per sec for the Mississippi River, and 300 000 cu ft per sec for the Atchafalaya River. In preparing the study for each gauge station, all gauges published by the Mississippi River Commission and measured with meters were used, falling within 25 000 cu ft per sec above or below the selected volume for the stations on the Mississippi River and 15 000 cu ft per sec above or below the selected volume for the Atchafalaya


MEMPHIS
250
Little Rock
WHITE RIVER
300
Helena
ARKANSAS RIVER
350
Pine Bluff
400
A R K
Arkansas City
450
Greenville
500
OUACHITA RIVER
BOEUF RIVER
550
YAZOO RIVER
Monroe
600
Vicksburg
650
700
Natchez
M I S S
Alexandria
RED RIVER
750
Old River
Simmesport
5
Angola (Red River Landing)
Melville
30
800
Krotz Springs
42
L O U I S I A N A
ATCHAFALAYA RIVER
Baton Rouge
850
Bonnet Carre
900
LAKE PONTCHARTRAIN
950
NEW ORLEANS
(Carrolton)
Caernavon
1000
Pointe La Hatche
Morgan City
1050
GULF OF MEXICO
10 5 0 10 20 30 40 50
Scale in Miles

Gauge Stations Analyzed:

Mississippi River	
Columbus, Ky.	Mile 22
Helena, Ark.	" 307
Arkansas City, Ark.	" 437
Red River Landing (Angola, La.)	" 773
Carrolton (New Orleans, La.)	" 964
Achafalaya River	
Simmesport, La.	Mile 5
Melville, La.	" 30
Krotz Springs, La.	" 42

FIG. 1

River. The gauge for these volumes was adjusted to give a gauge for the selected volume. Readings measured with rods and floats were not used. All recorded readings were checked as to influence by crevasses and levee changes. These volumes are close to bank-full stage and no effect from crevasses was found. At only one point was the gauge station affected by levee changes and the results are discussed subsequently. That crevasses and levee confinement have not had any measurable effect on the carrying capacity of the main river is substantiated by reports of the Army Engineers.[4] During the period of this analysis only two cut-offs occurred; one at Mile 680 (Waterproof), in 1884; the other at Mile 638 (Yucatan), in 1928. These cut-offs are approximately 100 miles away from the nearest gauge station used in this paper and are so distant that they would not have any measurable effect.

To analyze the gauge and discharge record at any station it is necessary to understand that a silt-bearing river is somewhat human in its ability to do more work at one time than another. It will discharge a given volume of water at different gauge heights, depending upon its physical condition and the movement of water. This phenomenon of the river's capacity, to do more work at one time than another, as reflected by the passage of a given volume of water at different gauge heights is, for the purpose of this paper, termed its "efficiency."

It is generally recognized that the system used in taking discharge observations of large rivers, especially during high stages, is subject to error, but that it is sufficiently accurate for practical purposes. It is impossible to separate the factor of error in discharge measurements and the factor of river efficiency; both must be considered in river work. Therefore, it is consistent to embody the two in the term, "efficiency," which is so used in this analysis.

The variation in gauge heights for the same volume of water due to the efficiency of the river is clearly shown at gauge stations not influenced by unrestricted diversion; that is, uncontrolled diversion of water from the parent river at all stages, such as that from the Mississippi River into the Atchafalaya River. Three gauge stations selected for this purpose are those at Columbus-Wickliffe, Ky., and Helena, and Arkansas City, Ark. These three stations were selected over any of the other gauge stations because they furnish more readings of the selected volume. An analysis of a discharge of 1 200 000 cu ft per sec at these three stations is afforded by reference to Tables 1(*a*), 1(*b*), and 1(*c*). A summary of these data is given in Tables 2(*a*), 2(*b*), and 2(*c*).

COLUMBUS, KENTUCKY, GAUGE

All the discharges used were measured with current meters and those varying by more than 25 000 cu ft per sec from a volume of 1 200 000 cu ft per sec were rejected. Discharges measured with float or rod were not used in any of the studies of this paper. At the Columbus gauge (see Table 1 (*a*)), the readings were adjusted to a basis of 1 200 000 cu ft per sec, using 60 000 cu ft per sec per ft of gauge, as shown by the 1929 discharges. Only actual observed discharges for 1929 were used for the analysis of this record and no crevasses that would affect the readings were reported.

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[4] "Spillways on the Lower Mississippi River", H. R. Doc. No. 95, 70th Cong., First Session, p. 4.

TABLE 1.—Study of Gauge Records

Date (1)	Gauge reading, in feet (2)	Discharge, in cubic feet per second (3)	Gauge reading,* in feet, adjusted to 1 200 000 cubic feet per second (4)
(*a*) Columbus (Ky.) Gauge, 22 Miles below Cairo, Ill. (Bank-full Stage, 40 Feet; Flood Stage, 43 Feet)			
April 15, 1892	41.2	1 203 000	41.2
March 3, 1897	39.2	1 188 000	39.4
February 1, 1898	40.5	1 228 000	40.0
February 3, 1898	40.1	1 181 000	40.4
February 4, 1898	39.6	1 177 000	40.0
March 29, 1907	41.4	1 213 000	41.2
April 24, 1911	41.1	1 176 000	41.5
March 3, 1923	43.0	1 219 000	42.7
March 24, 1923	42.8	1 213 000	42.6
April 29, 1929	42.5	1 174 000	42.9
April 30, 1929	43.0	1 191 000	43.2
(*b*) Helena (Ark.) Gauge, 307 Miles below Cairo, Ill. (Bank-full Stage, 39.0 Feet; Flood Stage, 44.0 Feet)			
April 6, 1882	43.3	1 210 000	43.1
May 3, 1892	44.5	1 209 000	44.3
May 4, 1892	44.6	1 206 000	44.5
June 2, 1892	44.7	1 194 000	44.8
June 3, 1892	44.7	1 177 000	45.1
June 4, 1892	44.6	1 180 000	45.0
June 6, 1892	44.6	1 185 000	44.9
June 7, 1892	44.6	1 177 000	45.0
June 8, 1892	44.6	1 198 000	44.6
June 9, 1892	44.7	1 203 000	44.7
May 9, 1893	43.1	1 190 000	43.3
May 10, 1893	43.3	1 210 000	43.1
May 11, 1893	43.5	1 197 000	43.5
March 25, 1908	45.1	1 228 000	44.6
March 12, 1929	42.9	1 195 000	43.0
March 13, 1929	43.7	1 219 000	43.4
May 3, 1929	45.9	1 222 000	45.5
May 4, 1929	45.9	1 218 000	45.6
(*c*) Arkansas City (Ark.) Gauge, 437 Miles below Cairo, Ill. (Bank-full Stage, 42 Feet; Flood Stage, 48 Feet)			
March 14, 1890	48.2	1 186 000	48.5
March 25, 1890	49.2	1 220 000	48.8
March 28, 1890	49.2	1 223 000	48.7
April 4, 1890	48.3	1 221 000	47.9
April 7, 1890	47.9	1 187 000	48.2
April 9, 1890	48.0	1 220 000	47.6
April 11, 1890	47.9	1 199 000	47.9
March 11, 1891	46.1	1 212 000	45.9
March 16, 1891	46.9	1 209 000	46.7
March 17, 1891	46.9	1 219 000	46.5
March 18, 1891	47.0	1 193 000	47.1
March 20, 1891	47.3	1 182 000	47.7
March 21, 1891	47.3	1 214 000	47.0
March 23, 1891	47.5	1 211 000	47.3
April 25, 1891	47.3	1 228 000	46.7
April 27, 1891	47.5	1 228 000	46.9
April 29, 1891	47.3	1 201 000	47.3
April 30, 1891	47.1	1 189 000	47.3
May 2, 1891	46.5	1 186 000	46.8
June 27, 1892	47.5	1 201 000	47.5
June 28, 1892	47.2	1 179 000	47.6
June 29, 1892	46.6	1 180 000	47.0
June 20, 1892	46.1	1 192 000	46.3
May 5, 1893	46.2	1 198 000	46.2
March 12, 1903	49.0	1 198 000	49.0
April 28, 1911	47.0	1 184 000	47.3
March 27, 1923	46.9	1 174 000	47.4
March 29, 1923	47.9	1 174 000	48.4
(*c*) Arkansas City (Ark.) Gauge.—(*Continued*)			
March 21, 1923	48.6	1 224 000	48.1
April 2, 1923	49.0	1 213 000	48.7
April 3, 1923	49.1	1 197 000	49.1
April 4, 1923	49.4	1 203 000	49.4
March 19, 1929	48.9	1 192 000	49.1
March 20, 1929	49.5	1 213 000	49.2
June 20, 1929	49.3	1 206 000	49.2
June 21, 1929	48.5	1 178 000	48.9
(*d*) Carrollton (La.) Gauge†, 964 Miles below Cairo, Ill. (Bank-full Stage, 9.0 Feet; Flood Stage, 17.0 Feet)			
April 7, 1883	15.4	1 086 000	15.6
February 2, 1885	13.5	1 102 000	13.5
March 4, 1890	14.9	1 110 000	14.7
March 7, 1890	15.2	1 128 000	14.7
March 14, 1890	Crevasse at Nita, Ky.		
March 22, 1890	15.9	1 124 000	15.5
March 28, 1890	15.5	1 106 000	15.4
March 31, 1890	15.2	1 082 000	15.5
April 3, 1890	15.6	1 106 000	15.5
April 4, 1890	15.4	1 092 000	15.5
April 5, 1890	15.3	1 102 000	15.3
April 7, 1890	15.3	1 096 000	15.4
April 8, 1890	15.2	1 091 000	15.4
March 4, 1891	14.4	1 100 000	14.4
March 9, 1891	14.6	1 093 000	14.7
March 10, 1891	14.9	1 114 000	14.7
March 12, 1891	15.3	1 103 000	15.2
May 18, 1892	16.2	1 083 000	16.5
May 19, 1892	16.1	1 111 000	15.9
June 4, 1892	16.6	1 091 000	16.8
June 6, 1892	16.7	1 088 000	16.9
June 8, 1892	16.7	1 121 000	16.3
June 10, 1892	17.0	1 079 000	17.4
June 12, 1892	Crevasse at Belmont, Ky., 911 miles below Cairo, Ill. Discharge, 140 000 cu ft per sec.		
June 14, 1892	16.7	1 092 000	16.8
June 19, 1893	17.0	1 089 000	17.2
June 21, 1893	17.2	1 120 000	16.9
June 22, 1893	17.3	1 113 000	17.1
June 23, 1893	17.5	1 106 000	17.4
June 26, 1893	17.0	1 089 000	17.2
May 3, 1898	15.6	1 096 000	15.7
May 4, 1898	15.5	1 087 000	15.7
April 23, 1904	15.7	1 091 000	15.9
February 2, 1907	17.1	1 091 000	17.2
February 5, 1907	17.4	1 100 000	17.4
February 6, 1907	17.5	1 090 000	17.7
February 7, 1907	17.6	1 114 000	17.4
March 29, 1909	15.9	1 110 000	15.7
March 30, 1909	16.2	1 093 000	16.3
March 31, 1909	16.2	1 107 000	16.1
April 1, 1909	16.4	1 095 000	16.5
April 2, 1909	16.4	1 102 000	16.4
April 3, 1909	16.4	1 100 000	16.4
April 4, 1909	16.4	1 108 000	16.3
April 5, 1909	16.4	1 089 000	16.6
April 7, 1909	16.4	1 118 000	16.1
April 8, 1909	16.5	1 089 000	16.7
April 9, 1912	16.5	1 091 000	16.7
April 10, 1912	16.8	1 110 000	16.6
April 11, 1912	17.2	1 104 000	17.1
February 12, 1913	16.0	1 095 000	16.1
February 13, 1913	16.2	1 099 000	16.2
February 14, 1913	16.4	1 099 000	16.4
February 15, 1913	16.4	1 099 000	16.4

* Except as noted in the case of Table 1 (*e*).

† Gauge reading, Column (4), adjusted to 1 100 000 cu. ft. per sec.

TABLE 1.—(*Continued*).

Date (1)	Gauge reading, in feet (2)	Velocity, in feet per second (3)	Discharge, in cubic feet per second (4)	Gauge reading,* in feet, adjusted to 1 200 000 cubic feet per second (5)
(*d*) CARROLLTON (LA.) GAUGE.— (*Continued*)				
February 16, 1913	16.5	...	1 091 000	16.7
February 17, 1913	16.6	...	1 085 000	16.9
February 18, 1913	16.7	...	1 084 000	17.1
February 19, 1913	17.0	...	1 101 000	17.0
February 20, 1913	17.0	...	1 088 000	17.2
February 21, 1913	17.0	...	1 112 000	16.8
February 23, 1913	17.0	...	1 106 000	16.9
February 24, 1913	17.0	...	1 112 000	16.8
February 25, 1913	16.9	...	1 087 000	17.1
February 26, 1913	16.7	...	1 074 000	17.1
April 20, 1913	17.9	...	1 124 000	17.5
April 22, 1913	18.3	...	1 124 000	17.9
April 29, 1913	18.8	...	1 109 000	18.6
May 6, 1913	19.0	...	1 118 000	18.7
May 18, 1913	18.5	...	1 102 000	18.5
May 19, 1913	18.2	...	1 080 000	18.5
May 20, 1913	17.9	...	1 089 000	18.1
February 1, 1916	16.6	...	1 123 000	16.2
April 27, 1917	16.6	...	1 111 000	16.4
April 23, 1920	17.6	...	1 072 000	18.1
April 24, 1920	17.7	...	1 086 000	17.9
April 25, 1920	17.8	...	1 085 000	18.1
April 4, 1922	17.3	...	1 073 000	17.8
April 8, 1922	18.6	...	1 091 000	18.8
April 11, 1922	19.3	...	1 128 000	18.8
April 11, 1929	17.3	...	1 109 000	17.1
April 12, 1929	17.4	...	1 078 000	17.8
April 15, 1929	17.7	...	1 102 000	17.7
April 16, 1929	17.7	...	1 110 000	17.5
April 18, 1929	17.7	...	1 108 000	17.6
April 19, 1929	17.7	...	1 097 000	17.8
April 20, 1929	18.0	...	1 124 000	17.6
April 23, 1929	18.2	...	1 118 000	17.9
April 24, 1929	18.3	...	1 111 000	18.1
April 26, 1929	18.3	...	1 122 000	17.9
April 30, 1929	18.3	...	1 118 000	18.0
June 26, 1929	18.4	...	1 108 000	18.3
February 8, 1932	16.7	...	1 104 000	16.6
February 10, 1932	17.0	...	1 091 000	17.2
February 11, 1932	17.1	...	1 107 000	17.0
February 16, 1932	17.6	...	1 117 000	17.3
March 16, 1932	17.4	...	1 092 000	17.5
(*e*) SIMMESPORT (LA.) GAUGE‡, 5 MILES BELOW THE HEAD OF ATCHAFALAYA RIVER.				
April 7, 1890	42.3	5.33	306 594	41.8
March 23, 1891	40.9	4.69	296 482	41.2
March 25, 1891	41.0	4.68	301 000	41.0
March 28, 1891	40.9	4.61	293 244	41.4
March 30, 1891	41.1	4.70	296 996	41.3
April 4, 1891	41.1	4.71	302 113	40.9
April 6, 1891	41.1	4.80	307 522	40.5
April 9, 1891	41.2	4.69	302 555	41.0
April 15, 1891	41.4	4.76	306 075	40.9
April 17, 1891	41.4	4.72	300 510	41.4
May 20, 1892	41.7	5.10	303 049	41.5
May 22, 1892	41.8	4.93	296 983	42.0
May 24, 1892	41.9	5.07	302 257	41.7
May 26, 1892	41.9	4.96	299 479	41.9
May 28, 1892	42.0	5.11	306 556	41.5
February 14, 1907	41.5	4.92	299 945	41.5
February 16, 1907	41.9	4.90	289 262	42.8
April 6, 1908	39.9	5.04	290 389	40.7
May 4, 1908	42.1	5.08	299 270	42.1
May 6, 1908	42.3	5.15	310 578	41.5
June 16, 1908	44.9	4.84	312 262	43.9
April 13, 1912	43.4	4.95	290 348	44.2
April 24, 1920	43.8	4.49	298 119	44.0
April 27, 1920	44.3	4.41	297 512	44.5
May 3, 1920	45.1	4.50	305 979	44.7
April 13, 1923	42.7	4.69	311 061	41.8

* Except as noted in the case of Table 1 (*e*).

‡ Gauge reading, Column (4) adjusted to 300 000 cu ft per sec.

TABLE 1—(*Continued*).

Date (1)	Gauge reading, in feet (2)	Velocity, in feet per second (3)	Discharge, in cubic feet per second (4)	Gauge reading,[*] in feet, adjusted to 1 200 000 cubic feet per second (5)
(*e*) SIMMESPORT (LA.) GAUGE[‡].—(*Continued*)				
July 17, 1928	37.8	4.40	287 379	38.8
July 21, 1928	37.8	4.39	290 914	38.6
April 6, 1929	39.9	4.29	301 995	39.9
April 10, 1929	40.5	4.42	310 596	39.7
April 21, 1933	37.9	4.34	290 457	38.6
April 22, 1933	38.8	4.39	298 232	39.0
April 24, 1933	39.0	4.33	299 665	39.0
April 26, 1933	39.3	4.37	297 111	39.5
April 27, 1933	39.6	4.34	302 736	39.4
May 1, 1933	40.1	4.48	315 364	39.0
May 3, 1933	40.6	4.50	315 306	39.5
May 5, 1933	40.9	4.43	315 515	39.8
May 13, 1933	41.0	4.45	311 273	40.1

[*] Except as noted in the case of Table 1 (*e*).

[‡] Gauge reading, Column (4) adjusted to 300 000 cu ft per sec.

TABLE 2.—COMPARATIVE GAUGE VARIATION FOR A DISCHARGE[*] OF 1 200 000 CUBIC FEET PER SECOND

Maximum[†] variation for discharging the same volume of water during: (1)	Period of record, in years (2)	GAUGE HEIGHTS, IN FEET				Variation, in feet (7)
		Lowest		Highest		
		Date (3)	Reading (4)	Date (5)	Reading (6)	
(*a*) COLUMBUS-WICKLIFFE (KY.) GAUGE STATION						
The earliest and latest recorded year[‡]	37	March 24, 1929	42.9	April 15, 1892	41.2	1.7
Any one year	37	April 29, 1929	42.9	April 30, 1929	43.2	0.3[**]
The 37-year period	37	March 3, 1897	39.4	April 30, 1929	43.2	3.8
(*b*) HELENA (ARK.) GAUGE STATION						
The earliest and latest recorded year [‡]	47	March 12, 1929	43.0	April 6, 1882	43.1	0.1
Any one year	47	March 12, 1929	43.0	May 4, 1929	45.6	2.6
The 47-year period	47	March 12, 1929	43.0	May 4, 1929	45.6	2.6
(*c*) ARKANSAS CITY (ARK.) GAUGE STATION						
The earliest and latest recorded year [‡]	39	June 21, 1929	48.9	March 25, 1890	48.8	0.1
Any one year	39	March 27, 1923	47.4	April 4, 1923	49.4	2.0
The 39-year period	39	May 5, 1893	46.2	April 4, 1923	49.4	3.2
(*d*) RED RIVER LANDING (ANGOLA, LA.) GAUGE STATION[§]						
The earliest and latest recorded year	50	February 17, 1932	49.0	June 20, 1882	42.1	6.9
Any one year	50	February 15, 1913	45.9	May 5, 1913	50.4	4.5
The 50-year period	50	January 21, 1885	40.2	May 5, 1913	50.4	10.2
(*e*) CARROLLTON (LA.) GAUGE STATION[§]						
The earliest and latest recorded year	49	February 8, 1932	16.6	April 7, 1883	15.6	1.0
Any one year	49	February 12, 1913	16.1	May 6, 1913	18.7	2.6
The 49-year period	49	February 2, 1885	13.5	April 6, 1922	18.8	5.3
(*f*) SIMMESPORT (LA.) DISCHARGE OBSERVATION STATION, ON THE ATCHAFALAYA RIVER[‖]						
The earliest and latest recorded year	43	April 7, 1890	41.8	May 13, 1933	40.1	1.7
Any one year	43	April 8, 1908	40.7	June 16, 1908	43.9	3.2
The 43-year period	43	June 21, 1928[¶]	38.6	May 3, 1920	44.7	6.1

[*] Except as noted in the case of Tables 2(*d*), 2(*e*), and 2(*f*).

[†] Except as noted.

[‡] Minimum.

[§] For a discharge of 1 100 000 cu ft per sec.

[‖] For a discharge of 300 000 cu ft per sec.

[¶] Also April 21, 1933.

[**] Sufficient readings not available to determine maximum variation during the same flood

Referring to Table 2 (*a*), the highest gauge of 1892 is 1.7 ft lower than the lowest gauge of 1929. This indicates that the two different floods, although rated at two different "efficiencies" at this station, were at one time discharging the same volume of water at the same gauge height. In other words, between 1892 and 1929, no appreciable silting occurred at this station. The limited number of observations during any one year at this station (Table 2 (*a*)) does not reflect the gauge variation that can be expected during a flood, due to "efficiency." For a 37-yr period, the greatest range on the gauge to discharge the same volume of water, is 3.8 ft (see Table 2 (*a*)) which is a measure of the effect of "efficiency" at this station, for all floods recorded (see Table 1 (*a*)).

HELENA, ARKANSAS, GAUGE

Referring to the Helena gauge (Table 2 (*b*)), 55 000 cu ft per sec per ft of gauge was used in adjusting readings to a discharge of 1 200 000 cu ft per sec. As in the case of the Columbus-Wickliffe gauge only actual observed discharges during 1929 were used.

Crevasses occurring within 50 miles above and below Helena prior to the discharge rates listed in Table 1 (*b*), in general did not affect the recorded gauges. Records of crevasses that may have occurred in 1882 were not available to the writer. The recorded gauges of 1893 that were not affected by crevasses, however, are practically the same as those for 1882. From this evidence and the fact that the discharge is close to bank-full stage, the discharge of 1882 is considered to be unaffected by crevasses. In other words, it would not affect the analysis if the discharges of 1882 or 1893 were used.

Reference to Table 2 (*b*) demonstrates that, in the two different floods, 47 yr apart, the same volume of water was passing at the same gauge height, which indicates that there has been no filling or silting in the channel at this station. The greatest variation in discharging the same quantity of water occurred in 1929. The difference of 2.6 ft between the lowest and highest gauge of this year reflects the effect of "efficiency" during the same flood. For the 47-yr period, the greatest range on the gauge for discharging 1 200 000 cu ft per sec is 2.6 ft, which value measures the influence of "efficiency" at the Helena gauge for all the floods recorded. It is to be noted (see Table 2 (*b*)) that the lowest and highest gauges of any of the years recorded occurred in 1929.

ARKANSAS CITY, ARKANSAS, GAUGE

Effect of Crevasses on the Gauge Study.—Prior to the discharge reading of March 14, 1890 (see Table 1 (*c*)), there was a crevasse of 35 000 cu ft per sec, 8 miles above the Arkansas City gauge on March 9, 1890, and another of 6 055 cu ft per sec, 35 miles below on March 7, 1890 (see Table 3). These crevasses were too small to affect the gauge reading as shown by the increase of 0.3 ft in the gauge height by March 25, 1890. In other words, the gauge height increased despite the occurrence of two additional small crevasses (19 901 cu ft per sec) 4 miles above, and two of 35 000 and 31 000 cu ft per sec, 7 and 40 miles, respectively, below the Arkansas City gauge. Prior to the gauge reading of April 11, 1890 (see Table 1 (*c*)), in the 50-mile reach above this gauge, nine crevasses occurred, discharging 515 901 cu ft per sec; and, in the reach 50 miles below, six crevasses occurred, having a discharge of 242 255 cu ft per sec, or a total of 758 156 cu ft per sec. A comparison of the gauge of April 11, 1890, with that of March 14, 1890, shows a reduction of only 0.6 ft to discharge the same volume of water (1 200 000 cu ft per sec) with more than 700 000 cu ft per sec of water diverted through crevasses within 50 miles above and 50 miles below the gauge. This small variation proves that there was little, if any, effect from crevasses, which conclusion is further supported by a comparison of the gauges of 1890 and 1891.

TABLE 3.—RECORD OF CREVASSE OCCURRENCE

Item	Date	Miles below Cairo, Ill.	Bank	Discharge, in cubic feet per second
(1)	(2)	(3)	(4)	(5)
(*a*) VICINITY OF ARKANSAS CITY GAUGE (437 MILES BELOW CAIRO, ILL.)				
1	March 9, 1890..	429	Right	35 000
2	March 7, 1890..	472	Right	6 055
	Total, prior to March 14, 1890	.....	.....	41 055
3	March 24, 1890..	433	Right	17 000
4	March 24, 1890..	433	Right	2 901
5	March 18, 1890..	444	Left	35 000
6	March 18, 1890..	467	Right	31 000
	Total, prior to March 25, 1890	.....	.....	85 901
7	March 27, 1890..	426	Right	17 000
8	March 27, 1890..	432	Right	10 000
	Total, prior to March 28, 1890	.....	.....	27 000
9	April 5, 1890....	426	Right	44 000
10	April 4, 1890....	432	Left	208 000
11	April 4, 1890....	431	Left	114 000
12	March 28, 1890..	435	Left	68 000
13	March 28, 1890..	438	Left	125 000
14	March 28, 1890..	470	Right	35 200
15	April 7, 1890....	474	Left	10 000
	Total, prior to April 1, 1890..	.....	.....	604 200
	Total, in 1890...	.....	.....	758 156
16	May 13, 1892...	452	Right	91 623
17	May 25, 1892...	484	Right	17 253
18	June 2, 1892....	439	Right	19 554
19	June 22, 1892...	470	Right	18 483
	Total in 1892, prior to July 1	.....	.....	146 913
(*b*) VICINITY OF RED RIVER LANDING GAUGE (773 MILES BELOW CAIRO, ILL.)				
20	March 14, 1884..	789	Right	211 000
21	March 14, 1890..	899	Left	....
22	March 16, 1891..	961	Right	91 000
23	March 14, 1903..	738	.....	....
(*c*) VICINITY OF CARROLLTON GAUGE (964 MILES BELOW CAIRO, ILL.)				
24	March 14, 1890..	902	.....	402 000
25	March 16, 1891..	961	Right	91 000
26	....	939	.....	116 000
...	....	.....	.....	....
...	....	.....	.....	....
...	....	.....	.....	....
...	....	.....	.....	....
...	....	.....	.....	....
...	....	.....	.....	....

Prior to July 1, 1892 (see Table 3 (*a*)), four crevasses, occurred within a reach of 50 miles below the Arkansas City gauge, with a total discharge of 146 913 cu ft per sec. The recorded readings in this year were not appreciably more or less than those for years that were not affected by crevasses.

The highest gauge of 1890 (see Table 2 (*c*)) is 0.1 ft lower than the lowest gauge of 1929, showing that at one time the two different floods were discharging the same volume of water at practically the same gauge. Again, this shows that no appreciable silting had occurred at this station in the 39-yr period. For the same discharges, the year 1923 showed the greatest variation in gauge heights. The difference of 2.0 ft (see Table 2 (*c*)) between the highest and lowest gauge of this year, for an adjusted discharge of 1 200 cu ft per sec, reflects the effect of "efficiency" during the same flood. For the entire 39-yr period, the greatest range on the gauge for discharging this same volume of water is 3.2 ft, which, likewise, measures the influence of "efficiency" at the Arkansas City gauge for all the floods on record.

SUMMARY: COLUMBUS-WICKLIFFE, HELENA, AND ARKANSAS CITY, GAUGES

Flood Efficiency.—The foregoing data show that the maximum variation is 2.6 ft for the same volume of water during the same flood at these three gauge stations. A variation in flood "efficiency" of approximately 3 ft can be expected at these stations. Floods divide themselves into the following three classes: (1) Those of poor "efficiency" (high gauge height for a given discharge); (2) those of normal "efficiency" (medium gauge height for a given discharge); and (3) those of high "efficiency" (low gauge height for a given discharge). Certain of the ordinary causes that influence the class of each flood for the same volume of discharge are: (*a*) The relationship between the velocity of the main stream and the sediment introduced by tributaries; (*b*) control of sediment carried by tributary streams; (*c*) channel stabilization on the entire river system; (*d*) extent to which the main channel has been flushed by intervening water stages; (*e*) the effect of winds in the direction of flow; (*f*) the effect, on a gauge in the main river close to the mouth of a tributary stream, of the discharge of a large volume of water; and (*g*) change in slope for rising and falling stages of the river.

There is a relationship between velocity and quantity of sediment carried (Cause (*a*)). If the velocity of the main river, required for carrying sediment, is not overtaxed by the sediment contributed by the tributaries the deficiency is picked up from the channel of the main river, causing lower gauge heights. On the other hand, if the sediment contributed is greater than can be transported by the parent river, it is deposited and causes a higher gauge. As for Cause (*b*), an attempt to control the sediment carried by the tributaries, in order to assist the parent river, by protecting it against erosion is advisable. This policy has been adopted, to some extent, for reasons far removed from flood-control purposes, such as the terracing of cultivated fields in hill farms required by the Federal Land Banks and vigorously pursued by the United States Department of Agriculture in erosion control and forest protection. Channel stabilization by protecting the banks on both the main river and the tributaries (Cause (*a*)) is also an aid in reducing the quantity of sediment. Prevailing winds in the direction of flow (Cause (*e*)) will lower the gauges for the same discharge, whereas prevailing winds in the opposite direction will raise them The change in rate of rising stages over falling stages (Cause (*g*)) affects the slope at corresponding gauge heights thereby producing different volumes of discharge. For slow falling stages the slope is flattened out from that produced by fast rising stages.

The extent to which Causes (*a*) to (*g*) act in unison controls, to a major degree, the class "efficiency" of the flood. It is not unusual for a combination of the adverse causes to occur at the same time, creating a flood of poor "efficiency."

The greatest variation at the three gauge stations between the earliest and latest flood-years is 1.7 ft, showing that the two floods are of different efficiency. Each flood, although 37 yr apart, passed the same volume of water at one time during the flood interval with a difference of 1.7 ft. As this is within the expected range of maximum gauge variation, due to the influence of "efficiency," the river is not silting up at these gauge stations and causing higher flood elevations. This is further substantiated by the cross-sections taken at various intervals by the Mississippi River Commission. Comparison of cross-section elements of the general survey of 1894 to 1904 with those of the survey of 1911 to 1913 covering the section of the river from Cairo to Red River shows that, at bank-full stage, the average area increased 2.3%, the average width increased 3.9%, and the average mean depth increased 1.3 per cent[5].

The levee systems at these gauge stations have been improved gradually by raising and strengthening. These levee changes have not affected the gauge readings recorded. As will be demonstrated subsequently for the Atchafalaya River the gauge height for a given volume of water without levees is increased after levee confinement at some point; however, after once confined, further levee extension down stream from this point does not influence the gauge for the same volume of water.

Overbank Diversion.—During 1921 the Cypress Creek gap in the levee line was closed immediately above Arkansas City. Previously, flood water did escape from the Mississippi River through this gap at overbank elevations. Subsequent to this date, flood waters have been confined and could only escape by crevassing. The overbank flood-water escape of this frequency did not have any noticeable effect on the efficiency of the Arkansas City gauge.

Red River Landing (Angola, La.) Gauge

The gauge station at Red River Landing is situated on the Mississippi River approximately 1 mile below Old River (see Fig. 2). All the discharges used (see Table 4) were those measured with current meters. Those varying more than 25 000 cu ft per sec from a volume of 1 100 000 cu ft per sec, were rejected. At this gauge, the readings were adjusted to 1 100 000 cu ft per sec, using 35 000 cu ft per sec per ft of gauge, as shown by the 1929 discharge, and only the actual discharges during that year were used.

The major crevasses that occurred within 50 miles above or below Red River Landing prior to the discharge dates in Table 4, are given in Table 3. Data concerning crevasse occurrence in 1882 were not available. The Morganza crevasse, 16 miles below the gauge station at Red River Landing, which occurred on April 16, 1874, was not closed until February, 1884. It breached again on March 14, 1884, and was closed in January, 1887; breached once more on April 22, 1890, was closed in March, 1891, and has remained closed since.[6] The levee breach at Morganza was open during the recorded

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[5] Basic Data, Mississippi River, Annex No. 5, H. R. Doc. No. 798, pp. 71–73.

[6] Rept of the Mississippi River Comm., 1894, p. 3067.

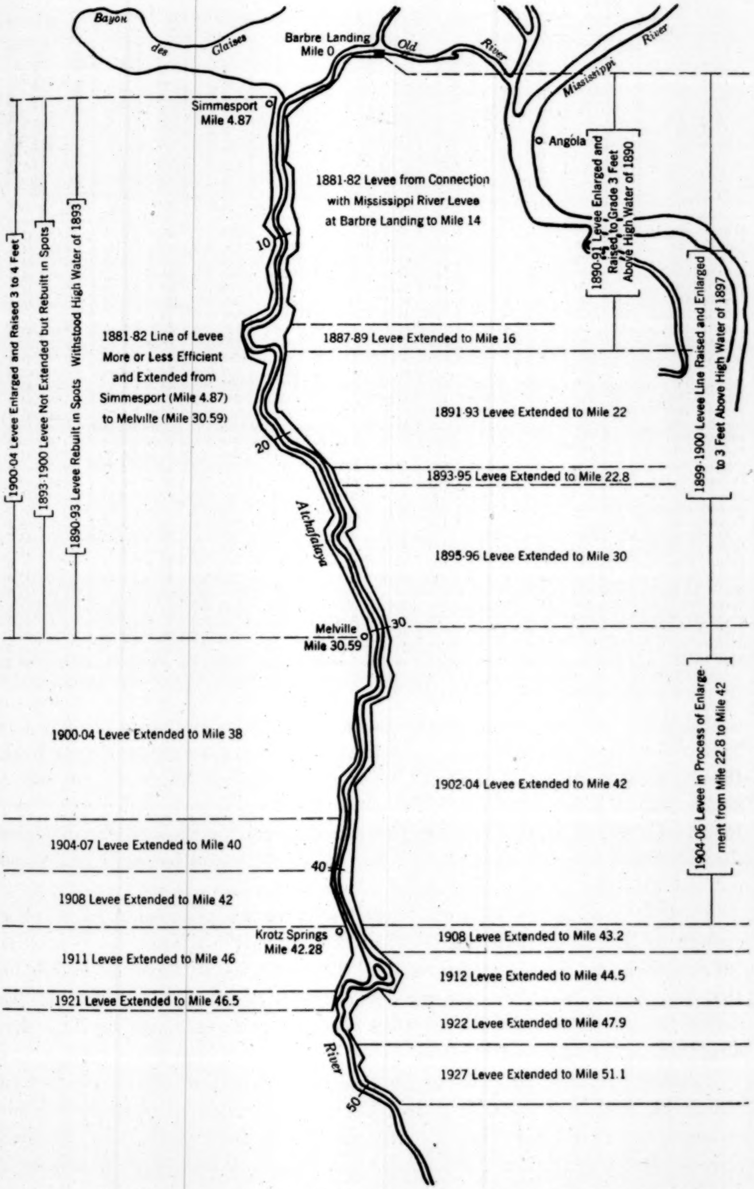
Bayou des Glaises
Barbre Landing
Mile 0
Old River
Mississippi River
Simmesport
Mile 4.87
Angola
1881-82 Levee from Connection with Mississippi River Levee at Barbre Landing to Mile 14
1890-91 Levee Enlarged and Raised to Grade 3 Feet Above High Water of 1890
1899-1900 Levee Line Raised and Enlarged to 3 Feet Above High Water of 1897
1900-04 Levee Enlarged and Raised 3 to 4 Feet
1893-1900 Levee Not Extended but Rebuilt in Spots
1890-93 Levee Rebuilt in Spots Withstood High Water of 1893
10
1881-82 Line of Levee More or Less Efficient and Extended from Simmesport (Mile 4.87) to Melville (Mile 30.59)
1887-89 Levee Extended to Mile 16
1891-93 Levee Extended to Mile 22
20
1893-95 Levee Extended to Mile 22.8
Atchafalaya
1895-96 Levee Extended to Mile 30
30
Melville
Mile 30.59
1900-04 Levee Extended to Mile 38
1902-04 Levee Extended to Mile 42
1904-06 Levee in Process of Enlargement from Mile 22.8 to Mile 42
1904-07 Levee Extended to Mile 40
40
1908 Levee Extended to Mile 42
Krotz Springs
Mile 42.28
1908 Levee Extended to Mile 43.2
1911 Levee Extended to Mile 46
1912 Levee Extended to Mile 44.5
1921 Levee Extended to Mile 46.5
1922 Levee Extended to Mile 47.9
River
1927 Levee Extended to Mile 51.1
50

FIG. 2.—PROGRESS OF LEVEE CONSTRUCTION, ATCHAFALAYA RIVER.

TABLE 4.—STUDY OF GAUGE RECORDS, RED RIVER LANDING

Date (1)	Area of cross-section, in square feet (2)	Gauge height, in feet (3)	Velocity, in feet per second (4)	Discharge, in thousands of cubic feet per second (5)	Gauge height adjusted to 1 100 000 cubic feet per second (6)
February 10, 1882	192 200	41.5	5.72	1 099	41.5
June 3, 1882.....	233 100	41.4	4.83	1 125	40.7
June 5, 1882.....	232 200	41.5	4.72	1 097	41.5
June 6, 1882.....	233 500	41.5	4.77	1 114	41.1
June 7, 1882.....	233 900	41.5	4.79	1 120	40.9
June 8, 1882.....	232 400	41.6	4.81	1 117	41.1
June 10, 1882....	234 500	41.6	4.77	1 119	41.1
June 12, 1882....	233 800	41.7	4.74	1 107	41.5
June 15, 1882....	236 300	41.8	4.72	1 115	41.4
June 19, 1882....	233 800	41.8	4.72	1 105	41.7
June 20, 1882....	234 400	41.8	4.67	1 094	42.0
June 21, 1882....	234 200	41.8	4.77	1 118	41.3
June 23, 1882....	234 000	41.8	4.80	1 123	41.1
June 24, 1882....	234 100	41.8	4.68	1 096	41.9
June 26, 1882....	235 200	41.8	4.67	1 099	41.8
June 28, 1882....	235 400	41.7	4.61	1 085	42.1
June 30, 1882....	234 100	41.6	4.73	1 106	41.4
July 1, 1882......	232 800	41.5	4.68	1 089	41.8
January 21, 1885..	226 700	40.2	4.86	1 101	40.2
January 26, 1885..	229 300	41.5	4.88	1 120	40.9
January 29, 1885..	228 700	41.8	4.93	1 127	41.0
February 10, 1885	235 300	41.4	4.73	1 113	41.0
Average gauge height for the first period........					41.3
March 28, 1890...	210 500	44.8	5.29	1 117	44.3
April 1, 1890.....	213 900	44.9	5.22	1 121	44.3
March 10, 1891...	210 800	43.2	5.13	1 082	43.7
March 16, 1891...	209 800	44.5	5.34	1 100	44.5
March 18, 1891...	210 100	44.8	5.23	1 102	44.7
March 20, 1891...	211 700	45.0	5.23	1 109	44.7
April 10, 1891....	216 900	45.4	5.14	1 118	44.9
April 13, 1891....	219 200	45.5	5.12	1 125	44.8
April 16, 1891....	218 200	45.4	5.08	1 112	45.1
April 20, 1891...	222 700	45.4	4.93	1 100	45.4
May 4, 1892....	199 600	43.8	5.42	1 082	44.3
May 23, 1892...	208 800	45.4	5.34	1 119	44.9
May 24, 1893...	207 900	44.3	5.27	1 100	44.3
May 26, 1893...	208 200	44.4	5.20	1 089	44.7
May 29, 1893...	208 200	44.5	5.11	1 070	45.4
June 7, 1893....	221 200	45.0	4.90	1 089	45.3
April 28, 1898...	192 600	44.3	5.63	1 084	44.8
April 10, 1903...	220 200	50.0	5.05	1 122	49.4
April 28, 1906...	219 500	44.4	5.08	1 116	43.9
April 30, 1906...	218 000	44.6	4.94	1 076	45.3
April 7, 1908....	222 500	45.3	4.98	1 108	45.1
April 13, 1908...	230 600	45.3	4.88	1 125	44.6
April 8, 1909....	237 100	45.9	4.63	1 103	45.8
Average gauge height for the second record......					45.0
February 15, 1913	245 800	46.4	4.53	1 117	45.9
February 20, 1913	243 100	47.0	4.60	1 122	46.4
May 2, 1913....	225 800	50.0	4.72	1 107	49.8
May 5, 1913....	227 500	50.4	4.65	1 101	50.4
April 17, 1920...	246 000	47.9	4.36	1 099	47.9
April 21, 1920...	241 600	48.6	4.39	1 098	48.7
April 22, 1920...	244 600	48.8	4.28	1 109	48.5
April 23, 1920...	243 500	49.0	4.39	1 110	48.7
April 27, 1920...	244 400	49.4	4.42	1 121	48.8
May 4, 1920....	243 000	49.6	4.44	1 122	49.0
April 11, 1923...	231 500	47.1	4.74	1 102	47.0
April 16, 1923...	236 800	48.0	4.73	1 126	47.3
April 8, 1929....	234 400	47.4	4.60	1 085	47.8
April 11, 1929...	232 400	47.8	4.72	1 106	47.6
April 18, 1929...	233 500	48.8	4.76	1 124	48.1
February 17, 1932	257 580	49.5	4.33	1 119	49.0
Average gauge height for the third period...					48.2

readings of 1882 and 1885. There are no data as to discharge through this breach for these two years. It is estimated that the crevasse of March 14, 1884, was discharging 211 000 cu ft per sec on March 25; the approximate gauge at Red River Landing on this date was 46.0. The recorded gauges for 1882 and 1885 are from 4.2 to 5.8 ft lower than this gauge, and the diversion through the Morganza opening at these low stages would be small and would not influence the gauge at Red River Landing 16 miles above.

In 1890 (see Table 3), there were no crevasses prior to April 1, close enough to Red River Landing to affect the gauge, the nearest being at Nita (899 miles below Cairo) on March 14. The crevasse at Ames on March 16, 1891 (see Table 3(*b*)), was too far down stream to affect this gauge. In the period from 1892 to 1903, no crevasses occurred prior to gauge readings that would affect the observed height.

Bougere crevasse, 35 miles above Red River Landing, which occurred on March 14, 1903, was 9 300 ft wide. There is no record of discharge of this crevasse, but it did not affect the recorded gauge readings because it served only to divert a portion of the main river into the back-water reservoir of Black River at a point about 30 miles up stream from where it would have been diverted into the reservoir at the end of the levee. In the period, 1904 to 1932, no crevasses occurred prior to readings that would affect the gauge at Red River Landing.

Referring to Table 2 (*d*), the variation of 6.9 ft for discharging the same volume of water in the two different floods, 50 yr apart, shows that excessive silting occurred in the river channel at this station. The year 1913 had the greatest variation for discharging the same volume of water. The difference of 4.5 ft between the highest and the lowest gauge of this year for discharging 1 100 000 cu ft per sec is due in part to "efficiency" influence and in part to conditions created by diversion. For the entire 50-yr period the greatest range in gauge heights for discharging the same volume of water is 10.2 ft (see Table 2 (*d*)). This large variation is due in part to "efficiency" influence and in part to silting in the channel.

SUMMARY: RED RIVER LANDING GAUGE

Unrestricted Diversion.—The effect of unrestricted diversion from the Mississippi River at the gauge station at Red River Landing (Angola, La.), creates a condition at variance with that at the stations previously analyzed. At this location the Mississippi River has two outlets of levee-confined discharge to the Gulf of Mexico; one past New Orleans into the Gulf, the other through Old River into the Atchafalaya River and thence to the Gulf. This latter outlet through the Atchafalaya permits unrestricted diversion of water from the Mississippi River at all stages.

To follow the changes occurring in the Mississippi River at Red River Landing, it is necessary to consider the same volume of water during each flood-year. For this purpose, a volume of 1 100 000 cu ft per sec was selected because, in the early years, it was fairly close to bank-full stage and, therefore, was not affected by crevasses. Furthermore, a greater number of gauge readings was available for study.

Shreve's cut-off occurred in 1831 and the Raccourci cut-off, in 1847, being, respectively, short distances above and below Red River Landing. The earliest reading used at this gauge station occurred during 1882, being 51 yr and 35 yr, respectively, later than these cut-offs. Therefore, the rise in gauge to discharge the same volume of water at this station since 1882 cannot be attributed to the effect of these cut-offs.

Records of the gauge station at Red River Landing since 1882 show that the gauge for discharging the same volume of water has been raised by permanent silting in of the channel (see Table 2 (*d*)). A further increase results from temporary silting occurring during the same flood, which is either removed by that flood or by intervening waters. The rise in gauge for discharging the same volume of water caused by the permanent silting in of the channel is the difference between the highest gauge of 1882 and the lowest gauge of 1932, being 6.9 ft. Reference to Table 4 shows that this rise in gauge height since 1882 is the result of permanent silting during three distinct periods, the average gauge for each period being as summarized in Table 5 (*a*).

The greatest gauge variation of 4.5 ft (see Table 2 (*d*)) for discharging the same volume of water during any one year occurred in 1913. This variation is greater than would be expected normally as the flood of 1913 is the end of the second period and the beginning of the third period of silting transition. The low gauges that occurred during the first days of the 1913 flood have never been repeated. The high gauges recorded during the latter days of the 1913 flood became normal for the third period. Since 1913, the lowest gauge for discharging 1 100 000 cu ft per sec is 47.1 as against 45.9 during the first part of 1913, and the highest gauge is 50.4. Therefore, the range of "efficiency" is from 47.1 to 50.4 ft on the gauge, or 3.3 ft.

TABLE 5.—Average Gauge Heights, Gauge at Red River Landing

Period No.	(*a*) Increase in Gauge Heights			(*b*) Peak-Flood Gauge Heights		
	Years	Average gauge, in feet, for a discharge of 1 100 000 cubic feet per second	Gauge increase, in feet	Years	Average peak-flood gauge, in feet	Increase for the period, in feet
1....	1882 to 1885	41.3	...	1867 to 1886	45.1	...
2....	1890 to 1909	45.0	3.7	1890 to 1907	48.7	3.6
3....	1913 to 1932	48.2	3.2	1912 to 1933	52.4	3.7

The greatest gauge variation for discharging the same volume of water is 10.2 ft for the 50-yr period of which 6.9 ft is the loss due to permanent silting and 3.3 ft reflects the influence of efficiency. A report of the Mississippi River Commission records the peak-flood gauges at Red River Landing.[7] The average of these peak-flood gauges, grouped as three periods in Table 5 (*a*), gives the increase in average peak floods for each of these three periods (see Table 5 (*b*)). This average period peak-flood gauge of 7.3 ft is practically the same gauge increase as the 6.9-ft increase in gauge height for discharging the same volume of water, showing a direct influence of flood height on the loss in discharge capacity below the point of diversion.

The actual changes in cross-section elements for the 2-mile reach below Old River, in the period, 1882 to 1924, are, as follows[8]: The bank-full width increased 535 ft; the bank-full area decreased 13 089 sq ft; and the bank-full mean depth decreased 9.6 ft.

A peak-flood height of 50.5 ft was never exceeded during the periods prior to 1912. During the period, 1912–1933, this gauge was exceeded eight times with the maximum peak gauge of 57.5 ft, which occurred in 1927. The main river at this point has adjusted itself to an average peak-flood gauge of 52.4 ft.

Loss in the cross-sectional area at Red River Landing during the same flood, and when diversion is occurring, was well illustrated during the high water of 1929. In this year the deposition of silt below the point of diversion restricted the cross-sectional area to the extent of holding it to the same area during the last 5-ft rise in flood height, as follows:

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[7] "Improvement of Lower Mississippi River for Flood Control and Navigation", Vol. 1, p. 104.

[8] *Loc. cit.*, Vol. 1, Table 6.

Gauge height, in feet	Area, in square feet
47.4	234 000
48.8	234 000
49.8	236 000
51.1	230 000
52.2	230 000
52.4	232 000

From the foregoing the deposition of the silt during the same flood is shown to act as a natural automatic valve to hold the cross-section to an area of about bank-full and against a rise in gauge, thus producing a steeper slope during the same flood. This silt is removed during the same flood or intervening waters. The gain in cross-sectional area at this station during the same flood, and when diversion from the Mississippi River is prevented by higher stages in the Red River, is shown by the 1932 flood in the Red River. During this flood a portion of Red River water was discharged into the Mississippi River and increased the volume carried by it. Discharge cross-sections during this year at Red River Landing show:

Gauge, in feet	Area, in square feet
46.6	245 000
48.8	260 000
50.9	267 000
52.1	271 000

The cross-sectional area of 271 000 sq ft at the peak gauge of 52.1 during 1932, as against a cross-sectional area of 232 000 sq ft at a peak gauge of 52.4 ft during 1929, shows a loss of 39 000 sq ft, or 17% of area, caused by diversion during the same flood at this gauge station.

The silting changes of these two floods were only local and did not materially affect the gauge for the same volume of water. The 1932 flood, with diversion eliminated, shows the power of the river to clean the channel and if given the chance, eventually to re-open the entire reach to its old cross-sectional area and produce lower stages for the same volume.

The changes at Red River Landing due to unrestricted diversion are illustrated at the two overbank diversions below New Orleans at Pointe a la Hache and Caernarvon, during the flood of 1927. Discharge observations were taken immediately above and below these diversions. At Pointe a la Hache, 57 miles below New Orleans, 10.5 miles of the Mississippi River levee was removed during 1924 in an effort to reduce flood heights at New Orleans. The 1927 flood discharge observations at Pointe a la Hache show that the velocity above the diversion increased over that below the diversion as the discharge through the levee gap increased. There was a maximum difference in velocity of 1.33 ft per sec, with the maximum diversion of 301 845 cu ft per sec. After diversion the cross-sectional area at the upper station was found to have increased 8 125 sq ft and that at the lower station had decreased 8 169 sq ft. The increase in area of the upper section was caused by an increase in velocity induced by the suction through the levee gap, thereby increasing the sediment-carrying capacity of the river, adding to its load by scour from the channel within the zone of influence of the diversion. Contrarily, the loss in area at the lower section was due to the decrease in velocity, thus causing deposition of sediment in the channel.

The induced draft through the levee gap at Pointe a la Hache during the 1927 flood did not reach far enough up stream to relieve New Orleans, making it necessary for a further levee breach nearer that city. An artificial crevasse was then made at Caernarvon twenty miles below New Orleans. The greatest difference in velocity between the upper and lower stations at the Caernarvon diversion was 1.13 cu ft per sec for the maximum diversion of 326 446 cu ft per sec. The discharge measurements were not carried through the entire period of diversion and, therefore, cross-sectional comparisons are not possible. These effects from the drop in velocity, and the gain and loss in channel area caused by overbank diversion are noticeable at Red River Landing in the greater channel depth immediately above, and the shallow channel depth immediately below, Old River.

As the peak-flood gauge increased 7.3 ft caused by higher flood stages, due to increased height and strength of levees, larger volumes of water were diverted from the Mississippi River and forced down the Atchafalaya. The higher the gauge at Red River Landing the larger the volume diverted into the Atchafalaya. The Mississippi River, below the point of diversion, adjusted itself with silt deposition, progressively, with increased flood heights and consequent increase in diversion. The loss in mean depth for bank-full stage from 1882 to 1924 was 9.6 ft. Under this unrestricted diversion, it is apparent that the Mississippi River is adjusting itself to the loss in volume. Diversion into the Atchafalaya River is dividing the Mississippi River, and this results in a velocity drop below the point of diversion as compared with that above it. The larger the volume of water diverted the greater the velocity difference. This velocity change is causing a deposition of silt which forms a dam below the point of diversion, thereby adjusting the river to a new slope. This change in slope varies with the volume subtracted. The "efficiency" of the stream down stream from this point is diminished, and, the river at this point is almost continuously subject to diversion and only infrequently is its volume augmented by waters from Red River. The preponderance of influence is due to diversion occurring at all stages from the Mississippi into the Atchafalaya; therefore, there is no opportunity for the sediment-formed dam at Red River Landing to be cleaned out. As the volume of the main river is reduced by increased diversion through the Atchafalaya, the Mississippi River adjusts itself to an increased slope by the deposition of silt below, thus causing (since 1882) a rise in the gauge of 6.9 ft to discharge the same volume of water.

The adjustment at Red River Landing conforms to the hydraulic theory that "whenever silt-bearing streams flow through alluvual deposits, other conditions being the same, the slope is least where the volume is greatest, and, conversely, the slope is found to be invariably increased as the volume is diminished."[9]

CARROLLTON, LA. (NEW ORLEANS), GAUGE STATION

All available discharges used were measured with current meters and those varying more than 25 000 cu ft per sec from a volume of 1 100 000 cu ft per sec were rejected. Readings were adjusted to the basis of 1 100 000 cu ft per sec, using 60 000 cu ft per sec per ft of gauge, as shown by the 1929 discharge. Only actual observed discharges during 1929 were used.

The record of three crevasses that have occurred within 50 miles up stream and down stream from the Carrollton gauge is given in Table 3 (*c*). No records are available of crevasses that occurred in 1883. Between February 13 and March 26, 1890, ten small crevasses occurred, between Miles 973 and 1 000, down stream from Cairo. These breaks varied in widths from 15 to 540 ft and were too small to affect the gauge readings. Furthermore, gauges adjusted for 1 100 000 cu ft per sec before and after the Nita crevasse, which occurred on March 14, 1890 (see Table 3 (*c*)), do not show any influence. The Ames crevasse on March 16, 1891 (see Table 3 (*c*)), occurred after the dates of the gauge readings under consideration. Between May 3 and June 13, 1892, seventeen crevasses are listed. All these breaks were small and closed within a few days of their occurrence, except Sarpy crevasse, 939 miles from Cairo (see Table 3 (*c*)). A comparison of the adjusted gauge readings for this year with those of 1893 (which were not affected by crevasses) shows no influence.

Referring to Table 2 (*e*), the same volume of water in the two different floods, 49 yr apart, was passing the gauge at heights only 1 ft apart, which shows that the channel had not silted appreciably in that time. The greatest variation for discharging the same volume of water in a single year, occurred in 1913. The difference of 2.6 ft between the highest and lowest gauges of this year, adjusted for a discharge of 1 100 000 cu ft per sec, reflects the effect of "efficiency" during the same period. For the 49-yr period, the greatest variation on the gauge for discharging the same volume of water, is 5.3 ft (Table 2 (*e*)). This large variation is due in part to "efficiency" influence, and in part to conditions created by unrestricted diversion at Red River Landing.

SUMMARY: CARROLLTON, LA. (NEW ORLEANS), GAUGE STATION

The study at the Carrollton gauge station is based on a discharge of 1 100 000 cu ft per sec, details of which are recorded in Table 1 (*d*). This volume of water is close to bank-full stage and recorded gauge readings are not affected by crevasses or levee changes.

The variation of 1 ft between the lowest gauge of 1932 and the highest gauge of 1883 (see Table 2 (*e*)) shows that at one time during these two floods, 49 yr apart, the same volume of water was passing within a foot on the gauge. This variation is within the range of "efficiency" influence and the channel has not silted. The greatest variation for discharging the same volume of water occurred in 1913. The difference of 2.6 ft between the highest and lowest gauge of this year for discharging 1 100 000 cu ft per sec, is due in part to "efficiency" influence. This variation of 2.6 ft is greater than can normally be expected due to efficiency influence, as the conditions set up during the flood of 1913 at Red River Landing caused an abnormal gauge fluctuation at Carrollton. Prior and subsequent to the flood of 1913 the greatest gauge variation during any one year for discharging this same volume of water is 1.5 ft. For the 49-yr period the greatest range in gauge for discharging this same volume of water is 5.3 ft. The increase in gauge for discharging the same volume of water at this station

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[9] Letter from the late James B. Eads, F. Am. Soc. C. E., to Mississippi River Comm. dated April 12, 1882.

TABLE 6.—COMPARISON OF CROSS-SECTION ELEMENTS, STAGE BANK-FULL: ATCHAFALAYA RIVER; SURVEYS OF 1904–05, 1916–17, 1931, AND 1932

Reach	STATIONS, IN MILES*		WIDTHS, IN FEET				AREAS, IN SQUARE FEET	
	From: (1)	To: (2)	1904–05 (3)	1916–17 (4)	1931† (5)	1932† (6)	1904–05 (7)	1916–17 (8)
0	0.0	3.3	1 147	1 173	1 395	1 420	58 456	67 937
1	3.3	8.1	1 177	1 143	1 277	1 304	52 792	58 717
2	8.1	13.1	1 135	1 251	1 423	1 469	58 018	65 483
3	13.1	18.1	1 116	1 152	1 326	1 370	55 966	63 556
4	18.1	23.0	1 017	1 044	1 204	1 216	54 588	57 382
5	23.0	28.0	1 044	1 119	1 200	1 210	58 529	68 368
6	28.0	32.9	1 044	1 086	1 163	1 170	55 222	58 803
7	32.9	37.9	920	976	1 052	1 062	46 479	53 859
8	37.9	42.9	719	834	980	987	31 769	40 553
9	42.9	47.8	579	872	1 148	1 158	19 418	41 126
10	47.8	52.8	527	751	1 094	1 097	19 121	39 749
11	52.8	57.9	434	572	870	887	15 107	27 041
12	57.9	63.0	367	390	596	592	12 604	15 729
13	63.0	68.0	494	509	640	638	15 184	16 819
Total	....	....	11 720	12 874	15 368	15 580	554 253	675 132
Average	....	....	837	920	1 098	1 113	39 590	48 224

Reach	AREAS, IN SQUARE FEET (*Continued*)		MAXIMUM DEPTHS, IN FEET				Stage elevation, in feet, above mean Gulf level (15)
	1931† (9)	1932† (10)	1904–05 (11)	1916–17 (12)	1931 (13)	1932 (14)	
0	64 724	79 056	88.2	91.5	77.9	100.5	47.72
1	59 449	70 462	69.9	78.8	76.3	88.3	45.71
2	67 498	74 362	79.7	83.5	83.4	91.8	43.65
3	66 572	72 542	80.7	86.0	87.4	92.5	41.52
4	70 812	71 829	77.4	84.5	94.2	94.8	38.79
5	73 063	74 019	93.0	98.1	97.3	99.5	35.59
6	57 701	57 519	87.6	87.6	83.5	82.5	32.14
7	58 544	59 573	81.4	86.2	85.5	84.1	29.07
8	53 714	54 737	68.4	78.2	84.5	85.7	26.59
9	57 657	58 210	54.6	80.6	81.4	83.3	24.30
10	57 438	58 830	58.5	80.4	88.8	90.7	21.23
11	43 263	45 447	58.3	70.4	83.3	85.5	17.92
12	24 804	26 140	56.2	59.2	69.0	69.7	15.38
13	22 665	23 716	51.8	50.3	57.3	58.4	13.29
Total	777 804	826 442	1005.7	1115.3	1149.8	1207.3	432.90
Average	55 557	59 032	71.8	79.7	82.1	86.2	30.92

* Station 0 is at Barbres Landing, La. (see Fig. 2.).

† Sections normal to stream computed where necessary.

is between 3 and 4 ft. This may be caused by conditions set up by unrestricted diversion at Red River Landing. Definite proof is not available at this time, however, but it is thought that future studies will throw additional light on this subject.

THE DISCHARGE OF THE ATCHAFALAYA RIVER

The cross-sectional elements of the Atchafalaya River for the upper 68 miles, as taken by the Mississippi River Commission for various years (Table 6), show that this stream has increased an average of 47% in area at bank-full stage from 1904 to 1932. (Levees have been constructed along the first 51 miles of this river.) In order to determine the effect, if any, that this large increase in cross-sectional area has had on the discharge capacity of the stream, it is necessary to consider the passage of the same volume of water through the various flood-years. To obtain a sufficient number of readings for consideration, a volume of 300 000 cu ft per sec is used as reported at the Simmesport Discharge Observation Station (see Table 1(*e*) and Table 2 (*f*)). As in the case of gauge records previously reported in this paper, all discharges in Table 1 (*e*) were measured with current meters and those varying more than 15 000 cu ft per sec from a volume of 300 000 cu ft per sec were rejected. At this gauge the readings were adjusted to a basis of 300 000 cu ft per sec, using 13 000 cu ft per sec per ft of gauge, as shown by the discharge rating curve issued by the Corps of Engineers, U. S. Army, for this station. Readings from 1890 to 1893, which are published with the zero of the gauge at 3.88 ft above mean Gulf level were corrected to a gauge with its zero 5.79 ft above mean Gulf level, thus referring all readings in Table 1 (*e*) to the same datum plane. A record of the more important crevasses along the Atchafalaya River is presented in Table 7.

TABLE 7.—RECORD OF IMPORTANT CREVASSES

Name (1)	Bank (2)	Date (3)	Width, in feet (4)	Name (1)	Bank (2)	Date (3)	Width, in feet (4)
Barbree	Left	April 21, 1890	215	Burton	Right	April 22, 1890	1 600
Cottage Point	Left	April 21, 1890	600	Churchville	Right	April 22, 1890	500
Yoist	Left	April 21, 1890	1 800	Bayou Marine	Left	April 15, 1890	550
School	Left	April 21, 1890	110	Pouncey	Left	1891	.....
Jacoba	Left	April 21, 1890	150	Deer Range	Right	1891	.....
Callahan	Left	April 21, 1890	180	Nelson-Eddy	.....	1893	.....
Mock	Left	April 21, 1890	480	Holloway	Left	1903	.....
Smith and Taylor	Left	April 21, 1890	50	Alto	Right	May 19, 1912	2 450
Harmanson	Right	April 8, 1890	300	Atkins	Right	May 17, 1912	.....
Norwood	Right	April 8, 1890	150	McCracken	Left	April 16, 1912	5 200
Yellow Bayou	Right	April 8, 1890	200	Coville	Left	April 25, 1913	400
Cason	Right	April 15, 1890	300	McCrea	Left	May 24, 1927	.....
Gordon	Right	April 18, 1890	400	Melville	Right	May 27, 1927	.....

There is wide variation in the "efficiency" of the Atchafalaya River. The difference of 6.1 ft between the 1920 and the 1928 readings, eight years apart (see Table 2 (*f*)), shows the influence on "efficiency" by the sediment deposited in the stream as a result of diversion. The flood of 1927 formed crevasses in the Atchafalaya River levees, flushing the river channel sediment at different points and allowing efficient handling of the 1928 water at low gauges. The flood in the Red River during 1932 established the highest gauge of record at Alexandria, La. This high stage of the Red River prevented the diversion of the Mississippi into the Atchafalaya and cleaned the Atchafalaya River to greater depths than recorded by surveys since 1880–1881. This flushing increased the Atchafalaya "efficiency," allowing the early part of the 1933 water to pass at low stages.

The gauge variation of 1.7 ft during the 43-yr period, 1890 to 1933 (see Table 2 (*f*)), develops the fact that no material change occurred in the discharge capacity of the stream, even in the face of a 47% increase in cross-sectional area. The velocities in Table 1 (*e*) show that as the river expanded in cross-sectional area there was a reduction in velocity from about 5 ft to less than 4.5 ft per sec caused by slope adjustment of the stream. The head is fixed by the water elevations in the Mississippi River at Red River Landing, and, therefore, the slope is reduced because of the increased silt deposits below the lower end of the levees. Proof of this silting is shown[10] in Table 8. Melville is at the lower end of Reach *B*; and the lower end of Reach *E* is approximately at the end of the east levee. No change in gauge for the discharge of the same volume of water has occurred since 1890; therefore, the increase in cross-sectional area has kept pace and has offset the slope reduction. The theory that with an increase in the volume of a silt-bearing stream the slope will diminish, is borne out by the actions of the Atchafalaya River.

TABLE 8.—ATCHAFALAYA RIVER: CHANGES IN AVERAGE BANK ELEVATION (IN FEET), AS SHOWN BY SURVEYS

Reach	Station, in miles	1880–81 to 1904–05	1904–05 to 1916–17	1916–17 to 1931
		(24 yr)	(12 yr)	(15 yr)
A..............	0–13.7	+1.0	—0.5	+0.8
B..............	13.7–29.8	+3.1	+1.0	+1.7
C..............	29.8–36.9	+3.0	+4.3	+4.0
D..............	36.9–43.0	+1.8	+6.3	+1.7
E..............	43.0–52.9	.....	+5.0	+3.3
F..............	52.9–63.7	.....	+2.5	+5.7
G..............	63.7–66.9	.....	+1.6	+4.8

SIMMESPORT, LA., GAUGE

In 1890, the east levee on the Atchafalaya River extended to 26 miles below Simmesport (see Fig. 2), the west levee extending to 11 miles below. Levee extensions since 1890 have not affected this gauge for the same volume of discharge. For the various flood-years when the high gauge of 50 ft was reached in the Mississippi at Red River Landing, the record of discharge in the Atchafalaya River at Simmesport for corresponding dates is given in Table 9. The discharges vary with the silted condition of the channel and are not upset or influenced by crevasses.

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[10] "The Improvement of the Lower Mississippi River for Flood Control and Navigation", May, 1932, prepared under the direction of the President of the Mississippi River Commission, Vol. 1, p. 126.

The Atchafalaya discharge of 376 000 cu ft per sec during 1932 at a gauge of 49.5 ft at Red River Landing (see Table 9, Item No. 7) is within reasonable limits of the 390 000 cu ft per sec discharged during 1897 at a gauge of 50.0 ft at Red River Landing. (The year, 1897, is the earliest on record in which a gauge of 50.0 ft was reached at Red River Landing.) This further substantiates the flattening of the slope in the Atchafalaya River; it offsets the 47% increase in cross-sectional area; and the volume of water diverted into the Atchafalaya is dependent upon water elevations in the Mississippi River. The increase in discharge of the Atchafalaya River due to the increase of 6.9 ft in the Mississippi for discharging the volume of 1 100 000 cu ft per sec is shown in Table 10.

TABLE 9.—DISCHARGE IN THE ATCHAFALAYA RIVER

Item No.	Date	Mississippi River gauge, at Red River Landing, in feet	Discharge, Atchafalaya River, at Simmesport, in cubic feet per second
1	May 10, 1897	50.0	390 000
2	April 8, 1903	50.0	388 000
3	April 20, 1912	50.1	331 000
4	May 2, 1913	50.0	390 000
5	May 4, 1920	49.6	306 000
6	May 2, 1929	49.9	346 000
7	February 17, 1932	49.5	376 000

TABLE 10.—INCREASE IN THE DISCHARGE OF THE ATCHAFALAYA RIVER

Period	Dates	Mississippi River, Red River Landing		Atchafalaya River, Simmesport discharge, in cubic feet per second
		Gauge height, in feet	Discharge, in cubic feet per second	
1	1882–1885	41.3	1 108 600	184 421
3	1913–1932	48.2	1 105 000	313 866
Increase	....	6.9 ft	....	129 445

MELVILLE AND KROTZ SPRINGS GAUGE STATIONS

Discharge observations are not made in the Atchafalaya River, at Melville and Krotz Springs, La. (see Fig. 2). To study the action of the river at these two points gauge heights are listed in Table 11 for the day following the approximate discharge of 300 000 cu ft per sec at Simmesport. The dates for the gauges at Melville and Krotz Springs are one day later than those for the gauges at Simmesport to correspond with a discharge of 300 000 cu ft per sec. The zero for 1933 values on the Melville gauge is 0.2 ft above mean Gulf level. To convert the gauge readings for previous years to the same datum plane as 1933, subtract 0.3 ft from the readings in 1890 and 1892, and add 0.2 ft to the readings in 1907 and 1929. The gauge readings in Table 11 are not adjusted to an even 300 000 cu ft per sec.

At Melville the gauges from 1907 to 1933 do not show any noticeable changes other than those due to "efficiency" influence. The average of the gauge readings prior to 1907 is 5.2 ft lower than the average of those after that year. This increase in gauge between 1892 and 1907 is due to levee confinement. In 1893 the west bank of the Atchafalaya River had a continuous levee line extending to Melville and the east levee line ended 8 miles up stream from this point. Between 1893 and 1907 the west levee was extended to 12 miles below Melville and the east levee extended to 13 miles below that point. This confinement caused the increase in gauge heights of approximately 5 ft at Melville.

TABLE 11.—GAUGE READINGS AT MELVILLE, LA., AND KROTZ SPRINGS, LA. (SEE FIG. 2)

Date	Gauge height, in feet, Melville, La.	Date	GAUGE HEIGHT, IN FEET		Date	GAUGE HEIGHT, IN FEET	
			Melville, La.	Krotz Springs, La.		Melville, La.	Krotz Springs, La.
April 8, 1890....	33.7	June 17, 1908..	39.8	....	April 22, 1933..	36.4	52.4
March 26, 1891..	33.3	April 14, 1912..	39.8	53.7	April 23, 1933..	36.6	52.5
April 18, 1891...	33.4	April 25, 1920..	41.3	54.9	April 25, 1933..	37.0	52.7
May 21, 1892...	33.4	April 28, 1920..	41.6	55.0	April 27, 1933..	37.3	52.9
May 29, 1892...	35.5	May 4, 1920...	41.9	55.2	April 28, 1933..	37.4	53.0
February 15, 1907	37.6	April 14, 1923..	39.8	....	May 2, 1933...	37.8	53.4
February 17, 1907	37.8	July 22, 1928..	37.7	51.9	May 4, 1933...	38.1	53.6
April 7, 1908....	41.1	April 7, 1929...	39.0	53.0	May 6, 1933...	38.3	53.7
May 5, 1908....	38.3	April 11, 1929..	39.6	53.3	May 14, 1933..	38.2	53.7
May 7, 1908....	38.5	....	....	....	....	....	....

The gauge at Krotz Springs from 1912 (the earliest gauge reading available) to 1933 does not show any noticeable change other than that due to "efficiency" influence. In 1912 the east levee ended 3.7 miles below this station and has since been extended to 4 miles below it (see Fig. 2). In 1908 the west levee ended 1 mile below this station and has since been extended to 9 miles below it. These levee extensions have not affected this gauge for the same volume of discharge.

HISTORY OF THE ATCHAFALAYA RIVER BASIN[11]

In 1804, the channel of the Atchafalaya River was choked with rafts of driftwood extending from bank to bank, beginning on the north at a point 30 miles from the Mississippi River and extending down the Atchafalaya about 20 miles. They clogged the river channel completely, making navigation impossible. Trees were growing around these rafts, the largest being about 10 in. in diameter. Driveways crossed them at different places. These rafts rose and fell with the water and, apparently, they were effective in checking any natural opening of this channel between Old River and the Gulf.

About 1840 the State of Louisiana, in order to open a navigable channel, began cutting channels through the rafts with a view to their complete removal. This work continued to the beginning of the Civil War period. Immediately upon the removal of the rafts the channel enlarged rapidly at bends in the river and on both sides of the narrow straight reaches. Lands in the Atchafalaya Basin which had been exempt from overflow were flooded annually until, in 1881, the Upper Atchafalaya Basin "had gone back to Nature."

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[11] Comp. from Repts. of the Mississippi River Commission.

Along the river were to be seen ruined dwellings and sugar houses, buried fence lines, broken levees, hardwood timber killed by standing water, and cypress swamp growth started. Navigation, however, was excellent in 1881. The change in the characteristics of the river at its head between 1851 and 1879, is illustrated by reference to Table 12.

TABLE 12.—CHANGES IN THE CHARACTERISTICS OF ATCHAFALAYA RIVER

Year	Cross-section area of channel, in square feet	DIMENSIONS OF CROSS-SECTION AT HIGH WATER, IN FEET	
		Width	Depth
1851	24 400	730	52
1859	28 700	830	63
1874	39 160	891	114
1879	52 100	940	130

In 1881, the channel of the Atchafalaya River had enlarged to a point at which it had a discharge nearly equal to that of the Red River and afforded that stream the line of least resistance for flow to the sea. In 1881, the elevation of the water surface at Red River Landing was almost constantly above that at the head of the Atchafalaya, being 7.3 ft above the latter point on October 13, 1891. At this time there was a marked tendency to increase the channel from the Mississippi to the head of the Atchafalaya. Quoting from the report of the Commission:[12]

"No decrease in flood heights in the Mississippi River has been observed, although a diversion of one-sixth of the Mississippi River discharge has taken place. However, there has been a decrease of flood heights on the Atchafalaya."

The report of the Mississippi River Commission of 1881 contained a recommendation that a sill dam be built across the mouth of Old River between Turnbull's Island and the Mississippi, and, also, that a study be made of the proposal to divorce completely the Mississippi River from the Red and Atchafalaya drainage. During 1882 actual work at the head of the Atchafalaya was confined to dredging bars at the mouth of the Red River and the head of the Atchafalaya, creating a 16-ft channel from the Red River through Old River and a 6-ft channel connecting the Red with the head of the Atchafalaya River, thus making navigation conditions very satisfactory. In 1884 the Commission reported[13] that the formulation of a comprehensive plan for the treatment of the Atchafalaya situation had been deferred from time to time on account of the necessity for further observation, experience, and study. It was prepared to present a plan which was stated to be the best that could be devised in point of practicability, safety, and economy; a plan that should include "the prevention of the diversion of the Mississippi into the Atchafalaya Basin and the closure of any depleting outlet in that point, either now existing or likely to be induced by the changes reasonably to be anticipated and also the preservation of navigation of the Red and the Atchafalaya Rivers."

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[12] Progress Rept. of the Mississippi River Comm., 1881, p. 13.

[13] Rept. of the Mississippi River Comm., 1884, p. 2554.

At that time Major Amos Stickney, Corps of Engineers, U. S. Army, was in charge of the lower district, and he recommended the construction of a series of brush dams to be placed in the Atchafalaya River, the upper one to be below the Bayou des Glaizes and the others at intervals not exceeding ¼ mile. These dams were to be built to a point just below low water; they were planned not to interfere with navigation, and to regulate the discharge of the Atchafalaya so that it would take care of the flood discharge of the Red River. These dams were to be supplemented by levees to prevent the overflow into the Atchafalaya Basin of water from the Mississippi. At this time a dam was also recommended across Old River between the Atchafalaya and the mouth of the Red near Turnbull's Island. This was in order to force the Red River down the Atchafalaya. Apparently, no action was taken by Congress on this recommendation and the report of 1885 again recommends the construction of these sill dams in the Atchafalaya as soon as the water stage permits and money is available. Quoting the report:[14]

> "The work to be done would have for its object the gradual construction and perhaps finally the complete closure of the Atchafalaya as an outlet of the Mississippi. Whether the treatment should be carried further than this, to the extent of making the Red a tributary of the Mississippi by the construction of a high dam across the head of the Atchafalaya River and its basin, thus cutting the latter off from navigation of the Red and Mississippi, is a consideration of great moment. Determination can be deferred. The Commission is distinctly committed to the idea of closing all outlets as part of the plan for the improvement of the Mississippi River, both low-water outlets and breaks in the levees, and has consistently opposed the fallacy known as the outlet system because it stands in sharp antagonism to all the fundamental laws of hydraulics. When the Atchafalaya shall cease to be an outlet the discharge of the Mississippi below the mouth of the Red will be decreased from 40 to 50 per cent. by the addition of volume now substracted by the outlet."

The report of the Commission of 1886 states that "no instructions have been received for carrying out the foregoing plan." During 1888 construction on the sill dams in the Atchafalaya below Simmesport, La., was started, in line with recommendations previously made by the Commission. The first sill dam was located 500 ft below the mouth of the Bayou des Glaizes. This report contains complete descriptions and analyses of a number of plans for handling the Red River-Atchafalaya Basin problem. These data appear to be segregated about as follows: (1) Major Stickney's plan (six submerged sill dams); (2) Eads' plan (to cut off the Atchafalaya River with a high dam at the head); (3) canal and lock plan (cut the canal, Mississippi to Atchafalya Rivers, with locks therein); (4) Mississippi River Commission plan (includes Plan (1) and also a plan to shut off the connection between the mouth of the Red and the Atchafalaya Rivers with a sill dam on the west side of Turnbull's Island); and (5) variation of Plan (4) (includes a long jetty from the lower point of Turnbull's Island extending into the Mississippi to prevent direct flow from the latter into Old River).

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[14] Annual Rept. of Mississippi River Comm., 1885, p. 2872; Appendix WW of the Annual Rept. of the Chf. of Engrs., 1885.

The report of 1889 indicates that Plan (5) had been adopted for handling the situation. During the year the upper sill dam in the Atchafalaya was practically completed and the second sill dam had been commenced. This second dam is located 1 750 ft down stream from Sill Dam No. 1. The report of 1890 shows the completion of the second sill dam on August 27. During the months of September, October, and November, 1890, the sill or lower course of a sill dam was constructed across Old River, ¾ mile down stream from Sugar House Chute. The reports of 1891 and 1892 show progress on the construction of this sill dam, which was practically completed in 1892.

To summarize, then, from 1804 to 1840, the Atchafalaya was completely blocked by rafts and received very little flood water from the Mississippi River. In 1852, according to an estimate by Charles Ellet, a Civil Engineer employed by the Government, it was handling one-twelfth of the Mississippi discharge. In 1882, Mr. Eads estimated that it was carrying approximately one-sixth of the Mississippi River outflow. At present (1935) the Atchafalaya is discharging, during flood periods, a levee-confined volume of about 500 000 cu ft per sec, or one-third of the discharge of the Mississippi River below Red River Landing. The sill dams, the construction of which has been mentioned previously, were designed and built in order to restrict the discharge of the Atchafalaya to about 200 000 cu ft per sec, which was estimated to be the discharge of the Red River while in flood at that time (1882). These dams did not restrict the Atchafalaya to a capacity of 200 000 cu ft per sec, due to later increased flood heights caused by the construction of higher and stronger levees. Both sill dams are alike in design and construction. The base or sill consists of a willow mattress, 3 ft thick, and 304 ft long, up stream and down stream, loaded with rock. Upon this sill were built three courses of willow mattress, 100 ft, 66 ft, and 30 ft in width, respectively, each course being covered with successive layers of hard clay and gravel. The up-stream edge of the dam is 20 ft below the upper edge of the sill. The entire structure was given a heavy covering of rock.

CONCLUSIONS

Except where indicated the gauge and discharge readings in this paper are not affected by crevasses, cut-offs, levee extensions, or enlargements. At the gauge stations along the Mississippi River, at Columbus, Ky., Arkansas City, Ark., and Helena, Ark., the flow is not subjected to unrestricted diversion and the same volume of water is now passing at the same elevations as in earlier years. At these points the river is not silting, but a variation in gauge due to "efficiency" influence of approximately 3 ft can be expected for the same volume of water. Overbank diversion occurred at the Arkansas City gauge station prior to 1921 through the gap at Cypress Creek, a short distance above, without affecting the gauge in passing the same volume of water.

At Red River Landing, the Mississippi River is divided into two levee-confined outlets to the Gulf, permitting diversion from the Mississippi into the Atchafalaya River at all stages, except when higher stages occur in the Red River. The higher stage in Red River is infrequent compared to diversion from the Mississippi. This division of the river causes a drop in velocity below the point of diversion over the velocity of that above, the difference being greater as the volume diverted increases. This checking of the current in flood time causes a deposition of some part of the silt load, and it is in this manner that Nature provides the means of the river's adjustment. The greater the diversion, the greater will be the difference between the velocity above and below, and, consequently, the greater the deposition of silt below the point of diversion. The levees on the main river were raised and strengthened, thus causing levee confinement to larger volumes of water. Increased flood heights resulted and raised the average flood peaks at Red River Landing 7.3 ft between the periods 1867 to 1886 and 1912 to 1933. This peak-flood increase caused larger volumes of water to be diverted from the Mississippi through the Atchafalaya. The Mississippi River below the point of diversion, adjusted itself progressively with increased flood heights and consequent increase in diversion by the river channel being partly closed by a silt-deposited dam. This dam increased the river slope for the loss in volume diverted and raised the gauge 6.9 ft to discharge the same volume of water since 1882. The close relationship between the increase in peak-flood heights and the increase in gauge for discharging the same volume of water is apparent as only a difference of 0.4 ft exists.

At Carrollton the gauge has been raised between 3 and 4 ft since 1882 in passing the same volume of water. Definite proof as to the cause of this increase is not available. The head of the Atchafalaya is fixed by the water elevation in the Mississippi River at Red River Landing. Consequently, the slope has been adjusted by silt deposited below the lower limit of the levee influence. Although the velocities have decreased in the Atchafalaya with the diminished slope the increase in bank-full cross-sectional area of 47% since 1904 has kept pace and offset the effect of slope reduction, so that the same volume of water at this time is passing the same gauge height as it was in 1890. This balancing of the lessened slope by the enlargement in cross-sectional area has made it possible to increase the volume of discharge through the Atchafalaya only by increased flood heights in the Mississippi River. This is definitely shown by the discharges of the Atchafalaya River at Simmesport for a high gauge of 50 in the Mississippi River (see Table 9, Items Nos. 1, 2, 4, and 7).

The distance from Red River to the sea *via* the Mississippi River is 310 miles and *via* the Atchafalaya River, 125 miles. The shorter distance and steeper slope afforded by the Atchafalaya appears to be a logical course for the Mississippi River to follow. This condition existed for centuries before Man made levees, giving every opportunity for the Mississippi River to take advantage of this short route to the sea if Nature so desired. Through the manner in which the river treats the silt load the shorter route to the sea is blocked and, in addition, makes it necessary to increase the head to increase the discharge. The discharge of the Mississippi River at Red River Landing is 35 000 cu ft per sec per ft of gauge. The increase of 6.9 ft in gauge since 1882 at Red River Landing for a discharge of 1 100 000 cu ft per sec means that the Mississippi River has lost 240 000 cu ft per sec in discharge capacity. At the same time, the discharge of the Atchafalaya River at Simmesport increased 129 000 cu ft per sec, due to this additional head of 6.9 ft. The combined loss in discharge capacity of 111 000 cu ft per sec results from unrestricted diversion.

The Mississippi River has conformed itself to the hydraulic theory and increased its slope as the volume has been diminished through diversion. Likewise, the Atchafalaya River has conformed itself to the hydraulic theory and flattened its slope as its volume increased through diversion. It is obvious that the vicious cycle of conditions set up through unrestricted diversion from the Mississippi into the Atchafalaya results in a large loss in "efficiency."

The treatment of the silt load by a silt-bearing stream is its powerful agency of adjustment to changed conditions. The detrimental effects resulting from this power to adjust itself to changed conditions is reflected clearly in the loss in discharge capacity of the Mississippi River at Red River Landing due to unrestricted diversion at all stages. By eliminating diversion below bank-full stage a major portion of this power of adjustment is overcome as it will disturb to a minor extent the greater portion of the silt load that is handled along the bed of the channel. Furthermore, the agency is provided for waters below bank-full stage, acting for 365 days to clean out any deposition of silt left after the short duration of over-bank diversion. This method of diversion will practically restore the channel to its original discharge capacity and so maintain it. The slight loss in cross-sectional area during overbank diversion at Pointe a la Hache and the fact that overbank diversion occurred at Arkansas City for a number of years did not reduce the discharge capacity of the river, substantiate the conclusion that overbank diversion will not cause the ill effects suffered from diversion at below bank-full stages.

DISCUSSION

Leo M. Odom,[15] Assoc. M. Am. Soc. C. E. (by letter).—Certain proofs and deductions are set forth in this paper, based on data and theory, which merit the consideration of students of the subject of the control of the Mississippi River and its tributaries. Among such students may be counted, to a greater or less extent, practically every engineer who resides in the Mississippi Valley.

Apparently, the author begins with the hypotheses: (1) That at points on the Mississippi River not affected by diversion operative at all stages, the same volume of water is now passing at the same elevation as in earlier years; (2) that the Mississippi River has conformed itself to "the" hydraulic theory and increased its slope as the volume has been diminished by diversion; and (3) that the Atchafalaya River has conformed itself to "the" hydraulic theory and has flattened its slope as its volume has been increased through diversion. The hydraulic theory from which the second and third premises were deduced is presented with discussion to show how it was interpreted to arrive at the hypotheses and to explain the causes underlying the action. Selected data from the records of the Mississippi River Commission for the years 1882 to 1932, inclusive, are presented and discussed. By consideration and interpretation of the data, the author concludes: (*a*) That the hypotheses are correct; (*b*) that the first is correct through simple consideration of the data presented; (*c*) that the data presented show that the Mississippi River has actually increased its slope and that the Atchafalaya River has actually decreased its slope during the period in which records were examined, as set forth in the second and third premises; and (*d*) from the fact that the first premise is true, and from consideration of the theory cited, that the cause of the noted phenomena in connection with the second and third premises is the fact that diversion is permitted from the Mississippi River into the Atchafalaya River at practically all stages.

Besides concluding that his hypotheses are proved, the author arrives at certain other conclusions and hypotheses. He deduces from data presented as to the Pointe a la Hache crevasse and study of the discharge measurements at Arkansas City, Ark., near which many crevasses have occurred, that overbank diversion does not cause the same action as "year-round diversion" and that if overbank diversion were substituted for the present continuous diversion through Old River, the main channel below that point would be practically restored to its original discharge capacity. He theorizes that the difference in effect between year-round diversion and the overbank diversion is due to the fact that (1) the greater portion of the silt load that is handled along the bed of the channel is accorded less disturbance with overbank diversion only; and (2) with diversion occurring only at stages above bank-full, an agency exists, acting for 365 days per yr, to clean out any deposition of silt left after the short duration of overbank diversion, which agency presumably does not exist in full force at points of year-round diversion.

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He concludes from the fact that the Mississippi River had not previously taken the shorter course to the sea offered by the Atchafalaya, that the normal action of the stream, determined by its silt load, caused the longer route to be followed and the shorter to be blocked. He deduces from comparative data that the total discharge of both rivers at equivalent gauge readings is less now than formerly, which he explains to be the result of the action set forth in his hypotheses.

The writer concludes from the remarks in the content of the paper that the first premise as stated in the "Synopsis" should also include cut-offs as well as the diversion referred to, as affecting the author's choice of gauge stations for analysis in this regard. Except for this fact, and the warning that the premise can hardly be supposed to be proved by consideration of so few stations in the total length of the river, no more will be said about this premise.

As to the last two premises, proof will be offered in this discussion that "the" hydraulic theory referred to therein is not at present generally accepted and that evidence is against its application to the Mississippi River. The possibility of other interpretation of the data presented will be advanced, and the probability of causes other than the fact of diversion for the bar below Red River Landing will be discussed.

The hydraulic theory referred to in the second and third premises is apparently the theorem of Guglielmini[2] cited under the heading, "Introduction." Commenting on this theorem, Humphreys and Abbot did not state as a fact the quotation attributed to them in the paper. The entire discussion in the "Report on Physics and Hydraulics of the Mississippi River", which contains the statement of Guglielmini's theorem and the aforementioned quotation is stated therein at the beginning of the paragraph to be an "extract from the writings of Maj. J. G. Barnard, Corps of Engineers, U. S. Army", and is given for the express purpose of refutation.

Commenting on this theorem and Major Barnard's remarks, Humphreys and Abbot state: "It will be noticed that two important assumptions are necessary to support this reasoning: First, that the bottom of the Mississippi River is composed of its own alluvion, which can be readily acted upon by the current; and, second, that its water is always charged to the maximum capacity allowed by its velocity."

The first assumption is then attacked on the grounds that the bed of the river is not composed of its own alluvion, but of a tough and stiff clay which they suppose to have had its origin in a more ancient geologic period than that of the river itself. This supposition as to the ancient geologic formation of this clay probably is incorrect, but it is still a fact that there is a very extensive clay stratum interspersed with shell and sand pockets throughout the entire valley from the ancient delta to the sea. This clay was more probably formed by the deposition at its mouth of the fines and colloids in the sediment carried by the river as it came into contact with the salt water of the ancient bay. Regardless of its origin this clay exists; it is quite compact and is not readily scoured, and forms the bed of both the Atchafalaya and the Mississippi Rivers. The existence of this difficultly erodible clay has been the cause of the failure of the Atchafalaya to cut itself an hydraulically efficient channel to the Gulf and has resulted in the spreading of that stream, past the confining influence of its levees, into numerous shallow branches (which in times of flood overflow over the entire basin), the frictional resistance of which system retards the velocity in the channel and prevents its discharge from increasing in the same ratio as if it, too, had a single channel to the sea.

The second assumption that Humphreys and Abbot state is necessary for the river action to conform to the hydraulic theory—that its water is always charged to the maximum capacity allowed by its velocity—is proved by them to be wrong by presentation of the analyses of sediment content of the river water at various velocities, which analyses were conducted in 1851-52 and 1858 at Carrollton, La., and Columbus, Ky. They state that, "at the date of highest water the river held in suspension little more sediment than at dead low water." They state[16], also, referring to tests made in 1851: "When the quantity of earthy matter held in suspension was greatest, the velocity was 2 feet per second less than the greatest velocity."

In a paper on "Permissible Canal Velocities" by the late Samuel Fortier, M. Am. Soc. C. E., and Fred C. Scobey, M. Am. Soc. C. E., which constitutes a final report of the Special Committee of the Society on Irrigation Hydraulics, the following statement[17] is made:

> "It is believed that there is a broad belt of velocities between the two 'critical' velocities, within which silt already loosened or brought in through a head-gate will remain in suspension while the bed nevertheless will remain undisturbed as regards scour. It is easy to show the absurdity of accepting the laws of silting as giving immediate answer to the laws of scour."

Any number of references and detailed sediment analyses disproving the theorem of Guglielmini could be cited from published documents of the U. S. Corps of Engineers, but it is believed that, with the complete citation of one of the author's own authorities and the foregoing additional reference, the writer has shown that part of the progress of the science of river hydraulics since the Seventeenth Century has consisted in the abandonment of the erroneous theorem.

The data presented are found to have been accurately copied from the published documents of the Mississippi River Commission. The choice of bank-full stage as the proving ground for the hypotheses and the "efficiency" assumption will be accepted although bank-full stage occurs too seldom in the 365 days to be sure that it is as logical a stage on which to base the proof as a lower one.

If the theory by which the author arrived at his hypotheses had been one generally accepted by the Engineering Profession and data had been presented for down-stream points which bore some relation to the data presented for Red River Landing, the writer would have been more inclined to have accepted the second hypotheses as borne out by the data. Without adequate theoretical basis, however, the statement that the Mississippi River has increased its slope as the volume has been diminished by diversion, rests entirely upon the gauge records at Red River Landing. At Carrollton—the only down-river point treated—the gauge height for bank-full discharge has not varied appreciably in the last 45 yr. As one outstanding example of a case wherein the data do not even show a continuous growth of bar at this point, the interpolated gauge height given for the chosen discharge at Red River Landing was 49.4 in 1903, whereas, in 1932, it was 49.0. The entire period of study was less than twice this interval. That a bar has formed and at times washed out below Red River Landing is evident and known to every one familiar with the river. The formation of this bar might as well be due to the angle of diversion or to the bend in the river, or both, as to the mere fact of diversion.

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[16] "Physics and Hydraulics of the Mississippi River", p. 674.

[17] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 941.

There are three points in the lower river where diversion occurs at all stages: Baptiste Collette's Bayou, 83 miles below New Orleans in the east bank; Cubit's Gap, 91 miles below New Orleans, in the east bank; and The Jump, 84 miles below New Orleans in the west bank. No records are kept on these streams ordinarily. Measurements during the flood of 1927 show a maximum discharge through Baptiste Collette of 13 689 sec-ft, through Cubit's Gap, of 134 288 sec-ft, and through The Jump, of 32 511 sec-ft. No shoaling of the main stream in the vicinity of these outlets has ever been noted.

At Head of Passes, 94½ miles below New Orleans, the river breaks up into three principal outlets: Pass a l'Outre, South Pass, and Southwest Pass. The average discharge through the passes in percentage of the total volume at Head of Passes is approximately: Pass a l'Outre, 42.5%; South Pass, 17.5% and Southwest Pass, 42 per cent. A point bar forms at the head of Southwest Pass and a tendency to scour is noted at the heads of the other two passes. Sills were placed across the head of Pass a l'Outre in 1876 and 1900 in an attempt to increase the flow through the other passes. The record of the action has been that the discharge through Pass a l'Outre was not reduced, but a turbulence was created which scoured the bottom of the channel to a depth of 122 ft between Pass a l'Outre and South Pass. A mat sill placed across the Head of South Pass in 1917 for the purpose of holding stationary the discharge in South Pass which had increased from 7% of the river's flow in 1891 to 14.7% in 1917, did not succeed in its purpose, but produced a hydraulic jump that scoured the bottom of the channel of the pass below the sill to a depth of 129 ft. The sill dams mentioned were similar in construction to that referred to by the author.

It is seen from the foregoing that the fact of diversion does not necessarily create bars in the main stream. The point bar which forms across the head of Southwest Pass is due to the direction of the thread of the current across Head of Passes and the fact that Southwest Pass takes off on a curve from the course of the river above this point.

It is also apparent that sill dams have not been found to be efficacious in the control of discharge. The percentage of discharge has been found to be regulated by the total resistance to flow offered by a channel and regulatory works offering resistance relatively minor in comparison to this total have little or no effect.

As to the third of the three premises, practically no proof is offered. Much is made of the fact that the average cross-sectional area of the stream increased 47% during the period embraced by the study. It is evident that practically all this large increase in average cross-sectional area is caused by the enlargement in the lower end due to extension of levees. At Reach 6, for instance, the enlargement has been only 4.17 per cent. The author is correct in stating that the capacity of the Atchafalaya has not been increased by its increased cross-section since this increased cross-section has not extended to a point where the stream can finally discharge freely. Increased cross-section will be of advantage only when and if levees are extended to Grand Lake and a pilot cut furnished between them, as the Atchafalaya probably never will be able to cut itself an efficient channel through the stiff clay of the basin. Its enlargement at its source, due to scour, when it acts as the outlet for the relatively clear back-water area of the upper basin, is also useless as far as getting any water out the other end is concerned.

The author concludes that the fact that the Mississippi has never taken the shorter course to the sea offered by the Atchafalaya proves that the longer route is the one determined by Nature in its handling of the silt load. Another hypothesis is offered herein taking into account observed action at the Passes, a study of the course of the Mississippi River below Red River Landing, and the conclusions of geologists.

Geologists state that at one time the Mississippi River discharged into an estuary which reached farther north than Red River Landing and that in time it has completely filled this estuary and extended its delta out into the Gulf. The material of which this fill is formed, to approximately mean Gulf level, is preponderantly a stiff silty clay interspersed with sand and shell pockets with apparent remains of ancient forests found at different levels below mean low Gulf level. The toughness of this clay is attributable to the flocculation of the colloids when the silt-laden waters came in contact with the Gulf. The remains of forests give rise to the assumption that the old fill has settled and compacted and that new fill has formed on top of it in a manner similar to that occurring at the mouths of the river to-day.

As the river built itself out of its estuary, it became more and more acted upon by the littoral currents and littoral drift and the prevailing winds. The effect of these agencies is quite apparent at the mouth of South Pass. There, the sea channel at the entrance makes an angle of 36° with the axis of the Pass, and the sediment of the river is piled up on the west bank of the channel. The river does not flow into this sea channel, which is kept clear by the littoral current, but spreads out in a fan-shaped area on top of the salt water. When the bar becomes so extensive to the west that the friction is greater than the force it would take to move the salt water out of the channel, the river water will penetrate farther into the channel and thus its bed is continued on out to sea.

From Baton Rouge, which is near the limit of the ancient estuary, to the Gulf, the easterly trend is particularly to be noted, as past this point the mouth of the river was subject to the full action of the winds and Gulf agencies. From Red River Landing to Baton Rouge it will be noted that the river is near the eastern limit of the estuary with the main delta to the west.

It is believed that this hypothesis offers a better explanation for the present location of the main river than that based on the theorem of Guglielmini offered by the author; especially since it is evident that it has not always been 310 miles from Red River Landing to the Gulf by way of the Mississippi River.

The land structure now existing between the head of the Atchafalaya River and the Gulf creates an entirely different condition than that which existed when the mouth of the Mississippi was in the vicinity of Red River Landing. The sea agencies and prevailing winds which closed off the westerly outlets at that time would now affect only the mouth of the Atchafalaya.

As to the conclusion of the author that overbank diversion offers a solution to the evils attributed to continuous diversion, it is remarkable that he should have chosen as his basis for argument against continuous diversion the same theory advanced generations ago against overbank diversion.

E. W. Lane,[18] M. Am. Soc. C. E. (by letter).—Briefly stated the author's contentions are:

(1) That the discharge capacity of the Mississippi River below Red River Landing has deteriorated considerably during the past fifty odd years.

(2) That this deterioration has resulted from a deposit of silt in this stretch which has reduced the channel area.

(3) That no corresponding deterioration has occurred in the channel capacity above Red River Landing.

(4) That the deposit below Red River Landing has occurred due to the diversion of water from the Mississippi at all stages through Old River, in conformity with the hypothesis of Guglielmini.

(5) That the slope of the Atchafalaya River has decreased in agreement with this same hypothesis.

The writer's discussion will cover these five points in order.

If, as the author contends, a reduction has occurred in the discharge capacity of the Mississippi River below Red River Landing in the 50 yr following 1882, of about 7 ft for a discharge of 1 100 000 sec-ft (which is equivalent to a reduction in discharge of about 250 000 cu ft per sec), it is a matter of major importance in connection with the flood control of the Mississippi River. Should this continue in the future, continuous construction will be necessary to prevent a progressive decrease in the degree of flood protection given by any system of works which does not remove the cause.

The method used by the author to prove this contention is sound, although so far as the writer knows it has not heretofore been used extensively, if at all.

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[18] Prof. of Hydr. Eng., State Univ. of Iowa, Iowa City, Iowa.

In order to check his results the writer has investigated the effect in a similar manner for the same discharge, and for discharges other than those used by the author. All current meter measurements, available to the writer, between 1 150 000 and 750 000 cu ft per sec have been used. The stages for each discharge within 50 000 of the even 100 000 cu ft per sec were adjusted to the nearest even 100 000 by using the discharge increment corresponding to that discharge, in a manner similar to that used by the author. All measurements that might have been affected by crevasses have been eliminated, except in 1882. Since the stages used in this year are so little, if any, above the bank level, the effect of crevasses on discharge would be negligible.

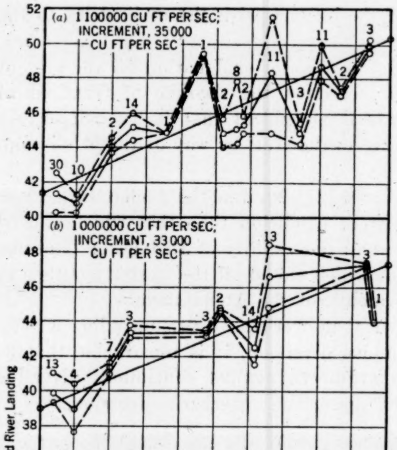

FIG. 3.

The results of these studies are shown on Fig. 3 in which the adjusted stages are plotted against their respective years for each of the four discharges. The solid line joins the points indicating the mean of the adjusted stages in each year. The lower dotted line joins points showing the lowest adjusted stage and the upper dotted line joins the points giving the highest adjusted stage. Above each set of points is given the number of observations on which the points were based. For each discharge a mean line has been drawn showing the general trend.

In each case the trend is unmistakably upward, the slope magnitude increasing somewhat with the discharge. This increase of stage for a given discharge is also indicated by plotting the discharge measurements near Red River Landing against stage, which is just another way of presenting the same data. A similar indication may be noticed in the Carrollton gaugings.

In Fig. 4 are plotted discharge rating curves for the years 1880 and 1930—based upon points taken from the mean lines of Fig. 3 at those years—for the four discharges investigated. These curves have been extended by dotted lines to zero discharge at Gulf level (Elevation − 3.57 on the gauge) to indicate approximately the difference at other discharges. A comparison of these rating curves indicates very strongly that a reduction of the discharge capacity has occurred. The curves indicate a decrease of discharge capacity in 50 yr, at a 40-ft stage, of about 250 000 cu ft per sec.

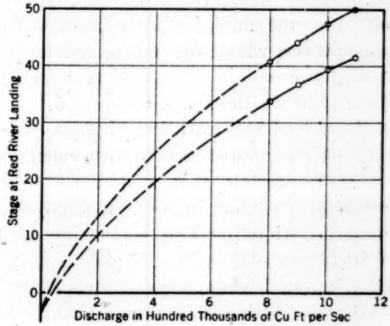

FIG. 4.—DISCHARGE RATING CURVES FOR 1880 TO 1930.

The evidence presented by the author that the maximum stages reached during the years since 1867 were tending to increase also is evidence pointing to deterioration, but is not so definite as the evidence based upon discharge, since the maximum stage reached in high-water years depended upon the strength of the levees, as the author states under "Conclusions." However, since the great flood years are few in number, the effect of the increasing strength of levees would probably be small. The value of 7.3 ft found by this method is not exactly comparable to the 6.9 ft found by the hydraulic method, since, assuming a uniform rate of deterioration, the 7.3-ft rise would occur between the middle of the first and the middle of the last period for which the stages were averaged, or 56 yr, whereas the period covered by the discharge method was 50 yr. The rise for 50 yr instead of 56 yr, assuming a uniform variation, would be 6.5 ft.

The author's second contention is that the deterioration of channel capacity below Red River Landing is due to a reduction of the channel area caused by a deposit of silt. A general form of equation for the discharge capacity of a river is:

$$Q = C\,A\,R^m\,S^n \qquad (1)$$

A reduction in capacity can result from a decrease in one or more of the elements, C, A, R, or S. For the Mississippi River below Red River Landing it can scarcely be due to a decrease in the slope, S, since the river discharges at a constant elevation into the Gulf of Mexico, and, for a given stage at Red River Landing, the average slope through this section of the river must be constant, both the fall and the length remaining the same. Small local or temporary changes of slope occur, but these changes tend to re-adjust themselves since the average slope cannot change. It seems improbable that a change in capacity could have resulted from a change in the value of C, the roughness factor. Except for the negligible increase in roughness, which no doubt occurred as a result of the construction of wharves and other works of Man, no reason is apparent to cause a permanent change of roughness in this section of the river, and so far as known, no one has ever suggested that one has taken place. Temporary changes of roughness, described by the author as changes of "efficiency" however, do occur. The writer believes that these changes are to a greater extent due to changes in roughness than to deposits of silt, since the silt load of the river is insufficient to cause enough deposit to produce the decreases in "efficiency" as rapidly as they occur. It is probable that the change of roughness is due to the formation and obliteration of sand waves, which can take place in a short time. These changes of roughness, however, are temporary and tend to equalize over a period of years. A large part of the space between the upper and lower dotted lines in Fig. 3, and some of the variation of the average line, is no doubt due to this cause, although inaccuracies of measurement also form a part of this difference.

It is possible that a decrease in discharge capacity could occur as a result of a change in *R* without any change in *A*. This could result from a material widening and shallowing of the river without a change in area. With levees close to the banks, as they are in this stretch of the river, however, no such change could escape notice, as it would endanger many miles of levee. Although a minor change in width may have occurred, since no general widening has taken place, it seems safe to conclude that no appreciable deterioration has resulted from a change of the hydraulic radius except as it may have occurred as a result of a decrease in the channel area. Since the other possibilities seem to be eliminated, it is very probable that any deterioration that has occurred in the channel capacity below Red River Landing has been caused by the deposit of silt in the river bed, causing a decrease in the area, *A*, and a corresponding decrease in the hydraulic radius, *R*.

The stage-discharge curves of Fig. 4 indicate that the increase in stage is different for each discharge. This shows that the amount of the filling of the bed cannot be determined by taking the difference in the stages at the ends of the period for any given discharge since the result obtained would depend upon the discharge selected. It should be possible, from the dimensions of the channel, and the slope at the beginning of the period, to compute the amount of the filling (assuming it to be distributed in proportion to the distance from the Gulf) which would cause the deterioration indicated, but it cannot be determined from the stage differences alone.

The third contention—that no material deterioration has occurred above Old River—seems well established, since it is supported not only by the discharge measurements but also by the measurement of great numbers of cross-sections, which show a small increase in area. Because of the variation that may occur at any one cross-section in the river (due to the passage of a sand wave or from a number of other causes) reasoning regarding the action of the river bed based upon a few measurements at one cross-section, or in a stretch of river, while useful as evidence, is not very conclusive. For this reason too much weight should not be given the data presented by the author showing the loss of cross-section at Red River Landing during the high waters of 1929 and 1932; nor to the measurements of the areas at the two ends of the Pointe a la Hache diversion. However, when arguments are based upon hundreds of cross-section measurements scattered over a stretch of river and, to a lesser extent, on many measurements at a given locality spread over a long period of time, they should carry considerable weight.

The fourth contention (that the deposit below Old River was caused by the diversion of water from the Mississippi at all stages through Old River, in conformity with the hypothesis of Guglielmini), is likely to provoke considerable discussion. Probably no engineering controversy has continued so long and resulted so inconclusively as that dealing with the Guglielmini hypothesis and its application to the Mississippi River. As early as 1816 this hypothesis was used as an argument against spillways for flood control. It was strenuously combatted by Humphreys and Abbot, as a result of their studies, and strongly supported by Eads, who applied it with great success to the deepening of the South Pass at the mouth of the Mississippi by jetties in the years 1875 to 1879. Largely as a result of this success, the hypothesis was adopted by the Mississippi River Commission as a basic principle; but as a practical means of flood control it proved nearly a complete failure. Nevertheless, it exerted a strong influence on the policy of the Mississippi River Commission until the 1927 flood and was a considerable obstacle in the path of flood control by spillways until it was forced into the background by the strength of the sentiment for spillways which developed as a result of that flood. With the author's paper it has come to the fore again. It is greatly to be hoped that the truth or fallacy and the bearing of it may be established, as the uncertainty has been a serious obstacle in the way of establishing a sound flood-control policy for this great river.

Personally, the writer is now inclined to accept the Guglielmini hypothesis, although at one time he was very skeptical of it because of the difficulty (which has since been removed) of reconciling the apparently conflicting evidence when applying it to the Mississippi River. Of course, the wording of the Seventeenth Century does not fit in with the modern more exact conceptions, but it should be remembered that no such accurate conceptions existed at that time. Moreover, part of the inexact expression may have been caused by inapt translation. Perhaps the greatest argument against the hypothesis, as far as the Mississippi River is concerned, is its apparent failure when applied by the Mississippi River Commission to flood protection. By plotting a mean relation of the silt load carried by the Mississippi and applying this relation to the discharge of past floods as they would have occurred had no levees been built, and again to these same floods under conditions of confinement actually produced by levees, it can be shown that the small enlargement which has taken place in the river is all that the Guglielmini hypothesis would indicate.

The error of the members of the original Mississippi River Commission, therefore, was not in believing in the Guglielmini hypothesis, but in not using the data available to them and applying the hypothesis quantitatively to their problem. It is believed that an application of the foregoing method to the conditions in the relatively short stretch at the mouth of the river covered by jetties (where the action went on for 365 days per yr) would show that it should succeed in that case when it failed in the case of flood control. A similar application to the reduction of flow in the Mississippi below Red River Landing, due to the greater discharge down the Atchafalaya, would throw valuable light on the action of the river in that stretch.

Another argument against the Guglielmini hypothesis is that used by Humphreys and Abbot based on the fact that they found no exact relation between the discharge and the silt load of the river. Recent studies[19] have shown why this is not necessarily a disproof of the theory. The writer believes that when applied quantitatively, with due regard to the time element involved, the Guglielmini hypothesis will be found to be a sound general relation as far as it goes, although of course many other factors may enter to prevent its working out in special cases. The hypothesis is incomplete, in that it does not consider the variation of the solids load as a factor, and to make accurate prediction this factor also must be added.

Independent, however, of the soundness of the Guglielmini hypothesis, if silting is progressing below Red River Landing and not above that point, it is logical to expect the cause of this silting to be something operating below that point and not above it. One of the strong arguments in favor of the applicability of the Guglielmini hypothesis to this case would seem to be the difficulty of explaining the apparent decrease of capacity below Old River without utilizing it.

Diversion through Old River, however, is not the only phenomenon that might cause the deposition below that point, according to the hypothesis of Guglielmini. Before the removal of the raft from the Atchafalaya River, probably the greater part of the flow of the Red River reached the Mississippi; now only a very small part of it reaches that river. This diversion to the Atchafalaya of the greater part of the flow of the Red River may have a greater effect on the Mississippi channel below Red River Landing than the flow that passes out of the Mississippi through Old River. Judging from the size of the trees said to be growing around these rafts and the damage that followed the removal of the raft, it is reasonable to suppose that the flow down the Atchafalaya had been small for many years, and, therefore, that the greater part of the Red River flow had passed down the Mississippi. In a long period of time the Mississippi would have adjusted itself to this flow and when it was practically discontinued as a result of the enlargement of the Atchafalaya, the effect would be much the same as if that quantity of water had been abstracted from the Mississippi through Old River. In considering the action of the Guglielmini hypothesis on the Mississippi below Red River Landing, therefore, it is necessary to consider not only the actual diversion of water from the river through Old River, but also the diversion of that part of the Red River flow which formerly entered the Mississippi, but now passes down the Atchafalaya. In this problem the character of the solids load of the Red River is a factor and would have to be considered to arrive at accurate quantitative results.

A silting up of the river might take place during a series of dry years, but this would be expected to occur along the length of the stream not just in one part. However, the bottom of the river in the section below Old

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[19] *Engineering News-Record*, Vol. 115, October 17, 1935, p. 538.

River is considerably below sea level and the flow section there would not decrease with decreased flow as rapidly as in sections of the river farther up stream. This would give rise to relatively lower velocities below Old River in low-flow periods, and hence more tendency to silt. Whatever the cause of the decreasing capacity, it seems to have been acting fairly continuously throughout the past fifty years, and if it is due to a long-continued drought it would be necessary to have longer rainfall records than are available to prove it. Some light might be cast on this question by tree-ring studies if they covered the entire drainage area of the river.

The author's proof of his fifth contention, that the slope of the Atchafalaya River has decreased in agreement with the Guglielmini hypothesis, is not very complete. He has shown that the Atchafalaya has enlarged materially in cross-section, that the bank elevations along the stream have become higher, and he has given data tending to prove that a discharge of 300 000 cu ft per sec occurred at substantially the same stage from 1890 to date. Since the area was increased and the discharge remained the same, the velocity has been reduced, and he concluded, therefore, that the slope had been reduced enough to cause this decreased velocity. As previously stated, the general discharge relation is $Q = C\ A\ R^m\ S^n$. If Q remains the same, the increase in area (and presumably the consequent increase in R) can be offset by a decrease in either, or both, C or S. Although it seems improbable that the value of C has changed in the Mississippi it is by no means improbable in a river scouring as actively as the Atchafalaya was during the period under consideration. The irregular cutting of the banks and bed which occurred under such circumstances might easily cause an increase in roughness. The great turbulence of the river is very noticeable at high flows, and the computed roughness factors show considerably higher values than an ordinary river channel.

The upper section of the Atchafalaya River has a comparatively steep slope and the lower section, which is composed largely of shallow lakes, has a very flat slope. The material scoured from the upper end as the river enlarged, and a considerable part of the silt load brought into the upper end of the river, has been deposited in the upper end of the shallow lakes and has formed a delta. This condition has caused a rise in the water level at this point and, to this extent, a decrease in the slope of the Atchafalaya. The rise in level has been strikingly shown by the tendency of the water about 1926 to flow to the Mississippi through the Plaquemine Lock, whereas, formerly, it always tended to flow in the opposite direction. It is probable, therefore, that there is some decrease in slope of the upper end of the Atchafalaya, due to its tendency to form a delta at the head of the lakes; but it seems doubtful whether this would be enough to offset the increase in both area and hydraulic radius which occurred as a result of the enlargement of the river bed. The late John Augustus Ockerson, Past-President, Am. Soc. C. E., has shown[30] that the enlargement of the river was due largely to the concentration of velocities that occurred as a result of the steep slopes where the water spread out at the end of the leveed section of the river, and the progressive moving down of this high-velocity section as the levees were extended. It is probable that the decrease of slope which offsets the increased area is due largely to the extension of the confinement of the river by levees, which occurred during the period of river enlargement. Immediately preceding Table 11 the author states that the increase in gauge height at Melville between 1892 and 1907 is due to levee confinement. An increase of stage at Melville, the level at the upper end of the river remaining the same, would result in a flatter slope.

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[30] *Transactions*, Am. Soc. C. E., Vol. LVIII (1907), p. 1.

On the whole the author has presented a strong argument to support the contention that the discharge capacity of the Mississippi below Old River is decreasing. As a result of his studies of Mississippi River hydraulics, however, the writer has learned that different, apparently equally good, lines of proof sometimes lead to directly opposite conclusions and, therefore, every available method should be used before a final conclusion is formed. It would seem that the best way to prove or to disprove a contention that the stretch of the Mississippi below Old River had been filling would be to make another cross-section survey similar to that made from 1893 to 1898. The writer could find no published record of such a survey since that date. This method and any other possible method of indicating the deterioration, should be investigated so that its existence can be established or disproved as conclusively as possible. Other possible investigations that might throw light on this subject are a study of the frequency of the various stages below Old River throughout the period of record and a comparison of the area below a certain datum in the cross-sections shown by the discharge measurements at Red River Landing. A comparison of stage relations below Red River Landing, however, would probably not indicate the deterioration as the stages at all points are likely to be changed proportionally. In this section of the river, also, a comparison of low-water stages would not indicate deterioration because of the great depth of the water, even at the lowest flows.

Assuming that the deterioration is proved conclusively, it would seem that serious thought should be given to Mr. Eads' proposal to place a dam across the upper end of the Atchafalaya and force the ordinary flow of the Red River down the Mississippi. The dam should be provided with gates which could be opened during floods to make available the discharge capacity of the river. A lock would have to be provided at this dam, and perhaps another lock and dam farther down stream would be required to take care of navigation.

If the deterioration is due to the working out of the Guglielmini hypothesis, any increase in the diversion down the Atchafalaya River at low or medium flows would increase the rate of deterioration and thus offset a part, or all, of the benefit from the increased diversion. Of course, it would be possible to increase the Atchafalaya capacity fast enough to keep ahead of the deterioration, but in the present state of the knowledge of such action it would be a race with an uncertain outcome.

The Guglielmini hypothesis has been used so often as an argument against flood control by spillways that an argument for the hypothesis might erroneously be considered to be an argument against spillways. The writer believes that much of the argument against flood-relief spillways based on the Guglielmini hypothesis has not considered the entire situation. When the first flood-control works were built on the Mississippi, the river had carved for itself a channel of a size which was a result of the action of a great variety of conditions in the past decades and probably did not change materially over a period of years. The construction of levees tended to confine the water and has caused an enlargement of the river channel. This enlargement is so much smaller than the proponents of the confinement theory expected that it has not been seriously considered, but it is all that the theory indicates should take place. A flood-relief spillway will cause some deposit in the river down stream from it, but here, again, the amount will probably be much smaller than the opponents of spillways anticipate. Since these spillways will operate only for short periods with considerable intervals between (during which time the channel will often carry increased flows due to the confinement by the levees of all the floods which do not bring the spillways into action) it is probable that the increased scouring caused by the levees will take care of the deposit caused by the spillway. Of course, the river below the spillway will not enlarge as rapidly as it would if no spillway was present, and the flow continued to be confined between levees; nor would it enlarge as fast as the river up stream from it, but there would probably not be a decrease in section. That the result would be a net enlargement is indicated by the increase in section area that has occurred in the river above Red River Landing in spite of the frequent occurrences of crevasses (which have the same effect as spillways).

The three effects, enlargement due to confinement by levees, deposit below spillways due to diversion through them, and deposit due to constant diversion or subtraction of tributaries, can be considered to act largely independently of each other. Whether the river fills or enlarges depends on the relative magnitude of the effects. Above Old River the enlargement due to confinement has been greater than the deposit resulting from crevasses. Below Old River the effect of the diversion and subtraction of tributaries, plus the deposits caused by crevasses, seems to have been greater than the enlargement due to confinement.

At present, it is not possible to give quantitative answers to what the effects of these various changes being made on the river will be, but with the rapid advance of knowledge of solids transportation in the last few years, and with the collection of a reasonable amount of data on the characteristics of the bed and suspended load of the Mississippi and its principal tributaries, it should soon be possible to make definite quantitative predictions based on adequate data and sound reasoning.

E. F. SALISBURY,[21] M. AM. SOC. C. E. (by letter).—The frank criticism and constructive comments in the discussions are appreciated. The discussion by Professor Lane assists materially in clarifying the paper and the writer is practically in complete agreement with his views.

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[21] Chf. Engr., L. & A. R. R., Minden, La.

Professor Lane points out that, prior to the removal of the rafts from the Atchafalaya River, probably the greater portion of the flow of the Red River reached the Mississippi River and that now (1936) only a very small part of it reaches the main river. This diversion to the Atchafalaya of a greater part of the flow of the Red River may have much more effect on the channel below Red River Landing than the flow that passes out of the Mississippi through Old River.

The writer is not completely in accord with this view. If, during the period of this analysis, increasing volumes of water from the Red River were being diverted through the Atchafalaya, a resulting detrimental effect on the main channel is not questioned. However, although the Atchafalaya River has greatly increased in cross-sectional area, during the period of this study, it has not increased materially in discharge capacity; therefore, the conditions controlling the draw from the Red River have not changed during the time under consideration. The removal of the rafts in the Atchafalaya began about 1840 and was completed in the early Sixties; so there were from 20 to 40 yr for any channel adjustment before the earliest date used in the paper if the removal of the rafts would have the effect of drawing off more of the Red River discharge through the Atchafalaya than formerly. Consequently, it is believed that deterioration in the main channel below Red River Landing is due solely to diversion of water from the Mississippi through Old River.

In commenting on the slope of the Atchafalaya River, Professor Lane, states in part:

> "Although it seems improbable that the value of C has changed in the Mississippi, it is by no means improbable in a river scouring as actively as the Atchafalaya was during the period under consideration. The irregular cutting of the banks and bed which occurred under such circumstances might easily cause an increase in roughness. The great turbulence of the river is very noticeable at high flows and the computed roughness factors show considerably higher values than an ordinary river channel."

For some time the writer has been of the opinion that when the silt load of a heavily laden stream is subjected to a major disturbance, causing either a considerable increase or a considerable loss in the silt load, such as that produced by a diversion of the flow, a material increase in roughness factor is immediately produced over that customarily used for ordinary river channels. Definite proof as to this thought is not available at this time. The observations of the effects at the cut-offs being made in the Mississippi River may produce valuable data in this direction.

Mr. Odom is correct that the reference to the Guglielmini hydraulic theory as mentioned in the Humphreys and Abbot report on the "Physics and Hydraulics of the Mississippi River" is an "extract from the writings of Maj. J. G. Barnard, Corps of Engineers, U. S. Army." The writer is indebted to Mr. Odom for clearing up this reference.

In preparing the paper it was the writer's intention to interpret correctly that which the Mississippi River is recording in so far as the data apply to the subject treated. To the writer's knowledge no data have been intentionally selected or omitted. With this premise the data presented cannot be considered as selected as was intimated by Mr. Odom. On the basis of interpreting that which the river has recorded, it is premature to conclude that the "Synopsis" should include cut-offs as well as diversion. The further recordings of the river will be the answer to this separate problem. The reasons advanced by Mr. Odom as to the existence of a compact, and not readily scoured, clay in the bed of the Atchafalaya and Mississippi Rivers, are generally accepted. Furthermore, they apply to other silt-bearing streams and canals under certain conditions.

Below Red River Landing, the Carrollton gauge is the only recorded gauge and discharge study that provides data for analysis. Mr. Odom's statement that "at Carrollton the gauge height for bank-full discharge has not varied appreciably in the last 45 yr", depends upon the limitations as defined by the word, "appreciably." That the gauge has increased for the same volume of water is apparent from the gauge study. The increase in gauge for the same volume of water at Carrollton could not be expected to be of the same magnitude as that at Red River Landing as it is about 191 miles down stream on the slope. Furthermore, it could be expected that the gauge increase at Baton Rouge for the same volume of water will be less than at Red River Landing as it is about 60 miles below, and yet greater than at New Orleans, which is 124 miles farther down stream. Discharge observations are not taken in the Mississippi River at Baton Rouge. To study the action of the river at this point, gauge heights were listed for the day prior to the discharge of 1 100 000 cu ft per sec at the Carrollton gauge station. Table 13 shows the result of the gauge study at Baton Rouge, together with the studies at Red River Landing and Carrollton. It demonstrates clearly, the increase in gauge for discharging the same volume of water at three points on the Mississippi River below the diversion through Old River.

TABLE 13.—COMPARATIVE GAUGE STUDIES

Location	Period	Years	Average gauge, in feet, for a discharge of 1 100 000 cu ft per sec.	Gauge increase, in feet
Red River Landing	1	1882–1885	41.3	...
	2	1890–1909	45.0	3.7
	3	1913–1932	48.2	3.2
Total	.....	....	....	6.9
Baton Rouge	1	1883–1891	34.0	...
	2	1892–1912	36.0	2.0
	3	1913–1932	38.3	2.3
Total	.....	....	....	4.3
Carrollton (New Orleans)	1	1883–1891	15.0	...
	2	1892–1912	16.6	1.6
	3	1913–1932	17.5	0.9
Total	.....	....	....	2.5

Prior to the flood of 1922 the largest reliable discharge recorded as passing New Orleans was 1 351 000 cu ft per sec, in 1897, at a gauge of 18.6 ft. The flood of 1922 produced a gauge of 21.2 ft for a smaller discharge of 1 281 000 cu ft per sec. During the flood of 1890, a volume of 1 292 000 cu ft per sec passed on the gauge at 16.0 ft, whereas, in 1922, a volume slightly less passed at a gauge of 21.2 ft.

The volume of water diverted into the Atchafalaya is dependent upon the flood height in the Mississippi River. The higher the gauge in the Mississippi at Red River Landing the larger will be the volume of diversion, and the greater the deposition of silt below the point of diversion. The growth or change in the bar below Old River, therefore, is regulated by flood heights and will increase or decrease in extent with the increase or decrease of the average of peak-flood heights. The average peak-flood heights for the period, 1890 to 1907, is 48.7 ft. The flood of 1903, which falls within this group, had a flood peak of 50.1 ft. at Red River Landing, the second highest flood of record to this time. It would be expected, therefore, that the gauge for discharging the same volume of water during this peak-flood year would be reflected by a high gauge.

The locations of diversion as mentioned by Mr. Odom where shoaling in the Mississippi River is not noticeable, are immediately adjacent to the Gulf and below the leveed section of the river, and are not comparable with the effects noted at Red River Landing where the river is confined by levees and subjected to a variation in water elevation of about 50 ft. Baptiste Collette's Bayou is immediately below the end of the East Levee line of the Mississippi; the Jump at the end of the West Levee; Cubit's Gap and Head of Passes are below the end of the levee system about 7 and 10 miles, respectively. The greater part of the variation in water levels at these points is due to Gulf tides and wind. The river channel has adjusted itself to this condition. On the other hand, if the maximum variation was due to flood heights and if flood heights were increased, causing consequent increase in diversion, shoals would occur. In 1922 the elevation of high water above Mean Gulf Level at these points, were[22]:

Baptiste Collette	4.3
The Jump	4.1
Cubit's Gap	2.0
Head of Passes	2.0

The data presented for the Atchafalaya, which are recorded by the river itself in substantiation of the hydraulic theory, are that since 1890 the same volume of water has been discharging at the same gauge heights as it was in 1933, although the leveed section of the river, including about 17 miles beyond, shows large increases in cross-sectional area. This is further substantiated by the fact that the discharges are the same for the period, 1897 to 1932, for a high gauge of 50 ft at Red River Landing. The velocity during this period shows a slight decrease. Mr. Odom considers that these data are practically no proof and substitutes therefor his opinion, that, when the stream is permitted to discharge freely into Grand Lake by the extension of the levees and the cutting of a pilot channel between them, the full advantage of the increased cross-section will be available. Evidently, he does not consider the inefficient channels below the end of the levee system as being due to the silt load deposited by the Atchafalaya River, and that, after this immediate section of hydraulically inefficient channels is eliminated by the work he suggests, no more will be formed beyond the levee extensions as proposed. If the river's power of adjustment in the handling of its silt load is changed by some method of disposal foreign to the river's treatment, it may create an entirely different condition.

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[22] Data furnished by Louisiana State Board of Engrs.

The Atchafalaya is similar to other silt-bearing streams in having numerous hydraulically inefficient channels beyond the end of the levee system. With the drop in velocity, silt is deposited at the levee end, filling in and building up the area below. Due to the presence of colloids, this freshly deposited silt, consolidates and hardens, and requires higher velocities to dislodge it again than it did to keep it moving. This process of land building below the levee end is the foundation utilized by the Louisiana State Board of Engineers in the levee extensions on the Atchafalaya. The levees have been extended in progression with the forming of this foundation by the deposition of silt. The confinement of waters between the extended levees produces sufficient velocity to pick up the consolidated clays again, thereby materially increasing the cross-sectional area of the stream, and transferring the land-building process down stream. If the past is any criterion, that which is now the portion of Grand Lake will, under Mr. Odom's plan, be the location of the numerous hydraulically inefficient shallow branches when the levees have been extended a sufficient distance and the problem of obtaining a single hydraulically efficient channel will still be present.

The greatest increases in cross-sectional areas of this river are in the lower reaches and are due to levee confinement, as stated by Mr. Odom. In the upper 28 miles of the leveed portion (Reach 0 to 5) the cross-sectional area in 1932 ranged from 70 000 to 79 000 sq ft, being an increase in area from 26 to 35% over that in 1904. In the next following 25 miles (Reach 6 to 10) the cross-sectional area in 1932 ranged from 55 000 to 59 000 sq ft, being an increase in area over that in 1904 from 4 to 208 per cent. The lower 16 miles (Reach 11 to 13) is below the levee system and, in 1932, the area for Reach 11 was 45 000 sq ft, diminishing to 24 000 sq ft for Reach 13, which indicates an area increase over 1904 of 201% and 56%, respectively. Cross-sectional areas are not available prior to 1904–05. Greater increases in area, than those listed, would occur for the period, 1890 to 1932, the period of gauge study. By 1896, the levee system ended within the limits of Reach 6 and was extended into the limits of Reach 8 prior to the cross-sectional survey of 1904–05. Because of this levee extension (which was made prior to the survey of 1904–05), Reach 6 would show a considerably larger increase in area for the prior period over that of 4% for the period since the survey of 1904–05. Therefore, it is logical to consider that the cross-sectional area of Reach 6 was greatly increased during the time of this gauge study (1890–1932) over that of the 4% recorded since 1904–05. This small increase of area for Reach 6, as pointed out by Mr. Odom, is not the total increase in cross-sectional area for the period of this study, and, therefore, does not disturb the conclusions reached as to the adjustment of the river.

It is not the writer's intention to deal with causes of the formation of the present channel of the Mississippi River, but the theory advanced by Mr. Odom on this feature is interesting. The paper deals with the condition of the existing rivers and offers the reason why the Mississippi River is blocked from diverting its flow from its present channel through the Atchafalaya River even if the Atchafalaya is the shortest distance with steeper slopes to the Gulf.

Although the views expressed by Mr. Odom are appreciated, it is thought he did not produce data and conclusions warranting a change in the original deductions.

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STABLE CHANNELS IN ERODIBLE MATERIAL

By E. W. Lane,[1] M. Am. Soc. C. E.

With Discussion by Messrs. R. C. Johnson, E. S. Lindley, J. C. Stevens, C. R. Pettis, Harry F. Blaney, Sigurd Eliassen, R. E. Ballester, Gerald Lacey, V. V. Tchikoff, W. M. Griffith, and E. W. Lane.

Synopsis

The design of the All-American Canal, which will divert 15 000 cu ft per sec from the Colorado River, required a thorough study of stable channel shapes. Data from various sources were conflicting and unsuitable for the unusual conditions on this canal. These data were analyzed and conclusions were drawn regarding the various factors controlling stable channel shapes, and the relation between them.

Introduction

For a canal to be stable the banks must not slough or slide, and the bottom and sides must neither silt nor scour. To establish these conditions the engineer must consider a number of factors.

The problem of controlling sloughing or sliding is not treated in detail in this paper, since stable slopes for the various soils are comparatively well known. To prevent scouring or the accumulation of silt in the bed, it is necessary that the velocity along the bed is sufficient to move all the material brought into the canal, and yet not be so high as to cause the subgrade of the canal to scour. Flowing water will not attack the subgrade unless its velocity is more than sufficient to move the material brought into the canal. The excess of velocity over that required to move this material, which will attack the canal subgrade, depends upon the material of which the subgrade is composed.

In order that the banks may neither silt nor scour, the velocities along them must be sufficient to prevent deposition but not sufficient to cause the material of which they are composed to scour. From a practical standpoint

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Note.—Published in November, 1935, *Proceedings*.

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a slight degree of silting on the sides is not especially detrimental, so that the important requirement is the prevention of scour and excessive deposition. The maximum allowable velocity along the banks depends upon the material of which they are composed. The material on the sides is also acted upon by the force of gravity, which assists the water in tending to cause motion. The sides, therefore, will scour under velocities less than would be permissible along the bottom.

The ratio between the velocity acting on the sides and that on the bottom depends upon the ratio of the bed width of the canal to the depth. The greater this latter ratio the greater will be the ratio of the velocity acting on the bottom to that acting on the sides. The bed width-depth ratio required for a stable channel is that which will bring the proper ratio of velocity acting on the bottom to that acting on the sides. Conditions that require high velocities acting on the bottom, as compared with those that may be permitted to act on the sides, require high ratios of bed width to depth. For example, canals that carry a heavy bed load in friable material require high velocities on the bed to move the load and low velocities along the banks to prevent cutting them; in other words, such channels require a high ratio of bottom velocity to side velocity, and, therefore, a high bed width-depth ratio. Canals with small bed loads in friable material do not require such high velocities along the bottom to transport the bed load and, therefore, the ratio of this lower velocity, V_B, to the permissible side velocity, V_S, can be smaller. The correct ratios for other conditions can be determined by the application of these principles. This paper records an attempt to outline the major principles that control stable channel shapes and velocities.

The All-American Canal

The All-American Canal is planned to take water from the Lower Colorado River and carry it to the lands lying in the Imperial and Coachella Valleys by a route lying entirely within the United States. The Imperial Valley is now (1935) irrigated from the Colorado River by a publicly owned canal system, of which a large part of the main canal lies within the Republic of Mexico. The difficulties of international administration, and the undesirable silt conditions connected with the existing canal has led to the instigation by the United States Bureau of Reclamation of the new All-American Canal project to be built entirely within the United States, which has been approved by Congress.

The Colorado River is a very silt-laden stream. It has a discharge varying from 2 500 to 190 000 cu ft per sec and a suspended silt content near the intake of the proposed canal averaging 0.90% by weight, and, at times, reaching 5.40 per cent. The suspended silt is extremely fine and the bed silt averages about 0.10 mm (0.004 in.) in diameter. The river slope is approximately 1.2 ft per mile. The use of this very silty water in the Imperial Valley has led to great difficulty and an expense estimated at approximately $1 400 000 per yr for dredging, canal cleaning, and land leveling.

Before the All-American Canal is finished, it is expected that the Boulder Dam, 303 miles above the proposed head-works, will be completed. This dam will be 725 ft high and will form a reservoir, with a maximum capacity of 30 500 000 acre-ft, in which all the silt brought down to the reservoir will be deposited. Another dam will be built 155 miles above the intake, which will stop practically all the silt coming from above that point. The silt that reaches the intake, therefore, will be only that picked up from the banks and bed of the river below the lower dam, and the small quantity brought in by the tributaries to the river between the lower dam and the intake. An elaborate desilting works will be built to remove the coarser part of the silt load which will be carried into the canal. Consequently, the silt load in the canal will differ greatly from that now (1935) carried, and will create different stable channel shapes. To determine the shape best adapted to the new condition a thorough study of the subject was made, the results of which are described herein.

NOTATION

The symbols introduced in this paper are defined as follows (the English system of units being used unless otherwise stated):

a = a subscript denoting "average";
d = depth of flow; d_a = average depth;
f = silt factor $= 8\sqrt{D}$;
m = a subscript denoting "mean";
n = exponent in a formula of the Kennedy type;
s = a subscript denoting "near the sides";
A = area of water cross-section;
B = breadth, or width of channel; B_B = width at the bottom; B_m = mean channel width; as a subscript, B, denotes "at or near the bottom";
C = coefficient in a formula of the Kennedy type;
D = diameter of particles;
L = length;
P = wetted perimeter (not including water surface);
Q = flow, or rate of discharge;
R = hydraulic radius $= \frac{A}{P}$;
S = slope $= \frac{\text{fall}}{L}$
V = velocity; average velocity in a section; V_B = velocity near the bottom; V_S = velocity near the sides; and V_0 = critical velocity from the standpoint of silting.

HISTORY OF NON-SILTING CANAL SECTION STUDIES

Most of the study of the problem of non-silting canal sections has been made by the British engineers in India, in connection with the large irrigation projects of that country. A certain amount has also been done in Egypt, in connection with the irrigation work on the Nile. Thus far, little has been contributed by the United States. In the last few years, however, a surprising interest in silt problems has developed in this country and it is now (1935) being attacked from many angles by a number of research engineers. From this interest, no doubt, future progress will become considerably more marked.

The first study of non-silting canal sections was made by Mr. R. G. Kennedy (1)[2]. His work is a classic in this field, and has resulted in the saving of millions of dollars in reducing the cost of cleaning irrigation canals in India and elsewhere. Unfortunately, like most outstanding studies, it came to have such prestige that for many years little further progress along this line was made.

Kennedy gave the result of measurements of bed widths and "full supply" depths on about twenty-two canals in the Lower Bari Doab Canal System, in which the channels had become stable and several more which had nearly reached this condition. He also gave the "full supply" discharge and the velocity computed from this discharge and the full supply area. From these data he developed a formula of the type:

$$V_0 = C\,d^n \qquad (1)$$

which expressed, with reasonable accuracy, the relation between the critical mean velocity, V_0, and the depth, d, as indicated by the results of the measurements. For the Lower Bari Doab Canal, C was 0.84 and n was 0.64. Kennedy expected C to vary with the quality and quantity of silt, but thought n would be nearly constant. On Fig. 1 (with reference to Table 1) is shown a line giving the velocities corresponding to the various depths according to Equation (1). The local conditions influencing these observations and Kennedy's conclusions will be given more in detail subsequently.

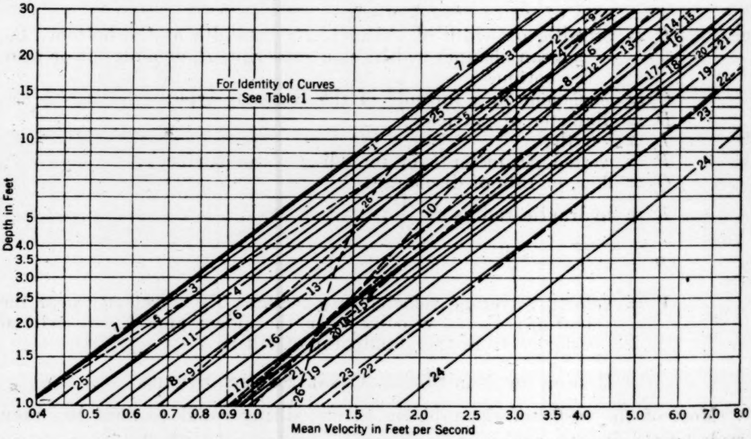

FIG. 1.—CRITICAL VELOCITY FORMULAS FOR NON-SILTING, NON-SCOURING VELOCITIES.

In 1895 Kennedy issued a set of hydraulic diagrams to aid in the design of non-silting channels. In 1904, he gave a rough rule for the relation of width to depth in non-silting canals (5). A second edition of "Hydraulic Diagrams" (6) was issued in 1907, in which Kennedy reprinted the original paper and added an extended discussion to clarify some of the obscure points and to give the results of his experience since the first paper was printed.

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[2] For reference to figures in parentheses see "Bibliography."

TABLE 1.—VALUES OF C AND n, EQUATION (1), FOR NON-SILTING, NON-SCOURING VELOCITIES

Curve No. (see Fig. 1)	FACTORS APPLYING TO EQUATION (1) (IN ENGLISH UNITS)		Locality	Authority	Limit *	Reference
	Coefficient C	Exponent n				
1	0.381	0.64	Egypt	Buckley	Lower	Irrig. Dept. of Egypt.
2	0.46	0.64	Egypt	Buckley	Upper	
3	0.39	2/3	Egypt	Molesworth and Yenidunia	Lower	" Irrigation Practice," p. 207.
4	0.475	2/3	Egypt		Upper	
5	0.391	0.727	Egypt	K. D. Ghaleb		*Minutes of Proceedings*, Inst.. C. E, p. 260, Vol. 229; also p. 285, Vol. 223.
6	0.56	0.64	Egypt			U. S. Dept. of Agri., *Technical Bulletin No. 67*, p. 44.
7	0.38	0.64	Mozaffargorih D. Punjab, India.	G. W. Duthy	Lower	*Proceedings*, Punjab Eng. Congress, pp. 44 and 48, 1919.
8	0.63	0.64		G. W. Duthy	Upper	
	0.63	0.64	Sind	F. W. Woods		*Engineer*, p. 648, Vol. 143; Parker, " Control of Water," p. 678.
9	0.67	0.55	Godavari Western Delta, Madras.	Kennedy		*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229; Madras Public Works Dept., Oct. 9, 1912, Dist. 1872.
10	1.01	0.44	Rio Negro, Argentina	R. E. Ballester		*Minutes of Proceedings*, Inst. C. E., p. 280, Vol. 223.
11	0.52	0.66	Siam			U. S. Dept. of Agri., *Technical Bulletin No. 67*, p. 44.
12	0.93	0.52	Madras (Kistna)	Kennedy		*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229.
13	0.67	0.64	Sutlej, India	F. W. Woods		*Engineer*, June 17, 1927, p. 648.
14	0.91†	0.57†	Burma (Shwebo)	Kennedy		*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229 (1929–30); Parker, " Control of Water."
15	0.95	0.57	Chenab, Punjab.	Lindley		*Minutes of Proceedings*, Inst. C. E., p. 260, Vol. 229; Punjab Eng. Congress, *Proceedings*, 1919, p. 63.
16	0.756	0.64	Sirhind, Punjab.	W. B. Harvey		*Proceedings* Punjab Eng. Congress, 1919, p. 58.
17	0.84	0.64	Bari Doab	Kennedy		Hydraulic Diagrams, Kennedy, Public Works Dept., India, 1907.
18	0.924	0.64	Penner River	J. M. Lacey	Lower	*Minutes of Proceedings*, Inst. C. E., p. 333, Vol. 229.
	0.924	0.64	Cauvery Delta	J. M. Lacey	Lower	
19	1.09	0.64	Penner River	J. M. Lacey	Upper	*Minutes of Proceedings*, Inst. C. E., p. 333, Vol. 229.
20	0.966	0.64	Cauvery Delta	J. M. Lacey	Upper	
21	0.98	0.64	Imperial Valley.	Rothery		*Minutes of Proceedings*, Inst. C. E., p. 179, Vol. CCXVI.
22	1.26	0.64	Cauvery Delta	J. M. Lacey	Extreme. Upper	*Minutes of Proceedings*, Inst. C. E., p. 333, Vol. 229.
23	1.33	0.61	Imperial Valley	Collings	Lower	*Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 549.
24	1.83	0.61	Imperial Valley	Collings	Upper	
25	0.42	0.64	Behera Delta	R. G. Kinder		*Proceedings*, Punjab Eng. Congress, 1919, p. 74j.
26	‡	‡	Jamrao, Sind	W. L. C. Trench.		*Minutes of Proceedings*, Inst. C. E., p. 307, Vol. 223, 1926–27.

* " Limit " refers to the upper or lower limit of the data observed, which may spread over a considerable range.

† Approximate.

‡ $V_o = (1.1+)\ 0.095\ d$.

Kennedy's work soon became extensively used throughout India; observations were made on the ditches of other irrigation systems and a number of other equations of the same type as those of Kennedy, were developed, suitable to the various local conditions. One of these was for the Godavari Western Delta and the Kistna Western Delta, in Madras (7). In 1913 a set of hydraulic diagrams for the design of channels was presented by Capt. A. Garrett which deals with non-silting channels (8), and which is used extensively in the United Provinces.

In 1917, Mr. F. W. Woods proposed (2) the use of definite ratios of depth to width, based on an analysis of data from the Lower Chenab Canal System. In 1919, the results of an extensive analysis of canal dimensions of the Lower Chenab Canal, by E. S. Lindley, M. Am. Soc. C. E., was published (3). For these canals, Mr. Lindley found a critical velocity relation such that, in Equation (1), $C = 0.95$ and $n = 0.57$. He also found a relation of bed width to depth of $B_B = 3.8\ d^{1.61}$.

In 1927, Woods (4) proposed a general formula covering velocity, average depth, mean width, and slope, as follows:

$$d_a = B_m^{0.434} \quad \ldots\ldots (2)$$

$$V_0 = 1.434 \log_{10} B_m \quad \ldots\ldots (3)$$

and,

$$S = \frac{1}{2 \times \log_{10} Q \times 1\,000} \quad \ldots\ldots (4)$$

Equations (2), (3), and (4) cover not only the depth and width, but also the discharge and slope. According to them for a given discharge there is a single condition of depth, width, and slope that will produce a stable channel.

In 1928, Mr. W. T. Bottomley (9) advanced the idea that irrigation channels would be non-silting and non-scouring if the slope of the canal was of the same order as that of the parent river, regardless of the relation of width to depth and the shape of the channel. In 1930, an excellent paper on this subject (18) was presented by Mr. Gerald Lacey in which he advanced the proposition that the wetted perimeter of stable channel was a simple function of the square root of the discharge; or,

$$P = 2.668\ Q^{0.5} \quad \ldots\ldots (5)$$

and that the shape of the section depended upon the fineness of the silt carried, coarse silt giving rise to wide, shallow sections and fine silt to narrow, deep ones. He developed the formulas,

$$Q\ f^2 = 3.8\ V_0^6 \quad \ldots\ldots (6)$$

and,

$$V_0 = 1.17 \sqrt{f\ R} \quad \ldots\ldots (7)$$

in which f is a silt factor, related to the diameter of the bed material by the expression:

$$f = 8 \sqrt{D} \quad \ldots\ldots (8)$$

In Equation (8) D is in inches. From Equations (6), (7), and (8), knowing the flow, Q, in the ditch, V_0, A, and R can be computed.

Lacey also stated that the shape of a stable channel approximated an ellipse, with its major axis horizontal, the ratio of the major to the minor axis being larger as the silt became coarser. Lacey's ideas have been widely accepted in India, and extensive observations are under way to study the effect of various conditions on his silt factor, f.

The aforementioned authorities have developed their ideas almost entirely from experience in India. The result of experience on canals in Egypt is given by Messrs. Molesworth and Yenidunia (10). They give a general formula in English units:

$$d = (9\,060\ S + 0.725) \sqrt{B_B} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

as developed from a careful examination of a large number of recognized good Egyptian canals. As a result of further experience, Mr. A. B. Buckley (11), develops the adjustment of Equation (9) for canals of depths of 1.6 m (5.26 ft) and less, as follows:

$$d = \frac{0.0025\ (100\,000\ S + 8)^2\ B_B}{1.62} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Equation (10) is in English units. In addition to the general formulas proposed by various investigators, a large number of special formulas of the Kennedy type have been developed. These formulas are listed on Table 1.

SUMMARY OF PREVIOUS STABLE-CHANNEL FORMULAS

The formulas developed for stable channels fall into two classifications: (*a*) Those giving an expression for velocity; and (*b*) those giving stable channel shapes. Those in the first class are similar to the Kennedy formula, Equation (1). In most of these formulas n has been taken as 0.64, the value developed by Kennedy. In all cases the value of C was constant for a given locality or canal system. Kennedy believed that C would vary with both the size and quantity of silt, but did not emphasize the effect of the quantity of silt as much as the quality, and, as a result, it has been largely lost sight of by other students of the subject. He did not believe that the value of n would change greatly.

A formula of the Kennedy type indicates that the critical velocity increases with the depth, but experience shows that as the depth is increased a velocity is finally reached at which the banks begin to erode. Kennedy believed that the limiting velocity was a matter of experience, and gave limiting values which correspond to depths of about 10 ft. This had the effect of limiting the depths of canals designed according to his rules to these values. No data on this limitation are available for the conditions under which any of the other formulas of the Kennedy type were developed.

Of the formulas of the second type, Lindley gives a relation of critical velocity to depth and bed width, but suggests no modifications for the quality or quantity of silt. Woods gives relations for mean depth, velocity, and slope, but like Lindley makes no suggestion that these relations might be influenced by the quantity or the quality of the silt. Lacey gives channel shapes and velocities, introducing the effect of the size of the silt grain, but does not consider the quantity of material to be transported.

COMPARISON OF CRITICAL VELOCITIES

In order to determine what velocities could be used safely in the All-American Canal, Fig. 1 was prepared, showing the relation of depth to critical velocity, as determined from all available observations on actual ditches. These show at a glance that for a given depth of flow there is a tremendous variation in critical velocity. The line representing Kennedy's data is shown heavier than the others. The variations range approximately from 46 to 208% of Kennedy's results, or the highest value over 450% of the lowest.

The local conditions under which most of these formulas were developed are not known in detail. In general, however, it is believed that the silt of the Nile River is finer than that of Ravi River, from which water is drawn for the Lower Bari Doab Canal, on which Kennedy's observations were made. The lower velocities found for the Egyptian canals, as compared with those given by Kennedy are, therefore, consistent with the relation of Lacey's formula (Equation (6)), that finer material results in lower critical velocities. It is also known, however, that the silt of the Colorado River and the tributary Imperial Valley canals is finer than that of the Ravi, but the critical velocities are higher in the case of the Imperial Valley canals. This is contrary to the relation given by Lacey.

COMPARISON OF FORMULAS FOR WIDTH-DEPTH RELATION

A comparison similar to that of the critical velocity relations was also made of the various formulas for the relation of bed width to depth. The results are shown on Fig. 2, which gives the relation of bed width for the principal formulas and some of the data. The Woods' formula was expressed in terms of mean width, and has been changed to terms of bed width by assuming side slopes of 1 on 2. The data for channels as proposed by Lacey (using side slopes of 1 on 2) for three sizes of material, are also shown. The finest of these, 0.0025 in. in diameter, is for material roughly corresponding in size to that composing the bottom of the Imperial Valley canals.

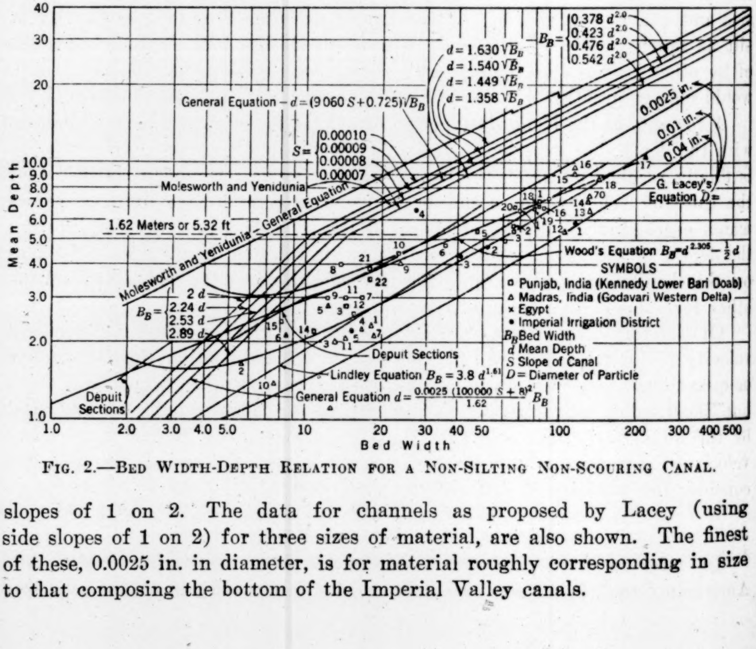


FIG. 2.—BED WIDTH-DEPTH RELATION FOR A NON-SILTING NON-SCOURING CANAL.

The Punjab (Kennedy) data are computed from those given by Kennedy for the Lower Bari Doab Canal, using vertical side slopes, as reported by him. Data are also given on canals in the Godavari Western Delta and some values obtained from canals in the Imperial Valley. The data from Egypt were in the form of a general equation by Molesworth and Yenidunia, which is independent of the slope, and an equation which is dependent on the slope, four slopes being given. For depths less than 1.62 m (5.32 ft), a modification of the Molesworth-Yenidunia, formula given by Buckley has been used, based on data which have been collected since the other formula was proposed. The Depuit Section, said to be widely used in Egypt, is also shown.

These data for the bed width-depth relation show even greater variation than the depth critical-velocity relations shown on Fig. 2. For a 5-ft depth, the Molesworth-Yenidunia formula without the slope factor, gives a bed width of 6.4 ft and the Lindley equation gives 50.0 ft, or a ratio of maximum to minimum of 781 per cent. Some of the Imperial Valley data indicate even higher ratios than those given by the Lindley formula. The wide range does not seem to be due to variation in the size of the silt because, although the Egyptian data are believed to be for finer silt than the Indian data of Woods and Lindley, most of the Imperial Valley data, which are also for fine silt, give even higher bed width-depth ratios than those of either Lindley or Wood.

Factors Affecting Stable Channel Shapes

As a result of the wide range of critical velocities and bed width-depth relations found on the canals in the different parts of the world, and the lack of any readily apparent consistency in the variations, it was clear that if the factors controlling this variation could not be determined it would not be safe to adopt any of the relations given by existing formulas for the design of the sections of the All-American Canal. Although these formulas no doubt provide workable relations for the conditions for which they were developed, these conditions have not been delineated sufficiently to enable them to be applied elsewhere. In general, also, they were developed empirically from a very limited range of conditions, and in most cases they omit important factors from consideration.

To develop a rational design for the sections of the All-American Canal, it was necessary, therefore, to attempt to go back to the fundamentals and try to make an analysis of the factors controlling the shape of a stream channel in erodible material, and their relations to each other.

The following is a list of factors that may enter into a determination of stable channel shapes: (*a*) Hydraulic factors (slope, roughness, hydraulic radius or depth, mean velocity, velocity distribution, and temperature); (*b*) channel shape (width, depth, and side slopes); (*c*) nature of material transported (size, shape, specific gravity, dispersion, quantity, and bank and subgrade material); and, (*d*) miscellaneous (alignment, uniformity of flow, and aging).

In arriving at a rational solution of the problem of stable channels it is necessary to consider all these factors, and to determine as accurately as possible which of them are of major importance, and which are minor or negligible. By determining first the relation between major factors, it may be possible later to study the effect of minor factors, but until the major relations are known, the data available are only a collection of miscellaneous facts of limited value.

Of the hydraulic factors, the slope, roughness, hydraulic radius, and mean velocity are interdependent and with reasonable certainty their relation is known quantitatively through the ordinary velocity formulas. It is true that the effect of the movement of material in suspension and by traction upon the roughness is not definitely known and more information on this point is needed, but compared to the uncertainty in other phases of the problem, the relations of these four items is so well known that for the purposes of this study further investigation along these lines was probably unjustified. The relation between these factors and scouring or transportation of solids in channels is not well established and must be studied further.

As will be shown subsequently, it is believed that the velocity distribution, as well as the mean velocity, is of primary importance to the problem and that it, together with the channel shape factors of width and depth, exercise an important influence on stable shapes. The side slopes are relatively unimportant except as regards sloughing.

Temperature has been suggested as having an important effect because it influences the viscosity of the water and, consequently, the rate at which solid particles settle. That temperature might have some effect cannot be questioned, but it is probably small, from the standpoint of stable channel shapes. Most of the data were collected in warm countries, comparable to the locality of the All-American Canal, and the temperature variation although it might cause some difference in settlement rate would not, ordinarily, be enough, when averaged over the year, to cause major effects. For many sizes of silt the effect of temperature on settling rate is small. Moreover, it is possible that the tractive force, which is not appreciably affected by temperature, is the most important factor in stable channel shapes, and, therefore, temperature effects are relatively unimportant. In any event temperature data, which would enable an analysis of its effect to be made, are not available.

Nearly all students of the problem have admitted that the size of the material transported is of major importance. The shape no doubt has an effect, but it is believed to be of secondary importance as laboratory experiments show that angular particles are moved by only slightly higher velocities than rounded ones. In any event, no data on it are available for any of the localities, so that its influence could not be investigated, even if it was desirable to do so. Specific gravity of the transported material also has its effect, but since it rarely varies much from 2.65 it is of secondary importance. No data on this subject would be available, even if it were desirable to study them. The dispersion of the material by virtue of the electrical charges carried by the particles is important in some phases of sedimentation, but it is probably active only in the case of very fine material which is ordinarily not much of a factor in stable channel sections. In this case, again, no data for further study are available.

The quantity of solids in motion is an important factor in the shape of stable channel shapes, and has not received the attention that its importance warrants. Cases illustrating its importance are numerous. For example, it is a common occurrence in some irrigation systems to have the upper section of the ditch fill during periods of high silt content in the streams from which they draw, and for this fill to scour out later during periods of clear-water flow. In other words, the ditch is unstable, at times being unable to transport all the material brought to it, hence filling up, and at other times transporting more material than brought to it, and, therefore, cutting down its bed. Over long periods the ditches are approximately stable because the two actions counteract one another. Another common example is the change from unstable to stable condition which results when an effective sand trap is applied to a ditch that is becoming filled. There are also numerous cases in which the channel of a natural stream is stable but begins to scour severely when a dam is built on it and cuts off the supply of solid material which formerly came down to renew that which was moved forward by the flowing water.

One of the most important factors controlling stable channel shapes is the nature of the material composing the banks and subgrade. If these materials are resistant to scour, higher velocities can be used than if the material is friable. Alignment is another factor to be considered, because bank scour is more likely to occur on curves. If a canal is apt to be operated a large proportion of the time at part capacity, this must also be considered in the design.

Another factor that influences the stability of an irrigation channel is what is commonly termed "aging." After water has run for some time in a channel, the particles composing the bed arrange themselves in such a manner that they are more difficult to move than when the water is first turned in. If the water is silty, this material forms a kind of weak cement which binds the bed material together and makes it more resistant.

In addition to the items listed herein under the heading "Factors Affecting Stable Channel Shapes," another set of relations enters into the selection of the best channel section in any instance; which depends upon the conditions which the canal is designed to meet.

Canals for conveying water for irrigation or power are usually designed to meet one of three sets of conditions. The first type is encountered when it is desired to use the lowest practicable velocity, in order that the slope may be reduced to a minimum. In the case of power canals this is done to obtain the greatest feasible power head, and, in irrigation canals, it is done to enable the ditch to command as much irrigable area as possible for a given length. A second type of conditions is met in both power and irrigation canals where it is desired to reduce the size of the canal to a minimum, in order to make the cost as small as possible without making the slope steeper than necessary. This requires that the velocity be made as great as can be carried without scouring the banks or bed. A third condition is met in irrigation canals where it is desired to carry the ditch on an alignment that has a slope as steep as possible, in order to reduce the cost of drops. The first of these conditions aims at securing the minimum practical velocity within the limitations of cost and silting. The second aims to secure the highest velocity that the ditch will stand with a shape which will convey the water with a reasonable loss of head. The third aims to dissipate as much head as possible by making the canal wide and shallow, thus reducing the hydraulic radius to a minimum and the slope for a given velocity to a maximum. The group into which any particular canal falls, therefore, indicates limitations which are likely to control the best channel shape, which must not be exceeded while still being subject to the influence of the aforementioned factors.

Conditions Required for Stable Channels

For a channel to be perfectly stable, it must not fill or scour on either the banks or bed. The banks must also be stable against sloughing or sliding. To meet the non-filling requirement, the velocity must be enough to flush away all the solid material brought into the section by the flowing water. To fulfill the non-scouring requirement, the velocity at the bed or at the banks must not be great enough to scour the material of which they are composed. To determine a stable section for a set of conditions it is necessary to determine the various relations which will cause velocities at the banks and along the bed that will bring about these conditions.

The silt carried into a section of canal may be composed entirely of fine material, which is easily moved by the water, or it may be composed entirely of coarse material, which is moved only at relatively high velocities. Usually, however, it has a graded composition varying from coarse to fine. If all the material is very fine, ordinarily it offers little practical difficulty because the velocities required in the ditch to meet conditions of economy are sufficient to keep it in motion. If the material is graded, the fine material moved depends upon the velocity near the bed, to obtain a stable channel the the material is coarse, all of it may be dragged along the bed, and little if any be carried in suspension. Since the quantity of the bed material that can be moved depends upon the velocity near the bed, to obtain a stable channel the velocity along the bed must be greater for larger bed loads. This may require a higher velocity along the bed than the material in which the channel was built would stand from clear water, the entire energy of the water on the bottom being expended in dragging along the bottom the material which has been brought down by the water from above. However, if the velocity along the bottom exceeds that necessary to move the bed load, it will act on the subgrade of the channel. To have a stable channel, the subgrade material must be sufficiently tenacious to resist this scour. Summarizing this relation, it may be stated that the velocity along the bottom of a stable channel must be sufficient to move the quantity of material supplied to it, but not so great as to scour the subgrade.

The material composing the banks of the canal is acted upon by two forces tending to produce motion. One of these is gravity, which tends to make the material roll or slide down the sides of the ditch. The effective gravity force is the component that acts downward along the side slopes of the ditch. The other force acting is that due to the motion of the water through the canal, which tends to drag or to push the material in a down-stream direction. The magnitude of this force depends upon the velocity adjacent to the bank

The force of gravity and the force of the stream both act together when water is flowing in the canal, and when the resultant of the two forces is sufficient to dislodge material from the sides, it moves in a diagonal direction to the bed of the stream, or if fine enough, is carried off in suspension by the water. The slope of the bank must be sufficiently flat so that the component along it, of the force of gravity, when combined with the force of the water, is insufficient to dislodge the particles. Since flat side slopes cause a smaller component of gravity, therefore, they have less tendency to scour from this cause.

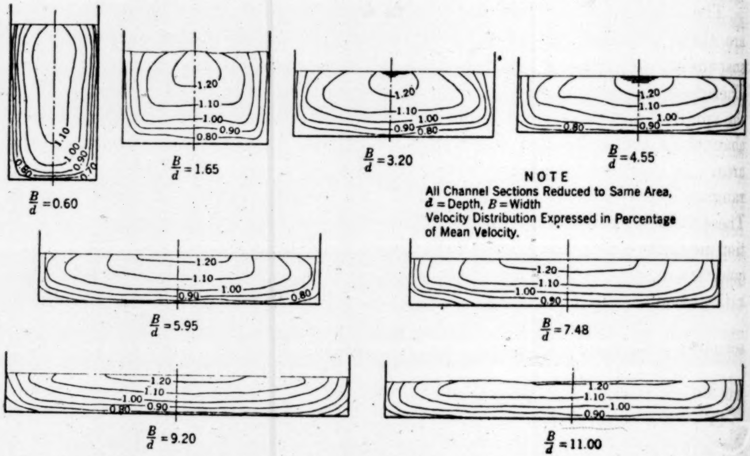
STABLE CHANNELS FOR CLEAR WATER

The simplest cases involving the determination of stable channel sections are those required to convey clear water. When the water carries silt in suspension or drags a load along the bottom, there are added complications. Therefore, the simplest cases, with clear water, will be considered first. When the smallest practical slope is desired, it is usually considered that the cheapest channel is secured when the wetted perimeter is least in proportion to the area. In trapezoidal channels this occurs with a ratio of bed width to depth ranging from 2.0 to 0.472 for side slopes between the vertical and 1 on 2. These values give the most efficient hydraulic section, but since this consideration neglects any excavation above the water line the flattest slope for a given quantity of excavation for a channel in cut is given by cross-sections where the ratio of bed width to depth is less than the foregoing. Thus, maximum economy will result from very low $\frac{B}{d}$-ratios. In earth, however, experience has shown that such channels are not stable.

A stable channel for clear water must have banks with sufficiently flat side slopes to keep the material from sloughing or rolling in, and sufficiently low velocities to keep the banks and bed from scouring. As previously stated, since the material on the banks is acted upon by the force of gravity, as well as that due to the motion of the water, it will not resist as high a force from the motion of the water as the bottom, where gravity does not tend to produce motion.

If the mean velocity in a narrow, deep channel is low enough so that the forces acting on the side material are insufficient to move it, and the sides are stable from sloughing, the channel will be stable. In other words, narrow, deep channels can be used with clear water and low velocities. Ordinarily, however, considerations of cost prevent the use of the large canals necessary to produce low velocities. For such a channel a cross-section must be selected that will give velocities along the bottom which will not move the bottom material, and velocities along the sides which will not move the side material. Since the side material, due to the action of gravity, will move at a lower velocity than that on the bottom, to obtain the maximum possible mean velocity without scour the velocity along the sides must be enough less than that along the bottom to offset the gravity effect. This reduction of side velocity, as compared with bottom velocity, is secured by increasing the ratio of bed width to depth.

In Fig. 3 are shown the velocity distributions in a number of rectangular channels having the same cross-sectional area. Most of these data were secured from the results of the experiments of D'Arcy and Bazin. The velocities are indicated by the "isovels" (also called "isotacks"), or lines of equal velocity expressed in terms of the mean velocity. (These data were obtained with different discharges for many of the examples. Although it is probable that the positions of the isovels would change somewhat with different velocities, such changes would be relatively small and would not change the main relations.) Since the areas of all water cross-sections are equal, the lines giving the same ratio to mean velocity in all the diagrams represent the same velocity for any given discharge. A study of the velocity distribution in these sections will show that high velocities extend closer toward the sides in the narrow, deep cross-sections than in the broad, shallow ones. The science of hydrodynamics has not yet progressed to the point where the relation between the velocity distribution adjacent to a surface can be related quantitatively to the drag of the water along the surface or to the velocity "gradient" adjacent to the surface; but progress along this line is rapid, and the near future may bring sufficient advancement in this field to enable more exact analysis to be made. For the present, however, it is sufficient to state that when the high velocities extend close to a surface the pushing or dragging force of water on the surface is greater than if these velocities close to the surface are low.



NOTE
All Channel Sections Reduced to Same Area,
d = Depth, B = Width
Velocity Distribution Expressed in Percentage of Mean Velocity.

FIG. 3.—RELATION OF WIDTH TO VELOCITY DISTRIBUTION IN RECTANGULAR CHANNELS.

In a very narrow, deep section, the velocities close to the sides are as high as, or higher than, those close to the bottom. If the velocity in such a channel is increased gradually, due to the added force of gravity on the side material, motion would occur first on the sides.

The design of a channel to convey clear water where it is desired to obtain the smallest practicable slope, therefore, consists, from the hydraulic standpoint, of securing the smallest ratio of bed width to depth that will not produce scour on the sides, provided such ratio is not less than that which will give the smallest wetted perimeter for a given earthwork quantity. This latter qualification will probably rarely control. The design of a channel for the second type of conditions, where the highest practicable mean velocity is to be secured, is obtained by proportioning the ratio of width to depth so that the forces tending to produce movement, both on the sides and bottom, are the maximum they will stand without motion. For canals in the third class, where it is desired to make the ditch as steep as possible without producing scour, it is customary to make the section wide and shallow in order to reduce the hydraulic radius and thus lower the velocity. For such very wide ditches the scour would be greatest on the bottom, and this condition would control the slope that might be used. Theoretically, there is no limit to the slope a ditch might be given because the velocity could be reduced to any desired value by making it sufficiently wide and shallow. As a practical matter, however, it has been found that when the depth is made very small, the irregularities of construction are such that scour starts in the slightly deeper parts of the channel and enlarges them, causing a progressively greater concentration and scour until the beneficial effect of the widening is lost. This action has been noted in ditches with steep slopes 8 to 10 ft wide and 0.6 to 0.8 ft deep.

CHANNELS CARRYING SOLIDS IN SUSPENSION

It is quite generally agreed that material can be carried in suspension by a stream because of the vertical currents that occur in flowing water and carry the solid particles upward at a greater rate than the force of gravity causes them to fall. A promising hypothesis for the capacity of a stream to transport material in suspension, therefore, should be that the capacity is proportional to its turbulence, which, in turn, is probably proportional to the energy expended. The concentration of a given quality of solids which a stream could support, therefore, would be proportional to the energy expended per unit of volume of the water. This energy is proportional to the rate of fall of the water, which is equal to the product of the velocity and the slope.

This is a kind of over-all relation, however, and silting may occur in one part of a ditch cross-section while other parts may be scouring. Because the velocity near the edge of a stream flowing in a trapezoidal section is low, in a channel carrying silt in suspension there is a tendency to deposit at that point. This is aided by the growth of vegetation, and usually a berm is formed which creates steeper sides to the section than were originally constructed. This material is quite resistant to scour, and forms more or less evenly even in ditches where there are high velocities. This action is not ordinarily very detrimental, and is often anticipated and allowed for by computing the capacity of the channel with the slopes which it is expected the silt will cause, rather than the slopes to which it is first excavated.

It is probably not feasible to prevent entirely the deposition of suspended material along the edges of channels in earth, although it may be reduced by using higher velocities. The greatest difficulties from suspended matter occur in ditches where the slopes are so slight that the energy being dissipated in the water is insufficient to prevent deposit. The remedy in these cases is to increase the velocity, or to remove the suspended material in some kind of desilting device.

Channels Carrying Bed Load

Irrigation channels frequently carry considerable solid matter by dragging or pushing it along the bed, with either clear or silt-laden water flowing above. The quantity of material depends upon the velocity near the bottom of the channel. Higher velocity is probably necessary to move the same quantity of coarse material as fine material. If a channel is supplied with a heavy bed load, in order to be stable it must move this load along; otherwise, the channel will become filled. This requires a high velocity along the bottom, as compared with a channel carrying clear water. For a given quality of material in the banks, the velocities that could act on the banks in the two cases would be the same. To be stable, the channel carrying bed loads, therefore, should have a higher velocity along the bed, but the same velocity along the banks, and this could only occur with a wider, shallower section. Heavily loaded channels in easily scoured material therefore, should have high ratios of bed width to depth. If the banks of the loaded channel are of material which is resistant to scour, the ratio of bed width to depth can be less than in friable material without scouring the banks.

Colloids

Colloids carried in the water exercise a considerable effect on the shape of the channel cross-section. They cement the fine particles which collect along the sides of the ditch and are responsible for the vertical or nearly vertical banks which exist in many canals. This colloid cemented material along the sides is more resistant to scour than the size of material would indicate, and thus permits higher velocities along the banks than would otherwise be allowable. To a certain extent also colloids may cement the particles composing the bed and make it more resistant to scour. It is believed, however, that some of the effects ascribed to colloids are really due to the presence of a high silt load. The ability of the canals of the Imperial Valley, which are constructed in fine silt, to carry velocities of 4 or 5 ft per sec without scour has been ascribed to colloids, but the writer believes that a large part of it is due to the presence of the high silt load. When these canals are supplied by the All-American Canal with desilted water, their beds will scour considerably, and this would not be prevented by the colloids.

Recent Conceptions of Flow not Used

To engineers who are familiar with the latest theories of stream mechanics and hydrodynamics, the foregoing analysis of the factors controlling stable channel shapes may seem somewhat crude, and in ignoring the drag theory of bed-load movement and the conception of velocity gradient, it may appear that the writer has not taken advantage of the best available information.

In making the study outlined herein, the most recent pertinent literature in stream mechanics and hydrodynamics has been analyzed to determine all the material that was applicable to the problem. A list of these references is included in the "Bibliography" contained in this paper. Since many engineers are not familiar with recent ideas, however and since a knowledge of them is not necessary to understand the relations developed regarding stable channel shapes, it was believed to be better to explain these relations in terms of conceptions with which all engineers are familiar, rather than to make it unnecessarily confusing to some by adding the other new ideas.

AGREEMENT OF SUGGESTED RELATIONS WITH OBSERVED DATA

The relations suggested in this paper seem to agree with the observed data, as shown on Figs. 1 and 2. The critical velocity shown for the Nile River on Fig 1 is much less than that found by Kennedy in India and also less than that for the Imperial Valley canals. The quantity as well as quality of silt is an important factor in these cases. The silt in both the Nile and Colorado Rivers is fine, but the Imperial canals require a much higher velocity than the canals of Egypt because the quantity of silt is much greater in the Imperial canals. The critical velocity for the canals observed by Kennedy is higher than those in Egypt, probably because the particles moved on the bed were larger, and, therefore, required higher velocities to move them. The Imperial canals require more velocity than the canals observed by Kennedy, although the latter have coarser loads, because the velocity required to transport the immense bed load of the fine Imperial Valley sand is greater than that necessary for the lesser quantity of coarser sand of the canals mentioned by Kennedy.

A similar agreement of the relations previously discussed in found in the data on bed width-depth ratios as shown on Fig. 2. In the canals of Egypt the velocities are low and, therefore, the velocities along the sides, although relatively high because the channels are narrow and deep, are still below those that will move the side material. The bed load is fine and small in quantity and, therefore, the low velocities along the bottom are sufficient to move it all.

The Indian canals shown on the diagram (Fig. 2) carry medium loads of rather coarse material and, therefore, require rather wide sections. The Imperial Valley canals carry immense loads of fine sand, which require high velocities to transport. In order that the velocities along the sides may be low enough so that the banks do not scour, the bed width-depth ratio must be high. In three of the four canals on which data are available, this relation is higher than that indicated by the equations of either Woods or Lindley. These had readily erodible sides. The fourth, which had a lower bottom width-depth ratio, had sides composed of material which had considerable resistance. The three sections with easily erodible banks gave widths considerably greater than is indicated by Lacey's formula for the type of silt which they contained. It is believed that Lacey's formula was based on data from canals which carry loads of considerably less magnitude.

Stable Channel Shapes

The only investigator who has attempted a closer definition of the shape of stable channels than the bed width-depth relation is Lacey. He states (18):

> "That natural silt-transporting channels have a tendency to assume a semi-elliptical section is confirmed by an inspection of a large number of channels in final regime and an examination of cross-sections of discharge sites of rivers in well-defined straight reaches of known stability."

He concluded that stable channels would be semi-elliptical, with the major axis horizontal, and with the ratio of the major axis to the semi-minor axis depending on the nature of the silt carried, being greater for coarser silt.

The results of the writer's investigations do not support this conclusion. A convenient way of comparing channel cross-sections is by means of a ratio, which may be called the "form factor," between the area of the channel section, up to the water surface, and the area of the enclosing rectangle. For an ellipse this ratio would be $\frac{\pi}{4} = 0.79$; for a parabola, 0.67; for a triangle, 0.50; and for a rectangle, 1.00. A study of a large number of cross-sections of channels has disclosed ratios from 0.56 to 0.92. The stable channels observed by Kennedy were reported to have practically vertical sides and horizontal bottoms, which would give a form factor of 1.00.

Just what shapes for stable channels are produced by all varieties of conditions has not yet been determined, but the writer has observed that for channels carrying a heavy load of graded silt, ranging from colloids to fine sand, the sections have nearly horizontal beds composed of the fine sand and nearly vertical sides of silts and clays. Such channels have form factors of about 0.90. This is the condition on the canals of the Imperial Valley. This difference between the composition of the bed and bank material has also been observed in India. Similar conditions result in channels carrying considerable bed load and a moderate quantity of fine silt. For a channel carrying water containing a small quantity of silt at high velocity in a material containing a considerable number of cobbles, the cross-section is distinctly saucer-shaped, most of the section being covered with cobbles, but with a small silt berm at each edge. This condition was observed on some of the ditches on the Uncompahgre Project, in Colorado, and in the San Luis Valley, in Colorado. One ditch that was carefully measured, had a form factor of 0.85. It is believed that further study will disclose typical shapes for a number of common conditions and the reasons therefor.

Variation in Discharge

The design of the best section for many canals is complicated by variation in the flow they are to carry. Some canals fill at one time and scour out at another, a substantially stable channel resulting from the balance of scour and fill. Many of the data which can be secured on stable shapes are complicated by this discharge variation. No rules can be given for the treatment of such cases. Until the problem of the simple case of relatively uniform flow is obtained, the more complex case of variable flow can be attacked only by the application of engineering judgment based on a knowledge of existing conditions and available information regarding shapes required for uniform flow.

ACKNOWLEDGMENTS

In the investigation leading to this paper many discussions of the problem were studied and many helpful suggestions were thus secured. With these the writer has combined his own ideas. It is not always possible to state definitely which were original and which were secured from literature. The papers on the subject which have been found most useful have been incorporated in the "Bibliography." To the authors and discussers of those papers the writer is particularly indebted. He is also indebted to C. A. Wright, H. F. Blaney, W. M. Griffith, F. C. Scobey, J. C. Stevens, and S. P. Wing, Members, Am. Soc. C. E., and to the General Board of Irrigation of India, for valuable data and suggestions.

As previously stated, this paper gives the results of studies for the selection of stable channels for the All-American Canal. This canal is being designed by, and constructed under, the direction of the U. S. Bureau of Reclamation. All designs and investigations of the Bureau of Reclamation are under the direction of J. L. Savage, M. Am. Soc. C. E. All engineering and construction work is under the general direction of R. F. Walter, M. Am. Soc. C. E., and all activities of the Bureau are under Elwood Mead, M. Am. Soc. C. E.[3] The writer wishes to express his appreciation to the authorities of the Bureau of Reclamation for permission to publish these data.

BIBLIOGRAPHY

(1) **Hydraulic Diagram for Canals in Earth,** by R. G. Kennedy, *Minutes of Proceedings,* Inst. C. E., Vol. 119 (1895), p. 281.

(2) **Normal Data of Design for Kennedy Channels,** by F. W. Woods, Chf. Engr., Irrig. Works, Punjab, July 28, 1917.

(3) **Regime Channels,** by E. S. Lindley, M. Am. Soc. C. E., *Proceedings,* Punjab Eng. Congress, 1919, p. 63.

(4) **A New Hydraulic Formula (for Silting Velocity—Kennedy's Data),** by F. W. Woods, *The Engineer,* Vol. 143, June 17, 1927, p. 646.

(5) **Instructions for Grading and Designing Irrigation Canals,** by R. G. Kennedy, *Punjab Irrigation Paper No. 10.*

(6) **Hydraulic Diagrams for Channels in Earth,** by R. G. Kennedy, Public Works Dept., India. (A new edition of this work is available.) Also, *Engineering News-Record,* Vol. 106, 1931, p. 370.

(7) **Critical Velocity Observations,** Madras Public Works Dept., October 9, 1912.

(8) **Hydraulic Diagrams for Design of Channels in Earth by Kutter's Formula,** by Capt. A. Garrett, Second Edition, 1913. (The data and contents of the first edition are not known.)

(9) **A New Theory of Silt and Scour,** by W. T. Bottomley, *Engineering,* Vol. 125, March 16, 1928, p. 307.

(10) **Irrigation Practice in Egypt,** by Molesworth and Yenidunia, 1922.

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[3] Dr. Mead died on January 26, 1936.

(11) **Silt Investigations,** by A. B. Buckley, *Paper No. 4,* Irrig. Projects Dept. of Egypt.

(12) **Some Problems Connected with Rivers and the Canals in Southern India,** by J. M. Lacey, *Minutes of Proceedings,* Inst. C. E., Vol. 216, p. 150.

(13) **The Influence of Silt on the Velocity of Water Flowing in Open Channels,** by A. B. Buckley, *Minutes of Proceedings,* Inst. C. E., Vol. 216, p. 183.

(14) **Irrigation Project of the Californias,** by S. L. Rothery, M. Am. Soc. C. E., *Minutes of Proceedings,* Inst. C. E., Vol. 216, p. 161.

(15) **Note on Silt Investigation No. 1,** by A. B. Buckley and Hughen, Cairo, 1919.

(16) **Influence of Silt on the Velocity of Water Flowing in Open Channels,** by R. B. Buckley, *Engineering,* Vol. 115, 1923, p. 311.

(17) **A Theory of Silt and Scour,** by W. M. Griffith, *Minutes of Proceedings,* Inst. C. E., Vol. 223, January 25, 1927. (Abstracted, *Engineering,* Vol. 123, January 21, 1927, p. 72; and Vol. 125, March 16, 1928, p. 307.)

(18) **Stable Channels in Alluvium,** by Gerald Lacey, *Minutes of Proceedings,* Inst. C. E., Vol. 229; also *Engineering,* Vol. 129, February 7, 1930, pp. 179–180.

(19) **The Transportation of Debris by Running Water,** by G. K. Gilbert, *Professional Paper No. 86,* U. S. Geological Survey.

MECHANICS AND HYDRODYNAMICS

(20) **Modern Development in Study of Turbulence,** by L. Prandtl, *Memorandum No. 720,* National Advisory Committee for Aeronautics.

(21) **The Present State of the Turbulence Problem,** *Applied Mechanics,* A. S. M. E., Vol. 1, No. 1.

(22) **Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation,** by M. P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Geophysical Union, April, 1933.

(23) **Towards a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams,** by J. B. Leighly, Univ, of California Geographical Publications, Vol. 6, No. 1, 1932.

(24) **Hydraulic and Sedimentary Characteristics of Rivers,** by L. G. Straub, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Geophysical Union, April, 1932.

DISCUSSION

R. C. JOHNSON,[4] M. AM. SOC. C. E. (by letter).—From a practical point of view it seems that the actual design of channels to prevent both erosion and silting must remain for some time to come largely a question of judgment. Although it is generally agreed that Kennedy's formula expresses a general law governing eroding velocities, the coefficient, *C*, will range through such wide variations for different soil types and other variable conditions, that the formula is of little value for use in the United States. The theoretical considerations, of course, are necessary for the designer as well as for those engaged in research, but such theory cannot supersede judgment based on a knowledge of local conditions.

The writer had the opportunity of observing the erosion and silting in channels constructed by the United States Soil Conservation Service in the Piedmont Section of South Carolina during 1935. These channels were constructed chiefly for the purpose of drainage and have the following general characteristics: (1) The predominating type of soil is Cecil clay high in colloidal content (ranging as high as 46%, as determined by the Bouyoucos hydrometer method); (2) the flows in the waterways vary through wide limits, ranging from no flow to the maximum designed capacity; (3) practically all flows carry large quantities of silt; (4) the depth of flow seldom exceeds 2 ft; and (5) the channels range through a great variety of slopes and shapes. The ratio of width to depth ranges from about 3:1 to 10:1.

It is true that such channels do not come within the same classification as channels of more or less uniform flow; yet their consideration may have some related value. Observations were made repeatedly on a number of these channels, in sufficient numbers to lead to the following conclusions:

(*a*) Kennedy's formula for critical velocity was found to be definitely unsuited for design purposes, especially with a value of *C* even approximately as low as that recommended for Indian conditions;

(*b*) The roughness of the bottom plays a most important part in influencing bottom erosion for shallow depths of flow;

(*c*) Critical slopes exist beyond which no width-depth ratio will prevent erosion of the bottom, regardless of theoretical calculations. (This may be due chiefly to irregularities in the bottom which cannot possibly be avoided in construction of the channel under field conditions); and,

(*d*) Permissible canal velocities as published by Fred C. Scobey, M. Am. Soc. C. E., and the late Samuel Fortier, M. Am. Soc. C. E.,[5] were found to be much better suited, as a rough guide, for general Piedmont conditions than the critical velocity given by Kennedy's formula.

At present, the problem of stable channels in erodible material is of major importance in soil-erosion control work. The problem is extremely complicated by the fact that the flow is most erratic and by the ever-present danger of excessive silting. It is believed, however, that it is possible to place the design on a more rational basis than can be done with existing knowledge of hydrodynamics and soil mechanics. The field for research along this line is practically unlimited.

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[4] Assoc. Prof., Civ. Eng., School of Eng., Univ. of South Carolina, Columbia, S. C.

[5] "Permissible Canal Velocities", *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 940.

E. S. LINDLEY,[6] M. AM. SOC. C. E. (by letter).—Throughout the paper, in the text, and on the diagrams, Kennedy's data are ascribed to the Lower Bari Doab Canal, whereas they were observed on the Bari Doab Canal, which became "Upper" on the construction of another canal many years later. The latter takes off from the same river about 160 miles lower in its course; it was originally designed to Kennedy velocities, but experience has shown that (writing from memory) about 0.80 of these would have been better.

The Garrett diagrams do not offer any additional silt theory, but are only an alternative tool for design with provision for following the Kennedy theory. They originated in the Central Provinces of India, before diversions of rivers for irrigation had been begun there.

As to the writer making "no suggestion that these relations might be influenced by the quantity or quality of silt", there was no need even to "suggest", in 1918, what was so generally known and accepted in Indian irrigation practice. The designing diagram in the writer's paper[7] provides for varying the relation of velocity both as to depth and to width, for local conditions.

Although it has since been superseded by Lacey's later work, the writer would like to refer to a development of his work quoted in the paper; this was published in the *Panjab Irrigation Technical Review* in 1925, and its diagrams (Plates 7 and 8) with a brief explanation, are given also in *Panjab Irrigation Nomograms.*[8] These diagrams increase the widths for very large channels in accordance with data obtained, as the former paper was practically complete. Whereas the former paper dealt mainly with the data examined, the latter considered more the points to be kept in mind in designing.

A fundamental point, now widely accepted in India, but not brought out by Professor Lane, is that a given volume of flowing water carrying a given proportion of a given quality of silt tends to form a channel of which the depth, width, and gradient are all fixed by the said volume and silt. In the writer's papers, attention is called to a number of factors that are likely to operate against this tendency, temporarily, but more powerfully.

This problem is too complicated for rational solution; and it is unlikely that the rational approach will yield more than an occasional "pointer" in the right direction. For example, it may confirm or may contradict Lacey's suggestion that velocity should be correlated with hydraulic mean radius instead of with depth. For effective advance by empirical methods it is almost essential to have a "yardstick" by which to test all obtainable data, and to use these data to improve the yardstick periodically. In India and in the irrigating countries in touch with it, such a rule was provided by the Kennedy theory, followed by the others mentioned in the author's paper.

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[6] Superintending Engr., Indian P. W. D., Panjab Irrig. Branch, (*Retired*), Wotton-under-Edge, Gloucester, England.

[7] "Regime Channels", by E. S. Lindley, *Proceedings*, Punjab Eng. Congress, 1919, Plate III.

[8] On file for reference in Engineering Societies Library, New York, N. Y.

To allow for the considerable volume of data needed to yield any real information, the labor of calculating factors from observed data, and one's weak and sinful human nature, it is further almost essential to put this yardstick into the form of diagrams to facilitate the examination of data and the design of new channels which can be watched. It is highly probable that Kennedy's theory would have been still-born, if he had not also provided diagrams which facilitated work that had to be done in any case.

If the paper conveys the impression that it is now anywhere possible to design a channel precisely to the dimensions which Nature will accept for local conditions, that impression is false. Where "design" is applied to an established channel, and consists of regularizing and accepting actual conditions, this will be accurate as long as the silt-draw of all intakes above, and of all offtakes, does not change materially. Where a new channel is being designed without data for the dimensions demanded by the silt, a value must be judged for the silt factor in the formula in use, and design must err on the side on which error causes the least inconvenience when Nature gives its decision. With designing diagrams available for the different formulas, it is not difficult to calculate a number of channels indicating the range of probability if the silt factors have been wisely chosen. For the choice of silt factors, the only help to experience and judgment is in a suggestion to equate Manning's flow formula and the silt formula, and insert values of observable data taken from the river. This suggestion is due to Mr. F. R. Burkitt, of the Panjab Irrigation Department; it has not been published by him, but is developed for Kennedy's silt formula in *Panjab Irrigation Nomograms* (Plate 6*a*).

The enormous silt charge described in the paper, and also in a paper[9] by the late Carl Ewald Grunsky, Past-President, Am. Soc. C. E., entitled "The Head-Works of the Imperial Canal", is striking: even if all channels are designed to pass it in suspension, it must cause difficulty when it reaches the irrigation ditches and the fields, and so must be reduced to reasonable quantities. This should be feasible if advantage is taken of the experience available in places at which considerable advance has been made in designing offtakes to exclude silt. When that has been done, it will soon be clear whether the channels depart so materially from the dimensions of those studied.

J. C. STEVENS,[10] M. AM. SOC. C. E. (by letter).—The anomalous behaviors noted between Egyptian, Indian, and American canals and rivers in alluvium are probably due to fundamental differences in the character of the alluvium. The author discounts the effect of colloids and thinks their apparent effect may be attributed to the silt load of the stream.

When segregated, colloids appear jelly-like, whereas the cementing effect attributed to them may well be given by coarser material, such as fine clay, that would not fall in the colloid group at all. It is a common observation that clay deposits that have had an opportunity to consolidate become highly resistant to erosion as long as the beds remain intact. Once the particles become loosened from the bed by exposure and weathering, however, they are readily moved by very low velocities. Many such examples of this fact may be cited in clear streams and canals of the West.

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[9] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 262.

[10] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

If the silt carried by a canal contains a considerable quantity of fine clays or colloids it begins to consolidate as soon as it is deposited, and once deposited it requires much higher velocities to dislodge it again than to keep it moving. The writer believes, therefore, that the colloids and fine clays in the alluvium have a great influence on the stability of the streams flowing through it and are among the important reasons for the aforementioned anomalous results. The relations between size of particles and velocities that will move them, given in most textbooks, do not apply even approximately to the case in hand. Moreover, it is not the average velocities near the bed and banks, as given in Fig. 3, that are to be reckoned with. Such "isovels", no doubt, were obtained by current meter and, therefore, represent mean velocities over a certain period of time. If an instantaneous velocity indicator, such as a Pitot tube or a Bentzel velocity tube, had been substituted for the current meter, momentary velocities greatly in excess (often two or three times) of the current meter velocity, would have been observed. For want of a better name the writer has termed them "stray currents." In glass-sided laboratory channels they may be observed in action. They are almost like, and have somewhat the effect of, a whip lash. It is these stray currents that move the bed material, rarely the mean velocity that can be measured with a current meter. These stray currents are induced by the irregularities of the channel bumps, holes, ripples, etc., and are phenomena of turbulence.

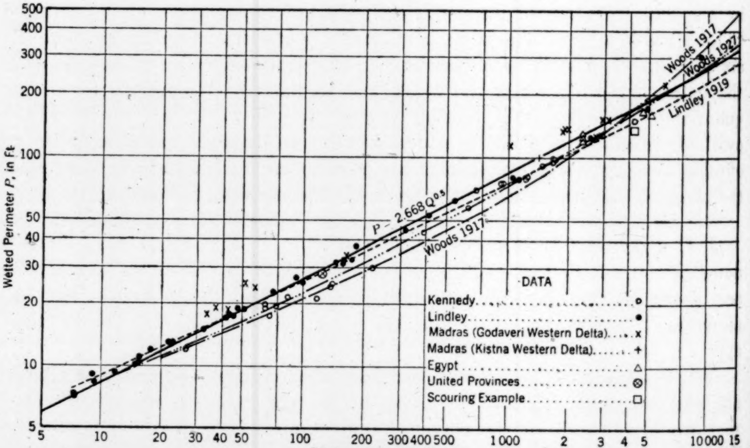

FIG. 4.—THE LACEY FORMULA, WITH SUPPORTING DATA.

If the Lacey formulas cited by the author are quite generally applicable, a stream flowing in its own alluvium would have the following remarkable properties:

(1) The cross-section on straight reaches tends to become semi-elliptical;

(2) The parameter of the ellipse (ratio of the major to one-half the minor axis or surface width to maximum depth) depends solely on the character of the silt as regards fineness;

(3) For a given discharge the wet perimeter is constant and independent of the character of silt; and

(4) The silt factor bears a definite relation to the roughness coefficient in the Kutter and Manning formulas for discharge.

Property (3) is expressed by the author's Equation (5). Fig. 4 is a reproduction of this formula from the Lacey paper[11]. Fig. 5 shows how certain alluvial channels not given by Lacey conform to this formula. The Imperial Valley canals do conform fairly well as credited by the author. The Rio Grande also conforms scatteringly. The Colorado River shows a splendid conformity to another law—not that of Lacey—as is also the case with the Huang-ho River.

It may be argued that the Lacey formula can only obtain where a given flow persists long enough to establish a stable regimen. The beds of the Colorado River and the Rio Grande respond readily to changes in flow. Figs. 6 and 7 show cross-sections of both streams during 1929. Periods were selected when the discharge was fairly constant.

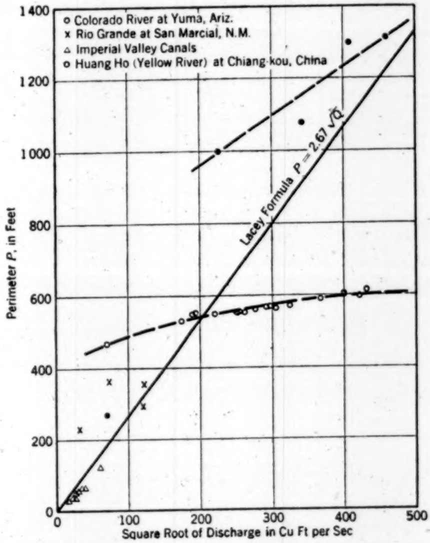

FIG. 5.—RELATION OF PERIMETER TO DISCHARGE, FOR ALLUVIAL STREAMS.

From data given by the author, particularly the range indicated by his Fig. 1, it is evident that the problem of designing canals in erodible material is an intricate and complex one in which the engineer's judgment and experience must be given a liberal measure of weight.

The writer believes, however, that there is a rational method of approach if the requisite data were in hand. The author could well have added a fourth set of conditions to his list, namely, silt-laden canals requiring hydraulic elements such that silt will be carried through while not scouring the channel—the very condition under discussion.

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[11] "Stable Channels in Alluvium", by Gerald Lacey, *Minutes of Proceedings*, Inst. C. E., Vol. 229, p. 259.

Among the thousands of canals throughout the western part of the United States, which carry silt in their alluvial valleys, will be found some that meet this condition, and do carry silt without appreciable deposition or scour.

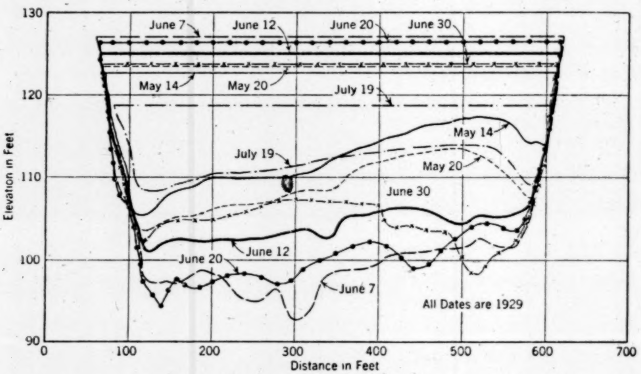

Fig. 6.—Typical Cross-Sections, Colorado River at Yuma, Arizona.

The former Special Committee of the Society on Irrigation Hydraulics attempted to gather data on such canals and channels, but with only limited success. To secure data of value, field study and measurements are required for which the Committee had no funds. Suppose, however, that complete data were in hand for canals of all sizes and slopes that are known to carry their silt load through, and that complete data were also available as to the mechan-

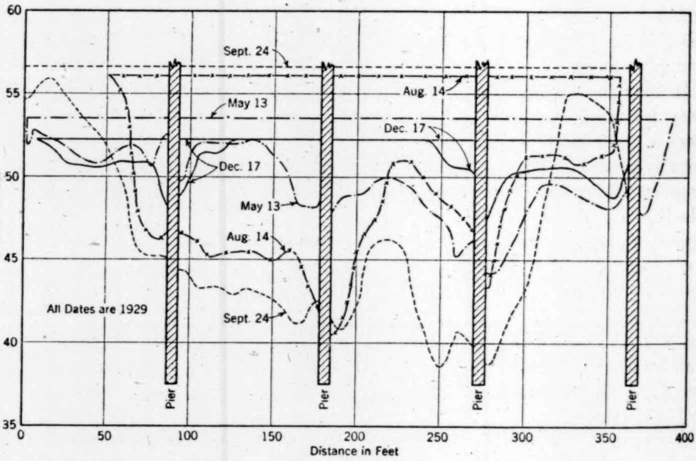

Fig. 7.—Typical Cross-Sections of Rio Grande, at San Marcial, New Mexico.

ical and physical properties of the silts so carried. Each such canal at once becomes an hydraulic model for the design of others to be constructed under similar conditions to which the principles of similitude will apply.

C. R. PETTIS,[12] M. AM. SOC. C. E. (by letter).—A clear and interesting discussion of the principles involved in the design of stable channels in erodible material is contained in this paper. Some statement as to the final types adopted for design would have been interesting.

Mr. Gerald Lacey, whose work is cited by Professor Lane, gave[11] certain equations from which the various elements of a stable channel in erodible material can be determined for any given value of discharge, Q; the various elements determined are the velocity, V, the cross-sectional area, A, the hydraulic radius, R, the width of stream, B, and the slope, S. He assumes that the stable channel has a semi-elliptical cross-section. The stable channel in the sense considered is in a straight reach, with uniform and steady flow, and of compact and fairly regular cross-section; if it is a canal it is of sufficient age to have adjusted itself to hydraulic conditions, and it is non-silting and non-scouring. Mr. Lacey's solution is unusually complete and comprehensive.

The writer has proposed an independent solution of the same problem, as applied to rivers[13], which was completed before he knew of Mr. Lacey's work. This solution was derived as a corollary to the width formula for floods: $Q \propto A^{1.25}$; from which $V \propto A^{0.25} \propto Q^{0.2}$. From data on the Miami River it was indicated that the proper value for the velocity in a stable channel is,

$$V = 0.8\,Q^{0.2} \qquad (11)$$

Attention is called to the fact that this is a main channel velocity, and that it is 2.5 times the average velocity of all the flood water, including overflow areas. Since $A = \frac{Q}{V}$,

$$A = 1.25\,Q^{0.8} \qquad (12)$$

From a Kennedy type of formula, assuming $C = 1$ for average conditions in alluvial sections of rivers, the maximum depth, d, of a cross-section will be,

$$d = V^{1.5} = 0.715\,Q^{0.3} \qquad (13)$$

The writer has investigated the shape of a natural stable channel which will best comply with known hydraulic conditions. The evidence indicates that the cross-section must be of the parabolic type indicated by $d\,B^k$; that $\propto$ k is not less than 2 nor more than 3; and that $d \propto B^{2.5}$ is probably the ideal shape for a stable channel. A semi-ellipse is not an ideal shape for the channel of a natural stream. All the writer's formulas were derived from natural streams with drainage areas between 100 and 40 000 sq miles. For such streams (but not for smaller ones) the ratio, $\frac{B}{d}$, will be sufficiently great so that it may be assumed that B is equal to P, the wetted perimeter. Then, $\frac{R}{d}$ will be the author's form factor. The form factor for the parabolic curve, $d \propto B^{2.5}$, is 0.715; for this type of curve, $R = 0.715\ d$. Consequently,

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[12] Col., Corps. of Engrs., U. S. A., Detroit, Mich.

[13] "Relation of Rainfall to Flood Run-off", *Military Engineer*, March–April, 1936, p. 94.

$$R = 0.715\ d = 0.511\ Q^{0.3} \qquad (14)$$

$$B = \frac{A}{R} = 2.45\ Q^{0.5} \qquad (15)$$

and,

$$\frac{B}{d} = 3.43\ Q^{0.2} \qquad (16)$$

Substituting the foregoing values of V and R in the Manning formula, with Kutter's $n = 0.0225$, $S = 0.00036$.

The ideal stable river cross-section is a curve of the type, $d \propto B^{2.5}$. It is a characteristic of rivers that they are subject to very variable flows. No given cross-section can be theoretically perfect for more than one value of Q. A natural channel will adapt its shape to correspond to the bank-full value of Q. Since there is a certain range of stability, such a channel will be safe for a certain amount of overflow. If the overflows are frequent or high, the original shape may be modified slightly to meet this condition. When the water falls below bank-full stage, such a channel will remain within the limits of stability for flows at medium stages. At low stages there will be a tendency to silt along the sides, which may cause some irregularity in flow, and which may be followed by some scouring. These effects will tend to correct themselves with succeeding high stages. The ideal stable channel will remain stable for all ordinary conditions, except for frequent minor and temporary instability near the thalweg.

Considering a given section of a river in connection with a given value of Q, it is evident that V must not be less than a certain minimum value, or silting will occur; and V must not be more than a certain maximum value, or erosion will occur. River channels are carved out of the bed material by erosion, and the writer's solution is based on the maximum value of V beyond which one cannot go and be certain of having a stable channel.

Mr. Lacey's formulas were derived largely from canals. Most of his values of Q were less than any the writer has considered, and his values of V are noticeably less than those of the writer. Mr. Lacey's solution represents the lower limit of safe velocities, which are primarily non-silting; and the writer's solution represents the upper limit of safe velocities, which are primarily non-scouring; and both solutions are necessary to an understanding of the problem as applied to rivers.

On the hypothesis that the Lacey solution and the writer's solution can be combined in the manner indicated, Table 2 gives the limits of stability for the various elements of a cross-section, for a given value of Q; and also intermediate values which are assumed to represent the most stable conditions.

It is assumed that V_0 may vary from $d^{0.67}$ to $d^{0.5}$, or between $d = V^{1.5}$ (non-scouring) and $d = V^{2.0}$ (non-silting). This relationship was used in obtaining a non-silting value of d, since Mr. Lacey did not include d in his solution.

TABLE 2.—ELEMENTS OF STABLE RIVER CHANNELS IN ALLUVIUM

Description	Pettis (non-eroding)	Most stable	Lacey (non-silting)
Velocity, V	$0.8\,Q^{0.2}$	$0.8\,Q^{0.183}$	$0.8\,Q^{0.167}$
Area, A	$1.25\,Q^{0.8}$	$1.25\,Q^{0.817}$	$1.25\,Q^{0.833}$
Depth, d	$0.715\,Q^{0.3}$	$0.683\,Q^{0.317}$	$0.64\,Q^{0.33}$
Hydraulic radius, R	$0.511\,Q^{0.3}$	$0.488\,Q^{0.317}$	$0.468\,Q^{0.33}$
Surface width, B	$2.45\,Q^{0.5}$	$2.56\,Q^{0.5}$	$2.67\,Q^{0.5}$
Kutter s coefficient, n	0.0225	0.0225	0.0225
Slope, S	0.00036	$\frac{0.00038}{Q^{0.055}}$	$\frac{0.0004}{Q^{0.11}}$
Ratio, $\frac{B}{d}$	$3.43\,Q^{0.2}$	$3.75\,Q^{0.183}$	$4.17\,Q^{0.167}$

Table 3 indicates values of the elements corresponding to given values of Q. The values for $Q = 1$ indicates that the Lacey and the writer's solutions are consistent for a terminal condition; $Q = 1\,000$ is fairly representative of the original data used by Mr. Lacey; and $Q = 100\,000$ is representative of the writer's data.

TABLE 3.—ELEMENTS OF STABLE CHANNELS FOR GIVEN VALUES OF Q

Description	$Q = 1$		$Q = 1\,000$		$Q = 100\,000$	
	Pettis	Lacey	Pettis	Lacey	Pettis	Lacey
Velocity, V	0.8	0.8	3.18	2.53	8	5.45
Area, A	1.25	1.25	314	395	12 500	18 340
Depth, d	0.715	0.64	5.7	6.4	22.6	29.7
Hydraulic radius, R	0.511	0.468	4.1	4.7	16.1	21.7
Surface width, B	2.45	2.67	78	84	775	844
Kutter s coefficient, n	0.0225	0.0225	0.0225	0.0225	0.0225	0.0225
Slope, S	0.00036	0.0004	0.00036	0.00019	0.00036	0.00011
Ratio, $\frac{B}{d}$	3.43	4.17	13.7	13.2	34.3	28.5

The writer's formulas are a special solution for indicated conditions. Mr. Lacey claims that his formulas are general. The two solutions have been checked with such river data as are available, and the foregoing explanation seems to be the most satisfactory hypothesis under which one can co-ordinate the Lacey assumption that V varies as the sixth root of Q, and the writer's assumption that V varies as the fifth root of Q.

The laws governing velocity distribution in stream cross-sections are not definitely known, but to a certain extent they can be inferred from a study of available diagrams. The following statements appear to be justified: (1) In the stable parabolic cross-section, the bottom velocity will decrease gradually from the thalweg to the top of the banks; (2) the gravity component described by the author will increase gradually from the thalweg to the top of the banks; and (3) the result will be a condition of equilibrium at all points of the cross-section, except at low stages. It is obvious that a semi-elliptical cross-section will not meet the conditions for equilibrium near the top of the banks. The assumption of an elliptical cross-section does not materially affect the solution if the value of d is omitted.

The writer's conclusion is that Mr. Lacey's solution, to which the author refers, is fundamentally sound to the extent indicated, but that his theories should be slightly modified and expanded.

Harry F. Blaney,[14] M. Am. Soc. C. E. (by letter).—A problem is analyzed in this paper which has not always been given serious consideration, and the author presents data and conclusions which should prove of great value. Professor Lane has illustrated some of the problems which confront the designer when he attempts to apply formulas by Kennedy and others to conditions in the Southwestern United States.

In connection with silt investigations[15] made some years ago, several efforts were made by the writer and others to co-ordinate the movement of silt in the Lower Colorado River and in canals in Imperial Valley with the laws and formula held to be applicable to the movement of silt in other streams; but although there was agreement in some features, there was disagreement in others, so that, on the whole, few satisfactory conclusions could be drawn, largely because of the character of the silt and the chemical activity of certain salts in the waters of the river. The preponderance of fine silt held in suspension, and the fact that, in its movement downward or parallel to the grade of the channel, it seems to obey physical laws different from those governing bed silt, led to the conclusion that any formula applicable to one kind of silt would not apply to the other. Furthermore, it is not generally feasible to apply two sets of laws or formulas to the same part of a channel, since silt suspended at one time and place may become bed silt elsewhere at another time. Conversely, more or less bed silt may become suspended silt.

It was not difficult to trace the relation between the movement of the finer silt and the velocity of the current. All velocities in excess of about $\frac{2}{3}$ ft per sec transported the finer silt, not only in the river, but also in the canals. The chief difficulty arose in determining the velocities and other hydraulic elements that would cause the transportation of bed silt and the heavier grade of suspended silt.

In the earlier silt investigations, experiments were conducted with the object of applying Kennedy's formula (Equation (1)), to the canals of the Imperial Valley. Kennedy states[16];

> "Strictly speaking, a separate value of V_0 should obtain at each season of the year, and for each class of sand carried in each reach of a canal, but this being impossible, the values of V_0 given must be taken as an all-round figure for what may be called the standard sand, found in the river beds near the hills. The question then arises how to define this sand, and a partial reply can be furnished, by taking as the criterion of coarseness, the rate of fall of the grains in still water. Thus, Grade 0.10 would consist entirely of grains which dropped at the rate of 0.10 feet per second; and Class $\frac{0.10}{0.15}$ would include atoms of all coarseness which fell between the rates of 0.10 and 0.15 feet per second."

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[14] Irrig. Engr., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Los Angeles, Calif.

[15] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and Harry F. Blaney, M. Am. Soc. C. E., *Technical Bulletin No. 67*, U. S. Dept. of Agriculture.

[16] "Hydraulic Diagrams for Channels in Earth", by R. G. Kennedy, Second Edition.

Kennedy's method of grading silt is described by Parker[17]. A typical analysis is shown in Table 4(*a*).

TABLE 4.—GRADING OF DEPOSITS

(*a*) SIRHIND CANAL, PUNJAB (PERCENTAGE BY VOLUME)					
Locality	$\frac{0.00}{0.10}$	$\frac{0.10}{0.20}$	$\frac{0.20}{0.30}$	$\frac{0.30}{0.40}$	$\frac{0.40}{0.60}$
Main Line:					
10 000 ft*......	4	50	30	9	7
15 000 ft*......	7	65	21	4	3
20 000 ft*......	5	53	28	7	7
Branch Line:					
150 000 ft*.....	28	56	12	4	0
170 000 ft*.....	29	57	11	3	0

(*b*) IMPERIAL CANALS, CALIFORNIA (PERCENTAGE BY WEIGHT)				
Locality	$\frac{0.000}{0.105}$	$\frac{0.105}{0.207}$	$\frac{0.207}{0.339}$	$\frac{0.339}{0.500}$
Alamo Canal:				
1 mile†.......	33.5	49.3	8.1	7.0
Eastside Canal:				
53 miles†.....	97.1	2.7	0.2	0.0
75 miles†.....	70.7	27.5	1.5	0.0
88 miles†.....	95.1	4.5	0.0	0.0

* Distance below heading. † Distance below Rockwood Heading.

Attempts to grade the Colorado River silts by Kennedy's method were not very successful as the particles would cling together and produce eddies. However, by making standard sieve analyses by weight and then determining the rate of dropping, in feet per second, for particles of various size, some results were obtained. Compilations for two canals in the Imperial Valley are shown in Table 4(*b*). Although not directly comparable, the results indicate that the silt deposits in the Imperial Canal are somewhat lighter and finer in texture than those of the Sirhind Canal in India.

TABLE 5.—BED WIDTH, DEPTH, AND MEAN, MAXIMUM, AND BED VELOCITIES FOR TYPICAL CANALS, IMPERIAL VALLEY, 1918–1919

Name of cana	Location	Bed width, in feet	Depth, in feet	$\frac{\text{Bed-width}}{\text{Depth}}$	Mean velocity, in feet per second	Maximum velocity, in feet per second	Bed velocity*, in feet per second
Date............	Meter Bridge.....	14	1.5	9.3	2.66	2.95	2.3
Westside.........	At boundary......	36	5.0	7.2	2.71	....	....
Lateral..........	Sharps Heading...	4	1.0	4.0	1.58	1.74	1.0
Central..........	Sharps Heading...	50	5.4	9.3	4.06	....	....
Brawley..........	El Centro Road...	14	6.0	2.3	3.66	4.12	1.8
No. 5 Main......	Yuma Road......	23	4.8	4.8	3.52	4.25	2.1
No. 5 Main......	Allison...........	30	4.7	6.4	4.04	5.10	2.5
Central..........	Boundary........	38	5.5	6.9	2.80	3.20	1.7
Dogwood.........	Meter Bridge.....	19	3.2	5.9	1.67	1.95	0.9
Alamitos.........	Sharps Heading...	24	3.3	7.3	2.54	2.75	1.9
Briar............	Ten foot Drop....	11	2.3	4.8	2.76	3.30	1.7
Evergreen........	Dahlia Heading...	10	1.5	6.7	1.56	1.88	0.9
Elder............	Five Gates........	10	3.1	3.2	3.00	3.40	2.2
Encino..........	Flume............	22	3.4	6.5	3.48	4.00	2.2

* Estimated.

The author indicates that additional data are desirable on the "bed width-depth ratio." Table 5 has been compiled from original records of the U. S. Bureau of Agricultural Engineering (1918–1919) and may be of use in future studies.

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[17] "The Control of Water", by the late Philip A Morley Parker, M. Am. Soc. C. E., Second Edition, p. 758.

SIGURD ELIASSEN,[18] ASSOC. M. AM. SOC. C. E. (by letter).—The subject of stable channels in erodible material is of great interest to engineers in North China, where the silt problem is extremely serious and, consequently, channel stability is difficult to attain. Measurements taken throughout 1934 show that, where the Yellow River enters its diked section, it carried in suspension the stupendous total of 1 500 000 000 cu m, or 2 400 000 000 tons, of silt as this would have been deposited on the plain in a natural dry state. It was more than an average flood-year, but far from an exceptional one. Silt measurements were taken every day throughout the entire year and twice each day during the freshet season, when the silt percentage runs high. The silt load at the place mentioned ran as high as 19% by weight at times, and at places where the river is confined between high banks, such as at Shanchow, in Western Honan, even as high as 38 per cent. On the King Ho, in the Province of Shensi, a tributary to the Wei Ho, which again is a tributary to the Yellow River, or Huang Ho, the writer has personally measured 48% by weight—that is, weight of dried silt over weight of silt plus water. Similar high percentages have been measured on other tributaries of the Yellow River. If the customary procedure for determining silt percentages is used (namely, weight of dried silt over weight of clean water), the percentages can be greater than 100 as the weight of the silt becomes heavier than the weight of the water. It does not seem logical to express it in this manner.

The measurements are dependable enough and have been observed by too many engineers to be doubted. The writer has mixed artificial silt percentages in a gasoline tin and finds that they remain quite fluid, up to 50% by weight. At 56% the "solution" begins to be sticky, and between 58% to 60% it becomes a brick-maker's paste. When fully analyzed, the measurements for 1935 are likely to show similar results as regards silt transportation.

The foregoing comments are offered only to demonstrate that American engineers have a "snap" compared to what confronts engineers in China when it comes to stability of river channels.

The author begins with the Kennedy rule for non-silting channels, $V_0 = C\ d^n$, and then reviews the work of other investigators who have included, also, the relation of width to depth, together with slope and silt character, in their formulas. He mentions that his investigations were prompted by the special conditions of the All-American Canal, for which it is claimed that the Kennedy rule does not apply. At any rate, considering the fine silt which this canal carries, surprisingly large constants have to be used to fit the Kennedy formula. Professor Lane, however, omits the hydraulic features of this canal, and the conclusions arrived at as regards its most satisfactory shape. The writer hopes that the author will include these in his closing discussion and also will show the character of silt in the form of an analysis curve, and the proportional quantity of suspended silt and "geschiebe", or bed load, carried by the canal. Professor Lane thinks that the varying quantity of suspended silt carried by the canal is a reason why the Kennedy rule does not apply. He also infers that the shape of the channel is a very important consideration when discussing stability.

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[18] Engr.-in-Chg., Survey Dept., The Yellow River Comm., Kaifeng, Honan, China.

Although the writer is in hearty agreement that channel shape is a very important factor, nevertheless, he thinks that the quantity and character of the silt transported as bed load is another highly important factor—more so than the varying quantity of the suspended silt, especially if this is fine. He has found that the Wei Pei Main Irrigation Canal, which has a slope of 1:2 330, a bed width of 6 m (19.7 ft), and side slopes of 1:1, carries fine sediment in suspension without silting the canal even when the load fluctuates from 0.1% to 10% by weight. Attempts have been made to use the canal when the silt ratio has been as high as 20 per cent. However, such attempts were discouraged not so much on account of silting of the main canal as the silting of the laterals. The silt is so fine that it all passes a 200-mesh sieve, the largest particles having a diameter of 0.03 mm. No coarse bed load is carried by this canal. It has been in use since 1933 and has not yet been cleaned out. The mean velocities in the canal, when it is running more than half-full, or depths from 1.5 m to 2.0 m (3.9 to 6.6 ft) is from 0.7 m to 1.0 m per sec. (2.3 to 3.3 ft per sec), and the discharge from 8.0 to 16 cu m per sec. (282 to 575 cu ft per sec); thus there is much fluctuation both in silt content and discharge.

From the foregoing it seems that as far as the All-American Canal is concerned there are other factors at work more important than silt variation, when considering the question of non-silting stability. As regards stability against scour the opinions expressed by the author seem very much to the point.

With regard to channel efficiency, a study of the roughness coefficient for the Yellow River shows that Kutter's *n* is fluctuating considerably at the same measuring section. Exactly what causes the fluctuation has not yet been fully discovered. The difficulties of securing good measurements for cross-section, velocity, and slope, under all conditions of flow and silt loads are obstacles to reliable conclusions. On the Yellow River the variation of silt, shape of section, slope, velocity, and discharge are so marked that it ought to be possible to come to definite conclusions in time; but from measurements which have been taken, it is quite evident that *n* is not constant. One of the reasons why it varies, perhaps, is that bottom waves of fine sand pass the measuring station from time to time. During the passage of such sand waves bottom irregularities are set up which increase the frictional resistance and raise the value of *n*. At other times there are no bottom sand waves and the frictional resistance is less with a consequent lowering of *n*. It is difficult to arrive at any conclusion as regards changes in *n*, due to a varying silt percentage.

Scour and re-fill of the bed or cross-sectional changes which proceed so rapidly on the Yellow River are, on the other hand, factors which definitely seem to influence *n*. This leads the writer to think that bed irregularities which slow up the velocity and raise the water level may cause deposits to occur, and he believes that the bed-load movement will create sufficient irregularities to cause deposition especially if the actual velocities under an ideal canal condition lie close to the silting velocities. On the other hand, under a heavy suspended silt load the coarser bed-load silt grains should be carried along more easily, since they are ‘buoyed up” by the finer silt particles in suspension, due to the effect of increased specific gravity which mainly affects particles coarser than the average.

With regard to the Kennedy formula the writer wishes to draw attention to feet and metric units. This seems very elementary; but recently he has several times encountered cases in which metric units have been introduced directly into the common form of Kennedy's formula for the Bari Doab System, $V_0 = 0.84\, d^{0.64}$, which is in English units. In metric units, this formula reads, $V_0 = 0.56\, d^{0.64}$.

In logarithmic form, the Kennedy formula reads:

$$\log V_0 = \log C + n \log d \dots\dots\dots\dots\dots\dots (17)$$

As far as can be judged, Kennedy plotted his results on logarithmic paper. The points fell rather scattered and he drew the best average straight line and obtained his constants from Equation (17). It is interesting to note that the ordinary discharge or velocity formula is also of this form when the gauge height, H, is substituted for the depth, d. Assuming cases in which the Kennedy rule is applicable, the non-silting channel should be one in which the actual velocities and the Kennedy velocities coincide, or better, the actual velocity curve lying a short distance to the right of, but parallel to, the Kennedy curve. In practice, however, the two curves seldom coincide, but cross each other at some definite velocity.

If computed velocity curves are plotted on logarithmic paper for various kinds of sections (such as triangular, trapezoidal, or rectangular sections), it will be found that the slope of the curves varies somewhat with the shapes and dimensions of the sections, especially when these sections are narrow. As the sections widen the curves become parallel. Hence, it is impossible to apply one set of Kennedy coefficients even to one irrigation system.

In order to compare the Kennedy formula with the Chezy-Kutter formula the latter may also be expressed in logarithmic form:

$$\log V = \log C + \frac{1}{2} \log S + \frac{1}{2} \log R \dots\dots\dots\dots (18)$$

Considering, first, the Chezy-Kutter formula, it seems evident that since the slope is constant for a given section of a canal, $\frac{1}{2}$ log S becomes a part of the intercept constant. If there is any change in S, the curve will be displaced parallel to itself.

Coefficient C is a function of R and also of the roughness constant, n; but C varies in the same manner throughout a considerable range in R-values for almost any value of n, and the effect on the curve due to a change in n is to displace it nearly parallel to itself. In other words, a change in n has little effect in changing the slope of the velocity curve. For the most part it changes the intercept value of the curve; but since C has also been made a function of R, it too should influence the slope of the curve.

Strictly, Coefficient C should be regarded as being made up of two parts, a constant curve intercept part and a curve slope factor. The constant intercept part, therefore, may be considered as being due to channel roughness, which is fixed within narrow limits, provided there is no bed-load movement and the channel keeps its shape otherwise. The slope factor part of C, for clear water, would then be a cross-section form factor which would change according to the shape of the cross-section. In the Chezy-Kutter formula, C has been made a variable depending to a certain extent upon R and then a general, fixed exponential factor, 0.5, made to represent the remainder of the form factor. To the writer it would have seemed more logical to have given C a fixed value according to the value of n, and then to have had an exponent of R which would change according to the shape of the cross-section. The Manning and Forschheimer formulas involve only n; but they also fix the exponent of R. It is true that form factors for areas of different shapes do not vary much, but it seems obvious that an improvement would result from making the exponent of R vary according to the shape of the section.

In the Kennedy formula, the disturbing element, silt, is introduced. Proceeding through the same line of argument it becomes obvious that the silt composition will affect C, or the intercept constant. Unfortunately, the silt composition is subject to change since floods from different tributaries may bring in different grades of silt. This is very noticeable on rivers in North China. If the silt composition is such that a part is carried as a bed load, this fact is likely to affect the roughness and disturb C still further. It may also affect the shape of the channel and thus change the exponential factor, d^n, which unquestionably can be regarded as a cross-section form factor. If the silt composition does not change much for varying silt percentages, then, up to the point where silting begins, the silt will be carried vertically and longitudinally very much in the same manner throughout the section, even when the water level fluctuates considerably, due to a varying discharge, and the channel is likely to have the same form factor and hence a constant exponent of d. In such cases both C and n can be determined reliably; but the difficulty in applying the Kennedy rule begins when the silt composition changes and a heavy "geschiebe" is introduced. For such cases it may be possible to determine the upper limit for C; but to find a reliable value for n is more difficult since anything may happen to the shape of the section when increasing the velocity to an extent which will make sure that the "geschiebe" is being transported without silting the canal. Most likely, in such cases, there will have to be desilting works. This seems to have been the case with the All-American Canal.

R. E. BALLESTER,[19] ESQ. (by letter).—Table 1 of the paper shows a wide variation in values of C and n for Equation (1) applied to non-silting and non-scouring velocities. The formula proposed by the writer (Curve No. 10, in Fig. 1 and Table 1), namely, $V_0 = 1.01\, d^{0.44}$, in English units, has a lower exponent, n, than the other formulas. The writer wishes to present some additional data, that has confirmed its application to the channel sections in which it was observed (Rio Negro, Argentina).

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[19] Prof. of Applied Hydraulics, Univ. of Buenos Aires, Buenos Aires, Argentine Republic.

In a stretch of 10 km (6.2 miles) of the main channel of the irrigation system, of 2.00-m (6.6-ft) depth, 32.75-m (107.4-ft) bottom width, and 45 cu m per sec (1 589 cu ft per sec) of discharge, it was observed in 1926 that the mean velocity was not sufficient to prevent silting and the formula, $V_0 = 1.01\ d^{0.44}$ (English), or, in metric units,

$$V_0 = 0.52\ d^{0.44} \qquad (19)$$

was then graphically established[20], taking this fact into consideration.

The channel silted gradually and, in 1933, the writer advised that the velocity in the channel be increased by widening the notched fall openings at the end of a 10-km stretch, that was causing some back-water. The advice was followed in 1934 and in 1936, after two years, the silting action has disappeared in the last 2 km (1¼ miles) of channel. The mean velocity arrived at is somewhat higher than would be necessary, because some scour has occurred along the bottom of the channel, the slopes being unaffected. The velocities registered before and after the widening of the fall are shown in Fig. 8, together with the Kennedy and Rio Negro formulas. If the Kennedy formula had been used, the writer is certain that the channel would have scoured seriously.

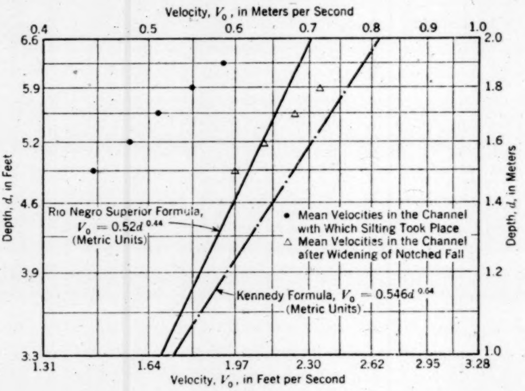

FIG. 8.

The channel cross-section now shows a tendency to scour along the bottom and to assume a semi-elliptical form. The bed-width-depth ratio of 16 for this cross-section is too high for this particular location. In alluvial valleys the soil is far from homogeneous, and there are great changes in the quality of soil, which are difficult to establish from the general topographical survey that precedes the design and alignment of the channel.

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[20] "Velocidades y Coeficientes de Aspereza para el Cálculo de Canales en el Rio Negro", Contribuciones al Estudio de Las Ciencias Físicas y Matemáticas, Série Tecnica, Vol. III, Pt. 5, p. 435, Buenos Aires, 1927.

Regarding the composition of soil in the channel of the Rio Negro System, the data in Table 6 may be of interest for comparison with other channels.

TABLE 6.—Analysis of Silt and Bottom Soil in the Main Channel of the Rio Negro System

Sample No.	Station, in kilometers	Depth	Clay contents (percentage)	Sieve Analysis: Percentage retained on Sieve No. 100	Sieve Analysis: Percentage retained on Sieve No. 200	Sieve Analysis: Percentage passing Sieve No. 200
1........	18	0.20 m below water	19.4	10.0	23.0	67.0*
2........	16	0.20 m below water	22.2	3.0	7.0	90.0*
3........	12	Bottom of channel	4.6	14.2	49.1	36.7†
4........	12.6	Bottom of channel	2.0	37.7	38.0	24.2†

* Samples of silt deposited on the side slopes.

† Samples of soil forming the channel bed (not silt).

Of course, there are many variables that combine to stabilize the shape of a channel and some of them do not depend on soil condition. In the Rio Negro System there are favorable conditions tending to minimize the troubles due to silting. Irrigation delivery is suspended in winter from the end of May to the middle of August (eighty days on the average), just when the river is in flood, carrying its heavier silt load. At the end of spring (November), there are some small floods, but the intakes are ordered closed during the peak of the flood (not more than a day), because the trouble caused by delaying the delivery of water to the 130 000 acres in the System is less than would be caused by a continuous silting of the distribution system. Special conditions such as these, which are peculiar to each system, may explain the wide variation in the critical velocity formulas in Fig. 1 and Table 1 of the paper.

Gerald Lacey,[21] **Esq.** (by letter).—Congratulations are due the author on his very able discussion of the various theories of silt transport extant when his paper was prepared. Engineers in India will immediately be struck by the very different conditions obtaining in Colorado and the great alluvial plains of India. The problem of regime flow is, in general, much simpler in India than on the Colorado River. Kennedy postulated for his channels that they should be flowing in their own self-silted beds and within self-silted side slopes; effectively they were regarded as flowing in an unlimited alluvial plain of the same silt grade as that transported. Recently, the writer has developed, on the same lines, "a general theory of flow applicable to channels flowing uniformly in incoherent alluvium"[22]. If a heavily charged channel is excavated in a medium other than the silt transported, or if the channel is afforded no opportunity of forming a reasonably thick boundary of the material transported, other factors will come into play. In certain circumstances the problem may merge from Gilbert's stream traction into flume traction[23].

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[21] Superintending Engr., Irrig. Secretariat, Lucknow, U. P., India.

[22] "Uniform Flow in Alluvial Rivers and Canals", by Gerald Lacey, *Minutes of Proceedings*, Inst. C. E., Paper No. 4893.

[23] "The Transportation of Débris by Running Water", by G. K. Gilbert, *Professional Paper No. 86*, U. S. Geological Survey.

In India canal cross-sections flowing within a boundary of silt deposit present an undeniable cup-shaped, curved, cross-section closely approximating a semi-ellipse. Professor Lane has quoted Kennedy's observations that channels carrying a heavy load had practically vertical sides and horizontal bottoms. This condition is prevalent where the channels are in "cut" and the wetted perimeter is less than the discharge can demand, or both. All recent observations have shown that when the channel is in "fill" (that is, formed by artificially constructed embankment) and the wetted perimeter assigned is sufficiently large, the cross-section tends to become semi-elliptical. Engineering practice favors the excavation of channels with horizontal bottoms, and for this reason many large channels with bottoms of stiff clay, or material more tenacious than the silt transported, preserve their horizontal beds to some extent. It may be easier for a limited depth of bed silt to slip and slide over a tenacious sub-stratum, than for the tenacious material to be picked up in the center. The difference between a scouring velocity and a silting velocity may be great. In certain embanked channels in India, with a bottom above ground level no attempt, from motives of economy, was made to raise the bed itself by "filling." It was anticipated that the defect would be corrected in time by silt deposits. It is these channels that now present the characteristic semi-elliptical cross-section. The Kheri Branch of the Sarda Canal in its lower reaches was excavated in fine sand that was almost incoherent. These reaches now present a curved cross-section in many instances of great regularity.

Professor Lane draws attention to the fact that on the Imperial Valley canals the non-silting velocity is considerably greater than the writer's formulas would indicate. The fine heavy silt charge demands a velocity of the same order as that demanded by a smaller charge of coarse silt in India. Equation (8) quoted by Professor Lane can apply only in the case of a regime charge. It was advanced by the writer as "a very rough qualitative formula for the diameter in inches of the predominant type of silt transported" and was intended mainly to assist in computing the critical velocities of natural streams in sand, shingle, or boulders, rather than as a basis of design. The silt factor is proportional to $\frac{V^2}{R}$, and it is this ratio which is required before channels can be designed. Mere measurement of the size of the particle without observations of the local velocity required to transport the charge is of little assistance. The silt charge in the Colorado River appears to be of great importance and abnormal as compared with conditions in India. The writer also feels that there are certain elements of flume traction that cannot be entirely dissociated from channel behavior on some, at least, of the Colorado canals. In channels with flume traction there is no limit to the surcharge that can be forced down provided slope is available and the banks possess some tenacity. In rivers flowing in deep alluvium, the surcharge is thrown down, and the river moves to a flank and picks up a fresh charge which it sweeps forward.

The remark by Professor Lane that the capacity of a stream to transport silt in suspension is probably proportional to its "turbulence" and to the energy expended, is illuminating. According to Gilbert the prime mode of transportation is saltation rather than suspension, and the difference is important. With flume traction and a heavy surcharge the turbulence necessary to drive the boundary layer forward would also occasion an abnormal suspended charge.

In the writer's view "turbulence" is susceptible of definition, and the ratio, $\frac{V^2}{R}$, for any grade of silt and irrespective of the charge, epitomizes "turbulence." It is thus found that fine silt with a heavy surcharge may well demand the same value of $\frac{V^2}{R}$ as a coarse silt with a smaller charge in another locality. Again, the ratio, $\frac{V^2}{R}$, is intimately connected with the energy concept.

Professor Lane has given a comprehensive list of factors entering into channel behavior, but the writer feels that it is preferable to write them down as dimensioned variables. The Buckingham theory[24] summarizes very conveniently modern practice in dimensional analysis. Briefly, the method consists of writing down all the known independent dimensioned variables, and determining dimensionless arguments therefrom. These arguments are then correlated by ordinary statistical methods. Results obtained in such a manner have the merit of dimensional homogeneity. Mere dimensional homogeneity, however, does not denote accuracy, and a poor degree of correlation may reveal that other important variables remain to be sought.

If the number of independent variables is a, the number of dimensionless arguments is equal to $a-3$. Dealing with the independent variables historically, Chezy selected, effectively, four independent variables which left him with $a - 3$ (equal to one) dimensionless arguments for which it remained only to ascertain the numerical value. Thus, $\frac{i\,R}{\rho\,V^2}$ = the Chezy number, in which, in addition to the notation of the paper, i = energy gradient; R = hydraulic radius; V = mean velocity; and ρ = density of water.

Experience showed that not only was the Chezy number a function of the roughness, but it was also a function of R and possibly, as Kutter thought, of S. An essential variable had been omitted. Attention is drawn to the manner in which the ratio, $\frac{V^2}{R}$, enters the Chezy number, a number that must survive with the addition of other dimensionless arguments as knowledge increases.

Osborne Reynolds added, as a fifth variable, the viscosity of water. This added another dimensionless argument, the classic Reynolds number; thus:

$$\frac{i\,R}{(\rho\,V^2)} = K\left(\frac{R\,V}{\nu}\right)^m \qquad \text{..........................(20)}$$

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[24] "On Physically Similar Systems; Illustration of Use of Dimensional Equations", by E. Buckingham, *Physical Review*, Vol. IV, Series 2, pp. 345–376, 1914; also, "Model Experiments and the Form of Empirical Equations", by E. Buckingham, *Transactions*, A. S. M. E., Vol. 37, pp. 263–296.

in which ν is the kinematic viscosity of water, and $\frac{R\,V}{\nu}$ is the dimensionless Reynolds number. It will be observed that the acceleration due to gravity, g, does not enter as an independent variable in Osborne Reynolds' relation. Osborne Reynolds' experiments were carried out with pipes. The boundaries were rigid and the shape was a constant.

In 1930, the writer suggested that the rugosity, or hydraulic roughness, of regime alluvial stream beds was implicit in the slope and the hydraulic mean depth which they assumed. In open channels, g enters as an independent variable. With the addition of this sixth variable a third dimensionless argument is evolved. This is none other than the Froude number, and,

$$\frac{i\,R}{(\rho\,V^2)} = K' \left(\frac{R\,V}{\nu}\right)^m \left(\frac{V^2}{g\,R}\right)^p \dots\dots\dots\dots\dots\dots (21)$$

Attention is again drawn to the manner in which the ratio, $\frac{V^2}{R}$, enters the Froude number. The silt factor is merely a simple proportion to the ratio, $\frac{V^2}{R}$, and, therefore, as long as dimensional analysis is used as an instrument in hydraulic analysis the ratio, $\frac{V^2}{R}$, must persist as a fundamental element, and with it the silt factor, which more appropriately could have been termed "turbulence." In regime channels, "turbulence" with dimensions, $\frac{L}{T^2}$, would appear to be as real a factor as kinematic viscosity with dimensions, $\frac{L^2}{T}$ (in which T = time).

The writer[22] has shown how these dimensioned arguments can be fitted to empirical relationships defined by him. Thus, substituting for the energy gradient in terms of the geometrical slope, density, and the acceleration due to gravity,

$$\frac{S\,g\,R}{V^2} = K'' \left(\frac{RV}{\nu}\right)^{-\frac{1}{3}} \left(\frac{V^2}{g\,R}\right)^{\frac{2}{3}} \dots\dots\dots\dots\dots\dots (22)$$

If the silt factor is treated as being proportional to $\frac{V^2}{R}$, Equation (22) may be solved and will prove the basis of all the writer's formulas other than those involving the discharge, thus leaving the silt factor in the ratio form, $V \propto g^{\frac{5}{9}}\ \nu^{-\frac{1}{9}}\ R^{\frac{2}{3}}\ S^{\frac{1}{3}}$; and this relationship is true for any regime channel in incoherent alluvium whether fine sand, coarse sand, shingle, or boulders. The very low power of the kinematic viscosity should be noted and the complete disappearance of the "silt factor", which is inherent in the dimensions assumed by the channel.

The formula may be re-cast in the form, $V \propto g^{\frac{5}{8}} \nu^{-\frac{1}{4}} \sqrt{\frac{R}{V}} \sqrt{RS}$, from which it follows if an equation of the Chezy type is required, $V \propto g^{\frac{5}{8}} \nu^{-\frac{1}{4}} f^{-\frac{1}{4}} R^{\frac{3}{4}} S^{\frac{1}{2}}$. The Manning power of $\frac{2}{3}$ becomes $\frac{3}{4}$, and the rugosity coefficient, n, is replaced by $f^{\frac{1}{4}}$.

The introduction of the wetted perimeter, P, as a variable gives the solution, $\frac{P}{R} \propto \left(\frac{RV}{\nu}\right)^{\frac{1}{3}} \left(\frac{V^2}{gR}\right)^{\frac{1}{3}}$, from which $P \propto Q^{\frac{1}{2}}$; and $\frac{P}{R} \propto V$. All the most recent work in India tends to confirm the expression, $P \propto Q^{\frac{1}{2}}$, as fundamental.

Since, by the writer's Equation (5)[22], $P = 2.668\ Q^{\frac{1}{2}}$:

$$P^2 = 7.12\ P\ R\ V \text{.......................} (23)$$

and, dividing through by $P\ R$,

$$\frac{P}{R} = 7.12\ V \text{...........................} (24)$$

This is the writer's empirical shape formula, and it is most unfortunate that so many of the data presented in the past are incomplete in respect of the wetted perimeter. Thus, a diagram showing $\frac{P}{R}$ plotted against V would be more illuminating than Professor Lane's Fig. 2, in which he has used the diameter of the silt as a basis for deriving the shape from the writer's formulas instead of the velocity. It is not clear from Professor Lane's paper whether or not the writer's shape formula is confirmed. The relation between the silt factor and the diameter clearly breaks down when there is a heavy silt charge, the fine silt behaving in the same manner as coarse silt in a normal channel. The writer would be very grateful if Professor Lane would plot the ratio, $\frac{P}{R}$, against V for all the Colorado River data.

Professor Lane states that the velocity along the bed should be sufficient to move all the material brought into the canal and yet not be so high as to cause the sub-grade of the canal to scour. He then goes on to state that the excess of velocity over that sufficient to move the transported material, which will attack the sub-grade, depends upon the material of the sub-grade. This concept clearly involves conditions approximating those of flume traction in which the transported material is pushed forward over a harder, more coherent, and possibly coarser sub-grade. When these conditions obtain, a horizontal bed is readily understandable. The maximum allowable velocity along the banks depends not only upon their material but also largely upon the shape factor. The contours of equal velocity ("isovels") plotted in Fig. 3, proceeding from a central nucleus of maximum velocity, show very clearly what a channel would choose to be, as opposed to what Man makes it. The suggestion of the semi-ellipse is plainly discernible and also the cramping warping effect of the vertical sides. With a tough sub-grade and a heavy surcharge, nearly all the work on the perimeter is done on the bed and little on the sides. The bed would then be nearly horizontal and the sides vertical.

If a channel is too narrow according to Equation (24) it will "kick" at the sides and this is the main reason for bank erosion. Furthermore, if the wetted perimeter is correct, but the bed as constructed is horizontal, the "velocity nucleus" of the isovel diagram will hunt about. With a channel properly constructed to a semi-elliptical trace the greatest velocity will be permanently located over the center of the channel. If a channel is constructed in material similar to that of the silt transported, or is a natural channel flowing in its own alluvium (for example, any sandy river), the central velocity will hammer the bed and curve it. Nature has nothing to say to horizontal beds when a river is flowing in a true alluvial plain.

The writer is of the opinion that a final solution of the problem of silt transport is to be found by investigation on the lines of dimensional analysis outlined by him. In India where the problem is simpler the writer's solution appears to fit a great mass of data. In the Colorado River, where the silt charge is excessive and there are many disturbing factors (notably the merging of stream traction into flume traction), practical design must depend largely on local conditions. The sub-grade as a factor is unknown in India, an unlimited bed of the same material as the silt transported being postulated. These are the conditions governing most great rivers in true alluvium. An examination of the Colorado Delta data should prove interesting.

V. V. Tchikoff,[25] M. Am. Soc. C. E. (by letter)—An excellent analysis of a complex problem is contained in this paper, and the author should be congratulated for his collection of the data accompanying it. The writer approves of most of the propositions suggested in the paper; they appear to be logical and convincing. The special consideration of the stability of the banks and the presentation of the cross-sections with the "isovels" are the most important additions to the study of the problem (see Fig. 3).

Isovels.—The purpose of a number of the rectangular cross-sections with the isovels presented by the author is to show "that high velocities extend closer toward the sides in the narrow, deep cross-sections than in the broad, shallow ones." However, an examination of Fig. 3 indicates that all cross-sections except one ($\frac{B}{d} = 0.60$, which is almost never used in practice), have the same isovel, 0.80, nearest the sides. The author's statement is perhaps more applicable to the bottom velocities, although there is a discrepancy in their distribution. The isovel, 0.80, for instance (shown in the sections with $\frac{B}{d} = 5.95$ and $\frac{B}{d} = 9.20$), disappears at the bottom of the intermediate section in which $\frac{B}{d} = 7.48$; but a more important observation is that both the side and the bottom isovels, being equal to 0.80 in most cases, do not vary much, especially if the section with $\frac{B}{d} = 0.60$ (which is not a very practical one) is disregarded.

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[25] Cons. Engr., Washington, D. C.

It is well known that the mean velocity and the corresponding side and bottom velocities vary considerably in stable channels of the various sizes carrying the same sediment. Kennedy's stable sections[26] serve as an example; the mean velocity in the smallest section is 1.31 ft per sec, whereas, in the largest section, the velocity is 2.86 ft per sec. This variation is still more noticeable in Lindley's sections[27], the mean velocities ranging from 1.03 to 3.61 ft per sec, or about 350 per cent. Table 7 further illustrates how the velocities are augmented with the growing of the width-bed ratio. These changes of velocity are incomparably larger than the rate of variation, if any, of the isovels near the side and bottom in the cross-section presented by the author.

TABLE 7.—DEPTH AND VELOCITY IN RECTANGULAR STABLE CHANNELS, CORRESPONDING TO VARIOUS WIDTH-DEPTH RATIOS

Item No.	Width-depth ratio, $\frac{B}{d} = r$	HYDRAULIC NUMBER, $N = 40$				HYDRAULIC NUMBER, $N = 20$			
		Kinetic Factor, $K = 0.04$		Kinetic Factor, $K = 0.16$		Kinetic Factor, $K = 0.04$		Kinetic Factor, $K = 0.16$	
		Depth, d, in feet	Velocity, V, in feet per second	Depth, d, in feet	Velocity, V, in feet per second	Depth, d, in feet	Velocity, V, in feet per second	Depth, d, in feet	Velocity, V, in feet per second
1.....	3.20	2.32	1.36	2.32	2.71	9.28	2.71	9.28	5.42
2.....	5.95	3.02	1.71	3.02	3.41	12.08	3.41	12.08	6.82
3.....	11.00	5.56	2.46	5.56	4.92	22.24	4.92	22.24	9.85
4.....	20.00	13.00	3.90	13.00	7.80	52.00	7.80	52.00	15.60

Velocity Gradient.—The approximate value of the velocity gradient for the middle vertical can be calculated by dividing the difference of the two neighboring velocities or isovels by the vertical distance between them. This distance, however, scarcely changes for the low isovels (0.80, 0.90, and 1.00) in the greater part of the sections shown in the paper. There is a minor irregularity in the position of the isovel, 1.10, and a more noticeable one in that of the isovel, 1.20. The maximum velocity gradient is usually found in the low parts of the cross-sections, but it remains practically constant in the cross-sections of the paper.

"Pressure Gradient."—The action of the sides of a channel is mainly typified by the depression of the filament of maximum velocity. This is due to the existence of the transverse current in the flow, which was observed and experimentally demonstrated by Dr. A. H. Gibson[28]. The double-spiral-motion theory and the existence of the cross-currents with the direction of their flow toward, and up along, the sides of a channel could explain, reasonably well, such observed phenomena as the undermining of the low parts of the sides of channels, the rounding up of the angles between the bottom and sides, and the silting of the upper parts of the sides.

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[26] "Stable Channels in Alluvium", by Gerald Lacey, p. 286.

[27] *Loc. cit.*, p. 287.

[28] "On the Depression of the Filament of Maximum Velocity in a Stream Flowing Through an Open Channel", by A. H. Gibson, *Proceedings*, Royal Soc. of London, Vol. 82-A, 1909.

The sum of the potential and kinetic energy is constant for all the points of the stream filament that lie on the same horizontal plane. Applying this condition, the "pressure gradient" that causes the transverse current may be expressed by the formula:

$$\frac{V_c^2 - V_s^2}{2g\,\frac{B}{2}} = \frac{V_c^2 - V_s^2}{g\,B} \qquad \text{.................... (25)}$$

in which V_c = a velocity at the center of the cross-section; and V_s = a velocity on the same horizontal plane at the side of a channel. The isovels do not vary much in the cross-section in Fig. 3, but the value of B changes considerably. The broader the channel, the smaller the "pressure gradient" and the transverse currents, and the more stable are its banks. The effect of the sides of a channel is to produce the curvilinear movement of the flow, the cross-sectional profile of the water surface usually being a curve. As a result, the actual vertical pressure differs from the hydrostatic-pressure triangle. This difference depends upon the rate of changes of the pressure heads corresponding to the near-by velocities.

The theoretical equation for the vertical velocity curve is a parabola of the following type:

$$V = a\,y^2 + b\,y + c \qquad \text{.................... (26)}$$

The equation for the corresponding pressure heads will be,

$$h_p = \frac{1}{2\,g}\,(a\,y^2 + b\,y + c)^2 \qquad \text{................ (27)}$$

and the rate of the changes of these pressure heads, S_p (= pressure gradient), is,

$$S_p = \frac{h_p}{dy} = \frac{V}{g}\,S_\epsilon \qquad \text{.................... (28)}$$

in which S_ϵ is the velocity gradient. Due to the combined effect of the isovels and the velocity gradient changes, both leaning in the same direction, the variation of the pressure gradient in Fig. 3 is greater than the change of each component, but even this double effect cannot explain the decidedly different property of the sections in Fig. 3 in regard to their relative stability. The reason for this, in the writer's opinion, is that each individual cross-section will fit a certain set of the hydraulic conditions upon which its stability depends. The condition for stability remaining unchanged, the width-depth ratio is determined according to the discharge of flow. For explanation of this statement, three general hydraulic factors should be considered: The kinetic factor; the coefficient of hydraulic similtude; and the hydraulic number; which have been introduced by the writer elsewhere.

Kinetic Factor.—The kinetic factor[30] is equal to,

$$K = \frac{V_m^2}{g\,R} \qquad \text{........................ (29)}$$

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[30] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 171, (Equation (73)); also, pp. 852–858.

It is a measure of the kineticity of flow, or turbulence. Equation (29) is similar to Mr. Lacey's formula expressed as Equation (7) in the paper, except that the kinetic factor has a definite physical significance. Equation (29) may be rewritten in the form:

$$V_m = (g\,K)^{0.5}\ R^{0.5} \qquad (30)$$

"Sediment Equivalent."—Lacey correlates his silting factor to the diameter of the bed material by Equation (8) which, expressed in terms of the kinetic factor, is as follows:

$$D = 8.6\,K \qquad (31)$$

Lacey gave no precise indication of the classification of the sediment; nor has he considered the quantity of the material transported. It should be noted, however, that both Kennedy and Lacey connected their formulas to the sediment taken from the bed of canals or rivers.

By designating the hydraulic value of a particle by ω (the settling velocity in the water) and the concentration of the sediment by η (percentage by weight), the work per second (power) produced by the settlement in the water of a given sample of sediment may be termed the "sediment equivalent" and expressed by:

$$E = \frac{\rho_s - \rho_w}{\rho_s} \sum (\omega\,\eta) \qquad (32)$$

in which ρ_s is the density of the particles; ρ_w is the density of the water; and $\Sigma\,(\omega\eta)$ = the sum of $\omega\eta$ for each class of particles according to mechanical analysis. If ω is measured in feet per second and η in pounds (in 100 lb of water), E is in foot-pound units. For computing E it is convenient to use the mean hydraulic value of a sample, ω_m, which is equal to $\frac{\Sigma\,(\omega\,\eta)}{\eta_t}$, in which η_t is the total concentration. If each class of particles in a mechanical

TABLE 8.—MECHANICAL ANALYSES, MEAN HYDRAULIC VALUES, AND SEDIMENT EQUIVALENTS, FOR TWO CANALS IN IMPERIAL VALLEY, CALIFORNIA

Canal	PERCENTAGE OF SILT PASSING A GIVEN SIEVE (SIEVE NUMBERS IN MESHES PER INCH)							
	10	20	40	60†	80	100	200	300
Hydraulic value, ω_a*	0.361	0.191	0.111	0.073	0.054	0.033	0.013	0.0036
Alamo...........	0	0	0.91	....	5.35	16.34	15 38	62 02
Brawley.........	0	0	0.54	....	8.86	13.58	17 38	59.64

Canal	Location	Depth to sample, in feet‡	Mean hydraulic value, ω_m, in feet per second	Concentration of sediment η (percentage of silt by weight)	Sediment equivalent, E	Kinetic factor, K	Ratio, e, $\frac{E}{K^2}$
Alamo...........	Hanlon §.	9.8	0.014	0.486	0.0043	0.075	0.76
Brawley..........	Central . .	4.2	0.015	0.542	0.0051	0.074	0.93

* Average hydraulic value of particles, corresponding to the sieve analysis, in feet per second. † 60-mesh sieve omitted. ‡ 0.2 ft above the canal bed, in each case. § 120 ft from the east bank.

analysis were indicated by the percentage, ρ, of the total concentration, the mean hydraulic value would be equal to $\frac{\Sigma\,(\omega\,\rho)}{100}$ (see Table 8). The "sediment equivalent" may be used as a criterion for the comparison of sediments, and it is rightly comparable to the kinetic factor.

Quantity and Quality of Sediment.—If two samples of the sediment, each consisting of only one size of particles of the same density, have the same value, E, the following equation is applicable: $\omega_1\,\eta_1 = \omega_2\,\eta_2$; or,

$$\frac{\eta_1}{\eta_2} = \frac{\omega_2}{\omega_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

Equation (33) means that the concentration of the sediment varies in reverse proportion to the hydraulic values of the particles. This relation appears to reveal the approximate general law of the transportation of sediment in suspension. The law may be expressed in general terms:

$$\frac{\eta'_t}{\eta''_t} = \frac{\omega''_m}{\omega'_m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

That is, if the "sediment equivalents" are equal, the total concentration of the sediment transported in suspension varies in reverse proportion to the mean hydraulic value of the samples. There is a certain limit for the maximum hydraulic value, $\omega_{max.}$, of the particles that are found in the samples, which have the same value as the "sediment equivalent."

Kinetic Factor and "Sediment Equivalent."—The condition for the transportation of the sedimentary material near the bed in a stable channel in erodible material is of primary importance. The suspended material and bed-load merge into each other near the bed. This is especially true concerning the flow which is heavily charged with fine sediment, as the bed-load differs but little from that carried in suspension. Even if the sand were presented in a noticeable quantity, the same merging effect would occur and the coarse particles are often found at a relatively high distance from the bottom. The kineticity of the flow is directly related to the transportation of the sedimentary material near the bed, especially if this material is not of a coarse nature. Since the kinetic factor is a measure of turbulence and the sediment equivalent serves as a silt criterion, they should be related to each other. On the basis of certain considerations, it appears that their relation may be stated by the following equation:

$$E = \alpha\,K^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

in which α is a coefficient. The kinetic factor, K, is a dimensionless number, and, therefore, α is in foot-pound units. The determination of α for the two Imperial Valley canals[30] is demonstrated in Table 8. The average hydraulic value for each class of particles (see Table 8), more or less corresponds to the Hazen formula[31]. One-half the hydraulic value of the particles corresponding to the 300-mesh sieve is assumed for the sediment passed through this sieve. Although this class of sediment constitutes a considerable portion of the total concentration, its work during the settlement is relatively small. The silt samples shown in Table 8 are the nearest to the bed of the canals, being in both cases 0.2 ft above the bottom. With the density of the sediment assumed as 2.65, the values of the coefficient, α, are equal to 0.76 and 0.93. The lack of information in regard to the stability, and especially in regard to "degree of concentration" of the sediment, makes it difficult to appreciate these values.

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[30] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and Harry F. Blaney, M. Am. Soc. C. E., Tables 55 and 56, p. 75.

Degree of Concentration.—Three states of the flow carrying the sediment are distinguishable: (*a*) The over-loaded flow; (*b*) the normally loaded flow; and (*c*) the sub-normally loaded flow. The criterion for this general classification is the relation of the sediment load to the kineticity of the flow.

The flow, with a certain kineticity, will only carry the definite sediment load, provided, of course, the necessary quantity of sediment is supplied. This will be a normal sediment-loaded flow. Any increase in the supply of the sediment will over-load the flow with the result that the extra supply will drop. On the contrary, the under-loaded flow tends to pick up the material from the bottom and will scour the channel, depending upon the bed resistance. If the sediment is non-cohesive, there would be a tendency toward the normally loaded flow.

The value of the coefficient, α, depends upon the degree of sediment concentration. For the normally loaded flowing water in a stable channel, it seems possible to suggest that the value of α would be equal approximately to a unit if the sediment at the bottom and the kinetic factor are considered. It is difficult to obtain a representative sample of the sediment transported at the bottom, due to the merging situation. If the sediment from the bottom is considered, the nominal sediment concentration may be computed from Equation (35) on the basis of mechanical analysis of the bed material and under the assumption that $\alpha = 1$.

Sediment Concentration at the Bottom.—Twelve mechanical analyses of bed sediment in the Imperial Valley canals for various stations, at distances of 48 to 104 miles from the river, are given by Messrs. Fortier and Blaney[32]. The result of the calculation of the mean hydraulic values, ω_m, for these samples ranges from 0.023 to 0.047. However, the several highest values of ω_m have been influenced by the local conditions, as the wind-blown sand changes the character of the bed deposit. The average value for all twelve stations is 0.0324, and it is reduced to 0.03 if the three highest values are disregarded. The value of the kinetic factor for the Imperial Valley canals is analyzed in another part of this discussion (see "Stable Canals of Imperial Valley Project"). Its average value is equal to 0.117. Substituting this value into Equation (35) and assuming $\rho_s = 2.65$, the concentration, η, is 0.73 (percentage by weight). This number corresponds to the mean hydraulic value of 0.03, and the concentration will vary directly with ω_m and K. More information on this point is needed, but the writer believes that the foregoing value is more or less comparable to the concentration of the normally sediment-loaded flow in the Imperial Valley canals. It should be noted that the foregoing consideration refers to the non-silting non-scouring canals.

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[31] "Distribution of Silt in Open Channels", by J. E. Christianson, *Transactions*, Am. Geophysical Union, 1935, Pt. II, p. 481.

[32] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and Harry F. Blaney, M. Am. Soc. C. E., Table 49, p. 72.

With reference to the canals of British India, the writer has in his possession[33] the analysis of the sediment from the bed of the Sutlej River which was considered by Kennedy as one of the typical analyses corresponding to his formula,

$$V_0 = 0.84\, d^{0.64} \qquad (36)$$

This analysis is as follows:

Hydraulic value, ω, in feet per second.	Percentage by volume	Hydraulic value, ω, in feet per second.	Percentage by volume
0.00 to 0.05	11	0.20 to 0.30	7
0.05 to 0.10	43	0.30 to 0.40	2
0.10 to 0.15	25	0.40 to 0.50	1
0.15 to 0.20	11		

The mean hydraulic value of this sample is equal to 0.114, or approximately four times larger than the bed sediment of the Imperial Valley canals. Substituting into Equation (35) and taking $K = 0.0415$ (for Equation (36)[34]), $\rho_s = 2.65$ and $a = 1$, the concentration is approximately equal to 0.0242% by weight, or approximately a ratio of 1 to 4 130 by weight. This quantity is comparable to the sediment concentration of the Sirhind Canal which takes its supply from the Sutlej River. The concentration in this canal ranges from 1 in 3 300 to 1 in 9 000 by volume[33]. The highest concentration causes the silting, whereas the scouring was observed at the time of low concentration. Therefore, it may be assumed that the average (approximately 1 to 4 000 by volume) is the normal concentration that is more or less comparable to 1 in 4 130 by weight. The foregoing calculations are approximate, but the purpose of their presentation is to indicate the existence of a quantitative relation between the kinetic factor and the sediment at the bottom of the stable channels. It is also possible to show that the kinetic factor and the "pressure gradient" near the bottom are directly related.

Coefficient of Hydraulic Similitude and Hydraulic Number.—The knowledge of the kinetic factor is not sufficient for determining the dimensions of a stable channel. In addition, one of the two other factors should be known: Either the coefficient of hydraulic similitude, denoted as Y:

$$Y = \frac{Q}{R^3} \qquad (37)$$

or, the hydraulic number, N, related to Y by the equation:

$$Y = N\, K^{0.5} \qquad (38)$$

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[33] The data are taken from the writer's publication on "Silting of the Irrigation Canals" (Petrograd, 1915), which was the result of his trip to British India.

[34] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 853.

The coefficient of hydraulic similitude defines the condition that the discharges of similar stable channels are in proportion to their hydraulic radii, raised to the third power. The hydraulic number gives the relation between the other two factors. Representing area of cross-section of a channel by $A = \beta R^2$ and substituting the values of K and Y from Equations (29) and (37), Equation (38) gives:

$$N = g^{0.5} \frac{\beta}{R^{0.5}} \quad \text{.......................(39)}$$

in which β is a "hydraulic" shape factor. The writer has presented the values of these factors elsewhere[35]. The rectangular cross-sections in this paper furnish an opportunity to examine this type of channel by applying the aforementioned factors. For a stable channel of rectangular form, the relation between depth, hydraulic number, and width-depth ratio, $r = \frac{B}{d}$, is expressed by the following equation:

$$d = \frac{g\,(r + 2)^5}{N^2\, r^3} \quad \text{.......................(40)}$$

which indicates that the depth in the rectangular non-silting non-scouring channels depends upon only the hydraulic number and the width-depth ratio. Equation (40) is presented by a series of diagrams on Fig. 9. By substituting for r, its equivalent, $\frac{B}{d}$, Equation (40) may be rewritten in a form, giving the direct relation between the depth and the width.

The paper includes two diagrams, Figs. 1 and 2, which give the velocity depth and the bed-width-depth relation for numerous formulas. It is natural that the critical velocity-depth relations (Fig. 1), being contingent on the kinetic factor and the character of the sediment, should vary considerably. The formulas of the type of Equation (1) are analyzed elsewhere in this discussion.

In regard to the equations for width-depth relation (Fig. 2), the author, taking the width corresponding to the 5-ft depth for two formulas, the Molesworth-Yenidunia formula and the Lindley formula, remarks that a ratio of maximum to minimum is of 781 per cent. It is also natural that the bed-width corresponding to the same depth must vary considerably, as it is dependent upon the hydraulic number, as illustrated by Fig. 9.

The width-depth formulas and the velocity-depth equations should be studied together if they were developed for the same conditions. Two previously mentioned formulas, the Molesworth-Yenidunia for $S = 0.00010$ and 0.00007, and the Lindley formula, are drawn on Fig. 9. Their positions are quite consistent with the remainder of the curves. An hydraulic number of about 20 is rather typical of the Egyptian canals, whereas that of the Lindley formula lies between 35 and 45. However, it should be repeated that the curves in Fig. 9 are for rectangular channels. Their application to other forms of channels, therefore, has some limitations.

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[35] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), pp. 853–855, Tables 5, 7, and 8.

Stability of Rectangular Cross-Sections.—Assuming the values of the kinetic factor, the hydraulic number, and the width-depth ratio, the other hydraulic elements can be calculated for the rectangular cross-sections. For the computation of the corresponding discharge, the following formula applies:

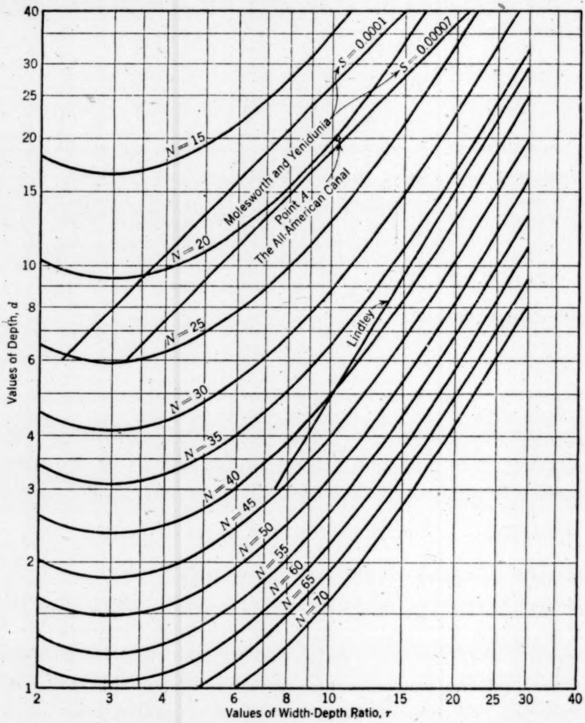

FIG. 9.—DEPTH, WIDTH-DEPTH, RATIO, AND HYDRAULIC NUMBER FOR RECTANGULAR, NON-SILTING, NON-SCOURING CHANNELS.

$$Q = g^3 \frac{K^{0.5}}{N^5} \left[\frac{(r + 2)^2}{r} \right]^6 \qquad (41)$$

Table 7 gives the depth and velocity for the various values of K, H, and r. It is obvious from Equation (30) that the velocity changes in proportion to the square root of the kinetic factor. For the same width-bed ratio, the influence of the hydraulic number upon the velocity is that the latter varies in reverse ratio to the hydraulic number.

The width-bed ratio of the cross-sections in Fig. 3 is given in Item Nos. 1, 2, and 3, of Table 7. Considerable variations of the velocity should be noted for the cross-section that has the same width-bed ratio. For example, the range of velocities in cross-section, $r = 5.95$, is from 1.71 to 6.82 ft per sec, depending upon the values of K and N. The examination of the velocities for the same value of K and N (vertical columns) also shows that they vary considerably They increase with the growing value of the width-depth ratio. There is a practical limit for the increase in the velocity, mainly because of irregularities in the bottom of the canal. However, for a given value of K, N, and Q, the only section of a definite width-depth ratio will be stable, at least theoretically, for the non-cohesive sediment.

For an illustration of the manner in which the foregoing relations could explain the condition of unstable channels, an example is presented. Among Lacey's data[36] there is a cross-section of the Upper Bari Doab Canal, which has "a velocity approximating Kennedy's regime velocity, yet it had scoured its bank for 20 yars past." The actual section of this canal is shown in the solid line on Fig 10. A "normal" section is drawn in dotted lines, and both sections are assumed to be of the rectangular form. The "normal" section is determined, using the same discharge and assuming that $K = 0.0415$ and $N = 35.3$, the values that are in Kennedy's formula for this project. Comparing the two sections, the canal scoured to the depth of 9.9 ft instead of the "normal" depth of 9.5 ft, but its "normal" width should be 16.2 ft wider. This condition naturally causes scouring of the banks, which apparently are more resistant than the "normal" ones should be.

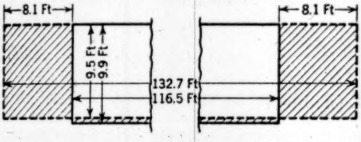

FIG. 10.—ACTUAL AND NORMAL CROSS-SECTIONS, UPPER BARI DOAB CANAL, MAIN BRANCH.

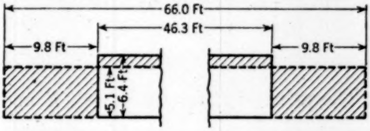

FIG. 11.—VARIATION OF NON-SILTING, NON-SCOURING SECTIONS, EAST HIGHLINE CANAL, AT B HEADING, IMPERIAL VALLEY, CALIFORNIA.

Formulas of the Type of Equation (1).—This type of formula has been used extensively for the expression of the relation between the critical velocity and depth, as is well illustrated by Table 1. The same kind of formula, however, can be developed from the data of Table 7. For example, for a velocity and depth corresponding to $K = 0.04$ and $N = 40$, four variations of Equation (1) may be written with two unknowns, C and n. Numbering these equations (A), (B), (C), (D), in the order of the bed-depth ratio values in Table 7, there are six combinations of these equations for finding the unknown value of the exponent, n. The results are as follows:

Combinations of Equations:	Values of n:	Combinations of Equations:	Values of n:
(A) and (B)	0.87	(B) and (C)	0.60
(A) and (C)	0.68	(B) and (D)	0.57
(A) and (D)	0.61	(C) and (D)	0.54

An examination of these and similar data for the other values of K and N, leads to several interesting conclusions.

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[36] "Stable Channels in Alluvium", by Gerald Lacey, p. 289.

The exponent, n, is not influenced much by the values of K and N; it depends primarily upon the width-depth ratio. The larger this ratio, the smaller the value of n will be. Its limit is equal to 0.50 for a channel of indefinite width. The range of the width-depth ratio in the canals that have been observed for the development of a specific formula of the type discussed, mainly if not primarily determines the value of n. The more channels with small width-depth ratios that are included, the greater will be the value of n. For instance, the width-depth ratio in the twenty-two cross-sections of the Bari Doab Canal which served for the development of Kennedy's formula, with $n = 0.64$, ranges from 3.5 to 12.7, the group from 3.5 to 5.0 constituting more than 40 per cent. For the data given previously, 0.66 is the average of all six values of n, and this average would be 0.57 if Equation (A) pertaining to the lowest value, $r = 3.20$, is excluded. Assuming 0.57 for the exponent, n, the values of Coefficient C would be as follows: 0.85, 0.92, and 0.93, the mean being 0.90. Consequently, the form of Equation (1) becomes:

$$V_o = 0.90\, d^{0.57} \quad \text{.........................(42)}$$

The value of C for the others, K and N (Table 7) in the formulas:

$$V = C\, R^{0.5} \quad \text{.........................(43)}$$

and

$$V = C\, d^{0.57} \quad \text{.........................(44)}$$

is given in Table 9.

TABLE 9.—COEFFICIENT, C, IN EQUATIONS FOR VELOCITY IN RECTANGULAR STABLE CHANNELS

Description	HYDRAULIC NUMBER, $N = 40$		HYDRAULIC NUMBER, $N = 20$	
	Kinetic factor, $K = 0.04$	Kinetic factor, $K = 0.16$	Kinetic factor, $K = 0.04$	Kinetic factor, $K = 0.16$
Equation (43). .	1.135	2.27	1.135	2.27
Equation (44). .	0.90	1.80	0.82	1.64

It is of interest to mention that Lorenz G. Straub, Assoc. M. Am. Soc. C. E., found the exponent to be equal to 0.56 for the non-silting, non-eroding condition in his study on the bed-sediment transportation.[87]

Unlike the exponent, n, which is practically independent of the kinetic factor and the hydraulic number, the value of the coefficient, C, depends upon both of them. For the same value of n and r (rectangular channels considered), the ratio of the C-values may be expressed by the equation:

$$\frac{C_2}{C_1} = \left(\frac{K_2}{K_1}\right)^{0.50} \left(\frac{N_2}{N_1}\right)^{3n-1} \quad \text{..................(45)}$$

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[87] "Hydraulic and Sedimentary Characteristics of Rivers", by Lorenz G. Straub, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, April, 1932.

A similar type of equation may be developed by using one-half, or the entire, width of the channels. If d in Equation (1), is equal to $\frac{B}{2}$, the coefficient, C, will change slightly, but the exponent, n, will be noticeably less than in an equation of the usual type.

Stable Canals of Imperial Valley Project.—William T. Collings, Jr., M. Am. Soc. C. E., has published the hydraulic elements for six canals of the Imperial Valley Project[38]. His data have been used by the writer in preparing Table 10, giving the general hydraulic factors for these canals. For the first four canals (Items Nos. 1, 2, 3, and 4), all the sections but one have been taken, which are considered by Mr. Collings as non-silting, non-scouring sections. Average values for the kinetic factor, coefficient of hydraulic similitude, and hydraulic number are presented in Columns (8), (9), and (10), of Table 10. These data permit the division of these six canals into two groups.

TABLE 10.—GENERAL HYDRAULIC FACTORS, CANALS IN THE IMPERIAL VALLEY, CALIFORNIA

Item No.	Canal (1)	Location (2)	Discharge, Q, in Cubic Feet per Second, Ranging:		Mean Velocity V_m, in Feet per Second, Ranging:		Hydraulic slope, S (7)	Mean kinetic factor, K_m (8)	Mean coefficient of hydraulic similitude, Y_m (9)	Mean hydraulic number, N_m (10)
			From (3)	To (4)	From (5)	To (6)				
1	Alamo	Alamo Mocho, Meter Station, in Mexico	2 900	3 600	4.6	5.0	0.000255	0.127	19.6	54.9
2	East Highline	"B" Heading	705	1 300	3.5	4.4	0.000322	0.108	12.1	37.0
3	Central Main Canal	Alamo Mocho Meter Station	500	750	3.5	4.0	0.000440	0.120	13.1	37.7
4	Briar	International Boundary Line	180	230	2.9	3.1	0.000382	0.115	15.3	44.5
5	West Side	Boundary Meter Station	400	750	2.8	3.3	0.000525	0.063	5.7	22.7
6	West Side Main Canal	Drain Meter Station, in California	450	650	2.9	3.4	0.000380	0.064	4.9	19.3

The first group consists of the four canals (Items Nos. 1, 2, 3, and 4) having practically the same kinetic factor, which ranges from 0.108 to 0.127, the average being 0.117 ± 8 per cent. The two remaining canals (Items Nos. 5 and 6) belong to the second group, having practically the same kinetic factor (0.063), but it appears that both canals are not stable and the material composing their cross-sections differs greatly from that made by the sediments of the Imperial Valley Canal. In regard to Item No. 5, Table 10 (West Side, Main Canal, Boundary Meter Station, California), Mr. Collings made a remark that "the sides and bottom show a hard and tight clay, and an eroded rough surface." The other canal (Item No. 6, Table 10) is called "self-maintaining" by Mr. Collings: "What silting may take place at low heads is relieved when these heads increase, and no cutting of the banks takes place at maximum heads." It should be noted that the hydraulic number is about

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[38] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 549.

20 in these two canals, or approximately one-half as much as in the canals of the first group. These values of kinetic factor and hydraulic number are comparable with those that have been found by the writer in eroding drainage channels.[39] Hence, the writer is inclined to believe that the second group belongs rather to the canals scoured in "earth", whereas the first group is typical for the Imperial Valley canals, which are built up, or strongly affected, by the sediment transported by the system. Accepting the kinetic factor, $K = 0.117$, as a typical one for this project, the equation for the mean critical velocity will take the form:

$$V_0 = 1.93\, R^{0.5} \qquad (46)$$

Equation (46) covers a considerable range of discharges, from 180 to 3 600 cu ft per sec, and of velocities, from 2.9 to 5.0 ft per sec.

Influence of Fine Particles.—For estimating the cohesive force between the particles, Dr. Herbert Chatley suggests the following formula[40]:

$$V_e = \frac{0.02}{D} \qquad (47)$$

in which D is a diameter of the particle, in centimeters; and V_e is a velocity in centimeters per second, which will just produce the necessary friction force to rupture the molecular bonds; that is, to erode the fine material.

Although the application of Equation (47) is perhaps limited, it offers a concrete idea as to the influence of the fine particles on scouring. This is illustrated by Table 11 in which the velocity, V_e, is computed by Equation (47). There is no definite limit to the size of colloidal particles, but it is generally assumed that particles with a diameter smaller than 0.001 mm (1 micron) are colloidal. For such particles, V_e = 6.56 ft per sec. On the other hand, the hydraulic value of a particle represents the gravity action in the water and is equal practically to zero for colloids. The relative values of cohesion and gravity for other sizes of particles are given in Table 11.

TABLE 11.—ERODING VELOCITY AND HYDRAULIC VALUE

Class of particle	Diameter D, in millimeters	ERODING VELOCITY, V_e		Hydraulic value, ω, in feet per second
		In centimeters per second	In feet per second	
Fine gravel to coarse sand	1.00	0.2	0.0066	0.3280
Fine sand to very fine sand	0.10	2.0	0.0656	0.0262
Very fine sand to silt	0.05	4.0	0.1312	0.0095
Silt to clay	0.005	40.0	1.3120	0.0001
Colloids	0.001	200.0	6.5600	0.00004
Clay	0.0001	.....	......	......

There is a large portion of the material in sediment and soils of the Imperial Valley Project, which passed a No. 300 sieve (D = about 0.0043 mm), that approximately separates silt and clay. The presence of colloids in this class makes the sedimentary material rather coherent and considerably affects the scouring resistance, especially of the banks of the Imperial Valley canals. The steep banks of the channels in many localities, including the Imperial Valley, are explained by the cohesion forces of the very fine particles.

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[39] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 857, Table 8.

[40] "Problems in the Theory of River Engineering", by Herbert Chatley, Lond., 1929, p. 17.

Silting and Scouring Limits.—There are two critical conditions or limits in a non-silting non-scouring channel. The low limit—the non-silting section—is the point at which the deposition of material stops. The upper limit —the non-scouring section—is the point at which the scouring begins. It has been established that even the sediment recently deposited would not scour again until the velocity was increased to the larger value of the silting velocity. The extent between these two limits might be considerable.

TABLE 12.—GENERAL HYDRAULIC FACTORS; EAST HIGHLINE CANAL AT "B" HEADING; IMPERIAL VALLEY, CALIFORNIA ($S = 0.000322$)

Item No.	Date	Hydraulic radius, R, in feet	Velocity, V, in feet per second	Discharge, Q, in cubic feet per second	Kinetic factor, K	Coefficient of hydraulic similitude, Y	Hydraulic number, N	Depth, d, in feet	Width, B, in feet	Width-depth ratio, r
1	1- 7-1928	3.42	2.16	405.6	0.042	10.1	49.3	4.0	46.5	11.6
2	12-25-1928	3.35	3.25	593.5	0.098	15.8	50.5	3.9	46.6	11.9
3*	11-19-1928	3.68	3.47	705.1	0.102	14.1	44.4	4.4	46.8	10.7
4*	1-28-1928	3.94	3.62	801.2	0.103	13.1	40.8	4.7	46.6	9.9
5*	10-10-1928	4.17	3.83	900.4	0.109	12.4	37.4	5.1	46.1	9.1
6*	10-27-1928	4.45	3.90	1 000.9	0.106	11.3	34.8	5.50	46.6	8.5
7*	10-21-1928	4.61	4.10	1 100.6	0.113	11.2	33.4	5.7	47.0	8.2
8	1-28-1928	4.66	4.45	1 201.6	0.132	11.8	32.6	5.8	46.4	8.0
9*	9- 4-1928	5.03	4.35	1 301.6	0.117	10.2	29.9	6.4	46.3	7.3
10	5-23-1928	5.00	4.69	1 399.7	0.137	11.2	30.2	6.3	47.0	7.4
11	7-28-1928	5.60	4.58	1 598.5	0.116	9.1	26.7	7.3	47.4	6.5

* Sections considered non-silting and non-scouring by William T. Collings, Jr., M. Am. Soc. C. E.

As a demonstrative example in Table 12, Mr. Collings' data for the East Highline Canal, at "B" Heading, are used once more. Among the eleven observations, six are considered by Mr. Collings as non-silting non-scouring cross-sections. It appears that the first of these sections (Item No. 3, Table 12) is for the conditions when the silting stops, and the last observation (Item No. 9) is the upper limit related to the beginning of scouring. Since the kinetic factor in all the six observations is approximately the same, indicating that the conditions at the bottom do not vary much, attention should be directed to the stability of the banks. The widths of cross-sections shown in Table 12 and calculated under the assumption that the channels are of rectangular form, remain constant, their average value being 46.5 ft ± 1 per cent. This fact proves the stability of the banks, although the velocity gradually becomes greater with the diminishing width-bed ratio, both conditions tending to intensify the scouring action of flowing water on the banks. The degree of sediment concentration in flow at the time the measurements were taken, and the character of material composing the banks, are not known to the writer However, an examination of Table 12 as well as consideration of other data, suggests that the value of the hydraulic number is a controlling factor of the stability of banks.

The hydraulic number becomes smaller in all the eleven cross-sections, ranging from 44.4 to 29.9 in the six stable sections. The other three stable canals indicated in Table 10 have the same diminishing tendency in regard to the hydraulic number. The study of canals in British India leads the writer to a suggestion[41] that their average hydraulic number is about 40. The average value of *N* for the four Imperial Valley canals, cited previously, is 43.5 and exactly 40, if the Alamo Canal is omitted. However, the range of these values is great, being approximately 65 to 30. The variation in flow when it extended beyond the non-silting and non-scouring limits, the large velocities, the difference in resistance of bed and banks, and the influence of original designs and methods of maintenance are probable reasons for such a range in the value of *N*. The hydraulic number in the two unstable canals of Table 10 has also the same diminishing tendency, approaching the value of 20. A reference has already been made to several erodible drainage canals[42] in which the hydraulic number varies from 15 to 22.50. The tendency for some of the foregoing values to coincide suggests that a hydraulic number equal to about 20 is the average upper limit for stable channels in "earth."

Variations in Discharge.—Examination of Table 12, and the previous remarks in regard to the non-silting and non-scouring limits for a cross-section, seem to indicate a method for the design of a canal with a varied flow. Two sections are drawn on Fig. 11. One section in solid lines is the actual section of Item No. 9, Table 12; that is, the upper limit mentioned previously. Another section in dotted lines is the non-silting section for the same discharge of 1 301.6 cu ft per sec, but the kinetic factor and hydraulic number are of Item No. 3 (Table 12); that is, the low limit. These two sections, or any intermediate section, might be taken for a prototype to compute and design the section corresponding to the given discharge. Each of these sections will be stable. Incidentally, that is one of the principal reasons for the large variation of width-depth ratio in the canals of the Imperial Valley.

If the discharge is variable, the upper limit may be used for the maximum discharge, or the low limit for the minimum discharge. The spread between these two limits will determine the possible variation in the discharges. If the spread could not include the given variation in discharge, it would be impossible to design a stable channel. There remains the possibility of devising a more or less "self-maintaining" channel with an unvarying tendency to silt or scour, the latter usually being preferable for irrigation canals.

All-American Canal.—Among the various factors introduced and discussed, the writer has intended mainly to bring out the importance of two factors, the kinetic factor and the hydraulic number, which, in his opinion, may be used as criteria for the stability of a channel in erodible material. Its bed stability is controlled by the kinetic factor, whereas the hydraulic number is related to the scouring resistance of banks.

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[41] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 853, Table 5.

[42] *Loc. cit.*, p. 857, Table 8.

The author presents a study based on the design of the All-American Canal, and it would be of interest to see how the writer's discussion might be applied to the cross-sections adopted for this canal. The hydraulic elements[43] are, as follows, for its largest section: $Q = 15\,155$ cu ft per sec; $A = 4\,041.3$ sq ft; $V = 3.75$ ft per sec; $R = 16.63$ ft; $d = 20.61$ ft; and, by computation, $K = 0.02626$ and $N = 20.33$.

Assuming a rectangular cross-section, the foregoing values of K and N yield values of $d = 19.9$ ft and $r = 10.2$. A point corresponding to this depth and width-depth ratio is shown in Fig. 9 (see Point A, the All-American Canal) near the curves for the Egyptian canals, indicating that conditions in such channels are more or less similar to those assumed for the design of the All-American Canal.

The foregoing kinetic factor in the All-American Canal is not comparable to its prevailing value in the Imperial Valley on account of the proposed desilting works. A proper procedure for checking its value by Equation (35) would be to determine the "sediment equivalent" from a mechanical analysis of silt that is assumed to enter the canal; but such an analysis has not been made as far as the writer is aware. The concentration of the sediment near the bed should be known. Assuming that this concentration would be 10% higher than the average and that the coefficient, α, = 1, the mean hydraulic value computed from Equation (35) is $\omega_m = 0.01$ sec-ft. This hydraulic value corresponds approximately to the 280-mesh sieve, but that gives only a general indication of predomination of the finest particles in the sediment. Further study is required to derive a quantitative expression of the relation between the hydraulic number and the resistance of the material composing the banks. However, it is of considerable interest to note that the hydraulic number in the All-American Canal ($N = 20.33$) coincides with the value of 20, which has been suggested in the discussion as the average hydraulic number for canals in "earth."

Assuming that the proposed cross-section of the All-American Canal would tend to adopt a shape corresponding to the kinetic factor and hydraulic number as calculated previously, its depth should be less, whereas the width should be greater, than in the proposed cross-section. In other words, there might be a slight tendency to scour especially along the low parts of banks. Assuming, furthermore, that the hydraulic number is correct, this tendency to scour would be slight and would occur only at the time of full flow in the canal. The smaller the flow and, consequently, the cross-section, the greater will be the tendency for the silting to develop. These conditions, as the writer understands them, are desirable for the Imperial Valley because the collection of the settling sediment in the main canal is preferable to its conveyance to the distribution system.

W. M. GRIFFITH,[44] ESQ. (by letter).—Because it gives no new data, or mathematical reasoning to support or refute theories already advanced on this interesting subject, this paper is disappointing.

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[43] *Engineering News-Record*, October 17, 1935, p. 539.

[44] Cambridge, England.

Fig. 3 was presented to show that high velocities extend closer toward the sides in the narrow, deep cross-sections than in the broad, shallow ones. This fact in itself would not appear to offer a satisfactory explanation to the well-known phenomenon that channels carrying heavy silt loads tend to adopt broad shallow cross-sections because, for any given discharge and surface slope, channels will be found to carry heavier silt loads if they are given broad shallow cross-sections than if they are given deep narrow ones, even if the sides are protected against erosion by a non-erodible lining.

The author quotes Mr. Kennedy's classic formulas (1) of relationship between velocities and depth of non-silting channels, and a large number of subsequent formulas, based on the same form of analysis of existing canal systems, in many parts of the world, which have values in many cases different from those of the channels cited by Mr. Kennedy. All these equations are in the form, $V_0 = C\,d^n$ (see Equation (1)) in which V_0 is the velocity that gives stability, and d is the depth of the channel section.

He also quotes Mr. Lacey's formula (18) (see Equation (7)), which may also be written in the form,

$$V_o = C\,R^n \qquad (48)$$

and the essential difference between it and those based on Kennedy's studies, is that it claims that the stable velocity is a function of the hydraulic mean depth of the channel, and not of the depth of the channel. It is a more logical formula, in that the principal function, R, is governed by the general shape of the section, whereas d, the depth, is not. In an irregular cross-section it might be difficult to decide what value to take for d; that is, the average depth of the bed, or the maximum depth of the bed.

In consequence, it is not surprising that the values of the indices in these formulas based on Kennedy's studies differ. Kennedy did not claim that his formula was a basic law applicable to all channel sections; it referred to the Lower Bari Doab channel sections, which had approximately the same type of cross-section; namely, side slopes averaging 1 on 0.5 and presumably a bed level across the cross-section.

It has been found that none of these formulas has a universal application, and opinion has been expressed that it is not possible to write formulas for silt transportation and the hydraulic conditions governing a stable section, that have more than a local application.

In a paper entitled "A Theory of Silt and Scour",[48] the writer outlined a theory of silt transportation and the hydraulic conditions governing stable channel sections, which he advanced as the result of experience both in river training work and canal maintenance. This theory was based on the assumption that the power to transport silt depended only on the relative strength of the vertical or resultant vertical eddies, and was independent of that of horizontal eddies or of resultant horizontal eddies.

The main observation on which the theory was based was as follows: At any point in a flowing cross-section heavily charged with silt, if the velocity is reduced artificially the depth is found to reduce automatically due to silting, and if the velocity at a point is increased, the depth is found to increase due to scour.

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[48] *Minutes of Proceedings*, Inst. C. E., Vol. 223 (1927) pp. 243 to 251.

In consequence, the writer was led to the conclusion that Kennedy's relationship between depths and velocities (Equation (1)) was in reality a basic law of silt transportation, not for channel sections considered as a whole, but giving the connection between the silt charge transported and the hydraulic conditions necessary for its transportation at a point.

It follows from this general assumption (that in a flowing section, equilibrium at all points requires that Equation (1) shall be satisfied), and from Chezy's basic law, that if the value of the exponent, n, is equal to 0.5, the section can be in equilibrium whatever its shape. On the other hand if the value of n is less than 0.5 the section must be deep and narrow, for equilibrium, and *vice versa*. As natural sections carrying heavy silt charges are invariably broad and shallow it follows that the value of n must be greater than 0.5.

In the writer's paper[45] referred to, the value of n was assumed to be 0.64. As the examples dealt with were for river beds of shingle and boulders this value was consistent. For sand, however, a lower value of n is indicated. Arguing mathematically from this basic assumption it follows, considering any cross-section, that for equilibrium as a whole, by integrating Equation (1) across the section the condition for equilibrium is that, necessarily,

$$V_m = C\,d^n_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

in which V_m is the mean velocity of the entire cross-section and d_m is the mean depth. In Equation (49), as in Lacey's formulas, the main function governing the critical velocity (that is, the mean depth) is controlled by the shape of the cross-section. On the other hand, while from this theory it follows mathematically that the bed of the cross-section for equilibrium must be level across the section, Lacey postulates a curved bed which, in its true form, he claims to be semi-elliptical.

The difference between Equation (48) and Equation (49) is fundamental because the sum of the eddies created by the friction of the flow is a function of R, the hydraulic mean depth, whereas the sum of the vertical eddies only, or their vertical components, is a function of d_m, the mean depth.

In other words Lacey postulates a semi-elliptical stable cross-section and a law of silt transportation depending on all eddies created by the flowing section, and the writer, a cross-section having a horizontal bed and a law of silt transportation depending on the vertical eddies or the vertical components of the eddies only.

The writer advanced his theory as a general theory of silt transportation, but, in reality, it relates to loose granular material, such as boulders, shingle, and sand, graded down to a size of the order of $\frac{1}{300}$ in. in diameter, but it does not relate to the transportation of a silt charge containing only such light material as is held in suspension and transportable under the action of all eddies.

In practise, normally, a silt charge will contain silt of both classes—heavy and light—but such a case is governed by the law that controls the heavier class only, because if the heavier class is transported, the lighter material will remain in suspension automatically. There are cases, however, in which the silt charge consists of very light material only (such as clay, organic matter, and sand of an extreme order of fineness), and in such cases the theory advanced by the writer[45] does not apply. The two different sets of conditions, perhaps, can best be illustrated by considering a canal or a canal system carrying at its head a heavy silt charge consisting of both classes of material. The sections of the channel or channels near the head will have horizontal beds and the beds will be of pure sand, the great majority of the grains being more than $\frac{1}{300}$ in. in diameter. If the system is a long one, the silt of a heavier order will leave it through the distributing outlets which being situated near the bed level draw off the heavier silt particles. At the tails of this system, then, only light material will be left to be transported and the type of cross-section, if left to form its natural section, will change from a level bed to a curved bed, possibly elliptical in shape as claimed by Lacey.

Now, in such a system the question of designing a non-silting section correctly is usually of importance only in the upper reaches carrying the silt of a heavy order, because the rate of settlement of silt of the lighter order is very small and the material settled on the perimeter at the tail of the system is not pure sand; it has binding properties and is useful for forming the channel banks.

If, therefore, as the writer suggests, there are, in fact, two different conditions of silt transportation, one depending on the ratio, $\frac{V}{d^n_m}$, and the other on $\frac{V}{R^n}$ or $\frac{V}{R^{\frac{1}{2}}}$, it would explain why relationships argued mathematically from the latter formulas would not be applicable to the conditions pertaining in the Imperial Valley Canals, which carry silt of the heavy order.

In Fig. 2 the author has plotted on logarithmic paper the mean depths against bed widths of a number of channels, including those considered to be in regime on different canal systems, apparently, with a view to determining whether any general relationship between bed width and mean depths can be established for non-silting channels. The result shows very clearly that no such relationship exists.

If the general formula, Equation (49), is correct, if B = width at water surface, and Q is the discharge, then as $V_m = \frac{Q}{B\,d_m}$: From Equation (49), $\frac{Q}{B\,d_m} = C\,d^n_m$. Therefore,

$$B = \frac{Q}{C\,d^{1+n}_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

Equation (50) would indicate that it is not possible to frame a general law of relationship between width and depth irrespective of the discharge or the silt load carried.

Table 1 gives values of C and n (in a general formula of the Kennedy type —Equation (1)) which are found to give values for non-silting channels in different canal systems in different countries.

The differences in C are understandable as the silt load will vary with the conditions of the different rivers feeding the canal systems. The big differences in value of n shown in Table 1, however, are illogical if all beds are of graded sand. In model experiments normal differences in the size of sand are found not to affect the coefficient of roughness. In the case of boulders, or boulders and shingle, where the vertical eddies are materially affected by the obstruction of particles of an entirely different order of size, a different value of n might be expected, but it is not reasonable to expect big differences in silt containing graded sand.

If, however, as the writer claims, Equation (1) is inapplicable to the channel section considered as a whole, and should be of the form of Equation (49), then these differences are explainable.

For example on the assumption of Kennedy's law (Equation (1)), the data of the channels cited by Kennedy are found to conform to the equation,

$$V = 0.84\ d^{0.64} \qquad \text{(51)}$$

(Table 1, Curve No. 17) and the Godavari Western Delta, Madras (Table 1, Curve No. 9),

$$V_0 = 0.67\ d^{0.55} \qquad \text{(52)}$$

This gives a wide difference in values of n. If, however, both sets of data are plotted in terms, not of d, the depth, but of d_m, the mean depth, Kennedy's data are found to conform to the law,

$$V_m = 0.97\ d^{0.57}_m \qquad \text{(53)}$$

and the Godavari Western Delta, Madras, to

$$V_m = 0.69\ d^{0.57}_m \qquad \text{(54)}$$

and this difference in the values of n has disappeared.[46]

In his research the author is presumably seeking for the most efficient type of cross-section, namely, that relationship of width and depth which will give the channel its greatest silt-carrying capacity for any given discharge and surface slope.

With Equation (49), the maximum silt-carrying capacity is attained when $\dfrac{V}{d^n_m}$ is a maximum. If B = the width at the water surface,

$$V = \frac{Q}{B\ d_m} \qquad \text{(55)}$$

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[46] "Stable Channels in Alluvium", *Minutes of Proceedings*, Inst. C. E., Vol. 229 (1930), p. 321.

and, therefore,

$$\frac{V}{d^n{}_m} = \frac{Q}{B\ d^{1+n}{}_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(56)$$

Consequently, for any given discharge, the maximum silt-carrying capacity is attained for the cross-section in which $B \times d^{1+n}{}_m$ is a minimum.

Accepting, for sand, the value of n obtained from the data used by both Kennedy and for the Godavari Western Delta, Madras (that is, $n = 0.57$) then the maximum silt is carried when $\frac{V}{d^{0.57}{}_m}$ has a maximum value, or for any fixed discharge when $B \times d^{1.57}{}_m$ has a minimum value. If Kennedy's equation is correct the maximum silt-carrying capacity is attained when $\frac{V}{d^{0.64}}$ has a maximum value.

To compare these two different equations of silt-carrying capacity or efficiency, it is necessary to take some particular case as, for example, a channel having a discharge of 100 cu ft per sec, a surface slope of $\frac{1}{4\,000}$, and side slopes of 1 on 0.5, carrying a heavy silt load.

TABLE 13.—Hydraulic Data Pertaining to Different Channel Sections

(Discharge = 100 cu ft per sec; surface slope = $\frac{1}{4\,000}$; side slopes, 1 on 0.5; and discharge is calculated by Kutter's formula for $n = 0.0225$)

Item No.	Bottom width, B_B, in feet	Depth of channel, d, in feet	Area of channel section, A, in square feet	Velocity, V, in feet per second	Width, B, of channel at the water surface, in feet	d_m = mean depth of channel section, in feet	Values of $d^{0.57}{}_m$	Silt-carrying capacity, $\frac{V}{d^{0.57}{}_m}$	Values of $d^{0.64}$	Silt-carrying capacity, $\frac{V}{d^{0.64}}$
1...	54.0	1.5	82.12	1.22	55.5	1.48	1.25	0.978	1.298	0.942
2...	47.0	1.6	76.98	1.30	48.6	1.64	1.305	0.999	1.343	0.968
3...	43.0	1.7	74.45	1.34	44.7	1.66	1.335	1.003	1.387	0.97
4...	33.0	2.0	68.0	1.47	35.0	1.94	1.46	1.008	1.56	0.944
5...	23.0	2.5	60.62	1.65	25.5	2.42	1.652	1.00	1.80	0.920
6...	17.0	3.0	55.5	1.81	20.0	2.725	1.77	0.99	2.045	0.886
7...	13.5	3.5	53.4	1.88	17.0	3.14	1.92	0.98	2.23	0.842
8...	10.9	4.0	51.6	1.94	14.9	3.46	2.03	0.96	2.43	0.80
9...	8.8	4.5	49.6	2.01	13.3	3.77	2.13	0.945	2.60	0.772
10...	7.4	5.0	49.5	2.02	12.4	4.00	2.20	0.92	2.80	0.720
11...	5.9	5.7	49.87	2.01	11.6	4.30	2.30	0.875	2.98	0.675

In Table 13 are given the calculated hydraulic data of eleven different designs of channel sections giving this discharge, ranging from a 1.5-ft depth, with 54-ft bed width, to 5.7-ft depth and 5.9-ft bed width. Values of $\frac{V}{d^{0.57}{}_m}$ and $\frac{V}{d^{0.64}}$ are also given.

Fig. 12, for this case, shows that if the silt-carrying capacity is correctly expressed in terms of $\frac{V}{d^{0.64}}$, as Kennedy suggests, the silt-carrying efficiency will increase uniformly and rapidly from a channel of 6-ft depth to 1.7-ft depth, at which time the maximum efficiency is reached, and a sudden change occurs in the efficiency curve.

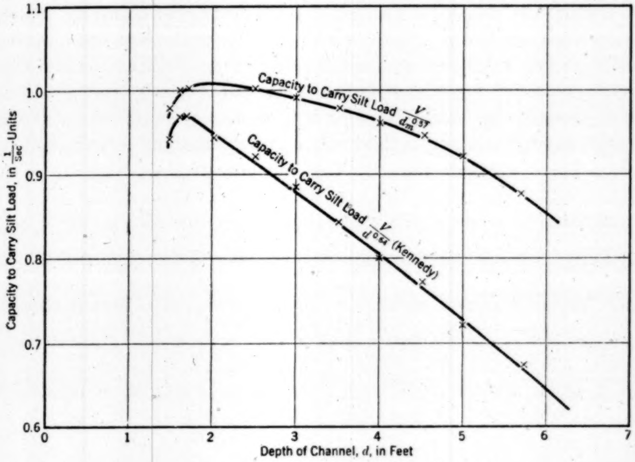

FIG. 12.

If, on the other hand, the correct expression is $\frac{V}{d^{0.57}_m}$, the silt-carrying efficiency at first increases rapidly as depths are reduced from 6 ft downward, but the curve flattens and shows little difference in silt-carrying capacity between a channel of 3-ft depth and one of 2.5-ft depth. The maximum efficiency appears to lie with a channel 2 ft deep.

The writer's experience with channels of this discharge and slope indicates that in practice the expression, $\frac{V}{d^{0.57}_m}$, gives, more correctly, the value of silt-carrying efficiency of the various types of section.

E. W. LANE,[47] M. AM. SOC. C. E. (by letter).—The constructive nature of the discussion of this paper is gratifying, and the writer feels that its value has been materially increased thereby. Although these discussions have shown considerable differences of opinion on minor phases, there seems to be little criticism of the major contentions of the paper, and no points were subject to a general attack. The importance of silt quantity as a factor in stable channel shape and the tendency of heavily silt-laden water to form wide, shallow channels seems to have been generally accepted.

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[47] Prof. of Hydr. Eng., State Univ. of Iowa, Iowa City, Iowa.

Since the items of criticism or questions raised by those discussing the paper cover a wide range of ideas and in few cases is the same question raised by more than one person, this closure can best be presented by dealing with the various discussions individually.

The empirical relations derived by Mr. Lacey seem to have been widely accepted in India. The writer believes that they are probably the best quantitative relations yet developed. The striking agreement with the results obtained by Colonel Pettis on an entirely independent basis indicates strongly the general accuracy of the conclusions of both men for average conditions. It is believed, however, that at least one important factor has been omitted, namely, the concentration of the silt load, and when the load is high, this becomes such an important factor that it must be considered.

Mr. Lacey suggests a plotting of the values, $\frac{P}{R}$, against V for the Imperial Valley canals, which, to agree with his fundamental shape formula, should show $\frac{P}{R} = 7.12\,V$. This has been done in Fig. 13. The solid black dots representing the results of sections of the canals reported by William T. Collings, Jr., M. Am. Soc. C. E., as stable[48] and the small open circles give the dimensions of ditches cited by Mr. Blaney in Table 5, assuming that the sides are vertical. The values given scatter considerably, even for the same ditch and station, but with one exception, all of them show the ratio, $\frac{P}{R}$, to be less than $7.12\,V$. Although the accuracy of individual observations may be questioned, the data very strongly indicate that the Lacey shape formula does not fit Imperial Valley conditions. This lack of agreement is probably the result of the high silt concentration in these canals.

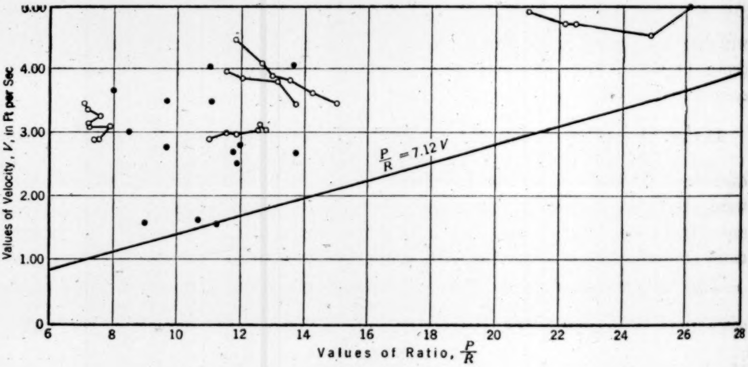

FIG. 13

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[48] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 554.

The writer does not believe that the factor, $\frac{V^2}{R}$, empirically derived by Mr. Lacey and also independently by Mr. Tchikoff, has been demonstrated to be a rational measure of turbulence, although it may be a very convenient empirical index of it. Some of the arguments advanced in support of it do not appear to be valid. For example, the fact that the ratio, $\frac{V^2}{R}$, is a valuable criterion in non-uniform flow seems to be pure coincidence and to bear no relation to the phenomena of stable channel shapes. Assuming that all the energy loss due to the friction of the water flowing in a ditch is dissipated by turbulence, the average turbulence of a stream (under conditions of steady, uniform flow), expressed in terms of the energy loss per unit time per unit of volume (occupied by a unit weight), is proportional to the product of the slope and the mean velocity of flow. This follows from the fact that if the water is not accelerating, the energy dissipated per unit of volume per unit of time must be equal to the work represented by the drop of the water in that time, which, in turn, is equal to the mean velocity of the water times the slope. Hence, the criterion of turbulence should be $S\,V$. Although some of the loss, no doubt, occurs in the laminar flow of the boundary layer on the sides and bottom of the ditch, it is believed that under ordinary conditions of an earth channel this is a small part of the total. If the Manning formula is assumed to evaluate, correctly, all the variable factors, this criterion can be transformed as follows:

$$S\,V = \frac{V^3}{\left(\frac{1.486}{n}\right)^2 R^{\frac{4}{3}}} = \left(\frac{n}{1.486}\right)^2 \left(\frac{V^2}{R^{\frac{8}{9}}}\right)^{\frac{3}{2}} \qquad \text{.............(57)}$$

or, in other words, $SV \infty \left(\frac{V^2}{R^{\frac{8}{9}}}\right)^{\frac{3}{2}}$. If the Lacey formula,

$$S = \left(\frac{N_a}{1.3458}\right)^2 \frac{V^2}{R^{\frac{3}{2}}} \qquad \text{......................(58)}$$

is used instead of that of Manning, the expression, $S\,V$, becomes $\left(\frac{N_a}{1.3458}\right)^2 \left(\frac{V^2}{R}\right)^{\frac{3}{2}}$. Thus, it will be seen that using the Lacey formula, the empirical Lacey (and Tchikoff) criterion, $\frac{V^2}{R}$, is brought into almost perfect agreement with the criterion, $S\,V$. This probably explains why a criterion, $\frac{V^2}{R}$, gives such good results, especially when used with the Lacey formula instead of the Manning formula.

Mr. Lacey advocates the use of the Buckingham method for future silt studies. This method, unquestionably, is a useful tool, but can only be used safely if all the important dimensioned variables are known, and it will indicate nothing as to the effect of the dimensionless factors.

It is not believed that the conditions in the Imperial Valley canals approach the condition of flume traction, as suggested by Mr. Lacey. In nearly all cases, the material through which these canals flow is composed of the same kind of silt as is carried by the ditches, since the material was formed by the deposits of the same river, and, therefore, the Imperial Valley canals are flowing in a self-borne alluvium in a manner similar to that of the canals of India. A good description of conditions in the Imperial Valley has been given by H. T. Cory, M. Am. Soc. C. E.[40] The writer's discussion of scour of the sub-grade when the flow in the channel is capable of carrying more material than enters at the head-gate, was included to cover the entire range of cases. It does not apply generally to the canals of the Imperial Valley.

The flow conditions in the Imperial canals are quite different from those found by Gilbert, and saltation is not believed to occur to an important extent. Even the coarser particles of the Imperial Valley material are much smaller than the finest material used by him. Observations were made in the laboratory of the U. S. Bureau of Reclamation on the flow in a long flume with glass panels in the side, using bed material brought from the Colorado River. This showed that at ordinary velocities there was a very slight motion of the material along the bottom as bed load, forming ripples similar to those which appear on the bottom of the Imperial Valley canals, but nothing resembling saltation was observed, and the greater part of the material traveled in suspension. At very high velocities the ripples disappeared and the bottom became smooth, with great quantities of material moving in suspension. This smoothing out of the bottom in Colorado River material at high velocities was also observed by C. A. Wright, M. Am. Soc. C. E.

Mr. Stevens gives an interesting comparison of the sections of several rivers with the Lacey formula, Equation (5), and notes the lack of agreement for the values of the Colorado River at Yuma, Ariz., and the Yellow River, at Chiang-kou, China. The conditions of the Colorado River at the Yuma gaging-station are very unusual, and do not conform to the requirement of banks and bed of alluvium postulated by the Lacey relation. This may account for the lack of agreement. A short distance above the gaging section at Yuma, the Colorado River contracts very suddenly from a wide, meandering stream and passes through a narrow gorge about 400 ft wide, with sides of very resistant material of clay and boulders. At the gaging section, a short distance down stream from this throat, it is about 500 ft wide, and also has very permanent banks, possibly formed of the sandstone rock which outcrops only a short distance away. To pass through the narrow gap during periods of high discharge the water of the river must have a high velocity, which scours the light bed material to a great depth. When the discharge falls, the channel refills. The writer does not believe that this is typical of the Colorado River as a whole, as soundings in an unconfined section made elsewhere show no change of the mean bed elevation with high or low water. Moreover, for such scour to occur along the entire river during high water would require the transportation by the river of much larger quantities of silt than are shown by any measurement. The conditions at the Yuma gaging station, therefore, are a very special case, and conclusions based upon what happens there, if generally applied, will be very misleading.

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[40] "Irrigation and River Control in the Colorado River Delta." *Transactions*, Am. Soc. C. E. Vol. LXXVI (1913), p. 1204.

The measurements on the Yellow River given by Mr. Stevens seem to be those recorded by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E.[80] At this section there was a rock ledge on the right bank and rock also appeared on the left bank some distance back from the river. It is doubtful, therefore, whether this station is in the class for which Mr. Lacey claims his relations to hold.

Mr. Stevens is probably correct in his opinion that it is the momentary "stray currents" which are most effective in moving bed material, and the same probably applies also to the movement of the material at the sides of the ditch. The writer does not believe that it is the difference in the velocity of the adjacent filaments of water which in itself causes the movement of the material, but that in those parts of the cross-section of a ditch where the isovels are close together the velocity differences within a small distance are great and, therefore, the turbulence is large. The spacing of the isovels, therefore, is an indication of turbulence, close spacing indicating high, and wide spacing indicating low, turbulence. Although this relation is believed to be true for the ordinary conditions of ditch flow where the turbulence is introduced by friction, it does not necessarily hold where the turbulence is the result, for example, of a hydraulic jump.

The data on the silt-carrying ability of the Wei Pei Main Canal which Mr. Eliassen presents, and that of Mr. Ballester for the Rio Negro Canals, are very valuable. Very little complete data of that kind are available and until it is collected, the carrying capacity of channels for solids will be only imperfectly known.

Mr. Eliassen's description of the heavy silt loads of the Chinese rivers is interesting. Unquestionably, he is correct in his remarks that a silt percentage should be a ratio of silt weight to the weight of silt plus water. In most conditions the silt concentrations are so small that to omit the silt weight from the silt-plus-water weight makes a negligible difference, but with the concentrations mentioned by Mr. Eliassen such a procedure leads to important errors.

The writer is not inclined to share with Mr. Eliassen the belief that bed load is an important factor in streams that carry a heavy suspended silt load. Measurements were made in the Colorado River on the crest of Laguna Dam where many years ago the river had filled the storage space with silt to the crest level, and, therefore, where the load measured was the combined bed and suspended loads. At the same time, measurements were made in a section five miles up stream, where only the suspended load was sampled. A comparison of the results over a considerable period of time showed very little difference,[51] indicating a relatively small bed load. That this is the case is confirmed by the observations in the flume previously mentioned. No bed-load measurements have been made for material comparable in size with that of the Colorado River, but the formulas derived for coarser material were extrapolated by Samuel Shulits, Assoc. M. Am. Soc. C. E., to the size of the Colorado River material, and the loads computed were found to be large, but in comparison with the very large suspended load, are relatively small. Still another reason for believing that the bed is not, to any great extent, in motion is that when sounded with a rod it is surprisingly hard, and does not have the much softer feeling which characterizes coarser material moving as bed load. As a result of these four independent lines of approach all of which are in agreement, the writer believes that in streams carrying high concentrations of fine material in suspension, the bed load, although perhaps of considerable absolute magnitude, is a small part of the total load.

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[80] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1437, Fig. 16.

The changes of roughness of the Yellow River channel mentioned by Mr. Eliassen may be due to the elimination of the ripples on the bottom observed at high velocities in the flume, as previously described. Such changes occur also in the Mississippi. In this case they are probably due to bed-load movement, as Mr. Eliassen suggested, as waves reaching more than 30 ft in height have been observed in the Mississippi River which, compared with the Colorado and Yellow Rivers carries a very small concentration of suspended material. For reasons previously stated, however, the writer does not believe that such waves form in rivers carrying heavy loads of suspended material.

Mr. Lindley objects to the writer's statement that he did not mention the quantity of silt as a factor in stable channel shape, explaining that the effect of the quantity of silt was so well known in India as to need no mention. There is no doubt that, in the literature from India, accessible to the writer when his paper was written, occasional mention was made of the fact that the quantity of silt was a factor, but in no place had he found any one who gave it the important place to which he believed it was entitled. The writer's emphasis of this phase was largely due to the fact that the Lacey relations, which were gaining wide acceptance in India, made no mention of the effect of the concentration of the silt load as a factor. Since preparing his paper the writer has secured a copy of the *Punjab Irrigation Technical Review* of 1925, and finds there a statement by Mr. Lindley (as well as others by Messrs. W. G. Quinton, A. R. Murray, and A. M. R. Montagu) that both quantity and quality are factors in determining the channel shape. Except for this publication (which is relatively inaccessible to American engineers) in no recent publication from India or elsewhere at the time this paper was written had the writer found the effect of silt concentration mentioned as an important factor.

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[51] "Why Desilting Works for the All-American Canal", by C. P. Vetter, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 118, March 4, 1937, p. 322.

Mr. Griffith states that it was well known that channels carrying heavy silt loads adopt broad shallow cross-sections. At this distance from India, it is possible for an American engineer to learn what was known there only from their written literature. From a very thorough study of extensive literature from India and other countries the writer found, as late as November, 1935, not a single mention of this phenomenon, and has found since only casual mention of it by Messrs. F. W. Woods and W. G. Quinton in the relatively inaccessible publication previously mentioned. Under these circumstances no apology for discussing it seems to be required.

Mr. Lindley suggests that the writer should have emphasized the fact that a given volume of flowing water carrying a given proportion of a given quality of silt tends to form a channel of which the depth, width, and gradient are all fixed by the said discharge and silt load. The writer's discussion suggested that a given discharge with a given quality and quantity of silt did fix a definite cross-section of channel The gradient would follow from these by the ordinary laws of flow in open channels and, therefore, could not be considered an independent variable. Fixing the shape of the cross-section for a given discharge, therefore, fixes the gradient which would be stable, and it is not necessary to discuss it separately.

Not much benefit will result from arguments over the relative merits of the mean depth and the hydraulic radius as the hydraulic property to which the critical velocity is related. Mr. Griffith plots the available data on the basis of mean depth and finds it follows, reasonably well, an exponential relation to the critical velocity. Mr. Lacey uses the same data and finds an equally good agreement using the hydraulic radius. Both claim this as important evidence of the correctness of their formulas. The facts probably are that, within the range of channel conditions ordinarily encountered, the ratio of the mean depth to the hydraulic radius varies so little that the discrepancy is much less than that due to other factors.

The writer was seeking to obtain the form of cross-section which would be stable for the various conditions of discharge and silt characteristics, not necessarily the depth-width relation which would give the channel its greatest silt-carrying capacity for a given discharge and surface slope, as Mr. Griffith concluded. His determination of the depth of channel that would give the greatest silt-carrying capacity for a given condition may have value, but it seems to rest entirely upon his assertion that the capacity is a maximum when the ratio, $\frac{V}{dm^n}$, reaches its highest value, since no proof to support this assertion was submitted.

Mr. Tchikoff's proposition of "sediment equivalent" is a very ingenious one which merits thorough experimental investigation and testing by comparison with observed data in natural streams. It has the advantage of reducing the very complex relation of the quantity and quality of the silt present in a stream at a given time, to a single number. It is unfortunate that more quantitative data are not available to check this and the other interesting hypotheses suggested by Mr. Tchikoff. The collection of quantitative data seems to be one of the greatest needs in this field of science, for until hypotheses can be adequately checked against observed relations it is unsafe to build upon them.

Referring to Mr. Tchikoff's criticism of the position of the isovels in the cross-section given in the paper, the fact that the isovel, 0.80, is nearest the sides does not mean that the isovels of the lower values were non-existent but merely that they could not be plotted because of the lack of space. These velocity distribution charts were drawn from data which were not taken originally to investigate the point under discussion. They are the result of a number of different experiments and, therefore, are not perfectly consistent. It is believed, however, that they do in general illustrate the fact that in narrow channels the high velocities come closer to the sides.

The writer is unable to accept Mr. Tchikoff's arguments regarding the velocity gradient. He states: "The sum of the potential and kinetic energy is constant for all the points of the stream filament that lie on the same horizontal plane." This would only be true in a frictionless channel. Where friction is present, the water particles near the sides of the channel are more retarded by friction and more of their energy is converted into heat, than those near the center. On a given horizontal plane, therefore, the sum of the potential and kinetic energy of the water at the sides of the channel may be less than near the center. This is demonstrated by measurements made in a chute near Montrose, Colo.[53] A velocity of 34.8 ft per sec was observed 0.4 ft below the surface near the center of the channel, and, at the same depth at the sides, velocities of 23.3 and 26.9 ft per sec were observed.

The water surface was level, and, hence, the pressure at the three points was the same. The kinetic energy, expressed in velocity head, at the sides was 10.4 and 7.6 ft, respectively, less than at the center. At another section of the same chute (not described in the paper cited[53]), float measurements showed a velocity of 20 ft per sec at the center and about 9 ft per sec as close to the sides as could be observed. The water surface was shown by measurements to be level, and the pressure was the same (atmospheric), but the sum of the potential and kinetic energies was 5 ft of head greater at the center than near the sides. By observations at a large number of cross-sections the water surface was shown to be level across the flume (except at the entrance where contraction effects entered), but the water in the center was obviously moving much more rapidly than near the edges and, therefore, the sum of the potential and kinetic energies at the center was greater than at the sides. The error in Mr. Tchikoff's statement arises from not considering the friction term in the Bernoulli theorem.

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[53] "Recent Studies on Flow Conditions in Steep Chutes," *Engineering News-Record*, Vol. 116, January 2, 1936, p. 5.

Mr. Blaney has called attention to the difficulties found in his investigations of the Lower Colorado River, in analyzing the data, because of the widely different size of silt particles carried. In the writer's studies made for the U. S. Bureau of Reclamation in 1934 and 1935, on the silt carried in the Colorado River, a discovery was made which is believed to be of great significance in understanding the transportation of silt in suspension. It has long been known that, in many rivers (including the Colorado) carrying heavy silt charges, there is no close relation between the discharge and the total silt load. In these studies analysis was made not only of the quantity but of the size range of the particles carried in the Colorado River, by means of bi-weekly samples taken over a period of more than a year. It was found that although there was no relation between the quantity of fine silt and the discharge there was a very definite relation between the load of coarser particles and the discharge. Some of the results of this study have been published.[51] The size range of the particles the load of which bore a definite relation to the discharge was the same as the range of sizes found in the bed material. The reason for this condition is believed to be the fact that the river, in general, always carries as great a load as it is able, of the material which is available in large quantities in its bed. Because of the eddies which impinge on the river bottom, particles from the bottom are thrown into suspension. At the same time, other particles from suspension are settling down again on the river bed. When the same number of particles are picked up as settle back to the bottom the load is as great as the river can carry. When fewer are picked up than fall back, the bed is being filled and when more are picked up the bed is being scoured. With a given size of material on the river bed, the river will pick up sufficient material to produce a condition in which the rate of re-depositing on the bottom is equal to the rate at which the material is picked up from the bottom. The load of very fine silt which the river is capable of carrying seems usually to be much greater than the supply available, which is limited by the quantity of fine material that is brought into the stream by its tributaries. The load of fine material, therefore, is dependent entirely on the load brought to the stream by the tributaries and, hence, is independent of the discharge carried by the stream. The stream picks up from the river bed all the fine material available, but is not able to obtain from this source sufficient material to saturate it with the finer particles. It is believed that many other of the phenomena of the transportation of silt in suspension may be cleared up as a result of this discovery, and that it can form the basis of a quantitative science in this field.

The dimensions adopted for the All-American Canal have been requested and are given in Table 14.[53]

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[53] Specifications 573 and 621, U. S. Bureau of Reclamation.

In selecting these sections several factors were unusually important. One was the great depth of cut in which much of the canal was built, which reached a maximum of more than 100 ft and for long stretches was more than 75 ft. Another was the fact that considerable time may elapse before the demand reaches the design capacity. Both these factors justify the use of narrower widths and higher velocities than would otherwise be the case. In comparing this canal with the Imperial Valley canals it should be kept in mind that the material traversed by them is not the same. The Imperial canals, in general, flow through a Colorado River deposit of very fine material, whereas the All-American Canal is nearly all located on higher land in coarser "mesa material" which seems to be an outwash deposit from the surrounding high land.

TABLE 14.—ALL-AMERICAN CANAL CROSS-SECTIONS*

Discharge, in cubic feet per second	Bottom width, in feet	Depth, in feet	Side slopes 1 on 1.75, etc.	Area, A, in square feet	Wetted perimeter, P, in feet	Hydraulic radius, R,
15 155	160	20.61	1.75:1	4 041.3	243	16.63
13 155	150	19.12	1.75:1	3 508.0	227	15.45
10 155	130	16.59	2:1	2 708.0	204.2	13.26
7 600	118	14.73	2:1	2 171	183.8	11.81
7 400	116	14.57	2:1	2 114	181.2	11.67
7 100	114	14.24	2:1	2 029	177.6	11.42
6 800	114	14.02	1.75:1	1 943	170.5	11.39
5 100	100	12.82	1.75:1	1 569	151.6	10.34

*Earth sections only.

In the acknowledgments accompanying the author's paper he inadvertently omitted mention of his indebtedness to Mr. W. M. Griffith, to H. F. Blaney, and F. C. Scobey, Members, Am. Soc. C. E., and to the Central Board of Irrigation of India, for valuable data and suggestions.

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TRANSACTIONS

Paper No. 1958

TRUSS DEFLECTIONS: THE PANEL DEFLECTION METHOD

By Louis H. Shoemaker[1], M. Am. Soc. C. E.

With Discussion by Messrs. David B. Hall, E. Mirabelli, William Bertwell, Robert H. Hurlbutt, A. A. Eremin, T. P. Noe, Jr., David A. Molitor, Glenn L. Enke, Fang-Yin Tsai, A. W. Fischer, L. E. Grinter, and Louis H. Shoemaker.

Synopsis

The purpose of this paper is to describe a new method for computing the deflections of trusses. In principle, the method is similar to that of computing the deflections of beams, use being made of the distortions of the individual panels. In this manner, the deflections of all the panel points of a truss can be computed with a considerable saving in work over other analytical methods. Therefore, it offers material advantages in computing the reactions of statically indeterminate structures.

Application of the Method

To compute the deflection of a point of a beam of constant moment of inertia with reference to a tangent to the elastic curve of that beam, the sum is taken of the products of the angular change of every vertical section, multiplied by its distance from the point. If the moment of inertia of the beam is not constant, it is necessary to divide the beam into longitudinal sections of lengths corresponding to the different moments of inertia. In this case, the deflection of the point is computed by taking the sum of the vertical distortions of the different longtitudinal sections, plus the sum of the products of the angular distortion of each section, multiplied by the distance of the section from the point. It is evident that the same method can be used to compute the deflections of a truss by utilizing the distortions of the individual panels of the truss.

Note.—Published in November, 1935, *Proceedings*.

[1] Santa Monica, Calif.

To apply the method to a truss, a convenient point is selected as the origin, from which all deflections are measured. The vertical and horizontal deflections and angular distortion of every panel are computed. In Fig. 1, *ABCD* represents any panel of a truss. The origin of deflections is considered as being at some panel point to the right. The point, *A*, and the side, *AB*, are taken as reference point and direction from which panel distortions are measured. The triangle, *ABC*, will be called the deflection triangle, and *C*, the deflection point. The point, *D*, will be called the secondary point. The vertical and horizontal deflections of *C* are caused by the changes of length of the members of the triangle, *ABC*. The vertical deflection at *D* is the vertical deflection at *C* plus or minus the change of length of *CD*, which is also a member of the deflection triangle of the panel to the left. The angular distortion of the panel is the angular rotation of *CD*, due to the horizontal deflections of *C* and *D*. The total vertical deflection of any deflection point is the sum of the vertical deflections of all the deflection points taken from the origin to and including the point in question plus the sum of the products of the angular distortion of each panel multiplied by its distance from the point in question taken in the same way. The total deflection of any secondary point is the deflection of the corresponding deflection point plus or minus the change of length of the connecting vertical.

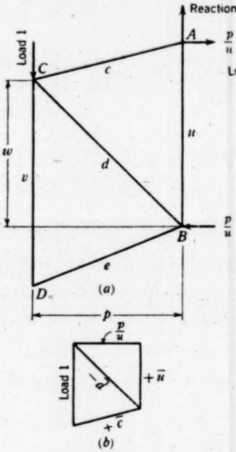
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FIG. 1.

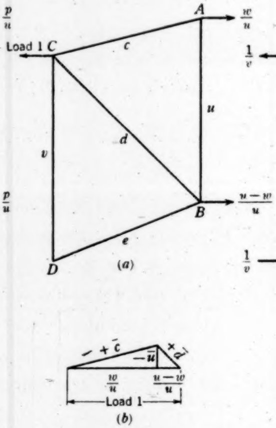
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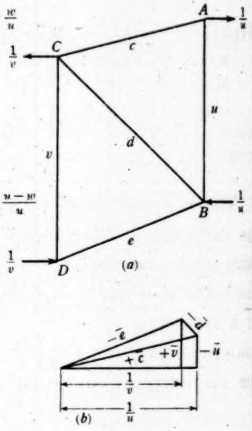
FIG. 2.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$\bar{d}$
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$-\bar{u}$
$\frac{1}{v}$
$\frac{1}{u}$
(b)

FIG. 3.

NOTATION

The following notation is introduced in this paper:

c = length of a chord member of a deflection triangle; c' = change in length, $+ c'$, denoting an increase and $- c'$, denoting a decrease; $\bar{c}$ = the stress in Member c resulting from a load of 1, or a bending moment of 1, applied to the panel;

d = length of a diagonal member of a deflection triangle; d' = a change in length, $+d'$ denoting an increase and $-d'$ denoting a decrease; $\overline{d}$ = the stress in Member d resulting from a load of 1, or a bending moment of 1, applied to the panel;

e = length of a chord member at a secondary point; e' = a change in length, $+e'$ denoting increase and $-e'$ denoting decrease;

n = number of panels, counted from the origin;

p = length of a panel;

u = length of a vertical member of a deflection triangle; u' = change in length, $+u'$ denoting an increase and $-u'$ denoting a decrease; $\overline{u}$ = the stress in Member u resulting from a load of 1, or a bending moment of 1, applied at the panel;

v = length of a vertical member between a deflection point and a secondary point; v' = change in length, $-v'$ denoting a decrease, and $+v'$ denoting an increase, in length;

w = length of a vertical projection of a diagonal member;

x = number of panels from the origin to any deflection point or secondary point between the origin and the nth panel point;

y = vertical distance of any panel point n panels from the origin above or below a deflection point x panels from the origin, $+y$ denoting distance downward and $-y$ distance upward from the xth panel point.

A = cross-section area;

E = modulus of elasticity;

H_1 = horizontal deflection of Deflection Point C with respect to Point A; H_T = total horizontal deflection of a deflection point or a secondary point, distant n panels from the origin;

L = length of a member:

R = angular distortion of a panel with reference to Member AB;

S = total axial stress applied to a member;

T = a subscript denoting "total";

Δ_1 = vertical deflection of Deflection Point C with reference to Point A; $\Delta_2 = \Delta_1 - (\pm u')$ = vertical deflection of Deflection Point C with reference to Point B; $\Delta_3 = \Delta_1 + (\pm v')$ = vertical deflection of Secondary Point D with reference to Point A; $\Delta_4 = \Delta_1 + (\pm v') - (\pm u')$ = vertical deflection of Secondary Point D with reference to Point B;

δ = change in length, c', d', etc.

GENERAL FORMULAS FOR PANEL DISTORTIONS

In developing formulas for panel distortions, the method of internal work will be utilized. To derive the formula for the vertical deflection at Point C, refer to Fig. 1, thus: by proportion, $\pm\overline{c} : 1 :: c : u$; $-\overline{d} : 1 :: d : u$; and $+\overline{u} : 1 :: w : u$. Therefore, $\overline{c} = \frac{c}{u}$; $\overline{d} = -\frac{d}{u}$; and, $\overline{u} = \frac{w}{u}$, respectively, and:

$$\Delta_1 = \frac{c}{u}(\pm c') - \frac{d}{u}(\pm d') + \frac{w}{u}(\pm u') = \frac{(\pm c'c) - (\pm d'd) + (\pm wu')}{u} \quad \ldots(1)$$

The horizontal deflection at C is derived by reference to Fig. 2, thus: $\overline{c} : \frac{w}{u} :: c : p$; $\overline{d} : \frac{u-w}{u} :: d : p$; and, $-\overline{u} : \frac{w}{u} :: u - w : p$. Therefore,

$\overline{c} = \frac{c\,w}{p\,u}$; $\overline{d} = \frac{d\,(u - w)}{p\,u}$; and, $\overline{u} = -\frac{w\,(u - w)}{p\,u}$, respectively, and,

$$H_1 = \frac{[(\pm c'\,c) - (\pm d'\,d) + (\pm w\,u')]\,w}{p\,u}$$

$$+ \frac{(\pm d'\,d)}{p} - \frac{(\pm w\,u')}{p} = \frac{w\,\Delta_1}{p} + \frac{(\pm d'\,d) - (\pm w\,u')}{p} \quad \ldots\ldots\ldots(2)$$

The angular distortion of Member *CD* is derived by reference to Fig. 3, thus:

$\overline{c} : \frac{1}{u} :: c : p$; $-\overline{e} : \frac{1}{v} :: e : p$; $-\overline{u} : \frac{1}{u} :: u - w : p$; $\overline{v} : \frac{1}{v} :: v - w : p$, and, $-\overline{d} : \frac{1}{u} - \frac{1}{v} :: d : p$. Therefore, $\overline{c} = \frac{c}{p\,u}$; $\overline{e} = -\frac{e}{p\,v}$; $\overline{u} = -\frac{u - w}{p\,u}$; $\overline{v} = \frac{v - w}{p\,v}$; and, $\overline{d} = -\frac{d\,(v - u)}{p\,u\,v}$, respectively, and:

$$R = \frac{c}{p\,u}(\pm c') - \frac{e}{p\,v}(\pm e') - \frac{u - w}{p\,u}(\pm u') + \frac{v - w}{p\,v}(\pm v') - \frac{d\,(v - u)}{p\,u\,v}(\pm d')$$

$$= \frac{1}{p}\left[\frac{(\pm c'c) - (\pm d'd) + (\pm w\,u')}{u} + \frac{(\pm d'd) + (v - w)\,(\pm v') - (\pm e'e)}{v} - (\pm u')\right]$$

$$= \frac{1}{p}\left[\Delta_1 + \frac{(\pm d'd) - (\pm e'e) - (\pm w\,v')}{v} + (\pm v') - (\pm u')\right] \quad \ldots.(3)$$

The total deflections are expressed, as follows:

$$\Delta_T = \sum_{x=0}^{x=n}\left[\Delta + R\,p\,(n - x)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

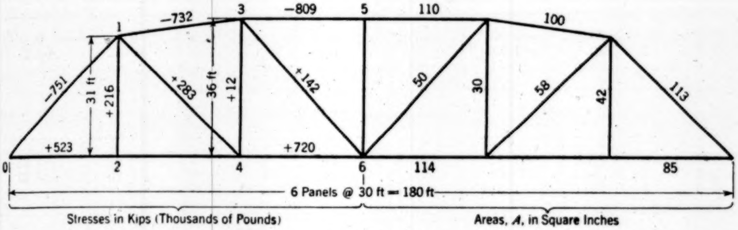
and,

$$H_T = \sum_{x=0}^{x=n}\left[H - R\,(\pm y)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

It is evident from the manner in which Equations (1) to (5) have been derived that the resultant signs of Δ, H, and R, have the following significance: $+\ \Delta$ indicates deflection inward and $-\ \Delta$, deflection outward with respect to the truss; $+\,H$ indicates deflection away from the reference point and $-\ H$, toward the reference point; and, $+\ R$ indicates left-handed rotation if acting to the left of the reference side and right-handed rotation if acting to the right of the reference side, and $-\ R$ is the reverse.

The formulas for panel distortions are very simple in form and composition. There are five terms involved for each panel, the products, $c'c$, $d'd$, etc. When these terms have been computed, the evaluation of the formulas to determine the total deflections, is a simple matter. Two short examples are offered to illustrate the application of the method. The loads indicated are in kips, 1 kip being equal to 1 000 lb.

Example 1.—Deflections of a Simple Truss.—In Table 1 the deflections for the 180-ft truss shown in Fig. 4 are calculated. The deflection points are 0, 1, and 3. The values of Δ are the deflections upward of 0 from 2, 1 from 3, and 3 from 5. The deflection of Point 2 is obtained by adding algebraically the change of length of Member 1-2 to the deflection of Point 1. The signs of Δ for 1 and 3 are minus as the deflections are outward, and that of 0 is plus for an inward deflection. In computing the changes of length, the formula,



1
3
5
−732
−809
110
100
−751
31 ft
+216
+283
36 ft
+12
+142
50
30
58
42
113
+523
+720
0
2
4
6
114
85
6 Panels @ 30 ft = 180 ft
Stresses in Kips (Thousands of Pounds)
Areas, A, in Square Inches

FIG. 4.

$$\delta = \frac{S\,L}{1\,000\,A} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

has been used, this being the variable factor. The constant factor, $\frac{12}{30\,000}$, is applied in the final calculations. The calculated deflections are upward from Panel Point 5. These are transposed in the last two columns to give deflections downward from Point 0, in inches.

TABLE 1.—DEFLECTIONS OF A 180-FOOT SIMPLE TRUSS.

Deflection points	Members	LENGTH		Change in length, Equation (6) (c', d', etc.)	Products: Column (4) × Column (5)	Vertical deflection, Δ, in inches $\times \frac{30\,000}{12}$	Values of n	$Rp\,(n-x)$		SUMMATION		TOTAL DEFLECTION, Δ_T		
		Symbol	In feet					Point 3 ($R = -11.4$ and $x = 1$)	Point 1 ($R = -10.9$ and $x = 2$)	Vertical deflection, Δ, in inches $\times \frac{30\,000}{12}$	$R\,p\,(n-x)$	In inches $\times \frac{30\,000}{12}$	Transposed	Multiplied by $\frac{12}{30\,000}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
5.6	...	...	...	...	...	0	...	...	...	...	...	0	2 366	0.95
3	3-5	*c*	30	−220.8	−6 624	−354	1	0	0	354	...	354	2 012	0.81
	3-6	*d*	46.8	+131	+6 127	...	...	...	...	...	...	...	...	...
	4-6	*e*	30	+189.3	+5 679	...	...	...	...	...	...	...	...	...
	5-6	*u*	36	0	0	...	...	...	...	...	...	...	...	...
	3-4	*v*	36	+14.4	+518	...	...	...	...	...	...	...	...	...
4	...	*w*	36	...	...	−340	...	...	...	340	...	340	2 026	0.82
1	1-3	*c*	30.4	−222.5	−6 764	−426	2	−342	0	780	342	1 122	1 244	0.50
	1-4	*d*	43	+210	+9 030	...	...	...	...	...	...	...	...	...
	2-4	*e*	30	+184.5	+5 535	...	...	...	...	...	...	...	...	...
	3-4	*u*	36	+14.4	+446	...	...	...	...	...	...	...	...	...
	1-2	*v*	31	+159.3	+4 938	...	...	...	...	...	...	...	...	...
2	...	*w*	31	...	...	−267	...	−342	...	621	342	963	1 408	0.57
0	0-2	*c*	30	+184.5	+5 535	+734	3	+684	+327	1 355	1 011	2 366	0	0
	0-1	*d*	43	−286	−12 300	...	...	...	...	...	...	...	...	...
	1-2	*u*	31	+159.3	+4 938	...	...	...	...	...	...	...	...	...
	...	*w*	31	...	...	...	...	...	...	...	...	...	...	...

TABLE 2.—DIMENSIONS, 200-FOOT ARCH (EXAMPLE 2).

Members (1)	LENGTH: Symbol (2)	LENGTH: In feet (3)	Change in length, Equation (6) (c′, d′, etc.) (4)	Products: Column (3) × Column (4) (5)	Members (6)	LENGTH: Symbol (7)	LENGTH: In feet (8)	Change in length, Equation (6) (c′, d′, etc.) (9)	Products: Column (8) × Column (9) (10)
(a) DEFLECTION POINT 7									
7–9	*c*	25.0	+2.33	+58.25	8–9	*u*	10.0	0	0
7–8	*d*	26.9	−1.38	−37.122	6–7	*v*	12.5	+0.42	+4.2
6–8	*e*	25.0	−2.51	−62.75	...	*w*	10.0	......	......
(b) DEFLECTION POINT 5									
5–7	*c*	25.0	+2.14	+53.5	6–7	*u*	12.5	+0.42	+5.25
5–6	*d*	28.0	−2.61	−73.08	4–5	*v*	20.0	+1.25	+15.63
4–6	*e*	26.1	−1.514	−39.52	...	*w*	12.5	......	......
(c) DEFLECTION POINT 3									
3–5	*c*	25.0	+1.39	+34.75	4–5	*u*	20.0	+1.25	+25.00
3–4	*d*	32.0	−2.62	−83.84	2–3	*v*	32.5	+1.66	+33.2
2–4	*e*	27.9	−0.857	−23.91	...	*w*	20.0	......	......
(d) DEFLECTION POINT 1									
1–3	*c*	25.0	+0.50	+12.5	2–3	*u*	32.5	+1.66	+53.95
1–2	*d*	41.0	−3.02	−123.82	0–1	*v*	50.0	+1.75	+56.88
0–2	*e*	30.5	−0.555	−16.93	...	*w*	32.5	......	......

Example 2.—Horizontal Reactions of a Two-Hinged Arch.—In Tables 2 and 3 the horizontal reactions of the 200-ft arch shown in Fig. 5 are calculated. The stresses in the half arch have been computed for a load of unity applied horizontally at 0. The formula,

$$\delta = \frac{S L}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

TABLE 3.—REACTIONS, 200-FOOT ARCH (EXAMPLE 2).

Deflection points (1)	Vertical deflection, Δ, in feet, multiplied by E (2)	Horizontal deflection, H, in feet, multiplied by E (3)	Angular distortion, R (4)	Product Ry, (Column (4) × 50) (5)	Values of n (6)	VALUES OF $Rp\,(n-x)$ AT PANEL POINTS: 7 ($R = +0.46$ and $x = 1$) (7)	5 ($R = +0.356$ and $x = 2$) (8)	3 ($R = +0.19$ and $x = 3$) (9)	SUMMATION: $Rp\,(n-x)$, in feet, multiplied by E (10)	Vertical deflection, Δ, in feet, multiplied by E (11)	TOTAL DEFLECTION, Δ_T: In feet, multiplied by E (12)	Transposed (13)	Horizontal reactions, at Point 0 (Column (13) ÷ 99.6) (14)
9	0	....	0	....	0	....	....	....	....	0	0	92.44	0.93
7	+9.54	+2.33	+0.467	+23.35	1	0	....	....	0	+9.54	+9.54	82.90	0.83
5	+10.54	+2.14	+0.356	+17.80	2	+11.68	0	....	+11.68	+20.08	+31.76	60.68	0.61
3	+7.18	+1.39	+0.190	+9.50	3	+23.35	+8.9	0	+32.25	+27.26	+59.51	32.93	0.33
1	+5.85	+0.49	+0.110	+5.50	4	+35.03	+17.8	+4.75	+57.58	+33.11	+90.69	1.75	......
0	+7.50	....	....	....	....	....	....	....	....	+34.86	+92.44	0	......
....	....	+6.35	....	+56.15		$\Sigma\,(H-R_y) = -49.8 \times 2 = -99.6$ for full span							......

has been used in computing the changes of length. The deflection points are 1, 3, 5, and 7. The values of Δ are the deflections downward of 1 from 3, 3 from 5, etc. The deflection of Point 0 is obtained by adding the elongation of Member 0-1 to the deflection of Point 1. The horizontal deflection of Point 0 is obtained by applying Equation (5). The calculated deflections downward from Panel Point 9 are transposed to give upward deflections from Point 0.

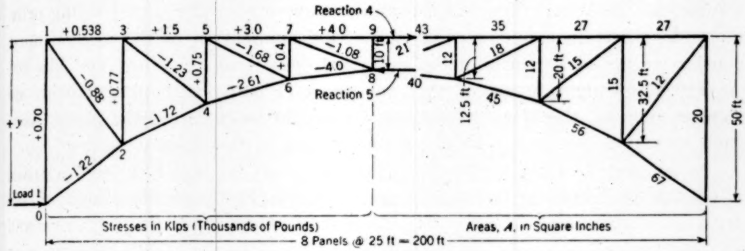

Reaction 4
1
+0.538
3
+1.5
5
+3.0
7
+4.0
9
43
35
27
27
-0.88
+0.77
-1.23
+0.75
-1.68
+0.4
-1.08
-4.0
10 ft
21
12
18
12
20 ft
15
15
32.5 ft
12
8
40
Reaction 5
-2.61
6
45
12.5 ft
-1.72
4
56
20
50 ft
+y
+0.70
-1.22
2
67
Load 1
0
Stresses in Kips (Thousands of Pounds)
Areas, A, in Square Inches
8 Panels @ 25 ft = 200 ft

FIG. 5.

CONCLUSION

The "panel deflection method" solves the problem of truss deflections in a simpler and more direct manner than other analytical methods. It has the advantage of effecting a large saving in the work of computation.

The principal distortions are expressed by means of formulas, which makes the computation easy and accurate. Two short examples have been given to illustrate the practical application of the method. It was not considered necessary to include the extended calculations of a large structure. The writer, however, has applied the method to the solution of most of the problems of indeterminate structures and to the computation of the stresses in a very large structure.

DISCUSSION

DAVID B. HALL,[2] ASSOC. M. AM. SOC. C. E. (by letter).—By well-chosen formulas, Mr. Shoemaker has unquestionably eliminated most of the superfluous operations and reduced the labor of calculating truss deflections to a minimum. However, these same formulas have obscured the underlying principles to a certain extent, so that the actual application of the method must consist mainly of performing the indicated operations faithfully and blindly.

The following solution of the arch truss (Fig. 5), although probably no shorter, may be, therefore, of some assistance in interpreting the paper, since each operation can be explained separately, and in fairly simple terms.[3]

In general, if a section cutting three members is passed through a truss, a change in length of one of these members causes the free side of the truss to rotate about the point of intersection of the other two members, and the amount of the rotation equals the change in length of the member divided by the perpendicular distance from the center of rotation to the member. Shortening Member 4–6 in this manner causes the left portion of the truss to pivot about Point 5, and the stretch in Member 4–5 causes it to rotate about the point where Member 4–6 (prolonged) intersects the top chord.

For chord members, since the center of rotation is at a panel point, the work is very simple and obvious. Thus, when Member 5–7 (Fig. 6) stretches 2.14 units, the deflection one panel length from Point 6 (the hinge for this member) is $2.14 \times \frac{25.0}{12.5} = 4.28$ units, and at more remote panels is a multiple of this value. In the case of lower chords, the lever arms of which are not given directly, it will be noted that the lever arm is equal to the height of the panel times the cosine of the chord inclination, and that the cosine of the chord inclination equals the panel length divided by the chord length. From this it readily follows that substitution of the chord length for the panel length is the only additional step necessary. Thus, the deflection factor for Member 2–4, for instance, is $\frac{27.9}{32.5}$, as given in Table 4.

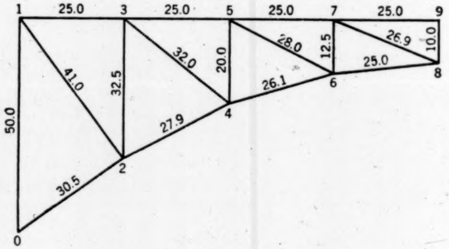


FIG. 6.—LENGTHS OF MEMBERS.

To avoid the inconvenience resulting from the imaginary centers of rotation for web members the following procedure appears to be the simplest to explain. The deflection at the end of a vertical web member is equal to the change in length of the member. For a diagonal, the change in length must be multiplied by the length of the diagonal divided by the height of the panel on the stationary side.

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[2] Asst. Engr., State Dept. of Public Works, Albany, N. Y.

[3] For a similar method see letter by William Bertwell, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, November 1, 1934, p. 565.

TABLE 4.—VERTICAL DEFLECTIONS

Member	Change of length	Factors		Deflections						Horizontal deflection factors	Horizontal deflection
		Base	Increment	Base	Increment	Panel Points					
						0 and 1	2 and 3	4 and 5	6 and 7		
7-9	2.33	....	25.0÷10.0	....	5.83	23.32	17.49	11.66	5.83	4.00	9.32
5-7	2.14	....	25.0÷12.5	....	4.28	12.84	8.56	4.28	....	3.00	6.42
3-5	1.39	....	25.0÷20.0	....	1.74	3.48	1.74	.....	....	1.50	2.09
1-3	0.50	....	25.0÷32.5	....	0.39	0.39	.....	.....	....	0.538	0.27
6-8	2.51	....	25.0+12.5	....	5.02	15.06	10.04	5.02	....	4.00	10.04
4-6	1.51	....	26.1÷20.0	....	1.97	3.94	1.97	.....	....	2.61	3.94
2-4	0.857	....	27.9÷32.5	....	0.74	0.74	.....	.....	....	1.72	1.47
0-2	0.555	....	....	....	....	.....	.....	.....	....	1.22	0.68
7-8	1.38	26.9÷10.0	2.5÷12.5	3.71	0.74	5.93	5.19	4.45	3.71	1.08	1.49
5-6	2.61	28.0÷12.5	7.5+20.0	5.85	2.19	10.23	8.04	5.85	....	1.68	4.39
3-4	2.62	32.0+20.0	12.5÷32.5	4.19	1.61	5.80	4.19	.....	....	1.23	3.22
1-2	3.02	41.0+32.5	....	3.80	....	3.80	.....	.....	....	0.88	2.66
6-7	0.42	1.0	2.5÷12.5	0.42	0.08	0.66	0.58	0.50	0.42*	0.40	0.17
4-5	1.25	1.0	7.5+20.0	1.25	0.47	2.19	1.72	1.25*	....	0.75	0.94
2.3	1.66	1.0	12.5÷32.5	1.66	0.64	2.30	1.66*	.....	....	0.77	1.28
0-1	1.75	1.0	....	1.75	....	1.75*	.....	.....	....	0.70	1.22
Upper panel points	.....	....	....	......	....	90.68	59.52	31.76	9.54	....	49.60
Lower panel points	.....	....	....	....	....	92.43	61.18	33.01	9.96	....	.....

* Lower panel points.

The following proof of this rule may be found easier to remember than the rule itself. In Fig. 7, if Member *BC* (or *a*) changes in length an amount, Δa, while Side *b* remains fixed, Side *c* will rotate about Point *A*. The rotation will be $\frac{\Delta a}{h_a}$, and the vertical movement at Point *B* will be $\Delta a \frac{h_b}{h_a}$. Now, $\frac{h_b}{h_a} = \frac{a}{b}$, since $b\frac{h_b}{2} = a\frac{h_a}{2}$ = area of triangle (also because $\frac{h_b}{a} = \frac{h_a}{b} = \sin C$).

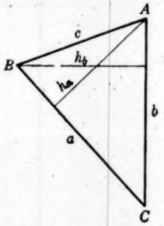

FIG. 7.—WEB MEMBERS.

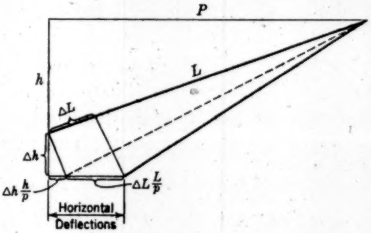

FIG. 8.—HORIZONTAL DEFLECTIONS.

Therefore, the deflection at Point $B = \Delta a \frac{a}{b}$. Thus, the deflection at

Point 5 for Member 5–6 is $2.61 \times \frac{28.0}{12.5} = 5.85$. Since the deflections at other points are proportional to their distances from the center of rotation, and since, in this case, the distances of Points 5 and 7 are in the same ratio as the panel heights at Points 5 and 7, it is seen that the difference per panel may be obtained by multiplying the deflection at Point 5 by $\frac{(20 - 12.5)}{20}$, making the panel increment equal to $2.61 \times \frac{28.0}{12.5} \times \frac{7.5}{20} = 2.19$. Thus, the deflection at Points 2 and 3 contributed by Member 5–6 is $5.85 + 2.19 = 8.04$, etc.

For a single horizontal deflection at the support as in this problem, a very simple method is available. The multiplying factor for each member of the truss is the stress due to a unit load at the support. These factors are already known, and the calculations are shown in the last two columns of Table 4. The remainder of the work, transposing the deflections and calculating the horizontal reactions, is the same as in the original paper and has not been repeated. For algebraic signs it is sufficient to examine the picture and note whether a given deformation deflects the truss up or down.

TABLE 5.—HORIZONTAL DEFLECTIONS, BOTTOM CHORD PANEL POINTS

Point	Δh-CONTRIBUTION				ΔL-CONTRIBUTION			Combined increment	Horizontal deflection
	Vertical deflection	Difference, Δh	Factor	Increment	ΔL	Factor	Increment		
8	0								0
		9.96	2.5 ÷ 25	1.00	2.51	1	2.51	3.51	
6	9.96								3.51
		23.05	7.5 ÷ 25	6.91	1.51	26.1 ÷ 25	1.58	8.49	
4	33.01								12.00
		28.17	12.5 ÷ 25	14.09	0.857	27.9 ÷ 25	0.95	15.04	
2	61.18								27.04
		31.25	17.5 ÷ 25	21.87	0.555	30.5 ÷ 25	0.68	22.55	
0	92.43								49.59

If complete horizontal deflections are required (as in the case of secondary stress calculations), the method illustrated in Table 5 may be used. Each bottom chord member has its ends displaced vertically a known amount and its length changed a known amount. The resulting horizontal movement is shown in Fig. 8.

E. MIRABELLI,[4] M. AM. SOC. C. E. (by letter).—The numerical examples chosen by the author to illustrate the application of the panel deflection method involve symmetrical loadings and symmetrical structures with vertical members at mid-span. In dealing with such cases it is possible to use the center vertical as a reference bar and thus to obtain deflections directly. When there is lack of symmetry it is necessary to apply corrections for rotation of the structure for the same reason that corrections are necessary when a Williot diagram is used. The necessity for such corrections when dealing with vertical deflections may be avoided by combining the panel deflection method with a method of elastic weights.

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[4] Asst. Prof., Structural Eng., Mass. Inst. Tech., Cambridge, Mass.

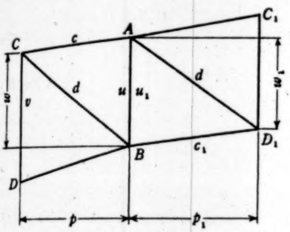

FIG. 9.

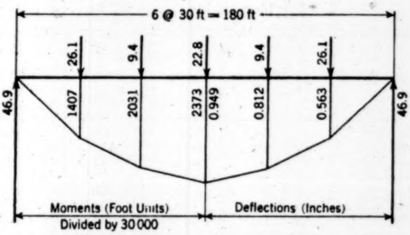

FIG. 10.—DEFLECTIONS OF A 180-FOOT SIMPLE TRUSS.

Referring to Fig. 9 which shows two adjacent panels of a truss, the elastic weight to be applied at Panel Point B may be expressed by the equation developed by Müller-Breslau[5]. In the notation used by the author, the elastic weight is:

$$W_B = -\frac{\Delta_D}{p} - \frac{\Delta_{D1}}{p_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which, Δ_D = downward deflection of Point D with respect to Point B; and, Δ_{D1} = downward deflection of Point D_1 with respect to Point B.

From the author's development (under the heading "General Formulas for Panel Distortions"):

$$\Delta_D = \frac{c'\,c - d'\,d + w\,u'}{u} + v' - u' \quad \ldots\ldots\ldots\ldots\ldots (9)$$

and,

$$\Delta_{D1} = -\frac{c'_1\,c_1 - d'_1\,d_1 + w_1\,u'_1}{u_1} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Equations (9) and (10) may be combined with Equation (8) to determine the elastic weight for any panel point of the truss. The deflections of the bottom chord panel points are then found by the usual method of calculating the bending moments in an auxiliary end-supported beam on which the elastic weights are applied as loads. It should be noted that Equation (9) is used for panels in which the diagonal is attached to the lower panel point, B, and Equation (10) is used for panels in which the diagonal is attached to the upper panel point, A.

The data from the author's Example 1 are used to illustrate the application of this method. The elastic weights are computed as shown in Table 6. The application of elastic weights to the auxiliary beam and the determination of bottom chord deflections are shown in Fig. 10.

It appears to the writer that the panel deflection method may lose some of its simplicity when applied to truss panels which differ from the typical one shown in the author's Fig. 1. For example, if the web system has no vertical members at the panel points it is necessary to insert phantom vertical bars at such points unless entirely different deflection equations are to be developed. For a truss with a K-web system, application of the method would seem to be even more difficult.

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[5] "Die graphische Statik der Baukonstruktionen", by H. Müller-Breslau, Fifth Edition, Vol. 2, Pt. 1, Stuttgart, 1922, p. 95, Equation (2).

TABLE 6.—ELASTIC WEIGHTS FOR A 180-FOOT SIMPLE TRUSS

Panel	Member	Length		Change in length	Products, $c'c$, $d'd$, wu'	Δ_D or Δ_{D1}	30 000 W_B
		Symbol	In feet				
2–0	0–2	c_1	30	+184.5	+5 530		
	0–1	d_1	43.1	−286.2	−12 340		
	1–2	u_1 (w_1)	31	+159.3	+4 940	−736	
2–4	2–4	c_1	30	+184.5	+5 530		
	1–4	d_1	43.1	+210.2	+9 060		
	1–2	u_1 (w_1)	31	+159.3	+4 940	—46	+26.1
4–2	1–3	c	30.4	−222.5	−6 770		
	1–4	d	43.1	+210.2	+9 060		
	3–4	u	36	+ 14.4	+ 446		
	1–2	v (w)	31	+159.3	....	−282	
4–6	4–6	c_1	30	+189.3	+5 680		
	3–6	d_1	46.8	+132.8	+6 220		
	3–4	u_1 (w_1)	36	+ 14.4	+ 518	+1	+ 9.4
6–4	3–5	c	30	−220.8	−6 620		
	3–6	d	46.8	+132.8	+6 220		
	5–6	u (w)	36	0	0		
	3–4	v	36	+ 14.4	....	−342	+22.8

WILLIAM BERTWELL,[6] ASSOC. M. AM. SOC. C. E. (by letter).—In calling attention to the process of computing truss deflections as the cumulative effect of movements of the component parts of the truss, this paper is interesting and valuable. The author states that the "'panel deflection method' solves the problem of truss deflections in a simpler and more direct manner than other analytical methods." By the use of set formulas, however, the writer believes that the simplicity of the process is somewhat lessened.

By treating a single member at a time, rather than an entire panel, there results what may be termed the "geometric" method.[7] The computations for the author's Examples 1 and 2, presented in Tables 7 and 8, respectively, demonstrate that this basis of analysis is simpler.

The basis of the geometric method is: Assuming the part of the structure to the right of the member under consideration fixed in position, any change in length of the given member causes the part of the structure to the left to rotate about the center of moments for the given member. It remains only to determine the angular movement caused by the member, a geometric problem.

The writer believes that this one fundamental relationship is sufficient to solve all such problems simply and directly. Reducing the principle to a group of equations, or introducing supplementary derivations of one sort or another, inevitably diminishes understanding of the physical action of the structure. Keeping this one principle in mind, a glance at the line diagram

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[6] Associate Civ. Engr., Regional Office, U. S. Bureau of Public Roads, San Francisco, Calif.

[7] *Engineering News-Record*, June 7, 1934, p. 746, and November 1, 1934, p. 565.

TABLE 7.—DEFLECTIONS OF A 180-FOOT SIMPLE TRUSS

Member	Change in length $\times \frac{30\,000}{12}$	Vertical component of movement at left end of member	Distance to center of moments (in panels)	Panel increment	Deflections of Panel Points					
					0	1	2	4	3	6
3-5	-220.8	....	1.200	184.0	552.0	368.0	....	184.0	184.0	0
4-6	189.3	....	1.200	157.8	315.6	157.8	....	....	....	....
1-3	-222.5	....	1.184	188.0	376.0	188.0	....	....	....	....
0-4	369.0	....	1.033	357.2	357.2	....	....	....	....	....
3-6	131.0	170.3	....	0	170.3	170.3	....	170.3	170.3	....
3-4	14.4	....	7.200	2.0	-10.4	-12.4	....	-14.4	....	....
1-4	210.0	250.8	6.200	40.5	210.3	250.8	....	....	....	....
0-1	-286.0	396.7	....	....	396.7	....	....	....	....	....
1-2	159.3	....	....	....	....	....	-159.3	....	....	....
Total deflection from Point 6................					2 367.7	1 122.5	....	339.9	354.3	0
Total deflection from Point 0................					0	1 245.2	1 404.5	2 027.8	2 013.4	2 367.7
Total deflection from Point 0, in inches.......					0	0.50	0.56	0.81	0.81	0.95

of the structure is sufficient to establish the direction of movement imparted to any point by a given member. The figures are then set down as easily as a Williot diagram is drawn.

TABLE 8.—INFLUENCE ORDINATES FOR HORIZONTAL REACTION OF 200-FOOT ARCH

Member	Change in length	Vertical component of movement at left end of member	Distance to center of moments (in panels)	Panel increment	Horizontal movement of Point 0	Vertical Deflection of Panel Points				
						0	3	5	7	9
7-9	2.33	....	0.400	5.825	9.32	23.30	17.47	11.65	5.83	0
6-8	-2.51	....	0.498	5.040	10.08	15.12	10.08	5.04	....	....
5-7	2.14	....	0.500	4.280	6.42	12.84	8.56	4.28	....	....
4-6	-1.514	....	0.766	1.975	3.95	3.95	1.97	....	....	....
3-5	1.39	....	0.800	1.737	2.09	3.47	1.74	....	....	....
2-4	-0.857	....	1.163	0.737	1.47	0.74	....	....	....	....
1-3	0.50	....	1.300	0.385	0.27	0.38	....	....	....	....
0.2	-0.555	....	1.639	0.339	0.68	....	....	....	....	....
7-8	-1.38	3.71	5.000	0.742	1.48	5.94	5.19	4.45	3.71	....
7-6	0.42	....	5.000	0.084	0.17	0.67	0.59	0.50	....	....
5-6	-2.61	5.85	2.667	2.195	4.39	10.24	8.05	5.85	....	....
5-4	1.25	....	2.667	0.469	0.94	2.19	1.72	....	....	....
3-4	-2.62	4.19	2.600	1.613	3.22	5.80	4.19	....	....	....
3-2	1.66	....	2.600	0.638	1.28	2.30	....	....	....	....
1-2	-3.02	3.81	2.857	1.333	2.67	3.81	....	....	....	....
1-0	1.75	....	2.857	0.613	1.22	....	....	....	....	....
Total horizontal movement of Point 0...........					49.65 × 2 99.30					
Total deflection from Point 9						92.50	59.56	31.77	9.54	0
Total deflection from Point 0............................						0	32.94	60.73	82.96	92.50
Horizontal reaction = $\frac{\text{vertical deflection}}{99.30}$						......	0.332	0.612	0.836	0.932

Every step of the geometric method appears in Tables 7 and 8; no formulas need be kept in mind. The figures for the chord members are obvious. The effect of each diagonal web member has been determined by finding the vertical component of motion at the left end of the member and dividing this by the horizontal distance to the center of moments. The vertical component is equal to the change in length of the diagonal multiplied by the ratio of its length to the height of the truss at its right end.

ROBERT H. HURLBUTT,[8] JUN. AM. SOC. C. E. (by letter).—The method described in this interesting paper is based upon the area-moment principle as applied to a beam of varying moment of inertia. The same relation between the panel distortions and panel-point deflections of a truss may also be derived through inspection, as follows.

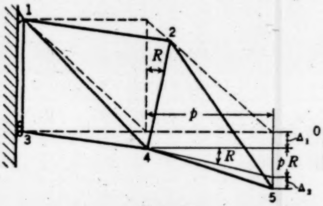

FIG. 11

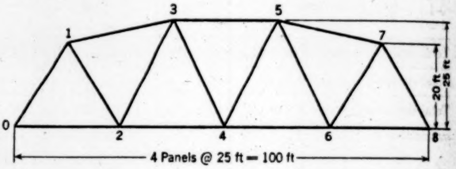

FIG. 12

In Fig. 11, a cantilever truss shown in a distorted position, the total deflection of Panel Point 5 is the sum of Δ_1, the vertical distortion of the first panel, plus R, the angular distortion of the first panel, multiplied by p, the length of the second panel, plus the vertical distortion, Δ_2, of the second panel.

The panel-deflection method has advantages over the Maxwell-Mohr method for trusses with rectangular panels. The application is simple and direct, each step is easily visualized, and, most important, the deflections of all points are found from one set of calculations, as by the Williot diagram. However, a rotational correction corresponding to that provided by the Mohr rotation diagram, is necessary when dealing with a truss in which every member changes its line of action when the truss deflects.

Table 9 gives the calculations for Δ in a problem of this type, an unsymmetrically loaded Warren truss. Assuming that Member 0–1, Fig. 12, does not rotate, all deflections are found upward from Point 0. The distorted truss is then rotated downward to produce the effect of the rotation of Member 0–1. Thus, referring to Table 9 for values of Δ and R:

$$30\,000\ V_8 = 811 + 2\,026 + 3\,155$$

$$- 3\,590 - 300\ (3 \times 1.824 + 2 \times 5.0 + 9.705) = - 5\,151$$

$$V_2 = \frac{1}{30\,000}\left(811 + \frac{5\,151}{4}\right) = +\ 0.070 \text{ in.}$$

$$V_4 = \frac{1}{30\,000}\left(811 + 2\,026 - 300 \times 1.824 + \frac{5\,151}{2}\right) = +\ 0.162 \text{ in.}$$

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and,

$$V_6 = \frac{1}{30\,000}\left[811 + 2\,026 + 3\,155 - 300\,(2 \times 1.824 + 5.0) + \frac{3 \times 5\,151}{4}\right] = +\ 0.242 \text{ in.}$$

TABLE 9.—DEFLECTIONS OF A 100-FOOT WARREN TRUSS

Member	Length, in inches	Area, in square inches	Stress, in thousands of pounds	Values of $\frac{SL}{A}$	Stress in member due to unit force, in pounds	Product of stress due to unit force and change of length	Stress in member due to unit couple, in pounds	Product of stress due to unit couple and change in length
0-1	283	20	−29.5	−416	−1.180	+493	0.0	....
1-2	283	15	+23.0	+434	+1.180	+512	+0.000874	+0.379
0-2	300	15	+15.5	+310	−0.625	−194	−0.00416	−1.290
1-3	306	20	−28.5	−436	0.0	....	+0.00377	−1.645
2-3	336	8	−21.0	−884	0.0	....	−0.00083	+0.732
Δ_2	......	......	....	....	....	+811	....	....
R_{2-3}	......	......	....	....	....	....	....	−1.824
2-3	336	8	−21.0	−884	−1.12	+990	0.0	....
3-4	336	8	+28.0	+1.260	+1.12	+1 411	0.0	....
2-4	300	15	+37.5	+750	−0.50	−375	−0.00333	−2.500
3-5	300	20	−50.0	−750	0.0	....	+0.00333	−2.500
4-5	336	8	−28.0	−1.260	0.0	....	0.0	....
Δ_4	......	......	....	....	....	+2 026	....	....
R_{4-5}	......	......	....	....	....	....	....	−5.0
4-5	336	8	−28.0	−1 260	−1.12	+1 411	0.0	....
5-6	336	8	+47.0	+2 115	+1.12	+2 369	−0.00083	−1.755
4-6	300	15	+62.5	+1 250	−0.50	−625	−0.00333	−4.167
5-7	306	20	−85.0	−1 300	0.0	....	+0.00377	−4.910
6-7	283	15	+68.5	+1 290	0.0	....	+0.000874	+1.127
Δ_6	......	......	....	....	....	+3 155	....	....
R_{6-7}	......	......	....	....	....	....	....	−9.705
6-7	288	15	+68.5	+1 290	−1.18	−1 522	....	....
7-8	283	20	−88.5	−1 255	+1.18	−1 481	....	....
6-8	300	15	+47.0	+940	−0.625	−587	....	....
Δ_8	......	......	....	....	....	−3 590	....	....

The advantages of the panel-deflection method for the solution of this problem are not particularly great, due to the increased work involved in applying the unit forces and couples to the irregularly shaped panels.

A. A. EREMIN,[9] ASSOC. M. AM. SOC. C. E. (by letter).—Equations for computing truss panel deflections have been developed by Mr. Shoemaker in this paper. His method is more direct than the least work method with unit loading at each panel joint. However, the panel deflections may also be computed in one operation by means of elastic weights.[10] For example, at some chord joint of a truss, such as Point N, Fig. 13, the elastic weight is expressed by:

$$W_n = \delta\,\alpha_n + \frac{s_{n1}}{E}\tan\phi_{n1} + \frac{s_{n2}}{E}\tan\phi_{n2} \ldots\ldots\ldots\ldots (11)$$

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[10] Hütte: Des Ingenieurs Tashchenbuch, Bd. III.

in which, $\delta \alpha_n$ = the deformation of a chord angle at Joint N; ϕ_{n1}, ϕ_{n2} = angles between the chord and a horizontal line to the left and to the right of a

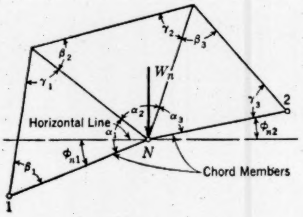

FIG. 13.—ELASTIC WEIGHT OF TRUSS CHORD JOINT.

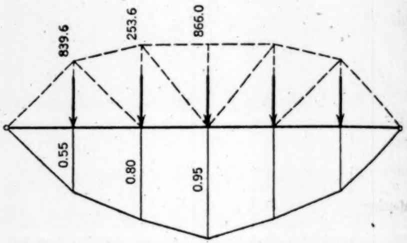

FIG. 14.—DEFLECTION OF TRUSS LOWER CHORD.

joint; s_{n1}, s_{n2} = unit stresses in the chord members to the left and to the right of Joint N; and E = modulus of elasticity of the chord in tension and compression.

If the chord is horizontal the angles, $\phi_{n1} = \phi_{n2} = 0$. Therefore, the elastic weight is equal to:

$$W_n = \delta \alpha_n \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The angular deformation of the chord at Point N is computed from deformations of the angles formed by members converging at that point. The deformation of an angle such as α_2 in Fig. 13, is expressed as,[11]

$$E \, \delta \, \alpha_2 = (s_3 - s_2) \cot \beta_2 + (s_3 - s_1) \cot \gamma_2 \ldots\ldots\ldots (13)$$

in which β_2 and γ_2 are angles in the triangular panel opposite Joint N.

The elastic weights are considered as vertical forces at the chord joints. The force polygon of the elastic weights constructed through the end joints of a truss represents a deflection polygon of chord panels. The force polygon may be determined either graphically, or analytically, as the bending moment diagram of a beam sustaining the elastic weights at the spaces corresponding to the horizontal projections of the joints (see Fig. 14).

In Example 1 of the paper the elastic weights at the lower chord computed with Equation (12) are: $W_2 = 839.6$; $W_4 = 253.6$; and $W_6 = 866.0$.

Deflections at the lower chord panels determined as ordinates of the bending moment diagram in terms of $\frac{12}{30\ 000}$ are: $\Delta_2 = 0.55$; $\Delta_4 = 0.80$; and, $\Delta_6 = 0.95$. These deflections are closely in agreement with those computed by the author (see Table 1). The elastic weights and deflections were computed with a standard 10-in. slide-rule. Various simplifications in the elastic weight method have been developed by Professor Müller-Breslau.[10,12]

The advantage of the elastic weight method over that introduced by Mr. Shoemaker is in the simplicity of the conventional rules for determining signs, which are the same as those for the stresses in the members of a truss. In the elastic weight method it is necessary to remember only two formulas (Equations (11) and (13)). Furthermore, the deformations of panel angles, computed in the elastic weight method may also be used for computing the secondary stresses in the truss.

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[11] "Modern Framed Structures", by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 1917, p. 427.

[12] "Graphische Statik", Bd. II.

T. P. NOE,[13] JR., JUN. AM. SOC. C. E. (by letter).—A clearer conception of the panel deflection method as described by Mr. Shoemaker is gained by developing the basic equations by means of the Williot diagram. C. A. Ellis, M. Am. Soc. C. E., has pointed out in two papers[14] that the geometrical relationships established by a Williot diagram may be expressed in the form of algebraic equations which will yield an algebraic solution for those values previously obtained graphically from a Williot diagram drawn only for known changes in length of the truss members. The writer has had several occasions to make use of such an algebraic solution of the Williot diagram, notably in the determination of secondary stresses, but heretofore has not developed any standard expressions for typical truss panels. However, there is much merit in doing so. For unusual cases, however, the writer believes that the Williot diagram offers the best solution.

In Fig. 15 is shown the Williot diagram drawn for the typical panel considered by the author This diagram is constructed on the basis of the assumptions made in the paper, namely, that Point A is fixed in position, Member AB is fixed in direction, and the change in length of each member is considered as a lengthening. From the geometry of Fig. 15 the following relations may be developed, Angles α, β, and θ referring to the slopes of the inclined members as indicated:

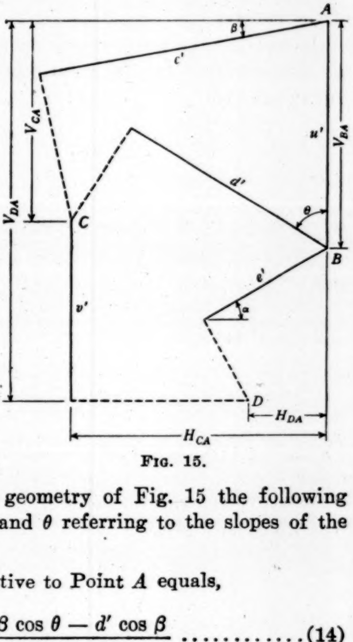

FIG. 15.

The vertical deflection of Point C relative to Point A equals,

$$V_{CA} = \frac{c' \sin \theta + u' \cos \beta \cos \theta - d' \cos \beta}{\sin \beta \sin \theta + \cos \beta \cos \theta} \quad \ldots\ldots\ldots\ldots (14)$$

the horizontal deflection of Point C relative to Point A equals,

$$H_{CA} = \frac{c' \cos \theta - u' \sin \beta \cos \theta + d' \sin \beta}{\sin \beta \sin \theta + \cos \beta \cos \theta} \quad \ldots\ldots\ldots\ldots (15)$$

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[13] Checker, Carolina Steel & Iron Co., Greensboro, N. C.

[14] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 580; also *Engineering News-Record*, Vol. 112, April 26, 1934, p. 534.

the vertical deflection of Point D relative to Point A equals,

$$V_{DA} = V_{CA} + v' \quad \ldots\ldots (16)$$

the horizontal deflection of Point D relative to Point A equals,

$$H_{DA} = (e' \sec \alpha - v' \tan \alpha) - \frac{(c' \sin \theta - u' \sin \theta \sin \beta - d' \cos \beta) \tan \alpha}{\sin \beta \sin \theta + \cos \beta \cos \theta} \ldots (17)$$

and the rotation of Member CD relative to Member AB equals,

$$R_{CD} = \frac{H_{CA} - H_{DA}}{\overline{C\,D}} = \frac{H_{CA} - H_{DA}}{v} \quad \ldots\ldots (18)$$

The sign convention and notation for change in length of a member are those of the paper.

Expressing the functions of the angles, α, β, and θ, in terms of the lengths of the members or their projections, and substituting these values in Equations (14), (15) and (17):

$$V_{CA} = \frac{c'\,c + u'\,w - d'\,d}{u} \quad \ldots\ldots (19)$$

$$H_{CA} = \frac{(c'\,c - d'\,d + u'\,w)\,w}{u\,p} + \frac{(d'\,d - u'\,w)}{p} = \left(\frac{w}{p}\right)(V_{CA}) + \frac{(d'\,d - u'\,w)}{p} \ldots (20)$$

and,

$$H_{DA} = \frac{(e'\,e - v'\,v + v'\,w)}{p} + \frac{(c'\,c - d'\,d + u'\,w)\,w}{u\,p} - \frac{u'\,w}{p} - \frac{(c'\,c - u'\,u + u'\,w - d'\,d)\,v}{u\,p} \quad \ldots\ldots (21)$$

Equations (19) and (20) are identical with Equations (1) and (2) of the paper.

Substituting the values of H_{CA} and H_{DA} given by Equations (20) and (21) into Equation (18):

$$R_{CD} = \left(\frac{1}{p}\right)\left(V_{CA} + \frac{d'\,d - e'\,e - v'\,w}{v} + v' - u'\right) \quad \ldots\ldots (22)$$

Equation (22) is identical with Equation (3) of the paper.

It is evident that each of the panel points, C, D, and B, will undergo an additional deflection, both horizontally and vertically, if the reference member, AB, rotates through some small angle denoted by R_{AB}. These rotational displacements are similar in nature to those given by a Mohr diagram. It is easily seen that for the case considered these additional deflections are given by the following expressions:

$$V'_{CA} = (c\,R_{AB}) \cos \beta = p\,R_{AB} \quad \ldots\ldots (23)$$

$$H'_{CA} = (c\,R_{AB}) \sin \beta = (u - w)\,R_{AB} \ldots\ldots\ldots\ldots\ldots (24)$$

$$V'_{DA} = (\overline{A\,D}\,R_{AB}) \left(\frac{p}{\overline{A\,D}}\right) = p\,R_{AB} \ldots\ldots\ldots\ldots\ldots (25)$$

and,

$$H'_{DA} = (\overline{AD}\,R_{AB})\,\frac{(u - w + v)}{\overline{A\,D}} = (u - w + v)\,R_{AB} \ldots\ldots (26)$$

In order to illustrate the application of the panel deflection method to the case of an unsymmetrically loaded truss, the writer has computed the vertical deflections of a truss having the same dimensions and cross-sectional areas as that of the author's Example 1, but with an unsymmetrical loading. The values of the stresses in the members due to the assumed loading are recorded in Table 10. Panel points on the right of Member 5–6 are denoted by the corresponding panel point numbers appearing on the left with the prime attached.

TABLE 10.—VERTICAL DEFLECTIONS OF UNSYMMETRICALLY LOADED PRATT TRUSS

Deflection point (1)	Member (2)	Total stress, in kips* (3)	Change in length, inches × $\frac{30\,000}{12}$ (4)	Relative vertical deflection given by Equation (19) (5)	Relative rotation of reference member given by Equation (22) (6)	Rotation relative to Member 5–6 = R (7)	Deflection due to rotation = p × Column (7) (8)	Total deflection with Member 5–6 fixed (9)	Transposed vertical deflections (10)	Deflections due to rotation of entire truss about Point O (11)	True vertical deflection Column (10) × Column (11) (12)	Vertical deflections, in inches = Column (12) × $\frac{12}{30\,000}$ (13)
0	0–2	+653	+230.5	—980.2	......	......	......	—2 309.2	0	0	0	0
	0–1	—939	—358.5									
2	1–2	+350	+258.3	......	—14.34	—26.58	—797.4	—531.6	+1 777.6	+28.6	1 806.2	0.722
	1–4	+259	+192.6									
	2–4	+653	+230.5									
1	1–3	—845	—257.0	—278.4	......	......	......	—789.9	+1 519.3	+28.6	1 547.9	0.619
4	3–4	+164	+196.8	......	—12.24	—12.24	—367.2	52.5	+2 361.7	+57.3	2 419.0	0.968
	3–6	— 33	— 30.9									
	4–6	+833	+219.2									
3	3–5	—812	—221.4	—144.3	......	......	......	—144.3	+2 164.9	+57.3	2 222.2	0.889
5, 6	5–6	0	0	0	0	0	0	0	+2 309.2	+85.9	2 395.1	0.958
3′	3′–5	—812	—221.4	—541.9	......	......	......	—541.9	+1 767.3	+114.5	1 881.8	0.756
	4′–6	+625	+164.5									
	3′–6	+293	+274.6									
4′	3′–4′	—121	—145.2	......	—10.72	—10.72	—321.6	—687.1	+1 622.1	+114.5	1 736.6	0.695
1′	1′–3′	—634	—192.8	—561.5	......	......	......	—1 425.0	+884.2	+143.2	1 027.4	0.411
	2′–4′	+411	+145.1									
	1′–4′	+307	+228.3									
2′	1′–2′	+100	+ 73.8	......	— 7.97	—18.69	—560.7	—1 351.2	+958.0	+143.2	1 101.2	0.440
	0′–1′	—591	—255.0									
0′	0′–2′	+411	+145.1	—569.1	......	......	......	—2 481.0	—171.8	+171.8	0	0

* 1 kip denotes "kilo-pounds," or 1 000 lb.

Since the writer has modified the author's arrangement of the computations a brief summary of the procedure used will be given. To save space the individual products entering into the solution of the basic equations have not been recorded in Table 10.

In Column (5) are recorded the values of the vertical deflections of the panel reference points as computed by Equation (19). It should be noted that the deflections of Points *O* and *O′* are referred to Points 2 and 2′, respectively. In Column (6) are recorded the values of the rotations of each reference member due to the distortion of the given panel, as computed by Equation (22). Column (7) gives the total rotation of each reference member referred to Member 5–6. Column (8) gives the vertical deflection of each reference point due to rotation of the panel reference member. This value is given by Equation (23). The values in Column (9) represent the vertical deflection of each point relative to Point 5, assuming that Member 5–6 does not rotate. For example, the deflection of Panel Point 1 equals the deflection of Point 1 relative to Point 3, plus the deflection of Point 3 relative to Point 5, plus the deflection of Point 1 due to the rotation of the panel reference Member 3–4. This sum is given by $(-278.4 - 144.3 - 367.2) = -789.9$. Column (10) gives the values of the transposed deflections referred to Point *O*. It is seen from Column (10) that the deflection of Point *O′* relative to Point *O* is not zero, due to the assumption that Member 5–6 does not rotate. This means that the truss as a whole must be rotated clockwise about Point *O* through such an angle as to make the deflection of Point *O′* zero. This rotation is identical to that assumed in the Mohr diagram. The vertical deflection of each point for such a rotation will be given by the product of the horizontal distance of the point in question from Point *O*, multiplied by the rotation angle (in this case $\frac{171.8}{180}$). Column (12) gives the true deflection of each point, which is the sum of the values in Columns (10) and (11). In Column (13) the final values are converted to inches.

It is the writer's opinion that the foregoing procedure for applying the basic equations is easier to follow than that given by the author.

DAVID A. MOLITOR,[15] M. AM. SOC. C. E. (by letter).—The method proposed by the author should scarcely be classified under such a broad title, because of its limited application to very simple truss forms.

In point of labor and clarity of presentation, the method of elastic weights affords equally simple solutions for the same cases, but with the advantage of being universally applicable to all kinds of structures, even to solid web-beams. The complications encountered in odd-shaped triangular truss systems without vertical members, are due to the more involved geometric relations, for which the new method becomes quite "out of bounds."

An exhaustive treatment of the method of elastic weights, based on Professor Otto Mohr's (1875) statement of the relation between deflection polygons and equilibrium polygons for simultaneous cases of loading, was presented by the writer in 1911.[16]

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[15] Structural Engr., Public Buildings Branch, U. S. Treasury Dept., Washington, D. C.

There is a decided danger that those who do not appraise the distinct limitations of this paper may misapply it, in view of the author's broad conclusion that the method "solves the problem of truss deflections in a simpler and more direct manner than other analytical methods."

GLENN L. ENKE,[17] JUN. AM. SOC. C. E. (by letter).—The analytical method for solving for truss deflections presented by Mr. Shoemaker is an interesting one. It appears much superior to what is referred to in most textbooks as the "method of internal work" wherein individual computations are made for any one panel point.

The writer prefers a graphical method utilizing Williot-Mohr diagrams to arrive at the vertical and horizontal displacement of all panel points simultaneously. An unsymmetrical or continuous structure is handled as easily as a symmetrical layout and throughout the process a rough idea of what is happening to any joint can be visualized; a valuable factor in eliminating blunders. By means of a drafting-machine, large-scale diagrams of sufficient accuracy are quickly prepared.

The writer's purpose in discussing this paper, however, is not to extol the merits of any particular method, but to point out the necessity of correctly applying results obtained by any method to the case in hand. Consideration of the various types of structures will be restricted to the modern types of highway and railway bridges.

Resident engineers on bridge construction from the writer's office have found, without exception, that actual truss deflections are uniformly less than the theoretical by 40 to 50%, and in setting header-boards for concrete surface finishing, they take this fact into consideration. Upon the accuracy of their estimate depends how level a roadway surface is obtained. This effect is a very common one, and is based upon a number of factors to be discussed subsequently. A specific instance is quoted: The Bridge Department of the State of California in 1935 completed a vertical lift structure over the Sacramento River, known as the M Street Bridge. The lift span is a symmetrical 202-ft, riveted, through truss span, 56.5 ft from center to center of trusses, with two unsymmetrical approach spans of 167.5 ft and 192.5 ft, respectively. A concrete deck with curbs 52 ft apart, over which four lanes of highway traffic and a single-track railroad pass, is supported by the usual system of floor-beams and stringers framed into each other and riveted throughout. Deflection diagrams were prepared for various conditions of dead load for each span. In erecting these trusses and pouring the concrete roadway slab, the resident engineer correctly estimated the deflection at about one-half the theoretical and set his grade line accordingly. A highly satisfactory profile was obtained, to the credit of the resident engineer. Many factors enter into a result such as the foregoing, as, for example: (1) Joint rigidity; (2) joint

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[16] "Kinetic Theory of Engineering Structures", by David A. Molitor, Chapter VII, 1911.

[17] Associate Bridge Designing Engr., Highway Div., State Dept. of Public Works, Sacramento, Calif.

"slippage"; (3) direct stress taken by lateral bracing and steel floor systems; (4) using gross section to compute total elongation; and (5) reduced strains due to heavy lacing-bars, stay-plates and long gusset-plates.

Factor (1).—Probably the most important cause of reduced truss deflection is joint rigidity. In a completely riveted truss, every member is practically fixed at the ends (end restraint is not unknown in pin-connected members), and the secondary stresses set up by a nominal amount of deflection will produce a certain degree of resistance to further deflection in addition to a resistance already created by direct stresses. Analysis for secondary stresses can be made, and from these a set of panel-point deflections secured. The work involved, however, is very laborious, and the assumptions made are even broader than those used in the original computations. For purposes of analysis, the distance between intersections of gravity axes is considered as effective length. With this assumption, the moment of inertia of an individual member is decidedly not uniform throughout its length, although it is ordinarily considered so for convenience.

Factor (2).—Another kind of distortion brought about by joint rigidity and the resulting end moments induced in every member can be termed, "joint slippage", wherein the outer rivets in any connection deform laterally under secondary stress and permit a slight end rotation of the member. This effect is usually slight, and acts to relieve secondary stress.

Factor (3).—Despite every effort to avoid it, lateral bracing and steel floor systems do take a part of the direct stress originally intended by analysis for the chord members only, the percentage taken by each being nearly proportional to the relative areas, positions, and stiffness of connections. In a narrow structure with heavy wind-bracing and relatively light main members, a considerable portion of the direct stress can be resisted by lower chord-bracing.

Distribution of stress among chord members and floor systems is very uncertain. The writer recently conducted a few strain-gauge measurements on a lower chord of the lift span of the M Street Bridge. Available loading consisted of two electric freight engines, which, theoretically, would stress this lower chord in tension to 2 400 lb per sq in. Within the range of accuracy of the measuring equipment, this lower chord was stressed almost to this value. The live load was symmetrically placed, longitudinally and laterally, to avoid any eccentric loading. Strain-gauge readings were also taken on the lower flange of a roadway stringer parallel to, but a considerable distance away from, the railroad stringers directly supporting the live load, and, theoretically, not subject to railroad loading. This stringer is framed between floor-beams by means of light connection angles, but is rigidly attached to the concrete deck because of the field welding of the main slab reinforcement directly to the top flange of all roadway stringers. Longitudinal continuity of the slab across floor-beams is prevented by the presence of a cold joint along the center line of floor-beams; yet, under the same live load, it was able to register a tensile stress of about 500 lb per sq in. Although no definite conclusions can be drawn from this brief investigation, it is of interest to note the presence of direct stress in members distinctly not a part of the lower chord, nor directly connected thereto.

Factor (4).—A designer usually considers the gross section of all members in arriving at the total elongation, or strain. Actually, this is somewhat in error for built-up tension members, as directly across a line of rivets, the unit strain is greater than across a line between rivets. A more accurate method is to use a "weighted" cross-section wherein the rivet pitch and percentage of holes out, are taken into consideration.

Factor (5).—Another important effect is the presence of much detail material on the member, such as heavy lacing-bars, stay-plates, long gusset-plates, and portal and sway-frame gussets, all of considerable cross-section, which materially reduce the unit strain wherever they occur. This effect becomes most pronounced at the joints themselves. A haphazard allowance for these variables is impossible. A scientific analysis may be attempted for every truss member throughout its length to determine the variation in unit strain, but the numerous assumptions made while doing it scarcely warrant any high regard for the result.

In consideration of these variables, the writer wishes to conclude his discussion by warning against too much accuracy in determining truss deflections by any method, however excellent it may be, if the actual structure under consideration will behave quite differently from a theoretically pin-connected truss. As the majority of highway and railway structures to-day are completely riveted throughout, in addition to being well braced and comparatively rigid, the aforementioned facts deserve full consideration when applying truss deflections in the field.

Undoubtedly, the author is well aware of these limitations, but there are many who need a gentle reminder from time to time to prevent their interest in the mathematics of a solution from obliterating other, and often more important, phases of a problem.

FANG-YIN TSAI,[18] ASSOC. M. AM. SOC. C. E. (by letter).—The method for computing truss deflections presented in this paper is certainly somewhat simpler and more convenient than the elastic weight methods of either the bar changes developed by Professors O. Mohr[19] and H. Müller-Breslau[20], or the angle changes (also known as bar-chain method) developed by Professor R. Land[21], which, as far as the writer knows, have been the only analytical methods available to date for computing the truss deflections for all the panel points in any chord by a continuous operation. The author is to be commended for his ingenuity.

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[18] Prof. of Structural Eng., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

[19] "Beitrag zur Theorie des Fachwerks", *Zeitschrift* des Architekten- und Ingenieur-Vereines zu Hannover, 1875, p. 17.

[20] "Beitrag zur Theorie des Fachwerks", *Zeitschrift* des Architekten- und Ingenieur-Vereines zu Hannover, 1885, p. 418.

[21] "Beitrag zur Ermittelung der Biegungslinien ebener und elastischer Stabwerke", *Zivilingenieur,* 1889. This method has been presented in the United States in the following three books and papers only: (1) "Kinetic Theory of Engineering Structures", by D. A. Molitor, M. Am. Soc. C. E., McGraw-Hill Book Co., New York, 1911, p. 107; (2) "Truss Deflections Accurately Determined by Angle Changes and Elastic Weights", by D. B. Steinman, M. Am. Soc. C. E., *Engineering Record,* May 13, 1916, p. 644; and (3) "Structural Theory", by H. Sutherland and H. L. Bowman, Members, Am. Soc. C. E., Second Edition, John Wiley & Sons, New York, 1935, p. 178.

It is regrettable, however, that the derivation of Equations (4) and (5) was not included in order to clarify their application and their limitation. These formulas are valid only for trusses having vertical web members and equal panel lengths. It is possible to modify the method, expressing it in a more general form, so as to make it applicable to trusses of any type.

Fig. 16 shows an intermediate oblique panel, $MN\,mn$, of a truss, with M and N denoting the upper panel points and m and n the corresponding lower panel points. The dimensions of the panel are designated by P, p, Y, y, h, and k, which will be considered as positive when measured upward from, or to the right of, a certain reference point (say, M, Fig. 16) and otherwise negative. The lengths of the five bars of the panel are designated, respectively, by L_{MN}, L_{mn}, L_{Mm}, L_{Nn}, and L_{Nm}. Let ΣR be the sum of the rotations of all the deformed panels to the left of this panel, ΣR being considered as positive when it is clockwise and negative when counter-clockwise. Considering the panel point, M, as the reference point (that is, considering M as fixed in position after it has been deflected by all the deformed panels to its left), the panel, as a fixed frame, will first be rotated about M through the angle, ΣR. Thus, the panel points N, n, and m, are deflected, respectively, to the positions of N', n', and m'. The deflection, NN', is evidently equal to $L_{MN}\,\Sigma R$, which may be resolved into the vertical component, $P\,\Sigma R$, and the horizontal component, $Y\,\Sigma R$. The deflection, nn', may be resolved into the two components, $L_{mn}\,\Sigma R$ and $L_{Mm}\,\Sigma R$, which again may be resolved, respectively, into the two vertical components, $p\,\Sigma R$ and $k\,\Sigma R$, and the two horizontal components, $y\,\Sigma R$ and $h\,\Sigma R$. All the deflections will be considered as positive when they are downward from, or to the right of, the original position of the panel point.

Assume the bar deformations, ΔL_{MN}, ΔL_{Mm}, and ΔL_{Nn}, to be the increases in length, and ΔL_{Nm} and ΔL_{mn}, the decreases. Considering M as a fixed reference point and, in addition, $M\,m'$ as a reference bar, fixed in direction, the deflected panel points, N', n' and m', due to those bar deformations, will be further deflected, respectively, to the positions of N'', n'', and m'', which are located by the Williot construction as shown in Fig. 16. Let V'_{NM}, H'_{NM}, and V'_{nM}, and H'_{nM} be the vertical and horizontal components of the deflections, $N'\,N''$ and $n'\,n''$, respectively, which are the panel deflections due to the bar deformations of the panel alone, and let V_{NM}, H_{NM}, V_{nM}, and H_{nM} be the vertical and horizontal components of the deflections, $N\,N''$ and nn'', respectively, which are the panel deflections due to both the bar deformations of the panel and the total rotation of all the deformed panels to the left of this panel. In the foregoing notation for deflections, the first subscript denotes the deflected panel points, whereas the second subscript denotes the reference point. From Fig. 16 the following relations are evident:

$$V_{NM} = V'_{NM} + P\,\Sigma R \quad \text{.......................} (27)$$

$$H_{NM} = H'_{NM} + Y\,\Sigma R \quad \text{.......................} (28)$$

$$V_{nM} = V'_{nM} + (p + k)\,\Sigma R \quad \text{.................} (29)$$

and,

$$H_{nM} = H'_{nM} + (y + h)\,\Sigma R \quad \text{.................} (30)$$

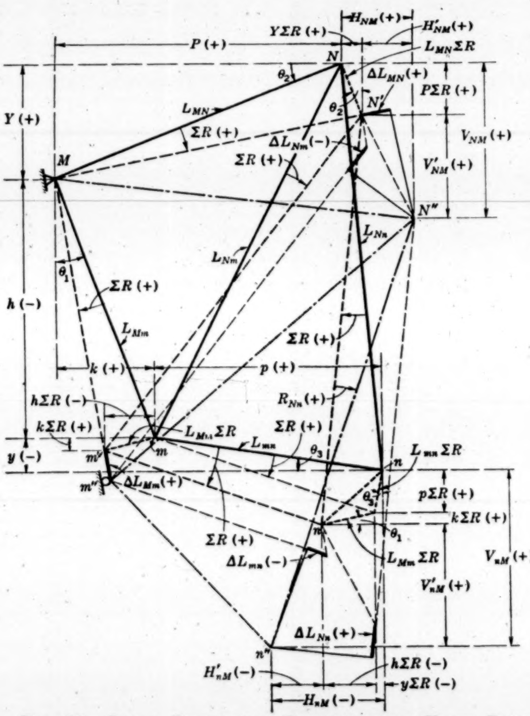

P (+)
YΣR (+)
H_{NM} (+)
H'_{NM} (+)
$L_{MN}ΣR$
N
$θ_2$
$ΔL_{MN}$ (+)
PΣR (+)
N′
Y (+)
L_{MN}
ΣR (+)
$ΔL_{Nm}$ (−)
V_{NM} (+)
V'_{NM} (+)
M
N″
L_{Nn}
L_{Nm}
$θ_1$
h (−)
L_{Mm}
k (+)
p (+)
hΣR (−)
R_{Nn} (+)
kΣR (+)
$L_{Mm}ΣR$
L_{mn}
$θ_3$
$L_{mn}ΣR$
m
n
y (−)
m′
m″
$ΔL_{Mm}$ (+)
pΣR (+)
kΣR (+)
$ΔL_{mn}$ (−)
$L_{Mm}ΣR$
V_{nM} (+)
V'_{nM} (+)
$ΔL_{Nn}$ (+)
n″
hΣR (−)
H'_{nM} (−)
yΣR (−)
H_{nM} (−)

FIG. 16.—PANEL DEFLECTION DIAGRAM WITH UPPER PANEL POINT *M* AS THE REFERENCE POINT.

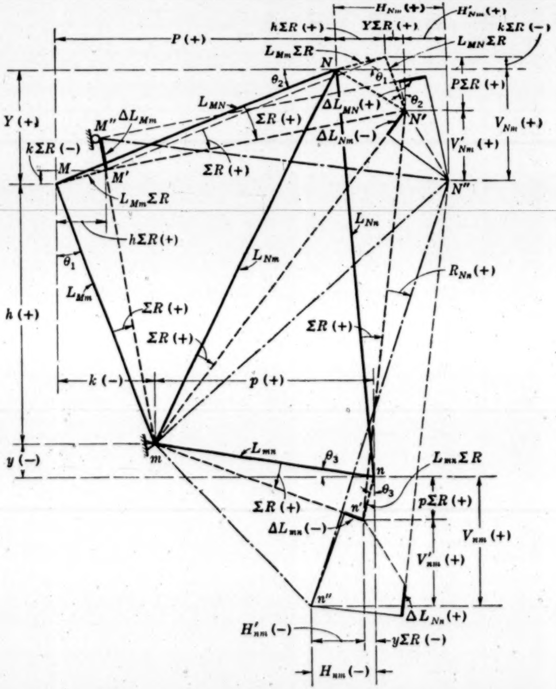

H_{Nm} (+)
H'_{Nm} (+)
hΣR (+)
YΣR (+)
kΣR (−)
P (+)
$L_{MN}ΣR$
$L_{Mm}ΣR$
N
$θ_1$
$θ_2$
PΣR (+)
Y (+)
M″
$ΔL_{Mm}$
L_{MN}
ΣR (+)
$ΔL_{MN}$ (+)
$ΔL_{Nm}$ (−)
N′
V_{Nm} (+)
V'_{Nm} (+)
kΣR (−)
M
M′
N″
$L_{Mm}ΣR$
L_{Nn}
hΣR (+)
L_{Nm}
R_{Nn} (+)
h (+)
k (−)
p (+)
y (−)
m
L_{mn}
$θ_3$
$L_{mn}ΣR$
n
n′
pΣR (+)
$ΔL_{mn}$ (−)
V_{nm} (+)
V'_{nm} (+)
n″
$ΔL_{Nn}$ (+)
H'_{nm} (−)
yΣR (−)
H_{nm} (−)

FIG. 17.—PANEL DEFLECTION DIAGRAM WITH LOWER PANEL POINT *m* AS THE REFERENCE POINT.

If the lower panel point, *m*, is considered as the reference point, as shown in Fig. 17, the various components of the panel deflections will be as follows:

$$V_{Nm} = V'_{Nm} + (P + k) \Sigma R \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

$$H_{Nm} = H'_{Nm} + (Y + h) \Sigma R \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

$$V_{nm} = V'_{nm} + p \Sigma R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

and,

$$H_{nm} = H'_{nm} + y \Sigma R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

In order to represent, very clearly, the various components of the panel deflections, the total rotation, ΣR, and the bar deformations are much exaggerated in proportion to the size of the panel in Figs. 16 and 17.

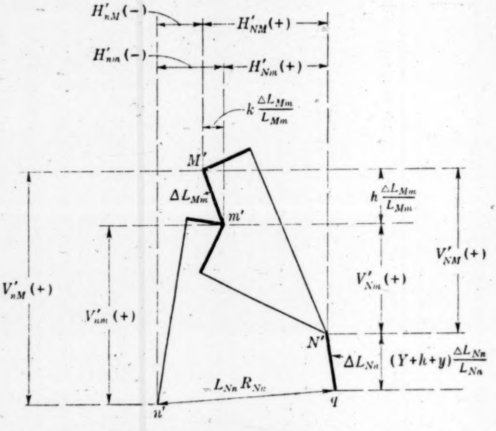


FIG. 18.—WILLIOT DEFLECTION DIAGRAM FOR THE PANEL SHOWN ON FIG. 16 AND FIG. 17.

From Fig. 18, which shows the Williot diagram constructed for both the two cases shown in Figs. 16 and 17, the following relations are evident:

$$V'_{NM} = V'_{Nm} + h \frac{\Delta L_{Mm}}{L_{Mm}} \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

$$V'_{nM} = V'_{nm} + h \frac{\Delta L_{Mm}}{L_{Mm}} \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

$$H'_{NM} = H'_{Nm} + k \frac{\Delta L_{Mm}}{L_{Mm}} \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

$$H'_{nM} = H'_{nm} + k \frac{\Delta L_{Mm}}{L_{Mm}} \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

$$V'_{nM} = V'_{NM} + (Y + h + y) \frac{\Delta L_{Nn}}{L_{Nn}} \ldots\ldots\ldots\ldots\ldots (39)$$

and,

$$V'_{nm} = V'_{Nm} + (Y + h + y) \frac{\Delta L_{Nn}}{L_{Nn}} \ldots\ldots\ldots\ldots\ldots (40)$$

In Equations (35) to (40), the dimensions, h, k, Y, and y, have numerical values only, and ΔL_{Mm} and ΔL_{Nn} will be positive when they are increased in length and negative when decreased. In the case of a rectangular panel, with vertical members, $Y = y = k = 0$, $P = p$, and $h = L_{Mm} = L_{Nn}$, Equations (27) to (40) will be much simplified. These formulas will suffice to indicate the fact that the values of the panel deflections, V, H, V', and H', may be altogether different in accordance with the reference point chosen and, for the sake of clearness, the latter should be preferably indicated in computing the former.

In Fig. 18, note also that $n'q = L_{Nn} \, R_{Nn}$, or $R_{Nn} = \dfrac{n' \, q}{L_{Nn}}$, which is the rotation of the panel due to its bar deformations, with Mm as the reference bar. If Bar Nn is a vertical member, the rotation, R_{Nn}, will be,

$$R_{Nn} = \frac{H'_{NM} - H'_{nM}}{L_{Nn}} = \frac{H'_{Nm} - H'_{nm}}{L_{Nn}} \ldots\ldots\ldots\ldots (41)$$

When both Nn and Mm are vertical, $H'_{NM} = H'_{Nm}$; and $H'_{nM} = H'_{nm}$, then,

$$R_{Nn} = \frac{H'_{NM} - H'_{nm}}{L_{Nn}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

Since the total rotation, ΣR, and the bar deformations, are always very small in comparison with the dimensions of the panel, the computation of the panel deflections, V' and H', due to its bar deformations may be made without including the effect of ΣR.

For the computation of the panel deflections, V', H' (which correspond to the author's V_1 and H_1), and the rotation, R, the author has presented Equations (1), (2), and (3), respectively, which are valid only for a panel with vertical members. It seems to the writer that those formulas are likely to cause confusion in their application, and this is indicated by the fact that a special column is provided in Tables 1 and 2 of the paper to "symbolize" each bar in applying the author's formulas, even to the simple examples. Therefore, for such computations, the writer would prefer the method of work, or even the Williot diagram, the latter being particularly expedient for the oblique panel shown in Figs. 16 and 17. With the construction of the simple Williot diagram shown in Fig. 18, all the eight panel deflections, V' and H', and the rotation, R, of the panel are obtained at once. It has been universally recognized that the Williot diagram is the best method when the deflections of all the panel points of a truss are desired, the only disadvantage being that the diagram increases in size, rapidly, as more panels are included in the construction, and, consequently, it is somewhat difficult to obtain the results with a high degree of precision. However, if applied to one panel only, the Williot diagram will have no disadvantage. By combining the Williot diagram for computing V', H', and R, and the author's method for summing up the panel deflections to obtain the deflection of the truss, a semi-analytical method is evolved which, perhaps, will be the best method of all.

With the panel deflections known, it will be very simple to compute the total deflections of the truss. Fig. 19 shows a non-deformed truss, with its deformed position, the end panel point, *a*, being a reference point and the bar, *ab*, a reference member, fixed in direction. The total deflection for any panel point is designated by the second subscript, *a*, which indicates the reference point; thus, V_{Da} is the total vertical deflection of the Panel Point *D* with Panel Point *a* as a reference point. From Fig. 19 it is evident that the total deflection of any panel point in any chord is the algebraic sum of the panel deflections of all the panel points to the left of, and including, the panel point in question.

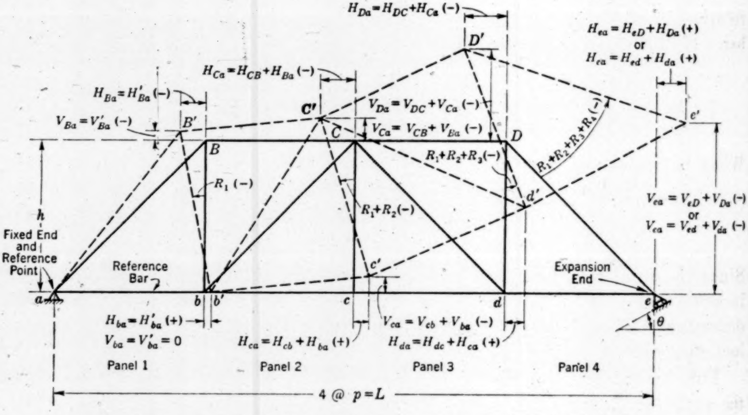

$H_{Da}=H_{DC}+H_{Ca}$ (−)
$H_{ea}=H_{eD}+H_{Da}$ (+) or $H_{ea}=H_{ed}+H_{da}$ (+)
$H_{Ca}=H_{CB}+H_{Ba}$ (−)
$H_{Ba}=H'_{Ba}$ (−)
$V_{Ba}=V'_{Ba}$ (−)
$V_{Da}=V_{DC}+V_{Ca}$ (−)
$V_{Ca}=V_{CB}+V_{Ba}$ (−)
D'
C'
B'
B
C
D
e'
d'
c'
R_1 (−)
R_1+R_2 (−)
$R_1+R_2+R_3$ (−)
$R_1+R_2+R_3+R_4$ (−)
h
Fixed End and Reference Point
Reference Bar
Expansion End
$V_{ea}=V_{eD}+V_{Da}$ (−) or $V_{ea}=V_{ed}+V_{da}$ (−)
a
b
b'
c
d
e
θ
$H_{ba}=H'_{ba}$ (+)
$V_{ba}=V'_{ba}=0$
$H_{ca}=H_{cb}+H_{ba}$ (+)
$V_{ca}=V_{cb}+V_{ba}$ (−)
$H_{da}=H_{dc}+H_{ca}$ (+)
Panel 1
Panel 2
Panel 3
Panel 4
4 @ p = L

FIG. 19.—RELATIONS OF TOTAL DEFLECTIONS WITH PANEL DEFLECTIONS.

The correctness of this statement may be proved by the following example, considering Panel Point *C*. Since V_{CB} is the vertical panel deflection of *C*, with *B* as the reference point, and if Panel Point *B* also has its own vertical deflection, V_{Ba}, with *a* as the reference point, the algebraic sum, $V_{CB} + V_{Ba} = V_{Ca}$, must be the total vertical deflection of *C* with *a* as the reference point. Of course, all these deflections are computed with the bar, *ab*, as the reference bar. It should be observed that in summing up the panel deflections, care must be taken regarding their reference points and, by using panel deflections with different reference points, the total deflection of any panel point may be obtained in various ways. Thus, the total vertical deflection, V_{da}, of the panel point, *d*, may be obtained in any of the following four ways:

$$V_{da} = V_{dc} + V_{CB} + V_{Ba} \dots\dots\dots\dots\dots\dots\dots (43)$$

$$V_{da} = V_{dc} + V_{cb} + V_{ba} \dots\dots\dots\dots\dots\dots\dots (44)$$

$$V_{da} = V_{dc} + V_{Cb} + V_{ba} \dots\dots\dots\dots\dots\dots\dots (45)$$

or,

$$V_{da} = V_{dc} + V_{CB} + V_{Ba} \dots\dots\dots\dots\dots\dots\dots (46)$$

The significance of Equations (43) to (46) is evident from Fig. 20, which shows the various panel deflections for different reference points and also the total deflections (shown barred; thus, $\overline{V_{ca}}$) with *a* as the reference point, for all the panel points of the truss in Fig. 19.

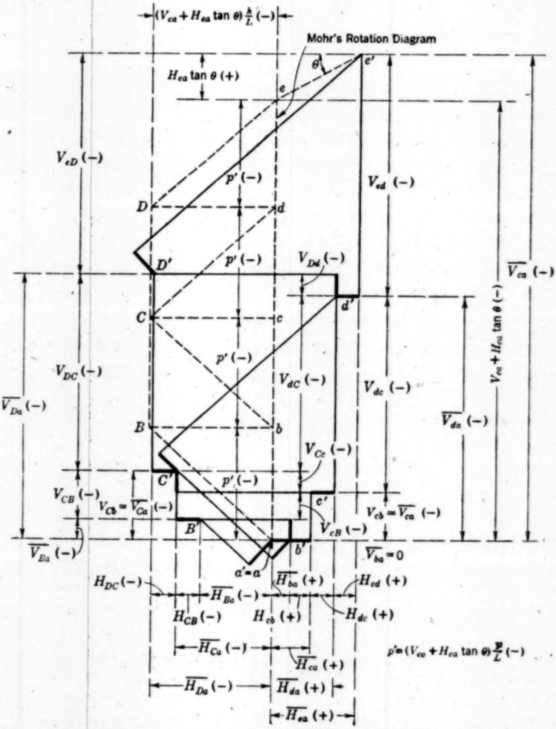

$(V_{ca} + H_{ea} \tan\theta)\frac{h}{L}(-)$
Mohr's Rotation Diagram
θ
e'
$H_{ea} \tan\theta\ (+)$
e
$V_{eD}(-)$
$p'(-)$
$V_{ed}\ (-)$
D
d
D'
$p'(-)$
$V_{Dd}(-)$
$\overline{V_{ca}}\ (-)$
d'
C
c
$V_{ca} + H_{ca} \tan\theta\ (-)$
$p'(-)$
$V_{DC}(-)$
$V_{dC}(-)$
$V_{dc}\ (-)$
$\overline{V_{Da}}(-)$
$\overline{V_{da}}\ (-)$
B
b
$V_{Cc}(-)$
C'
$p'(-)$
$V_{CB}(-)$
$V_{Cb} = \overline{V_{Ca}}(-)$
c'
$V_{cb} = \overline{V_{ca}}\ (-)$
B'
$V_{cB}(-)$
$\overline{V_{Ba}}\ (-)$
b'
$\overline{V_{ba}} = 0$
$a' = a$
$H_{DC}(-)$
$\overline{H_{ba}}\ (+)$
$H_{ed}\ (+)$
$\overline{H_{Ba}}(-)$
$H_{CB}(-)$
$H_{cb}\ (+)$
$H_{dc}\ (+)$
$\overline{H_{Ca}}\ (-)$
$p' = (V_{ca} + H_{ca} \tan\theta)\frac{p}{L}(-)$
$\overline{H_{ca}}\ (+)$
$\overline{H_{Da}}(-)$
$\overline{H_{da}}\ (+)$
$\overline{H_{ea}}\ (+)$

FIG. 20.—WILLIOT-MOHR DIAGRAMS SHOWING THE VARIOUS DEFLECTION COMPONENTS WITH DIFFERENT REFERENCE POINTS FOR THE TRUSS OF FIG. 19.

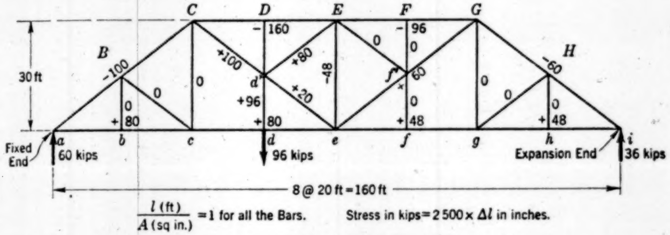
Since the computation of the total deflections is merely a matter of summation as soon as the panel deflections are known and since the summation may be somewhat varied according to the choice of the reference points on the basis of which the panel deflections are computed, it seems to the writer that formulas for computing the total deflections such as Equations (4) and (5) of the paper are not only unnecessary, but also undesirable.

The author has presented two numerical examples, in which both the trusses and their deformations are symmetrical with respect to the center lines. In computing the panel deflections, the center panel point and the center bar are taken as references (that is, they are assumed to be fixed) and, by so doing, the true deflections can be obtained by merely transposing the reference point from the center panel point to the end panel point, which is actually fixed. This procedure is certainly advantageous in these symmetrical cases. However, in cases in which either the truss or its deformations are asymmetrical, such a procedure will require corrections both for transposing the reference point and rotating the reference bar in order to obtain the true deflections from the total deflections, and the computations might become seriously complicated in an analytical method such as this. Therefore, in asymmetrical cases, it will always be advantageous to use the end panel point, (preferably the left end) and a convenient bar connected to it, for reference.

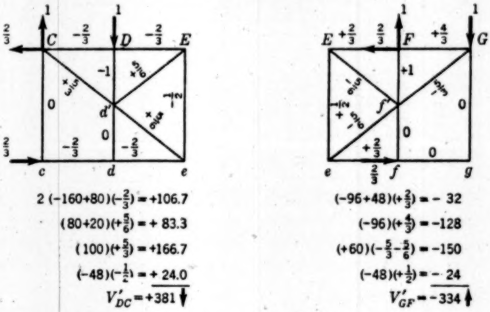
In this manner the computations may proceed from panel to panel toward the right, and only corrections for rotating the reference bar will be required to obtain the true deflections. Such corrections for rotation are similar to Mohr's rotation diagram in Williot's construction and are also very simple if applied analytically to this method. Fig. 20 shows Williot's and Mohr's diagrams for the truss of Fig. 19, from which the following general rule may be stated for obtaining the true deflections from the total deflections in a truss with its expansion end supported by rollers on any inclined plane, making an angle, θ, with the horizontal:

From the total panel deflection of any panel point subtract algebraically the quantity, $\frac{V_{ea} + H_{ea} \tan \theta}{L}$, multiplied by its vertical distance from Reference Point a when computing the horizontal deflection and by its horizontal distance from Reference Point a when computing its vertical deflection.

The vertical or horizontal distance of any panel point from Reference Point a will be positive when the panel point in question lies, respectively, above, or to the right of, a. The proper sign must also be assigned to V_{ea} and H_{ea}, which are, respectively, the total vertical and horizontal deflections of the expansion end (Panel Point e, Fig. 19). When the plane that supports the rollers is horizontal (that is, when $\theta = 0$), the quantity, $\frac{V_{ea} + H_{ea} \tan \theta}{L}$, is reduced to $\frac{V_{ea}}{L}$. Applying the foregoing general rule to the example of Figs. 19 and 20, it is noted that: (1) The horizontal deflections of all the lower panel points require no corrections; (2) for the horizontal deflections of all the upper panel points, the quantity, $(V_{ea} + H_{ea} \tan \theta) \frac{h}{L}$, must be subtracted algebraically from the respective total horizontal deflection; and (3) for the vertical deflections of all the upper and lower panel points, the quantity, $(V_{ea} + H_{ea} \tan \theta) \frac{p}{L}$, multiplied by n, the number of panels which the panel point in question is away from Reference Point a, must be subtracted algebraically from the respective total vertical deflection.


C
D
E
F
G
B
H
30 ft
−100
+100
+80
+20
−48
− 160
− 96
0
60
+96
+ 80
+ 48
d′
f′
a
b
c
d
e
f
g
h
i
Fixed End
60 kips
96 kips
Expansion End
36 kips
8 @ 20 ft = 160 ft
$\frac{l\,(\text{ft})}{A\,(\text{sq in.})} = 1$ for all the Bars.
Stress in kips = 2 500 × Δl in inches.

(a) TRUSS AND LOADING


1
1
2/3
C
−2/3
D
−2/3
E
−1
+5/3
+5/6
−1/2
0
d′
0
−2/3
−2/3
c
d
e
E
+2/3
2/3
F
+4/3
G
+1
−5/3
−5/6
+1/2
0
f′
0
0
+2/3
2/3
e
f
g
2(−160+80)(−2/3) = +106.7
(80+20)(+5/6) = + 83.3
(100)(+5/3) = +166.7
(−48)(−1/2) = + 24.0
V′_DC = +381
(−96+48)(+2/3) = − 32
(−96)(+4/3) = −128
(+60)(−5/3−5/6) = −150
(−48)(+1/2) = − 24
V′_GF = −334

(b) COMPUTATION OF V'_{DC} (c) COMPUTATION OF V'_{GF}

FIG. 21.

For a subdivided panel containing a quadrilateral such as that in the Pettit truss, it is evident that the computation of the panel deflections can not be made by considering the panel only, since it is unstable in itself. To obtain the correct results, the main panel containing such a quadrilateral must be considered as a free body in computing panel deflections, but the dummy unit load and its reactions are still considered as acting on the subdivided panel in question. This is clearly illustrated in Figs. 21(b) and 21(c).

In this case, again, it is noted that, for computing the panel deflections, V' and H', formulas of limited applicability, such as Equations (1) and (2) of the paper, will be of little value and it will also be futile to attempt devising such equations in a generally valid form.

In applying a method of this nature, systematic tabulation of computations and consistent convention for signs are both of the utmost importance; otherwise, confusions and errors are likely to occur. Even for the very simple example given in the paper, the writer experienced some difficulty at first

TABLE 11.—COMPUTATIONS FOR THE DEFLECTIONS OF ALL THE PANEL POINTS OF THE TRUSS SHOWN IN FIG. 21(*a*).
(With Panel Point *a* as Reference Point and Bar *ab* as Reference Bar).

Quantity	Step No.	UPPER AND LOWER PANEL POINTS							
		B–b	*C–c*	*D–d*	*E–e*	*F–f*	*G–g*	*H–h*	*i*
R (of the preceding panel)	(1)	....	*Bb* —13.7	*Cc* — 9.5	*Dd* — 8.0	*Ee* — 8.0	*Ff* — 4.8	*Gg* — 4.8	*Hh* — 5.7
Σ *R*	(2) = Σ (1)		—13.7	—23.2	—31.2	—39.2	—44.0	—48.8	—54.5
VERTICAL DEFLECTIONS, UPPER PANEL POINTS									
V′	(3)	*Ba* 0	*CB* — 107	*DC* + 381*	*ED* — 186	*FE* — 74	*GF* — 334†	*HG* — 50	*iH* — 164
PΣ *R* (*P* = 20 ft.)	(4) = 20 × (2)	0	— 274	— 464	— 624	— 784	— 880	— 976	—1 090
V = *V′* + *P* Σ *R*	(5) = (3) + (4)	0	— 381	— 83	— 810	— 858	—1 214	—1 026	—1 254
Total, *V* = Σ *V*	(6) = Σ (5)	*Ba* 0	*Ca* — 381	*Da* — 464	*Ea* —1 274	*Fa* —2 132	*Ga* —3 346	*Ha* —4 372	*ia* —5 626
Correction for rotation	(7)	— 703‖	—1 407	—2 110	—2 813	—3 516	—4 220	—4 923	—5 626
True *V*	(8) = (6) — (7)	*B* + 703	*C* +1 026	*D* +1 646	*E* +1 539	*F* +1 384	*G* + 874	*H* + 551	*i* 0
True *V* × 0.0004 (in.)	(9)	+0.28	+0.41	+0.66	+0.62	+0.55	+0.35	+0.22	0
VERTICAL DEFLECTIONS, LOWER PANEL POINTS									
Δ*L*, in inches	(10)	0	0	*d′ d* +0.04	*Ee* —0.02	0	0	0	0
True *V* × 0.0004 (in.)	(11) = (9) + (10)	*b* +0.28	*c* +0.41	*d* +0.70	*e* +0.60	*f* +0.55	*g* +0.35	*h* +0.22	*i* 0
HORIZONTAL DEFLECTIONS, UPPER PANEL POINTS									
H′	(12)	*Ba* — 125	*CB* — 205	*DC* — 160	*ED* — 160	*FE* — 96	*GF* — 96	*HG* — 38	*iH* + 48
Y Σ *R* ‡	(13)	0	— 205	0	0	0	0	+ 732	+ 817
H = *H′* + *Y* Σ *R*	(14) = (12) + (13)	— 125	— 410	— 160	— 160	— 96	— 96	+ 694	+ 865
Total *H* = Σ *H*	(15) = Σ (14)	*Ba* — 125	*Ca* — 535	*Da* — 695	*Ea* — 855	*Fa* — 951	*Ga* —1 047	*Ha* — 353	*ia* + 512
Correction for rotation	(16)	— 527‖	—1 054‖	—1 054	—1 054	—1 054	—1 054	— 527	0
True *H*	(17) = (15) — (16)	*B* + 402	*C* + 519	*D* + 359	*E* + 199	*F* + 103	*G* + 7	*H* + 174	*i* + 512
True *H* × 0.0004 (in.)	(18)	+0.16	+0.21	+0.14	+0.08	+0.04	+0.00	+0.07	+0.21
HORIZONTAL DEFLECTIONS, LOWER PANEL POINTS									
H = *H′* = Δ *L*	(19)	*ba* + 80	*cb* + 80	*dc* + 80	*ed* + 80	*fe* + 48	*gf* + 48	*hg* + 48	*ih* + 48
True *H* = Total *H* = Σ *H*	(20) = Σ (19)	*b* + 80	*c* + 160	*d* + 240	*e* + 320	*f* + 368	*g* + 416	*h* + 464	*i* + 512
True *H* × 0.0004 (in.)	(21)	+0.03	+0.07	+0.10	+0.13	+0.15	+0.17	+0.19	+0.21

* See the computations in Fig. 21 (*b*). † See the computations in Fig. 21 (*c*). ‡ $Y = +15$ ft for Panel *CB*; — 15 ft for Panels *HG* and *iH*; and 0 for all the remaining panels. ‖ $-703 = +20 \times \frac{-5\,626}{160}$; $-527 = +15 \times \frac{-5\,626}{160}$; and $-1\,054 = +30 \times \frac{-5\,626}{160}$

in following Tables 1 and 2. For instance, the author has not indicated the reference points for the vertical deflections in Table 1, Column (7), so it is somewhat difficult to understand how they can be added in Column (11). The value, 1 355, in Column (11) is evidently the numerical sum of the values, −354, −267, and +734 in Column (7), indicating, obviously, that the signs of the latter have been disregarded.

In order to illustrate the application of the method to an asymmetrically deformed truss with subdivided panels, and also to show the method of tabulating the computations without using Equations (4) and (5) (as the writer prefers), the example[22] shown in Fig. 21(*a*) is completely developed and tabulated systematically in Table 11. The deflections, V' (Step No. (3)), for all the panel points in the upper chord, and H' (Steps Nos. (12) and (19)), for all the panel points in both the chords are first computed and tabulated, most of the latter being merely the bar deformations of the chord members. The rotations, R, are computed by Equation (41) or Equation (42). With those values known, the remaining computations are merely a matter of arithmetic, the operations for obtaining the results in every row being also indicated. It may be noted that the true vertical deflection of any lower panel point (Step No. (11)) is equal to the algebraic sum of the same deflection of its corresponding upper panel point (Step No. (9)) and the bar deformation of the vertical member connecting the two panel points (Step No. (10)). This relation is always valid for trusses having vertical members, irrespective of the shape of the chords and the arrangement of the diagonals. The exact agreement between the values (+ 0.21 in.) of the true horizontal deflection of Panel Point i computed from both the upper and the lower chords serves as a check.

A. W. FISCHER,[23] ESQ. (by letter).—In this paper the author has added another method for calculating the deflections of a truss by the analytical method, which is stated to solve "the problem of truss deflections in a simpler and more direct manner than other analytical methods." To the writer it seems that there are several analytical methods for trusses of a certain type, that are as simple as that of the author, which is based upon set formulas. One solution which is very simple for computing the deflections of Warren trusses is the "method of elastic weights"; another is the "geometric method"; and still another (which is a very flexible method for solving the vertical and horizontal deflections of any shape truss) is the method of "relative deflections"[24].

To determine the deflections of panel points in the author's Example 1, for instance, relative to Member 5–6, assume the truss supported at Panel Points 6 and 5 instead of at the left and right ends, being hinged at Point 6 and supported horizontally at Point 5.

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[22] The example is taken from "Structural Theory", by H. Sutherland and H. L. Bowman, Members, Am. Soc. C. E., John Wiley & Sons, New York, Second Edition, 1935, p. 173, Figs. 7-12.

[23] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

[24] "Stresses in Statically Indeterminate Structures", by Prof. H. Yu, National Wuhan Univ., Wuchang, Hupeh, China, Second Edition, 1935, pp. 13 to 39.

TABLE 12.—VALUES OF u* FOR UNIT VERTICAL LOAD AT PANEL POINTS

Member	Unit Vertical Load at Panel Point 3	1	0
3-5	$+\frac{30}{36}\times 1$	$+\frac{30}{36}\times 2$	$+\frac{30}{36}\times 3$
3-6	$-\frac{46.8}{36}$	$-\frac{46.8}{36}$	$-\frac{46.8}{36}$
4-6	0	$-\frac{30}{36}\times 1$	$-\frac{30}{36}\times 2$
3-4	0	$+1-\frac{1}{7.2}$	$+1-\frac{1}{7.2}\times 2$
1-3	0	$+\frac{30}{35.52}\times 1$	$+\frac{30}{35.52}\times 2$
1-4	0	$-1.387+\frac{1}{5.191}$	$-1.387+\frac{1}{5.191}\times 2$
0-4	0	0	$-\frac{30}{31}$
0-1	0	0	$+\frac{43}{31}$

* The stresses in all members due to unit vertical loads at various panel points.

TABLE 13.—EVALUATION OF RELATIVE VERTICAL DEFLECTIONS OF PANEL POINTS

Member (1)	$\frac{S L}{1\,000\,A}$ (2)	$u_m = \left(\pm\frac{p}{r^*}\right)$ (3)	Moment center (4)	$g_{mn} = \frac{S L}{1\,000\,A} u_m$ (5)	u_s (6)	†$G_n = \Sigma g_{mn}$ (7)	ΣG_n (8)	$\frac{S L}{1\,000\,A} u_s$ (9)	Total vertical deflection from Point 6, in inches, $\times \frac{30\,000}{12}$ (10)	Point (11)	Total vertical deflection from Point O, in inches, $\times \frac{30\,000}{12}$ (12)	Total vertical deflection from Point O, in inches (13)
3-5	−220.8	$+\frac{30}{36}$	6	−184.0	......	......	......	......	....	6	2 367.7	0.95
3-6	+131.0	......	...	......	$-\frac{46.8}{30}$	−184.0	−184.0	−170.3	−354.3	3	2 013.4	0.81
3-4	+ 14.4	......	...	......	+1.0	......	......	+14.4	−339.9	4	2 027.8	0.81
3-4	+ 14.4	$-\frac{30}{216}$	(3)	−2.0	......	......	......	......	....	...	....	.....
4-6	+189.3	$-\frac{30}{36}$	3	−157.8	......	−159.8	......	......	....	...	....	.....
1-3	−222.5	$+\frac{30}{35.52}$	4	−188.0	......	......	......	......	....	...	....	.....
1-4	+210.0	$+\frac{30}{155.73}$	(4)	+ 40.5	−1.387	−147.5	−491.3	−291.3	−1 122.5	1	1 245.2	0.50
1-2	+159.3	......	...	......	+1.0	......	......	+159.3	(−963.2)	2	1 404.5	0.55
0-4	+369.0	$-\frac{30}{31}$	1	−357.2	......	......	......	......	....	...	....	.....
0-1	−286.0	......	(1)	......	$+\frac{43}{31}$	−357.2	−848.5	−396.7	−2 367.7	0	0.0	0.00

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* r = the lever arm of Member mn about its center of moments, expressed in feet.

† $G_n = \Sigma g_{mn}$ includes the summation of all the g_{mn}- values of all the members having a moment center at Panel Point n.

g_{mn} represents the quantity, $\frac{S\ L}{A\ E}\ u_m$, for Member mn.

Table 12 gives the values of u for the members for unit vertical load at various panel points. This table is not absolutely necessary, in the method of relative deflections, but is presented for a better understanding of certain values given in Table 13.

Using Example 1 and the elongations given by Mr. Shoemaker it is easy to enter the values as given in Table 13 from which the deflections can be computed. All the values given in Table 13 are readily determined and once the method is mastered it is easy to calculate the vertical deflections of a truss. Furthermore, the method of relative deflections may be readily applied to a truss bridge with inclined top and bottom chords and with unequal panel lengths, p.

The paper is based on analytical methods, but for solving the secondary stresses in a truss by the slope-deflection method or by distributing fixed-end moments, the relative vertical and horizontal deflections are only required for the value of Δ for each member, Δ being the displacement, at right angles to the axis of the member of one end with respect to the other end, and to get these values of Δ the analytical method is not as flexible as the simple Williot diagram. In the author's closure it might be well for him to demonstrate the calculation of the horizontal deflections for his Example 1 and then show how he would calculate the values of Δ (that is, the displacement at right angles to the axis of the member, of one end with respect to the other end) and compare the time against that required by using a Williot diagram.

In conclusion, the writer will state that he sees no particular advantage in using the "panel deflection" method for the solution of certain types of trusses. There are certain other types in which the method is satisfactory, but as certain set formulas must be available for use, it loses its simplicity against other analytical methods.

L. E. GRINTER,[25] ASSOC. M. AM. SOC. C. E. (by letter).—The summation process of deflection computation has long been used in the study of truss deflections. The innovation proposed by Mr. Shoemaker is that the contributions of the several panels, rather than the individual contributions of the separate members, be added to obtain the deflection. However, since the effect of each panel must be evaluated by considering the individual changes in lengths of the separate members, it is evident that the amount of computations involved in the two procedures must be about the same. If the method proposed by the author seems simpler than the procedure of summing, directly, the effects of the individual members, it is merely because the standard textbook treatment of the latter method (elastic or angle weights) is unnecessarily cumbersome. When the method of angle weights is reduced to its fundamental conceptions, it will be found to be essentially the same as the method suggested by Mr. Shoemaker.

There are two basic conceptions involved in the method of angle weights. First, the physical picture that deflection can be computed by summing the products of angle changes and lever arms; and, second, the conjugate-beam conception which cares for the effect of dissymmetry. The author has considered only symmetrical structures with symmetrical loading; and, accordingly, he has neglected consideration of the conjugate beam. Hence, his procedure for determining the deflection of a symmetrical truss is to find the upward deflection of the end relative to a fixed center-vertical member or relative to a fixed central-chord member. One notices at once the similarity to the use of the Williot diagram for those cases in which the use of the Mohr rotation diagram can be avoided.

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[25] Dept. of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

When the author discusses the correspondence of his method with the method of computing beam deflections in the following passage, "to compute the deflection of a point of a beam of constant moment of inertia with reference to a tangent to the elastic curve of that beam, the sum is taken of the products of the angular change of every vertical section, multiplied by its distance from the point", he is referring to relative and not to absolute deflections. If this method of computing relative deflections is to be used to compute the maximum deflection of a beam that is unsymmetrically loaded, the procedure illustrated by Fig. 22 becomes necessary. The deflection, Δ_1, is obtained as the statical moment of the $\dfrac{M}{EI}$ - area from A to B about the point, B, where the deflection, Δ_1, exists. Then, the value of θ_1 is Δ_1 divided by L, the span of the beam. The point of maximum deflection, or D, is located such that the area of the $\dfrac{M}{EI}$ - diagram from A to D is equal to θ_1. Then, finally, the maximum deflection Δ_2, is computed as Δ_3, the deflection of the beam at A from a tangent drawn to the elastic curve at D.

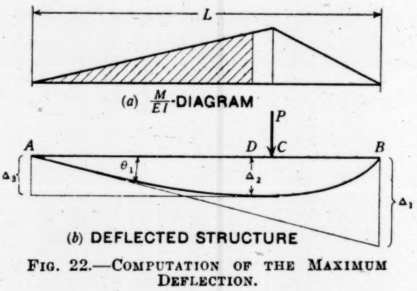

FIG. 22.—COMPUTATION OF THE MAXIMUM DEFLECTION.

The foregoing procedure, or a similar one, would be involved in the use of the author's method for the study of an unsymmetrical truss. Evidently, the difficulties involved would make the use of the method of panel deflections rather cumbersome for such trusses. The study of dissymmetry by the use of elastic weights (which involves the conjugate beam) would be much more satisfactory. The Williot-Mohr diagram is also a convenient method of determining the deflection of an unsymmetrical structure.

In the selection of a method for computing deflections, one should make a distinction between those methods that give a single deflection and those that give rise to the entire deflected load line. The author's procedure belongs to the second classification along with the method of elastic weights and the Williot-Mohr diagram. Castigliano's theorem and the method of virtual work give a single deflection for each separate computation. Hence, the determination of a single deflection or the calculation of a reaction (indeterminate structure) for fixed loads follows more simply by the use of these tools

On the other hand, a method that develops the entire elastic curve (deflected load line) is more satisfactory for the determination of an influence line as the shape of the deflected load line.

In summarizing, the writer feels that the panel-deflection method is an interesting and useful conception that in practice should be limited to the field of symmetry. It bears the same relation to the method of elastic weights that the moment-area method (statical moments of moment areas) bears to the conjugate-beam method. One should not lose sight of the fact that the deflections obtained are relative to a fixed tangent. The panel-deflection method belongs to the classification of methods that develop the entire elastic line. No known procedure can compete with the method of virtual work for the computation of a single truss deflection. The author has performed a service in calling this interesting procedure to the attention of engineers and educators.

LOUIS H. SHOEMAKER,[26] M. AM. SOC. C. E. (by letter).—A number of interesting suggestions have been made by discussers of this paper. Several have proposed, and explained in some detail, methods of calculation in which the effect of the deformation of each truss member is treated separately. Others have proposed special ways of treating certain problems which they appear to consider as offering difficulties if a solution is sought by the method under discussion. These methods, as well as the Williot diagram and the method of "elastic moments," are claimed to offer material advantages, especially as regards the work of computation and avoidance of the use of formulas.

The question of the comparative amount of work required by different methods of computation can only be decided by experience; and in this case there is no doubt as to the result. Formulas, as the name implies, provide a form for the computations to take and, consequently, expedite the work and contribute to its accuracy; and, although these formulas will not be memorized, the manner in which they are developed and applied is easily remembered. Other methods, although not requiring the use of formulas, are much more complicated in their application.

The "panel deflection method" treats the deflections of a truss in exactly the way in which they occur. There is no question but that the panel is the unit of deflection and the equations express the panel deflections in the simplest form.

It is evident that the typical form of panel used applies to all single braced panels with parallel sides, the deflections being taken in the direction of the sides. This type is used in practically all engineering structures.

In the case of sub-divided panels, the deflections of the main panel points are obtained from the deformations of the members of the main panel and the deflections of the sub-panel points from the deformations of the members of the triangle forming the sub-panel construction. In trusses having web systems composed of diagonals only, temporary verticals can be considered as applied at the panel points. As these verticals have no stress, the terms in the formulas involving their changes in length disappear. Formulas for the *K*-type of panel are easily developed by the method described in the paper.

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[26] Santa Monica, Calif.

Mr. Noe has made an excellent contribution to this discussion by developing the deflection formulas by the use of Williot equations. By applying the formulas to the deflections of an unsymmetrically loaded truss, he has answered the objection that has been raised, that the procedure in this case would necessarily be very complicated.

Mr. Molitor advances the statement, without offering supporting facts, that the method under discussion applies only to very simple truss forms. He refers to "odd-shaped triangular truss systems without vertical members" and complications "due to the more involved geometric relations, for which the new method becomes quite 'out of bounds'." Other than to disagree completely with Mr. Molitor's conclusions in so far as they are definite, it seems unnecessary to reply. However, it is confidently expected that thoughtful and discerning members, who study the facts as published, will conclude that the method offered has the advantages claimed for it.

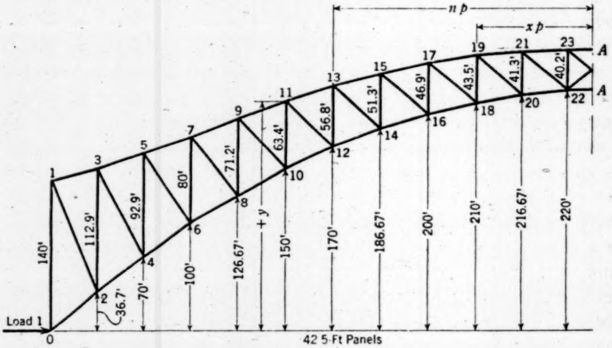

FIG. 23.—ELEVATION OF HALF ARCH, OVER HELL GATE, NEW YORK, N. Y.

An analysis of the steel arch bridge over Hell Gate, in New York City, (see Fig. 23) will serve to answer various criticisms offered in discussion. The computations for the reactions are given in Tables 14 and 15. The preliminary data (that is, changes of length and length of members) were taken from a paper[27] by O. H. Ammann, M. Am. Soc. C. E., published in 1918. Table 14 gives the results of the computations of the products $c' c$, $d' d$, etc., which are the terms composing the formulas. In Table 15 the final results are given. In computing the total deflection of a point Equation (4) gives the total deflection of each point from the origin. Another method of making these computations is to compute the deflection of each point with reference to the preceding point.

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[27] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 852.

Professor Tsai has developed equations for the deflections of an irregularly shaped panel by Williot diagrams. The writer appreciates his interesting contribution to this discussion. In developing the "panel deflection method," the writer had in mind its application to engineering structures, and derived the necessary formulas for that purpose in the most direct manner. A complete explanation of the data in Table 1 was included in Example 1 of the paper. Professor Tsai is in error in thinking this method does not apply to trusses with sub-divided panels. The writer believes that his explanation, given previously in this discussion, fully answers this question.

TABLE 14.—Computations for $c' c$, $d' d$, etc., Hell Gate Arch Bridge, New York, N. Y.

Deflection points (1)	Members (2)	Symbol (3)	Length, in feet (4)	Change in length, in feet × 1 000 *E* (5)	Products, Column (4) × Column (5) (6)	Deflection points (1)	Members (2)	Symbol (3)	Length in feet (4)	Change in length, in feet × 1 000 *E* (5)	Products, Column (4,) × Column (5) (6)
23	23-A	*c*	21.25	+347	....		.....	..	.....	....	....
	22-A	*e*	21.25	−150.2	....		.....	..	.....	....	....
22	22–23	*v*	40.2	−120.4	....		.....	..	.....	....	....
21	21–23	*c*	42.6	+708.6	+30 186	9	9–11	*c*	45.3	+296.3	+13 422
	21–22	*d*	57.0	− 88.7	− 5 056		9–10	*d*	64.0	−434	−27 776
	20·22	*e*	42.6	−287.3	−12 240		8–10	*e*	48.5	−132.2	− 6 412
	22–23	*u*	40.2	−120.4	− 4 575		10–11	*u*	63.4	+360	+17 244
	20–21	*v*	41.33	− 19.9	— 756		8–9	*v*	71.2	+405	+19 400
20	.....	*w*	38.0	....	....	8	.....	*w*	47.9	....	....
19	19–21	*c*	42.7	+585	+24 980	7	7–9	*c*	46.0	+270.4	+12 438
	19–20	*d*	56.3	−159.2	− 8 963		7–8	*d*	68.2	−447	−30 485
	18–20	*e*	43.0	−268	−11 524		6–8	*e*	50.2	− 99.7	− 5 005
	20–21	*u*	41.3	− 19.9	− 733		8–9	*u*	71.2	+405	+21 586
	18–19	*v*	43.5	+ 53.3	+ 1 961		6–7	*v*	80.0	+482	+25 690
18	.....	*w*	36.83	....	....	6	.....	*w*	53.3	....	....
17	17–19	*c*	43.0	+545.6	+23 461	5	5–7	*c*	45.8	+196	+ 8 977
	17–18	*d*	56.3	−244.4	−13 760		5–6	*d*	76.0	−334	−25 384
	16–18	*e*	43.7	−239	−10 444		4–6	*e*	52.0	− 88.3	− 4 591
	18–19	*u*	43.5	+53.3	+ 1 967		6–7	*u*	80.0	+482	+30 318
	16–17	*v*	46.9	+128	+ 4 723		4–5	*v*	92.9	+396	+24 908
16	.....	*w*	36.9	....	....	4	.....	*w*	62.9	....	....
15	15–17	*c*	43.4	+492	+21 353	3	3-5	*c*	44.5	+111.6	+ 4 966
	15–16	*d*	57.0	−284.3	−16 205		3–4	*d*	90.2	−359	−32 380
	14–16	*e*	44.5	—207	− 9 211		2–4	*e*	54.0	− 71	− 3 830
	16–17	*u*	46.9	+128	+ 4 864		4–5	*u*	92.9	+396	+31 520
	14–15	*v*	51.3	+204	+ 7 752		2–3	*v*	112.9	+428	+34 070
14	.....	*w*	38.0	....	....	2	.....	*w*	79.6	....	....
13	13–15	*c*	43.9	+429.5	+18 855	1	1–3	*c*	43.6	+ 46	+ 2 005
	13–14	*d*	58.5	−401.1	−23 464		1–2	*d*	111.7	−405	−45 240
	12–14	*e*	45.7	−181.3	− 8 285		0–2	*e*	56.1	+ 53.6	+ 3 010
	14–15	*u*	51.3	+204	+ 8 180		2–3	*u*	112.9	+428	+44 210
	12–13	*v*	56.8	+281.2	+11 276		0–1	*v*	140.0	+387	+39 980
12	.....	*w*	40.1	....	....	0	.....	*w*	103.3	....	....
11	11–13	*c*	44.6	+363.1	+16 194	....	.....	..	.....	....	....
	11–12	*d*	60.8	−420.4	−25 560	....	.....	..	.....	....	....
	10–12	*e*	47.0	−156.2	− 7 340	....	.....	..	.....	....	....
	12–13	*u*	56.8	+281.2	+12 204	....	.....	..	.....	....	....
	10–11	*v*	63.4	+360	+15 624	....	.....	..	.....	....	....
10	.....	*w*	43.4	....	....	....	.....	..	.....	....	....

Professor Grinter has misinterpreted the description of the method under discussion. The writer's reference to the deflections of a beam was for the purpose of calling attention to similarity of methods. A careful study of

TABLE 15.—FINAL COMPUTATIONS FOR REACTIONS OF HELL GATE ARCH BRIDGE, NEW YORK, N. Y.

Deflection points (1)	Vertical deflection, Δ, in feet, and angular distortion R (2)	Values of $\Sigma\ (Rp)$* and $\Delta + \Sigma\ (Rp)$ (3)	Horizontal deflection, H (4)	Values of y and Ry (5)	Total deflections, Δ_T (6)	Δ_T transposed (7)	Reactions (8)
23	0	....	+347	+260.2	0	54 915	$\frac{55\ 035}{65\ 400} = 0.8417$
22	....	....	....	....	−120	55 035	
	+12.37	....	....	+3 218	....	......	
21	+763	+ 526	+664	+ 258	+ 1 289	......	$\frac{53\ 646}{65\ 400} = 0.820$
20	....	....	....	....	+ 1 269	53 646	
	+ 24.9	+1 289	....	+6 424	....	......	
19	+804	+1 584	+502	+253.5	+ 3 677	......	$\frac{51\ 185}{65\ 400} = 0.783$
18	....	....	....	....	+ 3 730	51 185	
	+ 21.0	+2 388	....	+ 5 324	....	......	
17	+901	+2 477	+412	+246.9	+ 7 054	......	$\frac{47\ 733}{65\ 400} = 0.730$
16	....	....	....	....	+ 7 182	47 733	
	+ 19.0	+3 377	....	+4 691	....	......	
15	+905	+3 284	+314	+ 238	+11 243	......	$\frac{43\ 468}{65\ 400} = 0.665$
14	....	....	....	....	+11 447	43 468	
	+ 16.3	+4 189	....	+3 879	....	......	
13	+984	+3 978	+184	+226.8	+16 205	......	$\frac{38\ 425}{65\ 400} = 0.5876$
12	....	....	....	....	+16 486	38 429	
	+ 14	+4 962	....	+3 175	....	......	
11	+950	+4 572	+ 81	+213.4	+21 727	......	$\frac{32\ 828}{65\ 400} = 0.502$
10	....	....	....	....	+22 087	32 828	
	+ 11.6	+5 522	....	+2 475	....	......	
9	+922	+5 065	− 20	+197.9	+27 714	......	$\frac{26\ 796}{65\ 400} = 0.4096$
8	....	....	....	....	+28 119	26 796	
	+ 9.3	+5 987	....	+1 840	....	......	
7	+906	+5 460	− 89	+180	+34 080	......	$\frac{20\ 352}{65\ 400} = 0.311$
6	....	....	....	....	+34 562	20 352	
	+ 8.1	+6 366	....	+1 458	....	......	
5	+809	+5 804	−114	+162.9	+40 693	......	$\frac{13\ 825}{65\ 400} = 0.2112$
4	....	....	....	....	+41 089	13 825	Correction $\underline{0.0015}$
	+ 5.4	+6 613	....	+880	....	......	0.2127
3	+741	+6 034	−116	+149.6	+47 468	......	$\frac{7\ 018}{65\ 400} = 0.1072$
2	....	....	....	....	+47 896	7 018	Correction $\underline{0.0032}$
	+ 5.1	+6 775	....	+763	....	......	0.1104
1	+810	+6 250	−136	+140	+54 528	......	0
0	....	....	....	....	+54 915	0	Correction $\underline{0.004}$
	+ 4.3	+7 060	....	+602	....	......	0.004
			+ 2 029	+ 34 729			

By Equation (5). $H_T = 2\ 029 - 34\ 729 = 32\ 700$; $32\ 700 \times 2 = 65\ 400$

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* $p = 425$ ft.

the paper will reveal the fact that deflections are computed with reference to a point called the origin of deflections and a line through this point called the axis of deflections. The deflection of a single point of a structure, furthermore, can be computed as easily by the "panel deflection method" as by the method of virtual work.

The deflection of the *n*th panel point with reference to the $(n-1)$th panel point is,

$$\Delta_n = \Delta_1 + \sum_{x=0}^{x=n-1} (R\,p) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47$$

and,

$$\Delta_T = \Delta_T + \Delta_n \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

This method of computing Δ_T enables the results to be tabulated in more compact form. In Table 15, Δ and R are given together in Column (2), Δ on the upper line and R on the lower line of the panel space. In the same manner $\Sigma\,(R\,p)$ and $\Delta + \Sigma\,(R\,p)$ are given in Column (3) and y and $R\,y$ in Column (5). In obtaining the values of Δ_T, the value of $\Delta + \Sigma\,(Rp\,)$ on the bottom line of a given panel space is added to the value of Δ_T in the panel space above, and the result is the value of Δ_T for the given panel. The computations for horizontal deflections of Point *O* are written at the bottom of the table. It is believed that a comparison of Tables 14 and 15 with the tables of computations, made by Mohr's method of elastic moments,[27] is a sufficient answer to the various claims and criticisms that have appeared in this discussion.

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TRANSACTIONS

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LATERAL PILE-LOADING TESTS

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With Discussion by Messrs. A. E. Cummings, J. C. Meem, T. Kennard Thomson, August E. Niederhoff, Lazarus White, Y. L. Chang, D. P. Krynine, and Lawrence B. Feagin.

Synopsis

Tests conducted at Lock and Dam No. 26, Alton, Ill., to determine the resistance under lateral loads of timber and concrete piles, driven in Mississippi River sand, are described in this paper. The work includes descriptions of tests on single piles with heads not fixed, and on groups of four, twelve, and twenty piles with heads fixed in concrete test monoliths. The field data are presented in tabular or graphical form, and the results discussed. In view of the fact that these tests were conducted in only one type of soil, and in view of the many variables involved, no mathematical analysis is included. It is hoped, however, that the results of tests conducted by others may be presented in the discussion.

Introduction

Lock and Dam No. 26 on the Mississippi River, at Alton, Ill., is the farthest down stream of the twenty-six locks and dams included in the general program of canalization of the Upper Mississippi River. It is situated 23 miles up stream from St. Louis, Mo., 8 miles above the mouth of the Missouri River, and 15 miles below the mouth of the Illinois River. In addition to being a part of the 9-ft canalization of the 650-mile section of river below the "Twin Cities"—St. Paul and Minneapolis, Minn.—the pool above Lock and Dam No. 26 will also form a part of the waterway from the Great Lakes to the Gulf of Mexico.

The designs of twenty-two of the twenty-six locks and dams are entirely, or in part, on piles, driven in most cases in river sand of varying degrees

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Note.—Published in November 1935, *Proceedings*.

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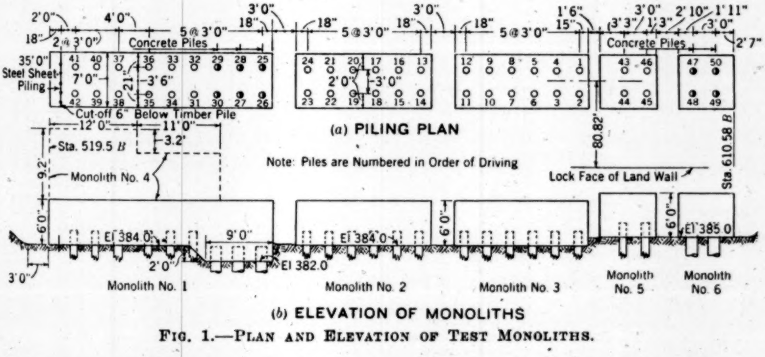
of coarseness. Lock and Dam No. 26 is founded entirely on piles, and its designed lift at extreme low water of 25.2 ft is nearly double that of any of the other projects founded on piles. Under the twin locks there are about 14 200 timber piles and nearly 5 000 concrete piles, and under the dam, about 13 100 timber piles. Although many vertical pile-loading tests have been made in various soils, it appears that comparatively few field tests have been made of the resistance of piles to movement under lateral loads; such, for example, as those resulting from back-fill behind the land-wall of a lock, or from water pressure on dams. It is hoped, however, that lateral loading tests made by others will be fully described in discussion of this paper.

PURPOSE OF TESTS

The general purpose of the tests herein described was to secure data on the movement of timber and concrete piles in groups of various sizes when subjected to lateral loads. In as much as the design lateral load assumed under certain conditions, for the piles beneath the river wall of the auxiliary lock, is 6.5 tons per pile (which, in general, may be regarded as rather high), it was particularly desirable, furthermore, to determine the degree of safety of this assumption. Information was also desired for use in designing the foundations for the piers of Dam No. 26, and for designing future locks and dams on similar foundations.

TEST MONOLITHS

In order to simulate, as nearly as practicable, the actual loading conditions that will occur in the completed structures, six concrete monoliths were constructed on timber and concrete piles driven along the toe of the slope of the Illinois shore behind the land-wall of the main lock, as shown in Figs. 1 and 2. The heads of the piles were fixed, as in the lock-walls, by being embedded 2 ft in the concrete. The cross-section of Monolith No. 1 was typical of the foundation pour of the river wall of the auxiliary lock between gate-bays. The section includes two rows of nine piles each and a 7-ft width of 35-ft, steel sheet-piling. There are six concrete, and


2'0"
4'0"
18"
2 @ 3'0"
5 @ 3'0"
Concrete Piles
3'0"
18"
18"
5 @ 3'0"
3'0"
18"
18"
5 @ 3'0"
1'6"
15"
3'0"
3'3"
1'3"
2'10"
1'11"
3'0"
2'7"
Concrete Piles
35'0" Steel Sheet-Piling
7'0"
2'1"
3'6"
41 40 37 36 33 32 29 28 25
42 39 38 35 34 31 30 27 26
24 21 20 17 16 13
23 22 19 18 15 14
2'0"
3'0"
12 9 8 5 4 1
11 10 7 6 3 2
43 46
44 45
47 50
48 49
Cut-off 6" Below Timber Pile
12'0"
11'0"
3.2'
Sta. 519.5 B
9.2'
Monolith No. 4
Note: Piles are Numbered in Order of Driving
80.82'
Lock Face of Land Wall
Sta. 610.58 B
6'0"
El 384.0
9'0"
2'0"
3'0"
El 382.0
El 384.0
6'0"
6'0"
El 385.0
Monolith No. 1
Monolith No. 2
Monolith No. 3
Monolith No. 5
Monolith No. 6

(a) PILING PLAN

(b) ELEVATION OF MONOLITHS

FIG. 1.—PLAN AND ELEVATION OF TEST MONOLITHS.

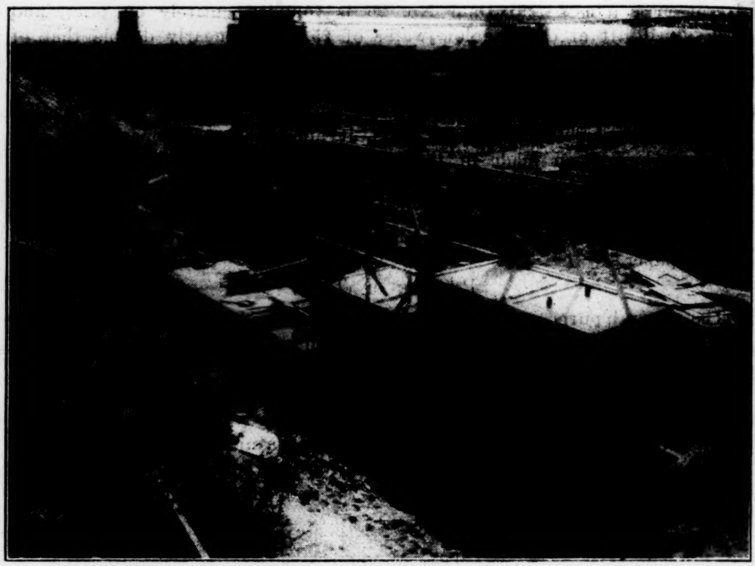
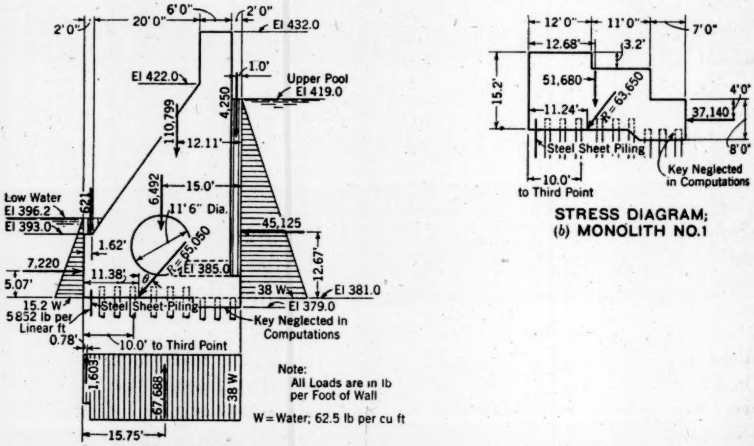


FIG. 2.—VIEW OF TEST MONOLITHS, UPPER MISSISSIPPI RIVER, LOCK NO. 26.

twelve timber, piles. Monolith No. 4 was superimposed on Monolith No. 1 to provide a vertical load approximately equal to that of the river wall under the eccentric loading that occurs when the auxiliary lock is filled


2'0"
20'0"
6'0"
2'0"
El 432.0
El 422.0
1.0'
Upper Pool
El 419.0
4,250
110,799
12.11'
15.0'
6,492
11'6" Dia.
Low Water
El 396.2
El 393.0
621
45,125
1.62'
R=65,050
12.67'
7,220
11.38'
El 385.0
θ
5.07'
38 W
El 381.0
15.2 W
Steel Sheet-Piling
El 379.0
5852 lb per Linear ft
Key Neglected in Computations
0.78'
10.0' to Third Point
1,603
67,688
38 W
15.75'
Note:
All Loads are in lb per Foot of Wall
W=Water; 62.5 lb per cu ft
12'0"
11'0"
7'0"
12.68'
3.2'
15.2'
51,680
R=63,650
4'0"
11.24'
37,140
Steel Sheet Piling
8'0"
Key Neglected in Computations
10.0' to Third Point

STRESS DIAGRAM;
(a) RIVER WALL BETWEEN GATES

STRESS DIAGRAM;
(b) MONOLITH NO.1

FIG. 3.—LOAD AND STRESS DIAGRAMS.

and the river below the dam is at extreme low water. A comparison of the loading conditions is shown in Fig. 3. Monoliths Nos. 2 and 3 are each on two rows of six timber piles each, or twelve piles to the monolith. Monolith No. 5 is supported by four timber piles and Monolith No. 6 by four concrete piles. Thus, provision was made for comparing the movement of monoliths on groups of four, twelve, and twenty piles, assuming the steel sheet-piling under Monolith No. 1 as equivalent to two piles. In addition to supplying the means of making lateral load tests, the monoliths were used for making studies of relative volume changes, heats of hydration, etc., with various types of cement and methods of curing.

SINGLE PILES

In order to compare the movement of groups of piles with heads fixed in the concrete to that of a single pile with head not fixed, lateral tests were also made on a single timber pile and on a concrete pile driven as a part of the foundation of the land-wall of the locks.

DRIVING AND PHYSICAL DATA

Table 1 contains pile-driving and physical data for each pile under the test monoliths, and Fig. 4, supported by Table 2, shows corresponding data for the single concrete and timber piles tested.

TABLE 1.—PILE-DRIVING DATA*

Pile No. (see Fig. 1)	Penetration, in feet	DIAMETER, IN INCHES		Total surface area of penetration, in square feet	Blows in the last foot of penetration	Depth penetrated without jetting, in feet	Indicated bearing value, in tons, by *Engineering News* formula	Pile No. (see Fig 1)	Penetration, in feet	DIAMETER, IN INCHES		Total surface area of penetration, in square feet	Blows in the last foot of penetration	Depth penetrated without jetting, in feet	Indicated bearing value, in tons, by *Engineering News* formula
		Of butt at the ground line	Of tip							Of butt at the ground line	Of tip				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.....	29.2	....	11	....	25	6	25.2	26†....	27.1	17½	10¾	99.9	82	17	59.2
2.....	29.2	13¼	9½	86.8	24	6	24.3	27†....	27.1	17½	10¾	99.9	23	7	23.8
3.....	29.2	13¼	9½	86.8	24	6	24.3	28†....	27.1	17½	10¾	99.0	13	8	14.2
4.....	29.2	12¼	9	81.2	42	6	37.9	29†....	22.1	16¼	10¾	78.0	63	11	50.2
5.....	29.2	15½	11½	103.5	30	7	29.2	30†....	25.5	17¼	10¾	93.3	150	6	81.0
6.....	29.2	13¾	9½	88.5	31	7	30.1	31.....	29.7	13¼	9¾	89.5	32	7	30.9
7.....	29.2	13	8¼	81.6	14	7	15.2	32.....	29.6	14¼	10¼	95.0	80	6	58.2
8.....	28.2	11½	9	75.8	26	6	26.2	33.....	29.6	12¼	7¾	77.2	24	7	24.3
9.....	29.1	12½	8¾	81.3	44	6	39.0	34.....	29.7	13½	9½	89.5	76	7	56.5
10.....	29.3	12	7¾	75.2	25	6	25.2	35.....	29.7	11½	7¾	74.8	37	7	34.5
11.....	29.1	12	8½	78.2	33	9	31.7	36.....	29.6	12½	9½	85.0	30	7	29.2
12.....	29.0	13	8½	81.5	29	9	28.6	37.....	30.2	11½	7¾	76.2	23	8	23.8
13.....	29.0	11¾	8¾	78.0	21	9	21.9	38	30.5	11½	8¼	78.7	20	7	20.9
14.....	29.0	13¼	9½	86.2	42	9	37.9	39.....	30.3	12¼	8	80.0	25	8	25.2
15.....	29.0	13¼	8½	82.2	28	7	27.8	40.....	30.3	12	7¾	78.3	41	8	37.1
16.....	29.0	12¾	9	82.2	30	6	29.2	41.....	30.2	12¾	9¾	89.2	26	6	26.1
17.....	28.9	12¾	8	80.9	36	6	33.8	42.....	30.3	11½	8	77.2	30	6	29.2
18.....	28.9	13	9	83.1	25	6	25.2	43.....	30.0	12	8¼	79.3	34	7	32.1
19.....	29.0	12¼	7¾	76.0	45	6	39.5	44.....	30.0	12¾	8½	82.5	59	13	48.0
20.....	28.9	12	8¾	78.5	35	6	33.0	45.....	30.0	12½	8½	82.5	45	5	39.5
21.....	28.1	12	8½	75.7	35	7	33.0	46.....	30.0	12½	8½	82.5	73.	6	55.1
22.....	28.9	12½	8¾	80.7	29	6	28.5	47†....	30.0	18	10¾	112.8	101	6	66.9
23.....	28.8	13½	9½	86.8	25	9	25.2	48†....	30.0	18	10¾	112.8	77	6	57.0
24.....	28.7	13¾	9¼	86.6	50	9	42.8	49†....	30.0	18	10¾	112.8	156	5	82.5
25†....	27.1	17½	10¾	99.9	65	11	51.2	50†....	30.0	18	10¾	112.8	47	5	40.9

* Measured stroke of ram, 35 in.; weight of ram, 5 000 lb; and jet pressure, 100 lb. All timber piles were oak, principally white oak.

† Concrete pile.

FOUNDATION CONDITIONS

A log of a hole, *O*, bored in the vicinity of Monolith No. 1, is shown in Fig. 5. It will be noted that the foundation consists of medium sand. The samples, of which sieve analyses are given in Fig. 5, were taken by jetting a 2-in. pipe about 3 ft below a 3-in. casing, stopping the jet, and permitting sand to enter the 2-in. pipe through side openings, and then withdrawing the pipe. Although this method affords a reasonably representative sample, the sample tends to be somewhat coarser than the material from which it is taken, due to the fact that some of the fines are washed away prior to stopping the jet. A small open pit dug during the progress of the tests indicated that the line of saturation was at Elevation 381.7, or approximately 2.33 ft below the base of Monoliths Nos. 2 and 3. A surcharge

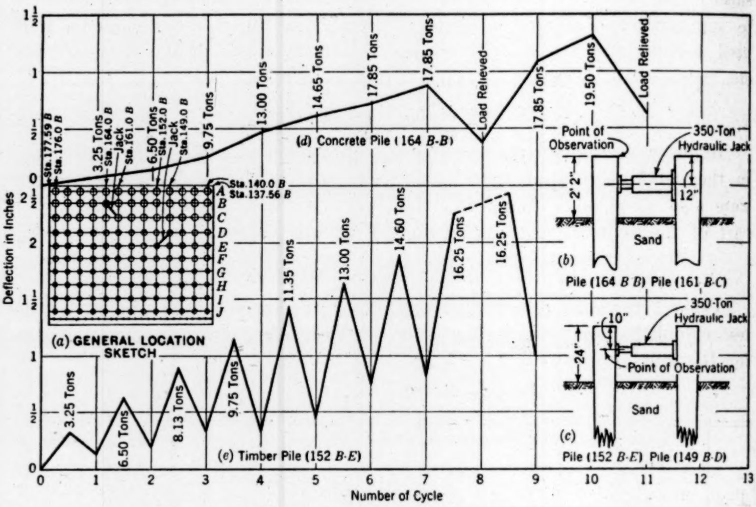
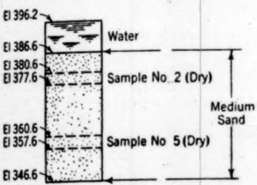


FIG. 4.—DEFLECTIONS OF A SINGLE CONCRETE PILE.

TABLE 2.—DRIVING DATA, PILES IN FIG. 4

Description (1)	Tapered concrete pile (Fig. 4 (a)) (2)	Timber pile (oak) (Fig. 4 (b)) (3)
Location (see Fig. 4 (*a*)):		
Pile number	*B*	*E*
Station	164.0 *B*	152.0 *B*
Total penetration, in feet	30	30
Diameter, in Inches:		
Of the butt, at the ground line	18	14
Of the tip	10.75	10
Total surface area, in square feet, below the ground line	112.8	94.2
Blows in the last foot of penetration	68	100
Penetration without using a jet	5	7
Engineering News Formula:		
Length of stroke, in inches	35	35
Indicated bearing, in tons	52.7	66.3

was created at each end of the group of monoliths by sand back-fill. At the up-stream end of Monolith No. 1, however, the back-fill above its base was removed for a distance of 3 ft, and at the down-stream end of Monolith No. 6, approximately 5 ft. In view of the small movement of the monoliths during the tests, and the fact that there was no perceptible upheaval of the sand between the monoliths and the toe of the slope of the back-fill, it is believed that the movement of the monoliths under test was not materially affected by the surcharge.



BORING AT STA. 500 *B*; 68 FT LANDWARD OF LOCK FACE OF LAND WALL

SIEVE ANALYSIS

SIEVE NUMBER	PERCENTAGE OF MATERIAL PASSING	
	SAMPLE NO. 2	SAMPLE NO. 5
4	98.7	99.3
8	97.1	97.5
16	89.2	68.1
28	56.5	7.7
48	3.2	0.2
100	0.1	0.0

FIG. 5.—TEST BORING IN VICINITY OF MONOLITH NO. 1.

EQUIPMENT

The tests under lateral load were made by placing a hand-operated hydraulic jack of 350 tons rated capacity between two monoliths and jacking them apart. The loads were determined from an accurately calibrated gauge. The jacking equipment is shown in Figs. 6 and 7. In order to secure sufficient resistance to move Monolith No. 1, heavy oak struts were placed between Monoliths Nos. 2, 3, 5, and 6. The point of application of the lateral load was 2.5 ft above the base of the monoliths for the test on Monoliths Nos. 5 and 6, and 2.0 ft above the base for the tests on Monoliths Nos. 1, 2, and 3. Thus, the point of application was at the general level of the tops of the piles.

OBSERVATIONS

Observations were taken by reading with a transit a rule held over the jack with its end against a point on the center line of the monolith and 2.25 ft below the top. Readings were taken to the nearest $\frac{1}{32}$ in. Additional observations were taken at the top and sides of each monolith to determine whether or not there was twisting or tilting of the monoliths. In general, however, it was found that there was no appreciable twisting or tilting. The movement of the single piles in the foundation of the land-wall was taken by jacking two piles apart and observing their spread.

SEQUENCE OF TESTS

The first tests, which were somewhat of a preliminary nature, were made on the single timber and concrete piles of the foundation of the land-wall, with butts not fixed. Details are shown in Figs. 4 (*b*) and 4 (*c*). The first test on the monoliths (see Fig. 8 (*a*)) was made by jacking

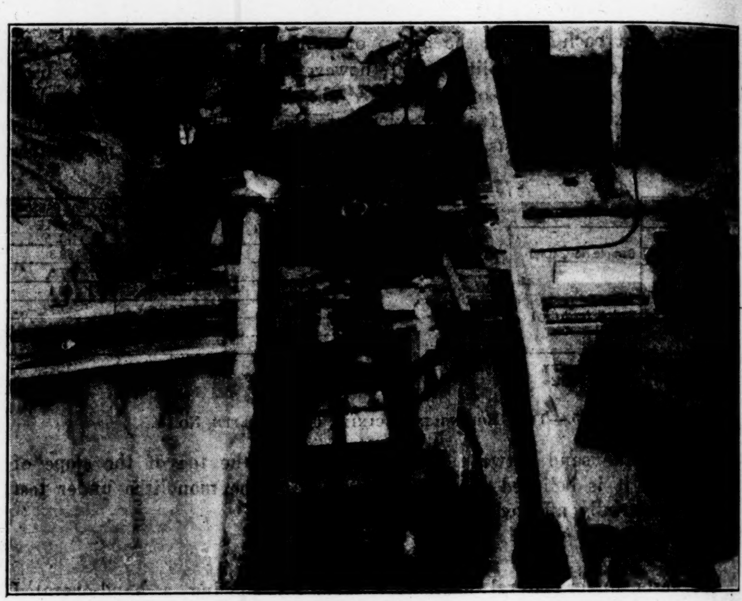


Fig. 6.—Horizontal Loading Test Between Monoliths.

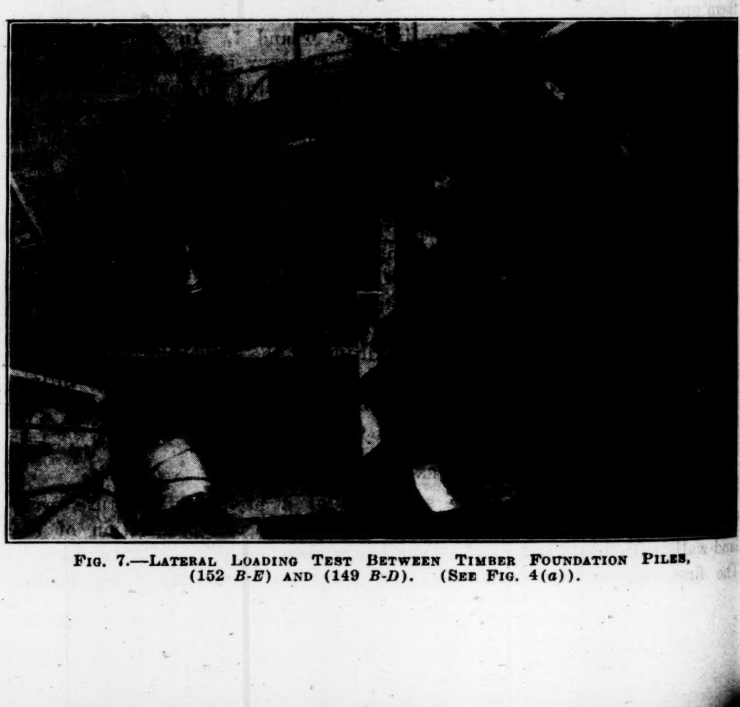
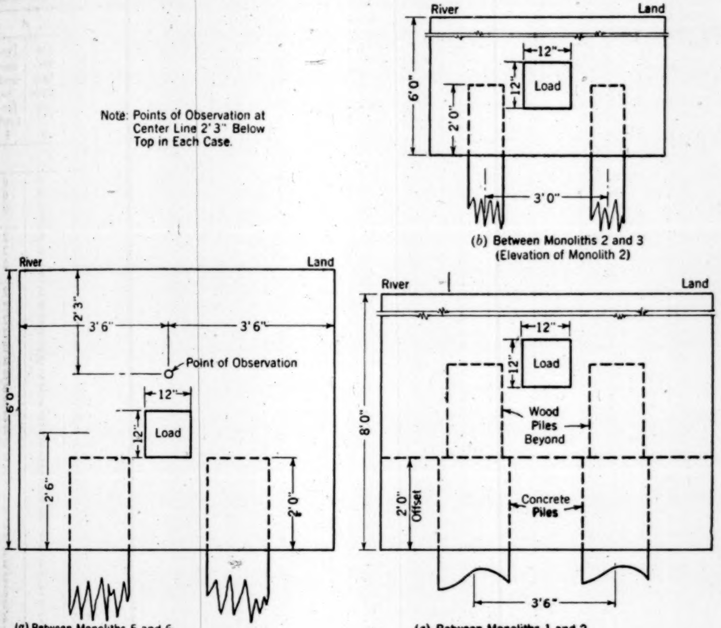


Fig. 7.—Lateral Loading Test Between Timber Foundation Piles, (152 *B-E*) and (149 *B-D*). (See Fig. 4(*a*)).

Monoliths Nos. 5 and 6 apart. The load was increased in increments of 1 ton per pile until a load of 5 tons per pile was reached. This load was maintained overnight, after which it was increased to 30 tons per pile.


Note: Points of Observation at Center Line 2' 3" Below Top in Each Case.
River
Land
12"
12"
Load
6'0"
2'0"
3'0"
River
Land
2'3"
3'6"
3'6"
Point of Observation
12"
12"
Load
6'0"
2'6"
2'0"
River
Land
12"
12"
Load
Wood Piles Beyond
8'0"
2'0" Offset
Concrete Piles
3'6"

(*b*) Between Monoliths 2 and 3 (Elevation of Monolith 2)

(*a*) Between Monoliths 5 and 6 (Elevation of Monolith 5)

(*c*) Between Monoliths 1 and 2 (Elevation of Monolith 1)

FIG. 8.—POSITION OF 350-TON, MANUALLY-OPERATED HYDRAULIC JACK IN LATERAL PILE-LOADING TESTS.

The results of this test are shown in Table 3 (*a*) and Fig. 9 (*a*). (It is to be noted that, in Fig. 9, the loads are in tons per pile. Each cycle comprises a variation from no load to the indicated load and back to no load.)

The second test was made by jacking Monoliths Nos. 2 and 3 apart, each of which is supported by twelve timber piles. The load (see Fig. 8 (*b*)) was increased in increments of 1 ton per pile until a load of 10 tons per pile was reached, which was then maintained overnight, after which it was increased to 20 tons per pile. Following each increment the load was released and the permanent deflection observed. The results of this test are shown in Table 3 (*b*) and Fig. 9 (*b*).

The third test was made by placing the jack between Monoliths Nos. 1 and 2 (see Fig. 8 (*c*)) and placing oak struts between Monoliths Nos. 2, 3, 5, and 6. A load of 4 tons per pile, assuming the steel sheet-piling as the equivalent of two piles, was maintained overnight. This load was then increased to 6.5 tons per pile (the design load of the river wall of the auxiliary lock) and maintained over the week-end. The load was then

TABLE 3.—Lateral Pile-Loading Tests

Date: September	Time	Load, in Tons		Deflection in Thirty-Seconds of an Inch			
				A		*B*	
		Total	Per pile	Deflection reading	Permanent deflection¶	Deflection reading	Permanent deflection¶
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*a*) *A* = Monolith No. 5; *B* = Monolith No. 6							
26.....	5:15 P.M.	0	0	0	0	0	0
26.....	....	4	1.0	0	0	0	0
26.....	....	8	2.0	0	0	0	0
26.....	....	12	3.0	1	0	0	0
26.....	....	12	3.0	1	0	0	0
26.....	....	12	3.0	1	0	0	0
26.....	....	16	4.0	2	0	1	0
26.....	....	16	4.0	2	1	1	1
26.....	....	16	4.0	2	1	1	....
26.....	5:30 P.M.	20	5.0	3	1	2	....
27.....	9:15 A.M.*	20	5.0	4	1	3	2
27.....	....	20	5.0	4	2	3	1
27.....	....	20	5.0	4	2	3	2
27.....	....	20	5.0	5	2	3	2
27.....	....	24	6.0	5	2	4	2
27.....	....	26	6.5	6	2	4	2
27.....	....	26	6.5	6	3	4	2
27.....	....	26	6.5	7	3	4	2
27.....	....	26	6.5	7	3	4	2
27.....	....	28	7.0	7	3	5	2
27.....	....	28	7.0	8	4	5	3
27.....	....	28	7.0	8	4	5	3
27.....	....	32	8.0	9	4	5	3
27.....	....	36	9.0	10	5	7	3
27.....	....	40	10.0	12	6	8	4
27.....	....	48	12.0	14	....	10	....
27.....	....	56	14.0	17	....	13	....
27.....	....	64	16.0	20	9	16	9
27.....	....	80	20.0	29	14	26	14
27.....	....	100	25.0	42	....	42	....
27.....	11:00 A.M.	120	30.0	56	30	56	....
(*b*) *A* = Monolith No. 2; *B* = Monolith No. 3							
27.....	2:00 P.M.	24	2.0	0	....	1	....
27.....	....	36	3.0	1	0	2	0
27.....	....	48	4.0	2	1	4	1
27.....	....	60	5.0	3	2	4	2
27.....	....	60	5.0	4	2	6	2
27.....	....	60	5.0	4	....	6	....
27.....	....	72	6.0	5	2	6	2
27.....	....	72	6.0	6	2	7	3
27.....	....	72	6.0	6	3	8	3
27.....	....	72	6.0	6	....	7	....
27.....	....	78	6.5	6	3	8	3
27.....	....	78	6.5	7	3	9	5
27.....	....	78	6.5	7	....	9	....
27.....	....	84	7.0	8	4	9	4
27.....	3:00 P.M.	84	7.0	8	4	9	4
27.....	....	96	8.0	9	4	11	5
27.....	....	96	8.0	9	5	11	6
27.....	....	96	8.0	10	....	12	....
27.....	....	108	9.0	10	5	13	7
27.....	....	108	9.0	11	5	14	7
27.....	....	108	9.0	11	....	16	....
27.....	....	120	10.0	11	6	16	8
27.....	....	120	10.0	12	6	16	9
27.....	3:05 P.M.	120	10.0	12	6	16	10
27.....	4:05 P.M.	120	10.0	12	....	16	....
28.....	9:40 A.M.	120	10.0	14	8	17	10
28.....	....	120	10.0	14	....	17	....
28.....	....	132	11.0	15	8	18	10
28.....	....	132	11.0	15	....	18	....
28.....	....	144	12.0	16	8	20	10
28.....	....	144	12.0	17	9	22	11
28.....	....	144	12.0	17	....	21	....
28.....	....	168	14.0	20	10	26	12
28.....	....	168	14.0	21	....	25	....
28.....	....	192	16.0	26	13	28	14
28.....	....	192	16.0	30	....	30	....
28.....	....	192	16.0	30	....	30	....
28.....	10:25 A.M.	240	20.0	38	18	39	20

Date: September and October	Time	Load, in Tons		Deflection, in Thirty-Seconds of an Inch	
				A	
		Total	Per pile	Deflection reading	Permanent deflection¶
(1)	(2)	(3)	(4)	5	6
(*c*) *A* = Test Monolith No. 1					
28	1:30 P.M.	0	0	0	0
28	....	80	4.0	2	0
28	....	80	4.0	2	0
28	1:50 P.M.	80	4.0	2	....
29	9:15 A.M.	80	4.0	4	2
29	....	100	5.0	4	2
29	....	120	6.0	4	2
29	9:45 A.M.	130	6.5	5	....
1	10:15 A.M.	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	2
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	5	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	3
1	....	130	6.5	7	4
1	....	130	6.5	7	4
1	10:50 A.M.	130	6.5	7	4
1	1:10 P.M.	130	6.5	7	4
1	....	130	6.5	8	4
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	....	130	6.5	8	4†
1	1:50 P.M.	130	6.5	8	4
1	3:00 P.M.	130	....	....	4
1	....	140	7.0	8	4
1	....	160	8.0	8	4
1	....	180	9.0	9	4
1	....	200	10.0	11	4
1	3:45 P.M.	200	10.0	12	‡
2	11:00 A.M.	200	10.0	15	6
2	12:00 A.M.	200	10.0	20	‡
4	9:20 A.M.	240	12.0	26	16
4	....	280	14.0	31	18
4	....	320	16.0	36	....
4	10:30 A.M.	366	18.3	45	26
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....
......	....	....	....	....	....

* Load of 5 tons per pile sustained overnight. † Five cycles of load. ‡ 25 cycles of load.

¶ Permanent deflection is deflection observed after indicated load was removed.

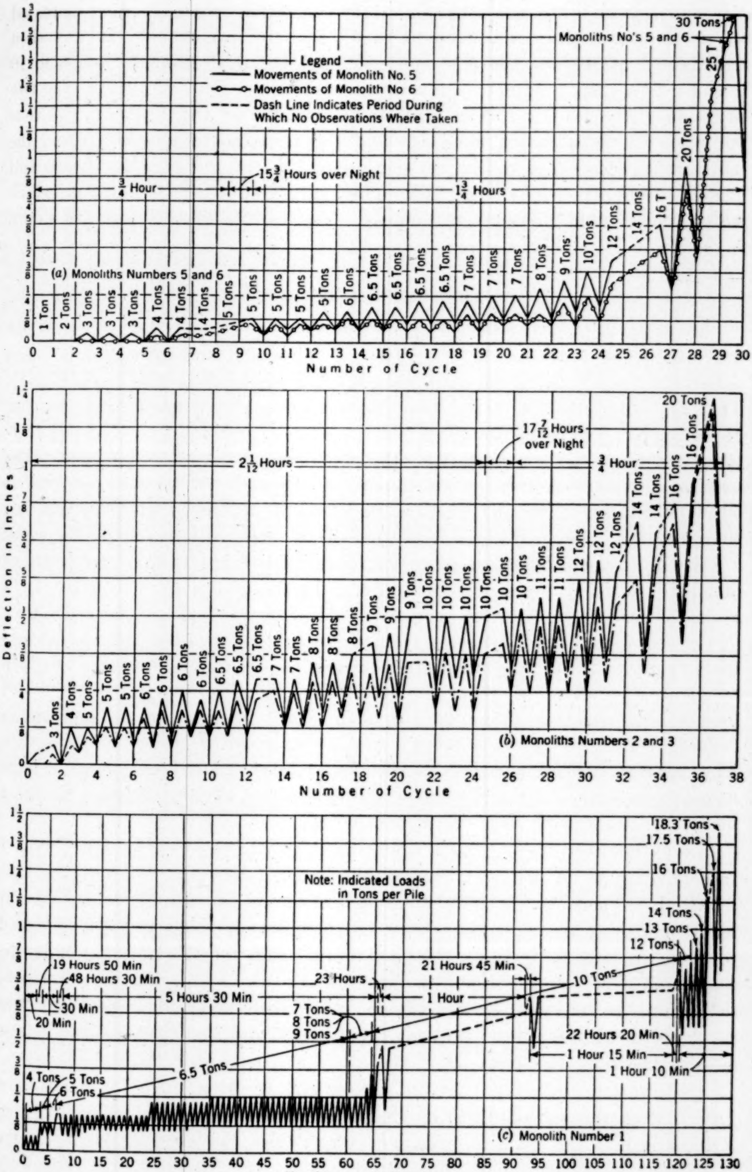


FIG. 9.—DEFLECTION OF TEST MONOLITHS.

alternated from 0 to 6.5 tons for 53 cycles, after which it was increased to 10 tons and alternated from 0 to 10 tons for 25 cycles. It was then maintained at 10 tons per pile overnight and again alternated from 0 to 10 tons for 25 cycles. After continuing to maintain a load of 10 tons for two more days, the load was then increased to a total of 366 tons, or 18.3 tons per pile—the capacity of the jack—and the test was completed. From the morning of October 1 to the completion of the test on October 4, the soil around Monolith No. 1 was thoroughly saturated by a hose from which water was allowed to run continuously. The results of this test are shown in Table 3 (*c*) and Fig. 9 (*c*).

DISCUSSION OF RESULTS

A comparison of the deflection of the single piles and the monoliths for different loads is given in Fig. 10 (*a*). It will be noted that the least deflection was shown by Monolith No. 6, on four concrete piles (see Fig. 1). That of Monolith No. 5 on four timber piles was greater than that of Monolith No. 6 by a maximum of $\frac{1}{8}$ in. at 10 tons, and, for loads from 25 to 30 tons per pile, the movement of the two monoliths was the same. The maximum deflection for any of the monoliths at 4 tons was $\frac{1}{8}$ in. and at 6.5 tons, $\frac{9}{32}$ in. For all practical purposes it appears that for loads less than about 6 tons per pile the deflection of all monoliths was about the same.

It will be noted by referring to Fig. 9 that sustained loads and repetitions of load resulted in progressive deflection. In the case of Monolith No. 1, 53 cycles of load from 0 to 6.5 tons resulted in an increase in deflection from $\frac{5}{32}$ in. to $\frac{1}{4}$ in., and 53 cycles from 0 to 10 tons resulted in an increase from $\frac{15}{32}$ in. to $\frac{13}{16}$ in. The effect of sustaining the load appeared to be less than frequent repetitions of the same load; for example, applying a sustained load of 6.5 tons for 48 hr on Monolith No. 1 resulted in no increase in deflection, whereas 53 cycles from 0 to 6.5 tons added $\frac{3}{32}$ in. to the deflection. The greater deflection of Monolith No. 1 may have been due, in large measure, to the fact that it was subjected to 128 cycles, whereas the other monoliths underwent less than 40 cycles.

Level readings taken on a point fixed in the top of Monolith No. 1 indicated that no vertical movement occurred during the test on that monolith. There was a permanent deflection or set for all monoliths for loads of 4 tons, or more. For loads of 6.5 tons this permanent set amounted to from $\frac{1}{16}$ in. to $\frac{1}{8}$ in. For maximum loads of 30 tons per pile the maximum deflection of Monoliths Nos. 5 and 6 was $1\frac{3}{4}$ in. and the permanent deflection, or set, was $\frac{15}{16}$ in. Even under such severe loading there was no definite failure, and subsequent examination did not disclose any distress in the fibers of the piles.

Observations taken on the up-stream end of Monolith No. 1 near the base and at the top of Monolith No. 4 indicated that there was no tilting. Tilting would not be expected, however, unless the vertical loads on the piles at the toe caused settlement, or unless the lateral loads were such that the piles at the heel might be pulled upward, or both. An examination

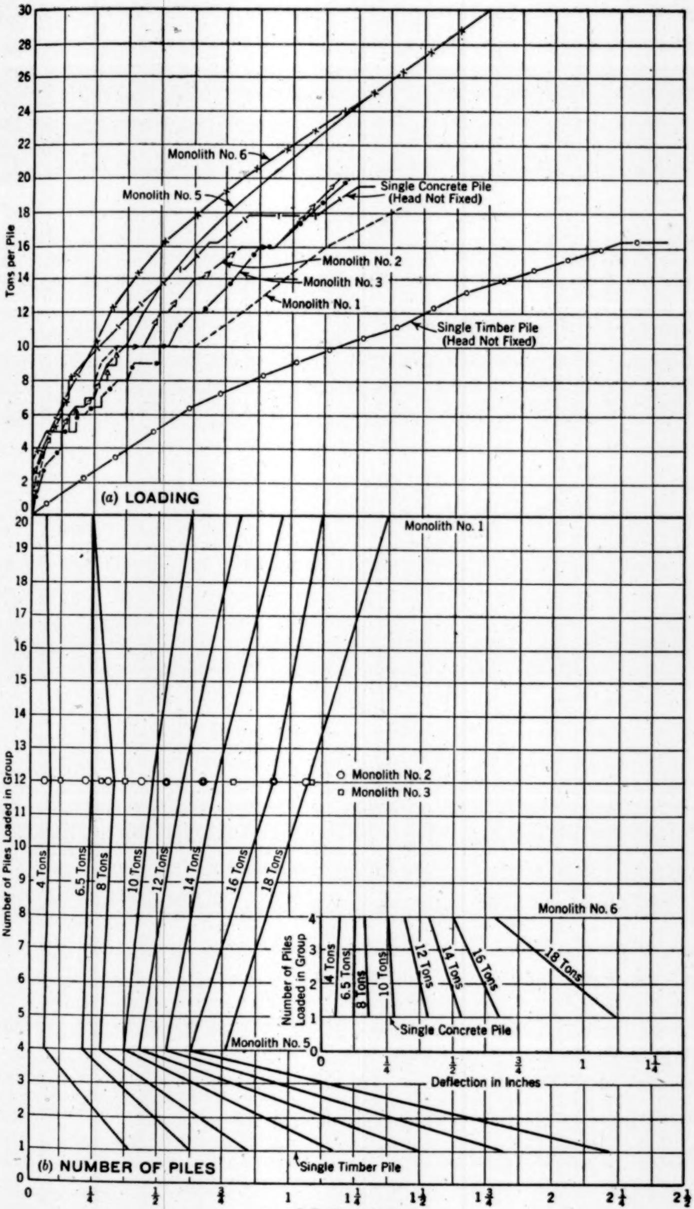


FIG. 10.—EFFECT OF LOADING AND NUMBER OF PILES, ON THE LATERAL DEFLECTION OF A FOOTING.

of the sizes of the piles indicates that the average shear for loads of 6.5 tons per pile was about 85 lb per sq in., which is not serious.

In order to determine the effect of the size of the group of piles loaded, the movement of each monolith was plotted against the number of piles in the group (see Fig. 10 (*b*)). For loads of less than about 6.5 tons per pile the size of the group appears to have no influence on the deflection, whereas, for higher loads, the deflection appears to be less for the smaller group.

Fiber Stress Investigation

Following the completion of the foregoing tests the sand was removed beneath the up-stream end of Monolith No. 2 to a depth of 2.17 ft, the level of the line of saturation, thereby exposing two of the timber piles. A careful examination of these piles failed to disclose any indication of failure in the fibers. Plugs were then inserted on 10-in. centers in the up-stream and down-stream sides of the two up-stream piles. The top plugs were 8 in. below the base of the monolith (see Fig. 11). By means of an extensometer, observations were then taken on the strain in the outside fibers of the piles both on the up-stream, or compression, side and on the down-stream, or tension, side, for various lateral loads ranging from 6.5 to 30 tons per pile.

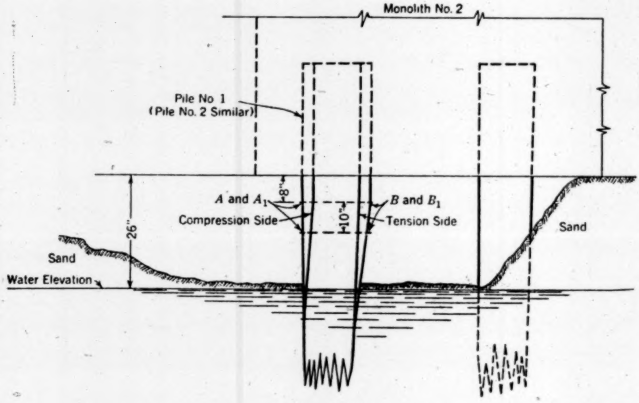


Fig. 11.

In order to determine the modulus of elasticity of the oak piles and thereby determine the fiber stress, the two piles were sawed off beneath the monolith, and blocks were tested in the compression machine in the Concrete Laboratory. The strain was determined by means of a compressometer as shown in Fig. 12. In Fig. 13 (*b*) is shown the derivation of the modulus of elasticity, which was determined as $\frac{432}{0.000230} = 1\,878\,260$, and in Fig. 13 (*a*), the fiber stress in bending for various loads as determined by the extensometer measurements both for the compression and tension side of the piles. It will be noted that the fiber stress in the compression side was only about 60% of that in the tension side. This would seem to indicate that there was a tendency to pull the pile upward which partly counterbalanced the compression from bending in the compression side and increased the tension on the tension side. It is probable that the strain measured may have been affected by the fact that the plugs were inserted between the point of fixation of the head of the pile and the point of contraflexure. The data shown in Fig. 13 are based on observations repeated with great care and are believed to be essentially correct. Prior to the time that they were taken, however, Monolith No. 2 had been subjected to lateral loads from both directions during the tests outlined previously herein (under the heading "Sequence of Tests") with loads as great as 20 tons per pile. Under the maximum load of 20 tons, with the jack between Monoliths Nos. 2 and 3, Monolith No. 2 had received a permanent deflection, or set, of $\frac{9}{16}$ in. These data are included, therefore, only as an indication of the extreme fiber stresses created above the point of contraflexure, following a series of reversals of load.

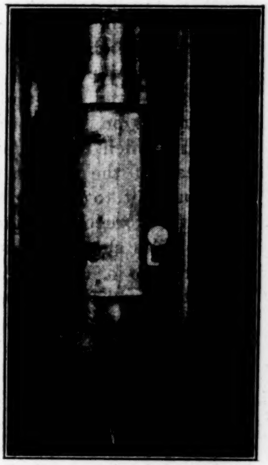


FIG. 12.—DETERMINING MODULUS OF ELASTICITY OF OAK BLOCK CUT FROM PILE BENEATH TEST MONOLITH NO. 2.

It is apparent that, although the observed fiber stresses under loads within the range normally used for design are quite low, and well within allowable working stresses, nevertheless a permanent deflection was produced by loads equal to or exceeding about 4 tons per pile. A possible explanation is that greater stresses may have been set up within the pile below

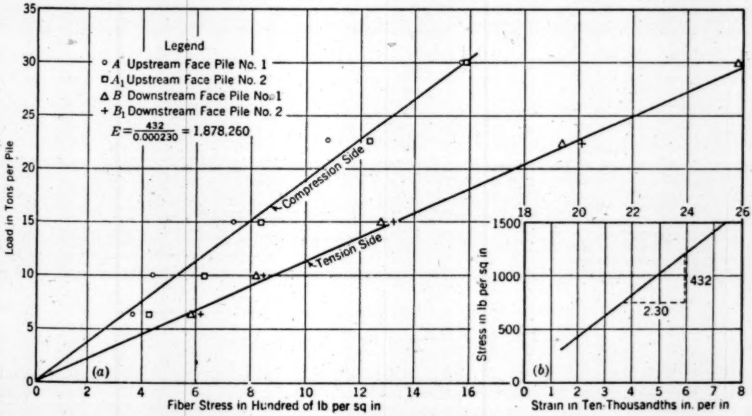


FIG. 13.—FIBER STRESS DETERMINATION.

the point of contraflexure not only in bending but in direct tension as a result of an upward force tending to pull up the pile. The point at which the elastic limit is reached is somewhat conjectural, as lateral movement of the pile may be accompanied by a displacement of the sand which might fill the space left on the back side of the pile, thereby preventing its full return to its original position. An examination of these piles at the base of the concrete showed that under load the bond between the pile and the concrete was broken on the down-stream, or tension, side of the pile, forming a crack in which the blade of a pocket-knife could be inserted. Nevertheless, the head of the pile was still firmly held in the concrete. The sand was also dug out to the water-table beneath a part of Monoliths Nos. 5 and 6 after the foregoing tests. A careful examination of both concrete and timber piles showed no indication of distress, although these piles had been subjected to a lateral load of 30 tons per pile.

Plugs were then inserted, on 10-in. centers, in the up-stream and down-stream sides of one of the concrete piles, with the top plugs 8 in. below the base of the monolith. The jack was again set between these monoliths and observations were taken as before with an extensometer. Again, it was found that the observed strain on the down-stream or tension side exceeded that in the compression side. The resulting computed stress did not reach 600 lb. per sq in. until the load was increased to 28.5 tons per pile. The load was then increased to 40 tons per pile. Between 30 and 40 tons per pile several circumferential cracks developed in the concrete piles, but there was no indication of distress in the timber piles, all of which were of oak. The concrete piles, which were typical of the concrete foundation piles driven under the lock walls (see Fig. 14), were manufactured by spinning a very rich mixture (9.6 sacks of cement per cu yd) in a horizontal cast-iron mould at 400 rpm, for 4 min. The result was a very dense concrete with 14-day strengths averaging about 5 200 lb. per sq in.

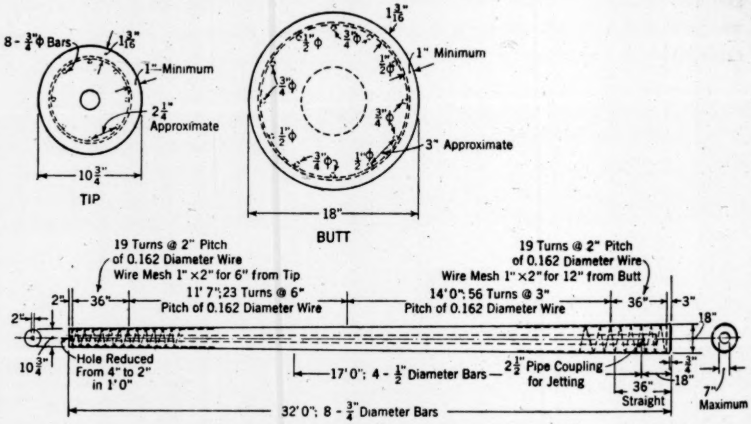

8 - $\frac{3}{4}$" Φ Bars
1$\frac{3}{16}$"
1" Minimum
2$\frac{1}{4}$" Approximate
10$\frac{3}{4}$"
TIP
1$\frac{3}{16}$"
1" Minimum
$\frac{1}{2}$" Φ
$\frac{3}{4}$" Φ
3" Approximate
18"
BUTT
19 Turns @ 2" Pitch of 0.162 Diameter Wire
Wire Mesh 1" × 2" for 6" from Tip
19 Turns @ 2" Pitch of 0.162 Diameter Wire
Wire Mesh 1" × 2" for 12" from Butt
2"
36"
11' 7"; 23 Turns @ 6" Pitch of 0.162 Diameter Wire
14' 0"; 56 Turns @ 3" Pitch of 0.162 Diameter Wire
36"
3"
2"
18"
10$\frac{3}{4}$"
Hole Reduced From 4" to 2" in 1' 0"
17' 0"; 4 - $\frac{1}{2}$" Diameter Bars
2$\frac{1}{2}$" Pipe Coupling for Jetting
$\frac{3}{4}$"
18"
36" Straight
7" Maximum
32' 0"; 8 - $\frac{3}{4}$" Diameter Bars

FIG. 14.—DETAILS OF CONCRETE TEST PILE.

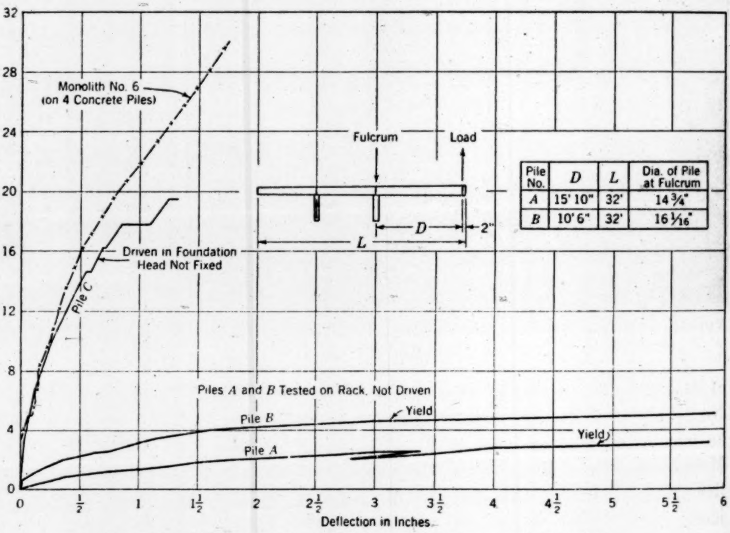
The movement of the monoliths was so rapid at the higher load that the resistance was insufficient to permit the building up of a load greater than 40 tons per pile. When the ram of the jack was fully extended, Monolith No. 6 had moved a total of 10 in. and Monolith No. 5, a total of 8 in., measured at a point 2 ft 3 in. below the top of the monoliths (see Fig. 15). In view of the tilting, the corresponding horizontal movement of Monolith No. 6 at the base was approximately 6 in., and that of Monolith No. 5 about 5 in. This difference in movement of the two monoliths may be accounted for, at least partly, by the probability that the piles beneath Monoliths Nos. 1, 2, and 3, contributed resistance to the movement of Monolith No. 5. Under this maximum load the two concrete piles nearest the jack were pulled upward about 3 in., and the two timber piles nearest the jack, about 2 in., resulting in corresponding tilting of the monoliths. It was again observed that under load the bond between the pile-head and the concrete was broken on the tension side of the concrete and timber piles, forming a crack in which the blade of a knife could be inserted. Nevertheless, the piles were still firmly held in the concrete.

FIG. 15.—TILTING OF TEST MONOLITHS NOS. 5 AT LEFT, AND NO. 6 AT RIGHT, UNDER LATERAL LOAD OF 40 TONS PER PILE, WITH RAM OF JACK FULLY EXTENDED.

RESISTANCE OF SOIL RELATIVE TO THAT OF PILES

In Fig. 16 are shown four curves of deflection of concrete piles plotted against load. Curve *A* represents the deflection of a pile tested in the open while lying in a horizontal position on a testing rack with the fulcrum at the mid-point and the load at the butt, but with the butt not fixed. Curve *B* is similar to Curve *A,* with the exception that the fulcrum is at the third point. The other two curves represent the deflection of piles driven in the foundation sand, one with head not fixed and the other with heads fixed in Test Monolith No. 6. All these piles were of the same size and were similarly reinforced. It will be noted, as might be expected, that the deflection at the yield point decreased quite rapidly as the moment arm was decreased.

When a load of approximately 30 tons per pile was being applied during the final test, it was found that a carpenter's rule could be inserted to a depth of 6 ft in the space between the pile and the adjacent foundation sand on the side of the hole nearest the jack. This would indicate that the moment arm on the pile was slightly more than 8 ft, as the sand had been excavated to a depth of 26 in. Probably the length of pile above the second point of contraflexure did not exceed about 10 ft, which is very nearly equal to the moment arm of the pile represented by Curve *B*. Bearing in mind the difference in application of load and of fixation, it is noted nevertheless that loads producing any given deflection up to $1\frac{3}{4}$ in. in the pile represented by Curve *B,* were uniformly only about 14% of the loads required to produce corresponding deflections in the piles under Monolith No. 6. It appears, therefore, that by far the greater part of the resistance to lateral movement was supplied by the passive pressure of the soil than by the resistance of the pile itself. In view of the importance of limiting lateral movements in structures to a minimum, it is believed,



Pile No.	*D*	*L*	Dia. of Pile at Fulcrum
A	15' 10"	32'	14¾"
B	10' 6"	32'	16 1/16"

FIG. 16.—DEFLECTIONS FOR VARIOUS FULCRUM LOCATIONS.

nevertheless, that the structural rigidity of the pile itself is of primary concern, especially for about 10 ft below the base of the concrete foundation, as the line of support of the soil lies several feet below the top of the soil.

EFFECT OF TESTS ON CONCRETE IN MONOLITHS

After completing all the tests a careful examination of all the monoliths failed to reveal a single crack in the concrete, with the exception of Monolith No. 5, in which diagonal cracks developed on one side near the base during application of the final load of 40 tons per pile. Incidentally, in order to determine (as a matter of interest) the resistance of the concrete to punching, the steel plate used throughout the tests to distribute the load from the jack, was removed after the lateral test between Monoliths Nos. 1 and 2 was completed. The load was again increased to 366 tons, with the 9-in. ram of the jack bearing directly on the concrete, thereby concentrating this load on an area of 63.6 sq in., or 11 500 lb per sq in., without giving any evidence of crushing the concrete.

CONCLUSIONS

It is recognized that conclusions of a general nature are warranted only by extensive and conclusive tests under a wide variety of conditions. The following conclusions, therefore, are confined to the soil conditions and piling arrangements under which the tests were made

(*a*) For structures built on timber piles in which lateral movement of not more than $\frac{1}{4}$ in. is allowable, a maximum lateral load of not more than 4 tons per pile should be allowed if the piles are subject to fatigue by frequent repetitions or reversals of load, or 4.5 tons per pile if the load is merely to be sustained;

(*b*) For structures built on timber piles in which a lateral movement of not more than $\frac{1}{2}$ in. is allowable, a maximum lateral load of not more than 6.5 tons per pile should be allowed if the piles are subject to fatigue by frequent repetitions or reversals of load, or 7.0 tons if the load is to be sustained;

(*c*) Walls supported by piles having adequate vertical bearing capacity, when subjected to lateral loads as great as 20 tons per pile, will remain vertical when moved in a horizontal direction;

(*d*) Fixing the heads of the piles is essential in order to determine, accurately, the movement of structures under lateral loads;

(*e*) Frequent repetition of lateral loads results in slightly greater lateral movement than a sustained load of the same magnitude;

(*f*) For lateral unit loads as great as about 6.5 tons per pile on a group of piles arranged in two rows, as under the test monoliths, the total resistance to movement increases in direct proportion to the number of piles. (As the unit lateral load increases above about 6.5 tons per pile, this no longer applies and the resistance per pile becomes less as the number of piles in the group increases); and (*g*) The use of concrete piles of the type tested under Monolith No. 6, will permit increasing the designed lateral loads given in Conclusions (*a*) and (*b*) by from 1 ton to 2 tons per pile.

ACKNOWLEDGMENTS

The tests herein described were made in connection with the construction by the United States Government of the Twin Locks of Lock and Dam No. 26, which were designed under the supervision of W. H. McAlpine, M. Am. Soc. C. E. The construction work is under the supervision of Captain B. M. Harloe, Corps of Engineers, U. S. Army, District Engineer, St. Louis District, with Captain W. W. Wanamaker, Corps of Engineers, U. S. Army, Military Assistant, in local charge.

The writer wishes to acknowledge the able assistance of Messrs. C. E. Wuerpel, Assistant Engineer, and G. A. Allen, Junior Engineer, in making the field observations and assembling data, and of J. A. Adams, Associate Engineer, in preparing the illustrations accompanying this paper. Acknowledgment is especially made of the helpful suggestions of Captain Wanamaker, who conceived the idea of making the tests. The test monoliths were constructed and the tests were performed by Bruce Gordon, Superintendent for the John Griffiths and Son Company, General Contractor for the construction of the Twin Locks.

DISCUSSION

A. E. Cummings,[2] M. Am. Soc. C. E. (by letter).—A considerable store of valuable information on the lateral deflection of foundation piles is contained in this paper. The writer is fortunate in having had a number of opportunities to discuss these tests with the author and the method of analysis herein proposed was developed for the particular purpose of attempting to check, theoretically, the experimental data presented.

The assumption is made that the upper end of the pile is embedded deeply enough in the foundation to be fixed against rotation. It is also assumed that the lower part of the pile is fixed and that some unknown length, L, of the upper part is bent into a deflection curve as shown in Fig. 17(a). The positive directions of the co-ordinate axes are taken as shown and the point of reverse curvature is assumed to be at a distance of $\frac{1}{2}$ L from the top, at which point the deflection is one-half the total deflection, f.

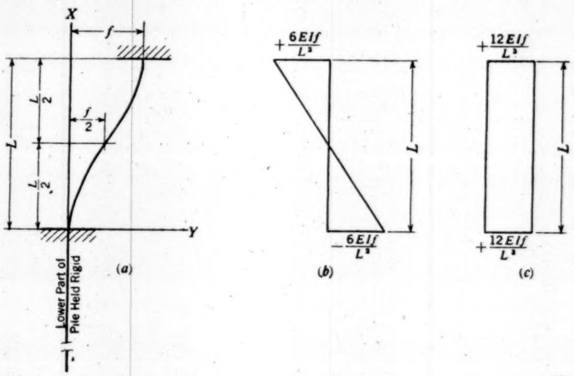


Fig. 17.

The pile is considered first as a free standing column and the surrounding soil is ignored temporarily. It is assumed that the equation of the central line of the deflected pile can be expressed by the power series[3]:

$$y = a_1\left(\frac{x}{L}\right)^2 + a_2\left(\frac{x}{L}\right)^3 + a_3\left(\frac{x}{L}\right)^4 \quad \ldots\ldots\ldots\ldots\ldots (1)$$

It is then necessary to determine the coefficients of the series in such a way that the boundary conditions of the problem will be satisfied and the resulting equation will represent the elastic curve of the deflected pile. The assumptions as to the shape of the deflection curve furnish the following boundary conditions, from which these coefficients are determined: (1) Since the deflection curve is to have a vertical tangent at the top and at the bottom, the first derivative of Equation (1) is zero at $x = 0$ and at $x = L$; (2) since the point of inflection is to be at the middle of the deformed part of the pile the second derivative of Equation (1) is zero at $x = \frac{L}{2}$; and (3) the deflection is f at $x = L$. These three sets of boundary conditions serve to establish the equation:

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[2] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

[3] "Drang und Zwang", von Föppl, Vol. 2, p. 312.

$$y = \frac{3\,f\,x^2}{L^2} - \frac{2\,f\,x^3}{L^3} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

which is the equation of the deflection curve of the bent pile.

Equation (2) is then differentiated, successively, three times. By using the differential equation of the elastic curve of a bent beam:

$$E\,I\,\frac{d^2y}{dx^2} = -\,M \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

and substituting the second derivative of Equation (2), the following moment equation is obtained:

$$-\,M_f = E\,I\left(\frac{6\,f}{L^2} - \frac{12\,f\,x}{L^3}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

In a similar manner, the shear is obtained from the third derivative of Equation (2), the result being,

$$V = +\,\frac{12\,E\,I\,f}{L^3} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The moment diagram from Equation (4) and the shear diagram from Equation (5) are shown in Fig. 17(*b*) and Fig. 17(*c*), respectively.

When the pile stands vertically in the soil, the earth pressures are approximately in equilibrium on all sides of it. When it is subjected to the horizontal deflection, f, a certain earth load is built up in front of the pile due to the passive resistance of the soil. It will be assumed that this earth load at any point is equal to the deflection times the elastic modulus of the soil. It will also be assumed that the elastic modulus of the sand increases directly with the depth.[4] With the positive directions of the co-ordinate axes chosen as shown in Fig. 18, the equation of the elastic modulus of the soil may be written:

$$E_s = \frac{U\,(L - x)}{\omega} \;\ldots\ldots\ldots(6)$$

in which U = the weight per unit volume of soil and ω = a dimensionless coefficient determined by the elastic property of the soil.

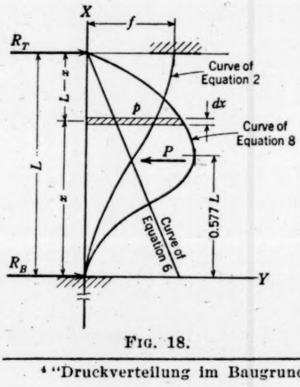

FIG. 18.

The unit earth pressure on the front side of the deflected part of the pile is then given by,

$$p = E_s\,y \;\ldots\ldots\ldots\ldots\ldots(7)$$

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[4] "Druckverteilung im Baugrunde", von O. K. Froehlich, p. 89.

in which y is the deflection curve of Equation (2) and the assumption is made that the shape of this curve is not materially changed by the earth load. This unit pressure is:

$$p = \frac{U(L-x)}{\omega}\left[\frac{3fx^2}{L^2} - \frac{2fx^3}{L^3}\right] \quad\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The total earth load is, then,

$$P = \int_0^L p\,dx = \frac{Uf}{\omega}\int_0^L\left[\frac{3x^2}{L} - \frac{5x^3}{L^2} + \frac{2x^4}{L^3}\right]dx \quad\ldots\ldots\ldots (9)$$

which when integrated, gives,

$$P = \frac{3}{20}\,\frac{UfL^2}{\omega} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The earth pressure curve in Fig. 18 is given by Equation (8). The point of maximum earth pressure can be found by differentiating Equation (8) with respect to x; setting this derivative equal to zero; and then solving for x. The maximum is slightly above the center (see Fig. 18).

The earth-load reactions are computed by taking moments about the top and bottom points of the deflected part of the pile. Referring to Fig. 18, the reaction at the top is:

$$R_T = \int_0^L\left(\frac{x}{L}\right)p\,dx = \frac{Uf}{\omega}\int_0^L\left[\frac{3x^3}{L^2} - \frac{5x^4}{L^3} + \frac{2x^5}{L^4}\right]dx\ldots (11)$$

or,

$$R_T = \frac{1}{12}\,\frac{UfL^2}{\omega} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The reaction at the bottom is:

$$R_B = \int_0^L\frac{(L-x)}{L}\,p\,dx = \frac{Uf}{\omega}\int_0^L\left[\frac{3x^2}{L} - \frac{8x^3}{L^2} + \frac{7x^4}{L^3} - \frac{2x^5}{L^4}\right]dx..(13)$$

or,

$$R_B = \frac{1}{15}\,\frac{UfL^2}{\omega} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The sum of these two reactions is equal to the total earth load as given by Equation (10).

To calculate the bending moments and deflections caused by the earth load, use will be made of the method of super-position.[5] Referring to Fig. 19(a), the moment due to the couple at the top is:

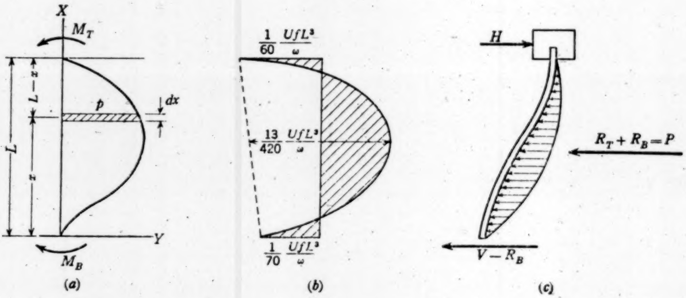
$$M_T = \int_0^L p\,dx\,\frac{x^2(L-x)}{L^2} \quad\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

---

[5] "Strength of Materials", by S. Timoshenko, Vol. 1, p. 209.

Substituting the value of p from Equation (8) and integrating gives,

$$M_T = \frac{1}{60}\,\frac{U\,f\,L^3}{\omega} \qquad \text{(16)}$$



X
M_T
$L-x$
p
dx
L
x
Y
M_B
(a)
$\frac{1}{60}\frac{UfL^3}{\omega}$
$\frac{13}{420}\frac{UfL^3}{\omega}$
$\frac{1}{70}\frac{UfL^3}{\omega}$
(b)
H
$R_T + R_B = P$
$V - R_B$
(c)

FIG. 19.

The moment due to the couple at the bottom is:

$$M_B = \int_0^L p\,dx\,\frac{x\,(L-x)^2}{L^2} \qquad \text{(17)}$$

Substituting the value of p and integrating gives:

$$M_B = \frac{1}{70}\,\frac{U\,f\,L^3}{\omega} \qquad \text{(18)}$$

The moment produced by the distributed earth load when the pile is considered as a simple beam is,

$$M_S = \int_0^L p\,dx\,\frac{x\,(L-x)}{L} \qquad \text{(19)}$$

Substituting for p and integrating,

$$M_S = \frac{13}{420}\,\frac{U\,f\,L^3}{\omega} \qquad \text{(20)}$$

Combining the moment diagram from Equation (20) with the diagram from the two end couples, produces the combined moment diagram shown in Fig. 19(b).

In computing the deflections due to the earth load, it will be assumed that the maximum deflection occurs at the center of the span although as has already been shown the earth load is not quite symmetrical. The equation[6] of the deflection curve due to the couples at the ends is,

$$\Delta_c = \frac{M_T\,(x)\,(L^2 - x^2)}{6\,E\,I\,L} + \frac{M_B\,(L-x)\,(2\,L\,x - x^2)}{6\,E\,I\,L} \qquad \text{(21)}$$

---

[6] "Strength of Materials", by S. Timoshenko, Vol. 1, p. 168.

Substituting $x = \frac{L}{2}$ and the values of M_T and M_B from Equations (16) and (18) gives, at $x = \frac{L}{2}$:

$$\Delta_c = \frac{13}{6\,720} \frac{U f L^5}{E I \omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

The deflection[7] at the center of the span due to the distributed earth load and considering the pile as a simple beam, for $x = \frac{L}{2}$, is:

$$\Delta_S = 2 \int_0^{\frac{L}{2}} p \, dx \, \frac{x}{48 E I} (3 L^2 - 4 x^2) \quad \ldots\ldots\ldots\ldots (23)$$

Substituting the value of p from Equation (8) and integrating $\left(\text{for } x = \frac{L}{2}\right)$:

$$\Delta_S = \frac{89}{43\,008} \frac{U f L^5}{E I \omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

It should be noted that these deflections are in opposite directions and that they are approximately equal since the numerical coefficient of Equation (22) is 0.00193, whereas that of Equation (24) is 0.00207.

One of the quantities measured in the experiments was the jack load. This is an external horizontal force applied near the top of the pile, and it is resisted partly by shear in the pile itself and partly by the earth load built up in front of the pile. Referring to Fig. 19(*c*), the equilibrium condition for the horizontal forces is,

$$H = (V - R_B) + (R_T + R_B) = V + R_T \quad \ldots\ldots\ldots\ldots (25)$$

Substituting Equations (5) and (12) into Equation (25) gives:

$$H = \frac{12 E I f}{L^3} + \frac{1}{12} \frac{U f L^2}{\omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Equation (26) indicates that the deflection is a linear function of the external load so that the work done during deflection may be expressed by,

$$W = \frac{H f}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Substituting Equation (26) into Equation (27) gives:

$$W = \frac{6 E I f^2}{L^3} + \frac{1}{24} \frac{U f^2 L^2}{\omega} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The assumption is then made that during deflection the pile follows the principle of least action and that the bent length, L, adjusts itself so as to make the total work of deformation a minimum. Differentiating Equation (28) with respect to L:

$$\frac{\partial W}{\partial L} = - \frac{18 E I f^2}{L^4} + \frac{1}{12} \frac{U f^2 L}{\omega} \quad \ldots\ldots\ldots\ldots\ldots (29)$$

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[7] "Strength of Materials", by S. Timoshenko, Vol. 1, p. 164.

The condition for a minimum is,

$$-\frac{18\,E\,I\,f^2}{L^4}+\frac{1}{12}\,\frac{U\,f^2\,L}{\omega}=0 \quad\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

Solving for L gives:

$$L=\sqrt[5]{\frac{216\,E\,I\,\omega}{U}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

This is a minimum since the second derivative of Equation (28) with respect to L is positive.

The foregoing theoretical equations will be used to check some of the test results on Monoliths Nos. 2, 3, and 5, of Mr. Feagin's paper. These monoliths contained only wood piles, for which a Young's modulus was determined and for which the moment of inertia can easily be computed. The average diameter at a point 5 ft below the heads of all the piles in these three monoliths is found to be 12.077 in., or 1.0064 ft. The moment of inertia of a circular area of this diameter is 0.0503 ft.^4 The Young modulus of one of the wood piles was determined as 1 878 260 lb per sq in., or 270 469 440 lb per sq ft. The average weight of the sand on this site is 112.5 lb per cu ft, dry. The percentage of voids varies from 30 to 37 so that in a thoroughly saturated state the sand surrounding these piles weighs about 133 lb per cu ft. This is a very dense sand[8] and the dimensionless coefficient, ω, will be taken as 0.005.

Using these numerical values in Equation (31), the calculated length of the deformed part of the pile is 10.2 ft. With this value of L and with the same values of E, I, U, and ω, a series of calculations was made with Equation (26). For loads of 4, 8, 12, 16, and 20 tons per pile, the theoretical deflections were computed and the results are plotted as a dotted line in Fig. 20. The observed load-deflection curve (which is the average of the

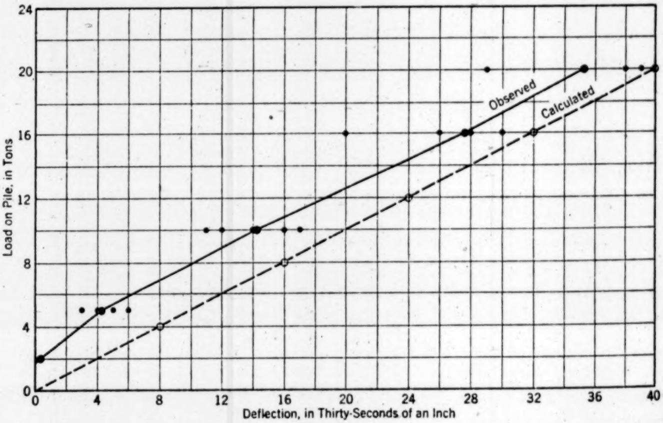

FIG. 20.

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[8] "Erdbaumechanik", by Charles Terzaghi, M. Am. Soc. C. E., pp. 11–12, and Table 24, p. 92.

data for Monoliths Nos. 2, 3, and 5, taken from Table 3) is plotted on the same diagram. It is seen that the computed curve falls somewhat below the observed curve, but, in general, the variations between the two are no greater than those among the observations themselves. The computed curve is on the safe side. In two other ways there are indications of agreement between observation and theory. Mr. Feagin mentions a measurement of the deformed length of one of the piles and reports this as about 10 ft, although the measurement was made on a concrete pile. He also mentions the fact that the earth in front of the pile provided the greater proportion of the resistance against deflection, which conclusion is checked by Equation (26).

Fig. 21(*a*) shows a combined moment diagram at a load of 16 tons per pile, which corresponds to a theoretical deflection of 1 in., the values of

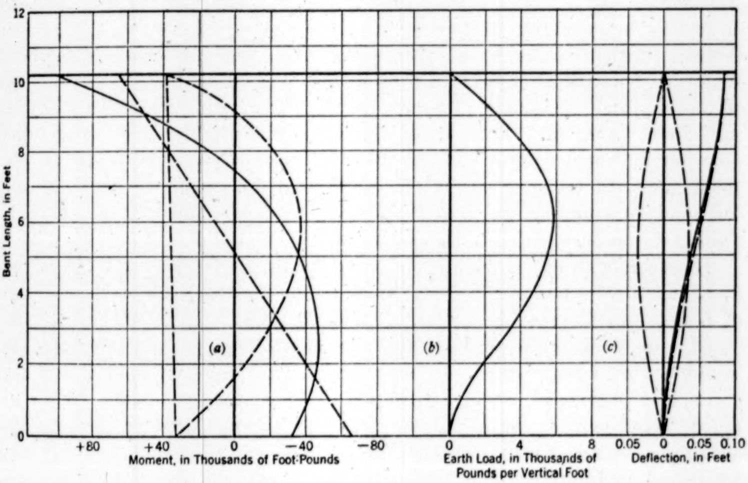

FIG. 21.

L, *E*, *I*, *U*, and ω being the same as before. Fig. 21(*b*) is an earth-load diagram for the same numerical values, and Fig. 21(*c*) is a combined deflection diagram for the same conditions, the solid line representing the final deflection.

In any analysis of this kind it is necessary to make some assumptions. The analysis having been completed and having been applied to experimental data it is usually desirable to reconsider the basic assumptions. Stated in general terms, this problem may be said to involve the determination of the stresses and deflections of an elastic rod of relatively high flexural rigidity submerged vertically in a more or less elastic medium of relatively low elastic resistance, the rod being moved horizontally at the top but fixed against rotation at that point so that the tangent to the elastic curve remains vertical at the surface. This general statement involves no assumptions, but it contains only one boundary condition which is the requirement that the tangent to the elastic curve shall remain vertical at the surface. This is the only known boundary condition of the problem and in order to develop any kind of analysis it is necessary to begin by making assumptions.

It is sometimes assumed[9] that a pile embedded in the ground and subjected to a horizontal force at the top, will pivot about a point somewhere down in the ground. The movement of the pile is approximately a rotation as a rigid body; but it would appear that this could not happen in the case of the monoliths described by Mr. Feagin because of the fixed-end condition at the top. The assumption can be made that the elastic curve is a wavy line of gradually diminishing amplitude similar to a damped vibration curve. This involves further assumptions as to the rate at which the amplitude is damped out and the number of wave lengths that shall be included in the pile length. An analysis of this kind is almost certain to become involved in serious mathematical difficulties. The analysis given in this discussion assumes fixation against rotation at some point along the length of the pile. Whether or not this condition can occur will depend on the flexural rigidity of the pile and the elastic properties of the surrounding soil. This can be seen from Equation (31) and, for the wood piles in the Alton tests, this equation indicated a length of the deflected part of about 10 ft, which was about one-third the pile length. It would seem reasonable to conclude that the sand surrounding the lower 20 ft of the pile could provide sufficient rigidity to produce the deflection curve of Fig. 21(*c*).

However, with a very rigid pile embedded in a soft loose soil—for which E, I, and ω were large, and U was small—Equation (31) might give a value of L that was as great as, or greater than, the pile length. When the computed value of L is greater than the pile length, it is probable that the pile would act as a long cantilever beam fixed at the top and loaded with the earth load developed during deflection. When the calculated length of the deflected part of the pile is of the order of two-thirds or three-fourths of the pile length, it is improbable that the lower end of the pile could remain fixed against rotation. The bottom of the pile would probably move in a direction opposite to that of the deflection at the surface. In either case it would be necessary to revise the fixed-end assumption and to repeat the analysis with some other assumed deflection curve.

It should be noted that Equation (31) provides a means of determining whether or not the fixed-end assumption is justified. It should also be noted that this equation does not contain the deflection, but it should not be concluded from this that the bent length is independent of the deflection. Equation (31) is a combination of Equations (5) and (12). Equation (5) is based on the usual theory of elastic structures in which the displacements are required to be very small in comparison with the dimensions of the structure. Equation (12) is based on similar assumptions. Equation (31), therefore, applies only to small deflections even if the deflection itself does not appear in the equation.

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[9] *Civil Engineering*, December, 1934, p. 622.

In order to determine the relative behavior of rods of different flexural rigidities embedded in a granular material, a simple experiment was performed on a small scale. The results are shown in Figs. 22 and 23. Three steel rods were placed in a vertical position inside the glass front of the box and dry sawdust was then poured in and tamped lightly. The sizes of the rods, from left to right, were $\frac{3}{32}$ in. round, $\frac{1}{8}$ in. square, and $\frac{1}{4}$ in. square. The upper ends of the rods were fixed against rotation by being clamped between two notched steel bars, and vertical lines were marked on the glass for comparison as shown in Fig. 22.

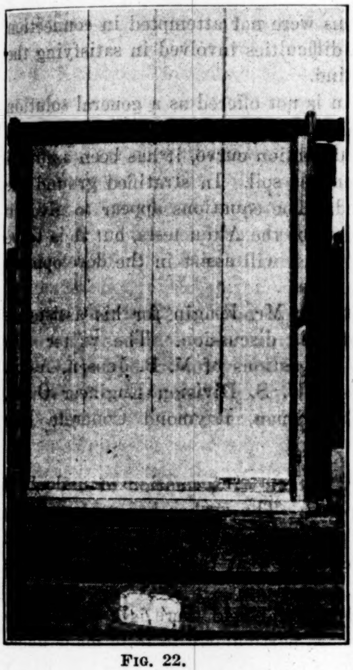

FIG. 22.

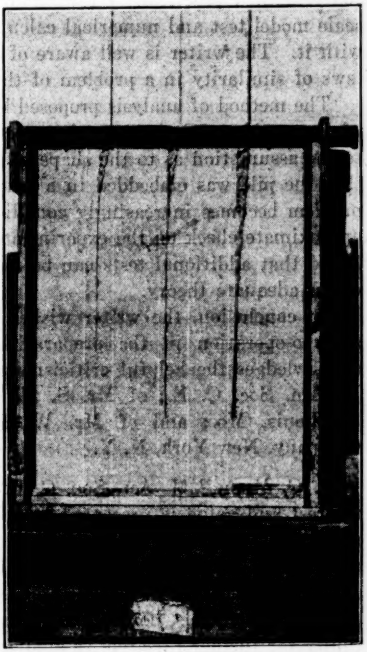

FIG. 23.

The steel bars holding the upper ends of the rods were then moved slowly about 1 in. to the right with the results shown in Fig. 23. This was an excessive deflection but it was used in order to produce sufficient movement to photograph. Each rod deflected in a reverse curve and the horizontal lines on the glass mark the bottoms of these curves. The smallest rod had a very short deflection curve, and there was no apparent lateral movement of the lower part of the rod. The middle rod had a longer deflection curve and just below the curve there was a slight movement of the rod toward the left. This movement was very small in comparison with the deflection at the top and it did not become apparent until the top deflection was nearly an inch. The deflection curve of the largest rod amounted to almost three-fourths of the embedded length of the rod. The lower part of the rod was incapable of maintaining its vertical position and the bottom of the rod moved about $\frac{1}{4}$ in. to the left. As nearly as could be determined the lower part of the rod remained practically straight. This experiment is not intended to be a scale model test and numerical calculations were not attempted in connection with it. The writer is well aware of the difficulties involved in satisfying the laws of similarity in a problem of this kind.

The method of analysis proposed herein is not offered as a general solution of the problem of the lateral deflection of a foundation pile. In addition to the assumption as to the shape of the deflection curve, it has been assumed that the pile was embedded in a homogeneous soil. In stratified ground the problem becomes increasingly complicated. The equations appear to give an approximate check on the experimental data of the Alton tests, but it is to be hoped that additional tests can be made which will assist in the development of an adequate theory.

In conclusion, the writer wishes to thank Mr. Feagin for his assistance and co-operation in the preparation of this discussion. The writer also acknowledges the helpful criticisms and suggestions of V. P. Jensen, Assoc. M. Am. Soc. C. E.; of Mr. S. M. Gleser, U. S. Division Engineer Office, St. Louis, Mo.; and of Mr. W. P. Kinneman, Raymond Concrete Pile Company, New York, N. Y.

J. C. Meem,[10] M. Am. Soc. C. E. (by letter).—Information of undoubted interest and value is presented in this paper. The author has presented the results in such a way as to make them practically applicable. As far as it goes, the paper covers the subject so thoroughly that there is little left to be added in the nature of actual discussion.

The writer feels, however, that in this general field there is a wide opportunity for further research, as applying more particularly to tests made under otherwise similar conditions on piles or foundations that bear on widely varying types of soils, such as clay, loam, sand, gravel, etc. For instance, in his preliminary paragraph of conclusions, the author states: "* * *. The following conclusions, therefore, are confined to the soil conditions and piling arrangements under which the tests were made."

Wherever it is possible to do so, measurements should be made and tabulated of lateral thrust against structures and of resistance to such thrust under conditions similar to those noted. After all, the resistance of the soil to lateral thrust is only secondary in a minor degree, if at all, to that of the pile or structure itself, and conditions exist, or may readily be conceived, in which the instability of the soil is such that tie-backs or lateral bracing must be introduced.

A case in point, which is of paramount interest, is that in which a monolithic concrete pier of a bridge at Grand Coulee Dam, in Washington, was tilted by movement of the glacial drift surrounding it. This movement[11] was such that it reached a maximum of 9 in. at the top, approximately 150 ft above the base.

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[10] Cons. Engr., Brooklyn, N. Y. Mr. Meem died June 24, 1936.

[11] *Engineering News-Record,* November 7, 1935.

Such glacial movement could only occur in soils having a predominating clay or equivalent content. It illustrates, however, the necessity for a full understanding of soil conditions where the structure is subject to lateral stresses and, more particularly, where there is a possibility that future operations may tend to change the status of the existing conditions.

T. KENNARD THOMSON,[12] M. AM. SOC. C. E. (by letter).—Experiments are always interesting, but care must be exercised against drawing unwarranted conclusions, especially in subsoils where borings may be inadequate. The author is very wise, therefore, in stating that: "In view of the fact that these tests were conducted in only one type of soil, and in view of the many variables involved, no mathematical analysis is included."

The writer has read many papers supported by beautifully drawn diagrams showing uniform curves of underground pressure, in which regularity might be imagined, but probably never exists. In the sense that no two blades of grass are identical, probably no two cubic feet of subsoil are identical either; conditions created by the material, the pressure exerted by Nature in compacting the mass, variations in the ground-water level, the roots of trees, boulders, etc., are probably different in each case.

Such uniform or symmetrical lines of pressure might possibly occur in man-made piles of sand—but never in foundations on original subsoil.

Many years ago, the writer received a letter from an architect about 300 miles away, containing a long and apparently full description of a proposed foundation, and was asked to telegraph his fee for giving his opinion as to the safe load per square foot permissible to be placed on the foundations. The proposition seemed so plain and simple that the writer was almost misled by it, but suddenly changed his mind and telegraphed that he would not give such an opinion without first examining the site. The architect made no reply, but some time afterward the writer, by mere chance, heard that the "proposed" building had been constructed in another city and had collapsed before the architect had tried to secure the writer's endorsement.

The only time that one can feel really sure about stresses underground is when they are purely hydrostatic—and even then the surrounding conditions must be understood thoroughly. In pneumatic caisson work for buildings in Manhattan, the writer has seldom noticed the air pressure to vary more than a few pounds from the theoretical head, with no allowance for weight of soil, adjoining buildings, etc.; but the horizontal Hudson River Tunnels are different, probably due to the great speed of construction as well as to the closed shields, where sometimes only one-half the anticipated air pressure is required. Slow progress would probably greatly increase the amount of air pressure involved in any case by giving the water above a chance to circulate.

The proportions of sand, clay, boulders, etc., vary in every location even a few feet apart. Sometimes, the engineer finds more than a dozen distinct layers, with different colors of sand and clay in a single vertical foot.

Clay, dry or mixed with sand, boulders, etc., often makes a natural concrete so compact that deep excavations may be made without any side pressures, and, consequently, without sheathing or shoring, but a small quantity of clay in sand under water, often makes the entire mass slide with disastrous results. The author's experiments apply the tests at or near the tops of the piles; but, in actual work, the stresses might occur near the bottom of the piles, or anywhere between the bottom and top, due to quicksand or wet clay in the subsoil stratification.

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[12] Cons. Engr., New York, N. Y.

Some of the reasons for observing the locations (not always made clear in the reports of borings) are to determine the location and character of any adjoining buildings, rivers, lakes, etc., and the heights and character of any adjacent hills. The borings themselves should give the ground-water level and whether or not it varies. Of course, "core" borings are more definite than auger or "wash" borings but in some cases where the engineer knows, the subsoil conditions at the particular site—"core" borings would simply be a waste of money.

Some years ago, about 12 acres of land dropped 20 ft, sliding down hill and carrying buildings, a 200-ft chimney, etc., with it in a few seconds. A geologist from the South and another from the North had each made twenty-five auger borings, and then the writer was consulted. As he could not find the cause from the borings, he decided to sink a 4-ft shaft, which penetrated good compact material for a depth of about 30 ft. At that depth, a stratum of very sloppy clay, almost soup, was encountered, which explained everything. The site of the factory was surrounded on two or three sides by high hills, and above the factory was a fair-sized lake.

There had recently been a very rainy season and some of the rain had found its way to the clay stratum, which was at the level of the creek at the foot of the slope. The slide resulted, moving the creek itself about 20 or 30 ft. The first day the writer saw the site he advised draining the lake (which was above the most important building left standing) and this was done at once. It had been claimed that a pile of broken stone, covering a comparatively small area was responsible for the damage; but the broken stone did not weigh 1 ton per sq ft on the small area it covered. Furthermore, a section of virgin forest near-by had also sunk.

That stresses may occur at any point along the entire length of a pile is demonstrated by still another case. A railroad company had constructed a high earth-fill leaving an opening for an existing railroad at right angles to its line. This opening was provided for by driving 1 600 wood piles, on top of which were built two side walls, 8½ ft thick, of concrete. These side walls were 33 ft apart and connected at the top by a concrete arch, supporting the new fill, but had no connection at the bottom above the wooden platform on the piles.

When the writer examined the site he found the concrete abutments badly shattered, having moved horizontally 10 ft at one end and 3 ft at the other. The elevation of the original ground was about 1 300 ft above sea level, and the hills near-by were about 700 ft higher.

It was obvious that the water must have found its way from these hills to a clay or quicksand stratum at a level somewhere above the bottom of the foundation piles. In this case the piles may have weakened instead of strengthened the structure. If a circular concrete tunnel had been constructed in the first place, it would not have required as much material as was actually used in the abutments or side walls, and could not have collapsed. It might even have stood without any pile foundation.

As each site requires a special study, an unlimited number of examples might be quoted.

AUGUST E. NIEDERHOFF,[13] JUN. AM. SOC. C. E. (by letter).—The lateral pile-loading tests in this paper shed additional light on a subject that has long been obscure. Mr. Feagin's excellent record of the test data and the evident care and methodical procedure in gathering and presenting the information deserve commendation from the profession. It is noted that he is careful to state the limitations under which his findings were obtained, and invites discussion that will contribute to the general store of information on this subject.

Early in the period for designing locks and dams on the Upper Mississippi River, it was realized that specific knowledge of the resistance of round, wood, bearing piles to lateral loads was necessary. The design of these structures was based upon the assumption that all vertical loads were to be transmitted to the sub-strata by piles, leaving nothing for surface soil bearing, and it was believed to be just as logical an assumption to resist all lateral loads by piles and to omit all frictional resistance between the bottom of the structure and the ground surface. The design analysis, therefore, had to be based upon a certain allowable value and to arrive at this value a series of tests were inaugurated about December 19, 1933, at the proposed site of Lock No. 3 on the Mississippi River, immediately up stream from Red Wing, Minn.

This site was chosen for conducting the tests because borings indicated that soil conditions for a foundation were less desirable than at other Mississippi River lock and dam sites within the St. Paul (Minn.) District Office of the U. S. Corps of Engineers. A boring made where the tests were conducted is shown in Fig. 24. Both piles subjected to lateral loads were of red oak, 30 ft long, and with the tip diameters about 7 in. and 8 in., respectively, and ground-line diameters of about 12 in. each. The piles were driven about 28 ft through a mixture of silt and clay with a 3 000-lb steam hammer with a 33-in. stroke under 110-lb steam pressure. Allowing a penetration of 0.63 and 1.35 in. under the last blow, the indicated bearing capacity by the *Engineering News* formula for Pile No. 2 was 10 tons and for Pile No. 6, 5 tons. This indicated bearing value is given only to show that the soil was not very compact. Some idea of the surface material can be obtained from the information that a man rolled up his sleeve and stuck his arm into the mud alongside one of the driven piles until it came above his elbow.

The equipment for the tests is shown in Fig. 25. The arrangement for testing Pile No. 1 at the upper end of the framework is shown in the plan view. Section *A–A* indicates the cable arrangement about the test pile. A load applied to the test pile by exerting a pull with a ratchet, *R*, at the other end of the lever beam, *A*, was measured on the 5 000-lb dynamometer to the left of, and attached to, the anchor pile, *X*. The lever beam, *A*, was pivoted on a case-hardened steel fulcrum supported by a frame work of 12 by 12-in. timbers. When sufficient load was applied by Ratchet *R*, the top of Test Pile No. 1 assumed an inclined position that was registered by an originally vertical flag attached firmly to the test specimen. To simulate the restraining action of the pile-butt embedded in concrete masonry, the flag was returned to a vertical position by pulling with Ratchet *S*. Transits checked the movement of the test specimen and kept the flag in a vertical position. The actual load on the pile, of course, was the difference in the readings of the two dynamometers on either side of the anchor pile, *X*, corrected by the ratio of the lever arms of Beams *A* and *B*. If the 5 000-lb gauge had been used to capacity, the maximum moment in Beams *A* and *B*, Fig. 25, would have been 1 620 000 in-lb and the extreme fiber stress, 27 500 lb per sq in., but because very few stiffeners had been provided the two beams showed a tendency toward diagonal buckling before they could be stressed to these values.

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[13] With Hydr. Eng. Dept., Aluminum Co. of America, Pittsburgh, Pa.

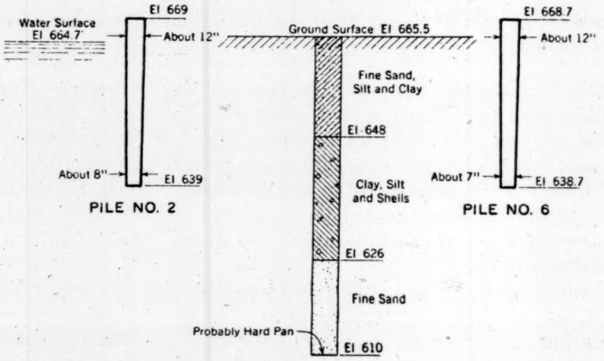

FIG. 24.

The nature of the equipment, therefore, limited the load that could be applied to the test specimen to 18 kips. Prior to running the tests, the soil around the test specimens had been frozen to a depth of approximately 6 in. When testing Piles Nos. 2 and 6, this frozen ground was removed from around the pile for a distance of 4 in. It was believed that the remaining frozen ground would prevent the surrounding soil from bulging during the tests just as effectively as a concrete footing would act in preventing soil bulge. To obtain some idea of the effect of frozen soil on the resistance of a vertical pile to lateral loads, Pile No. 1 was tested to the limit of the apparatus.

From the arrangement shown in Fig. 25 it was possible to measure three factors entering into the analyses of a vertical pile subjected to a lateral load. The deflection at the ground surface was measured to the nearest 0.02 in.; the applied load causing the deflection was measured on dynamometers graduated to 100 lb; and the moment in the pile produced by the arrangement of the two cables that simulated the restraining action of embedment of the butt in masonry was measured by the known force exerted by each cable and the measured moment arm between the cable attachments on the test specimen. The recorded observations and the computed moment in the pile are presented in Table 4. A load of 17 kips was applied on Pile No. 6 (see Table 4(*b*)) for a sustained 5-min interval. The pile continued to move under the influence of this load, thus substantiating the theory of plastic flow of the foundation soil.

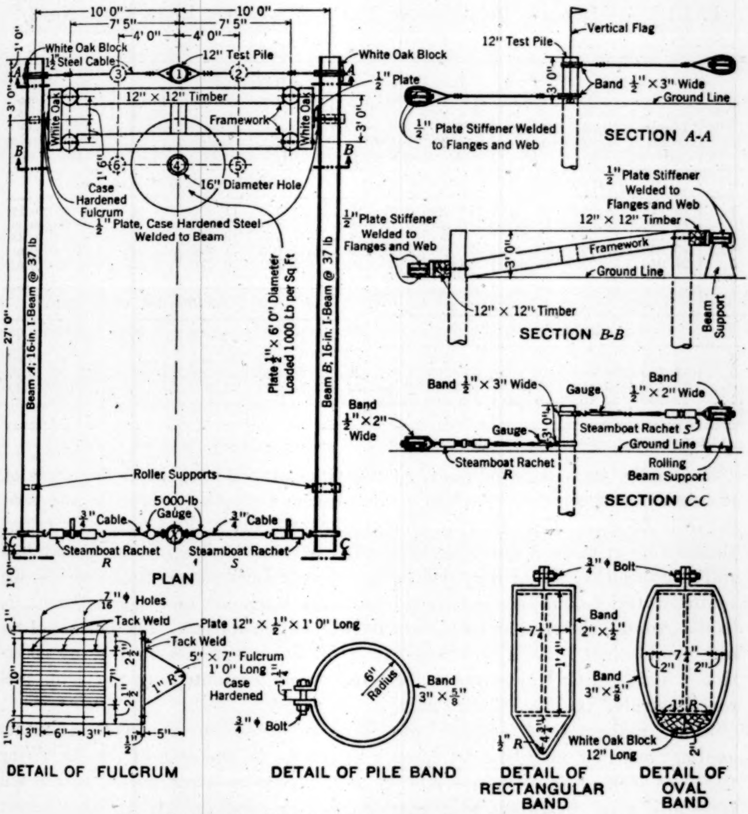
10' 0"
10' 0"
7' 5"
7' 5"
4' 0"
4' 0"
1' 0"
White Oak Block
$1\frac{1}{2}$" Steel Cable
12" Test Pile
White Oak Block
$\frac{1}{2}$" Plate
A
A
3' 0"
12" × 12" Timber
White Oak
White Oak
Framework
3' 0"
B
B
1' 6"
1
2
3
4
5
6
16" Diameter Hole
Case Hardened Fulcrum
$\frac{1}{2}$" Plate, Case Hardened Steel Welded to Beam
27' 0"
Beam A; 16-in. I-Beam @ 37 lb
Plate $\frac{1}{2}$" × 6' 0" Diameter Loaded 1 000 Lb per Sq Ft
Beam B; 16-in. I-Beam @ 37 lb
Roller Supports
5 000-lb Gauge
$\frac{3}{4}$" Cable
$\frac{3}{4}$" Cable
Steamboat Rachet R
Steamboat Rachet S
C
C
1' 0"
PLAN
12" Test Pile
Vertical Flag
3' 0"
Band $\frac{1}{2}$" × 3" Wide
Ground Line
$\frac{1}{2}$" Plate Stiffener Welded to Flanges and Web
SECTION A-A
$\frac{1}{2}$" Plate Stiffener Welded to Flanges and Web
12" × 12" Timber
$\frac{1}{2}$" Plate Stiffener Welded to Flanges and Web
3' 0"
Framework
Ground Line
12" × 12" Timber
Beam Support
SECTION B-B
Band $\frac{1}{2}$" × 3" Wide
Gauge
Band $\frac{1}{2}$" × 2" Wide
Band $\frac{1}{2}$" × 2" Wide
3' 0"
Gauge
Steamboat Rachet S
Ground Line
Steamboat Rachet R
Rolling Beam Support
SECTION C-C
$\frac{7}{16}$" ϕ Holes
Tack Weld
Plate 12" × $\frac{1}{2}$" × 1' 0" Long
Tack Weld
5" × 7" Fulcrum 1' 0" Long Case Hardened
1"
10"
7"
$2\frac{1}{2}$"
$2\frac{1}{2}$"
1" R
1"
3"
6"
3"
$\frac{1}{2}$"
5"
DETAIL OF FULCRUM
6" Radius
Band 3" × $\frac{5}{8}$"
$\frac{3}{4}$" ϕ Bolt
DETAIL OF PILE BAND
$\frac{3}{4}$" ϕ Bolt
$7\frac{1}{4}$"
1' 4"
Band 2" × $\frac{1}{2}$"
$4\frac{1}{4}$"
$\frac{1}{2}$" R
DETAIL OF RECTANGULAR BAND
$7\frac{1}{4}$"
2"
2"
Band 3" × $\frac{5}{8}$"
1" R
2"
White Oak Block 12" Long
DETAIL OF OVAL BAND

FIG. 25.—ARRANGEMENT FOR PILE TESTS.

Comparing the results of the Red Wing tests with the Alton, Ill., tests, it should be kept in mind that the former did not evaluate the effect of a lateral load applied to a group of piles. Recognizing this, it is remarkable that at Alton the maximum deflection of $\frac{9}{32}$ in. of a monolith when stressed to a lateral load per pile of 6.5 tons is comparable with the deflection of 0.28 and 0.39 in., respectively, of the piles at Red Wing when subjected to approximately an equivalent load. Among other considerations not evaluated by the Red Wing tests were: (*a*) The effect of confining soil around the pile in preventing or restraining plastic flow of foundation material; and (*b*) the effect of vertical load applied in conjunction with the lateral load.

TABLE 4.—LATERAL PILE-LOADING TESTS, MISSISSIPPI RIVER LOCK NO. 3, RED WING, MINN.

Test No.	READINGS		Load, P_o, in pounds (3)	Moment,* M_o, in inch-pounds (4)	Deflection, in inches (5)
	Lower scale (1)	Upper scale (2)			
(*a*) PILE NO. 2					
1...	10 720	4 940	5 780	118 600	0.07
2...	18 340	7 020	11 320	168 400	0.13
3...	27 700	13 040	14 660	313 500	0.28
4...	25 850	11 540	14 310	277 000	0.26†
5...	27 050	12 250	14 800	294 500	0.30
6...	35 950	20 120	15 830	483 000	0.39
7...	30 900	12 830	18 070	308 000	0.36‡
(*b*) PILE NO. 6					
8...	4 165	2 310	1 855	55 400	0.01
9...	7 720	4 240	3 480	101 700	0.05
10...	16 400	9 110	7 290	218 500	0.15
11...	22 900	12 450	10 450	299 400	0.23
(*b*) PILE NO. 6 (*Continued*)					
12...	22 050	11 800	10 250	283 500	0.23†
13...	30 850	17 340	13 510	416 000	0.39
14...	34 500	19 670	14 830	472 000	0.46
15...	37 400	21 800	15 600	523 000	0.61
16...	36 300	19 270	17 030	462 500	0.63†
(*c*) PILE NO. 1					
17...	10 640	4 950	5 690	118 800	0.06
18...	20 530	10 900	9 630	261 600	0.07
19...	28 100	15 560	12 540	373 440	0.10
20...	36 800	20 740	16 060	497 760	0.16
21...	39 170	21 980	17 190	527 520	0.20
......	....	....	....	....	......

* 24-in. between clamps. † 5-min. intervals. ‡ Load released and re-applied.

Logically, it is believed that, to the time of applying loads in excess of 12 kips, the specimens assumed a center-line deflection curve of the same general shape as Fig. 17(*a*). Loads in excess of 12 kips probably caused plastic flow of the soil governed by the law of viscous flow in plastic materials and produced a center-line deflection curve described as fixed at the top against rotation, but showing considerable displacement, and at some depth, *L*, in the ground, showing no displacement, but some rotation. The bottom of the pile would then show a movement in a direction opposite to the top deflection. This center-line curve is believed to be simulated by the $\frac{1}{4}$-in. steel rod illustrated in Figs. 22 and 23.

The mathematical analysis of either of these curves involves certain assumptions that may be considerably in error. In the case of the Red Wing tests the lack of specific information and laboratory examination of the soil forestalls even an attempt at mathematical reasoning. If it were known definitely just what the volume change of the soil would be when subjected to increased pressure and the permeability and cohesion of the foundation material, a satisfactory attempt at induction might be tried, based primarily on soil mechanics.

On the basis of the Red Wing tests the District Engineer, at St. Paul, concluded that lateral load resistance of vertical round, wood piles is determined to a great extent by the nature and characteristics of the soil rather than by the flexural rigidity of the pile itself. There are certain limitations to this statement, of course, but for round, wood piles longer than 30 ft and with butt and tip dimensions in accordance with the standards specified in the A. S. T. M. specifications, the foregoing statement is believed to be valid. The extreme fiber stress on the pile that carried a maximum moment was 2 417 lb per sq in., and there was no apparent distress in the fibers. In making comparisons between the results of the tests and the behavior of actual vertical pile foundations when subjected to lateral loads, it might be mentioned that a lateral movement of a lock-wall occurred at Lock No. 5*A* on the Mississippi River when the computed horizontal load per round, wood pile did not exceed 3.5 tons. This lock-wall movement was approximately 9 in. in a foundation soil composed principally of sand with a thin clay stratum lying at considerable depth below the structure. It is believed that the combined effect of slippery clay and pile-driving in the vicinity of the wall produced this extraordinary lateral movement. The arresting fact is that construction conditions and the hydrostatic head that helped to move the wall at Lock No. 5*A* had been previously duplicated (although perhaps not quite so severely) at Lock No. 5. These conditions at Lock No. 5 had probably stressed the piles by a horizontal load of approximately 3 tons per wood pile. The foundation soil at Lock No. 5 is principally sand with some indications of gravel far below the lock-walls. The fact that no lateral deflection took place confirms the belief that soil investigation is of primary importance, and arbitrary "safe" values for lateral loads on piles are figments of the imagination.

LAZARUS WHITE,[14] M. AM. SOC. C. E. (by letter).—The experiments made by Mr. Feagin and his associates are familiar to the writer and the paper describing them constitutes a valuable contribution in a field in which reliable information is scanty. The conditions under which the experiments were conducted are carefully described and the author's conclusions are properly limited to the soil conditions and piling arrangements under which the tests were made.

It will interest most engineers to note that the lateral resistance of the piles tested was less than the ratios ordinarily assumed in such designs. That piles about 30 ft long—of white oak and concrete, driven into firm sand, and capable of sustaining a vertical load of, say, 30 tons, with their ends firmly fixed into a solid concrete block—should have a resistance of 4 to $4\frac{1}{2}$ tons for a lateral movement of $\frac{1}{4}$ in., is rather startling. It is startling, furthermore, to discover that well made concrete piles have a lateral resistance of only 1 ton or 2 tons more. For a lateral movement of $\frac{1}{2}$ in. the indicated lateral load is only 7 tons. That the values are really low is borne out by observations on the movements of walls founded on wooden piles driven in sand (see Fig. 26). These walls moved several inches.

Tests made on the holding-down power of wooden piles indicate a high value in most cases. A vertical load of 20 to 75 tons is required to pull one pile out of a concrete block, so that it is probably correct to assume fixed-end conditions.

The writer wishes to call attention to the danger of extending observations or calculations made on single piles or on a small group, to a large group, such as is ordinarily done The piles of a large group may grip enough soil to act as a diaphram much as one may retain a bank with longitudinal or vertical sheeting with wide open joints. In this case the resistance against lateral movement is that of an approximately vertical plane of earth bounding the group, much the same as the resistance of a line of very stiff steel sheeting.

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[14] With Hydr. Eng. Dept., Aluminum Co. of America, Pittsburgh, Pa.

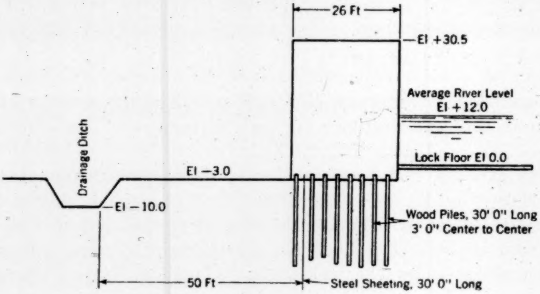

FIG. 26.

The writer has observed large movements of groups of piles serving as foundations to concrete walls. When the lateral resistance of the earth surrounding them has diminished due to near-by excavations, the walls move as a unit. These walls were also supported on lines of steel sheeting serving as cut-off walls. Much of this diminution of resistance, in the writer's opinion, was due to the lateral flow of water in the adjacent excavation, rendering the sand surrounding the piles partly quick. It is apparent, therefore, that under field conditions, the lateral resistance of piles may be highly variable, and, at times, it may be only a few tons per pile. The lesson is that under the conditions described it is wise to supplement the lateral resistance of vertical piles with other, more positive, means. Steel sheeting below the walls does not provide this resistance, as, in the cases quoted, the tops of the steel sheeting utilized as cut-offs were incorporated in the wall. The more positive means may be struts or ties or battered piles, advantageously placed.

Y. L. CHANG,[15] ESQ. (by letter).—These valuable tests on the lateral deflection of foundation piles give rise to much useful information on this subject not hitherto available. The need of some definite information and a method of analysis is certainly urgent, owing to the rapidly increasing use of piles and sheet-piling in foundation work. Mr. Feagin has taken the opportunity of conducting tests on piles driven in river sand, with a view to determining the resistance of such piles to movement under lateral loads caused by back-fill on a lock or by water pressure on a dam. The Huai River Commission in China is at present (1936) constructing three locks on the Grand Canal to improve navigation, and, at the same time, to maintain a definite slope to facilitate local irrigation. A general knowledge of the lateral deflection of piles, such as that presented by the author, will certainly be of great help in such a case.

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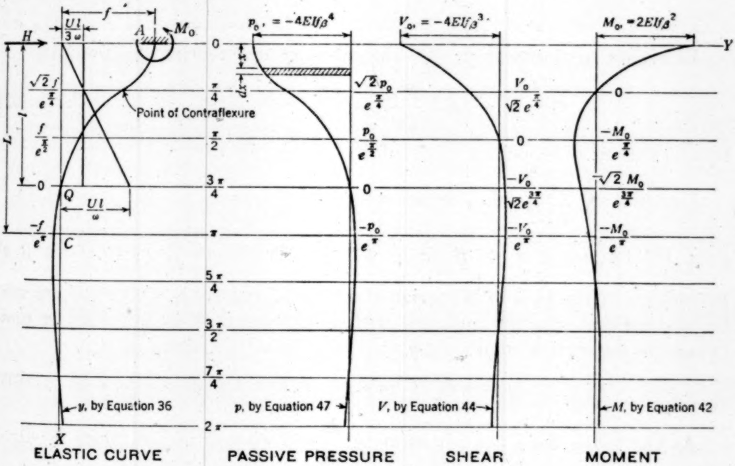
[15] Scholar of Board of Trustees, Nanking, China; Research Student, Victoria Univ., Manchester, England.

The problem is equally interesting in the case of a sheet-pile; and, since it is usually designed with a comparatively low section modulus, an investigation of its movement under lateral load is of special importance.

The author has mentioned that an exact mathematical analysis to verify the experimental data is not at all easy, owing to the fact that so many variables are involved and that conditions vary widely for different localities. An attempt at such an analysis, with the aid of model tests of the pile, would provide a close guide to apply in a given case. When small elastic rods are embedded in granular material and subjected to lateral pressure they will bend in the form of a reverse curve, and the bottom of the rod will move slightly in a direction opposite to that of the displacement at the top. At a certain intermediate depth there is rotation, but no displacement.

An approximate analysis is suggested herein, involving the assumption that the "elastic modulus of soil", E_s, is a constant, and it follows that the foregoing phenomena can be demonstrated quite rationally. By assuming an arbitrary value for E_s (the true value of which should be determined at the site locally), a curve can be plotted to agree quite satisfactorily with Mr. Feagin's experimental data.



H
$\frac{Ul}{3\omega}$
f
A
M_0
$\frac{\sqrt{2}f}{e^{\frac{\pi}{4}}}$
$\frac{f}{e^{\frac{\pi}{2}}}$
L
l
0
Q
$\frac{Ul}{\omega}$
$\frac{-f}{e^{\pi}}$
C
Point of Contraflexure
x
dx
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π
$\frac{5\pi}{4}$
$\frac{3\pi}{2}$
$\frac{7\pi}{4}$
2π
$p_0 = -4EI f \beta^4$
$\frac{\sqrt{2}\,p_0}{e^{\frac{\pi}{4}}}$
$\frac{p_0}{e^{\frac{\pi}{2}}}$
0
$\frac{-p_0}{e^{\pi}}$
$V_0 = -4EI f \beta^3$
$\frac{V_0}{\sqrt{2}\,e^{\frac{\pi}{4}}}$
0
$\frac{-V_0}{\sqrt{2}e^{\frac{3\pi}{4}}}$
$\frac{-V_0}{e^{\pi}}$
$M_0 = 2EI f \beta^2$
Y
0
$\frac{-M_0}{e^{\frac{\pi}{4}}}$
$\frac{-\sqrt{2}\,M_0}{e^{\frac{3\pi}{4}}}$
$\frac{-M_0}{e^{\pi}}$
y, by Equation 36
p, by Equation 47
V, by Equation 44
M, by Equation 42
X

ELASTIC CURVE PASSIVE PRESSURE SHEAR MOMENT

FIG. 27.

The assumptions upon which the analysis is based are as follows:

(1) The upper end of the pile is embedded deeply enough in the monolithic foundation to be fixed against rotation;

(2) The pile is infinitely long (as will be shown subsequently, the lateral movement of a pile at depths greater than l (Fig. 27), is so small that any pile of sufficient length may be assumed to be infinitely long);

(3) The elastic modulus of soil, E_s, is constant throughout the depth; and,

(4) The passive pressure, p, on a pile is proportional to its displacement; that is, $p = - E_s\, y$, in which y is the lateral deflection of the elastic curve.

Regarding Assumption (3), moreover, the soil itself will be compacted, more or less, to the same degree and uniformity, due to the overlapping of the pressure cones that are created when all the piles are driven to their proper depths. Since most foundations are designed on the assumption that the supporting power is distributed properly both on the piles and on the bearing area of the earth foundation, it follows that the earth must be quite compact even at the surface. Therefore, it is not necessary to assume that the "elastic modulus of soil" is a straight line, varying with the depth. As a matter of fact, its value depends on the properties of the soil at different strata and on the intensity and the distribution of the vertical loads. If it is assumed, tentatively, that this constant modulus is one-third of what the variable modulus would be at the depth, l (Fig. 27), a simple substitution can be made. The reason for making E_s less than the mean value of a triangular variation may be justified by considering that the upper part of a pile, being subjected to much more pronounced lateral movement, is surrounded by earth of comparatively low elasticity. It will be shown subsequently that l is proportional to $\frac{1}{\sqrt[4]{E_s}}$, so that if an error of 10% is made in the value of E_s, there would be only an error of 2% in the value of l.

Referring to Assumption (4), the ordinary differential equation[16] is,

$$EI \frac{d^4 y}{dx^4} = p = - E_s\, y \qquad (32)$$

in which E = the modulus of elasticity of the pile material.

Let, $\beta = \sqrt[4]{\frac{E_s}{4\,EI}}$ and, the general solution is,

$$y = e^{\beta x} (A \cos \beta\, x + B \sin \beta\, x) + e^{-\beta x} (C \cos \beta\, x + D \sin \beta\, x)$$

in which e is the base of Naperian logarithms, and A, B, C, and D are arbitrary constants. As $e^{\beta x}$ increases with x, Constants A and B must be equal to zero to satisfy the condition, $y = 0$ at $x = \infty$; therefore,

$$y = e^{-\beta_x} (C \cos \beta\, x + D \sin \beta\, x) \qquad (33)$$

As the top is fixed against rotation, $\frac{dy}{dx} = 0$ when $x = 0$; or, since $C - D = 0$,

$$\frac{dy}{dx} = - \beta\, e^{-\beta x} [(C - D) \cos \beta\, x + (C + D) \sin \beta\, x] = 0 \qquad (34)$$

and, therefore, Equation (33) becomes,

$$y = C\, e^{-\beta x} (\cos \beta\, x + \sin \beta\, x) \qquad (35)$$

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[16] "Strength of Materials", by S. Timoshenko, Pt. II, p. 402, Equation (1).

At $x = 0$, $y = f$, from which $C = f$; and,

$$y = f\ e^{-\beta x}\ (\cos \beta\ x + \sin \beta\ x) \ldots\ldots\ldots\ldots\ldots (36)$$

and,

$$\frac{dy}{dx} = -\ 2\ \beta\ f\ e^{-\beta x}\ \sin \beta\ x \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

The point of zero deflection (see Point Q, Fig. 27) can be found by making $y = 0$ and $x = l$ in Equation (36), from which,

$$e^{-\beta l}\ (\cos \beta\ l + \sin \beta\ l) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

One solution of Equation (38) is given by $l = \infty$; or, $\cos \beta\ l + \sin \beta\ l = 0$; $\tan \beta\ l = -\ 1$; and,

$$\beta\ l = \left(n - \frac{1}{4}\right) \pi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which $n = 1, 2, 3$, etc. The foregoing analysis demonstrates that the elastic curve for an infinitely long pile should be a harmonic wave, damping away rapidly after each successive wave length, $2\ l$. For the least value, $\beta\ l = \frac{3\ \pi}{4}$, and,

$$l = \frac{3\ \pi}{4\ \beta} = \frac{3}{4}\ \pi \sqrt[4]{\frac{4\ EI}{E_s}} \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

For the point of zero slope, $\frac{dy}{dx} = 0$, and $x = L$ in Equation (37); therefore, $e^{-\beta L} \sin \beta\ L = 0$. In one solution, $L = \infty$; or, $\sin \beta\ L = 0$; and, $\beta\ L = n\ \pi$. The least value is $\beta\ L = \pi$, and,

$$L = \frac{\pi}{\beta} = \pi \sqrt[4]{\frac{4\ EI}{E_s}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

By proportion, $l = \frac{3}{4} L$. It is to be noted that for $n = 2$, L is very nearly equal to l, showing that the pile is practically vertical and rigid at a depth of one wave length. At the depth, L (Fig. 27), the lateral movement in the opposite direction is a maximum. To find this deflection, substitute $\beta\ x = \pi$ into Equation (36), and $y = -\frac{f}{e^{\pi}}$, which amounts to 4.3% of the top deflection in the opposite direction.

For the point of maximum slope, differentiate Equation (37) with respect to x; thus: $\sin \beta\ x = \cos \beta\ x$; and $\beta\ x = \frac{\pi}{4}$. To compare the foregoing with Equations (40) and (41), $x = \frac{l}{3} = \frac{L}{4}$. To find an expression for moment, Equation (37) is differentiated further:

$$\frac{d^2 y}{dx^2} = 2\ f\ \beta^2\ e^{-\beta x}\ (\sin \beta\, x - \cos \beta\, x)$$

and,

$$M = -\,2\ E\ I\ f\ \beta^2\ e^{-\beta x}\ (\sin \beta\, x - \cos \beta\, x) \qquad (42)$$

At $x = 0$,

$$M_0 = 2\ E\ I\ f\ \beta^2 \qquad (43)$$

The point of zero moment, determined by $\sin \beta\, x = \cos \beta\, x$, occurs at the point of contraflexure. Differentiating again:

$$\frac{d^3 y}{dx^3} = 4\ f\ \beta^3\ e^{-\beta x}\ \cos\ \beta\ x \qquad (44)$$

At $x = 0$, the shear, V_0, equals:

$$V_0 = -\,4\ E\ I\ f\ \beta^3 = -\,H \qquad (45)$$

in which H = the external force applied at the top of a pile, and, at $x = L$,

$$V_L = 4\ E\ I\ f\ \beta^3\ e^{-\pi} = -\,\frac{V_0}{e^{\pi}} \qquad (46)$$

which is only 4.3% of V_0.

For the point of zero shear, or for that of maximum moment, $\cos \beta\, x = 0$; $\beta\, x = \left(n - \frac{1}{2}\right)\pi$; and the first point occurs at a depth of $\frac{L}{2}$.

The passive pressure on the pile is:

$$p = -\,E_s\ y = -\,E_s\ f\ e^{-\beta x}\ (\cos\ \beta\ x + \sin\ \beta\ x) \qquad (47)$$

Considering a pile of infinite length, the total passive pressure on the pile is:

$$P = \int_0^\infty p\ dx = -\,E_s f \left[\int_0^\infty e^{-\beta x} \cos \beta\, x\ dx + \int_0^\infty e^{-\beta x} \sin\ \beta\ x\ dx\right]$$
$$= -\,4\ E\ I\ f\ \beta^3 = -\,H \qquad (48)$$

If Equation (48) is integrated to the point of zero deflection,

$$P = \int_0^l p\ dx = 4\ E\ I\ f\ \beta^3\ e^{-\beta x}\ \cos\ \beta\ x \Big]_0^{\frac{3\pi}{4\beta}} = -\,4\ E\ I\ f\ \beta^3 \left(1 + \frac{1}{\sqrt{2}\, e^{\frac{3}{4}\pi}}\right)$$
$$= -\,1.067\ H \qquad (49)$$

The difference between the total passive pressure when assuming that the pile does not bend beyond the depth, l, is only 6.7% in excess of that on an infinitely long pile. This difference is due to the decreasing pressure intensity as the depth increases. The length of pile that is subject to passive pressure can also be determined by considering the minimum internal energy set up in a pile during elastic deformation, thus:

$$W = \frac{1}{2} \int_0^d p\ y\ dx = -\,\frac{1}{2} \int_0^d E_s\ f^2\ e^{-2\beta x}\ (\cos\ \beta\ x + \sin\ \beta\ x)^2\ dx \qquad (50)$$

in which the upper limit, d, denotes the depth, or, finally,

$$W = \frac{E_s f^2}{8\,\beta}\,[e^{-2\beta d}\,(\cos 2\,\beta\,d + \sin 2\,\beta\,d + 2)]\ldots\ldots\ldots(51)$$

As before,

$$\frac{\partial W}{\partial d} = -\,\frac{E_s f^2}{2}\,[e^{-2\beta d}\,(\sin 2\,\beta\,d + 1)] = 0\ldots\ldots\ldots\ldots(52)$$

In one solution, $d = \infty$; or, $\sin 2\,\beta\,d = -\,1$; that is,

$$2\,\beta\,d = \left(2\,n - \frac{1}{2}\right)\pi\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(53)$$

For $n = 1$, $2\,\beta\,d = \frac{3}{2}\,\pi$, and,

$$d = \frac{3\,\pi}{4\,\beta} = l = \frac{3}{4}\,\pi\,\sqrt[4]{\frac{4\,E\,I}{E_s}}\ \ldots\ldots\ldots\ldots\ldots\ldots(54)$$

which is also the expression for l in the case of zero deflection (see Equation (40)). Equation (54) shows that a similar expression for a definite bent length, l, can always be derived by considering either that the pile is rigid beyond this depth or that it is deformed in damping waves throughout the entire length.

Let E (the modulus of elasticity of the pile) $= 270\,469\,440$ lb per sq ft; $I = 0.0503$ ft^4; $U = 133$ lb per cu ft; and $\omega = 0.005$. Then, by the foregoing assumption, $E_s = \frac{U\,l}{3\,\omega} = \frac{133\,l}{3 \times 0.005} = 8\,866\,l$. These values, substituted in Equation (40) yield, $l = 11.35$ ft; or, $E_s = 100\,700$ lb per sq ft. Then, by substituting, in the expression, $\beta = \sqrt[4]{\frac{E_s}{4\,E\,I}}$, $\frac{1}{\beta} = \sqrt[4]{541}$; or $\frac{1}{\beta^3} = 112.2$. Equation (45) gives the relation between the horizontal load applied at the top of the pile and the lateral deflection of the top of the pile; that is, substituting the foregoing values and solving for f:

$$f = \frac{H}{4\,E\,I\,\beta^3} = \frac{H}{20.2}\ \ldots(55)$$

in which f is in inches and H is in tons.

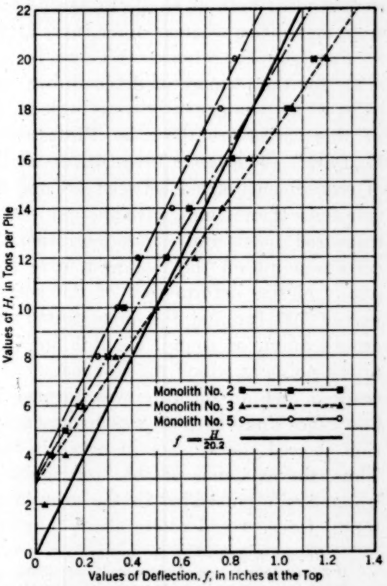

FIG. 28.

When Equation (55) is plotted (see Fig. 28) the curve has nearly the same slope as the experimental curves of M_2 and M_5 in Mr. Feagin's paper. In the author's "Conclusions", a value of 7 tons has been suggested as the maximum lateral load on the top of a pile if a lateral movement of not more than 0.5 in. is allowable. Adopting 0.5 in. as a practical limit, Fig. 28 gives 10 tons as the maximum lateral load. Reference to this curve shows that for deflections of less than 0.5 in., the experimental data for M_2, M_3, and M_5 follow lines of the same general slope as those defined by Equation (55); in fact, they are on the safe side.

The best empirical equation that will fit the field data is evidently of the form:

$$f = \frac{H}{a} - b \qquad (56)$$

in which a and b are constants.

In conclusion, the writer wishes to thank Professor A. H. Gibson and Mr. J. Allen, of the Victoria University of Manchester, England, and A. E. Cummings, M. Am. Soc. C. E., for their helpful criticism of this discussion.

D. P. Krynine,[17] M. Am. Soc. C. E. (by letter).—The lateral pile-driving tests at Alton, Ill., described in this paper, were made on a considerably larger scale than any other experiments known to the writer. Therefore, they approach the actual loading conditions more closely. Furthermore, the piles were studied both as single units and in groups, with heads embedded in concrete monoliths, whereas other investigators have dealt with isolated piles only.

Previous Experiments.—There is a series of older German experiments made for the purpose of determining the resistance of posts placed in narrow holes which subsequently were back-filled with earth. Among the experiments of this series are those by Professor Engel[18], H. Fröhlich[19], and those by H. Dörr[20]. The latter investigator used posts about 18 ft high, and the depth of embedment varied between 4.3 and 6.1 ft. The soil was Rhine sand, as a rule finer than 1 mm, with the angle of natural slope about 40 degrees. Dörr measured horizontal forces corresponding to horizontal displacements of a certain point on the test post, care being taken to substract therefrom the purely elastic deformation of the post itself. The horizontal forces did not exceed 700 lb; and the piles tested were rectangular, about 6 by 9 in. in cross-section, or round, about 8 in. in diameter. The tests were conducted to the point at which the soil mass failed so that the ground close to the pile cracked and an opening was formed behind it. This hole was filled with fine ashes and the post was hammered down somewhat. Subsequent excavation showed the absence of ashes in the lowest third of the embedded depth. Thus, the existence of the "zero point" was established experimentally. An isolated pile embedded in the ground and subjected to the action of a horizontal force, would rotate about that zero point.

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[17] Research Associate in Soil Mechanics, School of Eng., Yale Univ., New Haven, Conn.

[18] *Zentralblatt der Bauverwaltung*, 1903, p. 274.

[19] "Beitrag zur Berechnung der Mastfundamente", 1913.

[20] "Die Standsicherheit der Masten und Wände im Erdreich", 1922.

Experiments conducted at North Carolina State College, Raleigh, N. C., and by the State Highway Board of Georgia[21] have shown that the resistance of a pole to overturning is proportional to the square of the penetration depth. This fact induces one to believe that the resistance of a pile to the action of a lateral force is connected with the configuration of some body of rotation which develops within the earth mass close to the pile under consideration. The North Carolina tests were conducted in hard clay, and those in Georgia in clay containing about 50 to 60% of sand,

Introducing the following notations: P_o = an overturning force, in pounds; d = depth of embedment or penetration, in feet; and c = a coefficient. The overturning force may then be expressed as:

$$P_o = c\,d^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

The value of the coefficient, c, was found to be 1 250 in North Carolina and 300 in Georgia.

In 1928 the writer made some field and laboratory tests on pile models loaded horizontally. These tests were made in co-operation with Messrs. G. I. Pokrowski and N. V. Laletine[22] who later continued this work alone.[23] Wooden piles, 3 to $3\frac{1}{2}$ in. in diameter, ranging in length from 6 to $7\frac{1}{2}$ ft, were driven into a very uniform clay with 17 to 18% of natural moisture. Then the piles were overturned by applying a horizontal force. A cross-section through the center of the pile was excavated carefully, using shovels and finishing the vertical surface with a sharp knife. Fig. 29 is a sketch made from a photograph of that cross-section. The zero point, Z, located at a depth from

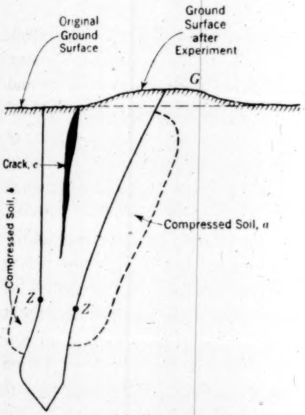

FIG. 29.

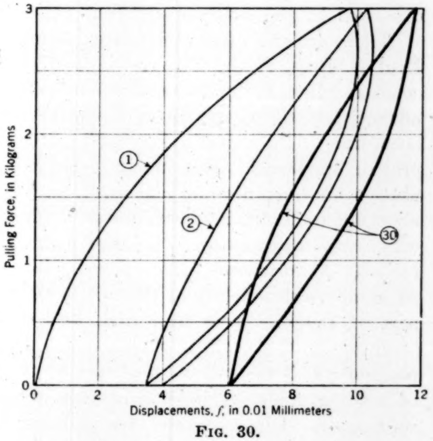

FIG. 30.

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[21] *Wood Preservation News*, November, 1932, Vol. 10, pp. 156–158.

[22] "Soil Investigations in 1928–1929" (printed in Russian), pp. 5–21, 65–67.

[23] "Soil Investigations in 1929–1930" (printed in Russian), pp. 15–33.

0.52 d to 0.69 d, is clearly seen; and it appears that the overturning process consists of two phases. In the first phase the pile rotates slightly about its zero point, Z, being resisted by both the compressive resistance of the soil (compressed zones, a and b) and by its tensile strength (Crack c). The second phase is characterized by the breaking down of the shear resistance. The center of rotation gradually moves from the zero point, Z, to the surface of the ground, G, following the curve, ZG; and, finally, an earth cone is pushed out. A weak pile breaks during this second phase.

It follows from these experiments that a structure founded on piles and subject to lateral forces, is kept in place by the compressive resistance of the adjacent earth mass. This compressive resistance, however, does not control the ultimate, or the limit of equilibrium of the structure. As soon as the acting force overcomes the value of the shearing resistance of the soil, the deformation takes the path of least resistance, and the mass fails by shear. Hence, the actual lateral force should not be greater than an nth part of the shearing resistance, in which n is the safety factor. Furthermore, there is a sharp difference between the failure of the earth mass and the failure of the structure itself, since owing to the excessive compressibility of an earth mass, the structure may move laterally and fail before the adjacent earth mass is disturbed by shear. A further development of these experiments has been discussed by the writer elsewhere.[24]

There are two more series of experiments of this kind. In 1928 and 1930, Paul E. Raes, of the University of Ghent, Belgium, made some tests using concrete piles, 12 by 12 in. and 14 by 14 in. He published his results and advanced a theory which is essentially that of a zero point, or "pivot" according to his terminology[25]. In 1933, M. Nakamura[26] made some very precise small-scale experiments concerning lateral pile action. He used wooden and metallic pile models (from $2\frac{5}{16}$ to $3\frac{5}{16}$ in. in diameter) embedded about 10 to 12 in. in a sand mass confined in a box, 24 by 24 in. by 16 in., which was supposed to be large enough to leave the model pile action undisturbed. A horizontal force up to 10 lb was applied at levels of about 22 to 28 in. above the surface of the soil in the box. Deformations of the embedded part of the pile were measured, and the zero point was located at about seven-tenths the depth of embedment. Fig. 30 represents the horizontal displacements of the pile at the level of the ground under the repeated action of a force gradually increasing from zero to about 6.6 lb and decreasing to zero again. Displacements consisted of both an irreversible and an elastic part; but after a certain number of loading and unloading cycles there was elastic action only. (Note the heavy lines in Fig. 30.)

Lateral Pile Action and Loading Tests Interrelated.—Mr. Feagin states that "frequent repetition of lateral loads results in slightly greater lateral movement than a sustained load of the same magnitude." This is because a repetition of load application tends to eliminate gradually irreversible lateral movements, as shown in Fig. 30. The case of a pile acted upon by a horizontal force is a special case of the loading of an earth mass; hence analogics should be sought between such an action and the action of a loaded plate placed at the surface of an earth mass. Fig. 31 shows the settlement curve in an experiment made by the writer[27]. A confined sand layer, 3 in. in diameter and about $1\frac{1}{4}$ in. thick, was subjected to compression, using loads ranging from 16 to 65 lb per sq in. applied to a metallic piston acting at the sand surface. Curves A and B, Fig. 31, correspond to the total settlement of the sample in the loaded and unloaded condition, respectively, so that the difference between them denotes the elastic rebound at a given stage of the experiment. Similar curves may be obtained by joining the upper and the lower points of the curves in Fig. 4(e), or in Fig. 9.

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[24] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), pp. 271–273.

[25] *Proceedings*, International Conference on Soil Mechanics (1936), Vol. 1, Paper H-1; also, *La Technique des Travaux*, No. 8, p. 505 (1928).

[26] "Ueber den Erdwiderstand der Maste", *Der Bauingenieur*, Vol. 16, pp. 269–275, 1935.

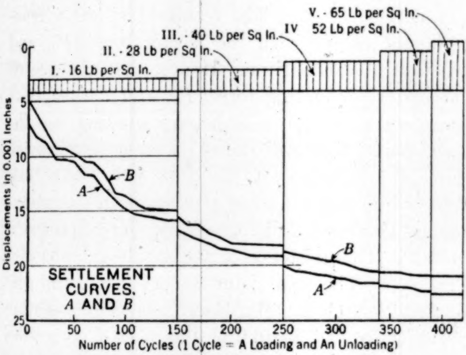

FIG. 31.

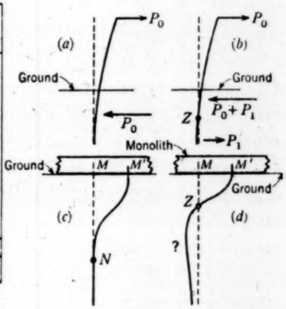

FIG. 32.

In connection with Fig. 30 it should be stated that each pair of curves corresponding to a loading and unloading cycle, is analogous to a hysteresis loop in loading experiments. Order numbers of loading and unloading cycles are indicated by the numbers in circles in Fig. 30. After a certain number of such cycles (thirty in Fig. 30) the hysteresis loops become closed, which means that the earth mass has attained perfect elasticity under the action of a given stress. This elasticity is not the same, however, as that required by Hooke's law, since displacements are not proportional to the stresses. It is also remarkable that this state of elasticity may be destroyed as soon as a stress greater than that used in compression is applied (see Fig. 31).

"Pre-Testing" of Piles by Lateral Pull.—If instead of a confined sand sample (Fig. 31) a plate is placed at the ground surface and loaded repeatedly, the results would be practically the same[28]. If a pile is subjected to repeated vertical loading, an analogous action occurs. Using this property, Lazarus White, M. Am. Soc. C. E., developed a so-called "pre-test" method of foundation construction applied principally to piles[29]. It is well known that the

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[27] *Proceedings*, International Conference on Soil Mechanics (1936), Vol. II, pp. 22–23.

[28] "Underpinning", by E. A. Prentis and Lazarus White, Members, Am Soc. C. E., p. 241, Fig. 128 (1931).

[29] *Loc. cit.*, p. 113, Fig. 61.

surface of an earth road that is reasonably moist becomes elastic under traffic; and this surface is easily destroyed should a few heavy loads pass over the road. All kinds of pavements are automatically "pre-tested" by traffic. Assembling all the foregoing facts, it may be stated that practically every earth structure, except perhaps the case of saturated plastic soils, can be "pre-tested" within the meaning of the term introduced by Mr. White. This suggests that irreversible deflections of piles subjected to lateral forces could be also eliminated by duly "pre-testing" them during construction. How this "pre-testing" could be done—pile by pile, or otherwise—and whether such a method is economically feasible are questions to be decided for each individual case.

Theory of the Zero Point.—Fig. 32 represents a hypothetical weightless pile loaded at its top with a horizontal force, P_0. If there is no zero point, the pile would deflect as shown in Fig. 32(*a*). The resultant of earth reaction, P_1, would form an unbalanced couple with the acting force, P_0, and this is incompatible with conditions of equilibrium. Hence, the existence of a zero point is a mechanical necessity, the acting force, P_0, and the reaction, P_1, of the earth at the bottom part of the pile being balanced by the reaction, $P_0 + P_1$, at the upper part (Fig. 32(*b*)). A most comprehensive mathematical theory concerning the location of the zero point has been advanced by K. Hayaschi[30] in describing the behavior of a beam on a continuous elastic support. The Russian Professor I. P. Prokofiev has advanced a simplified theory of the zero point[31]. The existence of the zero point was also proved mathematically by Prudon[32]. The zero-point theory has been discussed by C. C. Williams[33], M. Am. Soc. C. E. Mr. Raes[25] advanced a mathematical theory of the zero point ("pivot"), including the case of two piles with heads embedded in a concrete monolith. Mr. Raes comes to the conclusion that in the latter case one of the two piles is pulled, and the other pushed, and that the "pivot" exists in all cases.

In the experiments by Mr. Feagin there is no direct indication as to whether or not there is a "pivot" in the case of piles with embedded heads. The elastic line in this case may be visualized either according to Fig. 32(*c*) or Fig. 32(*d*). In the former case there are vertical tangents at Points *N* and *M′*, and the elastic line in the latter case has been drawn quite arbitrarily only to show that, in the opinion of the writer, there is a zero point located rather high. Mr. Feagin testifies that between the pile and the concrete (Point *M′*) there was "a crack in which the blade of a pocket-knife could be inserted". Hence, it is doubtful whether the tangent to the elastic line at Point *M′* was vertical. On the other hand, in Fig. 32(*c*), the line, *NM′*, is longer than the line, *NM*; and neglecting the deformations of the pile, one may conclude that the pile should be pulled out at Point *N*, which contradicts the assumption of ideal embedment at that point. The writer believes that only in the case of feebly rigid piles the assumption of Fig. 32(*c*) may lead to a rather satisfactory working theory. In a general case, however, the attention of the investigators should be directed toward the location of the zero point, Z (Fig. 32(d)). Mr. Feagin states: "It was found that a carpenter's rule could be inserted to a depth of 6 ft in the space between the pile and the adjacent foundation sand on the side of the hole nearest the jack." Probably that rule reached Point Z, as shown in Fig. 32(d).

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[30] "Theorie des Balkens auf elastischer Unterlage", Berlin, 1921, p. 228 *et seq.*

[31] "Theory of Structures", Pt. II, p. 232 *et seq* (printed in Russian in 1928).

[32] *Le Génie Civil*, No. 5, 1926, p. 117.

[33] "The Design of Masonry Structures and Foundations", Second Edition, 1930, p. 478 *et seq.*

Extension of the Experiments Suggested.—The excellent experiments by Mr. Feagin should be extended somewhat further: First, it seems necessary to investigate whether or not the "pre-test" method as applied to piles deflected laterally can decrease the lateral movement of locks; and, second, the shape of the elastic line of such piles should be investigated more closely. For economy, smaller pile models driven into uniform natural clay may be used in these particular field tests. It would be very important, furthermore, to study the characteristics of batter piles.

Additional Remarks on Irreversible Earth Deformations.—The following observation by Mr. Feagin is to be emphasized: A pile loaded horizontally never returns to its original position upon removal of the load. Actually, even a perfectly elastic pile is restrained from returning to its initial position owing to irreversible deformations of the earth mass. This fact may cause serious secondary stresses in a framework, such as a wooden bridge.

Action of Forces at a Lock.—If there were no piles, horizontal stresses due to the difference of levels in both the upper and the lower pool (Fig. 3(a)) would develop under the structure. The piles driven into the ground form, with the soil, a mass of "reinforced earth", and it is very probable that the horizontal stresses referred to, are taken up by that mass which works as a whole. A graphic method of computing these horizontal stresses has been advanced by the writer elsewhere[34]. Assuming the earth material under the structure to be elastically isotropic, the value of such a stress pushing the structure down stream, would be approximately: $0.5 \times 62.5 \times (38.0 - 15.2) = 713$ lb per ft of width of the structure and per lin ft of the length of the piles. This value should be less in the case of sands. The values given do not pretend to any degree of accuracy; the writer wishes only to state that an additional force pushing the lock down stream may exist and that this force is due to the difference of water levels in the pools.

Conclusion.—As a rule, the closer a soil model is to a full-sized structure, the more trustworthy are the results. In this respect Mr. Feagin's experiments with large models are unusually interesting. To increase the value of the experiments, it seems necessary to extend their scope somewhat, but even in their present form, Mr. Feagin's experiments are an important contribution to the field of soil mechanics and a valuable source of information for both the practical engineer and the theoretical worker in that field.

LAWRENCE B. FEAGIN,[35] M. AM. SOC. C. E. (by letter).—A wide range of thought and of methods of considering the problem of the reaction of piles to lateral loads is reflected in the discussions of the paper. The interest and consideration accorded it are gratifying and appreciated. The problem has been approached both from the standpoint of mathematical analysis and from that of experience.

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[34] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1292, Fig. 19.

[35] Senior Engr., U. S. Engr. Office, St. Louis, Mo.

Mr. Cummings and Mr. Chang have both contributed ingenious mathematical analyses. They have sought to derive equations whereby the following may be determined: (*a*) The curve of deflection of the central axis of a pile subjected to a lateral load; (*b*) the horizontal deflection of a pile subjected to a given lateral load; (*c*) the length of the pile from the top downward, which is moved, also referred to as the bent length; (*d*) the relationship of the portion of the load resisted by the rigidity of the pile itself to that resisted by the soil; and (*e*) the stresses in the pile.

The writer has studied, with interest, each of these discussions as well as the assumptions made in each, and believes that perhaps a brief comparison may be of assistance to those who may hereafter have occasion to refer to them.

Mr. Cummings has assumed that the equation of the central line of the deflected pile can be expressed by the power series, Equation (1), and assumes certain boundary conditions whereby the formula of the deflection curve of the bent pile is established in the form of his Equation (2).

Mr. Chang has applied the theory presented by Professor S. Timoshenko[36] for determining the deflection of a bar supported along its entire length by a continuous elastic foundation, and has arrived at a corresponding formula of the deflection curve of the pile (his Equation (36)).

It will be noted that Equation (36) is of a form which will produce a sinusoidal curve of diminishing amplitude. In general, this latter type of equation will give a curve which is more nearly in keeping with the actual movement of the pile.

After proceeding with his analysis far enough to determine an expression for the bent length of the pile, Mr. Chang, however, concludes that the pile is practically vertical and rigid at a depth of one wave length, thus indicating that Mr. Cummings was justified in assuming that for all practical purposes the deflection curve has a vertical tangent at depth, *L*.

In order, for comparison, that it may have the same unknowns as occur in Equation (36), by substituting from Equation (31), Equation (2) becomes:

$$y = \frac{3\,f\,x^2}{\left(\dfrac{216\,E\,I\,\omega}{U}\right)^{\frac{2}{5}}} - \frac{2\,f\,x^3}{\left(\dfrac{216\,E\,I\,\omega}{U}\right)^{\frac{3}{5}}} \dots\dots\dots\dots\dots (58)$$

Equations (36) and (58) both have two unknowns which must be determined by test; that is, f and E_s, the elastic modulus of the soil. Mr. Cummings assumes that the elastic modulus of the sand increases directly with the depth of the sand, whereas Mr. Chang assumes that the elastic modulus is a constant. The writer believes that the former assumption is more nearly correct, notwithstanding the possibility, as suggested by Mr. Chang, that the elastic modulus may be affected by the compaction of the soil in the driving of the piles.

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[36] "Strength of Materials", by S. Timoshenko, Pt. II, p. 402.

Mr. Cummings determined the relationship between the horizontal load, H, and the top deflection of the pile as expressed by his Equation (26). It is interesting to note that in this formula the first term, representing that part of the load carried due to rigidity of the pile, is much less in magnitude for large values of L, than the second term which represents the part carried by the soil. This indicates the important rôle played by the soil in resisting lateral loads. On the other hand, it will be observed that for small values of f, which it seems reasonable to assume will also be accompanied by correspondingly small values of the bent length, L, the first term will represent a large proportion of horizontal load, H. This indicates the importance of the structural rigidity of the top few feet of the pile if the top deflection, f, is to be kept small. It should be borne in mind, however, that the extent to which results from this equation can be relied on depends primarily upon the accuracy with which the bent length, L, and the elastic properties of the soil (represented by ω) are determined.

Mr. Chang has developed a corresponding expression (see his Equation (55)) which may be written as:

$$f = \frac{H}{4\,E\,I\left(\frac{E_s}{4\,E\,I}\right)^{\frac{3}{4}}} = \frac{H}{\sqrt[4]{4\,E\,I}\;(E_s)^{\frac{3}{4}}} \quad \ldots\ldots\ldots\ldots(59)$$

In Equation (55) he has combined in a single term the resistance due to the rigidity of the pile and to the passive resistance of the soil. The reliability of this equation is also dependent upon the accuracy of determination of E_s which, in turn, is dependent on the bent length of the pile, as well as the elastic properties of the soil.

It will be noted that both Mr. Cummings and Mr. Chang have developed a single value of bent length of pile and each has used his respective single value throughout in computing deflections for the various loads as a basis for comparison with the test data (see Fig. 20 and Fig. 28). This probably accounts in large measure for the fact that the calculated results by both equations show greater deflections for the lower range of loads than the test results. Mr. Cummings uses a computed bent length of 10.2 ft, and Mr. Chang uses $L = 11.35$ ft. Attention is invited to the fact that the writer stated in the paper that at a load of approximately 30 tons per pile and with a top deflection, f, of $1\frac{3}{4}$ in., it was estimated that the bent length of the pile was about 10 ft. For small loads, it is undoubtedly less.

It is believed to be apparent that the depth to which movement occurs in a pile subjected to a lateral force at the top is dependent upon: (*a*) The physical characteristics of the pile, such as E and I; (*b*) the elastic modulus of the soil; and (*c*) the amount of the horizontal force and the resulting lateral movement at the top of the pile.

It is believed, therefore, that any mathematical expression which purports to give the depth of movement should, either directly or indirectly, contain all these factors. The formula developed by Mr. Cummings is Equation (31) and the corresponding formula developed by Mr. Chang is Equation (40). In each of these two expressions it will be noted that terms for both the physical characteristics of the pile and the elastic modulus of the soil are included, but there is no term for either the amount of the horizontal force, H, or the deflection, f. The results from these formulas are largely dependent on the values assumed for the "dimensionless coefficient, ω," and E_s. Based on these formulas the depth of movement would be the same with a deflection, f, of either 0.25 in. or 6 in. at the top, or with a lateral load, H, of 5 tons or 40 tons. It is obvious that this is not correct. The small-scale model tests made by Mr. Cummings (see Figs. 22 and 23) indicate quite clearly that the large rods had a greater depth of movement than the smaller rods. The top deflection of all three rods was about the same (1 in.), but a greater force, H, was required to move the $\frac{1}{4}$-in. rod than the $\frac{3}{32}$-in. rod, thereby causing a greater depth of movement in the more rigid rod.

Mr. Cummings was evidently aware of this and in his discussion states that "it should not be concluded from this that the bent length is independent of the deflection." He then justifies Equation (31) on the grounds that it is a combination of Equations (5) and (12) both of which are "based on the usual theory of elastic structures in which the displacements are required to be very small in comparison with the dimensions of the structure."

The writer does not desire that the foregoing comments on these two excellent discussions be construed as implying that he does not believe that the mathematical analyses are of practical value; they mark an important beginning in a field in which much remains to be done. He does desire, however, to point out the limitations of certain of the equations, in order that they may be used with proper caution. The equations for deflection check the particular tests results described in the paper quite well and, for loads within the range normally pertinent to design problems, give results which, subject to the additional considerations given subsequently, appear, in general, to be on the side of safety. Furthermore, a careful consideration of the derivations of the bent length provides an approach for the problem of determining the length of pile required to be driven to resist a given lateral load.

Both Mr. Meem and Mr. Thomson have emphasized very properly the fact that the conclusions given in the paper were distinctly limited to the soil conditions and piling arrangements under which the tests were made. The writer recalls reading at one time that Darwin is reputed to have said in effect that "the greatest danger to scientific investigation is generalization based on limited experience." This statement is especially applicable to investigation of problems relating to soil foundations. It is apparent that when movement occurs in the entire foundation, such as that resulting from the movement of the glacial drift described by Mr. Meem, or from quicksand or wet clay in the subsoil stratification in the cases cited by Mr. Thomson, the results of the tests described in the paper are clearly not applicable. Each foundation should be treated as a distinct problem in itself, and the extensiveness of field investigations and office study made dependent upon the importance of the stability of the foundation.

Mr. Niederhoff has ably presented the results of lateral loading tests conducted under the direction of Major Dwight F. Johns, Corps of Engineers,

U. S. Army, District Engineer at St. Paul, Minn., in the vicinity of Lock No. 3, near Red Wing, Minn. An ingenious method simulating a condition of fixation of the top of the pile was used. It is rather surprising that in the soft material such as that described by Mr. Niederhoff no greater lateral movement occurred. This may have been due, in part, to the fact that except for a 4-in. space adjacent to the test pile the ground was frozen to a depth of 6 in., thus confining the soil. Furthermore, the low temperature may have greatly retarded or prevented plastic flow of the foundation soil which might otherwise have occurred. With the exception of Tests Nos. 12 and 16 for which notations of 5-min. intervals are given, the length of time that each test load was sustained is not given. If the loads had been sustained for several days it is possible that greater deflections might have occurred.

Mr. Niederhoff has referred briefly to the lateral movement of 9 in. of a portion of the river wall of Lock No. 5A on the Mississippi River, at a time when the lateral force computed by usual methods was only about 3.5 tons per pile. This force was caused by an unbalanced hydrostatic load which, in the writer's opinion, may have produced a partly quickened condition in the foundation sand, giving rise to a much greater actual active load on the foundation piling, especially on the steel sheet-pile cut-off wall, and, at the same time, reducing the passive resistance of the foundation sand. This condition, combined with the presence of slippery clay and the vibration from pile-driving in the immediate vicinity, can easily account for the movement. The presence of wet clay alone might have caused it. Under these conditions it is quite probable that the lateral movement may have extended to the tips of the piles. It is interesting to note that notwithstanding a movement of about 9 in. this wall remained vertical and did not settle appreciably.

Quite properly, Mr. White calls attention to the "danger of extending observations or calculations made on single piles or on a small group, to a large group, such as is ordinarily done." In some soils under certain conditions such an extension may be entirely satisfactory, whereas for other soils under the same conditions, or for the same soils under other conditions, extrapolation of observations on single piles or a small group to a large group, might result in failure. Mr. White very ably discusses large movements of groups of piles serving as foundations to concrete walls, and the writer is in accord with his suggestion that, in many instances, more positive means of insuring lateral stability by use of struts or ties or other precautionary measures may be highly desirable. The possible effect on structures subjected to lateral loads of vibration such as that resulting from driving piles within a radius of 50 or 75 ft, should be carefully considered, especially when the structure rests on a foundation soil which itself may be affected by vibration.

Professor Krynine has added to the value of the paper by citing and briefly discussing a number of other tests on this interesting subject, including tests made by himself on pile models. He has also given an interesting discussion of the "Theory of the Zero Point." The writer agrees that there is a zero point, but believes that, for piles of the length described in the paper, instead of rising from some point, *Z* (see Fig. 29), to the ground surface with increasing horizontal load, the zero or pivot point (or point about which rotation occurs, as others refer to it), is nearest the surface with small lateral loads and moves downward with increasing loads. The location of the zero point, which serves to define the bent length of the pile, is regarded by the writer as being a variable which depends on the relationship between the physical properties of the pile (such as size, shape, elastic modulus, and length) and the elastic properties of the soil, and upon the magnitude of the lateral load.

Professor Krynine suggests that in Fig. 32(*c*) the line, NM, is longer than the line, NM'; and concludes that the pile is pulled out at Point N, which contradicts the assumption of ideal embedment at that point. The writer agrees that, in order that lateral movement may occur, either the pile must be slightly pulled up, or the top of the pile slightly lowered as it is bent, or the pile must be elongated. That there is a tendency even at low loads for the pile to be pulled up is believed to be clearly indicated by the extensometer readings described in the paper. These readings indicated that the fiber stress in the compression side was only about 60% of that in the tension side (see Fig. 13). Furthermore, in the final test on Monolith No. 6, with an extreme load of about 40 tons per pile, the two concrete piles nearest the jack were actually pulled up about 3 in., whereas the lateral movement was about 6 in. For all practical purposes, however, within the range of lateral loads normally considered in designs, it is believed that it may be assumed that the lower part of the pile is fixed.

The writer agrees heartily with Professor Krynine that it is very desirable that the scope of the tests described in the paper be extended, particularly to other types of soils and other piling arrangements, including piles battered in both directions.

In conclusion, the writer wishes again to call attention to the fact that the conclusions given in the paper were confined to the soil conditions and piling arrangements under which the tests were made.

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SEDIMENTATION IN QUIESCENT AND TURBULENT BASINS

BY J. J. SLADE[1], JR., ESQ.

WITH DISCUSSION BY MESSRS. THOMAS R. CAMP, HARRY H. HATCH, HARRY H. MOSELEY, GEORGE J. SCHROEPFER, AND J. J. SLADE, JR.

SYNOPSIS

In a paper which has become a classic[2] the late Allen Hazen, M. Am. Soc. C. E., stated the problem of sedimentation with characteristic clearness. For sediment consisting of particles of constant hydraulic value in a completely turbulent basin (the term will be defined subsequently) Hazen's theory is complete; but actual sediment is usually far from uniform, and, therefore, Hazen's formulas are not always applicable.

The purpose of this paper is to continue the analysis and to develop a workable theory which takes into account the variation in hydraulic value of the consituent elements of actual sediment as well as the variation in the degree of turbulence of the basin. The procedure followed herein is such that the formulas obtained may be generalized to any desired degree; that is, only practicability need limit the number of constants entering into them and the consequent flexibility of the curves representing the processes.

The constants required will be found to fall into three distinct classes: (1) Those which represent the characteristics of the sediment; (2) those which represent the characteristics of the settling basin; and (3) those which depend on both the character of the sediment and the degree of agitation of the fluid. The separation of the constants into these three categories will, it is hoped, suggest experimental procedures for the study of settling-tank characteristics and be an aid in the understanding of this complex phenomenon.

Notation.—The symbols used in this paper are introduced as they occur and are presented, for convenience of reference, in the Appendix.

NOTE.—Published in December, 1935, *Proceedings*.

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[2] "On Sedimentation", *Transactions*, Am. Soc. C. E., Vol. LIII (1904), pp. 43–88.

THE QUIESCENT BASIN

It is necessary to investigate thoroughly the mechanics of sedimentation in the still basin before attempting to analyze the phenomenon in the turbulent tank. Assume, then, a tank of still water of constant depth, h, which holds in suspension a quantity, B, of solid matter, which has a constant hydraulic value, v. Assume also that at the beginning of the process the solid matter is uniformly distributed throughout the liquid and that there is a quantity, b, per unit of volume. The time required for a particle to fall from a height, y, to the bottom of the tank is:

$$t_y = \frac{y}{v} \quad \text{..............................} \quad (1)$$

and the time required for a particle to fall from the surface is:

$$t = \frac{h}{v} \quad \text{..............................} \quad (2)$$

Eliminating v between Equation (1) and Equation (2), the following relation is derived:

$$y = \frac{h t_y}{t} \quad \text{..............................} \quad (3)$$

The quantity of sediment that falls to the bottom in time, $a = t_y$, is $b\ y\ A$ (in which A is the horizontal area of the tank), and this is equal to $\frac{b\,A\,h\,a}{t}$, using the value obtained for y in Equation (3). Since $b\ A\ h = B$, however, the quantity of solid matter remaining in suspension at the end of time, a, is:

$$B_a = B\left(1 - \frac{a}{t}\right) \quad \text{..........................} \quad (4)$$

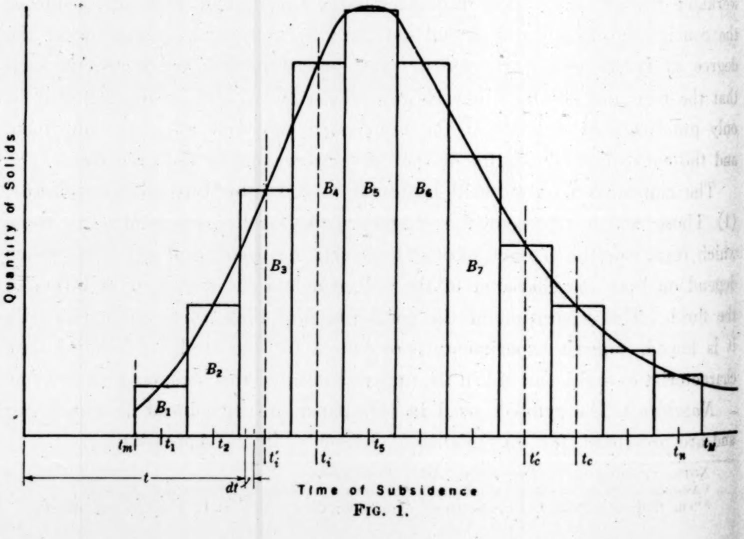

FIG. 1.

This is Hazen's first formula. It is of fundamental importance, all subsequent expressions being obtained from it by means of integrations and summations.

Assume now that, instead of a constant hydraulic value, the constituent particles have values, v, ranging continuously from a minimum, v_M, to a maximum, v_m, and let t be the time it takes the particles of hydraulic value, v, to fall from the surface to the bottom of the tank; t will then range from a minimum, t_m, to a maximum, t_M, since $t = \frac{h}{v}$. In general, it will not be known what proportion of the suspended solid matter has a given hydraulic value; but this unknown distribution function of the times of subsidence may be designated as $\phi(t)$; thus, $\phi(t)\ dt$ is the quantity of suspended matter having times of subsidence between t and $t + dt$; or, what amounts to the same thing, having hydraulic values lying between v and $v + dv$ (see Fig. 1). Before attempting to determine this function it is well to note some of its formal properties.

If it is required to find the quantity of sediment, dB_a, remaining in suspension at the end of time, dt, the hydraulic values of which lie between v and $v + dv$, Equation (4) and the definition of the distribution function yield:

$$dB_a = \phi(t)\left(1 - \frac{a}{t}\right)dt \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The total quantity remaining at the end of time, a, will be the integral of this expression; that is:

$$B_a = \int_{p(a)}^{t_M} \phi\,(t)\left(1 - \frac{a}{t}\right)dt \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The integration, being with respect to the times of subsidence, must range from t_m to t_M. The upper limit of this integral is certainly t_M. At first, the lower limit must be t_m, because particles of all hydraulic values are falling; but after a time, $a = t_m$, those particles of greatest hydraulic value will all have fallen. In general, at the end of time, $a = t_1$, say, all particles of hydraulic values greater than v_1 will have fallen to the bottom of the tank. Consequently, after a time of settling, $a = t_m$, the lower limit of the integral must be a.

Referring to Fig. 2, let $p(a)$ equal t_m for all values of a less than t_m; and a for all values of a between t_m and t_M. Then Equation (6) as it stands will give the quantity of suspended solids remaining at the end of time, a, for all

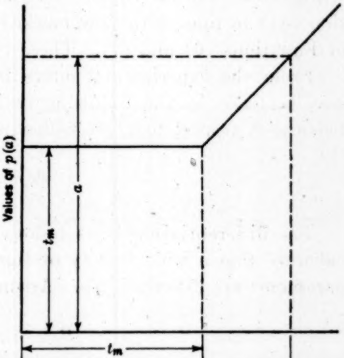

FIG. 2.

values of a from 0 to t_M, when all the solids will have settled. This formula applies, of course, to the settleable solids.

What is considered settleable and colloidal solids is probably an arbitrary, although convenient, classification. In practice, some upper limit for the time of subsidence, t_M, might be set, particles having a greater time of subsidence than this being classed with the colloids. If the quantity of this colloidal and near-colloidal material is B_c, Equation (6) becomes:

$$B_a = \int_{p(a)}^{t_M} \phi\ (t) \left(1 - \frac{a}{t}\right) dt + B_c \ldots\ldots\ldots\ldots\ldots (7)$$

From the definition of the distribution function it is also to be noted that:

$$\int_{t_m}^{t_M} \phi(t)\, dt = B - B_c = B_s \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Equation (8) merely states that the sum of all particles of all hydraulic values from v_m to v_M at the beginning of the process (that is, when $a = 0$) is equal to the quantity of settleable material.

THE DISTRIBUTION FUNCTION

In order to make use of Equations (6) and (7) as they stand it would be necessary to know the form of the function, $\phi(t)$, for the sediment in question, and, by experiment, this would be difficult. The direct procedure would consist in sorting the settleable solids according to size and applying Stokes' law (which gives the relation between size of particles and their hydraulic values) to the various classes found. The quantity in each class, plotted against the time of sedimentation thus found, would give $\phi(t)$.

It will now be shown, however, that $\phi(t)$ may be determined easily by using its formal properties, together with the experimental curve for B_a. This experimental sedimentation curve is readily determined; in fact, its determination constitutes a large part of the experimental work that has been done in the past on sedimentation investigations. Its shape varies from a straight line for the case of constant hydraulic value (represented by Equation (4)) to lines with very marked curvature given by formulas of the types of Equations (6) and (7). These types are sketched in Fig. 3.

From the experimental curve for B_a a graphical determination of $\phi(t)$ may be made by the following reasoning: If Equation (7) is differentiated twice with respect to a, the following relation is obtained,

$$\phi(a) = a \frac{d^2B_a}{da^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The differentiation is as follows: It is shown in treatises on the integral calculus[3] that a wide variety of functions defined by definite integrals with a parameter are differentiable. Assume that,

$$B_a = \int_{g(a)}^{f(a)} F(a, t)\, dt \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

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[3] See, for instance, Woods, "Advanced Calculus", Ginn & Co., 1926, pp. 141–143; also, Granville, "Differential and Integral Calculus", Ginn & Co., 1911, pp. 22–23.

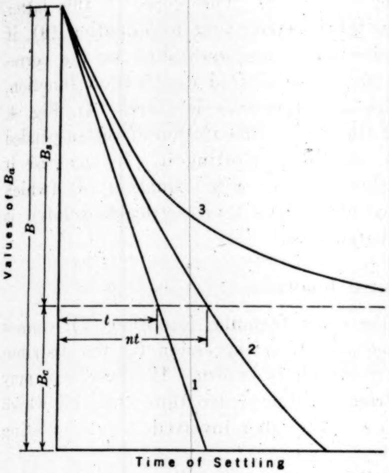

FIG. 3.

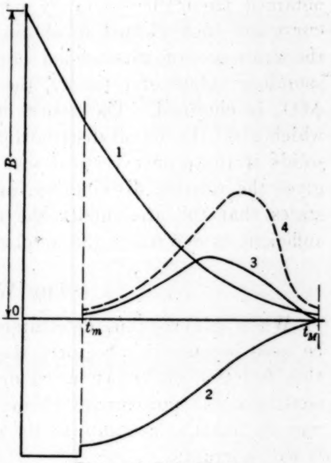

FIG. 4.

in which the limits are functions of the parameter, a, then the formula for differentiation is:

$$\frac{dBa}{da} = \int_{g(a)}^{f(a)} \frac{\partial F(a, t)}{\partial a}\, dt + F(a, f(a)) \frac{df(a)}{da} + F(a, g(a)) \frac{dg(a)}{da} \quad ..(11)$$

Applying Equation (11) to Equation (6) once:

$$\frac{dB_a}{da} = -\int_{p(a)}^{t_M} \frac{\phi(t)}{t}\, dt - \phi(a) \left[1 - \frac{a}{p(a)}\right] \frac{dp(a)}{da} \quad \ldots\ldots(12)$$

in which the middle term has disappeared because the derivative of the upper limit is zero. Now, the last term also vanishes because $\frac{dp(a)}{da} = 0$ for values of a less than t_m, and $\frac{a}{p(a)} = 1$ for values of a greater than t_m (see Fig. 2).

Applying Equation (11) to Equation (12):

$$\frac{d^2B_a}{da^2} = \frac{\phi[p(a)]}{p(a)} \frac{dp(a)}{da} = \frac{\phi(a)}{a} \quad \ldots\ldots\ldots\ldots\ldots(13)$$

which is Equation (9).

It is to be noted first that $\phi(a)$ is exactly the same thing as $\phi(t)$, because a and t assume the same values in the range, t_m to t_M. Since a derivative curve is the graph of the slopes of the original curve, Equation (9) indicates how to find $\phi(t)$ from the graph of B_a; that is, from the experimental sedimentation curve.

In Fig. 4, Curve (1) shows the quantity of solids remaining in suspension plotted against the time, as determined experimentally. Curve (2) has been obtained from Curve (1) by plotting its slopes. The slopes of this latter curve are then plotted to obtain the third. According to Equation (9), if the ordinates of this second derivative curve are multiplied by the corresponding values of t (or a), the graph of the desired distribution function, $\phi(t)$, is obtained. The result of this last operation is Curve (4), Fig. 4, which gives the actual distribution of times of sedimentation of the suspended solids if the proper vertical scale is selected in plotting it. In any case it gives the relative distribution, and this, together with Equation (8) (which states that the area under the curve equals the total settleable solids), is sufficient to determine the absolute distribution.

THE WORKING FORMULAS

When $\phi(t)$ is thus determined, the exact formula, Equation (7), cannot be used because this requires that the analytical expression for the distribution function be known when only its graph is known. However, one may construct an approximate formula from the knowledge thus obtained which may be made as accurate as the data and the labor involved in manipulating it will warrant.

To obtain this approximate equation break up the distribution function into a convenient number of parts, say n, as shown in Fig. 1, and take the average hydraulic value of each part. Equation (7) then becomes:

$$B_a = \sum_{r=1}^{n} B_r\left(1 - \frac{a}{t_r}\right) + B_c \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which B_r is the quantity of sediment with an average hydraulic value, v_r; t_r is the corresponding time of subsidence.

Attention is called to the fact that when $a = t_1$ all the sediment with a hydraulic value of v_1 has fallen to the bottom and that, therefore, in Equation (14) the first term appears only for values of a less than t_1. As a increases past the value of each t_r, each term of Equation (14) drops out, successively. This progressive dropping of terms corresponds to the variable lower limit in the exact Equation (7). Obviously,

$$\sum_{r=1}^{n} B_r = B_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

This is merely the approximate form of Equation (8).

DISCONTINUOUS DISTRIBUTIONS

It may happen that the sediment does not have a continuous distribution of hydraulic values, but that there are n kinds of particles of quantities, B_1, B_2, ..., B_n, with corresponding hydraulic values, v_1, v_2, ..., v_n. The experimental curve will then take the form of the broken line, Curve (1) of Fig. 5. In this case Equation (14) is the exact form, and the distribution function is the set of B-values. To obtain these quantities first differentiate B_a given by Equation (14) with respect to a. Remembering the progressive dropping of terms:

For $a < t_1$:

$$D_1 = -\left(\frac{B_1}{t_1} + \frac{B_2}{t_2} \ldots + \frac{B_n}{t_n}\right). \quad (16a)$$

for $t_1 < a < t_2$,

$$D_2 = -\left(\frac{B_2}{t_2} \ldots + \frac{B_n}{t_n}\right) \ldots . \quad (16b)$$

and for $t < a < t_n$:

$$D_n = -\frac{B_n}{t_n} \quad \ldots\ldots\ldots\ldots\ldots \quad (16c)$$

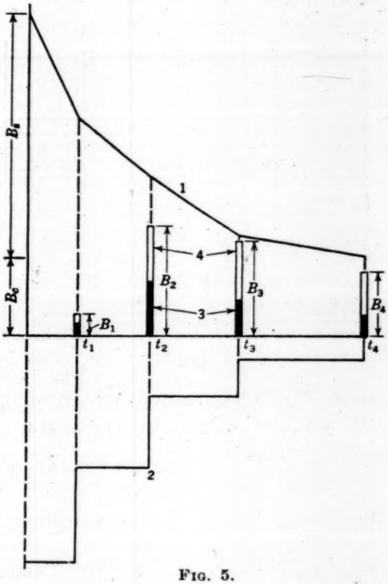

FIG. 5.

The step function (Curve (2), Fig. 5) is the result of plotting these quantities; that is, it is the graph of the slopes of the broken line, Curve (1), Fig. 5. The differences of every two successive values of D are expressed as:

$$D_2 - D_1 = \frac{B_1}{t_1} \quad \ldots . \quad (17a)$$

$$D_3 - D_2 = \frac{B_2}{t_2}, \text{ etc.} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17b)$$

or, generally,

$$B_r = t_r \, (D_{r+1} - D_r) \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

in which $r = 1, 2, \ldots, n$.

Equation (18) corresponds exactly to Equation (9), so that, although the step function given by Equation (16) cannot be properly said to possess a derivative other than zero, still the differences of the ordinates at the points of discontinuity are equal to the quantities, B, when multiplied by the corresponding values of t. These differences are shown by Ordinates 3 of Fig. 5, while Ordinates 4, obtained from Ordinates 3 by multiplying by the corresponding abscissas, represent the discontinuous distribution function; that is, they represent the set of Q-values.

STRATIFICATION OF SEDIMENT

It may be well to note here that quiescent sedimentation is characterized by the stratification that will occur in the upper regions of the basin. At the end of time, a, there will be a clear layer (except for fine colloids) of depth, $v_M \, a$, as shown in Fig. 6. From that level down to a depth, $v_m \, a$, the suspended material will be found in increasing density (in layers if the distribution of hydraulic values is discontinuous, and gradual if the distribution is continuous). The vertical distribution of the density of suspended matter in the liquid will be proportional to B_a in Equation (7) or Equation (14), as the case may be, because at a depth, z, below the surface there will be found only material with a hydraulic value less than $\frac{z}{a}$. At a depth greater than $v_m a$, the density will be what it was throughout the basin originally.

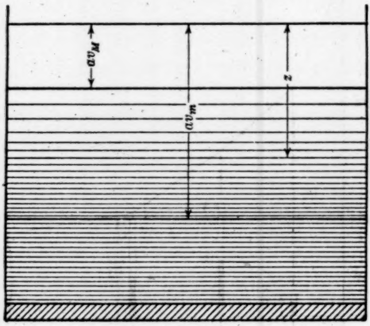

FIG. 6.

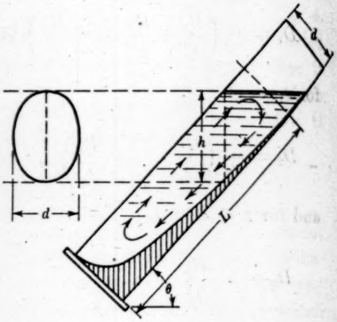

FIG. 7.

THE TURBULENT TANK

The foregoing analysis has been developed for the purpose of determining the hydraulic properties of the solid material suspended in the liquid of the settling tank, which is seldom still. The important problem of sedimentation is to determine the behavior of the suspended matter when a certain degree of turbulence is present in the tank.

Turbulence is defined herein as the average degree of agitation or circulatory motion of the water in the tank. It is conceivable that this turbulence might be defined as some function of the average velocity but, for the purpose of the following analysis, this correlation with velocity need not be made. Just as in the preceding development it was not necessary to determine the masses or sizes of the constituent particles, so now the turbulence in the tank will be considered only as it affects the hydraulic behavior of the suspended material.

As treated in this paper, therefore, turbulence is quite a relative term. A certain amount of circulation in a tank which would not affect, appreciably, the time required for grains of sand to subside, for instance, might be sufficient to pick silt from the bottom and keep it in suspension as long as the circulation persisted. From the writer's point of view this same circulation would present two distinct degrees of turbulence, in this example: One with respect to the sand; and the other with respect to the silt. The degree of turbulence is a difficult concept, but it seems reasonable to make the following simplifications (an extension of this classification will no doubt be made necessary by the results of future experiments).

Assume, first, the suspended matter in a basin of water to be uniform and of constant hydraulic value, v. If a slight circulatory motion be imparted to the water, the first appreciable effect which this agitation will have will be to lengthen the time of subsidence of the material from t to kt, say. With respect to this material the motion at this stage is termed "incomplete turbulence", the only appreciable effect of which is to lengthen the time of subsidence. Except for this increase, sedimentation in a tank in which the water is only slightly turbulent will be identical with quiescent sedimentation.

Assume, next, that the agitation is gradually increased. A state of turbulence will eventually arise in which the phenomenon of sedimentation will be radically different from that of the preceding state. The material that remains in suspension will be mixed thoroughly so that its density is uniform throughout the basin, and that which settles will remain at the bottom. This is the state of turbulence required in Hazen's theory. Herein it will be termed "complete turbulence".

Finally, if the agitation increases, a state of turbulence will be reached in which the material will not settle. The solids that may be at the bottom will be picked up and held in suspension, thoroughly mixed. This condition will be termed "critical turbulence". In all likelihood, the transition from one state of turbulence to another will not be sharply defined, and careful experiments may show transition stages that need to be taken into account. In what follows, however, only these three conditions will be considered.

Thus far, turbulence has been defined for material of constant hydraulic value, and, consequently, for variable circulation of the liquid. If, as is the usual case, a material of variable hydraulic values and fairly uniform circulation of the water is encountered, the foregoing notions may be applied in the following manner: Assume the material to have hydraulic values ranging from v_m to v_M, and assume the water to have a given uniform and constant degree of agitation; then there will be a value, v_i, such that for all particles with hydraulic values greater than v_i the turbulence will be incomplete. There will also be a value, v_c, such that the turbulence will be complete for particles with hydraulic values between v_i and v_c. This same agitation will be critical for all particles with hydraulic values less than v_c.

If the material in a tank is distributed with respect to its times of subsidence, as shown in Fig. 1, then, for a given agitation of the water, there will be two values of t — t_i and t_c, say—such that the turbulence is incomplete, complete, and critical in the intervals, t_m to t_i, t_i to t_c, and t_c to t_M, respectively. For some other degree of agitation these transition points will be t'_i and t'_c.

INCOMPLETE TURBULENCE

For the state of incomplete turbulence the formulas obtained for quiescent sedimentation will hold with slight modifications. The only alteration will occur in the time of sedimentation of the various particles, which will increase according to some law, $k(t)$, t being the time of settling in a quiescent basin. This law must be determined experimentally. Assuming it to be known, Equation (5) becomes:

$$dB_a = \phi\,(t)\left[1 - \frac{a}{k\,(t)}\right] dt \;\dots\dots\dots\dots\dots\dots (19)$$

Equation (19) gives the quantity of sediment remaining in suspension at the end of the time in which the hydraulic values (in a quiet tank) lie within the interval, t and $t + dt$. For the total quantity of sediment remaining at the end of this time, instead of Equation (6) the formula is,

$$B_a = \int_{p(a)}^{t_M} \phi\,(t)\left[1 - \frac{a}{k\,(t)}\right] dt \dots\dots\dots\dots\dots\dots (20)$$

If instead of the function, $k(t)$, a set of factors, k_1, k_2, ..., k_n, is determined which, for a given degree of agitation, gives the times of subsidence, $k_1\,t_1$, $k_2\,t_2$, ..., $k_n\,t_n$, of particles for which the times of subsidence in the still tank are t_1, t_2, ..., t_n, respectively; then the approximate Equation (7) becomes:

$$B_a = \sum_{r=1}^{n} B_r\left(1 - \frac{a}{k_r\,t_r}\right) \dots\dots\dots\dots\dots\dots (21)$$

These factors, k_r, depend both on the hydraulic values of the particles and on the degree of turbulence in the tank.

Complete Turbulence

It is with the state of complete turbulence, as defined herein, that Hazen was chiefly concerned.[2] The characteristic feature of this state is that the turbulence is sufficient to keep the sediment uniformly distributed throughout the tank, but not sufficient to pick up that which has once fallen to the bottom.

To determine the quantity of sediment (assumed at present to have a constant hydraulic value, v) that remains in suspension at the end of time, a, it is proposed to follow Hazen's procedure in its essentials: The time, a, is divided into n equal parts, each interval, $\frac{a}{n}$, being so small that the water in the tank may be considered quiet during that time. At the end of the first interval, the quantity, B_1, of sediment remaining will be, by Equation (4),

$$B_{1a} = B\left(1 - \frac{a}{n\;t}\right) \dots\dots\dots\dots\dots\dots (22)$$

At the end of the second interval the quantity remaining will be,

$$B_{2a} = B_1\left(1 - \frac{a}{n\;t}\right) = B\left(1 - \frac{a}{n\;t}\right)^2 \dots\dots\dots\dots (23)$$

Repeating the process n times the quantity at the end of time, a, will be:

$$B_a = B\left(1 - \frac{a}{n\,t}\right)^n \quad \text{(24)}$$

This is, substantially, Hazen's second formula. This is the form in which it was presented and has since been retained, but it is awkward to apply. However, it attains a definite limit as the number, n, of divisions is made indefinitely great, which is, of course, the actual condition.

The base of natural logarithms is usually defined[3] as:

$$e = \underset{(m=0)}{\text{limit}}\,(1 + m)^{\frac{1}{m}} \quad \text{(25)}$$

If $m = -\frac{a}{n\,t}$, Equation (24) becomes:

$$B_a = B\,(1 + m)^{-\frac{a}{m\,t}} = B\left[(1 + m)^{\frac{1}{m}}\right]^{-\frac{a}{t}} \quad \text{(26)}$$

and, in the limit, n becomes infinite, m becomes 0, and the expression in brackets becomes e (Equation (25)). Therefore (see Fig. 3, Curve (3)):

$$B_a = B\,e^{-\frac{a}{t}} \quad \text{(27)}$$

If, as before, instead of material with constant hydraulic value, the water carries in suspension solids with times of subsidence distributed according to the function, $\phi(t)$, then the quantity of material remaining in suspension at the end of time, a, is:

$$B_a = \int_{t_m}^{t_M} \phi\,(t)\,e^{-\frac{a}{t}}\,dt \quad \text{(28)}$$

It is to be noted that, in Equation (28), unlike the case of Equation (6), the lower limit is t_m throughout the process, because, since the turbulence is complete, none of the material ever settles completely. This expression does not yield a simple relation connecting the sedimentation curve with the distribution function analogous to that of Equation (9). It is for this reason that it is necessary to determine the function, $\phi(t)$, from the sedimentation curve for the quiescent state. The distribution function is a property of the sediment only, of course, and so is quite independent of the state of turbulence.

Mathematically, Equation (28) presents no great difficulty. By a slight transformation it becomes what is known as an integral equation of the Laplace-Fourier type. Such integral equations have wide application in physics and engineering. There are many useful relations known that exist between the sedimentation curve, B_a, and the distribution function, $\phi(t)$, connected as in Equation (28); but the writer has failed to find one with a simple graphical interpretation such as that of Equation (9).

From this formula (Equation (28)), an approximate formula is derived:

$$B_a = \sum_{r=1}^{n} B_r\,e^{-\frac{a}{t_r}} \quad \text{(29)}$$

which is the exact form when the distribution is discontinuous.

CRITICAL TURBULENCE

Material for which the turbulence in a settling tank is critical does not settle, because that which reaches the bottom is picked up again. For turbulence of this type, $B_a = B_c$, in which B_c is the material with time of subsidence greater than t_c as previously defined.

THE ACTUAL BASIN

In any settling tank there will ordinarily be material with a wide range of hydraulic values, so that the normal circulation of the water in the basin will be turbulence of one or another of the types described herein for some part of the sediment. As has already been stated, a proportion of the sediment will have times of subsidence between t_m and t_i for which the turbulence will be incomplete, another portion with times of subsidence between t_i and t_c for which the turbulence will be complete and, finally, a part with times of subsidence greater than t_c for which the turbulence will be critical (see Fig. 1).

The constants, t_i and t_c, are characteristic of the tank. They measure the turbulence of the fluid in it. They are taken into account in the general formula for the settling basin by noticing that they become limits of integration. Combining all results, the general expression for the sediment remaining in suspension at the end of time, a, is:

$$B_a = \int_{p(a)}^{t_i} \phi(t) \left[1 - \frac{a}{k(t)} \right] dt + \int_{t_i}^{t_c} \phi(t)\, e^{-\frac{a}{t}} dt + B_c \ldots\ldots (30)$$

From Equation (30) the approximate expression is obtained, which will be exact for the case of discontinuous distribution of hydraulic values, by dividing the range, t_m to t_c, into n parts, of which the first, s, will be in the range from t_m to t_i. Summing over these ranges:

$$B_a = \sum_{r=1}^{s} B_r \left(1 - \frac{a}{k_r\, t_r} \right) + \sum_{r=s+1}^{n} B_r\, e^{-\frac{a}{t_r}} + B_c \ \ldots\ldots\ldots (31)$$

This general expression may be carried to any degree of refinement, but in engineering practice a very few terms should suffice. It exhibits at once the constants of the theory and their categories: The B-values which are characteristic of the sediment; the limits of summation (or integration), which separate the expression into three groups, corresponding to the three kinds of turbulence, and which are characteristic of the basin; and the k-values, which depend both on the basin and the sediment.

EXAMPLES

A. W. Dilling, Assoc. M. Am. Soc. C. E., and Langdon Pearse, M. Am. Soc. C. E., report[4] the results of certain experiments which may be used to illustrate the theory developed herein. Samples of sludge were settled in glass cylinders and the sedimentation curves recorded. The cylinders were held upright and also inclined at various angles. These experiments were intended to show the effect of inclination on the rate of settling. What actually happens is that the height, L, through which the sediment falls in the upright cylinder is reduced to the height, $h = d \sec \theta$, as is shown in Fig. 7, when the cylinder is inclined at an angle, θ. A circulatory motion is steadily induced in the liquid, probably due to the downward slipping of the sediment on the side of the glass.

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[4] Rept. on Industrial Wastes from the Stockyards and Packingtown in Chicago, by A. W. Dilling and Langdon Pearse, Vol. II, January, 1921 (The Sanitary Dist. of Chicago), pp. 140–141.

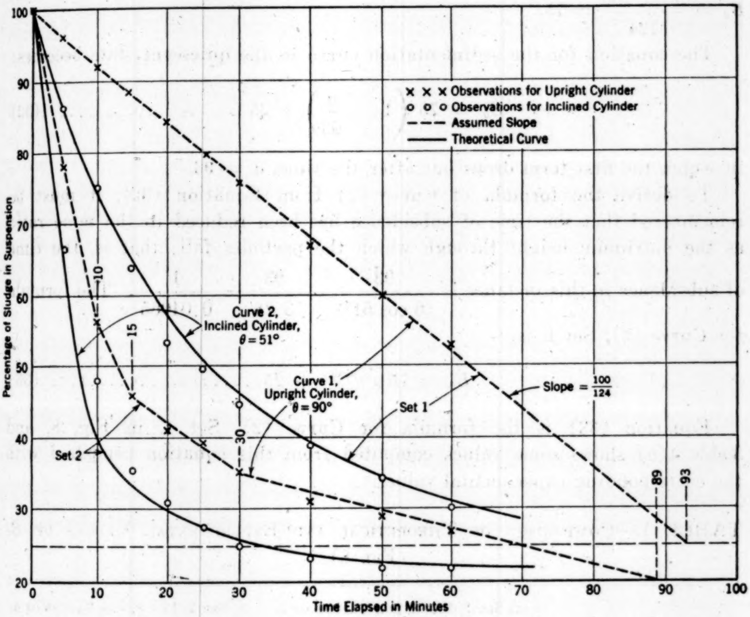

FIG. 8.

Consequently, there is: (1) Quiescent settling through the height, L, in the upright cylinder; and, then (2), turbulent settling through the height, $h = d \sec \theta$. Two sets of curves from the report by Messrs. Dilling and Pearse are shown in Fig. 8; in both sets, Curve (1) is for the upright cylinder ($\theta = 90°$), and Curve (2) is for the cylinder inclined at an angle of $\theta = 51$ degrees. Messrs. Dilling and Pearse do not give the dimensions of the cylinders. It will be assumed that $d = \frac{L}{6}$. From the description given of the circulation in the inclined cylinder, the turbulence may be assumed to be complete for the sludge of the experiments. If, from the curve for the upright cylinder, the distribution of hydraulic values of the sediment are determined, it should be possible to predict the behavior of the sediment in the inclined cylinder.

Curve (1), Set 1, for the upright cylinder, shows the sediment to have quite a uniform value, as far as the experiment has been run. The curves are not extended far enough to determine the quantity of colloidal and near-colloidal matter in the sediment, but all those for the inclined cylinders in this particular experiment seem to approach a horizontal line 25 units above the horizontal axis; for this reason the value of B_c was adopted. Since there is only one type of settleable material, $B_s = 100 - 25 = 75$. This value may also be determined by the use of Equation (18) (see Fig. 8, Set 1); thus,

$$B_s = \frac{93 \times 100}{124} = 75.$$

The equation for the sedimentation curve in the quiescent state becomes:

$$B_a = 75 \left(1 - \frac{a}{93}\right) + 25 \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which the first term drops out after the time, $a = 93$.

To derive the formula of Curve (2) from Equation (32), it must be remembered that the time of subsidence has been reduced in the same ratio as the maximum height through which the particles fall; that is, the time of subsidence in this instance is $\frac{93}{(6 \cos 51°)} = \frac{93}{3.78} = \frac{1}{0.04065}$. The formula for Curve (2), Set 1, is,

$$B_a = 75\, e^{-0.04065a} + 25 \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

Equation (33) is the formula for Curve (2), Set 1, in Fig. 8, and Table 1(*a*) shows some values computed from this equation compared with the corresponding experimental values.

TABLE 1.—COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF B_a (FIG. 8).

By:	(*a*) SET 1, FIG. 8, FOR VALUES OF *a*:					(*b*) SET 2, FIG. 8, FOR VALUES OF *a*:				
	0	10	20	40	60	0	10	20	40	60
Formula	100	75	58.3	39.8	31.5	100	39.6	29.9	23.7	21.5
Experiment	100	75	53	39	30	100	48	31	23	21.5

Curve (1), Set 2, Fig. 8, for the upright cylinder, represents sediment with a variety of hydraulic values. For reasons similar to the previous case, B_c is assumed as 20. As shown in Fig. 8, four slopes are taken, and their values are determined, by scaling, to be: $D_1 = 4.350$; $D_2 = 2.195$; $D_3 = 0.700$; and $D_4 = 0.224$, and the corresponding times of subsidence are (see Fig. 8, Set 2): $t_1 = 10$; $t_2 = 15$; $t_3 = 30$; and $t_4 = 89$.

From Equation (18): $B_1 = t_1 (D_1 - D_2) = 10 (4.350 - 2.195) = 21.55$; and, $B_2 = 22.50$; $B_3 = 14.30$; and $B_4 = 20$. If there were no errors in the process, these would be the actual values of B. The scaling has not been done very accurately, however, and the sum of these quantities (which, by

Equation (15), should be 80) is 78.35. Stating these quantities in round numbers: $B = 22, 23, 15,$ and 20, respectively. Consequently, the equation of Curve (2), Set 2, is:

$$B_a = 22\,e^{-\frac{3.78a}{10}} + 23\,e^{-\frac{3.78a}{15}} + 15\,e^{-\frac{3.78a}{30}} + 20\,e^{-\frac{3.78a}{89}} + 20 \ldots . \quad (34)$$

In this case, as in that of Equation (33), and for the same reason, the times of subsidence must be multiplied by the factor, $\frac{1}{3.78}$. This curve is plotted in Fig. 8, Set. 2. Table 1(*b*) shows values computed from Equation (34) compared with corresponding experimental values.

The writer is not acquainted with the results of other experiments which would illustrate these relations in greater detail. He has seen many curves of observations on actual tanks, but none with the corresponding curves for quiescent settling, and no experiments at all from which an estimate might be made of the transition points from one state of turbulence to another. As has been stated, the distribution function cannot be found from the curve for turbulent sedimentation, so that wherever the writer has applied the general formula, Equation (31), it has been a matter of mere curve fitting.

The next example is selected at random from data made available by H. N. Lendall, M. Am. Soc. C. E. The small circles in Fig. 9 represent observations on the rate at which untreated sewage settles.

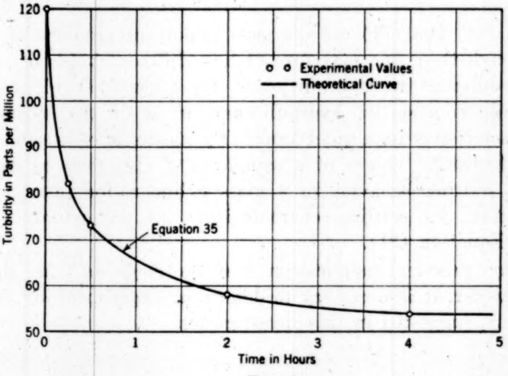

FIG. 9.

To fit a curve to these data by Equation (31) it was assumed that one term would be sufficient to take into account the portion of the sediment for which the turbulence in the basin was incomplete, and two terms for the portion for which the turbulence was complete; that is, Equation (31) became simply:

$$B_a = B_1\left(1 - \frac{a}{k_1\,t_1}\right) + B_2\,e^{-\frac{a}{t_2}} + B_3\,e^{-\frac{a}{t_3}} + B_c \ldots\ldots\ldots \quad (35)$$

To a certain extent the value one selects for the times of subsidence is a matter of choice. Fig. 1 shows the B-values determined from a certain distribution function for one choice of t-values, and Fig. 10 shows the B-values determined from the same distribution function for another choice of t-values. It was assumed that $k_1 t_1 = 15$ (since k_1 is not known, it is impossible to state the corresponding values, t_1); $t_2 = 30$; and $t_3 = 120$. By trial and error, or by any other method of curve fitting, the B-values are determined to be 25, 30, 13, and 52, respectively.

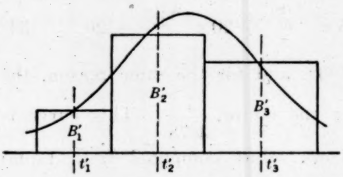

FIG. 10.

Equation (35) then, becomes,

$$B_a = 25\left(1 - \frac{a}{15}\right) + 30\,e^{-\frac{a}{30}} + 13\,e^{-\frac{a}{120}} + 52 \ldots\ldots\ldots\ldots (36)$$

in which it must be remembered that the first term drops out for values of a greater than 15.

In a way, of course, this curve-fitting procedure furnishes a means of determining the distribution function (the B-values), but since the distribution function is not what one usually wants, no importance should be attached to it.

CONCLUSIONS

Equations (18) and (31) are the most important products of this investigation. Through them one may predict how different kinds of sediment will settle under different tank conditions. Equation (18) gives the relation between the distribution of hydraulic properties of the sediment and its settling characteristics in a quiet tank. By means of this equation the distribution of hydraulic values of a sample of sludge may be determined by observing its settling behavior in a glass cylinder, for instance. Knowing this distribution, its settling characteristics in an actual tank may be predicted by Equation (31).

Before any practical application can be made of the theory herein developed, however, it is necessary to determine experimentally the constants, (t_t, t_c, k_r, etc.), suggested in this investigation.

APPENDIX

NOTATION

a = time during which sedimentation occurs; $p(a)$ = function of a;
b = quantity of solids, B, per unit volume;
c = a subscript denoting "colloidal", "complete", or "critical", as defined in each case;
d = inside diameter of a glass cylinder;

$\left.\begin{array}{r} f = \\ g = \end{array}\right\}$ functional symbols;

h = height; depth of a settling basin;

i = a subscript denoting "incomplete";

k = a coefficient; k_1, k_2, etc. = coefficients that produce the average times of subsidence, $k_1 t_1$, $k_2 t_2$, etc., under certain conditions of turbulence for particles with times of subsidence in a still basin equal to t_1, t_2, etc.;

m = a substitution factor $= -\dfrac{a}{nt}$; as a subscript denoting "minimum" when used with times of settling, and "maximum" when used with hydraulic values, $t_m = \dfrac{h}{v_m}$;

n = a number, such as a number of parts;

p = a functional symbol;

s = a number of parts, less than n; as a subscript, s denotes "settleable";

t = time of subsidence; t_m = minimum time corresponding to v_m; t_M = maximum time corresponding to v_M; t_1 and t_2 = times corresponding to v_1 and v_2, respectively; t_r = time corresponding to v_r; t_c = time required for complete or critical turbulence; $\phi(t)$ = distribution function of hydraulic values of suspended matter; as a subscript, t denotes "total";

v = a hydraulic value or the limiting velocity of a particle as it settles in a liquid; v_m = a maximum value: v_M = a minimum value; v_i = limit value for incomplete turbulence (turbulence is incomplete for $v_i < v$); v_c = limit value for complete or critical turbulence (turbulence is complete for $v_i < v < v_c$ and critical for $v < v_c$);

y = distance of a suspended particle above the floor of a settling basin;

z = distance from the water surface to a given depth;

A = horizontal area of a tank or stilling well;

B = quantity of solids; B_a = quantity remaining in suspension at the end of time, a; B_t = total quantity in a given basin; B_s = quantity of settleable matter in a basin; B = quantity in a colloidal state, including that part of the settleable solids with hydraulic values, v, so small that, for practical purposes, they may be considered as in colloidal suspension; B_1, B_2, etc., = quantities with hydraulic values, v_1, v_2, etc.; B_r = quantity with an average hydraulic value v_r;

D = a derivative of B_a in Equation (14);

F = a functional symbol;

L = length of a glass cylinder;

M = a subscript denoting "maximum", when used with t; "minimum" when used with v;

θ = inclination of a glass cylinder, with the horizontal;

ϕ = a function; $\phi(t)$ = distribution function of hydraulic values of suspended matter.

DISCUSSION

THOMAS R. CAMP,[5] M. AM. SOC. C. E. (by letter).—In his paper Professor Slade revives interest in the development of a theory which is much needed for the design of sedimentation basins. Considerable attention has been given to this subject for a great many years among those concerned with water and sewage clarification. Despite this interest, it appears that an effective attack upon the problem in a rational way has not yet been made. The author's attempt to use a "distribution function" to describe the suspension is a step in the right direction. It is to be regretted, however, that so much of the paper is premised upon the validity of the assumptions made by the late Allen Hazen, M. Am. Soc. C. E., in his paper[6], published in 1904, setting forth what has become known as "Hazen's theory of sedimentation". Although Hazen's paper contained something of value toward the development of a rational theory, it fell far short of presenting a complete theory of sedimentation. Moreover, Hazen's theory contains so many invalid assumptions that it is of little practical use to sanitary engineers in design.

In order to discuss the present paper, it will be necessary to revive the discussion of Hazen's paper and also to refer to the paper entitled "Cleaning Water by Settlement"[7] by the late James Alexander Seddon, M. Am. Soc. C. E., which influenced Hazen considerably in the development of his theory.

As pointed out by Professor Slade, the settling in still water of a suspension of discrete particles, each of which settles at a constant velocity, will result in a faster clearing near the top of the basin than below it. The concentration of suspended matter will be reduced continuously during the settling period, but it will be greater near the bottom than near the top. Seddon anticipated this stratification, but found in experimenting with the sedimentation basins of the St. Louis (Mo.) Water-Works that the density was practically the same at depths of 2, 4, 6, and 8 ft. He considered both vertical mixing and coagulation in order to explain this inconsistency, and he concluded that the uniform density distribution was due to vertical mixing and not to flocculation. No scientific observations were made to support this conclusion, and it was drawn in spite of the fact that an increase in wind velocity over the basin from zero to 11 miles per hr produced no measurable effect upon the settling.

Hazen accepted the observations of Seddon that the concentration of suspended matter did not vary appreciably with depth, and he also accepted Seddon's conclusion that the cause was vertical mixing. No experimental data were presented by Hazen to substantiate either Seddon's observations, or his conclusions, and coagulation was definitely excluded in Hazen's theory as a contributing cause of uniform concentration.

It is well known that practically all the suspensions dealt with by sanitary engineers are composed of particles which are subject to flocculation during settling. It is not well known, however, that flocculation takes place to so great an extent in most cases as to invalidate the assumption that each particle settles at a constant velocity. Even with clay suspensions, such as Seddon was dealing with, flocculation takes place. In making wet mechanical analyses of clays by the hydrometer method, it is necessary to treat the suspensions with a peptizing agent, such as sodium silicate, in order to avoid the effects of coagulation.

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[5] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[6] "On Sedimentation", *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 45.

[7] *Journal*, Assoc. of Eng. Societies, 1889, p. 477.

The writer has been studying sedimentation both theoretically and experimentally during the past several years. In some of the experiments, the settling of ferric hydrate floc was studied in glass tubes equipped with sampling cocks at various depths below the water surface. Some samples, drawn simultaneously during settling from different depths, have shown substantially the same iron concentration. Other simultaneous samples have indicated slightly higher concentrations near the bottom, and still others have indicated slightly higher concentrations near the top. Similar analyses made in terms of suspended solids upon the suspension in the coagulation basin of the Cambridge (Mass.) filter plant, where alum is used for coagulation, have indicated a very small increase in concentration with depth. In the glass-tube experiments protected from temperature changes residual mixing currents could be observed for a period of only about 20 min after filling, but substantially uniform concentration throughout the depth was observed in some of the samples withdrawn as late as 7 hr after the water became quiet. Coalescence of particles during settling could be seen at any time during the period. Larger particles were observed to overtake smaller ones, enmesh them, and thus increase their size and hydraulic value.

In his discussion of Hazen's paper, the late Galen W. Pearsons, M. Am. Soc. C. E., describes[8] some experiments made under his direction preparatory to the design of the Kansas City (Mo.) settling basins. Three glass tubes, 3 in. in diameter and 5, 10, and 15 ft high, respectively, were filled with Missouri River water which was allowed to settle. To quote Mr. Pearsons,

> "As it cleared gradually the writer was able to see particles descending near the bottom of the 15-ft tube; at times, something of the same could be discerned in the 10-ft tube, but with difficulty, and none at any time in the 5-ft tube.
>
> "These particles, by their uniform shape, explained their origin and action; they were pear-shaped, or rather like little tadpoles swimming head down, the tails tapering to invisibility; plainly some larger particle by its quicker descent had overtaken and joined smaller ones, and, increasing by constant addition, had at last become visible, their motion near the bottom being so rapid that, if it had been uniform, but a few minutes would have been required for the whole descent."

The writer does not conclude as a result of these observations that the effect of coagulation will be the same for all suspensions or the same throughout the settling period for any one suspension. There is every reason to suppose that coagulation may be completely absent with certain granular suspensions, and may exhibit its effects in varying degrees with other suspensions. In some cases the concentration may be greater near the bottom and in others it may be less, all due to the degree of flocculation without any consideration of vertical mixing.

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[8] *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 72.

It is not the writer's contention that no vertical mixing takes place in sedimentation basins. Eddy currents are obvious in most basins of the continuous-flow type at the influent end. These eddy currents do not occupy any appreciable portion of the volume of the basin, however. Eddy currents are also produced by obstructions in basins, such as baffles, and by sludge-removal equipment. Convection currents due to temperature changes are sometimes present. Eddy currents so uniform in type and distribution throughout the basin as to produce the uniform distribution of suspended matter throughout the depth which has been observed, certainly do not exist for any considerable time in any basin. By means of dyes, the writer has observed the time required to damp the eddies produced by small submerged jets with a velocity of 1 ft per sec, and has found few of the eddies to last more than about 15 min.

It should be clearly understood that Hazen assumed the existence of vertical mixing in order to account for the uniform vertical distribution of suspended particles. There was no other object for this assumption. If vertical mixing is the cause of uniform concentration, the clarification process will be less rapid than in the case of still water. If flocculation is the cause, clarification will be more rapid than with discrete particles in still water. The importance of this assumption is thus apparent. That flocculation is the cause is evidenced again by the much greater speeds with which clays which have not been peptized will settle out in wet, mechanical, soil analysis. If the concentration of suspended matter in a basin is the same from top to bottom, but decreases as the time of settling increases, it is difficult to escape the conclusion that the removal is a function of the detention period and is thus not independent of the basin depth.

Acceptance of the validity of Hazen's equations results in the conclusion that the percentage removal of sediment for a given discharge is a function of the surface area of the basin and independent of the depth. The introduction of the author's distribution function into Hazen's equations does not modify this conclusion. It is well to note that the author's equations apply to both fill and draw basins and continuous flow basins. As applied to continuous flow basins, the direction of flow is assumed to be horizontal and the velocity is assumed to be uniform over any cross-section of the basin. If the depth of the basin is reduced to $\frac{h}{2}$, for example, the settling period, a', will be equal to $\frac{a}{2}$; t' becomes $\frac{t}{2}$; and,

$$B'_a = B' e^{-\frac{a'}{t'}} = B' e^{-\frac{a}{t}} \quad \text{.....................(37)}$$

Since $B'_a = \frac{1}{2} B_a$ and $B' = \frac{1}{2} B$, the removal expressed as a ratio is,

$$\frac{B'_a}{B'} = \frac{B_a}{B} = e^{-\frac{a}{t}} \quad \text{.........................(38)}$$

The point is that Hazen's theory maintains that depth has no influence upon removal, a conclusion in support of which there is no experimental evidence from basins clarifying water and sewage.

The selection of the experiments of Dilling and Pearse[4] upon the consolidation of sewage sludges in glass cylinders, by Professor Slade to illustrate his theory was an unfortunate one. In the first place, flocculation was present in these tests to a marked degree and was described fully by the experimenters. In the second place, the portion of the liquid occupied by the sludge was observed by noting the position of the top of the sludge blanket. This percentage of sludge by volume as reported by the experimenters is a very different thing from the percentage of sludge in suspension, the interpretation given by the author. After a brief time, none of the sludge is in suspension. All of it is supported by the bottom, and each particle thereafter settles at a diminishing rate. Consolidation is a vastly different phenomenon from free sedimentation. In sedimentation, the resistance to settlement is due entirely to the viscosity of the fluid. In consolidation, this resistance is augmented by the supporting power of the particles below.

The writer has made use of a distribution function in a theory of sedimentation of discrete particles, flocculation being excluded. Although this theory cannot be used for most of the suspensions dealt with by sanitary engineers, nevertheless it may be of value toward the development of a complete theory including flocculation. The writer's distribution function has several advantages over that of Professor Slade, as will be apparent, and the theory has the advantage over that of Hazen in that mixing is specifically excluded. Both mixing and short-circuiting should be excluded as far as possible in an actual basin in the interest of efficiency, and a proper measure of the efficiency of a basin is the extent to which mixing and short-circuiting reduce the removal.

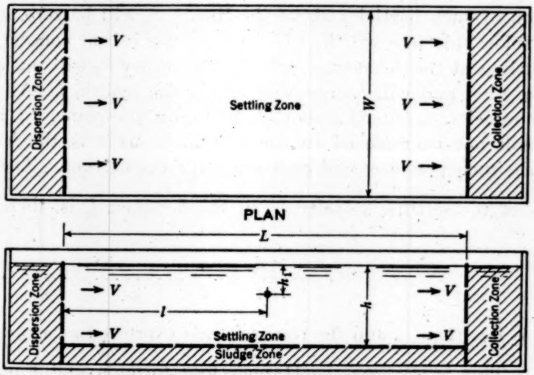

FIG. 11.—CONTINUOUS FLOW RECTANGULAR SETTLING BASIN.

The notation used by Hazen and by Professor Slade will be utilized as far as is convenient. The theory will be presented for a "perfect", continuous-flow, rectangular basin. In such a basin it will be assumed that all settling takes place in a "settling zone", as shown in Fig. 11. As conceived by Hazen, a particle will be assumed to be removed if it reaches the bottom of the basin, or the top of the sludge blanket within the settling zone. Flow in the settling zone is assumed to be uniformly horizontal and at the same velocity, V, and direction throughout. The concentration of all suspended particles is assumed to be the same at all points in the plane of entrance to the settling zone.

In view of the assumptions made, the paths of the particles will be straight lines and all particles settling at the same velocity will move in parallel lines. The velocity and direction of settling of each particle will be the vector sum of its own hydraulic value and the velocity of flow of the liquid. Moreover, the settling pattern will be the same in all longitudinal sections of the settling zone, and any longitudinal section may be considered as shown in Fig. 12, in order to study the removal.

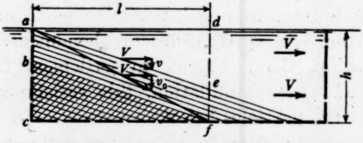

FIG. 12.—LONGITUDINAL SECTION, SHOWING PATHS OF PARTICLES IN SETTLING ZONE.

Consider any part of the length, l, of the settling zone. Particles which settle at the velocity, v, will move along paths parallel to the line, $a\,e$. The concentration of such particles above the line, $a\,e$, will be zero, and the concentration below this line will be the same as the initial concentration. All particles settling at the velocity, v, which enter below b, will settle within the shaded zone, $b\,c\,f$, and will be removed within the length, l. All particles of the settling velocity, v, entering above b, will cross the plane, $d\,f$, below e and, therefore, will not be removed in the length, l. If l is the length of the settling zone, such particles will pass out with the effluent. The removal, r, of particles of the settling velocity, v, in the distance, l, is, therefore, $\frac{b\,c}{h}$, or,

$$r = \frac{v\,a}{v\,t} = \frac{a}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which v is less than v_0 and the removal, r, is expressed as a ratio; r is equal to $1 - \frac{B_a}{B}$. This is the same as Hazen's first formula and Equation (4) of Professor Slade's paper. If v is equal to or greater than v_0, all particles will be removed and r is equal to unity. Now, since $a = \frac{A\,h}{Q}$ and $t = \frac{h}{v}$, in which A is the surface area of the basin; h, the depth; and Q, the discharge; the removal is, also,

$$r = \frac{A\,v}{Q} = \frac{v}{v_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Equation (40) is Hazen's formula[9] and the writer's fundamental equation for discrete particles. From this equation, it may be noted that the removal of particles which settle at any constant velocity, v, less than v_0, is directly proportional to the surface area for any given discharge and is independent of the depth; or that the removal of such particles is inversely proportional to the "overflow rate", v_0. It may be noted that v_0 is equal to $\frac{Q}{A}$. The overflow rate is equivalent to a velocity, although it is most frequently stated in terms of the discharge per unit of surface area. All particles that settle at velocities equal to or in excess of v_0 will be removed. Hence, the removal from a suspension of discrete particles of varying hydraulic value will be a function of the surface area and independent of the depth of the basin.

An analysis of the suspension in terms of the settling velocities (that is, a distribution function) is required in order to predict removal for suspensions composed of discrete particles of different hydraulic values. Professor Slade has chosen to state his distribution function in terms of the time of settling of each particle from top to bottom. Although the function thus stated does describe the suspension, it is not independent of the depth of the basin. The writer prefers to state the function in terms of the hydraulic values of the particles. This practice has another advantage in that the settling velocity of very fine particles approaches zero, and zero may be used as a limit for integration. The time of settling of such particles approaches infinity, and, as the author has discovered, involves difficulties in the selection of an upper limit for integration.

The writer prefers to state the distribution function in terms of a mass curve, such as Fig. 3 of the paper, rather than as a rate curve such as Fig. 1. The distribution function thus stated will be analogous to a sieve analysis curve for sand. The advantages of this practice are that quantities are shown as linear dimensions rather than as areas; such a distribution function is more easily determined experimentally; and the computations for the removal of suspended matter are simplified very much.

Using Hazen's practice, the concentration of all the sediment in the unsettled or raw water will be designated as unity. Let P be the ratio of the initial concentration of particles which have hydraulic values of v, or less, to the total initial concentration; and P_0, the value of P corresponding to v_0. A typical distribution function of this type is shown by the curve of Fig. 13.

Since all particles which settle faster than v_0 will be 100% removed in the time, a, and the length, l, the removal of such particles in terms of the initial suspension is $1 - P_0$. The removal of particles having any settling velocity, v, which is less than v_0 is $r = \frac{v}{v_0}$, and since the ratio of such particles to the total initial concentration is dP, the removal in terms of the total initial suspension is,

$$r\ dP = \frac{1}{v_0}\ v\ dP \qquad (41)$$

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[9] *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 55, Equation (7).

To obtain the total removal of particles which settle at velocities of less than v_o, it is necessary to integrate Equation (41) between the limits of zero and P_0. The total removal, R, of all particles during the time, a, is therefore expressed as follows:

$$R = 1 - P_0 + \frac{1}{v_0} \int_0^{P_0} v \, dP \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

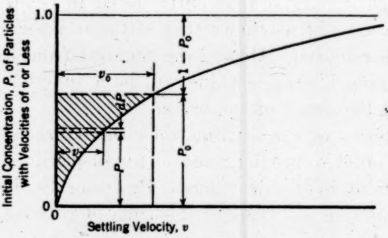

FIG. 13.—TYPICAL SETTLING VELOCITY ANALYSIS CURVE OF SUSPENSION.

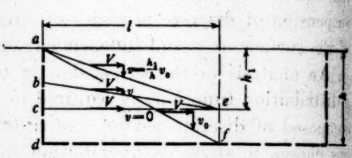

FIG. 14.—LONGITUDINAL SECTION OF SETTLING ZONE, SHOWING METHOD OF ESTIMATING CONCENTRATION AT A POINT.

If the equation of the analysis curve (that is, the distribution function) is known, the value of v in terms of P, or of dP in terms of dv, must be substituted in Equation (42), in order to solve the integral analytically. The expression under the integral sign, however, is the area on Fig. 13 shown shaded, and the last term of Equation (42) is the average vertical distance from the curve to the horizontal line at P_0. Hence, the removal may be measured graphically with sufficient accuracy by means of a planimeter, or with only a scale and slide-rule, without knowing the equation of the curve.

The concentration of suspended matter at any point in the settling zone, such as Point e on Fig. 14, may be estimated from the position of the point and the velocity-analysis curve of the original suspension.

Particles having any settling velocity, v, must cross the plane of entrance to the settling zone at b, in order to reach Point e. The concentration of such particles will be the same at Point e as at b, since they move along parallel paths, and the ratio to the total initial concentration, therefore, will be dP. Only particles that settle at velocities between zero and $\frac{h_1}{h} v_0$ can reach Point e. If the value of P on the velocity analysis curve which corresponds to the velocity, $\frac{h_1}{h} v_0$, is designated by P_1, the concentration of suspended matter, X, at any point, e, in terms of initial concentration will be,

$$X = \int_0^{P_1} dP = P_1 \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

The value of the integral is simply P_1, which may be read from the velocity analysis curve for any value of $\frac{h_1}{h} v_0$. The capital letter, X, has been used for the concentration at a point to distinguish from the average concentration in a vertical cross-section of the basin which Hazen denoted by the small letter, x. Now, since v_0 has one value at a given flow at any cross-section of the basin, it is obvious from Equation (43) that the density of suspended matter will be less near the top of the basin when settling discrete particles.

Equations (39) to (43) are also applicable to radial flow basins in settling discrete particles if the assumptions are made that flow is truly horizontal and radial at all points in the basin and that the velocity is the same at all points equi-distant from the center of the basin. The velocity, of course, will decrease, as the distance from the center increases, and all particles will settle along curved paths. Particles settling at the same velocity, however, will settle along parallel paths at any distance from the center of the tank.

If the frame of reference is transferred from the settling basin to the liquid moving within the settling zone, the velocity, V, becomes zero, and settling takes place as in still water. In other words, the settling which occurs in a vertical column of water as it moves through the basin is the same as that which occurs in an experimental tube, if all convection and eddy currents and wall effects are eliminated. Hence, experimental studies in tubes are an excellent means for determining the effects of coagulation in a "perfect" basin as defined previously. The difference between the observed concentration and that computed by means of Equation (43) is attributable to flocculation. It should be noted in this connection, that an experimental determination of the velocity analysis curve for discrete particles can be made readily; but no such curve exists for particles that flocculate since their velocities are increasing continuously.

In Fig. 11, the writer has shown a zone at the influent end of the basin for dispersion of the suspension uniformly over the cross-sectional area of the basin, and a similar zone at the effluent end for the collection of the clarified liquor uniformly over the cross-sectional area. It is very important in basin design to recognize the functions of these zones and to design them to perform their functions properly. This is an hydraulic problem which has not been effectively solved in the design of most existing basins. The penalties for inadequate solutions are vertical currents and short-circuiting, the latter being, undoubtedly, one of the major causes of basin inefficiency.

In the development of his theory, Hazen assumed the existence of vertical planes in the basin perpendicular to the direction of flow at which instantaneous decreases in concentration of sediment take place. To these planes, he gave the name, "baffles", and thereafter stated that real baffles such as vertical walls or partitions across the basin so as to divide it partially into two or more sections in series were equivalent in their function to his imaginary baffles. Such is not the case, and it is difficult to conceive of any physical device other than a fine screen or a filter placed in a vertical cross-section of the basin, which will produce a sudden decrease in concentration. All baffles across the direction of flow within the settling zone produce vertical currents which tend to carry particles upward, produce higher velocities, and produce eddy currents. They are objectionable, therefore, within the zone of settling.

Many reports have been published which indicate improvements in settling efficiency due to baffling. The increases in efficiency were doubtless observed, but the cause had nothing to do with Hazen's theory or the effectiveness of baffles generally in increasing basin efficiency. The cause was an inadvertent lessening by the baffles of the degree of short-circuiting previously present, and a corresponding increase in the flowing-through period. The cause was not that the basin was divided into two or more parts by baffling, but that more of the volume of the basin was made effective for settling. The increase in efficiency could have been made more effectively by improving the design of both influent and effluent zones with the definite aim of minimizing short-circuiting.

Harry H. Hatch,[10] M. Am. Soc. C. E. (by letter).—The author should be commended for his efforts to derive equations which will eliminate errors in the application of the Hazen formulas on sedimentation. Mr. Hazen applied the behavior of one particle size to an entire material in suspension. Obviously, this application can be justified only when the assumed value of t can satisfy the existing conditions.

In discussing Hazen's paper, "On Sedimentation", Robert Spurr Weston, M. Am. Soc. C. E.,[11] states:

> "Mr. Hazen bases a large number of propositions on one value of t. It would be interesting to substitute other values of t,—for instance, infinity. In the latter case, t would be a quantity vastly different from those mentioned in the paper."

Professor Slade has attempted to do this and more.

It would be interesting to know the difference, if any, between "time of settling", "time of sedimentation", and "time of subsidence", as used by the author. If they all mean the same, it would have been better for the sake of clearness to use only one of the expressions throughout the paper.

The plotting on logarithmic scale of observed data for Curve (2), Set 2, Fig. 8, did not result in a simple equation satisfying all the points. Had there been numerous observational data on the same material in suspension, an average condition could have been assumed and expressed easily. If it is absolutely essential to fit a curve that will pass through every observation point (or reasonably so), the graph on the logarithmic scale could be broken up into sections, and a simple equation for each section within the limits could be expressed. However, a logarithmic-probability scale plotting of the data of Curve (2), Set 2, Fig. 8, resulted in a flat and smooth curve passing practically through all the observation values. The experimental value of 48% sludge in suspension at the end of 10 min, falls well on this curve; for which point the author had computed a percentage of 39.6 (see Table 1(*b*)), or a variation of 17½% from the observed value.

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[10] Engr. in Chg., Cobble Mountain Reservoir, Springfield Water-Works, Springfield, Mass.

[11] *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 77.

With respect to Fig. 9, the author states:

"* * * the distribution function cannot be found from the curve for turbulent sedimentation, so that wherever the writer has applied the general formula, Equation (31), it has been a matter of mere curve fitting."

He then begins with the formidable Equation (31) and reduces it "simply" to Equation (35), which is no less formidable; and then assumes certain unknown values and "by trial and error, or by any other method of curve fitting", arrives at Equation (36), which is cumbersome and far from being simple for practical work. The author further states,

"In a way, of course, this curve-fitting procedure furnishes a means of determining the distribution function (the *B*-values), but since the distribution function is not what one usually wants, no importance should be attached to it."

The question, therefore, is why do all that work?

The following equation was readily obtained by the writer from logarithmic plotting of the observation data given in Fig. 9:

$$Y = \frac{66}{t^{0.156}} \qquad \text{.........................(44)}$$

in which Y = turbidity in parts per million, and t = time, in hours. Equation (44) will be near enough for all practical purposes. Its graph will vary slightly from that of Fig. 9 for values of t greater than 4 hr. There is no real justification, or no real objection to the path of the curve on Fig. 9 beyond the fourth hour, where it appears that turbidity does not decrease below 52 ppm, for practical purposes.

The writer really fails to discern any particular advantage of all the work done by the author to obtain Equation (36), when a simple equation answering the purpose could be obtained readily.

The settling time of a particle, practically speaking, depends on its velocity, and the velocity, in turn, depends upon the size of the particle. For the same sample material the sedimentation and gradation curves bear a definite relation. In fact, each depends on the other and one could be obtained from the other. No satisfactory general formula has been obtained for the distribution of particle sizes represented by a gradation curve, because of the complex nature of the material. It has been found more practical to make the necessary number of tests and plot their graphs. These test results are usually plotted on natural, semi-logarithmic, or logarithmic-probability scales. Often an error in observation can be detected in the plotted graph. Logarithmic-probability scale plottings result in a flatter curve and are better for the purpose.

It appears that, practically in all cases, without actual test runs, certain terms or factors of the equations proposed by the author can not be obtained. The author states the necessity of determining numerous constants experimentally before making any practical application of the theories developed in the paper. The characteristics of material in suspension are so complex that general application of any set of constants is questionable. To this uncertainty must be added the tedious work involved and probability of errors in solving complicated equations. The writer questions whether direct tests or simpler methods, similar to gradation curves, would not be more practical in every respect. However, the paper affords a good exercise in mathematics and Equation (27) is an improvement over Equation (24), Hazen's second formula.

HARRY H. MOSELEY,[12] ASSOC. M. AM. SOC. C. E. (by letter).—Much has been written about the design of sedimentation tanks, but the correct design still remains an open question. Professor Slade's paper presents a mathematical approach to this problem. He has attempted to establish a relation between the settling of similar solids in a quiescent basin and in a turbulent basin. He has developed a theory which is dependent upon a group of constants and suggests three sets of constants that effect the solution of his formulas: "(1) Those which represent the characteristics of the sediment; (2) those which represent the characteristics of the settling basin; and (3) those which depend on both the character of the sediment and the degree of agitation of the fluid."

Much experimentation and work has been done and many data have been collected on the constants pertaining to the characteristics of the sediment. The point, it seems, may be raised regarding the density of the solids in the liquid. Was it the author's intention to include this in the characteristics of the sediment and thus affect the first constant by a certain degree? There is a large store of data available for the second set of constants. Any settling tank is a potential source. The information desired probably should include: The type of tank inlet and tank outlet; the length and the width of tanks; the depth or length-depth ratio; the type of sediment-removal equipment; the type of operation of removal equipment; the baffling linear velocity and direction of flow of liquid (average and bottom if possible); the direction of flow; the characteristics of solids in the liquid; and the results obtained by the tank. It is appreciated that many of these data may seem to be superfluous, but they all have a bearing on the results obtained from a basin. In many respects, the third set of constants, may be absorbed by the first two sets. However, in the design of grit chambers, or like structures, this set of constants would have an effect on the solution of the problem.

Much of the information pertaining to settling tanks in sewage treatment works has been obtained by the use of the Imhoff cone. Under which classification would the author place these data? Would it be quiescent, the same as a vertical cylinder, incomplete turbulence, or complete turbulence? These available data in many cases will have to be used in the determination of the second set of constants.

At the end of his paragraph discussing "Stratification of Sediment" Professor Slade states that "at a depth greater than $v_m\, a$ [see Fig. 6], the density will be what it was throughout the basin originally." This statement is based on the fact that the sediment in the basin has variable hydraulic subsidence values ranging from v_m to v_M. With a sediment of constant hydraulic subsidence characteristics it is possible after time, a, to have that part of the basin below $a\,v_m$ or $a\,v_M$ ($a\,v_m = a\,v_M$ when the hydraulic subsidence characteristics are constant) of the same density as the original, throughout the basin. Theoretically, all particles of the same hydraulic subsidence characteristics are subsiding at the same rate; hence the absence of any flocculating action of the sediment. However, in a basin filled with sediment of variable hydraulic subsidence characteristics there is a flocculating action or the natural combination of suspended particles into larger aggregates. These larger aggregates also have hydraulic subsiding characteristics which are greater than the original particles and, consequently, settle more rapidly. Therefore, as soon as settling is started in this basin a portion of the liquid will have a density of suspended solids greater than that throughout the basin originally.

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[12] Asst. Engr., George B. Gascoigne, M. Am. Soc. C. E., Cons. San. Engr., Cleveland, Ohio.

This flocculating action, of course, increases as the density of the suspended solids increases, even though the hydraulic subsidence characteristics of the original suspended solids in each case are the same before the settling starts, because there is a greater possibility of the suspended solids combining with each other as they start settling, thereby making larger aggregates. For example, a settling tank having a detention period of 1.5 hr will remove roughly 60% of the suspended solids from a raw sewage of 400 ppm, whereas it will remove approximately 45% from one containing only 100 ppm. It is true that in the raw sewage of 400 ppm, some of the solids will have greater hydraulic subsidence characteristics than in the one of 100 ppm; yet it is very doubtful whether this increase in hydraulic subsidence characteristics of some of the suspended solids is great enough to account for an increase of one-third in the efficiency of the tank. This increase can be accounted for better by the flocculating action of the suspended solids in the tank. It seems that the statement made at the end of the paragraph on "Stratification of Sediment", previously referred to, is true mathematically, but due to the flocculating action of the solids, the actual approaches it only as a limit.

The two examples presented by the author based on his interpretation of the original data substantiate his theory in a marked degree. However, it will require agreement in many actual field tests to prove his theory fully. It is hoped that Professor Slade's paper will further stimulate research in the field of sedimentation.

GEORGE J. SCHROEPFER,[13] ASSOC. M. AM. SOC. C. E. (by letter).—An admirable attempt to set up, by means of equations and formulas, some of the factors affecting sedimentation of solids, is contained in this paper. In view of the importance of sedimentation in water purification and sewage treatment, and in the separation and classification of ores and chemicals, all investigations that contribute to the fund of knowledge on this subject are distinctly to be appreciated.

Although this discussion will be limited to the sedimentation of sewage solids, many of the principles of sewage sedimentation to which reference will be made, are equally applicable to the theory of sedimentation in general.

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[13] Asst. Chf. Engr., Minneapolis-St. Paul San. Dist., St. Paul, Minn.

The writer is inclined to favor the evaluation and determination of various engineering phenomena by means of formulas, but in the case of sedimentation, especially of sewage solids, the large number of influencing factors makes such a method of attack questionable when applied to an actual and practical situation.

Several times in the course of the paper the author makes reference to the application of his theory to sedimentation of sewage solids. Sewage is an extremely variable material, changing in quantity and quality, and in various characteristics which affect its settleability hourly, daily, and seasonally. The results of the application of a purely theoretical basis of analysis to a particular sewage under certain fixed or assumed conditions is valueless and even misleading when considered in terms of a varying material under varying external influences. A certain degree of control can be exercised over some of the variables, but in case of others it is necessary to accept the material as it exists, amenable to sedimentation, or otherwise.

In considering sewage sedimentation, the writer groups the various influencing factors under four headings, as affected by: (*a*) Characteristics of the liquid; (*b*) characteristics of the solids; (*c*) characteristics of the design; and, (*d*) miscellaneous effects.

(*a*).—The principal characteristics of the liquid which affect the settleability of solids are its specific gravity and viscosity. Of the two, it can be demonstrated that viscosity is the more important factor. With a liquid, such as water, the fluid characteristics, therefore, are closely affected by temperature. From a study of monthly average results at two large plants the writer has found a close correlation between the reduction accomplished and the sewage temperature. The reduction in suspended solids at the two plants closely approximates a 0.65% increase in reduction per degree Fahrenheit of temperature change; and in reduction of bio-chemical oxygen demand, 0.50% for every degree of temperature change. These results closely approximate those expected from purely theoretical considerations.

(*b*).—Under characteristics of the solids can be included such factors as size, shape, specific gravity, concentration of particles, natural flocculation, and artificial coagulation and coalescense. Sewage contains particles varying largely in size, shape, and specific gravity. In specific gravity alone particles vary from less then 1.0 to 2.65, or more. As an example of the importance of this factor as affecting a purely theoretical consideration, the hydraulic subsiding value of a particle 0.50 mm in size and having a specific gravity of 1.1, is 3.3 mm per sec; and 53.00 mm per sec for a particle of the same size having a specific gravity of 2.65.

Concentration of particles also plays an important part in the sedimentation of sewage solids, as evidenced by the fact that a study of the data collected on the effect of this factor indicates that, for a given detention period, a sewage containing 200 ppm of suspended solids will have its solids removal increased approximately 30% when compared with the sewage containing 50 ppm of suspended solids. Flocculation, coagulation, and coalescence exert a variable effect depending upon local conditions.

(*c*).—Under characteristics of the design can be included such factors as detention period, linear velocity of flow, depth and ratio of length and depth, inlet and outlet effects, shape of tank, baffling, and mechanism effects.

The first factor mentioned—namely, detention period—plays a very important part in the removal effected by a settling tank. The writer has collected data from more than forty settling-tank installations throughout the United States, which data, for those who desire to express the results of sedimentation data in the form of a equation, can be stated in the following formula:

$$R = C_1 - \frac{C_2}{t + C_3} \qquad \text{(45)}$$

in which R is the normal expectancy, in percentage removal of suspended solids; t is the detention period, in hours; and C_1, C_2, and C_3, are coefficients depending on the characteristics of the sewage and of the settling tanks under the particular conditions existing at that time. In view of the importance of the detention period provided, considerable thought should be given this factor in order to be assured that the desired period of detention is actually secured. Sewage treatment plants investigated indicate that the actual flowing-through time in the tanks may be as low as 10% of the theoretical detention period. This would indicate that, in some plants, a considerable expenditure of funds is, in effect, wasted by reason of inefficient design.

With respect to linear velocity of flow, a considerable variation in thought exists as to the desirable maximum velocities beyond which it is not advisable to go. Although this variation ranges from 4 to 59 mm per sec, the writer is inclined to believe that the linear velocity of flow for average flow conditions should preferably be not more than 2 ft per min (approximately 10 mm per sec), depending, however, on influencing conditions and requirements.

Inlet and outlet effects play a very important part in the length of tank actually effective for settling purposes and, in view of their importance, should be given considerable thought in design.

(*d*).—Under miscellaneous effects might be included such factors as currents caused by wind, eddies, and difference in temperature and biological activities. At one plant investigated the writer observed that with a wind velocity estimated at 25 miles per hr the velocity of the surface sewage in the 90-ft square tank was observed to be such as to cover the distance to the outlet in 2 min. The theoretical detention period in the tank was 1.5 hr. Methods effective in reducing this action consist of baffling, provision of free-boards, and the limitation of the size of tanks. When air and sewage temperatures are materially different, short-circuiting, mixing, and stratification may result.

Realizing the value of such investigations as the author reports, the writer has attempted to point out some of the factors which affect the evaluation of sedimentation data, but which, in the case of sewage solids subsidence, at least, are likely to make the methods of determination suggested by the author uncertain and possibly even misleading.

J. J. SLADE, JR.,[14] ESQ. (by letter).—In the analysis presented in this paper the writer was prompted by the realization that the problem of the settling of solids in suspension may be separated completely from consideration of the very complex relations that exist between the particles and the liquid. Homogeneity of material does not imply uniformity of settling, and it is this fact that led the writer to introduce a distribution function into the analysis. Since this distribution function is a function only of the hydraulic properties of the material, the homogeneity or heterogeneity of the material is irrelevant to the phenomenon of sedimentation considered only from the point of view of bulk that settles out in a given time.

That objections to the writer's development were made in the discussion on the grounds that coagulation takes place is, therefore, very surprising to the writer since it is precisely coagulation that his distribution function takes into account. It appears desirable, therefore, to explain this point in greater detail.

Consider, for instance, a clay suspension so homogeneous that all the particles may be taken as identical in size, shape, and density. These particles will not fall at a uniform rate through the liquid. The statistical-mechanical picture of subsidence will be somewhat as follows: Two particles falling freely will come close together and, because of the ionization potentials surrounding them, they will adhere. After that, they fall as a single particle with a different velocity. Farther down a third particle will adhere to these two and the three will fall together, with a still different velocity. These nuclei may grow to fairly large masses depending on the physico-chemical character and the concentration of the solids. These nuclei are not stable, however. As they go down a collision may disrupt them instead of adding more particles to the mass. This interchange is always going on, but in a large enough sample a statistical equilibrium is established, so that a certain percentage of the particles at any time will be falling singly, a certain percentage will be in pairs, etc. Each one of these groups is characterized by its individual hydraulic value, and it is this distribution of hydraulic values that is given by the writer's distribution function. Obviously, no matter how homogeneous or how complex the material may be, it will always be characterized by some distribution function of its hydraulic values.

The method given by the writer to determine the distribution function takes into account only the hydraulic behavior of the sub-groups into which the material arranges itself and, consequently, it leads to a statement only of the bulk that is removed in a given time; that is, it takes coagulation fully into account.

Professor Camp offers two severe criticisms: First, that it is unfortunate that the theory should be based on Mr. Hazen's work, because his formulas are known to fail; and, second, that in introducing the distribution function the basic assumptions on which the theory is established are completely violated.

These two objections may be answered as one. In the first place, the writer's development is scarcely a theory; it is more in the nature of a truism.

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In its more general form it may be stated thus: If the quantity of material which has hydraulic values between v and $v + dv$ is $\phi(t)\,dt$, and, if at the end of time, a, the quantity remaining in suspension is a fraction, $f(a, t)$ of the amount initially there, then for this small range in v the quantity remaining in suspension is $\phi(t)\,f(a, t)\,dt$, and the total quantity is $\int \phi(t)\,f(a, t)\,dt$. The integration extends, of course, over the complete range of values of t. Thus far, this is independent of Mr. Hazen's logic, and it is merely a truism stated in compact symbolic form. This statement is useful or useless, depending on whether or not $\phi(t)$ and $f(a, t)$ can be evaluated.

Consider, then, a particle (or globule of particles cohering as a single body) that is falling freely through the liquid. In this case the function, $f(a, t)$, takes the form of Equation (4). This is simply Hazen's statement of Stokes' law, which is known to be correct well within the errors of experiment. For material that has attained statistical equilibrium of coagulation, Equation (6) is true. It remains only to determine $\phi(t)$ and the writer has shown in detail how this may be done. Therefore this equation is a determinate, complete, and correct statement of sedimentation in a still liquid which takes fully into account, coagulation and every factor pertinent to the problem.

For the condition described as critical turbulence the form of $f(a, t)$ is also known. It is simply $f(a, t) = 1$. This is the condition in which turbulence is so great that the material does not settle out. From no turbulence to critical turbulence there is a steady transition of $f(a, t)$ from $\left(1 - \frac{a}{t}\right)$ to 1. This transition can be determined only through experiment. The writer has suggested the form, Equation (19), but this is really quite general because $k(t)$ must be determined experimentally. There is one particular condition of turbulence, which the writer has called complete, in which he has evaluated the function, $f(a, t)$. This evaluation is given in Equation (27) and in that case explicit use was made of Hazen's theory. Whether or not there does exist a condition of turbulence in which $f(a, t) = e^{-\frac{a}{t}}$, the writer is not qualified to state, but if under some condition of turbulence the suspended material is observed to have a fairly uniform vertical distribution, then this form is quite nearly correct. That this form is not a bad approximation to certain conditions of turbulence is shown by the results obtained by applying it to the experiments of Dilling and Pearse[4]. In view of the explanation that has been given of the distribution function it is seen that the choice of these experiments for illustration is not so unfortunate as Professor Camp considers it. However, except for the form of this particular evaluation, the writer's development is in no way dependent on Hazen's definition of turbulence.

It must be explicitly stated that the writer's development as presented herein holds only for the case in which statistical equilibrium of coagulation occurs in a time that is short compared to the period of retention. This may by no means be the case in practise. As Mr. Schroepfer points out, sewage solids change in character hourly, daily, and seasonally. Formally, this is quite easily taken into account by allowing the distribution function to be dependent both on *a* and *t*; practically, however, no advantage would result from this extension. The theory holds only where statistical equilibrium is obtained throughout the period of retention.

It is far from the writer's intention to offer an exercise in curve fitting in presenting this paper. He quite agrees with Mr. Hatch that if a short formula will do as well as a long one, the short formula is much the more desirable of the two.

In evaluating the constants of Equation (36) the writer desired merely to show that Equation (31) does represent the process accurately. Curve fitting after the process is complete is merely a waste of time. However, if the curve for the performance of a tank can be constructed accurately beforehand, then even a long computation is justified. For instance, if through experiments (such as will be indicated under Items (1) to (4) subsequently) it is possible to assign to a given tank a certain degree of turbulence for some given rate of discharge, and if, from a test-tube sample of the sludge, the distribution function is determined by the process represented in Figs. 4 and 5, the writer's contention is that Fig. 9 may be drawn before the tank is in operation; in other words, that it is possible to predict the performance of a tank for any rate of discharge and for any material that may be put through it.

If this is true, it is certainly possible to design a tank to operate in a given manner for a particular type of sludge, and to predict its operation for other types that may be put through it.

Mr. Moseley is quite right in stating that there already exists a vast amount of information regarding the operation of settling tanks. However, it is not the amount of information that is lacking, but the kind of experiments required to evaluate the constants presented in this paper.

Since the principal outcome of this investigation is the suggestion of a unified experimental procedure and standardization of terms, and since it is not likely that the writer will have the opportunity in the near future to undertake the experiments himself, he should like, in closing, to suggest an outline of what he thinks could be done to correlate this theory with practise:

(1) A standard set of materials should be selected for experimental purposes. Each material should be selected for the uniformity of its hydraulic behavior, and as many materials should be in this group as will give a fair range of hydraulic values.

(2) A standard cylinder should be adopted for these experiments. These cylinders should be large enough so that the walls will not appreciably modify the hydraulic behavior of the suspended solids and so that a sample will be able to attain the statistical equilibrium it reaches in a full-sized tank. These cylinders should be equipped with mechanical rotors. By means of these rotors it is possible to give turbulence precise values. For instance, the value, 1, may be assigned to the turbulence produced by a standard rotor in a standard cylinder of water when it rotates at the rate of 1 rpm, the value, 2, when it rotates at the rate of 2 rpm, etc.

(3) The hydraulic behavior of the standard materials of Item (1) may then be studied when subjected to various standard degrees of turbulence and, in this manner, the variations of the function, $f(a, t)$, may be determined for each.

(4) By the use of some suitable material of Item (1) a tank of standard design may be calibrated. Thus, for a given rate of discharge, the material will settle out in one or the other of the various ways determined in Item (3), so that, for that particular rate of discharge, the tank will have assigned to it one of the degrees of turbulence standardized in Item (2). Interpolations, of course, will be necessary.

When data such as Items (1) to (4) have been collected, it will be possible to make use of the theory presented in this paper. This is certainly a formidable set of experiments which requires a fine technique, but, in the writer's opinion, this suggestion represents only a small fraction of the work that has been done in the past and is now in progress in the field; and it has the advantage of a reasonable theory to unify the results. The much more difficult problem of taking into account slow time changes in the material may then be considered.

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WIND STRESSES IN REINFORCED CONCRETE ARCH BRIDGES

BY A. A. EREMIN[1], ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. LEON BLOG, FANG-YIN TSAI, PAUL ANDERSEN, LOUIS BLUME, AND A. A. EREMIN.

SYNOPSIS

Ordinarily, the effect of wind forces is slighted in the design of arch bridges despite the fact that wind stresses in these structures are often considerable. In the Ammer Arch Bridge,[2] near Echelsbach, Germany, with a span of 426 ft, the stresses due to wind constitute about 55% of the total, including live load, dead load, and temperature changes. On the other hand, the excessively heavy wind-bracing sometimes built into reinforced concrete arch bridges results in an uneconomic and unsightly design.

INTRODUCTION

The usual assumptions made in designing reinforced concrete rigid frames have been adopted in this paper. For example, the concrete was assumed to be elastic within the range of working stresses, from which it follows that strain is proportional to stress. The effect of plastic flow on the distribution of stress was not considered.

Notation.—The notation of this paper is defined where first introduced and summarized for reference, in the Appendix. A definition of the torsion factor, F, has been stated by J. Charles Rathbun, M. Am. Soc. C. E.[3] in the following form:

$$F = \frac{b^3 t^3}{3.58 (b^2 + t^2)} \qquad \ldots\ldots (1)$$

in which b is the width, and t is the thickness of the arch rib measured radially.

NOTE—Published in December, 1935, *Proceedings*.

[1] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[2] *Beton und Eisen*, March, 1930.

[3] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 38, Equation (28).

ARCH RIB WITHOUT BRACING

Obviously, the single arch rib, fixed at the abutments and subjected to lateral forces, is statically indeterminate and requires six equations for its solution. In the case of a symmetrical structure, however, the computation of wind stresses is simplified, and, furthermore, the effects of shear and direct stresses in the rib may be neglected without introducing serious error.

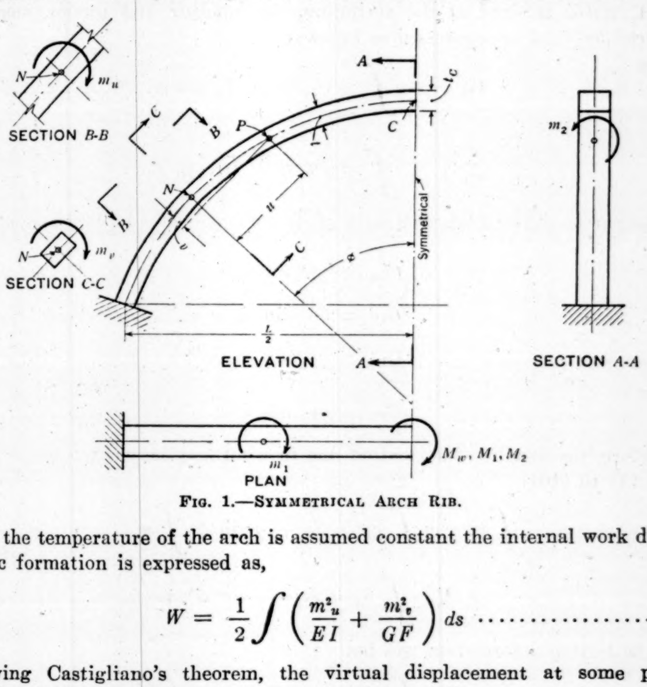

FIG. 1.—SYMMETRICAL ARCH RIB.

If the temperature of the arch is assumed constant the internal work due to elastic formation is expressed as,

$$W = \frac{1}{2}\int\left(\frac{m^2_u}{EI} + \frac{m^2_v}{GF}\right) ds \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Applying Castigliano's theorem, the virtual displacement at some point, N (Fig. 1), may be derived from Equation (2) as follows:

$$\theta = \int (m_u\, C_{1m} + \gamma\, m_v\, C_{1t})\, dw \ldots\ldots\ldots\ldots\ldots (3)$$

and,

$$\tau = \int (m_u\, C_{2m} + \gamma\, m_v\, C_{2t})\, dw \ldots\ldots\ldots\ldots\ldots (4)$$

in which $\frac{ds}{EI} = dw$ and C_{1m} and C_{1t} are, respectively, the partial differential coefficients representing the moment and torsion produced by a unit moment, $m_1 = 1$, applied at Point P, and C_{2m} and C_{2t} are, respectively the partial differential coefficients representing the moment and torsion produced by a unit moment, $m_2 = 1$ at Point P.

Professor E. Mörsch has shown[4] that, when a symmetrical arch rib is exposed to symmetrical lateral wind forces, such as an ideal uniform wind, the only bending moment that occurs at the crown is M_w about the vertical axis. It is obvious that the torsion and shear stresses at this point are equal to zero, as are the bending moments about the horizontal axis, since the direct stresses are assumed to be negligible. Consequently, for the structure in Fig. 1, which is fixed at the springing, the bending and torsion moments produced by wind are expressed as follows:

$$m_u = - \int w\ t\ u\ ds + M_w \cos\phi \ldots\ldots\ldots\ldots\ldots (5)$$

and,

$$m_v = + \int w\ t\ v\ ds - M_w \sin\phi \ldots\ldots\ldots\ldots\ldots (6)$$

The partial differential coefficients of the unit moments, $m_1 = 1$ and $m_2 = 1$, are:

$$C_{1m} = + \cos\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$C_{1t} = - \sin\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$C_{2m} = + \sin\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and,

$$C_{2t} = + \cos\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Therefore, the bending deformation due to wind may be written, from Equations (4) to (10):

$$\theta_w = - \int \left(\cos\phi \int w\ t\ u\ ds + \gamma \sin\phi \int w\ t\ v\ ds\right) dw$$
$$+ M_w \int (\cos^2\phi + \gamma \sin^2\phi)\ dw \ldots\ldots\ldots\ldots\ldots (11)$$

and the torsion deformation will be:

$$\tau_w = - \int \left(\sin\phi \int w\ t\ u\ ds - \gamma \cos\phi \int w\ t\ v\ ds\right) dw$$
$$+ M_w \int (1 - \gamma) \sin\phi \cos\phi\ dw \ldots\ldots\ldots\ldots\ldots (12)$$

The tangent at the crown does not move when a symmetrical arch rib is subjected to uniformly distributed lateral forces. Therefore, by equating the bending deformation at the crown to zero, M_w may be expressed:

$$M_w = \frac{\int \left(\cos\phi \int w\ t\ u\ ds + \gamma \sin\phi \int w\ t\ v\ ds\right) dw}{\int (\cos^2\phi + \gamma \sin^2\phi)\ dw} \ldots\ldots (13)$$

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[4] *Beton und Eisen*, 1923, p. 53.

In a similar manner the equations for bending and torsion deformations and the moment at the crown produced by the symmetrical moments, $m_1 = 1$, may be written:

$$\theta_1 = \int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw + M_1 \int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw \ldots (14)$$

$$\tau_1 = \int (1 - \gamma) \sin \phi \cos \phi \, dw + M_1 \int (1 - \gamma) \sin \phi \cos \phi \, dw . (15)$$

and,

$$M_1 = -\frac{\int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw}{\int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw} \ldots\ldots\ldots\ldots\ldots (16)$$

In a similar manner the equations for bending and torsion deformations, and moment at the crown produced by the symmetrical moments, $m_2 = 1$, are:

$$\theta_2 = \int (1 - \gamma) \sin \phi \cos \phi \, dw + M_2 \int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw . (17)$$

$$\tau_2 = \int (\sin^2 \phi + \gamma \cos^2 \phi)\, dw + M_2 \int (1 - \gamma) \sin \phi \cos \phi \, dw . (18).$$

and,

$$M_2 = \frac{\int (\gamma - 1) \sin \phi \cos \phi \, dw}{\int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw} \ldots\ldots\ldots\ldots\ldots (19)$$

The numerators of Equations (16) and (19) must be integrated from the points of application of the unit moments, m_1 and m_2, to the springing and the denominator from the crown to the springing section.

BRACED ARCH RIBS

Reinforced concrete arch bridges are generally constructed with lateral braces between the ribs. Each of these braces adds six more unknowns to the system of forces in the rib. When the structure is symmetrical, however, the number of unknowns is reduced; but in order to derive simple expressions for stresses in the bracing, certain assumptions are necessary. For example, it is assumed that each rib carries the same wind load. Furthermore, the intersections of the braces with the rib are assumed to be fixed in space and, therefore, the points of contraflexure in the braces will be at the mid-point in each case, and will also be fixed in space.

One-half of one arch rib is analyzed by substituting for the remainder of the structure the forces, R_1 and R_2, as shown in Fig. 2. Direct stresses in the members are neglected. Forces R_1 and R_2, therefore, will keep the mid-point of the brace fixed in position and the equations for these reactions can be developed by an application of Maxwell's theorem.

Two increments of displacement in the arch rib and bracing are considered: First, that produced by wind pressures on the half arch rib in Fig. 2, without the reactions, R_1 and R_2; and, second, that produced by the forces, R_1 and R_2, in order to translate the mid-point of the cross-brace back to its original position.

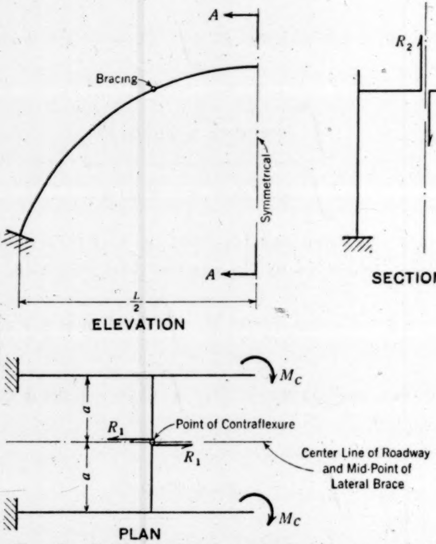
A
Bracing
Symmetrical
A
$\frac{L}{2}$
ELEVATION
R_2
R_2
SECTION A-A
M_C
a
R_1
Point of Contraflexure
R_1
a
Center Line of Roadway and Mid-Point of Lateral Brace
M_C
PLAN

FIG. 2.—SYMMETRICAL ARCH RIBS, WITH LATERAL BRACE.

In Fig. 2, with the forces, R_1 and R_2, removed, the displacement in the direction of the force, R_1, is $\theta_w a$, and in the direction of R_2, $\tau_w a$. If, instead of the uniform lateral wind load, the half arch in Fig. 2 were loaded with the force, R_1, the deflections would be, respectively, in the direction of R_1 $\left(\text{since the deflection of a cantilever beam is } \frac{R_1 a^3}{3 E I_a}\right)$:

$$\Delta = R_1 a \left(\theta_1 a + \frac{a^3}{3 E I_a}\right) \qquad (20)$$

and, in the direction of R_2;

$$\Delta = R_1 a \tau_1 a \qquad (21)$$

in which the products, $R_1 a$ and $R_2 a$, are moments of the forces, R_1 and R_2, about the intersection of the cross-brace and the arch rib.

If, instead of the reaction, R_1, the arch is loaded with the reaction, R_2, the corresponding displacements are:

In the direction of R_1:

$$\Delta = R_2 a \theta_2 a \qquad (22)$$

and, in the direction of R_2:

$$\Delta = R_2\, a \left(\tau_2\, a + \frac{a^2}{3\, E\, I_b} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Consequently, according to Maxwell's theorem, the displacements may be expressed as follows:

$$\theta_w = R_1 \left(a\, \theta_1 = \frac{a^2}{3\, E\, I_a} \right) + R_2\, a\, \theta_2 \ldots\ldots\ldots\ldots\ldots (24)$$

and,

$$\tau_w = R_1\, a\, \tau_1 + R_2 \left(a\, \tau_2 + \frac{a^2}{3\, E\, I_b} \right) \ldots\ldots\ldots\ldots\ldots (25)$$

Equations (24) and (25) were simplified by dividing through by the length, *a*. The deformations, θ and τ, are computed by means of Equations (11) to (18), inclusive, and with these values substituted in Equations (24) and (25), simultaneous solutions will yield the values of R_1 and R_2. Removing the right half of the arch has the effect of substituting a moment at the crown of the remaining left half equal to,

$$M_C = M_w + M_1 R_1\, a + M_2 R_2\, a \ldots\ldots\ldots\ldots\ldots (26)$$

The bending moment and torsion at any other section of the rib can be computed by statics. Formulas similar to Equations (24) and (25) may be written for any number of braces between the arch ribs.

ILLUSTRATIVE EXAMPLE

In order to illustrate the application of the foregoing equations, the wind load stresses in the arch shown in Fig. 3 will be computed. The ribs are fixed at the springing and braced at a distance of 26.28 ft measured hori-

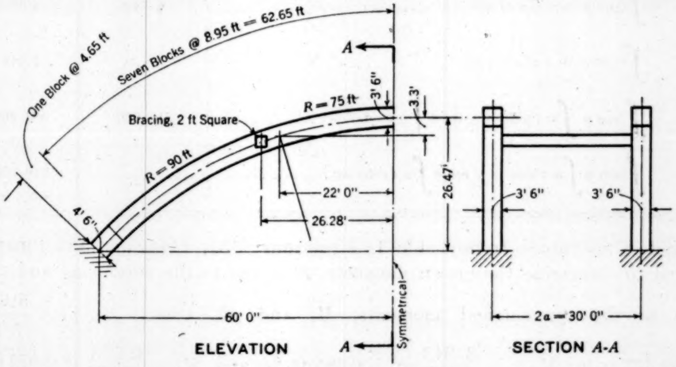

FIG. 3.—ILLUSTRATIVE EXAMPLE.

zontally from the crown. The theoretical span of the arch is 120 ft and the thickness of the arch rib varies according to the formula,

$$t = t_c\ (1 + 0.446\ \phi^2) \qquad \text{(27)}$$

in which t_c equals the thickness of the rib at the crown and ϕ equals the angle between the radial line and the vertical through the crown. Since the lateral cross-brace is 2 by 2 ft in section, $I_a = I_b = 1.33$ ft^4, and,

$$\frac{a^2}{3\ E I_a} = \frac{a^2}{3\ E I_b} = \frac{56.26}{E} \qquad \text{(28)}$$

The steel reinforcement is not taken into consideration in computing the moment of inertia. Such precision would be useless because the theory of wind load distribution on exposed surfaces is still so inadequate that extreme refinements in other factors are inconsistent.

Although the complete computations for this numerical example are not recorded herein, it is believed that they are given in sufficient detail for the guidance of those who are familiar with the design of arch bridges.

A wind load of $w = 45$ lb per sq ft is assumed to act horizontally and at right angles to the plane of the arch rib. In order to integrate the elastic and geometric properties, the rib was divided into eight blocks as indicated in Fig. 3, the values being given in Table 1. The modulus of elasticity of concrete in compression was assumed equal to 2.5 G, the modulus of elasticity of concrete in shear.

TABLE 1.—VALUES OF INTEGRATIONS IN TERMS OF $\frac{1}{E}$

Integrals	For half arch rib	From springing to bracing
$\int (\gamma - 1) \sin\phi \cos\phi \frac{dw}{2}$	0.543	0.374
$\int (\sin^2\phi + \gamma \cos\phi)\ dw$	6.542	3.461
$\int (\cos^2\phi + \gamma \sin^2\phi)\ dw$	5.199	3.043
$\int (\cos\phi \int w\ t\ u\ ds + \gamma \sin\phi \int w\ t\ v\ ds)\ dw$	516 660	481 360
$\int (\sin\phi \int w\ t\ u\ ds + \gamma \cos\phi \int w\ t\ v\ ds)\ dw$	....	246 720

Selecting values from Table 1, Equations (13), (14), and (19) may be solved to determine the crown moments, M_w, due to the wind load and those due to the symmetrical moments, M_1 and M_2, thus: $M_w = \frac{516\,660}{5.199} = 99\,377$ ft-lb; $M_1 = -\frac{3.043}{5.199} = -0.585$ ft-lb; and, $M_2 = \frac{0.374}{5.199} = 0.072$ ft-lb.

The angular deformations $\left(\text{multiplied by } \frac{1}{E}\right)$ are computed by means of Equations (11) to (18), as follows:

$$\theta_w = -\ 481\,360 + 99\,377 \times 3.043 = -\ 178\,956$$

$$\tau_w = -\ 246\,720 + 99\,377 \times 0.374 = -\ 209\,652$$

$$\theta_1 = +\ 3.043 - 0.585 \times 3.043 = +\ 1.263$$

$$\tau_1 = -\ 0.374 + 0.585 \times 0.374 = -\ 0.155$$

$$\theta_2 = -\ 0.374 + 0.072 \times 3.043 = -\ 0.155$$

and,

$$\tau_2 = +\ 3.461 - 0.072 \times 0.374 = +\ 3.434$$

It follows from Maxwell's theorem, furthermore, that $\tau_1 = \theta_2$. The bending moment and the torque were considered positive in the direction indicated in Fig. 1. The same is true of the angular deformations. The forces in the lateral cross-brace may be derived from Equations (24) and (25) by substituting the necessary angular deformations; thus:

$$(56.25 - 0.16 \times 15)\ R_1 + 3.434 \times 15\ R_2 = 209\,652 \ldots\ldots (29)$$

and,

$$1.263 \times 15\ R_1 + (56.26 - 0.16 \times 15)\ R_2 = 178\,956 \ldots\ldots\ldots (30)$$

which, solved simultaneously, yield $R_1 = 1.074$ lb and $R_2 = 2.946$ lb.

The bending moment about the vertical axis at the crown is computed by Equation (26), thus: $M_c = 99.377 - 0.585 \times 1.074 \times 15 + 0.072 \times 2\,946 \times 15 = 92\,785$ ft-lb. Therefore, the maximum unit compressive stress in the arch rib at the crown, produced by a wind-load bending moment, M_c, will be, $f_c = \frac{M}{S} = \frac{6 \times 92\,785 \times 12}{42 \times 42 \times 42} = 90.5$ lb per sq in., which is about 9% of the maximum compressive working stress (1 000 lb per sq in.) generally used in the design of reinforced concrete arch bridges.

The maximum wind-load bending moment at the end of the lateral cross-brace is $M = R_2 A = 2\,946 \times 15 = 44\,190$ ft-lb, which is about 126% of the dead load bending moment in the brace at that point. For comparison, the dead load bending moment is $M_d = \frac{D(2a - b)}{12} = \frac{600 \times 26.5 \times 26.5}{12}$ $= 35\,113$ ft-lb.

When this problem occurs in connection with a through bridge, hangers may be assumed flexible enough so that their influence on the rigidity of the rib may be neglected. If the roadway is poured monolithic with the abutment, it will transmit its own wind pressure to the abutment without materially affecting stresses in the arch ribs and the braces. Therefore, only the wind pressure on the side of the arch itself needs to be assumed to be carried by the ribs and the bracing.

The foregoing development may lead some readers to criticize the paper for lack of precision. However that may be, the illustrative example is offered merely for the purpose of demonstrating the application of the formulas developed in this paper. The relative accuracy can be controlled by the designer himself.

CONCLUSION

Although the wind stresses computed in this paper have not been verified by tests, the formulas were developed from the well-known elastic equations. Furthermore, the method of computing the torsion factor and the influence of torque on the deformation of an arch have been successfully proved by tests of skew arches reported in 1928 by Professor Rathbun.[5]

The effects of lateral wind forces on the forces in the plane of the arch rib are so slight as to be negligible and, therefore, the lateral forces may be treated separately.

In the case of arches subjected to unsymmetrical lateral forces, the equations in this paper may be modified as the particular case demands. Forces inclined to the plane of the arch rib may be resolved into their components. Furthermore, the eccentric forces applied to the arch through the roadway deck may be divided into components of force acting in the plane of the arch rib and a torque equal to the vertical force times eccentricity. In this case, the torque may also be treated as a moment produced by horizontal lateral forces.

It is important to study the practical application of the elastic equation to problems in which forces in space occur. Such study may eliminate many obsolete assumptions in time, thus resulting in further economic and æsthetic improvements in the design of such structures.

APPENDIX

NOTATION

The following notation is adopted for use in this paper:

a = one-half the length of a lateral csoss-brace, measured from its intersection with the axis of the arch rib; as a subscript, a denotes "vertical axis";
b = breadth; width of arch rib; as a subscript, b denotes "horizontal axis";
c = a subscript denoting "concrete";
d = a subscript denoting "dead load";
f = unit stress; f_c = unit compressive stress in concrete;
m = moment component; m_u = a component of the moment acting in a tangential plane through Point N (Fig. 1); m_v = a component of the moment acting in a radial plane through Point N.
ds = length of an element of the arch rib = $E\ I\ dw$;
t = thickness of the arch barrel, measured radially;

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[5] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

u = a co-ordinate of any point, P, measured at right angles to a radial line through Point N; as a subscript, u refers to a plane perpendicular to a radial line through Point N;

v = a co-ordinate of any point, P, referred to a Point N, measured parallel to the radial line through Point N (see Fig. 1); as a subscript, v refers to a radial plane;

w = uniformly distributed, lateral, wind load; $dw = \frac{ds}{EI}$; as a subscript, w denotes "due to wind load";

$\gamma = \frac{EI}{GF}$; or, if $E = 2.5\ G$, $\gamma = \frac{2.5I}{F}$;

C = a partial differential coefficient; C_{1m} and C_{2m} = bending moment coefficients corresponding to $m_1 = 1$ and $m_2 = 1$ applied at Point P; C_{1t} and C_{2t} = torque coefficients corresponding to moments, $m_1 = 1$ and $m_2 = 1$, applied at Point P; as a subscript, C denotes "at the crown";

D = total dead load;

E = modulus of elasticity of concrete in tension and compression;

F = a torsion factor;

G = modulus of elasticity of concrete in shear;

I = moment of inertia of the cross-section area of the arch cut by a radial plane; I_a = moment of inertia of a section of a cross-brace, about a vertical axis through the center of gravity; I_b = moment of inertia of a section of bracing about a horizontal axis through the center of gravity;

L = span length;

M = bending moment; M_C = bending moment at the crown; M_w = bending moment due to wind load; M_1 = bending moment at the crown due to the symmetrical moments, $m_1 = 1$; M_2 = bending moment at the crown due to the symmetrical moments, $m_2 = 1$; M_d = dead load bending moment;

R = reaction; R_1 = horizontal force applied to the center of a wind-brace; R_2 = vertical force applied to the center of a wind-brace;

S = section modulus;

T = torsion; T_1 = torque corresponding to a unit moment, $m_1 = 1$, applied at Point P;

W = internal work due to the elastic deformation of the arch rib;

Δ = displacement;

θ = bending deformation at Point N, about the vertical axis; θ_w = bending deformation produced by the wind; θ_1 = bending deformation produced by $m_1 = 1$; θ_2 = bending deformation produced by $m_2 = 1$;

τ = torsion deformation at Point N about the horizontal axis; τ_w = deformation produced by wind; τ_1 = deformation produced by $m_1 = \mathrm{v}$; τ_2 = deformation produced by a moment $m_2 = 1$;

ϕ = central angle locating any point, N, on the arch axis with relation to a radius through the crown.

DISCUSSION

Leon Blog,[6] **Assoc. M. Am. Soc. C. E.** (by letter).—An elegant method of analyzing an arch subjected to lateral stresses is presented in this paper which the author has explained by an application to a reinforced concrete arch loaded by horizontal wind forces. This method is elegant because it weaves together the theorems of Castigliano and Maxwell, but it is not so simple as the method devised by Professor Maurice Levy[7] who solved the problem by a graphic solution based upon a rigorous mathematical analysis. By his method an arch can be solved in one day at as many sections as desired and, furthermore, the section of the arch at which a strut subject to the least bending or torsion should be located, aside from the question of æsthetics, is clearly indicated. The author's method is not that speedy nor can the physical behavior of the arch be visualized without considerable work supplementary to the analysis for various sections.

Arch Ribs Without Bracing.—Although the introduction of so many types of moments is perfectly natural due to the use of the Castigliano and Maxwell theorems, the visualization of the contribution of each toward the solution of the problem is difficult. The use of the designation, "deformation", for the values, θ and τ, is confusing. Although an angular change in the position of a section is, in a sense, a deformation, the ordinary use of the term is to describe changes of length. It is only when the reader comes to the paragraph following Table 1 that he learns that θ and τ are angular changes unless he happens to recall that the basis of the terms in the parentheses of Formula (2) (which leads to the internal work, W) is that the angular change at any section of a member subjected to bending is the derivative of the internal work with respect to the external bending moment and the angular change has the same sign as that of the moment, that is,

$$\theta \text{ or } \tau = \frac{dW}{dM} \qquad \text{......................(31)}$$

By virtue of its derivation Equation (31) involves the concept that the angular change occurs either in the same plane, or in a plane parallel to the actuating bending moment. Although the accuracy of Equation (2) is admitted, it is not obvious to the writer that the torsional moment, m_v, contributes anything to the value of θ, or that the bending moment, m_u, contributes anything to the value of τ. If these views are correct, there will be no partial differential coefficients, Equations (3) and (4) will be modified, and Equations (7) to (10), inclusive, will be superfluous.

The formula for the angular rotation about the normal to the tangent at any section of an arch rib in terms of an external bending moment is,

$$\theta = -\int \frac{M_b \, ds}{E \, I_n} \qquad \text{......................(32)}$$

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[7] "La Statique Graphique", Pt. III, by Maurice Levy.

and that about such a tangent is,

$$\tau = + \int \frac{M_t \, ds}{G \, I_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

in which M_b and M_t are, respectively, the external bending and torsional moments at the section; ds is a small piece of the arch axis; E and G, respectively, are the moduli of elasticity in tension; and, in shear, I_n and I_t are the rectangular moment of inertia about the normal and the polar moment of inertia about the tangent, respectively.

The limits of the definite integrals are the springing line and the distance from it to the section of the arch rib measured along its geometric axis. The sign convention is that of the author, θ and τ are independent variables along the rib and are proved to be so by Professor Levy[8] from simple geometric considerations.

Assume for the moment that the torsional moment, m_v, has some effect upon θ and that the bending moment, m_u, has some effect upon τ. Then, C_{1m} is obtained by partial differentiation of Equation (5) with respect to m_u, which amounts to finding the first derivative of cos ϕ, which is $-$ sin ϕ. Likewise, C_{1t}, the first derivative of sin ϕ, is $+$ cos ϕ, allowing M_w to retain its negative sign. Similarly, C_{2m} and C_{2t} are, respectively, $-$ sin ϕ and $-$ cos ϕ. When substituted in Equations (11) to (13), inclusive, these values will alter their form. Equation (16), being derived from Equation (14), reveals that $M_1 = -1$ always, whereas its evaluation from Equation (15) would also $= -1$. Equations (14) and (15) express the fact that θ_1 always equals τ_1, which is doubtful. Similar doubt exists as to M_2 from Equations (17) to (19) which would yield two different values of M_2.

A statement should be made as to the evaluation of the lateral shear in the rib. In 1918, Professor Levy[9] stated that, for an arch with symmetrical shape and loading, the torsion and shearing stresses are zero at the crown. He also stated that the total external shear at any section is equal to the algebraic sum of all the lateral forces between the crown and the section in question. This external shear results in internal shearing stress at the section, which must be combined properly with similar stresses due to the torsional moment and the vertical shear due to gravity loads.

The author closes the development of the first caption with a statement as to the limits of the integrations for Equations (16) to (19). It would assist those not so familiar with the method if this were done for the formulas preceding Equation (16).

Before closing the discussion of the subject under the caption, "Arch Ribs Without Bracing", the writer directs attention to the statements by the author that direct stress is induced by wind in the rib. Since the arch subjected to lateral forces acts as a beam which is subjected to bending, torsional shear, and ordinary beam shear, and has no curvature in the horizontal plane, the writer does not see how direct stress without a prerequisite arch thrust can occur.

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[8] "La Statique Graphique", p. 183 *et seq.*

[9] *Loc. cit.*, Pt. III, p. 199, Paragraph *b*.

Another statement of theory which must be questioned is the second paragraph of the author's "Conclusion." Professor Levy[10] shows that the action of gravity forces leaves the vertical neutral plane undeformed, whereas the lateral forces produce displacements normal to that plane, and "each of these kinds of forces produce the same displacements as if the other did not exist." Therefore, the lateral wind forces can have absolutely no effect on the forces in the plane of the arch rib.

Braced Arch Ribs.—The author's assumption that each rib is subjected to the same wind load happens to fit in nicely with the development of his bracing theory for two symmetrical arches, but what would he do if the bridge were on a curve with one arch longer and thicker than the other? If there were three or more ribs in the bridge unequally spaced, or of different dimensions, or both? The correct guess as to wind-load distribution between the ribs would be difficult. Professor Levy has shown[11] how to treat this problem with only the generally accepted assumptions.

By Equations (29) and (30), the values of R_1 and R_2 are approximately 3 150 lb and 2 018 lb, which corresponds with the writer's experience that the horizontal bending moment and the shear it causes in the brace are more serious than the torsional bending moment and its resultant shear. The combined bending and direct wind stress is equal to ± 528 lb per sq in., which indicates a large factor of safety for the 24 by 24-in. brace. However, the brace should always be analyzed for shear due to bending and for other column considerations. If the underlying assumptions are accurate, this method of analysis is an elegant one.

Professor Levy's Method.—Professor Levy has presented analyses of arches subjected to lateral loads for all shapes and degrees of fixation, but the writer will merely touch upon the symmetrical shape and loads to indicate the rapidity, and the ease of visualizing the structure during such analyses. Fig. 4 shows a pair of such ribs in plan, and the elevation of the rib shows all the work required for the determination of the bending moment and the torsional moment at any section. The leeward rib receives its wind loads at Points 10, 8, 6, 4, and 2, and these loads are laid off in order, to yield the force polygon, *P–M–N*. For convenience, the pole distance, *P–M*, was taken as the sum of the loads, *M–N*, and the equilibrium polygon, *e–f*, was drawn through the Load Points 10 to 2. The closing line, *u–v*, of the arch considered as a fixed-ended beam is located by laying off from Line *g–f* the arithmetic mean of all the ordinates through Points 10 to 0 between the line, *g–f*, and the polygon, *e–f*, taken to the dimension scale of the layout. Then, the ordinate, v_a, between Lines *e–f* and *u–v*, multiplied by the pole distance, *P–M*, is the bending moment at Point 5 about the vertical plumb-line, *y–y*.

By drawing another force polygon for the same loads laid off on the half-chord, *N′–S*, and drawing an equilibrium polygon through horizontals drawn through the load points, 10, 8, 6, 4, and 2, the torsional moment about the horizontal through Point 5 is equal to h_a times the pole distance for the force polygon. These two values correspond in direction to the values obtained by the author when he multiplies R_1 by one-half the brace length in the problem and does the same with R_2.

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[10] "La Statique Graphique", Pt. III, p. 190, Lines 29 to 31.

[11] *Loc. cit.*, Pt. III, p. 211.

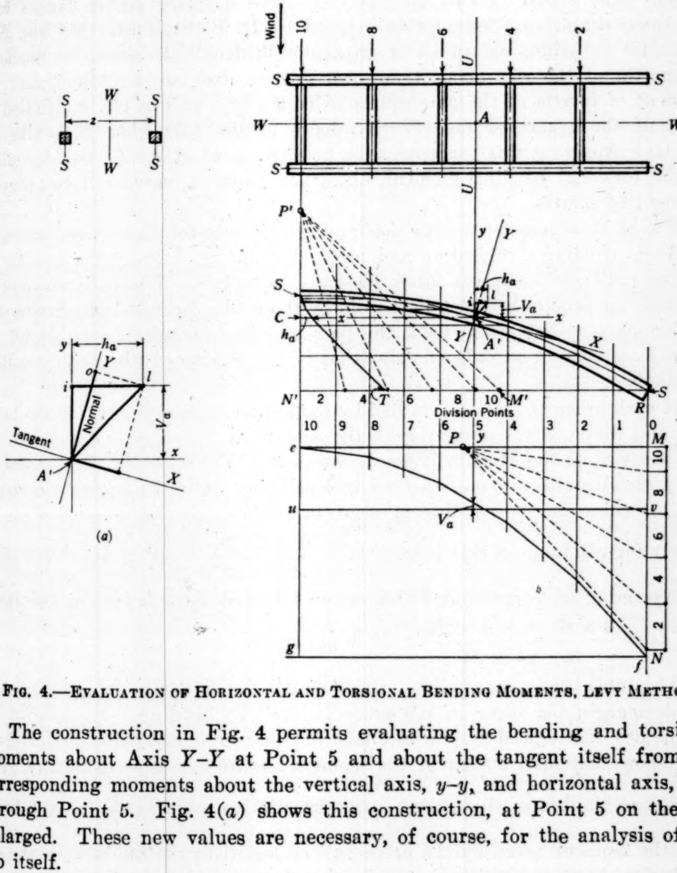

Fig. 4.—Evaluation of Horizontal and Torsional Bending Moments, Levy Method.

The construction in Fig. 4 permits evaluating the bending and torsional moments about Axis Y–Y at Point 5 and about the tangent itself from the corresponding moments about the vertical axis, y–y, and horizontal axis, x–x, through Point 5. Fig. 4(a) shows this construction, at Point 5 on the rib, enlarged. These new values are necessary, of course, for the analysis of the rib itself.

Most of the groundwork for this analysis having already been done in solving the problem of the gravity load, the solution of the arch for wind is rapid. The concepts of loads and force polygons are familiar to most engineers; relative magnitudes can be compared, and it is seen that the point where the polygon, e–f, crosses the line, u–v, is the point of minimum bending. It is also a point of relatively small torsional moment.

Rational Method for Analyzing a Braced Arch System.—The author's development of formulas and his "Illustrative Example" show clearly what he means by the assumption that each rib is subjected to the same wind load;

that is, the leeward rib of the system resists the same amount of wind as the windward rib. This assumption, made in order to arrive at a solution, penalizes the system too severely and is entirely unnecessary. Professor Levy has shown[12] that a pair of ribs may be considered as plate girder flanges with the same simplifying assumption utilized by D. B. Steinman, M. Am. Soc. C. E., for the stiffening trusses of suspension bridges.[13] In short, the moments of inertia of the ribs about their own gravity axes are neglected, and the moment of inertia of the pair equals twice the area of one rib, multiplied by one-half the square of the effective depth of the pair when both ribs are similar. Applying this concept to the polar moment of inertia corresponding to the torsional bending moment, the polar moment becomes a rectangular moment of inertia.

Use of these constants with the familiar formula for fiber stress in simple bending, utilizing the bending and torsional moments found for a specific rib section, will yield stresses fairly close to the actual. The only assumption made in the process has been held valid by those who discussed Mr. Steinman's classic paper. In this method, the pair of ribs is caused to withstand the wind force coming on only one rib (that is, the windward rib), with resultant economy.

If each brace of the type studied by the author were designed for the bending and direct stress occurring at the section at which it is framed to the rib, the size of the brace will not be excessive. For instance, if the arch in the author's problem were supplied with a brace, 24 by 24 in., at the crown, and his value for the bending moment, 92 785 ft-lb (which the writer checks closely for wind on one rib) is used, $f_b = \frac{6 \times 92\,785 \times 12}{24 \times 24 \times 24} = 485$ lb per sq in.

Assuming an average depth of rib of 4 ft and three braces in the entire rib: $f_c = \frac{2 \times 67.30 \times 4 \times 45}{3 \times 24 \times 24} = 24$ lb per sq in.

The total bending and direct stress equals 509 lb per sq in., which leaves ample margin for shear in the brace induced by bending. A study of the mechanics of a braced system of ribs, whether two or more ribs are involved, will show that distribution of the moment existing in the rib at any given section to the brace will depend upon the relative stiffness, $\frac{I}{L}$, of the brace, and the moment taken by the brace will be less than the moment in the rib. Esthetics, however, will not permit of a brace section commensurate with the stresses therein, so that the writer's severe but simple method used in connection with Professor Levy's method, will not result in excessively large braces.

The writer finds the bending moment at the crown to be 99 700 ft-lb; that in the rib at the brace, 44 700 ft-lb; and the torsional moment in the rib at the brace, 21 000 ft-lb. The values found by the author in the rib at the crown, were 92 785 ft-lb, and with corrected formulas the bending in the brace would be 47 250 ft-lb, with a torsional moment in the brace of 30 270 lb. The writer believes the author's values for the braces are too high.

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[12] "La Statique Graphique", p. 211 *et seq.*

[13] "A Generalized Deflection Theory for Suspension Bridges", *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1223, Lines 10–12.

Mr. Eremin has opened a discussion of the treatment of the simplest types of arch-rib bridges in which the ribs and the loading are symmetrical. When either of these items is unsymmetrical, the solution is not as simple as the author's conclusion indicates. It is to be hoped that still other methods of analyzing structures subjected to lateral loads will be developed, which are simpler, easier to grasp, and more rapid.

FANG-YIN TSAI,[14] ASSOC. M. AM. SOC. C. E. (by letter).—The problem of wind stresses in reinforced concrete arch bridges is similar to that of bow girders, which have been treated previously by many authors[15], the only difference being that the former are fixed-end beams, curved in vertical plan, and subjected to horizontal loads, whereas the latter are fixed-end beams curved in horizontal plan, and subjected to vertical loads. However, the part of the paper under the heading, "Braced Arch Ribs", seems to be quite original. Although the solution of the problem has been indicated previously by some authors[16], the scant attention which the problem has received thus far, as well as the tendency to increase the span lengths of such bridges, makes the presentation of this paper very timely.

It is most unfortunate that the demonstrations are rather too brief and also not very clear. There is also quite a number of ambiguities which cause considerable confusion when one wishes to follow the demonstrations closely. For example, as stated by the author, Equations (3) and (4) are derived from Equation (2) by applying Castigliano's theorem. It may be noted that, for computing displacements, the application of Castigliano's theorem is advantageous only in cases where there is a load at the point and also in the direction of the desired displacement. Otherwise, an arbitrary load must be first introduced at the said point and in the said direction, and then the load is set to zero after the partial differentiation[17]. Consequently, there is a considerably long step from Equation (2) to Equations (3) and (4). However, if the method of work with a dummy unit loading as developed by Maxwell, Mohr, and the late George Fillmore Swain[18], Past-President and Hon. M., Am. Soc. C. E., is used, Equations (3) and (4) can be written directly with the dummy unit moments, $m_1 = 1$ and $m_2 = 1$, applied, successively, at Point P. Therefore, it is much simpler and more straightforward than the application of Castigliano's theorem. Of course, the two methods differ only in conception and operation, and they give identical results.

It must be also noted that the quantities, m_u and m_v, in Equation (2) are not the same as those in Equations (3) and (4). Whereas, those in Equation

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[15] For a comprehensive bibliography on the subject, see *Journal*, Am. Concrete Inst., November, 1932, p. 158.

[16] See, for instance, discussion by Norman M. Steinman, *Journal*, Am. Concrete Inst., November, 1932, p. 153.

[17] See "Statically Indeterminate Stresses", by J. I. Parcel and G. A. Maney, Members Am. Soc. C. E., John Wiley, New York, First Edition, 1926, pp. 37–39.

[18] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 622.

(2) are, respectively, the bending and torsion moments at Point N due to both the actual load (wind, m_1 or m_2, as the case may be) and the dummy moments, m_1 and m_2, applied at Point P in the horizontal and profile planes, those in Equations (3) and (4) are, respectively, the bending and torsion moments at Point N due to the actual load only. The values of m_u and m_v, due to the actual load, to be substituted in Equations (3) and (4), will be, respectively, those given by: (*a*) Equations (5) and (6) for deriving Equations (11) and (12) due to wind load; (*b*) $+ \cos \phi + M_1 \cos \phi$ and $- \sin \phi - M_1 \sin \phi$, for deriving Equations (14) and (15), due to the symmetrical moments, $m_1 = 1$; and (*c*) $+ \sin \phi + M_2 \cos \phi$ and $+ \cos \phi - M_2 \sin \phi$, for deriving Equations (17) and (18) due to symmetrical moments, $m_2 = 1$. The foregoing distinction must be noted carefully in following the derivation of all the aforementioned equations in order to avoid confusion. Again, the complications in applying Castigliano's theorem to the problem are evident.

Following Equation (25) the author states that Equations (24) and (25) may be expressed by Maxwell's theorem. Apparently, he refers to the principle of superposition, which is not due to Maxwell, as pointed out elsewhere[19] by the writer.

Table 1 gives only the numerical results of the various terms of integration, without showing the detailed method of computing them. The writer hopes that, in his closing discussion, the author will show the detailed computations for all the terms in tabulated forms as A. J. S. Pippard[20], M. Am. Soc. C. E., did in solving a problem of bow girder. Such an example of tabulated computations will not only facilitate the application of the method, but will also help in understanding the various terms of integration. It will be certainly welcomed by all those who wish to use the method in practice.

Paul Andersen,[21] Assoc. M. Am. Soc. C. E. (by letter).—This paper is a valuable contribution to the theory of arch design. It is incomplete to the extent that it makes no mention of stresses in barrel arches with spandrel walls. Although the bending stresses, due to wind load, in an earth-filled arch are of no consequence, torsional stresses at the springing line may become quite large.

Using the author's notation and applying the principles of statics, the maximum torsional moment, M_T, which occurs at the springing line can be determined from,

$$M_c \cos a + M_T = \int w\, t\, n\, ds \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

in which a = the angle between the horizontal line and the tangent to the earth axis at the springing line; and n = the distance from the arch element to this tangent.

Using the values from the illustrative example in Fig. 3, Equation (34) becomes, $92\,785 \times 0.715 + M_T = 99\,470$; and $M_T = 33\,130$ ft-lb.

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[19] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 130.

[20] "Strain Energy Methods of Stress Analysis", by A. J. S. Pippard, Longmans, Green & Co., London, 1928, pp. 133–135.

[21] Balboa Heights, Canal Zone.

This moment produces a maximum shearing stress, at the center of the wide side, of $\frac{33\,129 \times 12}{0.225 \times 54 \times 42^2} = 19$ lb per sq in. To this stress should be added the shearing stress due to dead load and live load.

In case of an arch with a rigidly connected superstructure (Fig. 5), the arch rib will transmit to the springing lines considerably more stress than the wind load acting upon the rib itself. If the arch is of the barrel type with spandrel walls, a large area is exposed to lateral wind pressure and the arch section, although possessing high resistance to horizontal bending, has comparatively low torsional strength. Shearing stresses for this type of structure will be considerably greater than 19 lb per sq in.

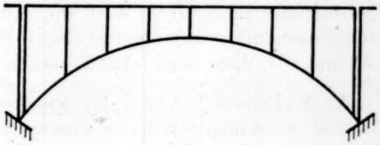

FIG. 5

It is unfortunate that the author chose the symbol, *w*, to indicate load intensity and *dw* to represent elastic weight; integration limits in the deflection terms and moment equations would also have been helpful.

LOUIS BLUME,[22] ESQ. (by letter).—Under the heading, "Arch Rib Without Bracing", the author presents an analysis that appears to lead to correct results. However, careful examination indicates that the method of approach is such as to involve superfluous work. To illustrate this, the following treatment of Equations (2), (5), and (6), is presented: Equation (2) can be written,

$$W = \frac{1}{2}\int m_u^2\,dw + \frac{\gamma}{2}\int m_v^2\,dw \;\ldots\ldots\ldots\ldots\ldots(35)$$

Applying the principle of least work:

$$\frac{dW}{dM_w} = \int m_u \frac{dm_u}{dM_w}\,dw + \gamma\int m_v \frac{dm_v}{dM_w}\,dw = 0 \;\ldots\ldots\ldots(36)$$

From Equations (5) and (6), $\frac{dm_u}{dM_w} = +\cos\phi$ and $\frac{dm_v}{dM_w} = -\sin\phi$.

By substitution:

$$\frac{dW}{dM_w} = \int M_w \cos^2\phi\,dw - \int\left[\cos\phi\,dw\int_c^N w\,t\,u\,ds\right]$$

$$+\,\gamma\int M_w \sin^2\phi - \gamma\int\left[\sin\phi\,dw\int_c^N w\,t\,v\,ds\right] = 0\ldots\ldots(37)$$

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[22] Structural Engr., City Engr.'s Office, Los Angeles, Calif.

Solving for M_w:

$$M_w = \frac{\int \left[\cos \phi \int_c^N w\ t\ u\ ds + \gamma \sin \phi \int_c^N w\ t\ v\ ds \right] dw}{\int [\cos^2 \phi + \gamma \sin^2 \phi]\ dw} \quad \ldots (38)$$

This value of M_w agrees with that of Equation (13) but is obtained by a less tortuous procedure. With the value of M_w found, the bending and torsional moments at any point in the rib may be obtained by means of Equations (5) and (6), dispensing with Equations (16) and (19).

A. A. EREMIN,[23] ASSOC. M. AM. SOC. C. E. (by letter).—An interesting method of computing wind stresses in arch bridges, devised by Professor Maurice Levy[7], was described by Mr. Blog who, in discussing its various advantages, has not shown its serious limitations. Equations for wind stresses developed by Professor Levy may be applied only to a single arch rib. In the numerical example of wind-stress computations in an arch bridge with two steel ribs, braced with San Andreas cross-braces, as solved by Professor Levy, it was assumed that the bracings were absolutely stiff and two arch ribs were considered as a single arch rib. A similar method of analyzing wind stresses in reinforced concrete arch ribs with elastic braces, such as that shown in Fig. 3, would involve a serious error.

The graphical construction for computing wind stresses in a symmetrical arch rib shown by Mr. Blog, was devised by Professor Levy for a case in which the product of the moment of inertia of the arch rib and the cosine of the angle between a tangent to the arch axis and the horizontal, is a constant. A similar graphical construction of moments in the arch rib in Fig. 3 would require complicated computations for locating the line, *u-v*, in Fig. 4. In this case, analytical computations are much shorter. With a mechanical calculating machine the coefficients in Table 1 may be computed in a few hours. Furthermore, they may be computed with a slide-rule with reasonable accuracy for practical purposes.

Mr. Blog could not understand why the torsion moment, m_v, contributes anything to the value of θ, or that the bending moment, m_u contributes anything to the value of τ. This may be proved with reference to the similarity between forces and deformations in the plane of the arch rib. Vertical forces in this plane produce vertical and horizontal displacements. Likewise, horizontal forces also produce vertical and horizontal displacements. Therefore, Equations (32) and (33) are incorrect if applied to arch ribs.

In evaluating the coefficients, C_{1m} and C_{1t}, Mr. Blog erroneously considers the angle, ϕ, as a variable, which precludes the possibility of verifying the writer's equations. Contrary to Mr. Blog, wind pressures do have an effect on direct stresses in arch ribs. Such stresses in Equation (2) were not considered because they are negligible. In straight beams with ends fixed, which sustain transverse loads the direct stresses are never considered. However, it does not indicate that, in a beam completely fixed at the ends, transverse forces have absolutely no effect on direct stresses. Mr. Blog cites the writer's assumption that, in the braced arch ribs in Fig. 3, both arch ribs resist the same amount of wind pressure, stating that this condition penalizes the system severely. The writer does not agree since, by means of rigid braces, wind forces from the windward rib are transferred to the leeward rib in such a manner as to involve both arch ribs equally. For all practical purposes this assumption is correct and facilitates the location of the points of contraflexure in the braces.

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[23] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

Professor Tsai states that in developing Equations (3) and (4) by means of Castigliano's theorem, the writer failed to show all the steps. This theorem is described in standard textbooks on strengths of materials[24] sufficiently to reveal all the steps between Equations (2) and (3). The method involving the idea of a "dummy" unit force at a displacement point also follows from Castigliano's theorem.

The values in Table 1 were computed by the method of approximate summation. The arch rib was divided into voussoirs as shown in Fig. 3. The physical properties of the voussoirs were computed at their centers of gravity. The limits of integration are shown in Table 1; and, for the half arch rib, the limits are from the springing to the crown section.

Mr. Andersen's Equation (34) is similar to Equation (5) developed for a single arch rib. It is to be noted that, in the braced arch rib shown in Fig. 3, Equation (34) cannot be used for computing the bending moment at the springing. From Equations (5) and (6) the bending moment, M_B, and the torsion moment M_T, at the springing, in a single arch rib, are:

$$M_B = -\int w\,t\,u\,ds + M_w \cos\alpha \ldots\ldots\ldots\ldots\ldots (39)$$

and,

$$M_T = +\int w\,t\,v\,ds - M_w \sin\alpha \ldots\ldots\ldots\ldots\ldots (40)$$

The bending moment, M_{BS}, and the torsion moment, M_{TS}, at the springing of the braced arch rib in Fig. 3 are:

$$M_{BS} = -\int w\,t\,u\,ds + M_c \cos\alpha - R_1\,a \sin\alpha + R_2\,a \cos\alpha \ldots (41)$$

and,

$$M_{TS} = +\int w\,t\,v\,ds - M_c \sin\alpha + R_1\,a \cos\alpha - R_2\,a \sin\alpha \ldots (42)$$

Equations (39) to (42) may be used for computing wind stresses in filled or spandrel open arch bridges such as that shown in Fig. 5, as stated by Mr. Andersen. Fig. 6 shows a symmetrical spandrel filled arch bridge, subjected to a wind pressure, P, on a vertical strip, dx. In this case,

$$P = w\,H\,dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

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[24] Interesting information regarding Castigliano's theorem and the originality of Maxwell's theorem may be found in "Strength of Materials", Pt. I, by S. Timoshenko.

The force, P, is concentrated at the center of the strip but it may be transferred to the arch axis as a force, P, and an overturning moment, thus:

$$M_P = P\ e = w\ H\ e\ dx \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

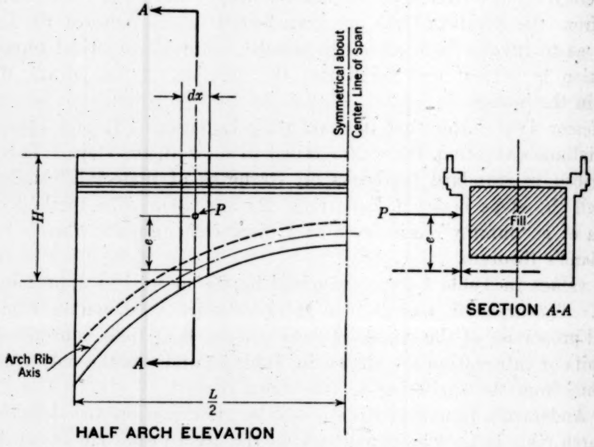

FIG. 6.—SPANDREL FILLED ARCH BRIDGE.

Therefore, the wind stresses in the arch rib in Fig. 6 are equal to the sum of the stresses computed with concentrated forces, P, acting along the arch axis, and the overturning moments, M_P. Wind stresses and deformations produced by Force P may be computed by means of Equations (1) to (13). The deformations and stresses in an arch rib sustaining overturning moments, M_P, may be developed by means of Equations (3) and (4).

The moments, m_u and m_v, in the arch rib due to an overturning moment, M_P, are:

$$m_u = - \int M_P \sin \phi\ ds + M_{mw} \cos \phi \ldots\ldots\ldots\ldots (45)$$

and,

$$m_v = + \int M_P \cos \phi\ ds + M_{mw} \sin \phi \ldots\ldots\ldots\ldots (46)$$

in which M_{mw} is the bending moment at the crown due to the symmetrical overturning moments, M_P. From Equations (45) and (46) and Equations (7) to (10), the deformations in an arch rib sustaining symmetrical moments, M_P, are found to be:

$$\theta_{mw} = - \int \left[\cos \phi \int M_P \sin \phi\ ds + \gamma \sin \phi \int M_P \cos \phi\ ds \right] dw$$
$$+ M_{mw} \int (\cos^2 \phi + \gamma \sin^2 \phi)\ dw \ldots\ldots\ldots\ldots (47)$$

and,

$$\tau_{mw} = \int \left[\sin \phi \int M_P \sin \phi \, ds - \gamma \cos \phi \int M_P \cos \phi \, ds \right] dw$$

$$+ M_{mw} \int (I - \gamma) \sin \phi \cos \phi \, dw \ldots\ldots\ldots\ldots\ldots (48)$$

In an arch rib with symmetrical wind pressures, the deformation, θ_{mw}, at the crown is equal to zero. Therefore, from Equation (47), the bending moment at the crown is:

$$M_{mw} = \frac{\int \left[\cos \phi \int M_P \sin \phi \, ds + \gamma \sin \phi \int M_P \cos \phi \, ds \right] dw}{\int (\cos^2 \phi + \gamma \sin^2 \phi) \, dw} \ldots (49)$$

For integrating Equations (47) to (49), the overturning moment, M_P, should be expressed as a continuous function. However, in practice the integration may be performed as an approximate summation without serious error. The arch rib is divided into a convenient number of vertical strips and forces, P, with overturning moments, M_P, computed for each strip. Strips that intersect the arch rib are considered as arch voussoirs and are treated in the manner shown in the illustrative example given in the paper (see Fig. 3).

Evidently, in the summation, the wind forces, P, the moments, M_P, and the physical properties of an arch rib must be considered at the centers of gravity of the voussoirs. An approximate method of summation is especially convenient in open spandrel arch bridges, in which wind-stress pressures vary abruptly along the arch axis, at the spandrel column.

Mr. Blume has shown the intermediate steps in developing Equation (13) from Equation (2). However, he was incorrect in considering the ratio, γ, to be constant. Obviously, it varies, along the arch axis, with the moment of inertia, I, and the torsion factor, F.

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SUCCESSIVE ELIMINATION OF UNKNOWNS IN THE SLOPE DEFLECTION METHOD

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WITH DISCUSSION BY MESSRS. C. A. WILLSON, PAUL ANDERSEN, R. W. STEWART, ADOLPHUS MITCHELL, FANG-YIN TSAI, A. FLORIS, A. W. FISCHER, L. E. GRINTER, L. T. WYLY, AND JOHN B. WILBUR.

SYNOPSIS

A method of applying the usual slope deflection equations[2] is presented in this paper, which by successively expressing all the unknown elements in terms of a few unknowns makes an exact solution possible without the formal solution of a large number of simultaneous equations.

The method is described by showing its application to three types of structures: Continuous beams, building frames with shallow wind-bracing, and the Vierendeel truss. Certain principles are then enumerated, which may be extended to other structures that may be solved by the slope deflection method.

NOTATION

Throughout this paper, it will be assumed that the reader is familiar with the slope deflection theory. The usual slope deflection notation will be used, as follows:

d = relative deflection measured normal to the axis of a member, between the ends of the member;
h = height of a column, or distance between floors; story height;
p = panel length;

NOTE.—Published in December, 1935, *Proceedings*.

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[2] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method", by W. M. Wilson, F. E. Richart, and Camillo Weiss, Members, Am. Soc. C. E., *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

C = fixed-end moment at Point B on Beam BC, due to any load, P; a resisting moment;
E = modulus of elasticity in tension and compression;
I = moment of inertia;
K = ratio of the moment of inertia, to the length of a structural member;
M = moment of an external couple; M_{AB} = moment at End A, of Beam AB, etc. (see, also, C);
P = a concentrated load;
R = ratio of the increment in horizontal deflection occurring in a story, to the story height, the floors being identified by subscripts;
θ = a change in the slope of the tangent to the elastic curve of a member.

Continuous Beams

For a slope deflection solution of the structure shown in Fig. 1(*a*), there are five unknowns: θ_A, θ_B, θ_C, θ_D, and θ_E. For the order of using the slope deflection equations to be outlined, θ_A will be termed the permanent unknown, since all the other values of θ will be expressed, successively, in terms of θ_A, which will then be evaluated. The slope deflections, θ, at the remaining supports will be termed the temporary unknowns.

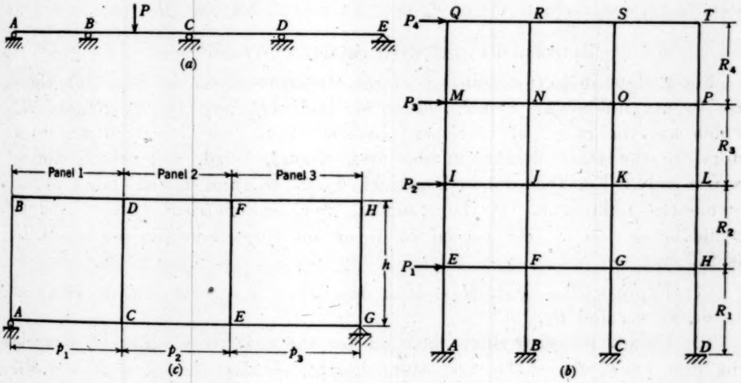

FIG. 1.

At either end support of a continuous beam, a known element exists; either the moment or the slope is zero, or the moment is known, as when the beam is a cantilever beyond the support. In Fig. 1(*a*) the moment at each end is zero. Thus, taking θ_A as the permanent unknown:

(1) Express θ_B in terms of θ_A by writing,

$$M_{AB} = 2\ E\ K_{AB}\ (2\ \theta_A + \theta_B) = 0 \ldots\ldots\ldots\ldots\ldots (1)$$

or,

$$\theta_B = -\ 2\ \theta_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

(2) Express θ_C in terms of θ_A by writing $M_{BA} + M_{BC} = 0$, in which,

$$M_{BA} = 2\ E\ K_{AB}\ (2\ \theta_B + \theta_A) = 2\ E\ K_{AB}\ (-\ 3\ \theta_A) \ldots\ldots\ldots (3)$$

and,

$$M_{BC} = 2\ E\ K_{BC}\ (2\ \theta_B + \theta_C) \pm C = 2\ E\ K_{BC}\ (-4\ \theta_A + \theta_C) \pm C. (4)$$

in which C is the fixed-end moment at Point B on Beam BC due to any load, P.

(3) Similarly, by $M_{CB} + M_{CD} = 0$ (that is, $\Sigma M = 0$ at Support C), express θ_D in terms of θ_A.

(4) Similarly, by $\Sigma M = 0$ at Support D, express θ_E in terms of θ_A.

(5) The fact that $M_{ED} = 0$ enables a solution for θ_A since $M_{ED} = E\ K_{DE}\ (2\ \theta_E + \theta_D) = 0$, in which θ_E and θ_D are functions of θ_A.

(6) With θ_A known, and all other values of θ in terms of θ_A, the moments in the beam are easily determined by the slope deflection equation.

Thus, the continuous beam shown in Fig. 1(*a*) is analyzed without the formal solution of simultaneous equations. This fact is not dependent upon the number of spans. Had the beam been fixed at Point A, θ_B would have been taken as the permanent unknown, and the first step would have been to express θ_C in terms of θ_B by $\Sigma M = 0$ at Support B.

BUILDING FRAMES WITH SHALLOW WIND-BRACES

For a slope deflection solution of the structure shown in Fig. 1(*b*), there are twenty unknowns, namely, θ_E to θ_T, inclusive, and R_1, R_2, R_3, and R_4, which are the ratios of increment in horizontal deflection occurring in a story, to the story height, in the first, second, third, and fourth stories, respectively. For the proposed method, θ_E, θ_F, θ_G, and θ_H, are taken as the permanent unknowns. If the structure were symmetrical only θ_E and θ_F would be so taken. The procedure, using the slope deflection equations, is, as follows:

(1) Express M_{AE}, M_{EA}, M_{BF}, M_{FB}, M_{CG}, M_{GC}, M_{DH}, and M_{HD} in terms of θ_E, θ_F, θ_G, θ_H, and R_1;

(2) Using the shear-story equation for the first story (that is, shear in the first story, times the first story height, $+ M_{AE} + M_{EA} + M_{BF} + M_{FB} + M_{CG} + M_{GC} + M_{DH} + M_{HD} = 0$), express R_1 in terms of θ_E, θ_F, θ_G, and θ_H;

(3) At Joint E, write $\Sigma M = 0$, where all moments may now be written in terms of θ_E, θ_F, θ_G, θ_H, θ_I, and R_2; from this equation express θ_I in terms of θ_E, θ_F, θ_G, θ_H, and R_2. Similarly, working from $\Sigma M = 0$ at Joints F, G, and H express θ_J, θ_K, and θ_L, respectively, in terms of θ_E, θ_F, θ_G, θ_H, and R_2;

(4) The moments in the ends of the second-story columns may now be written in terms of θ_E, θ_F, θ_G, θ_H, and R_2, and the application of the shear-story equation to the second story, enables the designer to express R_2, and, hence, θ_I, θ_J, θ_K, and θ_L, in terms of the permanent unknowns;

(5) Working progressively up the building in this manner, express all θ-values and all R-values in terms of the permanent unknowns; four equations, one for each unknown, will still be available, namely, $\Sigma M = 0$ at Joints Q, R, S, and T. The formal solution of these four simultaneous equations yields values of θ_E, θ_F, θ_G, and θ_H; and,

(6) With all the R-values and all the temporary θ-values previously expressed in terms of the permanent unknowns, the moments in the structure are easily determined by the slope deflection equation.

Thus, the building frame shown in Fig. 1(*b*) is solved with the formal solution of only four simultaneous equations, one equation for each row of columns. This fact is independent of the number of stories. Symmetry would further reduce the labor since for the structure shown in Fig. 1(*b*), $\theta_E = \theta_H$ and $\theta_F = \theta_G$. Thus, a twenty-story, four-column, symmetrical bent[3] could be analyzed with the formal solution of only two simultaneous equations.

Illustrative Problem

To demonstrate the application of the proposed method to the case of building frames with shallow wind-braces, the structure shown in Fig. 2 may be solved, as follows: The shear-story equation applied to the first floor yields:

$$2\,[2E\,(2\,\theta_A - 3\,R_1) + 2E\,(\theta_A - 3\,R_1)] + 20\,(20) = 0$$

or,

$$3\,R_1 = 1.5\,\theta_A + \frac{100}{2E}$$

For the condition, $\Sigma M = 0$ at Joint A:

$$2\,\theta_A - 1.5\,\theta_A - \frac{100}{2E} + 3\,\theta_A + 2\,\theta_A + \theta_C - 3\,R_2 = 0$$

or,

$$\theta_C = 3\,R_2 + \frac{100}{2E} - 5.5\,\theta_A$$

Applied to the second floor, the shear-story equation yields:

$$2\left[\,2E\left(3\,\theta_A + 9\,R_2 + \frac{300}{2E} - 16.5\,\theta_A - 6\,R_2\right)\right] + 10 \times 20 = 0$$

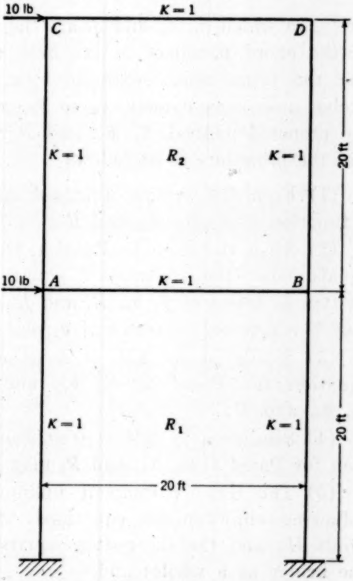

Fig. 2.

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[3] For example, see "Wind Stresses in the Steel Frames of Office Buildings", by W. M. Wilson and G. A. Maney, Members, Am. Soc. C. E., *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1915.

or,

$$3\ R_2 = 13.5\ \theta_A - \frac{400}{2\,E};\ \text{and,}\ \theta_C = 8\ \theta_A - \frac{300}{2\,E}$$

Similarly, for the condition, $\Sigma M = 0$ at Joint C:

$$2\left(8\ \theta_A - \frac{300}{2\,E}\right) + \theta_A - 13.5\ \theta_A + \frac{400}{2\,E} + 3\left(8\ \theta_A - \frac{300}{2\,E}\right) = 0$$

and $\theta_A = \frac{40}{2\,E}$. Therefore, the deflection ratio, R_1, is expressed by $3\ R_1 = \frac{160}{2\,E}$; and the moment at the column base is $M = 2\,E\left(\frac{40}{2\,E} - \frac{160}{2\,E}\right) = -120$ ft-lb, the minus sign denoting that the couple, M, acts contra-clockwise on the column.

Vierendeel Truss

For the structure shown in Fig. 1(*c*), let $R = \frac{d}{h}$, in which d is the relative horizontal deflection between the top and bottom of any of the vertical members of the truss. Let R_1, R_2, and R_3 equal $\frac{d_1}{p_1}$, $\frac{d_2}{p_2}$, and $\frac{d_3}{p_3}$, respectively, in which d_1, d_2, and d_3 are the relative vertical deflections of the ends of the chord members in the first, second, and third panels, respectively. For the usual slope deflection solution of this structure, there would be twelve unknowns namely, θ_A to θ_H, inclusive, and R, R_1, R_2, and R_3. For the proposed method, θ_A, θ_B, and R are taken as the permanent unknowns, and the procedure is as follows:

(1) From $\Sigma M = 0$ at Joints A and B, express θ_C and θ_D, respectively, as a function of θ_A, θ_B, R, and R_1;

(2) Since the shear in Panel 1, times p_1 plus $M_{AC} + M_{CA} + M_{BD} + M_{DB}$, equals zero (the shear-panel equation), and all these moments may be written in terms of θ_A θ_B, R, and R_1, it follows that R_1 and, thus, θ_C and θ_D may be expressed in terms of θ_A, θ_B, and R;

(3) Similarly, by $\Sigma M = 0$ at Joints C and D, and the shear-panel equation for Panel 2, θ_E, θ_F, and R_2 may be expressed in terms of θ_A, θ_B, and R;

(4) Similarly, by $\Sigma M = 0$ at Joints E and F, and the shear-panel equation for Panel 3, θ_G, θ_H, and R_3 may be expressed in terms of θ_A, θ_B, and R;

(5) The three permanent unknowns may now be evaluated from the following simultaneous equations: $\Sigma M = 0$ at Joint G; $\Sigma M = 0$ at Joint H; and the shear-story equation applied to the vertical members of the bridge as a whole; and,

(6) With θ_A, θ_B, and R known, and the temporary unknowns already expressed in terms of these permanent unknowns, the moments are easily evaluated by the slope deflection equation.

REMARKS

The foregoing examples show that it is necessary to take as permanent unknowns only one θ-value for each row of members parallel to the direction in which the solution is to progress across a structure, plus one R-value for each panel between such rows (if there is more than one row), whenever the construction is such that these R-values do not equal zero. For the formal solution of simultaneous equations, there will be as many equations as there are permanent unknowns. It should be evident that solution in this manner does not actually avoid the solution of simultaneous equations, but, rather, provides a means of eliminating the unknowns progressively. The usual slope deflection equations could be written simultaneously, and if solved in the order described in this paper, would lead to the same solution. It seems to the writer, however, that there is little likelihood of this being done without purposely following the line of reasoning herein outlined.

A portion of the underlying research which forms the basis of this paper was submitted to Massachusetts Institute of Technology by the writer in 1933 in partial fulfillment of the requirements for the degree of Doctor of Science. Other phases of the broader subject have been published elsewhere under the titles, "A New Method for Analyzing Stresses Due to Lateral Forces in Building Frames[4]", and "Distribution of Wind Loads to the Bents of a Building[5]."

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[4] *Journal*, Boston Soc. of Civ. Engrs., January, 1934, p. 45.

[5] *Loc. cit.*, October, 1935.

DISCUSSION

C. A. WILLSON,[6] M. AM. SOC. C. E. (by letter).—The slope deflection method of analyzing statically indeterminate structures is recognized by structural engineers as a valuable working tool. It is especially useful in the analysis of a complicated unsymmetrical framework. However, it is a fact that the solution of a large number of simultaneous equations can become very tiresome. Therefore, the method outlined by Professor Wilbur should be of real service to designing engineers who find it advisable to make complete analyses of structural frames.

The writer has found it convenient and sufficiently accurate in many cases to determine the bending moments in the members meeting at a particular joint by a consideration of sixteen members that radiate from it. In such a layout there are five unknown deflection angles, the one at the joint in question and four others. The writer has used the same idea advanced by Professor Wilbur of successive elimination of unknowns in the development of a simple solution.

The sixteen-member framework is preferable to the smaller frameworks frequently used because greater accuracy is obtained and because the effect of loading and unloading a number of members in the vicinity of the joint in question can be determined quickly.

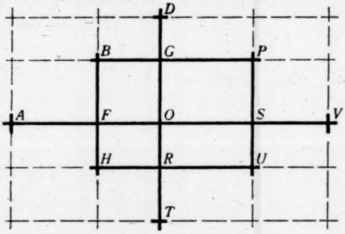

FIG. 3.

Let Fig. 3 represent a part of a building framework composed of beams and columns of different lengths, moments of inertia, and loadings. Assume that bending moments are to be determined at Joint O because of the unsymmetrical conditions that surround that joint. Assume fixed-end terminals at a distance of two joints removed from Joint O in all directions. The resulting partial structure of sixteen members is indicated by heavy lines in Fig. 3. To summarize, fixed ends are assumed at A, B, D, P, V, U, T, and H, and bending moments are to be determined at Joint O.

The moments in members meeting at Joint R will be as follows:

$$M_{RT} = 4\ E\ \theta_R\, K_{RT} \quad \text{.......................} (5)$$

$$M_{RH} = 4\ E\ \theta_R\, K_{RH} + C_{RH} \quad \text{..................} (6)$$

$$M_{RO} = 2\ E K_{OR}\,(2\ \theta_R + \theta_O) \quad \text{..................} (7)$$

and,

$$M_{RU} = 4\ E\ \theta_R\, K_{RU} - C_{RU} \quad \text{..................} (8)$$

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[6] Structural Engr., State Archt.'s Office, Madison, Wis.

Let $C_R = C_{RH} - C_{RU}$ and let $\Sigma K_R = K_{RT} + K_{RH} + K_{RO} + K_{RU}$. With $\Sigma M = 0$ at Joint R:

$$0 = 4 E \theta_R \Sigma K_R + 2 E \theta_O K_{OR} + C_R \dots\dots\dots\dots (9)$$

Then,

$$E \theta_R = -\frac{2 E \theta_O K_{OR} + C_R}{4 \Sigma K_R} \dots\dots\dots\dots (10)$$

Similarly,

$$E \theta_F = -\frac{2 E \theta_O K_{OF} + C_F}{4 \Sigma K_F} \dots\dots\dots\dots (11)$$

$$E \theta_G = -\frac{2 E \theta_O K_{OG} + C_G}{4 \Sigma K_G} \dots\dots\dots\dots (12)$$

and,

$$E \theta_S = -\frac{2 E \theta_O K_{OS} + C_S}{4 \Sigma K_S} \dots\dots\dots\dots (13)$$

The moments in members meeting at Joint O will be as follows:

$$M_{OR} = 2 E K_{OR} (2 \theta_O + \theta_R) = 4 E \theta_O K_{OR} - \frac{E \theta_O (K_{OR})^2}{\Sigma K_R} - \frac{C_R K_{OR}}{2 \Sigma K_R} \dots (14)$$

$$M_{OF} = 2 E K_{OF} (2 \theta_O + \theta_F) + C_{OF} = 4 E \theta_O K_{OF} - \frac{E \theta_O (K_{OF})^2}{\Sigma K_F}$$

$$- \frac{C_F K_{OF}}{2 \Sigma K_F} + C_{OF} \dots\dots\dots\dots (15)$$

$$M_{OG} = 2 E K_{OG} (2 \theta_O + \theta_G) = 4 E \theta_O K_{OG} - \frac{E \theta_O (K_{OG})^2}{\Sigma K_G} - \frac{C_G K_{OG}}{2 \Sigma K_G} \dots (16)$$

and,

$$M_{OS} = 2 E K_{OS} (2 \theta_O + \theta_S) - C_{OS} = 4 E \theta_O K_{OS} - \frac{E \theta_O (K_{OS})^2}{\Sigma K_S}$$

$$- \frac{C_S K_{OS}}{2 \Sigma K_S} - C_{OS} \dots\dots\dots\dots (17)$$

With $\Sigma M = 0$ at Joint O and $\Sigma K_O = K_{OR} + K_{OF} + K_{OG} + K_{OS}$:

$$E_O \left[4 \Sigma K_O - \frac{(K_{OR})^2}{\Sigma K_R} - \frac{(K_{OF})^2}{\Sigma K_F} - \frac{(K_{OG})^2}{\Sigma K_G} - \frac{(K_{OS})^2}{\Sigma K_S} \right]$$

$$= \frac{C_R K_{OR}}{2 \Sigma K_R} + \frac{C_F K_{OF}}{2 \Sigma K_F} + \frac{C_G K_{OG}}{2 \Sigma K_G} + \frac{C_S K_{OS}}{2 \Sigma K_S} - C_O \dots\dots (18)$$

and,

$$E\,\theta_O = \frac{\dfrac{C_R\,K_{OR}}{2\,\Sigma K_R} + \dfrac{C_F\,K_{OF}}{2\,\Sigma K_F} + \dfrac{C_G\,K_{OG}}{2\,\Sigma K_G} + \dfrac{C_S\,K_{OS}}{2\,\Sigma K_S} - C_O}{4\,\Sigma K_O - \dfrac{(K_{OR})^2}{\Sigma K_R} - \dfrac{(K_{OF})^2}{\Sigma K_F} - \dfrac{(K_{OG})^2}{\Sigma K_G} - \dfrac{(K_{OS})^2}{\Sigma K_S}} \quad \ldots\ldots\ldots(19)$$

When the value of the deflection angle at Joint O is determined by Equation (19), the values of the other four unknown deflection angles (Equations (10) to (13)) can be found easily. Then, with known values of the five deflection angles, it is a simple matter to calculate the desired bending moments.

PAUL ANDERSEN,[7] ASSOC. M. AM. SOC. C. E. (by letter).—A clear and interesting method of setting up the slope deflection equations necessary for the analysis of continuous structures, is contained in this paper. It should be noted, however, that the method of reducing the number of unknown quantities, as presented by the author, is one which suggests itself immediately after all the "joint" and "bent" equations are written, and as such the idea is not unknown to designers. Professor Wilbur's procedure is to eliminate unknown quantities as the equations are set up rather than afterward. In all cases the author's final equations can be derived by writing the total number of slope deflection relations and proceeding as outlined in the paper. Furthermore, designers who make use of the slope deflection method are not, as a rule, objecting to the large number of simultaneous equations; a careful tabulation and the use of two or three successive approximations will yield results which are very close to the actual rotations.

In the case of secondary stress analysis the author's method will involve more work than the usual process, the permanent and the temporary unknowns being the rotations of bottom chord joints and top chord joints, respectively.

Professor Wilbur's method can be used to advantage in dealing with a continuous beam; the slope deflection equations for this type of structure are so simple and few that much unnecessary writing is avoided by choosing one permanent unknown. It is interesting to note, however, that an application of the same principle to the theorem of three moments results in a still larger saving. Thus, referring to Fig. 1(a):

$$2\,M_B\left(\frac{1}{K_{AB}} + \frac{1}{K_{BC}}\right) + \frac{M_C}{K_{BC}} = A_{BC} \quad \ldots\ldots\ldots\ldots\ldots(20)$$

in which A_{BC} can be computed from the position and magnitude of the load, P, and K_{BC}. Express M_C in terms of M_B.

Applying the three-moment theorem to $B\,C\,D$:

$$\frac{M_B}{K_{BC}} + 2\,M_C\left(\frac{1}{K_{BC}} + \frac{1}{K_{CD}}\right) + \frac{M_D}{K_{CD}} = A_{CB} \ldots\ldots\ldots\ldots(21)$$

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[7] Balboa Heights, Canal Zone.

in which A_{CB} likewise is known. After substituting for M_C the value in Equation (20), it is seen that from Equation (21), M_D is obtained in terms of M_B.

Applying the three-moment theorem to $C\,D\,E$:

$$\frac{M_C}{K_{CD}} + 2\,M_D\left(\frac{1}{K_{CD}} + \frac{1}{K_{DE}}\right) = 0 \ldots\ldots\ldots\ldots\ldots (22)$$

In Equation (22) substitute the values of M_C and M_D in terms of M_B, and solve for M_B.

The application of the theorem of three moments has the advantage of leading directly to the bending moments without computing first the changes of slope in which the designer is generally not interested.

R. W. STEWART,[8] M. AM. SOC. C. E. (by letter).—Systemization is the essence of slope deflection and this is recognized in the author's paper. However, it appears that one opportunity for better systemization has been overlooked; that is, improvement in the rules for signs in the use of the fixed-end moment constants.

The author's Equation (4) terminates with the term, $\pm C$. Authoritative treatises[9] would invert this double sign, making it $\mp C$. A study of several available expositions of these sign rules has convinced the writer that the use of a single minus sign for the fixed-end moments throughout all the standard equations will operate more satisfactorily than the double sign usually published, or the alternate plus and minus signs sometimes used.[10] Furthermore, the usual explanatory statements can be curtailed with greater resulting clarity. To illustrate this in an introductory manner the following solution of the author's Fig. 1 is offered, assigning numerical values to the load, its position, and the relative stiffnesses $\left(\frac{I}{L}\text{- values}\right)$ of the members. In this illustration the fixed-end moments are considered as the activating moments which a loaded span would exert on its end joints if these joints were fixed.

The following rules of signs are offered as completely covering all conditions: (1) Clockwise rotations are taken as positive, the reverse as negative; (2) moments that tend to produce clockwise rotation are considered positive, the reverse negative; and (3) deflections that produce clockwise rotations are considered positive, the reverse negative. Any further statements the writer feels to be dangerous and likely to cause confusion, except that the reader may be asked to observe the obvious fact that a moment or a deflection at a point in a beam which tends to cause the beam on one side of it to rotate in a clockwise direction will tend to cause the part on the other side to rotate in a contra-clockwise direction. This is analogous to the shear changing from positive to negative under the load when a simple beam is loaded with a single concentrated load.

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[8] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

[9] *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.; also, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 75.

[10] "Modified Slope Deflection", by L. T. Evans, Assoc. M. Am. Soc. C. E., *Journal*, Am. Concrete Inst., October, 1931.

In the author's program for solving Fig. 1, the solution is carried from the left end of the beam to the right, thereby necessitating carrying the constants for the fixed-end moments through the equations for the two unloaded spans, $C\,D$ and $D\,E$. An easier program will be to work from right to left, as is done in the following solution, thereby carrying these constants through the equations for a lesser number of spans. The fixed-end moments with proper signs and directional arrows are, in accordance with the foregoing conventionality, indicated on Fig. 4. The standard equations used are the same as those in the various existing texts except that in all standard equations the fixed-end moment constants are preceded by a minus sign only—not a double sign, nor alternating plus and minus signs. Relative stiffness or $\frac{I}{L}$ - values are indicated. Referring to Fig. 4:

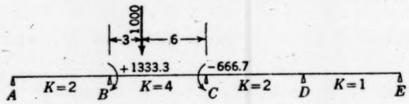

FIG. 4.

$$M_{ED} = 2\,(2\,\theta_E + \theta_D) = 0 \,\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

and, $\theta_D = -\,2\,\theta_E$;

$$M_{DE} + M_{DC} = 0 = 2\,(-\,4\,\theta_E + \theta_E) + 2 \times 2\,(-\,4\,\theta_E + \theta_C) \,\ldots \quad (24)$$

and, $\theta_C = 5.5\,\theta_E$;

$$M_{CD} + M_{CB} = 0 = 2 \times 2\,(11\,\theta_E - 2\,\theta_E) + 2 \times 4\,(11\,\theta_E + \theta_B) + 666.7 \,.. \quad (25)$$

and, $\theta_B = -\,15.5\,\theta_E - 83.33$;

$$\begin{aligned} M_{BC} + M_{BA} = 0 = 2 \times 4\,(-\,31\,\theta_E - 166.7 + 5.5\,\theta_E) \\ -\,1\,333.3 + 2 \times 2\,(-\,31\,\theta_E - 166.7 + \theta_A) \,\ldots\ldots\ldots\ldots \quad (26) \end{aligned}$$

and, $\theta_A = 82\,\theta_E + 833.3$;

$$M_{AB} = 0 = 2 \times 2\,(164\,\theta_E + 1\,666.7 - 15.5\,\theta_E - 83.33) \,\ldots\ldots \quad (27)$$

and, $\theta_E = -\,10.67$.

Comments.—In Equation (25) the fixed-end moment has the plus sign because it is negative in Fig. 4 and is preceded by a minus sign in the standard equation, and − × − = +. The fixed-end moment of 1 333.3 is plus in Fig. 4 and minus in the standard equation making it + × − = − in Equation (26).

If the load acted upward, the signs in Fig. 4 would be reversed, thereby reversing the signs for these terms in Equations (25) and (26) and yielding $\theta_E = +\,10.67$. Likewise, the constant, H_{AB}, familiar to users of slope deflection, may always be preceded by the minus sign only in the standard equations provided it is delineated on the diagram in accordance with the convention stated.

The author's numerical solution of Fig. 2, using θ_A and the first-story R-value as the permanent unknowns, is of interest in that, apparently, it displays the full possibilities of slope deflection for this problem, which involves side-sway and is not so conveniently solved by end-moment distribution.

However, it is of interest and probably useful to show the following solution for this problem in which the phenomena of flexure are so treated as to eliminate the R-terms (deflection terms) which are necessary in the author's solution, and, otherwise, to simplify and reduce materially the work.

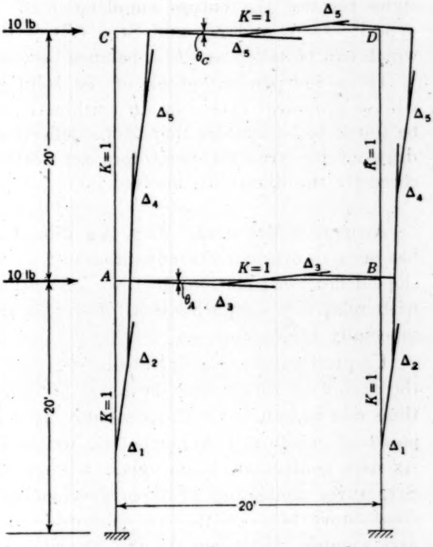

FIG. 5.

Referring to Fig. 5:

$$(A)\ldots.2\,(2\,\Delta_1 + 2\,\Delta_2) = 20 \times 20$$

and, $\Delta_1 = 100 - \Delta_2$;

$$(B)\ldots.\theta_A = 100 - \Delta_2 - \Delta_2 \ldots\ldots\ldots\ldots\ldots\ldots\Delta_2 = 50 \quad - \quad 0.5 \;\; \theta_A$$

$$(C)\ldots.2\,(2\,\Delta_4 + 2\,\Delta_5) = 10 \times 20$$

and, $\Delta_5 = 50 - \Delta_4$;

$$(D)\ldots.\theta_A = \tfrac{1}{3}\,(50 - \Delta_4) + 50 - \Delta_4 - \Delta_4 \ldots\ldots\Delta_4 = 28.6 - \;\; 0.43\;\theta_A$$

$$(E)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\underline{-\Delta_3 = \qquad\quad - \;\; 3.0 \;\; \theta_A}$$

$$(F)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\Sigma\,\Delta = 0 = 78.6 - \;\; 3.93\;\theta_A$$

and, $\theta_A = \dfrac{20}{E}$. The moment at the column base $= 2(100 - 50 + 10) = 120$.

Comments.—Since the stiffness factors of all members are equal and may be taken as unity, each moment will be double its corresponding Δ.

In Step (A), the sum of the internal moments is equated to the external moment for the first story; in Step (B), the column angles of the left lower column are added to obtain an expression for θ_A; Step (C) is the same as Step (A), except that it is for the second story; Step (D) is the same process as Step (B), except that it is for the left upper column; Step (E) is written directly from the geometry of the figure[11] and the terms are given negative signs because the obtuse supplement of Δ_3 indicates rotational activation opposite to Δ_3 and Δ_4; and Step (*F*) is the moment balance about Joint *A* which can be taken as the Δ-balance because all *K*-factors are equal.

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[11] "An Improved Method of Finding Beam Deflections", *Civil Engineering*, February, 1934, p. 88; also "Analysis of Continuous Structures by Traversing the Elastic Curves", *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

If an independent check of the solution is desired, the traverse method with a graphical check, or an arithmetical angle summation check, will also be found to be simpler than slope deflection for Fig. 4. If the procedure for drawing the traverse heretofore set forth[12] is followed, it is easy to draw correctly the diagrams involved.

ADOLPHUS MITCHELL,[13] JUN. AM. SOC. C. E. (by letter).—The recent trend has been to overlook the advantages of an algebraic procedure of applying the slope-deflection equations in favor of arithmetical methods even when dealing with relatively simple frames. For this reason Professor Wilbur's ideas are especially interesting.

Unquestionably, algebraic solutions are preferable when analyzing frames, the design of which must be made at frequent intervals, whenever the equations can be handled with reasonably little effort. Among the structures composed of constant *I* members, the writer has found that triple culverts and six-span continuous beams yield without difficulty to the algebraic solution. Structures consisting of tapering members are more difficult to solve, but single-story bents with two columns and three-span continuous beams are easy enough. However, in the writer's opinion, a skeleton type of building frame, such as that described by the author, should, unhesitatingly, be solved by fixation factors[14], or by moment distribution.[15] If the slope-deflection method is applied, the use of Gauss' normal equations[16] will be found valuable as it affords many checks as the solution proceeds.

The importance of arranging the solution so as to produce the greatest degree of accuracy cannot be over-emphasized. Although in the schoolroom it is sufficient to gain the principles involved, in the design office, the quantitative results must be correct as well. In the treatment of continuous beams and frames of limited complexity it is best to consider all members loaded and to solve the problem in such a manner that a symmetry of form in the equations is preserved throughout the analysis. One will soon find this an inestimable aid to accuracy.

Consider the problem of a continuous beam over five supports and with equal spans, as shown in Fig. 6. Let $A = -C_{1,2}$; $B = C_{2,1} - C_{2,3}$; $C = C_{3,2} - C_{3,4}$; $D = C_{4,3} - C_{4,5}$; and $E = +C_{5,4}$; and notice that for uniform loading over all spans the loading

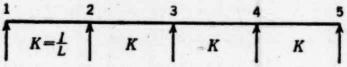

FIG. 6.

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[12] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 136.

[13] Santa Fe, N. Mex.

[14] "Structural Frameworks", by T. F. Hickerson, M. Am. Soc. C. E., Univ. of North Carolina Press., Chapel Hill, N. C.

[15] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[16] "Secondary Stresses in Bridges", by Cecil von Abo, Assoc. M. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 107; also, "Method of Least Squares", by Merriman, N. Y., John Wiley & Sons.

factors, B, C, and D, equal zero. Next, write the equation for each support, omitting EK for simplicity; thus:

$$M_{1,2} = 0 = 2\,\theta_1 + \theta_2 + A$$

$$M_{2,1} + M_{2,3} = 0 = 4\,\theta_2 + \theta_3 + \theta_1 + B$$

$$M_{3,2} + M_{3,4} = 0 = 4\,\theta_3 + \theta_4 + \theta_2 + C$$

$$M_{4,3} + M_{4,5} = 0 = 4\,\theta_4 + \theta_5 + \theta_3 + D$$

and,

$$M_{5,4} = 0 = 2\,\theta_5 + \theta_4 + E$$

To eliminate the unknowns, begin simultantously at each end, as indicated by the arrows:

$$\theta_1 = -\frac{1}{2}\,\theta_2 - \frac{1}{2}\,A$$

$$\frac{7}{2}\,\theta_2 + \theta_3 + B - \frac{1}{2}\,A = 0; \text{or } \theta_2 = -\frac{2}{7}\,\theta_3 - \frac{2}{7}\,B + \frac{1}{7}\,A$$

$$\frac{26}{7}\,\theta_3 + \theta_4 - \frac{2}{7}\,B + \frac{1}{7}\,A + C = 0;$$

$$\text{or, } \theta_3 = -\frac{7}{26}\,\theta_4 - \frac{7}{26}\,C + \frac{2}{26}\,B - \frac{1}{26}\,A$$

$$\theta_3 = \frac{2}{26}\,\theta_3 + \frac{2}{26}\,D - \frac{1}{26}\,E - \frac{7}{26}\,C + \frac{2}{26}\,B - \frac{1}{26}\,A \longrightarrow \theta_3$$

$$= \frac{1}{24}\Big[-A + 2\,B - 7\,C + 2\,D - E\Big]$$

$$\frac{7}{2}\,\theta_4 + \theta_3 + D - \frac{1}{2}\,E = 0; \text{or } \theta_4 = -\frac{2}{7}\,\theta_3 - \frac{2}{7}\,D + \frac{1}{7}\,E$$

$$\theta_5 = -\frac{1}{2}\,\theta_4 - \frac{1}{2}\,E$$

Finally:

$$\theta_2 = \frac{1}{84}\,[13\,A - 26\,B + 7\,C - 2\,D + E]$$

$$\theta_4 = \frac{1}{84}\,[A - 2\,B + 7\,C - 26\,D + 13\,E]$$

$$\theta_1 = -\frac{1}{168}[97\,A - 26\,B + 7\,C - 2\,D + E]$$

and,

$$\theta_5 = -\frac{1}{168}[A - 2B + 7C - 26D + 97E]$$

Substituting in the fundamental equations.

$$M_{2,1} = 2\,\theta_2 + \theta_1 + C_{2,1} = \frac{1}{56}[-15A - 26B + 7C - 2D + E] + C_{2,1}$$

$$M_{3,2} = 2\,\theta_3 + \theta_2 + C_{3,2} = \frac{1}{56}[+4A - 8B - 28C + 8D - 4E] + C_{3,2}$$

and,

$$M_{4,5} = 2\,\theta_4 + \theta_5 - C_{4,5} = \frac{1}{56}[+A - 2B + 7C - 26D - 15E] - C_{4,5}$$

Notice the symmetry preserved in each step. The writer now wishes to justify his claim that "it is best to consider all members loaded." There are almost always limiting cases for which the solution of a problem can be found. In this problem it is the condition of uniform load over all spans, which may be used as a check. For this case, $A = -\frac{1}{12}\,w\,L^2$; $B = 0$; $C = 0$; $D = 0$; $E = +\frac{1}{12}\,w\,L^2$; and $M_{3,2} = \frac{1}{56}\left(\frac{1}{12}w\,L^2\right)(-4\;-4) + \frac{1}{12}\,w\,L^2 = \frac{2}{28}\,w\,L^2$, which value can be found in any handbook. Although this check does not necessarily indicate that every part of a solution is correct, failure to obtain it does reveal that a mistake has been made. The thoroughness of this method of checking depends upon the number of known cases available. If there is a case requiring the retention of each loading term, the check may be regarded as absolute.

If the reader would write out a solution carrying θ_1 as the unknown throughout the evaluation of the rotations as suggested by the author, he would more readily appreciate the foregoing procedure

FANG-YIN TSAI,[17] ASSOC. M. AM. SOC. C. E. (by letter).—By a simple process the method presented by Professor Wilbur can be adjusted to apply to the case of continuous structures with variable moments of inertia.

Slope-deflection equations for the case of variable moments of inertia have been presented by L. T. Evans[18], Jun. Am. Soc. C. E., and also by the writer[19], both results being almost identical. These equations involve certain coefficients known as angle changes which express the effect of variable moment of inertia, and the forms of these equations are rather cumbersome, and, therefore, inconvenient for use in this method. The writer has deduced the following equations, using the various constants of the method of moment distribution developed by Hardy Cross[20], M. Am. Soc. C. E.:

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[17] Prof. of Structural Eng., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

[18] "The Modified Slope-Deflection Equations", by L. T. Evans, *Journal*, Am. Concrete Inst., October, 1931, p. 109.

[19] "Slope-Deflection Equations for the Analysis of Rigid Frames with Varying Moment of Inertia", by Fang-Yin Tsai, The Science Repts., National Tsing Hua Univ., Peiping, China, Series A, Vol. II, p. 75, July, 1933.

$$M_{ab} = S_a\,[\theta_a + C_{ab}\,\theta_b - (1 + C_{ab})\,R] \mp M_{ra} \dots\dots\dots (28)$$

and,

$$M_{ba} = S_b\,[\theta_b + C_{ba}\,\theta_a - (1 + C_{ba})\,R] \pm M_{rb} \dots\dots\dots (29)$$

in which C_{ab} and C_{ba} are factors to carry over the moment applied at Ends a and b, respectively, to the other ends, b and a, which are assumed fixed; S_a and S_b are stiffness factors when a unit moment is applied at the designated end with the other end fixed; and M_{ra} and M_{rb} are fixed-end moments at the designated ends, due to loading, with both ends fixed. For a member of which the moment of inertia varies symmetrically with respect to its center line:

$$C_{ab} = C_{ba} = C \dots\dots\dots\dots\dots\dots\dots\dots (30)$$

and,

$$S_a = S_b = S \dots\dots\dots\dots\dots\dots\dots\dots (31)$$

For a member of constant cross-section,

$$C = \frac{1}{2} \dots\dots\dots\dots\dots\dots\dots\dots (32)$$

and,

$$S = 4\,E\,K \dots\dots\dots\dots\dots\dots\dots\dots (33)$$

in which E and K are the same as the notation of the paper. With the values of C and S from Equations (32) and (33) substituted in Equations (28) and (29), the well-known ordinary slope-deflection equations for the case of constant moment of inertia are derived.

It may be noted that, according to the usual sign convention for moment adopted in the slope-deflection method, the carry-over factors, C_{ab} and C_{ba}, will always be positive, which is just opposite to that adopted in the method of moment distribution.

The determination of the values of the various constants in Equations (28) and (29) for members with moments of inertia varying in any manner and under any loading, the method of moment area, as shown by G. E. Large[21], Assoc. M. Am. Soc. C. E., will be found the most expedient. For certain special cases, there are many sets of tables and diagrams which give directly the values of those constants. Hence, with Equations (28) and (29) available, the application of this method to the case of variable moment of inertia will involve no difficulty or inconvenience.

A. FLORIS,[22] ESQ. (by letter).—This ingenious method of analyzing statically indeterminate structures by means of the slope-deflection equation reduces, considerably, the number of the unknowns arising in such problems.

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[20] "Analysis of Continuous Frames by Distributing Fixed-End Moments", by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[21] *Bulletin No. 66*, Ohio State Univ., Columbus, Ohio, November, 1932.

[22] Dipl.-Ing., Los Angeles, Calif.

It is elegant, direct, and does not involve approximations. Thus far, there are several methods available which render the analysis of structures with high redundancy comparatively easy. From the point of view of practical expediency simultaneous equations should be avoided, or their solution, at least, should be facilitated by special devices.

The avoidance of simultaneous equations by means of the moment distribution leads to tedious and indirect, if not difficult, processes. On the other hand, the method of iteration applied to the solution of simultaneous equations of a special kind, although convenient, is nevertheless time-consuming.

The author's method is a valuable contribution to the analysis of complicated structures. In principle, it is preferable to many widely used methods. However, whether it will replace them in practice is another matter. The question can be decided only by actual experience or comparative calculations.

A. W. FISCHER,[23] ESQ. (by letter).—The author has contributed a method for solving indeterminate structures by the slope-deflection method by what he terms "successive elimination", which seems to solve the simple examples he gives in a very short time. For solving secondary stresses in trusses by the slope-deflection method, however, the writer doubts if the author's method is any shorter than the use of a systematic table for solving the simultaneous equations.

Much has been written concerning the solution of rectangular frames by the slope-deflection method, but not much has been offered in regard to the solution of a symmetrical frame with inclined legs, supporting unsymmetrical loads. Consider, for example, the top story of the Kinzua Viaduct[24] with a horizontal load at the top (see Fig. 7). Assume the bases to be fixed and then compare the results with those that are obtained by the method of least work, which gives correct results. This bent is symmetrical and, using the author's method, sufficient equations can be written to solve it.

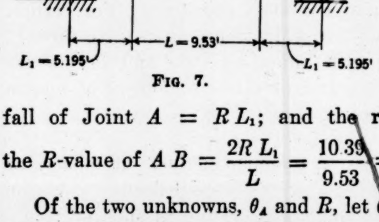

FIG. 7.

The change of length of members is considered zero; and, since the bent is symmetrical, $\theta_A = \theta_B$, $\theta_C = \theta_D = 0$. The horizontal movement at the top of Members AD and $BC = Rh$, and is to the right; the fall of Joint $A = R L_1$; and the rise of Joint $B = R L_1$. Therefore, the R-value of

$$A B = \frac{2R L_1}{L} = \frac{10.39}{9.53} = 1.09\,R.$$

Of the two unknowns, θ_A and R, let θ_A be the permanent unknown. As E is constant, it can be assumed equal to unity, which will shorten the equations.

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[23] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

[24] "The Kinzua Viaduct of the Erie Railroad Company", by the late Carl Robert Grimm, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVI (1901), p. 21.

For the condition, $\Sigma M = 0$, at Joint A, $M_{AD} + M_{AB} = 0$. Therefore, $2 \times 340.3\ (2\,\theta_A - 3\,R) + 2 \times 11\,280\ (3\,\theta_A + 3.27\,R) = 0$; from which,

$$R = -\,0.9625\ \theta_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

A second true equation can be developed by considering the equilibrium of Member AB. Let s_1 and s_2 be the direct stresses of Members AD and BC; then:

$$s_1 \sin\phi + \frac{680.6\ (3\,\theta_A - 6\,R)\cos\phi}{h\csc\phi} - \frac{22\,560\ (6\,\theta_A + 6.54\,R)}{L} = 0 \ldots (35)$$

$$s_2 \sin\phi - \frac{680.6\ (3\,\theta_A - 6\,R)\cos\phi}{h\csc\phi} + \frac{22\,560\ (6\,\theta_A + 6.54\,R)}{L} = 0 \ldots (36)$$

and,

$$s_1 \cos\phi - \frac{680.6\ (3\,\theta_A - 6\,R)\sin\phi}{h\csc\phi} - 26.8 - s_2\cos\phi$$

$$- \frac{680.6\ (3\,\theta_A - 6\,R)\sin\phi}{h\csc\phi} = 0 \ldots\ldots\ldots\ldots (37)$$

From Equation (35),

$$s_1 = \frac{22\,560\ (6\,\theta_A + 6.54\,R)}{L\sin\phi} - \frac{680.6\ (3\,\theta_A - 6\,R)\cos\phi}{h} \ldots\ldots (38)$$

and, from Equation (36),

$$s_2 = \frac{680.6\ (3\,\theta_A - 6\,R)\cos\phi}{h} - \frac{22\,560\ (6\,\theta_A + 6.54\,R)}{L\sin\phi} \ldots (39)$$

Substituting the values from Equations (38) and (39) in Equation (37) and reducing:

$$\frac{66.30\ (6\theta_A + 6.54\,R)\cos\phi}{L\sin\phi} - \frac{2\ (3\,\theta_A - 6\,R)}{h} - \frac{26.8}{680.6} = 0 \ldots (40)$$

Equation (40) also expresses the simple relation:

$$2\,M_{AD} + 2\,M_{DA} + \frac{4\,L_1}{L}\,M_{AD} + 26.8\,h = 0 \ldots\ldots\ldots (40a)$$

Substituting the value of R from Equation (34) and $\sin\phi$ and $\cos\phi$ from Fig. 7, and reducing: $\theta_A = -\,0.04343$; and, then, from Equation (34), $R = +\,0.04180$.

From the foregoing values of θ_A and R:

$$M_{AD} = 2 \times 340.3\ (-\,0.08686 - 0.12540) = -\,144.5 \text{ ft-kips}$$

$$M_{AB} = 2 \times 11\,280\ (-\,0.08686 - 0.04343 + 0.13670) = 144.6 \text{ ft-kips}$$

and,

$$M_{DA} = 2 \times 340.3\ (-\,0.04343 - 0.12540) = -\,114.9 \text{ ft-kips}$$

For the upper story, with fixed bases, Mr. Grimm[24], found that,

$$M_{AB} = 4.765 \times 30.4 = +144.9 \text{ ft-kips}$$

$$M_{AD} = -144.9 \text{ ft-kips}$$

and,

$$M_{DA} = 9.69 \times 30.4 - 31.16 \times 13.4 = -114.7 \text{ ft-kips}$$

On comparing the foregoing results it can be seen that the two different methods practically check. (All calculations in this discussion were made with a 20-in. slide-rule.)

The general equations for solving a symmetrical bent with inclined legs, fixed bases, and various loads derived by "relative deflections"[25] and modified to solve the foregoing example are:

$$M_{AD} = -\frac{P\,L\,n\,h\,(3\,L + 4\,L_1)}{2\left\{L^2\,(1 + 6\,n) + 12\,L\,L_1\,n + 8\,n\,L_1^2\right\}} \qquad \text{(41)}$$

and,

$$M_{DA} = -\frac{P\,L\,h\left\{L\,(1 + 3\,n) + 2\,L_1\,n\right\}}{2\left\{L^2\,(1 + 6\,n) + 12\,L\,L_1\,n + 8\,n\,L_1^2\right\}} \qquad \text{(42)}$$

in which $n = \frac{K_1}{K_2}$. Substituting the values given in Fig. 7 in Equations (41) and (42), and reducing: $M_{AD} = -144.7$ ft-kips; and, $M_{DA} = -115.1$ ft-kips. These values, again, practically agree with those determined by Mr. Grimm[24].

If $L_1 = 0$, Equations (41) and (42) can be used to solve a symmetrical rectangular bent fixed at the bases and a horizontal load, P, at the top.

The author's method can be used for solving many problems in indeterminate structures, but to the writer it is just one tool, of many, which solves such simple examples as those given by the author in a very short time. A problem such as the analysis of a Vierendeel truss, with inclined top chord, however, is not so simple, and if the author would analyze a three-panel Vierendeel truss with an inclined top chord, the analysis might supply additional data for solving indeterminate structures with inclined members.

L. E. GRINTER,[26] ASSOC. M. AM. SOC. C. E. (by letter).—There is no question but that the author has made an excellent case for his procedure of automatically eliminating the unknowns in the slope-deflection equations. In fact, Professor Wilbur has hit upon a major reason why the slope-deflection method has lost some of its original popularity. The lack of a standard procedure for solving the simultaneous equations involved had led to considerable dissatisfaction with the method itself. The truth is that a confused sign convention and the difficulties involved in the solution of the simultaneous equations are the major objections that have been offered to its use. A physical picture of the relation of moment, sign, and curvature will eliminate the confusion in regard to signs, and the automatic procedure of eliminating unknowns suggested by Professor Wilbur will aid in simplifying the solution of the simultaneous equations.

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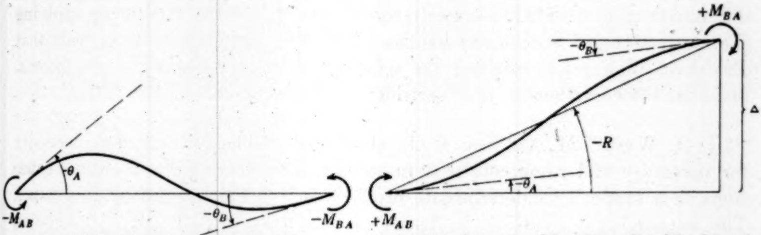
[25] "Stresses in Statically Indeterminate Structures", by Prof. H. Yu, National Wuhan Univ., Wuchang, Hupeh, China, Second Edition, 1935, pp. 471–474.

[26] Dept. of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

Physical Significance of Signs.—The question of signs naturally is involved in the illustrative example presented in the paper. This example ends with the statement, "and the moment at the column base is $M = 2\,E\left(\frac{40}{2\,E} - \frac{160}{2\,E}\right) = -120$ ft-lb, the minus sign denoting that the couple, M, acts contra-clockwise on the column." This statement without further explanation will not be understood by many readers. The signs of the end moments of an unloaded member will be made clear by Fig. 8. Such end moments are calculated by the standard slope-deflection equation, $M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R)$. Evidently, therefore, the sign of the moment will be dependent upon the signs and relative magnitudes of θ_A, θ_B, and R.



$M_{AB} = 2EK(-2\theta_A - \theta_B)$ (Negative)
$M_{BA} = 2EK(-2\theta_B - \theta_A)$ (Negative)

(*a*) END ROTATIONS WITHOUT RELATIVE TRANSLATION

$M_{AB} = 2EK(-2\theta_A - \theta_B + 3R)$ (Positive Since $3R > 2\theta_A + \theta_B$)
$M_{BA} = 2EK(-2\theta_B - \theta_A + 3R)$ (Positive Since $3R > 2\theta_B + \theta_A$)

(*b*) END ROTATIONS AND RELATIVE TRANSLATION

FIG. 8.—SIGNS OF END MOMENTS.

For the ordinary case of reversed curvature shown in Fig. 8(*a*), in which the value of R is zero, the sign of M is the same as the sign of the end slopes, θ_A and θ_B. The end rotations are counter-clockwise (negative) for the case illustrated, so that both the end moments, M_{AB} and M_{BA}, must be negative. Accordingly, the end moments represented by M_{AB} and M_{BA} must be the negative resisting moments external to the beam itself. Similarly, in Fig. 8(*b*), the geometry of the configuration determines the signs of the end moments. For the configuration shown, the value of $3\,R$ must be larger than $2\,\theta_A + \theta_B$ or $2\,\theta_B + \theta_A$. This is merely a geometric fact that any one can prove by attempting to re-sketch the diagram, assuming the reversed inequality. (For instance, when $3\,R = 2\,\theta_A + \theta_B = 2\,\theta_B + \theta_A$, the elastic line would be straight and the end moments would be zero.) Hence, for the case illustrated in Fig. 8(*b*), the end moments are of the same sign as the sign of R in the slope-deflection equations; that is, positive. Again, these end moments must be the resisting moments that act externally to the beam itself in order to agree with these signs.

Advantages of a Routine Procedure.—The usefulness of Professor Wilbur's procedure for the successive elimination of unknowns will depend very largely upon the number of simultaneous equations involved in the standard solution; that is, upon the number of unknowns. The continuous beam illustrated in Fig. 1(*a*) of the paper is indeterminate only to the third degree. The three simultaneous equations in any standard analysis can be solved so simply and conveniently that the successive elimination of unknowns would offer little advantage. However, the wind frame of Fig. 1(*b*) is a highly indeterminate structure the analysis of which would require the solution of twenty simultaneous equations, a tedious and wholly objectionable task. Certainly, there could be no justification of the use of the slope-deflection method for the analysis of this frame, unless some automatic procedure, such as that suggested by the author, was to be used.

Undoubtedly, many engineers have been applying the procedure suggested by the author, but in a rather haphazard fashion. The writer has found his students quite confused by the fact that different members of the class would seem to obtain the solution for a problem such as Fig. 1(*a*) by solving different numbers of simultaneous equations. In effect, they were applying the suggestions of the author haphazardly. The writer, therefore, feels that Professor Wilbur has clarified the solution of the slope-deflection equations, and that his contribution is of considerable importance.

L. T. WYLY,[27] M. AM. SOC. C. E. (by letter).—The oft-remarked necessity for ingenuity and resourcefulness in solving a number of simultaneous equations in a statically indeterminate problem is well emphasized by this paper. The author, by expressing in terms of one unknown all other unknowns, arrives at an exact value of each in turn. It is interesting to compare this method of attack with that of successive approximations[28] presented by G. A. Maney, M. Am. Soc. C. E., in his first publication of the slope-deflection relation, and with the moment-distribution method[29] presented by Hardy Cross, M. Am. Soc. C. E. In these latter procedures, the simultaneous solution of the equations is avoided and, by methods of approximations, the unknowns at each joint in succession are determined to any desired degree of refinement, the ultimate values, of course, being exactly the same as those obtained by the author in his exact solution of all equations simultaneously.

In order to compare the relative merits of these different methods of solution of the slope-deflection relation, it may be well to review, briefly, the history of the development of this very useful and general method of analysis. The essential steps are as follows:

(1) The first publication of the slope-deflection relation was in 1893 by Otto Mohr who used the method for the solution of secondary stress problems[30]. The form used by Mohr was limited to this type of problem; that is, to frames without loads between the joints.

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[27] With Bureau of Bridges, Div. of Highways, Springfield, Ill.

[28] Eng. Studies, *Bulletin No. 1*, Univ. of Minnesota, March, 1915.

[29] "Analysis of Continuous Structures by Distributing Fixed-End Moments," *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[30] *Zivilingenieur*, December, 1892–January, 1893.

(2) In May, 1914, Axel Bendixsen published in Denmark a monograph entitled, "Die Methode der Alpha-Gleichungen zur Berechnung von Rahmenkonstructionen", which gives the first general statement of the slope-deflection relation for any type of loading. This publication appears to have been generally overlooked even in Europe until about 1923 when Professor A. Ostenfeld amplified it somewhat[31].

(3) In 1914, Professor Maney evolved and, in March, 1915, published the first general statement of the slope-deflection relation to appear in America[28]. Professor Maney's work was original with him, as at this time he did not know of the work of Mohr and of Bendixsen on this problem. This presentation included any type of loading between joints and expressed the end moments of members in terms of the fixed-end moments, modified by the addition of moments arising from rotations and deflections of the joints. This method was not extended to members with variable moments of inertia at this time. Problems both in wind stresses in building frames and in secondary stresses in bridge trusses were solved; and, for the latter problem, a method of successive approximations was presented, which method did not require the simultaneous solution of the equations.

(4) In 1914, Professor Maney and W. M. Wilson, M. Am. Soc. C. E., developed, and, in June, 1915, published, the solution of wind stresses in a twenty-story office building by the slope-deflection method[32]. This solution was by the exact simultaneous method.

(5) In 1915 to 1918, Professor Maney and J. I. Parcel, M. Am. Soc. C. E., completed their analytical solution and experimental research in the field, and, in 1922, published their study of the secondary stresses on the Kenova Bridge[33]. This solution was made by the use of influence lines for the secondary stresses and involved the solution of forty-three equations for each case considered. The method of successive approximations was again used here in all cases. Eight cases were solved.

(6) In 1918 there was published by Professor Wilson, F. E. Richart, and C. Weiss, Members, Am. Soc. C. E., a bulletin giving general formulas for a large number of types of structures by the slope-deflection method[34].

(7) In 1923 Professor Ostenfeld published an amplification of Bendixsen's work, the presentation covering all cases for which solution is made for deformations rather than stresses[31].

(8) In 1930 Professor Cross published his moment-distribution method which he had been teaching for several years previously[29]. The Cross method also follows the general procedure of assuming fixed-end moments at ends of members and correcting them by a method of approximations that can be carried to any desired degree of refinement.

(9) In 1931, L. T. Evans, Assoc. M. Am. Soc. C. E., published a modification of the slope-deflection formulas, taking into account the variable moment of inertia of members for a number of typical cases[35].

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[31] "Die Deformationsmethode", by A. Ostenfeld, *Der Bauingenieur*, January 31, 1923.

[32] *Bulletin 80*, Eng. Experimental Station, Univ. of Illinois, June, 1915.

[33] Eng. Studies, *Bulletin No. 4*, Univ. of Minnesota, 1922.

[34] *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, 1918.

[35] *Journal*, Am. Concrete Inst., October, 1931.

The method of successive elimination of unknowns in the solution of secondary stress problems was used at least as early as 1911 by David A. Molitor, M. Am. Soc. C. E.[36] Mr. Molitor solved twenty-two equations for twenty-two moments, all expressed in terms of the first two unknown moments.

It is thus seen that, in America, a remarkable development has followed Professor Maney's derivation of the general method of slope deflection in 1914. E. J. Mehren, M. Am. Soc. C. E., lists slope deflection and moment distribution as the two analytical instruments which have vitally affected the advance of concrete structures in America.[37]

In order to compare the methods of approximate solution with the exact solution by successive elimination of unknowns illustrated by the author, attention is directed to the continuous girder of four spans illustrated in Fig. 1(*a*) of the paper. Assume the loading shown in the illustration, or any other loading. By Maney's method of approximations, the procedure is as follows:

(*a*) Fixed-beam end moments are computed on the assumption that all spans are fixed at the ends. Denote these moments as M_{FAB}, M_{FBC}, M_{FBA}, etc. Let ΣM_{FA} designate the sum of all fixed-beam moments at *A*.

(*b*) The first approximation of joint rotation values at a given joint are obtained by assuming that all joints at the far ends of the members in question have zero rotation. For purposes of illustration consider Joint *B*, Fig. 1(*a*). Then,

$$\theta_B = \frac{M_{FBA} + M_{FBC}}{2\,K_{BA} + 2\,K_{BC}} \qquad \text{(43)}$$

or, in general,

$$\theta_B = \frac{\Sigma\,M_{FB}}{2\,\Sigma\,K_B} \qquad \text{(44)}$$

Also,

$$\theta_C = \frac{M_{FCB} + M_{FCD}}{2\,K_{CB} + 2\,K_{CD}} \qquad \text{(45)}$$

or, in general,

$$\theta_C = \frac{\Sigma\,M_{FC}}{2\,\Sigma\,K_C} \qquad \text{(46)}$$

Substituting these values as found in the joint equations, a new and closer set of values for the joint rotations is obtained, and this can be continued as often as necessary, seldom more than three approximations being necessary.

(*c*) Placing the values of joint rotations as determined by Equations (44) and (46) in the joint equations, solution is made for the resulting end moments. As has been noted, this process consists in adding algebraically the end moments induced by the joint rotations to the fixed-beam end moments assumed at the start.

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[36] "Kinetic Theory of Engineering Structures", by David A. Molitor, 1911, p. 232 *et seq.*

[37] "Concrete, Yesterday, Today and Tomorrow", *Journal*, Am. Concrete Inst., March-April, 1935; also, *Proceedings*, Am. Concrete Inst., Vol. 31, p. 345.

By the Cross' moment-distribution method of approximations, the procedure is very similar. If the values of joint rotation, obtained by Equations (44) and (46), are substituted in the moment equation for a given member,

$$M_{BC} = M_{FBC} = \left(\frac{K_{BC}}{\Sigma K_B}\right) \Sigma M_{FB} - \frac{1}{2}\left(\frac{K_{BC}}{\Sigma K_C}\right) \Sigma M_{FC} \ldots\ldots (47)$$

It is seen at once that the second term on the right-hand side of the equality sign represents the "distribution factor" and that the third term is the "carry-over" factor. Distribution and carrying-over are continued until all unbalanced moments at the joints become negligible, or disappear, which is a parallel process to arriving at successively closer values of true joint rotations. Indeed, it is noteworthy that the reason the moment-distribution process converges so rapidly, is that a rotation at Joint *A*, say, will induce only one-half as much end moment at Joint *B* as at Joint *A*, etc. This relation is evident at once on inspection of the slope-deflection equation. By Maney's procedure, one solves for closer and closer values of the joint rotations, or deflections, and then obtains final moments from final values of the joint rotations. By Cross' procedure, one distributes moment differences until all unbalanced moments are negligible or disappear. A very interesting development in this regard was made in 1934 by C. E. Morgan, M. Am. Soc. C. E.[38]. Pursuing moment distribution to its logical conclusion mathematically, Mr. Morgan expressed the end moments in terms of a geometric series, and arriving at the limit of the series, obtained the true values of the moments—that is, the same values that would be obtained by Professor Wilbur's exact method of solution. In 1936, Mr. Morgan published formulas for moments for a four-span continuous structure similar to Fig. 1(*a*). These formulas were derived by Mr. Morgan's series method[38].

For frames such as those illustrated in Fig. 1(*b*) and Fig. 1(*c*), the procedure is essentially the same, and the relations between the foregoing methods of attack are similar. Methods of solution by approximations of these frames were published[39] by Professor Maney and J. E. Goldberg, Jun. Am. Soc. C. E., in 1932. In 1931, there was also published[40] a method of solving secondary stresses by moment distribution by L. S. Thompson and R. W. Cutler, Jun. Am. Soc. C. E.

With regard to the method of procedure for a given case, naturally the designer's preference will vary. Where not too many unknowns or joints are involved, it will frequently be desirable to use a procedure similar to that illustrated by the author. Where a large number of unknowns are involved, however, it is the writer's belief that most engineers will prefer one of the approximate methods. It should be noted in this connection that there is a difference in procedure between the slope-deflection method of approximation and the moment-distribution method. In both, any variation in the convergence indicates that an error has been made. In the moment-distribution method, however, it is possible to carry over an initial unbalanced moment of incorrect value, and it will be distributed and no check will appear. In the slope-deflection method, a complete set of static and elastic equations is set up, and when the true values of the unknowns are obtained, by whatever means, these unknowns must satisfy all equations simultaneously, and, hence, must be correct. In many cases the moment distribution method is undoubtedly most expeditious. On the other hand, it is the writer's belief, based upon a certain amount of observation, that there is a tendency for the designer to lose sight of the actual significance of each step of the process with moment distribution, and that this is not so likely to occur with slope deflection. Furthermore, in a number of cases, such as where correction is to be made during construction, it is necessary to know the computed value of the joint rotations or deflections, and in this case slope deflection is plainly necessary.

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[38] "Designing Concrete Girder Bridges for Continuity", *Engineering News-Record*, April 23, 1936, pp. 604–606.

[39] "Simplified Methods for Analysis of Multiple Joint Rigid Frames", *Bulletin No. 7*, Northwestern Univ., October 17, 1932.

[40] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 108.

In conclusion, it may be said that the progressive use of indeterminate structures seems to have been dependent, largely, upon the development of expeditious methods of solving simultaneous equations. This has probably been true—not because of the difficulties of the problem—but because simultaneous solution of a considerable number of equations has required more time than the procedure of most designing offices allows. This paper is of definite value in that it calls attention to means of expediting this analytical process.

JOHN B. WILBUR,[41] ASSOC. M. AM. SOC. C. E. (by letter).—The valuable additions that have been contributed by the discussers of this paper are appreciated. In one general respect the method of presenting the solution of a continuous beam by successive elimination of unknowns was unfortunate in that it was included as a pedagogical step in leading the reader gradually into the more complicated problem of a building frame, in which it was believed the proposed expedient might be of value. The writer agrees with those who suggest that other methods of solution are likely to be superior in analyzing continuous beams.

A method of determining, approximately, the bending moments in the members meeting at a particular joint of a building frame, is presented by Mr. Willson. He assumes that there is no side-sway in the building. Under loadings where side-sway would occur, his Equation (19) would be incorrect unless revised to include this effect. Such revision would seem to lead to a more cumbersome expression.

Mr. Andersen believes that the method of eliminating unknowns suggested by the writer would immediately suggest itself to one setting up the solution of a problem by the slope deflection method. Judging from his own experience, the writer is forced to disagree on this point. Mr. Andersen states moreover, that, in his opinion, designers do not object to the solution of simultaneous equations. Although this may be true, those who pay for the designer's time must certainly have an opposite point of view, especially if a large number of unknowns are involved.

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[41] Asst. Prof. Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

The necessity of understanding the sign convention fully in using the slope deflection method, is emphasized by Mr. Stewart, who gives an excellent statement of these conventions. Professor Grinter likewise shows, with examples, the conventions which the writer has followed. Both these discussions should be of value to those who may have difficulty with this important aspect. The modification of the writer's method as applied to a building frame, which Mr. Stewart suggests, is of genuine interest. Professor Grinter stresses the need of a physical conception of structural problems; and with this belief, the writer is in full accord.

Mr. Mitchell states that office practice should be such that, in so far as is practicable, it is self-checking. There can be no dispute on this point. Longer methods which have this advantage are often preferable to shorter approaches which leave one in doubt as to the accuracy of one's result. The method suggested by the writer, however, is self-checking. It yields certain values of R and θ, so that the solution corresponds to a possible deformation of the structure. The solution is thus consistent elastically. If the moments from these functions are in static equilibrium with the external forces acting on the structure, the solution is also consistent statically, and must be correct.

The equations contributed by Professor Tsai which take into account beams of variable moment of inertia, are of value in the general slope deflection solution and, of course, aid in the particular technique of solution suggested by the writer.

Mr. Fischer raises an interesting problem with respect to the solution by slope deflection of a frame with inclined legs. Inasmuch as the treatment of this problem as given in certain engineering literature is incorrect, in the writer's opinion, the treatment by Mr. Fischer is valuable.

The outline of the development of the slope deflection method given by Mr. Wyly is excellent, and will be of value to those investigating this field.

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REINFORCED CONCRETE MEMBERS UNDER DIRECT TENSION AND BENDING

By D. B. GUMENSKY[1], Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. A. W. Fischer, William E. Wilbur, F. C. Snow, Ralph E. Byrne, Jr., Carl H. Heilbron, Jr., F. E. Turneaure, H. E. Warrington, William A. Larsen, B. Kovediaeff, A. Floris, David B. Hall, Odd Albert, and D. B. Gumensky.

Synopsis

The combination of compression and bending stresses is a common occurrence in design and has been very well treated in standard books on reinforced concrete. However, the combination of direct tension and bending is a problem that has received less attention from authors of engineering texts. This problem is met with in the design of Vierendeel trusses or particularly in the design of closed water conduits under pressure, where the conduit barrel is horizontal and acts as a complete elastic ring. The following presentation is an attempt to analyze the stress distribution in reinforced concrete members subjected to a combined stress due to direct tension and bending, and to suggest an easy method of solving these problems.

Assumptions

The assumptions used in the derivation of the formulas are the same as those generally accepted for the derivation of working formulas for reinforced concrete. They are as follows: (1) A straight-line distribution of stress; (2) concrete is an elastic substance; (3) tension is resisted entirely by steel; (4) bond between concrete and steel is perfect; (5) there are no initial stresses in concrete or steel; (6) the symbols denote only a numerical value of the quantities they represent; (7) vertical distances measured downward from the axis of a member are negative and upward distances are

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Note.—Published in December, 1935, *Proceedings*.

[1] With Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

positive; (8) forces acting from right to left are positive and forces acting from left to right are negative; and (9) moments of forces about a point acting in a counter-clockwise direction are positive.

Notation.—The mathematical symbols are defined where they first appear in the paper and are summarized, for reference, in the Appendix.

ANALYSIS AND SOLUTION RELATED TO THE GEOMETRICAL AXIS OF THE CROSS-SECTION OF THE MEMBER

In analyzing indeterminate structures, the resultant moment, M, the direct pull, T, and the eccentricity of direct stress, $e \left(= \frac{M}{T}\right)$, at any one point, are usually related to the geometrical axis of the elastic ring or frame. This relation is maintained in the following presentation. It is generally accepted that the combination of an axial load and a couple will produce the same stresses as an equivalent eccentrically applied pull. In this paper, the latter will be used for the derivation of formulas. There are two distinct cases of stress distribution: (1) That in which the entire section is under tension; and (2) that in which part of the section is under compression.

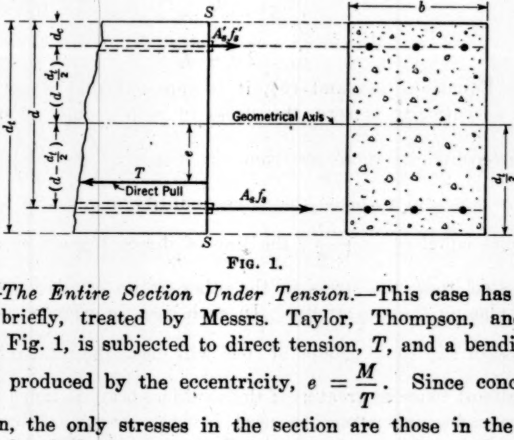

FIG. 1.

Case 1.—The Entire Section Under Tension.—This case has been excellently, but briefly, treated by Messrs. Taylor, Thompson, and Smulski.[3] Section S-S, Fig. 1, is subjected to direct tension, T, and a bending moment, M, which is produced by the eccentricity, $e = \frac{M}{T}$. Since concrete cannot resist tension, the only stresses in the section are those in the steel. For equilibrium, the algebraic sum of all forces acting on the section must equal zero. Therefore,

$$T = A_s f_s + A'_s f'_s \dots\dots\dots\dots\dots\dots (1)$$

Taking moments about the center of the upper steel rods,

$$A_s f_s = T \frac{\left(d - \frac{d_t}{2} + e\right)}{2\,d - d_t} \dots\dots\dots\dots\dots\dots (2)$$

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[3] "Concrete, Plain and Reinforced", by F. W. Taylor, S. E. Thompson, and Edward Smulski, Vol. 1, Edition 4, N. Y., John Wiley & Sons, Inc., 1925, p. 137.

In the same manner, taking moments about the center of the lower steel rods:

$$A'_s f'_s = T \frac{\left(d - \frac{d_t}{2} - e\right)}{2\,d - d_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Equations (1), (2), and (3) are used to determine the stresses in the two groups of steel. However, since there are, altogether, four unknowns—A, f_s, A'_s, and f'_s—and only three equations, it is necessary to make an additional assumption before these equations can be solved. Many designers arbitrarily make the area of steel in the upper and lower groups the same; or (which would be more economical), the unit stress in the steel of the upper and lower groups may be assumed to be equal.

If $A_s = A'_s$, the required area of steel in one group will be governed by the maximum allowable tensile stress in steel; thus:

$$A_s = \frac{T}{f_s}\,\frac{d - \frac{d_t}{2} + e}{2\,d - d_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which f_s is the maximum allowable stress in the steel. In the other group the steel will be under-stressed; thus:

$$f'_s = \frac{T}{A_s}\,\frac{d - \frac{d_t}{2} - e}{2\,d - d_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Comparing Equations (4) and (5), it is apparent that when the pull is axial and the eccentricity is zero, the stress in both groups of steel rods is equal. As the eccentricity increases from $e = 0$ to $e = \left(d - \frac{d_t}{2}\right)$, it becomes more and more wasteful to place the same quantity of steel in both groups. When e becomes equal to $d - \frac{d_t}{2}$, the line of direct tension coincides with one group of steel rods, the stress in the other group equals zero, and there is no need of placing the two groups. In such case, or when e nearly equals $d - \frac{d_t}{2}$, the use of two equal groups of steel will make the total reinforcement in a member almost twice as great as if the steel had been designed for $f_s = f'_s$.

Should the same unit stress be used in both groups of bars (that is, $f_s = f'_s$), A_s is the same as would be determined by Equation (4), and A'_s becomes:

$$A'_s = \frac{T}{f_s}\,\frac{d - \frac{d_t}{2} - e}{2\,d - d_t} = A_s\,\frac{d - \frac{d_t}{2} - e}{d - \frac{d_t}{2} + e} \quad \ldots\ldots\ldots\ldots (6)$$

Again, when $e = d - \frac{d_t}{2}$, the line of direct pull coincides with one group of rods, and no steel is required in the other group. When e is greater than $d - \frac{d_t}{2}$, Equations (1) to (6) are not applicable because part of the concrete begins to act in compression, and, if two groups of steel are used, one group also is in compression.

Case 2.—Part of the Section Under Compression.—This condition is created when the line of direct pull falls outside the steel reinforcement in the member.

The steel may be placed either in the tension face alone, or in both the tension and the compression faces of the member. By placing it only on the tension side, a design is produced in which steel acts in tension and concrete in compression, the combination that usually results in greatest economy.

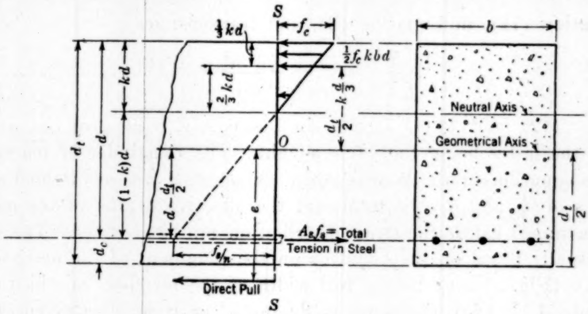

FIG. 2.

A thin section of concrete beam, S-S, in Fig. 2 (an independent structural element), is acted upon by the internal forces of compression in the concrete, the tension in the steel, and the external force of direct pull applied at a distance, e, from the geometrical center of the section.

Since Section S-S is in static equilibrium, the sum of the moments of all forces acting upon that section must be equal to zero. Taking moments about Point O:

$$A_s f_s \left(d - \frac{d_t}{2}\right) - T\,e + \frac{1}{2} f_c\, k\, b\, d \left(\frac{d_t}{2} - \frac{1}{3}\, k\, d\right) = 0 \;\ldots\ldots (7)$$

The sum of all horizontal forces acting on the section must equal zero:

$$T - A_s f_s + \frac{1}{2} f_c\, k\, b\, d = 0 \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Multiplying Equation (8) by e and equating to Equation (7):

$$A_s f_s \left(d - \frac{d_t}{2}\right) - T\,e + \frac{1}{2} f_c\, k\, b\, d \left(\frac{d_t}{2} - \frac{1}{3}\, k\, d\right)$$

$$= A_s f_s\, e - T\,e - \frac{1}{2} f_c\, k\, b\, d\, e \;\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Substituting in Equation (9):

$$f_s = n\, f_c\, \frac{1-k}{k} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

and,

$$A_s = p\ b\ d \qquad \text{(11)}$$

and solving for $\frac{e}{d}$:

$$\frac{e}{d} = \frac{2\,p\,n\,(1-k)\left(1-\frac{d_t}{2\,d}\right) + k^2\left(\frac{d_t}{2\,d} - \frac{k}{3}\right)}{2\,p\,n\,(1-k) - k^2} \qquad \text{(12)}$$

Substituting,

$$f_c = f_s \frac{k}{n\,(1-k)} \qquad \text{(13)}$$

and Equation (11), in Equation (8), and transposing:

$$\frac{T}{f_s\,b\,d} = p - \frac{k^2}{2\,n\,(1-k)} \qquad \text{(14)}$$

Equations (12) and (14), solved simultaneously, will give the values of k and p, the latter being the value sought. The remainder of the values in these two equations must be determined or assumed before the final solution.

Values of d_t and d—the total and the effective depths of the member—must be assumed before the stress distribution can be analyzed. This assumption is usually based on precedent or on the judgment of the designer. The value of b (Fig. 2) may be the full width of the member, in which event it is determined by trial, the same as d_t and d, or it may be taken as a unit width, usually 1 ft. Values of T and e are found from the analysis of stress distribution in the structure, thus, $e = \frac{M}{T}$.

The value of n which designates the ratio between the modulus of elasticity of steel and the modulus of elasticity of concrete, depends on the quality of concrete. Normally, it is assumed to vary between $n = 8$ and $n = 15$, which corresponds to $E_c = 3\,750\,000$ and $2\,000\,000$ lb per sq in., respectively. The value of f_s is determined by the grade of steel used and the purpose for which the structural member is designed, allowing an adequate factor of safety.

The two unknowns remaining are k and p. For any given condition of loading, dimensions, and stress distribution, the required amount of steel can be determined. However, the solution of Equations (12) and (14) is laborious, and, for this purpose, the writer has devised a set of diagrams, an example of which is reproduced on Fig. 3 for discussion. These diagrams were based on values of n equal to 15 and 10, respectively. In solving design problems by means of such curves the procedure is, as follows:

(1) Determine the bending moment, M, and the direct pull, T, for the member in question;

(2) Select trial values for d_t and d, unless they were tentatively chosen previously;

(3) Determine the value of $e = \frac{M}{T}$;

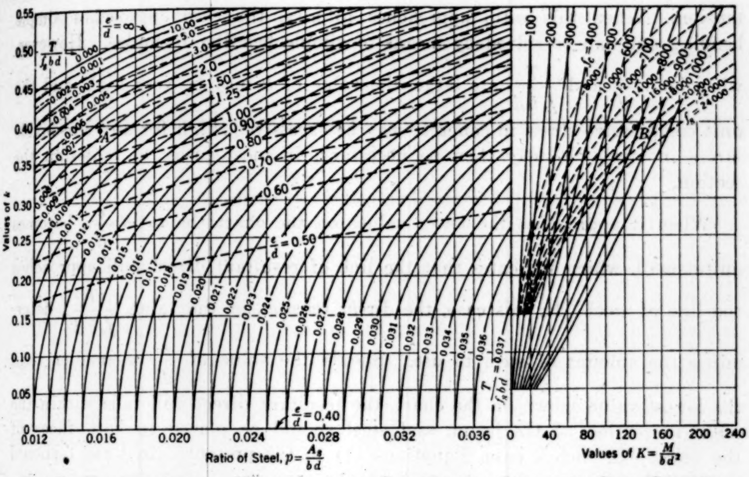

FIG. 3.—ECCENTRICITY, e, MEASURED FROM GEOMETRICAL CENTER OF CROSS-SECTION.

(4) Find the value of $\frac{e}{d}$;

(5) Assume a value of f_s, the unit stress in steel;

(6) Find the value of $\frac{T}{f_s\, b\, d}$;

(7) Find the ratio, $a = \frac{d_c}{d}$, of depth of cover on the steel to the effective depth of the beam;

(8) From a set of diagrams, such as Fig. 3, with the value of n in the problem, find the set of curves for a, closest to the value determined in Step (7);

(9) On Fig. 3 find the intersection of the two curves representing the values of $\frac{e}{d}$ and $\frac{T}{f_s\, b\, d}$ found in Steps (3) and (5);

(10) Directly below read the ratio of steel, p, to use for this particular condition; and

(11) Determine the area of steel, $A_s = p\, b\, d$, in section.

Incidentally, the value of k is determined by the same point of intersection as that found in Step (9). In order to find the unit compressive stress in concrete, follow a horizontal line from the point of intersection (found in Step (9)) to the right until its intersection with a curve representing the value of f_s assumed in Step (5). Finally, read or interpolate the value of f_c.

In most cases, the intersection of the curves for $\frac{e}{d}$ and $\frac{T}{f_s\, b\, d}$ will fall on the chart, which will give a definite solution of the problem. When the ratio, $\frac{e}{d}$, is very large the bending moment is very large in respect to direct tension. When this ratio approaches infinity, the case approaches simple bending and can be solved by means of simple bending formulas. However, when both $\frac{e}{d}$ and $\frac{T}{f_s b d}$ are large, the value of k may be very high, and the unit compressive stress in concrete may be excessively high for any reasonable stress in steel. It may then be necessary, in most cases, to re-design the section.

When it approaches the lower values on the chart the ratio, $\frac{e}{d}$, becomes quite small, which indicates that the line of resultant direct pull approaches the line of steel. In this case, the direct tension, or the value, $\frac{T}{f_s b d}$, determines the amount of steel to use. Should the value of $\frac{e}{d}$ be smaller than the lowest value shown on the chart, the resultant direct pull falls within the effective section of the beam, and steel should be placed in both faces of the beam. In such a case, Equations (1) to (6) presented in Case 1, should be applied.

Should the intersection of the two curves fall outside the limits of the chart, it would mean that the case is of an unusual character as far as the percentage of steel or the relationship of stresses is concerned and that further analyses or re-designing may be necessary.

Returning, again, to Equations (12) and (14): When $k = 0$, there is no stress in the extreme compression fiber of the concrete. Substituting $k = 0$ in Equation (12):

$$\frac{e}{d} = 1 - \frac{d_t}{2d} \quad \text{......................(15)}$$

or,

$$e = d - \frac{d_t}{2} \quad \text{......................(16)}$$

In other words, this condition occurs when direct pull coincides with the steel reinforcement (see Fig. 2). Substituting $k = 0$ in Equation (14):

$$\frac{T}{f_s b d} = p \quad \text{......................(17)}$$

or,

$$T = f_s p\ b\ d = f_s A_s \text{......................(18)}$$

that is, the quantity of steel is determined by the direct pull and the unit stress.

When the direct pull is zero, Equation (14) may be written thus:

$$k = \sqrt{2\ p\ n + (p\ n)^2} - p\ n \text{......................(19)}$$

which is a familiar relationship in the case of simple bending, when the eccentricity equals infinity, and the left side of Equation (12) is obviously equal to infinity; the right-hand expression of Equation (12) can be equal to infinity only in case the denominator is equal to zero, which again yields Equation (19). The use of the diagrams may best be illustrated by an example.

Example 1.—Determine the area of steel to use in a precast concrete pipe of 12.0-ft inside diameter. Under an internal pressure the top of the pipe is to be 10 ft beneath the surface and water will flow through it under a pressure head of 50 ft. The thickness, t, of the shell equals 12 in.; the steel reinforcement is covered with 2 in. of concrete, and the effective depth is 10 in. The pipe is embedded in concrete along the lower quadrant of its circumference. The severest condition of stress is at the bottom where $M = 19\,230$ ft-lb and $T = 16\,530$ lb. Assume $E_c = 2\,000\,000$ lb per sq in., which means that $n = 15$.

By following the procedure indicated previously: By Step (3) the eccentricity, $e = \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163$ ft $= 14$ in.; and by Step (4), $\frac{e}{d} = \frac{14}{10} = 1.40$; in the case of combined bending and tension where concrete is under compression, select $f_s = 18\,000$ lb per sq in. (Step (5)). Then by Step (6), $\frac{T}{f_s b d} = \frac{16\,530}{18\,000 \times 12 \times 10} = 0.0076$; by Step (7), $a = \frac{d'}{d} = \frac{2}{10} = 0.20.$ Steps (8), (9), (10), and (11) require the use of Fig. 3 for the case, $a = 0.200$, and $n = 15$. In this case (a member subjected to bending and direct tension, combined), the eccentricity, e, is measured from the geometrical center of the gross section. The intersection of two curves (Step 9), $\frac{e}{d} = 1.40$ and $\frac{T}{f_s b d} = 0.0076$, is at Point A, Fig. 3. Directly below the intersection (Step (10)) read the ratio of steel, $p = 0.016$. The area of transverse steel is $A_s = 0.016 \times 12 \times 10 = 1.92$ sq in. per lin ft (Step (11)).

The value of k for this condition is approximately 0.395. Following the horizontal line from Point A to the right until its intersection with the curve, $f_s = 18\,000$, $f_c = 795$ lb per sq in. at Point B, Fig. 3. Should it be desirable to use steel reinforcement in both the compression and the tension faces of the member, the compression steel will help the concrete under compression. Steel reinforcement could be used in the compression side to reduce the compression stress in the concrete or in order to provide for possible reversal of stresses.

ANALYSIS AND SOLUTION RELATED TO THE CENTER OF STEEL REINFORCEMENT IN TENSION FACE

The foregoing example, depending for quick solution on sets of diagrams of the type of Fig. 3, refers to the geometrical axis of the beam as the center of moments. This leads to consideration of a depth of cover on the steel and necessitates a number of charts for various ratios of the depth of cover on steel to the effective depth of beam.

It may be considered that the centroid of the tension steel is the true center of stress in the beam subjected to tension and bending. This consideration eliminates the necessity for several charts, but changes the value of eccentricity by an amount equal to $d - \frac{d_t}{2}$.

Fig. 4 illustrates a condition similar to that of Case 2, in which part of the section is under compression and steel is placed in the tension face only. All the notation and assumptions previously mentioned will apply in this case with one exception, namely, the eccentricity is measured from the center line of the steel in the tension face and is designated e'.

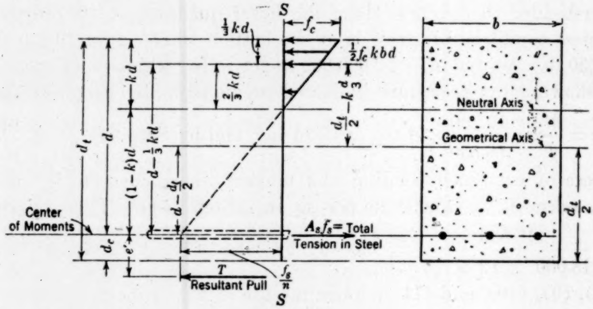
S
f_c
$\frac{1}{3}kd$
$\frac{1}{2}f_c kbd$
b
kd
$\frac{2}{3}kd$
$\frac{d_t}{2} - \frac{kd}{3}$
Neutral Axis
Geometrical Axis
d_t
d
$(1-k)d$
$d - \frac{1}{3}kd$
$d - \frac{d_t}{2}$
Center of Moments
$A_s f_s$ = Total Tension in Steel
$\frac{d_t}{2}$
d_c
e'
T
Resultant Pull
$\frac{f_s}{n}$
S

FIG. 4.

Taking moments about the centroid of the tension steel, for condition of equilibrium:

$$T\,e' = \frac{1}{2} f_c\,k\,b\,d \times (d - \frac{1}{3}\,k\,d) \qquad (20)$$

Since the sum of all horizontal forces must be equal to zero:

$$T - A_s f_s + \frac{1}{2} f_c\,k\,b\,d = 0 \qquad (21)$$

Solving for $\frac{e'}{d}$ after transpositions and substitutions:

$$\frac{e'}{d} = k^2 \frac{\left(1 - \frac{k}{3}\right)}{2\,p\,n\,(1-k) - k^2} \qquad (22)$$

Equation (21), with substitutions and transpositions similar to those of Case 2, could be rewritten:

$$\frac{T}{f_s\,b\,d} = p - \frac{k^2}{2\,n\,(1-k)} \qquad (23)$$

A simultaneous solution of Equations (22) and (23) will give the values of k and p.

A complete set of curves such as that shown in Fig. 5 obviates the necessity of laborious solutions and, similarly, to diagrams such as Fig. 3, gives a quick and accurate method of determining the percentage of steel reinforcement in a member subjected to combined tension and bending moment. The writer has constructed diagrams for values of n equal to 8, 10, 12, and 15. Only one set of curves is necessary for the solution of a problem, but one must be careful to use eccentricity relative to the center of reinforcement in tension. Following standard procedure, Steps (1), (2), (3), (5), (6), (10), and (11) are exactly as before.

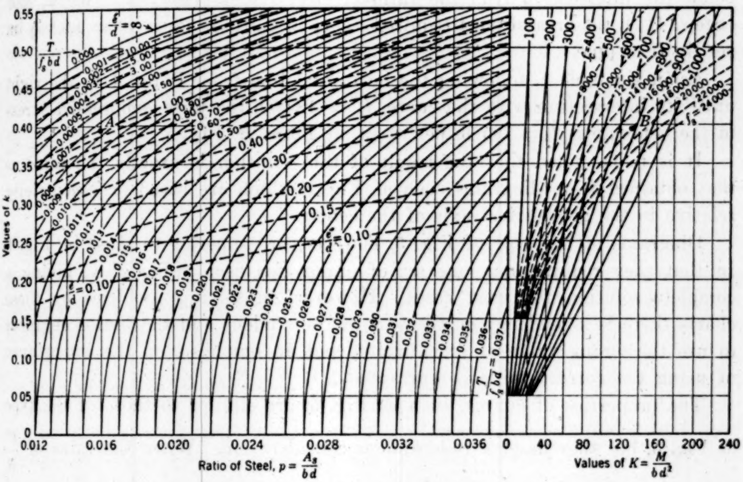

FIG. 5.—ECCENTRICITY, e', MEASURED FROM THE CENTER OF THE STEEL ON THE TENSION SIDE; $n = 15$.

In Step (4), determine $e' = e - d + \frac{d_t}{2}$; and from this find the ratio, $\frac{e'}{d}$. In this case Step (8) constitutes selecting a set of diagrams, such as Fig. 5, with the given or assumed value of n; and (see Step (9)), on this diagram, find the intersection of the two curves representing the values of $\frac{e'}{d}$ and $\frac{T}{f_s\, b\, d}$ found in Steps (4) and (6). The value of k and the unit compressive stress in the concrete are determined precisely as in Case 1.

Example 2.—Using the same problem as in Example 1, $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; and $d_c = 2$ in. As before, the eccentricity relative to the geometrical center of the section (Step (3)) is $e = \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163$ ft $= 14$ in. The eccentricity relative to the center of the steel reinforcement in tension (Step (4)) is $e' = e - d + \frac{d_t}{2} = 14 - 10 + 6 = 10$ in.; and, consequently, $\frac{e'}{d} = \frac{10}{10} = 1$. Assume a unit tensile stress in steel (Step (5)) of $f_s = 18\,000$ lb per sq in.; find the value, $\dfrac{T}{f_s\, b\, d}$ (Step (6)) $= \dfrac{16\,530}{18\,000 \times 12 \times 10} = 0.0076$; on Fig. 5 (Step (9)), find the intersection of two curves, $\dfrac{e'}{d} = 1.00$ and $\dfrac{T}{f_s\, b\, d} = 0.0076$ at Point A; directly below the intersection read the ratio of steel (Step (10)), $p = 0.016$; and (Step (11)), the area of steel required $= A_s = 0.016 \times 12 \times 10 = 1.92$ sq in. per lin ft of pipe.

As in Example 1, following the horizontal line from Point A to the right until its intersection with the curve, $f_s = 18\,000$, the unit compressive stress in the concrete is $f_c = 795$ lb per sq in. at Point B, Fig. 5.

It is to be noted that the result of this solution is exactly the same as that obtained from diagrams such as Fig. 3 which are based on eccentricity relative to the geometrical axis of the center.

Diagrams of the type of Fig. 5, however, are superior to those of Fig. 3 in that they require only one set of curves for each value of n, giving a complete solution for most cases. However, in solving problems by these charts there is a tendency on the part of the student and the junior engineer to use the wrong value of eccentricity, and an emphasis on the importance of using the correct value is warranted.

The properties of curves, such as Fig. 5, are similar to those of the type of Fig. 3, but they deserve independent consideration. When the value of $\dfrac{e'}{d}$ is very large, it indicates that the bending moment is very large in respect to direct tension and dominates the solution. The larger the value of $\dfrac{e'}{d}$, the more indefinite is the intersection of the two curves determining the ratio of steel, and, in extreme cases, it may be necessary actually to solve Equations (22) and (23).

When the value of $\dfrac{e'}{d}$ approaches infinity, the condition approaches that of simple bending moment, and the problem is solved either by the formulas or by the diagrams. In order to solve a case of simple bending moment by means of diagrams such as Fig. 5, proceed as follows: Having determined the value of the bending moment, M, the width, b, and the effective depth, d, of the member in question, determine the auxiliary value, $K = \dfrac{M}{b\, d^2}$. Enter the right side of the diagram for the proper value of n with the value of K found in the previous step. Follow the vertical line to its point of intersection with the desirable or assumed stress lines for f_c and f_s, or with the limiting stress as the case may be. From this point of intersection follow the horizontal line to the left to its intersection with the curve, $\dfrac{T}{f_s\, b\, d} = 0$.

Vertically below the last intersection read the steel ratio, p. Determine the area of steel, $A_s = p\ b\ d$. No example is necessary to illustrate this case.

When $\frac{e'}{d}$ and $\frac{T}{f_s\ b\ d}$ are both relatively large, the value of K may be very high and the unit compressive stress in concrete may be excessively high for any reasonable stress in the steel. In most cases, it would be necessary to re-design the section by increasing the over-all dimensions, b and d, of the member, or by placing steel in the compression side in order to relieve the stress in the concrete, or both.

When $\frac{e'}{d}$ is small it means that the line of direct pull approaches the line of steel. The direct tension, or the value, $\frac{T}{f_s\ b\ d}$, determines the amount of steel to use. It may happen that the value of e' is negative, which is possible when the value of e, by Step (3), is smaller than $d - \frac{d_t}{2}$. This would indicate that the line of direct pull falls within the effective section and that the two layers of steel are needed. Equations (1) to (6) in Case 1 provide the solution.

CONCLUSION

A full appreciation of the advantages of the procedure outlined herein, can be obtained only by using, in practice (and over a reasonable period of time), the complete set of curves upon which the arguments of the paper are based. In offering a condensed outline description of the curves it is hoped that discussion will develop their value in principle and that suggestions for their improvement, for example, such as the desirable intervals of the n-curves, will be advanced. If it is conceded that complete sets of curves such as those described in this paper, would be valuable to structural designers in this field, should they be of the type demonstrated by Fig. 3, or Fig. 5? These and other questions could be answered in discussion.

APPENDIX

NOTATION

The symbols introduced in the paper are summarized herein, for convenience of reference, as follows:

$a = \frac{d_c}{d}$ = ratio of the depth of cover on the steel, to the effective depth of the beam;

b = breadth of beam;

c = distance from the neutral axis of a beam to the extreme fiber in compression; as a subscript, c denotes "concrete";

d = depth; effective depth of a beam, or the depth from the extreme fiber in compression, to the centroid of the steel reinforcement on the tension side of the beam; d_t = total depth of a beam; d_c = depth of cover on the steel reinforcement, measured from the centroid of the rods; $k\ d$ = distance from the extreme fiber in compression, to the neutral axis;

e = eccentricity of the resultant force, T, in respect to the geometrical axis of the entire section $\left(e = \frac{M}{T}\right)$;

f = unit stress; f_c = unit compressive stress in the extreme fiber of a concrete section; f_s = unit tensile stress in steel; f'_s = unit stress in the upper steel rods;

k = ratio of the depth of the neutral axis from the extreme fiber in compression to the effective depth of the beam; $k\ d$ = the distance from the extreme fiber in compression to the neutral axis;

n = the ratio of the moduli of elasticity, $\left(n = \frac{E_s}{E_c}\right)$;

p = ratio of steel area to the effective area of a beam, $\left(p = \frac{A_s}{b\ d}\right)$;

s = a subscript denoting "steel";

t = thickness; as a subscript, t denotes "total";

A = area; A_s = cross-sectional area of steel ($A_s = p\ b\ d$); A'_s = cross-sectional area of the steel in the upper part of a beam;

C = a subscript denoting "depth of cover";

E = modulus of elasticity; E_c for concrete and E_s for steel;

I = rectangular normal of inertia;

K = a constant, $\left(K = \frac{M}{b\ d^2}\right)$;

M = moment; bending moment at the section under consideration;

T = total tension; resultant pull due to external forces.

DISCUSSION

A. W. Fischer,[3] Esq. (by letter).—The method of designing concrete members under direct tension and bending, as presented by the author, gives correct results. However, since values must be ascertained by using either Fig. 3 or Fig. 5 (which appear rather complicated) and generally must be interpolated, it seems that a more direct method can be used for solving such simple examples.

When e' is negative the author's method is as simple as any that the writer knows, but the suggested solution is not so simple when e' is positive. Consider Example 2 and Fig. 4 of the paper. Taking the center of moments about the centroid of the tension steel and solving for K, by Equation (20), $K = \dfrac{Te'}{bd^2} = \dfrac{16\,530 \times 10}{12 \times 10 \times 10} = 137.75$. Referring to a diagram in a standard textbook[4], with $n = 15$, $f_s = 18\,000$ lb per sq in., and $K = 137.75$; then, f_c is found to be equal to 796 lb per sq in. which practically agrees with the value derived by Mr. Gumensky. Furthermore, $k = \dfrac{n\,f_c}{f_s + n\,f_c} = \dfrac{796}{1\,200 + 796} = 0.3988$ (use 0.399), making $k\;d = 3.99$ in. The value of k can also be read directly from a diagram.[4]

Taking moments about the centroid of the compression area of the concrete (which is $\frac{1}{3}\;k\;d$ below the top of the beam in Fig. 4) the amount of tension steel required will be:

$$\frac{16\,530\left(e' + d - \dfrac{1}{3}\,k\,d\right)}{18\,000\left(d - \dfrac{1}{3}\,k\,d\right)} = \frac{16\,530\,(10 + 10 - 1.33)}{18\,000\,(10 - 1.33)} = 1.978 \text{ sq in. per lin ft,}$$

which corresponds with the results obtained by Equation (21).

If the stresses had been limited to $f_s = 18\,000$ lb per sq in., $n = 15$, and $f_c = 700$ lb per sq in., some compression steel would need to be added and, in that case, since the value of $K = 113.1$, the resisting moment in the concrete can be computed, and the remainder must be taken by compression steel.[5]

It seems to the writer that the foregoing procedure is more in line with that ordinarily used in beam design. The results can be derived very readily and all that is needed is a diagram giving the values of K and k, which are given in any standard textbook on reinforced concrete. All calculations have been made with a 20-in. slide-rule.

William E. Wilbur,[6] M. Am. Soc. C. E. (by letter).—It is a fact, frequently unnoticed by writers on structural design, that in determining the fiber stresses in a section of a beam to resist bending, the center of moments may be taken at any convenient point in the depth of the beam, and need not be at the neutral axis or at the geometrical axis. In a concrete beam, as the author shows in discussing his Case 2, it is advantageous to take the moment center at the location of the tensile steel. Using the nomenclature of Fig. 4, consider a beam subject to any bending moment, M, without direct stress, and take moments about the tensile steel; thus:

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[3] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

[4] See, for example, "Structural Members and Connections", by Hool and Kinne, p. 467.

[5] "Data on Design of Concrete", by A. W. Fischer, *Civil Engineering*, March, 1935, p. 180.

[6] Asst. Engr., Harrington & Cortelyou, Kansas City, Mo.

$$M = \frac{1}{2} f_c\, b\, k\, d \left(d - \frac{1}{3}\, k\, d\right) \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

which is the ordinary formula for beams subject to bending only. When the bending moment is that due to a force, T, with an eccentricity, e', the author's Equation (20) results. By carrying the foregoing analysis further, the author's process may be simplified considerably.

Referring again to Fig. 4, one may, without changing the conditions, add two equal opposite forces, each equal to T, in the plane of the tensile steel. The external forces to be resisted are then a couple, of amount, $T\, e'$, and a direct stress, T, in the plane of the tensile steel. The moment, $T\, e'$, as was demonstrated previously, is resisted in the same manner as a moment due to vertical forces; and the beam may be designed for this moment in the same manner as is the ordinary rectangular beam. This determines the concrete stress, f_c, and the quantity of steel required for moment only. The force, T, acts in the line of the tensile steel, and must be resisted by an additional steel area, $\frac{T}{f_s}$. Then one may write:

$$A_s = p_1\, b\, d + \frac{T}{f_s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which p_1 is the percentage of steel required for moment only; and A_s is the total cross-sectional area of steel. This completes the design. Equation (25) may be readily derived from Equations (20) and (21), if $f_s\, p_1$ is substituted for its equivalent, $\frac{1}{2} f_c\, k$. The value of p_1 is determined readily from any of the ordinary diagrams for rectangular beams given in textbooks on concrete design.[7]

The analysis of a beam subject to bending and direct tension then resolves into the following steps: (1) Determine the moment, $M = T\, e'$, in which e' is the distance from the tensile steel to the applied tension, T; (2) compute $K = \frac{M}{b\, d^2}$; (3) enter the diagram (for rectangular beam design) with the value of K, proceed to the desired value of f_s, and read directly the values of f_c and p_1; and (4) determine the total steel required, by Equation (25). This is seen to be much shorter than the author's method, and to require only the ordinary diagram for rectangular beam design found in textbooks.

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[7] For example, "Structural Members and Connections", by Hool and Kinne, p. 467.

To solve the example given by Mr. Gumensky: $T = 16\,530$ lb; $e' = 10$ in.; $n = 15$; $M = T\,e' = 165\,300$ in-lb; and, $K = \frac{M}{b\,d^2} = \frac{165\,300}{12 \times 10 \times 10} = 137.7$. Entering the diagram with this value of K, and reading at $f_s = 18\,000$, one finds at once, that $f_c = 800$, nearly, and $p_1 = 0.0088$. Then,

$$A_s = p_1\,b\,d = 0.0088 \times 120 = 1.056 \text{ sq in.}$$

$$+\frac{T}{f_s} = \frac{16\,530}{18\,000} = \underline{0.918}$$

$$1.974 \text{ sq in.}$$

The author finds a value of 1.92 sq in., the difference being no doubt due to differences in plotting and reading the diagrams.

An additional advantage of this method of analysis is that where the compressive stress is too high, compressive steel may be proportioned readily by the methods used in the design of ordinary rectangular beams reinforced for compression, proportioning for the bending moment, $T\,e'$.

The author treats of beams subject to bending and direct tension. As a corollary of this discussion, however, it may be noted that the foregoing analysis applies equally to the common case of beams under bending and direct compression, except that the area of the tensile steel is reduced, rather than increased, for the direct stress, the expression bcoming, $A_s = p_1\,b\,d - \frac{C}{f_s}$ in which C is the direct compression.

F. C. Snow,[8] M. Am. Soc. C. E. (by letter).—No attempt to design for compressive and tensile reinforcement is made by Mr. Gumensky, and this is often necessary in members of fixed size, subject to bending and direct stress. The following modifications of Mr. Gumensky's methods apply to reinforcement for tension only or for tension and compression if both are needed. Equation (10) can be written:

$$k_0 = \frac{1}{1 + \frac{f_s}{n\,f_c}} \quad \text{.......................(26)}$$

k_0 being the value of k which represents the limit of use of tensile steel alone, and demands the use of both compressive and tensile steel, if the f_c-stress is not to exceed a certain value.

If the value of f_c from Equation (10) is substituted in Equation (20):

$$\frac{T\,e'\,n}{f_s\,b\,d^2} = \frac{k^2(1 - \frac{1}{3}k)}{2(1 - k)} \quad \text{.....................(27)}$$

in which,

$$e' = \frac{M}{T} - d + \frac{d_t}{2} \quad \text{.....................(28)}$$

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[8] Prof. of Civ. Eng., Georgia School of Technology, Atlanta, Ga.

If the same value of f_c is substituted in Equation (21):

$$A_s = \frac{T}{f_s} + \left(\frac{k^2}{2\,(1-k)}\right)\frac{b\,d}{n} \;\dots\dots\dots\dots\dots\dots (29)$$

In Table 1 various values of k are computed in terms of $\frac{T e' n}{f_s\, b\, d^2}$ and $\frac{k^2}{2\,(1-k)}$.

TABLE 1.—CONSTANTS FOR USE IN REINFORCED CONCRETE BEAMS

k	$\frac{T e' n}{f_s\, b\, d^2}$	$\frac{k^2}{2\,(1-k)}$	k	$\frac{T e' n}{f_s\, b\, d^2}$	$\frac{k^2}{2\,(1-k)}$	k	$\frac{T e' n}{f_s\, b\, d^2}$	$\frac{k^2}{2\,(1-k)}$	k	$\frac{T e' n}{f_s\, b\, d^2}$	$\frac{k^2}{2\,(1-k)}$
0.10	0.0054	0.0055	0.27	0.0454	0.0499	0.44	0.1475	0.1729	0.60	0.3600	0.4500
0.11	0.0065	0.0068	0.28	0.0494	0.0544				0.61	0.3801	0.4771
0.12	0.0079	0.0082	0.29	0.0535	0.0592	0.45	0.1565	0.1841	0.62	0.4012	0.5058
0.13	0.0094	0.0098				0.46	0.1659	0.1959	0.63	0.4237	0.5364
0.14	0.0109	0.0114	0.30	0.0578	0.0643	0.47	0.1757	0.2084	0.64	0.4475	0.5689
			0.31	0.0624	0.0696	0.48	0.1861	0.2215			
0.15	0.0126	0.0132	0.32	0.0673	0.0753	0.49	0.1970	0.2354	0.65	0.4728	0.6036
0.16	0.0144	0.0152	0.33	0.0723	0.0813				0.66	0.5000	0.6406
0.17	0.0164	0.0174	0.34	0.0777	0.0876	0.50	0.2083	0.2500	0.67	0.5283	0.6802
0.18	0.0186	0.0198				0.51	0.2203	0.2654	0.68	0.5587	0.7225
0.19	0.0209	0.0222	0.35	0.0832	0.0942	0.52	0.2329	0.2817	0.69	0.5913	0.7679
			0.36	0.0891	0.1013	0.53	0.2460	0.2988			
0.20	0.0233	0.0250	0.37	0.0953	0.1087	0.54	0.2590	0.3159	0.70	0.6261	0.8167
0.21	0.0260	0.0279	0.38	0.1017	0.1165				0.71	0.6634	0.8692
0.22	0.0287	0.0310	0.39	0.1085	0.1247	0.55	0.2745	0.3361	0.72	0.7035	0.9257
0.23	0.0317	0.0344				0.56	0.2898	0.3564	0.73	0.7467	0.9869
0.24	0.0349	0.0379	0.40	0.1154	0.1333	0.57	0.3060	0.3778	0.74	0.7933	1.0531
			0.41	0.1230	0.1425	0.58	0.3230	0.4004			
0.25	0.0382	0.0417	0.42	0.1358	0.1521	0.59	0.3410	0.4245	0.75	0.8438	1.1250
0.26	0.0417	0.0457	0.43	0.1390	0.1622						

Use of Table 1 for Tensile Steel Only.—In this case k_0 is computed by Equation (26) and $\frac{T e' n}{f_s\, b\, d^2}$ is computed from the data of the beam. Then, k is taken from Table 1; if it is less than, or equal to, k_0 only tensile steel is needed. Area A_s is computed by Equation (29), taking the value of $\frac{k^2}{2\,(1-k)}$ from Table 1.

Applying the foregoing modifications to Example 1 of the paper, and assuming that f_c does not exceed 800 lb per sq in., and that f_s does not exceed 18 000 lb per sq in.: By Equation (26), $k_0 = 0.40$; by Equation (28), $e' = 10$ in.; and by Equation (27), $\frac{T e' n}{f_s\, b\, d^2} = 0.1148$. From Table 1, therefore, $k = 0.398$; and $\frac{k^2}{2\,(1-k)} = 0.1316$ by interpolation.

Since k is less than k_0 only tensile steel is needed and by Equation (29), $A_s = 1.97$ sq in.

Use of Table 1 for Compressive and Tensile Steel.—Assume that $M = 25\,000$ lb-ft, and that, otherwise, values are the same as in the foregoing case; then, by Equation (28), $e' = 14.14$ in.; and by Equation (27), $\frac{T e' n}{f_s\, b\, d^2} = 0.1623$. From Table 1, $k = 0.456$; and $\frac{k^2}{2\,(1-k)} = 0.1911$. Since k is greater than k_0 both compressive and tensile steel are needed.

If the reinforcement in the compressive side of a beam is, A'_s, placed a distance of d' in. in from the extreme fiber, the stress carried by this steel is,

$$A'_s f_c (n-1)\left(\frac{k\,d-d'}{k\,d}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

Equation (20) becomes,

$$T\,e' = \frac{1}{2} f_c\,k\,b\,d\left(d-\frac{k\,d}{3}\right) + A'_s f_c (n-1)\left(\frac{k\,d-d'}{k\,d}\right)(d-d')\,..(31)$$

and Equation (21) becomes,

$$T - A_s f_s + \frac{1}{2} f_c\,k\,b\,d + A'_s (n-1) f_c\left(\frac{k\,d-d'}{k\,d}\right) = 0 \;\ldots..(32)$$

Solving Equation (31) for A'_s,

$$A'_s = \frac{T\,e' - \dfrac{1}{2} f_c\,k\,b\,d\left(d-\dfrac{k\,d}{3}\right)}{(n-1) f_c\,\dfrac{(k\,d-d')}{k\,d}\,(d-d')} \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

Solving Equation (32) for A_s,

$$A_s = \frac{T}{f_s} + \frac{\dfrac{k^2}{2\,(1-k)}\left(\dfrac{b\,d}{n}\right)}{f_s} + \frac{A'_s (n-1)\left(\dfrac{k\,d-d'}{k\,d}\right)}{f_s} \ldots..(34)$$

Substituting f_s in terms of f_c from Equation (10) in Equation (34):

$$A_s = \frac{T}{f_s} + \frac{k^2}{2\,(1-k)}\left(\frac{b\,d}{n}\right) + A'_s (n-1)\left(\frac{k\,d-d'}{n\,(1-k)\,d}\right) \ldots(35)$$

If the value of A'_s from Equation (33) is substituted in Equation (34),

$$\begin{aligned} A_s &= \frac{T}{f_s} + \frac{k^2}{2\,(1-k)}\left(\frac{b\,d}{n}\right) + \frac{T\,e'}{f_s\,(d-d')} \\ &\quad - \frac{k^2\left(1-\dfrac{k}{3}\right)}{2\,(1-k)}\left(\frac{b\,d^2}{n\,(d-d')}\right) \ldots\ldots\ldots\ldots\ldots\ldots(36) \end{aligned}$$

Substituting f_c in terms of f_s and k from Equation (10):

$$\begin{aligned} A_s &= \frac{T}{f_s} + \frac{k^2}{2\,(1-k)}\left(\frac{b\,d}{n}\right) + \frac{T\,e'}{f_s\,(d-d')} \\ &\quad - \frac{k^2\left(1-\dfrac{k}{3}\right)}{2\,(1-k)}\left(\frac{b\,d^2}{n\,(d-d')}\right) \ldots\ldots\ldots\ldots\ldots\ldots(37) \end{aligned}$$

but $\dfrac{k^2\left(1-\dfrac{k}{3}\right)}{2\,(1-k)} = \dfrac{T\,e'\,n}{f_s\,b\,d^2}$ by Equation (27), so that Equation (37) is the same as Equation (29). In other words, the latter can be used to compute the tensile steel whether or not compressive steel is needed.

The foregoing substitution from Equation (27) involves the assumption that f_c can be made large enough so that no compressive steel is needed—namely, that $f_c = f_s \left(\frac{k}{n(1-k)}\right)$, k in this case being greater than k_0, making f_c greater than the allowable working stress. This statement indicates directly that Equation (29) applies both to tensile and compressive reinforcement conditions, but the foregoing proof is given as a further check.

Equation (35) will be used to compute A'_s and, since f_c cannot be increased beyond its allowable value, k_0 must be used instead of k. By Equation (35), substituting the value of A_s from Equation (29):

$$\frac{T}{f_s} + \left(\frac{k^2}{2(1-k)}\right)\frac{b\,d}{n} = \frac{T}{f_s} + \left(\frac{k_0^2}{2(1-k_0)}\right)\frac{b\,d}{n} + A'_s\,(n-1)\left(\frac{k_0\,d - d'}{n(1-k_0)\,d}\right)$$

from which,

$$A'_s = \left[\frac{k^2}{2(1-k)} - \frac{k_0^2}{2(1-k_0)}\right]\left(\frac{(1-k_0)\,b\,d^2}{(n-1)(k_0\,d - d')}\right) \quad \ldots\ldots(38)$$

Applying Equation (29) to the example for $k = 0.456$, Table 1 shows that $\frac{k^2}{2(1-k)} = 0.1911$; and for $k_0 = 0.400$, Table 1 shows that $\frac{k_0^2}{2(1-k_0)} = 0.1333$.

By Equation (29) $A_s = 2.45$ sq in.; and by Equation (38), $A'_s = 1.49$ sq in. In this case the compressive steel is not excessive, but suppose that $M = 21\,400$ ft-lb, $T = 3\,915$ lb, $b = 12$ in., and $d = 8$ in.; then $e' = 68.6$ in.; and $\frac{T\,e'\,n}{f_s\,b\,d^2} = 0.2913$. From Table 1, $k = 0.561$; and $\frac{k^2}{2(1-k)} = 0.3584$. With $k_0 = 0.4$, as before, $\frac{k_0^2}{2(1-k_0)} = 0.1333$, from Table 1. Then, by Equation (38), $A'_s = 6.17$ sq in. Should this steel area be considered excessive, it can be decreased by reducing f_s, keeping f_c constant. Assuming that f_s is 11 500: $\frac{T\,e'\,n}{f_s\,b\,d^2} = 0.4561$ (from the beam data). From Table 1, $k = 0.645$; and $\frac{k^2}{2(1-k)} = 0.5838$. By Equation (26), $k_0 = 0.510$; and by Table 1, $\frac{k_0^2}{2(1-k_0)} = 0.2654$. By Equation (38), $A'_s = 4.11$ sq in.; and by Equation (29), $A_s = 4.08$ sq in. This solution makes $A_s = A'_s$ and required three trials for varying values of f_s.

If f_s in terms of f_c from Equation (10), and A'_s for A_s, are substituted in Equation (35), and after solving for A'_s the resulting expression is equated to A'_s from Equation (33), a cubic equation in k_0 results, from which k_0 for $A_s = A'_s$ can be computed for any value of f_c. It is just as easy, however, to make three or four trial computations like the one just completed for different values of f_s until $A_s = A'_s$, as it is to solve the cubic equation in terms of k_0, should it be desired to make $A_s = A'_s$.

RALPH E. BYRNE, JR.,[9] JUN. AM. SOC. C. E. (by letter).—The problem of combined tension and bending has been presented very clearly in this paper. However, when part of the section is in compression, the case of tension and bending is one part only of the more general problem of bending and direct stress, the case of compression and bending being the other part. Furthermore, by re-arranging the terms in the equations derived by the author, these equations may be obtained in forms which lend themselves more readily to solution by means of curves.

Consider the case in which part of the section is in compression, and the solution is related to the center of the reinforcement in the tension face, there being no reinforcement in the compression face. The author's notation and sign convention will be used with the following exceptions: N will be used to denote the resultant direct stress, either tension or compression, and will be considered positive when the direct stress is compression. For the case being discussed, eccentricities are measured from the center of the tension steel, and will be assumed positive when measured toward the compression face of the member; that is, in Fig. 4 of the paper, positive eccentricities are measured upward from the steel. It should be noted that the part of the section in compression is always on the positive side of the steel as defined by this sign convention.

Taking moments about the center of the tension steel,

$$N e' = \frac{1}{2} f_c \, k \left(1 - \frac{k}{3}\right) b \, d^2 \quad \text{.................} (39)$$

Equating the sum of the horizontal forces to zero,

$$- N + \frac{1}{2} f_c \, k \, b \, d - A_s \, f_s = 0 \quad \text{................} (40)$$

Equation (40) differs from Equation (21) of the paper only in the sign of the direct stress; this difference is accounted for by the change in sign convention suggested by the writer. Equation (39) is identical with Equation (20). Since the right member of Equation (39) is always positive, it is evident that N and e' must always be of the same sign; that is, positive direct stress (compression) must always be accompanied by positive eccentricity, and negative direct stress (tension), by negative eccentricity. A third equation is obtained from the assumption of straight-line distribution of stress,

$$\frac{f_s}{n \, f_c} = \frac{1 - k}{k} \quad \text{......................} (41)$$

Equations (39), (40), and (41), solved simultaneously, will yield the same expressions as those obtained by the author, except that all terms containing e'

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[9] Structural Designer, Pacific Elec. Ry., Los Angeles, Calif.

will be of opposite sign. However, other forms of these equations more readily lend themselves to solution by means of curves.

Problems of bending and direct stress which fall into the case being considered may be divided roughly into two types: (*a*) Those in which the eccentricity is small compared to the depth of the member; and (*b*) those in which the eccentricity is large compared to the depth of the member. For the first type, curves plotted using values of the quantity, $\frac{e'}{d}$, will prove advantageous —that is, curves similar to Fig. 5. The chief objection to these particular curves, however, is that a separate set of curves is required for each value of n. This objection may be overcome as follows: Rewrite Equation (23):

$$\frac{T\,n}{f_s\,b\,d} = p\,n - \frac{k^2}{2\,(1-k)} \qquad \text{(42)}$$

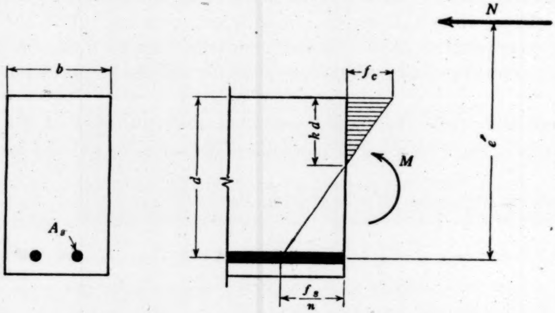

FIG. 6.

Using Equation (42), a set of curves similar to those in Fig. 5 could be plotted, using values of the quantity, $p\,n$, as abscissas, values of k as ordinates, and drawing curves of the quantities, $\frac{e'}{d}$ and $\frac{T\,n}{f_s\,b\,d}$. This single set of curves would then cover any problem that falls into the case being considered, and by adopting a sign convention similar to that offered in this discussion, the curves could be extended to cover the entire range of both tension and compression. For the second type, that is, problems in which the eccentricity is large compared to the depth of the member (see Fig. 6), curves plotted using values of the quantity, $\frac{d}{e'}$, will prove more advantageous. In Fig. 6, the quantities used in plotting the curves in Fig. 7 are defined; thus, $M = N\,e'$; $f_c = C\,\frac{M}{b\,d^2}$; and $R = \frac{f_s}{n\,f_c} = \frac{1-k}{k}$. It is to be noted that N and e are both positive when the direct stress is compression, and negative when the direct stress is tension. The moment, M, is always positive. The curves in Fig. 7 are for cases of large eccentricity, and were plotted from Equation (44), which was obtained as follows: Eliminating N between Equations (39) and (40) and letting $A_s = p\,b\,d$:

$$\frac{f_s}{f_c} = \frac{k}{2\,p}\left[1 - \left(1 - \frac{k}{3}\right)\frac{d}{e'}\right] \qquad \text{(43)}$$

Substituting the value of k from Equation (41) in Equation (43) and letting $R = \frac{f_s}{n f_c}$:

$$\frac{d}{e'} = \frac{3\,(R+1)}{3\,R+2}\,[1 - 2\,p\,n\,R\,(R+1)] \quad \ldots\ldots\ldots\ldots (44)$$

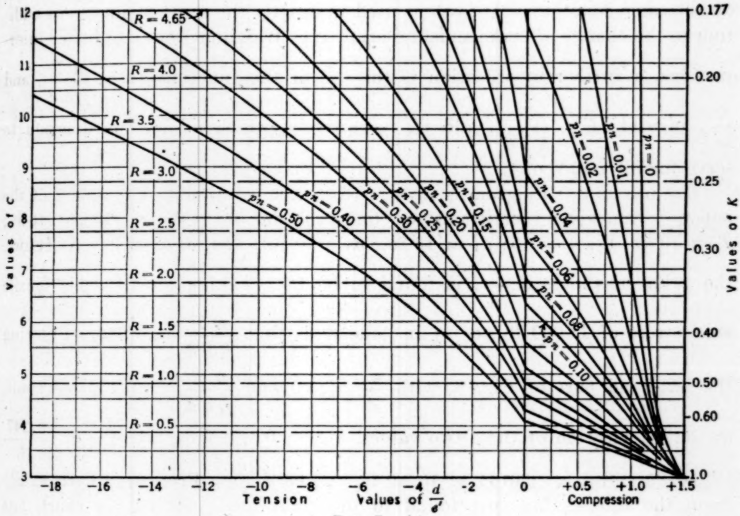

FIG. 7.

Probably of greater importance than any of the cases treated by the author is that of bending and direct stress in which both sides of the member are reinforced, and in which part of the section is in compression. In this case, again, the solution is general for both tension and compression. The case in which the direct stress is compression has already been amply treated[10], and need not be repeated here. Noting that direct tension must be accompanied by negative eccentricity, it is evident that all equations derived for the case of bending and compression can be applied directly to the case of bending and tension if the sign of every term which contains e is reversed; curves which cover the case of bending and tension are merely extensions of the curves which cover the case of bending and compression.

CARL H. HEILBRON, JR.,[11] ASSOC. M. AM. SOC. C. E. (by letter).—Useful and convenient charts are presented in this paper, for solving the usual type of problem of combined tension and bending in concrete members with reinforcement in a single layer. The writer believes that curves of the type of

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[10] "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. B. Maurer.

[11] Asst. Engr. (Design), Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

Fig. 5, rather than Fig. 3, will be of greater use, since a smaller number of diagrams will be required for the solution of all problems.

In using Fig. 5, it is rather difficult to follow some of the lines because of their curvature, and hard to determine the intersections accurately because of the sharp angles. To overcome these troubles the writer suggests the use of Fig. 8, which is fundamentally the same as Fig. 5, being based on the same equations. It is to be used in exactly the same manner. In addition to the change of arrangement, permitting straighter lines and better intersections, a further modification is the use of the functions, $n\ p$, $\frac{n\ T}{f_s\ b\ d}$, and $\frac{f_s}{n}$, instead of p, $\frac{T}{f_s\ b\ d}$, and f_s, respectively, which allows one diagram to serve for all values of n.

The use of this diagram is illustrated by the solution of Example 2 of the paper. The given conditions of the problem are: $M = 19\,230$ ft-lb.; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; and, $d_c = 2$ in. As before, the value of the ratio, $\frac{e'}{d}$, is found to be 1.0. Taking an allowable tensile steel stress of 18 000 lb per sq in., one finds that $\frac{n\ T}{f_s\ b\ d} = 0.115$. Entering the left-hand part of Fig. 8, with $\frac{e'}{d} = 1.0$ and $\frac{n\ T}{f_s\ b\ d} = 0.115$, one reads, at the intersection of these two values, $n\ p = 0.247$, from which $p = \frac{0.247}{15} = 0.0165$ and $a_s = 0.0165 \times 10 \times 12 = 1.98$ sq in. Projecting horizontally from the intersection just found to the right-hand part of the chart, one finds, at the intersection with the line, $\frac{f_s}{n} = \frac{18\,000}{15} = 1\,200$, the value, $f_c = 800$ lb per sq in.

The value found for a_s (1.98 sq in.) is very close to 1.973 sq in., as given by exact mathematical solution of the equations involved. The value obtained by the author ($a_s = 1.92$ sq in.), although sufficiently close for ordinary design purposes, is in error by an amount indicating inaccuracy in his diagrams.

Many engineers will probably prefer to use separate diagrams for various values of n. A diagram for any given value of n can easily be prepared from Fig. 8 by writing in the values of p, $\frac{T}{f_s\ b\ d}$, and f_s, for the value of n selected, on each of the lines for $n\ p$, $\frac{n\ T}{f_s\ b\ d}$, and $\frac{f_s}{n}$, respectively. Others may prefer diagrams of the type of Fig. 3, which can be modified in the same way as Fig. 5 was re-arranged to give Fig. 8, with the same resultant advantages.

The writer wishes to call attention to another method of solving the usual type of problem involving combined bending and tension (or bending and compression) where there is reinforcement on one side of the section

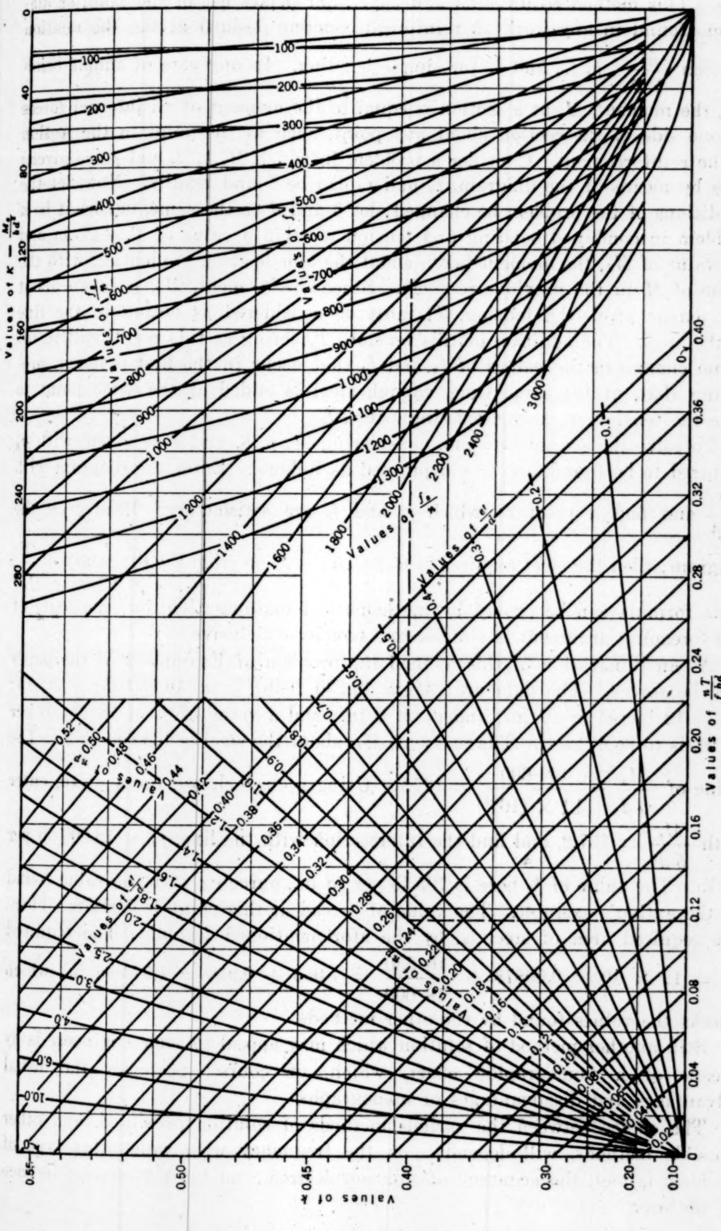

FIG. 8.—DIAGRAM FOR SOLUTION OF PROBLEMS IN REINFORCED CONCRETE, IN COMBINED BENDING AND TENSION WITH REINFORCEMENT IN A SINGLE LAYER. (MOMENT AND ECCENTRICITY ARE REFERRED TO THE CENTER OF GRAVITY OF THE TENSION STEEL.)

only. This method requires no new chart, but makes use of the familiar diagram (found in any work on reinforced concrete design) giving the relation between $\frac{M}{b\,d^2}$, p, f_c, and f_s for simple bending. In any case of simple bending, the moment, M, at a section is equal to the moment of all internal forces on one side of the section about any point, such as the point in the section at the reinforcement. Consider a problem in which M, b, d, and p are given; then by means of the diagram, f_s and f_c can be found readily. Now, let the conditions of the problem be changed; let b and d be the same, but let it be a problem in combined bending and tension wherein a value of T is given and the value of M_s, the moment taken about the tensile steel, is identical with the value of M in the previous problem. Since the moment, M_s, is taken about the tensile steel, the tension, T, must be considered as acting in the line of the steel. The addition of this tension, T, acting at this point, will result in no change in the values of f_s and f_c that occur in the first problem, providing that, at the same time, enough steel is added at the same point to take the tension, T, at the existing stress, f_s.

To solve the second problem, considering M, p, b, and d as given, with a_s required to be found, one would proceed as follows: Enter the diagram with $\frac{M}{b\,d^2}$ and find a point at which f_c and f_s are satisfactory. Read p on the diagram; then the steel required is $a_s = (b\,d) \times (p \text{ from the diagram}) + \frac{T}{f_s}$. This formula can be proved by mathematical manipulation, but the logic of the foregoing argument is simpler and equally conclusive.

As an illustration of this method the problem of Example 2 of the paper will be solved. The data are: $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; and $d_c = 2$ in.; and a value of f_s of 18 000 lb per sq in. is to be allowed. The moment, M_s, about the steel is 164 600 in-lb. The value of $\frac{M_s}{b\,d}$ is $\frac{164\,600}{12 \times (10)^2} = 137.2$. Using a chart drawn for $n = 15$, enter with $\frac{M_s}{b\,d^2} = 137.2$, and find the intersection with the line, $f_s = 18\,000$ lb per sq in. The value of f_c here is 795 lb per sq in., agreeing with the value found in the author's example. The value of p read at this point is 0.00878. Then, the required area of steel is, by the aforementioned formula: The required $a_s = 12 \times 10 \times 0.00878 + \frac{16\,530}{18\,000} = 1.055 + 0.918 = 1.973$ sq in., which checks the values found by the other methods.

Still another method of solution which may appeal to some engineers is by means of a nomographic chart, which, of course, possesses the usual advantages and disadvantages of nomographs.

The choice between the various methods of solution presented, and other possible methods, will depend upon the frequency with which this type of problem is met, the accuracy of solution desired, and the personal preference of the user.

F. E. TURNEAURE,[12] HON. M. AM. SOC. C. E. (by letter).—The author has presented a very complete analysis of the problem of combined bending and tension. His diagrams are particularly useful for determining the stresses in a given beam subjected to known forces, but in designing a beam for certain specified working stresses a simpler method can be used. For this purpose the tension steel should be taken as the moment center, as done by the author in his second solution. The forces acting on the section may then be represented as a moment, M_1, and a direct tension, T, applied at the level of the steel. The beam is then designed as an ordinary rectangular beam for M_1 and the resulting steel area increased by an amount equal to $\frac{T}{f_s}$.

Taking the problem used by the author, $M_1 = 165\,300$ in-lb; and, $K = \frac{M_1}{b\,d^2} = 137.7$: For this value of K and for $f_s = 18\,000$, the usual tables or diagrams for rectangular beams give $p = 0.88\%$ and $f_c = 795$. The total steel area $= 0.0088 \times 120 + \frac{16\ 530}{18\ 000} = 1.97$ sq in. If the concrete stress is too high, the design can be adjusted by determining the value of p from the allowed concrete stress, thus increasing the steel area, or by increasing the value of d, as in any rectangular beam. Any change in the value of d, of course, will modify M_1, but this is readily re-calculated. In analyzing a given design, diagrams may be used, or the problem must be solved by "cut-and-try" methods.

H. E. WARRINGTON,[13] M. AM. SOC. C. E. (by letter).—The problem treated in this paper was encountered in the design of the tunnel from the sewage treatment plant of the Los Angeles County (Calif.) Sanitation Districts to the Pacific Ocean. Diagrams such as those in the paper would have been of great service in that case. The same problem was treated quite fully by Professor Emil Mörsch in 1908.[14] The analysis, of course, is similar to that of the author, but the resulting diagrams are not. The requisite steel is not obtained directly as in this paper, hence, further computations are necessary.

In checking the results of the aforementioned studies with the author's diagrams, it happened, unfortunately, that $\frac{T}{f_s\,b\,d}$ and $\frac{e}{d}$ were far to the left upper corner, and their intersections were too indefinite, the curves being so nearly parallel. Mörsch's diagram does not suffer in this respect, although, in order to use it, a preliminary percentage of steel must be chosen. His graph then gives the value of k from that of $\frac{M}{T\,d_t}$, which is known. With k given, all stresses are determined at once. Therefore, the general formula,

$$e = \frac{M}{T} = \frac{x^2\,(3\,d_t - 2\,x) + 6\,n\,p\,d\,(d - x)\,(2\,d - d_t)}{12\,n\,p\,d\,(d - x) - 6\,x^2} \quad \ldots\ldots (45)$$

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[12] Cons. Engr.; Dean, Coll. of Mechanics and Eng., Univ. of Wisconsin, Madison, Wis.

[13] Asst. Engr., Los Angeles County San. Dists., Los Angeles, Calif.

[14] "Concrete Steel Construction", by Emil Mörsch; see, also, translation by E. P. Goodrich, M. Am. Soc. C. E., Engineering News Pub. Co., 1910.

was used, in which $x = k\ d_t$ and all other quantities are known. The cubic in x is best solved by trial approximation. Usually, two or three trials are all that are necessary. A check against numerical errors in the value of k is given by substituting the resulting foregoing data in the formula for,

$$M \text{ (in-lb)} = f_c \frac{b\ k\ d_t}{2} \left(\frac{d_t}{2} - \frac{k\ d_t}{3} \right) + f_s\ A_s \left(d - \frac{d_t}{2} \right) \ldots\ldots (46)$$

For preliminary results, Mörsch treats the rectangular section as a beam, without reinforcement, by the well-known formula,

$$f_c = \frac{T}{b\ d_t} \pm \frac{6\ M}{b\ d^2_t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

thus finding the edge stresses. Then, he assumes all the tension to be taken by the steel, determining a quantity,

$$Z = \frac{b^2\ d^3_t\ f^2_c}{24\ M} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

in which f_c is the tensile unit stress. This value of Z is the total steel stress, and the area required, of course, is $\frac{Z}{f_s}$. Trials of this method showed that the steel area thus found was a little larger than that given by the more accurate formulas. This method, by the way, is equally applicable to compression and bending. Possibly a diagram such as the valuable one for bending and compression, with single reinforcement, as presented[15] by F. E. Turneaure, Hon. M. Am. Soc. C. E., and Professor E. R. Maurer, could be made.

William A. Larsen,[16] Esq. (by letter).—The topic of this paper is one that has been peculiarly avoided in the past. If it has been one's experience to design members of reinforced concrete structures for direct tension and bending, one will realize how little is recorded concerning it, but one must also realize that work of this nature has been constructed for some time. The reason for omitting discussions and charts on this topic in the American texts on concrete design is not fully understood by the writer. Simple bending and compression is a common problem and is covered fully in many books. Bending and tension are so closely related to it that it should offer no difficulty, as is shown by the author. This is also verified by other works.

It was pointed out by the author that the curves of Fig. 3 are less desirable than those of Fig. 5. The writer concurs and will direct the succeeding discussion primarily to the method presented, using the tensile steel as a reference for the eccentricity of the normal force.

The use of charts for the solution of equations similar to those encountered in this case not only expedites the work but also provides a graphical picture of the variables assisting the designer to visualize which of them has the most pronounced effect. It is best, therefore, to make the charts as simple and clear as possible. It is also desirable to keep them similar to other reinforced concrete diagrams in current use. The angle of intersection of the curves on a chart should approximate 90° if accuracy is desired. Based on these restrictions, the writer suggests a few changes which would make Fig. 5 more desirable.

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[15] "Principles of Reinforced Concrete Construction", by F. E. Turneaure and E. R. Maurer, Fourth Edition, p. 434.

[16] With U. S. Bureau of Reclamation, Denver, Colo.

The author finds the value of k by use of $\frac{T}{f_s\,b\,d}$ and $\frac{e}{d}$. The intersection of the lines representing these values is not definite in the upper part of the chart. As $\frac{T}{f_s\,b\,d}$ approaches zero and $\frac{e}{d}$ approaches infinity, the angle of intersection of these curves approaches zero. The points of intersection are thus obscured, especially if interpolation is necessary. It would be better to use other curves to obtain better intersections. With this purpose in mind, re-arrange Equation (20) in the form,

$$\frac{T\,e'}{b\,d^2} = \frac{f_c}{2}\,k\left(1 - \frac{k}{3}\right) = \frac{f_c}{2}\,k\,j \qquad (49)$$

This relationship is the same as has been used to plot the curves of the right-hand part of Fig. 5, if $\frac{M}{b\,d^2} = \frac{T\,e'}{b\,d^2}$. If $K = \frac{T\,e'}{b\,d^2}$ is used as the abscissa, k can be found by entering the chart at the bottom for a known K-value, trace upward to the intersection of either f_c, or f_s, and a value of k is thus defined. With the determined value of k, follow horizontally to the intersections with the desired $\frac{e'}{d}$ - line and thence down to the required p-value. This gives the same results as those obtained by the author, but without the use of the $\frac{T}{f_s\,b\,d}$ - curves. This also gives the same procedure as that used for charts on compression and bending[17]. The one operation gives, directly, the values for f_c and f_s in addition to the p-value. It is suggested, therefore, that the $\frac{T}{f_s\,b\,d}$ - curves be omitted, and $\frac{M}{b\,d^2}$ be changed to read $\frac{T\,e'}{b\,d^2}$. This will simplify the charts as well as their use.

The writer believes the curves as now shown, with values, $K = \frac{M}{b\,d^2}$, are misleading, thus emphasizing the need to change to $\frac{T\,e'}{b\,d^2}$. For instance, from Example 2, $K = \frac{M}{b\,d^2} = \frac{19\,230 \times 12}{12 \times 100} = 192.3$. Entering the chart at the lower right and tracing vertically to $f_s = 18\,000$, $f_c = 1\,000$ lb per

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[17] "Concrete Designers Manual", by Hool and Whitney, pp. 179 and 180, McGraw-Hill Book Co., N. Y., 1926.

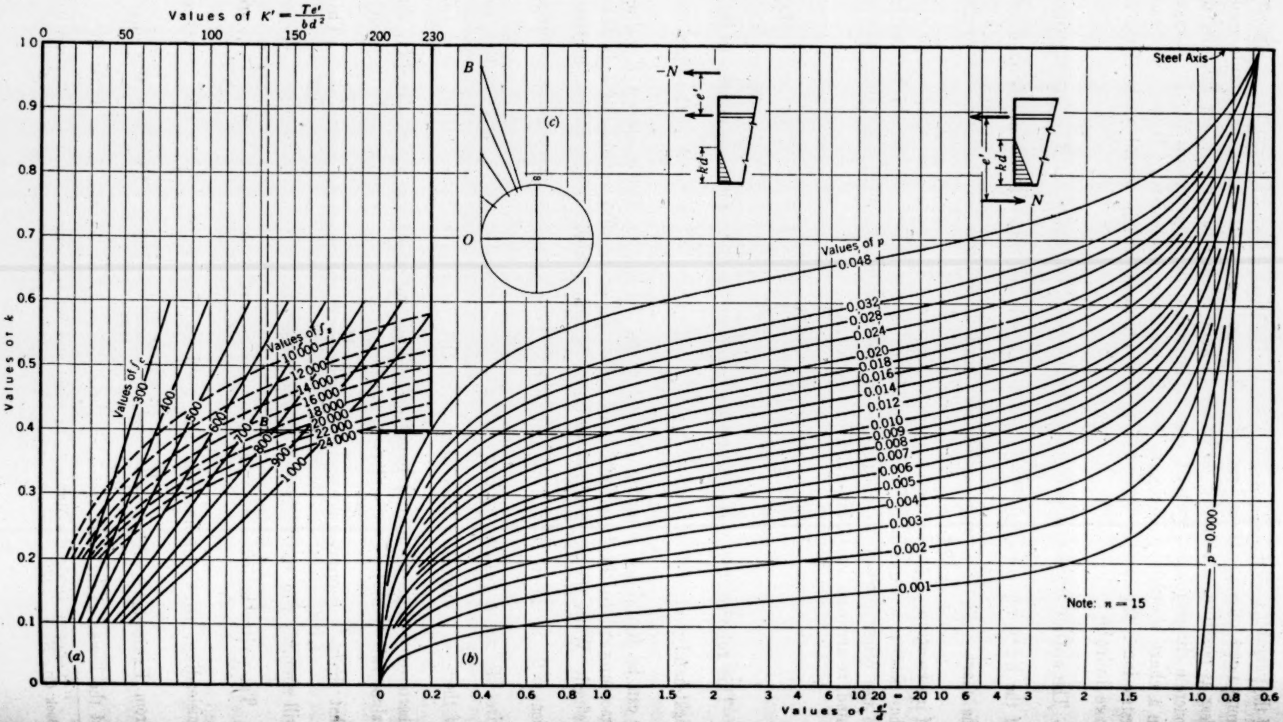
Values of $K' = \frac{Te'}{bd^2}$
0
50
100
150
200
230
Values of k
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Values of f_c
300
400
500
600
700
800
900
1000
Values of f_s
10 000
12 000
14 000
16 000
18 000
20 000
22 000
24 000
B
(a)
(b)
(c)
B
O
∞
$-N$
e'
kd
N
Steel Axis
Values of p
0.048
0.032
0.028
0.024
0.020
0.018
0.016
0.014
0.012
0.010
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
$p = 0.000$
Note: $n = 15$
0
0.2
0.4
0.6
0.8
1.0
1.5
2
3
4
6
10
20 ∞ 20
10
6
4
3
2
1.5
1.0
0.8
0.6
Values of $\frac{e'}{d}$

FIG. 9.

sq in. (approximately, which is erroneous). Thus, using $\frac{T\,e'}{b\,d^2} = 134$, Point B is located, giving the correct values for f_c and f_s.

It is interesting to note that in 1918, E. Suenson[18] published a chart similar to Fig. 9(b) dealing with tension and bending, giving the same results as Fig. 5, but plotted to different co-ordinates. It is simple and easily applied. By adding a set of K, f_c, and f_s-curves (Fig. 9(a)), the chart can be used as was suggested for Fig. 5.

Fig. 9 is unique in that it was plotted as a bending and direct stress diagram, using $-N$ for tension. It can be used for either bending and tension, or bending and compression. For that reason it may be of greater value than Fig. 5. Its use is illustrated by Example 3.

Example 3.—In Example 1 of the paper, $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; and $d = 10$ in. The eccentricity relative to the geometric center $= \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163 = 14$ in. The eccentricity relative to the tension steel is equal to $e' = e - d + \frac{d_c}{2} = 14 - 10 + 6 = 10$ in.; and, $\frac{e'}{d} = \frac{10}{10} = 1$. Furthermore, assume that $f_s = 18\,000$ lb per sq in., $\frac{T\,e'}{b\,d^2} = \frac{16\,530 \times 10}{12 \times 100} = 134$; then find the intersection of $\frac{T\,e'}{b\,d^2}$ and the f_s-curves, which gives $f_c = 795$ and $k = 0.395$ (Point B, Fig. 9(a)). Follow the horizontal line from Point B to the intersection of the vertical line, $\frac{e'}{d} = 1$, and p is found to be 0.016. These results are precisely the same as those found by the author in Examples 1 and 2.

One of the features of Fig. 9(b) is the plotting of the abscissas. The value of $\frac{e'}{d}$ ranging from 0 to ∞ necessitated a particular graph paper. A quadrant of a circle was used, the quarter circumference being assumed to represent a distance of 0 to ∞ (Fig. 9(c)). Points at equal distances were laid off on the tangent, $O\,B$, starting at Point O. Lines were drawn from these points to the center of the circle, the intercepts between these lines on the circumference being used as abscissa to plot the graph paper.

The necessity of using a different chart for each value of n is a serious disadvantage to diagrams of the type shown in Figs. 5 and 9(a). At present, there is a tendency to use n-values ranging from 6 to 15. To eliminate the need of so many charts J. A. Wineland, Jun. Am. Soc. C. E.,[19] in connection with his work at the Bureau of Reclamation, has prepared a diagram embodying n as a variable (see Fig. 10). Various co-ordinates were tried, using different combinations of variables until a diagram giving pronounced intersection of the curves was found. This diagram may be used for any value of n for tension and bending and, with a slight modification, for compression and bending. The diagram was plotted for members with a unit width of 12 in. Values of the ordinates are expressed in terms of,

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[18] "Jaernbeton—Teori og Praksis", by E. Suenson, 1918, P. E. Bluhmes Book Co., Copenhagen, Denmark.

[19] Not published.

$$\frac{e_1}{d} = \frac{k}{2}\left(\frac{2-k}{k+np}\right) + \frac{\left(1-\frac{k}{3}\right)\left(\frac{k^2}{1-k}\right)}{2\,np - \frac{k^2}{1-k}} \quad \ldots\ldots\ldots\ldots (50)$$

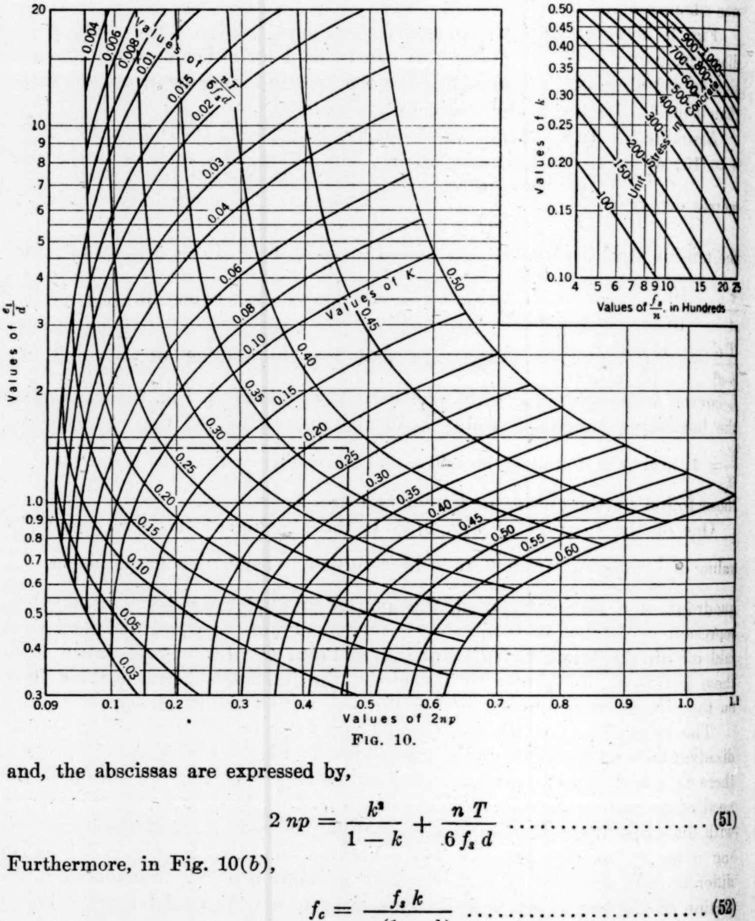

FIG. 10.

and, the abscissas are expressed by,

$$2\,np = \frac{k^2}{1-k} + \frac{n\,T}{6\,f_s\,d} \quad \ldots\ldots\ldots\ldots (51)$$

Furthermore, in Fig. 10(*b*),

$$f_c = \frac{f_s\,k}{n\,(1-k)} \quad \ldots\ldots\ldots\ldots (52)$$

in which, in addition to the notation of the paper, e_1 = eccentricity of the resultant force, T, with respect to the geometrical axis of the stressed areas.

The procedure in solving problems by Fig. 10 is very similar to that given by the author concerning the curves of Fig. 3:

(1) Knowing the moment, M, and the tension, T, determine the value, $e_1 = \frac{M}{T}$.

(2) With a trial value of d determine $\frac{e_1}{d}$.

(3) Using the desired value of n determine the value of $\frac{n\,T}{6 f_s d}$.

(4) Determine the relation of $\frac{f_s}{n}$.

(5) Using the values of $\frac{e_1}{d}$ and $\frac{n\,T}{6 f_s d}$, determine values for $2\,p\,n$ and k by use of Fig. 10(a). Enter the left side of the chart at the $\frac{e_1}{d}$-value and trace horizontally to the right to the intersection with the $\frac{n\,T}{6 f_s d}$-curve; k may be found by reading the curves practically 90° with the $\frac{n\,T}{6 f_s d}$-curves. The value of $2\,pn$ is found at the top of the chart directly above the point just located.

(6) With the value of k and $\frac{f_s}{n}$, Fig. 10(b) is used to find f_c.

(7) The necessary steel area is found by multiplying $2\,pn$ by $\frac{b\,d}{2\,n}$.

It is seen that it is as easy to use Fig. 10 as Fig. 5 and only the one chart is necessary for all values. This compact diagram should appeal to designers who are concerned with various values of n.

The solution of a problem by the use of Fig. 10, with the moment as given in Eample 1, will give slightly different results from those found by the use of Fig. 3 or Fig. 5, because of the assumptions used in deriving the formulas. The author assumed the working lines to go through the geometrical axis of the gross concrete area. The curves of Fig. 10 were made feasible by assuming the working lines to go through the center of gravity or the geometrical axis of the stressed areas of the reinforced concrete member. This means that the value of the moment, M, will vary slightly, depending upon the working lines used. The eccentricity, $e_1 = \frac{M}{T}$, should be referred to the same working lines to which the moment is referred. Usually, however, sufficiently accurate results may be obtained by using the center line of the member as the working line, thus referring e_1 to the center line of the member.

Example 4.—The solution of the problem in Example 1, by use of Fig. 10 will give results comparable to those already obtained. For example: $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; $b = 12$ in.;

and $f_s = 18\,000$. Finally, $e_1 = \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163$ ft $= 14$ in.; $\frac{e_1}{d} = \frac{14}{10} = 1.4$; $\frac{n\,T}{6 f_s d} = \frac{15 \times 16\,530}{6 \times 18\,000 \times 10} = 0.23$; and, $\frac{f_s}{n} = \frac{18\,000}{15} = 1200$. From Fig. 10(*a*), $2\,pn = 0.47$, or $p = 0.0157$; and, $k = 0.385$. From Fig. 10(*b*), $f_c = 750$ lb per sq in.

If the moment, M, is referred to the proper working lines for use in Fig. 10, exactly the same results as those obtained in the other examples will be found.

The geometrical axis of the stressed area is given by:

$$y = \frac{k\,d\left(1 - \frac{k}{2}\right)}{k + n\,p} \text{.......................} (53)$$

in which y = the distance to the geometrical axis of the stressed areas from the steel axis. From Example 2: $k = 0.395$; $p = 0.016$; $n = 15$; $y = 5$ in., or 0.0833 ft above the center line; and,

$$M_1 = M + T\ (0.0833) \text{...................} (54)$$

in which M_1 = the moment referred to the geometrical axis of the stressed areas. In other words, $M_1 = 19\,230 + 16\,530 \times 0.0833 = 20\,608$ ft-lb. Finally $e_1 = \frac{M'}{T} = \frac{20\,608}{16\,530} = 1.25$ ft, or 15 in.; $\frac{e_1}{d} = \frac{15}{10} = 1.5$; and by the same procedure as in Example 3, $p = 0.016$ and $f_c = 795$ lb per sq in.

Conclusion.—The preference of the individual designer for the use of different diagrams and charts, the problems at hand, and the general designing practice will determine which type of diagrams are to be used. It is hoped that this discussion has added constructive information to that already given by the author to clarify this topic, which has been so carefully avoided in concrete design texts.

There is one more important question that should be considered carefully although it has not thus far been mentioned: What are the bond stresses and how may they be calculated? After the designer has conducted a careful analysis and has determined the required steel area by any method similar to those presented, it is necessary that he obtain the correct bond values. The familiar, $u = \frac{V}{b\,j\,d}$, will not satisfy the necessary requirements for bending and direct stress. The direct pull, T, tends to increase the tensile force in the steel over that caused by pure bending.

The change in the total tension per unit length of bar is equal to the bond stress. If a curve is drawn representing the total tension in the steel at any point along the length of the bar, the bond may be determined by the slope of the tangent to that curve. The unit bond stress, being equal to the bond divided by the peripheral area, may then be found. This is an accurate and easily remembered method.

B. KOVEDIAEFF,[20] ESQ. (by letter).—The subject of direct tension combined with bending in reinforced concrete members has been discussed in the more modern textbooks on reinforced concrete. However, the Engineering Profession, and especially engineers interested in structural work, will more than welcome a consistent discussion covering such an important subject.

In the "Introduction", the author states that the paper is an attempt to analyze the stress distribution in reinforced concrete members under direct tension and bending, and yet his first assumption is that of a "straight-line distribution of stress." There seems to be some inconsistency between these two statements. Obviously, if the author is attempting to analyze the stress distribution in the section, he should continue to do so instead of assuming it.

Case 1, in which the entire section is under tension, has been very well treated by Messrs. Taylor, Thompson, and Smulski[8] and, therefore, no attempt need be made to carry the discussion further, since it deals with the reinforcement of both faces of the section, which is obviously not the nature of the problem undertaken by the author.

The first true attempt at an analysis is begun by the introduction of Equation (8). Except for a change of sign and slightly different notation, the same equation is presented in one of the standard handbooks in current use.[21] Furthermore, in Equation (12), the author introduces the term, d_t, and, later, d_c, primarily for the purpose of simplifying the matter to follow; but these terms led the author into difficulties which could be overcome only by means of the introduction of a great number of diagrams. As an example, each diagram is dependent upon n and upon $a = \frac{d_c}{d}$. Using four values for n (8, 10, 12, and 15), and at least five values for a (0.1, 0.15, 0.20, 0.25, and 0.30), the total number of diagrams required to solve problems falling within this scope is twenty, which can scarcely be termed simple. Any value of a within the 0.05 interval would have to be interpolated, which would further complicate the problem.

It is apparent that the value, d_c (depth of cover on the steel), as shown in Fig. 2, has no direct bearing on the solution of the problem, because no matter how large d_c becomes, the distribution of stress remains unchanged. For this reason all formulas used for the determination of steel reinforcement in a single face disregard this value of depth of cover.

It appears that the author finally realized the inconvenience of the numerous curves necessary for the solution of these problems and decided to change the lever arm of the force-producing moments, by introducing Equation (22). Taking Equation (23) and solving for p, the expression for bending and tension is expressed as:

$$p = \frac{k^2}{2\,n\,(1-k)} + \frac{T}{f_s\,b\,d} \qquad \text{..................(55)}$$

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[20] With U. S. Engr. Office, Los Angeles, Calif.

[21] Concrete Engineer's Handbook, by Hool and Johnson, McGraw-Hill Book Co., 1918, p. 403.

An identical equation can be written for bending and compression by changing the sign of the second term to minus; thus:

$$p = \frac{k^2}{2\,n\,(1-k)} - \frac{C}{f_s\,b\,d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

To obtain the expression for p required for the pure bending moment, the second term is omitted; thus,

$$p = \frac{k^2}{2\,n\,(1-k)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

Equations (55), (56), and (57) give three very simple expressions for steel percentage in single-face reinforcement. Determine $K = \frac{M}{b\,d^2}$, and from a set of diagrams of pure bending moments[22] (M being the transformed moment taken about the steel), read the value for p for pure bending as expressed by Equation (57). By one simple computation a value for $\frac{T}{f_s\,b\,d}$ is determined. Adding or subtracting this term to, or from, the value of p as found by Equation (57) will give the value of p for bending moment combined with tension or for bending moment combined with compression, respectively. Therefore, for the determination of the value of p in all three cases, the only diagrams required are those for the solution of pure bending. On the other hand, in the author's discussion, four diagrams for each case are required, or twelve in all.

This brief discussion points out the futility of a set of curves such as those suggested in the paper because of the simplicity of the relation existing between the forces, moments, and the steel ratio.

A. **Floris**,[23] Esq. (by letter).—In this elaborate paper the author derives formulas for the design of reinforced concrete members subjected to direct tension and bending. He states that the problem has received little attention from authors of engineering texts. This statement is rather broad and not quite correct because the subject is fully covered in several books on reinforced concrete. Suffice it to mention the well-known treatises by Moersch,[24], Manning[25], and others. Perhaps the author has in mind American textbooks in which the design of reinforced concrete structures under the combined action of direct tension and bending has been somewhat neglected. In addition to closed water conduits such a stress condition exists in tanks, silos, bunkers, etc.

The formulas derived by the author are based on the standard method which involves the ratio of modulus of elasticity of steel to that of concrete—

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[22] "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, Fourth Edition, John Wiley & Sons, Inc., N. Y., 1932, pp. 414–415.

[23] Dipl.-Ing., Los Angeles, Calif.

[24] "Der Eisenbetonbau, seine Theorie und Anwendung", von E. Moersch, Stuttgart, 1923, Vol. 1, Pt. 1, pp. 454–466.

[25] "Reinforced Concrete Design", by G. P. Manning, Lond., 1924, pp. 54–67.

the well-known *n*. It is becoming increasingly evident by tests and actual experience, however, that this method of designing reinforced concrete structures is fundamentally erroneous. The reasons are many and varied.

The strength of a member subjected to bending depends on the ultimate strength of steel and concrete and not on the value of *n* as the standard method appears to convey. Using this ratio in the calculations, the concrete and steel stresses of the structural member near the breaking point are considerably greater than the ultimate strength of the two materials determined separately. By applying the standard method, authorities permit far smaller stresses for concrete than steel in comparison to their ultimate strength. This leads to the absurdity that heavily reinforced members will resist external forces less than poorly reinforced members under the same conditions. This conclusion as to the standard method of designing structures is not backed by experience and tests. The introduction, in the analysis, of a constant or variable *n* does not improve matters, since it is not in agreement with the experimental evidence. Furthermore, the factor of safety determined by the standard method is not the same for various conditions.

All these facts indicate clearly the desirability of abandoning the present standard method of designing reinforced concrete structures. Attempts to develop methods without using the ratio, *n*, have been made by Stussi, M. and S. Steuermann, Saliger, Gebauer[26], and Bittner.

DAVID B. HALL,[27] ASSOC. M. AM. SOC. C. E. (by letter).—Whenever physical conceptions can be substituted for algebraic operations, understanding of any problem usually becomes clearer. Apparently, Case 1 of this paper, in which the resultant tension on a section falls between two layers of reinforcement, could have been covered completely by treating the two layers of steel as simple beam reactions and calculating the area of steel to furnish these reactions at the allowable working stress. The extreme simplicity of this case results from the fact that it is a statically determinate problem, and, therefore, no question of consistent deformations enters into it. Case 2 may also be a little easier to grasp[28] when viewed apart from its mathematical surroundings, and certain aspects can be treated in somewhat greater detail.

When it is possible to proportion a structure (in this case a concrete section) to resist the loads acting on it, instead of determining the stresses in an existing structure, then, as far as the designer is concerned, the structure ceases to be statically indeterminate. (This principle, incidentally, has many and varied applications; for instance, a two-span continuous truss with a jack placed under one support at the time of erection.) This will be the case in the design of concrete sections. It may be possible to vary the dimensions of the concrete, and it will practically always be possible to vary the area of the reinforcement.

When there is only one layer of steel, and moments are taken about the center of the steel, the moment of the concrete must be equal to the moment of all external forces (Equation (20)), and hence the concrete stress can be found either by calculation or from ordinary concrete tables. The unit stress in the steel is proportional to the distance from the neutral axis (Assumption (1) of the paper); and the total stress in the steel is the sum of the compression in the concrete and the external tension, since both these forces tend to push or pull the member apart while the steel tends to pull it together Although, in a sense, the foregoing is only a re-statement of Equations (20) and (21) in words, the fact is that once these principles are understood, the design of sections will actually be found as easy if the analysis stops at this point as if it is carried any further, as exemplified by Fig. 5. The steps, in brief, are as follows: (1) Determine a concrete section to resist the moment about the steel; (2) determine the steel for such a concrete section; and (3) determine also the steel to resist the tension (or compression) on the section, and add to (or subtract from) the steel found by Step (2).

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[26] "Das alte *n*-Verfahren und die neuen *n*-freien Berechnungsweisen des Eisenbetonbalkens; Kritische Besprechung und Schlussfolgerungen", von Franz Gebauer, *Beton u. Eisen*, January 20, 1936, p. 29.

[27] Asst. Engr., State Dept. of Public Works, Albany, N. Y.

[28] *Engineering News-Record*, July 26, 1928, p. 127.

It so happens that it is not always possible to fit the concrete section to the loads. In designing sections at intervals along a beam, many parts will be found to be understressed. On the other hand, in continuous beams with curved bottoms, there is usually an overstressed part near the middle which would receive more moment and would be stressed still more severely if made deeper. Thus, when the depth of the beam is fixed, two distinct cases may arise, depending on whether the moment about the tension steel is less or more than the resisting moment of the concrete in a "balanced section"; that is, one in which the steel and concrete stress are both the maximum allowable. The first case differs little from that in which the concrete as well as the steel can be controlled. It can be solved by means of the author's charts, or directly from the principles underlying Equations (20) and (21).

In connection with the latter method an expedient used by the writer may be found to be helpful: The function least sensitive to the variation of K (defined as $K = \frac{M}{b\,d^2} = \frac{1}{2}\,f_c\,k\,j$) is j, the fraction of the depth between the steel and the center of compression of the concrete. Since the steel area for moment may be calculated from the relation, $A_s = \frac{M}{f_s\,j\,d}$, the writer uses a table of values of $(f_s\,j)$ for the full range from zero moment to the moment for balanced steel. Since $(f_s\,j)$ varies only between 16 000 and 14 000 lb per sq in. (when $f_s = 16\,000$ lb per sq in.) in this entire range, a table with twenty entries is as accurate as a table of steel percentage value containing eight or ten times as many entries, and the preliminary determination of K can be very rough.

The second case (in which the strength of the concrete is the limiting factor and must be aided by shifting the neutral axis, adding compression steel, or more often both) is usually beyond the scope of the author's charts, and needs to be considered in some detail if the paper is to be reasonably complete. Unless there are restrictions on the amount of steel to be placed in either face of the beam, the most desirable solution will be the one which makes the total steel area in the two faces a minimum. Contrary to a widespread belief, this condition is usually not fulfilled by assuming maximum steel stress. Two causes can be mentioned which may make this fact easier to digest: (1) In cases of bending and compression, it can be seen that the use of a high steel stress, thereby crowding the neutral axis (and hence the area of concrete in compression) over to one side, will be inefficient whenever there is considerable direct compression and not much moment on a section; and (2) when a beam is not very deep and the compression steel is close to the neutral axis, this steel cannot be very effective. Its effectiveness, as well as that of the concrete, furthermore, is increased by shifting the neutral axis farther toward the tension side.

In terms of fundamental relations, only one additional operation is necessary in designing for compression steel, after a given position for the neutral axis has been assumed or established. Assumption (1) of the paper together with Hooke's law, establishes the stress in both layers of steel; compressive steel and additional tensile steel are added to provide a couple to care for the difference between the concrete moment and the external moment. With a good comprehensive table, made up for specified values of f_c and n, giving values of k, K, f_s, and p (with k varying by hundredths from the value corresponding to a balanced section up to 0.60 or 0.70), it is not especially difficult to make a few trials and obtain suitable steel areas. Even without such a table the foregoing constants are not very difficult to compute. Instead of by trial, the desired value of k can be found quite expeditiously with the aid of Fig. 11[29], which is self-explanatory.

It will be noted that the three quantities required for its use are all abstract numbers, so that the chart is independent of any particular units of force or distance. For a strict interpretation of Fig. 11 it is necessary to assume that the compression steel displaces no concrete, but the diagram will undoubtedly be found sufficiently accurate by designers who prefer to deduct the concrete (by using $n - 1$ instead of n for the compression steel). The construction of the chart involves considerable algebra, but not much mystery. An equation for the total area of steel can be written in terms of $k, \frac{d_c}{d}, \frac{d}{e'}$, and $\frac{f_c}{K}$, and can be differentiated with respect to k. The resulting derivative contains k and $\frac{d_c}{d}$ in a relatively complicated form, but can be arranged to contain $\frac{d}{e'}$ and $\frac{f_c}{K}$ as simple linear quantities. This is what makes the use of a nomogram possible. Examination of Fig. 11 will verify the statements made earlier concerning the causes for shifting the neutral axis. It will be observed: (1) That the compressive area is greater when there is thrust on a section than when there is tension; and (2) it is greater when the compression steel is deeply buried than when it is near the face.

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[29] "Calcul des armatures principales des pieces en béton armé a section rectangulaire sollictée en flexion composée", by Henri Dumontier, *La Tecnique des Travaux*, Vol. IV, No. 2, February, 1928, p. 135.

This chart will not always give usable results. It may sometimes call for steel stress higher than the allowable and, being purely algebraic, it can (and sometimes does) lead to the calculation of a negative area of steel, or less than would nominally be used, in one face or the other.

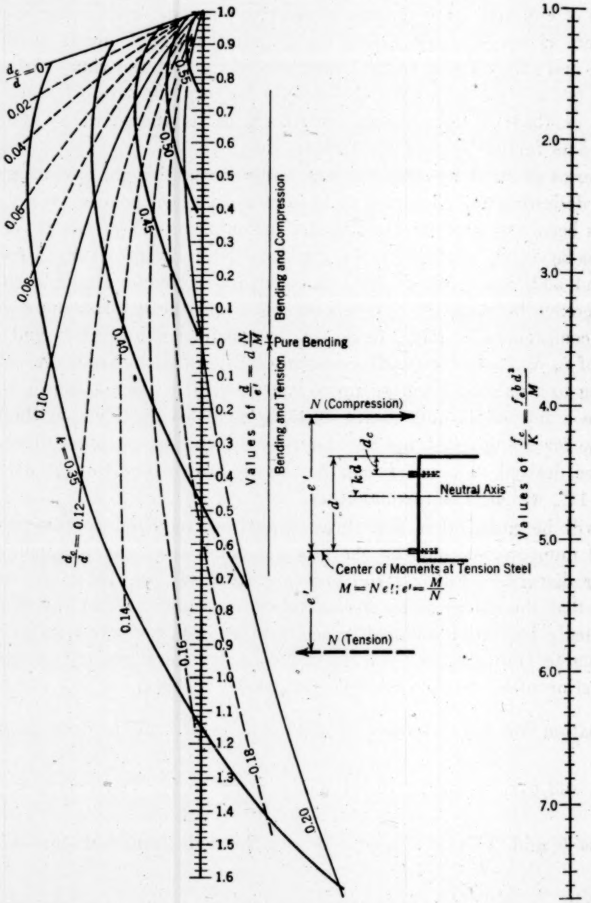

FIG. 11.—POSITION OF NEUTRAL AXIS FOR MINIMUM STEEL AREA. (GIVEN $\frac{f_s}{K}$, $\frac{d}{e'}$, $\frac{d_c}{d}$, THE CURVE GIVES k.)

In developing his special type of chart for direct tension and bending, the author has utilized principles which by themselves are as easy to apply as the chart (see Figs. 3 and 5), and which, furthermore, apply equally well to either tension or compression and bending, and may be applied to one important part of the subject of bending and direct stress to which the chart cannot be applied.

Odd Albert,[30] Assoc. M. Am. Soc. C. E. (by letter).—The statements made in this paper are correct, but the author has not treated the complete problem. There are three general cases in which part of the section is in compression; namely: (*a*) Tension steel only; (*b*) tension steel and compression steel; and (*c*) tension steel and compression steel alike.

The author treats only Case (*a*) in which steel is placed in the tension side. This produces a design in which steel acts in tension and concrete in compression, and results in some rather complicated diagrams, which involve eleven operations for each solution. He also states that, should the intersection of the curves fall outside the limits of the chart, a re-design may be necessary.

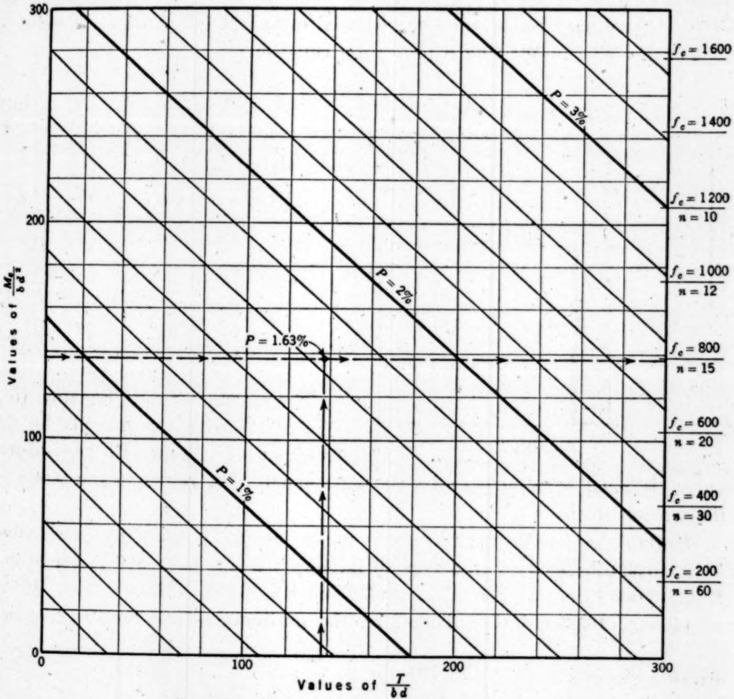

Fig. 12.—Column in Bending and Tension

The problem can be simplified considerably, and a chart, such as Fig. 12, can be produced with only straight lines, that will take care of all conditions.

Effective Depth.—Referring to Fig. 2 and taking moments about the center of the steel (see Fig. 13):

$$T\left(e - d + \frac{d_t}{2}\right) = \frac{f_c\, b\, d\, k}{2}\left(d - \frac{k\, d}{3}\right) \qquad (58)$$

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[30] Faculty Lecturer, Advanced Design of Structures, New York Univ.; Engr., Brick Mfrs. Assoc. of New York, New York, N. Y.

the moment, M_s, with reference to the tension steel is:

$$M_s = T\left(e - d + \frac{d_t}{2}\right) \quad \text{.....................} (59)$$

or, $M_s = 0.5\, b\, d^2\, k\, j\, f_c$. Solving for d:

$$d = \sqrt{\frac{M_s}{K\, b}} \quad \text{........................} (60)$$

Equation (60) is identical with the beam formula for tension steel only. The difference is that the moment, M_s, is taken with reference to the tension steel instead of to the center of the section. Therefore, after the change of M to M_s, ordinary beam tables can be used to find the dimensions, b and d.

Tension Steel.—Writing a projection equation:

$$A_s f_s - T - \frac{b\,k\,d}{2} f_c = 0 \quad \text{...................} (61)$$

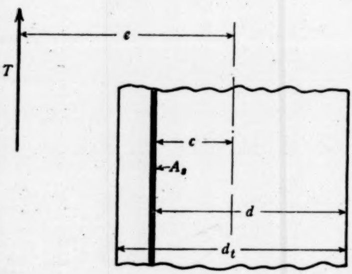

FIG. 13

and, solving for A_s,

$$A_s = \frac{b\,k\,d\,f_c}{2\,f_s} + \frac{T}{f_s} \quad \text{.....} (62)$$

Since $\dfrac{b\,k\,d\,f_c}{2\,f_s} = \dfrac{M_s}{f_s\,j\,d}$:

$$A_s = \frac{M_s}{f_s\,j\,d} + \frac{T}{f_s} \quad \text{.....} (63)$$

The second term of Equation (63) is identical with the formula for the steel area for a beam with tension steel only. It must be remembered that the moment, M_s, refers to the center of the tension steel.

Example A-1.—Assume the same values as in Example 1, namely, that $T = 16\,530$ lb; $e = 14$ in.; $d = 10$ in.; $d_c = 2$ in.; $f_c = 800$ lb per sq in.; $f_s = 18\,000$ lb per sq in.; $K = 138.7$; and $j = 0.867$: Hence, $M_s = 16\,530 \times 14–4 = 165\,300$ in-lb. Therefore, the minimum depth for the allowable stresses will be, by Equation (60): $d = \sqrt{\dfrac{165\,300}{138.7 \times 12}} = 9.93$ in.

The selection of 10 in. will correspond to a somewhat lower concrete stress. The author gives $f_c = 795$. The tension steel will be: $A_s = \dfrac{165\,300}{18\,000 \times 0.867 \times 10} + \dfrac{16\,530}{18\,000} = 1.97$, which is correct, although a little higher than the value obtained by the author.

Plotting the Chart.—Equation (63) indicates that the most economical section is obtained by using the maximum allowable steel stress. Therefore, it is scarcely necessary that variable steel stresses be introduced on a chart. It is better to begin with the allowable steel stress, and to let the concrete stress vary. From Equation (59):

$$\frac{M_s}{b\,d^2} = \frac{f_c\,k}{2}\left(1 - \frac{k}{3}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (64)$$

Introducing the quantity, $n\,f_c = 12\,000$ (that is, $n = 10$ for $f_c = 1\,200$; $n = 12$ for $f_c = 1\,000$, etc.), Equation (64) yields $k = 0.4$. Hence,

$$\frac{M_s}{b\,d^2} = 0.1733\,f_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

Also, for $k = 0.4$, $f_s = 18\,000$, and $A_s = p\,b\,d$, Equation (63) yields:

$$18\,000\,p = 0.20\,f_c + \frac{T}{b\,d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

and if f_c is eliminated from Equations (65) and (66):

$$15\,600\,p = \frac{M_s}{b\,d^2} + 0.8667\,\frac{N}{b\,d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (67)$$

Equation (67) gives the relation between p, M_s, and N. It will be noted that this formula represents a straight line, and, therefore, two points only are necessary to determine this line. For example, for $p = 1\%$:

$$156 = \frac{M_s}{b\,d^2} + \frac{26}{30}\,\frac{N}{b\,d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

For $M_s = 0$, $\frac{N}{b\,d} = 180$ in Equation (68); and, for $N = 0$, $\frac{M_s}{b\,d^2} = 156$. These two points connected, establish the line representing $p = 1\%$, etc.

Equation (65) demonstrates that the concrete stress varies in direct relation to the $\frac{M_s}{b\,d^2}$ - values, and, therefore, that they can be plotted on the right side of Fig. 12. These values represent Equation (60) graphically.

Example A-2.—Referring again to the data in Example 1: As before, $M_s = 165\,300$ in-lb, and $\frac{M_s}{b\,d^2} = \frac{165\,300}{12 \times 100} = 137.7$. Similarly, $\frac{N}{b\,d} = \frac{16\,500}{12 \times 10} = 137.7$. The intersections of these two lines (see the broken dotted lines in Fig. 12) give $p = 1.63\%$ and $f_c = 795$, which checks with previous results.

D. B. GUMENSKY,[31] ASSOC. M. AM. SOC. C. E. (by letter).—Many additional data and a further development of the subject were brought to light by those who discussed this paper. With the advancements made recently in the analyses of continuous frames and with the generally increasing scale of construction, particularly of hydraulic structures, the problem of designing reinforced concrete members subjected to combined stress of bending and direct tension is becoming more and more important.

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[31] With Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

By taking the moments about the tension steel, the problem can be divided into two distinct steps: (1) Determining the section to resist this moment; and (2) determining the additional steel in the tension face required by the direct tension; in case of direct compression, the additional amount is negative. This fact was demonstrated very ably in different ways by Messrs. Fischer and Wilbur.

Professor Snow takes the matter of design for compressive and tensile reinforcement and presents a complete solution of the problem. In connection with this question the writer would like to call attention to the fact that the seemingly simple solution of his original Case 1 in Fig. 1 may become more complicated when the direct pull is within the section, but still close enough to the tension steel to produce compression in concrete outside the farther steel layer. This situation recently presented itself to the writer in the interpretation of the experimental measurement of a double reinforced member subjected to a combined stress of direct tension and bending. Fig. 14 illustrates this condition which overlaps Case 1.

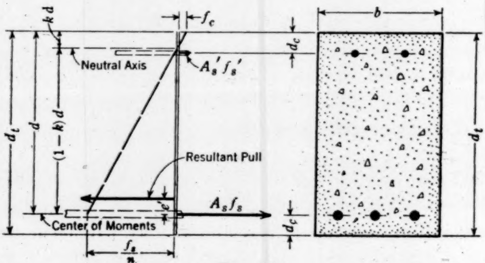

FIG. 14

Mr. Byrne treats the general problem of a direct stress and bending with very gratifying results. Mr. Heilbron, by clever modifications of the original charts and formulas, produces a new chart (Fig. 8) that has the advantage of definite intersections and increased general usefulness, as it can be used for any value of n. In his brief discussion Dean Turneaure contributes toward a clearer understanding of the problem. The "cut-and-try" method of solution mentioned by him seems to be favored by many designers.

The solution and the references presented by Mr. Warrington are of interest and were entirely unknown to the writer. Mr. Larsen should be congratulated on his elaborate and excellent discussion. His Fig. 9, and the charts presented by Mr. Wineland as shown in Fig. 10, should be of great interest to those interested in this broad problem.

The writer wishes to acknowledge his gratitude to Mr. Kovediaeff for his discussion and for his assistance in the preparation of the paper. He also appreciates the discussion by Mr. Floris and sincerely hopes that the comments thus offered will provide a stimulus for further investigations and a better understanding of reinforced concrete problems.

Thinking in terms of the physical action of structures and structural members rather than in mathematical terms, is what the designer strives to do. Mr. Hall recapitulates the problem of combined stresses in its true physical meaning. His discussion illuminates the aspects of the problem not covered by other writers, particularly the solution for a minimum total area of steel in a member reinforced in both faces.

The chart constructed by Mr. Albert possesses the advantage of extreme simplicity in construction and in use. This method and chart give a quick, simple means of designing a new member for a given condition of stresses. In analyzing an existing structural member, one would need to resort to "trial and error", or should have a series of such charts constructed for several unit stresses in steel covering a range, say, from 1 000 lb per sq in. to 25 000 lb per sq in.

Acknowledgments.—In closing, the writer wishes to express his appreciation of the interest displayed by those who commented on his paper. He also wishes to state that the material contained in the discussion represents a valuable contribution to engineering literature, and, in this light, his original presentation now seems justified. The paper had been prepared in connection with the design of hydraulic structures in the Colorado River Aqueduct and was submitted to the University of Southern California in partial fulfillment of requirements for the degree of Master of Science.

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TALL BUILDING FRAMES STUDIED BY MEANS OF MECHANICAL MODELS

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With Discussion by Messrs. L. C. Maugh, L. J. Mensch, Gilbert Morrison, Fang-Yin Tsai, George E. Large and Samuel T. Carpenter, A. W. Fischer, and Francis P. Witmer and Harry H. Bonner.

Synopsis

A study of tall building bents under the action of transverse forces, by the use of a mechanical model, is contained in this paper. Information is lacking as to the actual effects produced upon vertical reactions and direct stresses in columns from such transverse forces when the bent is varied by changing the relative width of bays, or the relative size of girders or columns in different parts of the frame. Such information can be obtained mathematically only by the application of laborious theoretical methods; but by the aid of a mechanical model results may be found quickly and with reliable accuracy so far as they concern the trend of reactions for assumed variations in design. The only theoretical consideration utilized is the well-known Maxwell reciprocal theorem.

The conclusions drawn from the study are believed to be of interest to engineers who are concerned with the designing of tall buildings and may also have practical value in guiding the design of such structures along lines which are more accurately in accord with true behavior than most of the admittedly approximate methods in common use, and without, at the same time, entailing any unnecessary sacrifice in economy of material.

Note.—Published in January, 1936, *Proceedings*.

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OUTLINE OF THE STUDY

The study herein described was confined to the determination of the vertical reactions from transverse forces acting upon bents of four and of six columns having small connections of a pure bending type, with no diagonal sway-braces or knee-braces.

Until quite recently the analysis of such a frame under wind action has generally followed one of two theories: (1) The portal theory; and (2) the cantilever theory. Both are based on fundamental assumptions that are not theoretically correct, but either of them results in satisfactory design for structures of ordinary proportions.

By the portal theory as herein understood the wind shear is assumed to be so distributed over the different columns of the bent that the direct compression in any interior column is equal to the direct tension, so that the resultant direct stresses in interior columns are all equal to zero. The direct wind stress is thus confined to the two exterior columns, being compression in the leeward, and tension in the windward, column.

By the cantilever theory as herein understood the wind shear is assumed to be so distributed that the unit direct stress from wind in any column will be in proportion to the distance of that column from the neutral axis of the bent, columns on the leeward side of this axis being in compression and columns on the windward side in tension. This distribution is analogous to the distribution of fiber stresses in a vertical cantilever beam fixed at the bottom.

Various modifications of these two methods have been devised and used with greater or less success. For analysis on a theoretically correct basis, two principal methods have been available: (1) The slope-deflection method; and (2) the method of moment distribution. In this case, again, modifications have been employed, usually in the nature of approximations for the purpose of abbreviating what otherwise would be a lengthy and laborious operation.

In 1930, Henry V. Spurr, M. Am. Soc. C. E., advanced a method of equal deflections[3], recommended specifically for very tall, narrow towers, in which a more correct theoretical basis is desired than is provided with either the portal or the cantilever method, but where the limitations of the slope deflection and the moment distribution methods, both theoretical and practical, render them undesirable, in the judgment of the designing engineer.

This method is based primarily upon the assumption that vertical reactions and direct column stresses from wind are in accordance with the cantilever theory, but with the further condition that sizes of girders and columns must be adjusted so as to render such an assumption justifiable. It is clear that, in general, this relation of stresses and reactions from wind loads will not be developed automatically and that, therefore, some adjustment of sizes will be required in order to produce the relation assumed.

As the time necessary to perform a complete analysis of a tall tower by any theoretically exact method precludes its application to a considerable number of frames of varying proportions, in order to determine convincingly the actual trend of reactions and stresses, the plan was conceived of determining vertical reactions by means of a mechanical model of simple and economical construction and, at the same time, capable of ready transformation from one set of proportions to another, so that a maximum number of cases might be considered in a minimum time. The scale ratio of model to prototype was 1 to 60 for 12-in. bents and 1 to 120 for 6-in. bents.

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[3] "Wind Bracing", by Henry V. Spurr, N. Y., McGraw-Hill Book Co., 1930.

Description and Use of Model

The columns and girders of this model were composed of strips of steel, $\frac{3}{4}$ in. wide and of varying gauge thickness, to produce the desired relative values of moment of inertia. At each intersection the column and the girder were notched half-way across, and the notches mutually fitted together. A gusset-plate connection was simulated by using a wooden block, $\frac{1}{2}$ in. square and 2 in. long, which was notched in two right-angled directions for a depth of $\frac{3}{4}$ in., and forced down upon the joint, one notch receiving the column and the other the girder so as to hold them firmly out to the edge of the block. The bending was thus confined to the clear length of column or girder between the blocks at two adjacent joints.

In operation, the bases of all columns except one were securely clamped to a table, the model being laid flat and being held clear of the table by two wires suspended from a gallows frame located at a point about three-fourths the distance from the base to the top. A special device was used to prevent rotation at the base of the column that was not clamped down. A micrometer screw was used to move this column longitudinally through a known distance (usually 0.005 in.) and the transverse movement induced at any desired floor was observed by a micrometer microscope. By the familiar Maxwell principle the ratio of the transverse movement to the vertical movement was equal to the vertical reaction produced at the column under observation by a transverse force of unity applied at the floor in question.

Fig. 1 illustrates the construction and use of the model. Obviously, by building up members of one or more properly selected thicknesses of material, any desired relation between values of moment of inertia can be reasonably well obtained.

For each case considered, vertical reactions were found for each column from a load of unity applied at each of several different floors. By varying the relation between moments of inertia, it was possible to find: (*A*) The actual vertical reactions produced with any assumed form of bent and any desired size of members; (*B*) the relative values of moment of inertia in members necessary to produce the cantilever relation of reactions; and (*C*) the relative values of the same function necessary to produce agreement with the portal theory.

The results, while not intended to be exact quantitatively, do show, unmistakably, the qualitative trend of vertical reactions for assumed variations in size of members and proportions of bays and, it is believed, give quantitative results closely enough to justify conclusions as to the relative values of vertical reactions for the various cases.

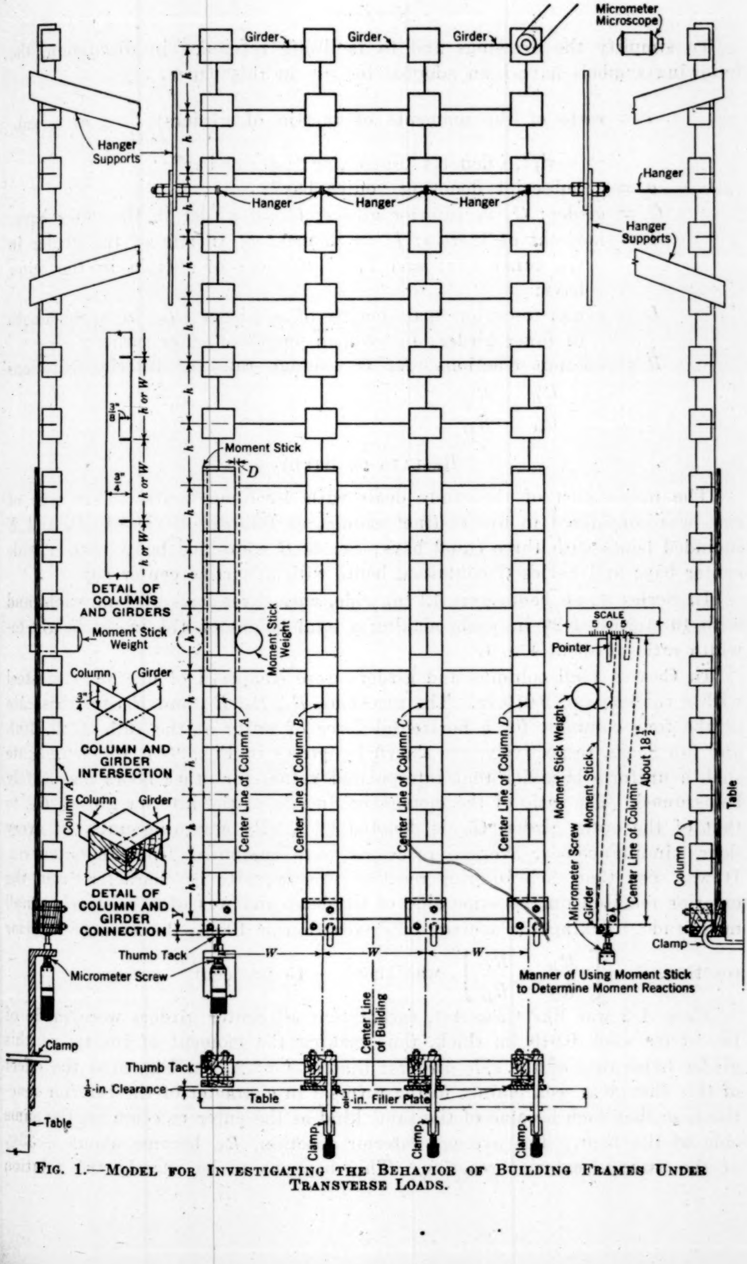

Girder
Girder
Girder
Micrometer Microscope
Hanger Supports
Hanger
Hanger
Hanger
Hanger
Hanger Supports
h
h or W
3/8"
3/4"
DETAIL OF COLUMNS AND GIRDERS
Moment Stick
D
Moment Stick Weight
Moment Stick Weight
Column
Girder
3/4"
COLUMN AND GIRDER INTERSECTION
Column A
Column
Girder
DETAIL OF COLUMN AND GIRDER CONNECTION
Y
Center Line of Column A
Center Line of Column B
Center Line of Column C
Center Line of Column D
SCALE
5 0 5
Pointer
Moment Stick Weight
Moment Stick
Micrometer Screw
Girder
Center Line of Column
About 18 1/2"
Table
Column D
Clamp
Thumb Tack
Micrometer Screw
W
W
W
Manner of Using Moment Stick to Determine Moment Reactions
Center Line of Building
Clamp
Thumb Tack
1/8-in. Clearance
Table
1/8-in. Filler Plate
Table
Clamp
Clamp
Clamp

FIG. 1.—MODEL FOR INVESTIGATING THE BEHAVIOR OF BUILDING FRAMES UNDER TRANSVERSE LOADS.

NOTATION

To simplify the drawings and to facilitate reference in discussion, the following symbols have been adopted for use in this paper:

i = ratio of the moments of inertia of girders: $\frac{G_i}{G_o}$; as a subscript, i denotes "inner", or "interior bay";
o = a subscript denoting "outer bay";
G = girder; G_i = interior girder; G_o = girder in the outer bays;
I = moment of inertia; I_i = moment of inertia of the girder in the inner bay; and I_o = moment of inertia in the outer bays;
L = center-to-center span length of a girder; L_i = span length of inner girder; L_o = span length of outer girder;
R = column reactions; R_i = average ratio of interior reactions $= \frac{R_B}{R_A} = \frac{R_C}{R_D}$.

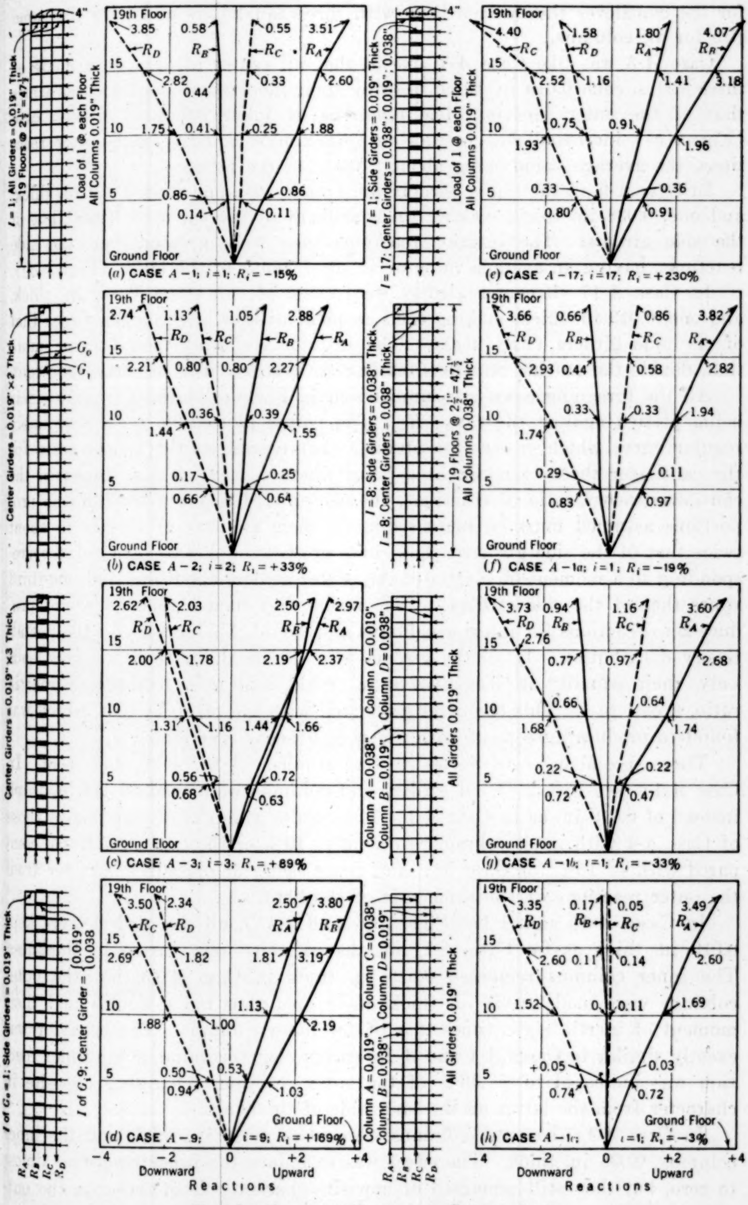
RESULTS OF STUDY

The major part of the study dealt with 4-column bents. This type of case was considered in five distinct groups, as follows: Series *A*, *D*, and *E* included bents with three equal bays; Series *B* contained bents with a wide center bay; and Series *C* contained bents with a narrow center bay.

In Series *A*, all bents were 12 in. wide, with three bays of 4 in. each, and with 19 stories of 2½ in. each, making a total height of 47½ in.—a height-to-width ratio of about 4 to 1.

In Case *A*-1 all columns and girders were composed of one strip of steel with a thickness of 0.019 in. The reactions, R_A, R_B, R_C, and R_D, for this case at the four columns, for a horizontal force of unity at the 19th, 15th, 10th and 5th floors, respectively, are shown by curves in Fig. 2(*a*), which indicate such a uniform behavior that interpolated values for other floors may safely be assumed. The ratio of the moment of inertia of the interior girder, G_i, to that of the outer girder, G_o, is denoted by i. By assuming unity at every floor, simultaneously, average reactions were computed from these curves. It was seen that each interior reaction was opposite in character from the exterior reaction on the same side of the bent and was of a relatively small magnitude. Taking the average outer reaction as 100%, the average interior reaction, $R_i = \left[\frac{R_B}{R_A} = \frac{R_C}{R_D}\right]$, was about -15 per cent.

Case *A*-2 was like Case *A*-1, except that all center girders were made of two strips, each 0.019 in. thick, thus making the moment of inertia of this girder twice that of the side girders; that is, $i = 2$. In Fig. 2(*b*) the effect of this change is seen plainly in the reversal in character of the interior reactions, so that each became of the same kind as the outer reaction on the same side of the bent. The average interior reaction, R_i, became about $+33\%$ of the average exterior reaction. This happens to be exactly the relation



(*a*) CASE $A-1$; $i = 1$; $R_1 = -15\%$

(*b*) CASE $A-2$; $i = 2$; $R_1 = +33\%$

(*c*) CASE $A-3$; $i = 3$; $R_1 = +89\%$

(*d*) CASE $A-9$; $i = 9$; $R_1 = +169\%$

(*e*) CASE $A-17$; $i = 17$; $R_1 = +230\%$

(*f*) CASE $A-1a$; $i = 1$; $R_1 = -19\%$

(*g*) CASE $A-1b$; $i = 1$; $R_1 = -33\%$

(*h*) CASE $A-1c$; $i = 1$; $R_1 = -3\%$

FIG. 2.—REACTION DIAGRAMS, SERIES *A*.

for the cantilever theory in a bent with three equal bays and with the same size for all columns.

Case *A*-3 was like Case *A*-1, except that all center girders were made of three strips, each 0.019 in. thick, making their moment of inertia three times that of the outer girders. The reactions of inner columns in this case (Fig. 2(*c*)) increased still further to approximate equality with outer reactions, the average value being about + 89% of the latter.

In Case *A*-9 all the center girders were made with one strip 0.019 in. thick, and one, 0.038 in. thick, making their moment of inertia nine times that of the side girders. The interior reactions then were greater than the side reactions, having an average value of about + 169% of the latter (Fig. 2(*d*)).

In Case *A*-17 all center girders were made of two strips 0.038 in. thick, and one, 0.019 in. thick, making their moment of inertia seventeen times that of the side girders (Fig. 2(*e*)). The interior reactions then became more than double the outside reactions, having an average ratio of + 230 per cent.

All the foregoing cases are summarized in Fig. 3, values of i as abscissas being plotted against the ratio, R_i. The points plotted lie on a reasonably regular curve which crosses the abscissa that represents the proper ratio for the cantilever theory relation at a point where $i = 2$. Thus, to obtain the cantilever-theory ratio of reactions, the center girders for a bent of the proportions assumed must be increased until their moment of inertia is about twice that of the side girders. The curve crosses the X-axis at a point corresponding to a moment of inertia of the center girders about one and one-third times that of the side girders, which means that such a relation would produce zero reactions for interior columns and would thus agree with the portal-theory distribution. If all the girders were designed for their vertical loads only, their equality in length of span would lead to a moment of inertia ratio, i, of 1 to 1. This compares quite well with the ratio, $1\frac{1}{3}$ to 1, which was found to produce agreement with the portal-theory reactions.

Three special cases of Series *A* were studied: *A*-1*a*, *A*-1*b*, and *A*-1*c*. In Case *A*-1*a* (see Fig. 2(*f*), all girders and columns were made 0.038 in. think instead of 0.019 in. as in Case *A*-1. The results were exactly similar to those of Case *A*-1 with inner column reactions — 19% of the outer reactions, compared with — 15% for Case *A*-1, and again being of opposite character from the outer reactions on the same side of the bent.

In Cases *A*-1*b* and *A*-1*c* (Figs. (2*g*) and 2(*h*)), all the girders were kept 0.019 in. thick as for Case *A*-1, but the columns were varied in thickness. The inner columns remained 0.019 in. thick in Case *A*-1*b*, but the outer columns were made 0.038 in. thick, their area thus being double and their moment of inertia eight times that of the inner columns. The behavior was exactly similar to Cases *A*-1 and *A*-1*a*, except that the value of the inner reaction was increased to — 33% of the outer reaction, remaining opposite in character from the latter on the same side of the bent.

In Case *A*-1*c* the outer columns were made 0.019 in. thick and the inner columns, 0.038 in. thick. The effect was to reduce the inner reactions almost to zero, but they still remained of opposite character from those on the outside of the bent. This relatively great increase in the size of inner columns

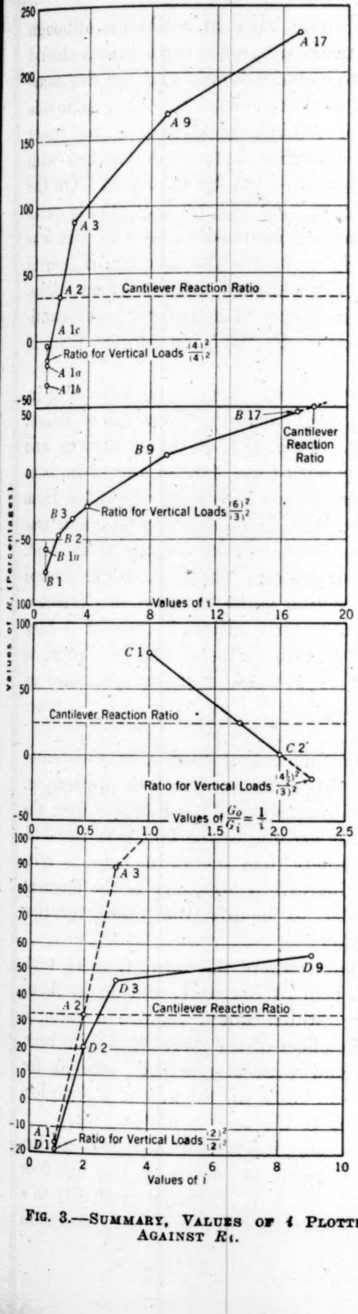


FIG. 3.—SUMMARY, VALUES OF i PLOTTED AGAINST R_i.

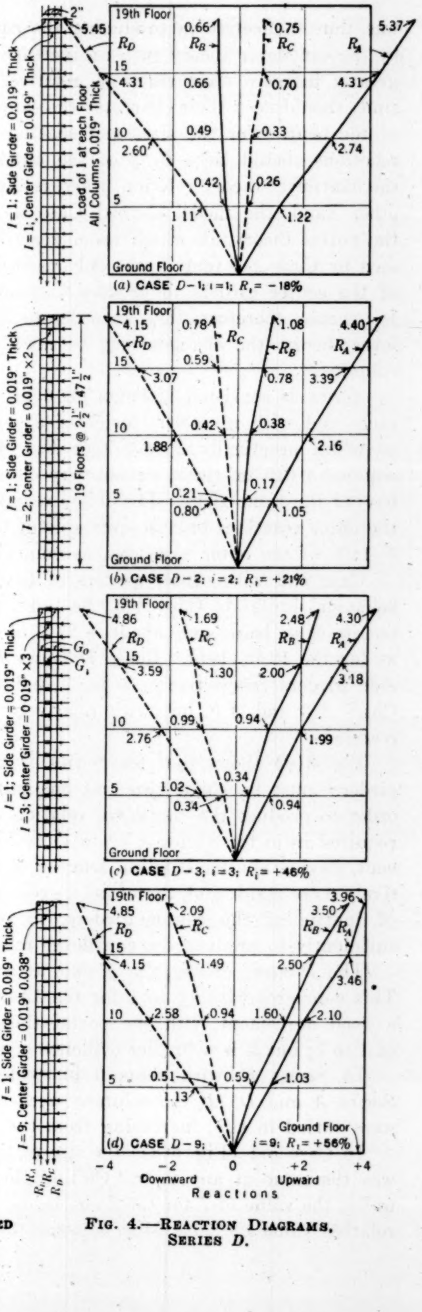

2"
I = 1; Side Girder = 0.019" Thick
I = 1; Center Girder = 0.019" Thick
Load of 1 at each Floor
All Columns 0.019" Thick
19th Floor
5.45
0.66
0.75
5.37
R_D
R_B
R_C
R_A
15
4.31
0.66
0.70
4.31
10
0.49
0.33
2.60
2.74
0.42
0.26
5
1.11
1.22
Ground Floor
(a) CASE D-1; i = 1; R_i = -18%
I = 1; Side Girder = 0.019" Thick
I = 2; Center Girder = 0.019" × 2
19 Floors @ $2\frac{1}{2}$" = $47\frac{1}{2}$"
19th Floor
4.15
0.78
1.08
4.40
R_D
R_C
R_B
R_A
15
0.59
3.07
0.78
3.39
10
0.42
0.38
1.88
2.16
0.17
5
0.21
0.80
1.05
Ground Floor
(b) CASE D-2; i = 2; R_i = +21%
I = 1; Side Girder = 0.019" Thick
I = 3; Center Girder = 0.019" × 3
G_o
G_i
19th Floor
4.86
1.69
2.48
4.30
R_D
R_C
R_B
R_A
15
3.59
1.30
2.00
3.18
10
0.99
0.94
2.76
1.99
0.34
5
1.02
0.34
0.94
Ground Floor
(c) CASE D-3; i = 3; R_i = +46%
I = 1; Side Girder = 0.019" Thick
I = 9; Center Girder = 0.019" 0.038"
19th Floor
4.85
2.09
3.96
3.50
R_D
R_C
R_B
R_A
15
4.15
1.49
2.50
3.46
10
2.58
0.94
1.60
2.10
5
0.51
0.59
1.03
1.13
Ground Floor
(d) CASE D-9; i = 9; R_i = +56%
R_A
R_B
R_C
R_D
-4
-2
0
+2
+4
Downward
Upward
Reactions

FIG. 4.—REACTION DIAGRAMS, SERIES D.

was thus ineffective in producing the cantilever ratio of reactions, although by the cantilever theory such an increase in the area of inner columns should greatly increase their reactions relative to outer columns and, at the same time, should keep their character the same as that of the latter on the same side of the bent. For the sizes and spacing of columns of Case *A*-1*c* the inner reactions should be + 67% of the outer reactions in order to accord with the cantilever-theory relation, whereas they were − 3% by the model. On the other hand, the model showed Case *A*-1*c* as agreeing almost exactly with the portal theory, in which inner reactions are assumed to be zero. It was seen by Case *A*-2 that it was only necessary to double the moment of inertia of the center girders to produce the cantilever-theory relation of reactions. It appears, therefore, that, in order to adjust sizes to accord with the cantilever theory, the changes may be made more effectively in girders than in columns.

Series *D* consisted of bents all 6 in. wide, with three equal bays of 2 in. each, and with nineteen stories of $2\frac{1}{2}$ in. each, making the ratio of height to width practically 8 to 1 (see Fig. 4). Case *D*-1, with all girders and columns 0.019 in. thick, agreed very closely with Case *A*-1 for the 12-in. bent having the same sizes. The inner reactions were of opposite character from the outer reactions on the same side of the bent, and their average value was − 18% of the outer reactions, as compared with − 15% for the 12-in. bent.

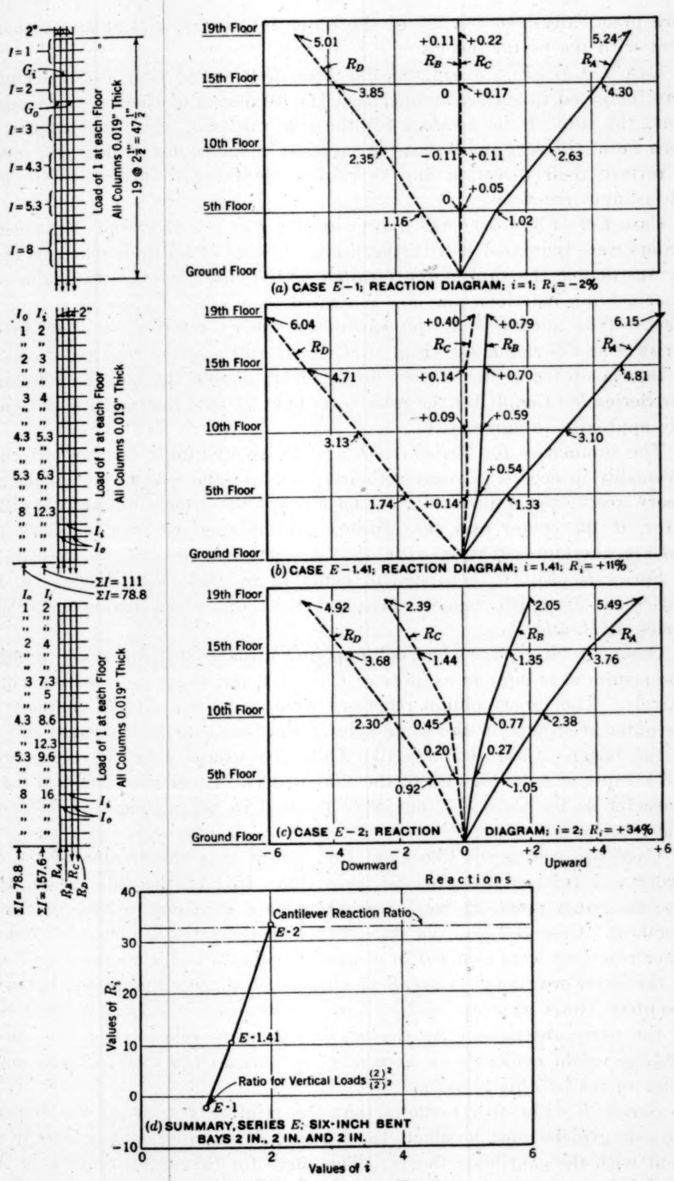
Case *D*-2 with center girders of two strips, each 0.019 in. thick, showed behavior similar to Case *A*-2, the inner reaction being changed in character, but its value being only about + 21% of the outer reaction, instead of + 33% as for the 12-in. bent. Cases *D*-3 and *D*-9, with ratios of center girders to side girders, respectively, 3 to 1 and 9 to 1, showed the same trend as Cases *A*-3 and *A*-9, but with relatively smaller average values of the inner reactions.

Fig. 3(*d*) shows that for Series *D* the moment of inertia of the center girders must be about two and one-half times that of the side girders, in order to produce the cantilever relation of reactions. This increase over the required ratio for Series *A* (2 to 1) is doubtless due to the fact that the 6-in. bent, having shorter bending lengths of girders than the 12-in. bent, is relatively more rigid; and, therefore, a greater increase is necessary in the moment of inertia of the center girders, in order to modify the inner reaction sufficiently to produce the cantilever relation.

The *D*-curve crosses the *X*-axis at a point where the ratio, *i*, is $1\frac{1}{2}$ to 1. This compares with $1\frac{1}{3}$ to 1 for the 12-in. bent of Series *A*, and again shows a good agreement with the portal theory, which would require a ratio, *i*, of 1 to 1; but it is a further evidence of the effect of the more rigid 6-in. bent.

In Series *E*, with bents 6 in. wide and with stories the same as for Series *A* and *D*, all the columns were made 0.019 in. thick, but the girders were varied in size, increasing from the top to the bottom of the bent.

In Case *E*-1 (Fig. 5(*a*)) the size of the side girders and the center girders was the same at any floor. Calling the moment of inertia of the top floor unity, the value of *I* for the floors below the top floor increased gradually to a relative value of 8.0 at the bottom. The resulting inner column reactions



(a) CASE $E-1$; REACTION DIAGRAM; $i=1$; $R_i=-2\%$

(b) CASE $E-1.41$; REACTION DIAGRAM; $i=1.41$; $R_i=+11\%$

(c) CASE $E-2$; REACTION DIAGRAM; $i=2$; $R_i=+34\%$

(d) SUMMARY, SERIES E; SIX-INCH BENT BAYS 2 IN., 2 IN. AND 2 IN.

FIG. 5.—SERIES *E*.

were practically zero (-2% of the outer reactions), almost an exact agreement with the portal theory.

Case *E*-1.41 (Fig. 5(*b*)) was like Case *E*-1, except that the center girders were increased in such a manner that the total sum of their *I*-values was 1.41 times the sum of the *I*-values for the side girders. The inner column reactions were still very small, but the increase in the center girders was sufficient to reverse their direction and to raise their value to about $+11\%$ of the side column reactions.

Case *E*-2 (Fig. 5(*c*)) was like Cases *E*-1 and *E*-1.41 except that the center girders were increased still further until the sum of all their moments of inertia was double the sum of the *I*-values of the side girders. The value of the inner column reactions was now increased to $+34\%$ of the side column reactions. This agreed with the cantilever theory relation so that, for both Series *A* and Series *E* (see Fig. 5(*d*)), the latter relation required the moment of inertia of the center girders to be about double that of the side girders. For Series *D* (Fig. 3(*d*)) the ratio was about $2\frac{1}{2}$ to 1 instead of 2 to 1—a very fair approximate agreement.

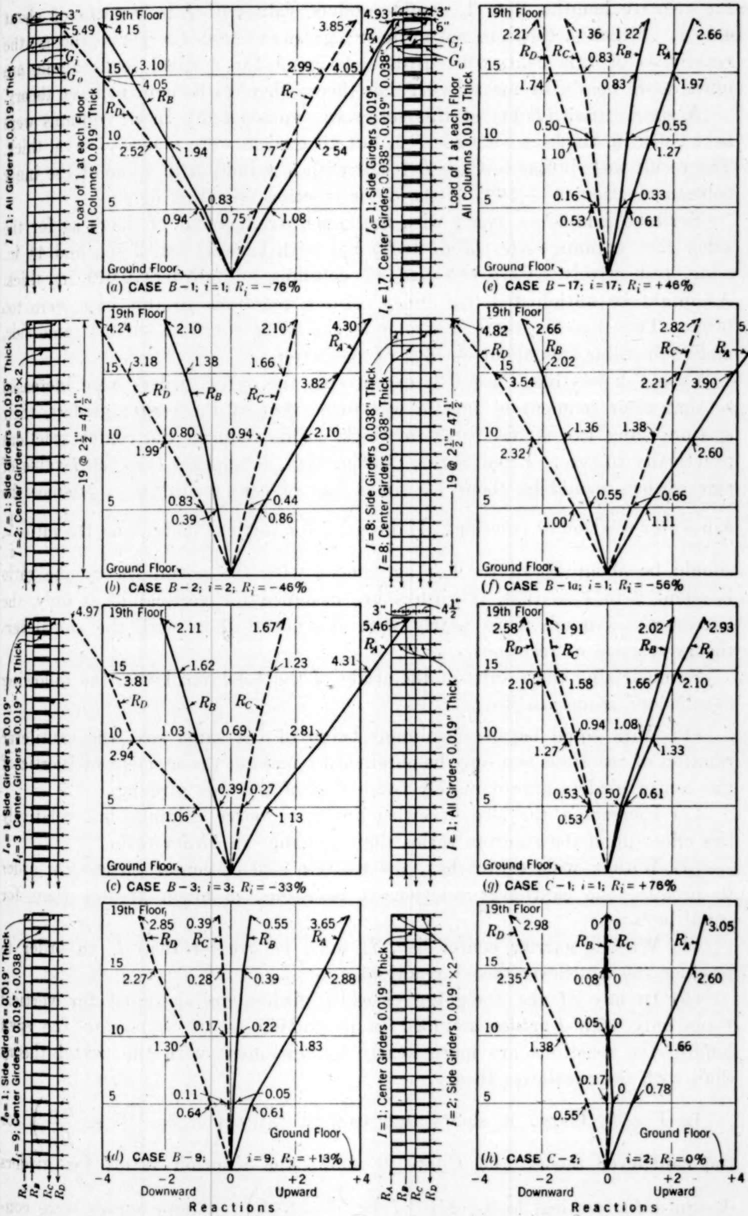
The summaries for Series *A*, *D*, and *E* are strikingly similar in form and reasonably in accord as regards their relation to the cantilever and the portal theory reactions, respectively, being much more nearly in accord with the latter, if the center and side girders are designed for vertical loads alone, and are, therefore, of equal size.

Series *B* (Fig. 6) consisted of bents 12 in. wide, with bays 3 in., 6 in., and 3 in. in width, respectively, and the same story arrangement as for Series *A*, *D*, and *E*.

Case *B*-1 (Fig. 6(*a*)) had columns and girders 0.019 in. thick throughout. The results were similar to those of Case *A*-1, but much more extreme quantitatively. The inner column reactions were again opposite in character from the outer reactions on the same side of the bent, but had a value of -76% of the latter. Case *B*-2 was like Case *B*-1 except that the center girders had a value of *I* double that of the side girders. Inner reactions had the same character as for Case *B*-1, but were reduced in magnitude to -46% of the side reactions.

Case *B*-3 was again like Case *B*-1, except that the *I*-value of the center girder was further increased to three times that of the side girder. In this case the inner reactions were further reduced to about -33% of the outer reactions. Case *B*-9 was the same, but with the ratio, *i*, increased to 9. The inner reactions were changed in character and attained a magnitude of $+13\%$ of the outer reactions. Case *B*-17 was like the preceding cases, but with I_1 seventeen times as great as I_0. The inner column reactions became $+46\%$ of the outer reactions. As the cantilever-theory relation for this case was 50%, it would evidently be necessary to increase the ratio, *i*, even more, in order to realize this relation.

Series *B* (Fig. 3(*b*)) shows that the relative *I*-value of the center and the side girders must be about 18 to 1 in order to develop reactions in agreement with the cantilever theory. The curve for this series crosses the *X*-axis at a girder ratio, *i*, equal to about 7 to 1. If designed for vertical loads alone,



(a) CASE B−1; i = 1; $R_i = -76\%$

(b) CASE B−2; i = 2; $R_i = -46\%$

(c) CASE B−3; i = 3; $R_i = -33\%$

(d) CASE B−9; i = 9; $R_i = +13\%$

(e) CASE B−17; i = 17; $R_i = +46\%$

(f) CASE B−1a; i = 1; $R_i = -56\%$

(g) CASE C−1; i = 1; $R_i = +78\%$

(h) CASE C−2; i = 2; $R_i = 0\%$

FIG. 6.—REACTION DIAGRAMS; SERIES B AND C.

the relative lengths, 2 to 1, would produce values of I in a ratio of about 4 to 1. Although they are not quite in agreement with the portal theory, the reactions for this bent, with girders designed for vertical loads alone, are much more nearly in accord with that theory than with the cantilever theory.

A special case (B-1a) of this series was considered. In it, columns were kept 0.019 in. thick as for Case B-1, but all girders were made 0.038 in. thick. The result was similar to Case B-1, but with a reduction in value of the inner column reaction to − 56% of the outer reaction (see Fig. 6(f)).

Series C had 12-in. bents with the same arrangement of floors as for the other four column series (Fig. 6(g)) but with bays 4½ in., 3 in., and 4½ in. wide, respectively. Case C-1 had all columns and girders 0.019 in. thick. As might be anticipated, the inner column reactions in this case were too large. They had the same character as the outer reactions on the same side, and their value was about + 76% of the latter.

Case C-2 was like Case C-1, except that the outer girders were increased so that their moment of inertia was twice that of the center girders, their section being two strips, 0.019 in. thick. This reduced the center reactions practically to zero. From Series C, Fig. 3(c), it appears that the I-value of side girders should be about 1.7 times that of center girders to produce the cantilever relation of reactions. If designed for vertical loads alone, the ratio, $\frac{1}{i}$, should be about 2¼ to 1. For agreement with the portal theory the ratio is about 2 to 1 so that, if girders are designed for vertical loads only, the agreement is much more nearly with the portal than with the cantilever-theory relation of reactions.

Summarizing the results of the study of the 4-column bents, the following conclusions seem justified:

(1) With equal bays, all columns being of the same size, the cantilever relation of reactions can only be obtained by making the moment of inertia of the center girders approximately twice that of the side girders.

(2) Increasing the size of either inner or outer columns has relatively less effect upon the reaction ratios than a change in girder ratio.

(3) With a wide center bay, the relative size of center girders, in order to produce the cantilever relation of reactions, is much greater than for equal bays.

(4) With a narrow center bay, I_i must be greater than I_o in order to produce the cantilever-theory reactions.

(5) In any of the foregoing cases if girders are designed for vertical loads only with I-values assumed in proportion to the square of the span length, the reactions are more nearly in agreement with the portal theory than with the cantilever theory.

In Fig. 7, Curve A shows the reaction ratios $\left(R_i = \frac{R_B}{R_A} = \frac{R_C}{R_D}\right)$ for the cantilever theory and Curve B shows the reaction ratios for girders designed for vertical loads only $\left(i = \frac{I_i}{I_o} = \frac{L^2_i}{L^2_o}\right)$. These curves were constructed from Series A, B, and C, as plotted in Fig. 3.

Abscissas are the ratios of inner to outer bay and ordinates are the ratios of inner to outer column reaction, R_i. Apparently, these curves can never intersect. Curve *A* is necessarily always positive and Curve *B* is always negative for the cases considered. Hence, for these cases, if girders are designed for vertical loads only, inner reactions will always be opposite in character from outer reactions on the same side, and cantilever theory reactions can never be developed except by increasing the moment of inertia, I_i, of the center girders. The increase required for the cases considered, is as follows:

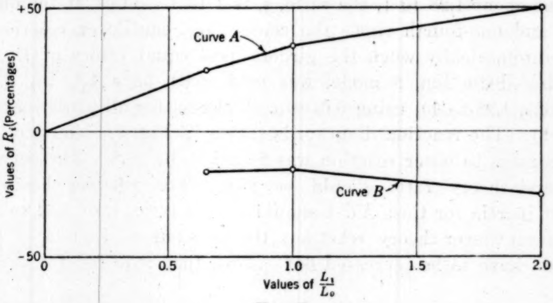


FIG. 7.

Series	Percentage increase
A	100
B	350
C	33
D	150
E	100

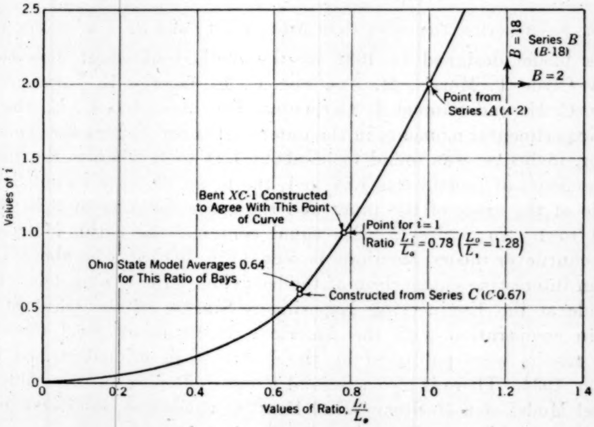


FIG. 8.—CURVE FOR CANTILEVER RATIOS.

It is seen from this tabulation that Series *C* is naturally most nearly in accord with the cantilever theory.

Fig. 8 shows the ratio, *i*, necessary to produce cantilever-theory reactions for a varying ratio of bays. It is seen that with a ratio, $\frac{L_o}{L_i}$, of outer bay to inner bay of about 1.28 to 1, the ratio, *i*, is 1 to 1. Thus, if the outer bay is about one and one-fourth times the center bay, cantilever reactions will be developed automatically when the girders have equal values of *I*. To verify the foregoing deduction, a model was made with bays $4\frac{5}{16}$ in., $3\frac{3}{8}$ in., and $4\frac{5}{16}$ in. (ratio, 1.28 to 1), using 0.019-in. thickness for all girders and columns (Case *XC*-1). The reaction diagram is shown in Fig. 9. The percentage, R_i, of inner reaction to outer reaction was found to be 28.8. The percentage for the cantilever-theory ratio should be 28.0. For vertical loads only the moments of inertia for Case *XC*-1 should have a ratio, *i*, of 1.64 to 1. Hence, to produce cantilever-theory reactions the moment of inertia of the center girder would have to be increased 64% above that required for vertical loads only.

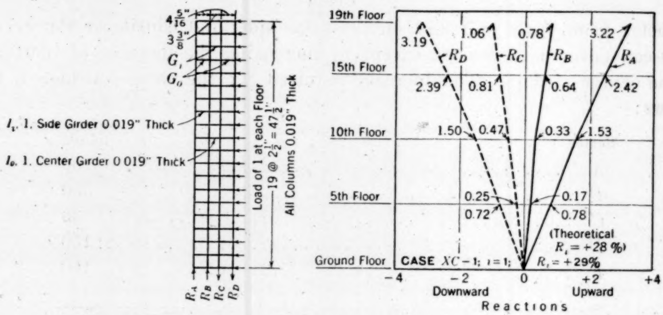


FIG. 9.—REACTION DIAGRAM; CASE *XC*-1; $i = 1$; and $R_i = +29$ PER CENT.

For a model designed in 1934 by the method of equal deflections and tested by Clyde T. Morris, M. Am. Soc. C. E., George E. Large, Assoc. M. Am. Soc. C. E., and Samuel T. Carpenter, Jun. Am. Soc. C. E., the average ratio of experimental moments in the outer and inner girders for the 3d to the 13th floor, inclusive, was found to be about 1.21 to 1, whereas the ratio, *i*, of actual moments of inertia was 1.88 to 1, the mean of these values being 1.55. The ratio of the areas of the inner and the outer columns in this test model was 1.14 to 1. In Series *C*, with equal columns, the ratio, *i*, necessary to produce cantilever-theory reactions is seen (Fig. 3(*c*)) to be about 1.68 to 1. This is an interesting comparison of the behavior of the two models. The tests were made at the Engineering Experiment Station of the Ohio State University in co-operation with the American Institute of Steel Construction, and the results were published by the Institute in mimeographed form, in November, 1934. There is also a Second Progress Report on this study ("Tests of a Steel Model of a 55-Story Wind Bent"), which was published in mimeographed form, in June, 1935.

STUDY OF SIX-COLUMN BENT

In addition to the foregoing study of 4-column bents, a rather limited study (Series *F*) was also made of 6-column bents.

In 1933, Professor Morris made a study of a 6-column bent in order to elucidate the method of equal deflections. He used a bent with bays of unequal widths, the center bay being wider than the others and these latter being equal. The columns were of different sizes. A height of about nine times the width of bent was used, 80 stories being assumed, and the girders in the different bays were adjusted in the application of the method of equal deflections in order to accord with the preliminary assumption of the cantilever-theory relation of column reactions.

In the model used for the study of 6-column bents, a width of $8\frac{1}{4}$ in., with bays of $1\frac{9}{16}$ in., $1\frac{9}{16}$ in., 2 in., $1\frac{9}{16}$ in., and $1\frac{9}{16}$ in., was assumed, and a height of $71\frac{11}{16}$ in. in 31 stories of $2\frac{5}{16}$ in. each. These proportions simulated those of the aforementioned 6-column bent, except as to the number of stories. Thicknesses of columns, from outside to center, were, respectively, 0.019 in., 0.026 in., and 0.030 in., agreeing as closely as was practical with the columns in the method of equal deflections.

In Case *F*-1 a single strip, 0.019 in. thick, was used for all girders; in Case *F*-2 the thickness of the center girders was increased to two strips of 0.019 in,; and in Case *F*-3 the thickness of the center girders was increased to three strips, 0.019 in. each, and the adjacent girders to two strips, 0.019 in. thick, leaving the outside girders of one strip, 0.019 in. thick. These sizes were reasonably in accord with the sizes in the method of equal deflections.

TABLE 1.—REACTION RATIOS FOR SIX-COLUMN BENT (PERCENTAGES)

Series *F*, Case:	Side column	Second column	Inner column
F-1	100	+32	−15
F-2	100	+50	+16
F-3	100	+91	+38
Cantilever-theory relation..............	100	+85	+41

Taking the outside column reactions at 100%, the results of the study for the three cases were as shown in Table 1. It is seen that for Case *F*-1 the inner column reactions are reversed in character from those of the other columns on the same side of the bent. For Case *F*-2, by increasing the girders in the center bay these inner reactions were changed in character and reached a value of + 16% of the outside columns. Case *F*-3, was made to simulate, as closely as practicable, the relation of girders and columns in the method of equal deflections. In this case the percentages for all column reactions agree surprisingly well with those of the cantilever theory, which was a fundamental assumption in the method of equal deflections, and thus verify the correctness of that method of adjustment of girder sizes to accord with the assumption of cantilever-theory reactions.

If girders were designed for vertical loads only, with moments of inertia in proportion to the square of the span length, the proportions of *I*-values for the side girder, second girder, and center girder, respectively, would be as 1:1:1.7. These values lie between the ratios for Cases *F*-1 and *F*-2.

By considering Table 1, the probable relation of column reactions for this relation of moments of inertia in the girders would be about as 100%:40%:0%. This seems to indicate that, if girders are designed for vertical loads only, the inner column reactions for this 6-column bent will accord very closely with those of the portal theory and will be quite out of agreement with the cantilever theory.

The following general conclusions appear to be warranted by the foregoing study:

(1) The natural tendency of both 4-column and 6-column bents in which girders are designed for vertical loads only is to develop vertical reactions much more nearly in agreement with the portal theory than with the cantilever theory.

(2) In order to make an assumption of the cantilever-theory distribution of vertical reactions justifiable, it will generally be necessary to increase the moments of inertia of certain girders by an appreciable percentage. For 4-column bents with a wide center bay this increase will be in the center girders and may be prohibitive from a standpoint of economy. For bents with equal bays, the center girder will require a moment of inertia approximately double that which would be required for girders designed for vertical loads only. For bents with a narrow center bay, the side girders must be greater than the center girders, but such bents naturally accord more nearly with the cantilever-theory relation than bents of the foregoing types.

(3) If the assumption of cantilever-theory reactions is desired for any reason, a carefully constructed model should furnish, *a priori,* information as to the probable percentage of increase in the *I*-value of any set of girders in order to develop the assumed relation of reactions.

(4) If a minimum adjustment of girder sizes for wind stress is desired, such a model should furnish a reasonably close relation between the reactions which will actually be developed, and these might be used as the basis of a rational design. This might necessitate assuming that direct stresses in the columns in any story, were in the same proportion as vertical column reactions, but this should not be very far from the truth.

Each column in the foregoing study was always assumed to have the same size from top to bottom. It is not believed that a normal variation in such size, increasing toward the bottom, would greatly vary the general relations discovered and, of course, the sizes of columns in different stories could readily be varied at will in a model of the kind herein described. It is to be remembered that the purpose of the study was to learn trends rather than true quantitative values of reactions for varying proportions of bents. It is thought, however, that, with great care in determining the thicknesses of material to agree with the sizes of an assumed design, and with care in assembling the model, particularly as regards the tightness of fit of joint blocks, reliable quantitative values may be obtained.

Where the model is made with considerable bending length of girders and is, therefore, relatively flexible, the results are most satisfactory. It is found that for such a case the micrometer screw actuates the columns through the entire distance indicated by the screw, and only the horizontal movement at a floor need be observed with the microscope. If the bent is narrow, with short girders, its greater rigidity may cause a backward movement of the screw where it is clamped so that the actual motion of the column is less than the nominal movement of the screw. In such a case, two microscopes may be used and both the horizontal movement at a floor and the vertical movement of the column observed simultaneously. Their ratio will give the true reaction. It is strongly recommended that models with excessively rigid members be avoided, as liberal a width of bay as practicable being adopted, so that the aforementioned loss will not occur. This will produce results having a maximum degree of reliability.

DISCUSSION

L. C. MAUGH,[4] ASSOC. M. AM. SOC. C. E. (by letter).—The use of small models in the study of complicated structural frames is undoubtedly a valuable adjunct to the various algebraic methods that are in current use. The results that are recorded must be considered as applying more to the conditions that exist in the usual hypothetical frame than in the actual tall building. From this viewpoint, the writer has made a comparison of the results that were obtained by the authors with those obtained by an approximate method that will be explained briefly in order that the comparison may be more complete.

In most regular building frames in which wind stresses are of any importance, the columns will be more rigid than the girders and will not vary greatly in stiffness from exterior to interior. For this condition, each bent can be considered as composed of a series of vertical Vierendeel trusses in which the columns are the chord members; the exterior columns act with only one truss whereas the interior columns are a part of two trusses. For Vierendeel trusses in which the chord members are approximately of equal rigidity, the writer has shown that the panel is the primary unit of the structure and can be used to provide a convenient method of solution by successive approximations.[5] The importance of the panel as a structural unit was also shown by Professor R. V. Southwell in his studies of stresses in rigid airships.[6]

To illustrate the foregoing, let Fig. 10(*a*) represent a typical building frame of three bays. Panel *abed* will then be a part of the vertical truss, *T*-1, and Panel *bcfe* will be part of Truss *T*–2. In Fig. 10(*b*) Panel *abed* is considered as a rigid frame that is acted upon by a shearing force, V_o, and is restrained by the forces, $\frac{V_o\, h}{L_o}$, at Joints *d* and *e*. The horizontal displacement of Joints *a* and *b* with respect to Joints *d* and *e* will be equal to,

$$\Delta = \frac{V_o\, h^3}{24\, E} \left(\frac{1 + n_o}{K}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which n_o equals the ratio of the rigidity of the columns to that of the girder, $\left(n_o = \frac{K_c}{K_o}\right)$, and should be greater than unity to provide the truss action shown; Panel *bcfe* is similarly shown in Fig. 10(*c*) and in the same manner,

$$\Delta = \frac{V_i\, h^3}{24\, E} \left(\frac{1 + n_i}{K}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

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[4] Asst. Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[5] "Analysis of Vierendeel Trusses by Successive Approximations", by L. C. Maugh, Publications of the International Assoc. of Bridge and Structural Eng., Vol. 3; also, "A Rapid and Concise Method of Analyzing Rigid Viaduct Bents", *Engineering News-Record*, March 14, 1935.

[6] "On the Calculation of Stresses in the Hulls of Rigid Airships", by R. V. Southwell, R and M No. 1057, Aeronautical Research Comm. of Great Britain.

in which V_i equals the shear taken by the panel and n_i equals the ratio of the rigidity of the columns to that of the girders $\left(n_i = \frac{K_c}{K_i}\right)$.

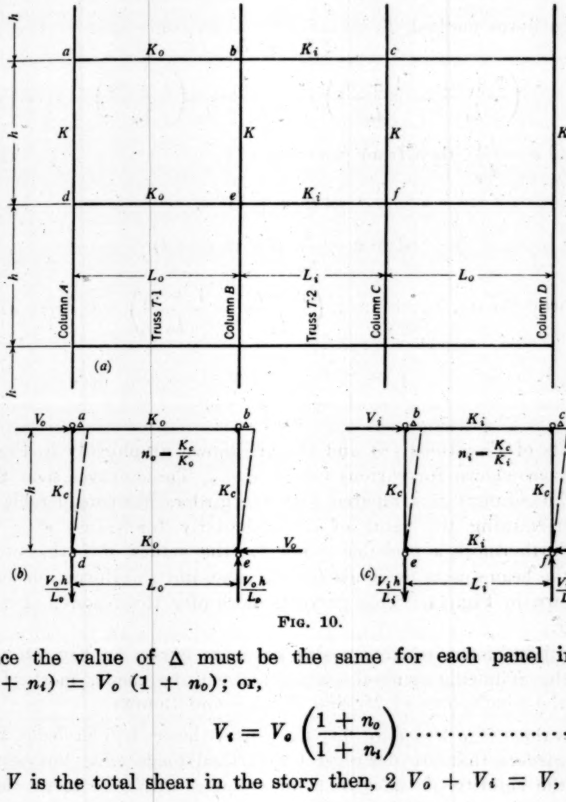
h
a
K_o
b
K_i
c
K
K
K
K
d
K_o
e
K_i
f
L_o
L_i
L_o
Column A
Truss T-1
Column B
Truss T-2
Column C
Column D
(a)
V_o
a
K_o
b
$n_o = \frac{K_c}{K_o}$
h
K_c
K_c
K_o
V_o
d
e
(b)
$\frac{V_o h}{L_o}$
L_o
$\frac{V_o h}{L_o}$
V_i
b
K_i
c
$n_i = \frac{K_c}{K_i}$
K_c
K_c
K_i
V_i
e
f
(c)
$\frac{V_i h}{L_i}$
L_i
$\frac{V_i h}{L_i}$

FIG. 10.

Since the value of Δ must be the same for each panel in the story, $V_i (1 + n_i) = V_o (1 + n_o)$; or,

$$V_i = V_o \left(\frac{1 + n_o}{1 + n_i}\right) \dots\dots\dots\dots\dots\dots (3)$$

and if V is the total shear in the story then, $2 V_o + V_i = V$, from which,

$$V_o = \frac{1 + n_i}{3 + n_o + 2 n_i} V \dots\dots\dots\dots\dots\dots (4a)$$

and,

$$V_i = \frac{1 + n_o}{3 + n_o + 2 n_i} V \dots\dots\dots\dots\dots\dots (4b)$$

By referring to Fig. 10(b) and Fig. 10(c) it can be seen that the increment of stress in Column A will be,

$$T_A = \frac{V_o h}{L_o} = \frac{V h}{L_o} \left(\frac{1 + n_i}{3 + n_o + 2 n_i}\right) \dots\dots\dots\dots (5)$$

and in Column B,

$$T_B = \frac{V_i\,h}{L_i} - \frac{V_o\,h}{L_o} = \frac{V\,h}{3 + n_o + 2\,n_i}\left(\frac{1 + n_o}{L_i} - \frac{1 + n_i}{L_o}\right) \ldots\ldots (6)$$

In the cantilever method, $T_B = \dfrac{L_i}{2\,L_o + L_i}\,T_A$; or,

$$\left(\frac{1 + n_o}{L_i} - \frac{1 + n_i}{L_o}\right) = \frac{L_i}{2\,L_o + L_i}\left(\frac{1 + n_i}{L_o}\right) \ldots\ldots\ldots\ldots (7)$$

If $i = \dfrac{I_i}{I_o}$ and $c = \dfrac{L_i}{L_o}$, then from Equation (7):

$$i = \frac{2\,c^2\,(c + 1)}{2 + c - \dfrac{1}{n_o}\,(2\,c^2 + c - 2)} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

In the portal theory, $T_B = 0$; or, $\left(\dfrac{1 + n_o}{L_i} - \dfrac{1 + n_i}{L_o}\right) = 0$, from which,

$$i = \frac{c^2}{1 - \dfrac{1}{n_o}\,(c - 1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The results of Equations (8) and (9) are shown graphically in Fig. 11 in which curves are shown for various values of n_o. These curves show that the rigidity of the columns as compared with the girders has considerable importance in determining the value of i, particularly for values of c greater than 0.8. Furthermore a probable curve for the values of i, that would be obtained if the beams were designed for the same unit, uniform, vertical load, has been shown in Fig. 11. This curve is more of a third power of c than a second power.

From a study of the relative positions of the curves in Fig. 11 it would seem that the following general statements can be made, most of which corroborate the conclusions of Messrs. Witmer and Bonner:

(1) For values of c less than 0.6, the portal theory will probably be most correct for girders that are designed for vertical loads only, but that small changes in the rigidity of the girders or columns will tend to produce considerable percentage changes in the results.

(2) For values of c between 0.6 and 1.2, the portal theory will be most correct for all rigidity ratios of girders and columns unless the center girders are arbitrarily increased in size.

(3) For values of c greater than 1.2, the cantilever theory will be most correct if the columns are much more rigid than the outer girders; otherwise, neither theory will give great accuracy.

(4) For values of n_o less than 1.5, the rate of change of i with respect to c is too rapid to make a numerical comparison of much use. The reason for this difference in accuracy can be most easily seen by referring to Fig. 12 in which the variation of i with respect to n_o is given for a particular value of c.

From this curve it can be seen that any conclusions that might be made from the portion of the curve for values of n_o less than 1.5 would be very uncertain in view of the practical difficulty of calculating the true value of n_o.

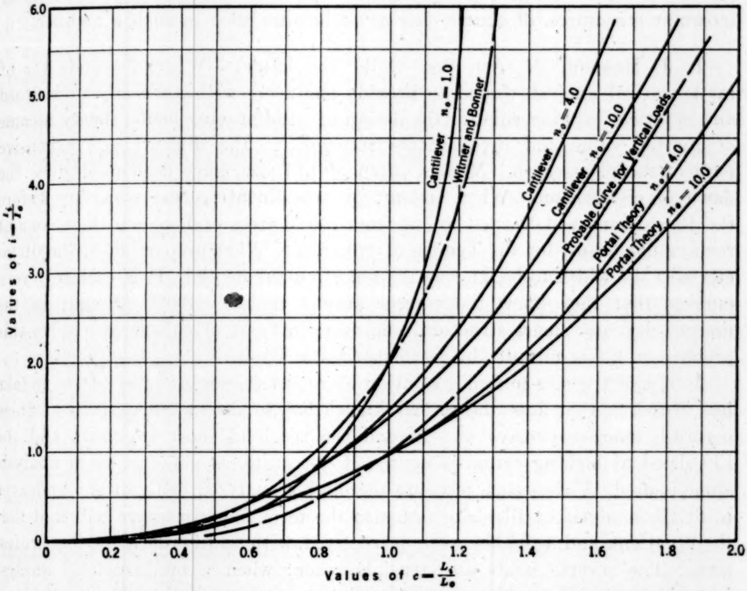

FIG. 11.—RELATION OF SPAN AND MOMENT OF INERTIA OF GIRDERS FOR CANTILEVER AND PARTIAL METHODS OF ANALYSIS.

In conclusion, the writer wishes to make the usual observation that the effect of the direct stress in the columns may modify the final results both for the hypothetical frame and for the model. In this respect a model that is proportioned according to the dimensions of the actual structure will have

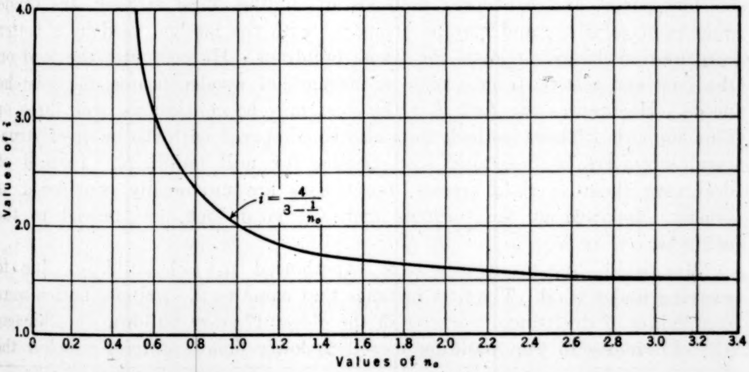

FIG. 12.—RELATION OF i AND n_o FOR CANTILEVER METHOD; $c = 1$.

considerable advantage, but there may be greater disadvantages. There is no general law that any type of structure will inherently follow, except the law of minimum energy, but perhaps certain regular types of structures can be forced into some definite group without undue loss of economy. Special or irregular structures, of course, can never be simplified in such a manner.

L. J. MENSCH,[7] M. AM. SOC. C. E. (by letter).—When the endeavor of mathematical analysts failed to provide engineers with an easily understood and practical working rule for the design of wind-bracing bents, theory became of limited value and investigators returned to the difficult and expensive path of model research. Models, often of full size, have been used since the dawn of civilization. What amount of experimenting was necessary before the Phœnicians had learned to cut from round trees of Lebanon the strongest rectangular beams for the Temple of Solomon? What proportion of the diameter was the width to be, and what proportion the depth? It is certainly very curious that they knew the correct answer probably 3 000 yr ago; in the present day any smart structural engineer, just out of college, can solve this problem in a few minutes by mathematical analysis.

Is it really easier to find a solution by model research alone? Does it take less perception or less theoretical knowledge to do so successfully? It is certainly more expensive and beyond the reach of most engineers and the likelihood of making errors is nearly, if not quite, as great as when analysis alone is used. Unless such tests are thoroughly analyzed, without any endeavor to fit the analysis by illogical means to the tests, experimenters will find that their patient and valuable work is received with indifference by the profession. The reverse holds also true, however; when a mathematical analyst investigates a new problem his endeavors are only of little value to the profession unless he substantiates his findings by careful tests.

The writer proposes to show that the models described in this paper were not as perfect as they should have been, a fact evidently known to the authors, as they wisely stated that their results should be accepted more to indicate trends of the reactions for the relative proportions of columns and girders.

The portal and cantilever methods of finding wind stresses are crude guesses at solutions and may be compared with the method used by a better carpenter in designing joists for frame buildings. He computes the load on the joist and selects, from a table of strength of wooden beams, the joist he needs. He ignores the fact that the joist may be continuous over three or four supports. These methods may also be compared with the habit of structural engineers in designing steel trusses for mill buildings. Instead of designing them as rigid frames, the trusses are universally considered as simply supported at the columns and, for good luck, are braced to the columns.

Meanwhile, inconveniences were experienced in tall buildings due to swaying under wind. The first instance that came to the writer's notice were complaints of draftsmen, working in the oldest 17-story building in Chicago, Ill. Engineers in that building moved to lower floors, preferably below the tenth. Investigations were made, and the amplitude of the vibrations were found to be of the order of $\frac{1}{16}$ in.; the building was strengthened by knee-braces at some of the columns and vibrations were greatly reduced. In another 16-story building draftsmen could not work on the tenth floor of the building when a high wind was blowing and the partitions in that building were vibrating so visibly that clerks moved their desks many feet away. When high hotel and apartment buildings came into general use, complaints were heard from occupants that they could not sleep on account of window weights hitting the window boxes and because water was spilling out of glasses and wash bowls, and pictures and chandeliers were swinging in a high wind.

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[7] Civ. Engr. and Constructor, Chicago, Ill.

It became apparent that a more accurate analysis was needed for the design of tall building frames, and it is to the credit of Albert Smith, M. Am. Soc. C. E., that he first published[8] a practical rule for the design of such frames, based on a laborious analysis by Castigliano's theorem of least work; his rule was a great improvement over the portal and cantilever methods, but was received indifferently by those engineers who did most of the designing of tall building frames, possibly on account of the labor involved in checking the basic calculations.

W. M. Wilson and G. A. Maney, Members, Am. Soc. C. E.[9], analyzed a 20-story building frame by assuming the rotations of the joints as unknowns in the elastic equations and gave their analysis the novel and rather unfortunate name of "slope-deflection theory." Their method and that of Mr. Smith were just as correct as Clapeyron's method of analyzing continuous beams, but were not used as much as the latter, because, in the nature of the case, these methods are too laborious for practical use. Furthermore, Messrs. Wilson and Maney deduced from their research a rule for simplifying the analysis, which is more correct for varying sections and spans than Mr. Smith's simple rule, but still too complicated for most practising engineers. The result of these two most important endeavors was that most engineers used the method of least work, namely, the crude and easily understood portal and cantilever methods.

Although it was long known that longitudinal deformation of the columns under wind, and also under dead load and live load, might produce considerable changes of stress in the girders of tall frames, and was found by Messrs. Wilson and Maney to amount to only a small percentage in a 20-story building, Henry V. Spurr, M. Am. Soc. C. E., in 1930, introduced an improved cantilever method[8] in which the longitudinal deformation of the columns was made the corner stone of his method of computing wind moments, neglecting at the same time other important factors, such as the law of least cost of a structure.

In 1931, the writer undertook to write a paper[10] on deflection and vibrations of tall building frames under wind and, at the same time, to give a simple working rule for the computation of wind stresses which the busy engineer could use. He first faced this problem in 1910 when he designed column supports for high elevated tanks of reinforced concrete without diagonal braces. The frames were, in fact, two column bents of several stories. When computing the deflection and rotations of the joints he noticed that the members far away from a joint had little influence on stresses produced there, and that

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[8] *Journal*, Western Soc. of Engrs., April, 1915.

[9] "Wind Stresses in the Steel Frames of Office Buildings", *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, 1915.

[10] "Deflections and Vibrations in High Buildings", by L. J. Mensch, *Proceedings*, Am. Concrete Inst., Vol. 28, 1932, p. 387.

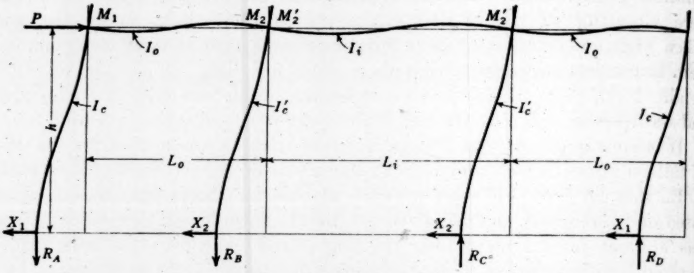

FIG. 13.—$\left(n_o = \frac{I_o}{L_o} \div \frac{I_c}{h}\right)$; AND $n_i = \frac{I_o}{L_o} \div \frac{I'_c}{h}$)

fairly accurate results could be obtained for bents with any number of spans and stories by the following assumptions:

(1) The point of contraflexure of all columns, except in the lowest story, is in the center of the story height;

(2) Where the values of stiffness of the girders, $\frac{I}{L}$, adjoining an inside column are alike, the wind moments in the girders at each side of the column at the joint are alike;

(3) Where the values of stiffness of the girders adjoining inside columns are not alike, the moments in the girders induced by the deflection of the columns are proportionate to the stiffness of the girders.

For the case shown in Fig. 13, which embraces all cases with four columns treated in the present paper, the writer deduced the formulas in the paper* mentioned:

$$X_1 = \frac{P}{4}\left(n_i + \frac{6}{1+N}\right) \div \left(\frac{n_o + n_i}{2} + \frac{6 + 3\,N}{1 + N}\right) \ldots\ldots\ldots (10)$$

$$X_2 = \frac{P}{2} - X_1 \ldots\ldots\ldots (11)$$

$$M_1 = X_1\,h \ldots\ldots\ldots (12)$$

$$M_2 = \frac{X_2\,h}{1 + N} \ldots\ldots\ldots (13)$$

$$M'_2 = \frac{X_2\,h\,N}{1 + N} \ldots\ldots\ldots (14)$$

$$R_A = \frac{M_1 + M_2}{L_o} \ldots\ldots\ldots (15)$$

and,

$$R_A + R_B = \frac{2\,M'_2}{L_i} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which, in addition to the notation of the main paper and the symbols illustrated in Fig. 13, P = the entire wind load above the middle of any story considered; n = ratio of stiffness between girders and columns; and,

$$N = \frac{L_o\,I_i}{L_i\,I_o} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

When all columns are alike and when the values of stiffness of the girders are all alike, Equation (10) reduces to,

$$X_1 = \frac{P}{4}\,\frac{n_o + 3}{n_o + 4.5} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

and,

$$M_2 = M'_2 = \frac{X_2\,h}{2} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

In the lower part of very tall buildings the stiffness of the columns is often ten times as great as that of the girders, and no great error is made by assuming n_o and n_i as zero in Equations (10) and (18), and Equation (10) becomes,

$$X_1 = \frac{P}{4}\,\frac{2}{2 + N} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

which means that the shear and reactions in the columns are mainly dependent on the relative stiffness of the girders.

When it is desired to find the relative stiffness of the girders, which would satisfy Mr. Spurr's idea of proportionate column stress reactions, one can proceed in this case as follows: Substitute Equation (20) in Equation (11),

$$X_2 = \frac{P\,(2 + 2\,N)}{4\,(2 + N)} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

substituting Equation (20) in Equation (12), and Equation (21) in Equation (13),

$$M_1 = M_2 = \frac{2\,P\,h}{(2 + N)\,4} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

which means that the point of inflection of the girders, where very heavy columns are used, is in the center of the girders. Consequently, Equations (14), (15), and (16) become:

$$M'_2 = \frac{X_2\,h\,N}{1 + N} = \frac{2\,N}{(2 + N)}\,\frac{P\,h}{4} \;\ldots\ldots\ldots\ldots\ldots (23)$$

$$R_A = \frac{M_1 + M_2}{L_o} = \frac{4}{(2 + N)}\,\frac{P\,h}{4\,L_o} \;\ldots\ldots\ldots\ldots\ldots (24)$$

and,

$$R_A + R_B = \frac{2\,M'_2}{L'_i} = \frac{4\,N}{(2 + N)}\,\frac{P\,h}{4\,L_i} \;\ldots\ldots\ldots\ldots\ldots (25)$$

When $L_o = L_i$, $R_A + R_B = N\ R_A$; and,

$$R_B = (N - 1)\ R_A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Mr. Spurr's assumptions of a straight-line deformation of a transverse section of a frame may be expressed mathematically in this case of equal spans by $\frac{R_A}{A_o} \div \frac{R_B}{A_i} = 3$; but, from Equation (26), $\frac{R_A}{R_B} = \frac{1}{N-1}$; and, therefore,

$$N = 1 + \frac{A_i}{3\ A_o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which A_o = the cross-section area of the outside column; and A_i = the cross-section area of the inside column. In other words, regardless of the requirements of dead and live load and wind-bending stresses, the section of the central girder must be increased which in most cases invites a waste of materials, especially in the lower part of the frames. The authors' models did not show this relation, as their columns were not much stiffer than the girders.

For young engineers who have done much research in the analysis of wind-bracing bents and who may be still doubtful as to the validity of Equation (10) on account of the writer's Assumptions (2) and (3), the following formula is presented, but is not needed in actual practice:

$$X_1 = \frac{P}{4} \left\{ \frac{2 + \frac{n_i}{3} + \frac{2\ n_i\ N}{3}}{2 + \frac{n_o}{6}\left(1 + \frac{I_c}{I'_c}\right) + N\left[1 + \frac{n_o}{3}\left(1 + \frac{I_c}{I'_c}\right)\right]} \right\} \ldots (28)$$

The results of Equation (10) vary only a small percentage from those of Equation (28), whereas Equation (18) will give results that may vary 5% in the lower part of a tall building and as much as 20% at the very top.

Clear spans are the governing factors[10] in the design of building frames and the use of theoretical spans causes the computed stresses to be often 10% to 30% greater than they appear from tests to destruction. When mathematical analysts, using an "exact" theory, start with an assumption which misleads them from 10 to 30% is it to be wondered that practical engineers do not pay much attention to their work? Equations (10), (18), (20), (21), and (28) do not, as a rule, give results that differ greatly whether clear or center-to-center spans are used. The moments in the columns and girders, however, vary greatly because of the shorter lengths of the clear spans, and the deflections and vibrations are also much smaller.

The writer wishes to repeat that he has every reason to believe that his formulas give results as reliable as the continuous beam formulas, provided, of course, that the joints are properly designed. In fact, he measured the deflection and rotation of joints on two model frames of 6-story and 13-story buildings, made of brass rods, soldered at the joints, and found a very good agreement with his analysis; and when his formulas give results that differ with the tests of the authors, he does not hesitate to state that their models did not have proper joints.

It will be interesting to check Cases A-1, A-2, A-1a, and A-1c of the paper by Equations (10) to (28). For Cases A-1 and A-1a, all columns are alike, all girders are alike, $h = 2\frac{1}{2}$ in., and $L_o = 4$ in. for center-to-center spans and, according to the authors, the clear spans are to be taken as $\frac{1}{2}$ in. less. The ratio, n, is $\frac{2.5}{4} = 0.625$ when theoretical spans are used, and n is $\frac{2}{3.5} = 0.571$ when clear spans are used. In both cases, $N = 1$. From Equation (28), $X_1 = \frac{P}{4} \frac{2 + n}{2 + \frac{n}{3} + 1 + \frac{2n}{3}} = 0.181\ P$ and $0.18\ P$, respectively. Equations (10) and (18) give identical values ($0.177\ P$ and $0.176\ P$, respectively).

Adopting the value of $0.18\ P$ for X_1, $X_2 = \frac{P}{2} - 0.18\ P = 0.32\ P$. From Equation (12), $M_1 = 0.18\ P\,h$ for center-to-center spans and $0.18 \frac{3.5 P h}{4}$ for clear spans. Furthermore, from Equations (13) and (14), $M_2 = M'_2 = \frac{0.32\ P\,h}{2}$ and $\frac{0.32\ P\,h}{2} \frac{3.5}{4}$, respectively. Equation (15) yields: $R_A = \frac{P\,h \times (0.18 + 0.16)}{3.5} \times \frac{3.5}{4} = 0.85\ P\,h$ $= 0.2125\ P$, and Equation (16), $R_A + R_B = \frac{0.32\ P\ h}{3.5} \times \frac{3.5}{4} = 0.08\ P\,h$ $= 0.08\ P \times 2.5 = 0.200\ P$; or, $R_B = 0.200\ P - 0.2125\ P = -\ 0.0125\ P$ $= -0.0588\ R_A$, against $-0.15\ R_A$ and $-0.19\ R_A$ from Models A-1, and A-1a.

Case A-2 was like Case A-1 except that all center girders were twice as stiff as the outside girders; Hence, $N = 2$, whereas $n_o = 0.571$, as before. From Equations (10) to (16), respectively: $X_1 = \frac{P}{4} \times \frac{0.571 + 2}{0.571 + 4} = 0.1406\ P$; $X_2 = 0.3594\ P$; $M_1 = X_1 h \times \frac{3.5}{4} = 0.1406\ P\ h \times \frac{3.5}{4}$; $M_2 = \frac{X_2 h}{1 + N} \times \frac{3.5}{4} = 0.1198\ P\ h \times \frac{3.5}{4}$; $M'_2 = \frac{X_2 h N}{1 + N} \times \frac{3.5}{4} = 0.2396\ P\ h\ \frac{3.5}{4}$; $R_A = \frac{0.1406 + 0.1198}{3.5} \times P\ h \times \frac{3.5}{4} = 0.163\ P$; $R_A + R_B = \frac{2 \times 0.2396}{3.5} \times P\,h \times \frac{3.5}{4} = 0.299\ P$; and $R_B = 0.299\ P - 0.163\ P = 0.136\ P = 0.835\ R_A$. The model for this case showed that $R_B = 0.33\ R_A$, which is a proof in the writer's mind that the connections in this case were not very effective.

In Case A-1c all girders and outside columns were as in Case A-1 except that the inside columns were twice as thick and their stiffness eight times as great; hence, $n_o = 0.571$; $n_i = \frac{0.571}{8} = 0.0714$; and $N = 1$.

From Equations (10) to (16), respectively: $X_1 = \frac{P}{4} \times \frac{0.571 + 3}{0.321 + 4.5}$ $= 0.1855\ P$; $X_2 = 0.3144\ P$; $M_1 = 0.1855\ P\ h \times \frac{3.5}{4}$; $M_2 = M'_2 = 0.1572\ P\ h \times \frac{3.5}{4}$; $R_A = \frac{0.1855 + 0.1572}{3.5} \times P\ h \times \frac{3.5}{4} = 0.0857\ P\ h$; $R_A + R_B = \frac{2 \times 0.1572}{3.5} \times P\ h \times \frac{3.5}{4} = 0.0786\ P\ h$; and $R_B = 0.0786\ P\ h - 0.0857\ P\ h = -0.0071\ P\ h = -0.0828\ R_A$, which is very close to the result found by the writer for Cases *A*-1 and *A*-1*a*, and, this time, more in agreement with the model test reported in the paper.

From the foregoing examples one may conclude that the authors were correct in their statement that the tests on the models showed the trend of reactions, but their values differed considerably with that obtained by analysis. No answer was given for the reactions for a horizontal load applied at the first-floor level which often varies considerably from the upper floors.

It would have been of great interest if, on one welded model, tests had been made on the period and amplitudes of vibrations and if the amplitudes had been compared with those of a solid bar of the same material having the same period of vibration.

GILBERT MORRISON,[11] M. AM. SOC. C. E. (by letter).—Information of much interest to designing engineers, on the subject of lateral forces, on tall building bents, has been presented in this paper. The results of the behavior of the many different combinations of bent structures, under tests shown by the authors, should be of material assistance in predetermining how similar structures are likely to act under lateral forces.

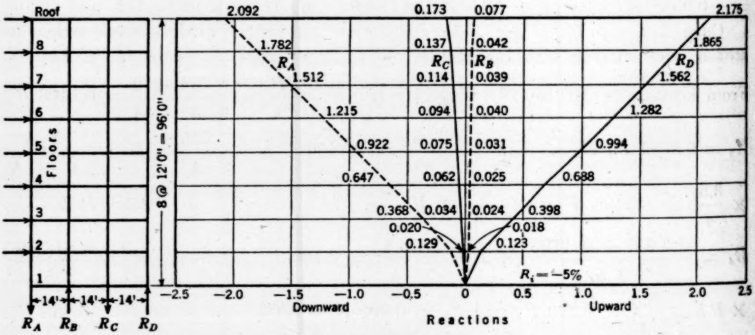

FIG. 14.—CASE *M*.

An investigation of the effect of lateral forces on tall buildings was made by the writer in 1933. Although this investigation did not cover as wide a range of bent structures as that reported by the authors, a comparison of the results is of interest. The first bent tested (see Fig. 14) was a four-column frame with equal, 14-ft bays and eight stories, each 12 ft high, giving a slenderness ratio of 2.3 to 1. The model was made of celluloid, to a scale of 1 in. = 4.0 ft. The apparatus used in the test was a Beggs deformeter. In this bent all girders were of the same size, having a moment of inertia of 1 728. The inner and outer columns of any floor were the same; but were increased in size downward for each two stories. The value of I for columns in the first and second stories was 2 920; for the third and fourth-story columns, $I = 2.835$; for the fifth and sixth-story columns, $I = 1\,728$; and for the seventh and eighth-story columns, $I = 1\,028$. All members of the model were proportioned to the cube root of their moment of inertia and the aforementioned scale. A lateral force of unity was acting at each floor above the first.

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[11] In Chg. of Structural Design, Cooper Union Inst. of Technology, New York, N. Y.

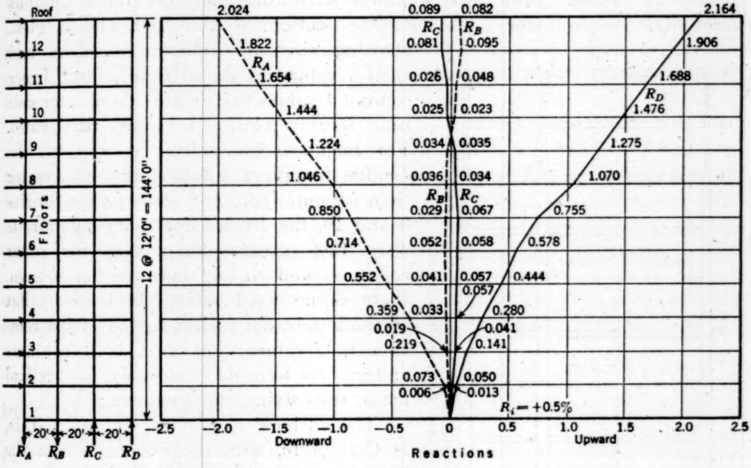

FIG. 15.—CASE *N*.

In Fig. 14, negative reactions, acting downward, are plotted to the left and positive reactions, acting upward, are plotted to the right. The summation of the abscissas from the top down to any floor, for any column, indicates the value of the vertical reaction for that column, for the unit loads applied at those floors. Hence, the summation of all the abscissas, from top to bottom, gives the value of the vertical reaction for that column, with lateral forces acting at all floors simultaneously, proper care being given to positive and negative signs. This shows that the vertical reaction for Column $A = -\,8.667$, for Column $B = +\,0.296$, for Column $C = -\,0.709$, and for Column $D = +\,9.087$, indicating an almost exact balance.

In this case, as in that of the paper, it is seen that the interior reaction is opposite in character from the exterior reaction on the same side of the bent, and also of relatively small magnitude, the average interior reaction being approximately − 5% of the exterior reaction. This is less than the − 15% shown for the authors' Case *A*-1. It should be noted that the only difference between the bents treated by the authors and the writer, is that in the former all the columns were kept the same size throughout the full height of the structure, whereas, in the latter, the column sizes were changed every two floors.

The results of the second bent tested by the writer (see Fig. 15) was also a four-column frame with equal, 20-ft bays and 12 stories each 12 ft high, giving a slenderness ratio of 2.4 to 1. This model was also made of celluloid, to a scale of 1 in. = 4 ft and tested with the Begg's deformeter. In this case all floor girders were of the same section, being 18-in. I-beams at 55 lb, with $I = 889.9$. However, the roof girders were 16-in. I-beams at 45 lb., with $I = 583.3$. Like Case *M* column sections were changed at each two succeeding stories. Both outer columns were alike and both inner columns were alike in any story. A complete section of the bent and a plan, showing sizes of different members and their *I*-values, is shown in Fig. 16. Inner column *I*-values are greater than outer column *I*-values; but not by any fixed ratio. For instance, for Stories 11 and 12, the *I*-value for inner columns is 33% greater than for outer columns, whereas, for Stories 9 and 10, the *I*-value for inner columns is only 12% greater than that for outer columns—and so on, down to the bottom, there is no fixed ratio. It is seen that sections were not picked for any fixed relation to *I*-values; but that both columns and girders were designed only for vertical loads, thus fixing the *I*-values.

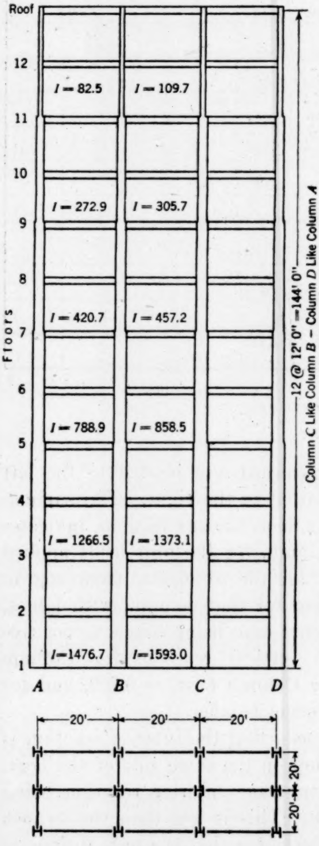

FIG. 16.

The authors' case most similar to this, is Case *A*-1*c*, with the exception that the latter carries the same size of columns for the entire height of the bent and keeps the relation of the *I*-values of the inner columns eight times the *I*-value of the outer columns; the writer's case shows actual sizes in an existing typical building bent. However, a study of Fig. 15, will show that the character of the reactions was the same on each side of the bent, contrary to that shown for Case *A*-1*c*. This test shows an almost exact agreement with the distribution indicated by the portal theory. It should be noted that the reaction for both inner columns changes direction in the ninth story.

The upward and downward reactions in Column *B* balance each other within

0.002, whereas the same items in Column *C* balance each other within 0.141. It should also be noted that the average interior reaction is + 0.5% of the exterior reaction. The total vertical reactions for Column $A = -11.981$, for Column *B*, − 0.002, for Column *C*, + 0.141, and for Column *D*, + 11.827, again showing an almost exact balance.

In view of these tests and the tests described in the paper, the writer is in agreement with the authors' statement that, if a structure is designed for vertical loads only, with *I*-values of girders assumed in proportion to the square of the span length, the reactions are in very close agreement with distribution determined by the portal theory. However, it is believed that a bent designed for vertical loads only, is not a typical frame from which to verify the application of any theory as to the distribution of lateral forces because a bent designed for both vertical and lateral loading, would undergo considerable change in structural requirements over one designed for vertical loads only. The writer does not feel that he would care to support, without reservations, the authors' statement that a normal variation in column sizes, increasing toward the bottom, would not greatly vary the general relations shown.

It should be noted that Cases *A*-1, *A*-2, *A*-3, *A*-9, and *A*-17, of the paper (in which *I*-values of center bays are increased by 2, 3, 9, and 17, respectively) indicate an increase in R_t, which was to be anticipated, because of the additional stiffness given; but the additional increases in R_t are not proportional to the increases in moments of inertia, which is clearly shown in Fig. 3.

In Case *E*-1.41 it would appear that the reactions, *A*, *B*, and *C*, act upward, whereas Reaction *D* acts downward. If this is correct, there appears to be about 20% variation between upward and downward reactions.

FANG-YIN TSAI,[12] ASSOC. M. AM. SOC. C. E. (by letter).—In Figs. 3, 5, 7, and 8, of this paper, the curves are plotted with *i*, or $\frac{L_t}{L_o}$, assumed to be variable as if it were the only factor that governs the value of R_t for the various bents. W. M. Wilson and G. A. Maney, Members, Am. Soc. C. E., have shown[13] that the distribution of moments in columns and girders of a symmetrical 4-column bent depends upon the ratios of the *K*-values of its members, in addition to the layout and loading. The writer has found that, for bents of the type of the authors' Series *A*, *B*, *C*, and *D*, the value of R_t depends upon the story number and the following factors:

$$\alpha = \frac{I'_o}{I'_t} \quad \text{.........................} (29)$$

$$\beta = \frac{I'_o}{I_o} \times \frac{L_o}{h} \quad \text{.....................} (30)$$

$$r = \frac{L_t}{L_o} \quad \text{.........................} (31)$$

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[12] Prof. of Structural Eng., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

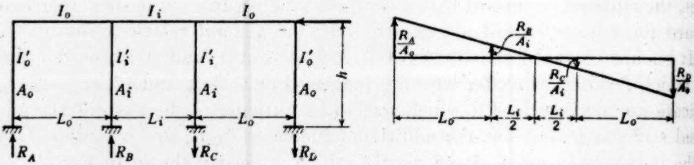
[13] "Wind Stresses in the Steel Frames of Office Buildings", *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, 1915, p. 26.

and,

$$i = \frac{I_i}{I_o} \text{ ....(32)}$$

in which I'_o and I'_i are, respectively, the moment of inertia of the outer and inner column; h is the story height; and the remaining symbols are the same as those used in the paper. The values of α, β, r, and i, together with the story number, may be considered as the elastic properties of such bents, which govern the value of R_i due to lateral loads.

An attempt has been made to verify the authors' results analytically and to deduce a general expression for R_i in terms of α, β, r, and i. Since such work for a 19-story, 4-column bent involves a prohibitive amount of time and difficulty, the writer selected an expression for such bent of one-story only (Fig. 17(*a*)). (He is indebted to several students in his class in Advanced Structural Theory for developing the formula by the slope-deflection method.) The result is as follows:



(*a*) 1-STORY 4-COLUMN SYMMETRICAL BENT (*b*) CANTILEVER RELATION OF REACTIONS

FIG. 17

$$R_i = \frac{2\,i\,(2\,\beta - \alpha + 2)}{r^2(4\,\beta + \alpha + 1) + 3\,\alpha\,r\,i} - 1 \text{ ....(33)}$$

TABLE 2.—COMPARISON OF THE VALUES OF R_i OBTAINED BY THE AUTHORS FOR 19-STORY BENTS WITH THOSE COMPUTED FOR ONE-STORY BENTS OF THE SAME ELASTIC PROPERTIES

Case	ELASTIC PROPERTIES OF BENTS				VALUES OF R_i, IN PERCENTAGES		Case	ELASTIC PROPERTIES OF BENTS				VALUES OF R_i, IN PERCENTAGES	
	α	β	r	i	The authors' (for 19-story)	Computed (for one-story)		α	β	r	i	The authors' (for 19-story)	Computed (for one-story)
A-1....	1	1.6	1	1	− 15	− 26.4	*B*-1....	1	1.2	2	1	− 76	− 79.5
A-2....	1	1.6	1	2	+ 33	+ 16.7	*B*-2....	1	1.2	2	2	− 46	− 65.3
A-3....	1	1.6	1	3	+ 89	+ 44.9	*B*-3....	1	1.2	2	3	− 33	− 54.9
A-9....	1	1.6	1	9	+169	+113.5	*B*-9....	1	1.2	2	9	+ 13	− 24.6
A -17...	1	1.6	1	17	+230	+140.0	*B*-17...	1	1.2	2	17	+ 46	− 10.5
A-1*a*...	1	1.6	1	1	− 19	− 26.4	*B*-1*a*...	1	0.15	2	1	− 56	− 84.2
A-1*b*...	8	12.8	1	1	− 33	− 68.4	*C*-1....	1	1.8	0.67	1	+ 76	+ 51.0
A-1*c*...	0.125	1.6	1	1	− 3	+ 28.5	*C*-2....	1	0.9	0.67	0.5	± 0	− 19.8
D-1....	1	0.8	1	1	− 18	— 36.6	*XC*-1..	1	1.72	0.78	1	+ 29	+ 14.6
D-2....	1	0.8	1	2	+ 21	− 7.2	....	....	...	..	..	.....	......
D-3....	1	0.8	1	3	+ 46	+ 9.9	....	....	...	..	..	.....	......
D-9....	1	0.8	1	9	+ 56	+ 45.4	....	....	...	..	..	.....	......

The elastic properties and the authors' values of R_i for the 19-story bents of Series A, B, C, and D, together with the computed values of R_i for one-story bents of the same elastic properties, are given in Table 2, in which it is seen that, with the exception of Case A-1c, all the computed values of R_i are consistently smaller (algebraically) than the corresponding values obtained by the authors. Both sets of the values of R_i are plotted against the values of i for Series A, B, and D in Fig. 18, which is similar to the authors' Fig. 3. It shows, not only the consistency in the trends of the curves and the correctness of the authors' results in general, but also that, with α, β, and r kept constant, the values of R_i increase algebraically as the story number increases for any value of i, and that the larger the value of i, the greater will be the increase in the value of R_i.

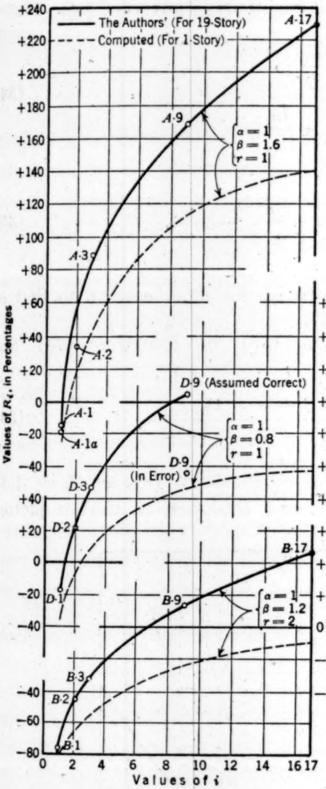


FIG. 18.—RELATION BETWEEN R_i AND i

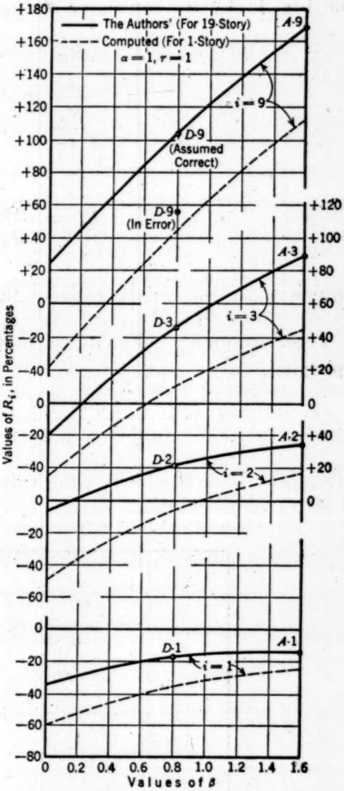


FIG. 19.—RELATION BETWEEN R_i AND β

Fig. 19 shows the relation between R_i and β, with α, r, and i kept constant, comparing the authors' values for the 19-story bent with those computed for the one-story bent. It is seen that, for this case, the values of R_i increase algebraically as the story number increases for any value of β, and that the larger the value of i, the greater the increase in the value of R_i.

The authors' Fig. 8 shows the relation between i and r for the cantilever relation of R_i, neglecting again the other elastic properties of the bents. From Fig. 17(b), the cantilever relation of reactions is governed by;

$$\frac{\dfrac{R_B}{A_i}}{\dfrac{R_A}{A_o}} = \frac{\dfrac{L_i}{2}}{L_o + \dfrac{L_i}{2}} \qquad \text{.......................(34)}$$

or,

$$R_i = \frac{R_B}{R_A} = \frac{q\,r}{2 + r} \qquad \text{.....................(35)}$$

in which $q = \dfrac{A_i}{A_o}$ = the ratio of the area of the inner column to that of the outer column. Equating Equation (35) to Equation (33) and simplifying,

$$i = \frac{q\,r^2\,(1 + r)\,(4\,\beta + \alpha + 1)}{(2\,\beta - \alpha + 2)\,(2 + r) - 3\,q\,\alpha\,r\,(1 + r)} \qquad \text{...........(36)}$$

Letting $\alpha = q = 1$, which is the value for all the bents (except Cases A-1b and A-1c) of the authors' Series A, B, C, and D, Equation (36) is plotted

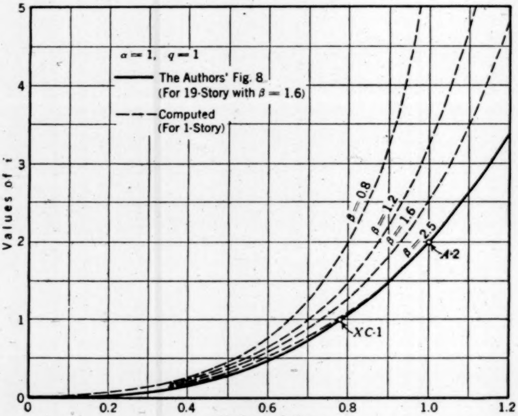


FIG. 20

with $\beta = 0.8$; 1.2; 1.6; and 2.5 in Fig. 20, in which it is seen that the curve with $\beta = 2.5$ coincides practically with the authors' Fig. 8. Fig. 20 also shows that, for a cantilever-theory value of R_i in one-story bents, with $\alpha = q = 1$, i decreases as β increases for any value of r, and that with β kept constant, i increases rapidly as r increases. Assuming the authors' Fig. 8 to be valid for 19-story bents with $\beta = 1.6$, the increase in the story number will have the same effect on i as the increase in β, and the girder length of Case XC–1 should be adjusted to make $\beta = 1.6$. Thus, by Equation (30),

$$\beta = 1.6 = \frac{I'_o}{I_o} \times \frac{L_o}{h} = \frac{I_o}{2.5} \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

in which $I'_o = I_o$. Hence, $L_o = 1.6 \times 2.5 = 4$ in.; and $L_i = i\, L_o = 0.78 \times 4 = 3.12$ in. If the foregoing values were used for L_o and L_i in Case XC–1 instead of $4\frac{5}{16}$ and $3\frac{3}{8}$ in., the value of R_i would probably be much closer to the cantilever-theory ratio than that obtained by the authors.

For a portal-theory value of R_i in one-story bents, equating Equation (33) to zero and simplifying:

$$i = \frac{r^2\,(4\,\beta + \alpha + 1)}{4\,(\beta + 1) - \alpha\,(2 + 3\,r)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

In order to avoid any negative value in i, which will be meaningless, α must always be less than $\dfrac{4\,(\beta + 1)}{2 + 3\,r}$.

The purpose of the foregoing discussion is mainly to verify the authors' results by comparing them with those computed for one-story bents of the same elastic properties and also to indicate the trend in the effect of the various factors, particularly the story number, upon the value of R_i.

The authors state that "the purpose of the study was to learn trends rather than true quantitative values of reactions for varying proportions of bents." Inasmuch as the value of R_i depends upon the numerous factors indicated herein, and the layout and the proportion of bents in any practical case will differ widely from those studied in the paper, it is doubtful whether the trends of any actual bent will conform to those presented by the authors. The writer thinks that the real value of the paper lies in introducing a simple and practical method of study, which, when applied carefully to an actual bent, may eventually lead to its proper proportion and design.

The authors use the title, "Reaction Diagram" for their Figs. 2, 4, etc., but the writer thinks that the diagrams will be better understood if the title is changed to read, "Reaction Influence Diagram." In the authors' Fig. 8 there are indicated Cases B–18 and C–0.67, both of which are apparently not described in the paper.

From the trends of the curves in Figs. 18 and 19, the writer concludes that the authors' value of R_i for Case D–9 is inaccurate, possibly due to the great inflexibility of the center girders of the bent. This is also indicated by the great asymmetry of the authors' Fig. 4(*d*). Hence, the curve is plotted according to an assumed correct value of R_i for Case *D*–9.

George E. Large,[14] Assoc. M. Am. Soc. C. E., and Samuel T. Carpenter,[15] Jun. Am. Soc. C. E. (by letter).—Much painstaking work is represented in the long series of tests on wind-bent models described in this paper. Although the writers cannot agree that all the conclusions of the study have general application to modern wind-bent design, they believe that this paper may be the means of defining the fields of at least two well-known methods of design with which they have experimented since 1933.

The Spurr adaptation[16] of the Cross moment distribution method has been used, as in Fig. 21, to verify the authors' experimental results. The regularity of the model framing will throw the contraflexure points at mid-story of the columns, as assumed in checking, except in a few of the lower stories where, because of the fixed column bases, the points of contraflexure will be above the half-story level, and, in some cases, absent entirely. Nevertheless, the ratio of girder shears across a floor will be substantially the same throughout all stories, as will be the ratio of column reactions, R_i, which is the crux of the argument.

TABLE 3.—Comparison of Experimental and Computed Results

Case	R_i, Inner Column Reaction Found, in Percentage of Outer Columns.			Customary Assumptions as to Column Reaction (R_i)		$\frac{\text{Column}}{\text{Girder}}$-stiffness, ratio, $\frac{\Sigma \text{ (columns)}}{\Sigma \text{ (girders)}}$
	Experimental result	Calculated as a check by moment distribution	Remarks	(Portal)	(Cantilever)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A–1	−15	−14	"O.K"	0	+33	1.75
C–2	0	+0.3	Equals portal performance	0	+25	1.05
C–0.5*	+28	+27.0	Integral connections, 55 stories	0	Designed +28	Lower 11.2 story
B–9	+13	+3	Approximately portal performance	0	+50	0.62
B–17	+46	+36	Extreme girder proportions	0	+50	0.39
A–2	+33	+50	Questionable cantilever performance	0	+33	1.31
A–17	+230	+340	Extreme girder proportions	0	+33	0.28
11–*C*–2	None	+20	Approximately cantilever performance	0	+25	11.6
5–*B*–9	None	+62	Approximately cantilever performance	0	+50	3.1

* Model tested by writers.

To compensate for the vertical displacement of girder ends due to floor-joint rotation, Mr. Spurr uses a reduced length for girder bending equal to

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[14] Associate Prof., Civ. Eng., Ohio State Univ., Columbus, Ohio.

[15] Instructor in Civ. Eng., Swarthmore Coll., Swarthmore, Pa.

[16] "Wind Bracing of Steel Buildings." Second Progress Rept. of Sub-Committee No. 31, Committee on Steel of the Structural Division, Discussion by H. V. Spurr, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., August, 1932, p. 1133, and Fig. 26.

$\sqrt{\frac{F^3}{F + D}}$ in which F is the distance face to face of columns, and D is the depth of column section. For a 4-in. model span it is $\sqrt{\frac{(3.5)^3}{3.5 + 0.5}} = 3.28$ in.[17] In computing the stiffness, $\frac{I}{L}$, of the members, the center-to-center lengths of columns and the foregoing "Spurr lengths" of girders were utilized, since their use has theoretical foundation and, ordinarily, gave results nearer to experimental values both on the authors' models and on those studied by the writers.

Columns (2) and (3), Table 3, show a comparison of the principal experimental and calculated results. The upper group of three cases check very closely, indicating the correctness of the calculation method as well. Serious discrepancies occur in the next group of four cases. Some extreme cases of panel-to-panel girder proportioning exist in this group, and it seems very likely that the connection blocks were inadequate for holding the very stiff girders then used, causing experimental error. It is the writers' experience that one cannot make a theoretically rigid connection, as assumed in the calculation methods, except by developing tremendous frictional forces or by making an integral connection. Perhaps the authors, thinking of practical wind connections, did not desire complete rigidity. Nevertheless, the connections are variables which obscure the picture of girder performance.

The principal comments which the writers wish to make are as follows:

(*a*) In constructing models of tall building bents, the stiffness of the column system relative to the girder system must be reproduced faithfully, since their ratio has tremendous effect upon the distribution of girder shears and column reactions across a bent.

(*b*) The model columns in the authors' tests were so flexible relative to the model girders that most of the models did not represent practical wind bents. The experimental data, therefore, were not sound bases for drawing general conclusions in regard to the distribution of column reactions.

(*c*) A distinction should be made between wind bents upon the basis of slenderness. Design methods and assumptions which are good enough for 20-story bents can be actually dangerous when applied to a 60-story bent of the same base width.

(*d*) For a fair comparison[18] of the several methods of assigning girder shears and column "chord pulls", with economy in view, one must contemplate designs having the same calculated total lateral deflection, including that due to lengthening and shortening of columns.

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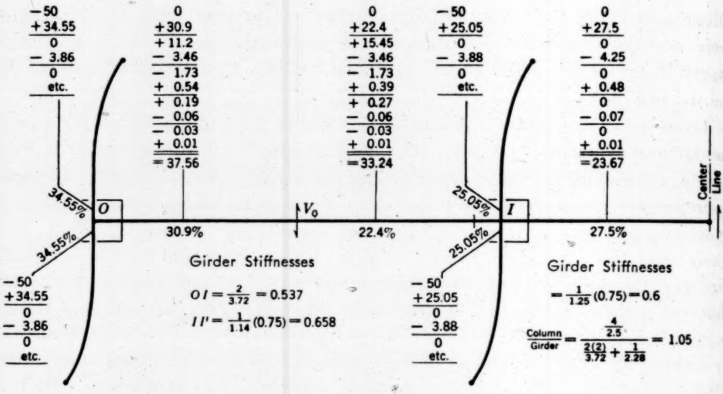
[17] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Assoc. M. Am. Soc. C. E., Samuel T. Carpenter, Jun. Am. Soc. C. E. and Clyde T. Morris, M. Am. Soc. C. E., *Bulletin No. 93*, Appendix, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

[18] "Wind Stresses and the Tall Building," by Samuel T. Carpenter, Jun. Am. Soc. C. E., presented at the meeting of the Philadelphia Section of the Society, April 15, 1936. (Not published.)

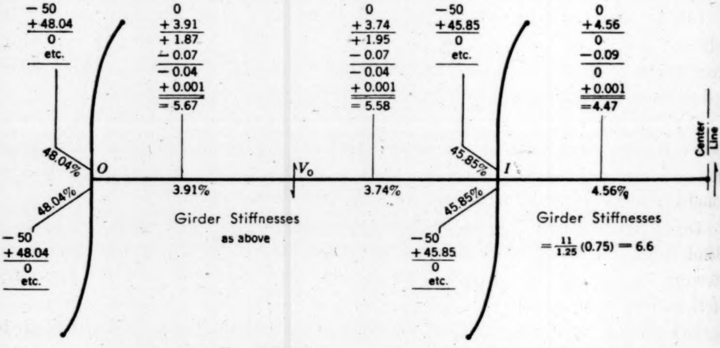
(*e*) Girder systems selected for strength only, without regard for stiffness, permit too much flexibility in a modern slender tower.

Fig. 21 is submitted in support of Item (*a*). The moment-distribution solution shown in Fig. 21(*a*) verifies the portal performance of Model *C*–2, as given in Table 3. When the columns were made eleven times as stiff, as in Fig. 21(*b*), the model dimensions conformed with the lower story of the writers' model, and the solution approximated the cantilever performance of that model.



−50
+34.55
0
−3.86
0
etc.
0
+30.9
+11.2
−3.46
−1.73
+0.54
+0.19
−0.06
−0.03
+0.01
=37.56
0
+22.4
+15.45
−3.46
−1.73
+0.39
+0.27
−0.06
−0.03
+0.01
=33.24
−50
+25.05
0
−3.88
0
etc.
0
+27.5
0
−4.25
0
+0.48
0
−0.07
0
+0.01
=23.67
Center Line
34.55%
O
V_0
30.9%
22.4%
25.05%
I
27.5%
V_1
34.55%
25.05%
Girder Stiffnesses
$OI = \frac{2}{3.72} = 0.537$
$II' = \frac{1}{1.14}(0.75) = 0.658$
−50
+34.55
0
−3.86
0
etc.
−50
+25.05
0
−3.88
0
etc.
Girder Stiffnesses
$= \frac{1}{1.25}(0.75) = 0.6$
$\frac{\text{Column}}{\text{Girder}} = \frac{\frac{4}{2.5}}{\frac{2(2)}{3.72} + \frac{1}{2.28}} = 1.05$

(*a*) CASE *C*-2: $\frac{V_1}{V_0} = \frac{23.67\ (4.5)}{1.5\ (70.8)} = 1.003;\ R_1 = +0.3\%$



−50
+48.04
0
etc.
0
+3.91
+1.87
−0.07
−0.04
+0.001
=5.67
0
+3.74
+1.95
−0.07
−0.04
+0.001
=5.58
−50
+45.85
0
etc.
0
+4.56
0
−0.09
0
+0.001
=4.47
Center Line
48.04%
O
V_0
3.91%
3.74%
45.85%
I
4.56%
V_1
48.04%
45.85%
Girder Stiffnesses
as above
−50
+48.04
0
etc.
−50
+45.85
0
etc.
Girder Stiffnesses
$= \frac{11}{1.25}(0.75) = 6.6$

(*b*) CASE 11-*C*-2: $\frac{V_1}{V_0} = \frac{4.47\ (4.5)}{1.5\ (11.25)} = 1.20;\ R_1 = +20\%$, Cantilever = +25%, Ohio State University Model = +28%

FIG. 21

Fig. 22 shows some actual wind-bent designs. Note how widely the $\frac{\text{column}}{\text{girder}}$-stiffness ratio may vary. This ratio is found by dividing the sum of the $\frac{I}{L}$-values of the four columns by the sum of the $\frac{I}{L}$-values of the three girders, the lower stories being of most interest in wind design. Most of the stiffness ratios of the authors' model bents shown in Table 3 fall outside the design range of Fig. 22.

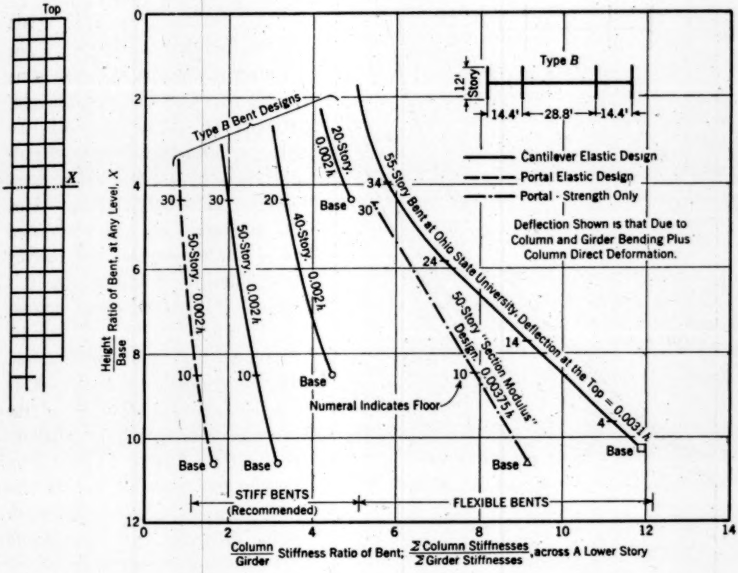


FIG. 22.

From Table 3, the authors' Case *B*–9 had approximately a portal performance. However, upon increasing the stiffness of the columns in Model *B*–9 five times, it conformed closely with the lower story of the 50-story cantilever design, in which the $\frac{\text{column}}{\text{girder}}$-stiffness ratio is 3.16, and had a cantilever performance. It is designated as Case 5–*B*–9 in Table 3. (The increased size of the inside columns of the practical designs prevents perfect checks.)

Similarly, by increasing the columns of (portal) Case *B*–9 eight times, the cantilever performance of the 20-story cantilever design can be verified approximately. Obviously, column stiffnesses have considerable to do with performance.

Fig. 23 is a summary of the design of several bents of varying $\frac{\text{height}}{\text{base}}$-ratios. The spans taken were chosen because they are generally con-

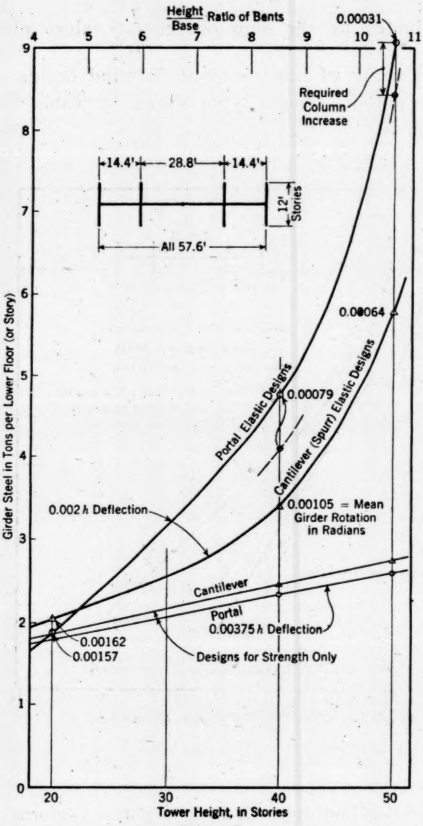


FIG. 23.

sidered favorable to the portal method. Both cantilever and portal designs were made for each height. All the bents represented by the two upper curves were designed to have a total deflection at the top of $\frac{1}{500}$ of the height, or 0.002 *h*. The column selections were the same in all bents of the same height, except in the case of the 40-story and the 50-story portal designed bents, in which extra column area had to be provided on account of the increased "wind-pull", carried entirely by the outside columns in this case. This extra was considered chargeable to the girder system, for fair comparison with the cantilever design. Note the economic advantage of the cantilever method for shallow-connection bents higher than twenty-five stories. It is also most desirable from the standpoint of avoiding column chord deformation accumulation.

These girders were designed as follows: With columns set, the portion of the entire 0.002 *h* deflection which was due to chord deformation of columns was calculated quickly by the following formula[19] for chord deflection:

$$y_T = \frac{2\, s_w\, h^2}{3\, E\, B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

in which s_w is the wind unit direct stress in the outside basement column; *h* is the full height of the bent; *E*, the elastic modulus; and *B*, the base width of the bent. The chord deflection was deducted from 0.002 *h* to find how much deflection was to be contributed by the bending of girders and columns, and girders were selected on the basis of required moment of inertia by the formula:

$$I_G = \frac{V\, b^2}{3\, E\, \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

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[19] For derivation of Equations (39) and (40) see *Bulletin No. 93*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio.

in which V is the assigned girder shear; b is the half-length of the girder; and θ equals the remaining deflection assignable to girders, divided by h, equals the joint rotation angle.

The point to be emphasized is that the chord deflection of the portal design was greater than that of the corresponding cantilever design, which meant that the girders designed by the portal method had to have smaller angular deformations, θ, at their ends if the bents were to have the same total deflection, and had to be stiffer and (usually) heavier than the girders designed by the cantilever method, as shown in Fig. 23. The small numerals near each curve point show in each case the required girder rotation angle, θ, in radians.

The two straight lines near the bottom of the graph show girders selected on the basis of section modulus solely to meet allowable stress, without regard for stiffness as defined by moment of inertia. As seen in Figs. 22 and 23 such a 50-story portal girder design has about one-half the weight, but almost twice the deflection, of the corresponding cantilever elastic design. What priced wind-bracing will you have?

In its 1933 report[30], the Wind-Bracing Sub-Committee, Committee on Steel of the Structural Division of the Society, recognized that limiting the design deflection to $0.002\,h$ has produced satisfactory results[31] in very slender towers, and suggested that fully twice that value may be allowable in less extreme bents. This is rather evident from Fig. 23 if one compares bents of heights to base ratios of 10 and 4, respectively.

It has been found by a moment-distribution check that the 50-story portal design for strength only has reversed interior column reactions due to the disregard of elastic requirements in designing. On this account the chord deflection of this bent had to be calculated by a more general formula than Equation (39), and was found to be $0.00185\,h$, which is greater than in the portal elastic design where it was $0.00163\,h$. In order to have the bent under consideration deflect only $0.002\,h$ for fair comparison with the elastic designs, the girder contribution would have to be reduced to about $0.00009\,h$, since the columns will contribute about $0.00006\,h$. This would require ridiculously large girders, twenty times as stiff, and at least five times as heavy, as those represented in Fig. 23, exceeding even the portal elastic design in weight.

Allowable values for deflection should be established as in the case of stress in order to insure against the application of 20-story methods to 50-story towers. Deflection data from the authors' models would be enlightening.

A. W. Fischer,[32] Esq. (by letter).—A considerable mass of experimental data as to the values of the vertical reactions of the columns at the first story of three-bay structures with equal vertical heights, is presented in this paper, and the authors' results seem to agree very well with those obtained by mathematical analysis. Additional experiments should be made, however, to determine the ratio of the vertical reactions at other stories (especially at the top story) before a general conclusion can be made. The authors' state that for equal bays (all columns being of the same size), the cantilever relation can be obtained by making the moment of inertia of the center girder approximately twice that of the side girders, or that if the moment of inertia of the center girders was about one and one-third times that of the side girders, zero reaction would result for interior columns and would thus agree with the portal-theory distribution.

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[30] "Wind Bracing in Steel Buildings," Third Progress Rept. of Sub-Committee No. 31, Committee on Steel of the Structural Division on Wind-Bracing for Steel Buildings, *Proceedings*, Am. Soc. C. E., December, 1933, p. 1614.

[31] The Ohio State University experimental bent was designed to have 50% greater deflection in order to facilitate the detection of joint rotations (θ). (See Fig. 22.)

[32] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

The writer applied the slope deflection method to a two-story structure with sizes of girders and columns as given for the authors' Case A-1 and with a horizontal unit load of 1 lb applied at the top of the second story. In this manner it was found that $R_A = + 0.2976$ lb and $R_B = - 0.05065$ lb, which makes R_i for the first story equal to -17%, and this value agrees fairly well with the results reported in the paper. If more stories had been used the results would be changed somewhat, but still the value of R_A would have an opposite sign from R_B. For the second story, $R_i = - 21$ per cent.

The writer also applied the slope deflection method to a two-story structure with the sizes of girders and columns as given for Case A-17 and with a horizontal unit load of 1 lb applied at the top of the second story. For the first story, $R_A = + 0.14985$ lb, and $R_B = + 0.43935$ lb, which makes R_i for the first story equal to $+ 293$ per cent. This result does not agree so well with that of the paper. If more stories had been used, no doubt the results would change somewhat, but still the values of R_A and R_B would be of the same sign, as the authors' experiments show. For the second story, $R_i = + 243$ per cent.

With the so-called exact methods of analysis the sizes of the girders and columns must be known before the analysis can be started, especially in the case of the slope deflection method and the Cross method, and, therefore, the methods are rather tedious for the design of a new structure.

In his analysis of a three-span structure (equal spans) by relative deflections, Professor H. Yu[23] assumes that the ratio of the columns to the girders is 1.5, the story height being equal to 1.0 and the span length to $\frac{4}{3}$. For a new design, three bays wide and with equal story heights, Professor Yu's method is very flexible because, with the span and story height given, the ratio of the moment of inertia of the columns to that of the girders (the moment of inertia is the same for the columns in each story, but for different stories it will be different) can be selected so as to fit approximately the relation assumed by Professor Yu in deriving his general equations. This assumed relation cannot be adhered to exactly, but if the sizes of the girders at each floor are increased gradually from the top of the structure to the bottom, the final results will agree very closely with any of the results obtained by the so-called exact methods.

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[23] "Wind Stresses in Tall Buildings in Stresses in Statically Indeterminate Structures", by H. Yu, National Wuhan Univ., Wuchang, Hupeh, China, Second Edition, 1935, pp. 70 to 84.

Case A-1 of the paper agrees very closely with the conditions assumed by Professor Yu. If the moment of inertia of the girders were 1.25 times that of the columns, the size of members, the story heights, and the span lengths will agree with these assumed conditions.

Applying the general equations introduced by Professor Yu to the fifth story of a 19-story structure of three equal bays with a unit horizontal load at each floor, the value of R_A for the fifth story is equal to $-$ 29.1579 lb and R_B is equal to $+$ 2.945 lb, which makes R_i for the fifth story equal to $-$10.10% and this result agrees very closely with the experimental data as given for Case A-1. In all stories the analytical sign values of R_A and R_B are of opposite sign, which agrees with the experimental data as given for Case A-1 of the paper.

Francis P. Witmer,[24] M. Am. Soc. C. E., and **Harry H. Bonner,**[25] Esq. (by letter).—The writers are gratified at the satisfactory verification of their results on the part of several who have discussed this paper. It must be borne in mind that their intent was not to produce exact quantitative values, but only to determine trends of reaction ratios resulting from assumed variations in the design of the bents. Agreement with the writers' results is particularly noticeable in the two upper curves of Fig. 11, in the discussion by Professor Maugh, and also in the celluloid model tests of Mr. Morrison. Professor Tsai obtains very interesting relations for single-story bents, which are also confirmatory of the writers' results, and Mr. Fischer shows a similar agreement with theory.

The failure of total upward and downward reactions to be equal in some cases, as mentioned by Mr. Morrison, is not surprising, since actual readings were recorded in all cases without any attempt, arbitrarily, to correct them in order to produce the agreement demanded by statics. It is believed that, in most cases, the discrepancies will not be great. The model has since been improved by the use of positive metal clamps, tightened by screws, instead of the wooden joint blocks, and with these the equality of upward and downward reactions is much more nearly obtained in all tests. It is interesting to note, however, that, with these metal connections, repetition of tests previously made with wooden blocks produces reaction ratios in very close agreement with those first found, thus indicating a surprising degree of efficiency on the part of the wooden blocks.

The discussion by Professor Large and Mr. Carpenter is very interesting and to a considerable extent confirmatory. It is felt, however, that they have expected rather too great a degree of exactness in some cases where they find what they term "serious discrepancies." The extension of their study to cases with relatively heavy columns is not necessarily contradictory of the writers' results. In subsequent tests, the writers have found that, when all columns are increased in size, but are still kept equal to each other, the reaction ratio will be increased algebraically. Some results in this connection are indicated in Table 4. The last ratio, + 23%, for Case C-2 with heavier columns, compares well with + 20% for Case 11-C-2 in Table 3.

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Professor Large and Mr. Carpenter state that, in computing the stiffness of the members, reduced lengths of girders ("Spurr lengths") were used instead of center-to-center lengths, as they ordinarily "gave results nearer to experimental values." In their original report[17] they state that the use of "center-to-center lengths of members in computing stiffness corresponds more nearly to the actual performance so far as girder shears are concerned." These two statements appear to be contradictory. The improved metal connections used by the writers in their later tests approximate closely the center-to-center length relation, whereas the original wooden blocks caused a reduced length of all members. The close agreement in results obtained with the two types of connections made it appear that the reaction ratios were not greatly affected by the shortened lengths. If further model study could demonstrate that this is true, the use of center-to-center lengths would greatly simplify the determination of reactions. Of course, reduced lengths are necessary in computing moments in girders.

TABLE 4.—COMPARISON OF REACTION RATIOS

Case	Reaction Ratios, $\frac{R_t}{R_o}$	
Relative column areas	1	2
Relative moments of inertia	1	8
A – 1	−15%	0%
C – 1	+76%	+92%
C – 2	0%	+23%

The writers would not expect their conclusions to apply to relations not reasonably within the range comprehended in their tests. Considerable increase in the size of columns might naturally call for some modification. They believe, however, that the qualitative trends which they endeavored to determine have been well substantiated by the various discussions.

The reference by Professor Large and Mr. Carpenter to the impropriety of applying "20-story methods to 50-story towers", of course, is unquestioned, as is also the proper consideration of deflection in the case of tall towers. The writers' models, in some cases, had a height-to-width ratio as great as 8.7 to 1, and were thus properly in the tower class. The number of stories is not so significant as the height-to-width ratio. No attempt was made to determine whether considerations of deflection should demand one method of computation rather than another in order to maintain a maximum degree of economy. The purpose was to ascertain the facts as to how frames of certain assumed proportions will naturally behave and what kind of modification is necessary to compel them to behave in a certain predetermined manner. The question of economy consistent with strength and stiffness is a compelling one, of course, but the relations among these three essential factors were not considered as a part of this study.

With regard to Conclusion (1) of the paper, the writers have subsequently shown that, for any bent of the type under consideration, for which both girders and columns are of the proportions required for vertical floor loads (assuming girders to be non-continuous and neglecting effect of column distortion from direct wind stress), the following relations are generally true, regardless of the number of columns, the number and height of stories, the length of girders, and the values of horizontal forces at the different floors:

(1) Girder moments are equal at both ends of any girder;
(2) Girder moments are proportional to their length;
(3) Girder shears are equal in all girders of any floor; and
(4) Direct stresses and vertical reactions for all interior columns are equal to zero.

These conditions were practically fulfilled for two assumed cases, applying Equations (10) to (17), inclusive, in the discussion by Mr. Mensch.

Thus, assuming $L_0 = 1$; $L_i = 2$; and $h = 1$, Equations (12) to (16), inclusive, become:

$$M_1 = \frac{4\,P}{29} \tag{41a}$$

$$M_2 = \frac{3.5\,P}{29} \tag{41b}$$

$$M'_2 = \frac{7\,P}{29} \tag{41c}$$

$$R_A = \frac{7.5\,P}{29} \tag{41d}$$

and,

$$R_B = -\frac{0.5\,P}{29} \tag{41e}$$

Furthermore, assuming $L_0 = 3$; $L_i = 1$; and $h = 1$, the corresponding equations are:

$$M_1 = \frac{19\,P}{94} \tag{42a}$$

$$M_2 = \frac{21\,P}{94} \tag{42b}$$

$$M'_2 = \frac{7\,P}{94} \tag{42c}$$

$$R_A = \frac{13.3\,P}{94} \tag{42d}$$

and,

$$R_B = \frac{0.7\,P}{94} \tag{42e}$$

Relation (4) was also practically fulfilled by the second model test reported by Mr. Morrison, and by Case *A*-1*c* of the paper, although in proportioning the frame for this latter case the writers did not have in mind the fact that the areas of the columns were proportional to those required for vertical floor loads. In these two cases the moments of inertia of columns were not in proportion to their areas, but the resultant reaction ratio in each case was approximately zero, thus practically agreeing with the portal theory.

The probability of such relations as those in Equations (41) and (42) was indicated by experimental results and recorded in the Fifth Progress Report of Sub-Committee No. 31 of the Structural Division, on Wind-Bracing in Steel Buildings[26], the proof being given in a discussion of the report.[27]

These relations are in exact agreement with the fundamental portal theory. It is thus interesting to discover that this theory has the backing of mathematical proof in the case of wind action on bents proportioned for vertical floor loads in the manner described in the aforementioned report.[26] The economic aspects of this conclusion would seem to be of considerable importance.

It is further of interest to note that if girders are assumed to act continuously, the interior column loads will be relatively increased and the other loads decreased, but an allowance for weight of outer walls will increase outer column loads and thus tend to offset the continuous girder effect. Under these conditions the relation between column loads across the bent will tend to approximate that for vertical floor loads alone with non-continuous girders.

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[26] *Proceedings,* Am. Soc. C. E., March, 1936, pp. 410–411.

[27] *Loc. cit.,* November, 1936, p. 1496.

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MODERN CONCEPTIONS OF THE MECHANICS OF FLUID TURBULENCE

By Hunter Rouse,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs Chesley J. Posey, S. Franz Yasines, Benjamin Miller, Ralph W. Powell, Joe W. Johnson, Warren E. Wilson, Theodor von Kármán, and Hunter Rouse.

Synopsis

Throughout the past century, numerous efforts have been made to understand the problem of resistance in turbulent flow, but only in recent years has a satisfactory analytical basis been laid for such research. The purpose of this paper is to summarize and interpret the modern scientific methods of approach that have made such progress possible, using the subject of flow in circular pipes as the simplest and most pertinent means of illustration.

After a brief discussion of pipe resistance in terms of general dimensionless parameters, the physical nature of turbulence is described qualitatively; the inner mechanism of flow is then followed from the laminar stage through the critical zone and well up into the region of high Reynolds numbers for both smooth and rough pipes. The Prandtl-von Kármán theory of the mixing length, the significance of the laminar boundary layer with relation to surface roughness, the latest experiments on artificial roughness, and von Kármán's universal relationship between the velocity distribution and resistance to flow, are finally treated in terms of the foregoing discussion of the mechanics of fluid motion. The notation used in the paper is summarized in the Appendix.

Dimensional Analysis of Pipe Resistance

For many decades the complexities of turbulent motion in fluids remained so baffling to engineers and scientists alike, despite great progress in other

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Note.—Published in January, 1936, *Proceedings*.

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fields of mechanics, that until recent years little had been accomplished toward placing the study of turbulent phenomena upon an analytical basis. As a result, hydraulicians have been forced to rely to a great extent upon empirical relationships for the expression of energy loss due to turbulence; and as empiricism is at best a rather haphazard means to an end, such relationships in themselves are seldom likely to suggest the correct nature of underlying physical principles. Thus it is that the subject of hydraulics now finds itself somewhat hobbled in its thought by those very methods which contributed greatly to its prestige in past generations.

Hence, it is only logical that the first successful approach to the analysis of turbulence should have come from scientists endeavoring to further the development of a new engineering field. Aeronautics recently found itself confronted with a vast assortment of problems in fluid motion, not the least of which dealt with turbulent formations; and as the new profession was unhampered by tradition, it turned eagerly to the scientific world for much-needed assistance. Astonished as many engineers may have been who had learned to mistrust the ideal notions of classical hydrodynamics, a new and intensely practicable science of fluid mechanics has been developed to meet these needs.

Since turbulence is completely general in its nature, its study through the methods of fluid mechanics is fully as applicable to problems of hydraulics as to problems of airplane design. Those scientists who helped place aeronautics on a sound physical basis have also turned with unabated zeal to the field of pipe flow, and it is only to the advantage of hydraulic engineers that they give serious heed to recent progress in various parts of the world. As the purpose of this paper is to summarize and interpret the most important of these advances in their application to pipe resistance, no attempt has been made at a proper historical treatment; however, abundant references are given in the several texts and papers listed in the footnotes, of which probably the most lucid and complete account is that presented by L. Schiller[2], in 1932.

The first great forward strides were made by Osborne Reynolds in the early Eighties of the Nineteenth Century through his investigations of laminar and turbulent resistance. Although these contributions to the science were forgotten for many years, his discovery of the dimensionless parameter which now bears his name has given the world perhaps its most valuable means of flow analysis. The Reynolds number is one of three dimensionless parameters applicable to the general movement of fluids in which the factor of compressibility—or elasticity—may be ignored[3]. The quality of these numbers is such that any two geometrically similar systems of flow are also dynamically similar if the corresponding dimensionless numbers, named for Froude, Reynolds, and Weber, are numerically equal in each case. Dynamic similarity, in turn, is that condition which prevails if all corresponding forces acting in the two systems are in the same ratio to each other as the inertial qualities of the existing state of motion. That is, according to the Newtonian principle of momentum, the action of any force upon a given mass of matter will produce upon that mass a certain rate of change of momentum:

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[2] "Strömung in Rohren". Handbuch der Experimentalphysik, Band IV, 4. Teil, Akademische Verlagsgesellschaft m. b. H., Leipzig, 1932.

[3] "The Analytical Approach to Experimental Hydraulics—On the Use of Dimensional Numbers", by H. Rouse, *Civil Engineering*, November, 1934.

$$F = \frac{d(m\,v)}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which F denotes the active force and $\frac{d(m\,v)}{dt}$, the equal and opposite inertial reaction of the mass. In the motion of any fluid, in which the change of density is negligible, only three basic and independent forces—namely, those due to gravity, viscosity, and surface tension—may act to produce such a rate of change of momentum; of these, only the action of viscosity may influence the character of flow entirely enclosed by pipe walls, so that the other two play no further part in this paper. In order for flow through any two geometrically similar pipes of uniform diameter to be dynamically similar, all corresponding viscous forces in the two systems must then bear the same relation to each other as do the inertial characteristics of flow in the two pipes; according to the Newtonian equation, these inertial characteristics must be dimensionally equivalent to the right-hand side of the equation—that is, instead of rate of change of momentum, this inertial term characteristic of uniform flow is the rate of passage of momentum, or the momentum of a unit volume of fluid multiplied by the volume passing a representative cross-section per unit of time. Expressing this inertial quality by the symbol, I, it becomes, in general dimensions of density, velocity, length, and time:

$$I = \rho\, V \frac{L^3}{t} = L^2\, \rho\, V^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Similarly, any viscous force, since the intensity of viscous shear equals $\mu \frac{dv}{dy}$, must have the following dimensional character:

$$F_v = L^2\, \mu \frac{V}{L} = L\, \mu\, V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The essential equality of force and inertial ratios,

$$\frac{F_{v1}}{F_{v2}} = \frac{I_1}{I_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

then becomes, through substitution of Equations (2) and (3):

$$\frac{L_1\, \mu_1\, V_1}{L_2\, \mu_2\, V_2} = \frac{L^2_1\, \rho_1\, V^2_1}{L^2_2\, \rho_2\, V^2_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and,

$$\frac{L_1\, V_1\, \rho_1}{\mu_1} = \frac{L_2\, V_2\, \rho_2}{\mu_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Thus, in order for dynamic similarity to obtain between the two geometrically similar systems of flow, it is essential that the dimensionless quantity, $\frac{L\, V\, \rho}{\mu}$, be identical numerically in both systems. This quantity, the Reynolds number, is usually given the symbol, **R**:

$$\mathbf{R} = \frac{L\,V\,\rho}{\mu} \quad \text{.........................(7)}$$

The length parameter, L, may have any value, characteristic of the flow, which the investigator may care to select, so long as he is consistent in its use throughout the investigation. Obviously, the characteristic length which he selects will determine the final numerical magnitude of the Reynolds number for a given set of conditions; otherwise, the number will be fixed, regardless of whether metric, English, or any other units are used. For purposes of uniformity it is customary, in pipe studies, to use the diameter of the pipe as the length factor.

The Reynolds number is thus seen to represent in simplified form the ratio between the inertial and the viscous properties of flow. If the rate of passage of momentum is increased (by increasing the velocity, density, or pipe diameter), the Reynolds number will grow in magnitude; if the viscosity is increased, the Reynolds number will diminish. As will be shown directly, the larger the value of **R**, the larger will be the tendency toward instability of flow; conversely, the smaller the value of the Reynolds number, the larger will be the stabilizing effect of viscosity. The magnitude of this number is then a direct indication of the flow characteristics, once a basic value has been established for purposes of comparison. Furthermore, if **R** has the same numerical value for movement in any two geometrically similar pipes, one may be certain that general conditions of flow in both pipes are identical, regardless of wide variation in diameter, velocity, density, and viscosity between the two systems. It is customary to write the Reynolds number in the form, $\mathbf{R} = \frac{V\,D}{\nu}$; one must remember, however, that the kinematic viscosity, $\nu_4 = \frac{\mu}{\rho}$, is the ratio of two independent fluid properties, each of which must be regarded as distinct from the other.

The motion of a fluid passing through a pipe is resisted by a certain tangential stress between the fluid and the pipe walls. Denoting this wall stress per unit area by the customary symbol, τ_0, it is reasonable to presume that the magnitude of the resistance will be a function of pipe diameter, density, viscosity, and average velocity; wall roughness will be omitted for the time being, on the assumption that the pipe is very smooth; that is,

$$\tau_0 = \phi\,(D, \rho, \mu, V) \quad \text{.....................(8)}$$

or, assuming for the present that the function is exponential:

$$\tau_0 = C\;D^{x}\;\rho^{y}\;\mu^{z}\;V^{n} \quad \text{.....................(9)}$$

Through the principle of dimensional analysis, the form of this function may be found by expressing the terms of Equation (9) in general M-L-T units (M = mass, L = length, and T = time):

$$\frac{M}{L\,T^2} = (L)^x \left(\frac{M}{L^3}\right)^y \left(\frac{M}{L\,T}\right)^z \left(\frac{L}{T}\right)^n$$

The three unknown exponents, x, y, and z, are found in terms of the fourth unknown exponent, n, by the solution of three simultaneous equations, thus:

For M,

$$1 = y + z$$

for L,

$$-1 = x - 3\,y - z + n$$

and for T,

$$-2 = -z - n$$

Hence: $x = n - 2$; $y = n - 1$; and $z = 2 - n$. By substituting these values for x, y, and z in Equation (9),

$$\tau_0 = C \times D^{n-2} \times \rho^{n-1} \times \mu^{2-n} \times V^n \ldots\ldots\ldots\ldots\ldots (10)$$

which may also be written in the form,

$$\tau_0 = \frac{C'}{\left(\frac{V\,D\,\rho}{\mu}\right)^{2-n}}\,\rho\,\frac{V^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In Equation (11) the first fraction is seen to be a numerical constant, C', divided by the Reynolds number to the power, $(2 - n)$. This fraction is equal to the customary resistance coefficient, f, divided by the factor, 4; that is,

$$\frac{f}{4} = \frac{C'}{\mathbf{R}^{2-n}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and, consequently, Equation (11) becomes:

$$\tau_0 = \frac{f}{4}\,\rho\,\frac{V^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Inasmuch as the resistance exerted upon any portion of the moving fluid must be counterbalanced by a drop in pressure intensity from one section to the next, the pressure gradient for steady flow in a horizontal pipe of uniform diameter may be written in terms of the total shearing force along the surface of the pipe between the two normal sections:

$$(p_1 - p_2)\,\frac{\pi\,D^2}{4} = \tau_0\,\pi\,D\,L \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and,

$$-\frac{d\,p}{d\,L} = \frac{4\,\tau_0}{D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

from which the familiar equation for lost head in pipes may be derived, through substitution of Equation (13):

$$-\frac{d\,p}{d\,L} = \frac{f}{D}\,\rho\,\frac{V^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and,

$$-\frac{\Delta p}{\gamma} = -\Delta h = \frac{f L}{D} \frac{V^2}{2 g} \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

If the pipe slopes, a further change in pressure intensity from one section to the next is essential to balance the longitudinal component of the fluid weight between the two normal sections. It will be clear from Fig. 1, however, that this occasions no change in the foregoing relationship between wall shear and that portion of the pressure drop with which it is in equilibrium; hence the assumption of a horizontal pipe merely eliminates two equal and opposite force components from the equation. That is, it makes no difference how steeply the pipe slopes, or whether the flow is the result of pumping or of direct feed from a reservoir; the state of flow at a given section is completely defined once the Reynolds number is known, and is independent of the magnitude of the static pressure load upon the system.

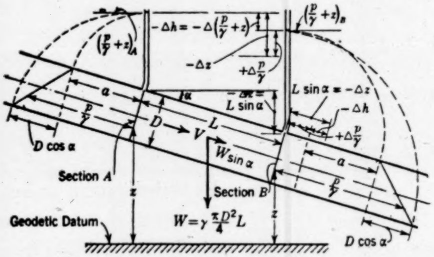
$-\Delta h = -\Delta\left(\frac{p}{\gamma}+z\right)$
$\left(\frac{p}{\gamma}+z\right)_B$
$\left(\frac{p}{\gamma}+z\right)_A$
$-\Delta z$
$+\Delta\frac{p}{\gamma}$
α
$-\Delta z = L \sin\alpha$
$L \sin\alpha = -\Delta z$
$-\Delta h$
$+\Delta\frac{p}{\gamma}$
a
$\frac{p}{\gamma}$
D
V
L
D cos α
$W_{\sin\alpha}$
Section A
Section B
z
z
a
$\frac{p}{\gamma}$
Geodetic Datum
$W=\gamma\frac{\pi D^2}{4}L$
D cos α

FIG. 1.

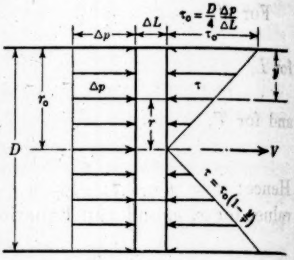
$\tau_0 = \frac{D}{4}\frac{\Delta p}{\Delta L}$
ΔL
Δp
τ_0
Δp
τ
r_0
r
y
D
V
$\tau = \tau_0\left(1-\frac{y}{r}\right)$

FIG. 2.

General use of Equation (17) is to be avoided, therefore, unless one is completely familiar with its significance. Involving as it does the specific weight of the fluid (note the open manometer columns in Fig. 1, in a system presumed to be entirely confined by pipe walls), this again brings into the picture the force of gravity, which has just been shown to have no influence upon the flow itself. Unfortunately, the slope of the hydraulic gradient is a function not only of the Reynolds number, but of the Froude number as well—and only if both numbers are the same, respectively, will the grade lines of two different systems be geometrically similar. Since the Froude number refers to flow with a free surface, and actually has no relationship whatever with the inner mechanism of confined flow, no further mention of specific weight will be made in this paper, and Equations (13) and (16) will be considered the basic expressions for pipe resistance.

In developing Equation (15) the assumption is made that the pressure intensity varies statically across any normal section of the pipe. That this must be true will be seen from the fact that neither shear nor acceleration can occur as a continuous process in a direction normal to the pipe axis, for the general movement of the fluid must be in the longitudinal direction. Hence, the only variation in pressure intensity along a pipe diameter must be that necessary to compensate for the change in elevation from one point to the next; that is, the pressure distribution must be static. Thus, the difference in pressure per unit length for any cylinder of fluid within the pipe (see Fig. 2) must vary directly with the square of the radius, r, of the cylinder. Since this difference in pressure must be exactly counterbalanced by the shear along the outer surface of the fluid cylinder, Equation (14) may be written in terms of any radius, r:

$$(p_1 - p_2)\,\pi\, r^2 = 2\,\pi\, r\, L\,\tau \qquad (18)$$

Equation (15) will then become:

$$-\frac{d\,p}{d\,L} = \frac{4\,\tau_0}{D} = \frac{2\,\tau}{r} \qquad (19)$$

Expressing the pipe diameter, D, as twice the radius, r_0,

$$\tau = \tau_0\,\frac{r}{r_0} = \tau_0\left(1 - \frac{y}{r_0}\right) \qquad (20)$$

In other words, the intensity of shear between adjacent layers of fluid co-axial with the pipe wall will vary directly with the distance from the axis of the pipe, as plotted in Fig. 2. It is upon this very general relationship, which holds for laminar and turbulent flow in either smooth or rough pipes, that further analysis of pipe resistance is based.

If the flow is laminar (that is, if the shearing stress is purely viscous, according to the relationship, $\tau = \mu\,\frac{dv}{dy}$), means are now at hand for determining the velocity distribution in terms of the wall shear. Introducing the relationship,

$$\tau = \mu\,\frac{dv}{dy} = -\,\mu\,\frac{dv}{dr} \qquad (21)$$

in Equation (20),

$$dv = -\,\frac{\tau_0}{\mu\, r_0}\, r\, dr \qquad (22)$$

and integrating,

$$v = -\,\frac{1}{2}\,\frac{\tau_0}{\mu\, r_0}\, r^2 + C \qquad (23)$$

The value of the constant of integration may be found through use of the fact that when $r = r_0$, $v = 0$; that is, the velocity at the pipe wall is zero. Under these conditions,

$$C = \frac{1}{2}\,\frac{\tau_0\, r_0}{\mu} \qquad (24)$$

and the equation for velocity distribution then becomes:

$$v = \frac{1}{2}\,\frac{\tau_0}{\mu}\left(\frac{r_0^2 - r^2}{r_0}\right) \qquad (25)$$

Inspection will show that the distribution curve is parabolic, attaining its maximum value at the center, where,

$$v_{\max} = \frac{1}{2}\,\frac{\tau_0 r_0}{\mu} \qquad (26)$$

Since the volume of a paraboloid of revolution is one-half that of a circumscribing cylinder, the mean velocity must be one-half the maximum:

$$V = \frac{1}{2}\, v_{\max} = \frac{1}{4}\,\frac{\tau_0\, r_0}{\mu} \quad \text{.................... (27)}$$

and,

$$\tau_0 = \frac{8\,\mu\, V}{D} \quad \text{.......................... (28)}$$

Through substitution in Equation (15),

$$-\frac{d\,p}{d\,L} = \frac{32\,\mu\, V}{D^2} \quad \text{.......................... (29)}$$

Similarly, substitution of Equation (28) in Equation (13) yields the fundamental dimensionless relationship for laminar flow,

$$f = \frac{64}{\mathbf{R}} \quad \text{............................. (30)}$$

in which case the exponent, n, must have a value of unity. The problem of laminar flow is thus fully solvable mathematically.

Abundant experimental data exist to vindicate these expressions. For instance, a logarithmic plot of τ_0 against V, for a given pipe and a given fluid, results in a single straight line with a slope of unity for the entire laminar range. Once turbulent flow has begun, however, the slope of the line changes (see Fig. 3) from unity to a value varying from $n = 1.75$ for extremely smooth pipes to a maximum of $n = 2$ for pipes that are very rough. Aside from this elementary although highly instructive knowledge of the significance of the exponent, n, such a plot is of little aid in drawing general conclusions about the relationships between various systems of flow. The curve is not applicable to more than the one pipe and the one fluid for which it was determined; nor is it of use in other dimensional systems without bothersome revision of scale.

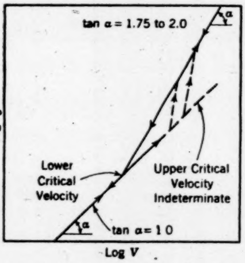

FIG. 3.

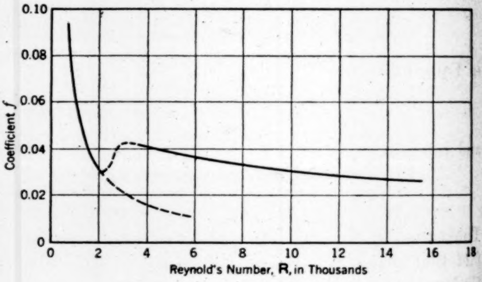

FIG. 4.

Those engineers who have followed the publications of laboratories in which the metric system is used will readily admit the superiority of dimensionless co-ordinates, if only from a utilitarian point of view. Moreover, the use of dimensionless parameters often enables the investigator to discover very general principles which otherwise would be obscured in the maze of experimental data plotted in dimensional form. For instance, Equation (16) may be rewritten in dimensionless form through solving for f and combining with Equation (12):

$$f = -2 \frac{dp}{dL} \frac{D}{\rho V^2} = \frac{C''}{\mathsf{R}^{2-n}} \qquad (31)$$

This expression for f as a function of the Reynolds number, another dimensionless quantity, at once suggests the use of these two flow characteristics as ordinates and abscissa for plotting on a single dimensionless graph the variation of the coefficient over the widest possible range of pipe diameter, velocity, viscosity, and density. The form of such a curve for smooth pipes (Fig. 4) is probably already quite familiar to the majority of engineers, for its efficacy in co-ordinating the results of many investigators is undeniable.

Despite the many points in favor of this method, engineers are wont to protest that although such a curve is excellent for very smooth pipes, almost no pipes used in engineering projects are smooth enough to fall within this class. Moreover, the few attempts that have been made to bring commercial rough pipes into such a general relationship have not met with complete success[4]. As a result, the treatment of roughness by engineers has of necessity remained almost purely empirical.

In order to answer such an argument emphatically, it is only necessary to cite the progress that has been made in the present century in this very attempt at generalization. Although the results are quite well known, so far as smooth pipes are concerned, it should prove encouraging to the Engineering Profession to know that progress in the study of rough pipes is continuing successfully at a fully equivalent rate. Research of the present day is emphasizing more and more the general nature of the problem, in an attempt to determine universal relationships which will include every type of pipe that is known, under all possible conditions. It is with the present status of this research that the remainder of this paper will deal.

PHYSICAL CHARACTERISTICS OF TURBULENCE

Attention has already been called to the fact that the Reynolds number is a direct indication of the relationship between the inertial qualities of the flow, which tend toward instability, and the viscous qualities, which have a stabilizing influence. Reynolds' original visual experiments to determine the upper critical velocity—that point at which laminar motion changes to turbulent motion—were splendid illustrations of the growth of instability. Reynolds' equipment consisted of a large tank of still liquid, from which led a glass tube with well-rounded entrance, at the end of which was a valve to regulate the discharge. By introducing a thin filament of dye at the entrance of the tube during discharge, the condition of the flow could be observed by eye over the entire length of the liquid column. So long as the flow remained laminar, the ribbon of color continued as a straight line without diffusing; but the moment the velocity passed above the critical, the ribbon began to waver and finally to diffuse over the entire cross-section.

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[4] "Elemente der technischen Hydromechanik", von R. von Mises, I. Teil, B. G. Teubner, Leipzig und Berlin, 1914; "Die Messung der hydraulischen Rauhigkeit", von L. Hopf, *Zeitschrift für angewandte Mathematik und Mechanik*, Band 3, S. 329, 1923; and "A Study of the Data on the Flow of Fluids in Pipes", by E. Kemler, *Transactions*, A. S. M. E., Hydraulics Div., August 31, 1933.

At the point of initial wavering of the colored filament Reynolds found a mean value of R ranging from 12 000 to 14 000. It is very important to note that later experimenters managed to increase this value of the Reynolds number by improving the entrance, eliminating all vibration, and allowing the liquid in the tank to stand for a much longer period—for several days, in fact. With Reynolds' original equipment, for instance, a value of more than 40 000 has been obtained[5]. The obvious conclusion to be drawn from these experiments is that for perfectly smooth pipes the limiting upper critical point is indeterminate (see Fig. 3), for it depends largely upon the care with which the experiments are conducted. In every case the initial turbulence was a result of some irregularity of flow that was too minute to be discovered beforehand, yet which gradually upset the equilibrium within the fluid.

On the other hand, below a fairly well established point called the lower critical velocity—the point of change from turbulent to laminar flow—the viscous forces are sufficiently large to quell any disturbances that may be present and thus prevent general instability of the flow. Above this lower critical point, which has been almost universally accepted to be about $\mathsf{R} = 2\,000$, the slightest disturbance in the oncoming flow is enough to cause a rapid growth of that irregularity of motion called turbulence.

The word "turbulence" has its distinct meaning to those scientists who are now responsible for its analytical investigation, yet it has various independent and often conflicting connotations in the engineering world. To the practical hydraulician engaged in irrigation and power development, for instance, the word instantly suggests the picture of large swirls and eddies which have no relation whatever in his mind with the tiny irregularities of flow in Reynolds' glass tube. Because of this apparently ambiguous usage of the word, the writer submits the following brief picturization of the turbulent phenomenon.

On the small scale of Reynolds' experiments, or on the large scale of the hydraulician's practical experience, fully developed turbulence is basically the same. Superimposed upon the translatory movement of the fluid is a complex internal agitation of the fluid mass, differing from the kinetic motion of gaseous molecules, however, in that finite portions of the fluid rather than infinitesimal particles are involved in the mixing process. Although the action is apparently fully disordered, a statistical treatment of the conditions at any point in a "steady" flow will yield a temporal mean value of the velocity which is invariable so long as the general conditions of flow remain unchanged.

In addition to this temporal mean, the statistical method introduces the conception of instantaneous velocity fluctuations at a given point, as components to be added to the temporal mean velocity measured at the point in question; that is, at any instant there may be superimposed upon the translatory movement of the fluid past that point either positive or negative velocity components in each of the three co-ordinate directions. Thus, there is a variable movement of small fluid masses in a direction normal to the general flow, these masses carrying into other regions increments of momentum or energy from the region from which they came—as a result, the turbulent mixing process involves a constant transfer of energy among the various portions of the flow.

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[5] "On the Change from Steady to Turbulent Motion of Liquids," by V. W. Ekman, *Arkiv für matematik, astronomi och fysik*, 6, Nr. 12, 1911.

Although this statistical interpretation of the turbulence phenomenon is of advantage in the mathematical treatment of energy distribution, only to the mathematicians can it offer a sufficiently complete picture of the internal fluid mechanism. In order to broaden the viewpoint, recourse is often taken to the concept of vortex motion as the root of fluid turbulence. The vortex —briefly, a fluid mass rotating about a central axis and inducing a velocity field in the surrounding fluid medium—is often the source of turbulence, and its rotational motion is fairly typical of the swirls and eddies in a moving stream[6]; yet while turbulent agitation might be thought of as an intricate system of vortices, large and small—some spinning violently and some very slowly, and each influencing the velocity of the neighboring fluid—such a schematic representation gives somewhat too ideal an impression of true turbulence.

Perhaps the best picture is a combination of the two extremes, in which from the statistical representation is taken the notion of continuous—but not abrupt—fluctuation at a given point, and from the vortex conception the visualization of irregular rotational movement of small masses of the fluid in addition to the general movement of the stream. As illustrative of the final picture, imagine dense smoke clouds issuing from a chimney; the motion is obviously changing from instant to instant, but it exhibits on the other hand a very typical sequence of rolling billows of smoke, the general movement of which is by no means the individual helter-skelter of tiny particles of soot; that is, one must not draw from the statistical method the conclusion that each fluid particle describes its own irregular path irrespective of the motion of its neighbors; instead, the behavior of every particle is completely dependent upon that of the others in its immediate vicinity. Free flight and resulting collision between gaseous molecules, furthermore, is not the basic movement in turbulent agitation of fluid masses; nor may the notion of molecular impact, or even of intermittent impact of fluid masses, be considered generally typical of turbulence. On the contrary, the swirling masses are always in complete contact with neighboring masses, and the consequent energy losses are entirely viscous in nature as a result of the variable velocity gradient across any section of the agitated fluid.

A fact commonly ignored at the present time is the existence of considerable energy of turbulence in a moving fluid. The familiar Bernoulli theorem includes only a single kinetic term, velocity head, which is computed from the mean velocity of flow over the entire cross-section. Needless to say, the existence of velocity fluctuations indicates further kinetic energy in the fluid, which, unlike the components of fluctuation, has by no means a temporal mean value of zero. This energy of turbulence is perhaps best illustrated by referring once again to the notion of vortices superimposed upon the general flow; however much the individual vortices may be distorted in the actual flow pattern, it is apparent that each single vortex represents a certain amount of rotational energy in addition to its kinetic energy of translation.

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[6] Refer to the photographs at the end of the Engineering Societies Monograph, "Applied Hydro- and Aeromechanics", by Prandtl-Tietjens, McGraw-Hill, 1934.

The hydraulic engineer has long been familiar with such turbulent energy, under perhaps a different name. The so-called "energy loss" in the hydraulic jump, at "stilling" racks, in tumble bays, at pipe valves, and in various other devices of a similar nature, is really a conversion from the energy of flow embodied in the three terms of the Bernoulli theorem into energy of turbulence. That this transformation is not directly into heat is shown by the fact that the resulting turbulence, of however fine a type (that just after a stilling rack, for instance) is still apparent quite some distance down stream. Eventually, of course, viscous action produced by the turbulent mixing process accomplishes the mechanical transformation into heat, but this is a gradual and not an abrupt process; consider, as illustration, the time necessary to still the water in Reynolds' experiments, or the distance down stream the spiral effect of elbows makes itself known. To be sure, such energy of turbulence is lost to the engineer, for it can never be regained in useful form; but as turbulent energy it can still play insidious havoc as it proceeds down the pipe or channel. C. Camichel[7], for instance, produced artificial turbulence in normal laminar motion by placing a screen in the path of the flow. The resistance then followed the turbulent law until the stabilizing viscous action had restored the flow to its original character some distance down stream. Thus, the presence of abnormal turbulence can change completely the general characteristics of flow.

The causes of turbulence are several in number, a clear understanding of which would depend upon a finer perception of the physics of fluid motion than can be presented in this paper[8]. Mention has already been made of the steady growth of any small irregularity of the flow at Reynolds numbers above the critical, resulting eventually in fully developed turbulence. Furthermore, under certain conditions, roughness of the boundary surface may produce a turbulent "wake" in much the same fashion as will the motion through the air of a vehicle that has not been stream-lined; thus, the action of wall roughness may serve to augment the normal turbulence of the flow. In the latter case the action at the wall will be in effect a constant transformation of potential energy into turbulent energy, which produces as a result an apparent shearing stress between wall and fluid.

Many investigations to determine the "friction factor" for different types of pipe have been of little direct value to the Engineering Profession because insufficient effort was made to distinguish between actual wall resistance and induced resistance caused by turbulence resulting from valves, pipe-joints, or by too great proximity of the measured section to entrance or exit.

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[7] "Notice sur les Travaux Scientifiques", Toulouse, 1929.

[8] See, for instance, "Entstehung der Turbulenz", von L. Prandtl, *Zeitschrift für angewandte Mathematik und Mechanik*, Band 11, 1931.

Although the total loss in commercial piping is of direct interest to the designing engineer, it is futile to attempt an analysis of each separate influence if these influences cannot be isolated one at a time. Hence all mention of normal pipe turbulence in this paper refers only to that fully developed turbulent state which will result from the interaction between the fluid itself and the walls of a uniform pipe. It is possible experimentally to make joints of such a nature that they will have no appreciable effect upon the flow, and it has been found that a distance of about 50 pipe diameters from a symmetrical entrance (short elbows or valves may cause so great a disturbance as to require a longer approach section) will be sufficient to permit the full development of normal turbulence.

RESISTANCE AS A FUNCTION OF THE REYNOLDS NUMBER

According to the principle of dynamic similarity, which has had sufficient experimental justification to warrant the complete confidence of both the scientist and the engineer, any single Reynolds number denotes a very definite state of turbulence. Although the flow picture may change from instant to instant because of the complexity of the vortex pattern superimposed upon the general translatory movement, a dimensionless temporal average of velocity conditions at any given section of flow will yield identical results for a given Reynolds number, regardless of fluid properties and pipe diameter. At the same time, it must be remembered that dynamic similarity is dependent upon geometrical similarity, which signifies that the roughness must play a definite part in similitude. Despite the prevalent custom of distinguishing between smooth and rough pipes, it is obvious that roughness is purely a relative characteristic, for even the smoothest surface is rough under the microscope. Recent experimental studies have very clearly demonstrated this fact.

As a general example for the following discussion, assume a pipe that is relatively smooth, remembering that its walls still possess finite surface irregularities. As a given fluid, whether it is a liquid or a gas, flows at a very low Reynolds number through this pipe, the velocity will vary parabolically from zero at the wall to a maximum at the center of the pipe. The fact that the wall velocity is zero is purely a viscous—not an adhesive—effect; for mercury, which does not "wet" the wall, will have just the same flow characteristics as air or water[9].

As the discharge through this pipe is increased, the velocity distribution will remain parabolic, varying only in scale; the shearing stress at the wall will increase in direct proportion to the average velocity (one-half the maximum velocity), as will also the pressure gradient in the direction of flow. Hence, for the given fluid and the given pipe, the frictional factor, f, will be in inverse ratio to the average velocity. Generally speaking, for any fluid and any pipe, so long as the flow is laminar, this friction factor is equal to $f = \frac{64}{\mathsf{R}}$. This condition holds until the critical zone is reached (where the

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[9] "Unsere Kenntnis der Zähigkeit von Quecksilber", von S. Erk, *Zeitschrift für Physik*, Bd. 47, 1928.

Reynolds number is approximately 2 000), at which point the viscous forces are no longer sufficiently great to counterbalance completely the inertial tendency toward instability. Since these viscous stresses are greatest near the walls of the pipe where the velocity gradient attains its highest value, and lowest in the central regions where the velocity gradient approaches zero, it is reasonable to presume that the first signs of instability will appear at some distance from the wall; this presumption may be vindicated experimentally. That wall roughness (when not excessive) is not the cause of the initial turbulence, will be seen from the following discussion of the boundary layer.

Within the critical zone (from R = 2 000 to about R = 3 000), under normal conditions a state of complete equilibrium is difficult to produce. The initial turbulence is so closely dependent upon slight disturbances within the pipe (sharp entrance, elbows, a valve, etc.), that it is difficult to distinguish a definite transition curve among the numerous plots of data that exist. Generalized transition curves (after Schiller) are shown in Fig. 5 in which the initial disturbance is extremely far away (Fig. 5(*a*)), a short distance away (Fig. 5(*b*)), and, finally, very close to the measured section (Fig. 5(*c*)). Curves marked I represent a large disturbance and II, a small disturbance. Obviously, the smaller and more remote the existing disturbances, the higher the value of R at which the change takes place.

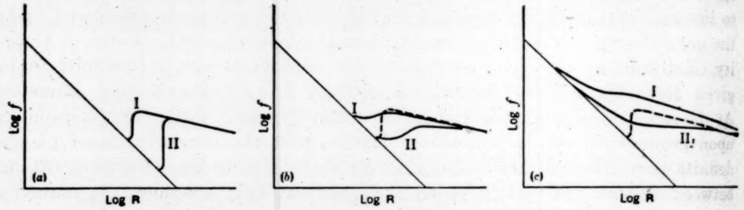

FIG. 5.—GENERALIZED TRANSITION CURVES.

However, once the critical zone has been passed, the curve of *f* against R assumes a very definite trend. The earliest measurements of lasting importance are those by Augustus V. Saph, Assoc. M. Am. Soc. C. E., and Ernest W. Schoder, M. Am. Soc. C. E.[10]—the true value of which was realized only after analysis by H. Blasius[11], When plotted by the latter in the now customary form of log *f* against log R, all results in the turbulent region for the relatively smooth (drawn brass) pipes of different diameters lay very close to a straight line with the slope, $-\frac{1}{4}$ (see Fig. 6). Hence, *f* may be written as inversely proportional to the fourth root of R, or, as determined by Blasius:

$$f = \frac{C''}{R^{(2-n)}} = \frac{0.3164}{R^{\frac{1}{4}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

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[10] *Transactions*, Am. Soc. C. E., Vol. LI (1903), p. 253.

[11] "Das Aehnlichkeitsgesetz bei Reibungsvorgängen in Flüssigkeiten", *Forschungsarbeiten* auf dem Gebiete des Ingenieurwesens, 131, 1913.

This relationship thus corresponds to the exponential form—for a given pipe and a given fluid—of $\tau_0 \propto V^{1.75}$. The logical conclusion to be drawn from this apparently simple result was that all very smooth pipes have an identical effect upon fluid motion, regardless of the degree of turbulence, and that such flow may always be expressed as an exponential function in which $n = 1.75$. (It is to be noted that Reynolds had previously determined this value as 1.732, within surprisingly close range of that of Blasius.)

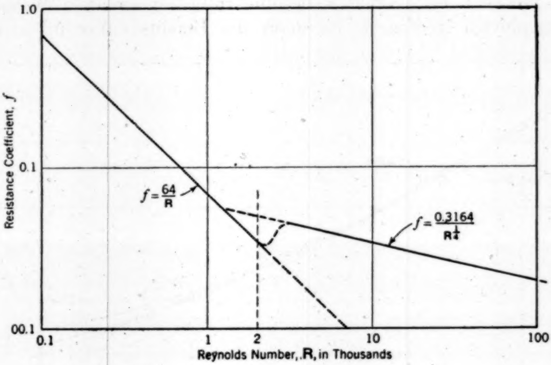

FIG. 6.

Nevertheless, research by Messrs. T. E. Stanton and J. R. Pannell[12] soon exceeded the range covered by the curve of Blasius (up to about $R = 105\,000$) by attaining a maximum Reynolds number greater than 400 000, using both air and water. Based upon these results, C. H. Lees[13] developed the following formula, which differs from that of Blasius in the upper range:

$$f = 0.00714 + \frac{0.6104}{R^{0.35}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

About fifteen years later, R. Hermann, after reaching a Reynolds number of nearly 2 000 000, in which range both the Blasius and the Lees formulas were found to deviate from experimental results, proposed the more accurate empirical relationship[14]:

$$f = 0.0054 + \frac{0.396}{R^{0.3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

This limit has since been surpassed by J. Nikuradse[15], who attained a value of $R = 3\,240\,000$. These investigations have proved conclusively that the magnitude of n is not constant even for very smooth pipes, but that beyond the Blasius range ($R = 105\,000$) it increases slowly with increasing values of **R**. Thus, flow through an absolutely smooth pipe—which may never be realized—would probably become less and less dependent upon the Reynolds number as the latter increased, attaining complete independence ($n = 2$), however, only when **R** became infinitely great. Measurements by Nikuradse, plotted in Fig. 7, lie upon the Blasius curve up to a Reynolds number of about 100 000, beyond which point they follow well the curve of Schiller-Hermann. For reasons which will be explained subsequently, Nikuradse has proposed the relationship,

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[12] "Similarity of Motion in Relation to the Surface Friction of Fluids", *Philosophical Transactions*, Royal Soc. of London, (A) 214, 1914.

[13] *Proceedings*, Royal Soc. of London, (A) 91, 46, 1915.

[14] See "Rohrwiderstand bei hohen Reynoldsschen Zahlen", von L. Schiller, Aachener Vorträge, S. 69, Springer, Berlin, 1930.

[15] "Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren", V.D.I. *Forschungsheft* 356, 1932.

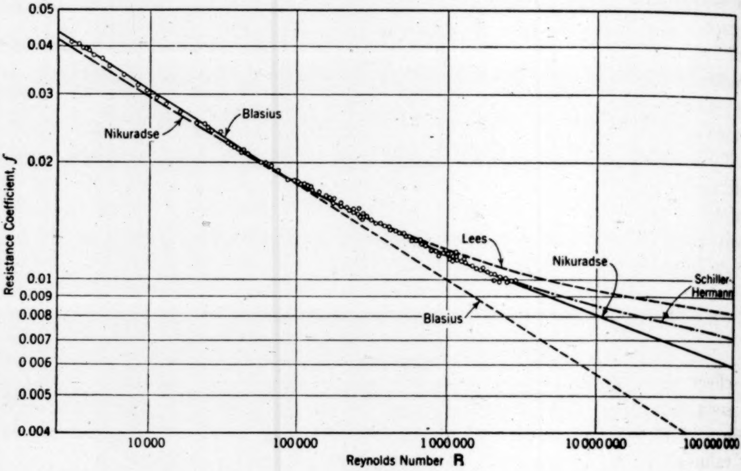

FIG. 7.

$$f = 0.0032 + \frac{0.221}{\mathbf{R}^{0.237}} \quad \text{.....................(35)}$$

which may be expected to apply to Reynolds numbers as high as 10 000 000. For purposes of comparison, the extrapolated curves of Blasius, Lees, Schiller-Hermann, and Nikuradse are all shown in Fig. 7.

Whereas in laminar flow the resistance was seen to be directly proportional to the average velocity, the turbulent state entails far greater resistance than such a function would provide, since the stress at the walls is now proportional to the average velocity raised to a power varying between 1.75 and 2. It is at once obvious that a redistribution of energy must occur within the flow to cause this change in resistance, since a complete parabolic curve of velocity will no longer provide a sufficiently high value of τ_0. Attention has already been called to the fact that turbulence within the moving body of fluid, schematically illustrated by innumerable vortices superimposed upon the translatory movement, causes a continuous transfer of fluid matter between neighboring layers. Thus, fluid masses from the central regions, possessing a higher velocity in the direction of flow, are whirled into regions in which the velocity is lower, and *vice versa*. In other words, there is a steady passage of energy from the inner regions of flow in the direction of the pipe walls, resulting in a more uniform velocity distribution over the central portion of the pipe. Hence, the temporal mean velocity curve in turbulent flow, as compared with that of laminar flow, is much flatter at the pipe axis, and accordingly much steeper near the walls (see Fig. 8(*a*)). Obviously, the greater the degree of turbulent mixing (that is, the higher the Reynolds number) the flatter the curve will become.

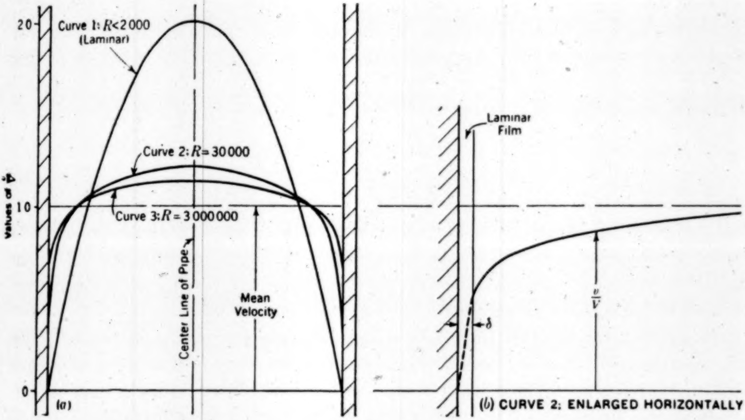

FIG. 8.—VELOCITY DISTRIBUTION IN A VERY SMOOTH PIPE.

Careful measurements of the velocity distribution have long seemed to indicate that the velocity at the wall is by no means zero, as in laminar flow. However, continued improvements in the design of Pitot tubes, enabling a closer approach to the wall, have lowered repeatedly the apparent wall velocity. Despite this indication of a finite velocity at the boundary, analytical investigators have always claimed that the velocity curve must pass through zero at a smooth wall. Based upon this hypothesis, Prandtl developed the theory of the laminar boundary film to account for the high shearing stress at the wall, at which point there is obviously no further turbulent interchange of momentum.

This theory of the boundary layer assumes not only that the wall velocity is zero, but also that the layer of fluid next to the wall contains only a laminar movement of the fluid particles. Since the velocity gradient here will be high —that is, of such magnitude as to provide the shearing stress required by the existing conditions of flow ($\tau_0 = \mu \frac{dv}{dy}$)—the error in assuming a linear velocity distribution, rather than a parabolic distribution, will be quite negligible (see Fig. 8(*b*)). It is evident that the boundary layer must become increasingly thin with ascending values of **R**, so that when the Reynolds number is high a rough idea of its thickness may be found from the following simple approximation. The shearing stress at the wall for turbulent flow may be written: $\tau_0 = f\,\rho\,\frac{V^2}{8}$ (see Equation (13)). This stress must be equivalent to the viscous stress at the wall caused by the velocity gradient, which is assumed to be linear over the small normal distance, δ, denoting the thickness of the laminar film. As an arbitrary value for the velocity at the edge of this film may be taken the apparent wall velocity as noted in measurements. Needless to say, the magnitude of this value is open to debate, since the ratio between the apparent wall velocity, v_w, and the average velocity of flow, V, increases with increasing Reynolds number and decreases with improved methods of measurement. Expressing the foregoing equality in terms of this arbitrary ratio: $\tau_0 = f\,\rho\,\frac{V^2}{8} = \mu\,\frac{dv}{dy} = \mu\,\frac{v_w}{\delta} = \mu\,\frac{v_w}{V}\,\frac{V}{\delta}$, and,

$$\delta = 8\,\frac{v_w}{V}\,\frac{\mu}{f V \rho} = 8\,\frac{v_w}{V}\,\frac{D}{f\,\mathbf{R}} \qquad \text{(36)}$$

For purposes of illustration, assuming the ratio of apparent wall velocity to mean velocity as about 0.5, the boundary film in a smooth pipe 1 ft in diameter, carrying water at an average velocity of 10 ft per sec at a temperature of 60° F ($\mathbf{R} = 840\,000$) would then be about 0.0004 ft.

In order to verify this theory, Stanton conducted a very painstaking series of experiments[16] with a special Pitot tube of rectangular cross-section, one side of which was formed by the pipe wall itself. Since this side remained stationary, movement of the outer side of the tube away from the pipe wall simply increased the size of the opening and changed the position of its center. The tube was rated first in laminar flow, in which the velocity distribution was definitely known, these corrections then being applied to measurements in the neighborhood of the wall under turbulent conditions. Nikuradse has found similar values through use of Pitot tubes of several different diameters[15], thus determining the error as a function of tube diameter; by extrapolation, the correct velocity (as would be measured with a tube of zero diameter) was readily available. The results of these experiments served as a striking verification of the theory of the laminar boundary film, as have also more recent tests conducted in wind tunnels of large diameter.

It is erroneous, of course, to consider the laminar film as adjacent to the turbulent region, yet sharply divided from it. Actually there is only an arbitrary limit to the boundary layer, for a transition zone must exist between the two regions, in which zone the motion is neither laminar nor completely turbulent. Thus, while within the laminar film the resistance is purely viscous, it is not until well within the turbulent region (where the velocity gradient is low) that the action of viscous shear finally becomes secondary.

It is now possible to draw a valuable comparison between the laminar and the turbulent régimes. In the former case, the laminar boundary film extends to the very center of the pipe; as the critical zone is reached, this laminar portion of the flow decreases rapidly in radial dimension, and then continues to diminish gradually with increasing values of the Reynolds number. The thickness, δ, becomes infinitesimal, if the pipe is extremely smooth, only as n approaches the limit 2 (that is, as $\mathsf{R} \rightarrow \infty$). The previous statement that initial turbulence in relatively smooth pipes does not occur at the walls now becomes clear, since the wall region, except in very rough pipes, is the last stronghold of laminar conditions.

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[16] "On the Conditions at the Boundary of a Fluid in Turbulent Motion", by T. E. Stanton, D. Marshall, and C. N. Bryant, *Proceedings*, Royal Soc. of London, (A) 97, 1920.

With the realization that a laminar boundary layer actually exists even in turbulent flow, a comprehension of the part roughness plays in influencing turbulent resistance becomes greatly simplified. Roughness may be described in part by comparing with the radius, r_0, the average distance, ϵ, over which the individual projections extend into the flow section from the pipe wall; thus, the term "relative roughness" signifies the ratio of ϵ to the pipe radius, $\frac{\epsilon}{r_0}$. Obviously, this ratio approaches—but never attains—the limit zero in the case of extremely smooth pipes, and on the other hand in rare instances may approach unity for badly corroded surfaces in which the pipe has become almost completely choked through age; the former class comprises very smooth commercial piping (glass, drawn brass, and lead), but the latter case is so extreme as to make its inclusion in such a paper quite reasonless. In other words, normal relative roughness of pipes can be assumed to vary from nearly zero to perhaps one-fifteenth.

With this partial definition of roughness, it now is possible to subdivide the turbulent region into two distinct zones, separated by a region of transition. It has often been proved that in the laminar régime slight surface roughness has no effect whatever upon either velocity distribution or wall resistance. Similarly, it may be reasoned that so long as the laminar boundary film is of such thickness as to cover all roughness protuberances at the wall, such roughness will continue to have no effect upon the conditions of flow. Thus it is that smooth pipes—despite the fact that they are actually rough to a certain degree—appear to follow a different resistance law from that of commercial rough pipes, for the relative roughness is so slight that the unevennesses are still enveloped in the thin laminar film; yet many pipes, ordinarily classed as smooth, behave as such only for comparatively low Reynolds numbers, since the relative roughness is such that the boundary layer at high Reynolds numbers is not sufficiently thick to cover the roughness particles. Other pipes, in which the relative roughness is high, at no time follow the curve of resistance for smooth pipes beyond the critical zone, because the thickness of the boundary film is never great enough to include all roughness particles in a continuous enveloping layer.

The data plotted by Kemler, previously cited[4], are not in general sufficiently systematic to warrant more than broad statements as to the nature of the absolute action of roughness in the sense already defined. Two conclusions are of great importance, however. One, long since recognized, is that once roughness extends inside the laminar film, the resistance to flow is greatly increased; the second is that roughness may not be defined completely by a ratio of radial dimension to pipe radius, for the plots of certain rough pipes show complete independence of R, whereas others run approximately parallel to the minimum curve for smooth pipes (Fig. 9).

The conditions which must exist for dynamic similarity, already stated in this paper, include first that of strict geometrical similarity. Thus, in order for flow through any two pipes to be dynamically similar, not only must the Reynolds numbers be identical, but the entire physical character of the inside of one pipe must be similar geometrically to the other; yet this means that not only the relative roughness, but also the general form and spacing of the irregularities themselves, must be the same in each case. Obviously, the relative roughness of rivets inside a penstock may equal the relative roughness of rust tubercles or that of unevennesses in commercial asphalt-lined pipe; yet there is no reason whatever to suppose that the action of all such forms of roughness should be the same. As a matter of fact, a surface may be further defined (refer to Hopf[4]) by distinguishing between waviness and actual jagged roughness; whereas jagged roughness that breaks the continuity of the laminar film results in a constant resistance coefficient for all values of R, the waviness (corrugated, asphalt, or galvanized lining) serves to increase the resistance in such a way that it is still a function of R. Of course, it is not always possible to differentiate between the two forms, as the wall material in some cases may be a combination of both, the resistance curves then lying between the two extremes. Typical plots may be seen in Fig. 9.

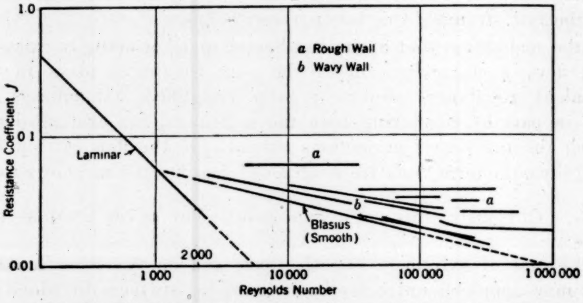

FIG. 9.—TYPICAL RESISTANCE CURVES FOR COMMERCIAL PIPE.

As a suggested explanation of the difference in behavior between rough and wavy surfaces, the writer offers the following: When bodies of very angular profile are tested in a wind tunnel, their resistance coefficients are found to be nearly independent of the Reynolds number characterizing the flow; the turbulent wake behind such a body is then determined in size largely by the form and position of the outermost irregularities of the profile. As the angles of the body are softened, approaching the ideal stream-line form, a profile is reached for which the turbulent wake is smaller and not of constant form, actually decreasing in size with increasing Reynolds number, the resistance factor thereby dropping appreciably in magnitude. This phenomenon is explained by the fact that an increase in velocity decreases the thickness of the laminar film surrounding the front of the body to such an extent that the point of separation (now a function of velocity distribution along the softened curves rather than of abrupt angularity) moves toward the rear of the body. In the completely stream-lined profile, separation and the formation of a wake are obviated. Whereas the body of irregular profile corresponds to the projections on a very rough wall and the stream-line profile typifies the smooth wall, there may be irregularities in pipe surface between the two extremes which exhibit much the same variation in turbulent "wake" as the transition profile in the foregoing illustration.

What is considered by several authorities to be a more scientific viewpoint than the notion of relative thickness of laminar film and roughness particles is the following: Every roughness protuberance may be considered to act as an individual body, to which a lower critical Reynolds number may be assigned in much the same way as is done for the plate or sphere; this number is composed of the dimension, ϵ, the kinematic viscosity of the fluid, and the velocity existing at the outermost part of the roughness projection. The latter may be computed on the basis of a linear velocity gradient in the boundary layer, with a velocity of zero at the wall itself. Thus, when the critical number for the projection in question is exceeded, separation occurs, with a resulting turbulent wake. Although it is doubtful whether roughness particles may behave in such an independent fashion unless their spacing is relatively great, several investigators have been able to compute, with fair approximation, the point at which roughness projections of known magnitude will begin to produce wall turbulence.

A great advantage of this viewpoint is the clarity it lends to the action of roughness in the laminar régime. Although the fact has already been mentioned that slight roughness has no influence in purely laminar flow, it is still possible that the local critical velocity for projections of large magnitude may be reached shortly before the critical value of R for the entire pipe. Although such individual disturbances may be dampened out before affecting the general flow, it is obvious that roughness of large magnitude may provide the initial disturbance leading to fully developed turbulence in the critical zone.

It has been stated that the resistance in the case of either roughness or waviness is a function not only of relative roughness, but also of the form and spacing of the individual particles. Not only is this complex general function still to be determined, but it is also difficult to classify the wide variety of commercial pipes accurately without further knowledge of the function itself. Such knowledge cannot come from the plentiful existing data on commercial pipes, because of the lack of systematic investigation already mentioned. However, a commendable start has been made by Nikuradse[11] and others in the investigation of controlled, artificial roughness. By means of uniform sand grains attached to the pipe wall with a thin coat of varnish, Nikuradse studied not only the similarity of flow with different pipe diameters and a constant relative roughness, but also the variation of f with roughness of six different relative magnitudes, over a wide range of the Reynolds number. So systematic was his investigation that the flow behavior in all the different zones discussed previously was clearly demonstrated. Reference to the plot (Fig. 10) taken from Nikuradse's experiments will show distinctly

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[11] **"Strömungsgesetze in rauhen Rohren", V. D. I., *Forschungsheft* 361, 1933.**

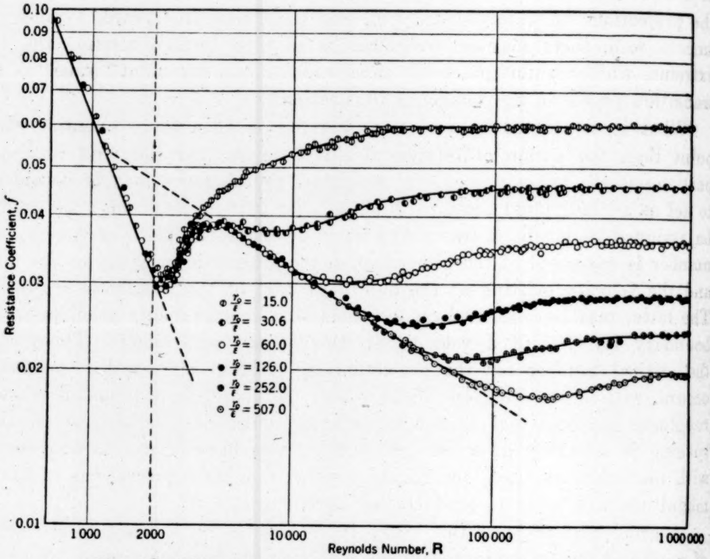

FIG. 10.—MEASUREMENTS ON ARTIFICIALLY ROUGHENED PIPE.

the regions in which the roughness particles were enveloped by the laminar film, the flow then being similar to that through a very smooth pipe. Just as distinct is the transition region, wherein the roughness particles begin to disrupt the boundary layer. Finally, Fig. 10 shows the zone in which the continuous laminar film has disappeared entirely, the friction coefficient then being dependent upon the relative roughness alone. For the last of the three zones Nikuradse has found the relationship:

$$f = \frac{1}{\left(2 \log_{10} \frac{r_0}{\epsilon} + 1.74\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

It is to be noted that Equation (37) may be expected to apply generally only in the event that the roughness in question is geometrically similar to that of the sand coatings in Nikuradse's investigation. Any departure in the form of roughness will most certainly require different constants in this relationship, and in some cases (waviness) even a different form of equation.

Although, to a certain extent, Fig. 10 shows the systematic variation of the coefficient with a change in relative roughness, Nikuradse has indicated their very close relationship by plotting them in such a way that all points in the third region (where f is independent of R) are superposed. As Equation (37) may be changed to read,

$$\frac{1}{\sqrt{f}} - 2\log_{10}\frac{r_0}{\epsilon} = 1.74 \text{ ....} (38)$$

which is simply a statement of the constant relationship between f and $\frac{\epsilon}{r_0}$ in this range, the use of the left-hand side of this equation as ordinates for all three regions would result in a series of points which might be expected to coincide only in the region of constant f. As abscissa was selected the quantity, $\sqrt{\frac{\tau_0}{\rho}}\ \frac{\epsilon}{\nu}$, which indicates as a dimensionless number of the Reynolds type the flow characteristics in the vicinity of the wall; that is, $\sqrt{\frac{\tau_0}{\rho}}$ has the dimension of a velocity (see Equation (13)) while ϵ is the appropriate length parameter.

A glance at this plot as reproduced in Fig. 11 will indicate at once that within the range of experimental error the resistance coefficient will lie upon a single dimensionless curve, regardless of relative roughness, so long as the roughness is of a similar nature. It is with the investigation of other types of roughness in this systematic fashion that laboratories are now engaged, with the ultimate goal of determining the proper functional relationship for major types of commercial pipe.

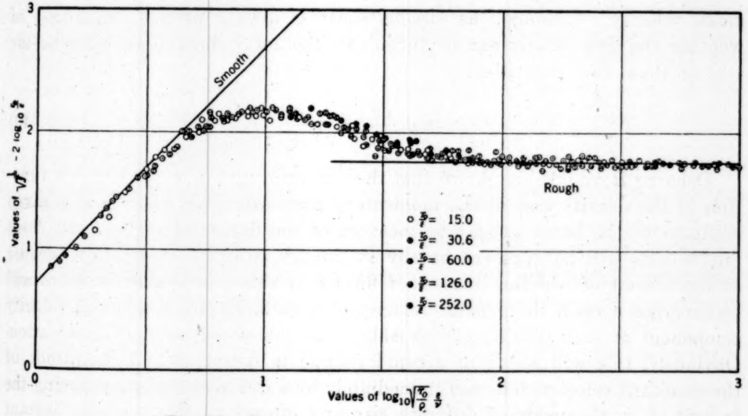

FIG. 11.—COMPOSITE PLOT OF CURVES IN FIG. 10.

VELOCITY DISTRIBUTION AND THE MIXING LENGTH

Notwithstanding the fact that numerous attempts have been made during the past hundred years to formularize the velocity distribution in turbulent flow, very few expressions have been derived which apply correctly to much more than the limited range of Reynolds numbers for which they were developed. Moreover, nearly all such empirical attempts have included only the central portions of the flow, disregarding entirely the region in the immediate neighborhood of the wall. By far the most scientific approach to the problem of energy distribution in turbulent flow is that proceeding from a general relationship developed by J. Boussinesq[18]; since the shearing stress at any point in purely laminar motion may be expressed as the product of the fluid viscosity and the velocity gradient (Equation (21)), Boussinesq reasoned that the intensity of shear in purely turbulent motion may be written in a closely parallel form,

$$\tau = \eta \frac{dv}{dy} \quad \ldots\ldots\ldots\ldots (39)$$

In Equation (39), however, the factor, η, unlike μ, is not a constant for a given fluid, for it expresses the dynamic quality of the turbulent process at a given point; η will thus depend not only upon the fluid density, but upon the general state of flow and upon the point in the flow which is being considered. The gradient, $\frac{dv}{dy}$, as shown by Boussinesq, will now be that of the temporal mean velocity. Inasmuch as viscous shear is always present, regardless of whether the flow is laminar or turbulent, the total shear must then be the sum of these two quantities:

$$\tau = \mu \frac{dv}{dy} + \eta \frac{dv}{dy} \quad \ldots\ldots\ldots\ldots (40)$$

Osborne Reynolds has shown that the last term may be expressed as a function of the density and of the momentary fluctuations in velocity at a given point, since the latter are a true measure of the degree of turbulence. Such fluctuations will be apparent at any stationary point of observation because of the transit of whirling masses of fluid past this point; that is, there will be superposed upon the general velocity of translation the tangential velocity component of each portion of the eddy as it passes the point of observation. Obviously, this will result in a rapid change in direction and magnitude of the resultant velocity, followed immediately by a different variation during the transit of another eddy of different size and intensity. Thus, at any instant there must be added, vectorially, to the temporal mean velocity, v, at the point in question the velocity of fluctuation, v', which varies constantly in magnitude and direction, having two essential components, v'_x, parallel to, and v'_y, normal to, the pipe wall; the third component, normal to the other two, is of no consequence in this development. It is evident that both v'_x and v'_y must yield a temporal mean value of zero; the temporal mean of the product of the two essential components, $\overline{v'_x v'_y}$, however, will always have a finite value, except at the pipe wall and pipe axis, because of a certain correlation which exists between the two components in the normal turbulent pattern.

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[18] "Essai sur la théorie des eaux courantes", Mémoires présentés par divers savants a l'Académie des Sciences de l'Institut de France, Tome 23, 1877.

As a result of these velocity fluctuations, at any instant through a small increment of area, Δa, normal to the pipe radius, there will be a rate of flow equal in magnitude to $\Delta a\, v'_y$. A positive magnitude of this value denotes an instantaneous rate of flow toward the pipe axis—that is, into a region where the velocity is higher—and *vice versa*. Because of this movement into a region of different velocity, at that instant there is being transported into the neighboring region of flow either a positive or a negative increment of momentum. The component of this increment in the direction of average flow is proportional to the density, ρ, and to the velocity fluctuation in this direction, v'_x. The rate of passage in the y-direction of this axial component of the increment of momentum is then given by the quantity, $-\Delta a\, v'_y\, \rho\, v'_x$, which has the dimension of a force. The negative sign signifies that fluid moving in the negative y-direction (toward the wall) into a region of lower velocity exerts a positive force. Hence, the force per unit of area, or the apparent shearing stress due to turbulence, must have a temporal mean value of $-\rho\, \overline{v'_x\, v'_y}$, and,

$$\tau = \mu \frac{dv}{dy} - \rho\, \overline{v'_x\, v'_y} \qquad \text{.....................(41)}$$

Since viscous shear within a given fluid must depend entirely upon the velocity gradient, it will be evident that such viscous action has two quite different functions: Owing to the presence of innumerable eddies, the local velocity gradient at a given section may be quite high across each of these eddies at the instant of transit, the curve of instantaneous velocity distribution then differing considerably from its temporal mean form. The function of this relatively intense local shear is to dissipate the energy of turbulence as rapidly as it forms, for only through viscous action can energy of flow finally be converted into heat. In addition to such local stress within the eddy itself, however, there must be a continuous viscous resistance to general translatory movement through the pipe, the latter resistance being proportional to the gradient of the temporal mean velocity curve. Obviously, the local dissipation of energy of turbulence is of secondary importance in this discussion, however vital the rôle it must play in maintaining the essential equilibrium of the fully developed turbulent state.

It so happens, moreover, that the magnitude of the viscous resistance to the general translatory motion becomes, at high Reynolds numbers, of inappreciable magnitude in comparison with that resulting from turbulent transfer of momentum. This will be seen from Fig. 12, in which are plotted, against dimensionless co-ordinates, data found by Nikuradse for smooth pipes at Reynolds numbers ranging from slightly above the critical to a maximum of more than 3 000 000. Not only does the slope of the velocity curve in the turbulent portion of the flow decrease with increasing Reynolds number, but the total resistance as evidenced by the slope at the wall becomes ever greater as the average viscous shear in the central regions diminishes. This trend is shown in a somewhat different form by Fig. 13, in which is plotted the ratio of the average velocity to the maximum velocity as a function of the

Reynolds number, based upon the results obtained by various investigators for smooth pipes; as is readily apparent from Fig. 13, the higher the values of R, the more nearly equal the average and maximum velocities become.

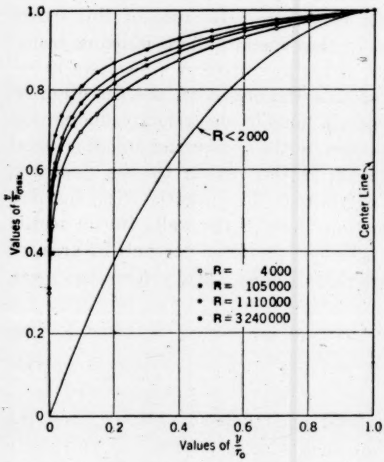

FIG. 12.—VELOCITY CURVES FOR SMOOTH PIPES.

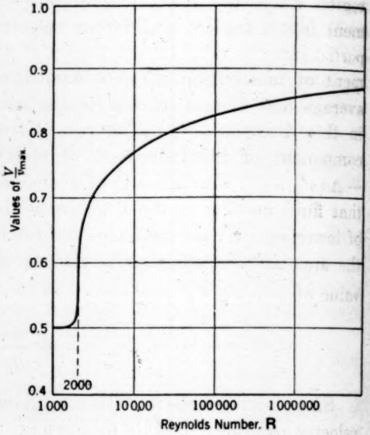

FIG. 13.—RATIO OF MEAN TO MAXIMUM VELOCITY AS A FUNCTION OF R.

Experimental results show that beyond the range of the Blasius equation for smooth pipes the term for average viscous shear may be omitted entirely without introducing noticeable error, the region of predominant viscous influence being restricted entirely to the vicinity of the pipe wall. In the case of rough pipes in the range of constant f, the turbulent action is so excessive that viscous resistance as a function of the mean velocity gradient is completely negligible in all parts of the flow. Thus, the Boussinesq concept of purely turbulent resistance in the central regions of flow is practically realized throughout the realm of high Reynolds numbers for smooth and rough pipes alike.

Before general use can be made of the Reynolds relationship for shear, it is necessary to find a means of expressing the quantity, $\overline{v'_x\ v'_y}$, in terms of known characteristics of flow. Prandtl made the first essential step in this direction by assuming a certain mixing length—analogous to the mean free path in the kinetic theory of gases—to denote the average transverse distance, in any region of the flow, over which a small fluid mass is carried by the turbulent mixing process. It then follows that the difference in average velocity between the beginning and the end of this distance (approximately equal to the product of the mixing length and the velocity gradient, $l\,\frac{dv}{dy}$) must be a measure of the component of fluctuation, v'_x, in the axial direction; furthermore, the existence of correlation between v'_x and v'_y denotes a general proportionality between these two components; that is, the difference between the original average velocity of the particle and that of the region into which it comes must be proportional to the magnitude of the velocity fluctuations involved in this transverse motion. This proportionality may be written: $l \dfrac{dv}{dy} \propto v'_x \propto v'_y$, and $\tau \propto \rho\, l^2 \left(\dfrac{dv}{dy}\right)^2$. By changing this proportionality to an equality, l will absorb the proportionality factor, and will then be completely defined:

$$\tau = \rho\, l^2 \left(\frac{dv}{dy}\right)^2 \quad \text{........................(42)}$$

At this point it is interesting to note that Boussinesq's coefficient of turbulence, η, through division by the density, ρ, will take on dimensional characteristics similar to those of the kinematic viscosity, ν, the ratio then denoting the kinematic characteristics of the turbulent process in a manner quite independent of the fluid properties. By combining Equations (39) and (42), there results the following significant expression for this kinematic parameter:

$$\frac{\eta}{\rho} = l^2 \frac{dv}{dy} \quad \text{..........................(43)}$$

Since the shearing stress at any point within the flow is inversely proportional to the distance from the wall (Equation (20)), Equation (42) then becomes:

$$\tau_0 \left(1 - \frac{y}{r_0}\right) = \rho\, l^2 \left(\frac{dv}{dy}\right)^2 \quad \text{..................(44)}$$

It is to be noted that Equations (42) and (44) do not include viscous shear; hence, they cannot be expected to apply to those conditions of flow in which viscous resistance is appreciable.

Nikuradse has computed the magnitude of l from measured values of wall resistance and measured curves of velocity gradient for smooth pipes[15] over a great range of R, and for rough pipes[17] with various values of $\dfrac{\epsilon}{r_0}$ within the region of constant f; these values are plotted in dimensionless form in Fig. 14. In every case the mixing length has a value of zero at the wall (actually at the edge of the boundary layer, for at this point there is no further velocity fluctuation), rising to a maximum at the center. As is to be expected, the computed mixing length for smaller values of R is greater in magnitude than is actually the case, since Equation (44) ignores the viscous influence, which is still appreciable in this range of R. Beyond the limit of $\mathsf{R} = 105\,000$, however, l is practically independent of R, apparently depending only upon the relative distance from the wall. Comparison with the single curve drawn through the points for rough pipes would seem to indicate that this function is invariable, regardless of the magnitude of the relative roughness.

Thus, although viscous shear still plays an important rôle throughout the moving fluid even well past the critical zone, influencing the internal mechanism of motion (and, hence, the velocity distribution and wall resistance) this general influence decreases steadily with rising values of R, until finally a limit is reached beyond which viscous resistance to flow is completely negligible throughout the greater portion of the pipe, being restricted to a relatively thin boundary region. Furthermore, experiments indicate that the variation of the mixing length for smooth pipes beyond this limit is identical with that for rough pipes in a corresponding range of R, showing that the internal mechanism of flow must be quite similar in all such cases.

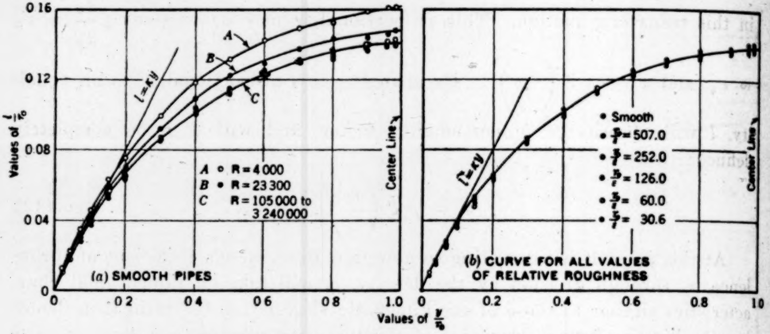

FIG. 14.—DIMENSIONLESS PLOT OF THE MIXING LENGTH.

UNIVERSAL CHARACTERISTICS OF TURBULENT FLOW

Through a statistical analysis of the velocity fluctuations at any point within turbulent movement past a fixed surface, Theodor von Kármán[19], M. Am. Soc. C. E., has concluded that (where viscous resistance to flow is negligible) there must exist a certain correlation between the components, v'_x and v'_y, regardless of the distance from this surface (excluding, of course, the boundary region). Without venturing into a detailed discussion of the theory of correlations, the following reasoning will serve to illustrate this relationship: Particles of fluid moving away from the wall into a region of higher velocity as the result of a positive v'_y-fluctuation will have an average velocity, v, smaller than that of this new region. Since this difference will appear as a negative v'_x-fluctuation at this point, it is obvious that positive values of v'_y are generally associated with negative values of v'_x and *vice versa.* Experiments made in the United States and in Europe with a hot wire anemometer capable of recording rapid fluctuations have verified von Kármán's conclusion that correlation must exist, these investigations indicating a constant correlation factor from the wall region almost to the center of the pipe.

As was shown herein under the heading "Velocity Distribution and the Mixing Length", there exists experimental evidence that the ratio of the mixing length to the pipe radius is a definite function of relative distance from the wall, and of no other characteristic of the flow. Hence, there appears to be a certain relationship between the turbulent patterns in any two pipes, regardless of pipe diameter, relative roughness, velocity, and fluid properties. As pointed out by von Kármán, the constant correlation of the components of velocity fluctuation indicates, furthermore, a similarity of the turbulent mechanism over the larger part of the cross-section of any given pipe; that is, not only should corresponding regions in two pipes show similar movement, but flow in any region in the one pipe should be similar to that in any region in the other pipe, the turbulent pattern differing only in time and length scales. (The variation in the length scale, for instance, may be typified by the variation of the mixing length, while the time scale is associated with the ratio, $\frac{dv}{dy}$.) Moreover, because of this similarity of the turbulent pattern, the conditions at any point of any flow may be expressed entirely in terms of their variation with y in the immediate vicinity of the point in question.

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[19] "Mechanische Aehnlichkeit und Turbulenz", *Göttinger Nachrichten*, Math. Phys. Kl., 1930; see, also, by the same author, "Turbulence and Skin Friction", *Journal of the Aeronautical Sciences*, January, 1984.

von Kármán has thus expressed the mean velocity, v, at a given point in terms of its variation with increments of y through expansion in a series, omitting, however, all terms after the second derivative. Then writing a general stream function in terms of this velocity series and introducing the proper time and length scales, by means of certain principles of classical hydrodynamics he has deduced the following relationship for the mixing length in terms of a universal constant:

$$l = \kappa \frac{\frac{dv}{dy}}{\frac{d^2v}{dy^2}} \dots\dots\dots\dots\dots\dots\dots\dots (45)$$

The merit of this expression lies in the fact that the value of l may now be written for any point in the turbulent region without regard for distance from the wall or center line of the pipe. It is to be noted that A. Betz[30] has developed an identical equation in a different manner, working in terms of the spacing and rotational speed of an ideal group of vortices superimposed upon the general translatory motion of the fluid. This fact is quite significant in correlating the statistical method with the physical picture of the turbulent mixing process.

A simple schematic digest of the complex mathematical reasoning leading to Equation (45) is given in the following development presented by B. A. Bakhmeteff, M. Am. Soc. C. E., in his lectures on fluid mechanics. The mean velocity, v, at any two points of the flow, expressed in terms of variation with increments of y according to Taylor's theorem, will be as follows:

$$v = \phi\,(y + \Delta y)$$

$$v_{[y_1+\Delta y]} = v_1 + \frac{dv_1}{dy}\,\Delta y + \frac{d^2v_1}{d\,y^2}\,\frac{\overline{\Delta y}^2}{2} + \frac{d^3v_1}{dy^3}\,\frac{\overline{\Delta y}^3}{6} + \dots$$

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[30] "Die v. Kármánsche Aehnlichkeitsüberlegung für turbulente Vorgänge in physikalischer Auffassung", *Zeitschrift für angewandte Mathematik und Mechanik*, Band 11, 1931.

and,

$$v_{[y_2+\Delta y]} = v_2 + \frac{dv_2}{dy}\,\Delta\,y + \frac{d^2v_2}{dy^2}\,\frac{\overline{\Delta y}^2}{2} + \frac{d^3v_2}{dy^3}\,\frac{\overline{\Delta y}^3}{6} + \cdots$$

Owing to the similarity of the flow pattern at Points 1 and 2, the following proportionality must exist between corresponding derivatives of the expansion:

$$\frac{\dfrac{dv_1}{dy}}{\dfrac{dv_2}{dy}} \propto \frac{\dfrac{d^2v_1}{dy^2}}{\dfrac{d^2v_2}{dy^2}}\,;\quad \frac{\dfrac{d^2v_1}{dy^2}}{\dfrac{d^2v_2}{dy^2}} \propto \frac{\dfrac{d^3v_1}{dy^3}}{\dfrac{d^3v_2}{dy^3}}\,;\ \text{etc.}$$

Omitting all terms of higher order than the second, this proportionality may then be written:

$$\frac{\dfrac{dv_1}{dy}}{\dfrac{d^2v_1}{dy^2}} \propto \frac{\dfrac{dv_2}{dy}}{\dfrac{d^2v_2}{dy^2}}$$

Obviously, each ratio has the dimension of a length, which means only that the magnitude of the ratio varies directly with the length scale of the pattern; that is, the mixing length is equal to a universal constant times the ratio of the first and second derivatives of the velocity with respect to y, as in Equation (45). In the wall region where y is very small, increments of y will be of the same order as y itself; in this region the mixing length will vary directly with y, as may be seen from Fig. 14; thus $l = \kappa y$.

The magnitude of the universal constant has been found experimentally to lie between the limits of 0.36 and slightly more than 0.40, the small discrepancy resulting to some extent from unavoidable experimental error. Variation within such narrow limits is surely sufficient reason to recognize the existence of a universal characteristic of the flow.

Darcy was probably the first to notice the resulting similarity between curves of velocity distribution in turbulent flow. Such similarity is strikingly displayed in the group of curves shown in Fig. 15(a), taken from measurements by Nikuradse. Each of these curves represents a different magnitude of relative roughness, the similarity arising from the fact that the dimensionless ordinates have been reduced to a common denominator, $\sqrt{\frac{\tau_0}{\rho}}$. The physical significance of this method of plotting is as follows: Equation (13) may be rewritten in the form:

$$\sqrt{\frac{\tau_0}{\rho}} = V\sqrt{\frac{f}{8}} \text{ and } \frac{1}{\sqrt{f}} \propto \frac{V}{\sqrt{\frac{\tau_0}{\rho}}}$$

in which the term, $\sqrt{\frac{\tau_0}{\rho}}$, has the dimension of a velocity and is known in fluid mechanics as the "friction velocity". The foregoing proportionality shows that the dimensionless ratio of mean velocity to friction velocity will vary inversely with the square root of f, regardless of whether the wall resistance is caused by viscous shear in the laminar film of smooth pipes or by the turbulent "drag" along the walls of rough pipes. In the case of smooth pipes this ratio will increase with decreasing values of f; referred to a common friction velocity, this would mean that the central portion of the distribution curve would merely shift in one direction or the other, the velocity gradient at the walls thereby remaining constant. In the case of rough pipes in the range of constant f, this ratio would of necessity be constant for any given relative roughness, but would decrease with increasing values of $\frac{\epsilon}{r_0}$.

Furthermore, the use of the friction velocity to indicate average conditions at once establishes a similar common denominator for all portions of the flow: Mention has already been made of the fact that since the pressure is statically distributed over the pipe cross-section, the intensity of shear must be a linear function of relative distance from the wall (see Fig. 15(b)). In flow at high Reynolds numbers, in which viscous resistance in the central portion of the flow is negligible, the shearing stress must be due entirely to the turbulent exchange of momentum—and this turbulent exchange of momentum at the same time determines the velocity distribution in all regions except the immediate vicinity of the wall. Thus, regardless of what the distribution curve may be in the wall region, the ratio of velocity to friction velocity must display a similar variation over the entire central region of

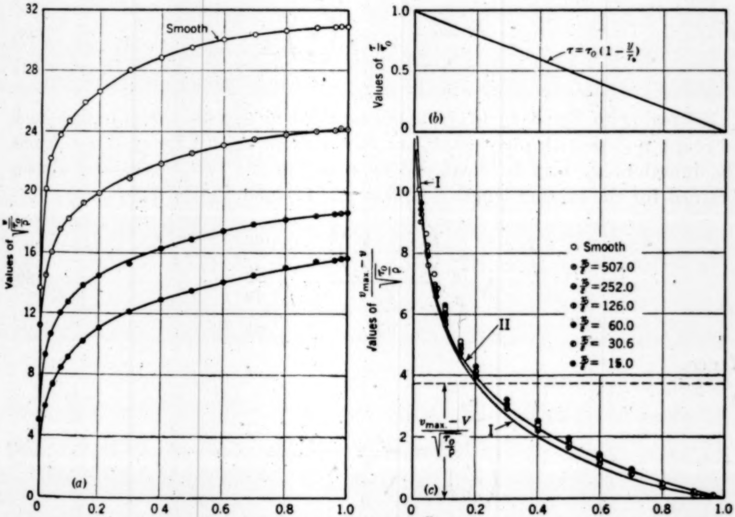

FIG. 15.—DIMENSIONLESS PLOT OF VELOCITY DISTRIBUTION FOR VARIOUS VALUES OF RELATIVE ROUGHNESS.

the flow, for smooth pipes as well as for all values of relative roughness. Hence, if the central portions of all curves in Fig. 15(*a*) are superposed, a single curve will result.

In other words, within the purely turbulent portion of the movement, the difference between the maximum velocity at the center and that at any other point (that is, the "velocity defect"), divided by the friction velocity, is purely a function of the relative distance from the wall. Stanton was the first to formulate this universal function:

$$\frac{v_{\max} - v}{\sqrt{\frac{\tau_0}{\rho}}} = \phi_1\left(\frac{y}{r_0}\right) \dots\dots\dots\dots\dots\dots\dots (46)$$

The conditions near the wall, however, are still dependent upon other influences. The velocity near a smooth surface, where viscous shear predominates, depends upon τ_0, ρ, μ, and y. A completely general expression for this universal relationship must, for dimensional reasons, have the form,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = \phi_2\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{y}{\nu}\right) \dots\dots\dots\dots\dots\dots\dots (47)$$

Similarly, the velocity in the vicinity of a rough wall, where viscous influences do not exist, will depend entirely upon τ_0, ρ, y, and ϵ. The corresponding universal function must then be:

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = \phi_3\left(\frac{y}{\epsilon}\right) \dots\dots\dots\dots\dots\dots\dots (48)$$

Returning to Equation (44), it will be recalled that this relationship holds for rough and smooth pipes alike, entirely without regard for wall conditions. The function, ϕ_1, may be developed by equating the two expressions already obtained for the mixing length (Equations (44) and (45)), thus:

$$l = \sqrt{\frac{\tau_0}{\rho}}\,\frac{\left(1 - \frac{y}{r_0}\right)^{\frac{1}{2}}}{\frac{dv}{dy}} = \kappa\,\frac{\frac{dv}{dy}}{\frac{d^2v}{dy^2}} \dots\dots\dots\dots\dots\dots (49)$$

Hence,

$$\frac{\frac{d^2v}{dy^2}}{\left(\frac{dv}{dy}\right)^2} = \frac{\kappa}{\sqrt{\frac{\tau_0}{\rho}}}\,\frac{\sqrt{\tau_0}}{\sqrt{r_0 - y}} \dots\dots\dots\dots\dots\dots (50)$$

Equation (50) may be integrated directly, the constant of integration being found by assuming an infinite velocity gradient at the wall (this is of no serious consequence, as the wall conditions play no part in the central regions for which the relationship is valid):

$$\frac{dv}{dy} = \frac{1}{2\kappa}\,\frac{\sqrt{\frac{\tau_0}{\rho}}}{\sqrt{r_0}}\,\frac{1}{\sqrt{r_0 - y} - \sqrt{r_0}} \quad\ldots\ldots\ldots\ldots\ldots(51)$$

Integrating once again between the limits, $y = y$ and $y = r_0$, the final relationship is obtained, in accord with Equation (46):

$$\frac{v_{\max} - v}{\sqrt{\frac{\tau_0}{\rho}}} = -\frac{1}{\kappa}\left[\log_e\left(1 - \sqrt{1 - \frac{y}{r_0}}\right) + \sqrt{1 - \frac{y}{r_0}}\right] \quad\ldots.(52)$$

A plot of the velocity defect divided by the friction velocity yields a general curve (I in Fig. 15(c)) which fits fairly well all measured values of the velocity within the central regions of flow. In order to take into consideration the outer regions, it is necessary to return to Equations (47) and (48); the only relationship commensurate with these expressions and the similarity of the turbulent pattern must be the following, first derived by von Kármán:

For smooth pipes,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = C_1 + \frac{1}{\kappa}\log_e\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{y}{\nu}\right) \quad\ldots\ldots\ldots\ldots\ldots(53)$$

and, for rough pipes,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = C_2 + \frac{1}{\kappa}\log_e\frac{y}{\epsilon} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(54)$$

With Constants C_1 and C_2 determined from Nikuradse's measurements, Equations (53) and (54) become:

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = 5.5 + 5.75\log_{10}\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{y}{\nu}\right) \quad\ldots\ldots\ldots\ldots\ldots(55)$$

and,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = 5.85 + 5.75\log_{10}\frac{y}{\epsilon} \quad\ldots\ldots\ldots\ldots\ldots\ldots(56)$$

in which the value, 5.75, corresponds to $\kappa = 0.40$, and includes the conversion factor, 2.3026, between the natural and common systems of logarithms.

Writing each of these equations in terms of the maximum velocity at the distance, $y = r_0$, from the wall:

$$\frac{v_{\max}}{\sqrt{\frac{\tau_0}{\rho}}} = 5.5 + 5.75\log_{10}\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{r_0}{\nu}\right) \quad\ldots\ldots\ldots\ldots\ldots(57)$$

and,

$$\frac{v_{\max}}{\sqrt{\frac{\tau_0}{\rho}}} = 5.85 + 5.75 \log_{10} \frac{r_0}{\epsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

and subtracting Equations (55) and (56), respectively, there results the single general relationship for the velocity defect,

$$\frac{v_{\max} - v}{\sqrt{\frac{\tau_0}{\rho}}} = 5.75 \log_{10} \frac{r_0}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

which is valid for the wall regions in either case, and, moreover, gives good results over the entire central portion of the flow (see Curve II in Fig. 15(*c*)).

The full significance of Equation (55) will be revealed by careful study of Figs. 16 and 17 based upon measurements by Nikuradse. In Fig. 16 the ratio of velocity to friction velocity has been plotted as a function of the "friction-distance parameter", $\sqrt{\frac{\tau_0}{\rho}}\frac{y}{\nu}$, which is structurally equivalent to the Reynolds number; Fig. 17 is the same plot in semi-logarithmic form. In Fig. 16 the wall itself is represented by the ordinate axis, whereas the distance to the pipe center, $y = r_0$, is different for each different set of conditions, and depends entirely upon the magnitude of the friction-distance parameter. The horizontal scale of this plot has been given two different values, showing twice

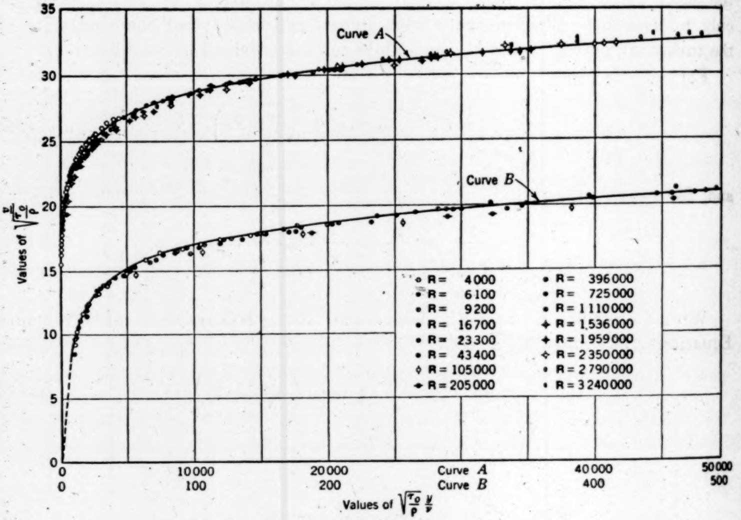

FIG. 16.

the same general curve drawn through the plotted points; the more extended scale shows to good advantage the existence of the laminar film, in which the velocity distribution is practically linear.

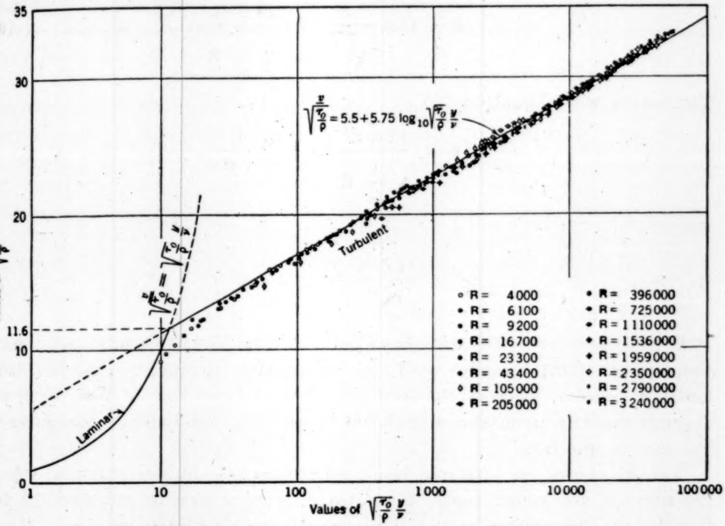

FIG. 17.—UNIVERSAL VELOCITY DISTRIBUTION FOR SMOOTH PIPES.

Equation (55) plotted in semi-logarithmic form (Fig. 17) must obviously be a straight line. Points measured by Nikuradse on smooth pipes for various values of R are seen to fall upon this linear plot throughout an extreme range of the friction-distance parameter. Even at the pipe centers (the last groups of plotted points) only a slight deviation is noticeable. The departure in the wall region is obviously the result of viscous action as the boundary film is approached; this may be further clarified by constructing the curve of laminar distribution in this film, in which the following equality will be seen to hold:

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = \sqrt{\frac{\tau_0}{\rho}}\,\frac{y}{\nu} \quad \text{......................(60)}$$

since $v = \frac{\tau_0 y}{\mu}$. Thus, the lowest measured points are seen to be in the transition zone between the laminar and the turbulent regions. Not only may the approximate thickness of the laminar film be found from this plot, but also the extent of the viscous influence in the turbulent region. By combining Equations (55) and (60), at the point of intersection,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = \sqrt{\frac{\tau_0}{\rho}}\,\frac{y}{\nu} = 11.6 \quad \text{..................(61)}$$

Since this intersection of the laminar and turbulent velocity distribution curves may be chosen arbitrarily as the border of the laminar film, y will then equal δ, and the thickness of the laminar film will be given by,

$$\delta = 11.6 \frac{\nu}{\sqrt{\frac{\tau_0}{\rho}}} = \frac{11.6\, D}{\sqrt{\frac{f}{8}}\, \mathsf{R}} \dots\dots\dots\dots\dots\dots\dots (62)$$

Comparing with Equation (36),

$$\delta = \frac{11.6\, D}{\sqrt{\frac{f}{8}}\, \mathsf{R}} = 8\, \frac{v_w}{V}\, \frac{D}{f\, \mathsf{R}}$$

whence,

$$\frac{v_w}{V} = \frac{11.6 \sqrt{f}}{\sqrt{8}} = 4.1 \sqrt{f} \quad \dots\dots\dots\dots\dots\dots\dots (63)$$

It is to be noted that the only fixed point in Fig. 17 is this arbitrary limit of the laminar film; the pipe wall lies an infinite distance to the left (since log 0 = $-\infty$), whereas the location of the center line of any given pipe depends entirely upon the magnitude of the friction-distance parameter for the case in question.

Fig. 18 shows the corresponding logarithmic plot of the universal velocity distribution for rough pipes, the ratio of y to ϵ having replaced the friction-distance parameter as the abscissa scale. This plot is similar to that of Fig. 17 in all respects, except for the absence of the laminar boundary film

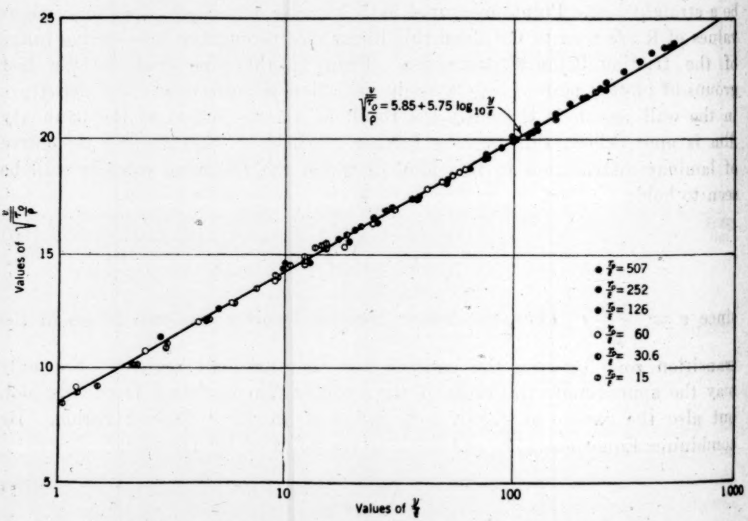

FIG. 18.—UNIVERSAL VELOCITY DISTRIBUTION FOR ROUGH PIPES.

and the accompanying viscous influence in the wall region. It will be noted that points determined by Nikuradse for various values of relative roughness fall upon a straight line even as close to the wall as the outer edges of the roughness particles (where $y = \epsilon$). Moreover, inspection of Equations (55) and (56) will show a striking similarity between the respective constants. It will be seen that, since the factor, 5.75, is the same in both expressions, the two curves have identical slopes and are simply shifted on the plot by the difference between 5.5 in the one case and 5.85 in the other. Different forms of roughness, however, will surely require a change in one or both constants; at any rate, the term, ϵ, must then denote far more than merely the extent of linear projection from the wall.

The fact has already been mentioned that the value, 5.75, corresponds to $\kappa = 0.40$ (combined with the conversion factor between the two logarithmic systems). If Equations (52) and (59) are correct for the regions for which they were developed, then they must coincide in the zone where the two regions overlap. This they will never do, however, so long as the same value of κ is used for both—as must be done if this is a true universal constant. This explains more fully why von Kármán found that κ must be about 0.36 for the central regions, and Nikuradse was forced to choose the higher magnitude of 0.40 for the outer portions of the flow. The two curves plotted in Fig. 15(*c*) are based upon these two different values of κ.

Furthermore, owing to the linear characteristics of Equations (55) and (56) when plotted in semi-logarithmic form, it will be evident that the velocity distribution curves computed from these relationships must have a finite slope at the pipe axis, although actual velocity curves all reach a zero slope at this point. While the discrepancy is so slight as to be entirely negligible in practice, it points to a fallacy in the fundamental assumptions upon which these expressions were based. That is, the application of Equation (45) to the entire central region of flow presumes a constant correlation of the two essential components of velocity fluctuation; actually, there is no reason to expect such correlation near the pipe axis, for points in this vicinity are practically equi-distant from either side of the pipe wall. If the assumed variation of the mixing length as expressed in Equation (45) is plotted after the manner of Fig. 14, these computed values will follow the actual curve over the greater portion of the abscissa scale, but near the center line will drop away rapidly, approaching the value of zero at the axis. Obviously, the analytical study of the velocity distribution has not quite reached perfection.

Since similarity of the turbulent mechanism has been found to result in a general function for the velocity distribution, the close relation existing between velocity distribution and shear would indicate that there is also a universal function for the resistance coefficient. Equation (55) written for the maximum velocity at the distance, $y = r_0$, and in terms of the universal constant, κ, will have the form:

$$\frac{v_{\max}}{\sqrt{\frac{\tau_0}{\rho}}} = 5.5 + \frac{1}{\kappa}\log_e\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{r_0}{\nu}\right) \qquad \text{(64)}$$

As shown in Fig. 15(c), furthermore, the general nature of the velocity-defect curve must permit a direct computation of the difference between the maximum and mean velocities in terms of the common denominator, $\sqrt{\frac{\tau_0}{\rho}}$. This is nothing more than the average ordinate of the velocity-defect curve, when the latter is considered the cross-section of a surface of revolution about the center line of the pipe. The average ordinate of Curve II, therefore, may be determined as follows, as shown by Professor Bakhmeteff, through integration of Equation (59):

$$\frac{v_{\max} - V}{\sqrt{\frac{\tau_0}{\rho}}} = \frac{\int_{r_0}^{0} 2\,\pi\,(r_0 - y)\,\frac{1}{\kappa}\log_e \frac{r_0}{y}\,dy}{\pi\,r_0^2} = \frac{3}{2\,\kappa} = 3.75\ldots(65)$$

Although Nikuradse found that a value of 4.07 would better fit experimental measurements of $v_{\max}$ and V, Equation (65) will be used without modification in the following development; it is this value which has been plotted as the mean in Fig. 15(c).

Subtracting Equation (65) from Equation (64), there results an expression for the average velocity of flow as a function of the friction velocity, the universal constant, and the friction-distance parameter:

$$\frac{V}{\sqrt{\frac{\tau_0}{\rho}}} = -3.75 + 5.5 + \frac{1}{\kappa}\log_e\left(\sqrt{\frac{\tau_0}{\rho}}\,\frac{r_0}{\nu}\right)\ldots\ldots\ldots\ldots(66)$$

Equation (66) may now be reduced to a more significant form through introduction of Equation (13) in the forms,

$$\frac{V}{\sqrt{\frac{\tau_0}{\rho}}} = \sqrt{\frac{8}{f}} \text{ and } \sqrt{\frac{\tau_0}{\rho}} = V\sqrt{\frac{f}{8}}:$$

$$\frac{\sqrt{8}}{\sqrt{f}} = 1.75 + \frac{1}{\kappa}\log_e\left(\frac{\sqrt{f}}{\sqrt{8}}\,\frac{VD}{2\,\nu}\right)$$

$$= 1.75 + \frac{l}{\kappa}\log_e\frac{1}{4\sqrt{2}} + \frac{1}{\kappa}\log_e(\mathsf{R}\sqrt{f})$$

Hence,

$$\frac{1}{\sqrt{f}} = -0.91 + 2.03\log_{10}(\mathsf{R}\sqrt{f})\ \ldots\ldots\ldots\ldots(67)$$

Thus, a very simple relationship for the resistance factor in terms of the universal constant and the Reynolds number has been developed directly from the general expression for the velocity distribution. This basic relationship was first derived by von Kármán in terms of the maximum velocity, and later put into the form of Equation (67) by Prandtl. With slight modification of constants according to the experimental data of Nikuradse, this basic equation finally becomes:

$$\frac{1}{\sqrt{f}} = -0.8 + 2 \log_{10} (\mathsf{R} \sqrt{f}) \quad \ldots\ldots\ldots\ldots\ldots (68)$$

Inasmuch as Equation (68) is based upon physical analysis, it might be expected to be valid (with correct constants) from $\mathsf{R} = 100\,000$ to extremely high, if not to infinite, values of R. The only drawback to its use for general purposes is the appearance of the square root of f on both sides of the equation. Although this disadvantage may be offset through use of a plotted curve, Nikuradse has suggested the empirical Equation (35), which is quite accurate for all ordinary values of R above the Blasius range, and which, moreover, may be extrapolated safely a considerable distance beyond the present limit of experimental measurements. As seen from Fig. 19, in which the curves of Equations (68) and (35) are practically identical, previous empirical curves deviate considerably from the linear form.

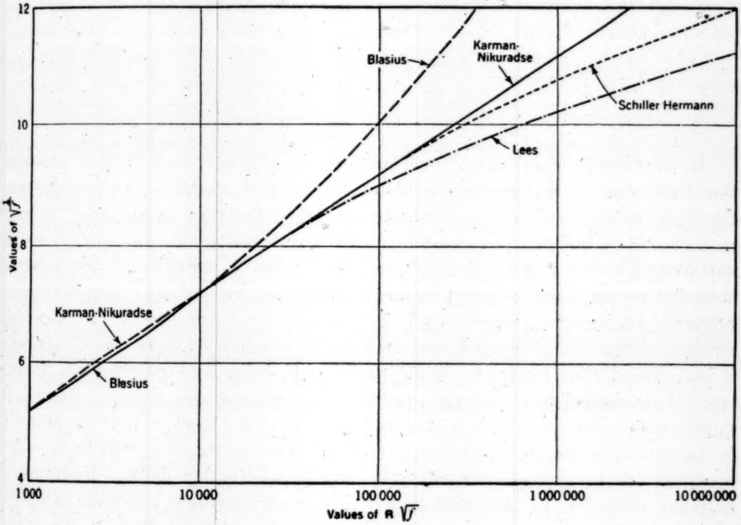

FIG. 19.

A similar operation performed on Equation (56), with slight correction of constants to conform with Nikuradse's data, results in the expression already given for the resistance in rough pipes (see Equation (38)):

$$\frac{1}{\sqrt{f}} = 1.74 + 2 \log_{10} \frac{r_0}{\epsilon}$$

The excellent agreement between Nikuradse's measurements and the linear quality of the analytical expression is shown in Fig. 20. Once again the curves for smooth and rough pipes are seen to have the same slope, differing only in position as determined by the constants, − 0.8 and 1.74, at least for the type of roughness studied by Nikuradse.

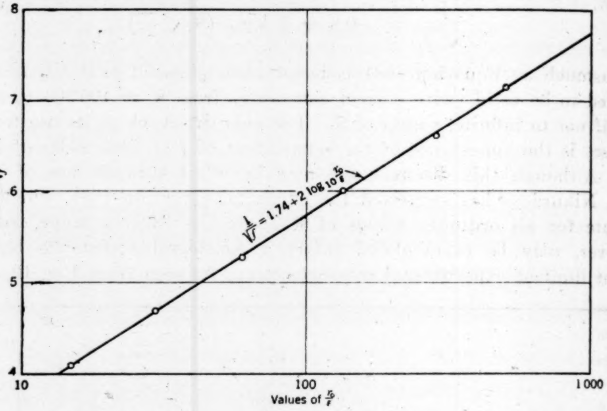

FIG. 20.

In developing a universal function for rough pipes, one is forced to adopt the dimension, ε, for want of a better measure of roughness characteristics. Obviously, this term does not completely define the type of surface. It may prove feasible, however, to adapt a linear characteristic of this kind to rough surfaces of other types, in which case the term, ε, would indicate relative behavior rather than an actual measure of radial projection. In this respect, reference is made to a paper[21] by V. L. Streeter, Jun. Am. Soc. C. E., in which further experimental results for artificially roughened pipes are described; it was found that roughness differing in both form and relative magnitude from that studied by Nikuradse yielded similar resistance curves. Hence, it might be concluded that this general function for the flow pattern actually is universal in its application to artificial and commercial roughness alike, provided the roughness characteristic may be adequately defined in terms of a linear quantity, ε. For the time being, wavy surfaces must be excepted, of course, since the form of the function may not be the same. At any rate, it has been proved conclusively that roughness dimensions alone are not sufficient indication of the resistance to flow, but must always be considered with relation to the dimensions of the pipe.

Mention must be made of the fact that, whereas the mathematical genius of von Kármán was first responsible for the development of these universal functions, several other scientists of note have deduced similar relationships in various ways; Prandtl[22] has continued to hold a foremost place in this field and it was under the latter's directorship of the Kaiser Wilhelm Institut für

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[21] "Frictional Resistance in Artificially Roughened Pipes", *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 681.

[22] "Neuere Ergebnisse der Turbulenzforschung", *Zeitschrift* des Vereines Deutscher Ingenieure, Band 77, Nr. 5, 1933.

Strömungsforschung, in Göttingen, that Nikuradse conducted his extensive and systematic measurements of turbulent flow in pipes. Also from the Prandtl school has come the latest effort to study the problem of turbulence in the light of probability and statistical physics.[28]

Although such experimental data as herein cited were limited by laboratory conditions so far as wall surface and pipe diameter are concerned, a splendid beginning has been made in the experimental verification of such broad analytical deductions.

RESEARCH OF THE FUTURE

Throughout this paper constant emphasis has been laid upon the necessity of systematic, physically sound investigation of fluid motion. Various schools in Europe and several of note in the United States are engaged upon such fluid research of an advanced nature, but both co-operation and correlation of effort are essential to continued progress. It is noteworthy that within the past year (1934–35), at the instigation of the World Power Conference, a committee on fluid resistance has been created in the United States with the object of planning and sponsoring organized research in this field.

What problems still remain unsolved in the domain of fluid resistance should be quite apparent. Fluid motion may no longer be classified satisfactorily simply as laminar or turbulent flow, since turbulence is not always free from appreciable viscous resistance to flow. Whereas on the one hand laminar or purely viscous motion is practically a closed book so far as general pipe resistance is concerned, on the other hand, the problem of purely turbulent motion at very high Reynolds numbers is not yet fully solved; a complete rational analysis of the inner mechanism of turbulent movement is still to be achieved, despite the success of the methods herein described; and no successful attempt has been made as yet to analyze the combined effect of viscous and turbulent shear within the lower range in which the Blasius equation applies.

Of primary interest to the engineer, of course, is the correlation between artificial roughness as produced in the laboratory, and the actual roughness of commercial pipe, if only for the range of constant resistance coefficient. Not only does this leave unexplained the entire transition region between the smooth and rough régimes, but it fails completely to consider that type of pipe of which the behavior does not fall in either category—the pipe with "wavy" surface. It is obvious that neither the existing data on commercial pipe nor the recent experiments on a few types of artificial roughness provides sufficient evidence for the formulation of general relationships including every type of roughness under every possible condition of flow.

Once the realm of pipe resistance is left, the problem becomes even more complex. Open channels present a further variable in the form of the free surface, and the question of geometrical similarity of the profile and cross-section becomes of major importance; it is still a debatable question whether the hydraulic radius may continue to prove satisfactory as a general length parameter. Doubtless, however, open channel flow is of so highly turbulent a nature that viscous influences are probably quite negligible; yet this again introduces a question in the mind of the experimenter, for open flow is often simulated in the laboratory by small-scale models, in which the viscous action is most assuredly of appreciable magnitude—even far above the laminar régime.

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[28] "Turbulenz" von H. Gebelein, Springer, Berlin, 1935.

In other fields of engineering similar problems must be encountered, for the designers of air and water craft are in much the same difficult situation in their efforts to understand the complexities of skin and form resistance. This very universality of resistance phenomena may well continue to prove an impetus toward the advancement of knowledge in this field.

ACKNOWLEDGMENT

For many hours of stimulating discussion and criticism during the preparation of this paper, the writer expresses his sincere gratitude to Boris A. Bakhmeteff, M. Am. Soc. C. E.

APPENDIX

LIST OF GENERAL SYMBOLS

$\left.\begin{matrix}M\\ L\\ T\end{matrix}\right\}$ = mass, length, and time as general dimensional terms.
F = any acting force.
I = generally, the inertial reaction to any acting force; specifically, the inertial character of flow; that is, the momentum passing a section per unit of time.
m = mass.
t = time.
V = the average velocity of flow for the entire cross-section.
v = in general, any instantaneous velocity vector; for pipe study, the temporal average velocity at any point in the cross-section.
$v_{max.}$ = the maximum value of v, occurring at the pipe axis.
p = pressure intensity.
R = Reynolds number.
f = resistance coefficient for pipes.
n = exponent in the general equation, $f = C\ \mathsf{R}^{(n-2)}$.
h = manometric head.
L = length of pipe.
D = diameter of pipe.
r_0 = radius of pipe.
r = radial distance from pipe axis.
y = normal distance from pipe wall.
ρ = density of fluid, or mass per unit of volume.

γ = specific weight of fluid, or weight per unit of volume.
g = gravitational acceleration; $g = \frac{\gamma}{\rho}$.
μ = viscosity of a fluid.
ν = kinematic viscosity of a fluid; $\nu = \frac{\mu}{\rho}$.
τ = intensity of shear at any point in the cross-section.
τ_0 = intensity of shear at the pipe wall.
ϵ = radial dimension of roughness particles.
δ = thickness of the boundary layer.
l = the mixing length.
κ = a universal constant relating to flow at high values of **R**.
ϕ = a function of.

DISCUSSION

Chesley J. Posey,[24] Jun. Am. Soc. C. E. (by letter).—In publishing this excellent paper the author has performed a service to the Engineering Profession. It summarizes, in a lucid manner, the progress made upon the important problem of fluid turbulence. The writer has recently developed an analytical approach to certain of the unsolved phases of the problem to which the author calls particular attention. Although the writer's studies are as yet in a tentative stage, it seems worth while to outline them briefly in discussion.

The author points out that one of the chief difficulties in the study of fluid resistance in rough pipes is the lack of a satisfactory measure of the roughness. The measure, ϵ, representing the radial dimension of roughness particles, is apparently only satisfactory when the particles are geometrically similar. The curves shown in Fig. 10, for artificially roughened pipe, cannot be expected to apply to other pipe as the friction factor, at large values of R, is a function not only of $\frac{r_0}{\epsilon}$, but also of other unknown factors dependent upon the geometry of the roughness. In mathematical terms:

$$f = \Phi\left(\mathsf{R}, \frac{r_0}{\epsilon}, ?, ?, ..\right) \qquad (69)$$

It is the writer's intention to point out what may be the nature of the unknown factors. The form of the function must remain to be determined experimentally.

Fig. 21 is a diagrammatic longitudinal section through the wall of the pipe, showing the inside surface. The section is long enough to include a representative sample of the "roughness." The dimension, ϵ, is of the extreme radial variation in the pipe's inner surface. Two curves are plotted above the "roughness" curve, representing its first and second derivatives with respect to the distance along the pipe. The maximum ranges of the two curves may be termed ϵ' and ϵ'', as shown. The first is obviously dimensionless, whereas

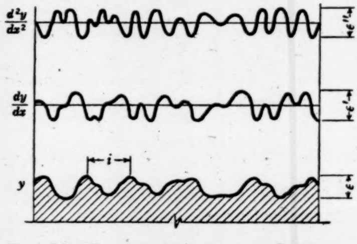

Fig. 21.—Longitudinal Section Through Pipe Wall, Showing Roughness, and Derivative Curves.

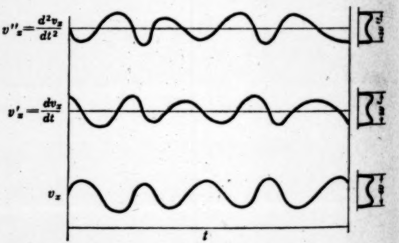

Fig. 22.—Instantaneous Velocity as a Function of Time, and Derivative Curves.

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[24] Asst. Prof. of Mechanics and Hydraulics, State Univ. of Iowa; and Assoc. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

the second must be multiplied by a length parameter to be dimensionless. The appropriate parameter would seem to be the average distance between successive roughness particles. Higher derivatives might be taken, but when it is noted that ϵ' is a measure of the maximum slopes of the roughness, and that ϵ'' is a measure of the sharpness of its points, it seems unlikely that higher derivatives will be needed to characterize the roughness satisfactorily. Thus:

$$f = \Phi\left(\mathsf{R}, \frac{r_0}{\epsilon}, \epsilon', i_w\ \epsilon''\right) \ldots\ldots\ldots\ldots\ldots\ldots (70)$$

It is questionable whether experiments to determine the form of Φ should include unnatural geometrical roughnesses, but whether artificial or natural roughnesses are investigated the experiments should include a measurement of the roughness as outlined herein. The technique of making such a measurement would require taking a precise profile along a longitudinal section of the pipe by use of the recently developed "profilograph", or by other means. The derivative curves could then be constructed graphically. In commercial pipes the roughness measured in a transverse direction might be different from that measured in a longitudinal direction. It is to be hoped that the transverse roughness, where different, will have a comparatively minor effect upon the frictional resistance to flow in the pipe.

In the more general problem, there is a need for a definite measure of turbulence. It is evident that the complete picture of velocity distribution in a turbulent region at any instant can never be exactly repeated; yet the turbulent stream may be "steady" in the sense that certain measures do remain constant, statistically. For example, the average velocity at a point during successive intervals of time may remain constant if the intervals of time are long enough. The average velocity, however, is unsatisfactory as a measure of the turbulence. To arrive at a definition of the statistical measures which may be expected to provide a sufficiently precise measure of the turbulence, consider the variation of the x-component of the velocity at a point in a turbulent stream. The x-direction is understood to be the direction of the mean velocity. Fig. 22 shows an imaginary curve for v_x as a function of t. Above it are the first two derivative curves, v'_x and v''_x. At the right end of each curve is represented the frequency distribution of ordinates during equal infinitesimal increments of time. The x-component of the turbulence at one point may be said to be identical with that at another if these distributions of v_x, v'_x, and v''_x are identical at each. Distributions from higher order curves are omitted arbitrarily; perhaps that of the second order is unnecessary. The same procedure can be applied to v_y and v_z. The use of six or nine frequency distributions to measure the turbulence at a single point seems hopelessly complicated, but in many cases correlations will reduce the number, and it may be that the double-moded distributions of Fig. 22 can be represented with sufficient accuracy for experimental work by the ranges, w, w', and w''. These ranges can be reduced to dimensionless form by dividing by appropriate parameters, but this should not be done except in comparisons of geometrically similar cases.

The measure of turbulence suggested by the writer bears somewhat the same relation to von Kármán's universal relation for the mixing length as does the Eulerian viewpoint to the Lagrangian viewpoint in classical hydrodynamics. The concept of mixing length is perhaps better adapted to studies of energy loss than that of the writer, but the writer's concept would seem susceptible to more direct experimental application. Study of each may yield information valuable in the development of the other.

S. Franz Yasines,[35] Assoc. M. Am. Soc. C. E. (by letter).—An excellent résumé of the investigations that have been made in this field of fluid turbulence since the middle of the Nineteenth Century, is contained in this paper.

Because the current mathematical, as well as the physical, knowledge of many fluid-flow problems is limited, present-day investigators are forced to idealize the conditions in such problems, in order to save time and labor in actual research in the laboratories; and in this manner they find at least a partial solution of a problem in an empirical form. When an attempt is made to imitate Nature in the laboratory, in performing tests on models, the investigator frequently resorts to such guides as Froude's or Reynolds' numbers. Often he finds that they fail to perform their functions. Is this because their true meanings (and hence their limitations in application to actual problems) are not understood?

Some hydraulicians maintain that Reynolds' number is the ratio of inertial force to that of frictional force in a given flow. Such a definition is confusing when flow in a straight pipe of uniform cross-section is under consideration. The Newtonian principle of momentum may be applied to a flow confined in such a conduit, but at a constant rate of flow the rate change of momentum is zero; hence, the inertial force in this particular flow must also be zero. Therefore, if Reynolds' number is to be defined as the ratio of inertial force to frictional force, it must be zero for any constant rate of flow in a conduit of uniform cross-section.

The brief derivation of Reynolds' number, presented by the author, leads to the conclusion that the only forces governing the flow of fluids in pipes are frictional and a certain fictitious force which he terms "the rate of passage of momentum." What is the latter force? Why not consider pressure force, since it is known definitely that the governing forces in the problem of motion under consideration are friction and pressure? The rigorous mathematical derivation of Reynolds' number is by no means easy when a physical conception of the problem is to be formulated. Considering, for example, the application of dimensional analysis (π-theorem) to the flow (at constant rate) in a conduit of uniform cross-section, it is possible to obtain the proper relation among the variables which indicates the presence of Reynolds' number, provided the independent variables are properly selected. On the other hand, how is one to know at the beginning of the analysis what variables are to be considered as being independent if no experimental data are available?

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[35] Instr. in Civ. Eng., Coll. of Eng., New York Univ., New York, N. Y.

By using the Navier-Stokes equation of motion and the Newtonian principle of dynamic similarity, it can be proved that the ratio of inertial force to that of frictional force may be equal to zero or to unity (depending upon the assumption made in the physical analysis of the flow), and yet Reynolds' number does appear as a governing factor for similarity of flow.

The reliability of investigations of the boundary layer with a thickness of 0.001 ft, by means of specially designed Pitot tubes, is questionable. Instruments based on electrical rather than physical principles would be more reliable in the study of such complex problems.

The writer agrees with the author that a more specific definition of surface roughnesses should be established. This can be done since apparatus for this purpose is already available.

Since the contemporary hydraulician has inherited a large amount of information from his predecessors and has the opportunity of resorting to efficient physical tools for the study of various types of fluid flow, there is good reason to believe that he may solve many intricate problems; but, in order to do so, a closer co-ordination among hydraulicians and physicists is essential.

Benjamin Miller,[38] Esq. (by letter).—The study of turbulent flow in commercial pipe has been conducted along both analytical and empirical lines. The author has referred to work of both types, and gives the impression that whereas great strides have been made on the analytical side, the empirical studies have not been of much help, and that as yet the engineer cannot use the results of recent work for design. The writer feels that the picture is rather different, that the analytical methods described are far from satisfactory, but that fluid-flow rate can now be predicted with great accuracy by means of the empirical relationship given in the paper. That empirical relationship is due entirely to experiment, and owes nothing to analytical work. A rational analysis of turbulent flow should lead to the same relationship as that found empirically. The methods described in the paper do not, but improved methods of the future may.

Before giving reasons for these statements, the writer wishes to point out that the law of viscous flow was discovered by Hagen in 1839 and established beyond doubt by Poiseuille in 1842, but it was not until 1856 that Wiedemann derived the law analytically. The law of turbulent flow was first published in 1932, although experimental work was available previously on which it might have been based. Perhaps, by 1946, an analytical derivation of it may have been developed.

The law of turbulent flow to which the writer refers is the author's Equation (68). Mr. Rouse states that the appearance of the square root of f on both sides of the equation is a drawback to its use for general purposes; but this is a drawback only because of the habit of thinking in terms of f, which in turn, comes about through thinking of the pressure drop required to maintain a given flow rate. If, however, one thinks in terms of the flow rate which can be maintained by a given pressure drop, one may transform Equation (68) into,

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[38] With Cities Service Co., New York, N. Y.

$$\frac{m}{t} = \frac{\pi}{\sqrt{8}}\left(D^5 \rho \frac{dp}{dL}\right)^{0.5}\left(\log_{10} \frac{D^3 \rho \dfrac{dp}{dL}}{\mu^2} - 0.5\right) \dots\dots (71)$$

Introducing the following symbols:

For the flow rate,

$$Q = \frac{m}{\rho t} \dots\dots\dots\dots\dots\dots\dots\dots (72)$$

for the pressure gradient,

$$G = \frac{dp}{dL} \dots\dots\dots\dots\dots\dots\dots\dots (73)$$

and, for the von Kármán number[27],

$$K = \frac{D^{1.5} \rho^{0.5} G^{0.5}}{\mu} \dots\dots\dots\dots\dots\dots\dots (74)$$

Equation (71) becomes,

$$Q = \frac{\pi}{\sqrt{2}}\left(D^5 \frac{G}{\rho}\right)^{0.5} (\log_{10} K - 0.25) \dots\dots\dots\dots (75)$$

Equation (75) is suitable for general use. It describes, accurately, the turbulent flow in commercial steel pipe. The flow through pipe lines will be less because of the pressure drops at joints, bends, fittings, etc.

The development of Equation (75) is detailed in the paper previously cited.[27] The author's Equation (8) may be used as a starting point, but it is written instead,

$$V = \phi_1 (D, \rho, \mu, \tau_0) \dots\dots\dots\dots\dots\dots\dots (76)$$

Since Q is proportional to V,

$$Q = \phi_2 (D, \rho, \mu, \tau_0) \dots\dots\dots\dots\dots\dots\dots (77)$$

By dimensional analysis Equation (77) transforms to,

$$Q = \left(D^5 \frac{G}{\rho}\right)^{0.5} \phi_3 (K) \dots\dots\dots\dots\dots\dots\dots (78)$$

and Equation (78) is put into the explicit form of Equation (75) by the empirical method of plotting the results of the various investigators, such as Stanton and Pannell, Nikuradse, etc.

Equation (75) is represented well by the author's Equation (35) between Reynolds numbers of 10^5 to 10^7, but it is not restricted to this region. Actually, it coincides with experiment down to the beginning of the turbulent régime. As mentioned previously, it applies to pipe, and not necessarily to pipe lines. However, there are commercial pipe lines in which the flow is more than 95% of that calculated by Equation (75), and it is an unusual line in which the flow is less than 90% of that so calculated.

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[27] *Transactions*, Am. Inst. Chem. Engrs., Vol. 32 (1936), p. 1.

The author does not consider that the methods of analysis described in the paper are perfect. He does indicate that, although there is a fallacy in the fundamental assumptions, the discrepancy thus introduced is so slight as to be entirely negligible in practice. Rather than pick out the discrepancies, the writer suggests that the problem be re-examined in the light of what is known.

In turbulent flow, as in viscous flow, there must be maximum velocity at the center and zero velocity at the wall. Furthermore, there must be a viscous drag proportional to the rate of change of velocity.

These statements may be expressed as follows: $v = 0$ at $r = r_0$; $\frac{dv}{dr} = 0$ at $r = 0$; $\mu \frac{dv}{dr} = \tau_0$ at $r = r_0$; and,

$$\tau = \mu \frac{dv}{dr} + f(r) = \frac{\tau_0 r}{r_0} \qquad \text{.....................(79)}$$

The law of turbulent flow (Equation (75)), is:

$$2\pi \int_0^{r_0} r\, v\, dr = \frac{\pi}{\sqrt{2}} \left(D^5 \frac{G}{\rho} \right)^{0.5} (\log_{10} K - 0.25) \qquad \text{........(80)}$$

Any analytical expression for v as a function of r must meet all these conditions. Assumptions which lead to infinite velocity gradient at the wall and infinite velocity at the center may be interesting, but they can not be correct.

Ralph W. Powell,[28] M. Am. Soc. C. E. (by letter).—The title of this paper is perhaps too broad, as it scarcely covers the entire subject of fluid turbulence. The question of turbulence as it affects problems in aeronautics[29] and the transportation of sediment in streams[30], for instance, is scarcely touched upon. As a résumé of present knowledge on the flow in pipes, however, the paper is excellent. The only other adequate treatment of the subject in English, as far as the writer knows, is that given by Prandtl and Tietjens[31], and the author has improved upon this study at several points. (However, he has not included all the interesting material given therein as, for instance, the "length of transition" at the entrance, the intermittent occurrence of turbulence in the critical range, the effect of convergent and divergent flow, and the seventh-root law of velocity distribution.)

In studying through parts of the paper with a graduate class in fluid mechanics, one idea has occurred to the writer which is perhaps worth noting. In the paragraphs following Fig. 8 the author assumes a linear distribution of velocity in the boundary layer. This simplifies the mathematics, but is contrary to the fact shown in Equation (25) that for stream-line flow the velocity distribution is parabolic. As a matter of fact, the assumption is unnecessary.

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[28] Assoc. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[29] See, for example, "Applied Hydro- and Aeromechanics", by Prandtl and Tietjens, Chapters IV, V, and VI, N. Y., McGraw-Hill Pub. Co., 1934.

[30] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation", by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, April, 1933, pp. 487–491.

[31] "Applied Hydro- and Aeromechanics", Chapters III and IV, N. Y., McGraw-Hill Pub. Co., 1934.

One can assume that the flow in the boundary layer obeys exactly the same laws as in the case of laminar flow, but that τ_0, the intensity of shear at the pipe wall, has the value of turbulent flow. Combining Equations (13) and (25):

$$v = \frac{f \rho V^2}{16 \mu} \left(\frac{r^2_0 - r^2}{r_0} \right) \ldots\ldots\ldots\ldots (81)$$

which will hold within the boundary layer only. Taking, as the author does, v_w as the velocity at the inner surface of the boundary layer, and δ as the thickness of the boundary layer, and substituting $v = v_w$, $r = \frac{D}{2} - \delta$, and $r_0 = \frac{D}{2}$ in Equation (81), and solving for δ:

$$\delta = \frac{D}{2} \left\{ 1 - \left(1 - \frac{32}{f \mathsf{R}} \frac{v_w}{V} \right)^{\frac{1}{2}} \right\} \ldots\ldots\ldots\ldots (82)$$

Expanding Equation (82) by the binomial theorem and dropping all except the first term gives Equation (36). The difference between the two expressions will be largest when $f \mathsf{R}$ is smallest; that is, just above the critical region, say, when $\mathsf{R} = 3\,000$. Then Equation (32) gives $f = 0.0428$ and $f \mathsf{R} = 128$. If $\frac{v_w}{V} = \frac{1}{2}$, as assumed by the author in his example, Equation (36) gives $\delta = 0.03125\, D$ and Equation (82) gives $\delta = 0.03229\, D$, or 3.3% greater thickness of boundary layer. For larger values of R the relative error is less, and the absolute error very much less. Thus, the author's assumption is satisfactory for purposes of calculation, but it is not necessary to think of the velocity in the boundary layer as varying uniformly.

One other fact may be added. If Equations (32) and (35) are solved simultaneously, $\mathsf{R} = 117\,500$; that is, Equation (32) may be taken as the law of fluid friction in smooth pipes up to a Reynolds' number of 117 500 and Equation (36) beyond that point. As both these equations are empirical, it is probable that there is actually no sudden change of law.

JOE W. JOHNSON,[82] JUN. AM. SOC. C. E. (by letter).—An extremely timely summary of the modern conceptions and theories of the mechanics of fluid turbulence is offered in this paper. The presentation is clear and orderly and contains a consistent and logical system of notation. The application of these principles affords new methods of approach to many complex hydraulic problems.

One such application, for example, is the problem of silt transportation advanced by W. Schmidt[83] and outlined by Morrough P. O'Brien[84], Assoc. M. Am. Soc. C. E., in 1932, in a review of the theory of turbulent flow and its relation to silt transportation in open channels. In later papers, Messrs. J. B. Leighly[35] and J. E. Christiansen[36] discussed Professor O'Brien's paper and reported encouraging results in comparing measured silt distribution and theory. The silt referred to is the solid matter transported in suspension, but not that which moves as bed-load. It is recognized, of course, that there is no dividing line between the classifications. Therefore, in view of the fact that the difference between suspended load and bed-load is purely arbitrary, it is fairly reasonable to believe that the theories discussed by the author and by Professor O'Brien may ultimately provide an adequate and accurate relation between bed-load transportation and certain hydraulic elements.

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[82] Care, SCS, U. S. Dept. of Agriculture, Washington, D. C.

[83] "Die Massenaustauch in freien Luft und verwandte Erscheinungen", von W. Schmidt, H. Grand, Hamburg, 1925.

[84] "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transportation", by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Hydrology Section, 1933.

The Prandtl-von Kármán theory of velocity distribution in turbulent flow can also be applied to open channels of great widths where the lines of equal velocity are parallel to the channel bed and viscous resistances are assumed to be negligible. Equation (52) as developed for pipes, is a general equation for turbulent flow in either tubes or open channels. For open channels, the expression for the distribution of velocity in the central region of the flow is:

$$\frac{v_{\max} - v}{\sqrt{\frac{\tau_0}{\rho}}} = -\frac{1}{\kappa}\left[\log_e\left(1 - \sqrt{1 - \frac{y}{d_0}}\right) + \sqrt{1 - \frac{y}{d_0}}\right] \ldots\ldots (83)$$

or,

$$\frac{v_{\max} - v}{\sqrt{\frac{\tau_0}{\rho}}} = \phi_1\left(\frac{y}{d_0}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

in which τ_0 is the intensity of shear at the channel bottom; d_0, the depth of flow, or the distance from the bottom to the point of maximum velocity when the maximum velocity is below the water surface; and y, the distance of the velocity measurement, v, from the channel bottom (see Fig. 23).

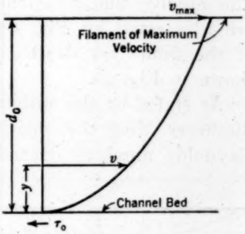

FIG. 23.—VERTICAL VELOCITY CURVE IN OPEN CHANNEL.

Equation (84), derived without regard to wall conditions, gives a theoretical velocity distribution only in the central regions of flow. Another equation must now be derived for the velocity distribution near the wall. Substitution of Prandtl's mixing length as given by the relation,

$$l = \kappa\, y \ldots\ldots\ldots\ldots\ldots (85)$$

into Equation (44) gives the expression,

$$\frac{dv}{dy} = \frac{1}{\kappa\, y}\sqrt{\frac{\tau_0}{\rho}}\sqrt{\frac{d_0 - y}{d_0}} \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

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[35] "Turbulence and Transportation of Rock Débris by Streams", by J. B. Leighly, *The Geographical Review*, Vol. XXIV, No. 3, July, 1934.

[36] "Distribution of Silt in Open Channels", by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Hydrology Section, 1935, p. 478.

Integration of Equation (86) with the condition that $v = 0$ when $y = 0$, gives,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = \frac{2}{\kappa}\left[4 + \sqrt{1 - \frac{y}{d_0}} - \tan h^{-1}\sqrt{1 - \frac{y}{d_0}}\right] \text{.......}(87)$$

or,

$$\frac{v}{\sqrt{\frac{\tau_0}{\rho}}} = \phi_2\left(\frac{y}{d_0}\right) \text{.....................}(88)$$

Of primary interest to the hydraulic engineer, of course, is the agreement between the foregoing theoretical formulas and experimental data. For illustration in this respect, vertical velocity curves will be used, that were measured by the writer in connection with research work at the U. S. Waterways Experiment Station, at Vicksburg, Miss. These experiments consisted of complete velocity traverses at a section near the center of a tilting rectangular flume, 45 ft long, 2.313 ft wide, and 1.3 ft deep, with smooth cement lining. The flume was given slopes of 0.0002, 0.0005, 0.0010, 0.0015, 0.0020, and 0.0040. At all times the water-surface slope was maintained parallel to the channel bottom. Velocities were measured by an accurately calibrated Pitot tube, which was mounted on the lower end of an ordinary laboratory gage which, in turn, was mounted on a rack that traveled upon a horizontal bar. The horizontal bar support was graduated along the top side to 0.01-ft intervals, and the laboratory gage read to 0.001 ft. With this precise horizontal and vertical control, it was possible to place the Pitot tube at identical locations for all tests and check measurements. The number and location of velocity observations in a typical cross-section are illustrated by Fig. 24. Average vertical velocity curves in the center of the flume for depths of 0.10 ft and 0.20 ft and for various slopes are shown in Fig. 25.

As stated by the author in referring to straight tubes, it was an important discovery that the relative motion of the particles at moderately large Reynolds numbers depends on the fall in pressure and not at all on the

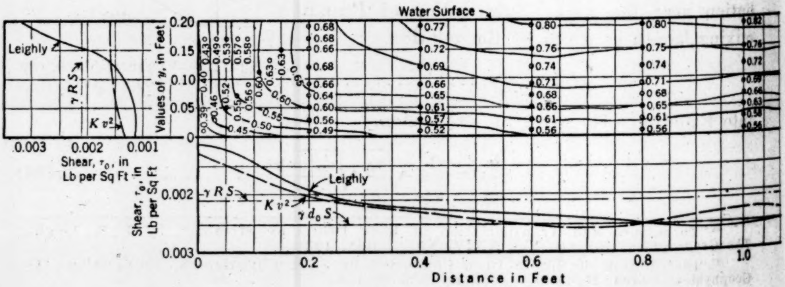
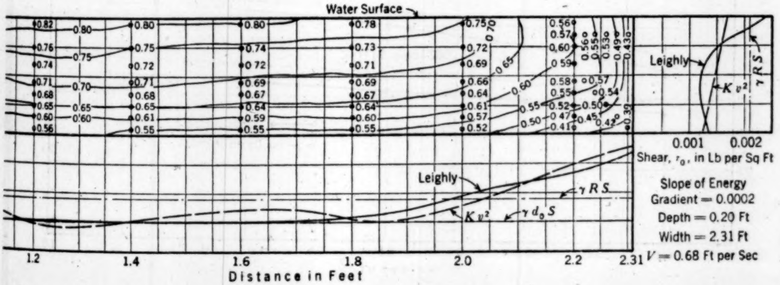


Fig. 24.—Distribution of Velocity and

character of the wall, so that, therefore, with constant fall in pressure, the velocity distribution curves in tubes with different values of relative roughness can be brought into conformity by shifting them along the velocity axis. The single curve resulting from superimposing the central parts of the curves in Fig. 15(*a*) of the paper is evidence of this fact; but it is not true, of course, for the layer in immediate contact with the wall, where the velocity increase is greater on a smooth surface than on a rough one. Comparable to this for open channels would be the vertical distributions in channels of infinite width in which surface roughness, depth of flow, and the slope of the energy gradient have been adjusted so that the bottom shearing force given by $\gamma\ d_0\ S$ is a constant. In channels of finite width, however, the question of the effect of shearing force along the side walls, as well as the bottom, becomes important. In this case the bottom shear is no longer equal to $\gamma\ d_0\ S$. Where flow resistance at the side walls is appreciable, the average shearing force on the side walls and bottom is given by $\gamma\ R\ S$, in which R is the hydraulic radius and S = slope of the energy gradient. As illustrated by Fig. 24, the shearing force on the bottom exceeds the average value in most cases and approaches, but should not equal, $\gamma\ d_0\ S$. A dimensionless plot similar to Fig. 15(*a*) is not attempted in this discussion, but a plot of velocity distribution is presented (Fig. 26) for those cases in which the bottom shearing force, as given by equal values of $\gamma\ d_0\ S$, is approximately constant. These velocity distribution curves differ slightly due to the fact, no doubt, that the bottom shearing forces may be (and are, perhaps, not) constant for any two pair of curves (analogous to this in pipe flow would be the condition in which fall in pressure is not constant); nevertheless, by superimposing those curves in which the values of the bottom shearing forces are approximately equal, it is evident that similar flow conditions will occur in the central region of the flow when the value of the bottom shearing force is a constant.

As a means of further comparison, the dimensionless plot defined by Equation (84), and presented in Fig. 27, is made for the same velocity distribution curves as those in Fig. 26. Since the value of the bottom shearing force, τ_0, is not accurately known, the mean value, $\gamma\ R\ S$, is used for illustrative purposes; also included in Fig. 27 for comparison are plots of the theoretical formulas, Equations (83) and (59). It is obvious from an exami-



SHEARING FORCES IN RECTANGULAR FLUME.

nation of Fig. 27 that the experimental curves differ from the theoretical curves by amounts too large to be attributed to experimental errors. This difference is due to several fundamental and important factors. The assumption that the bottom shearing force, τ_0, is equal to $\gamma R S$, or the fact that the value of the universal constant may be different from 0.40, does not alone account for the departure of the experimental velocity distribution from the theoretical one. Of perhaps more importance is the fact that the velocity distributions presented in Fig. 25 were observed in a channel of finite width, where the frictional resistance exerted by the side walls is appreciable.

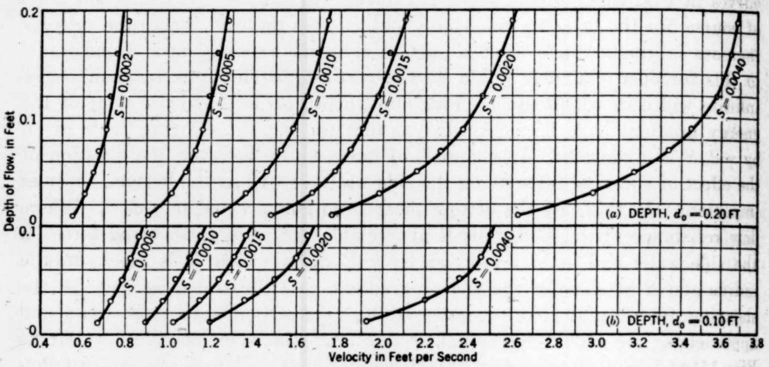


FIG. 25.—VERTICAL VELOCITY CURVES ALONG THE CENTER LINE OF A RECTANGULAR FLUME, 2.31 FEET WIDE.

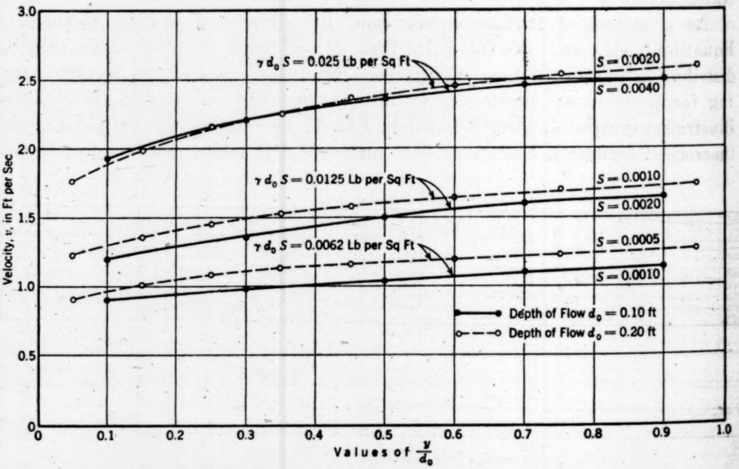


FIG. 26.—VELOCITY DISTRIBUTION IN OPEN CHANNELS WHERE BOTTOM SHEARING FORCES ARE APPROXIMATELY CONSTANT.

Therefore, the curves are the resultant of shearing forces exerted by both the bottom and the side walls and are not the velocity distributions resulting from shearing forces exerted only by the bottom. It is to be anticipated that frictional forces exerted by side walls tend to present experimental curves that are more erect than theoretical curves.

Although not considered in the foregoing discussion, it is of primary importance to recognize that the energy of water flowing in open channels is not dissipated by frictional resistances only. In the general case energy is used in several different ways, the most important of which are: (*a*) In overcoming frictional resistances of drag along the channel walls; (*b*) in overcoming resistances due to purely viscous shearing forces; (*c*) in transporting solids; (*d*) in the alteration of the channel cross-section; (*e*) in creating and maintaining waves on the surface of the stream; and (*f*) in creating and maintaining sand ripples. Although Factor (*a*) is perhaps the most important energy loss in ordinary open-channel flow, it is necessary that the existence of the other factors be recognized. An example in this latter respect is a movable-bed model with sand banks and sand bed. In this case the loss of energy due to Factors (*c*), (*d*), and (*f*) probably assumes a value in excess of that of Factor (*a*).

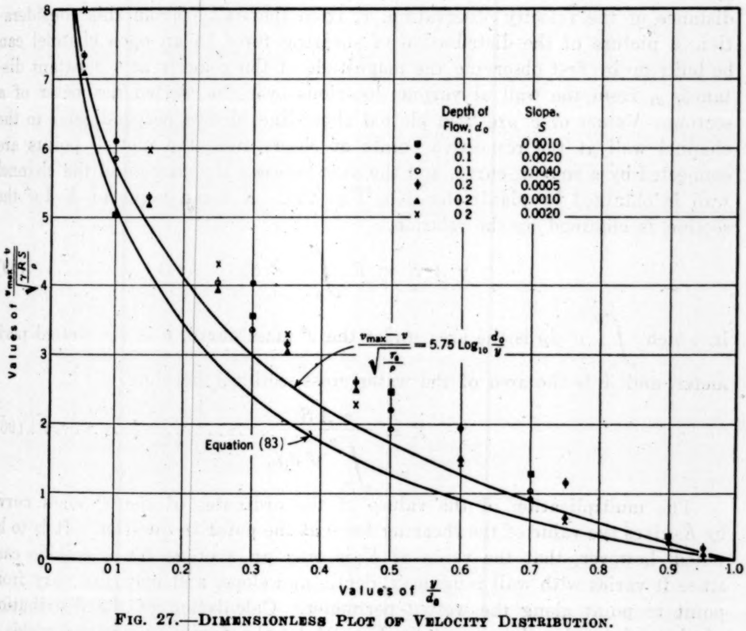


FIG. 27.—DIMENSIONLESS PLOT OF VELOCITY DISTRIBUTION.

The assumption that the bottom shearing force is evenly distributed over the wetted perimeter (that is, $\tau_0 = \gamma \, R \, S$) is merely an approximation as is shown by the irregularities in the isovels. That the trace of the isovels in a stream cross-section and the distribution of shearing force over the wetted perimeter are inextricably related was recognized by Leighly[37]. In 1932, he described a European method of approximating the distribution of shearing force, which consists in dividing the wetted perimeter into equal lengths and drawing from the dividing points between these areas, lines, perpendicular to the equi-velocity lines. The areas bounded by the perimeter, the line of maximum velocity, and the orthogonal lines are proportional to the shear at the bottom. The result of such an analysis for the cross-section shown in Fig. 24 is plotted below the section. For comparison, a plot of the average distribution of shear, as given by $\gamma\ R\ S$, and the bottom shear, as given by $\gamma\ d_0\ S$, are also given. Another approach to approximating the distribution of shear along the wetted perimeter is from the consideration that the shear is proportional to the square of the velocity measured at a distance, y, from the wall; that is, τ_0 varies as $K\ v^2$, in which K is a function of the distance of the velocity observation, v, from the wall. From this consideration, a picture of the distribution of shearing force in an open channel can be built up by first observing the magnitude of the velocity at a constant distance, y, from the wall at various locations over the wetted perimeter of a section. Values of v^2 are then plotted along lines drawn perpendicular to the channel wall at the respective points of observation, the plotted points are connected by a smooth curve, and the area between the curve and the channel wall is obtained by planimeter (see Fig. 24). A mean value of K for the section is obtained by the relation,

$$\gamma\ A\ S = K \int_0^p v^2\ dp \qquad \text{(89)}$$

in which $\int_0^p v^2\ dp$ is the area under the v^2 mass curve, p is the wetted perimeter, and A is the area of the water cross-section; therefore,

$$K = \frac{\gamma A S}{\int_0^p v^2\ dp} \qquad \text{(90)}$$

The multiplication of the values of the ordinates of the v^2 mass curve by K gives the value of the shearing force at the point in question. It is to be noted, however, that the value of K is only an average for a specific case since it varies with wall roughness, depth, and slope, and may also vary from point to point along the wetted perimeter. Calculation of the distribution of shear force for the section in Fig. 24 by the foregoing method yields a distribution that compares quite well with the distribution obtained by applying Leighly's method. Of particular interest is the variation in shearing force distribution resulting when various depths and slopes were introduced into the flume, calculation of shearing force being made by the v^2 mass-curve method. The total shearing force exerted by the side walls is equal to the areas under those parts of the shearing-force distribution diagram which have the side walls as bases. The percentage of the total shearing force exerted by the two side walls was found to vary with depth and to be a constant for a particular depth, regardless of slope. Fig. 28 shows a plot of the percentage of shearing force exerted by the side walls for various depths of flow in the flume.

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[37] "Toward a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams", by J. B. Leighly, Univ. of California, *Publications in Geography*, Vol. 6, No. 1, 1932, Univ. of California Press, Berkeley, Calif.

The information presented in Fig. 28 gives a picture of the magnitude of the error introduced when it is assumed that the total energy of a flowing stream is dissipated in frictional resistances along only the channel bottom. This erroneous assumption is particularly serious in flume studies of the transportation of bed-load material. The usual procedure in such work has been to attempt to correlate the rate of bed-load movement (in pounds per hour per foot width of flume) with the tractive force (in pounds per square foot) as calculated by the term, $\gamma\ d_0\ S$.

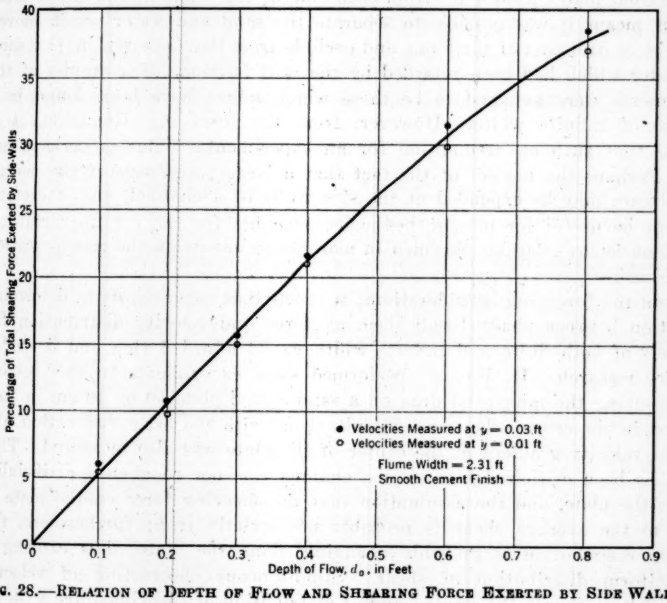

FIG. 28.—RELATION OF DEPTH OF FLOW AND SHEARING FORCE EXERTED BY SIDE WALLS.

Examination of Fig. 28 shows clearly that the difference between the true value of the bottom shearing force and the value of $\gamma\ d_0\ S$ varies with the depth of flow. Of particular importance is the fact that for a certain depth, d_0, this difference becomes less than is shown in Fig. 28 if the bottom roughness is made rougher than the side walls; that is, for example, in the case of the sand bed in the ordinary, smooth-wall, experimental flume the bed roughness increases with increased height of sand ripple, the result being a decrease in the percentage of the total shearing force exerted by the side walls (depth and slope, of course, are held constant as sand ripples change form). In his experiments on the transportation of bed-load, Hans Kramer, M. Am. Soc. C. E.[38], attempted to correct the condition of unequal bottom and wall roughness by roughening the side walls artificially with sand sprinkled on a coat of wet oil paint to which the grains adhered after the paint dried. Although this procedure is fundamentally sound, it is doubtful whether this type of roughness is effective in reducing the percentage of the shearing force that is exerted on the side walls to the value where the bottom shear can be considered equal to $\gamma\ d_0\ S$ as in a channel of infinite width. In this case the use of the average shearing force, as given by the term, $\gamma\ R\ S$, perhaps, gives a better approximation of the value of the bottom shear. Another method of reducing the effect of frictional resistances at the side walls is an arrangement used by C. H. MacDougall[39] for diverting the sand and the water which flowed adjacent to the walls of the channel. This was done by two longitudinal plates fixed 4 in. from each side of the flume at the outlet tank. By this means it was possible to separate the sand and water which moved down the central part of the flume and exclude from the measurement that sand and water which had been retarded by the wall friction. The results of the experiments were assumed to be those which might have been found in a channel of infinite width. However, from the foregoing discussion, it is obvious that such an assumption for an experimental flume is perhaps not true. Perhaps the neglect of the fact that a large percentage of the energy of a stream may be expended at the side walls of a channel, and, therefore, may not be useful for moving bed-loads, accounts for the erratic variations and inconsistent relations obtained in many experiments on the transportation of bed-load.

From the foregoing considerations, it seems that experiments to determine a relation between channel-wall shearing force and velocity distribution for channels of both finite and infinite width would afford a rich and desirable field for research. H. Engels[40] performed some experiments in this respect by measuring the tangential drag on a sand-coated plate 50 by 10 cm in size, recessed in the channel bottom. Simultaneous with the drag observation, the vertical velocity gradient at the center of the plate was also measured. This work has been criticized because the channel was not roughened artificially, as was the plate, and the assumption that the shearing force on the plate is equal to the average shear is probably not strictly true; furthermore, the velocity distribution is probably changing along the plate, thus causing a non-uniform distribution of shear. Simultaneous observation of velocity gradient by accurate Pitot tubes and of distribution of shearing force with an instrument perhaps of the type recently used by Fage[41] would prove of great value to a better understanding of the mechanics of flow in open channels.

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[38] "Sand Mixtures and Sand Movement in Fluvial Models", by Hans Kramer, *Transactions*. Am. Soc. C. E., Vol. 100 (1935). p. 798.

[39] "Bed Sediment Transportation in Open Channels", by C. H. MacDougall, *Transactions*. Am. Geophysical Union. 1933.

[40] "Versuche über den Reibungswiderstand zwischen stromenden Wasser und Bettsohle", von H. Engels, *Zeitschrift für Bauwesen*, p. 473, 1912.

[41] "An Experimental Determination of the Intensity of Friction on the Surface of an Aerofoil", by A. Fage and V. M. Falkner, *Proceedings*, Royal Soc., Series A, Vol. 129. 1930, p. 378; or for description, see "Aerodynamic Theory", Vol. 111, Pt. G, W. F. Durand, Editor. (Published in English by Julius Springer, Berlin, 1935.)

Warren E. Wilson,[42] **Jun. Am. Soc. C. E.** (by letter).—In presenting a useful summary of many of the modern developments of fluid mechanics the author has performed a valuable service to the Engineering Profession.

In the analysis of data on flow in open channels the Reynolds' number has been used to some extent and is expressed thus,

$$\mathsf{R} = \frac{V R}{\nu} \text{ .... } (91)$$

in which the nomenclature is that of the paper. The author quite rightly questions the use of the hydraulic radius in this case.

The success which has been reported in such usage may be due in part at least to the fact that most experimental work on open channels covers a comparatively limited depth range. Furthermore, most experiments have been conducted with a small depth-width ratio, thereby giving a flow which approximates that in an open channel of infinite width.

Lindquist has suggested[43] plotting the dimensionless quantities, $\frac{V^2}{R g}$ and $\frac{V R}{\nu}$, as ordinates and abscissas, respectively, on logarithmic paper. The results obtained for several sets of experimental data, notably those of Darcy and Bazin, indicate an equation of the form:

$$\frac{V^2}{R g} = C \left(\frac{V R}{\nu}\right)^n \text{ .... } (92)$$

However, if one were to plot a series in which the depth-width ratio varied over a range, such as 0.1 to 1.0, it seems entirely probable that the foregoing simple relation (Equation (92)) would no longer hold.

With small values of the depth-width ratio, the problem involves wide channels approximately similar, geometrically. With successive cross-sections of a stream in a series covering widely differing values of the depth-width ratio there are channels that do not even approximate a similar cross-section. In a wide channel the bottom shear is by far the predominant retarding force. In a square cross-section the effect of side-wall shear is of the same order of magnitude as that of bottom shear.

For wide channels the ratio of unit shear on the bottom to that on the sides may be nearly the same throughout a series of experiments. Such is not the case if the depth-width ratio is increased to any great extent, thus leaving the range of wide channels.

It is impossible, therefore, to express the unit shearing stress at the boundary as a simple function of the velocity and, until a completely satisfactory theory of turbulent flow is available, an experimental approach to the problem seems in order. A determination of the distribution of shearing stress in the boundary surface as a function of the depth-width ratio would provide useful information for a further analysis of the flow in open channels.

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[42] Asst. Prof., Civ. Eng., South Dakota State School of Mines, Rapid City, S. Dak.

[43] "Hydraulic Laboratory Practice", by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., p. 819.

THEODOR VON KÁRMÁN,[44] M. AM. SOC. C. E. (by letter).—It appears that this paper has two main objectives: First, to present the results of a series of experimental and theoretical investigations on the mechanism of turbulent flow to practical engineers; and, second, to suggest the use of rational formulas resulting from those investigations in design problems.

As far as the first objective is concerned, even the most practical engineer will agree that a better understanding of the underlying phenomena is useful also for the practical work. The only question is, whether in some cases total ignorance of some facts is not better than half-knowledge. Authorities disagree, however, on the second point. The writer has often been asked: "Are the new formulas free from any empirical element—that is, empirical assumptions and empirical constants? Why not use purely empirical formulas which have been carefully adjusted to the real conditions by experimental determination of the coefficients involved?" From a very competent source the writer learns that the formulas used for fluid resistance in actual practice are so exact that predictions agree with final results within 3 to 5%, whereas, unavoidable changes in the condition of the structures cause even larger differences.

Assuming that this statement is correct, nevertheless it appears to the writer that the rational formulas have at least three strong points:

First.—The rational formula is always dimensionally correct. This means that, in so far as the underlying assumptions are satisfied, it will give correct results independent of scale, absolute magnitude of speed, etc. Quite a few of the empirical formulas used in hydraulics are dimensionally incorrect. It seems that their success is based on the fact that experienced engineers use them only within a relatively narrow range.

Second.—Empirical formulas are good for interpolation, whereas rational formulas offer a fair chance for extrapolation. There is no reason why an empirical formula will give correct results beyond the limits of actual experiments. If, for instance, the friction coefficient, which is supposed to be a function of Reynolds number, was found, within a certain range, proportional to a certain power of that number, there is no justification to continue this law beyond the maximum Reynolds number used in the experiments. It is a common experience that almost every empirical function plotted in a suitable chosen scale on a logarithmic paper appears as a straight line within a small range. The theoretical formula for fluid friction in a smooth pipe suggested by the writer and given in the paper is deduced under the assumption that the influence of viscosity is negligible, except within a small range near the walls. Hence, it is probable that the formula will agree with the facts more and more, the larger the Reynolds number corresponding to the actual construction.

Third.—The theoretical formulas facilitate the transfer of results from one field to other related fields of research. Fluid friction and heat transfer in fluids are good examples for this point. Both phenomena are affected by the same mechanism of the turbulent flow. The problem of heat transfer is much more complex than the problem of friction. For instance, the friction coefficient for a smooth plate is a function of one parameter (Reynolds number) only, whereas the heat transfer depends on two parameters—namely, in addition to the Reynolds number, the ratio between the heat conduction coefficient and the product of heat capacity and viscosity. Hence, the flow conditions in oil and water will be similar if the corresponding Reynolds numbers are the same; but the heat flow will be different due to the relatively much smaller heat conduction coefficient of the oil. For this reason the heat transfer problem appears confusingly complicated from the standpoint of pure empirical research, and very elaborate series of research work in the past have yielded only information relating to a small section of the entire problem.

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The use of the results of the turbulence theory determines the general shape of the heat transfer formula, indicates the manner in which the Reynolds number will enter into the formula, and reduces the problem to the determination of one single function of one variable which is the ratio involving the aforementioned physical constant. In a similar manner the writer believes that the turbulence theory and the use of its results will facilitate the systematic solution of many other complex problems, as, for instance, that of silt transportation, which is now the center of interest.

For the reasons presented herein, papers such as that by Mr. Rouse are extremely useful, also, from a merely practical standpoint. The difficulty is to decide at which stage a certain theory is ripe enough to be injected into the pulsating veins of engineering routine practice. Handicapped, perhaps, by the fact that he had some opportunity to work on the theory presented, the writer has been somewhat cautious about making such a proposition. However, the results obtained in aeronautical practice, with the corresponding skin-friction formulas, are so satisfactory that it now seems to the writer that the introduction of rational formulas in hydraulics should be encouraged.

HUNTER ROUSE,[45] ASSOC. M. AM. SOC. C. E. (by letter).—Offered primarily as an interpretative review for hydraulic engineers of present-day knowledge of fluid turbulence, the paper was at the same time an attempt to emphasize basic methods of attack that have proved invaluable in the applied science of fluid mechanics. The interest that readers have shown for this subject through printed discussion, correspondence, and verbal comment is indeed encouraging. In this closing discussion, therefore, the writer seeks to clarify certain aspects of the problem that were questioned in the discussions, introducing as further illustration of the basic thesis several related phases having direct bearing upon the contents of the paper.

Professor Posey's statistical approach to the study of turbulent fluctuations is well adapted to research of a fundamental nature, but it would seem to the writer to be of questionable practical value. Evidently the behavior of the three components of fluctuation is a sufficient measure of the degree of turbulence and hence of all phenomena depending upon the mixing process. Although leading surely to a more thorough understanding of the turbulence mechanism, such analysis, divorced as it is from the basic pattern of mean flow, would scarcely tend to relate the degree of turbulence to flow characteristics readily measurable. The fact must be emphasized that the success of von Kármán's method lies in proving the existence of correlation at every point and in recognizing the similarity of the turbulent pattern from point to point, but above all in determining the degree of mixing as a universal function of the mean velocity gradient.

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Although it is not altogether likely to provide a simple mathematical statement of commercial pipe resistance, the analysis of roughness suggested by Professor Posey would surely help to clarify the basic action of surface irregularities. Systematic experimentation on controlled artificial roughness of an elementary nature, to which this type of analysis is admirably suited, may well replace such hit-and-miss investigations of internally threaded pipe as have been conducted in past decades. The roughness parameters used by Professor Posey—ϵ, ϵ', and $i_{av}\,\epsilon''$—are excellent criteria on which to base such research.

For the present, however, it would seem not only necessary but practicable to rely upon experimental determination of a single characteristic length parameter to designate the effective absolute roughness of a surface, whether referred to the sand grains used by Nikuradse or to some standard otherwise determined. Nikuradse was the first to make systematic studies of artificial roughness covering a sufficient range to be generally significant and, therefore, his studies would seem to merit use as a reference for comparative rating of other roughness types. Nevertheless, such adoption would be regrettable if only for the fact that the sand grains in his analysis, and his method of applying them to the surface, do not lend themselves to ease in reproduction. Sand of a given mean grain diameter undeniably varies greatly in shape and hence in roughness action; and even a change in the grade of lacquer used as binder might alter the absolute roughness appreciably. It would be better for one to select arbitrarily, as a standard reference, a numerical relative roughness yielding some convenient resistance coefficient; for instance, fix $\frac{r_0}{\epsilon}$ at 500 for a value of $f = 0.02$. This point lies close enough to Nikuradse's values to provide ease in comparison, but its arbitrary selection renders it forever independent of a standard sand, lacquer, and technique of application. This value is then readily convertible to a single length parameter, ϵ, designating the effective absolute roughness of commercial surfaces; the sole drawback lies in the unavoidable fact that this length parameter is not dimensionless, and hence tables of effective absolute roughness of different materials must necessarily vary among the several dimensional systems.

In this connection, a paper by H. Schlichting[46] that appeared almost simultaneously with the writer's paper, describes further research on artificial roughness which merits notice at this point. In an endeavor to develop a means of testing actual samples of commercial surfaces for effective absolute roughness, Schlichting modified Nikuradse's apparatus to include a wide, shallow, rectangular test section of considerable length, one side of which was composed of the surface to be investigated, the remaining three sides being very smooth. Piezometer openings at frequent intervals were located along only the wide smooth side, and the test specimen was of sufficient length to permit full establishment of flow before the sections of pressure measurement were reached; Pitot tube traverses were possible at several locations.

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[46] "Experimentelle Untersuchungen zum Rauhigkeitsproblem", *Ingenieur-Archiv*, Band VII, Heft 1, 1936.

Beginning, for purposes of comparison, with sand surfaces similar to those of Nikuradse, Schlichting thereby proved that combined measurements of longitudinal pressure gradient and transverse velocity gradient were sufficient to determine the intensity of shear along the rough surface. This was accomplished through use of the expression for the universal velocity gradient near a smooth wall (Equation (55)), and the relationship:

$$(\tau_o)_s + (\tau_o)_r = -b\,\frac{dp}{dL} \qquad \text{(93)}$$

in which b is the distance between the smooth wall and the rough test specimen, and $(\tau_o)_s$ and $(\tau_o)_r$ designate the intensity of shear at the smooth and rough walls, respectively.

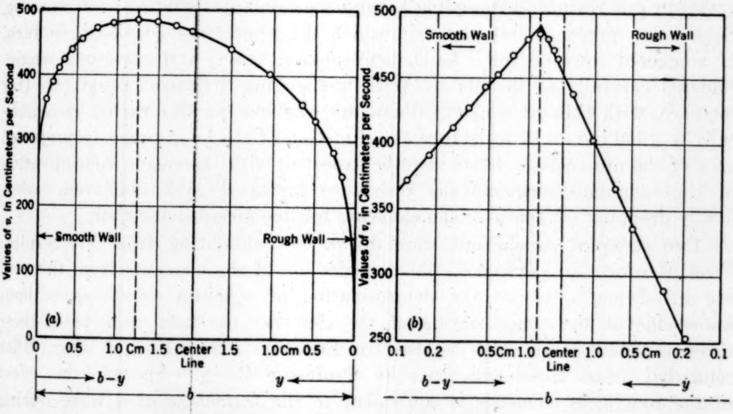

FIG. 29.—VELOCITY DISTRIBUTION BETWEEN SMOOTH AND ROUGH WALLS.

Referring to the paper it will be recalled that the universal velocity distribution curves for both smooth and rough walls were based upon the fact that the entire pattern of turbulent flow in the vicinity of the wall is a function of distance from the wall and hence is completely independent of any effect of the wall on the other side of the center line. Thus, even if the two opposite walls are of a different nature, it is still to be expected that only in the centermost regions of flow will the influence of both walls be felt. Fig. 29(b), taken from Schlichting's paper, is excellent vindication of this reasoning, the velocity distribution on either side of the maximum following a logarithmic function corresponding to the universal plots of Figs. 17 and 18. Furthermore, the constants of the several distribution curves for sanded surfaces determined in this way checked Nikuradse's measured values within a few per cent.

Use of such an apparatus for practical purposes (in Schlichting's case primarily for investigation of the roughness of ship hulls) is straighforward and simple: The test specimen in the form of a long flat plate is mounted in the rectangular conduit; measurements are made of pressure gradient and velocity distribution at several different flow rates (only one, however, being necessary theoretically). From the semi-logarithmic distribution curves Equation (55) enables the direct determination of $(\tau_0)_s$, and $(\tau_0)_r$ is then found at once from Equation (93). Equation (56), finally, provides a means of determining the effective absolute roughness referred to the sand used by Nikuradse; it is evident that to standardize some arbitrary value of ϵ requires simply a change in the constant, 5.85, of Equation (56). It should also be apparent that this procedure is correct only for that type of roughness which results in a resistance coefficient that is independent of the Reynolds number.

Other salient features of Schlichting's paper are equally deserving of notice in this review. Proceeding beyond the roughness studied by Nikuradse, Schlichting prepared test plates in which the projections consisted, in turn, of staggered rows of small balls, hemispheres, cones, and structural angles soldered carefully to the plate. With unchanging "relative" roughness (for instance, with balls of constant diameter), the spacing was varied systematically to yield curves of resistance as a function of the longitudinally projected area of the protrusions. As was to be expected, with increasing concentration of the roughness elements the resistance increased to a maximum value, finally dropping off again as the elements became crowded together.

Two essential conclusions were drawn by Schlichting from his results: First, the form and extent of linear projection of the elements from the wall are insufficient data for the determination of absolute roughness without knowledge of the concentration of the elements per unit wall area; these three variables might also be used in Equation (69), although they differ somewhat from those suggested by Professor Posey. Second, the effect of the roughness elements is equivalent to the formation of a wake during flow past an immersed body because the limiting resistance of each individual element investigated was of practically the same magnitude as that of a similar body moving through a fluid with a velocity equal to that at the outer edge of the roughness element (by "limiting resistance" is meant that of a single element as the concentration approached a minimum). As the elements become more crowded, it is clear that their effectiveness as individuals should decrease, the over-all effect still increasing until mutual interference finally results in a gradual reduction of the total resistance. Obviously, the limit of crowding is reached when the elements are packed so closely as to produce, in effect, a smooth wall.

The relative ease of experimentation, once such an apparatus for commercial testing is made available, and the evident dependability of results for widely different types of roughness, lead the writer to believe that such a method may lend itself to other purposes than simply the testing of commercial plate material. For instance, since the effect of roughness on turbulent resistance is identical for both closed and open conduits, such technique is easily adapted to the study of canal linings, whether of timber, concrete, or brick. Simply by building a test section of the material in question, and sealing the apparatus on top of it in inverted position, a conduit is formed exactly as in Schlichting's original tests. This would necessitate rebuilding the Chezy equation into a form such as that of Equation (16) or Equation (17) since the Kutter and Manning n used to determine the Chezy C is not a length parameter equivalent to the effective absolute roughness, ϵ. The idea has also occurred to the writer that similar methods might possibly be extended to the case of movable beds, eliminating the free surface by means of a smooth upper boundary. This might well permit investigation of the true shear and mean velocity distribution in the vicinity of the bottom, for through Schlichting's method both may be determined by indirect measurement.

Mr. Miller's comments on empirical *versus* analytical investigation must be examined more closely, for there are actually four, rather than two, methods of arriving at expressions for fluid motion. One must distinguish, first, between experimental measurement (which the writer by no means scorns) and empirical formulation of working rules for design—two processes which Mr. Miller seems to bulk as one. Experimental measurement is the mechanical determination of one or more flow characteristics for a given state of motion. Pure empiricism, on the other hand, includes the effort to develop a practical generalization of the results of experimental measurement for a number of different conditions; it may provide simple working formulas for design within the range of available data—but, although the data may be accurate, there is no guaranty whatever for the physical truth of the empirical statement, for only by chance are natural laws discovered in this way. Were this still the only recourse of the hydraulician, the subject of pipe resistance would probably be in the same jumbled state as it was before Blasius published his analysis of Schoder's experimental data.

On a considerably higher plane is the second method, that of Blasius and actually that which Mr. Miller is upholding—one which is partly analytical in character since it is based upon physically sound dimensional analysis; although experimental measurements are still necessary to determine the type of function and the numerical constants relating the several dimensionless parameters, the investigator is well beyond the chance of pure empiricism when he interprets his results. Still further advanced, it would seem to the writer, is the method of fluid mechanics presented in the paper, because it combines with dimensional analysis a reasonable and closely approximate physical analysis, thus determining the probable form of the function and leaving only the numerical constants to experimental investigation. The fourth method—complete rational analysis—leaves nothing to be found experimentally; it is the ultimate goal of every field of science, but as yet only in isolated cases has it been fully a success.

Although Mr. Miller looks forward to the attainment of this goal, he seems to frown upon present efforts in that direction. He must remember that the "law of turbulent flow" which he uses to emphasize the advantages of "empirical" relationships, and which "might" have been discovered earlier from experimental data, actually was far more the result of careful physical analysis than of empirical luck. Mr. Miller evidently attributes the law to Nikuradse's experimental work published in 1932,[15] whereas the latter was an outgrowth of von Kármán's original analysis published in 1930[19], [47]. As Mr. Miller finally remarks, an "infinite velocity at the center" would indeed be interesting, but the writer can recall no mention of such an assumption. The infinite velocity gradient at the wall, on the other hand, was not assumed to portray wall conditions, but to provide the necessary mathematical limit for a curve of which the characteristics are used only in the central regions of the flow. If ever found, a single analytic function to include both the laminar film and the zone of highly turbulent mixing would very likely prove too complex to serve any practical purpose.

As Mr. Yasines looks upon the application of dimensional analysis with such apparent skepticism, further discussion to supplement the original statements involving Equations (8) to (17) and (46) to (48) may possibly prevent future misunderstanding of this basic factor in the investigation of fluid motion. Mr. Yasines asks what means the investigator has of knowing which characteristics to include in the original expression. Briefly stated, the only variables belonging in any correct statement of flow conditions at constant temperature are as follows: The several linear and angular dimensions describing the geometry of the flow; two flow characteristics, a pressure gradient (or a shearing stress), and a velocity; and the fluid properties of density, specific weight, absolute viscosity, surface tension, and elastic modulus. Of these, either pressure gradient or velocity is usually considered the dependent variable. Use of the Π-theorem will group these variables in a number of dimensionless terms, each including four variables, three of which appear in every Π-term. It is most reasonable to select velocity, density, and a significant length as the three repeating variables, since this grouping yields as Π-terms the Froude, Reynolds, Weber, and Cauchy numbers.

If the flow is completely enclosed, specific weight and surface tension (and, hence, the Froude and Weber numbers) will disappear from the relationship. For cases of constant density, the modulus of elasticity (and, hence, the Cauchy number) may also be dropped. It is evident that use of wall shear instead of pressure gradient is perfectly sound, since the two are definitely related through Equation (15); although pressure is more easily measured, intensity of shear is often more significant. In open-channel flow the pressure gradient must be replaced by either boundary shear or surface slope. Little experience is necessary to reach these conclusions. However, if experience is completely lacking, the infallible method is to include every one of these variables in the basic statement, afterward eliminating from the equation those Π-terms which, through experimental trial, prove to have no effect upon the dependent variable.

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[47] "Mechanische Aehnlichkeit und Turbulenz", Verhandlungen des III. Internationalen Kongresses für technische Mechanik, Stockholm, 1930.

It must be remarked at this point that the Froude and the Reynolds numbers cannot "fail to perform their functions", despite Mr. Yasines' remark to the contrary. Only when they are misused will the investigator be disappointed, and then only he is to blame. It is futile, for instance, to expect the Froude number to bear the brunt of boundary roughness and viscous action without any assistance whatever from roughness and viscosity parameters, a condition too often encountered in model studies. "'Scale effect" and "limitations of similitude" are expressions often used to imply that correction factors are necessary to compensate for inherent weakness in theory. Such is by no means true; scale effect is fully covered by the several dimensionless parameters, and the limitations are not analytical, but practical.

Although the foregoing choice of the three repeating variables is the most logical, in that it separates the fluid force characteristics (specific weight, viscosity, surface tension, and elastic modulus) and thus leads to the most fundamental dimensionless parameters, it is sometimes expedient to use other groupings for the purpose of emphasizing some significant functional relationship not otherwise apparent. For example, the study of turbulence in smooth pipes involves only the measurable variables, V, D, ρ, $\frac{dp}{dL}$, and μ. Customary grouping leads to the expression,

$$\phi\left[\frac{\rho V^2}{D\,\frac{dp}{dL}},\ \frac{V D}{\frac{\mu}{\rho}}\right] = 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

which yields the familiar equation,

$$\frac{dp}{dL} = \phi'(\mathsf{R})\,\frac{\rho V^2}{D} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (95)$$

Mr. Miller has shown, implicitly, that by choosing D, ρ, and $\frac{dp}{dL}$ as the three repeating characteristics there will result,

$$\phi\left[\frac{\rho^{\frac{1}{2}} V}{D^{\frac{1}{2}}\left(\frac{dp}{dL}\right)^{\frac{1}{2}}},\ \frac{D^{\frac{3}{2}}\rho^{\frac{1}{2}}\left(\frac{dp}{dL}\right)^{\frac{1}{2}}}{\mu}\right] = 0 \;\ldots\ldots\ldots\ldots (96)$$

or, using more familiar parameters (see Equation (16)),

$$\phi\left(\frac{1}{\sqrt{f}},\ \mathsf{R}\sqrt{f}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (97)$$

A plot of $\frac{1}{\sqrt{f}}$ against log $\mathsf{R}\sqrt{f}$ yields a straight line, thereby disclosing the apparent fact that this line may be extrapolated without probable error. The writer cannot refrain from pointing out, however, that a logarithmic plot of Equation (95), first published by Blasius, also led to a straight line, and only after further research was it proved that extrapolation would introduce serious error. Assurance that Equation (75) may safely be extrapolated beyond the range of experimental data is not the outcome of Mr. Miller's dimensional analysis but of Professor von Kármán's analytical determination of the fact that the functional relationship actually is linear as long as viscous shear has a negligible effect upon the turbulent pattern. Since Equation (75) submitted by Mr. Miller states the same relationship as Equation (68), it does not eliminate the drawback mentioned in the text preceding Fig. 19—one can conveniently solve for **R** (or Q), with $f \left(\text{or } \dfrac{dp}{dL}\right)$ given, but not *vice versa*.

Mr. Yasines' use of the Navier-Stokes equations to illustrate his remarks on the dubious significance of the Reynolds number surely warrants further investigation. These equations are as follows:

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial x}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 v_x \ldots (98)$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial y}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 v_y \ldots (99)$$

and,

$$\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial z}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 v_z . (100)$$

in which v_x, v_y, and v_z, are the components of the instantaneous velocity vector, and,

$$\nabla^2 v_x = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}, \text{ etc.} \ldots\ldots\ldots\ldots\ldots (101)$$

The left side of each equation denotes the component of acceleration at any point with time and with distance in three-dimensional space; the right side represents the sum of all force components per unit mass which produce this acceleration. If the first term at the left of each equation is zero, the flow is steady (that is, unchanging with time); if the second, third, and fourth terms are zero, the flow is uniform (that is, unchanging with distance in the direction of motion). If the flow is both steady and uniform, there is no acceleration; the terms at the left disappear, and the terms at the right then indicate the equilibrium of pressure, weight, and viscous shear.

Obviously, only in this last case can the term for fluid density be dropped. Hence, Mr. Yasines' statement that in viscous flow the density plays no part holds true only if the flow is both uniform and steady. Under these conditions the density should not be included in the original group of variables for dimensional analysis; since there are then only four variables (pressure gradient, velocity, diameter, and viscosity), there will be only one Π-term, as seen from Equation (29); only the numerical constant, 32, cannot be found by this method. Equation (30) is derived by introducing the density at two compensating points, which produces a useful equation without altering the fundamental expression. Laminar flow, however, is uniform only if the stream lines are parallel, and it must be remembered that any boundary curvature or rapid change of flow section will involve mass acceleration, under which conditions appreciable velocity will make the density of decided importance.

The Navier-Stokes equations, contrary to popular opinion, apply just as accurately to turbulent flow as to laminar, for the velocity components used in the equations refer to the actual instantaneous velocity at any point in question. Whether the flow is laminar or turbulent, viscous action is the only means of reducing the total energy of flow, through conversion into heat. Although it is possible for laminar flow to be both uniform and steady, turbulent flow is always unsteady and non-uniform in the strict sense of the word, for the velocity fluctuations vary at a given point with time, and at a given instant with distance in any direction. Therefore, it is evident that, however correct the Navier-Stokes equations may be, it is futile to use them for a practical study of turbulence as long as they remain in their present form.

Statistically speaking, it is reasonable to consider each velocity component to be composed of a temporal mean value and a secondary fluctuating value:

$$v_x = \overline{v_x} + v'_x \dots\dots\dots\dots\dots\dots\dots (102a)$$

$$v_y = \overline{v_y} + v'_y \dots\dots\dots\dots\dots\dots\dots (102b)$$

and,

$$v_z = \overline{v_z} + v'_z \dots\dots\dots\dots\dots\dots\dots (102c)$$

Turbulent flow is then considered both steady and uniform if the values of the mean components do not vary with either time or space. By substituting these values for the velocity components in the original Navier-Stokes equations, one obtains parallel expressions for the Newtonian acceleration-force-mass equilibrium. Remembering that, while the temporal mean of any component of fluctuation is zero, the product of any two components (owing to the existence of correlation) will have a temporal average of finite magnitude, these parallel expressions may be written in terms of the mean velocity components and mean products of the components of fluctuation, as follows:[48]

$$\frac{\partial \overline{v_x}}{\partial t} + \overline{v_x}\frac{\partial \overline{v_x}}{\partial x} + \overline{v_y}\frac{\partial \overline{v_x}}{\partial y} + \overline{v_z}\frac{\partial \overline{v_x}}{\partial z}$$

$$= -\frac{1}{\rho}\frac{\partial}{\partial x}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 \overline{v_x} - \left(\frac{\partial \overline{(v'_x)^2}}{\partial x} + \frac{\partial \overline{v'_x v'_y}}{\partial y} + \frac{\partial \overline{v'_x v'_z}}{\partial z}\right) \dots (103)$$

$$\frac{\partial \overline{v_y}}{\partial t} + \overline{v_x}\frac{\partial \overline{v_y}}{\partial x} + \overline{v_y}\frac{\partial \overline{v_y}}{\partial y} + \overline{v_z}\frac{\partial \overline{v_y}}{\partial z}$$

$$= -\frac{1}{\rho}\frac{\partial}{\partial y}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 \overline{v_y} - \left(\frac{\partial \overline{(v'_y)^2}}{\partial y} + \frac{\partial \overline{v'_y v'_z}}{\partial z} + \frac{\partial \overline{v'_y v'_x}}{\partial x}\right) \dots (104)$$

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[48] "Turbulente Strömungen", by W. Tollmien, Handbuch der Experimentalphysik, Band IV, Teil 1, Leipzig, 1931.

and,

$$\frac{\partial \bar{v}_z}{\partial t} + \bar{v}_x \frac{\partial \bar{v}_z}{\partial x} + \bar{v}_y \frac{\partial \bar{v}_z}{\partial y} + \bar{v}_z \frac{\partial \bar{v}_z}{\partial z}$$

$$= - \frac{1}{\rho} \frac{\partial}{\partial t} (p + \gamma h) + \frac{\mu}{\rho} \nabla^2 \bar{v}_z - \left(\frac{\partial \overline{(v'_z)^2}}{\partial z} + \frac{\partial \overline{v'_z v'_x}}{\partial x} + \frac{\partial \overline{v'_z v'_y}}{\partial y} \right) \quad \text{..(105)}$$

Under these circumstances, the differential equation for steady uniform turbulent flow in a pipe will become, from Equation (103),

$$\frac{\partial (p + \gamma h)}{\partial x} = \mu \frac{\partial^2 \bar{v}}{\partial y^2} - \rho \frac{\partial \overline{v'_x v'_y}}{\partial y} \quad \text{..............(106)}$$

Mr. Yasines will note that, contrary to steady uniform laminar flow, there is now no possible way of eliminating the fluid density. In other words, once turbulent flow is described in terms of the mean, rather than instantaneous, velocity components (similar to the mean velocity, V, in the Reynolds number), then density must play a very essential rôle in any correct expression. (It is to be remarked that, for simplicity, the bar denoting temporal mean velocity was consistently omitted in the paper.)

From these considerations, Mr. Yasines' query about the writer's "rate of passage of momentum" should find a satisfactory answer. Since the components of fluctuation vary directly with the average velocity of flow, the rate of passage of momentum is a convenient means of representing, in measurable quantities, the magnitude of the momentum transport in the mixing process; and since the effect of viscous shear in terms of the mean velocity gradient varies inversely with the average velocity of flow, the Reynolds number for turbulent flow in pipes really denotes the ratio between the shearing intensity due to momentum transport, on the one hand, and that due to viscous shear in terms of the mean velocity gradient, on the other.

Similar conclusions will result from the following reasoning: In the Navier-Stokes equations the three components of the shearing stress, τ, are simply those due to viscous action, regardless of whether the flow is laminar or turbulent:

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad \text{..............(107)}$$

$$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \quad \text{..............(108)}$$

and,

$$\tau_{zx} = \tau_{xz} = \mu \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) \quad \text{..............(109)}$$

Once the instantantous velocity gradient is replaced by the mean velocity gradient, if the flow is turbulent there must exist additional terms to express the high viscous stresses in the turbulent eddies—that is, the effective shear of the momentum transport; introducing Boussinesq's turbulence coefficient, η, these become:

$$\bar{\tau}_{xy} = \bar{\tau}_{yx} = \mu \left(\frac{\partial \bar{v}_x}{\partial y} + \frac{\partial \bar{v}_y}{\partial x} \right) -- \rho \, \overline{v'_x v'_o} = (\mu + \eta) \left(\frac{\partial \bar{v}_x}{\partial y} + \frac{\partial \bar{v}_y}{\partial x} \right) \quad ..(110)$$

$$\bar{\tau}_{yz} = \bar{\tau}_{zy} = \mu \left(\frac{\partial \bar{v}_y}{\partial z} + \frac{\partial \bar{v}_z}{\partial y} \right) - \rho \, \overline{v'_y v'_z} = (\mu + \eta) \left(\frac{\partial \bar{v}_y}{\partial z} + \frac{\partial \bar{v}_z}{\partial y} \right) \quad ..(111)$$

and,

$$\bar{\tau}_{zx} = \bar{\tau}_{xz} = \mu \left(\frac{\partial \bar{v}_z}{\partial x} + \frac{\partial \bar{v}_x}{\partial z} \right) - \rho \, \overline{v'_z v'_x} = (\mu + \eta) \left(\frac{\partial \bar{v}_z}{\partial x} + \frac{\partial \bar{v}_x}{\partial z} \right) \quad ..(112)$$

It must be realized that Equations (107) to (109) and (110) to (112) are merely different ways of stating the same basic effect of viscous shear.

Reducing Equation (110) to the case of flow in a pipe:

$$\bar{\tau} = \mu \frac{d\bar{v}}{dy} - \rho \, \overline{v'_x v'_y} = (\mu + \eta) \frac{d\bar{v}}{dy} \quad \ldots\ldots\ldots\ldots(113)$$

In the paper it was shown that with increasing Reynolds number the magnitude of η becomes increasingly greater than that of μ, and that above the approximate Reynolds number of 100 000, the magnitude of μ is comparatively insignificant. In the light of the foregoing discussion, the Reynolds number might also be considered to denote the relative magnitude of η and μ, the former being averaged over the flow section.

Since in highly turbulent motion $\mu \dfrac{d\bar{v}}{dy}$ is generally negligible when compared with $\eta \dfrac{d\bar{v}}{dy}$, Equation (113) may then be written in the form,

$$\bar{\tau} = - \rho \, \overline{v'_x v'_y} = \eta \frac{d\bar{v}}{dy} \quad \ldots\ldots\ldots\ldots\ldots\ldots(114)$$

Introducing the expression, $| v'_x | = l \dfrac{d\bar{v}}{dy}$ (refer to the development of Equation (42)):

$$\frac{\eta}{\rho} = | v'_y | \, l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(115)$$

in which the vertical bars denote mean absolute magnitude, regardless of sign.

Equation (115) is of very basic importance, for it not only lends added clarity to Boussinesq's coefficient of turbulence, but is a direct measure of the transporting power of the mixing process. The writer stressed only the transport of momentum by this means, but the fact must be appreciated that it is the same mechanism which results in a transportation of heat, salinity, or suspended matter from one region of turbulent flow to another. The rate of transport of momentum has been shown to depend upon only the factor, $\frac{\eta}{\rho}$. and the mean velocity gradient, thus producing an effective shearing stress:

$$\bar{\tau} = \rho \, | v'_y | \, l \, \frac{d\bar{v}}{dy} \quad \text{.....................(116)}$$

The problem of suspended load in a stream may be attacked in similar fashion. If at any depth of flow there are n particles per unit volume of fluid, the concentration gradient being written $\frac{dn}{dy}$, then, due to the transverse fluctuations, fluid masses bearing n particles per unit volume will be carried the mean distance, l, across the flow to regions where the concentration differs by the amount, $l \, \frac{dn}{dy}$. Thus, the temporal rate of passage of sediment per unit area, N, will be the product of the rate per unit area of transverse flow, $| v'_y |$, and the difference between the sediment concentration in the traveling fluid mass and that of the region into which it comes, $- \, l \, \frac{dn}{dy}$ —in other words, the product of $\frac{\eta}{\rho}$ and the sediment gradient:

$$N = - \, | v'_y | \, l \, \frac{dn}{dy} \quad \text{.....................(117)}$$

Identical reasoning applies to the transportation of salinity, and similar considerations show that the rate of heat transfer per unit area, q, will depend only upon $\frac{\eta}{\rho}$, the specific heat, c, and the mean temperature gradient across the flow, $\frac{d\,\theta}{dy}$:

$$q = - \, | v'_y | \, l \, c \, \frac{d\,\theta}{dy} \quad \text{.....................(118)}$$

Such heat transfer is purely convective, and must be distinguished from the process of conduction in the laminar film.

The parallel development and structure of Equations (116) to (118) was shown to excellent advantage by von Kármán in a résumé of turbulence theories[49]. Since the problem of suspended load in a stream is of paramount interest in river hydraulics, amplification of this method of treatment should be of value at this time. If the sediment transportation of the stream as a whole is in a uniform state of equilibrium, the rate of upward transfer by the mixing process must equal the downward movement due to settling. Designating the settling velocity of a given particle size by w, then N, the temporal rate of transportation per unit area, must equal the number of particles per unit volume multiplied by their rate of fall:

$$N = w \, n = - \, | v'_y | \, l \, \frac{dn}{dy} \quad \text{................(119)}$$

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[49] "Some Aspects of the Turbulence Problem", *Proceedings*, Fourth International Congress for Applied Mechanics, Cambridge, England, 1934.

This expression may be integrated in the form,

$$\log_e \frac{n}{n_a} = - w \int_a^y \frac{dy}{|v'_y|\, l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (120)$$

in which $\frac{n}{n_a}$ is the relative concentration at any point, referred to some arbitrary elevation, a, above the bottom. The evaluation of the last term in the expression depends only upon knowledge of the kinematic turbulence factor, $\frac{\eta}{\rho} = |\, v'_y \,|\; l$, as a function of the depth.

As shown by von Kármán, introduction of Equation (116) yields an integrable expression,

$$\frac{dy}{|v'_y|\, l} = \frac{dy}{\dfrac{\tau}{l\,\dfrac{d\bar{v}}{dy}}} = \frac{l\,\dfrac{d\bar{v}}{dy}}{\tau_0\left(1 - \dfrac{y}{d}\right)}\, dy \quad \ldots\ldots\ldots\ldots\ldots (121)$$

in terms of the mean velocity gradient. If the latter is of the universal logarithmic type, then from Equation (59),

$$\frac{d\bar{v}}{dy} = \frac{1}{\kappa}\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (122)$$

and Equation (120) becomes:

$$\frac{n}{n_a} = \left[\frac{1 - \dfrac{y}{d}}{\dfrac{y}{d}} \times \frac{\dfrac{a}{d}}{1 - \dfrac{a}{d}}\right]^z = \left[\frac{1 - \dfrac{y'}{d'}}{1 + \dfrac{y'}{a}}\right]^z \quad \ldots\ldots (123)$$

in which $z = \dfrac{w}{\kappa\sqrt{\dfrac{\tau_0}{\rho}}}$. Two pertinent facts are to be noted in Equation (123): The relative sediment distribution, for a given particle size (that is, for a given settling velocity), is a function of relative depth only. It cannot, however, be extended to the channel bottom, but only to some arbitrary depth, $d' = d - a$, and then only if the concentration at that depth is not so great as to change either the density of the fluid mixture or the assumed velocity distribution. If the bottom is smooth, the logarithmic velocity curve does not extend to the bottom, and there is no mixing in the final layer of fluid at the boundary. If the bottom is rough, knowledge of the total suspended load would permit complete solution of the sediment distribution (provided the logarithmic law could be extended to the free surface), but this expression in no way permits determination of the total transporting capacity of the stream. The latter probably depends upon conditions in the transition region between movable bed-load and completely suspended matter, a problem still awaiting a satisfactory physical analysis.

On the assumption that the logarithmic velocity curve actually extends to the free surface of a very wide stream, Equation (123) enables one to construct a plot of sediment distribution curves (Fig. 30) for various values of the suspended load parameter, $z = \dfrac{w}{\kappa \sqrt{\dfrac{\tau_0}{\rho}}}$, indicating, for a given state of flow, the relative distribution of the different particle sizes based upon their settling velocities. Such a diagram was first developed by Arthur T. Ippen, Jun. Am. Soc. C. E., at the suggestion of Professor von Kármán. An interesting comparison may be made with a group of measured curves published by L. G. Straub[50], Assoc. M. Am. Soc. C. E.

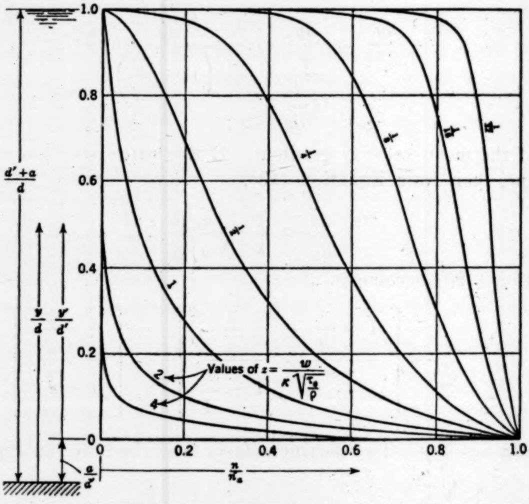

FIG. 30.—DIMENSIONLESS PLOT OF SUSPENDED LOAD DISTRIBUTION.

Professor Powell's remarks on the continuity of the f- versus R-function and on the actual curvature of the velocity gradient in the laminar film are quite true. It is not only a probability, but a physical certainty, that there is no sudden change of law at about $\mathsf{R} = 100\,000$. Viscous shear in terms of the mean velocity gradient diminishes gradually from the critical region on, but never disappears. It is a simple matter to construct a smooth curve through the measured points, somewhat less simple to express its variation in a brief empirical formula, and, to date, quite impossible to determine this function analytically. Neither is there any abrupt limit to the laminar film—despite the obvious convenience of designating an arbitrary point of juncture between turbulent and laminar regions. As Professor Powell will have noticed, Equation (36) was offered to give a preliminary, "rough idea" of boundary layer thickness "when the Reynolds number is high." To paraphrase his remark, it is not necessary to think of the velocity in the boundary layer as varying non-uniformly in this region, since the departure from uniform variation is quite insignificant—particularly when compared with the obviously arbitrary magnitude of v_w. Equation (62) completely avoids this latter discrepancy, and still introduces no appreciable error. Obviously, however, it does not apply to the region chosen by Professor Powell for illustration ($R = 3\,000$), for as yet no successful analysis has been made of the Reynolds number range from 2 000 to 100 000.

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[50] "Transportation of Sediment in Suspension", *Civil Engineering*, May, 1936.

Were the mechanism of turbulence basically different in the various phenomena in which it plays a leading rôle, Professor Powell's criticism that the title of the paper is too broad would be more fully justified. The writer's endeavor was not to present a complete text upon the subject, but to make available to hydraulic engineers the essential features in compact and comprehensible form, with pipe flow the most pertinent kind of illustration possible. It is only to be regretted that hydraulicians have previously had to rely upon aeronautical literature for such material, for the objectives of the two professions are by no means identical. On the assumption that criticism of this nature indicates imperfect success on the writer's part, a further effort was made in the foregoing relation of sediment transportation to the turbulent mechanism. It still remains to point out more explicitly the significance of the boundary layer to turbulence in general, particularly as the latter really includes the phenomenon of flow in pipes. However, while pipe flow is ordinarily a case of established uniform motion, primary interest in the boundary layer has to do with its rate of growth in the direction of flow; that is, the basic problem is essentially one of non-uniform movement.

Consider a smooth, thin plate moved at a steady rate through a fluid originally at rest. Since the fluid in immediate contact with the plate will have the same velocity as the plate itself, the resulting shearing stresses must cause movement of the surrounding fluid at a rate decreasing with distance away from the plate and increasing with distance back from the leading edge. Although this movement, mathematically speaking, extends an infinite distance away from the plate, it is convenient to designate that region in which the motion is appreciable as the boundary layer; in this sense, the boundary layer begins with a zero thickness at the leading edge and increases gradually from section to section. It is possible, analytically[51], to compute the thickness at any section as a function of the relative velocity of plate and undisturbed fluid, the kinematic viscosity, and the distance from the leading edge, proceeding on the assumption that the velocity distribution curve is a similar function at all sections; this assumption has been vindicated experimentally except in the immediate vicinity of the leading edge. (It should be evident that the kinematic, rather than the absolute, viscosity must be used, since non-uniform flow is a case of mass acceleration even when the motion is laminar.) Such analysis as this has led to basic expressions for the shearing stress and the velocity at any point in the fluid near the moving plate.

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[51] "Grenzschichten in Flüssigkeiten mit kleiner Reibung", von H. Blasius, *Zeitschrift für Mathematik und Physik*, Band 60, 1912.

Since the fluid was originally at rest and the surface of the plate very smooth, the resulting movement is necessarily a laminar one; but as the boundary layer becomes increasingly thicker the farther back it extends, some distance from the leading edge an unstable condition is reached (marked by a fairly definite critical Reynolds number, $\mathbf{R}_\delta$, composed of the boundary layer thickness, the relative velocity, and the kinematic viscosity) at which movement within the boundary layer abruptly becomes turbulent. From this point, there exists a turbulent boundary layer distinguished by a different type of velocity distribution and a more pronounced shearing stress; but since along the surface of the smooth plate the velocity of the fluid must be identical with that of the plate, there must still be a thin laminar region directly at the boundary, in which no turbulence exists; this region is called the laminar sub-layer. The flow pattern in the turbulent boundary layer may be determined as a function either of a Reynolds number composed of the relative velocity, the kinematic viscosity, and the distance from the leading edge, or of another Reynolds number in which the thickness of the boundary layer replaces the distance from the leading edge[19]. A simple proportionality always exists between these two Reynolds numbers in the turbulent region.

Had the leading edge of the plate, or the plate surface, been sufficiently rough, the boundary layer would have been a turbulent one from the outset. The result would be similar if the fluid were not originally at rest but slightly disturbed, since only a small degree of instability is sufficient to bring about fully developed turbulence at a very early stage of boundary layer growth.

The relation between this phase of boundary layer study and the case of flow in pipes should be quite evident. At the beginning of a pipe line, if the entrance is well rounded and the supply of fluid completely at rest, a laminar boundary layer will begin to form at the entrance around the entire inside of the pipe, growing in thickness with distance from the entrance. The limit of growth is attained when the effective thickness of the layer is equal to the pipe radius (that is, when the layer has penetrated to the center line of the flow), beyond which section the flow is fully established.

If the critical Reynolds number of the boundary layer is reached before the flow is thus established, this layer will become turbulent and then continue to grow as such until it reaches the central regions of the flow. Under these conditions the initial onset of turbulence will obviously not be in the central, undisturbed core of fluid. If, on the other hand, the supply of fluid is not in a state of rest, or if the pipe is rough or the entrance poor, the boundary layer will be turbulent from the outset, and its growth will soon bring it to the center line of flow. From these facts it will be clear that what is popularly called the boundary layer in pipe flow is actually the laminar sub-layer, whereas the true boundary layer extends to the very middle of the pipe.

Although the equations of boundary layer growth are at present primarily of value to aeronautical engineers and naval architects for use in determining skin resistance of air and water craft, of interest to the hydraulic engineer should be the fact that the equations for established flow in pipes have almost identical counterparts for boundary layer phenomena, since the fundamental mechanism of flow is the same. The coefficient of resistance, f, has its parallel in the local coefficient of resistance for the boundary layer, c_f, which necessarily varies with τ_0 from section to section:

$$c_f = \frac{\tau_0}{\rho \frac{V^2}{2}} \tag{124}$$

If τ_0 is integrated over the distance, x, from the leading edge to yield the total resisting force, F_x, up to that section, then a mean coefficient of resistance may be used, having the form,

$$C_f = \frac{F_x}{b\, x\, \rho \frac{V^2}{2}} \tag{125}$$

In the range of the laminar boundary layer, Equations (124) and (125) were shown analytically by Prandtl and Blasius to equal,

$$c_f = \frac{0.664}{\left(\frac{V x}{\nu}\right)^{\frac{1}{2}}} \tag{126a}$$

and,

$$C_f = 2\, c_f = \frac{1.328}{\left(\frac{V x}{\nu}\right)^{\frac{1}{2}}} \tag{126b}$$

Over a limited range past the critical point, corresponding to the Blasius range for pipe flow, Prandtl and von Kármán have shown the functional relationship to be closely exponential, just as in the case of pipes:

$$c_f = \frac{0.059}{\left(\frac{V x}{\nu}\right)^{\frac{1}{5}}} \tag{127a}$$

and,

$$C_f = \frac{0.074}{\left(\frac{V x}{\nu}\right)^{\frac{1}{5}}} \tag{127b}$$

For larger Reynolds numbers, however, at which viscous action need no longer be considered in the mixing process, the universal relationship for smooth pipe resistance (refer to Equation (68)) has a close double in von Kármán's universal equation for the boundary layer:

$$\frac{1}{\sqrt{c_f}} = 1.7 + 4.15 \log_{10} (\mathbf{R}_x\, c_f) \tag{128a}$$

and,

$$\frac{1}{\sqrt{C_f}} = 4.15 \log_{10} (\mathbf{R}_x\, C_f) \tag{128b}$$

in which R_x is the Reynolds number based upon distance from the leading edge. The three regions are shown in the logarithmic plot of Fig. 31, bearing striking similarity to Figs. 6 and 7.

Boundary layer growth has not yet begun to claim the attention of hydraulic engineers, in part because their interests, as opposed to those of aeronautical and naval designers, are still focused on problems of uniform flow; but uniform flow is by no means the sole type with which hydraulicians must deal, and such instances as siphons, draft-tubes, turbines, spillways, and even open channels, are only a few of the many cases in which occurrences within the boundary layer may have a decided influence upon the flow pattern. It has long been known that acceleration tends to make the velocity distribution more uniform, whereas deceleration (whether due to curvature or to local or general increase of flow section) not only hastens the spread of the boundary layer, but may even bring about a reversal of flow at the boundary surface. Under such conditions separation will occur, with a high rate of loss in the accompanying turbulent wake. Needless to say, unless the engineer remains content with rule-of-thumb empiricism, a clear understanding of boundary layer phenomena may soon become essential to healthy progress in hydraulic design.

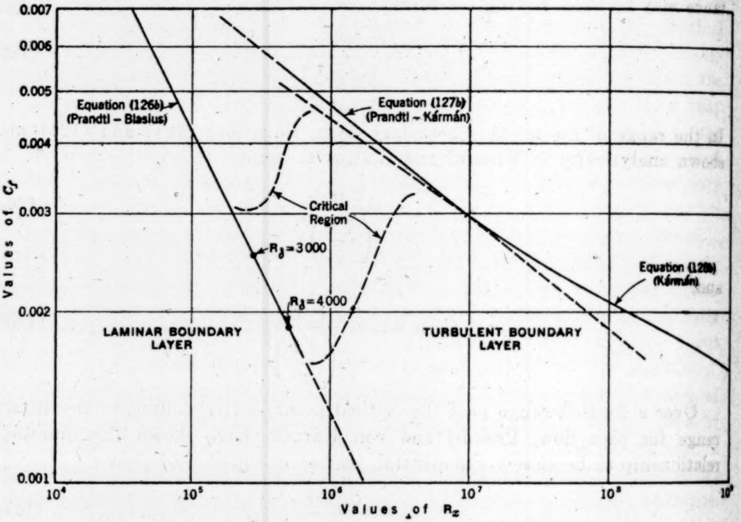

FIG. 31.—RESISTANCE CURVES FOR THE BOUNDARY LAYER.

Mr. Johnson has the writer's full appreciation for his excellent endeavor to extend the theories of universal velocity distribution to the difficult case of open channel flow—a very apt contribution at this time. Although, for many years, the velocity distribution has been held by certain engineers to be logarithmic[62], complexities resulting from the existence of a free surface, non-uniformity of the flow section, secondary currents, variable shear over the boundary, and the presence of bed and suspended loads have continued to prove obstacles to a general rational analysis.

It is evident that at the free surface of a stream the mixing length cannot have its maximum value as in the case of pipe flow. In the latter instance, however, the discontinuity in the universal velocity distribution along the center line of the pipe, owing to lack of correlation in the fluctuations in this region, still does not introduce serious error in this relationship. It is possible that discontinuity at the free surface is no more serious, as indicated by Mr. Johnson's measurements. The question of secondary currents, however, cannot be ignored unless the channel width is extremely great. Prandtl has explained this secondary motion in closed conduits as the resultant action of the tangential components of fluctuation at points of maximum curvature of the isovels. Although further analysis may be necessary in applying this reasoning to the case of a free surface, one fact is clear: Since secondary currents depress the region of maximum velocity farther and farther below the surface as the width-depth ratio decreases, it is evident that such motion cannot be ignored in deriving a universal velocity relationship. Introduction of the hydraulic radius to average the different intensities of boundary shear (resulting, as do the secondary currents, from the shape and variable roughness of the cross-section) is still to be proved fully justifiable in a general expression; any given hydraulic radius may apply to a wide range of channel proportions, and surely the change in geometrical cross-section will influence the flow pattern. Mr. Johnson's empirical expression, $T = k\,v^2$, depends entirely upon judicious choice of k—which is at best a variable quantity Needless to state, movable-bed studies must continue to retain their empirical nature as long as uniform open channel flow itself has not been mastered.

Mr. Wilson will note that he has omitted an essential variable in Equation (92)—the slope, S, properly included by Lindquist[43]. However, it is questionable whether Equation (92) even then is complete (or exponential, as assumed), except for small, smooth-walled channels of approximately similar form. In the light of the foregoing discussion, uniform open channel flow with fixed bottom should be completely described by a relationship of the form,

$$\phi\left(\frac{V^2}{d_0\,\dfrac{\gamma}{\rho}},\ \frac{V\,d_0}{\dfrac{\mu}{\rho}},\ S,\ \frac{\epsilon}{d_0},\ \beta\right) = 0 \quad \ldots\ldots\ldots\ldots\ldots (129)$$

in which the terms are, respectively, the Froude number, the Reynolds number (if $d_0 < b$), the relative roughness, and some dimensionless shape factor describing the proportions of the cross-section of flow.

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[62] See, for instance, R. Jasmund, *Zeitschrift für Bauwesen*, 43, 1893; "Die Quergeschwindigkeitskurve bei turbulenter Strömung", von H. Krey, *Zeitschrift für angewandte Mathematik und Mechanik*, 7, 1927; and "Hydraulik", von P. Forchheimer, B. G. Teubner, Leipzig und Berlin, 1930. S. 179.

Either S or $\dfrac{V^2}{d_0 \dfrac{\gamma}{\rho}}$ might be selected as the dependent variable, but it is evident that the resulting relationship, although physically correct, is too cumbersome to be practicable. However, by a development parallel to that leading to Equations (13) and (15), introduction of the mean wall shear, $(\tau_0)_m$, will lead to the expression,

$$(\tau_0)_m = \gamma R S = \frac{f}{4}\,\rho \frac{V^2}{2} \qquad \text{(130)}$$

or,

$$f = 4\,\frac{\gamma R S}{\rho \dfrac{V^2}{2}} = 4\,\frac{R S}{\dfrac{V^2}{2 g}} \qquad \text{(131)}$$

whence, in the Chezy form,

$$V = C_c \sqrt{R S} = \sqrt{\frac{8 g}{f}}\,\sqrt{R S} \qquad \text{(132)}$$

The Chezy coefficient, C_c, is different numerically and dimensionally from the C in Equation (92).

Assuming that the hydraulic radius, R, is a suitable length to be used in the Reynolds number and the term for relative roughness,

$$f = \phi'\left(\frac{V R}{\nu}, \frac{\epsilon}{R}, \beta\right) \qquad \text{(133)}$$

These three independent parameters will vary considerably in their relative influence on the resistance coefficient, f. The Reynolds number probably has marked significance only in small channels (such as are encountered in model studies) or those with very smooth boundaries, being subordinate to $\frac{\epsilon}{R}$ in appreciably rough channels, just as in the case of pipes. Of course, β becomes of no importance only as the width-depth ratio becomes great, unless the side slopes extend far into the central regions of flow. Some appropriate factor similar to the width-depth ratio, such as $\frac{d_0}{R}$, might conveniently replace β, as long as the channel remains of a given general form (that is, elliptical, trapezoidal, rectangular, etc.), but this evidently cannot define the relative proportions of all such forms at one and the same time.

Once non-uniform flow is considered, the problem becomes even more difficult. This phase has been approximately solved, as far as the immediate needs of practical design are concerned, by a combination of empirical data on resistance with rational analysis, through the use of certain simplifying assumptions. The empirical data involve the magnitude of the Chezy C as a function of channel roughness, hydraulic radius, and mean velocity, and were largely determined for cases of uniform flow. It is generally realized that non-uniform flow—whether accelerative or decelerative—will result in a rate of energy loss (and, hence, boundary shear) differing from that in uniform flow at the same depth by more than a negligible amount.

The writer stresses these difficulties not from any sense of discouragement, but in order that full understanding of the obstacles to be surmounted may prevent investigators from plowing blindly into a field so demanding of careful cultivation. It is hoped, therefore, that the combination of the basic thesis as presented in the paper, the discussion, and the closure may prove a stimulus to further research in a subject that is most essential to hydraulic engineering. To Professor von Kármán the writer expresses his appreciation, not only for contributing to the discussion of the paper, but for critically examining the contents of this closure.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 1966

COMPARISON OF SLUICE-GATE DISCHARGE IN MODEL AND PROTOTYPE

By Fred William Blaisdell[1], Jun. Am. Soc. C. E.

With Discussion by Messrs. Raymond Boucher, H. E. Hurst, G. H. Hickox, and Fred William Blaisdell

Synopsis

There is a need for experimental data to support hydraulic model theory because confidence in the use of models hinges upon this checking of model experimental results with prototype experiments. It is seldom possible to check model experiments with prototype experiments although this condition has shown considerable improvement in the last few years. In this paper the aim is to present the results of experiments on the prototype and the model of sluice-gates.

Notation

The symbols used in this paper are defined, as follows:

c = a coefficient; c_d = a coefficient of discharge;

g = acceleration due to gravity;

l = scale ratio of length = $\frac{L_n}{L_m}$;

m = a subscript denoting model;

n = an exponent in Equation (1); as a subscript, n denotes "in Nature" or "in prototype";

q = scale ratio of flow ($q = \frac{Q_n}{Q_m}$);

v = velocity;

A = area;

F = Froude's number;

H = head;

L = length;

Q = rate of flow; Q_n = rate of flow for prototype; Q_m = rate of flow for model.

Note.—Published in January, 1936, *Proceedings*.

[1] Junior Soil Conservationist, Hydr. Laboratory, National Bureau of Standards, Washington, D. C.

INTRODUCTION

A thorough search of technical literature reveals that the United States Bureau of Reclamation has made several experiments on canal head-gates and that the Ministry of Public Works of Egypt has made very thorough prototype and model experiments on the Assuan Dam on the Nile River. The experiments[2] made by the U. S. Bureau of Reclamation were on the prototype only. The experiments on the Assuan Dam[3] were continued for a period of 20 yr and include experiments on sluices 2 m wide by 3.5 m high (6 ft 6¾ in. by 11 ft 5¾ in.) and 2 m wide by 7 m high (6 ft 6¾ in. by 22 ft 11½ in.) through the dam under heads (measured from the gate-sill) from 0.3 to 13.5 m (1.0 ft to 44.3 ft) for the smaller sluices, ten of which discharged into a large tank where the water was measured volumetrically. Six models were built of these smaller sluices and approximately 1 500 experiments were made. The comparison of model and prototype results for all experiments shows that the ratio of $\frac{Q_n}{Q_m}$ varies as $l^{2.5}$, or that Froude's model law will apply to discharge through sluices. The mean departure of $\frac{Q_n}{Q_m}$ from $l^{2.5}$ was found to range from − 4.6% to + 4.5%, with an average departure of + 0.4 per cent.

PROTOTYPE

Plans of the Tremont gates at Lowell, Mass., with copies of data on experiments performed on these gates in June, 1868, by the late James B. Francis, Past-President and Hon. M. Am. Soc. C. E., were made available by Arthur T. Safford, M. Am. Soc. C. E. These gates were set in a canal and were apparently calibrated by Mr. Francis in order that they might be used to measure the quantity of water passing them. There are two sluices, of ashlar masonry, 9 ft wide by 7.12 ft high, to the top of the arch (see Fig. 1). Each sluice is closed by wooden gates 10 in. thick; the floors are set above the bottom of the canal; and the openings are flared. When in operation both gates were opened the same amount and the head-water was maintained at about the level of the top of the sluice. The head-water elevation was measured in a well at the Tremont cotton house after being conveyed from the apron by a 1½-in. pipe through one of the sluices (see Fig. 1). The tail-water elevation was measured on a wooden gauge in the same well. The actual quantity of water passing the gates was measured with a rectangular weir and computed by means of the Francis weir formula. Correction for velocity of approach was made to the head measured at the weir, but such correction was not made to the head measured at the gates. This latter omission on the part of Mr. Francis was undoubtedly due to the fact that the gates were calibrated for practical purposes only. As this was the manner in which

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[2] "Discharge Coefficients for Canal Head-Gates", by J. S. Longwell and Julian Hinds, Members, Am. Soc. C. E., *Reclamation Record*, Vol. 10 (1919), p. 475: and "Rating Curves for Canal Headgates", by Julian Hinds, M. Am. Soc. C. E., *Reclamation Record*, Vol. 13 (1922), p. 98.

[3] *Minutes of Proceedings*, Inst. C. E., Vol. 212, Pt. II (1920–21), p. 228; also, Vol. 218, Pt. II (1923–24), pp. 113 and 72.

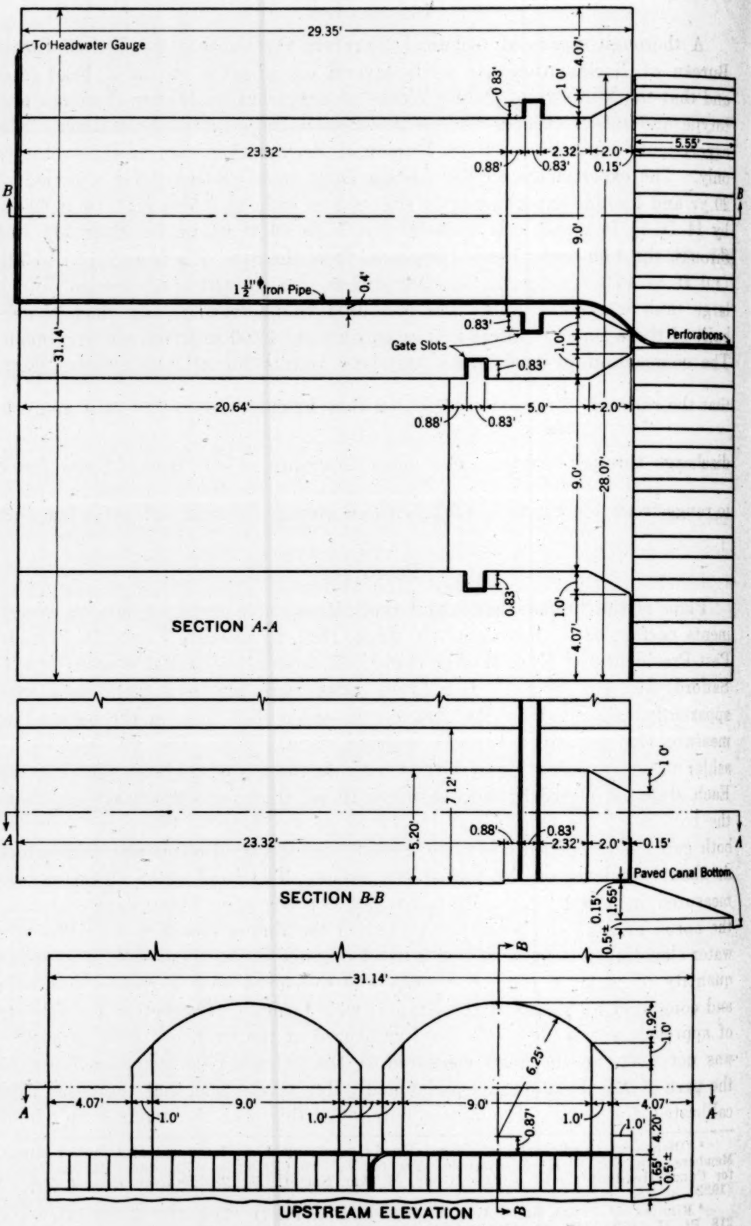
29.35'
To Headwater Gauge
4.07'
1.0'
0.83'
23.32'
2.32'
2.0'
5.55'
0.88'
0.83'
0.15'
B
B
9.0'
1½" Iron Pipe
0.4'
31.14'
0.83'
1.0'
Perforations
Gate Slots
0.83'
20.64'
5.0'
2.0'
0.88'
0.83'
9.0'
28.07'
0.83'
1.0'
4.07'
SECTION A-A
1.0'
7.12'
5.20'
A
A
0.88'
0.83'
23.32'
2.32'
2.0'
0.15'
Paved Canal Bottom
SECTION B-B
0.15'
1.65'
0.5'±
B
31.14'
1.92'
1.0'
6.25'
A
A
4.07'
9.0'
9.0'
4.07'
1.0'
1.0'
1.0'
1.0'
0.87'
4.20'
1.65'
0.5'±
UPSTREAM ELEVATION
B

FIG. 1.—TREMONT GATES, LOWELL, MASS.

head would be measured for future discharge measurements, no error would result in computing discharges.

The results obtained by Mr. Francis were plotted on logarithmic paper. It will be noted in Fig. 2 that the resulting straight lines are not quite parallel for the several gate-openings, which would indicate that n in the formula,

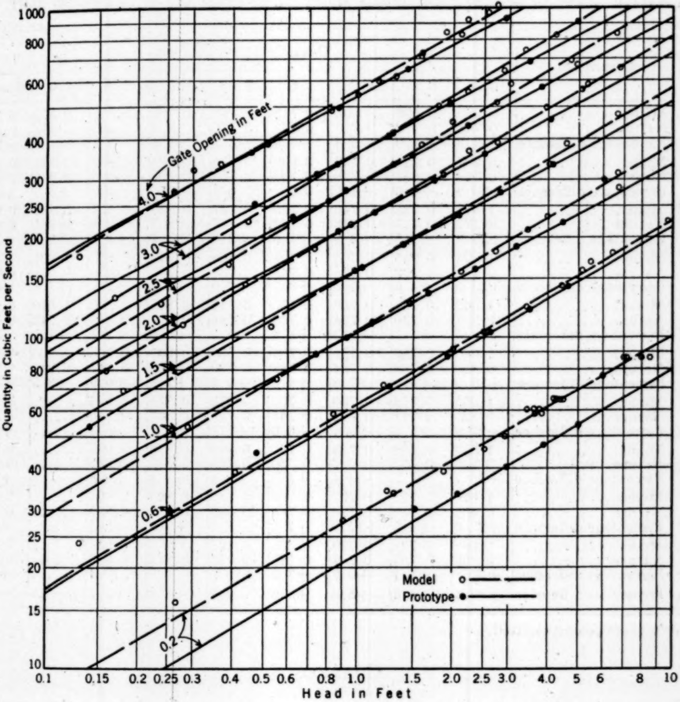

FIG. 2.—DISCHARGE CURVES; MODELS AND PROTOTYPE EXPERIMENTS.

$$Q = c_d\, A \sqrt{2\,g}\; H^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

TABLE 1.—CURVE CONSTANTS, c_d AND n, IN EQUATION (1)

Gate-opening, in feet	n			c_d		
	Model	Nature	Percentage difference	Model	Nature	Percentage difference
0.2	0.537	0.557	−3.6	1.007	0.762	+32.2
0.6	0.558	0.551	+1.3	0.721	0.698	+3.3
1.0	0.562	0.505	+11.3	0.723	0.716	+1.0
1.5	0.550	0.510	+7.8	0.736	0.740	−0.5
2.0	0.560	0.508	+10.2	0.778	0.771	+0.8
2.5	0.565	0.506	+11.7	0.804	0.800	+0.5
3.0	0.570	0.504	+13.1	0.838	0.841	−0.4
4.0	0.543	0.506	+7.3	0.971	0.933	+4.1
Mean *	0.558	0.506	+9.0	0.808	0.800	+1.3

* 0.2-ft gate-opening omitted

is not exactly equal in each case. The coefficient, c_d, is also not equal for all openings or heads. Reference is made to Table 1 for a comparison of values of n and c_d. Table 2 gives values of c_d for various heads and gate-openings.

TABLE 2.—COEFFICIENT OF DISCHARGE, c_d, IN EQUATION (1) ($n = 0.5$)

Head, in feet	Description	GATE-OPENING, IN FEET								
		0.2	0.6	1.0	1.5	2.0	2.5	3.0	4.0	Mean*
0.5	Model	0.980	0.699	0.695	0.713	0.745	0.765	0.795	0.950	0.777
	Nature	0.735	0.672	0.715	0.739	0.770	0.800	0.834	0.932	0.798
	Percentage difference	+33.3	+4.0	−2.8	−3.5	−3.2	−4.4	−4.7	+1.9	−1.8
0.0	Model	0.997	0.721	0.727	0.739	0.775	0.803	0.832	0.968	0.807
	Nature	0.762	0.693	0.720	0.739	0.775	0.803	0.836	0.934	0.801
	Percentage difference	+30.8	+4.0	+1.0	0.0	0.0	0.0	—0.3	+3.6	+1.3
1.5	Model	1.017	0.735	0.746	0.755	0.797	0.833	0.857	0.990	0.830
	Nature	0.777	0.717	0.718	0.744	0.777	0.808	0.838	0.934	0.803
	Percentage difference	+30.8	+2.5	+3.9	+1.5	+2.6	+3.1	+2.3	+6.0	+3.1
2.0	Model	1.028	0.755	0.760	0.768	0.808	0.843	0.873	1.004	0.843
	Nature	0.797	0.722	0.725	0.745	0.785	0.805	0.850	0.931	0.807
	Percentage difference	+29.0	+4.6	+4.8	+3.1	+2.9	+4.7	+2.7	+7.8	+4.5
2.5	Model	1.042	0.767	0.767	0.775	0.818	0.860	0.890	1.010	0.853
	Nature	0.801	0.730	0.724	0.746	0.785	0.807	0.846	0.943	0.808
	Percentage difference	+30.1	+5.1	+6.0	+3.9	+4.2	+6.6	+5.2	+7.1	+5.4
3.0	Model	1.050	0.766	0.775	0.784	0.830	0.865	0.905	1.020	0.863
	Nature	0.820	0.734	0.724	0.752	0.780	0.808	0.847	0.940	0.808
	Percentage difference	+28.0	+4.4	+7.0	+4.3	+6.4	+7.1	+6.9	+8.5	+6.3
3.5	Model	1.055	0.778	0.790	0.790	0.833	0.875	0.915	....	0.841
	Nature	0.825	0.741	0.722	0.745	0.778	0.807	0.840	....	0.788
	Percentage difference	+27.9	+5.0	+9.4	+6.0	+7.1	+8.4	+8.9	....	+7.5
4.0	Model	1.063	0.786	0.788	0.790	0.848	0.878	0.923	....	0.845
	Nature	0.830	0.752	0.727	0.750	0.783	0.810	0.842	....	0.789
	Percentage difference	+28.1	+4.5	+8.4	+5.3	+8.3	+8.4	+9.6	....	+7.4
4.5	Model	1.070	0.792	0.797	0.799	0.850	0.890	0.925	....	0.852
	Nature	0.834	0.760	0.720	0.745	0.785	0.810	0.850	....	0.783
	Percentage difference	+28.3	+4.2	+10.7	+7.2	+8.3	+9.9	+8.8	....	+8.3
5.0	Model	1.068	0.794	0.799	0.805	0.860	0.893	0.933	....	0.856
	Nature	0.836	0.763	0.727	0.746	0.782	0.812	0.845	....	0.782
	Percentage difference	+27.8	+4.1	+9.9	+7.9	+10.0	+10.0	+10.4	....	+8.7
Mean	Percentage difference	+30.4	+4.2	+5.8	+3.6	+4.7	+5.4	+5.1	+5.8	+5.1

*0.2-ft gate-opening omitted.

CHANNEL

The channel in which the model of the gates was set, was constructed especially for this experiment. It consisted of four sections—a weir tank in which the water was measured; an entrance box in which the turbulence of the water due to passing over the weir was absorbed; the channel proper with plate-glass sides; and the exit box. The weir tank was 3 ft 6 in. square, 5 ft deep, and contained a sharp-edged weir 1 ft deep, with a 90° notch, which was calibrated by means of volumetric tanks immediately before being put into use on this experiment.

The water flowed horizontally, and with marked turbulence, from the weir tank into the entrance box. A vertical baffle placed near the exit of the weir box deflected the water downward and dissipated all the horizontal velocity. Another baffle set in the entrance box served to accelerate the water gradually so that it entered the channel with a minimum of turbulence.

The channel was 2 ft wide, and the bottom (a 4-in. non-reinforced concrete slab, 5 ft 3 in. long, supported on two 3-in. 5.7-lb I-beams) was set 3 ft 5 in. above the floor of the laboratory. It was designed so that there would be practically no deflection under the heaviest possible loading. The sides consisted of $\frac{3}{4}$-in. plate-glass windows supported in structural steel frames. A head measuring gauge, reading to 0.01 cm, ran on adjustable rails, $1\frac{1}{16}$ in. in diameter, which were supported by $\frac{3}{8}$-in. bolts and double nuts spaced 9 in., center to center, from the steel framework for the glass channel sides. This gauge was movable in three directions so that measurements could be obtained in any part of the channel. No deflection was observed in the carriage rails when the gauge was in operation.

Water is collected in the outlet box, from which it is conveyed back to the main storage reservoir. It passes from the reservoir through a centrifugal pump and into a skimming tank in which a constant surface elevation is maintained under all conditions. Finally, it is distributed from this point to all parts of the laboratory under conditions insuring a constant rate of flow at all times.

MODEL

A scale ratio of 1 to 15 was determined upon as being the most satisfactory. The model was constructed in two parts which were grouted together after placing the model in the channel. A base section comprising that part of the model up to the floors of the sluices was constructed of sand-cement mortar. The top section containing the sluices and gates was constructed of cement and white pine sawdust. This use of sawdust resulted in a model approximately one-third as heavy as the sand-cement mixture, and no noticeable swelling or shrinking occurred after it had set. The model was smoothed (not polished) with an emery stone. The gates were of redwood, 0.66 in. thick; they were square-edged, and were held in place by wedges.

The elevation of the up-stream water surface was measured, as in the prototype (see Fig. 1), by means of a perforated pipe lying on the apron in front of the pier and leading to a glass well at the down-stream end of the sluices. The down-stream water-surface elevation was measured in a stilling-well, a vertical shutter arrangement controlling the tail-water elevation.

EXPERIMENTAL PROCEDURE

In making an experimental test the gates were first set to the desired opening with the aid of the traveling gauge and were wedged in position. A predetermined rate of flow was allowed through the model, and the tail-water elevation was controlled by means of the shutters so that the head-water elevation would be at approximately the top of the sluices.

After conditions had become constant, readings were taken of water-surface elevations above and below the gates and at the measuring weir. Any unusual conditions of flow were noted. The water approached the model with little velocity and an absence of turbulence. After passing the gates there was considerable turbulence noticeable in the sluices accompanied by a horizontal surface roller near the gates. Under some conditions a hydraulic jump would form in the sluices and when this occurred the tail-water elevation was measured down stream from the jump.

COEFFICIENT OF DISCHARGE

The experimental results are shown by broken lines in Fig. 2 and correspond to the solid curves for the prototype. For different gate-openings these curves are not parallel, as was the case with the curves for the prototype. This is also demonstrated by Table 1 in that n is not a constant for all gate-openings. The value of n in Equation (1), as obtained from the model experiments, is seen to vary from + 1.3% to + 13.1% of the values obtained from the prototype experiments with an average deviation of + 9.0 per cent. The values of c_d obtained from the model experiments for use in Equation (1) can be seen to vary from − 0.5% to + 4.1% of the values obtained from the prototype experiments with an average deviation of + 1.3%, with the 0.2-ft gate opening omitted. With $n = 0.5$ (which is the value commonly assigned to n) the values of c_d computed from the model experiments were found to vary from − 4.7% to + 10.7% of the values obtained from the prototype experiments with an average deviation of + 5.1 per cent.

It will be noted that the results obtained for the 0.2-ft gate-opening were omitted from the averages. This was done because these experiments support the recommendation of Messrs. Hurst and Watt[3] that,

> "Until further experiments on models of other structures define the limiting conditions more closely, it will be well to keep the product of velocity, in centimeters per second, and the smallest dimension of the orifice, in centimeters, above, say, 100 [cm^2 per sec = 0.11 ft per sec], and in general not to use orifices of less than 3 centimeters [$1\frac{1}{8}$ in.] in their smallest dimension."

The 0.2-ft gate-opening corresponds to a 0.41-cm ($\frac{5}{32}$ - in.) model gate-opening and the velocity through the gates for a 5-ft head is 151 cm per sec (5 ft per sec) which gives 62 as the product of velocity and least dimension for the highest head. For the 0.6-ft gate-opening, the model gate-opening is 1.22 cm ($\frac{15}{32}$ in.) and the velocity is 31 cm per sec, or 1 ft per sec ($H = 0.5$ ft) which gives 38 as the product of velocity and smallest opening. In spite of the fact that the 0.6-ft gate-opening had values of gate-opening times velocity of considerably less than 100, the results were included in the averages because of the good agreement between model and prototype results.

The model gate-openings are not greater than 3 cm ($1\frac{1}{8}$ in.) until the 1.5-ft prototype gate-opening is reached, and the product of velocity, in centimeters per second, and the smallest dimension, in centimeters, for all heads is not greater than 100 until this same gate-opening is reached. Because of the fairly good agreement of the results obtained for model and prototype, it was felt that all results for gate-openings, of 0.6 ft, or larger, should be included in the averages. The average percentage deviation of values of c_d (see Fig. 3(a)), as obtained from model experiments, from values of c_d obtained from prototype experiments for different heads, was found to increase as the head increases. The reason for this increase is perhaps due to some peculiarity of the sluice which was not copied exactly or to the lessening effect of friction as the velocity increases.

The recommendations of Messrs. Hurst and Watt[3] were checked roughly. Good results were obtained for smaller values of least dimension and the product of velocity and least dimension. The lowest satisfactory values of

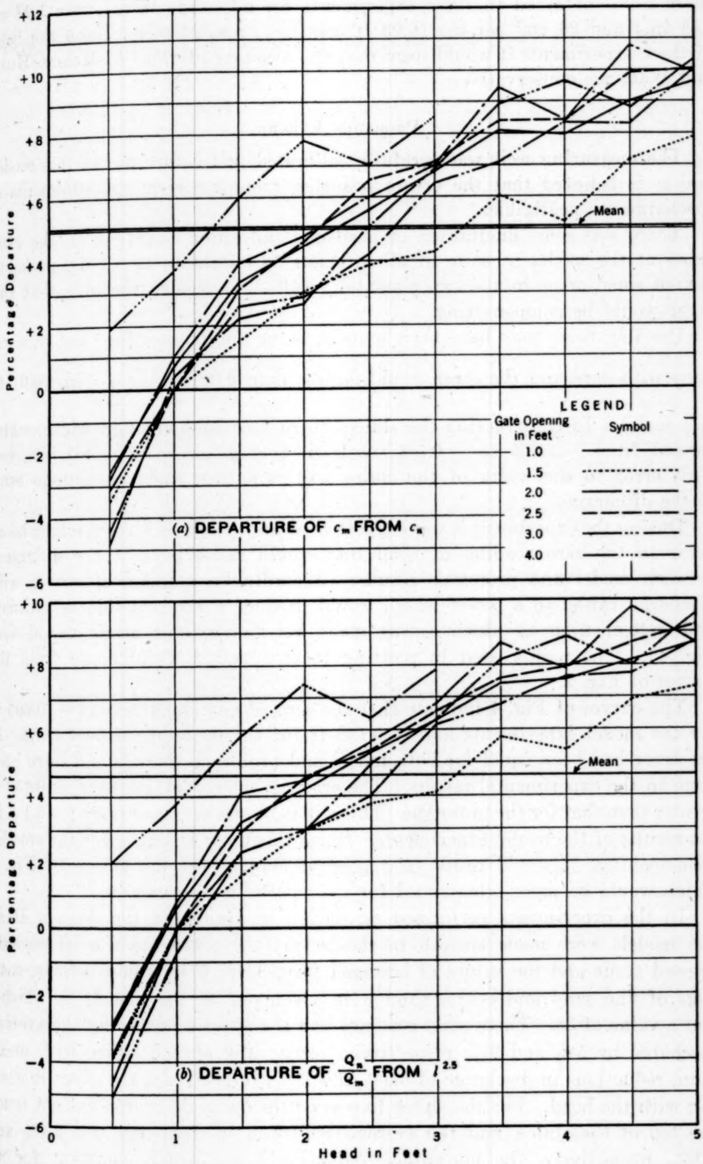
+12
+10
+8
+6
+4
+2
0
−2
−4
−6
Percentage Departure
Mean
LEGEND
Gate Opening in Feet
Symbol
1.0
1.5
2.0
2.5
3.0
4.0
(a) DEPARTURE OF c_m FROM c_n
+10
+8
+6
+4
+2
0
−2
−4
−6
Percentage Departure
Mean
(b) DEPARTURE OF $\frac{Q_n}{Q_m}$ FROM $l^{2.5}$
0
1
2
3
4
5
Head in Feet

FIG. 3.

these items obtained in these experiments for all heads tested were 1.22 cm ($\frac{15}{32}$ in.) and 28 cm^2 per sec (0.03 ft^2 per sec), respectively. Upon the basis of these experiments it would seem that the recommendations of Messrs. Hurst and Watt are conservative.

PROBABLE ERRORS

The measuring weir was carefully calibrated just before use in this model, and it is believed that the errors resulting from incorrect measurements of discharge are negligible.

There was some fluctuation of head and tail-water elevations in the glass tubes at the model in spite of all attempts to eliminate it. This, probably, caused some error in the experimental results. It is also probable that this error would be compensating.

Possibly there may have been some error in the gate-setting; but this was done with care and the error would be less than 0.02 cm, or $\frac{1}{128}$ in. (0.01 ft, full scale). In constructing the sluices there was some error in width, which ranged from -1.5% to -0.5%, with an average error of -1.0 per cent. This error in the width of the sluice will cause the same percentage error in the discharge.

During the experiment it was suggested to the writer that there might be an error in the zero reading of head that would cause part of the difference between model and prototype results. Accordingly, a series of curves with discharge raised to a power which would produce a straight line as ordinate and gate-opening as abscissa was made for the several heads tested (see Fig. 4). The points used in plotting these curves were obtained from the curves of Fig. 2.

The curves of Fig. 4 indicate that the zero of gate-opening was $-$ 0.265 ft for the model experiments and $-$ 0.135 ft for the prototype experiments. If the error had been equal for both model and prototype there would have been none in the experimental results. The error in the model, however, is 0.130 ft greater than that for the prototype. This will cause an average error of $+5.1\%$ in the results of the model experiments. Since this error is larger for the smaller gate-openings it would result in a general lowering of the curves of Fig. 3 which would be more pronounced for the smaller gate-openings.

In the experiments performed on models of sluices of the Assuan Dam' the models were made smooth to the touch (the prototype was of smooth-dressed stone and the values of l ranged from 33 to 67). Subsequently, coatings of fine sand and coarse sand were given one of the models in which l had a value of 33. These sand coatings had the effect of reducing the average discharge by 5% and 9%, respectively, for a fully opened sluice with maximum reductions in discharge of 6.5% and 11%, respectively, the effect increasing with the head. For the sluice four-sevenths open, the water did not touch the top of the sluice, and the average reduction in discharge was 2.4% and 3.1%, respectively, the percentage reduction being nearly constant for all heads. A reduction in discharge of the model will cause positive percentage departures of $\frac{1}{l^{2.5}}\left(\frac{Q_n}{Q_m} - l^{2.5}\right)$, and negative percentage departures of $\frac{c_{dm} - c_{dn}}{c_{dn}}$. Messrs. Hurst and Watt declare[3] that, "* * * if the frictional resistance is not exactly proportional to the square of the velocity, the relation, $Q = q\, n^{2.5}$ [$Q_n = Q_m\, l^{2.5}$] is departed from. * * * The effect of frictional resistance becomes less as velocity and dimensions increase." All these factors are conducive to positive percentage departures from Froude's law. On the basis of the results obtained the writer assumes that the roughness of the model of the Tremont gates simulated the roughness of the prototype fairly well since the latter was fairly rough, the value of l was small, and the model had the roughness of about No. 0 sand paper.

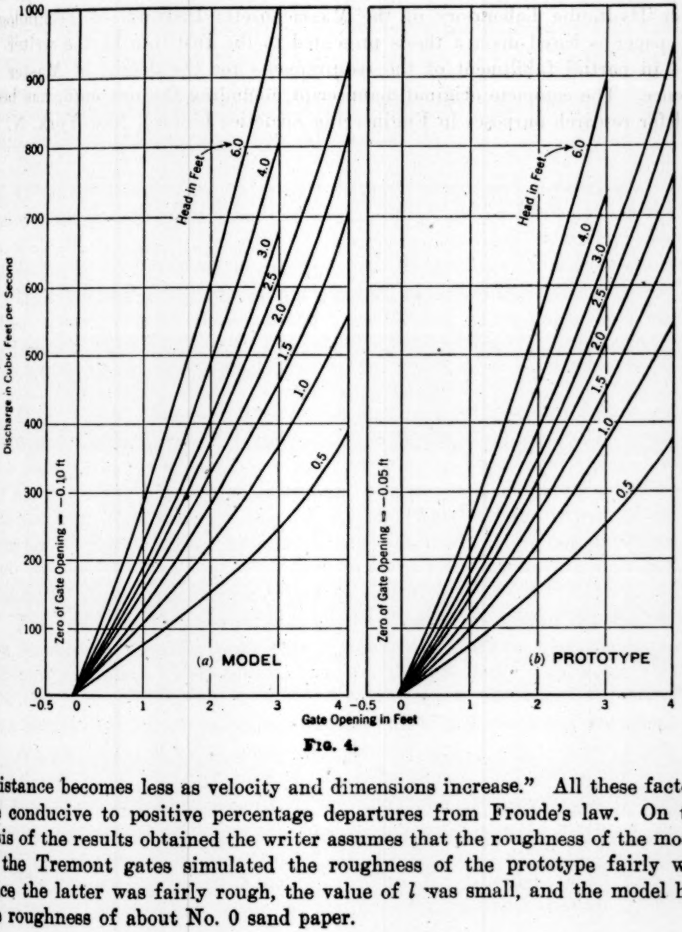

FIG. 4.

CONCLUSIONS

In this paper the writer has attempted to show that: (1) Models of sluice-gates can be depended upon to predict the discharge of their prototypes with reasonable accuracy; (2) Froude's model law will apply to the discharge of sluice-gates; (3) roughness of both model and prototype must be given consideration in the construction of the model; and (4) good results should not be expected for small gate-openings and low velocities.

ACKNOWLEDGMENTS

The experiments on the model of the Tremont gates were conducted at the River Hydraulic Laboratory of the Massachusetts Institute of Technology. The paper is based upon a thesis presented to the Institute by the writer in 1934 in partial fulfillment of the requirements for the degree of Master of Science. The complete original manuscript, including the test data, has been filed for research purposes in Engineering Societies Library, New York, N. Y.

DISCUSSION

Raymond Boucher,[4] Jun. Am. Soc. C. E. (by letter).—More than ever the hydraulic engineer is in need of proofs that model experiments are trustworthy. Mr. Blaisdell's paper is an excellent contribution toward that end.

The author states that "the coefficient, c_d, is not equal for all openings or heads." The statement leaves the reader under the impression that the head depends on the gate-opening, which is not the case in these experiments because by varying the discharge the gate-opening and the head were adjusted independently. It would be preferable to write that "the coefficient, c_d, varies with different gate-openings and different heads."

Referring to Table 1, the coefficient, c_d, corresponding to a gate-opening of 4.0 ft, has a difference of + 4.1% between model and Nature, whereas for gate-openings between 1.0 and 3.0 ft the difference in coefficient varies only from − 0.5 to + 1.0 per cent. The existence of that relatively abnormal difference in c_d, for the gate-opening of 4.0 ft, is not quite clear to the writer.

It is remarkable that the values obtained by model experiments are in very close agreement with those obtained in Nature. This paper is a valuable addition to the science of hydraulics and is another step in proving the reliability of model experiments.

H. E. Hurst,[5] Esq. (by letter).—In making a comparison of the discharge of a model of the Tremont sluice-gates with the discharge of its prototype, the author has done valuable work. It is to be hoped that the publication of his paper will lead to the recording of other comparisons of a like nature. He mentions the comparisons, made by Mr. Watt and the writer, of the discharges of models of the Assuan sluices with those of the actual sluices. In this connection it will be of interest to summarize some work that has been published[6] since the papers to which he refers.

The full-scale sluice discharge measurements at Assuan depend initially on volumetric measurements with a tank, and various methods have been adopted to extend these data to measure the discharge of sluices for which the tank cannot be used. As a result of this work, the discharge of the Nile can be measured by means of the sluices with a high degree of precision.

A comparison of these measurements with simultaneous measurements of the discharge by means of current meters has shown that during the low stage of the river the difference between the sluice and the current meter measurements is negligible. During the flood period the discharge given by the current meters was about 5% greater than the discharge given by the sluices.

The Assuan model experiments to which Mr. Blaisdell refers were made on sluices high above the river bed and discharging freely into air. For these experiments there was very good agreement between model and prototype. Since then experiments have been made on the type of sluice, 7 m (23.0 ft) high by 2 m (6.56 ft) wide, through which the river passes under a low head in flood time.

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[4] Asst. Prof. of Hydraulics, Ecole Polytechnique, Montreal, Que., Canada.

[5] Director-General, Physical Dept., Ministry of Public Works, Cairo, Egypt.

[6] "The Measurement of the Discharge of the Nile through the Sluices of the Aswan Dam", by H. E. Hurst and D. A. F. Watt, *Physical Dept. Paper No. 24*, Ministry of Public Works, Egypt; also, "Further Experiments on the Discharge of Models of Sluices", by H. E. Hurst, *Physical Dept. Paper No. 25*, Ministry of Public Works, Egypt. (A copy of *Papers Nos. 24* and *25* are available for reference, at Engineering Societies Library, 33 West 39th St., New York, N. Y.)

For these sluices the conditions are very different. They are not very high above the river bed and, therefore, are affected by differences on the river bed up stream of them. They are used when the velocity of approach is high and the direction of approach is different for different sluices. They are partly submerged down stream and the down-stream level is very difficult to measure in the turbulent condition of the water. It is also very different at different places owing to the large variation in the level of the bed down stream of the dam.

The mean discharge of six similar sluices is 6% less than the discharge inferred from the model. Two of these sluices are more similar in condition to the majority of the sluices than the other four, and for these two the mean discharge is only 1.7% less than that inferred from the model. The experiments on the model were very complete and covered a large range of up-stream and down-stream levels, and all measurements were made directly by means of a measuring tank.

As a result of these experiments three conditions of flow were found: (1) Free flow into the air; (2) submerged flow; and (3) intermediate types between Conditions (1) and (2).

(*1*).—Free flow into air persisted until the down-stream level was considerably above the sill of the sluice. In these conditions, the flow was represented by,

$$Q = cA\sqrt{2g(H - F)} \qquad (2)$$

in which Q is the discharge; A, the area of the gate-opening; H, the head above the sill of the sluice; and c and F are constants for any particular gate-opening.

(*2*).—The submerged condition begins when the down-stream level is well above the top of the sluice-opening, in which case the discharge is represented by,

$$Q = cA\sqrt{2g(H - h)} \qquad (3)$$

in which h is the head down stream above the sill.

(*3*).—The transition condition lies between the free and submerged conditions. The down-stream level affects the discharge, which, however, is not given by Equation (3). These conditions are best illustrated by reference to Fig. 5[7] in which a form of representation is adopted that is very useful in the study of the discharge of sluices or weirs.

In Equations (2) and (3), $\frac{Q^2}{A^2}$ is a linear function of H and, therefore, in Fig. 5, the two are plotted against each other. The result shows immediately the type of flow under any conditions of up-stream and down-stream level. The actual observation points are not shown, but they were numerous and fell very closely on the lines drawn in the diagram.

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[7] *Physical Dept. Paper No. 25*, Ministry of Public Works, Egypt, Pl. 3.

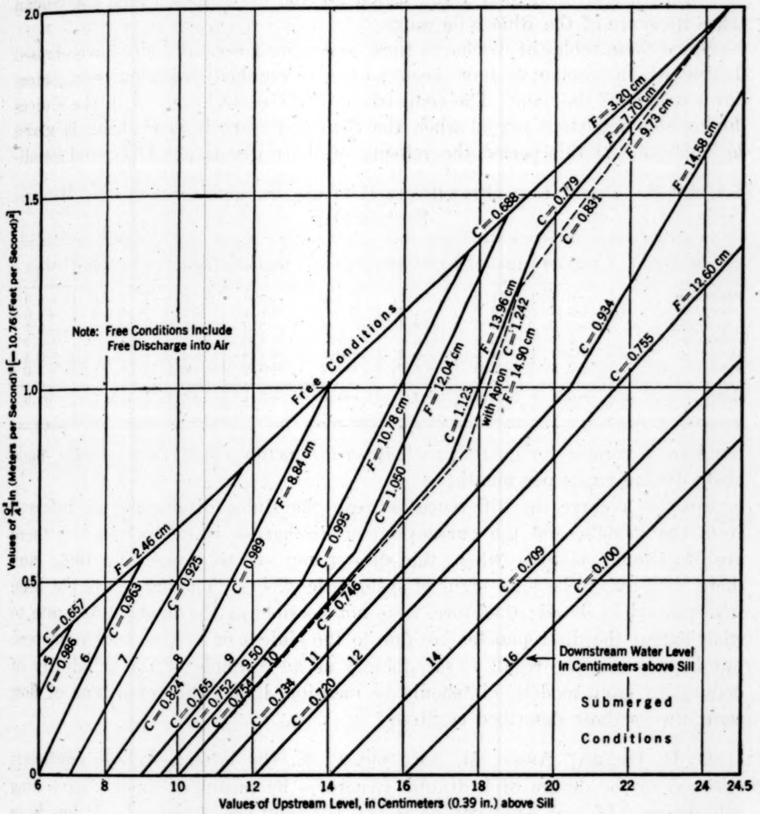

FIG. 5.—RELATION BETWEEN UP-STREAM LEVEL AND $\left(\frac{Q}{A}\right)^2$. (SLUICE, 4 CENTIMETERS, OR 1.57 INCHES, OPEN; MODEL SCALE IS ONE-FIFTIETH OF ASSUAN TYPE 4 SLUICE; AND SLUICE, 14 CENTIMETERS, OR 5.51 INCHES, BY 4 CENTIMETERS, OR 1.57 INCHES.)

The conditions of flow at Assuan are usually those described as "free", and it is to these that the comparisons of model and prototype refer. It would seem advisable, if possible, to avoid the complicated transition conditions in using a model sluice as a means of inferring the discharge of its prototype, as it may be that the discontinuities do not occur always at the same points on both. For example, one of the Assuan type of sluices showed a condition in which there were two separate values of the discharge for the same head where only one could be obtained on the model, although this condition was actually in the region of free discharge into air.

The results of the Assuan model experiments led to the use of models as a means of inferring the discharge through the sluices of the dam on the

Blue Nile, near Sennar. The results of this work are described in *Paper No. 25* previously mentioned, and from these results are constructed the tables of discharge for the Sennar Dam, which are the basis upon which the Sudan takes its share of the Blue Nile water.

Since these tables of discharge were constructed several years have elapsed during which discharges have been measured regularly with current meters down stream of the dam. The comparison of these discharges with the sluices during the important period when the Sennar Reservoir is working is given in Table 3. At this period the velocity of the water is not high, and conditions are favorable for current meter measurements which were usually made about twelve times per month.

TABLE 3.—PERCENTAGE DIFFERENCES BETWEEN SLUICES AND CURRENT METERS, SENNAR DAM

Month	1929–30	1930–31	1931–32	1932–33	1933–34	1934–35	1935–36	Mean
November.......	—0.6	+1.0	+4.3	+4.0	+5.9	+10.4	+9.8	+5.0
December........	+4.3	+0.2	+1.4	+2.1	+6.1	+9.8	+4.4	+4.0
January..........	+3.3	—3.5	—3.4	—2.3	+3.8	+3.6	+3.6	+0.7
February.........	—6.2	+2.6	—8.6	—7.3	—1.6	—2.6	....	—4.0
March...........	—5.9	+4.5	—5.3	—3.5	+1.0	—4.9	....	—2.4
April............	+2.4	+9.0	—6.2	+1.2	+3.6	+8.2	....	+3.0
Means...........	—0.4	+2.3	—3.0	—1.0	+3.1	+4.1	....	+1.0

On the average the difference between the sluice discharges, as inferred from the models, and the current meter discharges is negligible, but there are occasional months when the discrepancy is as much as 10%, and there seem also to be some signs of systematic effects. The data have not been investigated in detail; therefore, it is not possible at the moment to state to what extent the discrepancies are due to the sluices or to the current meters, but in any case the result is satisfactory as an example of the reliability of deductions from models. It should be mentioned that the conditions of flow were always those described as "free."

G. H. HICKOX,[8] ASSOC. M. AM. SOC. C. E. (by letter).—Those engineers engaged in the design of hydraulic structures by means of models have long felt the need of such comparisons as Mr. Blaisdell has presented. It has been assumed for too long a time that the actions of model and prototype were necessarily identical. In the majority of cases where large models have been properly constructed and where friction forces are negligible the correspondence between model and prototype is probably good, but, in general, quantitative confirmation has been lacking.

The author's test results show a gratifying correspondence with the observations on the prototype. Such differences as exist are not readily accounted for, but may be due partly to the fact that the model does not represent the prototype accurately. The model was placed in a straight flume and the water approached it directly, whereas in the prototype, the channel supplying water to the gate approached at right angles and immediately above it. The channel in the prototype is also somewhat wider above the gates than below them, whereas in the model, both parts of the channel were represented by the same straight flume. Part of the differences found may be due to the consequent difference in the effect of the velocity of approach. It would seem difficult to reproduce the up-stream head with certainty in the straight channel used. The measurement of tail-water elevation may also be open to some question. Although it is by no means certain that they are the only causes of discrepancy, it seems very probable that at least they are contributing factors.

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[8] Hydr. Engr., TVA Hydr. Laboratory, Norris, Tenn.

In models in which both gravity and friction forces are effective, the friction requirement may be satisfied by considering both Froude's law and Manning's equation. For similarity with respect to the gravity forces, by Froude's law,

$$v = \sqrt{l g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which v, l, and g (in accord with the author's notation) are the scale ratios of velocities, lengths, and accelerations due to gravity, respectively. Since g is commonly unity, Equation (4) may be written,

$$v = \sqrt{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

For similarity with respect to friction forces, Manning's equation may be utilized thus:

$$v = \frac{l^{\frac{2}{3}}\, s^{\frac{1}{2}}}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which s and n are the scale ratios of slopes and roughnesses, respectively. Where geometrical similarity exists between model and prototype, s = unity, and Equation (6) becomes:

$$v = \frac{l^{\frac{2}{3}}}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

In order to satisfy the two conditions simultaneously, Equations (5) and (7) must give the same value for v; that is,

$$l^{\frac{1}{2}} = \frac{l^{\frac{2}{3}}}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

from which $n = l^{\frac{1}{6}}$. In the case of the Tremont gates, the sluices were made of ashlar masonry with a probable value of n equal to 0.017. For correspondence, therefore, the n of the model should be: $\frac{0.017}{15^{\frac{1}{6}}} \doteq \frac{0.017}{1.57} = 0.0108$. This value of n can be obtained with smooth cement surfaces so that, in this case, it seems probable that the requirement for similarity was nearly satisfied.

FRED WILLIAM BLAISDELL,[9] JUN. AM. SOC. C. E. (by letter).—It has been suggested by Mr. Boucher that the statement, "the coefficient, c_d, is not equal for all openings or heads", be changed to "the coefficient, c_d, varies with different gate-openings and different heads." In Table 1 the coefficient, c_d, varies with different gate-openings but is constant for each gate-opening. In Table 2, where $n = 0.5$, the coefficient, c_d, varies with both gate-opening and head.

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[9] Junior Soil Conservationist, Hydr. Laboratory, National Bureau of Standards, Washington, D. C.

At present, the writer is unable to find a good reason for the erratic value of c_d in Table 1 when the gate-opening is 4.0 ft. It will be noticed in Fig. 2, however, that the experimental points obtained for the model gate-opening of 4.0 ft do not follow the curve as well as the points for other gate-openings.

The discussion by Mr. Hurst was of the type which the writer had hoped his paper would inspire. The accuracy of the results of tests made under the direction of Mr. Hurst indicates what can be expected from carefully constructed and carefully tested models. The fact that water is distributed in Egypt on the basis of model experiments indicates the confidence placed in these tests.

Equation (2), given by Mr. Hurst, applies only to free flow into air or to free-flow conditions. In this equation, F is usually assumed to be one-half the gate-opening, or $H - F$ is the head to the center of the gate-opening. In one of the publications mentioned by Mr. Hurst values of F are given for the sluices of the Assuan Dam[10]. These values of F range from 73 to 95% of the opening, which indicates that for the Assuan Dam the head should not be measured from the center of the gate-opening, but from some point above the center.

Mr. Hurst states[11] that:

> "This 'Free Condition' with the down-stream level above the level of the sill of the sluice is always marked by the occurrence of a standing wave in the culvert downstream of the sluice-gate"; and, "Observation of the flow shows a well-marked standing wave when the flow is similar to free into air. As the upstream level is lowered to the point where free-flow conditions cease, the wave gradually works back along the culvert to the sluice-gate. While the standing wave exists the flow is steady. When the standing wave begins to disappear the flow is pulsating and the wave oscillates between the sluice-gate and a point a few centimeters down stream of it, appearing and disappearing."

In the model sluices of the Tremont gates, a hydraulic jump was noticed under some conditions. If the standing wave mentioned in the quotation can be called a hydraulic jump, then free-flow conditions, as defined by Mr. Hurst, undoubtedly existed at times in the Tremont gates model.

Equation (3) corresponds to the writer's Equation (1). Submerged flow is the type of flow that existed most of the time in the Tremont model, although it is possible that intermediate flow is present at times. An attempt was made to construct a plot for the Tremont gates experimental results similar to Fig. 5 but there were too few points to define the curves properly.

The derivation of a value for the roughness factor, n, for use in the Manning equation, by Mr. Hickox, is of interest. In this model of the Tremont gates the roughness factor does not assume major importance because of the relatively rough prototype, the large scale ratio, and the ease of building a model having a roughness factor of about 0.108 as computed by Mr. Hickox. In many models, however, the effect of roughness can not be disposed of so easily.

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[10] *Physical Dept. Paper No. 24*, Ministry of Public Works, Egypt, p. 5, Table 1.

[11] *Physical Dept. Paper No. 25*, Ministry of Public Works, Egypt, pp. 4 and 14.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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A DIRECT METHOD OF MOMENT DISTRIBUTION

By T. Y. Lin,[1] Jun. Am. Soc. C. E.

With Discussion by Messrs. L. E. Grinter, W. H. Huang, Felix H. Spitzer, Harold E. Wessman, Egor P. Popoff, L. T. Evans, G. S. Salter, Leon Blog, Austin H. Reeves, E. J. Bednarski, John T. Howell, I. Oesterblom, W. P. Li, A. A. Eremin, and T. Y. Lin.

Synopsis

The Cross method of distributing fixed-end moments in the analysis of continuous frames[2] has become recognized as such a convenient tool of structural design that it is unnecessary to state its importance, or its advantages over the older, classical methods. However, the necessity of going through a series of approximations is still disliked by many engineers.

The purpose of this paper is to present a direct method[3] of moment distribution, derived from the fundamental conceptions of the original method, but applied without the series of approximations. Designers of continuous frames will find that the method, being a direct corollary of the moment-distribution principle, is easy to learn. The time saved is considerable as compared with other methods, especially for frames subject to several conditions of loading, frames requiring the use of influence lines, frames with members of variable section, and frames in which convergence is slow by the original method and in which greater accuracy is desired. This method also affords a direct visualization of the "transfer" of moments in continuous frames, and thus throws new light on the economy of proportioning, the effect of fixation at the supports, of continuity, etc., which is true of the original method only indirectly.

Note.—Published in December, 1934, *Proceedings*.

[1] Engr., Cheng-yii Ry., Chungking, Szechnan, China.

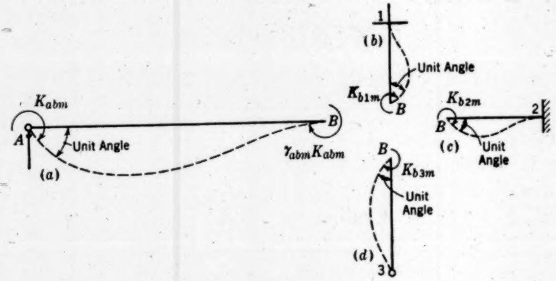
[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments", by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 1–156.

[3] A more complete treatment is given in a thesis by T. Y. Lin, presented to the Univ. of California in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

DEFINITIONS AND NOTATION

The following notation is used: K = stiffness of an end of a member; γ = carry-over factor of an end of a member; M' = fixed-end moment at an end of a member; and, M = actual moment at an end of a member.

The subscripts have the following significance: *ab* denotes End *A* of Member *AB*; *ba* denotes End *B* of Member *AB*; *b*2 denotes End *B* of Member *B*2; *dc* denotes End *D* of Member *CD*, etc.; and *m* denotes a quantity "modified", due to the actual restraint of the support. For example, K_{ab} = stiffness of End *A* in Member *AB*; and γ_{3bm} = a modified carry-over factor of End 3 in Member *B*3, or the moment at End *B* due to a unit moment at End 3 with End *B* under the actual condition of restraint. Furthermore, (see Fig. 1), let,

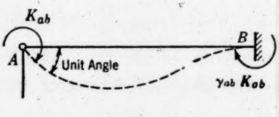


K_{abm}
A
Unit Angle
(a)
B
$\gamma_{abm}K_{abm}$
1
(b)
Unit Angle
K_{b1m}
B
K_{b2m}
2
B
(c)
Unit Angle
B
K_{b3m}
Unit Angle
(d)
3

FIG. 1

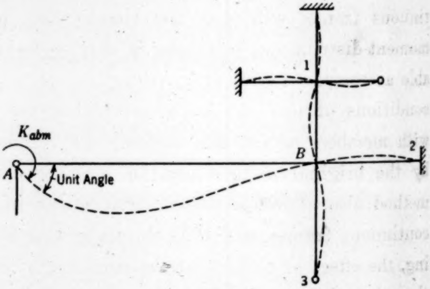
$$R_{ba} = \frac{K_{ba} + K_{b1m} + K_{b2m} + K_{b3m} + \ldots}{K_{ba}} = \frac{K_{ba} + \Sigma K_{bnm}}{K_{ba}} \ldots\ldots (1)$$

In Fig. 2, Member *AB* is simply supported at End *A* and fixed at Point *B*. Apply a moment at *A* to produce a unit rotation at that end of the Member *AB*. Then, by definition, K_{ab} = the moment applied at End *A* = stiffness at End *A* of Member *AB*; $\gamma_{ab}\ K_{ab}$ = the moment induced at End *B* due to a moment K_{ab} at End *A*; and γ_{ab} = the carry-over factor, from End *A* to End *B*.



K_{ab}
A
Unit Angle
B
$\gamma_{ab}\ K_{ab}$

FIG. 2



K_{abm}
A
Unit Angle
1
B
2
3

FIG. 3

In Fig. 3, Member AB is simply supported at End A and is connected to several members at Joint B—$B1$, $B2$, $B3$, etc. Apply a moment at A to produce a unit rotation at that end of the member. Then, K_{abm} = the moment applied at A = the modified stiffness at End A of Member AB, with End B held by its restraining members; $\gamma_{abm}\,K_{abm}$ = the moment induced at End B in Member AB, due to the moment, K_{abm}, at End A; and γ_{abm} = the modified carry-over factor from End A to End B.

Sign Conventions

A moment at an end of a member is considered positive when the external moment acting at that end is clockwise. Therefore, a moment carried over does not change in sign; and the algebraic sum of all the end moments at any joint equals the external moment applied at that joint, which is usually zero. (See Fig. 4.)

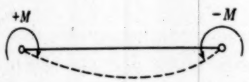


Fig. 4

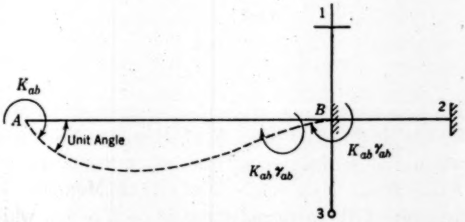


Fig. 5

Theory and Derivation of Equations

In the original method of moment distribution, the stiffness and the carry-over factor of an end of a member are calculated on the assumption that the other end is fixed. Hence, it is necessary to release one joint at a time, holding the joints at the other end of the connected members fixed, in order that the unbalanced moment at a joint may be distributed to the connected members in proportion to their stiffnesses and the distributed moments may be transferred by the carry-over factors. In the method herein presented, however, the modified values of the stiffness and carry-over factor of each end of each member are calculated in accordance with the actual condition of restraint at the other end. Hence, it is possible to release all the joints simultaneously and to distribute the unbalanced moments once only.

Only two formulas are necessary for the application of this method: One for the value of K_{abm}, the other for the value of γ_{abm}. These formulas will be derived from the fundamental concepts of the moment-distribution method as follows:

Given: The values, K_{ab}, K_{ba}, γ_{ab}, and γ_{ba}, of Member AB, and the values of K_{bnm} (see Equation (1)) for the members connected to AB at End B.

Required: The modified moment, K_{abm}, and the modified carry-over factor, γ_{abm}, for End A of Member AB.

Solution:

Step 1.—In Fig. 5, fix Joint *B*. Apply a moment at End A equal to K_{ab} to produce a unit rotation of that end. Since End *B* is fixed, a carry-over moment will be produced at End *B* of Member *AB* equal to $K_{ab}\ \gamma_{ab}$, which is held in equilibrium by the external moment, $K_{ab}\ \gamma_{ab}$.

Step 2.—Fix End *A* in this rotated position, as shown in Fig. 6. Release Joint *B* by applying an external moment of $-K_{ab}\ \gamma_{ab}$, which will rotate all the members meeting at that joint through an equal angle. Hence, this external moment will be distributed among the members meeting at the joint in proportion to their stiffnesses as existing under this condition; that is, the stiffness at End *B* of Member *AB* will be K_{ba} (End *A* being fixed), while the stiffnesses of the other members will be their modified values, K_{b1m}, K_{b2m}, K_{b3m}, etc. End *B* of Member *AB* will take its proportional amount of the external moment, $-K_{ab}\ \gamma_{ab}$, which is,

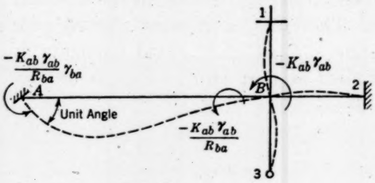

$-\frac{K_{ab}\gamma_{ab}}{R_{ba}}\gamma_{ba}$
A
Unit Angle
$-\frac{K_{ab}\gamma_{ab}}{R_{ba}}$
1
$-K_{ab}\gamma_{ab}$
B
2
3

FIG. 6

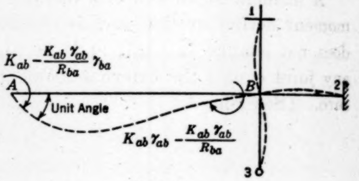

$K_{ab}-\frac{K_{ab}\gamma_{ab}}{R_{ba}}\gamma_{ba}$
A
Unit Angle
1
B
2
$K_{ab}\gamma_{ab}-\frac{K_{ab}\gamma_{ab}}{R_{ba}}$
3

FIG. 7

$$-\,K_{ab}\ \gamma_{ab}\left(\frac{K_{ba}}{K_{ba}+K_{b1m}+K_{b2m}+K_{b3m}+\ldots}\right)$$

$$=-\,K_{ab}\ \gamma_{ab}\left(\frac{K_{ba}}{K_{ba}+\Sigma\,K_{bnm}}\right)$$

or,

$$M=\frac{-\,K_{ab}\ \gamma_{ab}}{R_{ba}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Since End *A* is fixed, the distributed moment (Equation (2)), of End *B* will be carried over to End *A* by the carry-over factor, γ_{ba}. Hence, a moment will be produced at End *A* of Member *AB* equal to $\frac{-\,K_{ab}\ \gamma_{ab}}{R_{ba}}\gamma_{ba}$.

Step 3.—Next, examine the resulting elastic and static condition of Member *AB* at the end of Steps 1 and 2. (See Fig. 7.) End *A* of the member is rotated a unit angle and End *B* is connected to its adjoining members without any external restraint. The resulting moment at End *A* is the sum of the moments produced in the two steps; that is, $K_{ab}-\frac{K_{ab}\ \gamma_{ab}}{R_{ba}}\gamma_{ba}$. This is the moment at *A* necessary to produce a unit rotation at End *A* of Member *AB*, with End *B* under its actual condition of restraint. By definition, it is the modified stiffness at End *A* of Member *AB*. Therefore,

$$K_{abm} = K_{ab} - \frac{K_{ab}\,\gamma_{ab}}{R_{ba}}\,\gamma_{ba} = K_{ab}\left(1 - \frac{\gamma_{ab}\,\gamma_{ba}}{R_{ba}}\right) \text{ .... (3)}$$

The resulting moment at End B of Member AB, $K_{ab}\,\gamma_{ab} - \frac{K_{ab}\,\gamma_{ab}}{R_{ba}}$, is that produced at that end due to a unit rotation of End A, with End B under its actual condition of restraint. Its ratio to the resulting moment applied at A is, by definition, the modified carry-over factor at that end of the member, AB. Therefore,

$$\gamma_{abm} = \frac{K_{ab}\,\gamma_{ab} - \dfrac{K_{ab}\,\gamma_{ab}}{R_{ba}}}{K_{ab}\left(1 - \dfrac{\gamma_{ab}\,\gamma_{ba}}{R_{ba}}\right)} = \gamma_{ab}\,\frac{R_{ba} - 1}{R_{ba} - \gamma_{ab}\,\gamma_{ba}} \text{ .... (4)}$$

Equations (3) and (4) are the only ones necessary in this method. They may be simplified for special cases, as follows: (1) For a fixed end, R_{ba} equals infinity; (2) for a hinged end, R_{ba} equals unity; (3) for members of uniform moment of inertia, $\gamma_{ab}\,\gamma_{ba} = \frac{1}{4}$; and (4) for symmetrical members, $\gamma_{ab} = \gamma_{ba}$.

APPLICATION OF METHOD

The general procedure consists of two distinct steps: First, to analyze the structure under no load; that is, to find the K_m and γ_m-values; and, second, to distribute the unbalanced fixed-end moments.

Step 1.—Find the values of K and γ of both ends of all members by the usual methods. When the R-value of one end of a member is known, the K_m and γ_m-values of the other end can be found from Equations (3) and (4), respectively. For a hinged end, R is unity; for a fixed end, R is infinity; and, for all other cases, R will have values between the two, and can be either calculated or estimated from the modified stiffnesses of the connected members.

Step 2.—Find the fixed-end moments of each loaded or deformed member by the usual methods. Distribute the unbalanced moments at each joint to all the members meeting at the joint in proportion to their modified stiffnesses. Carry over the distributed moment of each member to the other end by the corresponding modified carry-over factor. Distribute each carried-over moment among the connected members in proportion to their modified stiffnesses. Continue this process of carrying over and distributing to the supports of the structure. Add the fixed-end moment, the distributed moments, and the carried-over moments, to find the resulting moment at each end of each member.

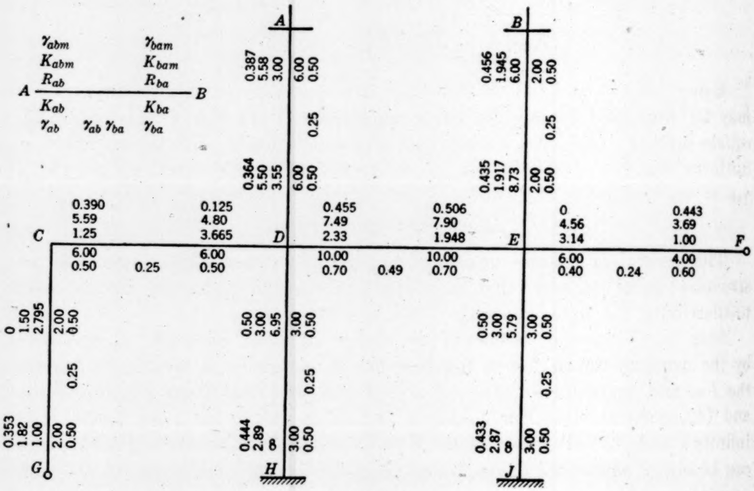
Steps 1 and 2 explain the general procedure only. They can be modified easily to suit individual cases. Some of the important variations will be shown in the subsequent illustrations.

LIMITATION OF THE METHOD

Since the two fundamental equations (Equations (3) and (4)) are derived from the first principles of the moment-distribution method, this modified method has the same limitations as the original. Problems such as sidesway,[4] that can be solved by the original method, can be solved similarly by the writer's method.

EXAMPLES

Example 1.—Consider the frame shown in Fig. 8. Ends *G* and *F* are hinged; Ends *H* and *J* are fixed; and Ends *A* and *B* are connected to members above the frame (not shown in the diagram), such that End *A* of Member *AD* has a value of $R_{ad} = 3.00$ and End *B* of Member *BE* has a value of $R_{be} = 6.00$.



γ_{abm}
K_{abm}
R_{ab}
γ_{bam}
K_{bam}
R_{ba}
A
B
K_{ab}
γ_{ab}
$\gamma_{ab}\,\gamma_{ba}$
K_{ba}
γ_{ba}
A
0.387
5.58
3.00
6.00
0.50
0.25
0.364
5.50
3.55
6.00
0.50
B
0.456
1.945
6.00
2.00
0.50
0.25
0.435
1.917
8.73
2.00
0.50
C
0.390
5.59
1.25
6.00
0.50
0.25
0.125
4.80
3.665
6.00
0.50
D
0.455
7.49
2.33
10.00
0.70
0.49
0.506
7.90
1.948
10.00
0.70
E
0
4.56
3.14
6.00
0.40
0.24
0.443
3.69
1.00
4.00
0.60
F
0
1.50
2.795
2.00
0.50
0.25
0.353
1.82
1.00
2.00
0.50
G
0.50
3.00
6.95
3.00
0.50
0.25
0.444
2.89
∞
3.00
0.50
H
0.50
3.00
5.79
3.00
0.50
0.25
0.433
2.87
∞
3.00
0.50
J

FIG. 8

Step 1.—The *K* and γ - values of all the members being given, determine the R, K_m, and γ_m-values from Equations (1), (3), and (4). Arrange these values for each member as shown in the key sketch at the top of Fig. 8. In Member *GC*, End *G* is hinged and $R_{gc} = 1$. Then, from Equation (3),

$$K_{cgm} = K_{cg}\left(1 - \frac{\gamma_{cg}\,\gamma_{gc}}{R_{gc}}\right) = 2\left(1 - \frac{\frac{1}{2} \times \frac{1}{2}}{1}\right) = 1.50$$

and, from Equation (4),

$$\gamma_{cgm} = \gamma_{cg}\,\frac{R_{gc} - 1}{R_{gc} - \gamma_{gc}\,\gamma_{cg}} = \frac{1}{2}\,\frac{1 - 1}{1 - \frac{1}{4}} = 0$$

Similarly, in Member *CD*, $R_{cd} = \dfrac{6 + 1.5}{6} = 1.25$; $K_{dcm} = 6\left(1 - \dfrac{0.25}{1.25}\right) = 4.80$; and, $\gamma_{dcm} = \dfrac{1}{2}\,\dfrac{0.25}{1.00} = 0.125$. In the case of Member *HD*, R_{hd} = infinity; $K_{dhm} = K_{dh} = 3$; and, $\gamma_{dhm} = \gamma_{dh} = \frac{1}{2}$. In Member

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[4] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 9.

AD, $R_{ad} = 3$; $K_{dam} = 6\left(1 - \dfrac{0.25}{3}\right) = 5.50$; and $\gamma_{dam} = \dfrac{1}{2}\left(\dfrac{3-1}{3-0.25}\right) = 0.364$. In Member *DE*, $R_{de} = \dfrac{10.0 + 5.5 + 4.8 + 3}{10.0} = 2.33$; $K_{edm} = 10\left(1 - \dfrac{0.7 \times 0.7}{2.33}\right) = 7.90$; and $\gamma_{edm} = 0.7\,\dfrac{1.33}{1.84} = 0.506$.

Consider Members *BE*, *FE*, and *JE* as a group. Determine the values of R_{ed}, K_{dem}, γ_{dem}, etc., and record all these values on the frame (Fig. 8). Evidently, many of these values of R, K_m, and γ_m are not needed.

Step 2.—Only some of the values calculated in Step 1 will be used in Step 2. Assuming an external moment of 100 at Joint *D*, distribute it among the members meeting at that joint in proportion to their K_{dm} - values (see Fig. 8), as follows:

$$\begin{array}{rcl}
M_{dc} = 4.80 & \times \dfrac{100}{20.79} = & 23.08 \\
M_{da} = 5.50 & \times\ 4.810 = & 26.45 \\
M_{de} = 7.49 & \times\ 4.810 = & 36.04 \\
M_{dh} = 3.00 & \times\ 4.810 = & 14.43 \\
\hline
20.79 & & 100.00
\end{array}$$

Carry over the distributed moments by the modified carry-over factors; thus:

$$\begin{aligned}
M_{cd} = M_{dc} \times \gamma_{dcm} &= 23.08 \times 0.125 = 2.88 \\
M_{ad} &= 26.45 \times 0.364 = 9.62 \\
M_{ed} &= 36.04 \times 0.455 = 16.40 \\
M_{hd} &= 14.43 \times 0.500 = 7.21
\end{aligned}$$

Distribute each moment thus carried over among the connected members at the joint, in proportion to their modified stiffnesses: At Joint *C*, $M_{cg} = -M_{cd} = -2.88$; at Joints *A* and *H*, the frame ends, and no further distribution will be necessary; and at Joint *E*, the moment, $M_{ed} = 16.40$, will be distributed to the connected members, *EB*, *EF*, and *EJ*, thus:

$$\begin{aligned}
M_{eb} &= 1.917 \times \frac{-16.40}{1.917 + 4.56 + 3.0} = -3.31 \\
M_{ef} &= 4.56 \times -1.73 = -7.90 \\
M_{ej} &= 3.00 \times -1.73 = -5.19 \\
\hline
& \qquad\qquad -16.40
\end{aligned}$$

The foregoing distributed moments will again be carried to the far end of each member, thus, $M_{be} = M_{eb} \times \gamma_{ebm} = 3.31 \times 0.435 = 1.44$, etc. The resulting moments are shown in Fig. 9.

Example 2.—In Fig. 8, let the fixed-end moments in Member *CD* be $M'_{cd} = -1\,000$; and $M'_{dc} = 2\,000$ (see Fig. 10). The unbalanced moment at Joint *C* is then $-1\,000$; and at Joint *D*, it is $+2\,000$. Instead of distributing the unbalanced moments separately, a better procedure is as follows: (1) Release all the joints; (2) distribute the unbalanced moment of $-1\,000$ at Joint *C* to End *C* of Member *CD*, or, $1\,000 \times \dfrac{5.59}{5.59 + 1.50} = 788$;

(3) carry over 788 to End *D* of Member *DC* by the modified carry-over factor, γ_{cdm}, or, $788 \times 0.390 = 308$; (4) distribute the unbalanced moment of $+ 2\,000$ at Joint *D* to End *D* of Member, *DC*, or, $- 2\,000 \times \dfrac{4.80}{4.80 + 5.50 + 7.49 + 3.00} = - 462$; and, (5) carry over $- 462$ to End *C* by the modified carry-over factor, γ_{dcm}, or, $- 462 \times 0.125 = - 58$.

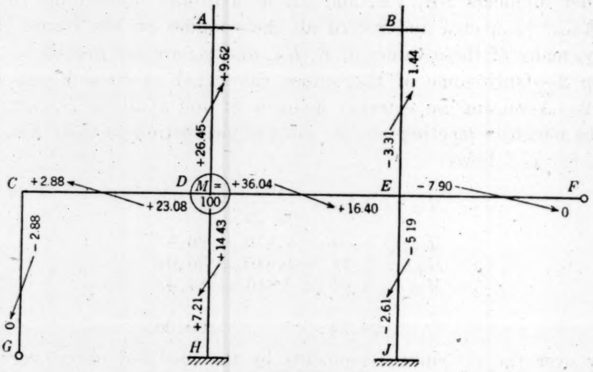
A
B
+9.62
+26.45
-1.44
-3.31
C
+2.88
D
M
=
100
+36.04
E
-7.90
F
+23.08
+16.40
0
-2.88
+14.43
-5.19
0
+7.21
-2.61
G
H
J

FIG. 9

The total moment at End *C* of Member *CD* will then be, $- 1\,000 + 788 - 58 = - 270$; and the total moment at End *D* of Member *CD* will be, $2\,000 - 462 + 308 = 1\,846$. These moments will be entirely resisted by their respective supporting or connected members. They are distributed and carried over to the ends of the frame, as shown in Fig. 10.

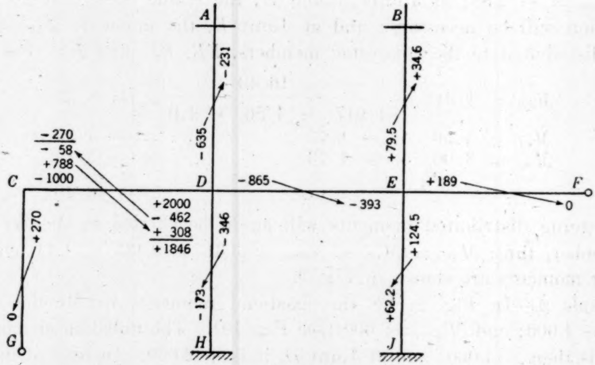
A
B
-231
+34.6
-635
+79.5
-270
- 58
+788
-1000
C
D
-865
E
+189
F
+2000
- 462
+ 308
+1846
-393
0
+270
-346
+124.5
0
-173
+62.2
G
H
J

FIG. 10

Example 3.—Consider the frame in Example 1, with fixed-end moments on Girders *CD*, *ED*, and *EF*, as shown in Fig. 11 (identifying symbol, (*f*)). It is possible to distribute the unbalanced moments at the joints separately, proceeding as in Example 1, or to find the end moments of each loaded member and proceed as in Example 2. A better method, however, is shown in Fig. 11: (1) Find the unbalanced moment at each joint and distribute it to all the members meeting at that joint, identifying each distributed moment

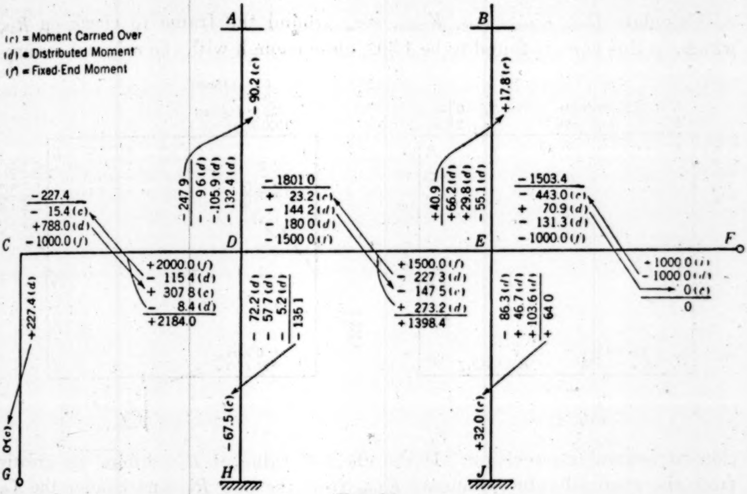
(c) = Moment Carried Over
(d) = Distributed Moment
(f) = Fixed-End Moment
A
B
−90.2 (c)
+17.8 (c)
−247.9
− 9.6 (d)
−105.9 (d)
−132.4 (d)
+40.9
+66.2 (d)
+29.8 (d)
−55.1 (d)
−227.4
− 15.4 (c)
+788.0 (d)
−1000.0 (f)
−1801.0
+ 23.2 (c)
− 144.2 (d)
− 180.0 (d)
−1500.0 (f)
−1503.4
− 443.0 (c)
+ 70.9 (d)
− 131.3 (d)
−1000.0 (f)
C
D
E
F
+2000.0 (f)
− 115.4 (d)
+ 307.8 (c)
− 8.4 (d)
+2184.0
+1500.0 (f)
− 227.3 (d)
− 147.5 (c)
+ 273.2 (d)
+1398.4
+1000.0 (f)
−1000.0 (d)
0 (c)
0
+227.4 (d)
− 72.2 (d)
− 57.7 (d)
− 5.2 (d)
−135.1
− 86.3 (d)
+ 46.7 (d)
+103.6 (d)
+ 64.0
0 (c)
−67.5 (c)
+32.0 (c)
G
H
J

FIG. 11

by the symbol, (*d*); (2) after distributing the unbalanced moments at all joints, start from one end of the frame, such as Member *CD*, and carry over the value, 788(*d*), at End *C* to End *D*, producing a moment at End *D* equal to 307.8(*c*), identifying each moment carried over by the symbol, (*c*); (3) distribute this value, 307.8(*c*), among the supporting members, so that

$$M_{de} = -307.8 \times \frac{7.49}{7.49 + 5.5 + 3.0} = -144.2\ (d), \text{ etc.; and (4) combine}$$

the value, − 144.2(*d*), with the first distributed moment, − 180(*d*), at End *D* of Member *DE* and carry the sum over to End *E*, producing a carried-over moment at End *E* equal to (− 144.2 − 180.0) 0.455 = − 147.5(*c*). Continue the foregoing steps until all the unbalanced moments are carried to the ends of the frame. Time is saved in this way by combining the moments carried over.

Example 4.—Consider the closed frame shown in Fig. 12. The fixed-end moments are $M'_{ab} = -1\,000$; and $M'_{ba} = 1\,000$. This box is made up of members of uniform moments of inertia and, therefore, only the *K*-values need be shown in Fig. 12.

Step 1.—Since the frame is closed, it is necessary, first, to assume the *R*-value of an end of some member. It can be seen by inspection that

$$R_{ab} = \frac{3 + 1\,(-)}{3} = 1.3, \text{ approximately.}$$

Then, from Equation (3), $K_{bam} = 3\left(1 - \frac{\frac{1}{4}}{1.3}\right) = 2.423$; $R_{bcm} = \frac{1 + 2.423}{1} = 3.423$; and, $K_{cbm} = 1\left(1 - \frac{\frac{1}{4}}{3.423}\right) = 0.927$.

Calculate R_{cd}, K_{dcm}, R_{da}, K_{adm}, etc., around the frame to check on R_{ab}, which, in this case, is found to be 1.302, close enough with the original assumption to warrant no revision. If the checked value of R_{ab} differs too greatly from the assumed value, calculate K_{bam} from the new R_{ab} and change the K_m and R-values as far as necessary. Then, calculate the γ_m-values from the values of R finally adopted.

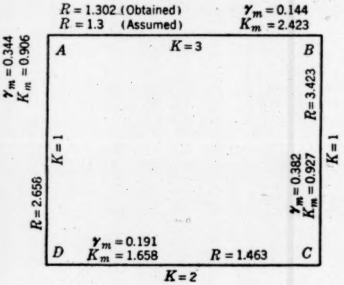
R = 1.302 (Obtained)
R = 1.3 (Assumed)
γm = 0.144
Km = 2.423
γm = 0.344
Km = 0.906
A
K = 3
B
R = 3.423
K = 1
K = 1
R = 2.656
γm = 0.382
Km = 0.927
γm = 0.191
D
Km = 1.658
R = 1.463
C
K = 2

FIG. 12

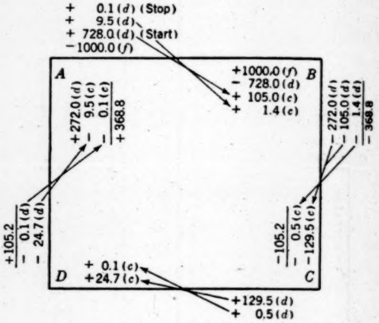
+ 0.1 (d) (Stop)
+ 9.5 (d)
+ 728.0 (d) (Start)
− 1000.0 (f)
A
+1000.0 (f)
− 728.0 (d)
+ 105.0 (c)
+ 1.4 (c)
B
+ 272.0 (d)
− 9.5 (c)
− 0.1 (c)
+ 368.8
− 272.0 (d)
− 105.0 (d)
− 1.4 (d)
− 368.8
+105.2
− 0.1 (d)
− 24.7 (d)
−105.2
− 0.5 (c)
− 129.5 (c)
+ 0.1 (c)
D
+24.7 (c)
C
+129.5 (d)
+ 0.5 (d)

FIG. 13

Step 2.—Distribute the unbalanced moments at Joints A and B (see Fig. 13). Carry over the distributed moment, 728(d), at End A of Member AB to End B, producing 105(c), which is resisted by − 105(d) at End B of Member BC. The moment, − 105(d), is combined with the moment, − 272(d), at End B of Member BC, and carried over to End C of Member BC. Continue the cycle until the amount to be carried is negligible. For this case of a symmetrical frame and symmetrical loading, if the moment, − 728(d), at End B of Member AB is carried over to End A, and the cycle of distribution is performed in the other direction, the moments would evidently be the same, except with reversed signs. Hence, it can be seen that the resulting moments are, $M_{ad} = 272 - 9.5 - 0.1 + 105 + 1.4 = 368.8$; and, $M_{da} = -0.1 - 24.7 + 0.5 + 129.5 = 105.2$.

CONCLUSION

The greatest advantage of this method is its simplicity and directness. Only two formulas need be remembered in addition to the fundamental concepts of moment distribution. Equations (3) and (4) can be derived, and remembered, so easily that an experienced designer can estimate the modified beam factors, mentally, with sufficient accuracy. In these formulas, the effect of fixity of supports is shown by the value, R_{ba}, and the effect of haunching is shown by the value, γ_{ab} γ_{ba}.

It is considered best to limit this paper to a presentation of the method only. Its special adaptability, such as its application to continuous beams of varying sections, has not been discussed. The most interesting application is sometimes found in the approximate but sufficiently accurate estimate of moment by this method. However, it is not recommended as an invariable substitute for the original Cross method. Each has its own advantages, and it is left to the designer to choose for himself. Certainly, it will be worth while for engineers dealing with continuous frames to spend the little time necessary to learn it, so as to be able to utilize its obvious advantages.

ACKNOWLEDGMENTS

The writer wishes to express his utmost gratitude to Bruce Jameyson, Assoc. M. Am. Soc. C. E., under whose guidance the study was made. Thanks are also due to John T. Howell, Jun. Am. Soc. C. E., for his valuable suggestions in the preparation of the paper.

DISCUSSION

L. E. Grinter,[5] Assoc. M. Am. Soc. C. E. (by letter).—In this paper the Engineering Profession is given a clear and concise statement of another method of analyzing continuous frames that is based upon the Cross method of balancing fixed-end joint moments. This method has been in use by associates of Professor Cross since about 1926, and a brief discussion of it has been published.[6] Several other papers have appeared, describing similar procedures. The writer has published two references to the method. The first will be found in his discussion of the paper[7] by Professor Cross where mention is made of an "approximate method of balancing moments where the carry-over moment is determined from the restraint at the far end of the member." The second occurs in a discussion on methods of checking rigid frame analysis.[8]

The foregoing references are mentioned, not with the idea of establishing prior authorship of the method (which obviously goes back to the original work of Professor Cross), but rather to establish the fact that the writer speaks from a background of experience when suggesting that the method described by Mr. Lin is of limited usefulness. It was introduced into the writer's graduate and undergraduate classes from 1929 to 1932. It has since been withdrawn from the undergraduate classes because experience has shown that its usefulness is limited and because it proved confusing to the student.

A study and comparison of Figs. 9, 10, and 11 of the paper, reveals a rapid increase in the volume of recorded calculations on successive illustrations. There are sixteen recorded results on Fig. 9, twenty-two on Fig. 10, and fifty on Fig. 11. The explanation is evidently found in the fact that the number of loaded spans has been increased from one to three. There would be a far smaller rate of increase in the number of recorded results by the original Cross method. The total number in the most complex case usually could be reduced to less than fifty, and still yield satisfactory accuracy. This fact is well illustrated by Fig. 14 which is the analysis of a frame similar to that of Fig. 11 by the procedure of balancing moments used by the writer. Incidentally, the five-place results recorded in Fig. 11 seem to be entirely unjustified and, in fact, their use is in conflict with the basic idea of the method which is most useful as an approximate method of moment distribution.

A comparison of the number of tabulated results as discussed in the previous paragraph is unfair to the original Cross method because the preliminary results recorded by Mr. Lin in Fig. 8 require repeated substitutions into formulas, whereas each result recorded by Professor Cross is obtained by a single setting of a slide-rule—or mentally by dividing a number in half.

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[5] Dept. of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

[6] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., see Section Heading, "Moment Distribution Using a Constant for End Rotation", p. 118.

[7] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 13.

[8] *Engineering News-Record*, August 23, 1934, p. 249.

Hence, in the majority of cases, the writer has found the standard method of balancing moments to be the simpler procedure, and invariably so when loads occur on several spans. The method described by Mr. Lin is advantageous when one is "running out" moments into unloaded spans in the manner illustrated by Fig. 9.

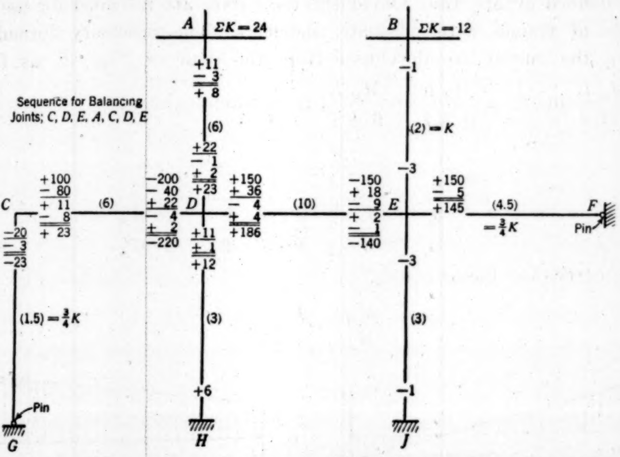

A
ΣK = 24
B
ΣK = 12
+11
− 3
+ 8
−1
Sequence for Balancing Joints; C, D, E, A, C, D, E
(6)
(2) = K
+22
− 1
± 2
+23
−3
+100
− 80
+ 11
− 8
+ 23
C
−20
− 3
−23
(6)
−200
− 40
+ 22
− 4
± 2
−220
D
+150
+ 36
− 4
± 4
+186
+11
± 1
+12
(10)
−150
+ 18
− 9
+ 2
− 1
−140
E
+150
− 5
+145
(4.5)
= ¾K
F
Pin
−3
(1.5) = ¾K
(3)
(3)
+6
−1
Pin
G
H
J

FIG. 14.—ANALYSIS OF A FRAME WITH THREE LOADED SPANS BALANCING FIXED-END MOMENTS.

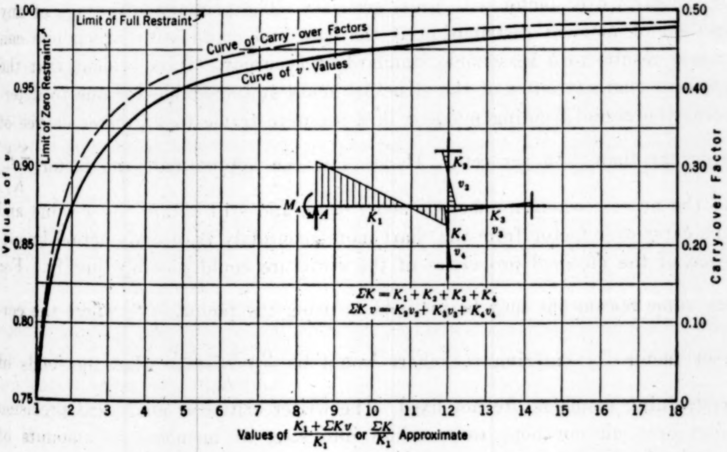


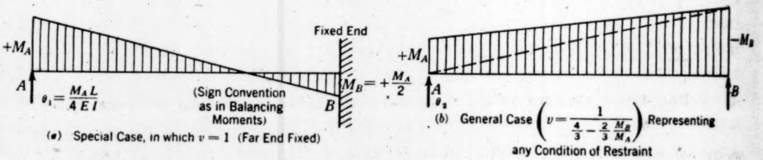
FIG. 15.—CARRY-OVER FACTORS AND v-VALUES FOR MOMENT DISTRIBUTION.

In order to make the paper as useful as possible, Fig. 15 is presented, which the writer developed several years ago for the use of his students. The nomenclature will be clear if it is understood that v is a factor such that Kv represents the relative resistance of a member to end rotation taking into account its condition of restraint at the far end. Hence, Kv is K_m as defined by Mr. Lin. Obviously, the curves are intended for use in the analysis of frames with prismatic members. The necessary formulas for plotting the curves are developed from the data of Fig. 16, as follows: $\theta_1 = \frac{M_A L}{4 E I}$ and $\theta_2 = \frac{M_A L}{3 E I} - \frac{M_B L}{6 E I}$, from which,

$$v = \frac{\theta_1}{\theta_2} = \frac{\frac{M_A}{4}}{\frac{M_A}{3} - \frac{M_B}{6}} = \frac{1}{\frac{4}{3} - \frac{2}{3}\frac{M_B}{M_A}} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

or, the carry-over factor equals,

$$\frac{M_B}{M_A} = \frac{4v - 3}{2v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$



$+M_A$
A
$\theta_1 = \frac{M_A L}{4 E I}$
(Sign Convention as in Balancing Moments)
Fixed End
B
$M_B = +\frac{M_A}{2}$
$+M_A$
$-M_B$
A
θ_2
B

(a) Special Case, in which $v = 1$ (Far End Fixed)

(b) General Case $\left(v = \frac{1}{\frac{4}{3} - \frac{2}{3}\frac{M_B}{M_A}}\right)$ Representing any Condition of Restraint

FIG. 16.—SIGNIFICANCE OF v-VALUES.

In using the curves of Fig. 15 one need not attempt to obtain the v-value or the carry-over factor with great accuracy. The primary advantage of any method of moment distribution is the ease and speed with which one can obtain results to a reasonable standard of accuracy. It is evident that the physical characteristics of the structure make it impossible for one to determine the actual bending moment in a concrete frame to a greater degree of accuracy than ± 5 per cent. Hence, one can approximate the factor, $\frac{\Sigma K}{K_1}$, to the nearest whole number in many cases and still obtain the v-value and the carry-over factor from the chart more accurately than one's actual knowledge of the physical properties of the structure could possibly justify. For the same reason one may be justified in using the factor, $\frac{\Sigma K}{K_1}$, where the correct factor for entering the chart would be $\frac{K_1 + \Sigma K v}{K_1}$ when far ends of restraining members are not fixed. The writer criticizes analytical precision that one can not hope to translate into sizes of members or amounts of reinforcing steel.

It is interesting to consider the effect of haunching of beams upon the moments in a continuous frame. It should be clear that the designer makes no effort to obtain better than a crude approximation of the Kv-value, or of the carry-over factor, for a reinforced concrete beam as usually constructed monolithically with a slab in a rigid frame structure. Actually, the moment of inertia, I, varies widely for such a beam and is dependent on the amount and sign of the bending moment because of the change from a rectangular section to a T-section at the point of contraflexure. It seems almost absurd to involve one's calculations to care for the effect of a small haunch—which may change a carry-over factor from 0.5 to 0.47—when an equal irregularity has been passed over without consideration. A study of the discussions of the writer's paper[9] on "Wind Stress Analysis Simplified," will make this observation clear.

Mr. Lin's idea of direct moment distribution can be used as a check upon the original Cross method of balancing moments. Since moments must be distributed according to the Kv-values, it is possible to check the total values of the distributed moments (balancing moments plus carry-over moments, but not including original fixed-end moment) by comparing their relative values to the relative Kv-values at the joint. For this purpose, it is convenient to revise the expression for v by removing a constant factor of $\frac{4}{3}$ from the denominator to obtain:

$$v = \frac{1}{\frac{4}{3} - \frac{2}{3}\frac{M_B}{M_A}} = \frac{1}{\frac{4}{3}\left(1 - \frac{1}{2}\frac{M_B}{M_A}\right)} = \frac{3}{4}\ V \dots\dots\dots\dots (7)$$

in which, $V = \frac{1}{1 - \frac{1}{2}\frac{M_B}{M_A}}$. Hence, the distributed joint moments must also be in proportion to the KV-values. This is a complete check upon the balancing procedure, but is not a check upon the original fixed-end moments. Neither does it check the moments where there is a joint translation in the structure. The writer prefers to check his results by statics and then to draw the deflected structure to see that the requirements of continuity are satisfied. In one particular case a serious error in the analysis of a wind bent was located[10] by plotting the column deflections.

This discussion has pointed out some of the applications of this paper that may be found useful. The individual preferences of the reader may extend its application to a broader field. However, the extreme simplicity of the original Cross method recommends its use by those engineers who have an occasional need for such a tool, but do not intend to become specialists in the field of indeterminate structures. The writer places his undergraduate students in that classification.

W. H. Huang,[11] Esq. (by letter).—The fundamental equations for modified stiffnesses (which the writer has elsewhere[12] termed "conjugate" stiffnesses) and carry-over factors for members of constant moment of inertia may be given in the following forms:

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[9] *Transactions*, Am. Soc. C. E., Vol. 98 (1934), pp. 610–669, Fig. 9 and Table 11.

[10] *Loc. cit.*, p. 663.

[11] Scholar of National Tsing Hua Univ., Peiping, China; Graduate Student, State Univ. of Iowa, Iowa City, Iowa.

[12] *Journal*, The Chinese Inst. of Engrs., Vol. 9, No. 5.

$$R_{ab} = S_{ab}\left(1 - \frac{S_{ab}}{4\,(S_{ba} + R_{bc} + R_{bd} + R_{be})}\right) \ldots\ldots\ldots\ldots (8)$$

and,

$$\gamma = \frac{1}{2 + 1.5\,\dfrac{S_{ab}}{R_b - R_{ba}}} \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which R_{ab} = the modified stiffness of Member AB at End A; $S_{ab} = S_{ba}$ = the stiffness of Member AB or BA; R_b = the summation of the modified stiffnesses of Members BA, BC, BD, and BE, at End B; and γ = the carry-over factor of Member AB at End A.

Instead of using the cumbersome Equation (8), or the author's Equation (3), which in certain cases can only be solved by successive approximations, the writer suggests[13] the use of the following approximate equation;

$$R_{ab} = S_{ab}\left(1 - \frac{S_{ab}}{4\,S_b}\right) \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which, $S_b = S_{ba} + S_{bc} + S_{bd} + S_{be}$. Equation (10) yields an error of less than 2.08 per cent. As the stiffness of a member usually cannot be determined within an accuracy of 2%, and as the effect of the errors on the final moment distribution is compensating, Equation (10) is recommended for general application.

The advantage of this method depends largely upon the ease with which the modified stiffnesses and the carry-over factors can be obtained. Therefore, a graphical method for solution is of value, and the writer uses nomographs in connection with Equations (9) and (10).

Felix H. Spitzer,[13] Assoc. M. Am. Soc. C. E. (by letter).—Justly, the author may claim for his presentation the advantage of superior convenience as a tool of structural design in comparison with the original Cross method; but he cannot maintain his claim when his method is compared with the modification of the Cross method introduced by L. E. Grinter, Assoc. M. Am. Soc. C. E.,[14] which has reduced labor to such an extent that frequently the calculation of stiffnesses of members and joints and other preliminary work common to all methods, becomes the major part of the work to be performed.

Due to the application of "modified" coefficients and of additional equations, although very simple, Mr. Lin's modification has lost much of the simplicity so characteristic of the Cross method. This desirable quality of a simple physical process easily visualized and remembered, is even more apparent in the Grinter modification, where the joints, which visual inspection finds to be most unbalanced, are successively released and balanced until an accuracy sufficient for the purpose is reached.

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[13] Structural Engr., Dept. of Public Works, San Francisco, Calif.

[14] Discussion of the paper by Hardy Cross, M. Am. Soc. C. E., entitled, "Analyses of Continuous Frames by Distributing Fixed-End Moments", *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 11.

There is scarcely need to sacrifice simplicity to exactness when for all practical purposes the difference is small enough to be of no importance.

HAROLD E. WESSMAN,[15] ASSOC. M. AM. SOC. C. E. (by letter).—The so-called "direct" method of moment distribution is presented in this paper in a very satisfactory manner. Several engineers have previously indicated their preference for a single, direct distribution and have given general equations which evaluate actual end restraints and lead directly to the final moments.

Of the various forms, however, the writer prefers Mr. Lin's presentation of equations for correct stiffness and carry-over factors. His derivation proceeds quite logically in accordance with the basic sequence of operations so nicely outlined by Professor Cross in his fundamental method, a treatment which emphasized concepts in terms of simple words and numbers rather than of general algebraic equations.

A designer, analyzing a structural framework with a variety of members subject to a number of loading conditions, will find it worth while to become familiar with this direct method, or some other abbreviated procedure[16]; but in classroom practice, the writer feels that better results are obtained when moment distribution is taught with a minimum number of equations. That, of course, means teaching the basic method. The two equations proposed by the author, although simple, are not easily remembered unless one is using this tool of analysis constantly. Incidentally, Mr. Lin should have stated that it is necessary to remember three equations, rather than two. One must also keep Equation (1) for R_{ba} in mind.

In the "Synopsis" of his paper, the author might have qualified the term, "exact." The method is exact for any structure of the type in Fig. 17, which does not contain closed structural panels and in which the conditions of restraint of all external terminals, such as those at *A, B, D, E, G,* and *H,* are completely known. The method is not exact for any structure of the type in Fig. 18, which contains a closed structural panel, *CDGF*. One must assume an initial value for R_{dc} or R_{gf}. The author notes a similar case in his Example 4. It is further illustrated by his assumption of values for R_{ad} and R_{be} in Example 1.

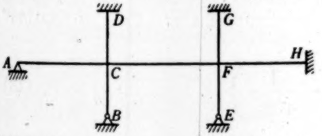

FIG. 17.

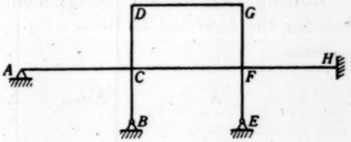

FIG. 18.

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[15] Associate Prof. of Structural Eng. and Mechanics, Coll. of Eng., State Univ. of Iowa, Iowa City, Iowa.

[16] See methods proposed in "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E.

Ordinarily, one can estimate these values closely enough so that the error involved in final moments is of no practical significance. It is also well to keep in mind that the error involved in moment distribution by terminating computations at the end of two cycles of distribution is ordinarily of no consequence in design.

Egor P. Popoff,[17] Jun. Am. Soc. C. E. (by letter).—The modification, presented by Mr. Lin, of the well-known Cross method of analysis contains material of considerable interest. It is timely to note that quite similar modifications have been proposed by a number of writers.

Mr. Lin uses the method of moment distribution, entirely, for the derivation of his equations for the modified stiffnesses of the members and for the "carry-over" factors. Other writers use the slope deflection method; but fundamentally the two methods are closely related.

Analyzing the work of three writers in this field,[18] it may be noted that a mere algebraic transformation is sufficient to put all the results into an identical form, if a common definition can be written for the "factor of restraint," R_{ba}. The expression used by Mr. Lin in Equation (1) is the restraining effect of the several members at a joint on the member under consideration. The factor of restraint is variously defined: Mr. Lin defines it by Equation (1), whereas others define it by $\sum \frac{K_{bnm}}{K_{ab}}$, and $\frac{\Sigma K_{bnm}}{K_{ab} + \Sigma K_{bnm}}$ (expressed in Mr. Lin's notation). Mr. E. B. Russell,[18] defines it as,

$$R'_{ba} = \sum \frac{K_{bnm}}{K_{ab}} \qquad \text{(11)}$$

whereas, L. T. Evans,[18] Jun. Am. Soc. C. E., and T. F. Hickerson,[18] M. Am. Soc. C. E., define it as,

$$f_{ba} = \frac{\Sigma K_{bnm}}{K_{ab} + \Sigma K_{bnm}} \qquad \text{(12)}$$

Of the foregoing definitions, perhaps Equation (12) is the most logical, because it has a value of 1 for the fixed condition and 0 for the hinged condition.

Considering, for simplicity, members of uniform cross-section, the expression for the modified stiffness of a member in terms of the degree of fixation[19] becomes,

$$K_{bnm} = K_{bn}\left(1 - \frac{1}{4\,R_{nb}}\right) \qquad \text{(13)}$$

which is Equation (3) developed by Mr. Lin for members with constant moment of inertia. The extension to a more general case is obvious.

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[17] Structural Eng. Designer, Fullerton Union High School, Fullerton, Calif.

[18] "Handbook of Rigid Frame Analysis", by L. T. Evans, Jun. Am. Soc. C. E., Edwards Brothers, 1934; "Structural Frameworks," by T. F. Hickerson, M. Am. Soc. C. E., Univ. of North Carolina Press, 1934; and "Analysis of Continuous Frames," by E. B. Russell, Ellison and Russell, 1934.

[19] "Handbook of Rigid Frame Analysis", by L. T. Evans, Jun. Am. Soc. C. E., Edwards Brothers, 1934 (In Equation (174), let $f = 1 - \frac{1}{R}$).

The idea of the modified stiffness of a member reveals a new concept. Thus, in addition to the dependence on the length and cross-section, the true stiffness is also dependent upon the degree of restraint at the far end of the beam.

For the special case of a member with a uniform cross-section, the following has been offered.[20]

$$\gamma_{abm} = \frac{1}{2} \frac{R_{ba} - 1}{R_{ba} - \frac{1}{4}} \qquad (14)$$

which is Equation (4) developed by Mr. Lin.

In the modifications cited for comparison herein, the equations developed are for the general cases, and many graphs and tables are prepared; but still other writers define some similar coefficients, such as R, differently, and thus the transformation into the form of equations as given by Mr. Lin is not quite as obvious as for the case shown, which is by far the most important one.

The method seems to be somewhat longer for the simpler problems than the usual Cross method. It appears to be especially inconvenient for closed frames as in Example 4 where several trials may be necessary to check the value of R originally assumed. However, this group of modifications is of practical significance for the cases in which there are several types of loading for the same structure. The method is also satisfactory for the simpler cases with the assistance of the extensive tables now available.[18]

L. T. Evans,[21] Assoc. M. Am. Soc. C. E. (by letter).—An interesting addition to engineering literature is contained in this paper, but it is to be regretted that Equations (1), (2), and (3) are incorrect. The first error is introduced in the assumption that a moment equal to $K_{ab} = \frac{I}{L}$ applied at End A will produce a unit angular deflection at A. Applying the modified slope-deflection[22] equations to End A:

$$M_{AB} = E\,K_c\,(C_1\,\theta_A + C_2\,\theta_B - [C_1 + C_2]\,R) - C_{AB} \qquad (15)$$

which, for the conditions of the problem, reduces to:

$$M_{AB} = C_1\,E\,K_c \qquad (16)$$

in which, K_c represents $\frac{I_c}{L}$ for a specified point of the beam, and C_1 is the beam coefficient, the value of which is determined by the variation of moment of inertia of the member. For a constant moment of inertia, $C_1 = 4$.

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[20] "Structural Frameworks", by T. F. Hickerson, M. Am. Soc. C. E., Univ. of North Carolina Press, 1934, Equation (13), where, again, let $f = 1 - \frac{1}{R}$.

[21] Cons. Structural Engr., Long Beach, Calif.

[22] See "Modified Slope-Deflection Equations", by L. T. Evans *Journal*, Am. Concrete Inst., October, 1931. pp. 109–130.

Since an external moment applied to a joint will be resisted by the moments in the members which will be in direct ratio to the "modified stiffness" of each member, Equation (1) should be:

$$R_{ba} = \frac{C_1 K_{ba} + \Sigma K_{bnm}}{C_1 K_{ba}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

The author has been inconsistent in using modified stiffness for Members $B1$, $B2$, and $B3$, but not for Member BA. In order to obtain the correct results for the problem as stated, the modified stiffness of Member BA must also be used. Under the assumed condition of full restraint[23] at End A in Step 2 the modified stiffness would be: $K_{ban} = C_1 K_{ab}$ and Equation (2) becomes:

$$M = -\frac{C_1 K_{ab} \gamma_{ab}}{R_{ba}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

which changes Equation (3) to:

$$K_{abn} = C_1 K_{ab} \left(1 - \frac{\gamma_{ab} \gamma_{ba}}{R_{ba}} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Expressed in terms of the degree of restraint at End B of Member AB (that is, f_{ba}) Equation (19) should read:

$$K_{abn} = K_{AB} \left[C_1 - (1 - f_{ba}) \frac{C^2_2}{C_3} \right] \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

which reduces to:

$$K_{abn} = K_{AB} (3 + f_{ba}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

for constant moment of inertia, and Equation (4) for constant moment of inertia is:

$$\gamma_{abm} = \frac{2 f_{ba}}{3 + f_{ba}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

It is to be noted also that the author's formulas, Equations (1), (2), and (3), apply only when the far end of Member AB is fixed, which introduces another error when applied to continuous structures. Equations (20) to (22) are general and apply for any degree of restraint at the far ends of the members.

The value, $R_{cd} = 1.25$, in the frame of Fig. 8 is equivalent to:

$$R = \frac{K_{CD} + \dfrac{K_{CG} (3 + f_{gc})}{C_1}}{K_{CD}} = \frac{6 + \dfrac{2 \times 3}{4}}{6} = 1.25$$

The correct value is:

$$R = \frac{6 (3 + f_{dc}) + 2 \times 3}{6 (3 + f_{dc})}$$

---

[23] For a complete development and application of the method of restraints, see "Handbook of Rigid Frame Analysis", by L. T. Evans.

which could yield $R = 1.25$ only for the special case of $f_{dc} = 1$. Members *DA, DE,* and *DH* do not fix End *D* of *CD*; in other words, f_{dc} is less than unity and, therefore, the correct value of this *R* is greater than 1.25. The author does not give sufficient data concerning Member *DE* to allow the computation of f_{dc}.

The correct expression for R_{de} is:

$$R_{de} = \frac{C_1\,K_c\,(0.51 + 0.49\,f_{ed}) + 6 \times 3.2 + 3 \times 4 + 6\,(3 + f_{ad})}{C_1\,K_c\,(0.51 + 0.49\,f_{ed})} \ldots (23)$$

Equation (23) takes into consideration both the beam coefficients and the actual degree of restraint at the far ends of the members.

Although one may begin with the fixed-end moments and make a correction, it seems more direct and logical to use a moment that takes into consideration the actual degree of restraint at both ends of the member. For example, consider the frame shown in Fig. 8. Let it be required to compute the moments, M_{AB} and M_{BA}, due to a concentrated load of 1 000 lb applied at the center of Span *AB*. The fixed-end moment is $\frac{P\,L}{8} = 3\,750$ ft-lb. The degree of restraint at End *A* of *AB* is $f_{ab} = \frac{3 \times 3.8}{3 \times 3.8 + 5 \times 4} = 0.363$, and, in a similar manner, $f_{ba} = 0.566$.

Now, let $D_4 = 3 + f_{ab} + f_{ba} - f_{ab}\,f_{ba} = 3.723$; and,

$$M_{AB} = -\,\frac{2\,f_{ab}\,(3 - f_{ba})}{D_4} \times C_{AB} = -\,1\,780 \text{ ft-lb}$$

and,

$$M_{BA} = +\,\frac{2\,f_{ba}\,(3 - f_{ab})}{D_4} \times C_{BA} = +\,3\,005 \text{ ft-lb}$$

These moments have the same values that would be obtained by the distribution methods after all joints have been "unlocked," all distributions completed, and the results totaled. It is seen that the "method of restraints" provides an easy, exact, and rapid solution and is especially adapted to problems of partial restraint, such as indicated at the bases on the columns of Fig. 8.

In the author's "Conclusion" it is stated that "the effect of fixity of supports is shown by the value, R_{ba}, and the effect of haunching is shown by the value, $\gamma_{ab}\,\gamma_{ba}$." This statement is in error because both the values, R_{ba} and $\gamma_{ab}\,\gamma_{ba}$, are influenced by both the degree of restraint at the ends of the members and by the moment of inertia variation.

Although the greater part of this discussion has been devoted to pointing out inconsistencies, the writer wishes to make it clear that this has been done to help Mr. Lin build up his work and not to destroy. The writer realizes from his own experience that the author has used considerable time in the development of the material of the paper and deserves credit. Advancement of the profession can occur only through the hard work of the members.

G. S. SALTER,[24] M. AM. SOC. C. E. (by letter).—Several papers have been presented recently, on various modifications of the direct method of moment distribution. These variations are interesting if for no other reason than that they show that the profession is becoming increasingly conscious of the adaptability of the moment-distribution method of analysis as presented by Professor Cross, and as designers become better acquainted with the original they devise modifications which they find adapted to their particular use. Too often, most of these simplifications and "short cuts" are such only to the person who devises them and other analysts are slow to adopt even the best of the methods because they prefer to use those with which they are more familiar even though they may be more laborious.

For frames subjected to several conditions of loading or other complexities, time could probably be saved by the use of the author's or some similar method, but for the usual frame and loading conditions, successive balancing of the joints is as simple as could be desired and involves less chance of error. It may be, as Mr. Lin states, that the special value of his modification is its application to continuous beams of varying section; but if so, it would seem that the paper would have been of much greater value if this application had been demonstrated. This probably would necessitate the inclusion of several tables or diagrams before it would have been usable.

Mr. Lin's concept is similar to that offered by Professor Cross[25] in 1929. It is more complete in that Mr. Lin uses modified carry-over factors as well as the modified $\frac{I}{L}$ (K)-values; but Professor Cross secures the same results by using the regular carry-over values of $\frac{1}{2}$ and by distributing further at successive joints.

As one application of his method Mr. Lin presents the analysis of a symmetrical closed frame (Example 4). Such a frame (which is of common occurrence in the design of water and sewage treatment plants) can scarcely be included in the category of structures which are of sufficient complexity to warrant the use of the author's method because more time would be required in determining the frame constants than would be necessary to make the regular moment distribution. However, he could have simplified the analysis materially by utilizing a chart similar to Fig. 19 for the determination of necessary constants.

For example, in the regular moment distribution for a symmetrical frame it may readily be seen that, if the first balancing moments at the two ends of the side members are equal (and opposite in sign) the distribution is

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[24] Structural Engr., San. Dist. of Chicago, Chicago, Ill.

[25] "Exact Method of Analysis", in his paper, entitled "Continuity as a Factor in Reinforced Concrete Design," presented before the Am. Concrete Inst.

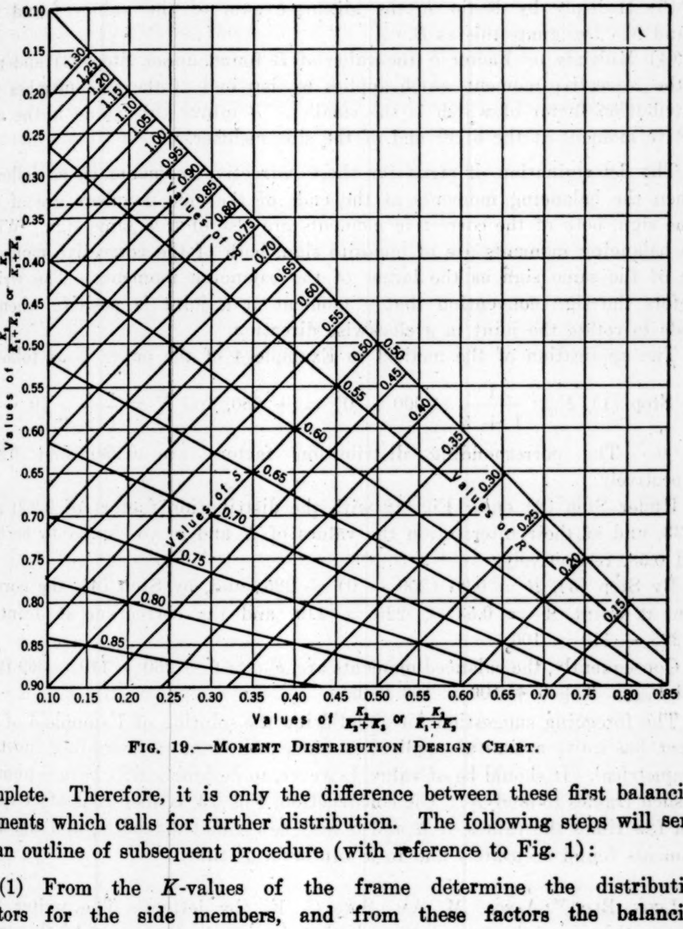

FIG. 19.—MOMENT DISTRIBUTION DESIGN CHART.

complete. Therefore, it is only the difference between these first balancing moments which calls for further distribution. The following steps will serve as an outline of subsequent procedure (with reference to Fig. 1):

(1) From the *K*-values of the frame determine the distribution factors for the side members, and from these factors the balancing moments at *A* and *D*. Designate these moments as $F = \frac{K_3}{K_1 + K_3} (M_{ab} - M_{ad})$ and $N = \frac{K_2}{K_2 + K_3} (M_{dc} - M_{aa})$, respectively. This follows the regular moment distribution procedure. The variation is, as follows:

(2) With the distribution factors as found in Step (1) for the side member enter Fig. 19 using either factor as the abscissa and the other as the ordinate so that their intersection is within the diagram area. At their intersection find the value, *S* (a stiffness factor), and *R*, the ratio of stiffness factors.

(3) Multiply by Ratio R the algebraic sum of the values found for F and N. Designate this as B.

(4) Multiply by Factor S the value of B found under Step (3) and this is the corrective moment which applies to that end of the side member the distribution factor of which is the smaller. B minus this value is the corrective moment at the other end of the side member.

The determination of signs for these corrective moments is, as follows: When the balancing moments at the ends of the side members are of the same sign, both of the corrective moments are also of this same sign. When the balancing moments are of opposite sign, both of the corrective moments are of the same sign as the larger of the balancing moments. The writer prefers the sign convention that a moment at a joint is positive when it tends to rotate the joint in a clockwise direction.

The application of the method to Example 4 of the paper is as follows:

By Step (1), $F = \frac{1}{1 + 3} (1\,000 - 0) = + 250$, and $N = \frac{1}{1 \times 2} (0 - 0) = 0$. The corresponding distribution factors are 0.250 and 0.333, respectively.

Under Step (2) enter Fig. 19 with the distribution factors of 0.250 and 0.333, and at their intersection the values of R and S are found to be 0.90 and 0.53, respectively.

By Step (3), $B = 0.90\ (250 - 0) = 225$; and, by Step (4), the correction at Joint $A = 0.53 \times 225 = 119$, and the correction at Joint $D = 225 - 119 = 106$.

Consequently, the balanced moments are, $F_{AD} = 0 + 250 + 119 = 369$ ft-lb, and $F_{DA} = 0 + 0 + 106 = 106$ ft-lb.

The foregoing suggestion for simplifying the solution of Example 4 of the paper has quite a limited application since the frame and loading must be symmetrical. It should be of value, however, to designers who have a number of such frames to analyze. The construction, Fig. 19, is such that after using it a few times the values of R and S may be estimated readily, and the final moments found accurately and in a minimum of time.

LEON BLOG,[26] ASSOC. M. AM. SOC. C. E. (by letter).—The writer will discuss this paper in two sections both referring to that part of the paper headed "Limitation of the Method."

Section I.—Limitation of the Lin Modification as Applied to Structures for Which Any of the Values of R as Defined by the Author Are Other Than 1 for a Hinged Condition and ∞ for a Completely Fixed-End Condition.—Hardy Cross, M. Am. Soc. C. E., demonstrated the basic method which the author proposes to modify by utilizing a structure that has its reaction ends of members either truly hinged, truly fixed, or truly cantilevered.[27] There were no approximations as to the degree of restraint in any of the members.

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[26] Asst. Chf. Structural Engr., Div. of Bridges and Structures, City of Los Angeles, Los Angeles, Calif.

[27] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 5.

In his examples, Mr. Lin makes assumptions as to the degree of restraint of the second-story ends of the columns in Fig. 8 and for one end of the upper horizontal member of the closed frame of Figs. 12 and 13.

To check the effect of such restraint assumptions upon the accuracy of the solution for moments in the structure as well as the accuracy of the assumptions which must be made in order to begin the solution with values of R reasonably close to the true ones, the writer performed a parallel distribution of a moment of 100 at D in Fig. 9 by giving R_{ad} and R_{be} simultaneous values of 3.5 and 6.5, and repeating the distribution with values of 2.5 and 5.5. The significant resultant moments are given in Fig. 20, and the relative values stated in percentages of the author's moments assumed as the true ones are given in Table 1, which also includes the modified carry-over and stiffness factors for End D, of Member DA, for reasons which follow.

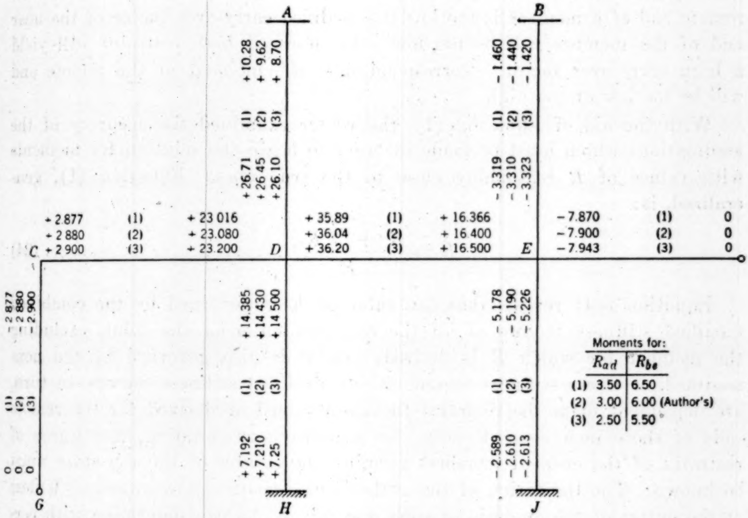

A
B
(1) + 10.28
(2) + 9.62
(3) + 8.70
(1) − 1.460
(2) − 1.440
(3) − 1.420
+ 26.71
+ 26.45
+ 26.10
− 3.319
− 3.310
− 3.323
+ 2.877 (1) + 23.016
+ 2.880 (2) + 23.080
C + 2.900 (3) + 23.200 D
+ 35.89 (1) + 16.366
+ 36.04 (2) + 16.400
+ 36.20 (3) + 16.500 E
− 7.870 (1) 0
− 7.900 (2) 0
− 7.943 (3) 0
− 2.877
− 2.880
− 2.900
(1)
(2)
(3)
0
0
0
+ 14.385
+ 14.430
+ 14.500
(1)
(2)
(3)
+ 7.192
+ 7.210
+ 7.25
− 5.178
− 5.190
− 5.226
(1)
(2)
(3)
− 2.589
− 2.610
− 2.613
G
H
J
Moments for:
R_{ad} R_{be}
(1) 3.50 6.50
(2) 3.00 6.00 (Author's)
(3) 2.50 5.50

Fig. 20.—Distribution of an External Moment of 100 Applied at D Using Step 2 of the Lin Paper.

TABLE 1.—Relative Values of Certain Moments of Fig. 20 (Expressed as Percentages) Based on the Moments for $R_{ad} = 3.00$ and $R_{be} = 6.00$

R_{ad}	R_{be}	M_{dc}	M_{dh}	M_{da}	M_{ad}	M_{de}	M_{ed}	γ_{dam}	K_{dam}
3.5	6.5	99.74	99.21	100.98	106.86	99.58	99.79	105.78	101.27
3.0	6.5	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2.5	5.5	100.91	100.48	98.68	90.43	100.46	100.41	91.48	98.18

From Table 1, observe that the greatest moment variations from the author's values occur for M_{ad}. For the case of restraint factors assumed as

0.50 more than those of the author, the surplus for M_{ad} is roughly 6% and for the case of 0.50 less than the author's, the deficit is about 9 per cent.

For the restraints assumed by the writer, the modified stiffness factor, K_{dam}, does not vary much from Mr. Lin's value so that K_{dam} cannot be the explanation for the large variations in M_{ad}; but the modified carry-over factor, γ_{dam}, varies almost as much as the moment, M_{ad}, and is almost directly responsible for the variations from the author's value of M_{ad}.

Inspection of Equation (4) reveals that when the restraint assumed at the remote end of a member is too low the modified carry-over factor of the near end of the member will be too low. An assumed high restraint will yield a high carry-over factor. Correspondingly, the moment at the remote end will be too low or too high.

With the aid of Equation (1), the writer examined the accuracy of the assumptions which must be made in order to begin the solution for moments with values of R reasonably close to the true ones. Equation (1), generalized, is:

$$R = \frac{K + \Sigma K_m}{K} \qquad (24)$$

Equation (24) reveals that the value of R is governed by the combined modified stiffness factors of all the members entering the joint, excluding the member for which R is desired; and it is also governed by the non-modified, or Cross stiffness factor. These modified stiffness factors, in turn, are dependent upon the restraint factors assumed or derived for the remote ends of those members. Finally, for a multi-story building, the degree of restraint of the ends of members forming the ceiling of the top story must be known. The limitation of the author's modification now appears. Unless at the outset of the analysis of such a structure, he can determine with certainty the degree of restraint that will prevail in the top-story members, the writer is unable to understand how Mr. Lin can claim that he has presented a rigid method having the same limitations as that presented by Professor Cross.

It is true that similar definiteness as to the degree of restraint or fixity for the ceiling members of the top story is requisite to the analysis of a multistoried structure by the pure Cross method; but Professor Cross did not find it necessary to introduce the concept of restraint factors intermediate in value between a hinged condition and complete fixity which the author evaluates, respectively, as $R = 1$ and $R = \alpha$. Having introduced the concept of partial restraint in connection with the Cross method, modified, Mr. Lin should give some indication as to the accuracy of estimating such restraint.

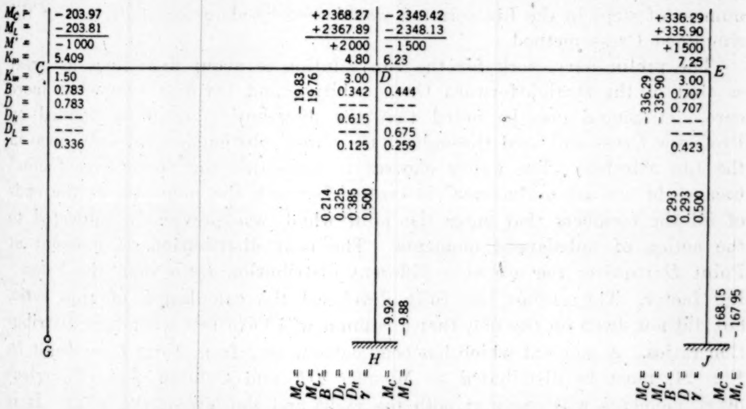
Table 2 shows how much error could be allowed in obtaining the combined modified stiffness factors of Equation (1) for the values of R which the writer assumed in his solution of Fig. 9, whereby he obtained the moment values of Table 1. Whether it is possible to stay within the limits of this error for a multi-storied structure, the writer is not prepared to state.

He has checked the closed structure of the author's Fig. 12 and Fig. 13 with the aid of a method presented[18] by L. T. Evans, Jun. Am. Soc. C. E. This is a method of determining the angular rotations produced in a structure when a unit moment is applied at one end of a member. However, it involves the idea of restraint at the end of the member opposite the one at which the unit moment is applied and is thus similar in that respect to the author's concept which is based on the amount of moment or stiffness which will produce a unit moment at the rotating end while the other end remains fixed. Mr. Evans gives a table by means of which such restraint may be estimated with reasonable precision. Were Mr. Lin to develop his modification so as to permit an estimate of the restraints which are considered the limitation of the method, the writer would use it for multi-storied structures for reasons which are enumerated in Section II of this discussion.

TABLE 2.—ALLOWABLE ERROR IN OBTAINING COMBINED MODIFIED STIFFNESS FACTORS OF EQUATION (1)

R_{ad}	K_{ad}	K_m of adjoining members	Allowable error	R_{be}	K_{be}	K_m of adjoining members	Allowable error
2.5	6	9	....	5.5	2	9	....
			3				1
3.0	6	12		6.0	2	10	
			3				1
3.5	6	15	....	6.5	2	11	....

Section II.—Application of Lin Paper to Such Structures for Which the Reaction Ends of Members Are Definitely Either Hinged or Fixed-Ended and for Which, Therefore, the Values of R, According to the Author, Are Respectively, 1 and ∞.—Fig. 21 shows a part of Fig. 8, the upper story columns and Span *EF* having been deleted. The symbols, including those introduced by Mr. Lin, are defined as: M_C = moment by the Cross method, without


M_C =
M_L =
M' =
K_m =
K_w =
B =
D =
D_H =
D_L =
γ =
-203.97
-203.81
-1000
5.409
C
1.50
0.783
0.783

0.336
+2368.27
+2367.89
+2000
4.80
-19.83
-19.76
3.00
0.342

0.615

0.125
-2349.42
-2348.13
-1500
6.23
D
0.444

0.675
0.259
+336.29
+335.90
+1500
7.25
E
-336.29
-335.90
3.00
0.707
0.707

0.423
0.214
0.325
0.385
0.500
0.293
0.293
0.500
-9.92
-9.88
-168.15
-167.95
G
H
J
M_C =
M_L =
B =
D_L =
D_H =
γ =
M_C =
M_L =
M_C =
M_L =
B =
D =
γ =
M_C =
M_L =

FIG. 21.—COMPARISON OF METHODS, PART OF FIG. 8.

simplifications; M_L = moment by the Lin modification, according to Example 3; M' = fixed-end moment; K_m = stiffness factors; B = balancing factor applied to an unbalanced moment derived from the fixed-end moments, M'; D = distribution factor; D_R = value of D to be applied to an unbalanced moment which occurs to the right of Column DH; D_L = value of D to be applied to an unbalanced moment which occurs to the left of Column DH; and γ = modified carry-over factor. The members of this abbreviated structure have been given the same unmodified stiffness and carry-over factors, and the assumed fixed-end moments distributed are the same as those in Fig. 11. The restraints assumed for the reaction ends of the lower story columns are the same as those of Fig. 11. Since there is no upper story, the question of estimating the restraint of any member does not enter the solution of the problem. A comparison between the Cross method, without simplifications, and the author's modification can then be made. Fig. 21 shows how closely the two solutions check each other, utilizing an ordinary slide-rule.

TABLE 3.—COMPARATIVE WORK INVOLVED IN SOLUTION OF FIG. 21

Method	Preliminary distribution ratios and carry-over factor calculations	Items written in solution	Total steps	Balancing cycles
Cross.......	7	120	127	6
Lin.........	36	21	57	1

Table 3 shows the relative labor involved in these solutions. The number of written items could have been reduced to 108 by using $\gamma = 0.75$ for M_{cg}, thus eliminating twelve items at End G in the Cross solution. The total number of steps in the Lin solution would then be about one-half that of the simplified Cross method.

The preliminary work for the Lin solution is about five times as great as that of the straightforward Cross solution and involves somewhat more care. It should also be noted that the distribution ratio as defined by Professor Cross and used throughout the Cross solution is used only once in the Lin solution. The writer chooses to term this the "balancing factor" because by its use a "balance" is created between the moments at the ends of all the members that enter the joint which was previously subjected to the action of unbalanced moments. The next distribution of moment at Point D requires the use of a different distribution ratio than the balancing factor. The author has fully developed the calculation of this ratio, but did not dwell on the fact that a column will have two secondary distribution ratios. A moment which has been carried over from Joint C to Joint D, Fig. 21, must be distributed to Member DE and Column DH. "Carried-over" moments will occur at both the right and the left of the joint. It is easy to remember that the member which supplies the carried-over moment does not share in the distribution and that its modified stiffness factor is not used when evaluating the distribution factor. The writer has also found it convenient to erase the balancing factor temporarily in order that it shall not be used after the first distribution.

To some readers who have noted that the calculations preliminary to actual distribution will be the most numerous for the Lin modification, the writer indicates the extreme rapidity of convergence of this technique. This quality results in a minimum of distributing and of carry-over items which are quickly added for a result. This part of the solution can be checked quickly. In the case of the pure Cross method, should the initial fixed-end moments be large or, should the convergence be slow, periodic summations of numerous moments must be made to determine when to cease distributing. Through an inadvertent interchange of distribution ratios between two members, joint moments may be erroneously balanced. This will not be discovered except through common-sense check when possible; but the writer prefers a technique which permits of speedy checking. He believes that the Lin procedure, when based on definitely determined restraint factors, affords such speed.

In conclusion, this modification of the Cross method is commended to the structural engineer for one-story structures with truly fixed or pin-ended reaction ends of members. If the author will develop a rapid method of estimating restraints for intermediate degrees of fixity, he will have made a valuable labor-saving contribution toward the solution of framed structures by the Hardy Cross method.

Austin H. Reeves,[28] Assoc. M. Soc. C. E. (by letter).—The "Conclusion" of this paper contains this sentence: "However, it is not recommended as an invariable substitute for the original Cross method."[3] It is neither advantageous nor advisable to substitute the author's method for the Cross method; the probable outcome of such substitution would be confusion.

When thoroughly understood, the Cross method is applied easily to any type of frame with the greatest simplicity, speed, and accuracy. The writer was much better able to utilize the Cross method because of a previous working knowledge of a few other methods, the most helpful of which was the conjugate point method.[29] This helpfulness was especially evident in the design of frames containing girders or columns with variable moments of inertia. A designer of rigid frames of all types is almost perfectly equipped if he has a thorough understanding of the application of the original Cross method, supplemented by an equally good working knowledge of the conjugate point method.

Mr. Lin could have explained his method much more easily by a statement that it is one of the special cases of the conjugate point method adjusted in a new and more complicated form to appear as a modification of the Cross method.

For example, to find the author's carry-over factors for any span, *AB*, by the conjugate point method, note where the *U*-line (Fig. 22) cuts the span at Point U and where the V-line cuts the span at Point V. Now, $-\frac{AU}{UB} = \gamma_{ba}$ and $-\frac{VB}{VA} = \gamma_{ab}$. To find the author's "modified" carry-over factors for any span, AB, note where the J and K fixed points are located. Then, $-\frac{AJ}{JB} = \gamma_{bam}$ and $-\frac{KB}{KA} = \gamma_{abm}$. After obtaining γ_{abm} and γ_{bam} for all members, it is easy to find the author's K_{abm} and K_{bam} in each case. There would then be no necessity for calculating or tabulating R_{ab} and R_{ba} as the author has done.

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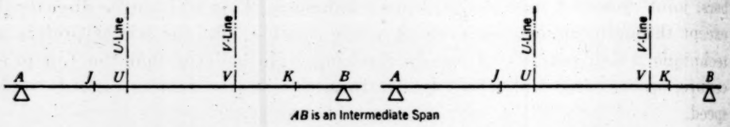
[28] Newark, N. J.

[29] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.



AB is an Intermediate Span

Constant I

Variable I

FIG. 22.

This leads up to what the writer considers the most unfortunate feature of the paper; namely, the signs. The carry-over factors should not be positive, but should have negative signs. To prove this statement, consider a horizontal continuous beam of three spans with simply supported ends. Twist the right-hand end with a moment, M, in a vertical plane through the longitudinal axis of the beam, there being no other loads on the continuous beam. The moment closing line will pass leftward from Point M, distance up or down on the R_4 line, and through J_3 until it intersects the reaction line, R_3; from this point on R_3 it will pass through J_2 in the middle span until it hits the reaction line, R_2; from which last named point it will pass through J_1, which is on the R_1-line. This demonstrates clearly that when a moment closing line passes through a point of inflection, the sign of the moment changes either from plus to minus or from minus to plus. Therefore, any carry-over factor should have a minus sign.

Furthermore, to finish a problem as in Fig. 11, Corner C, and have $+ 127.4$ on one side of the joint and $- 127.4$ on the other side, is at variance with both American and foreign texts, with customary practice (except in the slope deflection method),[30] and with common sense. It has been a source of deep regret to the writer that so many brilliant professors and engineers have seen fit to reverse Professor Cross on the matter of signs.

The writer agrees with Professor Cross as regards signs except in the following slight deviation in complicated frames; namely, the writer looks at all girders, truss chords, or arches from the bottom, at all outer columns from inside the structure, and at interior columns or web members of trusses from either direction. The moments are written on any member of a framework in such a manner that there can be no error as to which way the designer faced the member.

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[30] *Bulletin No. 108*, Univ. of Illinois, Eng. Experiment Station, Urbana, Ill.

After solving numerous complicated problems of various types by the Cross method, the writer believes the Cross sign conventions eventually will be in general use, except for minor variations. In a paper entitled, "Sign Conventions for Bending Moments in Rigid Frames,"[31] Mr. Robins Fleming, of the American Bridge Company, states: "Notwithstanding the vigorous exceptions that have been taken, the writer has a preference for the convention used by Professor Cross * * *."

Some of the disadvantages of Mr. Lin's modification are:

1.—Considerable preliminary work is required before a beginning can be made on the actual determination of moments;

2.—Side-sway, when present due to (*a*) unsymmetrical loads; (*b*) an unsymmetrical frame; or (*c*) the more apparent overturning moments, would further increase the necessary preliminary work;

3.—More concentration of thought is required during the determination of moments, due to the necessity of handling two rigidities and two carry-over factors in each span instead of one;

4.—It is subject to erroneous interpretation by any one but an expert designer; and

5.—The work necessary to obtain the final moments increases rapidly with the number of members that are loaded.

It seems proper to conclude by using the fifth disadvantage to disprove one of the author's concluding statements: "The greatest advantage of this method is its simplicity and directness." Fig. 10 appears simple, but only one member was loaded and to the labor on this simple drawing must be added the work of preparing the data on Fig. 8. Fig. 11, with three members of this same frame loaded, certainly is much more complicated than Fig. 10 and, furthermore, the data on Fig. 8 are necessary for the preliminary analysis. Imagine how Fig. 11 would look if each of the eight members had been loaded. It would then be highly involved; but in addition it would have charged against it the time spent on Fig. 8 which contains a mass of data. More than one-half the data on Fig. 8 is unnecessary in the original Cross method, and it is this unnecessary majority which requires a large amount of work to produce. Besides being a highly involved method, lacking in simplicity, it is not a direct method. Imagine, in a speed contest, two designers given a complicated rigid-frame problem which neither had seen before with all necessary data to solve it by the original Cross method, but with no limitations on the method of solution. Assume that Designer No. 1 elects to stake all on the author's method whereas Designer No. 2 chooses to stake all on the original Cross method. At the expiration of 30 min of calculations, a halt is called and each contestant is then informed that there will now be available only 5 min in which to estimate on the basis of the work already done the final size and sign of all end moments. Designer No. 1 probably would be in no position to make this estimate as he would scarcely have finished with all the preliminary data such as those shown on Fig. 8. On the other hand, Designer No. 2 probably would be about through with at least the second balancing and could make a fairly accurate forecast of the answer. This becomes important in design work where it may require two or more trials before the final moments of inertias of the members can be determined. Thus, which method would seem to be more simple and more direct?

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[31] *Engineering News-Record*, February 14, 1935, p. 258.

E. J. BEDNARSKI,[32] ASSOC. M. AM. SOC. C. E. (by letter).—The classical universal method requires a solution of n simultaneous equations with n unknowns in each equation. When the value of n is fairly large—as, for example, 9—the work required is theoretically 60 480 times more than is necessary for the solution of one equation with one unknown. This gives an approximate idea of the work necessary in computing the stresses by the classical method in a multiple-story structure with several bays, such as the Empire State Building, in New York City.

Thus, the development of methods of analysis which save work is of great importance. The author solves the problem by developing original formulas which are simple and constitute the inherent properties of a given structure and which permit the distribution of any moment around any joint of the structure and the reaction to this moment by the other end of this member. Development of these relations is the most important and valuable result of the author's endeavor. Actually, the determination of the values of the factors, K_m and γ_m, constitutes the essence of the computation. (In certain structures the process of determination of K_m and γ_m will require one, or not more than two, repetitions of the computation for all practical purposes.) They are valid for any loading, in any structure that does not sway sidewise Unfortunately, in the analysis of ordinary structures, lateral forces and unsymmetrical loads that act on a structure which is not capable of resisting side-sway, are the most common cases encountered. The method is an excellent one, however, as long as its application is limited to symmetrical structures under symmetrical loading, continuous portals secured against sidesway, and continous beams. The writer checked a continuous portal given as an example by the late Professor A. Ostenfeld[33] and found the results identical with those given in the paper.

In a simple structure, such as Example 1, an application of other methods may prove just as efficient. The force polygon method introduced by J. D. Gedo, M. Am. Soc. C. E., for instance, has certain advantages in producing a direct exact answer to the problem by means of the solution of two simple simultaneous equations.[34] This corroborates the statement made by Professor Ostenfeld in his life work on statically undeterminate structures,[35] that each particular problem may be solved best by a certain particular method, although it may require a special study in itself.

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[32] Structural Engr., Los Angeles, Calif.

[33] *Der Eisenbau*, November 8, 1921, p. 285.

[34] "Theory of Superstatic Structures," by J. D. Gedo, New York, 1935, Equations 117 and 118, p. 46.

[35] "Die Deformations Methode," by A. Ostenfeld, Berlin, 1926, Verlag Julius Springer.

The development of the "short cuts" in the analysis of statically indeterminate structures has recently gained a tremendous impetus. The writer wishes to express his hope that Mr. Lin may develop a solution of the problem that will include lateral and unsymmetrical forces acting on structures not secured against side-sway.

JOHN T. HOWELL,[36] JUN. AM. SOC. C. E. (by letter).—The method proposed by the author has proved to be of real practical value in the analysis of continuous frames. Engineers familiar with Professor Cross' method of moment distribution will be pleased with the manner in which the original concepts and definitions have been used to set up a direct method for determining moments. For those not yet familiar with this direct method a short general explanation will be given as the best way of comparing it with other modern methods.

The method offered is based entirely on Professor Cross' definitions of stiffness, carry-over factors, and fixed-end moments; and in a straightforward ingenious manner Mr. Lin develops simple relations for "modified" stiffness and carry-over factors.

The usual fixed-end moments are then distributed and "carried over" quite as one would visualize the restraining action of the actual frame. This "travel" of a moment to the ends of the frame is novel and interesting. Such a procedure is of importance to the designer as it affords valuable training in the art of estimating, accurately, the proportion and distribution of moment in any certain part of a structure, as controlled by the sizes and shapes of component members. This is true, since final moments are not determined by equations or formulas, or by repeated distributions, but are "written" on the frame in a regular routine manner by the use of simple arithmetic.

The method may be said to be of a type between the original method of moment distribution and various methods in which final moments are obtained directly by formulas. The analyst who prefers the simplicity of moment distribution and its easy application, but who wishes to avoid the numerous cycles of distribution necessary in many cases, will find great use for the direct method. After some practice one may estimate, with accuracy, the modified beam factors from the original stiffness and carry-over factors in particular cases. It is thus possible to "write in" the final moments in members of such indeterminate structures as multi-storied building frames with a degree of accuracy that is usually sufficient.

The most interesting feature of the method provides the greatest advantage or disadvantage, as the case may be, in comparing it with the method presented by Professor Cross. Since modified beam factors are determined before the fixed-end moments are distributed, and this preliminary calculation usually requires the major portion of the total time of analysis, it is important to consider the purpose and extent of any given analysis. In those cases where there is only one load, or a group of different loads taken together, it is quicker to use ordinary moment distribution. However, in cases where it is preferable to study the load effects separately, and in particular when influence lines are to be plotted, the direct method is much quicker and more satisfactory. This comparison shows that each method has its own field of use. The writer has found from experience that Professor Cross' method is quicker, and, of course, easier to apply in cases where one or two sets of joint moments are desired. The sign convention commonly attributed to L. E. Grinter, Assoc. M. Am. Soc. C. E.,"[37] and also his "short-cut" in which large unbalanced moments are distributed and carried over before starting the regular distribution cycle, save some time.

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[36] Structural Engr., Structural Bureau, Portland Cement Assoc., Chicago, Ill.

In other types of detailed analysis previously mentioned Mr. Lin's modification is thought to be preferable. Considerable time will also be saved if the two fundamental equations for the modified beam factors, K_{abm} and C_{abm} (Equations (3) and (4)) are set up in the form of nomograms.

The writer has analyzed 8, 9, and 10-bbl siphon structures by both methods, and has found that in the calculation of moments due to several load conditions where such effects are considered separately, and for the necessary analyses for shear correction coefficients, Mr. Lin's method is not longer than that of Professor Cross. A little practice in the determination of the "modified" beam factors for such frames will prove the procedure far easier than anticipated.

On the surface Mr. Lin's method and the alternate Cross method in which the "end-rotation constant"[38] is used, would seem somewhat similar. This is true in a superficial sense only, however, as the author's method is perfectly general whereas in the reference given "the end-rotation constant" method can only be used in those particular cases where the ratio of the changes in moment at the ends of the members are known.

During two years of practical use of the author's method the writer has noted a few points which might be mentioned. Under the heading "Definitions and Notation" the author has defined K as the stiffness of the end of a member. "Relative stiffness" values are most commonly used, the "absolute stiffness," that is, $K \propto \frac{EI}{L}$ being used where different materials make up the structure, or in the determination of fixed-end moments of haunched members subject to the lateral displacement of one end relative to the other, with no joint rotation.

Stiffness values in published tables or curves are usually in terms of $\frac{I}{L}$ or $\frac{4I}{L}$. In general, any numerical value may be given to the K-values as long as they are consistent and proportionate to the respective $\frac{I}{L}$ -values. The modified stiffnesses (Equation (3)) change by this same proportion, and the constants are cancelled in Equations (1) and (4).

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[37] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 610.

[38] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E.

To clarify any possible slight confusion, it should be noted that in Mr. Lin's terminology the carry-over factor placed at an end of a member is that one which carries over distributed moments to the opposite end. In many methods the opposite is true.

The actual degree of restraint provided at the joints of a structure has been the object of much research in recent years. Riveted connections of various types have been shown to have different restraint ratios.[39] To obtain the most accurate analyses, correlated and consistent experimental data will be used to indicate the "degree of fixity" provided at the ends of critical members. For such analyses Mr. Lin's method is ideal. The *R*-term (Equation (1)) denotes the end restraint of a member, and may be assigned any value indicated by such research. The modified beam factors and, lastly, the joint moments are then found as before.

The fundamental formulas in the direct method may be further modified to make them correspond to formulas in other methods. The final moments at the ends of a member can be given by those altered formulas, but it is the opinion of those familiar with the author's method that one of its chief advantages lies in the fact that it retains some of the features of moment distribution.

The writer found the direct method easy to learn, and its application has afforded real pleasure. Mr. Lin has made a worthwhile contribution to structural engineering literature.

I. Oesterblom,[40] M. Am. Soc. C. E. (by letter).—This paper seems to have its principal value in demonstrating that a framework has elastic properties entirely apart from any system of loads which may be applied (in the same manner as a beam), and that these properties are definitely ascertainable by simple and elementary analysis.

This is not to say that Mr. Lin is the first to develop this idea. Presumably, it has been studied by many investigators, but no one has stressed the significance sufficiently to cause much quotation or comment. An exception might be Dr. Hermann Zimmermann, who long ago saw the significance of segregation and presented papers on the subject as early as 1907, before the Akademie der Wissenshaften,[41] in Berlin, Germany, and other scientific bodies. Unfortunately, Dr. Zimmermann did not realize that his many contributions assembled into one unit would serve as a powerful tool of analysis and design, until quite recently, and when his book on "Knickfestigkeit der Stabverbindungen" was published in 1925, it caused little or no comment.

To Mr. Lin, therefore, belongs the honor of bringing before thinking engineers an important fact, and, if anything, it merely adds to its value that the presentation has been made in terms of Professor Cross' brilliant moment distribution concept.

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[39] "Elastic Properties of Riveted Connections," by J. Charles Rathbun, M. Am. Soc. C. E. *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 524.

[40] Civ. Engr., Chicago, Ill.

[41] *Sitzungsberichte*, 1907, 1909, 1921, 1923, 1924, and 1925.

It is to be regretted, however, that Mr. Lin did not confine himself to "elastic properties of a framework." That subject is so broad in itself as to provide material for many an academic thesis, and it needs emphasis because it has been so sadly neglected as a separate issue.

On the other hand, the writer is not convinced that Mr. Lin's procedure for finding the moments due to a system of loads will lead to results as quickly as Professor Cross' original procedure. For example, Equation (1) is most excellent for the analysis of important structures, where it is necessary to show graphically and clearly the elastic relations of the whole, but it should not be forgotten that in the usual framework four such equations must be solved for every nodal point. This means considerable work and is only a part of the total analytical task. The time required as compared to that allowable, for the usual run of designs, would automatically bar the method.

Professor Cross aimed at a series of progressive approximations leading ultimately to any degree of mathematical accuracy and all of a nature to be performable, even if not performed, by slide-rule. In ruling out the progressive approximations and bringing back a series of equations, it would seem that, on that point, Mr. Lin is taking a step backward. This criticism, however, is directed only toward over-enthusiasm in regard to the most vulgar applications and in no way lessens the writer's admiration for the fundamental idea of the paper.

In view of its importance the paper is regrettably brief and for that reason is not fully convincing at all points. Unfortunately, most engineers would like to examine the mathematical demonstrations in full, and thus avoid spending extra time to provide "the missing pieces in the game." A few more pertinent details would have been helpful.

W. P. LI,[42] JUN. AM. SOC. C. E. (by letter).—The difference between the original Cross method of moment distribution and the Lin modification may be visualized readily by the following formularized method of procedure:

Assuming an end of a member as being $\frac{\text{in a natural condition of restraint}}{\text{fixed}}$, when the joint at that end is released, the unbalanced fixed-end moment is distributed among the members in proportion to the $\frac{\text{modified } (K_m)}{\text{direct } (K)}$ stiffness (that is, the $\frac{\text{inter-related}}{\text{separate, } \frac{I}{L},}$ stiffness). (Note that, in the modified method, for unbalanced "carried-over" moments the distribution is only among the connecting members.) The moment thus distributed, is carried over to the other ends of the members in accordance with $\frac{\text{a modified}}{\text{the Cross}}$ carry-over factor. For beams of uniform cross-section the carry-over factor in the Cross method is -0.5.

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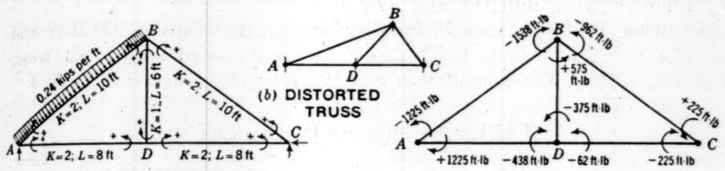
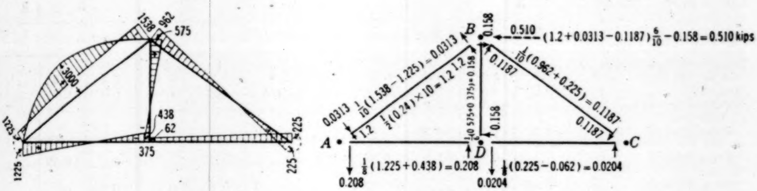
[42] Chf., Designing Dept., Kwangtung River Conservancy Bureau, Canton, China.

In the foregoing, compound rule, the inserts in the "numerator" refer to the modified method and those in the "denominator" to the Cross method. It is evident that the modified method is not entirely direct in its application since it requires two steps instead of one for the solution of the problem. The first step, that of computing the modified stiffness and the modified carry-over factor, is usually lengthy, especially when the R-value must be assumed at the beginning, as for the closed frames. Thus, it seems that the modified method is only a variant of the original method. One can

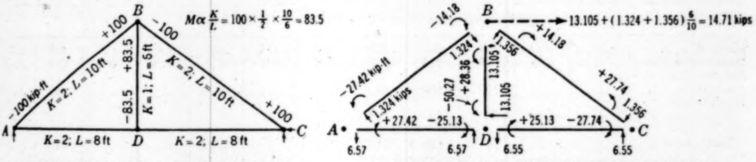

B
0.24 kips per ft
K=2; L=10ft
K=1; L=6ft
K=2; L=10ft
A
K=2; L=8ft
D
K=2; L=8ft
C
A
B
D
C
−1538 ft-lb
B
−962 ft-lb
+575 ft-lb
−375 ft-lb
−1225 ft-lb
+225 ft-lb
A
C
+1225 ft-lb
−438 ft-lb
D
−62 ft-lb
−225 ft-lb

(a) NON-DISTORTED TRUSS (b) DISTORTED TRUSS (c) SIGN OF MOMENT

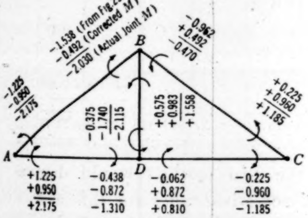

1538
962
575
300
438
62
1225
375
225
225
1225
B
0.158
0.510
(1.2+0.0313−0.1187) 6/10 − 0.158 = 0.510 kips
0.0313
1/10 (1.538−1.225) = 0.0313
1/2 (0.24)×10 = 1.2
1.2
1.2
0.1187
1/10 (0.962+0.225) = 0.1187
0.1187
1/10 (0.575+0.375) = 0.158
0.158
A
D
C
1/8 (1.225+0.438) = 0.208
1/8 (0.225−0.062) = 0.0204
0.208
0.0204

(d) MOMENT DIAGRAM (e) COMPUTATION OF SHEAR

FIG. 23


B
$M \propto \frac{K}{L} = 100 \times \frac{1}{2} \times \frac{10}{6} = 83.5$
+100
−100
+83.5
−83.5
K=2; L=10ft
K=1; L=6ft
K=2; L=10ft
−100 kip-ft
+100
A
K=2; L=8ft
D
K=2; L=8ft
C
−14.18
B
13.105 + (1.324 + 1.356) 6/10 = 14.71 kips
+14.18
1.324
1.356
−27.42 kip-ft
+28.36
−50.27
13.105
13.105
+27.74
1.324 kips
1.356
A
+27.42
−25.13
D
+25.13
−27.74
C
6.57
6.57
6.55
6.55

(a) ASSUMED FIXED-END MOMENT (b) MOMENT AND THE CORRESPONDING SHEAR RESULTING FROM THE ASSUMED FIXED-END MOMENT


−1.538 (From Fig 23c)
−0.492 (Corrected M)
−2.030 (Actual Joint M)
B
−0.962
+0.492
−0.470
−1.225
−0.950
−2.175
−0.375
−1.740
−2.115
+0.575
+0.983
+1.558
+0.225
+0.960
+1.185
A
D
C
+1.225
+0.950
+2.175
−0.438
−0.872
−1.310
−0.062
+0.872
+0.810
−0.225
−0.960
−1.185

Note: The Moment in Fig. 24 b to be Corrected = $\frac{0.51}{14.71}$ and the Actual Joint Moment Equals the Moments in Fig. 23 c Plus the Corrected Moments.

(c) ACTUAL JOINT MOMENTS

FIG. 24.

scarcely claim that the modified method is more direct than the original; ordinarily, the reverse is true. A closed frame such as that in Example 4 of the paper and in the truss in Fig. 23, and in Fig. 24, can be solved more easily and directly by the original method. It is difficult to assume a closed R-value for one of the truss members, say, AB (Fig. 23), and check it along two different paths, $ABCDA$ and $ABDA$.

Adopting Professor Cross' sign convention, the conventional left and right for each joint can be easily seen from the distorted form of the truss, as in Fig. 23(b). The computation may be arranged as shown in Table 4.

TABLE 4.—MOMENT DISTRIBUTION

Joint	*A*			*B*			*D*			*C*		
Left, or right	L	R		L		R	L	R		L		R
a) For Problem in Fig. 23.												
Relative stiffness factor	....	2	2	2	1	2	2	1	2	2	2	.. :
Distribution factor	....	0.5	0.5	0.4	0.2	0.4	4	0.2	0.4	0.5	0.5	....
Moment	....	*AD*	*AB*	*BA*	*BD*	*BC*	*DA*	*DB*	*DC*	*CD*	*CB*	....
Fixed-end moment	....	....	−2.0	−2.0	....	....	....	....	....	....	....	....
Distribution:												
From Joint *A*	....	+1.0	+1.0	−0.50	....	....	−0.50	....	....	....	....	....
From Joint *B*	....	....	−0.50	+1.0	+0.50	−1.0	....	−0.25	....	....	+0.50	....
From Joint *D*	....	−0.05	....	....	+0.025	....	+0.10	−0.05	−0.10	+0.05	....	....
From Joint *C*	....	....	....	....	....	+0.138	....	....	+0.138	−0.275	−0.275	....
From Joint *A*	....	+0.275	+0.275	−0.138	....	....	−0.138	....	....	....	....	....
From Joint *B*	....	....	....	+0.10	+0.05	−0.10	....	−0.025	....	....	+0.05	....
							+0.10	−0.05	−0.10	+0.05	....	....
From Joint *D*	....	....	....	....	....	....						....
From Joint *C*	....	....	....	....	....	....	....	....	....	−0.05	−0.05	....
Final moment	....	+1.225	−1.225	−1.538	+0.575	−0.962	−0.438	−0.375	−0.062	−0.225	+0.225	....
(b) For Assumed Fixed-End Moments in Fig. 24(a)												
Moment	....	*AD*	*AB*	*BA*	*BD*	*BC*	*DA*	*DB*	*DC*	*CD*	*CB*	....
Distribution factor	....	0.5	0.5	0.4	0.2	0.4	0.4	0.2	0.4	0.5	0.5	....
Assumed fixed-end moment	....	....	−100	+100	+83.5	−100	....	−83.5	....	....	+100	....
Distribution:												
From Joint *A*	....	+50	+50	−25	....	....	−25	....	....	....	....	....
From Joint *B*	....	....	+51.7	−103.4	−51.7	+103.4	....	+25.8	....	....	−51.7	....
From Joint *D*	....	+6.54	....	....	−3.27	....	−13.08	+6.54	+13.08	−6.54	....	....
From Joint *C*	....	....	....	....	....	+10.44	....	....	+10.44	−20.88	−20.88	....
From Joint *A*	....	−29.12	−29.12	+14.56	....	....	+14.56	....	....	....	....	....
From Joint *B*	....	....	....	−0.34	−0.17	+0.34	....	+0.085	....	....	−0.17	....
From Joint *D*	....	....	....	....	....	....	−1.614	+0.807	+1.614	−0.807	....	....
From Joint *C*	....	....	....	....	....	....	....	....	....	+0.488	+0.488	....
Final moment	....	+27.42	−27.42	−14.18	+28.36	+14.18	−25.13	−50.27	+25.13	−27.74	+27.74	....

An attempt to solve these problems by the Lin method would demonstrate the fact that the Cross method has distinct advantages for this case.

Since the truss in Fig. 23(a) is unsymmetrically loaded with wind pressure, Joint B will undergo side-sway if subjected to wind pressure alone;

however, Joint D is held against side-sway by the horizontal member, ADC, which is twice as stiff as Member BD. The shear correction for the side-sway of Joint B can be computed as follows: (1) Find the horizontal force to be applied at Joint B in order to hold it against side-sway; (2) assume a system of convenient fixed-end moments and find the final moment and the corresponding horizontal shear at Joint B; (3) by proportioning the force and the shear thus computed, find the moments to be corrected at the joints due to side-sway; and (4), combine the corrected moments with the moments obtained by simple moment distribution (as in Fig. 23(*c*)) to obtain the actual joint moments.

A. A. **Eremin**,[48] Assoc. M. Am. Soc. C. E. (by letter).—The method of distributing bending moments in rigid frames described by Mr. Lin has considerable merit. It gives direct distribution of bending stresses from the moment at one joint. Likewise, the method is convenient for the construction of influence lines for bending stresses in rigid frames.

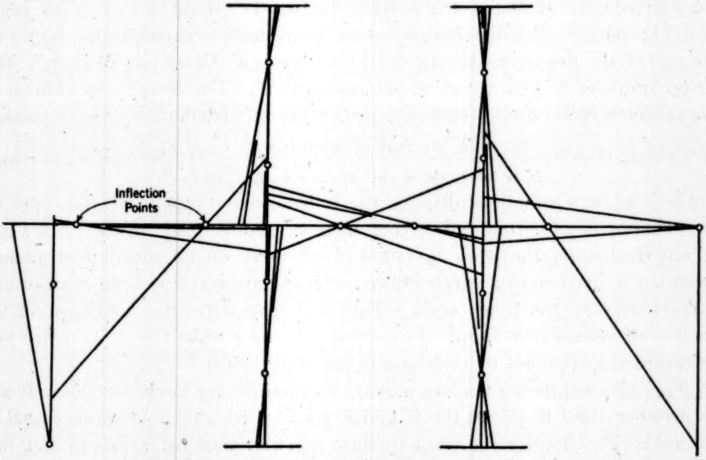


Fig. 25.—Distribution of Bending Moments.

The computation of bending moment stresses in a frame with two or more spans loaded may be simplified by applying graphical methods. For example, consider Example 3 of the paper. In Fig. 25 the lengths of beams and columns are assumed equal to 1. Points of contraflexure were determined with the modified carry-over factors shown by the author in Fig. 11. Bending moments were distributed at joints using the modified stiffness factors shown by the author. Carried-over bending moments were determined graphically by means of the points of contraflexure in Fig. 25. The resulting end bending moments in the members will be equal to the algebraic sum of moments scaled from Fig. 25 and fixed-end moments.

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[48] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works. Sacramento, Calif.

Conventional rules for signs of moments in Fig. 25 are similar to those introduced in the paper. Positive bending moments at joints are considered clockwise and negative moments, counter-clockwise. The use of Fig 25 helps to avoid an error in the signs of bending moments. By graphical methods, the distribution of moments can be visualized readily. Furthermore, graphical construction reduces the work of distributing moments.

T. Y. LIN,[44] JUN. AM. SOC. C. E. (by letter).—Judging from the interesting discussions of this paper, many engineers have evidently found it valuable. Professor Grinter makes a thoughtful comparison with the original method; but he does not compare the two on the same basis. The members in Fig. 14 are all of uniform moment of inertia, whereas in Figs. 8 to 11, some are not. To make a fair comparison, the writer will solve Example 3 by a simplified Cross method which, he believes, requires the least amount of written work possible with that method. Instead of writing both the distributed and carried-over moments at the same time, as Professor Grinter does, only the carried-over moments are written during the process of releasing joints (see Fig. 26(*b*)); the distributed moments are to be computed and written at the end of the procedure as suggested by Professors Cross and Morgan.[45] The writer proposes to give values of the product, $K\gamma$ (Fig. 26(*a*)), in addition to the stiffness ratio, K (which is the proportion of distribution of each member at a joint; that is, $\dfrac{K \text{ of end of member}}{\Sigma K \text{ of ends of all members at joint}}$), so that when releasing a joint one simply multiplies the unbalanced moments at that joint by the value of $K\gamma$ for that end of each member and writes it at the farther end. It also facilitates the work to put a check mark on an unbalanced moment (whether a fixed-end or carried-over moment) after it has been released and carried over to the other ends. This will eliminate any confusion in the process of releasing a joint. To release the most unbalanced joint first and to record the sequence of releasing is also very helpful.

This also requires a certain amount of preliminary work, however. It will be necessary first to record the K and $K\gamma$-values for each end which should be released. To simplify the work further, find the modified values of K_{cg}, K_{ef}, K_{da}, K_{eb}, γ_{da}, and γ_{eb} by Equations (3) and (4). Then, for the unbalanced moments, one begins by releasing Joint C, multiplying the -100 (reversed in sign) by the value of $K\gamma$ for CD and writing $+40$ at End D. Then, at Joint D, the unbalanced moment $= -150 + 200 + 40 = +90$, which is multiplied by the corresponding $K\gamma$-values and written at the farther ends of members concerned. For the terminal joints, A, B, F, G, H, and J, the moments will be carried over at the end of the procedure. Continuing in the sequence shown, it is found that the values are converging rapidly, giving close results at the end of the ninth cycle. The unbalanced moment at each joint is then added and distributed to the ends of members meeting at that joint in proportion to the K-values. Thus, at Joint D, the unbalanced moment is $245 - 135 = 110$, which gives -25 to Column DA, -45 to Girder DE, etc. The final moments are then as shown in Fig. 26(c). Moments

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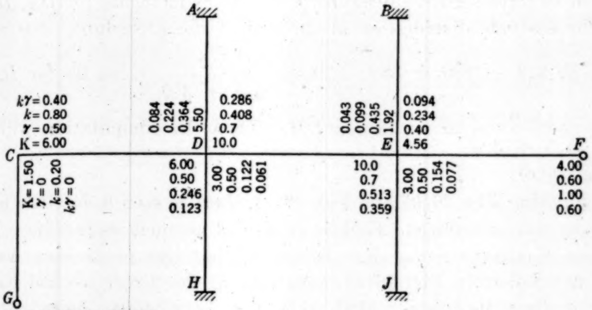
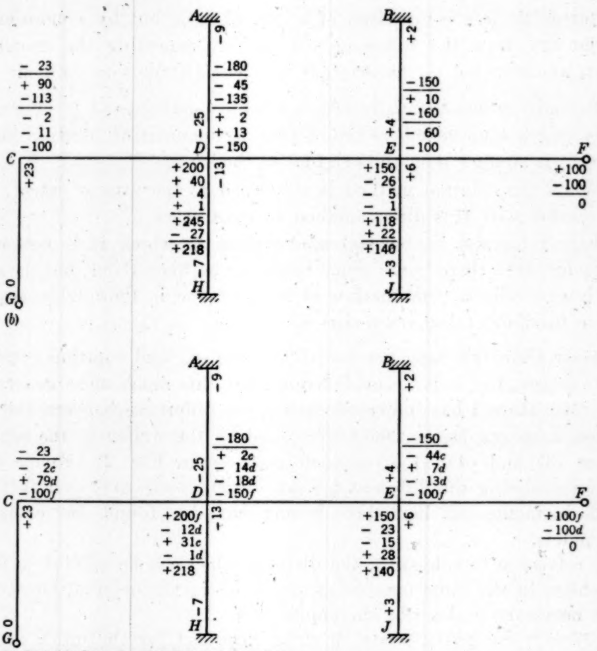
[44] Engr., Cheng-yil Ry., Chungking, Szechuan, China.

[45] "Continuous Frames of Reinforced Concrete", by Hardy Cross and C. E. Morgan, Members, Am. Soc. C. E., p. 104.


A
B
kγ=0.40
k=0.80
γ=0.50
K=6.00
C
K=1.50
γ=0
k=0.20
kγ=0
0.084
0.224
0.364
5.50
0.286
0.408
0.7
10.0
D
0.043
0.099
0.435
1.92
0.094
0.234
0.40
4.56
E
F
6.00
0.50
0.246
0.123
3.00
0.50
0.122
0.061
10.0
0.7
0.513
0.359
3.00
0.50
0.154
0.077
4.00
0.60
1.00
0.60
H
J
G

(a) BASIC FACTORS


A
B
−9
+2
− 23
+ 90
−113
− 2
− 11
−100
−180
− 45
−135
+ 2
+ 13
−150
−25
−150
+ 10
−160
− 60
−100
+4
C
D
E
F
+23
+200
+ 40
+ 4
+ 1
+245
− 27
+218
−13
+150
− 26
− 5
− 1
+118
+ 22
+140
+6
+100
−100
0
0
−7
+3
G
H
J
A
B
−9
+2
− 23
− 2c
+ 79d
−100f
−180
+ 2c
− 14d
− 18d
−150f
−25
−150
− 44c
+ 7d
− 13d
−100f
+4
C
D
E
F
+23
+200f
− 12d
+ 31c
− 1d
+218
−13
+150
− 23
− 15
+ 28
+140
+6
+100f
−100d
0
0
−7
+3
G
H
J

(b)

(c) SOLUTION BY DIRECT METHOD

FIG. 26.—SIMPLIFIED SOLUTION OF EXAMPLE 3.

at Point A of Column AD and Point H of Column HD can now be found to be -7 and -9, respectively.

To do justice to the direct method, it should be noted that Fig. 11 has been made voluminous purposely in order to give the reader an unmistakable understanding of the procedure. It can easily be simplified if one carries the calculation to three significant figures and obtains moments for *DA*, *DH*, *EB*, and *EJ* by a single distribution at the end of the procedure. For example, at Joint *D*, $218 - 180 = 38$; $-38 \times \dfrac{5.5}{5.5 + 3.0} = -25$ for *DA*; and, $-38 \times \dfrac{3.0}{5.5 + 3.0} = -13$ for *DH*. Then the computation will appear as in Fig. 26(*c*).

By comparing Fig. 26(*b*) and Fig. 26(*c*), there is seen to be little to choose between the two procedures. Each is simplified as much as possible. Perhaps the former requires a little more balancing, but less preliminary work. It must be noted that in Fig. 26(*b*), equations of the direct method have been applied to four members. Had this not been done, more calculation would appear.

The foregoing is a comparison of a typical case, but by a more extensive study, one can draw the following conclusions regarding the simplicity of the direct, as compared to the original, moment-distribution method:

(1) In many cases, especially where several conditions of loading are considered or where some members are of varying moment of inertia, this direct modification is simpler than the original method.

(2) When the original method is simplified to the utmost extent, it compares favorably with this direct method in most cases.

(3) Either method has special applications to which it is best adapted, and, in many cases, there is not much to choose between them; but the simplest method is generally a combination of both, if one is thoroughly acquainted with their fundamental characteristics.

Professor Grinter's experience with the use of "end rotation constant" is probably correct; but it is no evidence against this much more general direct method. Mr. Howell has indicated clearly the difference between the two.

Curves, nomographs, or tables were used by the writer in the solution of Equations (3) and (4). Curves such as those in Fig. 27 provide a better means of visualizing the different factors. Stiffnesses, carry-over factors, and fixed-end moments of haunched beams can be found in many books and pamphlets.

With reference to side-sway, the direct method can be applied in the side-sway problem in the same manner as was done in the original Cross method. It is not necessary to describe the application.

Fig. 27 is a composite curve diagram designed for the quick determination of modified stiffness and carry-over factors.

Assume, as given, the product of the original carry-over factors ($\gamma_{ab}\ \gamma_{ba}$), and also the R_{ba} - value (from the equation, $R_{ab} = \dfrac{K_{ba} + K_{bnm}}{K_{ba}}$.) The intersection of these values—using the solid R_{ba} curves—gives, at the bottom of the diagram the numerical value of $\left(1 - \frac{\gamma_{ab}\,\gamma_{ba}}{R_{ba}}\right)$ for use in the equation,

$$K_{abm} = K_{ab}\left(1 - \frac{\gamma_{ab}\,\gamma_{ba}}{R_{ba}}\right)$$

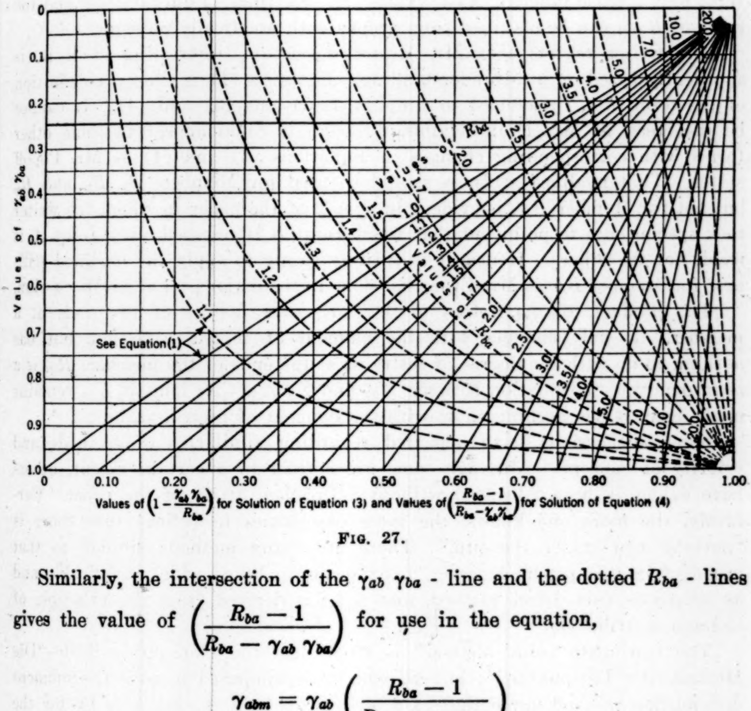


FIG. 27.

Similarly, the intersection of the $\gamma_{ab}\,\gamma_{ba}$ - line and the dotted R_{ba} - lines gives the value of $\left(\frac{R_{ba} - 1}{R_{ba} - \gamma_{ab}\,\gamma_{ba}}\right)$ for use in the equation,

$$\gamma_{abm} = \gamma_{ab}\left(\frac{R_{ba} - 1}{R_{ba} - \gamma_{ab}\,\gamma_{ba}}\right)$$

For example:

To find: K_{abm} and γ_{abm};

Given: $\gamma_{ab} = 0.50$; $\gamma_{ba} = 0.50$; $R_{ba} = 3.0$; $K_{ab} = 1.2$; and, $\gamma_{ab}\,\gamma_{ba} = 0.50 \times 0.50 = 0.25$.

In Fig. 27 follow the heavy line, $\gamma_{ab}\,\gamma_{ba} = 0.25$, to its intersection with $R_{ba} = 3.0$ on solid lines. Read at the bottom of the diagram the value 0.92. At the intersection with $R_{ba} = 3.0$, on dotted lines, read at the bottom the value, 0.73. Then, $K_{abm} = K_{ab}\,(0.92) = 1.2 \times 0.92 = 1.10$; and $\gamma_{abm} = \gamma_{ab}\,(0.73) = 0.50 \times 0.73 = 0.36$.

Professor Grinter and Mr. Huang both suggested the idea of using $\frac{K_{ba} + \Sigma K_{bn}}{K_{ba}}$ instead of $\frac{K_{ba} + \Sigma K_{bnm}}{K_{ba}}$. Mr. Huang has further found the resulting error in the value of K_{abm} to be less than 2.08 per cent. However, the writer finds that for haunched members the $\gamma_{ab}\,\gamma_{ba}$-value of which is greater than 0.25, the resulting error can be large, perhaps even more than 25 per cent. Hence, the writer prefers the use of estimated values of K_{bnm}. With only a little practice, K_{bnm}-values can be estimated quite closely and the value of K_{abm} can be obtained from graphs with practically no error.

The writer regrets that Mr. Evans' discussion, interesting as it is, is apparently based on a misunderstanding. Equation (1) is simply a definition of the term, R_{ba}, introduced to simplify Equations (3) and (4). It cannot be incorrect, as Mr. Evans claims, although it could appear in some other forms with corresponding changes in Equations (3) and (4) as Mr. Popoff states. That modified stiffness should be used for Members B_1, B_2, and B_3, but not for Member *BA*, is evident in Step 2 of the paper in which the writer took special care to point out that when Joint *B* is released, only Joint *A* is fixed, not the others. Hence, it is entirely wrong to apply the modified stiffness for Member *BA* as Mr. Evans has done in the major part of his discussion.

The product, $\gamma_{ab}\,\gamma_{ba}$, refers to the carry-over factors of two ends of a member. It only changes with the moment of inertia variation, but has nothing to do with the degree of restraint at the ends of the member; R_{ba} is a term denoting the relation between K_{ba} and ΣK_{bnm}. As long as K_{ba} remains unchanged, it has nothing to do with the moment of inertia variation.

Many discussions have appeared regarding similarity of methods and priority of concept. Although some are correct to a certain extent, others have evidently been advanced without a sufficient study of the paper. Certainly, the more one knows, the more one comes to believe that there is "nothing new under the sun." There are many methods similar to that proposed in this paper; however, none seems to be as widely applicable and as simple as this direct method, and none is derived from the principle of moment distribution.

The conjugate point method[39] is much like the German method—"Die Methode der Festpunkte"[40]; but all who have compared it with the moment distribution method agree that as a tool of analysis the latter is by far the better. This means that it cannot compare with this direct method, which can be much more easily learned and more widely applied.

It is to be regretted that the writer cannot obtain Dr. Zimmermann's book, mentioned by Mr. Oesterblom, and the thesis presented by Mr. A. Efsen to the University of Copenhagen in 1930 in fulfillment of the requirements for the degree of Doctor Technices, to which Theodore B. Host, Assoc. M. Am. Soc. C. E., has called the writer's attention by correspondence; but he infers from the opinions of the discussers that the methods are not derived in the same manner and probably cannot be as easily and generally applied.

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[40] "Berechnung der Statisch-unbestimmte Systeme", von A. Strassner, Berlin, 1921.

Mr. Huang[19] has a paper published in Chinese that advances a similar method derived from slope-deflection equations. It is similar to other methods derived therefrom, but is more cumbersome. Probably the method presented by Mr. E. B. Russell[47] is most similar to that proposed by the writer. This method is developed from slope-deflection equations and introduces two rather complicated equations for finding the end moments. The same is true of the method[48] of T. F. Hickerson, M. Am. Soc. C. E., and that[49] proposed by H. M. Hadley, Assoc. M. Am. Soc. C. E.

In the original thesis,[3] there is a brief, but rather extensive, comparison of all methods of continuous frame analysis. It might prove interesting to one who is scholarly enough to make further investigations. After long study of, and experience with, the subject, the writer has come to the conclusion that although each particular problem may be solved best by a certain particular method, it suffices for all purposes to have learned the Cross method and this direct method. The latter is most valuable in the visualization and study of elastic properties of continuous frames and in the approximate but sufficiently accurate estimate of moments in them. Any one who has learned the fundamentals of this method should be able to "write in" moments to a sufficient degree of accuracy. He should be able to see mistakes in analysis. He should be able to visualize the effect of haunching; the effect of restraint; and to see a continuous frame as he sees a simple truss. He should know what approximation can be applied in analysis, and where changes in design can be made for economy.

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[47] "Analysis of Continuous Frames", by Earle B. Russell, San Francisco, Calif., 1934.

[48] "Structural Frameworks", by T. F. Hickerson, Univ. of North Carolina Press, 1934.

[49] "The Assembly Stiffness Method of Stress Analysis", 8 pages of notes by H. M. Hadley.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1968

BEHAVIOR OF STATIONARY WIRE ROPES IN TENSION AND BENDING

By Douglas M. Stewart[1], Jun. Am. Soc. C. E.

With Discussion by Messrs. C. D. Meals, G. P. Boomsliter, Ingvald E. Madsen, and Douglas M. Stewart.

Synopsis

This experimental investigation of the behavior of wire ropes in tension and bending was undertaken in order to determine their strengths and the stresses produced under load, and to compare these values with those given by the several formulas in common use. Altogether, nine tension and thirty-six bending specimens were tested over sheaves of four diameters. The ropes selected were 1 in. in diameter, with hemp centers, and the tests included studies of regular and Lang lay ropes, of 6 × 7 and 6 × 19 construction preformed and non-preformed types, and of two different grades of steel. Important results are contained in the curves for loss of strength in bending and for the variation in modulus of elasticity of the rope under pre-stressing, and a comparative summary is given of stresses and strengths as observed and as computed by several formulas.

Introduction

Since wire ropes were first produced in the early part of the Nineteenth Century, with a view to obtaining high strength combined with flexibility over sheaves, the question of the stresses set up by bending them has been a subject of sharp controversy. Literally dozens of formulas have been developed to evaluate this bending stress, most of them of an empirical nature, and each wire rope user, in the past, has given preference to one or another in

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Note.—Published in February, 1936, *Proceedings*.

[1] Engr., Ingersoll-Rand Co., New York, N. Y., formerly Garrett Linderman Hoppes Research Fellow in Civ. Eng., Lehigh Univ., Bethlehem, Pa.

the light of his practical experience with ropes in service. In some cases, the formula was merely an expression of the results of a series of tests on specimens to determine the loss of strength over various sizes of sheaves, from which the bending stress could be evaluated in some measure. This has led recently to the expression of formulas for loss of strength in bending, which, in the end, is a more practical concept than that of the stresses to which this loss is due.

It was for the purpose of investigating the merits of these numerous bending formulas that this test program was originally conceived. Undoubtedly, there is a marked difference between the stress conditions in a stationary wire rope bent over a sheave and in one which is in rapid motion over the same sheave and possibly subjected to reverse bending as well. The scope of this investigation has been limited to a study of stationary ropes only, and while they hold admittedly a relatively minor place in wire-rope usage, the results may point the way to a clearer understanding of stress conditions in moving ropes as well as in stationary ones.

A program of tests of ropes over sheaves was planned accordingly, and a means devised for measuring the stress in any of the outer wires. For purposes of comparison a tension specimen of each type of rope was needed, and further stress observations were taken on these specimens. Because of the need in certain stress formulas for a value of the modulus of elasticity of the rope as a whole, numerous observations of this property were made, and this determination soon became one of the major branches of the investigation. Considerable data have been collected also on the untwisting effect in wire ropes under tension, on their shrinkage in diameter as their hemp centers are consolidated, and on the coefficient of friction between rope and sheave.

Notation.—The symbols used in this paper are summarized for reference in Appendix I.

The Problem

Review.—Probably the first and simplest formula that has been derived for the purpose of expressing the stress in a wire rope bent over a sheave was that of Reuleaux[3],

$$s = E \frac{d}{D} \qquad (1)$$

which was derived from the expression for bending stress in a slightly curved beam, by substituting instead of the diameter of the rod acting as a beam, the diameter of one wire (presumably in the outer layer) used in the rope. D is the diameter of the sheave and E, the modulus of elasticity of the wire, generally assumed to be about 28 500 000 lb per sq in. This formula is given by the late Robert Charles Strachan, M. Am. Soc. C. E. (4)[4], F. C. Carstarphen, M. Am. Soc. C. E. (1), and by numerous other writers.

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[3] Given by Reuleaux in his book "The Constructor", 1893 Edition, and probably as early as 1876 in other writings.

[4] For reference to figures in parentheses, see "Bibliography", Appendix II.

It was soon found that Equation (1) gave values of the stress which were far too high to be practicable, in many cases even exceeding the ultimate strength of the wire for small sheave sizes. Accordingly, attempts were made to modify the formula by empirical and semi-empirical means. Shortridge Hardesty, M. Am. Soc. C. E. (2), has derived the formula (4):

$$s = E \frac{d}{D} \cos a \cos b \quad \ldots\ldots (2)$$

in which a is the angle between a helical wire and the axis of the strand and b is the angle between a strand and the axis of the rope.

Mr. R. W. Chapman (5) modified Equation (2) by expressing the formula as:

$$s = E \frac{d}{D} \cos^2 a \cos^2 b \quad \ldots\ldots (3)$$

which gives values for the stress lower than those given by Equations (1) and (2).

B. R. Leffler, M. Am. Soc. C. E. (3) gives an empirical modification of this formula as adopted by the New York Central Railroad Company in 1928,

$$s = \frac{2\,Ed}{3\,D} \cos^2 a \cos^2 b \quad \ldots\ldots (4)$$

Mr. Carstarphen (1) makes mention of an empirical formula of even simpler form,

$$s = 0.44\ \mathrm{E} \frac{d}{D} \quad \ldots\ldots (5)$$

although it is not mentioned on what test results this formula is based.

All the preceding formulas have involved the use of the modulus of elasticity of the wire, and the tendency has been to reduce the abnormally high stress values by some coefficient. In 1918, Mr. James F. Howe (2) suggested that the proper value of E to use in a formula of the general type of Equations (1) to (5) was the modulus of elasticity of the rope as a whole, E_r; thus,

$$s = E_r \frac{d}{D} \quad \ldots\ldots (6)$$

or, as it is often used,

$$s = \frac{E_r\, d}{D + d_r} \quad \ldots\ldots (7)$$

This formula gave values considerably lower than those of Equation (2) or Equation (3), when using a value of 12 000 000 lb per sq in. as the modulus of elasticity of the rope.

In 1933, Mr. Carstarphen (1) approached the problem from an analytical standpoint, and on the basis of a wire rope consisting of a double set of open-coiled helical springs, in turn bent around a constant radius, arrived at an expression for the loss of strength of a wire due to such bending:

$$P = \frac{\pi\, (d)^4\, E\, G}{16\, R\, r_s\, [2\, G\, (1 + \sin^2 \alpha) + E \cos^2 \alpha]} \quad \ldots\ldots (8)$$

in which P = loss of strength in a given wire; d = the diameter of a given wire; E = modulus of elasticity of a given wire; G = modulus of rigidity of a given wire, $\left[G = \frac{E}{2\,(1 + \mu)}\right]$; μ = Poisson's ratio; R = radius of a sheave, $\left[R = \frac{1}{2}(D + d_r)\right]$; r_s = radius from the center of the strand to the center of the wire in question; and, α = the angle between the perpendicular to the axis of a rope and the tangent to the center line of the wire.

In Equation (8), when $r_s = 0$, substitute r_r which is defined as the radius from the center of a wire rope to the center wire of a strand.

The total loss of strength in a rope is equal to the value of P multiplied by the number of wires, or if the strand consists of a number of layers of different sized wires, a value of P must be computed for each layer and multiplied by the number of wires of that size in the layer. If a value of the bending stress were desired, this could presumably be obtained by dividing $\Sigma\, P$ by the net area of steel in the rope. According to Mr. Carstarphen, Equation (8) "takes into account the diameter of the wires, the rope, the radius of curvature, the angle of lay, the modulus of elasticity in tension, and the modulus of rigidity." The test results reported in the same paper seemed to support this method of computing loss of strength.

Any of the preceding formulas, for bending stress, f, may be adopted to give loss of strength, or ultimate strength in bending, S, by inserting them in the general form of the equation:

$$S = A\,(t - s)\,\epsilon \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which A is the net area of steel in a wire rope; t is the ultimate unit tensile strength of a wire; and ϵ is the efficiency of the rope in plain tension. Equation (8) is given by C. D. Meals, Assoc. M. Am. Soc. C. E. (1), and others, using Equation (7) to determine a value for f. Mr. Meals further developed an equation for the strength of a wire rope in tension, from which the efficiency, ϵ, might be computed:

$$S_t = n_s \cos b\,\left(\sum_{1}^{n} n_w\,S_i \cos^2 a_i\right) \;\ldots\ldots\ldots\ldots\ldots (10)$$

in which n_s is the number of strands in a rope; n_w is the number of wires of a given diameter in the ith layer; S_i is the tensile strength of a wire in the ith layer; a_i is the angle of pitch of the wires in the ith layer; and n_l is the number of layers of wires in a strand.

J. H. Griffith, M. Am. Soc. C. E., and Mr. J. E. Bragg (6) gave both Equation (9) and an empirical formula for tensile load based on the minimum results of tests, as:

$$T = C \times 75\,000\; d^2_r \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which, D is the diameter of the cable, in inches; and C is a constant for various constructions (see Table 1).

TABLE 1.—VALUES OF C FOR MEAN IN EQUATION (11)

Rope	Range		Mean*
	From:	To:	
6 × 19 plow steel	0.9	1.1	1.0
8 × 19 plow steel	0.8	1.00	0.85
6 × 19 cast steel	0.8	1.00	0.85
6 × 42 tiller rope	0.3	0.45	0.35
6 × 7 guy rope	0.3	0.45	0.35

* Approximate.

On the assumption that slipping does not occur between the straight and curved wires, Mr. Carstarphen gave the following formula (1) for the tensile strength of a rope,

$$S = \cos (a + b) \; A \; S_w \qquad (12)$$

in which S_w is the ultimate strength of the wire.

The foregoing equations for stresses and strengths in bending and in tension are only a few of the many that can be found in engineering literature. They were chosen as representative of current usage, and the range in values given by them demonstrates clearly the uncertainty that still exists as to the ultimate effect of bending stresses on the strength of a wire rope. Each of these formulas has been applied to the wire ropes used in this investigation, and a table of the results is included herein, under the heading, "Summary".

The Present Investigation.—The most logical manner in which to determine the bending stresses and loss of strength in wire ropes, seemed to be a series of tests on ropes on which the stresses could be measured by some standard extensometer. Fortunately, such equipment was available in the form of four tensometers that could be mounted on individual outer wires at different points around the sheave. From these readings unit strains were recorded directly, by multiplying by the predetermined constant for each instrument. To convert these values to unit stresses, it was necessary to draw, from auxiliary samples of the wires used, stress-strain curves for each size of outer wires encountered. From these curves the observed strains could then be transformed readily to their corresponding stress values, thus giving values of the stress in the outer wires at any point along a sheave or on a straight tension specimen.

This same principle is made use of time and time again in laboratory work on mild steel specimens, where extensometer readings of strain below the elastic limit are multiplied by 29 000 000 lb per sq in. in order to give the stress at these points. The difference lies in the fact that steel, such as is used in wire-rope manufacture, does not have a sharp, well-defined elastic limit since it is heat-treated, with the result that the stress-strain curve shows a proportional limit of about 40% of the ultimate strength. Furthermore, in the tests to destruction, strains were recorded on the ropes in most cases to about 90% of the ultimate load, the uniformity of the stress-strain curves of the wire even at these high loads permitting such readings to be made with considerable accuracy.

On the tension tests, a means was sought to determine the modulus of elasticity of the rope as a whole, in addition to the stresses in individual outer wires. One of the most satisfactory methods in use in the past has been described by G. P. Boomsliter, M. Am. Soc. C. E. (8). He utilized an 8-in. strain-gauge set in holes on brass rings soldered to the rope. Considerable difficulty was encountered due to the untwisting effect of the rope under load, which caused errors in his readings. In order to adapt this method to the present tests, and to minimize such errors, it was decided to use a 10-in. strain-gauge, with holes located on $\frac{1}{2}$-in. square brass lugs, curved to fit the rope and soldered to it. In this manner, the tendency of the rig to tear away as the rope shrinks under load was eliminated. To compensate for twist, two scales reading to hundredths of an inch were placed 10 in. apart on the rope, and read with the vertical hair of a surveyor's transit; the proper corrections to the measured gauge lengths were then computed after the completion of the test. The stress-strain curve of the rope could then be drawn readily and the modulus of elasticity obtained in the usual manner.

The tests reported by Professor Boomsliter showed quite definitely that the modulus of elasticity of a wire rope, especially one with a hemp center, is a decidedly variable quantity, and tends to increase as the number of loadings increases and as the stress to which the rope is loaded each time is raised. In view of these results, it was decided to investigate more fully this property of wire ropes with hemp centers and to load each tension specimen seven times to approximately 50% of its ultimate load, taking readings on the first, third, fifth, and seventh loadings. As the tests proceeded, it was found advisable to observe also the second loading. All bending specimens were similarly loaded seven times before finally fracturing them, both to insure conditions similar to those in their companion tension specimens and to work the individual wires so as to equalize the stress in them, as indicated by the tensometer readings.

PROGRAM OF TESTS

Because of the large number of variables involved in an investigation of this nature, it was decided to restrict as many of them as possible, and to confine the study to a determination of basic relationships. For this reason, the size of the rope to be tested was set at 1 in., since this was the minimum on which the outer wires extended along the surface far enough to admit attaching a tensometer on a $\frac{1}{2}$-in. gauge length. Similarly, a rope with a hemp center was selected as being more typical than one with an independent wire rope center, and less likely to be confusing in any analysis.

The variables to be investigated were: (1) Sheave diameter; (2) construction; (3) lay; (4) preforming; and (5) grade of steel.

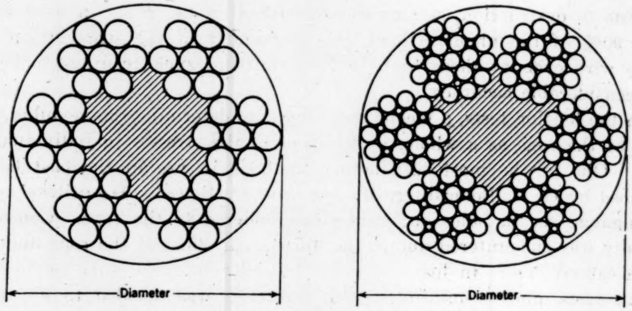

FIG. 1.—6 × 7 AND 6 × 19 WIRE-ROPE CONSTRUCTION.

(*1*) *Sheave Diameter.*—Four values of *D* were selected: 18 in., 14 in., 10 in., and 7 in., each, measured at the root of the groove.

(*2*) *Construction.*—Ropes of both 6 × 7 and 6 × 19 construction were tested (see Fig. 1). The 6 × 19 ropes contained six filler wires of the same grade of steel as the main wires, added to give a smoother surface to the strand.

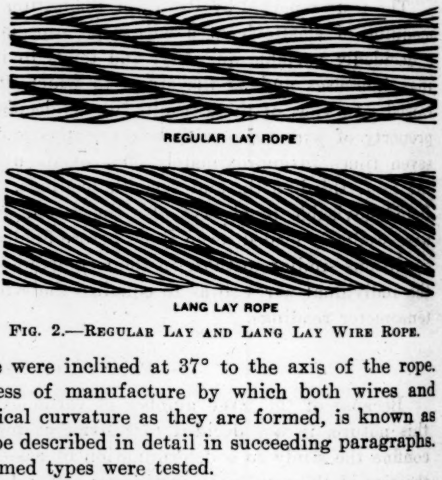

FIG. 2.—REGULAR LAY AND LANG LAY WIRE ROPE.

(*3*) *Lay.*— Both regular lay and Lang lay ropes were included (see Fig. 2). In regular lay ropes the angle of lay of the wire in the strand is equal and opposite to that of the strand in the rope, with the result that the outer wires lie parallel to the axis of the rope. In Lang lay construction, both wires and strands are twisted in the same direction. As the angle of lay was in all cases very close to 18½°, the outer wires in a Lang lay rope were inclined at 37° to the axis of the rope.

(*4*) *Preforming.*—The process of manufacture by which both wires and strands are given an initial helical curvature as they are formed, is known as preforming. This process will be described in detail in succeeding paragraphs. Both preformed and non-preformed types were tested.

(*5*) *Grade of Steel.*—Most of the specimens tested were of cast steel, of the grade produced by almost all wire rope manufacturers, with a specified ultimate strength of 205 000 to 220 000 lb per sq in. A few tests were made for correlation on specimens of plow steel, with an ultimate strength of 235 000 to 250 000 lb per sq in.

Specimens.—Five specimens constituted a set. Of these, one was a tension specimen, 4 ft 6 in. long, and four were bending specimens, 7 ft long, for the four sheave sizes. All the ropes were socketed by means of molten zinc in forged steel open sockets, and all the foregoing dimensions were taken from inside to inside of sockets. These sets were numbered as shown in Table 2.

TABLE 2.—PROPERTIES OF TEST SPECIMENS (HEMP CENTER; d_r = 1 INCH)

No. of set	Type	Lay	Forming
(a) 6×7 CONSTRUCTION			
1	Cast steel	Regular	Non-preformed
2	Cast steel	Regular	Preformed
3	Cast steel	Lang	Non-preformed
4	Cast steel	Lang	Preformed
(b) 6×19 CONSTRUCTION (SIX FILLER WIRES)			
9	Cast steel	Regular	Non-preformed
10	Cast steel	Regular	Preformed
11	Cast steel	Lang	Non-preformed
12	Cast steel	Lang	Preformed
13	Plow steel	Regular	Non-preformed

For determining the physical properties of the wires which made up these ropes, tensile tests were made on ten samples of each size of wire entering each construction, both of cast and plow steel. These sizes were as shown in Table 3.

TABLE 3.—SIZES OF WIRES

Construction	Number of wires in one round	Diameter, *d*, in inches
6×7	One core wire	0.115
	Six outer wires	0.105
6×19	One core wire	0.073
	Six intermediate wires	0.068
	Six filler wires	0.028
	Twelve outer wires	0.065

Observations were taken on these single wire specimens, of proportional limit, ultimate strength, modulus of elasticity, and location of fracture, and an average stress-strain curve was plotted for each size up to about 85% of the ultimate strength. Average values of these observations were used for determining the physical constants of the wire, and the probable error from this mean was noted.

MANUFACTURING PROCESSES

The various manufacturing processes and machines used in the production of wire and wire rope have been rather fully described elsewhere, notably by Messrs. Carstarphen (1) and Meals (1). The process is reduced essentially to three steps: First, the drawing and treating of the wire; second, the spinning of these wires into a strand of the desired size and construction; and, third, the closing of several strands around a hemp or wire rope center to form a wire rope. It is during this last step that the ropes are preformed, if so desired, so that the wires and strands are permanently deformed and lie in the finished rope with no tendency to unravel or kink.

Fig. 3 shows a large vertical closing machine at the point where six strands are drawn through a die over a lubricated hemp center to form a non-preformed, regular lay rope. The frame and die are held stationary, and the slotted cone and spools from which the strands are drawn rotate. In addition, the spools are given a planetary motion so that the strands are laid into the rope without any twist, and their lay is controlled by the speed with which the finished rope is withdrawn. For forming Lang lay rope, the frame is rotated in the reverse direction, and the spools given a back turn to minimize the untwisting effect

The contrast between this method and that used in making preformed ropes is illustrated in Fig. 4. The frame and die are the same, but, in this case, the smooth cone is replaced by one on which are mounted three small sheaves for each strand. These sheaves are placed as shown in Fig. 4 and the strands threaded around them so that a helical permanent set is imparted to them. This set is noticeable in the section of the strands just as they enter the closing die. The proper position of the small sheaves must be determined very exactly in order that the helix will be of the exact size required for forming the desired rope. In preforming Lang lay ropes, these small sheaves are replaced by spiral holes through which the strands are drawn, in order to minimize the twisting action to which this type is subject.

TESTING APPARATUS

All specimens both in tension and in bending were tested in a 300 000-lb testing machine, which was calibrated to 200 000 lb and found to be correct within 0.25 per cent. For the tension tests, the sockets were passed through the holes in the two heads of the machine and secured by steel plates, in which $1\frac{3}{4}$ - in. holes had been drilled to receive the socket pins. Brass gauge points for the gauge were cut from a section of 1-in. brass pipe, and when properly cleaned with emery paper were soldered to the rope. In all cases except the first test, where six gauge lengths were provided, two gauge lengths were used, directly opposite each other at about the center of the specimen. By exercising proper care in soldering, only one of all the brass gauge points broke away in the course of testing, and the fracture of the rope was never traceable to the heat treatment of wires in the vicinity of the soldering operations. For measuring the twist, two paper scales graduated to 0.02 in., were affixed to one side of the cable, just under each gauge point, by rubber bands, and these scales were read to 0.01 in. on the vertical hair of a surveyor's transit set up on a near-by table. The gauge holes in the brass plugs were also aligned vertically with this transit when drilled.

The operation of attaching the tensometers proved to be the most difficult part of the tension set-up, as the ropes contracted appreciably under load, causing the gauges to become loose. However, after some experimentation, it was found that for regular lay ropes a pair of tensometers could be mounted opposite each other on a standard gauge-holder, and could be held in place

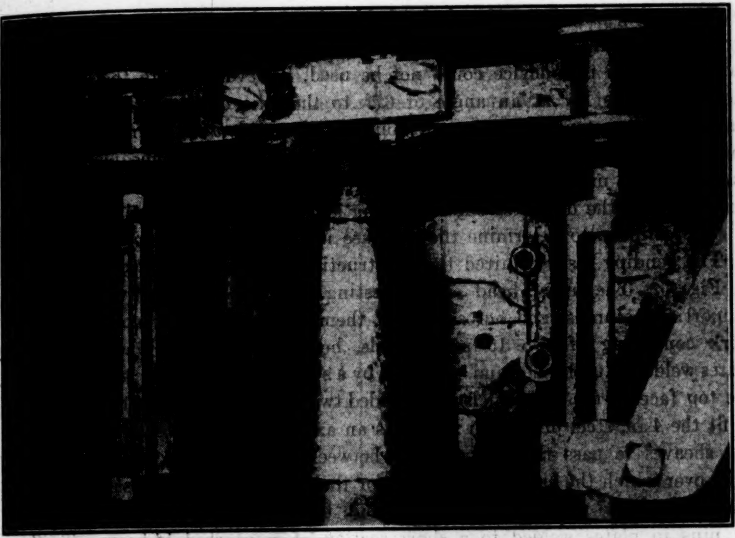

FIG. 3.—WIRE ROPE MACHINE FABRICATING A REGULAR LAY, NON-PREFORMED ROPE.

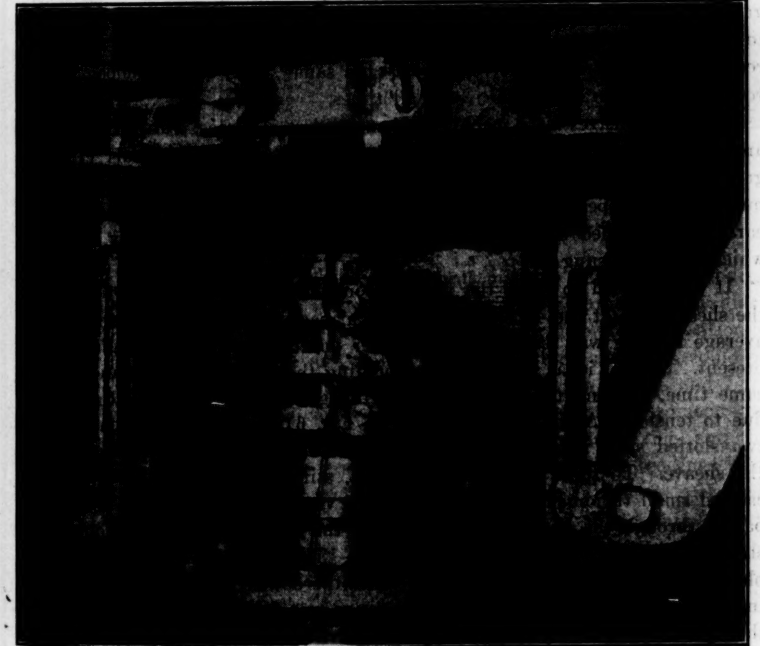

FIG. 4.—WIRE ROPE MACHINE MODIFIED FOR FABRICATING A REGULAR LAY, PREFORMED ROPE.

by connecting the far ends of the holder by a short strong spring. This arrangement gave consistently good readings even at very high loads. For Lang lay ropes this device could not be used, however, due to the fact that the outer wires lay at an angle of 37° to the axis of the cable. For this reason the two tensometers were mounted separately, on fittings especially built to hold the gauges at the required angle and allow some play as this angle changed under load. On many of the tension specimens measurements were taken of the diameter before loading and at nearly full load, with a pair of slide calipers, to determine the decrease in diameter under load.

The bending tests required the construction of a special testing rig shown in Fig. 5. The upper head of the testing machine was removed from its supporting columns, and across two of them was placed diagonally a framework consisting of two 15-in. channels, held vertically in place by $\frac{1}{2}$-in. plates welded to their ends and separated by a slot $2\frac{1}{2}$ in. wide. At the middle of the top face of these channels were welded two semi-circular bearing blocks, cut to fit the 4-in. steel pin which served as an axle for the sheaves. This allowed the sheaves to pass through the slot between the channels, leaving the top half, over which the cable passed, free for the mounting of gauges. The lower ends of the rope passed downward through the slot and the sockets were held by pins in plates welded to a short section of reinforced H-beam. To the lower face of this beam was welded a vertical steel piece which, in turn, was gripped by the jaws of the testing machine. The sheave at all times was free to rotate on its pin and the pin in its bearing-blocks; and from the gauge readings it is believed that very nearly the same stress was developed in the rope on each side of the sheave at all times.

The four sheaves were machined from solid steel plates, 2 in. in thickness, and in accordance with modern practice, as noted by Mr. Meals (7), the grooves were made $1\frac{1}{16}$ in. in diameter, and semi-circular to facilitate measurements on the ropes. A 4-in. hole was cut in the center of each sheave, and carefully machined so that the steel axle could be inserted easily by hand which insured a snug fit.

It was decided to place the four tensometers available, one at the top of the sheave, one at the 45° point, and one at each tangent point, since a good average reading was required at the latter point because of the high stresses present. Several ideas were tried for holding the gauges in place and, at the same time, meeting the problems of shrinkage and sliding along the sheave due to tension. As finally arranged (see Fig. 6), the apparatus consisted of two slotted steel rings which were slipped over the axle on either side of the sheave. To these were attached short, stiff springs, and these, in turn, carried small turnbuckles and wire loops of varying length. Small rods were passed through the holes in the gauges, in the case of regular lay ropes, and slipped through these wire loops (Fig. 6). The gauges were held firmly in place by tightening the turnbuckles until there was an appreciable tension in the springs on either side, and the gauge was free to move slightly along the sheave as the rope stretched. Various combinations of length of springs and wire loops enabled this apparatus to be used on all four sheave sizes.

For the Lang lay ropes the rig used was identical except that special holders of welded construction had to be devised for keeping the gauges fixed at an angle of 37 degrees. Although it was a rather delicate matter to set up the gauges for a test, this apparatus gave consistently good results. In some cases, the gauge could not be set directly at the tangent points, as no strand came to the surface there, and in this case they were set on the next strand above the tangent point, the stretch causing them to pull down slightly during the test.

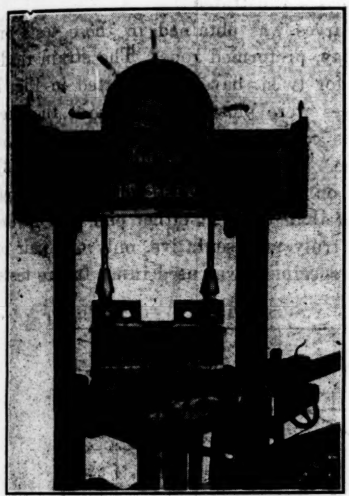

FIG. 5.—TESTING RIG FOR WIRE ROPE SPECIMEN OVER 18-INCH SHEAVE.

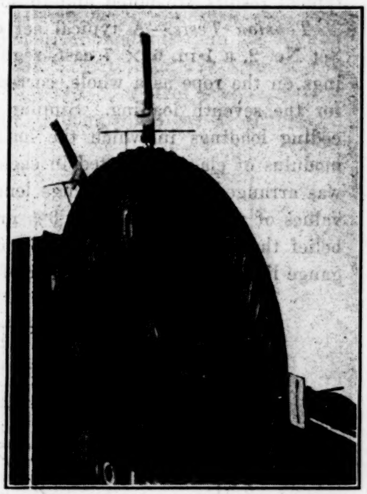

FIG. 6.—CLOSE-UP VIEW OF BENDING SPECIMEN, SHOWING MEANS OF ATTACHING TENSOMETERS.

On all the regular lay ropes, an attempt was made to evaluate the bending stress by a plain bending test without tension. For this purpose an auxiliary rig was devised, consisting of a steel plate bolted fast in a horizontal position to a heavy table. In this plate were drilled holes into which steel pins could be inserted to simulate sheave diameters of 50, 25, 18, 14, 12, 10, 8½, and 7 in. A space was left clear in the center to allow placing a pair of tensometers on the rope, one on the compression side and one on the tension side, and the rope was bent over these diameters in succession by hand, while readings were taken of the strains that occurred. This apparatus was inherently awkward, and only by averaging a great number of results could any definite trends be established. The rig was not adapted for Lang lay ropes, as the attachments necessary for holding the gauges in place would not fit in the space allowed.

Single wire tests for determining the stress-strain curve were made on a 2 000-lb, hand-power testing machine, with specimens about 15 in. long.

Discussion of Test Data

The great mass of data taken during these tests does not permit the inclusion of all the test results. Accordingly, an attempt has been made to follow through the procedure in one particular typical instance, giving all the results and curves obtained, and to summarize the results in the form of curves for the remaining sets of ropes. Particular exceptions or variations from the typical results are noted and in some cases illustrated.

Tension Tests.—A typical set of curves was obtained in these tests on Set No. 2, a 1-in. 6 × 7 cast, regular lay, preformed rope. The strain readings on the rope as a whole, corrected for twist, have been plotted in Fig. 7 for the seventh loading. Similar curves were plotted for each of the preceding loadings in which the load was carried to only 35 000 lb, and the modulus of elasticity noted in each case. The first rope tested (Set No. 13) was arranged with three gauge lengths on each side. The center set showed values of the modulus about 3% greater than those at either end, and in the belief that this center value was more truly representative, only one pair of gauge lines, located at the center of the specimen, was used in all future tests.

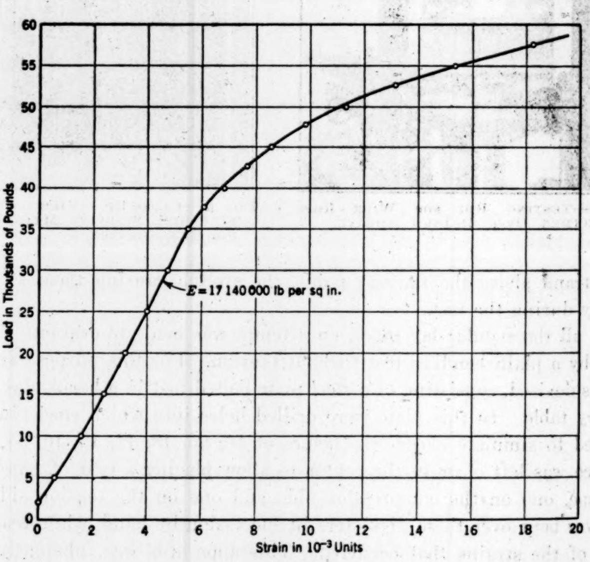

Fig. 7.—Set No. 2: Load Strain Curve of 1-Inch, 6 × 7, Cast-Steel Regular Lay, Preformed Wire Rope; Seventh Loading; Net Area = 0.374 Inch².

The reasons for the initial curvature of the load-strain curve, shown plainly in Fig. 7, are discussed by Messrs. Griffith and Bragg (6). Their conclusion is that at low loads the elongation under stress is not wholly elastic, due to the presence of initial curvature in the strands and wires from the laying and a certain "slack" or curvature in the rope itself.

Fig. 8 presents curves showing the rise in modulus of elasticity with repeated loadings for all sets of ropes tested (see Table 2). The sharp initial rise in all these curves after the first loading is due to the large initial consolidation of the hemp center, and the compacting of the wires and strands. It will be noted that, in general, the 6 × 7 ropes (Sets Nos. 1 to 4) show values greater than the 6 × 19 construction. Furthermore, the Lang lay ropes (Sets Nos. 3, 4, 11, and 12) show a slightly higher modulus than the regular lay ropes, and the preformed ropes (Sets Nos. 2, 4, 10, and 12) seem to run higher than the non-preformed types. The one plow-steel specimen (Set No. 13) had a slightly higher modulus than its companion cast-steel rope.

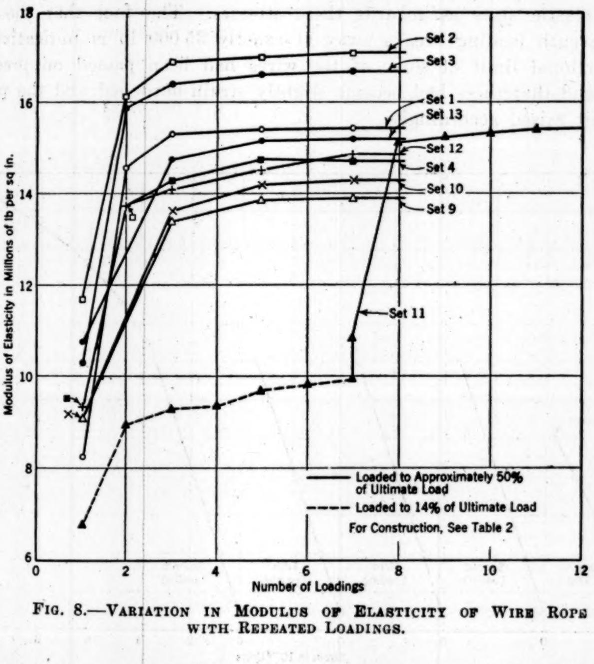

FIG. 8.—VARIATION IN MODULUS OF ELASTICITY OF WIRE ROPE WITH REPEATED LOADINGS.

The 1-in., 6 × 19 cast, Lang lay, non-preformed rope (Set No. 11) was tested for seven loadings to 10 000 lb (14% of the ultimate load), with a view to determining whether the same increase in modulus occurred at working loads as at relatively high loads. The seventh loading was then continued to 35 000 lb, or 49% of the ultimate, which was repeated until the eleventh loading, when the test was carried to destruction. Fig. 8 shows that at working loads a rise in modulus of elasticity does occur, but that the values are much lower than when the load is increased to about the proportional limit. This is explained by the fact that at these low loads the stress-strain curve was still concave upward. The value selected as the modulus in such cases was the slope of the tangent to the curve at the maximum load. In every case, the slope was less than that found when the load was further increased, indicating quite definitely that the rope had not yet, at the low loads, reached a period of elastic behavior.

For the tension test of Set No. 2, Fig. 9 shows the load-strain curves for the average of two individual wires, as shown by the tensometers, for each loading. On the first loading, both gauges ran off the scale early in the test, but succeeding loadings show the wires to be taking stress in a very uniform manner. It is evident that the first loading tends to redistribute and equalize the stress in the several wires and further loadings bring the rope to an almost perfectly elastic state as regards these stresses. The fact that the curve for the seventh loading breaks away at exactly 35 000 lb is indicative that the proportional limit of some of the wires had been passed on preceding loadings, and that these had become slightly strain-hardened and the proportional limit raised accordingly.

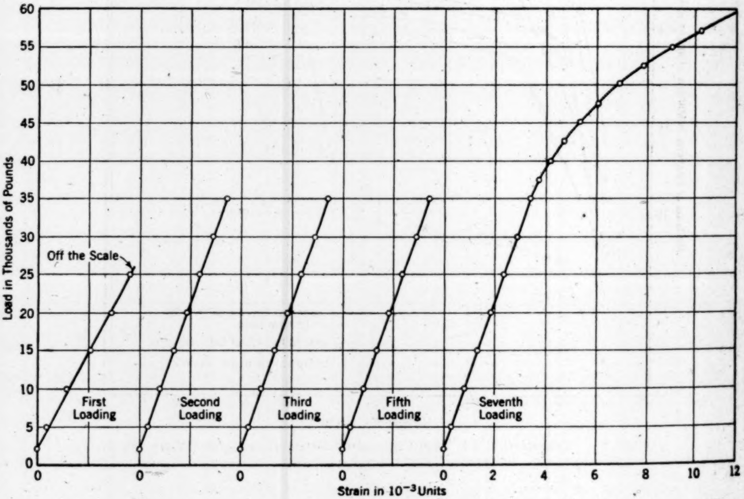

FIG. 9.—TENSION TEST, SET NO. 2; LOAD-STRAIN CURVES OF A SINGLE WIRE; 6 × 7 CAST-STEEL, REGULAR LAY PREFORMED WIRE ROPE.

The strains for the seventh loading (Fig. 9) were transformed into stresses as explained previously by use of a stress-strain curve for the single wire. Fig. 10 shows the stress-strain relations for this particular size of wire, 0.105 in. in diameter, cast steel.

The results of this transformation are expressed in the form of a load-stress curve, as shown in Fig. 11(*a*). It will be noted that up to the proportional limit the curve follows very closely the dotted line representing the load divided by the net area of steel, which is the curve for a homogeneous bar of the same area of cross-section. Beyond this point there is a reverse curve (although this is not present in every case). When extrapolated to a value of the load equal to the observed ultimate load on the rope, this reverse curve shows a stress value of 219 500 lb per sq in., which checks very closely the average single-wire strength for this grade, namely, 219 000 lb per sq in. As always, in extrapolating curves, there is a chance for error, but as every curve showed this ultimate stress to be close to 219 000 lb per sq in., these load-stress curves seem to be well established.

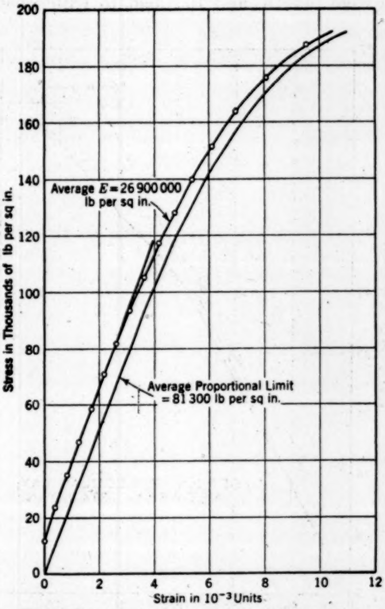

FIG. 10.—AVERAGE STRESS-STRAIN CURVE FOR SINGLE, CAST-STEEL WIRE; SPECIMENS 1 TO 10; AVERAGE AREA, 0.008544 SQUARE INCH.

The curve in Fig. 11(*a*) is typical of all those obtained on regular lay ropes. On Lang lay ropes, a different kind of curve was obtained, as is illustrated by Fig. 11(*b*), for 1-in., 6 × 19 cast-steel, Lang lay, preformed rope. In this case the stress in the outer wires is considerably lessened, and falls well below the dotted line for load divided by net area for the greater part of the test, but picks up rapidly at the end. In the case of Fig. 11(*b*), the stress below the proportional limit is 0.803 times that given by the dotted line, which may be assumed to be that for the regular lay rope. Theoretically, this factor should be cos 37°, or 0.799, since the outer wires are inclined at 37° to the axis of the rope. The average value of this ratio for all the Lang lay ropes tested was 0.783, and this agreement is within the limits of experimental error.

The difference in load-stress curves for the tension tests of preformed and non-preformed ropes is obscured by the incidental variations of each test. There seems to be no appreciable difference in the stress conditions for the two cases when loaded up to seven times, although some tendency for a quicker equalization of stress in the individual wires has been noted for the preformed type. The 6 × 7 ropes show a less steep load-stress curve than those of the 6 × 19 construction, indicating the presence of higher stresses, but this increase is in inverse proportion to the net area of section, and the foregoing facts are modified only in this proportion.

A summary of the untwisting effect observed for every rope is presented in Table 4. The values given are in inches of circumferential twist in a 10-in. gauge length, and the load to which each loading was taken is also recorded.

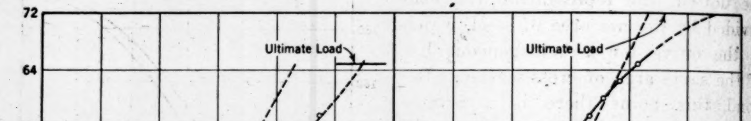

FIG. 11.—LOAD STRESS FOR TENSION TEST; 1-INCH, CAST-STEEL WIRE ROPE, PREFORMED.

These values show the decrease in twist with repeated loadings and are useful for purposes of comparison. The preformed ropes (Sets Nos. 2, 4, and 10) in general show less twist than the non-preformed ropes, with the exception of Set No. 10, which is about the same as Set No. 9. The Lang lay ropes

TABLE 4.—INCHES OF CIRCUMFERENTIAL TWIST IN 10-INCH GAUGE LENGTH

Set No.	Load, in thousands of pounds	Number of Loading												
		First	Second	Third	Fourth	Fifth	Sixth	Seventh		Eighth	Ninth	Tenth	Eleventh	
								Final load, in thousands of pounds	Twist, in inches				Final load, in thousands of pounds	Twist, in inches
1	35	0.11	0.06	0.05	....	0.05	....	60	0.10	....	....	....	....	....
2	35	0.04	0.01	0.01	....	0.02	....	57.5	0.02	....	....	....	....	....
3	35	0.09	0.07	0.06	....	0.03	....	60	0.09	....	....	....	....	....
4	35	0.03	0.02	0.02	....	0.01	....	57.5	0.08	....	....	....	....	....
9	35	0.08	....	0.05	....	0.06	....	65	0.12	....	....	....	....	....
10	40	0.15	....	0.07	....	0.08	....	60	0.13	....	....	....	....	....
11	10	0.03	0.02	0.02	0.02	0.02	0.02	....	0.03	....	....	....	....	....
11	35	....	....	....	....	....	....	....	0.09	0.09	0.07	0.08	65	0.13
12	35	0.08	0.04	0.05	....	0.02	....	65	0.06	....	....	....	....	....
13	50	0.14	....	0.08	....	0.07	....	64	0.10	....	....	....	....	....

(Sets Nos. 3, 4, 11, and 12) do not show any greater untwisting than regular lay ropes; in fact, Set No. 12 shows much less twist than its companion specimen, Set. No. 10. It must be remembered, however, that the ends of these specimens were totally fixed against rotation under load, by the frictional forces acting on the heads of the testing machine. Ropes of 6 × 7 construction (Sets Nos. 1, 2, 3, and 4) seem to untwist about as much as those of the 6 × 19 construction.

Data on the shrinkage of wire ropes due to consolidation of the hemp center are available for most of the specimens tested. These data are assembled in Table 5, the diameters being recorded to the nearest 0.01 in., and show that a rope will acquire a permanent decrease in diameter of 1% to 2% when loaded to 50% of its ultimate load, and that the total decrease at fracture is about 5 to 6% of the original diameter.

TABLE 5.—Decrease in Diameter (Inches) of Ropes Under Load

Set No.	Average original	At beginning of final loading	During Final Loading	
			Load, in pounds	Diameter, in inches
1	1.01	....	60 000	0.96
2	1.02	....	57 500	0.97
3	1.00	0.98	62 500	0.95
4	1.00	0.98	60 000	0.95
9	1.02	....	68 000	0.99
11	1.02	1.01	65 000	0.98
12	1.04	1.02	65 000	0.99

The efficiency of a wire rope in tension is its ultimate load divided by the product of the net area by the ultimate strength of the wire, the denominator being the theoretical maximum load that a homogeneous rod could attain. Values of the efficiencies obtained in these tests are tabulated in the "Summary".

Bending Tests.—Set No. 1 (Table 2), has been selected as typical of the bending test results obtained. Observations were made of the strains at the top of the sheave for the first, third, and fifth loadings, and of the strains at the top, 45° point, and the two tangent points for the seventh loading to destruction. The load-strain curves for these loadings of the specimen bent over the 18-in. sheave, are shown in Fig. 12(*a*); very similar curves were obtained over the other three sheave sizes. The friction effect on stress at the top of the sheave is well illustrated by the curves for the first, third, and fifth loadings. As all the stresses are below the proportional limit, these are also load-stress curves to another scale. Upon release of the load, the stress at the top point remained constant until the friction load caused by stretching the cable decreased to zero, and built up until slipping occurred along the sheave in the reverse direction. Thus, this drop is a measure of twice the frictional force present. According to the foregoing reasoning, the values of strain at the tangent points should follow back the original curve as the load is removed, with no hysteresis, and this has been found to be the case. On practically every one of the Lang lay ropes, an initial compression was observed on the first loading, which is evidently due to poor initial stress distribution inherent in this type of construction.

The transfer of strain values to stresses as described for the tension test was made for the seventh loading of Fig. 12(*a*), giving the set of load-stress curves shown in Fig. 12(*b*). These curves show very definitely that the maximum stress lies at the tangent point, where the rope meets the sheave, and that the stress decreases progressively up and around the sheave, due to the increasing frictional forces present, to a minimum value at the top. The coefficient of friction of rope on the sheave was determined in each instance by the formula[4]:

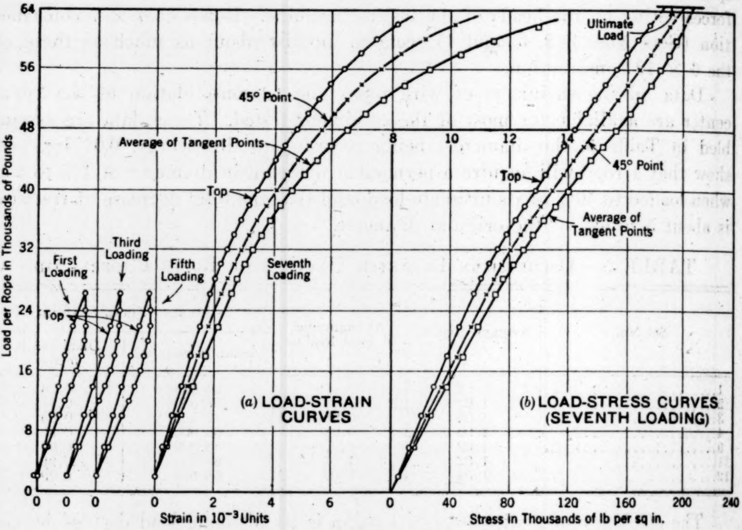

FIG. 12.—SET NO. 1; BENDING TEST OVER 18-INCH SHEAVE.

$$\frac{s_g}{s_p} = e^{0.5\pi f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Two values of the stress ratio for each sheave size were taken from the load-stress curves, one at the proportional limit and one near the ultimate load; in most cases these values were in reasonable agreement. The values of *f* for all four sheave sizes of each set were then averaged, with the results shown in Table 6. The average values for regular lay ropes is about 0.14, and for Lang lay ropes the value is raised to 0.38, due to the fact that the inclined wires on the surface offer much greater resistance to slippage than the wires of regular lay construction, which are parallel to the direction of motion.

In eleven cases out of the thirty-six bending tests, the load-stress curve for the 45° point at the lower loads fell to the left of the curve for the top point, indicating a lower stress to be present. It seems likely that the stresses at the top and at the 45° point for low loads are not very different, and that differences in the individual wires on which the gauges were set account for this seeming inconsistency. It might be noted that in every case where this occurred, the curves crossed and showed higher stress at the 45° point before the gauges were removed from the rope prior to failure.

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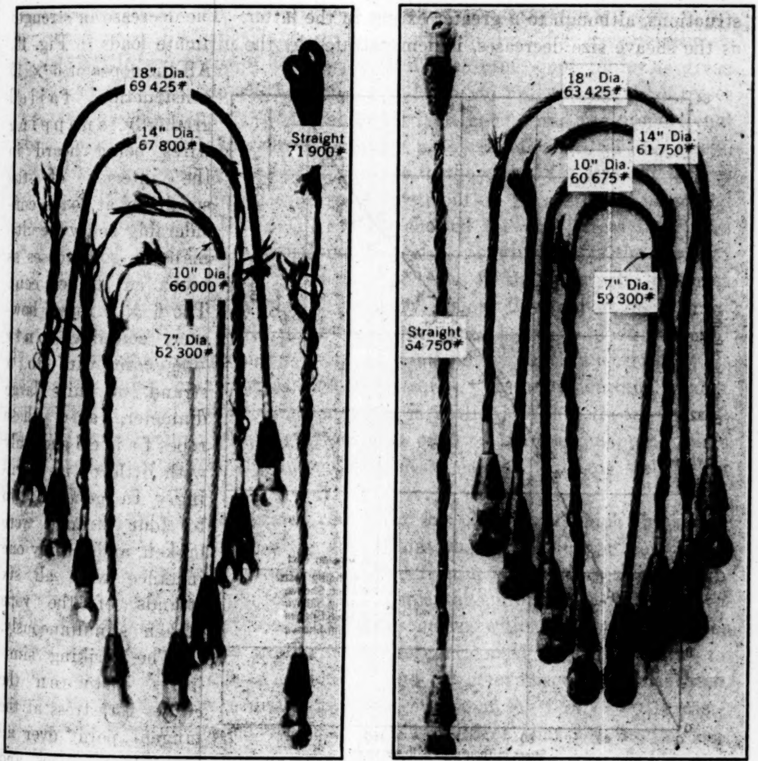
[4] "Applied Mechanics", by A. P. Poorman, M. Am. Soc. C. E., Second Edition (1932), p. 132.

TABLE 6.—COEFFICIENT OF FRICTION

Regular Lay Ropes		Lang Lay Ropes	
Set No. (see Table 2):	Coefficient, *f*	Set No. (see Table 2):	Coefficient, *f*
1	0.146	3	0.432
2	0.156	4	0.344
9	0.101	11	0.392
10	0.149	12	0.348
13	0.133		.....
Average	0.137	Average	0.379

The Lang lay ropes showed load-strain and load-stress curves similar in form to the specimens of regular lay ropes. These curves, as in the tension test, lie to the left of the corresponding curves for regular lay rope, and show


18" Dia. 69 425#
14" Dia. 67 800#
Straight 71 900#
10" Dia. 66 000#
7" Dia. 62 300#
18" Dia. 63,425#
14" Dia. 61 750#
10" Dia. 60 675#
7" Dia. 59 300#
Straight 54 750#

(*a*) Non-Preformed. (*b*) Preformed.

Fig. 13.—Typical Failures of Tension and Bending Specimens.

on an average 80% of the stress values. Again, the difference between stress conditions in preformed and non-preformed ropes was not marked after seven loadings, and the 6 × 7 ropes showed stresses higher than the 6 × 19 ropes in inverse proportion to their net areas. The plow-steel specimens showed exactly similar effects, with loads and stresses raised in proportion to the ultimate strengths of the single wires.

The manner in which the bending specimens failed supports the observations drawn from the preceding curves that the greatest stress occurs at the tangent points. With a very few exceptions, all bending specimens failed at one of the tangent points. The only exceptions were those that failed at the socket, which failure occurred for only two specimens of Set No. 12. This fact is demonstrated clearly in Fig. 13, which also illustrates the difference in the type of failure of preformed and non-preformed ropes. Fig. 13(*a*) shows Set No. 9, non-preformed, while Fig. 13(*b*) shows Set No. 2, preformed, after fracture of each rope. The scattering of the wires in the non-preformed rope and the relatively little disintegration of the preformed type, are distinctly shown. This effect was noticeable in both the 6 × 7 and the 6 × 19 constructions, although to a greater extent in the latter. The decrease in strength as the sheave size decreases, is demonstrated by the ultimate loads in Fig. 13. All the ropes of 6 × 19 construction failed gradually, snapping wires being heard in the interior of the specimen at loads considerably below the ultimate, in some cases as much as 10 per cent. The 6 × 7 ropes, however, contained only one core wire to a strand, of quite large diameter, and these ropes failed suddenly with little or no warning. In general, two to four strands were broken, and in only one instance were all six strands of the rope broken simultaneously.

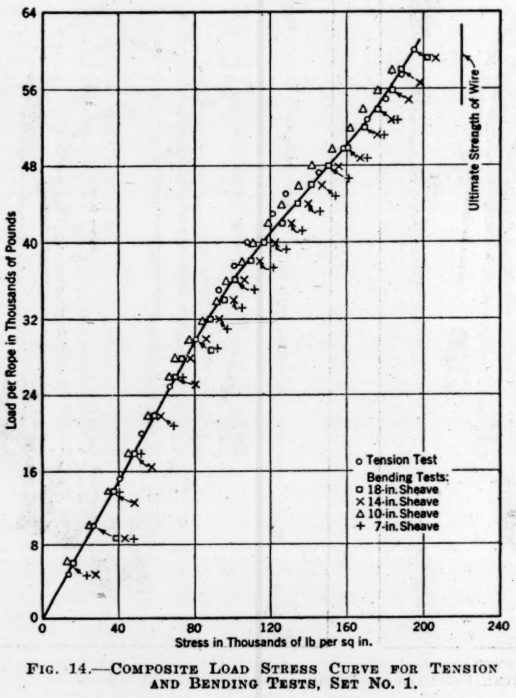


FIG. 14.—COMPOSITE LOAD STRESS CURVE FOR TENSION AND BENDING TESTS, SET NO. 1.

The striking similarity between the curves for stress at the tangent point over all four sheave sizes and the corresponding curve for the tension test, led to the plotting of these values for Set No. 1 on the same co-ordinates, as shown in Fig. 14. All the curves are seen to coincide within the range of experimental errors, and this was found to be the case for all the ropes tested, although the agreement was not so perfect for the Lang lay ropes. This demonstrates that the bending stress is not increased after the rope is initially bent over the sheave, but that thereafter the rope behaves exactly as in a tension test. Bending over a given sheave, therefore, is equivalent to shifting the curve of Fig. 14 to the right by a constant amount, thus causing the curves to intersect the vertical line representing their ultimate strength at successively lower values of the load as the bending stress increases. This fact served as a basis for a graphical determination of the bending stress, by extrapolating the load-stress curve to the breaking load and subtracting the stress there observed from the known ultimate strength of 219 000 lb per sq in. Although this method is admittedly crude, its accuracy in determining bending stress may be judged from the values recorded in the table on correlation of bending stress formulas, given in the "Summary".

The shape of the load-stress curve in all cases is essentially the same. One would normally expect this to be a straight line for a homogeneous material, but in a composite body, such as a wire rope, opportunity is given for some wires to yield more than others and thus to redistribute the stresses in a strand. An explanation of the observed fact that beyond the proportional limit the outer wires take more than their proportional share of the stress, as shown by the breaking away of the curves to the right of a straight line, is found in a theoretical analysis of stress distribution in a strand, presented by Messrs. Griffith and Bragg (6). They showed that the stress in a wire of a given ring is directly proportional to $\cos^2 a_n$, in which a_n is the angle of lay of the wires in the strand. The ratio of stress in the outer wire to stress in the core wire, therefore, should be 0.899, below the proportional limit. When the inner wire or wires reach this point, they begin to yield first, and this ratio will rise, approaching unity as a maximum, until theoretically at fracture the stress in all wires should be equal. Actually, certain weaker inner wires will fail before this condition is realized. This theory is supported by the experimental observations that on 6 × 19 ropes snapping wires were invariably heard in the interior of the specimen at loads well below the ultimate.

For illustration an exaggerated condition was chosen, in which the strain in the outer wire was assumed as 0.74 times the strain in the core wire. Proper values of the stress were then selected from a typical wire stress-strain diagram, and these were plotted against percentage of total load, as shown in Fig. 15, together with the average stress curve, which is a straight line, as would be expected. The similarity of the curve marked "Outer Wire" to the observed load-stress curves is notable, and for comparison there has been included on this plot the results of the tension test of the 1-in., 6 × 19 cast-steel, regular lay, preformed specimen. It is evident that the assumption that the outer wire takes 74% of the stress in the core wire, below the proportional limit, was none too extreme.

The results of the plain bending test without tension, in which the regular lay ropes were bent by hand over steel pins, are represented typically by Fig. 16, for 1-in., cast-steel, non-preformed ropes of 6 × 7 and 6 × 19 construction. Each point is the average of two readings on each of four ropes, a total of eight readings. These curves are interesting in that they show that the stress on both the tension and the compression sides of the rope increases nearly linearly with the ratio of rope diameter to sheave diameter at the root. Although in every case the compressive stress exceeds the tensile stress, this is of little importance, as the addition of direct stress in the form of a pull will decrease this stress and soon bring these wires into tension as well. When these strains are transformed into stresses, another value is obtained for the bending stress, and these results have also been listed in tabular form in the "Summary". The values for the 6 × 7 ropes were much higher than those for the 6 × 19 construction, and the method broke down completely for these values. However, the trend is notable, and, as will be shown subsequently, the loss of strength of a rope varies in exactly this manner with the ratio of rope diameter to sheave diameter.

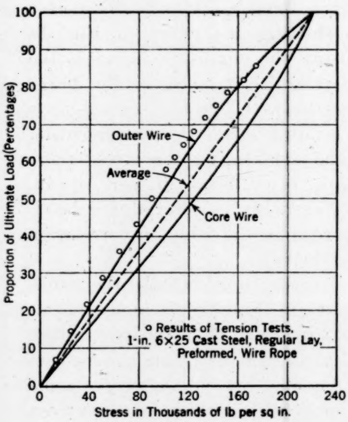


FIG. 15.—THEORETICAL STRESS DISTRIBUTION IN A STRAND (STRAIN IN OUTER WIRE = 0.74 STRAIN IN CORE WIRE.)

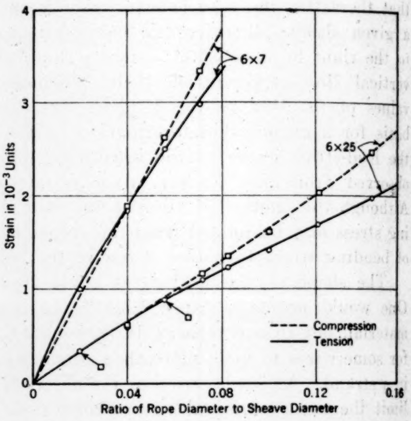


FIG. 16.—STRAIN SET UP BY PLAIN BENDING WITHOUT TENSION; 1-INCH CAST-STEEL REGULAR, NON-PREFORMED WIRE ROPE.

A summary is presented in Fig. 17 of the loss in strength in bending plotted against the ratio of rope diameter to sheave diameter for every set of ropes tested. A curve presented by Mr. A. S. Rairden (1) for tests on ⅝-in. 6 × 19 plow-steel, regular lay, non-preformed ropes, is also included with the reciprocal of his ordinate scale used in this case. It is believed that by inverting this ratio (thus converting a hyperbolic curve into a straight line), the curve may be fitted much more easily to a number of fairly erratic points, as the origin is fixed and only one degree of freedom is allowed in locating the curve. The curved relationship shown for some of the sets may very well be due to an inaccurate value for the tension test, which affects all the points, and is equivalent to shifting the curve up or down. Such a shift has been made in the case of Set No. 11, where the tension specimen failed at the socket at a load of 71 650 lb, without developing its full strength. This load was so low as to indicate an actual increase in strength over the 18-in. and the 14-in. sheaves, and in view of the relationships amply demonstrated in the other ropes, the value has been raised to 75 000 lb by shifting the straight line to pass through the origin, and the loss in strength computed on this basis. The short arrows on this plot serve merely to identify each point with its proper curve, and do not indicate that the point itself has been moved.

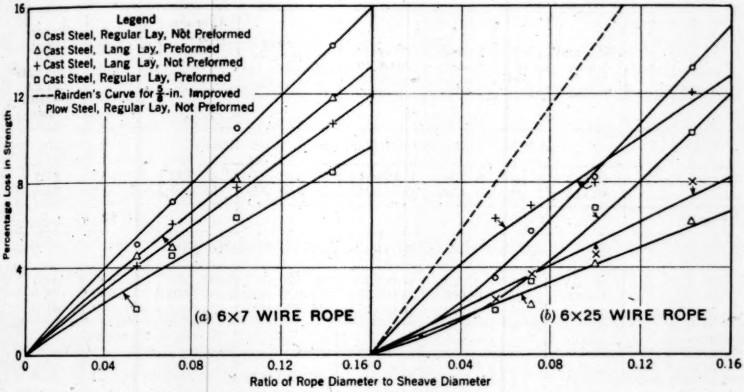


FIG. 17.—LOSS IN STRENGTH OF WIRE ROPES IN BENDING.

It may readily be seen that the non-preformed ropes show a greater loss in strength in every case than the preformed types. Similarly, in all but one instance, the Lang lay ropes show less loss in strength than their corresponding specimens of regular lay, due to the lower bending stresses present. There seems to be very little difference between the results for the 6 × 7 and 6 × 19 constructions and for the cast and plow grades of steel, when the loss is considered on a percentage basis. Considerably more data are needed, however, before any definite conclusions can be reached on these points.

A third method used in determining the bending stress in the rope is to divide the loss in strength by the product of net area and efficiency in tension, the denominator being the effective net area resisting bending. The results are shown in the "Summary".

Single-Wire Test Data.—A summary of test data on the various specimens of single wire tested is given in Table 7. On the basis of these results, an average ultimate strength of cast steel used in these ropes was selected as 219 000 lb per sq in., and for plow steel as 246 500 lb per sq in. A modulus of elasticity of 26 500 000 lb per sq in. was selected as characteristic of both the cast-steel and plow-steel specimens. Fig. 10 is an example of the average stress-strain relation for the ten samples of cast-steel wire 0.105 in. in diameter. As an example of the consistency of these data, for the plow-steel specimens, all had ultimate strengths within 3.2% of the average value, whereas 95% had values of modulus of elasticity within 7% of the average. The cast steel showed similar consistency with minor exceptions, notably the modulus values for the specimens 0.115 in. in diameter. Although the diameters of all test specimens were measured to 0.0001 in., the nominal diameters were used in computing net areas in all further calculations, on the basis that they represented an average condition of manufacture.

TABLE 7.—SINGLE-WIRE TEST DATA

Set No	Nominal diameter, in inches	No. of specimens	Average diameter, in inches	Average area, in square inches	Average modulus of elasticity, in pounds per square inch	Average proportional limit, in pounds per square inch	Average ultimate strength, in pounds, per square inch	Proportional limit + ultimate strength (percentages)
(*a*) WIRES USED IN 6×7 CAST-STEEL WIRE ROPES								
3	0.105	10	0.1043	0.008544	26 900 000	81 300	215 400	37.7
3	0.115	10	0.1142	0.010243	28 910 000	53 900	220 200	24.5
Average	......	..	......	....	....	......	217 800	....
(*b*) WIRES USED IN 6×19 CAST-STEEL WIRE ROPES								
3	0.065	10	0.0649	0.003308	26 820 000	91 000	221 600	41.1
3	0.068	10	0.0671	0.003536	25 090 000	82 800	212 600	38.9
3	0.073	10	0.0729	0.004174	26 830 000	83 900	228 800	37.5
Average	.....	..	......	....	....	......	219 300	....
Average, all cast-steel specimens	.....	..	......	....	26 910 000	78 600	218 700	35.9
(*c*) WIRES USED IN 6×19 PLOW-STEEL, WIRE ROPES								
5	0.028	10	0.0283	0.000629	....	....	248 000	....
5	0.065	10	0.0641	0.003227	26 050 000	107 000	245 800	43.5
5	0.068	10	0.0678	0.003610	26 840 000	88 700	250 600	35.4
5	0.073	10	0.0734	0.004231	26 390 000	87 000	241 700	36.2
Average	.....	..	......	....	....	....	246 500	....
Average, plow-steel specimens	.....	..	......	....	26 430 000	94 200	246 500	38.2

SIGNIFICANCE OF RESULTS

The most significant results of the tension tests on the various wire ropes studied are contained in the plots for increase in the modulus of elasticity of the rope as a whole with repeated loadings. In the past a value of 12 000 000 lb per sq in. has often been considered the maximum modulus that a wire rope would attain, and this value has frequently been termed conservative when used in bending stress formulas. From the data presented herein it may be noted that one excessive loading up to about 50% of the ultimate load will raise the modulus frequently greater than 14 000 000 lb per sq in., and even with working loads as low as 14% of the ultimate, a definite increase in modulus occurs, although it is probable that the value would never reach as high a figure as when overloaded once. For running ropes, it is considered desirable to have a fairly low modulus, so that the rope can "give" and absorb some of the shocks of sudden starting, whereas for stationary installations the reverse is true. In the case of suspender cables and guy-work for which accurate lengths are needed, the value of several pre-stressings to a fairly high load may be seen very plainly. This procedure has been followed in the past, and values for the modulus greater than 19 000 000 lb per sq in. have been obtained on large suspender cables for suspension bridges.

Of equal significance are the load-stress curves for both tension and bending over sheaves. It has been shown that the bending stress is not increased after the rope is once bent over a sheave. and that thereafter the rope at the tangent point behaves as if it were in pure tension. The point of maximum stress has been shown to be at the tangent point, falling off to a minimum at the top of the sheave due to the frictional forces acting. Probably the true point of maximum stress is just slightly above the tangent point, since it takes a short distance for the bending stresses to come into action and the frictional loss is low at this point. The stress distribution in the strand itself has been pointed out, and indications are that the inner wires take considerably higher stresses than the outer wires at ordinary working loads. The beneficial effect of Lang lay rope in reducing stresses in the wires both in tension and in bending is notable, the reduction being proportional to the cosine of the angle which the surface wires make with the axis of the rope, in this case, 37 degrees. A disadvantage of this type of cable, however, lies in its greater tendency to kink and untwist, and the ends should always be rigidly fixed against rotation.

None of the formulas dealing with wire ropes takes preforming into account. It is significant that for the ropes tested the summary of ultimate loads which follows shows that the preformed ropes are about 4% to 5% weaker in straight tension due to the process of manufacture. However, they are shown to develop less loss of strength due to bending, in some instances by quite appreciable amounts. The initial stress distribution among the individual wires of a preformed rope does not seem to be greatly improved over a non-preformed rope, but there is some tendency for re-adjustment and equalization of stress to occur more quickly in a preformed specimen. The modulus of elasticity of the preformed ropes seems to run slightly higher than that of the non-preformed types. The chief advantages to the use of preformed ropes seems to be the case in handling, cutting, and splicing them, the elimination of kinking to a large extent, and the manner in which they tend to remain closed when several wires are broken rather than bristling with jagged ends.

The curves for loss of strength over sheaves are of primary importance, and a new and simpler method of plotting these curves has been shown. The various features of these curves have been discussed previously, but it is worth noting here that Mr. Rairden's (1) curve based on tests of $\frac{5}{8}$-in. wire ropes does not agree with the results herein obtained, although a ratio of the diameters has been used. This leads to the speculation that possibly the ratio of rope to sheave diameter may not be the proper one to use in such a plot, and indicates the need of further experimentation on ropes of different diameters to discover whether results on one diameter can be transformed to another diameter by a simple ratio.

SUMMARY

Three tables are presented to summarize the wire-rope test results and the formulas with which they were compared. The first, Table 8, presents seven bending-stress formulas, and observed values obtained in most cases by three

TABLE 8.—SUMMARY OF BENDING-STRESS FORMULAS (ALL STRESSES IN THOUSANDS OF POUNDS PER SQUARE INCH)

Set No. (see Table 2):	Equation (1)	Equation (2)	Equation (3)	Equation (4)	Equation (5)	Equation (6)	Equation (8)	OBSERVED VALUES		
								Loss of strength divided by A_e	From load-stress curve at breaking point	From plain bending test (tension values)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(a) 18-INCH SHEAVE										
1.....	154.58	139.01	125.01	83.34	68.02	85.66	15.69	11.80	12.3	65.9
2.....	154.58	139.01	125.01	83.34	68.02	94.78	15.69	4.49	14.3	82.1
3.....	154.58	139.01	125.01	83.34	68.02	92.57	14.94	9.00	....	....
4.....	154.58	139.01	125.01	83.34	68.02	81.79	14.94	8.29	5.5	....
9.....	95.69	86.06	77.38	51.59	42.10	47.60	6.91	7.55	19.9	22.5
10.....	95.69	86.06	77.38	51.59	42.10	48.99	6.91	4.52	20.3	28.2
11.....	95.69	86.06	77.38	51.59	42.10	53.03	6.57	5.70*	2.1	....
12.....	95.69	86.06	77.38	51.59	42.10	50.97	6.57	14.78†	12.0†	....
13.....	95.69	86.06	77.38	51.59	42.10	52.20	6.91	15.43	25.0	21.7
(b) 14-INCH SHEAVE										
1.....	198.75	178.73	160.73	107.15	87.45	108.50	19.87	15.57	15.8	81.8
2.....	198.75	178.73	160.73	107.15	87.45	120.05	19.87	10.17	17.6	98.6
3.....	198.75	178.73	160.73	107.15	87.45	117.25	18.92	13.18	13.5	....
4.....	198.75	178.73	160.73	107.15	87.45	103.60	18.92	8.96	7.0	....
9.....	123.04	110.65	99.50	66.34	54.14	60.20	8.75	12.51	21.7	28.4
10.....	123.04	110.65	99.50	66.34	54.14	62.05	8.75	7.64	22.0	36.2
11.....	123.04	110.65	99.50	66.34	54.14	67.17	8.32	8.18*	7.0	....
12.....	123.04	110.65	99.50	66.34	54.14	64.57	8.32	4.80†	5.8†	....
13.....	123.04	110.65	99.50	66.34	54.14	66.13	8.75	17.19	33.5	27.1
(c) 10-INCH SHEAVE										
1.....	278.25	250.22	225.02	150.01	122.43	147.95	27.09	22.95	22.2	104.2
2.....	278.25	250.22	225.02	150.01	122.43	163.70	27.09	13.81	15.1	133.0
3.....	278.25	250.22	225.02	150.01	122.43	159.89	25.80	17.05	15.0	....
4.....	278.25	250.22	225.02	150.01	122.43	141.27	25.80	14.64	15.4	....
9.....	172.25	154.90	139.30	92.87	75.79	82.10	11.94	18.00	23.5	38.4
10.....	172.25	154.90	139.30	92.87	75.79	84.62	11.94	14.81	35.6	50.4
11.....	172.25	154.90	139.30	92.87	75.79	91.59	11.34	10.37*	10.8	....
12.....	172.25	154.90	139.30	92.87	75.79	88.05	11.34	8.99	10.0	....
13.....	172.25	154.90	139.30	92.87	75.79	90.17	11.94	19.54	29.8	35.7
(d) 7-INCH SHEAVE										
1.....	397.50	357.46	321.45	214.30	174.90	203.44	37.25	31.22	28.2	....
2.....	397.50	357.46	321.45	214.30	174.90	225.09	37.25	18.47	24.8	....
3.....	397.50	357.46	321.45	214.30	174.90	219.84	35.48	23.68	15.3	....
4.....	397.50	357.46	321.45	214.30	174.90	194.25	35.48	21.32	23.3	....
9.....	246.07	221.28	198.99	132.66	108.27	112.90	16.41	29.29	31.1	52.5
10.....	246.07	221.28	198.99	132.66	108.27	116.35	16.41	22.45	47.5	72.4
11.....	246.07	221.28	198.99	132.66	108.27	125.94	15.59	17.89*	18.8	....
12.....	246.07	221.28	198.99	132.66	108.27	121.06	15.59	13.94	21.0	....
13.....	246.07	221.28	198.99	132.66	108.27	123.99	16.41	29.50	45.0	47.4

* On basis of ultimate load in tension of 75 000 lb. †Fractured at socket.

different methods, as explained previously. Values that are underlined exceed the ultimate strength of the wire, even with no tensile load applied. The second, Table 9, summarizes the observed values of ultimate load and efficiency in tension as well as those predicted by the three formulas. The third, Table 10, presents similar predicted values of the ultimate load in bending over each of the four sheave sizes by two formulas, and the observed values for comparison.

TABLE 9.—SUMMARY OF PREDICTED AND OBSERVED VALUES OF ULTIMATE LOAD IN TENSION

Set No. (see Table 2):	Equation (10)		Equation (11)	Equation (12)		Observed load, in pounds	Values, efficient (percentage)
	Load, in pounds	Efficient (percentage)		Load, in pounds	Efficient (percentage)		
1	68 240	83.2	63 750*	65 530	79.9	68 350	83.3
2	68 240	83.2	63 750*	65 530	79.9	64 750	78.9
3	68 240	83.2	......	65 530	79.9	69 500	84.7
4	68 240	83.2	......	65 530	79.9	67 400	82.2
9	77 940	85.2	63 750	73 030	79.9	71 900	78.6
10	77 940	85.2	63 750	73 030	79.9	69 800	76.3
11	77 940	85.2	......	73 030	79.9	71 650†	78.5†
12	77 940	85.2	......	73 030	79.9	72 000	78.7
13	87 600	85.2	75 000	82 090	79.9	82 850	80.6

* Using $C = 0.85$ for 6×19 cast-steel ropes.

† Values should be 75 000 lb and 82.1% efficient as explained in test.

Which of the three observed values of bending stress is the most nearly correct is a doubtful question. From the foregoing discussion, the limitations of the plain bending test without tension and the method of extrapolating the load-stress curve to the breaking point are apparent, and the most logical method seems to be the first one presented in Table 8 Column (9)—that of dividing the loss of strength by the product of net area and efficiency in tension. Either on this basis or by striking an average of all three methods. one must exclude all the formulas except Equation (8) as giving values too conservative for use on stationary ropes. Although Equation (8) frequently does not come very close to the observed values, in view of the doubt as to

TABLE 10.—SUMMARY OF PREDICTED AND OBSERVED VALUES OF ULTIMATE LOAD IN BENDING OVER SHEAVES

Set No. (see Table 2):	Equation (9)*:				Equation (8):				Observed Values:			
	Sheave Diameter in Inches:				Sheave Diameter, in Inches:				Sheave Diameter, in Inches:			
	18	14	10	7	18	14	10	7	18	14	10	7
1	41 630	34 520	22 220	4 940	62 480	60 920	58 220	54 420	64 675	63 500	61 200	58 625
2	36 740	29 290	16 400	Minus	58 880	57 320	54 620	50 820	63 425	61 750	60 675	59 300
3	40 150	32 330	18 820	Minus	63 910	62 420	59 850	56 230	66 650	65 325	64 100	62 000
4	42 270	35 570	23 990	7 700	61 810	60 320	57 750	54 130	64 300	64 050	61 925	59 425
9	56 300	52 150	45 000	34 900	69 020	68 250	66 920	65 060	69 425	67 800	66 000	62 300
10	54 600	50 400	43 200	33 000	66 920	66 150	64 820	62 960	68 350	67 350	65 050	62 600
11	56 920	52 080	43 720	31 960	72 260	71 530	70 270	68 500	73 050	72 200	71 450	68 875
12	55 240	50 780	43 070	32 240	69 260	68 530	67 270	65 500	67 150†	70 425†	69 050	67 425
13	66 100	61 400	53 200	41 700	79 970	79 200	77 870	76 010	77 600	77 000	76 200	72 800

* In which $s = \frac{E_r\, d}{D + d_r}$.

† Fractured at socket.

the accuracy of these latter values, the discrepancy is not nearly as large as for any of the other formulas, and Equation (8), although unwieldy, gives results most comparable with the test data.

For the tension test predictions (Table 9), Equation (11) is admittedly based on minimum values, which are low. Equation (10) gives values that are in most cases slightly too high, whereas Equation (12) fits the test data very well on the average. According to this latter formula, all the ropes tested should show efficiencies of 79.9%, whereas the average for the seven ropes was 80.6 per cent. One factor, however, which the equation does not take into account is the slight loss of strength due to preforming.

Ultimate loads in bending could be predicted for any of the formulas listed in Table 8, but the last two only have been selected as giving possibly reasonable values. Mr. Meals states (1) that for his formula (a modification of Mr. Leffler's formula), the error is a maximum for the lower ratios of $\frac{D}{d_r}$. This statement is fully confirmed and values of ultimate load over the 7-in. sheave are far too low, in two cases even being a minus quantity. For large-sized sheaves as recommended in modern practice, Equation (10) tends to approach the observed values, but is consistently conservative. Equation (12), from which the expression for bending stress was derived, might be expected to give equally good predictions for strength, and observation will disclose that predictions based on this formula vary from the observed values in no case by more than 14%, in this case on the safe side. The average error on the unsafe side in only 2.5% and on the safe side, 4.5 per cent.

It should be remarked that all the formulas that have been found herein to vary from the observed data, have varied on the safe side, and that those which best fit the data vary sometimes on the safe side, but almost as often on the unsafe side, although the percentage error is very small in comparison with all the other formulas. Undoubtedly, velocity and reverse bending affect the ultimate load and the bending stress adversely, and until more tests are made of wire rope in motion under load, it is preferable on such installations to err on the safe side in stress computations.

CONCLUSIONS

From a study of the data obtained in this investigation, the following conclusions have been drawn, applying to stationary wire ropes with hemp centers in tension and in bending over sheaves:

(1) The modulus of elasticity of a rope as a whole was increased about 50% by one loading to 50% of the ultimate load, and continued to rise slowly upon further repetitions of the load. Even for ordinary working loads such a rise took place, but the values for modulus were only from 60 to 70% of the values when overloaded by pre-stressing.

(2) For tension tests of regular lay ropes below the proportional limit of the wires, the stress in the outer wires coincided very closely with that obtained by dividing the load by the net area of cross-section.

(3) The variation in stress with load for ropes bent over sheaves was exactly the same as for the same ropes in tension, except that a definite

bending stress, the magnitude of which depended on the sheave size, was added at the time of bending, and this did not vary as the load increased.

(4) The maximum stress in a wire rope bent over a sheave occurred at or immediately above the point of tangency to the sheave, and the rope might be expected to fracture at this point. The minimum stress in the rope occurred at the top of the sheave.

(5) Initial fracture in ropes of 6 × 19 construction, and probably also in the case of those of 6 × 7 construction occurred in the interior wires of the strands, the stress in the core wire being always greater than that in the outer wires up to the point of initial fracture.

(6) The percentage loss of strength of a rope bent over a sheave varied linearly with the ratio of rope diameter to sheave diameter at the root.

(7) The coefficient of friction of a regular lay rope on a steel sheave was roughly 0.14, and of a Lang lay rope, roughly, 0.38.

(8) The stresses in the outer wires of a Lang lay rope were reduced in proportion to the cosine of the angle of inclination with the axis of the rope, a reduction of very nearly 20% for ordinary construction.

(9) A preformed rope showed less loss of strength in bending over sheaves, but also about 5% lower tensile strength and efficiency than a non-preformed rope.

(10) The most satisfactory formulas found for the prediction of bending stress, tensile strength, and loss of strength due to bending, were those presented by Mr. Carstarphen (1).

ACKNOWLEDGMENTS

The testing program was conducted as a co-operative investigation with the Wickwire Spencer Steel Company, which Company furnished all the wire ropes tested and fitted the sockets. The tests were made in the Fritz Engineering Laboratory of Lehigh University, at Bethlehem, Pa., between November 1934, and May, 1935. The writer is indebted to Messrs. A. S. Rairden and Carl King, of the Wickwire Spencer Steel Company, for their valuable aid and suggestions, and to Inge Lyse, M. Am. Soc. C. E., Research Associate Professor of Engineering Materials, for his aid in supervising the tests and interpreting the results. Acknowledgment is also made to the members of the Laboratory Research Staff who have assisted materially in the conducting of these tests. A number of the illustrations used have been loaned by Mr. Rairden.

APPENDIX I

NOTATION

The symbols used in this paper are defined as follows:

a = angle that a helical wire makes with the axis of the strand; a_i = Angle a of the wires in the ith layer; a_n = angle of lay of Wire No. n;

b = angle that a strand makes with the axis of the rope;
d = diameter of a wire in a rope; d_r = diameter of a wire rope;
e = base of Naperian logarithms;
f = coefficient of friction;
g = a subscript denoting "at the point of tangency";
i = a subscript denoting the *i*th layer of strands in a rope;
l = a subscript denoting "layers";
n = number; n_s = number of strands in a rope; n_w = number of wires in a given diameter in the *i*th layer; n_l = number of layers of wires in a strand; as a subscript, n denotes "number";
p = a subscript denoting "at the top";
r = radius of a wire in a rope; r_s = radius from the center of the strand to the center of the wire in question; r_r = radius from the center of a wire rope to the center wire of a strand; as a subscript, r denotes "rope";
s = unit bending stress; as a subscript, s denotes "strand";
t = ultimate unit tensile strength of a wire; as a subscript, t denotes "tension";
w = a subscript denoting "wire";
A = area of a wire rope;
C = a constant representing a relation between the total tension on a wire rope, and its diameter, for various constructions;
D = diameter of sheave;
E = modulus of elasticity of a wire in a rope; E_r = modulus of elasticity of a wire rope;
G = modulus of rigidity in a rope $= \frac{E}{2}(1 + \mu)$;
P = loss of strength in a wire due to bending;
R = radius of a sheave $= \frac{1}{2}(D + d_r)$;
S = ultimate strength of a wire rope in bending; S_i = strength of a wire in the *i*th layer; S_t = strength of a wire rope in tension; S_w = ultimate strength of a wire;
α = angle between the perpendicular to the axis of a rope and the tangent to the center line of a wire;
ε = efficiency of a rope in plain tension;
μ = Poisson's ratio.

APPENDIX II

BIBLIOGRAPHY

(1) **Effect of Bending Wire Rope,** by F. C. Carstarphen, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 98 (1933), p. 562. Discussion by Messrs. C. D. Meals, A. S. Rairden, F. W. Deck, and others.

(2) **Determination of Stresses in Wire Rope as Applied to Modern Engineering Problems,** by James F. Howe, *Transactions,* A. S. M. E.. Vol. 40 (1918), p. 1043.

(3) **Heavy Duty Wire Ropes and Sheaves**, by B. R. Leffler, M. Am. Soc. C. E., *Civil Engineering,* Vol. 1, No. 2, p. 107.

(4) **Six Formulas Compared**, by the late R. C. Strachan, M. Am. Soc. C. E., *Civil Engineering,* Vol. 1, No. 2, p. 111.

(5) **R. W. Chapman**, in *Engineering Review,* October, 1908, Vol. 19, p. 258.

(6) **Strength and Other Properties of Wire Rope**, by J. H. Griffith, M. Am. Soc. C. E., and J. E. Bragg, *Technological Paper No. 121,* National Bureau of Standards.

(7) **Bending Stresses in Wire Rope**, by C. D. Meals, Assoc. M. Am. Soc. C. E., *Transactions,* A. S. M. E., Vol. 51 (1929) II MH–51-5-21.

(8) **Acceleration Stresses in Wire Hoisting Ropes**, by G. P. Boomsliter, M. Am. Soc. C. E., *Transactions,* Am. Inst. of Min. and Met. Engrs., Vol. LXXV (1927), p. 74.

(9) **Some Tests of Steel Wire Rope on Sheaves**, by Edward Skillman, *Technological Paper No. 229,* National Bureau of Standards, Vol. 17.

(10) **The Testing of Rope Wire and Wire Rope**, by A. V. deForest and L. W. Hopkins, *Proceedings,* A. S. T. M., Vol. 32 (1932), Pt. II, p. 398.

DISCUSSION

C. D. **Meals,**[5] Assoc. M. Am. Soc. C. E. (by letter).—Another "link in the chain" has been wrought by Mr. Stewart in his splendid paper pertaining to the strength of stationary wire ropes looped over sheaves. Of the bending stress formulas, Equations (1) to (7) inclusive, it may be noted that Chapman's (5)[3] formula antedated Hardesty's (2)[3] formula; the former was published in 1908 and the latter in 1918; consequently, it should not be implied that Chapman modified Equation (3).

Equation (5) was developed by Josef Hrabak[6] and published in 1902. Hrabak's writings on the subject are frequently ignored, and yet he presented the first logical theory on the subject as compared to the Reuleaux formula in vogue in 1902. Howe's formula, Equation (6), was first published in 1907; although credit is generally given to his 1918 paper (2)[3]. The years 1902 to 1918 saw the publication of many formulas for the calculation of bending stresses in operating wire ropes, and there may have been some justification for the consideration given the subject, as ropes did break before being worn appreciably, which was considered as indicative of abnormal bending stresses.

With the present knowledge of designing wire ropes, it is appreciated that improper proportioning of the wires lead to their premature breaking. If less time had been spent on bending stress theories and more time devoted to the engineering design of the rope, the troubles experienced would have been greatly eliminated.

In discussing bending stresses in wire rope, the author of an article[7] published in 1930, noted that "the intensity of stress due to bending varies inversely as the radius of curvature; consideration of this fundamental fact leads to the conclusion that the wires in contact with the sheave or drum, *i.e.*, those bent to the least radius of curvature, are subjected to the greatest bending stress." An extensive experience in testing moving wire ropes, under load, over one sheave and under another sheave, subjecting the rope to a reverse bending, has verified the foregoing statement.

In a discussion of Leffler's paper (3)[3], the writer noted that "the maximum bending stress [in an operating wire rope] is not necessarily in the outer wires of the strand farthermost from the axis of the rope." With some types of wire ropes and under certain conditions of loading and operation, however, the outer wires of the strand break next to the manila center of the rope where they are not susceptible to inspection.

For the determination of the strength of a wire rope bent over a sheave and subject to a static load, in a recent paper[8], the writer modified Equation (9) as follows,

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[5] Wire Rope Engr., The B. Greening Wire Co., Ltd., Hamilton, Ont., Canada.

[6] "Die Drahtseile", by Josef Hrabak.

[7] "Instructions for the Design of Wire Rope Installations", U. S. Navy Dept., Bureau of Construction and Repair, *Technical Bulletin No. 1*, p. 30.

[8] "Main Cables and Suspenders for Suspension Bridges", by C. D. Meals, Assoc. M. Am. Soc. C. E., *Journal*, Eng. Inst. of Canada, August, 1934.

$$S = k_1 A \epsilon \left\{ t - \frac{E_r D}{D + d_r} \right\} \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which k_1 is a correction factor with the following values:

$\frac{D}{d_r}$	k_1	$\frac{D}{d_r}$	k_1
3	1.150	8	1.080
4	1.135	10	1.055
5	1.120	12	1.03
6	1.105	14	1.00
7	1.095		

and E_r is the modulus of elasticity of the rope as manufactured and as determined by the first run loading on it, the load not to exceed 30% of the strength of the rope.

The use of Equation (14) will increase the values given in Table 10, under the heading, "Equation (9)", and using the first-run modulus values given in Fig. 8. The changes in tabular values are given separately in Table 11. These values show an appreciable advance compared with the data given in Table 10 for Equation (9); if the loadings on the ropes had not been so abnormally high, lower first-run moduli would have resulted, with a corresponding increase in the values of Table 11.

TABLE 11.—STRENGTHS OF ROPES BENT OVER SHEAVES

Set No.	EQUATION (14)			
	SHEAVE DIAMETERS, IN INCHES			
	18	14	10	7
1	53 600	49 800	45 600	37 200
2	47 600	42 200	34 800	21 700
3	50 200	45 500	39 450	28 450
4	51 700	47 400	42 200	32 300
9	66 600	63 700	61 900	56 400
10	66 600	63 700	61 900	56 400
11	65 700	62 500	60 100	53 800
12	66 300	63 300	61 200	55 500
13	74 400	70 800	68 400	61 700

It should be appreciated that Equation (14) is only an approximation, and yet it gives values quite closely in accord with the results of Skillman's (9)[3] and Rairden's (1)[3] series of tests and also with the test results of many 6 × 19 and 6 × 37, steel-center, suspender ropes as used on recent suspension bridges; although, as indicated previously, it is not in as close agreement with Mr. Stewart's test results.

For 6 × 19 ropes with manila centers, the efficiencies of the ropes reported by Mr. Stewart are higher than those of Skillman's (9)[3] tests for 6 × 19 Warrington plow-steel ropes and of Rairden's (1)[3] tests of 6 × 19 filler-wire improved plow-steel ropes, and it appears from a comparison of these three series of tests that the efficiencies may vary for the different types and grades of 6 × 19 ropes and even for ropes of the same type as made by the different manufacturers; consequently, too much reliance must not be placed on any particular formula until more tests are conducted to verify their accuracy, although no brief is being held for Equation (14) as the writer appreciates its limitations.

Equation (10) has been used for a number of years to determine the strengths of special wire ropes and has found to be more accurate than is indicated in Table 9. To verify this statement, tests were made of $\frac{7}{8}$-in. and 1-in. ropes with manila centers, the data pertaining to the ropes being noted in Table 12, and the results of these tests in Table 13.

TABLE 12.—DESCRIPTION OF $\frac{7}{8}$-INCH AND 1-INCH WIRE ROPES

Set No.	Description of ropes	Metallic area, in square inches	ANGLES		
			a	a_s	b
14.....	$\frac{7}{8}$-in., 6 × 7 Lang lay plow-steel, non-preformed	0.31975	14° 15′	12° 13′	15° 13′
15.....	1-in., 6 × 9 filler-wire regular lay cast-steel, non-preformed..............................	0.41268	17° 28′	14° 4′	18° 50′
16.....	1-in., 6 × 19 filler-wire regular lay plow-steel, non-preformed..........................	0.41268	17° 28′	14° 4′	18° 50′
17.....	1-in., 6 × 19 filler-wire regular lay plow-steel, preformed.............................	0.41268	16° 44′	13° 28′	19° 10′

The ropes were tested by the Ontario Department of Mines, in Toronto, Ont., Canada, and the individual wires of the ropes were check-tested by the Steel Company of Canada, Limited, at Hamilton, Ont. It will be seen from Table 13 that Equation (10) does agree quite closely with test values, and it is difficult to reconcile these efficiencies with those noted by Mr. Stewart in Table 9.

TABLE 13.—ACTUAL AND CALCULATED BREAKING STRENGTHS AND EFFICIENCIES OF WIRE ROPES

Set No.	ACTUAL TESTS		EQUATION (10)		EQUATION (12)		EQUATION (16)	
	Load, in pounds	Efficiency (percentage)	Load, in pounds	Efficiency (percentage)	Load, in pounds	Efficiency (percentage)	Load, in pounds	Efficiency (percentage)
14.....	66 050	90.2	65 300	89.1	63 750	87.1	65 000	88.8
15.....	67 285	85.6	67 500	86.0	63 300	80.6	66 000	84.0
16.....	83 425	85.4	83 700	85.7	78 800	80.6	82 000	84.0
17.....	81 225	85.9	81 900	86.6	76 600	81.0	79 600	84.2

One criticism of Equation (12) is that it does not take into consideration the difference in the angles of lay of the various wires in a strand. For example, it will give the same efficiency for a 6 × 19 two-operation strand, a strand having 12 wires laid over 7, as for a 6 × 19 one-operation strand rope as a filler-wire construction; and yet it must be obvious that the efficiency of the latter is greater than that of the former construction. Equation (10) makes this differentiation, whereas Equation (12) does not; also, the latter part of Equation (10) will give the breaking strength of the individual strands of the rope quite accurately.

Equation (12) may be modified to take into consideration the varying lays of wires in the strand by taking a as the average angle of lay of all the wires in the strand, or $a_a = \sum \frac{na}{n}$, which, for a 19-filler wire strand, becomes,

$$a_a = \frac{6\,a_1 + 6\,a_2 + 12\,a_3}{25} \qquad (15)$$

and, accordingly, Equation (12) may be written:

$$S = A\,S_w \cos\,(a_a + b) \qquad (16)$$

Values in accordance with Equation (16) are noted in Table 13.

It is regrettable that certain pitfalls were not avoided in Mr. Stewart's tests inasmuch as they detract somewhat from the value of the paper. Among others, four may be noted, as follows: (*a*) Loadings at 49% of the rope strength for the determination of the modulus of elasticity values; (*b*) a decidedly short gauge length of 10 in. for the measurement of the stretch of the rope under load; (*c*) the use of fixed clamps on the rope; and (*d*) the adoption of ropes with manila centers.

(a).—Loadings at 49% of the Rope Strength for the Determination of the Modulus of Elasticity Values.—Most certainly this is an abnormally high loading and not representative of any engineering or commercial practice pertaining to wire rope; lower loadings are more in keeping with actual practice and would result in lower modulus values. Such abnormally high loads result in a permanent breaking down of the structure of the manila center, nicking of strand against strand, and a high modulus value that is unreal in so far as standard practice is concerned.

(b).—A Decidedly Short Gauge Length of 10 Inches for the Measurement of the Stretch of the Rope Under Load.—It has been general practice in modulus tests of wire ropes, to use a gauge length as long as possible; Skillman (9)[3] used a gauge length of 50 in., and for most of the suspender ropes as used on the large suspension bridges built in recent years, the gauge length has been 80 in. The merit of the longer gauge length is that any irregularities or errors in measurements or in the behavior of the rope are not proportionately of much consequence as they must be in the shorter gauge lengths; those experienced in such testing will appreciate this point.

(c).—The Use of Fixed Clamps on the Rope.—Special swivel clamps have been used that are free to swivel or rotate, and, consequently, to eliminate any twisting of the measuring apparatus due to the untwisting of the rope.

(d).—The Adoption of Ropes with Manila Centers.—That such ropes are used to avoid confusion in the mathematical analyses is appreciated, but they are not as typical as those with an independent wire-rope center (IWRC); particularly for the consideration of the loss in strength due to bending, as this applies to suspender ropes for suspension bridges and such ropes are always made with an independent wire rope center.

It is a fact fairly well known to wire-rope engineers, that the tensile strengths of preformed wire ropes are from 3 to 5% lower than the strengths of non-preformed ropes, but sales policies have conveniently "glossed over" this fact. For the preforming of Lang lay wire ropes, the use of quills as described by Mr. Stewart is not necessary, as the roller head shown in Fig. 4 may be used; in fact, roller heads only are used in the making of all types and diameters of preformed Lang lay wire ropes by the writer's Company.

Relative to the coefficients of friction given in Table 6, it is presumed that these are for ropes that were dry—that is, devoid of any heavy lubricant. Mr. William Hewitt published data[9] pertaining to this subject in 1905, and it may be interesting to compare his values with those of the author.

That the bending stress in a Lang lay rope is approximately 20% less than that in a regular lay rope verifies a statement that the writer[10] made in 1928 regarding such ropes.

Mr. Stewart shows the same confusion as did Carstarphen and Rairden in the paper cited (1)[8] in considering that the loss of strength of a stationary wire rope looped over a sheave is the same as the bending stress in a moving wire rope operating over a sheave; the former is more susceptible of mathematical analyses than the latter. Reasons and examples were cited by the writer in his discussion of Carstarphen's paper (1)[8] to indicate that the latter was not susceptible to such an analysis and surely not to the extent that a "prediction of the bending stress" could be satisfactorily assured, as noted in Conclusion (10) of the paper. It would be a boon to the wire-rope users as well as to the manufacturers if such a prediction was possible.

G. P. BOOMSLITER,[11] M. AM. SOC. C. E. (by letter).—In calling attention to the increase in the modulus of elasticity of a wire rope under successive applications of load, Mr. Stewart has rendered a valuable service. A value of *E* of 18 000 000 lb per sq in. is not too great. As the author has shown, this value is perhaps high for repetitions of load under the proportional limit but, at some time or other during their period of use, most hoisting ropes are stressed beyond the calculated load. The writer is reminded of two such cases in his brief experience with wire ropes.

In one case a mine cage was customarily left all night at the bottom of a 250-ft shaft down which came the fresh air draft to a mine. One cold night the cage froze fast to the floor. It was finally pulled loose by stressing the hoisting rope, but the rope was 8 ft longer after pulling it loose than it was before. In another case, the circuit breaker on a hoist went out as a load of coal was being lifted in a shaft. A telemeter attached to the hoisting rope immediately above the cage showed that the action of the safety devices in stopping the cage caused stresses which were 2.29 times the dead load stresses. Many other conditions result in occasional applications of high stress to a rope so that after a short period of service its modulus of elasticity has been increased beyond that of a new rope. Indeed, calculations made in the telemeter test referred to, indicated a modulus of elasticity for the rope there tested of between 19 000 000 and 20 000 000 lb per sq in.

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[9] "Elements of Machine Design", by O. A. Leutwiler.

[10] "Aerial Tramways", by F. S. Carstarphen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 964.

[11] Prof. of Mechanics, West Virginia Univ., Morgantown, W. Va.

Mr. Stewart deserves congratulations for his clever method of attaching the tensometers to his wire rope when it was bent about a sheave. He has pointed the way for further investigations of bending stress in wire rope. However, the results of bending tests such as those of this paper are likely to be misleading. Undoubtedly, they determine the stresses due to bending a wire rope about a thimble, or in a stationary rope bent over a sheave while in an unstressed condition, since there is no constraint as the wires adjust themselves to the curved position about the sheave, but a heavily loaded rope running over a sheave will have other stresses which the author has not considered. These stresses are due to the frictional resistance to sliding of the wires upon each other when the loaded straight rope bends about the sheave. To illustrate, consider an axial load of 30 000 lb on one of Mr. Stewart's 1-in. ropes of regular lay as it passes over a sheave. Each strand of 19 wires will be assumed to take one-sixth of the load, or 5 000 lb.

The lay length of the strand will be taken as 93 d and d as $\frac{d_r}{15}$, in which d_r = the diameter of the rope; and d the diameter of the individual wires. The lay length will then be $\frac{93}{15}$, or 6.2 in. and the length of half a lay will be 3.1 in. The lay angle is 18° 39′. The component of stress in a strand normal to the axis is 5 000 tan 18° 39′ = 1 688 lb.

Let Fig. 18(b) represent a half lay length of the strand. Fig. 18(a) shows the components of the stresses at the two ends of the length which are normal to the strand length. These components are held in equilibrium by pressures of the other strands, assumed normal to the strand in question. By analogy with the pressures and tensions in a cylindrical vessel, the total lateral pressure in a length of a half lay will be 2 × 1 688 = 3 375 lb. Now, assume that the lower end of this length is in contact with the sheave. The upper end will be at the outside of the rope. The part in contact with the sheave will shorten and the part at the outside of the rope will lengthen. This is done by the slipping of the strand on its neighbors, but this slipping will be done against a friction. Assuming a coefficient of friction of 0.15, the frictional force set up to oppose this motion in a half lay length will be 506 lb. This force will be a measure of the difference between the stress in this

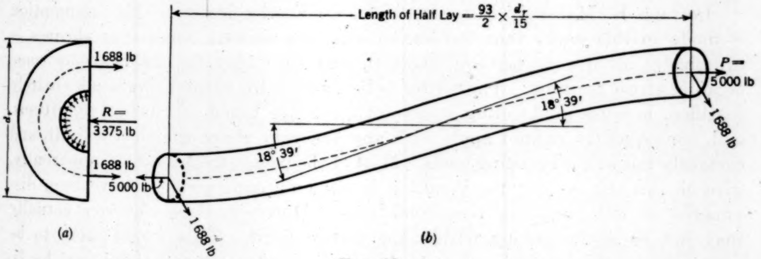

FIG. 18.

strand at contact with the sheave and at the outside of the rope. Note that this is 10.3% of the axial stress in the strand.

If the normal length of the rope is maintained at its line of contact with the sheave, all this frictional restraint (506 lb), will measure the increase in strand tension at the outside of the rope. If there is slip on the sheave the normal length of the rope is maintained along a line somewhere between the sheave and the outside of the rope. Assuming that this line coincides with the rope center, the stress at the inside is decreased and that at the outside increased, each by one-half the frictional restraint, or 253 lb. The area of the wire in one strand, as taken from Table 3 of the paper, would be 0.0694 sq in. The unit stress due to direct load would be 72 000 lb per sq in., and the frictional bending stress according to the first assumption would be 7 490 lb per sq in., and 3 745 lb per sq in., according to the second. These stresses, of course, are in addition to the stresses due to flexural bending. They would be independent of the ratio of the sheave diameter to the rope diameter. The formula expressing this stress would be:

$$s_f = 2\, s_R \tan \alpha\, f \quad\text{.......................}(17)$$

in which s_R is the axial unit stress in the rope; α is the angle of lay; and f is the coefficient of friction between strands. Since the same condition exists between the wires in a strand, Equation (17) is decidedly approximate and is given simply to indicate the effect of frictional resistance on sliding. The coefficient of friction is also assumed. Only further tests will indicate what it actually is, but the writer is firmly of the opinion that tests such as those presented by Mr. Stewart are likely to be misleading if assumed for a heavily stressed rope passing over a sheave. If a rope were rusted so that it could not slip, it would bend as a unit, of course, and Equation (7) would be a proper formula for bending stress. If it were so well lubricated that the friction was that of an oil layer on oil, this stress could be neglected. It is very doubtful whether this last condition would exist in a rope under heavy service. It is more likely that neglect would make f more than 0.15.

INGVALD E. MADSEN,[12] JUN. AM. SOC. C. E. (by letter).—The assumption is made in this paper that the loss of strength of wire ropes over sheaves is practically all due to bending stresses, and thus may be evaluated by some bending stress formula. The writer believes that this assumption is not entirely justified, because all bending stress formulas are based on elastic conditions, and, consequently, cannot apply with any accuracy above the elastic limit, and certainly not at the breaking loads. Most of the usual stress theories, apparently, give absurd stresses for the breaking loads, and, consequently, have been disparaged as not picturing true conditions. However, these theories actually may not be so far wrong within the elastic limit. This would seem to be borne out by the breaking of ropes in service under comparatively low loads, because the bending of the rope over sheaves causes stresses far above the fatigue limit, and it takes a relatively small number of repetitions of load to cause some of the wires to break.

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[12] Apprentice Engr., M. of W., P. R. R., Pittsburgh, Pa.

The same conditions are present in the usual structural research problems. In order to determine the stresses in a structure, experimentally, the strains are measured and multiplied by the modulus of elasticity of the material (usually 30 000 000 lb per sq in. for steel). However, if the material has exceeded its yield point, large strains are present, and if the method is then applied, calculated stresses result which are far greater than the breaking stress for the material. Analogously, the same thing occurs when the stress theories for bending in wire rope, are extrapolated to the breaking loads.

Actually, when a wire is bent over a sheave, large bending stresses occur at first; but as soon as the yield point of the wire is reached the wire yields, resulting in a re-adjustment of stress. Since practically all the wires used in cables have no definite yield point, this is a gradual process.

There are several other causes that will weaken a rope when tested over a sheave. One of the most important is the nicking effect which occurs between the outside wires of the strands. As the strands are wrapped around the center core, the outside wires of the strands bear on each other, and since these individual wires cross each other at a fairly sharp angle on the inside of the strand, they nick each other as the rope compresses under load. These nicks reduce the cross-sectional area of the individual wires, and, consequently, weaken the wire. When a rope is bent over a sheave, the nicking effect is more pronounced, since the bottom of the rope bears on the sheave, and the normal force between the wires will be augmented by the radial force of the rope on the sheave and its reaction. This radial force[13] is equal to the tension in the rope divided by the radius, or, in other words, is inversely proportional to the diameter of the rope. No nicks of any magnitude occur between the wires of the same strand, since these wires cross each other at a small angle, and have a long bearing surface on each other.

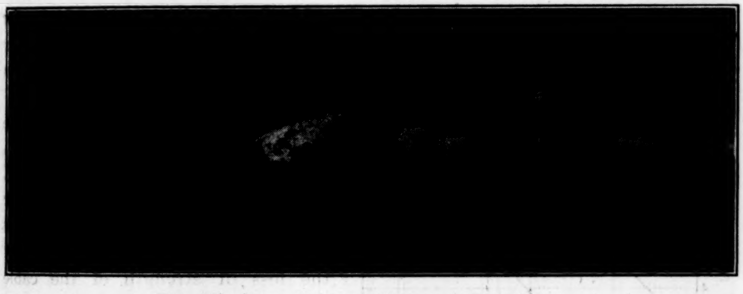

Fig. 19.—View Showing Nicks in Wire Rope.

Thus, the wires which will break first are not the inside wires as has been usually assumed, but the outside wires of the strands which break at the nicks formed by the crossing wires. These breaks are not visible to the eye, and were determined only by unraveling several cables which had broken over a sheave at one of the tangent points. When this was done, it was discovered that a large number of the outside wires of the strands, and only these, were already broken at the other tangent point, showing that these wires were the first to break.

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[13] "Applied Mechanics", by C. E. Fuller and W. A. Johnston, Vol. 1, p. 268.

The writer continued Mr. Stewart's work, and tested 1-in., 6 × 21, 1-in. 6 × 19 Seale, and 1-in. 6 × 19 Warrington, regular lay, non-preformed cables, with just about the same results obtained by the author. However, it was felt that if the wires were nicked by testing them over sheaves, these wires, of course, would be weakened permanently, which would be revealed in a simple tension test. After the several cables had been tested in tension, and over the 18, 14, 10, and 7-in. sheaves, they were unraveled, and ten individual wires of each size in the rope were taken from the part of the rope which lay over the sheave. These wires were carefully straightened by hand. Nicks were noticeable in all the outside wires, and a view of a typical nick is shown in Fig. 19. The reduction in cross-section is clearly shown. On the inside wires, there were no appreciable nicks and only a very small decrease in strength was revealed in the tension tests.

The nicks in the outer wires were larger in the specimens broken over the smaller sheaves, as would be expected, and, consequently, these wires were weaker than those taken from the cables which had been tested over the larger sheaves. This loss in strength of individual wires (diameter, 0.0795 in.) is shown by Curve *A*, Fig. 20. A graph of the loss in strength of the cables (cast-steel, regular lay, non-preformed) over the various sheaves is shown by Curve *B*, Fig. 20, and it is seen that the loss of strength of the cable over the various sheaves, and the loss of strength due to nicking are quite similar. The results shown, are for the 1-in. 6 × 19 Seale, and the results for the other cables were essentially the same. These results show that a large proportion of the loss of strength in cables is due to the nicking effect.

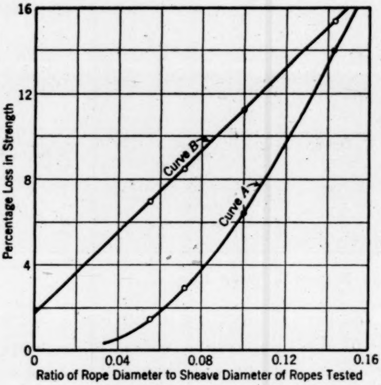

FIG. 20.

The occurrence of these nicks is an added explanation of the failure of cables running over sheaves at fairly low loads. Under the repetition of loads, the wires chafe on each other, enlarging the nicks and reducing the strength until the wire breaks, and this effect is much more pronounced when reverse bends are present.

Stress-strain curves were also drawn of the wires from the broken cables. For some reason, which is most likely the change in cross-section of the individual wires due to abrasion and squeezing in the cable, the stress-strain relationship of the wire seems to change during the testing of the rope. In

Fig. 21 are shown the stress-strain curves for a single wire in the rope. The dotted line is obtained from the wires which went into the rope. The full line is obtained from individual wires taken from a broken rope, and this is the curve that should be used in transposing strain data to stress data. If this is done, the load stress curves in the author's Fig. 11, will approach the load-divided-by-net-area line. Similar curves demonstrating this fact for the 1-in., 6 × 19 Seale are shown in Fig. 22. Curve *E* is obtained by using the dotted line in Fig. 21. This curve is seen to be similar to the 1-in. 6 × 7 regular lay, shown in Fig. 11. Curve *C,* in Fig. 22, is obtained by using the solid line in Fig. 21, and, at the breaking load, the stress in the wires is about equal to the average stress. This is what one would expect, since the plastic flow of the wires above the yield point, re-adjusts the stress throughout the various wires so that they are nearly equally stressed, and at the breaking load they have the same stress.

This condition is contrary to the author's theoretical stress distribution shown in Fig. 15. The writer believes that this distribution is in error for several reasons: First, the wrong stress-strain diagram for the single wires was used; second, the instruments used to measure strain, measured it along the chord of the curved wire, and not the actual strain; and, third, the stress

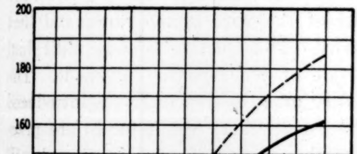

FIG. 21.—AVERAGE STRESS-STRAIN CURVE FOR SINGLE WIRE; 0.0795 INCH IN DIAMETER, CAST-STEEL SPECIMENS 1 to 10.

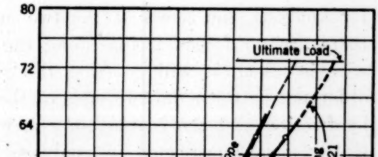

FIG. 22.—LOAD STRESS CURVE FOR TENSION TEST, 1-INCH, 6 x 19 SEALE, CAST STEEL, REGULAR, NON-PREFORMED.

in the outer wire is balanced against the assumed stress in the core wire, whereas actually there are enough more outside wires than core wires so that the average stress in the cable is not far from the stress in the outer wires. If these factors were taken into account, the stress in all the wires would be seen to be equal to the average stress as shown in Fig. 22.

A few more remarks on the general behavior of wire ropes may be appropriate. A wire rope may be considered as a single solid bar, or an assembly of individual wires, each acting separately. If thought of as a bar, the usual theories for curved beams would apply. Actually, the behavior of a wire rope is somewhere between these two conditions, the exact degree depending on the relative friction between the wires, the lubrication of the rope, the resistance of the individual wires to abrasion, and similar factors. The indeterminacy and the variation in these factors would tend to remove wire ropes from the fields of mathematical analysis. All bending stress theories are based on the assumption that the stress of a rope over a sheave varies from a maximum at the outside of the rope to a lesser stress on the sheave; and yet these theories assume that if an individual wire is taken out as a free body for analysis, the stress at the two ends of the wire is the same, and that there are no forces acting along the wire. If a section of any individual wire is taken, it will go from the outside of the rope to the inside. The difference between the two ends of the wire must be due to forces introduced by friction and the bearing of one wire on another, which forces are practically always neglected in analysis, but they cannot be neglected and still give a true picture of what actually occurs in the rope.

Douglas M. Stewart,[14] Jun. Am. Soc. C. E. (by letter).—Various discussers of this paper have stated certain of the limitations in the testing and analysis of stationary wire ropes in bending over sheaves. For these contributions the writer is especially grateful.

Mr. Meals has assisted in clearing up the chronology of the various formulas for bending stress listed. His previous conclusion that the maximum bending stress does not necessarily lie in the outer wires of the strand was supported in the paper. Equations (14) and (16) give values much more consistent with the test results obtained than those considered in the paper, and indicate that for sheaves of a diameter, such as that normally used in practice, Equation (14) can be used safely and will give fairly accurate predictions of strength.

Loadings of the tensile specimens too close to 50% of the ultimate strength, in determining the modulus of elasticity, were admittedly abnormally high when considered in the light of conventional factors of safety; and yet, as pointed out by Professor Boomsliter, overloads of this amount are frequently experienced in ordinary hoisting ropes, and when pre-stressing suspender cables for bridges to raise the modulus, such loads are commonly used. As shown by Set No. 11, in the ordinary range of working loads, a value of

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[14] Engr., Ingersoll-Rand Co., New York, N. Y.

$E_r = 10\,000\,000$ lb per sq in. may be expected after a few loadings, and it is a matter for further investigation to determine whether such a rope in normal service will attain modulus values comparable with those tested.

The general consistency of test results for each specimen, and the agreement between measurements made on opposite sides of the specimen, indicate that a 10-in. gage length was adequate, although a greater length is undoubtedly to be preferred when physical limitations permit. On the specimens on which six 10-in gage lengths were used, a difference of only 3% in modulus between those nearest the sockets and those at the center shows this consistency, and the writer believed that accuracy of this order was within the experimental limits of such testing. Special swivel clamps would no doubt have been of assistance in eliminating twisting of the measuring apparatus, but corrections for twist using the transit and scale were of very small magnitude except for loads near the breaking point. The use of ropes with hemp centers for these tests was dictated by practical considerations, and the question of whether this type, or that with an independent wire-rope center, is the more typical seems debatable. An extended research into the properties of such ropes would clarify many questionable points in the analysis.

Professor Boomsliter has stated clearly one of the chief causes of variation in any mathematical analysis of stresses in a wire rope. There can be no question that forces of the nature described actually exist in a rope, but since so many assumptions must be made in considering them, such frictional stresses are usually neglected. That these forces may have an appreciable effect on the strength of a rope, in certain cases, is shown in the numerical example considered. Variation in the quantity of lubricant used in the ropes was not attempted; all tests were made on new commercial ropes and represent current practice in this respect. Mr. Madsen makes mention of such stresses in the last paragraph of his discussion, but the very fact that strengths in both bending and tension may be predicted with fair accuracy by the formulas given, whether empirical or otherwise, indicates that too careful consideration is not justified.

Mr. Madsen has had the opportunity of carrying on the writer's work on various other types of wire rope specimens, and his conclusions merit careful consideration. It is to be regretted that complete results of his tests were not available for his discussion.

Specifically referring to Mr. Madsen's discussion, it is stated that the usual stress theories, as represnted by Equations (1) to (7), inclusive, may not be far from wrong below the elastic limit. These formulas actually indicate that the elastic limit (if a wire rope may be said to possess such a property) is reached long before the point shown by actual tests. Furthermore, breaking of stationary ropes in service under low loads almost never occurs, although no one will deny that fatigue will cause breakage in moving ropes at values of load far less than that given by any of these formulas. Since the bending stress is constant regardless of the load on the rope, it is obvious that if exceptionally large stresses are indicated by such formulas at the breaking load, the error will be even greater proportionally at lower values of the direct stress.

The nicking effect of one wire on another has been emphasized clearly, and the reduction in area (and, hence, in ultimate load) is appreciable. Although care must be taken in interpreting tests on specimens of this nature after being stressed almost to failure, it is apparent that the loss in the strength of the rope must be affected by this nicking of certain of the outer wires. In any cross-section, however, the number of such wires is not large, being limited to those in the outer layer of a strand where it touches another strand. The use of a stress-strain curve for a wire that has been loaded practically to failure, over the entire range of a tensile test, does not seem to be justified. From Fig. 20, the percentage loss in strength of the wire in a tensile test is very close to zero, which means that, at failure, the wire should show an ultimate strength of about 219 000 lb per sq in. This strength is very close to that obtained in Curve *E* of Fig. 22, whereas Curve *C* shows a stress of only about 185 000 lb per sq in. at failure, or about 15% lower strength. Fig. 21 cannot be based on wires taken from a tension specimen and still be consistent with Fig. 20, and values taken from such a curve based on nicked wires from a bending specimen (which themselves are not typical of conditions prevailing over the cross-section) are of little help in correcting Fig. 22.

The correct stress-strain curve of the wire to use would be one based on a gradual nicking, such as takes place in the rope, but lacking this, the original stress-strain curve seems to give the best results. The difference in length between the chord length measured in a $\frac{1}{2}$-in. gage length on the wire and the actual center-line length is so small that it may safely be neglected. That the stress in the inner wires is greater than the average over the cross-section, is appreciated by wire rope manufacturers, who make the inner wires slightly larger to withstand this effect.

The need for more extensive research in this field, to explain the apparent inconsistencies of various experiments has been mentioned previously, and is emphasized by the discussions of this paper.

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VARIED FLOW IN OPEN CHANNELS OF ADVERSE SLOPE

By Arthur E. Matzke[1], Jun. Am. Soc. C. E.

With Discussion by Messrs H. E. von Bergen, W. E. Howland, Arno T. Lenz, J. C. Stevens, F. T. Mavis, Hunter Rouse and Merit P. White, and Arthur E. Matzke.

Synopsis

A canal may be said to possess an adverse slope, if its bottom rises in the direction of the flow. When, as in the usual case, the bottom falls in the direction of flow, a channel may be said to possess a sustaining slope. An example of a layout with an adverse slope is a canal connecting a tidal basin, or an area to be drained, with the sea. Another case is the approach to a low spillway with a bed gently sloping over a considerable length. Although cases of this kind occur rather infrequently, there are instances in which a knowledge of the flow and methods of numerical computations is essential.

The general equations for varied flow in channels of adverse slope were originally developed by B. A. Bakhmeteff, M. Am. Soc. C. E., about 1910, the method of treatment being in general similar to that which he presented[2] in 1932. Practical application in engineering design, however, depends on knowing the numerical values of a function, similar to the "varied flow function", for canals of sustaining slopes.[3] The required function values were determined by graphical integration and are offered in the form of appropriate tables, together with a brief survey of the underlying theory and some numerical examples making clear the methods of practical applications.

General Equation of Flow in Canals of Adverse Slope

In general, the reasoning leading to the establishment of the general equation of flow in canals of adverse slope follows that developed by Professor

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Note.—Published in February, 1936, *Proceedings*.

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[2] "Hydraulics of Open Channels", Eng. Societies Monograph, McGraw-Hill Co., 1932.

[3] *Loc. cit.*, pp. 308–311.

Bakhmeteff[4] for the case of sustaining slope. Since the derivation of the varied flow equation for canals of adverse slope is practically unknown, an outline of the development will be presented herein. The symbols used are defined as they occur and are summarized in the Appendix for convenience of reference.

With reference to Fig. 1(*a*), a discharge, Q, in cubic feet per second, is assumed to flow in a canal of given cross-section. The adverse bottom slope is denoted by the negative sign, thus, $-S_0$. The discharge is referred to a parametric depth value, y_0, which is the depth of uniform flow for the same discharge in an identical canal with a sustaining slope, S_0. The variable depth of flow is denoted by y, and x is a distance measured in the direction of flow. Therefore, referring to Fig. 1(*b*), the slope of the water surface is:

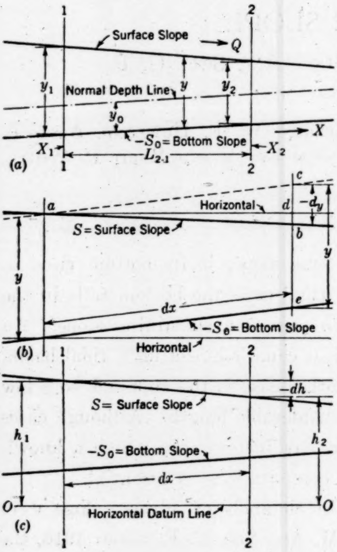
1
2
Surface Slope
Q
y_1
Normal Depth Line
y
y_2
y_0
$-S_0$= Bottom Slope
X
X_1
$L_{2\cdot1}$
X_2
(a)
1
2
c
a
Horizontal
d
$-d_y$
S= Surface Slope
b
y
y
dx
e
$-S_0$= Bottom Slope
Horizontal
(b)
1
2
$-dh$
S= Surface Slope
h_1
h_2
$-S_0$= Bottom Slope
dx
O
O
Horizontal Datum Line
(c)
1
2

FIG. 1.

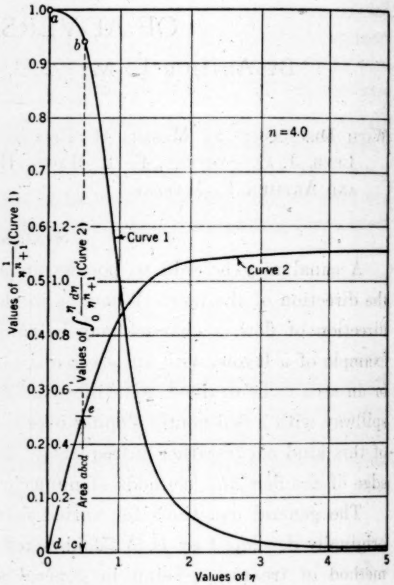

FIG. 2.

$$S = \sin \alpha = \frac{\overline{bd}}{\overline{ab}} = \frac{\overline{bc} - \overline{cd}}{\overline{ab}} = -\frac{dy}{dx} - S_0 \quad \text{..........(1)}$$

Referring to Fig. 1(*c*) and bearing Equation (1) in mind, the loss of energy head, de_r, over the distance, dx, becomes[4]:

$$\frac{de_r}{dx} = -\left(S_0 + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2\,g}\right)\right) \quad \text{..............(2)}$$

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[4] "Hydraulics of Open Channels", Paragraph 12, Eng. Societies Monograph, McGraw-Hill Co., 1932.

Whereas[5], $\frac{de_r}{dx} = \frac{Q^2}{K^2(y)}$, in which $K(y)$ is the value of the conveyance or carrying capacity of a given canal cross-section for any surface depth, y, and[6] $\frac{d}{dx}\left(\frac{V^2}{2\,g}\right) = -\frac{Q^2}{g} \times \frac{b}{A^3} \times \frac{dy}{dx}$ in which A is the cross-sectional area of flow and b is the top width of the cross-sectional area of flow. Therefore, Equation (2) becomes:

$$-S_0 - \frac{dy}{dx} = \frac{Q^2}{K^2(y)} - \frac{Q^2}{g} \times \frac{b}{A^3} \times \frac{dy}{dx} \qquad (3)$$

Using the parametric depth, y_0[7], explained previously, and the hydraulic exponent, n[8], it follows that $\frac{Q^2}{K^2(y)} = S_0 \left(\frac{y_0}{y}\right)^n$. Furthermore, as $\frac{Q^2}{g} \times \frac{b}{A^3}$ in Equation (3) may be expressed as being equal to $\beta \frac{K^2(y_0)}{K^2(y)}$ in which $\beta = \frac{S_0}{S_c}$[9] and S_c[10] is the critical slope at the varying depth, y, Equation (3) may be written as follows:

$$\frac{dy}{dx} = -S_0 \frac{1 + \left(\frac{K_0}{K}\right)^2}{1 - \beta\left(\frac{K_0}{K}\right)^2} = -S_0 \frac{1 + \left(\frac{y_0}{y}\right)^n}{1 - \beta\left(\frac{y_0}{y}\right)^n} \qquad (4)$$

Equation (4) is the general differential equation of varied flow in canals with adverse slope. Separating the variables and designating $\eta = \frac{y}{y_0}$ (that is, $dy = y_0\, d\,\eta$), Equation (4) becomes:

$$\frac{S_0}{y_0}\, dx = -d\,\eta + (1 + \beta)\,\frac{d\,\eta}{\eta^n + 1} \qquad (5)$$

The distance, $x_2 - x_1 = l_{2-1}$, between two sections with the respective depths, y_2 and y_1, which corresponds to $\eta_2 = \frac{y_2}{y_0}$ and $\eta_1 = \frac{y_1}{y_0}$, is obtained by integrating Equation (5); thus:

$$\frac{S_0}{y_0}\left(x_2 - x_1\right) = \frac{S_0}{y_0}\, l_{2-1} = -(\eta_2 - \eta_1) + \int_{\eta_1}^{\eta_2} (1 + \beta)\,\frac{d\,\eta}{\eta^n + 1} \qquad (6)$$

Assuming that the value of β may be taken as a constant average value for the range of integration, and designating $\int_0^{\eta} \frac{d\,\eta}{\eta^n + 1} = B'(\eta)$ (the

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[5] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, McGraw-Hill Co., 1932, Equation (14), p. 26.

[6] *Loc. cit.*, Equation (20), p. 31.

[7] *Loc. cit.*, p. 22.

[8] *Loc. cit.*, p. 84.

[9] *Loc. cit.*, p. 52.

[10] *Loc. cit.*, p. 47.

"varied flow function" for canals of adverse slope) Equation (6) becomes:

$$l_{2-1} = \frac{y_0}{S_0}\left[-(\eta_2 - \eta_1) + (1 + \beta)\left\{B'(\eta_2) - B'(\eta_1)\right\}\right] \ldots\ldots(7)$$

which is the general equation of varied flow for channels of adverse slope.

For comparison, in the case of sustaining bottom slope:

$$S = S_0 - \frac{dy}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

the varied flow equation is[11]:

$$l_{2-1} = \frac{y_0}{S_0}\left\{(\eta_2 - \eta_1) - (1 - \beta)\left[B(\eta_2) - B(\eta_1)\right]\right\} \ldots\ldots(9)$$

In other words, the formulas for adverse flow (Equations (1) and (7)) differ from those for sustaining flow (Equations (8) and (9)) only in the signs and in the form of the varied flow function, the essential structure of both being similar.

INTEGRATION PROCEDURE

For practical computations it is imperative to know the value of the varied flow function,

$$B'(\eta) = \int_0^{\eta} \frac{d\eta}{\eta^n + 1} \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

within the practical range of the hydraulic exponents, which usually lies between $n = 3$ and $n = 4$. For the particular value of $n = 3$ and $n = 4$, $B'(\eta)$ may be obtained analytically. The forms of the respective quadratures are:

For $n = 3$, substitute $z = -\eta$ so that $\int \frac{d\eta}{\eta^3 + 1} = \int \frac{dz}{z^3 - 1}$. Then,

$$\int \frac{dz}{z^3 - 1} = \frac{1}{6}\log_e \frac{(z - 1)^2}{z^2 + z + 1} + \frac{1}{\sqrt{3}}\operatorname{arccot}\frac{2z + 1}{\sqrt{3}} \ldots\ldots(11)$$

For $n = 4$,

$$\int \frac{d\eta}{\eta^4 + 1} = \frac{1}{4\sqrt{2}}\log_e\left(\frac{\eta^2 + \eta\sqrt{2} + 1}{\eta^2 - \eta\sqrt{2} + 1}\right) + \frac{1}{2\sqrt{2}}\left[\arctan\left\{\eta\sqrt{2} + 1\right\} + \arctan\left\{\eta\sqrt{2} - 1\right\}\right] \ldots\ldots(12)$$

For intermediate values of n, one could take recourse to computation by series, but this is a laborious and lengthy procedure scarcely warranted by the character of the problem. It seemed that results sufficiently accurate for engineering practice might be obtained by graphical integration. The method is illustrated by Fig. 2 which gives the curves for the particular case of $n = 4$. For values of η, the corresponding values of $\frac{1}{\eta^n + 1}$ are found, which when plotted against η give Curve 1.

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[11] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, Equation (84), p. 87.

For each η-ordinate, the value of $\int_0^\eta \frac{d\eta}{\eta^n + 1}$ is the numerical value of the area under the curve to the left of that ordinate. These areas may be computed by a planimeter or simply by counting squares. Curve 2 is drawn so as to represent for each η-ordinate, the corresponding area to an appropriate scale as Ordinate *ce* = Area *abcd*. The writer compared values of $B'(\eta)$ determined graphically for $n = 3$ and $n = 4$ with those computed analytically by substituting numerical values for η in Equation (11) and Equation (12), respectively (see Table 1). In no case did the discrepancy exceed 0.33 per cent.

TABLE 1.—COMPARISON OF GRAPHICAL AND ANALYTICAL VALUES OF INTEGRAL

$$\int_0^\eta \frac{d\,\eta}{\eta^n + 1}$$

Values of η	VALUES OF THE INTEGRAL		Percentage of error
	Graphical	Analytical	
0.500............	0.4999	0.4940	—0.02
2.500............	1.0880	1.0898	—0.16
5.000............	1.1045	1.1081	—0.33

TABLE 2.—VALUES OF THE VARIED FLOW FUNCTION, $B'(\eta)$, FOR CANALS OF ADVERSE SLOPE

Values of η	HYDRAULIC EXPONENTS, n:					
	3.0	3.2	3.4	3.6	3.8	4.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.00	0.000	0.000	0.000	0.000	0.000	0.000
0.05	0.050	0.050	0.050	0.050	0.050	0.050
0.10	0.100	0.100	0.100	0.100	0,100	0.100
0.15	0.150	0.150	0.150	0.150	0.150	0.150
0.20	0.200	0.200	0.200	0.200	0.200	0.200
0.25	0.249	0.249	0.249	0.250	0.250	0.250
0.30	0.298	0.299	0.299	0.300	0.301	0.302
0.35	0.345	0.346	0.347	0.348	0.349	0.350
0.40	0.393	0.394	0.396	0.397	0.398	0.399
0.45	0.439	0.440	0.441	0.443	0.445	0.447
0.50	0.485	0.488	0.490	0.491	0.493	0.494
0.55	0.529	0.532	0.535	0.537	0.539	0.540
0.60	0.572	0.575	0.578	0.581	0.584	0.586
0.65	0.611	0.615	0.620	0.624	0.628	0.630
0.70	0.650	0.655	0.661	0.665	0.668	0.672
0.75	0.685	0.692	0.697	0.700	0.706	0.709
0.80	0.719	0.726	0.732	0.737	0.742	0.746
0.85	0.752	0.758	0.764	0.770	0.775	0.780
0.90	0.782	0.789	0.796	0.802	0.807	0.812
0.95	0.809	0.816	0.822	0.829	0.835	0.840
1.00	0.836	0.843	0.850	0.854	0.862	0.866
1.05	0.859	0.867	0.873	0.878	0.885	0.890
1.10	0.881	0.889	0.896	0.901	0.907	0.911
1.15	0.901	0.908	0.915	0.918	0.925	0.928
1.20	0.920	0.927	0.935	0.938	0.944	0.948
1.25	0.938	0.945	0.950	0.955	0.958	0.963
1.30	0.955	0.960	0.964	0.968	0.973	0.976
1.35	0.971	0.975	0.978	0.982	0.986	0.989
1.40	0.985	0.988	0.991	0.994	0.997	1,000
1.45	0.997	1.000	1.003	1.006	1.008	1.010
1.50	1.010	1.012	1.014	1.015	1.017	1.019
1.55	1.020	1.022	1.023	1.024	1.025	1.027
1.60	1.028	1.030	1.031	1.032	1.033	1.034
1.65	1.038	1.039	1.040	1.040	1.041	1.041
1.70	1.048	1.048	1.047	1.047	1.046	1.046
1.75	1.057	1.056	1.055	1.054	1.053	1.052
1.80	1.064	1.062	1.061	1.060	1.059	1.056
1.85	1.071	1.068	1.066	1.064	1.062	1.060
1.90	1.078	1.075	1.072	1.069	1.066	1.063
1.95	1.083	1.080	1.077	1.073	1.070	1.066
2.00	1.090	1.086	1.083	1.076	1.074	1.070
2.10	1.100	1.095	1.090	1.082	1.079	1.075
2.20	1.109	1.102	1.097	1.088	1.085	1.080
2.30	1.117	1.110	1.102	1.093	1.090	1.084
2.40	1.124	1.116	1.108	1.097	1.094	1.087
2.50	1.131	1.121	1.113	1.101	1.097	1.090
2.60	1.136	1.126	1.116	1.104	1.100	1.092
2.70	1.141	1.130	1.120	1.108	1.102	1.094
2.80	1.146	1.134	1.124	1.110	1.104	1.096
2.90	1.150	1.137	1.126	1.112	1.106	1.097
3.00	1.154	1.140	1.128	1.114	1.108	1.099
3.10	1.158	1.143	1.131	1.116	1.109	1.100
3.20	1.161	1.145	1.133	1.118	1.110	1.100
3.30	1.164	1.147	1.134	1.119	1.111	1.101
3.40	1.166	1.150	1.136	1.121	1.112	1.102
3.50	1.169	1.151	1.137	1.122	1.113	1.103
3.60	1.171	1.153	1.139	1.123	1.114	1.104
3.70	1.173	1.155	1.140	1.124	1.115	1.104
3.80	1.175	1.156	1.141	1.125	1.116	1.105
3.90	1.177	1.157	1.142	1.126	1.116	1.105
4.00	1.178	1.159	1.143	1.126	1.117	1.105
4.10	1.180	1.160	1.144	1.127	1.118	1.106
4.20	1.181	1.161	1.145	1.128	1.118	1.107
4.30	1.183	1.162	1.145	1.128	1.119	1.107
4.40	1.184	1.163	1.146	1.129	1.119	1.107
4.50	1.185	1.164	1.147	1.129	1.119	1.107
4.60	1.186	1.165	1.147	1.130	1.120	1.108
4.70	1.187	1.165	1.148	1.130	1.120	1.108
4.80	1.188	1.166	1.148	1.130	1.120	1.108
4.90	1.188	1.167	1.149	1.131	1.120	1.108
5.00	1.189	1.167	1.149	1.131	1.120	1.108
...	....	....	....	....	....	....

Curves were made then for intermediary exponents at intervals of 0.2 and the results summarized in Table 2. Values of $B'(\eta)$ for exponents between the tabular values can either be taken by interpolation or, if desired, in each particular case, a special curve of $\frac{1}{\eta^n+1}$ and $\int_0^\eta \frac{d\eta}{\eta^n+1}$ may be computed and traced.

Such curves and tabular values allow a simple and rapid solution of practical problems. With reference to Fig. 1(*a*) the distance, l_{2-1}, between two selected depths, with $\eta_2 = \frac{y_2}{y_0}$ and $\eta_1 = \frac{y_1}{y_0}$, is determined from Equation (2) by inserting the values of $B'(\eta_2)$ and $B'(\eta_1)$ from Table 2.

TYPES OF SURFACE CURVES

Due to the direction of the slope, the motion in the canal takes place at the expense of the specific energy available in the initial section, which is dissipated in the course of the movement. Only two types of surface curves are possible: (1) For depths above the critical ($y > y_c$), a drop curve (see Fig. 3) of the M_2-type[13] which is characterized by a decreasing depth, y, terminating at y_c; and (2) in the range of depths below the critical ($y < y_c$), a rising curve (of the M_3-type)[13] illustrated by Fig. 4 and also terminating at y_c.

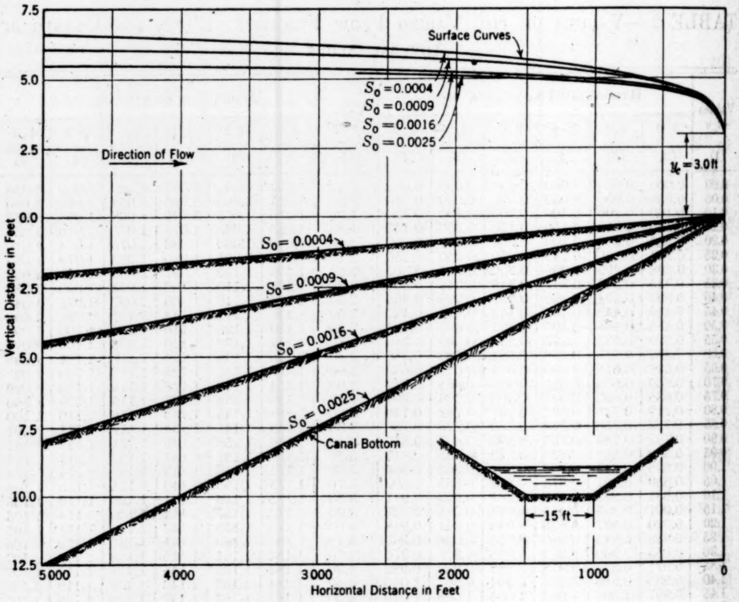

FIG. 3.

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[13] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, Fig. 60, p. 76.

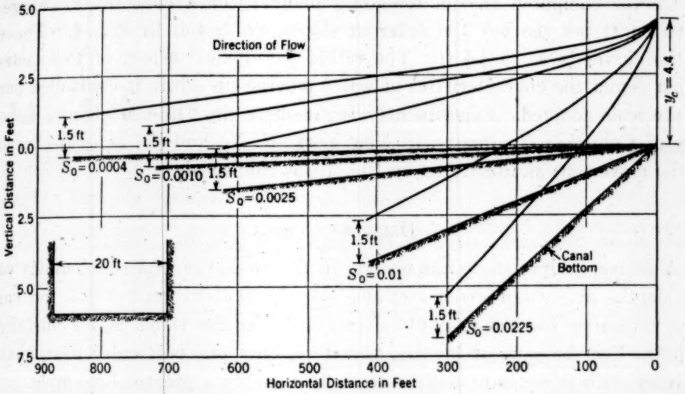

FIG. 4.

PRACTICAL EXAMPLES

Example 1.—M'_2-Curve.—Assuming a discharge of $Q = 519$ cu ft per sec, and a slope, $S_o = 0.0004$, following the usual procedure, the value of $y_0 = 6.56$ ft[13]. The critical depth is $y_c = 3$ ft. The hydraulic exponent is taken at $n = 3.80$; and the average value of β is 0.06. At the point of critical depth, $\eta_c = \dfrac{3}{6.56} = 0.457$, and, by interpolation, from Table 2, $B'(\eta_c) = 0.452$.

Calculating the distance from $y_c = y_2$ to a point at which the depth is $y_1 = 5.0$ ft, with $\eta_1 = \dfrac{5.0}{6.56} = 0.762$ and $B'\ (0.762) = 0.715$, by Equation (7) $l_{2\text{-}1} = \dfrac{6.56}{0.0004}\ [-\ (0.457 - 0.762) + 1.06\ (0.452 - 0.715)] = 459.$

A similar procedure determines the respective distance from $y_c = 3$ ft to a section with $y_1 = 4$ ft, 6 ft, etc. Thus, the entire surface curve may be traced. Fig. 3 gives the curves for the same discharge and cross-section, but for different values of the adverse slope, ranging from $-\ 0.0004$ to $-\ 0.0025$.

Example 2.—M'_2-Curve (Fig. 4).—Assume a discharge of $Q = 1\,080$ cu ft per sec, and $y_c = 4.4$ ft[14]; and, with $-\ S_o = 0.0004$, $y_o = 8.65$ ft. Furthermore, let $n = 3$ and $1 + \beta = 1.17$; and assume that a sluice-gate maintains an initial depth, $y_1 = 1.5$ ft. The rising curve is to be computed from these data. To illustrate, determine the distance from the section with $y_1 = 3.0$ ft to a section with $y_2 = 3.50$ ft; thus: $\eta_1 = \dfrac{3.0}{8.65} = 0.347$; and $\eta_2 = \dfrac{3.50}{8.65} = 0.405.$ Accordingly, from Table 2, $B'(\eta_1) = 0.342$; and $B'(\eta_2) = 0.398$. Substituting these values into Equation (7),

$$l_{2\text{-}1} = \frac{8.65}{0.0004}\ [-\ (0.405 - 0.347) + 1.17\ (0.398 - 0.342)] = 160 \text{ ft}$$

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[13] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, Canal Type *D*, p. 321.

[14] *Loc. cit.*, p. 320, Canal Type *C*.

Curves computed in the foregoing manner for a constant discharge of 1 080 cu ft per sec but for different slopes, are given in Fig. 4. The end of the curves is $y_c = 4.4$ ft. The rather paradoxical aspect of these curves, which depict the characteristics of water moving up a hill, is controlled partly by the scale adopted. Experiments were made in the Fluid Mechanics Laboratory of Columbia University, in New York, N. Y., and the general character of the movement as illustrated by Fig. 4 was confirmed.

DELIVERY CURVES

A delivery curve shows the change in the discharge in a canal under varying depths at certain given sections—usually the extremities—of the canal. The reasoning for the case of adverse slope is the same as for sustaining slope[15]. For the sake of brevity, therefore, only one particular case, that of delivery with y_2 constant will be treated to supply a practical example.

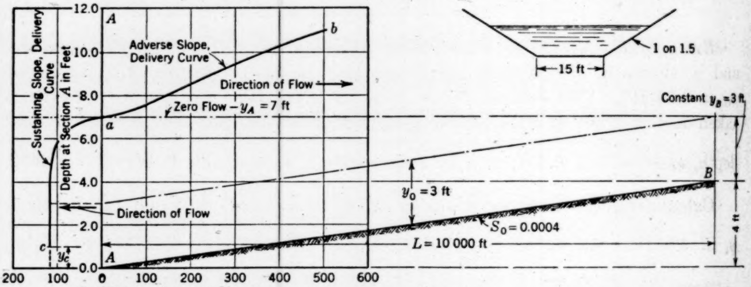
12.0
10.0
8.0
6.0
4.0
2.0
0.0
A
b
Adverse Slope, Delivery Curve
Direction of Flow
Zero Flow – y_A = 7 ft
a
Sustaining Slope, Delivery Curve
Depth at Section A in Feet
Direction of Flow
c
y_c
A
200
100
0
100
200
300
400
500
600
Delivery in cu ft per sec
1 on 1.5
15 ft
Constant y_B = 3 ft
y_0 = 3 ft
B
S_0 = 0.0004
L = 10 000 ft
4 ft

FIG. 5.

Example 3.—Assume that a canal of the cross-section shown in Fig. 5[15], has a length of 10 000 ft and connects the open sea at Section A with a large inland reservoir at Section B. The level of the latter is taken to be invariable, which gives a constant depth at Section B assumed to be $y_B = 3$ ft. The bottom slope is 0.0004. Obviously, when y_A happens to be 7 ft the surface is level and there is no flow through the canal. When the depth at Section A exceeds 7 ft there will be flow from Section A to Section B, the motion taking place against an adverse slope. A delivery curve (Curve ab, Fig. 5) indicates the value of the discharge, Q, for corresponding values of the variable depth, y_A.

To find a point of the delivery curve assume $y_0 = 5$ ft, which with K (5 ft) = 15 200, corresponds to a discharge, $Q = K \text{ (5 ft)} \sqrt{S_0} = 304$ cu ft per sec. Assume that $1 + \beta = 1.06$ and $n = 3.80$. The problem now is reduced to that of Fig. 3, namely, to find the depth, $y_A = y_1$, which, with

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[15] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, p. 143.

$l_{2-1} = 10\,000$ and $y_0 = 5$ ft, will correspond to $y_2 = y_B = 3$ ft, as given. Equation (7) may be rewritten in the form:

$$l_{2-1}\ \frac{S_0}{y_0} = [-\eta_2 + (1+\beta)\,B'(\eta_2)] - [-\eta_1 + (1+\beta)\,B'(\eta_1)]$$
$$= z'(\eta_2) - z'(\eta_1) \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which $z'(\eta)$ is a function of η of the form,

$$z'(\eta) = -\eta + (1+\beta)\,B'(\eta) \ldots\ldots\ldots\ldots\ldots (14)$$

In the present case, with y_2 (and thus η_2) given, the only unknown in Equation (13) is $\eta = \frac{y_1}{y_0}$ implicitly contained in the function, $z'(\eta_1)$.

For $y_0 = 5$ ft, therefore, $l_{2-1} = \frac{S_0}{y_0} = 10\,000 \times \frac{4 \times 10^{-4}}{5} = 0.8$; $\eta_2 = \frac{3}{5} = 0.60$; and $B'(\eta_2) = 0.584$. Consequently, by Equation (14), $z'(\eta_2) = -0.600 + 1.06 \times 0.584 = 0.019$. The solution of the problem involves finding the value of $\eta_1 = \frac{y_A}{y_0}$ which satisfies the equation:

$$z'(\eta_1) = z'(\eta_2) - l_{2-1}\ \frac{S_0}{y_0} = -0.781 \ldots\ldots\ldots\ldots (15)$$

The most practical way to solve Equation (14) is to build an auxiliary curve of $z'(\eta)$ plotted against η.

The value of η_1 which is sought is 1.912 and, thus, $y_A = \eta_1 \times y_0 = 1.912 \times 5 = 9.56$ ft. Hence, a point of the delivery curve is established with $y = 9.56$ ft and the corresponding discharge of 304 cu ft per sec. By assigning other values to the normal depth, y_0, other points of the delivery curve may be obtained, resulting in the curve, *ab* (Fig. 5).

When the depth at Section *A* becomes less than 7 ft, the flow will be in the direction of the sustaining slope. The corresponding delivery curve is *ac*.

CONCLUSIONS

The case of flow in channels of adverse slope completes the scope of application of the Belanger equation of varied flow based on the assumption of parallel flow to practical engineering design. It is hoped that the varied flow equation (Equation (7)) and the tables of values of the varied flow function will provide a direct and convenient method of solving some of the hydraulic problems connected with the design of open channels of adverse slope.

ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation to Boris A. Bakhmeteff, M. Am. Soc. C. E., for aid and guidance during the course of the work.

APPENDIX

NOTATION

c = a subscript denoting "critical";

l = length; distance between two given sections; $l_{2\text{-}1}$ = distance between Section 2 and Section 1;

n = an hydraulic exponent;

S = slope of the water surface; S_o = sustaining slope of a channel bottom; $-S$ = adverse slope of a channel bottom; S_c = critical slope of water surface;

x = a variable distance measured in the direction of flow;

y = a parameter denoting variable depth of flow; y_0 = normal depth, or the depth in the case of uniform movement; y_c = critical depth;

z = a function; $z'(\eta)$ = a function of η referred to the case of adverse slope;

A = area;

B = a function; $B(\eta)$ = the varied flow function $= \int_0^\eta \frac{d\eta}{\eta^n - 1}$, for sustaining slope; $B'(\eta)$ = varied flow factor for adverse slope;

M = a type of surface curve: M_2-curve, M_3-curve, etc.;

Q = rate of discharge, or flow;

V = average velocity in a section;

α = slope angle $= \sin^{-1} S$;

β = the relation between the bottom slope, S_o, and the critical slope, S_c;

η = the relation of the varying depth of flow, y, to the normal depth, y_0 $\left(\text{that is, } \eta = \frac{y}{y_0}\right)$.

DISCUSSION

H. E. von Bergen,[16] Assoc. M. Am. Soc. C. E. (by letter).—The science of obtaining the depth directly at any point in a channel of adverse grade and uniform section, for steady non-uniform flow, has been advanced appreciably by this paper. In the field of hydraulics, however, this case is rather rare and in the event of flow below critical depth the hydraulic jump is produced. Furthermore, unless the engineer has occasion to make numerous computations, using the author's formulas and "varied flow functions", he may find the method somewhat involved and cumbersome.

The writer has found it more practical and convenient to dispense with the author's analytical treatment beyond his Equation (3) and, solving for $\frac{dy}{dx}$, to make a graphical integration of the resulting differential equation as suggested[17] by Harold A. Thomas, M. Am. Soc. C. E. For example, using the notation of the paper:

$$\frac{dy}{dx} = \frac{-S_0 - \frac{Q^2}{K^2(y)}}{1 - \frac{Q^2 b}{g a^3}} \quad \text{......................} (16)$$

and, since $\frac{Q^2}{K^2(y)} = S$, which may be obtained quickly from various prepared charts or tables, and $\frac{Q^2 b}{g a^3} = \frac{V^2 b}{g a}$ (and for rectangular channels may be further simplified) Equation (16) may be reduced to:

$$\frac{dy}{dx} = \frac{-S_0 - S}{1 - \frac{V^2 b}{g a}} \quad \text{......................} (17)$$

To integrate, graphically, use the reciprocal of Equation (17), compute four or five values of $\frac{dx}{dy}$ over the desired range of depths, and plot the depth, y, as ordinate and $\frac{dx}{dy}$ as abscissa. Then, by graphical summation of the area to the left of the curve, the water-surface profile may be drawn over the desired range of depths.

The negative sign of S_0 was introduced from Equation (3), but for the general case the sign of S_0 is positive and, by substituting a minus value for "adverse" slope and a positive value for "sustaining" slope, the sign of S_0 will take care of itself automatically.

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[16] Water Master, Div. of Water Resources, State Dept. of Water Rights, Sacramento, Calif.

[17] "Hydraulics of Flood Movements in Rivers", by Harold A. Thomas, *Engineering Bulletin*, Carnegie Inst. of Technology, 1934, p. 13.

The writer believes the graphical solution is more practical and efficacious and, except for accidental errors in plotting, is more correct than the elaborate analytical treatment given by the author.

W. E. Howland,[18] Assoc. M. Am. Soc. C. E. (by letter).—This extension of the theory of varied flow to open channels of "adverse" slope furnishes additional proof of the power of the fundamental methods presented by Professor Bakhmeteff[2], and is, in itself, a valuable contribution to the practical tools of the hydraulic engineer. The concise presentation of theory and the numerous illustrative problems serve to make the methods available to students and practitioners alike.

A few suggestions may be helpful in gaining familiarity with the terminology. When the depth is y, the term, $K(y)$, is defined as $C A \sqrt{R}$, in which C is the constant in the familiar Chezy formula, $V = C \sqrt{R S}$; R is the hydraulic radius; and A is the area of the cross-section of the moving stream. It is thus a part of the total expression for the carrying capacity of the channel, but not all of it. The term, "conveyance, as a name for $K(y)$ is a good one, because it is not used for any other quantity, but the expression, "carrying capacity", seems to mean Q, or an expression for Q, and, therefore, may be misleading as applied to $K(y)$.

The term, "hydraulic exponent", or n, can be defined by the equation $K^2(y) =$ a constant $\times\, y^n$. It is obtained by plotting the logarithms of the computed values of $K^2(y)$, for a given cross-section against y, the depth, as abscissa. The slope of the line so plotted is the value of n.

There are two assumptions made in Mr. Matzke's study which might introduce small errors. A useful addition to the paper would be an estimation of the approximate limits of these errors as affecting computed depths in one or more of the illustrative problems. These two assumptions are: (1) That of the constancy of n previously mentioned; and (2), that of the constancy of η defined in the paragraph following Equation (4).

It is interesting to note that a more elementary method of solving problems involving changing velocities does sometimes yield results of sufficient precision with little labor. The writer has used one such method to check the results of Example 1. It involves the consideration of the canal as made up of six sections of variable length, but of constant difference in surface elevation of 1 ft. The effect on the surface curve due to the differences in velocity head can be considered exactly. The effect on the surface curve of friction in each section was obtained by computing the value of the energy gradient at the extremity of each section and then using the harmonic average (a reciprocal average of reciprocals) of the two extreme values in estimating the effect of friction in that section. The variable length of each section is then computed from the following self-evident expression (the nomenclature used is that of the paper):

$$l_{1\text{-}2} = \frac{\text{Difference in depths (1-2)} - \text{difference in velocity heads (2-1)}}{\text{Bottom slope} + \text{energy gradient}}$$

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[18] Associate Prof., Civ. Eng., School of Civ. Eng., Purdue Univ., West Lafayette, Ind.

or,

$$l_{1\text{-}2} = \frac{(y_1 - y_2) - \dfrac{Q^2}{2\,g}\left\{\dfrac{1}{(a_2)^2} - \dfrac{1}{(a_1)^2}\right\}}{s + \dfrac{Q^2}{k^2_{(\text{ave.})}}} \quad \ldots\ldots\ldots\ldots\ldots(18)$$

For Example 1, Equation (18) becomes:

$$l_{1\text{-}2} = \frac{1 - \dfrac{519^2}{64.4}\left\{\dfrac{1}{(a_2)^2} - \dfrac{1}{(a_1)^2}\right\}}{0.0004 + \dfrac{519^2}{k^2_{(\text{ave.})}}} \quad \ldots\ldots\ldots\ldots\ldots(19)$$

The values of a and k have been obtained for this section from Professor Bakhmeteff's work[13]. Substituting the values of $a_2 = 58.5$ ft² when $y = 3$ ft. and $a_1 = 84.0$ ft² when $y = 4$ ft, the numerator of Equation (19) becomes 0.373. Using the average of the values of $k^2 = (58.2 \times 10^2)^2$, which is the value when $y = 3$ ft, and of $k^2 = (99.4 \times 10^2)^2$, which is the corresponding value when $y = 4$ ft, the denominator of Equation (19) becomes 0.004062 and $l_{1\text{-}2}$ is 84 ft. Owing to the very small differences in the functions given in Table 2, it is almost impossible to check this value thereby.

Likewise, the writer has computed the distance from the place where $y = 4$ ft to where $y = 5$ ft, etc., and then, by addition, the distance from where $y = 3$ ft to where $y = 5$ ft, etc., and obtained the results listed in Table 3. The value, 426 ft, does not agree with the one given by Mr. Matzke, but it is more closely in agreement with the arithmetic values he himself has given; that is, $\dfrac{6.56}{0.0004}\left\{-(0.457 - 0.762) + 1.06\,(0.452 - 0.715)\right\} = 426.$ The agreement between the results of the two methods is striking and really surprising, considering that the approximate method involves so little work.

TABLE 3.—COMPARISON OF HORIZONTAL DISTANCES (UNITS IN FEET)

DEPTH AT SECTIONS BETWEEN WHICH HORIZONTAL DISTANCES ARE GIVEN:		COMPUTED HORIZONTAL DISTANCES	
From:	To:	By approximate method:	By Matzke's method:
3	4	84	.....
3	5	425	426
3	6	1 170	1 180
3	7	2 320	2 335
3	8	3 830	3 860

It is also interesting to note that if the arithmetic average of energy gradients (or what amounts to the same thing, the average of $\dfrac{1}{k^2}$) were used in the foregoing computation instead of the harmonic average of energy gradients (or average value of k^2) at the two extremities of a given section, the agreement would not have been nearly so good. This would suggest that, at least for this type of problem, the harmonic average should be used.

From the application of this particular approximate method to this problem and also from studying the somewhat similar graphical method presented by H. Addison[19], the writer is led to believe that the harmonic average is preferable to the arithmetic average of slopes in approximate methods of this nature. If further study on this point were thought to be desirable, Mr. Matzke's method would serve as an exact standard of comparison on regular channels. In natural channels with irregular sections, in which some kind of an approximate method must be used, great precision could be secured by the use of a large number of sections or small increments of depth.

The writer has attempted to apply this approximate method to Example 2, but has obtained a result of 127 ft instead of 160 ft. Mr. Matzke's result might be in error almost to this extent. A change in the last significant figure of one of the numbers from Table 2 would change the result of his computation as much as 13 per cent. It seems that this table is not satisfactory when applied to sections so close to one another. Indeed, this lack of precision in computations with close sections is the most serious limitation to the application of the method. The approximate method increases in precision as the sections become closer and would appear to be more satisfactory for these cases.

Applying the approximate method to the third problem the writer has obtained a value of $l = 11\,150$ ft from the right-hand end of the canal to the place where $y = 10$ ft. (In making these computations a value of $k^2 = 19.35 \times 10^8$, when $y = 9$ ft., and $k^2 = 23.95 \times 10^8$, when $y = 10$ ft., was used, in addition to the values for the lower depths given by Bakhmeteff.) Then, subtracting from 10 ft the value, 11 150 − 10 000, times the value of $\frac{dy}{dx}$ at the 10-ft depth, or $1\,150 \times 0.000425 = 0.49$, the writer obtains 9.51 ft as the depth of the water at the lower end of the canal. (Here, the water-surface slopes are very small and are very nearly constant.) This agrees very well with the values of 9.56 ft given by the author. This method probably involves no more labor than that of Mr. Matzke in solving this problem.

In the statement of this problem it is to be noted that the upper end of the canal level is held fixed at 3 ft; and that the reservoir surface elevation at Section *B* is invariable. It is impossible to maintain both conditions when the water is flowing to the left since the water line in the canal must be at least one velocity head lower than that in the reservoir. This velocity head at the highest flow amounts to 0.61 ft. The solution given is consistent with the assumption that the levels indicated at Sections *A* and *B* are canal levels and not reservoir or ocean levels, unless, in some case, they might happen to be the same.

ARNO T. LENZ,[20] JUN. AM. SOC. C. E. (by letter).—An interesting extension of the work of B. A. Bakhmeteff, M. Am. Soc. C. E.[2], on open channels is contained in this paper. The examples show its practical application in an excellent manner. In mathematical derivations of this type, however, there are certain basic assumptions which must be made. These assumptions may be said to be the foundation upon which the structure is built, and they must be sound. They should be checked in every manner possible.

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[19] "Hydraulics", by H. Addison, N. Y., John Wiley & Sons, 1934, p. 135.

[20] Instr., Dept. of Hydr. and San. Eng., Univ. of Wisconsin, Madison, Wis.

In the derivation of the general differential equation for varied flow, the hydraulic exponent, n, is used, based on the fact that "the conveyance function, $K = a\,C\,\sqrt{R}$, within a reasonable range of depths, follows sufficiently close the exponential relations: $K^2\,(y) = a^2\,C^2\,R =$ constant y^{n}"[8]. This exponential relation is of extreme importance in all that follows.

It is evident that the best check of this relation would be to plot the conveyance, K, against the depth, y, on logarithmic paper, which plotting should give a straight line. Unfortunately, practically all published experimental discharge measurements omit the slope measurement necessary to determine the value of K from the discharge, $Q = K\,s^{\frac{1}{2}}$.

An approximate check might be made, however, by considering that when a number of stream gagings are taken, some will be made at times when the surface slope is greater than normal and at others when it is less than normal, with the average of a number probably about normal. Furthermore, the gage height as recorded is a measure of the depth, y. With these approximations in mind, discharge was plotted against gage height for ten streams picked at random from the *Water Supply Papers* of the U. S. Geological Survey, from all sections of the country. The curves are plotted on Fig. 6.

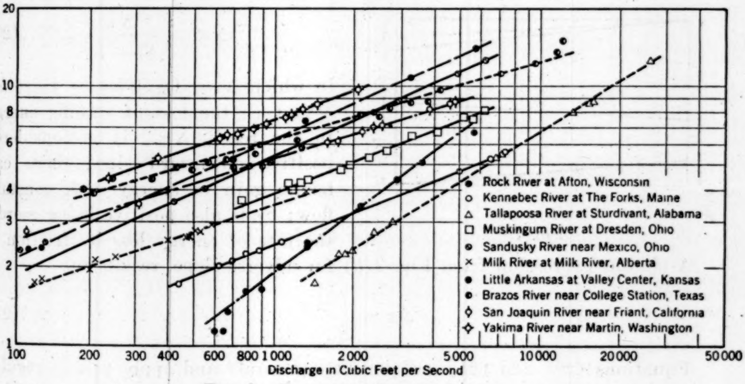

FIG. 6.—DISCHARGE RATING CURVES.

The writer was surprised that the points plotted did approximate straight lines. It is recognized that for canals it is likely that the exponential law holds even more closely than for natural channels. Therefore, the curves are a logical proof of the validity of the use of the hydraulic exponent as assumed by the author.

When flow spreads out over flood-plains there will be a sharp break in the line, and care must be exercised under such conditions. A check such as that presented herein is relatively easy to make and will show quickly whether the theory may be applied to any particular river or canal.

The author is to be congratulated on his extension of the theory presented by Professor Bakhmeteff, which should be a tool of great usefulness to those working on the hydraulics of open channels.

J. C. STEVENS,[21] M. AM. SOC. C. E. (by letter).—There seems to be little use for injecting the "varied flow function" into the computations of surface curves. Such functions are not only more laborious to apply, but they are full of inaccuracies as a result of the many basic assumptions and approximations entering into their development

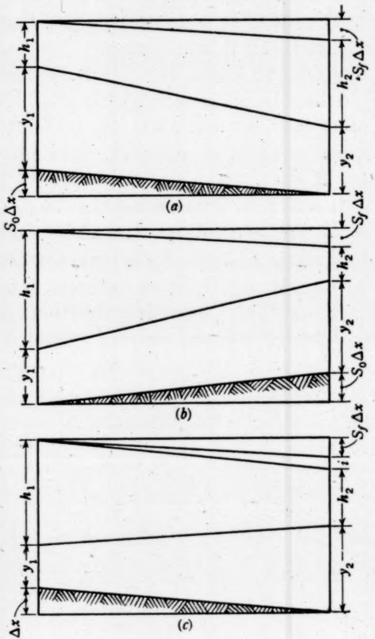

FIG. 7.

Fig. 7 shows the relationships that must obtain between two adjacent sections a distance, Δx, apart. The distance, Δx, is finite, the only limitation being that the surface curve is considered to have a uniform slope in that length. For a sustaining bed slope, Fig. 7(*a*) yields $S_0 \Delta x + y_1 + h_1 = y_2 + h_2 + S_f \Delta x$; whence,

$$\Delta x = \frac{\Delta \epsilon}{S_f - S_0} \quad \ldots\ldots (20)$$

in which $\Delta \epsilon = (y_1 + h_1) - (y_2 + h_2)$ —that is, the loss of specific energy in the reach, Δx. It is considered positive downward since there can be no gain in energy with constant flow; S_f is also positive being merely the rate of energy loss by friction. A similar expression from Fig. 7(*b*) for adverse slope, results in,

$$\Delta x = \frac{\Delta \epsilon}{S_f + S_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

Equations (20) and (21) are perfectly general[22] and apply to any regular channel in which the area is a function of the depth; that is, rectangular, trapezoidal, circular, triangular, parabolic, etc.

If there are energy losses in addition to friction, such as impact or eddy losses, they are deducted from the total energy losses. Thus, the general surface curve formula as indicated in Fig. 7(*c*), may be written,

$$\Delta x = \frac{\Delta \epsilon - i}{S_f \mp S_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

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[21] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[22] "New Method of Computing Backwater and Drop Down Curves", by Alva G. Husted, *Engineering News-Record*, April 24, 1924, p. 719.

depending on whether the bed slope is sustaining or adverse. In Equation (22), i = head losses due to impact and eddies.

Whenever velocities are diminishing as in expanding conduits, there is an inherent eddy loss, i, in addition to the normal channel friction. The varied flow functions necessarily assume full recovery of velocity head which is never realized.

TABLE 4.—COMPUTATIONS FOR EXAMPLE 1, APPLYING EQUATION (21)

Depth, y (1)	Area, A (2)	Velocity, V (3)	Velocity head, H (4)	Specific energy, e* (5)	Loss of specific energy, Δe (6)	Wet perimeter, p (7)	Hydraulic radius, R (8)	Kutter's coefficient, c† (9)	Friction slope, S_f, at the section commanded by y (10)	Average friction slope, S'_f (11)	Friction slope, S_f, plus bed slope, S_o, (12)	Length between sections, Δx (13)	Distance, x, from starting point (14)
(*a*) EQUATIONS (20), (21), AND (22) APPLIED TO EXAMPLE 1													
3.0	58.5	8.88	1.226	4 226	.....	25.8	2.26	67	0.00775	....	....	..	0
3.2	63.4	8.20	1.045	4.245	0.019	26.5	2.39	68	0.00610	0.00692	0.00732	2	2
3.4	68.3	7.61	0.900	4.300	0.055	27.2	2.51	68	0.00500	0.00550	0.00590	9	11
3.6	73.4	7.07	0.777	4.379	0.079	28.0	2.62	69	0.00400	0.00450	0.00490	16	27
3.8	78.7	6.60	0.672	4.472	0.093	28.7	2.74	70	0.00324	0.00362	0.00402	23	50
4.0	84.0	6.18	0.594	4.594	0.122	29.4	2.86	70	0.00263	0.00293	0.00333	37	87
4.2	89.5	5.81	0.525	4.725	0.131	30.1	2.97	71	0.00226	0.00245	0.00285	53	130
4.4	95.0	5.47	0.465	4.865	0.140	30.8	3.09	71	0.00193	0.00209	0.00249	56	186
4.6	100.7	5.16	0.414	5.014	0.149	31.6	3.18	72	0.00162	0.00158	0.00198	75	261
4.8	106.6	4.86	0.367	5.167	0.153	32.3	3.30	72	0.00138	0.00150	0.00190	81	342
5.0	112.5	4.61	0.332	5.332	0.165	33.0	3.41	73	0.00117	0.00128	0.00168	98	440
5.5	127.9	4.06	0.256	5.756	0.424	34.8	3.68	74	0.000818	0.000994	0.00139	305	745
6.0	144.0	3.60	0.202	6.202	0.446	36.6	3.94	75	0.000584	0.000701	0.00110	405	1 150
6.5	160.9	3.23	0.162	6.662	0.460	38.4	4.19	76	0.000431	0.000507	0.00091	500	1 650
7.0	178.5	2.91	0.132	7.132	0.470	40.2	4.44	77	0.000320	0.000375	0.00077	610	2 260
(*b*) COMPUTATIONS FOR EXAMPLE 2													
1.5	30.0	36.0	20.15	21.65	.....	23	1.30	123	0.0660	......	......	..	0
2.0	40.0	27.0	11.33	13.33	8.32	24	1.67	127	0.0270	0.0465	0.0469	178	178
2.5	50.0	21.6	7.25	9.75	3.58	25	2.00	130	0.0138	0.0204	0.0208	172	350
3.0	60.0	18.0	5.04	8.04	1.71	26	2.30	133	0.0080	0.0109	0.0113	151	501
3.5	70.0	15.4	3.69	7.19	0.85	27	2.59	135	0.0050	0.0065	0.0069	123	624
4.0	80.0	13.5	2.83	6.83	0.38	28	2.86	137	0.0034	0.0042	0.0046	83	707
4.5	90.0	12.0	2.24	6.74	0.09	29	3.10	139	0.0024	0.0029	0.0033	27	734

* Energy head above the channel bed. † For $n = 0.025$.

Equations (20), (21), and (22) are applied step by step, computing directly without trial—the length, Δx, corresponding to successive increments or decrements of depth. All that is required is a starting point; the computations may proceed either up stream or down stream. Table 4(*a*) shows the computations for the author's Example 1.

There are no "conveyance" factors involved; nor are normal depths, y_0, normal slopes, S_0, or critical slopes, S_c, required. Before the varied flow functions can be applied, all the various functions of the cross-section for all depths within the range must be computed[23] and curves must be drawn in order to determine the normal depth parameter; the normal and critical slopes, from which to find the constant, β, must also be obtained. The exponent, n, must be found or assumed, and, therefore, a set of computations as complete as those of Table 4(*a*) must be made before one begins to apply the varied flow function—and then one must have the varied flow tables.

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[23] See "Hydraulics of Open Channels", by B. A. Bahkmeteff, M. Am. Soc. C. E., p. 321, for the system of curves required for Example 1.

All that is required for Table 4(*a*) is a slide-rule and a table of Kutter coefficients. The distance between depths of 3.0 and 5.0 ft by the method herein outlined, 440 ft—the author gives 459 ft—a fair agreement.

The curves of Fig. 4 of the paper are certainly in error because the varied flow functions assume complete recovery of velocity head in the expanding water prisms from the gate to the end of the critical depth. Moreover, for the channel and flow given, the critical depth is 4.5 ft instead of 4.4 ft as given by the author.

Table 4(*b*) shows the computations by the method herein outlined. The total length between depths of 1.5 and 4.5 was found to be 734 ft, whereas between depths of 3.0 and 3.5, it is 123 ft. This length, using smaller depth increments, was found to be:

Depth increment, in feet	Distance, in feet, between depths of 3.0 and 3.5 feet
0.5	123
0.2	125
0.1	126

In Example 2 the author gives 160 ft for that distance. In searching for these discrepancies, the writer finds the varied flow functions very sensitive, even to dropping fourth place decimals in interpolation. He also finds[24] that the exponent, *n*, for this channel is 3.1 for depths less than 4.0 ft, and 2.75 for depths greater than 4 ft. The author used 3.0 throughout.

Table 5 shows the writer's findings for the distances from the gate to the critical depth in the author's Example 2, using the varied flow function. The last value (756 ft) compares favorably enough with the 734 ft of Table 4(*b*).

TABLE 5.—DISTANCES FROM THE GATE TO THE CRITICAL DEPTH IN EXAMPLE 2

Exponent, n	Critical depth, y_2, in feet	Distance, in feet, from gate to critical depth
3.0	4.4	821
3.0	4.5	842
3.1 and 2.8	4.5	756

The writer found the distance between depths of 3.0 and 3.5 and $n = 3.1$ by differently interpolating the function, but confining the results to three decimal places: $y_1 = 3.0$; $\eta_1 = 0.374$; $B'(\eta_1) = 0.342$ or 0.343; $y_2 = 3.5$; $\eta_2 = 0.405$; and, $B'(\eta_2) = 0.398$ or 0.399; hence, by the author's Equation (13),

$$l_{2-1} = \frac{8.65}{0.0004}\left[-(0.405 - 0.347) + 1.17\,(0.398 - 0.343)\right] = 130 \text{ ft}$$

or,

$$l_{2-1} = \frac{8.65}{0.0004}\left[-(0.405 - 0.347) + 1.17\,(0.399 - 0.342)\right] = 195 \text{ ft}$$

This shows a variation of 50% in the computed length.

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[24] See "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., p. 318.

The author gives this length as 160 ft, whereas the writer found 126 ft by his method. All methods (including those of the writer) are in error, however, because complete velocity-head recovery was assumed, and such recovery is impossible of realization.

The simple method outlined herein can readily take into account these eddy losses. They are usually estimated to be some percentage of the negative change in velocity head between adjacent sections. This percentage may be any value from 10 to 100, depending on the character of the channel and the judgment of the engineer; 10% to 20% may be used for artificial channels such as are treated herein, whereas 50% is a fair value for ordinary river channels. The effect of including eddy losses is to shorten the length between given depths.

If the varied flow function is to be used, the tables should be extended to at least four decimal places in order to avoid discrepancies due to interpolation.

F. T. Mavis,[25] M. Am. Soc. C. E. (by letter).—Depending upon the reader's point of view this paper may be considered either as a valuable supplement to previous discussions of steady, non-uniform (or varied) flow in open channels, or as an academic treatment of a special case of the broadly general back-water problem. In the first category the paper is so much a part of its antecedent[2] as to be scarcely legible except by frequent reference to it. In his practical examples the author leaves his reader to glean from other sources that, in Fig. 3, Kutter's *n* is 0.025 and, in Fig. 4, 0.013. Needless reference to authority for definitions of such terms as "the hydraulic exponent, *n*", and the critical slope, S_c, seems to be unwarranted when these terms are so readily definable. (For a given depth of flow, S_c is the normal (or neutral) slope required to produce a velocity head equal to one-half the hydraulic mean depth; $S_c = \frac{gp}{C^2 t}$, in which g = acceleration of gravity; p = wetted perimeter; C = the Chezy coefficient; and t = width of water surface; n is twice the slope of the curve on logarithmic paper, showing $K = CA\sqrt{R} = \frac{Q}{\sqrt{S}}$ as ordinates, and the corresponding depth of flow, y, as abscissas.)

In the "Synopsis" the author states that "practical application [of the general equations for varied flow in channels of adverse slope] depends on knowing the numerical values of a function, similar to the 'varied flow function', for canals of sustaining slopes."

It is the writer's opinion that the practical solution of back-water problems cannot be safely discussed in the abstract. In important practical problems one may well be wary of the better-known back-water functions, except as general guide-posts. Ignoring the simple, academic problems, the practical solutions withstand the tests of future observations and the Courts, rather on the strength of reliable basic data and the experience and sound judgment of a competent engineer than on the details of the methods of computation.

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Among the essentials to the practical solution of back-water problems it may be stated that:

(1) The engineer must be thoroughly acquainted with the conduit. Given adequate profiles, cross-sections, and field data, he must draw on a rich background of experience in the assignment of roughness factors and effective flow sections in curved and irregular conduits.

(2) The relation between flow and corresponding head losses must be assumed. In the simpler cases this is usually expressed as a formula for the mean velocity of flow in terms of the roughness factor, hydraulic radius, and the friction slope.

(3) Obviously, the computer must be well-grounded in the fundamentals of hydro-mechanics—work and energy (Bernoulli's theorem), and impulse and momentum—and in their application. It is desirable that he possess an aptitude for simplifying and idealizing a given problem for purposes of computation without introducing significant or uncertain errors.

In practice, those who must solve back-water problems of importance are likely to develop their own methods of analysis—based generally on the equivalent of the author's Equation (2) and applied to the effective conduit.

In back-water problems of rivers the distances between valley cross-sections are usually fixed by the surveys. Therefore, explicit relations for length of reach, similar to Equation (7), may not be as convenient as a more elementary form of equation, based on steady flow within a short reach, solved by trial.

Fundamentally, there is no difference between the author's method of computing back-water curves by the so-called "varied flow functions" and methods of successive approximation applied to short reaches of a stream. For a short reach, a regular channel, and steady flow, Fig. 1 and Equation (2) apply equally well to all methods.

In the method of successive approximations each reach presents a definite problem of determining for a given discharge the relation between water-surface elevation and slope at the mean section. Furthermore, the water-surface elevations at the common sections of adjacent reaches must coincide. A consistent water profile is obtained by successive trial computations. The difference in water-surface elevations at two sections is found simply by the numerical addition of the products of each intervening length of reach and its corresponding surface slope.

In the author's method the problem which involves definite physical factors, such as discharge, in cubic feet per second, elevations (or depths), in feet, and slopes of water surfaces, is early obscured by the introduction of the dimensionless ratios, $\eta = \frac{y}{y_0}$, $\beta = \frac{S_0}{S_c}$, and $n = 2\,\frac{y}{K}\left(\frac{dK}{dy}\right)$. Each of these ratios is a function of y, the depth of flow in the conduit, although there may be no significant error introduced by assuming β and n to be constant for purposes of integration. Although integration is often a convenient method of finding the sum of a column of figures that represent some regular, analytical (or graphical) array of numbers, the integration leading to the

$B'(\eta)$-functions replaces, in part, the numerical addition of products of each length of reach, and the corresponding surface slope aforementioned—nothing more.

For the solution of back-water problems in regular, artificial conduits, such as canals, sewers, etc., it may be that the so-called "varied flow functions" may effect a saving in time over the more elementary method of successive approximations. However, for practical river problems in which back-water curves are important, the author's method seems to be distinctly inferior to the method of successive approximations in which there is no necessity for overlooking such factors as roughness and effective sections, and obscuring such physical quantities as elevation, slope, and discharge, by introducing dimensionless ratios.

HUNTER ROUSE,[26] ASSOC. M. AM. SOC. C. E., AND MERIT P. WHITE,[27] JUN. AM. SOC. C. E. (by letter).—An effort such as this, to make available to hydraulic engineers additional means of simplifying open-channel design, should be welcomed by the hydraulic world. Although the subject of sustaining slopes has been covered amply elsewhere, this is the first successful attempt to provide both an outline of procedure and tabular values for the solution of problems of adverse slope. The labor of determining the numerous values of the $B'(\eta)$-function is not to be under-estimated, although the many hours given by the author to this tedious computation have been well spent.

It seems to the writers that the author's method of arriving at what should be a general equation of varied flow does not lend full clarity to the physical principles involved, for the procedure adopted with respect to signs tends to conceal the basic nature of the problem rather than to explain it. Indeed, the use of an upward—and hence negative—slope at times as a positive value is often confusing.

Mr. Matzke has designated a downward slope as positive and an upward slope as negative; to be consistent, this notation must apply as well to the slope of the water surface and of the energy line as to the slope of the channel bottom. The symbol for the slope of the water surface, S_w, for instance, therefore, denotes either a downward slope or an upward slope, depending upon whether its value is greater or less than zero. Referring to Fig. 8, the elevation of the free surface at any section, regardless of slope, is equal to the depth, y, plus the elevation of the channel bottom, h_0. Letting H_p represent the total potential head, or the vertical distance from the assumed geodetic datum to the water surface:

$$H_p = y + h_0 \qquad \text{(23)}$$

The rate of change of this surface elevation in the positive direction of flow is then simply the derivative of the potential head with respect to x:

$$\frac{dH_p}{dx} = \frac{dy}{dx} + \frac{dh_0}{dx} \qquad \text{(24)}$$

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[26] Asst. Prof. of Fluid Mechanics, SCS, California Inst. of Technology, Pasadena, Calif.

[27] Asst. Engr., SCS, Iowa Inst. for Hydr. Research, Iowa Univ., Iowa City, Iowa.

Since a downward slope is positive (that is, greater than zero), and signifies a negative rate of change of elevation,

$$\frac{dH_p}{dx} = -S_w = \frac{dy}{dx} - S_0 \dots\dots\dots\dots\dots\dots (25)$$

Equation (25) is offered in preference to Equation (1), for it is mathematically correct for both positive and negative slopes of the water surface and for both sustaining and adverse slopes of the channel bottom.

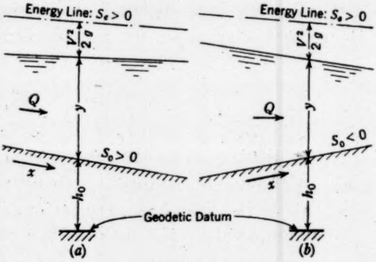

FIG. 8.

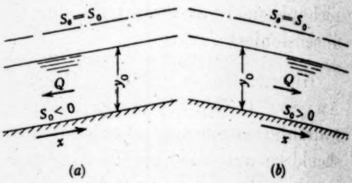

FIG. 9.

Proceeding in a similar fashion, the total head, H, above the arbitrary geodetic datum at any section is equal to the sum of the velocity head and the potential head (refer to Fig. 8):

$$H = \frac{V^2}{2\,g} + y + h_0 \dots\dots\dots\dots\dots\dots (26)$$

The rate of change of total head in the positive direction of flow is then simply the derivative of this expression, with respect to x:

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{V^2}{2\,g}\right) + \frac{dy}{dx} + \frac{dh_0}{dx} \dots\dots\dots\dots (27)$$

Since the positive rate of change of total head is equal to the negative rate of loss of total head, $\frac{dH}{dx} = -\frac{d\,e_r}{dx}$, the latter derivative being used in Equation (2); and since a positive rate of loss corresponds to a positive or downward slope,

$$\frac{dH}{dx} = -S_e = -\frac{d\,e_r}{dx} = \frac{d}{dx}\left(\frac{V^2}{2\,g}\right) + \frac{dy}{dx} - S_0 \dots\dots (28)$$

Equation (28) is offered to replace Equation (2); it is quite as general as Equation (25).

The Chezy equation,

$$Q = A\;C\,\sqrt{R\;S_e} \dots\dots\dots\dots\dots\dots (29)$$

expresses the rate of discharge, Q, in terms of the cross-sectional area of flow, A; a coefficient, C; the hydraulic radius, R; and the slope of the energy line, S_e. The first three terms, A, C, and R, depend only upon the cross-sectional shape and roughness of the channel and the variable depth of flow, and may all be determined as functions of y for a given channel. As shown by Bakhmeteff[3], the product, $A\,C\sqrt{R}$, as a function of y may conveniently be given the symbol, K, to denote the "conveyance" of the channel; hence,

$$Q = K\sqrt{S_e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

If the discharge and the conveyance are such that for the given bottom slope, S_0, the depth of flow is uniform over the entire channel, then $y = y_0$ and the slope of the energy line, S_e, is equal to both the surface slope, S_w, and the bottom slope, S_0. For such conditions,

$$Q = K_0\sqrt{S_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

The term, K_0^2, is used as a convenient relative parameter for flow on sustaining slopes, and has been adopted by the author for adverse slopes as well. Since uniform flow on adverse slopes is an artificial condition, such usage may well bear further analysis.

Although S_w and S_0 may be either positive or negative with flow in the positive x-direction, a negative value of S_e can only signify flow in the negative x-direction, for the energy line must invariably slope downward in the direction of flow. Hence, the equation of uniform flow,

$$Q = A\,C\sqrt{R}\sqrt{S_0} = K_0\sqrt{S_0} \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

when used as a flow parameter, must be such that for negative values of S_0 the discharge will also be negative (that is, it will still be in the direction of decreasing energy and downward bottom slope (see Fig. 9)). Thus, Q^2 and $K_0^2\,S_0$ are invariably positive, regardless of whether the numerical value of S_0 is positive or negative.

A plot of $S_0 = \dfrac{Q^2}{K_0^2} = \dfrac{\text{constant}}{K_0^2}$, shown schematically in Fig. 10, indicates clearly the fact that for negative values of S_0 the conveyance, K_0, must be imaginary, since its square is negative. This may be true only if the coefficient, C, becomes imaginary for all negative values of S_0, since A and R are invariably positive. In the relationship, $Q^2 = K_0^2\,S_0$, used by the author following the procedure outlined by Bakhmeteff[3], the imaginary root, $i = \sqrt{-1}$, contained in K_0 for negative values of S_0, therefore, cannot be neglected.

Such discontinuity in the conveyance function will be more clearly understood if one recalls that as the slope of a channel approaches zero, the discharge thereby remaining constant, the uniform depth must approach infinity in order that the slope of the energy line may also approach zero. If the slope of the bottom continues to decrease, once it is less than zero uniform conditions are quite impossible unless the flow reverses in direction.

Continued use of the uniform flow relationship of Equation (32) as a parameter for varied flow in the positive x-direction naturally requires that this equation be mathematically consistent.

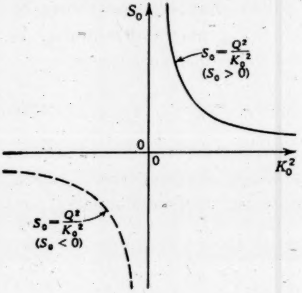

FIG. 10.

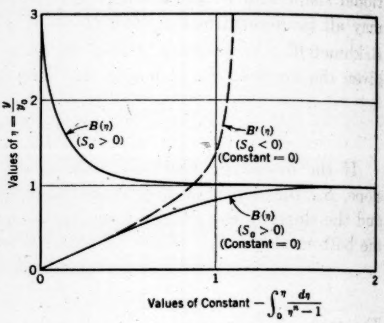

FIG. 11.

The following substitutions are now in order, after the method taken by the author from the cited text[2]:

$$S_e = \frac{Q^2}{K^2} = S_0 \frac{K_0^2}{K^2} \quad \text{.......................(33)}$$

$$\frac{d}{dx}\left(\frac{V^2}{2\,g}\right) = -\frac{Q^2}{g}\frac{b}{A^3}\frac{dy}{dx} = -\frac{S_0\,K_0^2}{g}\frac{g}{S_c\,K^2}\frac{dy}{dx} = -\frac{S_0}{S_c}\frac{K_0^2}{K^2}\frac{dy}{dx} \quad \text{..(34)}$$

and,

$$\beta = \frac{S_0}{S_c} \quad \text{..............................(35)}$$

Introducing Equations (33), (34), and (35) in Equation (28),

$$-S_0\frac{K_0^2}{K^2} = -\beta\frac{K_0^2}{K^2}\frac{dy}{dx} + \frac{dy}{dx} - S_0 \quad \text{.............(36)}$$

and, since K^2 varies as y^n:

$$\frac{dy}{dx} = S_0\,\frac{1 - \dfrac{K_0^2}{K^2}}{1 - \beta\,\dfrac{K_0^2}{K^2}} = S_0\,\frac{1 - \left(\dfrac{y_0}{y}\right)^n}{1 - \beta\left(\dfrac{y_0}{y}\right)^n} \quad \text{............(37)}$$

Equation (37) is quite general, applying to positive and negative slopes of the water surface and sustaining and adverse slopes of the channel bottom; as such it is offered as an improvement on Equation (4). One need only remember that when S_0 is negative, K_0^2, and hence, y_0^n, must have negative numerical values owing to the square of the imaginary i. That β will, of itself, become negative is quite evident, since it then represents the quotient of two slopes of unlike sign, S_c always being positive.

From the relationships, $\eta = \frac{y}{y_o}$ and $dy = y_o\, d\eta$:

$$\frac{y_o}{S_o} \frac{d\eta}{dx} = \frac{1 - \frac{1}{\eta^n}}{1 - \beta \frac{1}{\eta^n}} = \frac{\eta^n - 1}{\eta^n - \beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

which becomes, through division,

$$\frac{S_o}{y_o}\, dx = \frac{\eta^n - \beta}{\eta^n - 1}\, d\eta = d\eta + (1 - \beta)\, \frac{d\eta}{\eta^n - 1} \quad \ldots\ldots\ldots (39)$$

Equation (39) is offered to replace Equation (5). The term, η^n, now becomes negative with negative values of S_o, again owing to the imaginary value of C.

Integration of Equation (39) between the limits, x_1 and x_2, and η_1 and η_2, will be as follows:

$$\frac{S_o}{y_o}\,(x_2 - x_1) = \frac{S_o}{y_o}\, l_{2-1} = \eta_2 - \eta_1 + \int_{\eta_1}^{\eta_2} (1 - \beta)\, \frac{d\eta}{\eta^n - 1} \quad \ldots\ldots (40)$$

Equation (40) is completely general, and as such is to be recommended instead of Equation (6). For positive values of S_o, after Bakhmeteff[2].

$$\int_0^{\eta} \frac{d\eta}{\eta^n - 1} = \text{constant} - B\,(\eta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

When $\eta < 1$, the constant has been taken as zero in computing the values of $B\,(\eta)$ given by Bakhmeteff[2]. Equation (40) then becomes, for $S_o > 0$,

$$l_{2-1} = \frac{y_o}{S_o}\,[\eta_2 - \eta_1 - (1 - \beta)\,(B\,(\eta_2) - B\,(\eta_1))] \quad \ldots\ldots\ldots\ldots (42)$$

For negative values of S_o and hence of η^n, following the same designation as to sign,

$$\int_0^{\eta} \frac{d\eta}{\eta^n - 1} = -\, B'\,(\eta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

in which the numerical values of the function, $B'\,(\eta)$, are correctly given in Table 2. Thus, when $S_o < 0$,

$$l_{2-1} = \frac{y_o}{S_o}\,[\eta_2 - \eta_1 - (1 - \beta)\,(B'\,(\eta_2) - B'\,(\eta_1))] \quad \ldots\ldots\ldots (44)$$

which is fully commensurate with Equation (42), and as such is offered in preference to Equation (7). The form of the function, $-\int_0^{\eta} \frac{d\eta}{\eta^n - 1}$, for values of S_o and η^n greater and less than zero is indicated in Fig. 11.

The writers believe that adherence to the foregoing rigorous and general development will avoid confusion in the future treatment of varied flow; this development will also explain the apparent dissimilarity in the equations for adverse and sustaining slopes in the paper.

In closing, two essential points already discussed at length by Professor Bakhmeteff[2] might well be emphasized. First, the development of the varied flow relationships is based entirely upon the assumption of motion in which the curvature of the stream lines is not appreciable, and in which the pressure distribution, therefore, is hydrostatic. The surface curve is thus a function of varying resistance and is independent of the dynamic effects of rapid acceleration or deceleration. In the neighborhood of the critical depth, however, curvature actually becomes of major importance, and only because of the great length of channel involved in the resistance curve can this phenomenon of local transition be disregarded.[28] Second, success in applying these relationships in the accurate design of channels depends entirely upon the proper determination of the Chezy coefficient, C. At present, this must depend upon one or another of the several empirical relationships for C, not one of which can stand a rational analysis. Only when resistance in open channels is as well understood as that in circular pipes can the treatment of open-channel flow be improved by more sound physical relationships; however, although pipe resistance depends largely upon wall roughness, the open channel introduces as further variables the cross-sectional form and the non-uniformity of longitudinal profile, both of which can influence C to an appreciable degree. This is probably the most fertile field of present-day hydraulic research—surely the most imperative.

Arthur E. Matzke,[29] Jun. Am. Soc. C. E. (by letter).—The case of varied flow in open channels of adverse slope occurs rather seldom in practice. The extent of the discussion, therefore, was rather surprising and most pleasing. The rarity of occurrence also made the fact that the method presented in the paper was applied in such an important structure as the Cape Cod Canal even more gratifying. The kind reference made by Captain E. C. Harwood[30], Corps of Engineers, U. S. Army, would seem to offer ample compensation for the time spent.

The approach of most of the discussers was from a rather general point of view without direct reference to any particular practical application. In fact, much of the discussion extended beyond the scope of the problem actually treated and compared methods to be used in handling general cases of varied flow. Messrs. Stevens and von Bergen centered their remarks on the relative merits of computations by means of the varied flow function by comparison with the method of successive approximations.

Historically, the method of successive approximations begins with the work of Belanger (1827). In fact, his treatment is so complete that, fundamentally, nothing has been added since. It is interesting to note, however, that the tendency and the endeavor in the past hundred years on the part of Belanger's pupils and legatees was to substitute for the treatment by approximations a more analytical solution obtained by integrating the varied flow equation. The work of Dupuit (1848), Bresse (1860), Rühlmann (1880),

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[28] See "Discharge Characteristics of the Free Overfall", by Hunter Rouse, *Civil Engineering,* April, 1936, p. 257.

[29] Research Asst., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[30] "Proposed Improvement of the Cape Cod Canal", *Transactions,* Am. Soc. C. E., Vol. 101 (1936), p. 1473.

Tolkmitt (1892), Schaffernak (1914), and Professor Bakhmeteff (1932), might be viewed as the gradual development of this idea. Indeed, Professor Mavis seems to characterize this application of integral calculus as nothing more than a convenient method of summation of a column of figures that represent some regular, analytical (or graphical) array of numbers.

There is another point which should be mentioned in the hope of clearing up a misconception. The comparison of methods by Messrs. Stevens and Mavis is presented principally with regard to natural watercourses. As far as the paper is concerned, natural watercourses were not mentioned. In fact, Professor Bakhmeteff specifically mentions the method of successive approximations as particularly applicable to computing back-water curves in rivers. It may be interesting to remember that, in the past, back-water curves in rivers constituted the principal practical subject in the realm of varied flow. However, at present, the complex cases arising from irrigation, water power, and navigation projects, with canals working under different levels and discharges, have practically transferred the problems connected with them into a different field. As an example, one may cite the delivery problem (Example 3). Any one who would try to solve it by approximations instead of the varied flow function would soon realize the actual state of affairs.

The writer is grateful to Messrs. Stevens and Howland for demonstrating the inaccuracy in part of the tables of values of the varied flow function. The simple graphical method which was used in obtaining these tables is presented in the paper. This was done to enable those who found the tables inadequate for their purposes to obtain satisfactory values. Since the method was graphical, the limit of accuracy of the values is a function of the scale of the drawing.

In deriving Equation (21), which is described as "perfectly general", Mr. Stevens refers to Fig. 7(a) which depicts a sustaining slope. He then writes a formula (Equation (20)) in which $\Delta\epsilon$ is defined as "the loss of specific energy in the reach, Δx." Consideration of the specific energy curve and the fact of the existence of surface curves of the type referred to by Professor Bakhmeteff as S_1, S_2, S_3, and M_3 shows conclusively that it is possible for $\Delta\epsilon$ to be a gain in specific energy as well as a loss. Wherever the work done by gravity in transporting a quantity of water over the distance, Δx, is greater than the work done by friction, etc., in the same process, it follows necessarily that the difference between the two must be stored in the water in the form of an increase of specific energy. This is fundamental and highly important. In Fig. 1 and throughout the paper, S_0 is a symbol for bottom slope just as it is in Mr. Stevens' discussion, and does not refer to a "normal slope."

Thanks are due Mr. Lenz for his investigation and verification of the exponential relation between K and y. The regular behavior of his discharge rating curve is most convincing. It answers in part, at least, the doubts concerning the behavior of n expressed in other discussions. The writer is also deeply grateful to Messrs. Rouse and White for their truly rigorous development and generalization of the solution.

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Paper No. 1970

SURFACE AND SUB-SURFACE INVESTIGATIONS, QUABBIN DAMS AND AQUEDUCT

A SYMPOSIUM

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Note.—Published in March, 1936, *Proceedings*.

FOREWORD

By Frank E. Winsor[1], M. Am. Soc. C. E.

The Metropolitan Water District is one of the oldest districts of its kind in the United States, having been established in 1895, following previous investigations which had shown conclusively that many of the communities of which Boston, Mass., is the center could no longer cope with the problem of independent water supplies. Metropolitan Boston is the fourth most populous center in this country, and probably has more independent political units than any other. In addition to 35 cities and towns within a 10-mile radius of the State House in Boston, there are many other communities with populations of from 15 000 to 100 000 within 30 miles. The Metropolitan Water District originally included Boston and thirteen other municipalities to which seven have since been added. Since 1919 this District has been administered by the Metropolitan District Commission. A population of about 1 500 000 is being supplied with Metropolitan water, 15 communities with a population of about 400 000 are entitled legally to join the District, and it may be reasonably expected that other communities with substantial populations will be permitted to join when their needs demand.

From 1895 to 1905 the District acquired the collecting sources and aqueducts owned by the City of Boston and extended the water supply for the District from the Sudbury River sources westward to the Wachusett Reservoir on the Nashua River, at Clinton, Mass., this reservoir having, since completion, furnished the major portion of the water for the District. Water is supplied by wholesale to each community and the community distributes it, fixes its own water rates, and pays its own bills. Payment for the water is based, two-thirds on the metered quantity used and one-third on assessed valuation, all the water being metered.

In 1926, following about eight years of investigation and reports, the Legislature created the Metropolitan District Water Supply Commission, charged mainly with the construction of a major addition to the water supply to be obtained from the Ware and Swift Rivers, which rivers form a part of the drainage area of the Connecticut River. The diversion from these rivers was opposed by the State of Connecticut, and following several years of litigation the Supreme Court of the United States, in March, 1931, fixed certain conditions under which the diversion is being effected with due regard to interstate rights.

The main features of the additional supply, the estimated cost of which was $65 000 000, are: (1) A tunnel, known as Quabbin Aqueduct, 24.6 miles in length, of the horseshoe type, with a waterway of 127 sq ft, extending westerly from Wachusett Reservoir to the Swift River; and (2) a reservoir, known as Quabbin Reservoir, in the valley of the Swift River, which reservoir will flood 25 200 acres to an average depth of about 50 ft, the capacity being 415 000 000 000 gal, or 1 270 000 acre-ft. At about midway of its length the

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[1] Chf. Engr., Met. Dist. Water Supply Comm., Commonwealth of Massachusetts, Boston, Mass.

Quabbin Aqueduct passes 260 ft below the Ware River where diversion is effected through a shaft. After the completion of Quabbin Reservoir, water from the Ware River may be diverted either easterly or westerly and stored in either the Wachusett Reservoir or the Quabbin Reservoir. The Quabbin Aqueduct will be controlled by gates at Wachusett Reservoir, the relative elevations, in feet, above tide-water being: Wachusett, 395; Ware, 656; and Quabbin, 530.

The Ware River was first diverted in March, 1931, and the Quabbin Aqueduct was completed in 1935 sufficiently to permit diversion of the Swift River although such diversion is not contemplated unless the needs of the District require it, until the Quabbin Reservoir is completed probably late in 1939. The construction program has been planned to keep pace with the increasing demands for water and to delay capital expenditure and consequent increase in carrying charges as long as prudently possible.

The paper by Stanley M. Dore, M. Am. Soc. C. E., relates mainly to the foundations of the two earth dams of the Quabbin Reservoir, one known as the Dike with an embankment of about 2 500 000 cu yd, and the other known as the Main Dam with an embankment of about 4 000 000 cu yd. The valleys in which these dams are located are both filled with porous glacial drift to a maximum depth of 125 to 130 ft, and it was deemed necessary to build cut-off walls to rock to prevent seepage. The work of building these cut-off walls by the sinking of concrete caissons under air pressure, and the lowering of the ground-water by pumping, to reduce the air pressure required, are described in detail in Mr. Dore's paper. The work on these two dams illustrates the importance of thorough preliminary exploration. The feasibility of lowering the ground-water was demonstrated by exploratory caissons at both sites before bids were invited for building these structures. The uncertainties as to air pressure which would be required to sink the caissons, were eliminated to a great extent, resulting in more dependable and lower bids from contractors and a consequent material saving in the cost of the work to the Commission. So far as the writer is aware, unwatering of the ground on the scale and to the depths accomplished at these two dams is somewhat unique. The paper is a valuable contribution to the knowledge of the flow of ground-water through materials of varying permeability.

The paper by Frank E. Fahlquist, Assoc. M. Am. Soc. C. E., relates mainly to the geological and engineering investigations which were made preliminary to the location of the tunnel and to the tunnel construction. Adequate geological investigations coupled with intelligent sub-surface exploration are essential as a guide to the engineers in the location of such structures as the Quabbin Aqueduct and the dams of Quabbin Reservoir. The location of Quabbin Aqueduct was determined after a most thorough geological investigation supplemented by borings to determine the location and character of ledge rock. Several pitfalls which might have been very costly were thereby avoided, and in the tunnel excavation excellent rock was encountered for the entire 25 miles. with a negligible amount of timber support.

PERMEABILITY DETERMINATIONS, QUABBIN DAMS

By Stanley M. Dore[2], M. Am. Soc. C. E.

Synopsis

Several methods of determining the permeability of an earth over-burden which forms the foundation for a proposed dam are outlined in this paper. They are based on the assumption that either or both of two general sources of information is available: (1) Dry samples from bore-hole investigations; and (2) the effect of pumping upon the ground-water conditions in that overburden. These methods were developed in connection with permeability determinations at the sites of the Main Dam and Dike of Quabbin Reservoir,[3] 65 miles west of Boston, Mass., so that in order to describe fully and clearly the methods and their practicable applications much of the text deals with the particular conditions at those sites. The methods were developed primarily for use with materials of glacial origin, in cases where the earth cover is thick and the ground-water surface high in that earth cover, and if carefully applied, they can be adapted to give reasonable determinations under similar conditions elsewhere. Moreover, the principles involved may be adapted for use in cases where the materials are of a different nature, or where some of the other conditions are dissimilar.

Introduction

A major item in the construction of the Ware River and Swift River developments[3] for increasing the water supply for the Massachusetts Metropolitan Water District, which is comprised of the City of Boston and nineteen neighboring cities and towns, is the Quabbin Reservoir, formed by the construction of two similar earth dams named, for convenience, the Main Dam and the Dike. These two dams have been located in the low portions of the rim of the storage basin, the former being across the valley of the Swift River and the latter across the valley of Beaver Brook, a branch of the Ware River. Both of them are designed as earth dams to be placed by hydraulic methods, the Main Dam being about 2 640 ft long, 160 ft high above the original ground surface, and containing 4 000 000 cu yd of embankment, and the Dike being about 2 140 ft long, 135 ft high above the original ground surface, and containing 2 500 000 cu yd of embankment.

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[2] Associate Civ. Engr., Massachusetts Met. Dist. Water Supply Comm., Boston, Mass.

[3] Described by Karl R. Kennison, M. Am. Soc. C. E., in "Boston Metropolitan Water Supply Extension", *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2, and by Frank E. Winsor, M. Am. Soc. C. E., in "Boston's New Water Supply", *Civil Engineering*, June, 1934, p. 283.

The foundations for these dams are in glacial materials, the deepest point in the underlying rock gorge at the Main Dam being about 120 ft below the river level and the Dike, 130 ft below the brook level. The ground-water surface in the valleys is approximately at the river or brook level for some distance on either side of the stream.

The location of bed-rock and the character of the over-burden were investigated at both sites by a large number of bore holes, which were made by driving $2\frac{1}{2}$-in. pipes by wash-boring methods supplemented by core-boring methods through the larger boulders or into ledge. So-called dry samples of the over-burden were obtained about every 5 ft by driving a sampling tool into the materials below the bottom of the pipe-casing. Cores of boulders and ledge also were secured.

The rock valley cross-sections (shown in Figs. 1 and 2) are found to be the usual smooth, rounded, inverted-arch shape of valleys eroded by glacial action. In general, the rock is not decayed; nor is it badly seamed or broken. The types of rock appear to be gneisses, ranging from hornblendic to granitic types, but generally hard and sound.

The dry samples taken show the over-burden to be a glacial drift accumulation, mostly silts, sands, gravels, and boulders or mixtures of them. Later excavations show that these materials lie in pockets or lenses generally horizontal, but absolutely void of any stratification or orderly relation. Occasional streaks or pockets of rock flour were present usually running through the finer sandy deposits, and streaks, pockets, and lenses of very coarse gravel exist elsewhere. Geological sections are shown in Figs. 3 and 4. The names given to the numbered classes, which are those of the Kendorco classification described subsequently herein, in these sections differentiate them according to their relative pervious qualities, and each "variable" class contains a considerable number of larger particles. The meaning of the term, "uniform", is that only a small percentage of the material has particles larger or smaller than the average. The meaning of the term, "variable", on the other hand, is that a considerable number of particles is larger and smaller than the average. These materials are such that they would offer firm and stable support for the earth dams, but the desirability of a cut-off to ledge through these deep over-burdens depends upon their water-tightness. Studies were made to investigate their permeable qualities. The rock in the valley floors, being sound, is assumed to be water-tight, as its water-carrying capacity is negligible compared to the capacity of any of the materials in the over-burdens.

Preliminary investigations indicated that a water-tight cut-off would probably be desirable. Various types of cut-offs were considered, including open-cut trench to ledge, the driving of steel sheet-piling to ledge, the grouting of the foundation materials with cement grout or dobie, a concrete core wall to ledge by sheeted trench methods, and a concrete core wall to ledge by open or by pneumatic caisson construction. Preliminary estimates indicated that some kind of caisson core-wall construction would probably furnish the most economical cut-off.

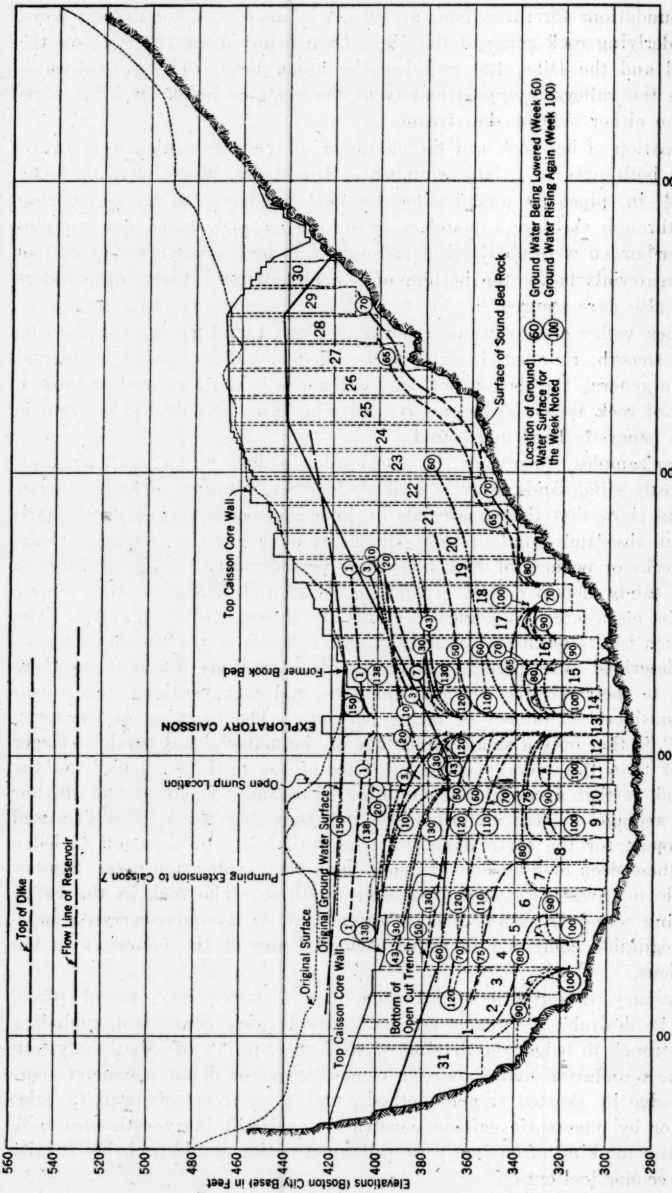
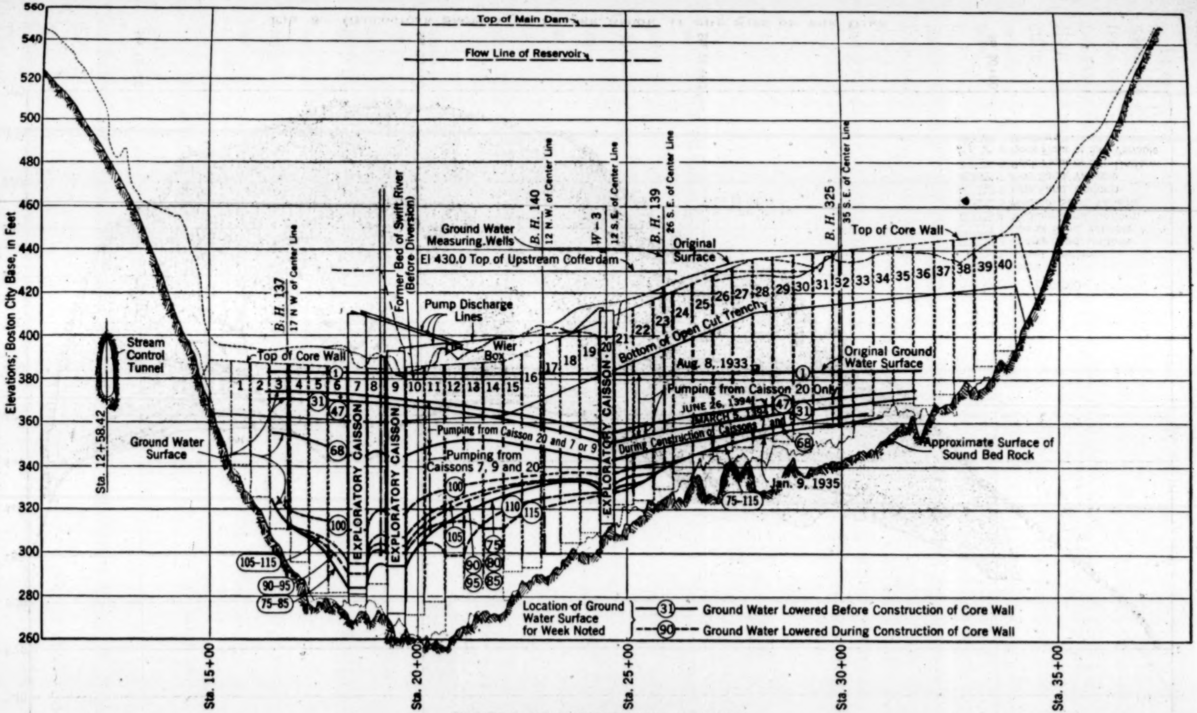


FIG. 1—CROSS-SECTION OF BEAVER BROOK AT THE SITE OF QUABBIN DIKE


Top of Main Dam
Flow Line of Reservoir
Elevations; Boston City Base, in Feet
560
540
520
500
480
460
440
420
400
380
360
340
320
300
280
260
Stream Control Tunnel
Sta. 12+58.42
Ground Water Surface
Top of Core Wall
B. H. 137
17 N W of Center Line
Former Bed of Swift River (Before Diversion)
Pump Discharge Lines
Wier Box
Ground Water Measuring Wells
El 430.0 Top of Upstream Cofferdam
B. H. 140
12 N.W. of Center Line
W – 3
12 S.E. of Center Line
B. H. 139
26 S.E. of Center Line
Original Surface
B. H. 325
35 S.E. of Center Line
Top of Core Wall
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
EXPLORATORY CAISSON
EXPLORATORY CAISSON
EXPLORATORY CAISSON
Bottom of Open Cut Trench
Aug. 8, 1933
Original Ground Water Surface
Pumping from Caisson 20 Only
JUNE 26, 1394
MARCH 5, 1934
During Construction of Caissons 7 and 9
Pumping from Caisson 20 and 7 or 9
Pumping from Caissons 7, 9 and 20
Jan. 9, 1935
Approximate Surface of Sound Bed Rock
①
31
47
68
100
105
110
115
75
80
90
95
85
105–115
90–95
75–85
75–115
Location of Ground Water Surface for Week Noted
31 Ground Water Lowered Before Construction of Core Wall
90 Ground Water Lowered During Construction of Core Wall
Sta. 15+00
Sta. 20+00
Sta. 25+00
Sta. 30+00
Sta. 35+00

PROFILE ON CENTER LINE OF DAM
LOOKING UPSTREAM

FIG. 2.—CROSS-SECTION OF SWIFT RIVER VALLEY AT THE SITE OF THE MAIN DAM

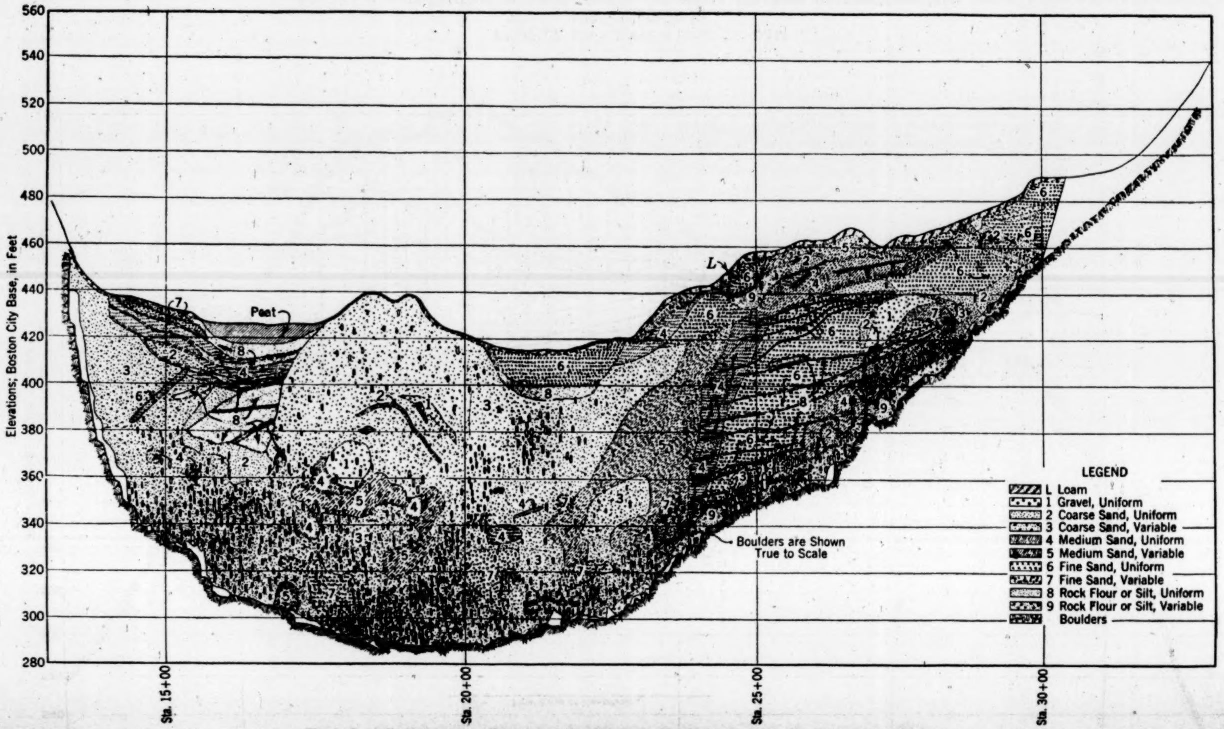

Elevations; Boston City Base, in Feet
560
540
520
500
480
460
440
420
400
380
360
340
320
300
280
Sta. 15+00
Sta. 20+00
Sta. 25+00
Sta. 30+00
Peat
L
Boulders are Shown True to Scale
LEGEND
L Loam
1 Gravel, Uniform
2 Coarse Sand, Uniform
3 Coarse Sand, Variable
4 Medium Sand, Uniform
5 Medium Sand, Variable
6 Fine Sand, Uniform
7 Fine Sand, Variable
8 Rock Flour or Silt, Uniform
9 Rock Flour or Silt, Variable
Boulders

FIG. 3.—GEOLOGICAL SECTION OF BEAVER BROOK AT THE SITE OF THE DIKE

In order to investigate, further, the water-tightness and other qualities of materials in the over-burden, and in order to study the ground-water conditions and the feasibility and cost of various types of caisson construction, two exploratory reinforced concrete caissons were sunk, one at the site of the Main Dam and the other at the Dike site, in such positions that each could be utilized as a part of the finished core-wall. The materials excavated from these caissons were examined very thoroughly, and it was possible to obtain much more satisfactory information regarding the characteristics of the over-burden, including data concerning the number and sizes of boulders and cobbles, than it was from the small bore-hole samples. Later, after the core wall of the Dike had been completed, two additional exploratory caissons were sunk at the site of the Main Dam to assist in the exploratory studies of over-burden and ground-water pumping. Figs. 1 and 2 show the location of the exploratory caissons and Fig. 4 shows a chart of the materials excavated from them.

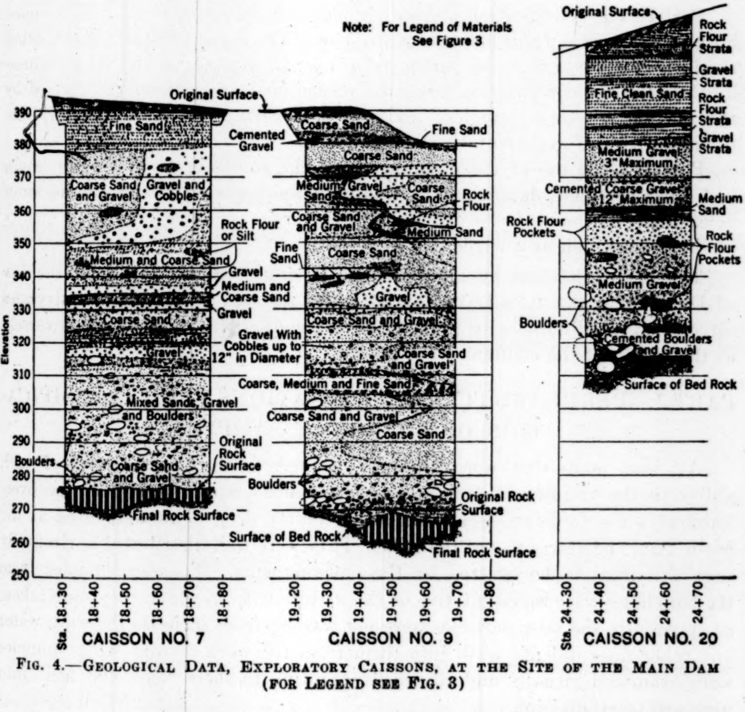

Note: For Legend of Materials See Figure 3
Elevation
390
380
370
360
350
340
330
320
310
300
290
280
270
260
250
Original Surface
Fine Sand
Coarse Sand and Gravel
Gravel and Cobbles
Rock Flour or Silt
Medium and Coarse Sand
Gravel
Medium and Coarse Sand
Gravel
Coarse Sand
Gravel
Mixed Sands, Gravel and Boulders
Boulders
Coarse Sand and Gravel
Original Rock Surface
Final Rock Surface
Sta. 18+30
18+40
18+50
18+60
18+70
18+80
CAISSON NO. 7
Original Surface
Cemented Gravel
Coarse Sand
Fine Sand
Coarse Sand
Medium Gravel
Sand
Coarse Sand
Rock Flour
Coarse Sand and Gravel
Medium Sand
Fine Sand
Coarse Sand
Gravel
Coarse Sand and Gravel
Gravel With Cobbles up to 12" in Diameter
Coarse Sand and Gravel
Coarse, Medium and Fine Sand
Coarse Sand and Gravel
Coarse Sand
Boulders
Original Rock Surface
Surface of Bed Rock
Final Rock Surface
19+20
19+30
19+40
19+50
19+60
19+70
19+80
CAISSON NO. 9
Original Surface
Rock Flour Strata
Gravel Strata
Fine Clean Sand
Rock Flour Strata
Gravel Strata
Medium Gravel 3" Maximum
Cemented Coarse Gravel 12" Maximum
Medium Sand
Rock Flour Pockets
Rock Flour Pockets
Medium Gravel
Boulders
Cemented Boulders and Gravel
Surface of Bed Rock
Sta. 24+30
24+40
24+50
24+60
24+70
CAISSON NO. 20

FIG. 4.—GEOLOGICAL DATA, EXPLORATORY CAISSONS, AT THE SITE OF THE MAIN DAM (FOR LEGEND SEE FIG. 3)

Pumps were installed in caissons at each site, and the effect of pumping the surrounding ground-water from these caissons was noted by reading the water levels in 2½ - in. pipe wells driven by wash-boring methods at designated locations in the valley up stream and down stream from the pumping plants. The levels of the water were read several times each week by lowering a cup-shaped washer into the wells. In addition to aiding in the determination of the effective water-tightness of the glacial materials in the valley over-burden, the effect of pumping also demonstrated the feasibility of unwatering the over-burden, and the extent to which it would be practicable and economical so to do.

As used in this paper, the term, "permeability", measured in units of million gallons per acre per day at 50° F, for a slope of unity, denotes the rate at which soils, sands, and gravels will transmit water. "Porosity" is the expression used to denote the percentage of a given volume composed of air and water pockets or "voids" in the materials in that volume.

CHARACTERISTICS OF THE OVER-BURDENS

The permeability of the over-burden at each site is determined by two general methods given in Parts I and II, as follows:

Part I: The classification of bore samples and the estimation of the permeability of each class from laboratory analyses. There are two methods of using the laboratory data to make partly independent estimates of the class permeabilities: (*a*) By computation from the mechanical analysis curves derived by the sieving of class samples; and (*b*) by testing in the laboratory the permeabilities of representative class samples.

Part II: The use of the pumping rates, the resulting ground-water levels, and other pertinent data in connection with the lowering of the ground-water at each site by pumping to determine the average effective permeable quality of the over-burden as a whole.

The determinations by all the methods described are based on the law of Darcy[4] that the velocity of flow of water through a column of soil varies directly as the difference in pressure on the ends of the column and inversely as the length of the column.

PART I.—PERMEABILITY DETERMINATIONS FROM CLASSIFICATION OF BORE-HOLE SAMPLES

All bore holes that were properly situated in the deeper parts of each valley in the vicinity of each site were taken as representative of the over-burden in the foundations at that site. Thirty deep holes were used at the Main Dam and thirty-five at the Dike. They were well distributed horizontally over the areas to be covered by the embankments. The dry samples from the bore-holes were representative of the materials from which they were taken, as apparently the finer particles were not washed from them by the wash water and other fines did not wash into them from the work above. These samples were examined visually and classified according to their "look and feel" into nine arbitrary divisions.

THE CLASSIFICATION

This classification is named the "Kendorco Classification" and was developed for use on this project to denote and describe roughly the general kinds of the glacial soils in the vicinity. Of course, there are no distinct divisions between the classes, but attention is called to the general differences in physical characteristics given in Table 1 and to the arbitrary limits set for the classes as shown in Fig. 5. The types of material in Table 1 may be further described, as follows:

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[4] "Les fontaines publiques de la ville de Dijon", by H. Darcy, 1856.

TABLE 1.—PHYSICAL CHARACTERISTICS OF SOIL SAMPLES

Class	Name of class	AVERAGE SIZES OF A TYPICAL SAMPLE, IN MILLIMETERS			Computed permeability, in million gallons daily per acre, at 50° F for slope of unity	AVERAGE OF LABORATORY DETERMINATIONS		
		Range of bulk of the sample	Effective size; 10% of the sample	Smaller size		Porosity (percentages)	Permeability in Million Gallons Daily per Acre, at 50° F (in Natural State)	
							Main Dam	Dike
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Clean gravel......	10.0 to 2.0	1.50	4 %<0.3	540	32	540	....
2	Coarse sand, uniform...........	1.5 to 0.3	0.31	4 %<0.15	70	46	110	....
3	Coarse sand, variable...........	Variable	0.35	4 %<0.15	12	25	15	12
4	Medium sand, uniform...........	0.7 to 0.2	0.15	4 %<0.07	20	49	38	14
5	Medium sand, variable...........	Variable	0.175	4 %<0.07	4	27	3.0	3.5
6	Fine sand, uniform	0.3 to 0.1	0.065	6 %<0.03	2.5	43	2.0	1.5
7	Fine sand, variable	Variable	0.075	6 %<0.03	0.8	27	0.8	0.7
8	Flour, uniform....	0.1 to 0.02	0.013	8 %<0.01	0.1	41	0.15	0.2
9	Flour, variable....	Variable	0.025	8 %<0.01	<0.1	24	<0.1	<0.1

Class Description

1.—A material that contains large particles of gravel and very coarse sand, and very few fines.
2.—A uniform coarse sand that contains only a few fines and no gravel or stones.
3.—A material of which the part controlling the permeability is coarse sand, but in which there are a few fines, some gravel, or both.
4.—A uniform medium sand containing but few fines and no gravel or stone.
5.—A material of which the part controlling the permeability is medium, but in which there are finer and coarser sands and possibly gravel or stones.
6.—A uniform fine sand containing only a small percentage of larger or smaller grains than the average.
7.—A material of which the part controlling the permeability is a fine sand but in which there is a large percentage of coarser materials, possibly gravel or stone.
8.—A rock flour or silt containing only a small percentage of grains larger than the average.
9.—A material of which there is a large quantity of rock flour or silt but in which there is a large percentage of coarser materials, possibly gravel or stone.

The values in Column (6), Table 1, were computed for the tested porosity shown in Column (7). The average of all determinations for porosity was 35%; the straight average porosity of the nine classes was also 35%;

the weighted mean porosity for material at the Main Dam was 37% and at the Dike, 39%; and the mean effective porosity (obtained from the permeability of the over-burden as a whole) at the Main Dam was 33%, and at the Dike, 37 per cent.

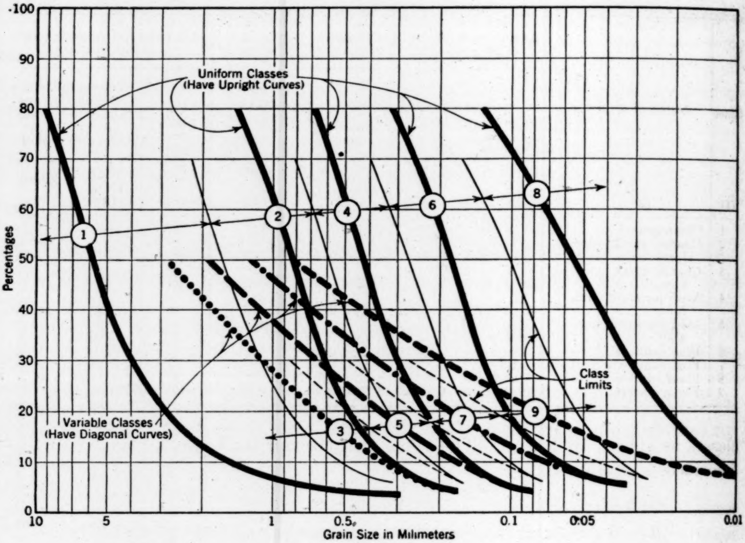

FIG. 5.—DEFINITICNS OF LIMITS, KENDORCO SOIL CLASSIFICATION

When the classification of all dry samples was complete, five were chosen carefully from each class at each site to show the range of all the samples of that general class. In choosing these samples, three were selected to represent the average of the entire classification, a fourth the coarser extreme, and the fifth the finer extreme. Sieve analyses were made on the five samples of each class (forty-five samples for each site). Such data were used as the basis for the diagram of class curves shown in Fig. 5. This system of general classification is easy to apply and fairly accurate in its results, even when applied by those relatively inexperienced with it, although there are always certain to be differences in opinion regarding border-line samples.

CLASSIFICATION OF THE OVER-BURDENS

Materials in the over-burdens at the Main Dam and at the Dike were assigned in both plan and elevation to the nine classes. This assignment was made according to the number of linear feet of the bore holes of each class of material encountered, each hole being weighted according to its plan location so that it represents its share horizontally of the entire valley.

PERMEABILITY OF THE CLASSES (COMPUTED FROM SIEVE ANALYSES)

The permeability of each class of material was computed from the mechanical analyses curves representing the averages of those classes by use of the formula introduced by Charles S. Slichter.[5] In applying Slichter's formula for determining ground-water flow, it has been the practice of many engineers to use erroneously in place of the "mean diameter" defined by Slichter, the effective size (10% size) defined by the late Allen Hazen,[6] M. Am. Soc. C. E., although the two, by definition, are different. Stearns,[7] King,[7] Melcher,[7] Slichter, and others, failed to find any consistent agreement between the 10% sieve size and the coefficient of permeability for most types of mixed sands and gravels. It is believed that the 10% size used by Hazen is applicable only to the filter sands with which he worked, and that for "variable" glacial materials the controlling size depends upon local characteristics of the materials, such as the shape of grains and the combination of sizes.

If permeability of a material or of an over-burden is to be computed by this formula it is recommended that, if practicable, many permeability tests be made on samples from that over-burden to determine the size, from sieve analyses curves, that will give dependable results when used instead of the mean diameter in Slichter's formula, otherwise, the determination may be very misleading.

Several hundred permeability tests were made on materials from overburdens of the Main Dam and the Dike, at known porosities and temperatures, and a sieve analysis curve of each of the samples was plotted. The size producing the tested permeability was obtained by substituting the permeability result in Slichter's formula in each case. The percentage grain size was chosen by inspecting the sieve analysis curve of that sample. From the large number of tests and inspections made, the size to be substituted in the formula to produce the tested permeabilities varied for these sites from the 5% to the 26% size, although the majority of the results were confined to closer limits, between the 8% and the 15% size. For these sands and gravels, the reasonable mean of all results seems to be the 12% size.

In order to compute the permeability of any material from its mechanical analyses curve, its porosity must be known. A large number of field tests were conducted at many locations in the excavations at the sites of the Main Dam and the Dike to determine the porosity of the materials in the overburden in general and particularly to determine the proper porosity that will be applicable to each classification. The results of these tests are shown in Table 1.

The temperature of the ground-water for any locality must also be known, and for these locations it was approximately 50° F. Fifty-six observations of ground-water flowing into the Quabbin Tunnel, which runs about parallel to the sites and about 6 miles to the north, were made at locations 80 to 600 ft below the ground surface and at intervals in a length of 10 miles of the tunnel. The average of all observations was 51° F, the lowest being 48° F, and the highest, 53½° F. The water pumped from the caissons at both the Main Dam and the Dike maintained a fairly constant temperature very close to 50° F, throughout the year (between 48° and 52°). Therefore, 50° F would be the proper value to use, especially as it is approximately the mean annual temperature for the climate of this region.

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[5] *Water Supply Papers Nos. 67* and *140*, U. S. Geological Survey.

[6] Rept., Mass. State Board of Health, by Allen Hazen, 1892.

[7] See *Water Supply Paper 596-F*, by N. D. Stearns.

The permeabilities of the nine classes were then computed for the tested porosities at a temperature of 50° F, using Slichter's formula and the 12% size from the theoretical curve for each class as the mean diameter (such curve representing the average of materials in that class). The results of such computations are shown in Table 1.

TESTED PERMEABILITY OF THE CLASSES

A large number of samples of materials belonging to each of the nine classes was taken directly from open-cut, or caisson, excavations in the over-burden of the Main Dam and Dike and were tested for permeability. Care was taken to select samples representing the class averages. For the most part these were small can tests made at 68° F, the material being packed by hand to known porosities with as little mixing as possible.

Several permeability tests of each sample were made at different porosities, and a permeability-porosity curve for that sample was plotted. From this curve, the permeability was read for the average porosity of that class previously determined, and the result was corrected[8] to a temperature at 50° F. The permeabilities thus obtained are shown in Table 1.

PERMEABILITY OF THE OVER-BURDEN FROM THE CLASSIFICATION PERMEABILITIES

The over-burdens were composed of heterogeneous glacial materials, in the valley inverts being forms of modified drift. The various classes of materials were not laid in strata or layers or with any order at all, but seem to be deposited in random lenses or pockets. In view of this lack of order, the question arises (assuming that the permeabilities of the respective classes and quantities of each class of material in the over-burden are satisfactorily computed or known) as to how best to obtain the permeability of the over-burden as a whole. For instance, the maximum total permeability of a given quantity made up of an equal quantity of each class would be for a condition in which the amounts of each class are laid in horizontal layers parallel to the flow, and the minimum would be for a condition in which the quantities are placed in vertical layers perpendicular to the flow. In the former case, herein termed "in parallel", the direct sum of the quantities carried by each layer will give the total carried by the mass, and from this the unit permeability of the mass can be computed. The average effective unit permeability of the whole mass, for that case, will be equal to the unit permeabilities of the component parts averaged and weighted in accordance with the relative quantities of the parts. In the latter case, herein called "in series", the average effective permeability of the mass is only a fraction of the "parallel" quantity, as the finer layers control the amount of flow through the coarser layers and prevent them from carrying larger quantities. However, any contact or linking of coarser classes, although by links of relatively smaller volumes, will so increase the effective permeability of the entire mass over that obtained by the "series" method that such permeability will be found to be a very large percentage of the "parallel" method, and except for extremely small linking volumes, this percentage is very close to 100. In random glacial deposits there is usually no uniform tendency for complete separation of the coarser parts by the finer. On the other hand, during the period of deposition and in the following washing periods, water tends to open passages of flow between pockets, if they do not already exist. Therefore, it is believed that approximately close results can be obtained for glacial materials if the average effective permeability is computed from the unit class permeabilities by the "parallel" combination. Such an approximation is on the safe side as far as percolation or seepage quantities are concerned, and probably is not far from being representative of true conditions in the over-burden.

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[8] Correction factors are given in "Smithsonian Physical Tables—Viscosity of Water in Centipoises *vs.* Temperature Variation", the permeability being varied inversely with the viscosity.

As the permeability of the over-burden as a whole is so dependent upon the manner in which the permeabilities of the various classes are combined, too much emphasis cannot be placed upon the importance of obtaining as much information as practicable regarding the existence and manner of occurrence of the various classes in the over-burden.

In the case of these sites, investigations of existing excavations in the open and in the caissons, of the bore-hole data, and of other geological data, indicate that probably numerous interconnections and intermeshings of the coarser lenses, streaks and pockets with the finer exist, and that there is no tendency toward segregation of lenses in such manner that the finer would separate and isolate the coarser. Thus, the resulting permeabilities at the sites of the Main Dam and Dike are estimated, using 100% of the results obtained by the "parallel" method.

Although the results of these classification methods may be open to criticism owing to the many arbitrary assumptions necessary to obtain them, it is believed that such results are adequate in this case and will be in similar cases, as the results for both these sites are confirmed by the determinations of permeability made by the pumping methods described subsequently herein, under Part II. The pumping results do not depend upon arbitrary assumptions or decisions and, therefore, this confirmation of results may be used as an independent check of those made for the bore-hole classification methods. The over-burdens at these sites have no conditions which are unusual or uncommon to glacial-formed valley deposits and, consequently, there seems to be no reason why such similar methods will not give equally good results elsewhere.

Although such methods are approximations in many respects and although the computed results cannot be treated with too much exactness, they can be used to obtain more comprehensive opinions of the permeability of any extensive glacial materials than any other methods known to the writer except the pumping methods described subsequently. Any determination should be used within the limits of accuracy considered reasonable, and for permeability of an over-burden these limits are quite elastic. For instance, limits between 50% and 200% of a result computed by these methods may not be unreasonable, although the writer believes that the results for these sites and for other similar sites may be computed within much closer limits. However, even a result with wide elastic limits is much better than no determination and to the engineer considering seepage possibilities, such limits are usually close enough for his purposes.

PART II.—PERMEABILITY DETERMINATIONS FROM EFFECTS OF GROUND-WATER PUMPING OPERATIONS

Pumping at the Dike Site

For examining the materials in the over-burden and for experimenting with pumping of ground-water, Caisson 13 was sunk at the Dike in a location where the ground-water surface was practically at the original surface, which is about 130 ft above ledge (see Fig. 1). During the sinking, if material below the ground-water level was being excavated under pneumatic pressures, it was impracticable to lower the ground-water outside the caisson by pumping from within, for two reasons: (1) The presence of air in the pores of the surrounding soils materially reduced the volume of the pores available for the flow of ground-water toward the caisson; and (2) the use of air (probably because it tended to break up and separate the fines) seemed to accelerate the flow of the fine particles near the pumping intakes into the intakes, and thereby clogging them quickly. As a head of water of 130 ft would give unreasonably high pressures for pneumatic work (50 lb per sq in. being the legal limit in Massachusetts), it was necessary to lower the ground-water level near the caisson in order to sink it completely.

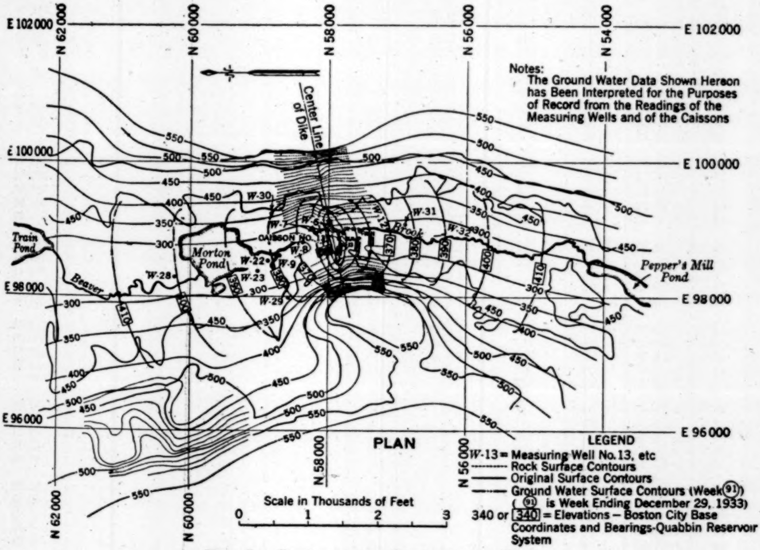
When the caisson had been sunk 63 ft below the brook level, sinking operations were suspended and pumps were installed in the working chamber of the caisson in order that the ground-water level might be lowered to provide for the sinking of an open external sump, to be used for pumping while the caisson was being sunk. The water was pumped from the working chamber of the caisson at that depth at an average rate of about 1 600 gal per min. for a period of about seven weeks. The sump, about 26 ft square, of wooden sheet-piling, was sunk at a location about 100 ft from the caisson, to below the lowered water level, the depth being about 50 ft below the brook level. In that period of seven weeks the water level had been lowered to a depth of about 38 ft below the brook level, the sump being completed in the wet below that depth by additional pumping at a small rate and by driving the sheeting as far ahead of the excavation as boulders would permit.

The pumps were then removed from the caisson and installed in the open sump until after the caisson was completed. Pumping intakes were built under the cutting-edge in the underpinning foundations of the caisson. During this period of about thirteen weeks the water drawn from the open sump averaged about 1 100 gal per min; in the sump, the minimum water level was about 45 ft below, and at the caisson about 30 ft below, the brook level. The highest air pressure used in sealing the caisson to ledge was about 48 lb per sq in. Pumps were then installed in the working chamber of the caisson utilizing the intakes previously built, and the ground-water was drawn from the caisson and the sump jointly for a period of five months at an average rate of about 1 600 gal per min.

At the start of the core-wall construction (January, 1933), greater pumping capacity was installed in Caisson 13 and pumping from the open sump was abandoned. Pumping continued from that caisson for about four months at the increased rate of about 3 200 gal per min, at which time Caisson 18 (see Fig. 1) had been sunk to a depth near ledge. Pumps were installed in the working chamber of Caisson 18 to supplement those of Caisson 13 and water was drawn at an average of about 3 000 gal per min from both caissons (about 2 600 gal from Caisson 13 and 400 gal from Caisson 18). This work was continued for ten weeks more.

During the sinking of Caisson 13, to aid the pumping operations, Beaver Brook was carried past the site in a wooden flume extending from a point 200 ft up stream from the center line of the Dike to a point about 160 ft down stream. During the construction of the core-wall this brook was dammed off about 500 ft up stream, the pumping discharge being deposited 400 ft down stream. The subsequent pumping operations dried up the brook up stream and lowered the level of Morton Pond, which is about 1 500 ft up stream, a maximum of about 9 ft (see Fig. 6).

Other caissons were sunk below the ground-water gradient and more water was pumped from the working chambers at intervals when no material was being excavated. When they were sealed to ledge many of the caissons were equipped with pumping intakes which were also used to lower the ground-water while sinking the other caissons. Ground-water was pumped in this manner from Caissons 18, 16, 8, 6, 4, 10, and 7, at the intervals and in the quantities shown in Table 2(*a*).


E 102000
N 62000
N 60000
N 58000
N 56000
N 54000
E 102000
Center Line of Dike
E 100000
E 100000
Train Pond
Morton Pond
Beaver
Brook
Caisson No.
W-30
W-31
W-7
W-6
W-12
W-32
W-8
W-22
W-9
W-23
W-28
W-29
Pepper's Mill Pond
E 98000
E 98000
E 96000
E 96000
550
500
450
400
350
300
370
380
390
400
410
PLAN
Scale in Thousands of Feet
0
1
2
3
N 62000
N 60000
N 58000
N 56000

Notes:
The Ground Water Data Shown Hereon has Been Interpreted for the Purposes of Record from the Readings of the Measuring Wells and of the Caissons

LEGEND
W-13 = Measuring Well No. 13, etc
------- Rock Surface Contours
——— Original Surface Contours
—·—·— Ground Water Surface Contours (Week (91))
((91) is Week Ending December 29, 1933)
340 or $\boxed{340}$ = Elevations – Boston City Base Coordinates and Bearings-Quabbin Reservoir System

FIG. 6.—PLAN AT THE SITE OF THE DIKE

TABLE 2.—Observed Rainfall and Pumping Rates

Week No.	(a) At the Dike					(b) At the Main Dam				
	Week ending	Total rain-fall, in inches	Average pumping rate, in gallons per minute	Location of pumping; Caisson*:	Elevation of water surface, Morton Pond, in feet	Week ending	Average pumping rate, in gallons per minute	Location of pumping; Caisson†:	Elevation of Water Surface, Swift River, in Feet	
									Up stream	Down stream
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	4-23-32	....	995	13	416.97	8-11-33	......	..	......	......
2	4-28-32	0.38	1 275	13	416.87	8-18-33	1 158	20	......	......
3	5- 5-32	0.80	1 953	13	416.86	8-25-33	2 220	20	383.47	381.57
4	5-11-32	0.39	2 016	13	416.74	9- 1-33	2 224	20	383.24	381.39
5	5-17-32	....	1 820	13	416.61	9- 8-33	2 190	20	383.94	381.89
6	5-23-32	....	1 785	13	416.51	9-15-33	2 269	20	383.40	381.52
7	5-27-32	0.30	1 655	13	416.47	9-22-33	2 245	20	388.12	385.69
8	6- 2-32	0.14	1 332	*S*	416.47	9-29-33	2 262	20	385.20	383.54
9	6- 8-32	0.59	1 370	*S*	416.46	10- 6-33	2 260	20	384.30	382.94
10	6-14-32	0.10	1 360	*S*	416.39	10-13-33	2 260	20	384.79	383.26
11	6-20-32	1.47	1 268	*S*	416.51	10-20-33	2 260	20	384.48	383.04
12	6-29-32	0.43	1 246	*S*	416.42	10-27-33	2 257	20	385.22	383.54
13	7- 5-32	2.03	1 016	*S*	416.48	11- 3-33	2 249	20	385.11	383.46
14	7-11-32	0.77	1 022	*S*	416.46	11-10-33	2 240	20	384.34	382.91
15	7-17-32	0.06	1 032	*S*	416.33	11-17-33	2 210	20	384.38	382.95
16	7-23-32	0.34	1 012	*S*	416.26	11-24-33	1 844	20	384.17	382.81
17	7-29-32	0.51	984	*S*	416.25	12- 1-33	1 796	20	384.40	382.96
18	8- 4-32	1.74	950	*S*	416.27	12- 8-33	1 894	20	384.34	382.92
19	8-10-32	0.38	827	*S*	416.36	12-15-33	1 803	20	384.53	382.82
20	8-16-32	0.25	837	*S*	416.26	12-22-33	2 143	20	......	382.74
21	8-22-32	0.76	1 000	13, *S*	416.19	12-29-33	2 159	20	......	382.5±
22	8-28-32	0.24	1 332	13, *S*	416.13	1- 5-34	2 170	20	......	382.65
23	9- 6-32	1.75	1 189‡	13, *S*	416.05	1-12-34	2 210	20	385.27	383.28
24	9-15-32	....	1 144	13, *S*	415.92	1-19-34	2 087	20	384.59	383.03
25	9-23-32	2.43	2 137	13, *S*	415.90	1-26-34	.....	..	......	......
26	9-30-32	1.37	1 898	13, *S*	415.74	2- 2-34	2 080	20	384.57	......
27	10- 7-32	1.41	1 895	13, *S*	415.55	2- 9-34	2 080	20	384.00	......
28	10-14-32	0.09	1 860	13, *S*	415.32	2-16-34	2 074	20	383.70	......
29	10-21-32	1.81	1 855	13, *S*	415.15	2-23-34	2 070	20	383.60	......
30	10-28-32	0.70	1 855	13, *S*	415.14	3- 2-34	2 058	20	383.64	......
31	11- 4-32	0.80	1 829	13, *S*	415.07	3- 9-34	2 050	20	386.69	384.91
32	11-11-32	2.14	1 778	13, *S*	415.17	3-16-34	2 021	20	385.28	383.62
33	11-18-32	0.54	1 748	13, *S*	415.58	3-23-34	1 940	20	385.94	383.96
34	11-25-32	0.89	1 740	13, *S*	415.74	3-30-34	1 999	20	386.71	384.56
35	12- 2-32	....	1 726	13, *S*	415.59	4- 6-34	2 042	20	388.30	385.83
36	12- 9-32	0.28	1 721	13, *S*	415.36	4-13-34	2 106	20	386.78	384.74
37	12-16-32	0.54	1 723	13, *S*	415.23	4-20-34	2 120	20	387.98	385.61
38	12-23-32	0.33	1 727	13, *S*	414.93	4-27-34	2 036	20	386.39	384.44
39	12-30-32	0.74	1 719	13, *S*	415.06	5- 4-34	2 009	20	385.36	383.61
40	1- 6-33	....	1 694	13, *S*	415.32	5-11-34	1 999	20	386.21	384.37
41	1-13-33	0.97	1 699	13, *S*	415.27	5-18-34	1 901	20	385.88	384.00
42	1-20-33	0.86	1 817	13, *S*	415.33	5-25-34	1 900	20	384.84	383.30
43	1-27-33	0.59	1 334	13, *S*	415.68	6- 1-34	1 900	20	384.63	383.15
44	2- 3-33	0.22	1 262	13	415.82	6- 8-34	1 983	20	384.28	382.88
45	2-10-33	1.48	1 517	13	415.87	6-15-34	1 917	20	383.95	382.63
46	2-17-33	0.54	2 353	13	416.19	6-22-34	1 933	20	384.53	382.93
47	2-24-33	1.01	2 661	13	416.34	6-29-34	1 335	9, 20	384.57	383.01
48	3- 3-33	1.23	2 583	13	416.64	7- 6-34	2 814	9, 20	383.47	382.27
49	3-10-33	1.09	2 558	13	416.69	7-13-34	2 583	7, 9, 20	383.20	382.10
50	3-17-33	0.85	2 909	13	416.91	7-20-34	2 846	9, 20	382.85	381.94
51	3-24-33	2.02	3 108	13	417.40	7-27-34	3 213	7, 9, 20	382.74	381.92
52	3-31-33	0.84	3 206	13	417.95	8- 3-34	3 133	9, 20	383.74	382.49
53	4- 7-33	1.48	3 249	13	418.61	8-10-34	3 030	9, 20	382.91	382.02
54	4-14-33	3.76	3 181	13	419.00	8-17-34	2 825	9, 20	382.72	381.92
55	4-21-33	1.48	3 200	13	419.14	8-24-34	3 428	7, 9, 20	382.79	381.95
56	4-28-33	0.59	3 189	13	418.46	8-31-34	3 979	7, 20	382.97	382.03
57	5- 5-33	0.52	3 228	13	417.76	9- 7-34	4 046	7, 20	382.78	381.93
58	5-12-33	0.35	3 214	13	418.00	9-14-34	2 364	7, 9, 20	384.34	383.06
59	5-19-33	....	3 200	13	417.10	9-21-34	3 643	7, 9, 20	385.41	383.62
60	5-26-33	0.98	3 337	13, 18	417.14	9-28-34	4 258	7, 20	383.97	382.70

* See Fig. 1, *S* denotes "Open Sump". † See Fig. 2. ‡ On one day there were no records.

TABLE 2.—(*Continued*)

Week No. (1)	(a) At the Dike: Week ending (2)	Total rain-fall, in inches (3)	Average pumping rate, in gallons per minute (4)	Location of pumping Caisson*: (5)	Elevation of water surface, Morton Pond, in feet (6)	(b) At the Main Dam: Week ending (7)	Average pumping rate, in gallons per minute (8)	Location of pumping; Caisson†: (9)	Elevation of Water Surface, Swift River, in Feet: Up Stream (10)	Down stream (11)
61	6- 2-33	0.52	3 428	13, 18	417.03	10- 5-34	4 027	7, 20	384.92	383.41
62	6- 9-33	1.70	3 336	13, 18	416.83	10-12-34	4 172	7, 20	384.46	383.04
63	6-16-33	0.11	3 171	13, 18	416.52	10-19-34	3 861	7, 20	383.84	382.56
64	6-23-33	0.76	2 976	13, 18	416.14	10-26-34	2 378	7, 9, 20	383.82	382.60
65	6-30-33	0.06	3 138	13, 18	415.61	11- 2-34	2 332	9, 20	384.40	383.01
66	7- 7-33	0.48	3 041	13, 18	415.07	11- 9-34	2 546	9, 20	385.26	383.63
67	7-14-33	0.45	2 964	13, 18	414.30	11-16-34	2 236	9, 20	384.75	383.11
68	7-21-33	0.17	2 877	13, 18	413.69	11-23-34	2 637	9, 20	384.17	382.80
69	7-28-33	1.60	2 952	13, 18	413.19	11-30-34	5 271	7, 9, 20	384.25	382.84
70	8- 4-33	0.97	2 948	13, 16, 18	412.55	12- 7-34	5 015	7, 9, 20	386.48	384.42
71	8-11-33	0.30	3 182	8, 13, 16	411.85	12-14-34	4 617	7, 9, 20	384.50	383.07
72	8-18-33	2.35	3 079	6, 13, 16	411.29	12-21-34	4 319	7, 9, 20	384.25	382.98
73	8-25-33	2.17	3.253	6, 8, 13, 16	410.80	12-28-34	4 223	7, 9, 20	384.64	383.04
74	9- 1-33	0.30	3 352	8, 13, 16	410.38	1- 4-35	4 098	7, 9, 20	384.24	382.88
75	9- 8-33	3.99	3 230	8, 13, 16	410.01	1-11-35	3 772	7, 9, 20	385.82	384.00
76	9-15-33	3.60	3 028	8, 13, 16	409.71	1-18-35	3 662	7, 9, 20	386.27	384.33
77	9-22-33	2.69	3 252	8, 13, 16	412.17	1-25-35	3 590	7, 9, 20	384.93	383.38
78	9-29-33	0.22	3 446	6, 8, 16, 13	412.17	2- 1-35	3 483	7, 9, 20	384.53	383.07
79	10- 6-33	1.43	3 094	6, 16, 13	411.29	2- 8-35	3 430	7, 9, 20	384.22	382.92
80	10-13-33	0.43	3 053	6, 16, 13	410.94	2-15-35	3 382	7, 9, 20	384.12	382.82
81	10-20-33	1.01	3 218	6, 16, 13	410.45	2-22-35	3 359	7, 9, 20	384.96	383.42
82	10-27-33	1.80	3 062	6, 10, 13, 16	410.31	3- 1-35	3 341	7, 9, 20	385.05	383.48
83	11- 3-33	0.03	3 250	6, 10, 13, 16	410.52	3- 8-35	3 264	7, 9, 20	385.43	383.68
84	11-10-33	0.35	2 400	6, 10, 13, 16	410.24	3-15-35	3 292	7, 9, 20	386.38	384.35
85	11-17-33	0.64	3 123	8, 13, 16	410.06	3-22-35	3 363	7, 9, 20	386.75	384.62
86	11-24-33	0.02	3 063	8, 13, 16	409.73	3-29-35	3 425	7, 9, 20	386.17	384.15
87	12- 1-33	0.89	3 290	8, 13, 16	409.59	4- 5-35	3 434	7, 9, 20	386.05	384.07
88	12- 8-33	0.63	3 321	4, 8, 13, 16	409.47	4-12-35	3 467	7, 9, 20	385.50	383.67
89	12-15-33	0.86	3 145	4, 8, 13, 16	409.26	4-19-35	3 436	7, 9, 20	386.82	384.64
90	12-22-33	1.22	2 842	4, 8, 13, 16	408.72	4-26-35	3 469	7, 9, 20	385.68	383.85
91	12-29-33	0.71	2 742	4, 8, 13, 16	408.77	5- 3-35	3 512	7, 9, 20,	384.97	383.32
92	1- 5-34	0.75	2 724	4, 8, 10, 13, 16	408.52	5-10-35	3 509	7, 9, 20	385.63	383.78
93	1-12-34	1.18	2 739	4, 8, 10, 13, 16	408.94	5-17-35	3 533	7, 9, 20	385.12	383.48
94	1-19-34	0.42	3 240	4, 8, 10, 13, 16	409.41	5-24-35	3 578	7, 9, 20	384.23	382.92
95	1-26-34	0.98	3 043	4, 8, 13, 16	409.24	5-31-35	3 530	7, 9, 20	383.78	382.66
96	2- 2-34	0.86	3 049	4, 8, 10, 13, 16	409.54	6- 7-35	3 456	7, 9, 20	384.13	382.83
97	2- 9-34	0.14	2 862	4, 8, 10, 13, 16	409.57	6-14-35	3 738	7, 9, 20	385.30	383.57
98	2-16-34	0.02	2 813	4, 8, 10, 13, 16	409.51	6-21-35	3 755	7, 9, 20	384.67	383.13
99	2-23-34	1.71	3 086	4, 7, 8, 13, 16	409.00	6-28-35	3 089	7, 9, 20	384.85	383.22
100	3- 2-34	1.27	3 094	4,7,8,10,13,16	408.20	7- 5-35	2 959	7, 9, 20	383.80	382.64
101	3- 9-34	0.83	3 206	4,7,8,10,13,16	409.67	7-12-35	3 699	7, 9, 20	383.52	382.47
102	3-16-34	0.37	3 120	4,7,8,10,13,16	411.49	7-19-35	3 696	7, 9, 20, 11	383.38	382.38
103	3-23-34	0.00	3 165	4, 7, 10, 13, 16	411.59	7-26-35	3 397	7, 9, 20, 11	383.33	382.37
104	3-30-34	1.10	3 168	4, 7, 10, 15, 16	411.95	8- 2-35	3 475	7, 9, 20	383.07	382.18
105	4- 6-34	0.71	3 127	4, 7, 10, 13, 16	413.24	8- 9-35	3 424	7, 9, 20	382.82	382.00
106	4-13-34	1.76	2 862	4, 7, 10, 13, 16	413.55	8-16-35	3 261	7, 9, 20	382.68	381.92
107	4-20-34	1.78	2 646	4, 7, 10, 13	414.72	8-23-35	3 584	7, 9, 20, 12	382.50	381.87
108	4-27-34	0.31	2 567	4, 7 10, 13	415.03	8-30-35	3 568	7, 9, 12	382.40	381.80
109	5- 4-34	1.76	2 648	4, 7, 10, 13	414.83	9- 6-35	3 484	7, 9, 12	382.68	381.98
110	5-11-34	1.46	2 560	4, 7, 10, 13	415.16	9-13-35	3 404	7, 9, 12	383.38	382.38
111	5-18-34	0.42	2 781	4, 7, 10, 13	415.52	9-20-35	3 423	7, 9, 12	382.90	382.10
112	5-25-34	1.11	2 786	4, 7, 10, 13	415.35	9-27-35	2 968	7, 9, 12, 14	382.47	381.90
113	6- 1-34	0.04	2 780	4, 7, 10, 13	415.19	10- 4-35	3 582	7, 9, 12, 14	382.67	382.02
114	6- 8-34	0.93	2 816	4, 7, 10, 13	414.97	10-11-35	3 312	7, 9, 12, 14	382.53	381.97
115	6-15-34	1.89	2 762	4, 7, 10, 13	414.58	10-18-35	2 748	7, 9, 14	382.40	381.88
116	6-22-34	2.23	2 516	4, 7, 10, 13	414.70	10-25-35	2 010	7, 9, 14	382.52	381.95
117	6-29-34	0.38	2 841	4, 7, 10, 13	414.85	11- 1-35	3 307	7, 9, 14	382.65	382.02
118	7- 6-34	0.35	2 866	4, 7, 10, 13	414.10	....	....	....	......	....
119	7-13-34	0.56	2 742	4, 7, 10, 13	413.40	....	....	....	......	....
120	7-20-34	0.00	2 621	4, 7, 10, 13	412.77	....	....	....	......	....

* See Fig. 1. † See Fig. 2.

TABLE 2.—(*Continued*)

Week No.	(a) At the Dike (*Continued*)					Week No.	(a) At the Dike (*Continued*)				
	Week ending (2)	Total rain-fall, in inches (3)	Average pumping rate, in gallons per minute (4)	Location of pumping: Caisson*: (5)	Elevation of of water surface, Morton Pond, in feet (6)		Week ending (2)	Total rain-fall, in inches (3)	Average pumping rate, in gallons per minute (4)	Location of pump-ing: Caisson:† (5)	Elevation of of water surface, Morton Pond, in feet (6)
(1)						(1)					
121	7-27-34	0.23	2 792	4, 7, 10, 13	412.32	136	11- 9-34	1.02	430	7	413.97
122	8- 3-34	2.08	2 801	4, 7, 10, 13	411.93	137	11-16-34	0.46	442	7	414.50
123	8-10-34	0.00	2 762	4, 7, 10, 13	411.54	138	11-23-34	0.37	275	7	414.67
124	8-17-34	0.83	2 660	4, 7, 10, 13	411.06	139	11-30-34	1.50	....	None	415.08
125	8-24-34	1.93	2 127	7, 13	410.56	140	12- 7-34	1.06	....	None	415.94
126	8-31-34	0.19	1 832	7, 13	410.58	141	12-14-34	0.07	....	None	416.31
127	9- 7-34	1.09	1 819	7, 13	410.05	142	12-21-34	0.72	....	None	416.51
128	9-14-34	3.12	1 478	7, 13	411.12	143	12-28-34	0.91	....	None	417.00
129	9-21-34	2.89	1 668	7, 13	411.81	144	1- 4-35	0.94	....	None	417.25
130	9-28-34	0.54	1 362	7, 13	411.94	145	1-11-35	2.06	....	None	417.82
131	10- 5-34	2.26	1 165	7, 13	412.60	146	1-18-35	0.76	....	None	417.57
132	10-12-34	0.44	1 229	7, 13	412.90	147	1-25-35	1.98	....	None	417.42
133	10-19-34	0.08	1 148	7, 13	412.88	148	2- 1-35	0.11	....	None	417.35
134	10-26-34	1.38	744	7, 13	413.03	149	2- 8-35	......	....	None	417.27
135	11- 2-34	0.85	686	7, 13	......	150	2-15-35	......	....	None	....

* See Fig. 1. † See Fig. 2.

The extent to which the ground-water was lowered, as read in the measuring wells and in the caissons, can be seen by inspection of Figs. 1 and 7 and of the data given in Table 2. In general, the ground-water was lowered more than 90 ft below the brook level over the deeper parts of the valley. The maximum air pressure used in sinking caissons other than Caisson 13 was 25 lb per sq in., only about 2% of the sinking requiring pressures in excess of 18 lb per sq in. (the legal limit in Massachusetts for 8-hr work). Altogether, 70% of the caisson sinking was accomplished in free air, and an additional 20% under pressures of less than 10 lb per sq in.

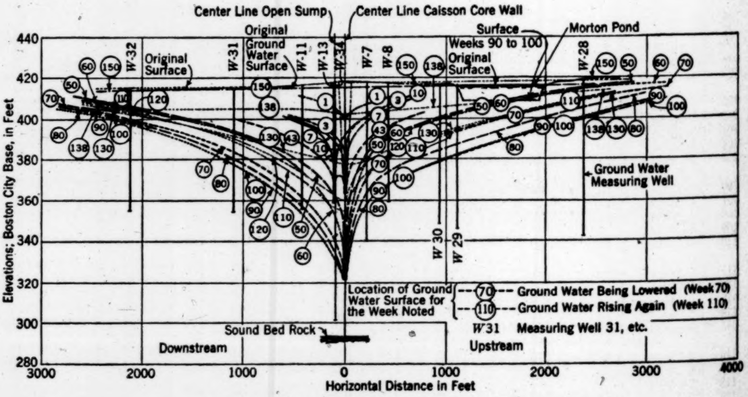


FIG. 7.—LONGITUDINAL SECTION OF BEAVER BROOK

As the core-wall of the concrete cassions neared completion, the pumping rate was decreased and the water level was allowed to rise again slowly while work progressed on the re-fill in the open trench excavated for caisson construction (see Figs. 1 and 7). In November, 1934, pumping was abandoned, and, in February, 1935, the water level was very close to its original position.

PERMEABILITY DETERMINATIONS FOR OVER-BURDEN FROM PUMPING OPERATIONS AT THE DIKE SITE

Several determinations of the permeability of the materials in the over-burden were made from the data pertaining to pumping and ground-water lowering. The permeability was computed in various ways, but all determinations depended basically on the formula,

$$q = k\,p\,A\,S \qquad (1)$$

in which q = quantity of water flowing past any cross-sectional area, in cubic feet per day; k = the velocity of the water through the pores, in feet per day; and p = the porosity of the over-burden. Then $0.3267\,k\,p = K$, the permeability coefficient, in million gallons daily per acre for a slope of unity; A = area of cross-section, in square feet; and S = slope of the ground-water gradient (expressed as a decimal).

The value of q used in all determinations is based on the pumping rate. The total rate was composed of two general parts—infiltration and reduction of storage. The former consisted of surface infiltration of water from rainfall and from surface supplies, such as brooks, ponds, etc., and of sub-surface infiltration supplied by slow percolation from underground volumes previously unwatered of all that would flow out relatively quickly and easily. The latter consisted of the quantities taken from storage, which would immediately affect the readings of the ground-water measuring wells. (Estimates for pumping at the Main Dam and Dike indicated that about 90% of the water in the pores was taken quickly from storage by the pumping. Of the remainder, part would stay permanently in the pores by capillary action and part would be slowly underdrained.) The value of the flow, Q (in gallons per minute), at any location varies from the total quantity pumped at the station to practically zero at the radius of influence. The true value of the flow at the radius of influence depends upon the natural ground-water slope and the permeability (which is being determined). In some problems it may be necessary to make a preliminary assumption of this value to obtain a reasonable value of Q, but for this site the actual figure for the permeability finally obtained and for the slope would be less than 100 gal per min, so that the accuracy of the determinations will not be noticeably affected by the assumption.

The location of the radius of influence can be obtained by estimating visually, from the ground-water data, the distance from the pumping center at which the lowered ground-water surface will be tangent to the original ground-water surface. The value thus obtained, although approximate, is usable, because the exact location does not materially affect the permeability results. The values of Q at various locations between the pumping center and the radius of influence must be estimated as the first step in making a determination. Although, theoretically, an exact determination depends upon the exact variation in flow estimated, actually equivalent results will be obtained for any reasonable distribution of these differences of flow. The most reasonable distribution that was used in the Dike determinations was that Q was assumed to vary proportionately as the two factors—infiltration and storage reduction. The first varied as the surface drainage area, weighted for ponds and watercourses; and the second according to the rate at which the increment volumes of over-burden were being depleted of ground-water storage. Other arbitrary variations of Q also gave good results; for instance, Q can be varied from P to zero according to the distance from the pumping center, without seriously affecting the value of the determination. The empirical formula,

$$Q = P\left(1 - \frac{r^2}{r_i^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

seems to give a distribution of differences in flow very close to those obtained by the more careful and logical distribution first described. In Equation (2), P = the total pumping rate, in gallons per minute; r = a radius measured from the weighted location of pumping operations; and r_i = the radius that defines the influence of pumping and is the distance from the weighted pumping center to the limit where the pumped ground-water surface is tangent to the normal ground-water surface.

The symbol, A, denotes the area of the surface of the section through which the water flows, this section being curved to be taken perpendicular to the lines of flow. Near the pumping locations, this area has a bulbous shape due to the fact that the flow of water is not approximately horizontal near the intakes because: (1) There are several pumping intakes; and (2) these intakes are in the deeper parts of the valley. However, as the distance from the pumping center increases, these surfaces approximate very closely the surfaces of concentric vertical cylinders with axes passing through, and with radii taken from, the weighted center of pumping operations (herein termed the "pumping center"). The upper limits of these surfaces in determining the value of A are taken at the ground-water surface, as shown by readings in the measuring wells and the lower limits at the surface of the rock valley beneath, as determined from bore-hole investigations (see Figs. 1, 6, and 7). The use of these cylindrical areas does not introduce an approximation influencing the results, as the permeability determinations do not include volumes close to the pumping center.

The slope, S, is read directly from the ground-water curve used and is taken as a straight line between adjacent points on the curve. To avoid the introduction of errors (due to the use of the slope as a straight line, to the use of the average of end areas as the area of the volume considered, etc.), each determination was made by computing the permeability of sufficiently small increment volumes of the over-burden. Cylindrical cross-sections were

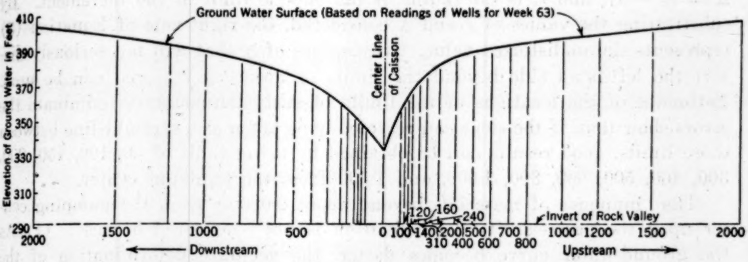


FIG. 8.—DEFINITION OF INCREMENTS FOR PERMEABILITY DETERMINATIONS

chosen at certain radii, as shown in Fig. 8, and each increment is the volume between cylindrical surfaces (see Table 3(*a*)). The effect of the size of the increment can be seen from the mathematical relation,

$$\frac{2\,\Delta}{2\,r + \Delta} = 2.3026\left(\log\frac{r_1 + \Delta}{r_1}\right)\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

in which r_1 = the radius of the inside limit of the volume increment;

TABLE 3.—COMPUTATION OF PERMEABILITY BY "SLOPE QUANTITY" METHOD FOR WEEK 63 FOR BEAVER BROOK VALLEY (SEE FIG. 8)

(See Fig. 8) Distance from pumping center, in feet	(*a*) UP STREAM					(*b*) DOWN STREAM				
	Areas,* in thousands of square feet	Flow, *Q*,* in gallons per minute	Volume, *V*, in thousands of cubic feet	Slope, *S*	Permeability coefficient, *K*†	Areas,* in thousands of square feet	Flow, *Q*,* in gallons per minute	Volume, *V*, in thousands of cubic feet	Slope, *S*	Permeability coefficient, *K*
0	......	......	......	......	......	......	......	......	......	......
100	20.0	1 910	......	......	......	16.7	1 446	......	......	......
	(22.5)	(1 908)	450	0.2000	26.7	(18.6)	(1 445.5)	372	0.1000	48.9
120	25.0	1 906	......	......	......	20.5	1 445	......	......	......
	(28.0)	(1 904)	560	0.1900	22.5	(23.0)	(1 444.5)	460	0.1000	39.5
140	31.0	1 902	......	......	......	25.5	1 444	......	......	......
	(34.25)	(1 899)	685	0.1750	19.9	(27.75)	(1 443.5)	555	0.1100	29.7
160	37.5	1 897	......	......	......	30.0	1 443	......	......	......
	(43.25)	(1 893)	1 730	0.1425	19.3	(33.8)	(1 442)	1 352	0.1050	25.5
200	49.0	1 889	......	......	......	37.6	1 441	......	......	......
	(56.0)	(1 881)	2 240	0.1150	18.3	(41.8)	(1 438)	1 672	0.0925	23.3
240	63.0	1 873	......	......	......	46.0	1 436	......	......	......
	(74.5)	(1 859)	5 215	0.0743	20.6	(56.0)	(1 432)	3 290	0.0857	18.8
310	86.0	1 845	......	......	......	66.0	1 428	......	......	......
	(99.5)	(1 827)	8 950	0.0444	26.0	(79.5)	(1 423)	7 160	0.0588	19.2
400	113.0	1 809	......	......	......	93.0	1 418	......	......	......
	(123.5)	(1 777)	12 350	0.0250	36.2	(102.5)	(1 411)	10 250	0.0430	20.1
500	134.0	1 746	......	......	......	112.0	1 405	......	......	......
	(138.0)	(1 714)	13 800	0.0180	43.5	(116.0)	(1 398)	11 600	0.0360	21.0
600	142.0	1 683	......	......	......	120.0	1 392	......	......	......
	(145.0)	(1 645)	14 500	0.0190	37.5	(126.0)	(1 379)	12 000	0.0280	24.5
700	148.0	1 608	......	......	......	132.0	1 366	......	......	......
	(151.0)	(1 571)	15 100	0.0150	43.5	(132.5)	(1 354)	13 250	0.0240	26.9
800	154.0	1 534	......	......	......	133.0	1 341	......	......	......
	(159.0)	(1 464)	31 800	0.0145	40.0	(134.5)	(1 289)	26 900	0.0210	28.6
1 000	164.0	1 394	......	......	......	136.0	1 238	......	......	......
	(169.5)	(1 312)	33 900	0.0120	40.5	(141.5)	(1 164)	28 300	0.0185	28.0
1 200	175.0	1 229	......	......	......	147.0	1 090	......	......	......
	(183.0)	(1 112)	54 900	0.0110	34.8	(152.5)	(983)	45 700	0.0166	24.3
1 500	191.0	994	......	......	......	158.0	876.5	......	......	......
	(204.5)	(815)	102 200	0.0114	22.0	(169.0)	(704)	100 500	0.0119	21.9
2 000	218.0	636	......	......	......	180.0	532	......	......	......

* Values in parentheses are averages.

† In millions of gallons per acre per day at 50° F for a slope of 1.

$\Delta = r_2 - r_1$; and r_2 is the radius of the outside limit of the increment. By substituting the values of r and Δ considered, the right side of Equation (3) represents the undistorted value. Any values of Δ that will not seriously distort the left-hand side beyond the limits of accuracy required can be used. Estimates of the locations of the limits of such increments to eliminate the errors show that, if the slope can reasonably be taken as a straight line between those limits, good results can be obtained by using radii of 50, 100, 150, 200, 300, 400, 500, 600, 800, 1 000, and 1 300 from the pumping center.

The influence of materials beyond about 2 000 ft from the pumping center upon the permeability determination is not considered because: (1) As the ground-water curve becomes flatter, the accurate determination of the permeability for that region becomes more difficult, slight differences in elevations of readings making material differences in results; (2) such materials are out of range of the immediate site; (3) the ground-water level near the radius of influence is more sensitive to fluctuations due to rainfall and seasonal influences than those due to the pumping; and (4) for the foregoing reasons, ground-water elevations were not read at this site.

Furthermore, volumes close to the caissons were not considered for four reasons: (1) There were no ground-water measuring wells near the caissons because of the open-cut trench and, therefore, the elevation of the ground-water surface at that point was not determined accurately; (2) cylindrical cross-sectional areas cannot be used accurately as the areas of flow; (3) the pumping operations and the compressed-air work during the sinking of the caissons so distributed the permeability of the materials immediately surrounding the caissons that those portions of the over-burden should not be considered as representative in obtaining the effective average; and (4) these portions represent relatively small volumes, and as the permeabilities of increments are weighted according to their volumes, the resulting determination would not be seriously influenced by the values obtained for these volumes in any event.

The shape of the rock valley at this site naturally divides the ground-water flow toward the pumps into two parts, that from up stream and that from down stream. A study of the rainfall and run-off conditions at the site and of the storage depletions up stream and down stream from the pumping center indicated that on the average about 57% of the total pumpage comes from the valley over-burden up stream and the remaining 43% from the valley over-burden down stream. Whether or not this indication is true, the result for the average effective permeability of the valley as a whole is affected but slightly by it, due to the compensating effect of the averaging of the two values to obtain the result, as can be seen from an examination of Table 4.

Using the factors, q, A, and S, the permeabilities of increment volumes of the over-burden can be obtained for any ground-water surface conditions and the corresponding pumping rate. By combining the permeabilities of the increments thus obtained, and weighting each increment according to its volume, the effective average permeability of the whole material is obtained, this method being called herein the "slope quantity" method.

TABLE 4.—COEFFICIENTS OF PERMEABILITY

Determination for:	ASSUMED PERCENTAGES OF TOTAL PUMPING		K=COEFFICIENT OF PERMEABILITY		
	From up-stream volumes	From down-stream volumes	Up stream	Down stream	Weighted average
Week 63	70	30	37.7	16.7	27.9
	57	43	31.9	24.0	28.2
	50	50	29.0	27.9	28.0
Week 91	77	23	45.1	16.2	32.0
	70	30	43.3	17.8	31.8
	57	43	35.2	25.5	30.9
	50	50	31.0	29.7	30.4

Fig. 8, with Table 3, shows the details of such a determination for Week 63 of the pumping. The best results can be obtained by choosing a week among those for which conditions (such as the pumping rates, depletions of storage, and uniform surface infiltrations due to normal rainfalls and run-offs) have been running constant or varying uniformly. The measuring wells register sensitively the changes in ground-water surface. For a period when the rainfall is uniformly distributed, the quantity of infiltration is probably constant. If the pumping plant operates at a constant rate, the reduction in storage is a constant volume from week to week. For such periods accurate estimates of the quantity of water furnished by storage depletion and by surface infiltration can be made. Such estimates can be used in determining accurately, the quantity of water flowing toward the pumps at various vertical cross-sectional areas (see Fig. 8 and Table 3). As the quantity flowing between any two vertical cross-sections and the loss of head are then known, the permeability of any corresponding increment volume can be determined accurately. Weeks 63 and 91 are of that nature and, therefore, are used. Table 5, introduced subsequently, gives certain of the better determinations made as described herein. That the value of K is constantly increasing as the water level recedes is accounted for by the fact that the upper portions of the glacial valley over-burden consist of finer sands containing streaks of rock flour and the lower portions contain the coarser lenses. Thus, as the ground-water level is lowered beneath the finer portions, the resulting average permeability of portions under the ground-water surface is greater.

It is interesting to note that in all determinations practically the same result can be obtained by using $Q =$ three-fourths of the up-stream pumpage, P_u, as the flow for the up-stream increments, and three-fourths of the down-stream pumpage, and P_d, as the flow for the down-stream increments. This is probably due to the facts that: (*a*) The K-values of the increments do not vary to such degrees as to upset the results; (*b*) the manner for distributing the infiltration is approximately a formula the equation of which is of the second degree; and (*c*) the storage depletion increments vary roughly as the square of the distance from the caisson (the rock valley being not far from a constant cross-section). It can be inferred, therefore, that reasonable approximations can be made for like conditions by adopting a constant value of Q instead of the variable one used in the more exact determinations.

The results of the work done by this increment permeability ("slope-quantity") method can be checked by a second cut-and-try method described subsequently as the "graphical method". In the former an estimate of the portions of the total pumpage coming from up stream and down stream is necessary. Although the accuracy of such an estimate does not seem to affect the average effective permeability of the over-burden as a whole, it will affect the relative values obtained for the up-stream and down-stream permeabilities. In the latter an assumption is made that the average effective permeabilities of the up-stream and down-stream portions are equal, which may or may not be true for any location considered. However, the results by either method are confirmatory, showing that the manipulation of these considerations within the reasonable limits indicated will not seriously affect the workability of the methods.

Another more approximate method of obtaining the permeability from ground-water pumping data is by an adaptation of the formula[9] used generally for determining available ground-water yield from wells:

$$y^2 = \frac{q}{k\,p} \log_e \frac{x}{r} + h^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which x and y are co-ordinate locations of any point on the ground-water curve in reference to co-ordinate axes, one axis being made to pass vertically through the pumping center and the other horizontally at the average elevation of rock. Furthermore, h = the y-co-ordinate at the pumping well; p = porosity; and k = the velocity of the ground-water through the pores, in feet per day, for a slope of 1 on 1 at 50° F.

Equation (4) can be used only in its present form for an accurate determination in a case where the pumping curve has reached a stable position, q being a constant flow from beyond the radius of influence to the pumps with no depletion in storage and with no infiltration within the radius of influence, and for a condition which considers a level rock floor under the over-burden extending 360° in plan around the pumps and to the radius of influence. (Even then the formula is for well-points and assumes horizontal flow near the pumping intakes.) Approximations in the field, of such theoretical conditions, for any dam location, or for any reasonable pumping or unwatering program, are extremely unlikely.

Approximate and useful results, however, might be obtained for some cases by adapting the principles of the formula, as follows: The average of the up-stream and down-stream ground-water curves can be obtained, as illustrated in Fig. 9. An equivalent rock floor, which will extend 360° around the pumping center, can be estimated by summing the areas up stream and down stream for various radii and dividing by $2\,\pi\,r$; and the horizontal plane which best averages the location of this rock floor so determined can be estimated roughly by eye, as shown in Fig. 9. A constant Q-value of three-fourths P can be used. Then, by substituting values of x and y (of any two points on the average ground-water curve in relation to the base fixed) twice in Equation (4), two equations, with values, $K p$ and h, unknown, can be obtained. These equations can be solved simultaneously for $K p$ by subtraction. A weighted mean of a number of such determinations would give the average effective permeability. In Equation (4) h is usually a difficult factor to obtain accurately, especially in cases of pumping from many locations, but where it can be obtained, permeability determinations can be made for as many points on the average curve as seem desirable without the necessity of solving simultaneous equations.

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[9] For derivation see "Public Water Supplies", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and H. L. Russell, M. Am. Soc. C. E., Second Edition, p. 279.

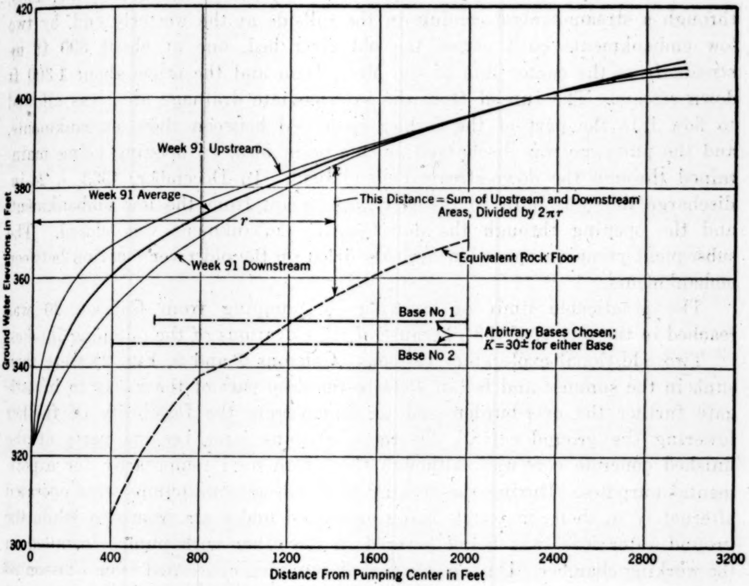


FIG. 9.—GROUND-WATER ELEVATIONS AT THE DIKE

PUMPING OF GROUND-WATER AT THE SITE OF THE MAIN DAM

The first of the exploratory caissons (Caisson 20, see Fig. 2) was sunk to a depth of 75 ft below the river level, at a location which was not in the deeper part of the rock valley. This caisson was equipped with intakes for pumping and two pumps with capacities of 300 gal per min were installed. Ground-water was pumped for two months in 1932 at an average rate of about 650 gal per min and the ground-water surface was lowered only about 7 ft in the vicinity of this caisson, because of the lack in capacity of the pumping intakes. Pumping was then abandoned at this site for about a year, and the ground-water surface returned to its original position. The capacity of the pumping intakes was increased in July, 1933, mainly by the forcing of well-points out into the surrounding materials through holes drilled in the sides of the caisson. Two additional pumps (capacity, 800 gal per min) were installed, and pumping was started at an increased rate. An average of about 2 100 gal per min was pumped over the period of the next eleven months, the water level being lowered about 30 ft near the caisson. The pumping rates and the progressive lowering of the ground-water may be seen in Fig. 2 and Table 2(*b*).

The Swift River, which flows past the site, was diverted in June, 1933, through a stream-control conduit in the hillside at the westerly end, by two low embankments built across the old river bed, one at about 800 ft up stream from the center line of the Main Dam and the other about 1 300 ft down stream. The run-off from the intermediate drainage area was allowed to flow into the part of the former river bed between these embankments, and the pumpage was discharged at this point also, an opening being maintained through the down-stream embankment. In December, 1933, a 24-in. discharge line was built to the river up stream from the low embankment, and the opening through the down-stream embankment was closed. The subsequent pumping operations quickly dried up the old river location between embankments.

The practicable limit of lowering by pumping from Caisson 20 was reached in the spring of 1934, because of the elevations of the pumping intakes.

Two additional exploratory caissons (Caissons 7 and 9, Fig. 2) were then sunk in the summer and fall of 1934 in the deep part of the valley to investigate further the over-burden and to demonstrate the feasibility of further lowering the ground-water. All these caissons later became parts of the finished concrete core-wall, although they were used temporarily for experimental purposes. During the sinking of these caissons pumps were operated alternately in them, materials being excavated under air from one, while the ground-water level was being lowered in the other with pumps installed in the working chamber. During this period, pumping continued from Caisson 20 at a diminishing rate, averaging about 1 600 gal per min at the start and decreasing gradually, until at the completion of these caissons, five months later, it was about 1 200 gal per min. In addition, pumping from Caisson 9, when used as a pumping plant, averaged about 1 200 gal per min, and from Caisson 7 about 2 500 to 3 000 gal per min. The effect of this pumping upon the ground-water level is shown in Fig. 2 and Table 2(*b*).

In this manner, these two caissons were sunk to ledge approximately 120 ft below the river level and about 100 ft below the ground-water level as lowered at the beginning of sinking, the maximum air pressure used in Caisson 7 being 37.5 lb per sq in. and in Caisson 9, 34 lb per sq in. When these caissons had been completed, pumping was continued from them and from Caisson 20. The average total rate at the start was 5 000 gal per min (3 100 gal per min from Caisson 7, 1 000 gal per min from Caisson 9, and 900 gal per min from Caisson 20). Two months later, in February, 1935, the rate had decreased to 3 400 gal per min (2 200 gal per min from Caisson 7, 1 000 gal per min from Caisson 9, and 200 gal per min from Caisson 20), and the ground-water level had been lowered about 75 ft in the vicinity of Caissons 7 and 9, as shown in the curves of Fig. 2.

Pumping was continued at approximately this rate for several months, maintaining the ground-water in its lowered position. The work of constructing the caisson core-wall began in May, 1935, and was practically completed by December, 1935, during which period the contractor pumped an average total of about 3 400 gal per min from Caissons 7 and 9 and other caissons, and such pumping did not materially lower the ground-water gradients, except that in the center of the rock valley, as shown in Fig. 2, further local lowering was secured near Caissons 12 and 14. The maximum air pressure used in sinking caissons other than Caissons 7, 9, and 20, was a little in excess of 25 lb per sq in., although nearly all caissons were sunk under pressures of less than 18 lb per sq in. About 82% of the sinking was accomplished in free air, Caissons 1, 2, and 32 to 40, inclusive, being entirely sunk and sealed without the use of air under pressure.

PERMEABILITY DETERMINATIONS FROM PUMPING AT THE MAIN DAM

Determinations of the permeability of the over-burden at the Main Dam can be made from the curves plotted showing the ground-water surfaces at various intervals during the pumping. The determinations can be made in addition to the "slope quantity" method described previously by "the graphical" (a cut-and-try) method. The procedure may be demonstrated by reference to Fig. 10, in which Curve *A* is plotted from measuring-well readings; Curves *B*, *C*, and *D*, for the values of *K* indicated, represent the total pumpage divided by 54% from up stream and 46% from down stream; and Curve *E*, for $K = 30$, represents the total pumpage divided by 60% from up stream and 40% from down stream. Preparation for a determination by this method consists of plotting in plan over the rock contours of the valley floor, the con-

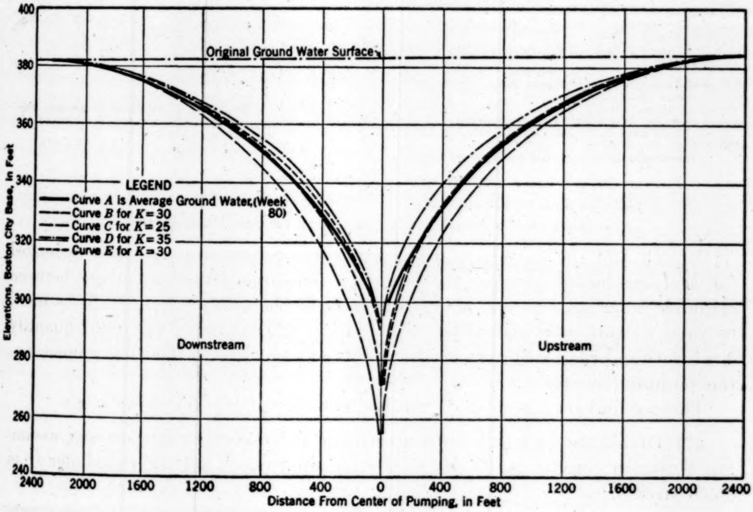
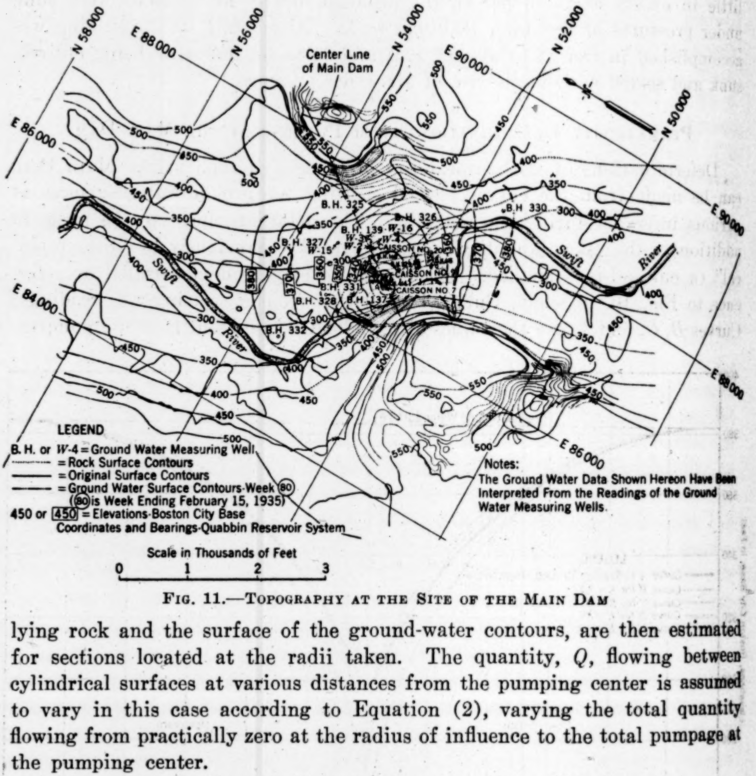


FIG. 10.—PUMPING AT THE SITE OF THE DAM

tours of the ground-water level from the readings of the ground-water measuring wells (see Fig. 11). A pumping center is chosen as in the other methods, in this case a point about 140 ft northeast of Caisson 7 on the center line of the Main Dam being taken. With this center, radii of 150, 200, 300, 400, 500, 600, 800, 1 000, and 1 300 ft are used, these radii being taken so as to give increments small enough to eliminate errors as described previously. The cylindrical surface, cross-sectional areas between the surface of the underlying rock and the surface of the ground-water contours, are then estimated for sections located at the radii taken. The quantity, Q, flowing between cylindrical surfaces at various distances from the pumping center is assumed to vary in this case according to Equation (2), varying the total quantity flowing from practically zero at the radius of influence to the total pumpage at the pumping center.


N 58 000
E 88 000
N 56 000
Center Line of Main Dam
N 54 000
E 90 000
N 52 000
N 50 000
E 86 000
E 84 000
E 90 000
E 88 000
E 86 000
Swift River
Swift River
B.H. 325
B.H. 326
B.H. 330
B.H. 139
W-16
B.H. 327
W-15
CAISSON NO. 30
CAISSON NO. 6
CAISSON NO 7
B.H. 331
B.H. 328
B.H. 137
B.H. 332
LEGEND
B. H. or W-4 = Ground Water Measuring Well
= Rock Surface Contours
= Original Surface Contours
= Ground Water Surface Contours-Week (80)
(80) is Week Ending February 15, 1935
450 or 450 = Elevations-Boston City Base
Coordinates and Bearings-Quabbin Reservoir System
Scale in Thousands of Feet
0
1
2
3

Notes: The Ground Water Data Shown Hereon Have Been Interpreted From the Readings of the Ground Water Measuring Wells.

FIG. 11.—TOPOGRAPHY AT THE SITE OF THE MAIN DAM

The cut-and-try "graphical" method consists of the following steps:

(1) Divide the pumpage into up-stream and down-stream portions, assuming a certain percentage of the total from up stream and the remainder from down stream.

(2) Assume that the average effective permeabilities of the down-stream and up-stream over-burdens are the same.

(3) Assume a value of K which seems reasonable.

(4) Using these assumed factors, and the computed cross-sectional areas obtained from the ground-water surface for which the determination is made, theoretical ground-water curves up stream and down stream can be plotted

in a vertical plane running longitudinally to the valley, starting at the radii of influence (as determined from the pumping curve data) and plotting in toward the pumping center (see Fig. 10). These curves are found by computing the lost head between each set of cylindrical surfaces by use of the formula:

$$H = \frac{62.88\, Q\, (r_2 - r_1)}{\frac{a_2 + a_1}{2} K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

(5) The curves of Step (4) are then superimposed on the curve of the ground-water surface formed by the intersection of the vertical longitudinal plane with the ground-water surface contours plotted (see Fig. 10).

(6) If all the points so plotted come above the observed ground-water curves, the permeability assumed is too large; if they are below, the permeability is too small. If the up-stream points are plotted below, and the down-stream points above, the division of the percentage of flow is incorrectly assumed.

(7) By repeating assumptions, a closer and closer solution for the permeability can be obtained, as well as a better idea of the division of flow coming from up stream and down stream. The nearness of the solution to the correct average effective permeability will be demonstrated graphically by the manner in which the computed curves coincide with the observed ground-water curves.

In all cases where determinations were made by the two methods, the results obtained with the "graphical" method check very nicely those obtained by the analytical "slope quantity" method (see Table 5).

TABLE 5.—PERMEABILITY RESULTS, IN MILLION GALLONS DAILY PER ACRE, FOR A SLOPE OF 1 AT 50° F.

Site	Part I, Bore-Hole Classification		Part II, Pumping Data				
	From sieve analyses	From tested permeabilities	Week No.	Slope Quantity Method		Graphical Method	Well Formula
				Approximate $\left(\frac{3}{4}Q\right)$	More Exact (Q Computed) (See Fig. 8 and Table 3)	(See Fig. 10)	(See Fig. 9)
Dike	19	21	5	..	..	..	16
			36	..	..	21	..
			63	24	28	..	..
			91	28	32	30	30
			133	23	..	..	..
Main Dam	24	28	30	29	..	33	..
			62	40	..	44	..
			80	29	..	31	..

COMPARISON OF PERMEABILITY DETERMINATIONS

An inspection of Table 5 shows that the coefficient of permeability at 50° F of the entire over-burden of the Dike is about 20 mgd per acre and of the lower parts of the over-burden about 30 mgd per acre; and that it is about

30 to 35 mgd per acre for the entire over-burden of the Main Dam. The result of all methods, whether obtained by the use of the bore-hole samples and laboratory work, or by the use of the pumping and ground-water data, substantiates these figures. Considering that a permeability determination within the limits of accuracy of 50% to 200% is extremely useful and sufficiently close for most design purposes, the results in Table 5 demonstrate the adequacy of the different methods for making determinations well within desired limits.

The accuracy of determinations made from the classification of bore-hole samples depends to a great extent upon the manner in which bore-holes are located and the samples taken and collected, upon the skill with which the field and laboratory work is conducted, and upon the judgment as to the manner in which the various classes of material exist in the over-burden. From experiences with the work described herein, the writer is confident that a similar application of the "bore-hole sample classification" method will give valuable results at other locations if reasonable care is exercised in the application.

The use of ground-water pumping to secure a permeability determination is much simpler, provided there is sufficient information regarding the pumping rates, the location of the ground-water surface, and the location of the rock valley. The ground-water surface is extremely sensitive and measures accurately the loss of head between points in the valley. This registration of lost head makes a determination by any of the foregoing methods a comparatively simple and accurate one, because very small differences in permeability will give large differences in readings of the ground-water surface in the measuring wells. Unreasonable determinations can easily be proved unworthy because the loss of head for such conditions will not agree with those observed. These pumping methods can be used elsewhere where the rock and ground-water surfaces are known, to determine the effective permeability of foundations below the ground-water surface.

Conclusions

There seems to be a scarcity of data concerning the permeable characteristics of soils and there are few, if any, definite methods in common use for making satisfactory determinations of the permeability of an over-burden. In many cases permeable qualities of foundations have been a matter of extremely rough estimate, of guess, or merely of conjecture. The aforementioned methods, although not entirely satisfactory, will be found useful in that connection for conditions similar to the ones described, and such methods or adaptations of them may prove helpful elsewhere where conditions are dissimilar. Although the results of determination by such methods may not stand too rigid tests for complete exactness, they will prove their value if only they can be used to serve as satisfactory guides in formulating the opinions or judgment of the engineer confronted by the problem of how large is the permeability of the foundations under consideration.

ACKNOWLEDGMENTS

Most of the information used for the preparation of this paper was obtained from data secured by the Engineering Department of the Metropolitan District Water Supply Commission of Massachusetts, of which Frank E. Winsor, M. Am. Soc. C. E., is Chief Engineer and Karl R. Kennison, M. Am. Soc. C. E., is Assistant Chief Engineer. The writer is indebted to many members of this organization for aid in preparing and compiling much of the information and for constructive suggestions on, and pertinent criticisms of, the substance of the paper itself.

GEOLOGIC FEATURES, QUABBIN AQUEDUCT

By Frank E. Fahlquist[10], Assoc. M. Am. Soc. C. E.

Synopsis

The geological studies which were of importance in constructing the Quabbin Aqueduct, of the Metropolitan Water Supply of Boston, Mass., are presented in this paper. The general geology of the area, including that of the surface and that of the underground, is discussed in relation to the several practical problems encountered during the location studies. Data pertaining to the behavior of the different rocks during construction and the cost and progress of excavation through them, are also included.

Introduction

Construction of the Quabbin Aqueduct is one of several steps in extending the water supply system of the Boston Metropolitan District beyond the Wachusett water-shed in Central Massachusetts (see Fig. 12). In the past water has been obtained from several minor sources and the major supply of Wachusett Reservoir, formed by impounding the flow of the Nashua River, at Clinton, Mass. In extending this system westward to the Swift River Valley additional storage and supply are made available. Completion of the easterly portion of the aqueduct, extending between the Wachusett Reservoir and the Ware River, near Coldbrook, Mass., marked the first step in this project. Construction of this section was sufficiently advanced by March, 1931, to permit diversion of the water of the Ware River to the Wachusett Reservoir. Construction of the western portion of the aqueduct, extending from the terminus of the East Tunnel section to the portal and intake at the Quabbin Reservoir, was completed during the summer of 1935. The result

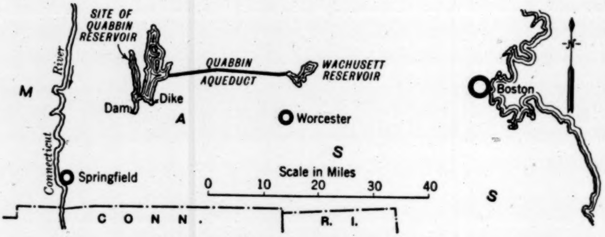


Fig. 12.—Location of Quabbin Aqueduct and Site of Quabbin Reservoir.

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[10] Geologists, War Dept. at Large, U. S. Engr. Office, Providence, R. I.

is a continuous aqueduct, 24.6 miles long, connecting the Quabbin Reservoir with the Wachusett Reservoir.[11]

Geologic Setting

The tunnel crosses a part of the central upland of Massachusetts known as the Worcester County Plateau, a large part of which stands between 1 000 and 1 200 ft above sea level. Geologists[12] have noted an unusual uniformity in the horizon line of the district, and have recognized in the general correspondence of the summits of the parallel ridges the former existence of an ancient and nearly uniform surface, or peneplain, sloping gently toward the southeast.

In general, the hills and valleys are oriented in a north-south direction, which is closely parallel to the folds of the rock formations of the area. There are exceptions, however, which are sufficiently numerous to indicate that dissection or erosion of the ancient surface, or peneplain, was not controlled entirely by the distribution and structure of the underlying rock formations. An explanation for this irregularity has been advanced by W. O. Crosby[13], who concluded that, at the time of its fullest development, the peneplain was covered by a thick mantle of sediment, upon which a drainage pattern was developed without much regard for the structure and distribution of the underlying hard and soft rocks. At this stage the stream gradients were steepened and the rivers rejuvenated as the result of crustal disturbances. As a result the rivers became deeply entrenched in the thick mantle of unconsolidated deposits. By the time these deposits were removed, the streams had been superimposed on the older crystalline rock floor. The pattern of this ancient drainage system can be traced and reconstructed, in part, except in those areas where the valleys have been completely blocked, and where features of the bed-rock topography have been buried too deeply by glacial deposits.

As the glacier advanced during the succeeding epoch known as the Glacial Period, several of these valleys were slightly deepened and widened, and the topography was modified somewhat by ice erosion. As it retreated from the area large quantities of rock material, which had been gathered by, and frozen into, the ice, were deposited. In general, this glacial drift material was deposited to rather shallow depths in the higher elevations, but accumulated to considerable thickness within the valleys.

Pre-Glacial Drainage Courses

Many of the streams of this section are too small in size to have excavated the broad and deep depressions which they now occupy. There are also numerous notches or gaps, situated across divides which are now unoccupied by streams. These features, which are duplicated many times throughout New England and other sections of the United States, have long been recognized by geologists as being related to the pre-historic or pre-glacial river systems developed before the Glacial Period.

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[11] Accounts by Frank E. Winsor, M. Am. Soc. C. E., and Karl R. Kennison, M. Am. Soc. C. E., have appeared recently in engineering literature, as follows: "Boston's New Metropolitan Water Supply", by Frank E. Winsor, *Civil Engineering*, Vol. 4, No. 6, June, 1934; "Ware River Intake Shaft and Diversion Works", by Karl R. Kennison, *Civil Engineering*, Vol. 4, No. 8, August, 1934; and "Boston Metropolitan Water Supply Extension", by Karl R. Kennison, *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2, June, 1934.

[12] "Physical Features of Central Massachusetts", by William C. Alden, *Bulletin 760*, U. S. Geological Survey, 1924, pp. 13–105.

[13] "Geological History of the Nashua Valley During the Tertiary and Quaternary Periods", by W. O. Crosby, *Technology Quarterly*, Vol. 12, 1899, pp. 288–324.

Reconstructing the pre-glacial drainage pattern was important in the tunnel location studies in indicating areas where the underlying rock floor might be lower in elevation than was apparent from a study of surface geological conditions. The limits of many of the pre-glacial valleys are obscured by extensive glacial drift deposits of uncertain thickness. Because of the uncertainty in the position of the rock floor, such areas were recognized as being more important than others, where geologic conditions were more easily interpreted. These preliminary investigations outlined the critical areas throughout which more detailed investigations should be made by borings. Certain areas thus outlined were eliminated from serious consideration at an early stage of the investigations, as the gaps or notches situated within some of the valleys occur at such high elevations that there was little possibility of a gorge extending to the tunnel grade. The most important pre-glacial drainage courses are shown in Fig. 13, together with the boundaries of the two principal types of glacial drift deposits.

Glacial Drift Deposits

The greater part of the area traversed by the tunnel is covered by a glacial soil known to the geologist as a ground moraine and sometimes to the engineer as hardpan. This deposit occurs as a heterogeneous mixture of rock flour, sand, gravel, and boulders. In general, it is found in areas of higher elevation, and is usually deposited to relatively shallow depths. Recognition of the fact that, in general, the ground moraine is a shallow deposit, was found helpful in estimating bed-rock configuration, and also in delineating the less critical areas where explorations by borings were not necessary.

When the glacier was receding, many of the valleys were blocked periodically by glacial drift and ice. Water derived from the melting ice was thus ponded, forming temporary glacial lakes which received large quantities of well assorted deposits. This phase of the glacial history may be recognized in the character and mode of occurrence of the deposits, which consist largely of stratified fine sand and banded clay or rock flour. The surface topography within these lake areas is generally flat. One of the most notable examples of such a temporary lake was the glacial Lake Nashua, cited by Crosby[11], which covered a part of the Nashua River drainage area. The present Wachusett Reservoir occupies a part of the bed of this lake.

Valleys with outlets that were kept open throughout the period of glacial recession contain deposits of a modified character, the irregular structure and coarseness of which indicate an origin other than that in the quiet waters of a glacial lake. The lower sides and bottoms of these valleys contain irregularities in the general surface topography in the form of ridges and terraces composed chiefly of sand and gravel. These deposits indicate deposition by glacial stream action. Some were deposited in front of the retreating glacier

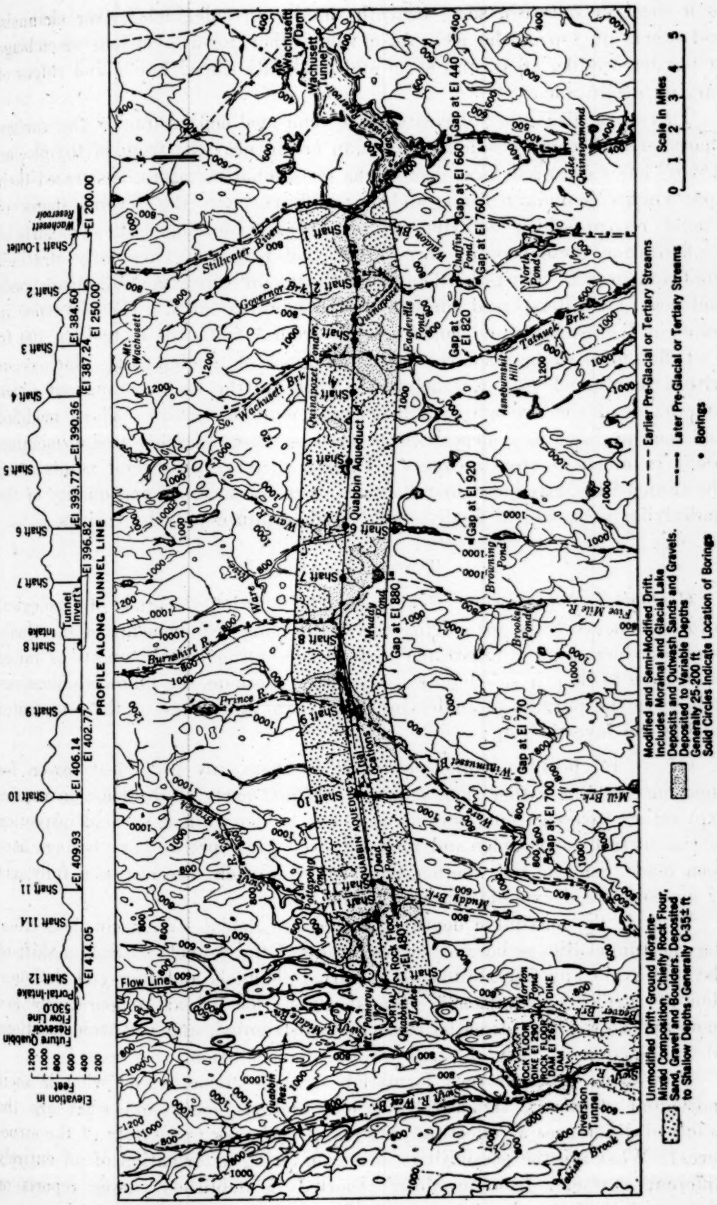
PROFILE ALONG TUNNEL LINE
Elevation in Feet
1200
1000
800
600
400
200
Future Quabbin Reservoir Flow Line 530.0
Portal-Intake
Shaft 12
El 414.05
Shaft 11A
Shaft 11
El 409.93
Shaft 10
El 406.14
El 402.77
Shaft 9
Shaft 8 Intake
Tunnel Invert
Shaft 7
El 396.82
Shaft 6
El 393.77
Shaft 5
El 390.36
Shaft 4
El 387.24
Shaft 3
El 384.60
El 250.00
Shaft 2
Shaft 1-Outlet
Wachusett Reservoir
El 200.00
Wachusett Dam
Wachusett Tunnel
Wachusett Reservoir
Stillwater River
Governor Brk.
Mt. Wachusett
So. Wachusett Brk.
Quinapoxet Pond
Quinapoxet R.
Quabbin Aqueduct
Waushacum Brk.
Eagleville Pond
Chaffin Pond
Gap at El 440
Gap at El 660
Gap at El 760
Gap at El 820
Gap at El 920
Gap at El 880
Gap at El 770
Gap at El 700
North Pond
Lake Quinsigamond
Tatnuck Brk.
Asnebumskit Hill
Browning Pond
Brooks Pond
Five Mile R.
Muddy Pond
Ware R.
Burnshirt R.
Prince R.
Initial Locations
Winimusset Brk.
Mill Brk.
Ware R.
Swift R. East Br.
Pottapaug Pond
Dead Pond
Muddy Brk.
QUABBIN AQUEDUCT
Rock Floor El 480±
Quabbin Lake
Flow Line
Mt. Pomeroy
PORTAL
Swift R. Middle Br.
Morton Pond
DIKE
ROCK FLOOR DIKE El 290
ROCK FLOOR DAM El 267
DAM
Beaver Brk.
Swift R.
Diversion Tunnel
Jabish Brook
Swift R. West Br.
Quabbin Res.
Shaft 1
Shaft 2
Shaft 3
Shaft 4
Shaft 5
Shaft 6
Shaft 7
Shaft 8
Shaft 9
Shaft 10
Shaft 11
Shaft 11A
Shaft 12
Unmodified Drift-Ground Moraine-Mixed Composition, Chiefly Rock Flour, Sand, Gravel and Boulders. Deposited to Shallow Depths-Generally 0-35± ft
Modified and Semi-Modified Drift. Includes Morainal and Glacial Lake Deposits and Outwash Sands and Gravels. Deposited to Variable Depths Generally 25-200 ft Solid Circles Indicate Location of Borings
Earlier Pre-Glacial or Tertiary Streams
Later Pre-Glacial or Tertiary Streams
Borings
Scale in Miles
0
1
2
3
4
5

FIG. 13.—DISTRIBUTION OF GLACIAL DRIFT AND PRE-GLACIAL DRAINAGE COURSES.

as a sheet or outwash, some beneath the ice in sub-glacial river channels, and others on top of the ice. As a whole, they form a curious assemblage of flat terrace-like areas, and steep and gradually sloped knolls and ridges of varying magnitude.

There are also other deposits of a semi-modified character the surface character and form of which indicate an origin near the front of the glacier. Alden[12] has recognized these deposits as recessional moraines, has traced their approximate boundary lines, and has differentiated the several stages of glacial retreat which contributed toward their development. In surficial configuration these deposits form knolls and ridges of irregularly stratified sand and gravel. Bowl-like depressions, many of which contain small ponds and bogs, are interspersed throughout the morainal area. The difference in relief of these humps and hollows range from 5 ft to not more than 100 ft.

Differentiating between the foregoing types of glacial drift, which occur within the tunnel area, was helpful in choosing the areas throughout which knowledge of the geologic conditions was more uncertain. These modified and semi-modified drift deposits, compared to the unmodified ground moraine, occur in much greater thickness. Their occurrence at several points along the tunnel line introduced uncertainties as to the location and quality of the underlying rock, which, finally, could be determined only by borings.

TUNNEL LOCATIONS

Geological Information Required.—The particular kind of geological information necessary for solving the engineering requirements of the Quabbin Aqueduct may be illustrated best by presenting certain questions raised from time to time in developing the project. Perhaps the following questions will serve the purpose of indicating the nature and range of those presented during the investigation:

Of several possible tunnel locations, is there any geological reason for choosing one line rather than another? Which parts of the location require exploratory investigations beyond what can be determined by field inspection of the surface topography and rock ledges? After preliminary borings have been made, can one tell whether the purpose in each case has been fully met or are additional explorations necessary?

What is the maximum depth of buried pre-glacial stream channels crossing the tunnel line as bearing on the grade of the tunnel? Would a shift of the line avoid any uncertainty as to buried channels and thus end the question of further explorations? Can shafts be located at comparatively low spots and at the same time be reasonably distributed, and yet avoid locations where the cover is heavy or where troublesome conditions may occur?

How many different rock formations will be encountered? what is their individual character, structural relation, and sequence, and what are the practical differences as affecting progress of the work and safety of the structures? What are the possibilities of encountering a formation of an entirely different character from anything reported in published survey reports or discovered in the exploratory program?

What are the possibilities of deep decay extending from the surface to tunnel grade, and of large inflows of water? How much timber is likely to be required? When exposed during the excavation of the tunnel, is the rock of any of these formations likely to cause trouble in the form of "popping rock", slaking, disintegration, or oxidation?

Is it likely that great faults or fault zones will be encountered in the tunnel, and, if so, are they likely to give particular trouble either on account of the quality of rock or the quantity of water? Is the rate of progress, due to changes in rock quality, likely to be different in certain sections of the tunnel?

Is there any feature, quality, difficulty, or combination of features that could make the tunnel project impractical; in other words, is the tunnel project feasible from a geological point of view?

Limitations for Suitable Shaft Sites.—The earlier investigational stages were concerned primarily with the location of the tunnel line and the selection of suitable shaft sites. In this connection it was important to recognize the limits of practicability of the information found in published reports, and to distinguish between those areas where the geological conditions could be reasonably and accurately inferred, and those where additional exploratory investigations would have to be made. Of chief concern was the thickness of glacial drift, the depth of buried channels, and their bearing on the location of the tunnel grade. Studies of the topographic map of the area, and more especially of the topographic forms in the field, revealed significant and helpful information which was used to advantage in the studies of tunnel alignment.

The engineering requirements of the project placed certain limits on the location of shafts and tunnel. These may be briefly summarized, as follows:

(1) The easterly terminus of the tunnel (Shaft 1, Fig. 13) was limited to some point adjacent to the Wachusett Reservoir, the westerly terminus (the portal at Shaft 12), to a suitable point along the east side of the proposed Quabbin Reservoir in the vicinity of Quabbin Lake.

(2) An intake shaft (Shaft 8) was to be constructed along the Ware River, near Coldbrook, the location of which was restricted not only by natural and engineering requirements, but also by legal limitations.

(3) The location of a shaft (Shaft 2) was selected near the junction of Trout Brook and the Quinapoxet River, in order that the combined flows of these streams could be diverted westerly for storage in the Quabbin Reservoir at such times as the Wachusett Reservoir was full or was being used for power purposes, if this was found desirable at some future time.

(4) As it was desired to have the easterly section, between Shafts 1 and part way to Shaft 9, completed by the early part of 1931, the tunnel was designed to be excavated from eight shafts, or fifteen headings. Shafts 3 to 7, inclusive, therefore, were located at points where the topography and bed-rock conditions were favorable to their economical construction. The shafts were spaced at as nearly equal intervals as the topography would allow. As finally arranged most of this section was excavated from twelve headings, or six shafts, there being only a small length of tunnel driven from Shafts 1 and 3.

(5) Fewer working headings, and consequently fewer shafts, were necessary for the westerly section as compared with the easterly section, because of the shorter tunnel length, and especially because more time was available for its construction. Because of this fact the westerly tunnel section was laid out to be constructed from only four shafts, or seven headings, Shaft 11-*A* being constructed as a waterway shaft.

PROBLEMS OF THE ROCK FLOOR

The district through which the Quabbin Aqueduct is located is heavily covered with glacial drift and a large part of the bed-rock geology is obscured. Rock outcrops are sufficiently numerous, however, to determine certain major features concerning the underlying geologic structure. Geologic information previously published in Government *Bulletins*[14] was accepted as a working basis. These *Bulletins* proved to be immensely helpful, especially in the beginning of the applied studies; in reality, they became the ready-at-hand guide in the early reconnaissance studies. The scarcity of rock outcrops added to the difficulty of stating the problems in exact and quantitative terms. This feature made it important to re-study the entire related field and re-state the geology in terms suited to the special purposes of the project.

Information gained through geological investigations made it possible to answer, completely, some of the questions previously presented. It was readily determined, for instance, that there were at least seven important geologic units sufficiently uniform within their own boundaries, and yet varying enough, one from another, to be regarded as different geologic formations. It was also possible to determine the formational sequence from one end of the tunnel to the other, and to indicate the approximate distance of the tunnel within each formation. It was impossible, however, to make a definite statement as to the possibility of faulting, as no important faults were discovered in the exploratory work. However, it was necessary to take this feature into account in making an estimate of the amount of ground likely to exhibit unusual behavior, which might require timbering. Because of the crystalline character of the rock as a whole, there was no apparent reason to expect any unusual or rare features that might cause trouble.

It was apparent from the beginning of the project that the tunnel could be constructed successfully along any of the alternative lines as far as the rock formations themselves were concerned. Such structural weaknesses as might be encountered were expected to cross the tunnel line, and these would be encountered no matter what line was followed. The choice of lines was dependent, therefore, not upon the question of rock character, but upon the topography of the rock floor, especially in regard to the depths of buried channels.

SPECIFIC GEOLOGICAL INVESTIGATIONS

Geological investigations included briefly: (*a*) Field work which was required in the construction of an outcrop map; (*b*) delineating areas where the glacial drift was so thick that knowledge concerning the topography and character of the rock floor was uncertain; (*c*) investigation of pre-glacial drainage courses; and (*d*) explorations by borings. Borings to, and into, rock were made in the conventional manner of doing such work. Particular care was taken, however, in being certain of the location of the underlying rock. It was recognized, in this connection, that boulders 5 ft or more in diameter are not unusual occurrences in the glacial drift covering crystalline rock formation. In order to avoid accepting an incorrect impression of the elevation of sound rock, borings were penetrated into ledge for distances varying between 10 and 20 ft. At several of the shaft sites, in addition, the rock formations were penetrated to a depth of 100 ft.

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[14] **"Geology of Massachusetts and Rhode Island", by B. K. Emerson, *Bulletin No.* 597, U. S. Geological Survey.**

Quinapoxet Valley Investigations.—Great quantities of modified and semi-modified drift deposits occur within the Quinapoxet Valley between Shafts 1 and 2. These deposits form a continuous cover of an uncertain and variable thickness. Such outcrops as were located in the valley indicated that the present stream does not flow in its original channel. Not only was the exact depth of the drift unknown, but it was equally impossible to tell where the deepest channel in the old rock floor should be. A more extensive boring program than usual, therefore, had to be planned to secure the necessary data. The greatest depth of drift penetrated by any of the borings (see **Fig. 14**) was found to be 114 ft. At points within this valley, there is reason to believe that glacial drift is deposited to depths exceeding 175 ft. Finding the rock floor so low (approximately, Elevation 400) raised serious concern as to the thickness of rock cover over the tunnel as it was then located, and was the chief reason for depressing this section of the tunnel approximately 200 ft. Another indirect reason was that by building the tunnel at this lower elevation, the easterly end could be put under full pressure and could serve as a penstock in the development of power at Shaft 1.

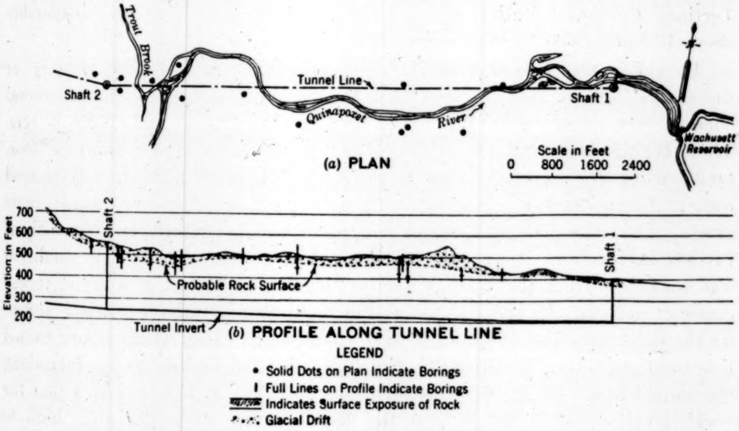

Trout Brook
Tunnel Line
Shaft 2
Shaft 1
Quinapoxet River
Wachusett Reservoir
(a) PLAN
Scale in Feet
0 800 1600 2400
Elevation in Feet
700
600
500
400
300
200
Shaft 2
Shaft 1
Probable Rock Surface
Tunnel Invert
(b) PROFILE ALONG TUNNEL LINE
LEGEND
• Solid Dots on Plan Indicate Borings
I Full Lines on Profile Indicate Borings
▨ Indicates Surface Exposure of Rock
∴ Glacial Drift

FIG. 14.—LOCATION OF BORINGS AND APPROXIMATE ROCK FLOOR ALONG TUNNEL LINE BETWEEN SHAFTS 1 AND 2, QUINAPOXET VALLEY.

This change of tunnel grade eliminated the question of precise depth of cover and the exact location of the deepest channel, for the greater depth fixed for the tunnel left a large margin of safety. The boring program was thus terminated by a modification of design.

Ware-Prince Valley Investigations.—The abrupt change of the Ware Valley, at Coldbrook, from a north-south to an east-west course, is a striking deviation from the normal trend of valleys throughout the district. One explanation of this behavior is based on the assumption that the Burnshirt-Ware and Prince-Ware River Valleys were originally separate systems. In the early stages of its history, the Burnshirt River flowed by way of the present Burnshirt-Ware Valley as far as Coldbrook, and thence southerly out of the district. The course of the Prince-Ware River at that time was directed the same as at present, but later the Burnshirt-Ware River was diverted into it.

A series of seven borings across the Ware River on the site of the diversion dam at Shaft 8, located the lowest point in the valley floor at about Elevation 607, which is approximately 40 ft lower than the flow line of the present stream. The high elevation of the floor at this point indicates that when the pre-glacial Burnshirt River was diverted, its valley floor was left at an elevation higher than 600 ft above sea level. This conclusion was checked by a boring placed ½ mile southeast of Coldbrook Springs, at a point within the Tertiary Burnshirt Valley, which located rock at an elevation considerably above Elevation 600.

To check the reasoned conclusions concerning the geologic history of these valleys, and also to determine the rock profiles across them, several borings were made near the confluence of the Prince and Ware Rivers. These borings indicated a great thickness of glacial drift over the rock floor. One boring, for example, after penetrating 161 ft of glacial drift showed rock at Elevation 434, which was only a few feet above the projected tunnel grade. That the floor was low at this point had been foreseen, but without further explorations it was not possible to determine whether this condition was confined within the Ware Valley or the Prince Valley, or whether it was common to both. In order to determine the extent of this depression, and to fix the point at which it would be safe to cross the valley along a new tunnel line location, several additional borings were located farther to the east along the tunnel line. These investigations (see profile, Fig. 15) indicated that the rock elevation along the line in the Ware Valley was sufficiently high to assure adequate rock cover over the tunnel, and also that the low floor section is confined to the Prince Valley. It became necessary then to select a crossing under the Prince Valley where a higher rock floor could be found. From the study of the erosional processes, river and glacial, which were responsible for the formation of this valley, it seemed evident that the floor of the valley would be appreciably higher farther north. Consequently, several borings were placed along a new location of the tunnel line and the results confirmed that conclusion. Subsequent investigations indicated a rock valley buried to a maximum depth of approximately 110 ft, with the lowest point of the valley floor at about Elevation 480.

One possible interpretation of the facts disclosed by these investigations, is that the low floor at the confluence of the Prince and Ware Rivers is a result of erosion of the Ware River alone. However, since there is a drop in the rock floor of at least 200 ft in less than 3 miles between Coldbrook and Barre Plains, Mass., it is scarcely likely that the Ware River alone could have been responsible for such an abnormal gradient. Further explorations within the Ware Valley between Barre Plains and Coldbrook, showed rock to be considerably higher than would have been expected had the valley been eroded solely by the river. The most reasonable interpretation seems to be that local over-deepening by glacial scour is responsible for the low rock floor in the vicinity of Barre Plains.

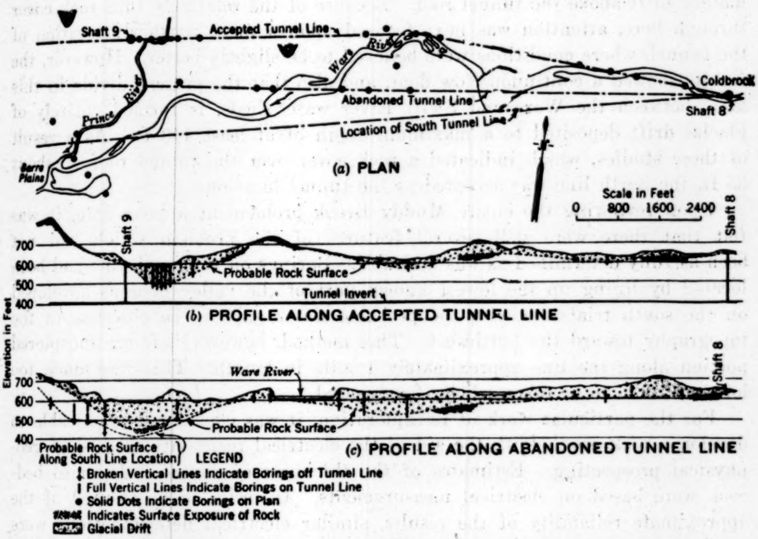

Shaft 9
Accepted Tunnel Line
Ware River
Prince River
Coldbrook
Abandoned Tunnel Line
Shaft 8
Location of South Tunnel Line
Barre Plains
(a) PLAN
Scale in Feet
0 800 1600 2400
Shaft 9
Shaft 8
700
600
500
400
Probable Rock Surface
Tunnel Invert
(b) PROFILE ALONG ACCEPTED TUNNEL LINE
Elevation in Feet
Shaft 8
Ware River
700
600
500
400
Probable Rock Surface
Probable Rock Surface Along South Line Location
(c) PROFILE ALONG ABANDONED TUNNEL LINE
LEGEND
Broken Vertical Lines Indicate Borings off Tunnel Line
Full Vertical Lines Indicate Borings on Tunnel Line
Solid Dots Indicate Borings on Plan
Indicates Surface Exposure of Rock
Glacial Drift

FIG. 15.—RELATION OF ALTERNATIVE TUNNEL LOCATIONS, TOGETHER WITH DISTRIBUTION OF EXPLORATORY BORINGS AND PROFILES, WARE-PRINCE VALLEYS.

Muddy Brook Valley Investigations.—Muddy Brook Valley is a fine example of a large valley now occupied by a stream apparently much too small to have accomplished such an extensive erosional effect. For more than $1\frac{1}{2}$ miles, the broad floor of this valley is covered with low sand and gravel ridges of glacial origin. Most of the drift is assorted and could be accumulated to almost any depth. There is nothing whatever to be seen in the valley bottom, from surface inspection, that would indicate either the form of floor profile or the quality of rock beneath it.

Preliminary investigation showed an irregular distribution of glacial drift, and suggested the conclusion that Muddy Brook Valley was the result of preglacial stream erosion, but that considerable deepening and widening may have been accomplished by glacial abrasion. How much of an effect these processes had on the final configuration of the rock floor it was impossible to foretell.

The first boring explorations in the valley were made along a trial location designated as the south line. A rock profile developed from several borings placed across a part of the valley near this line revealed a deeply buried valley. The lowest point of the floor along this line was determined at approximately 45 ft above the tunnel roof. Because of the relatively thin rock cover through here, attention was next focused upon a more northerly location of the tunnel, where conditions were believed to be slightly better. However, the results showed a continuous low floor, and also that the present divide in this area, between the Ware and Swift River water-sheds, is formed entirely of glacial drift deposited to a maximum depth of at least 140 ft. As a result of these studies, which indicated a rock cover over the tunnel roof of about 55 ft, the north line was accepted as the tunnel location.

Upon reviewing the entire Muddy Brook problem at a later date, it was felt that there were still several features of the problem which had not been as fully determined as was desirable. Borings on the north line had been located by lining up the lowest known part of the valley floor as developed on the south trial line with the probable water gaps to be observed in the topography toward the northwest. This method, however, left an unexplored portion along the line approximately 1 mile in length. This was much too long a stretch to be taken wholly for granted.

For the particular work of re-exploration, it was considered practicable to develop a rock profile with the aid of the electrical resistivity method of geophysical prospecting. Estimates of the thickness of cover or depth to bedrock were based on electrical measurements. In order to be assured of the approximate reliability of the results, similar electrical determinations were made at two points where the thickness of cover was already known from borings. In both cases the concordance of results was close.

A rock profile was thus developed across the valley floor, which considerably extended the reliable information. In Fig. 16, the profile is drawn along the north line (the final location) and defines the approximate rock floor as indicated by borings and geo-physical explorations. The group of borings farther south were located on a preliminary trial line which was abandoned in favor of a location farther north. Throughout the re-explored section in question, Fig. 16(*b*), the mantle of glacial drift was shown to be rather thick, but the elevation of the rock floor was considerably higher than in the adjoining section on the west. As a result of these investigations, successful excavation of the tunnel through the rock floor of this valley was made more certain.

Portal Site Investigations.—The first steps in the sub-surface investigations of the west portion of the Quabbin Aqueduct were made in Muddy Brook Valley, along a line referred to as the "south line." In the beginning, location studies of the west terminus of the tunnel were confined to the selection of a suitable portal site on the East Branch of the Swift River, somewhere south of Quabbin Lake. The portal of the south tunnel line was located about $\frac{1}{4}$ mile south of the lake at a natural site. It was noted, however, that the terrain along the east side of the lake would allow for considerable saving, in the length of tunnel—about $\frac{1}{3}$ mile—for a portal located farther north. Preliminary studies narrowed the list of possible sites to two,

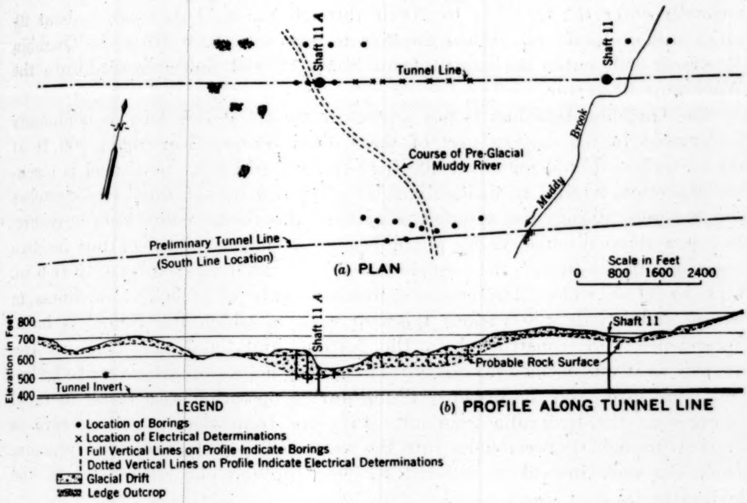

Shaft 11 A
Shaft 11
Tunnel Line
N
Course of Pre-Glacial Muddy River
Muddy Brook
Preliminary Tunnel Line (South Line Location)
(a) PLAN
Scale in Feet
0 800 1600 2400
Elevation in Feet
800
700
600
500
400
Shaft 11 A
Shaft 11
Probable Rock Surface
Tunnel Invert
LEGEND
(b) PROFILE ALONG TUNNEL LINE
● Location of Borings
× Location of Electrical Determinations
Full Vertical Lines on Profile Indicate Borings
Dotted Vertical Lines on Profile Indicate Electrical Determinations
Glacial Drift
Ledge Outcrop

FIG. 16.—BORINGS IN MUDDY BROOK VALLEY.

one at the extreme north end of Quabbin Lake, and the other midway along the east section. After careful investigations by borings, the most northerly site was finally chosen as the portal. This choice made it possible to take advantage of the shortest possible tunnel location with its consequent saving in construction cost.

TUNNEL DESIGN AND LOCATION ADOPTED

As finally located, the Quabbin Aqueduct is oriented in a general east-west direction and extends from the northwest side of Wachusett Reservoir to Quabbin Lake (see Fig 13). Excavation was carried through rock for its entire length of 24.6 miles. The quantity of rock cover above the tunnel, as nearly as could be determined by borings, varies between 55 and 800 ft. In the easterly 3½ miles, the tunnel level or invert lies between Elevations 200 and 385. For the remaining length it continues in a slightly inclined grade to the portal, where the invert is at Elevation 414. All elevations given herein and on the diagrams refer to the Boston City Base datum plane.

The completed tunnel will serve as an aqueduct which, because of the surface topography of the area and its design, will fulfill several functions: (*a*) It will furnish a means of bringing water from the Quabbin Reservoir which will have a maximum flow-line at Elevation 530; (*b*) at Shaft 8, at Elevation 656, it will permit diversion by gravity of the Ware River into the Quabbin Reservoir, or the Wachusett Reservoir; and (*c*) at Shaft 2, at Elevation 540, it will permit diversion of a considerable part of the Quinapoxet River flows into the Quabbin Reservoir if, and as, this is desired. The tunnel flow controlled by gates at the east, or the Wachusett Reservoir, end will normally enter the Quabbin Reservoir through Shaft 11-*A*, located about 2½ miles east of Shaft 12. When the flow is reversed, water from the Quabbin Reservoir will enter the tunnel from Shaft 12 and flow eastward into the Wachusett Reservoir.

The Quabbin Aqueduct is not a pressure tunnel as this term is commonly understood in the eastern part of the United States. For about 500 ft at the easterly end, and for about 200 ft at the westerly end, the tunnel is circular in section, having an inside diameter of 12 ft 9 in. In these two stretches the concrete lining was grouted solidly to the rock, under high pressure. Between these circular ends, which behave as plugs in preventing leakage from or into the tunnel, the tunnel section is of the horseshoe type, 12 ft 9 in. high by 11 ft wide. The concrete lining is only of sufficient thickness to insure stability in the various types of rock in which the tunnel is built. Weep-holes were constructed in the bottom, arch, and elsewhere, as frequently as it was deemed necessary. These weep-holes prevent excessive hydrostatic pressure upon the masonry lining during operation as a result of rapid changes in the hydraulic gradient. Pressure from the tunnel outward is carried through the weep-holes into the surrounding rock, and water pressure from the rock inward is relieved by flow through the weep-holes as the hydraulic gradient drops.

The tunnel portion from Shaft 8 to the Wachusett Reservoir, about 14 miles in length, has been operated under extreme conditions at intervals since 1931 without any signs of structural or other defects.

To determine the payment for rock excavation in the tunnel, certain reference lines were provided and designated as the *A* and *B* lines. The *A*-line was defined as that within which no excavated material of any kind was permitted to remain. In the normal tunnel cross-section, this line was placed at a minimum distance of 3 in. outside the neat line of the concrete side-walls, and of 8 in. above the neat line of the arch. The *B*-line was arbitrarily placed 12 in. outside the *A*-line in the side-walls and arch, and 2 in. below the *A*-line in the invert. Slight modifications of these distances were made where permanent timbering was required. All payment for excavation in the untimbered portions was for the volume bounded by the *B*-line, or normally about 6.8 cu yd per lin ft.

All shafts, except Shafts 1, 8, and 12, had an inside diameter of 14 ft The three exceptions were larger than this and were designed to serve not only for construction purposes, but also as waterway shafts. Shaft 1 was excavated as one shaft, but completed as two units, one unit being used as a water outlet, the other as a pump-well for dewatering the depressed tunnel section between Shafts 1 and 3.

TUNNEL EXCAVATION

Rock Formations Encountered.—From an engineering viewpoint, the rocks of the tunnel area can be divided into two major classes, schist and granite. Both these varieties are highly crystalline, the granite being of an igneous origin, and the schist being originally of sedimentary origin. Excavations

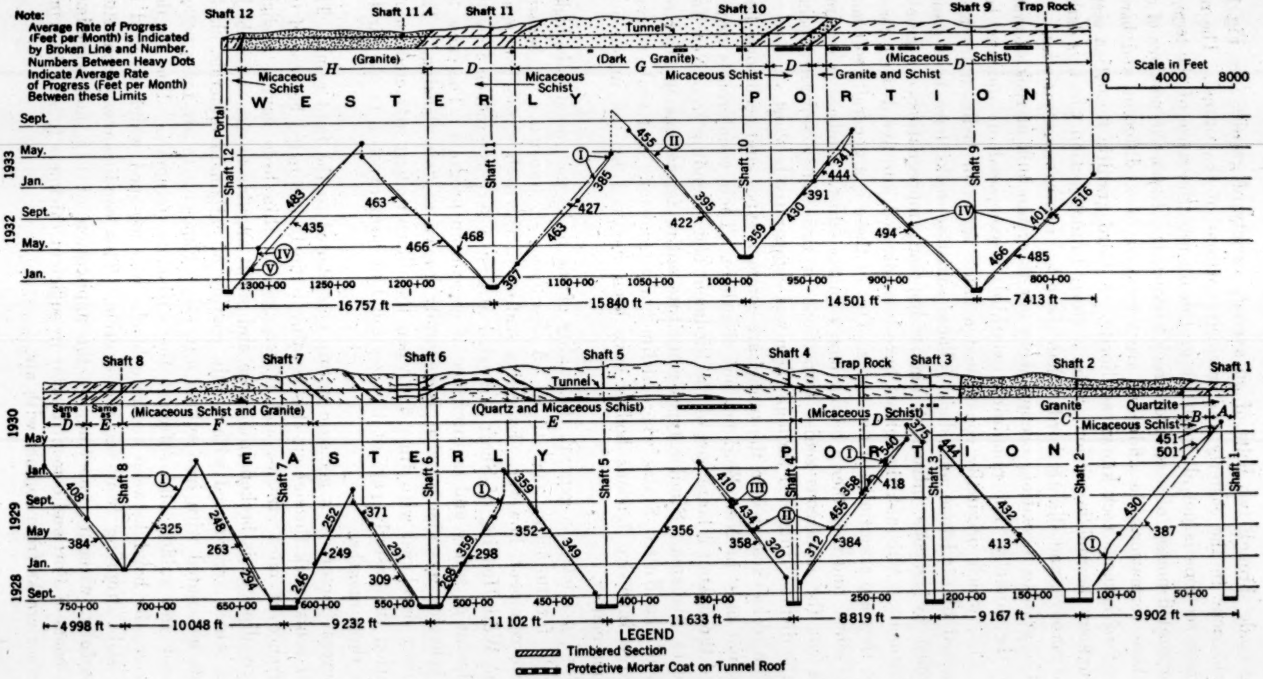


FIG. 17.—PROGRESS OF TUNNEL EXCAVATION AND GEOLOGICAL SECTION.

were made through two belts of an average variety of granite (see Fig. 17) which occur in the easterly and westerly portions of the tunnel. The rock in these sections is not all intrusive granite in the true sense of the term, as incorporated masses of a schistose rock also occur. Extensive portions of one of these formations, in addition, are composed of a different and harder variety of igneous rock known as diorite. A third granitic formation occurs throughout the middle portion of the westerly tunnel section which is darker, more massive, and slightly harder than the other two average varieties.

The granite formations occupy about one-third the bulk of rock excavated from the tunnels. The remaining two-thirds is composed of laminated beds of schist, which originally constituted a thick series of sedimentary rocks, composed of sandstone and shale. These schistose rocks have been greatly modified by igneous intrusions and widespread impregnations of granitic constituents. Although the schists have been divided into five distinct formations, they are essentially of two types. One of these is a generally soft micaceous schist, and the other a harder flaggy quartz schist. Diabase or trap-rock dikes cut the micaceous schist formations at two widely separated points, but the distance tunneled through them was comparatively short.

The rocks excavated in the tunnel are common types, and occur over large areas not only in New England, but elsewhere. Their counterparts may be traced throughout a large part of the Piedmont belt extending from New York State to Alabama. Similar types constitute entire formations in other sections of the United States where sedimentary sandstone and shale adjacent to large intrusive igneous bodies have been modified by the injection of igneous matter.

Scaling Rock.—In general, the quality of the rocks was found to be highly suitable for tunneling operations. A few cases of unusual rock behavior deserve comment, however. Excessive scaling in the arch, throughout much of the West Heading of Shaft 4 created at first a serious problem from the standpoint of safety and maintenance of excavation progress. The cause for this behavior was due to what is generally known as "air slacking." This condition developed with time, for when the difficulty first attracted serious attention the easterly part of this section had been exposed to the air for about three months.

Several factors of a geologic nature contributed to the final result. The quartz schist rock throughout this heading is finely laminated because of the occurrence of mica streaks, and is arranged in thin beds having a slight easterly dip. It is broken into large and small angular blocks and slabs by joints, which are covered with a thin coating of carbonate minerals, chiefly calcite, associated with iron sulfide minerals. Exposure to air for a period, oxidized the sulfide and softened the carbonate minerals. Due to this behavior, the bond between separate blocks was loosened which resulted in large slabs dropping from the roof section. This condition was combatted successfully, without timbering, by the application of two, and sometimes three, coats of Portland cement mortar in sufficient thickness to seal the joints and other cracks from contact with air.

The granite, throughout a short stretch in the easterly tunnel portion, also scaled, but this behavior resulted from a geologic condition which differed from that causing "air slacking." The grain of the granite and the orientation of incorporated masses of schist indicate a strong geologic structure, the rock lying in a nearly vertical position. The entire formation was under strain which, upon being released by excavation of the tunnel, developed new fractures causing slabs to fall from the arch. This condition commonly known as "popping rock" required constant attention and the frequent scaling of rock in the arch by the contractor. Covering the arch section with mortar would not have improved the situation in this case, and the condition was not sufficiently serious to require timbering.

Timbering.—A question of practical importance, always raised on a project of this kind, concerns the amount of timbering likely to be required, and which particular sections of the tunnel are most likely to require such construction.

No definite answer could be given in advance to this question, but it was thought that the rock would require timbering throughout possibly a maximum of 10% of the tunnel. In the estimates it was assumed that about 5% would require some form of timber support. The actual figures, including those for correcting the condition of "air slacking" in the arch, are as follows:

	Feet		Percentage
Permanent timber support.........	1 971	....	1.5
Temporary timber support.........	320	....	0.3
Protective mortar coat............	12 050	....	9.3

Many intrusions of granite in the micaceous schist formations established conditions that were followed by alteration and gradual decay of the adjacent rock. Partings along bedding lines served as watercourses, so that the entire formation was affected, some sections more so than others. This condition caused the rock to scale and crumble, which made it necessary to construct timbering throughout the stretches where the formations were softest. The protective coat was sufficient to stabilize the rock in other sections where alteration and consequent decay were not so prominent.

Excavation was carried through several zones of crushing and close jointing without any abnormal rock behavior. At some points, however, overbreakage on the sides and in the arch resulted from such conditions. Timbering was constructed in some of these sections, especially where the removal of loose blocks did not appear to remedy the unstable situation. In a short stretch, near Shaft 12, the rock had a tendency to swell and move into the excavated space. This behavior was due to the occurrence of large quantities of clay derived by alteration of an intrusive igneous body containing much feldspar.

PROGRESS OF EXCAVATION IN RELATION TO GEOLOGIC CONDITIONS

Schedule of Required Progress.—The first contracts, awarded in 1927, were those for the construction of Shafts 2 to 7 in the easterly tunnel portion. The award of two major contracts during the latter part of 1927, for constructing the easterly tunnel section from and including Shaft 1, to 4 000 ft beyond and including Shaft 8, was delayed about five months by unforeseen legal difficulties. During this time the construction of six shafts was completed, and a considerable length of tunnel, in all 8 238 ft, was excavated with equipment used in sinking the shafts. The schedule of minimum progress in the first major contracts, required that excavation of 65 664 ft of tunnel should be completed within 26 months. The third major contract, for constructing the westerly portion of the tunnel, including four shafts and appurtenant structures, was awarded during the early part of 1931. The schedule of progress on this contract required all excavation in shafts and 54 845 ft of tunnel to be completed in 34 months. The contract requirements as to the minimum performance in excavating these tunnels may be summarized as shown in Table 6. As actually constructed, all but 836 ft of the tunnel between Shafts 1 and 2 was excavated from Shaft 2, and the actual length of tunnel driven west of Shaft 8 was 5 000 ft. Furthermore, after being excavated, Shaft 3 was not used extensively as a construction shaft, and practically all the heading between Shafts 3 and 4 was excavated from Shaft 4.

TABLE 6.—CONTRACT REQUIREMENT FOR MINIMUM PERFORMANCE

Shaft (see Fig. 13) (1)	Length of tunnel to be excavated, in feet (2)	Minimum requirement, in feet per month* (3)	Shaft (see Fig. 13) (1)	Length of tunnel to be excavated, in feet (2)	Minimum requirement, in feet per month* (3)	Shaft (see Fig. 13) (1)	Length of tunnel to be excavated, in feet (2)	Minimum requirement, in feet per month* (3)
(*a*) EASTERN PORTION			(*a*) EASTERN PORTION (*Con.*)			(*b*) WESTERN PORTION		
1†	4 000	275	5 to 6	9 734	425	9‡	7 414	260
2‡	5 034	220	6 to 7	7 702	335	9 to 10	14 501	550
2 to 3	7 634	330	7 to 8	9 188	440	10 to 11	15 840	620
3 to 4	7 811	340	8†	4 000	275	11 to 12	16 757	580
4 to 5	10 561	460	......	....	....	12§	319	....

* Minimum progress required in one or more headings to complete tunnel excavation in the specified time. † West Shaft. ‡ East Shaft. § Shaft 12 to West Portal.

The rates indicated in Table 6, especially those in Table 6(*a*), were estimated from the average performance attained on other tunnel projects throughout the eastern part of the United States. Those in Table 6(*b*) were based upon the average performance attained in excavating the eastern part of the Quabbin Aqueduct with liberal allowance for unforeseen construction difficulties. A few of these expected difficulties, which are essentially of a geologic nature, and of which complete information could not be obtained in advance may be thus summarized:

1.—As the tunnel crosses the general trend of several geologic formations, over a belt 25 miles wide, it was expected that some of the headings would pass through several faults and crush zones requiring timber support and in other ways creating serious difficulties. The results of studies of surface outcrops and core borings, made at critical and intermediate points, did not reveal the occurrence of such zones of weakness. It was realized, however, that this was not unusual because of the extent to which glacial drift deposits had buried the underlying rock formations. It was believed expedient, therefore, to allow for the possibility that this condition might occur at least as frequently as once in every mile or so.

2.—An allowance was also made for possible occurrence of deep decay extending from the surface to the tunnel grade, or disintegration or slacking of the rock when exposed to air. The least troublesome of these conditions would require only precautionary measures during excavation, and if they were sufficiently extensive, timbering, cement mortar lining, or some other form of construction.

3.—The occurrence of rocks of a crystalline character was recognized, and also that they would not contain water, except within the space between joints, and other fractures and fissures. The tunnel passes under several river valleys and areas containing porous glacial deposits, and considerable leakage into the tunnel from these was anticipated, together with the attendant hazards of inflows temporarily beyond pumping facilities.

4.—Changes were made in the original location of the aqueduct in order to cross under two deeply buried valleys. The possibility of encountering serious difficulties in these stretches, because of the presence of faults or crush zones, deep decay, large inflows of water, or because other combinations of adverse considerations made it expedient to allow for considerable retardation of progress.

Methods of Excavation.—A description of the methods used in tunnel excavation is included in order to supplement the information presented in Fig. 17 and Table 7. The data concern only the performance attained after permanent tunnel equipment had been installed. The shorter tunnel stretches excavated with the equipment used in shaft sinking are not considered. Shaft 3 was eliminated as a construction shaft soon after construction began under the first major tunnel contracts, but access to it was maintained as insurance against difficulties encountered in excavating from Shaft 2. Only a short stretch of tunnel was excavated from Shaft 1.

The numerals within circles on the progress chart in Fig. 17 refer to geological or other conditions which appreciably affected the performance of the work, as follows: (I) Progress retarded by water; (II) increase in progress due to changes in organization; (III) progress retarded by "air-slacking" rock; (IV) progress retarded by unstable rock, requiring timbering; and (V) slow progress due to experiments on the dragline mucking machine.

The full-face method of excavation was used at Shafts 2, 4, and 9 to 12, inclusive. The headings at the first two shafts were advanced continuously by drilling and mucking crews, which worked on a schedule of three 8-hr shifts. By this plan the contractor hoped to obtain four advances in one day and five the next. This objective was realized at times, but not to the extent anticipated. Two 10-hr shifts were employed at Shafts 9 to 12, inclusive. The number of advances in one day was seldom less than four, and five advances per day were attained at times, except at Shaft 12, where there was one heading. Three advances were often made at Shaft 12, but in order to do this, the crews were required to work overtime.

TABLE 7.—Data Relating to Excavation and Character of Rock

Description	Quartzite	Micaceous schist	Granite	Micaceous Schist			Quartz Schist		Micaceous schist and granite	Dark Granite		Granite	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(a) Progress Data													
Location in Geologic Section (see Fig. 17)	*A*	*B*	*C*	*D*	*D*	*D*	*E*	*E*	*F*	*G*	*G*	*H*	*H*
Method of excavation	*	*	*	*	*	*	*	†	†	*	*	*	*
Heading in Which Formation Occurred; Shaft	2‡	2‡	2§	4‡	9§	11§	4§	5‡	8‡	10§	11‡	11§	12‡
Average Advance per Shot, in Feet: Monthly	7.3	7.7	7.0	6.5 and 7.3	8.8	8.3	6.8 and 7.3	6.2	6.5	7.3 and 7.8	7.8	8.0	7.7
Range: From	7.1	7.4	6.1	6.1‖	7.2	6.6	6.3¶	5.8	5.8	6.8**	6.0	7.6	7.0
To	7.4	7.9	7.4	6.9	9.5	9.4	7.8	6.7	7.5	8.2	8.5	8.8	8.9
Number of Days in Which There Was no Advance, for the Following Reasons: Timbering	0	0	0	5	7	0	0	0	0	0	8	0	0
Poor roof	0	0	0	2	0	0	25	1	0	2	0	0	0
Water	0	0	0	18	0	0	0	0	32	0	4	0	0
Holidays	0	0	2	4	5	5	4	2	2	5	4	0	4
Repairs to plant, etc.	0	0	3	7	7	4	11	2	2	4	3	0	6
Number of Shots Daily: One-round shots	4	2	51	75	35	24	102	36	62	61	38	17	23
Two-round shots	37	76	362	350	392	216	285	393	306	497	372	252	300
Three-round shots	7	21	37	54	4	0	30	0	0	0	0	0	78
(b) Drilling and Blasting Data													
Range in Number of Holes in Heading: From	30	31	30	30	38	29	32	24††	26††	36	31	29	30
To	32	35	40	40	40	36	40	32††	34††	44	36	36	42
Typical heading: Number of holes	31	31	37	37	38	32	37	28†† 6‡‡	29†† 7‡‡	40	32	36	36
Average total drilling, in feet	287	287	345	345	386	346	345	224†† 48‡‡	244†† 56‡‡	440	288	396	396
Average Depth of Individual Holes, in Feet: Cut	9	9	9	9	9 and 11	10+	9	8	10	9 and 11	9+	9+	9 and 11+
Relief	8 and 9	8 and 9	8 and 9	8 and 9	9 and 11	10	8 and 9	8	8	9 and 11	9	9	9 and 11
Trim	8 and 9	8 and 9	8 and 9	8 and 9	9 and 11	10	8 and 9	8	8	9 and 11	9	9	9 and 11
Speed of Drilling, in Inches per Minute per Drill: Typical	7	10	7	12	16	13	9	9	11	6	6	7	7
Ranging: From	5	5	5	7	10	7	7	7	7	5	5	5	5
To	9	12	9	15	20	20	11	11	14	12	12	9	9
Number of drills at heading	5	5	5	5	4	4	5	2	2	4	4	4	4
Average Time Required to Drill a Heading, in Hours: From	2	1.5	2	1.5	1.5	2	2	5††	5††	3	3	2.5	2.5
To	3.5	3.5	3.5	2.5	2.5	3	3	7††	7††	4	4	3.5	3.5
Average Time Required for Drilling - Blasting Cycle, in Hours: From	4	3	4	3	2.5	3	3.5	9	9	4	4	3.5	3.5
To	5	4	5	4	3.5	4	4.5	10	10	5	5	4.5	4.5
Average Unit per Cubic Yard of Excavation: Linear feet of holes	6.0	5.7	7.0	7.0	6.5	6.5	7.2	6.5	6.5	7.0	6.7	6.5	7.3
Pounds of explosive	5.0	4.5	6.5	6.0	3.8	4.9	6.2	6.0	6.0	5.7	6.0	5.3	5.1

* Full face. † Top heading and bench. ‡ East of shaft. § West of shaft. ‖ Also from 6.7 to 7.8. ¶ Also from 6.7 to 7.9. ** Also from 7.5 to 8.2. †† Top heading. ‡‡ Bench.

The heading and bench method was used in the headings of Shafts 5 to 8, inclusive. At these points the work was organized on a basis of two 10-hr shifts in such a manner as to produce two rounds of excavation per day in each heading. This arrangement required drilling and mucking to be done together.

At Shaft 7 a different plan was used in which drilling and blasting of each heading and bench was completed during one shift and mucking during the other. The depth of holes was increased to 10 ft. The heading was blasted in two operations, the cut and bench holes being loaded and shot first.

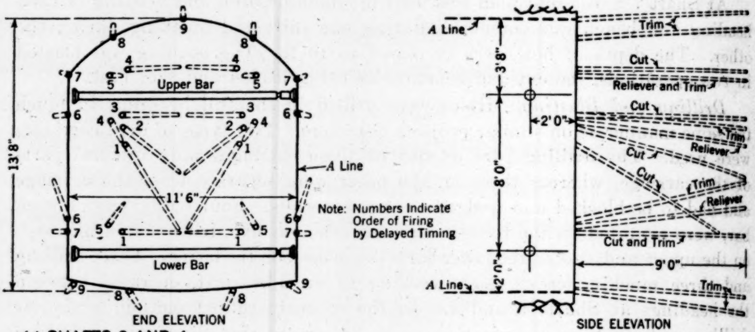
Drilling and Blasting.—Holes were drilled in the full headings in much the same manner as on similar projects elsewhere. Two types of drill carriages were used. The drilling bars on one of these carriages were integral parts of the carriage, whereas those on the other were separate from the carriage and had to be blocked and wedged to the side-walls. Four drills, one on each bar, were mounted on the former carriages whereas five drills—three mounted on the upper and two on the lower bars—were used on the latter. Three drilling and three mucking crews, each working in an 8-hr shift, operated between the headings at Shafts 2 and 4. In the westerly tunnel section, a similar drilling and mucking organization was used, except that the crews worked on only a 10-hr per shift schedule.

As good, and in several cases even better, progress was attained in excavating the easterly section, under similar rock conditions. The integral type of drill carriage, compared to the other type, proved to be a time-saver and in this way contributed toward a better performance record. Holes in the top headings were drilled by two machines mounted on horizontal arms clamped to two vertical columns, which were blocked and wedged to the tunnel roof and bench. Bench holes were drilled by the same machines, mounted on one horizontal bar located in front of the bench, and wedged to the tunnel side-walls. Drilling and mucking were simultaneous operations in these headings.

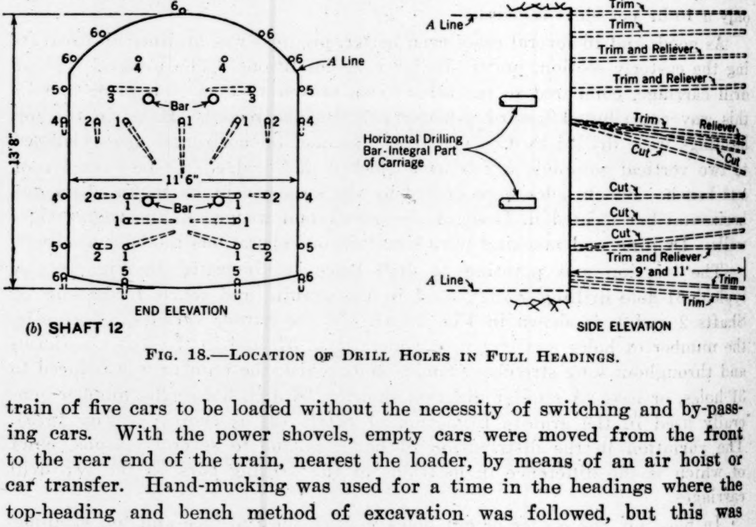
The location and pointing of drill holes is illustrated in Fig. 18. A typical 31-hole drilling round, used in the granite and schist formations of Shafts 2 and 4, is shown in Fig. 18(*a*). In the harder varieties of granite, the number of holes was increased generally to 37, and in the softer varieties and throughout long stretches of micaceous schist, the number was reduced to 31 holes, or less. A similar diagram showing 36 drill holes (the number generally used in the granite formation of Shaft 12) is shown in Fig. 18(*b*). The variation in the distribution of holes, is due to several reasons, chief of which is the difference in locations of the drilling bars on the two drill carriages.

In blasting, the center or cut holes in both the full-face and top headings were generally loaded with 60% gelatine dynamite, and the remaining holes with dynamite of 40% strength. At times, dynamite of 40% strength was used in all holes, especially in stretches of soft rock. Throughout the westerly tunnel portion the contractor used a new explosive product, in strengths of 40, 47, and 60 per cent. The quantity of explosive used per cubic yard of excavation in the different kinds of rock is indicated in Table 7.

Mucking.—In all the full headings, except the single one at Shaft 12, blasted rock was loaded by means of electric power shovels. At Shaft 12, the contractor installed a dragline excavator of his own construction. This machine was equipped with a conveyor belt of sufficient length to allow a full train of five cars to be loaded without the necessity of switching and by-passing cars. With the power shovels, empty cars were moved from the front to the rear end of the train, nearest the loader, by means of an air hoist or car transfer. Hand-mucking was used for a time in the headings where the top-heading and bench method of excavation was followed, but this was abandoned later in favor of mechanical mucking by air shovels.


Upper Bar
13'8"
A Line
11'6"
Lower Bar
END ELEVATION
Note: Numbers Indicate Order of Firing by Delayed Timing
A Line
3'8"
2'0"
8'0"
2'0"
A Line
Trim
Cut
Reliever and Trim
Cut
Trim
Reliever
Cut
Cut
Trim
Reliever
Cut and Trim
9'0"
Trim
SIDE ELEVATION

(a) SHAFTS 2 AND 4


A Line
13'8"
Bar
Bar
11'6"
END ELEVATION
A Line
Horizontal Drilling Bar-Integral Part of Carriage
Trim
Trim
Trim and Reliever
Trim and Reliever
Trim
Reliever
Cut
Cut
Cut
Cut
Cut
Trim and Reliever
9' and 11'
Trim
Trim
A Line
SIDE ELEVATION

(b) SHAFT 12

FIG. 18.—LOCATION OF DRILL HOLES IN FULL HEADINGS.

The time required to remove rock from the headings varied. In the full headings, except those at Shafts 11 and 12, the average time was between 2.5 and 3.5 hr. At Shafts 11 and 12 it was between 3 and 5 hr. Where the top-heading and bench method was used the average time was between 5.5 and 6.5 hr.

Throughout the easterly portion, loaded muck cars were hoisted from the tunnel by means of cages, whereas, in the westerly portion, skips were used. Provision was made at Shafts 9 to 12, inclusive, for stocking rock of suitable quality and size for use as concrete aggregate. This was accomplished by wasting rock in those headings where the formations were soft, and by dumping rock from hard formations over a coarse screen or "grizzly." The portion passing through this separator was then crushed and conveyed by belt to the stock-pile. From experience on the Quabbin project, the method of stocking suitable rock material and wasting that of inferior quality is believed to be superior to that of dumping excavated rock on a spoil bank regardless of quality. This is especially true where tunnels are excavated through different geological formations, some of which may be soft or in other respects may have objectionable characteristics.

Hardness of Formations.—The relative hardness of the rocks is shown in Table 7(*b*) under the heading, "Speed of Drilling." A wide range of drilling behavior is indicated. The micaceous schist formations were the softest, the high mica content and soft condition, due chiefly to alteration of minerals, being conducive to fast drilling. Slower drilling in these formations was due to encountering quartzite beds and granite bodies.

The hardest rocks to drill, except trap-rock, were those contained in the granite formations. In general, the granitic rocks were about three times as hard to drill as the micaceous schists. Between the extremes represented by these formations there was a rather uniform hardness represented in the flaggy quartz schist types and the micaceous schists containing much granitic material. The widespread intrusions of granite in portions of the micaceous formations rendered the rock harder and more stable than would otherwise have been the case.

EXCAVATION COSTS

Information pertaining to the cost of excavation in the several rock formations is shown in Table 8. Partial information relating to this particular item has been published[15] previously, although not in the form presented herein. The values given are the costs for excavating to approximate line and grade only. Before lining, the contractors were required to remove rock which projected beyond the arbitrary limits on the side-walls and the arch, and also to shape the tunnel invert to grade. The cost of this operation is not included in Table 8.

The much lower cost indicated in Column (10), Table 8(*b*), is due mostly to the use of second-hand equipment, purchased from the contractors who constructed the easterly portion. This equipment, which had been maintained in excellent condition, was obtained at greatly reduced prices, compared to the original costs. Comparison of costs in the easterly tunnel portion (Table 8(*a*)) with those in the westerly portion (Table 8(*b*)) indicate striking differences

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[15] "Construction of Wachusett-Coldbrook Tunnels", by Douglas C. Corner, U. S. Bureau of Mines; "Boston Metropolitan Water Supply Extension", by Karl R. Kennison, M. Am. Soc. C. E., *Journal*, New England Water Works Assoc., Vol. XLVIII, No. 2.

TABLE 8.—Approximate Operating Cost of Excavation, Quabbin Aqueduct, After the Installation of the Permanent Tunnel Equipment

(Values in Parentheses Are Dollars per Cubic Yard; the Remaining Values Are Dollars per Linear Foot)

Item No. (1)	Shaft (see Fig. 13) (2)	Rock formation (see Fig. 17) (3)	Location (see Fig. 17) (4)	Labor cost for all conditions, geologic or otherwise (5)	Labor cost for average conditions in good rock (6)	Labor cost for both headings for all conditions, geologic or otherwise (7)	Power (8)	Explosives and detonators (9)	Material and supplies (plant, etc.) (10)	Cost of overhead (11)	Total operation cost (12)
(*a*) Eastern Portion											
1	2	Quartzite.......	*A*	$23.80 (3.50)	$23.80 (3.50)	$27.50 (4.05)	$5.10 (0.75)	$6.20 (0.90)	$11.30 (1.65)	$4.10* (0.60)	$54.20 (7.95)
2	2	Micaceous schist	*B*	21.70 (3.20)	21.70 (3.20)						
3	2	Granite........	*C*	28.20 (4.15)	24.30 (3.60)						
4	2	Micaceous schist	*D*	24.30 (3.60)	23.60 (3.45)						
5	4	Micaceous schist	*D*	29.10 (4.28)	23.70 (3.50)	$32.50 (4.80)	$6.00 (0.90)	$5.60 (0.80)	$14.80 (2.15)	$3.80* (0.55)	$62.70 (9.20)
6	4	Trap-rock......	......	32.80 (4.85)	32.80 (4.85)						
7	4	Quartz schist....	*E*	33.20 (4.90)	27.80 (4.10)						
	5	Quartz schist....	*E*	28.20 (4.15)	26.10 (3.85)	$28.20 (4.15)	$4.40 (0.65)	$5.80 (0.85)	$8.90 (1.30)	$2.10* (0.30)	$49.40 (7.25)
(*b*) Western Portion											
9	9	Trap-rock......	......	$20.50 (3.00)	$20.50 (3.00)	$16.00 (2.35)	$2.80 (0.40)	$3.70 (0.55)	$5.50§ (0.80)	$4.40§ (0.65)	$32.40 (4.75)
10	9	Micaceous schist	*D*	15.60 (2.30)	14.10 (2.10)						
11	10	Micaceous schist	*D*	17.90 (2.60)	14.50 (2.15)	$18.10 (2.65)	$5.00 (0.75)	$4.80 (0.70)	$6.10§ (0.90)	$4.60§ (0.65)	$38.60 (5.65)
12	10	Dark granite....	*G*	18.20 (2.70)	15.40 (2.25)						
13	11	Dark granite....	*G*	16.90 (2.50)	14.70 (2.15)	$15.90 (2.35)	$4.10 (0.60)	$4.50 (0.65)	$5.50§ (0.80)	$4.40§ (0.65)	$34.40 (5.05)
14	11	Micaceous schist	*D*	13.90 (2.05)	13.90 (2.05)						
15	11	Granite........	*H*	16.10 (2.35)	16.10 (2.35)						
16	12†	Granite........	*H*	21.30‡ (3.15)	17.50‡ (2.60)	$21.30‡ (3.15)	$4.90 (0.70)	$4.90 (0.70)	$9.10§ (1.35)	$9.00§ (1.35)	$49.20 (7.25)

* Taken from *Information Circular*, entitled "Construction of the Wachusetts-Coldbrook Tunnel", U. S. Bureau of Mines. † Cost data for one shaft only. ‡ Labor costs of excavation after development of dragline excavator and after passing through a section that required timbering. § More approximate than other cost data.

of excavation costs in formations of similar rock character and behavior. This may be partly accounted for, as follows:

(1) The easterly tunnel portion was excavated during 1928–30, and the westerly portion during the period, 1931–33. During the latter period, the wage scale of the several classes of labor was from 5% to 25% lower than during the former years.

(2) Better progress as a whole was made in the westerly portion, where the full-face method of excavation was used entirely, than in the easterly portion, where only one-third of the headings were excavated by the full-face method and two-thirds by the slower top-heading and bench method.

(3) The number of man-hours per day, in the easterly portion, was generally from 10% to 20% greater than those in the westerly portion.

(4) Micaceous schist formations extended through longer stretches of the westerly tunnel portion than of the easterly portion. In addition, these schists were considerably softer than their counterparts farther east. These two conditions are partly responsible for the better progress throughout the westerly portion.

(5) Disregarding the single heading at Shaft 12, better progress was made in the granite formations of the westerly portion as compared to that made in granite of similar quality in the easterly portion. This advantage was largely the result of longer advances per shot, which, in turn, were due to an improved type of explosive, and deeper, and possibly better directed, drill holes.

Rock Breakage.—High and wide over-breakage at several places was caused by closely spaced and intersecting joints. This condition had some effect upon the actual volume of rock excavated as compared to the volume within the theoretical payment lines. The payment line for excavation was fixed arbitrarily 12 in. outside the *A*-line, the mimimum required line of excavation in the side-walls and arch. As excavated, the average rock breakage beyond the *A*-line, considering the total length of tunnel, was about 10½ in.

CONCLUSIONS

An attempt has been made in this paper, to correlate data of a geologic nature with that related to engineering and construction problems. Construction of the Quabbin Aqueduct, without encountering serious difficulties, may be attributed partly to the sound character of the rock formations through which the tunnel extends. It can be emphasized, however, that many of the difficulties that might have been encountered were eliminated by careful geologic investigations, which reduced construction uncertainties to a minimum, and conversely determined the favorable geologic factors.

Several items of interest may be emphasized in conclusion, as follows:

1.—The kind of geological investigations described herein are also applicable to the study of similar problems in tunnel location in other sections of the United States. Valleys which have been deeply buried by glacial drift deposits are not new topographic features to the engineer. Numerous accounts of the crossing of such valleys by tunnels may be found in engineering literature. In presenting details concerning the surface geologic features of the

Quabbin Aqueduct the writer wishes to suggest that an understanding of the genetic relation of correspondingly formed valleys may prove helpful elsewhere in the study of similar location problems.

2.—The engineer may obtain valuable data from published geological reports. These reports are generally of a scientific nature, but often contain information that can be used in a practical manner. They are related generally to the geology of a large area, and because of this may be helpful to the engineer in obtaining a broad perspective of the geology of a region, in some particular section of which an engineering project is proposed. The engineer must rely upon his own investigations for the geological information required within and adjacent to the construction lines of a structure.

3.—The data on excavation progress are important mainly in their manner of presentation. Because the geology of this project was followed so closely it was possible to draw an accurate geologic section covering the entire 24.6 miles of the tunnel. This fact, together with other related construction data and costs, correlates the geology with construction in considerable detail.

ACKNOWLEDGMENTS

Some parts of this paper have been taken from reports to the Chief Engineer prepared jointly by Charles P. Berkey, M. Am. Soc. C. E., and the writer. Dr. Berkey was Consulting Geologist on the work. The writer also wishes to express his indebtedness to the Chief Engineer of the Metropolitan District Water Supply Commission, Frank E. Winsor, M. Am. Soc. C. E., who has shown a keen interest in the application of geology to engineering works of this kind.

DISCUSSION

Bayard F. Snow,[16] **M. Am. Soc. C. E. (by letter).**—The operations preliminary to the construction of the Quabbin Dam and Dike afforded an excellent opportunity for the collection and study of many data relating to the exhaustive pumping of ground-water. Such data are of intense interest to many engineers whose work concerns the flow of ground-water, and Mr. Dore deserves much credit for the manner in which he has collected and presented the facts. Undoubtedly, the problem of estimating the expected leakage through the earth over-burden under a dam is nearer a rational and reasonable solution as a result of this paper. There is a question, however, as to whether the data and study are of most value in connection with estimating the flow under proposed dams. It is true that, in the case of the Quabbin works, the bore-holes were made and some of the pumping was done prior to the actual beginning of core-wall construction even if such holes may not have preceded the decision to construct such a water-stop. There is no doubt that they assisted greatly in the construction of the core-wall. However, much of the information was gathered during the sinking of the caissons and, therefore, can have had no part in reaching the decision that caissons were necessary.

It would be of considerable value to the Engineering Profession if Mr. Dore would present a modification of his study of these basic records to assist in answering a series of questions frequently met in the study of the flow of ground-waters; for example:

(*a*) What is the probable safe yield and the optimum spacing of wells under given geologic and topographic conditions?

(*b*) What portion of the pumpage is depletion of storage, what part is infiltration from within the area of the circle of influence, and what part a ground-water flow from outside that circle? and,

(*c*) With soil of a given porosity, what part of those pores is filled with water and how much of that water can be drawn as the water level is lowered by pumping?

Fig. 10 indicates that there was no change in water level at points ½ mile up stream and down stream after eighty weeks of pumping, although at the caisson it had dropped 100 ft below its original level. Pumping rates for a year had been about 3 mgd, after which they had increased so as to average slightly more than 5 mgd. One may assume that, at a point ½ mile above the dam, there was no change in the direction and velocity of the flow of ground-water and surface water as the result of the pumping. The easiest hydraulic channel for water precipitated on the uplands above this point was by surface and underground courses to the river, and this condition was unchanged by the pumping. It seems obvious, therefore, that in such materials the watershed from which a ground-water supply could be taken by wells near the site of the dam was less than a mile in diameter and that the safe yield would be not more than one-tenth the rate of pumping actually used in dewatering the soil, and even that only by excessive draw-down. It is probable that with a slight change in average size of grain, the radius of influence would be extended greatly. The real question is whether a circle of influence or area contributing to the yield of a ground-water supply can be predicted by analysis of soil samples. If Mr. Dore can utilize his data to this end and give the Engineering Profession a workable means of determining yield of wells under given geologic conditions, a noteworthy contribution to water supply engineering will have been made.

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[16] Pres., X. Henry Goodnough, Inc., Boston, Mass.

The determination of permeability by borings is certain to be, in a large measure, a matter of luck. Mr. Dore calls attention to the random nature of the deposits and refers to the connection of the coarser deposits by links of relatively small cross-section, but which serve very effectively in conducting the flow from one coarse lens to another. In seeking a suitable site for an additional ground-water supply, the writer has had the experience of finding what appeared to be a pocket of coarse material completely surrounded by a very fine sand. As far as borings could reveal any information, an acre or two of coarse sand was available, which would serve as a collecting and storing reservoir, surrounded by material through which water would flow at very slow rates. The area between the old well field and the new site was apparently an almost impervious barrier. On a pumping test, however, it was shown that hydraulic communication between the two areas was free and that each well field responded promptly to pumping from the other. It would appear, therefore, that the zone of influence was much larger than would have been indicated by computations based on examination of boring samples.

In this area the writer saw an interesting demonstration of the transmission of pressure through ground-water. On one well a stroke of a hand-pump caused an immediate temporary drop of an inch or more in the water level at the other well, 80 ft, or more, away. This calls attention to the fact that ground-water flow is affected not only by the permeability of the water-bearing strata, but also by the resistance to transmission of air through the overlying material. If the water-bearing material adjacent to the first well had been freely open to atmospheric pressure, there would have been little discernible effect of the vacuum a few feet away from the well. Many test wells in the writer's experience have disclosed nothing which would indicate a potential water supply, all the material above ledge being nearly impervious. Frequently, however, a thin stratum of coarse material lies at the rock surface and serves to form a channel through which the water flows freely and perhaps, by reason of the nature of the overlying material, transmits a differential pressure for considerable distances to points where the over-burden is less in amount or more pervious. When deposits of coarser material are connected by such devious channels the total effective water-shed to a well may be much greater than study of the most carefully selected boring samples would indicate was possible

The ratio of permeability between the material of such a very thin coarse stratum and the overlying fine deposit may be 100 to 1 so that it would appear to be important to measure the water-bearing stratum with extreme care. The ordinary methods of wash-boring and sampling do not permit of such refinement, even if by luck the boring penetrates a representative section of such a stratum. This being so, although the writer agrees that water tends to open passages of flow between pockets of coarser materials and feels that such passages may have a carrying capacity equal to that of the deposits of coarse sand they connect, nevertheless, the determination of the average permeability of a cross-section by boring samples is subject to great errors, especially where the borings miss the larger masses of relatively coarse material and may or may not penetrate a typical cross-section of the connecting channel. It is not more simple and more accurate to utilize the test borings that would be necessary for soil sampling, pumping from some and observing the slope of the water surface at others, rather than to depend upon samples obtained by more or less blind groping?

OLE SINGSTAD,[17] M. AM. SOC. C. E. (by letter).—Being especially well prepared the valuable paper by Mr. Fahlquist presents an excellent picture of the geologic features of an extensive project and their relation to its construction operations and cost. The most impressive aspect of the paper is the manner in which the geologic features have been taken into consideration in the planning of the tunnel project. It illustrates the importance and value of the co-operation between the engineer and the geologist in the proper planning of an underground project of this magnitude. It is quite evident that this thorough and scientific consideration of the geological features in the planning was in large measure responsible for the successful construction operations carried on without untoward happenings. This complete preliminary information undoubtedly removed from the project much of the uncertainty attached to the bidding on underground projects where the preliminary exploratory work is less thorough, resulting in economy to the owner and a greater degree of certainty that the contractor can complete the work within the bid price and the stipulated time.

The paper also presents in convenient tabular form much valuable information as to cost and progress, and shows the variation of these items in relation to the geologic conditions. It should be of great value to engineers dealing with underground construction work, in stimulating more attention to the geologic features and more thorough exploratory work than has sometimes been the case in the past.

VERNE GONGWER,[18] M. AM. SOC. C. E. (by letter).—In the "Synopsis" of Mr. Dore's very interesting and valuable paper, two general sources of information for use in determining the permeability of earth over-burden are mentioned: (1) Dry samples from bore-hole investigations; and (2) the effect of pumping upon the ground-water conditions in the over-burden.

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[18] Project Engr., Guntersville Dam, TVA, Guntersville Dam, Ala.

Mr. Dore has described a very careful and almost exhaustive application of these methods to the study of conditions at Quabbin Reservoir, and is to be commended for so thoroughly presenting it in his paper, because the characteristics of the underlying formations, where a dam is to be built on earth foundations, warrants the gravest study. This phase of the problem is too often given little or only perfunctory attention, usually being confined to the drilling of a few holes, sinking of a few test pits, and possibly doing a little perfunctory pumping from them; or confined to the application of some well known formula to a conjectural distance for seepage or "creep" by means of an effective size determined in the laboratory from a greater or less number of usually questionable samples recovered from the explorations. The formulas seldom have been developed for the conditions prevailing at the site under investigation, and the results of laboratory analyses and tests can be no better than the samples submitted to the technicians; in fact, the usual unwitting inaccuracies in the recovering of samples, even where great care is intended, often render the laboratory results seriously misleading.

The writer fully agrees with Mr. Dore's "Conclusions," and desires, for emphasis, to quote from them:

> "There seems to be a scarcity of data concerning the permeable characteristics of soils and there are few, if any, definite methods in common use for making satisfactory determinations of the permeability of an overburden. In many cases permeable qualities of foundations have been a matter of extremely rough estimate, of guess, or merely of conjecture."

Very pertinent also is a comment made to the writer by E. W. Lane, M. Am. Soc. C. E., that engineers have advanced greatly in their study of soils and the development of methods for building excellent and water-tight earth-fill dams; but that too little attention is paid to the materials and foundations upon which such structures are built and that they are woefully lacking in means and methods of attack.

The writer has found attempts to determine the permeability of most soils (other than uniform silts, clay, and fine sand), from samples, to be very unsatisfactory and misleading. In fact, they can be said to be almost invariably misleading, due to the difficulty of recovering, in place, truly representative samples of streamers, lenses, or veins of the more open gravels without admixture from adjoining layers of materials, above or below. This condition vitiates the accuracy of either mechanical analysis curves or direct testing of "so-called dry samples" for permeability.

Observations of the variations in, and effect on, the ground-water levels from pumping from open test pits, casings, or well-points is believed to furnish much more illuminating and dependable data in regard to permeability; however, the writer has never been able to arrive by this method at anything very tangible, other than a general indication of relative water-tightness. There have been certain surprising results, in that test pits in areas previously considered quite tight from borings and mechanical analyses of samples, have produced water under the pumping test almost as freely as other pits that have been expected from mechanical analyses of samples to produce water freely. In this connection, the writer's experience coincides remarkably with certain conclusions by Mr. Dore; for example, in the making of certain calculations of the quantity of water which would flow horizontally under a given head through a vertical 1-ft slice of the materials found in a test pit from surface to bed-rock, it was found that, theoretically, on the basis of flow constants corresponding to results of mechanical analyses, from 90% to 95% of the total water passing will be transmitted through, or by, the coarsest stratum penetrated by the test pit, even if that stratum is relatively insignificant in thickness and the "effective size" of the particles is not particularly, or startlingly, large. This lends considerable importance to a conclusion by Mr. Dore, which is also heartily concurred in by the writer, that although the coarser materials are found or are usually considered as lying in random assortment or lenses, they are usually in glacial soils—and almost invariably in alluvial soils—either connected or continuous in the general direction of the valley, or should be considered continuous for safety.

When a dam is proposed to rest on earth foundations, with or without underlying rock at shallow or greater depths, two main questions arise, which must be answered in advance:

(1) Will the volume of the total leakage through the sub-strata be excessive from the standpoint of loss of water, unsightliness, possible damage to property, or for any of a number of possible reasons, other than as affecting the safety of the dam itself?

(2) Are the materials, or disposition of the sub-strata, such as to render at all possible the disastrous phenomenon resulting from concentrated flow at one or more points, at high velocity, known as "piping"?

A positive answer to Question (1) or to both questions, of course, will call for a positive cut-off. This may apply either to the over-burden or to the underlying rock, where it is cavernous or seamy. In some few cases it may be permissible to "take a chance" and determine the answer to Question (2) by constructing the dam and awaiting results, trusting that correction will be then possible, but in the vast majority of cases too much is at stake.

From Mr. Dore's paper it is assumed that, at the Quabbin Dams, the preliminary data and investigation somewhat indicated the need of a positive cut-off, and that the first caissons constructed, although located and designed for possible incorporation in a cut-off wall, were by way of pioneering for positive proof that the completion of such a cut-off was actually necessary. The analysis of results of subsequent pumping from these caissons and sumps furnished a definite answer to Question (1) but not necessarily to Question (2), upon which the need of a cut-off could be decided. It is usually difficult to provide such an ideal pumping arrangement as that described by Mr. Dore except where caissons have been installed as at Quabbin, and large capacity pumps are usually necessary.

With the cut-off once decided upon, and assuming that it will be tight in itself, tightly sealed upon the rock, and that the rock itself is tight, an answer to Question (2) is not essential. However, since extreme difficulty of construction or other previously unknown hazards sometimes leaves doubt as to the existence of possible flaws in the cut-off, or in the seal to the rock, and due to the difficulty or cost of detection and correction, an answer to Question (2) is very desirable, both before and after the construction of the cut-off. This would scarcely be necessary with the type of cut-off adopted at Quabbin, but it is sometimes the case where a steel sheet-pile cut-off is adopted, or even more positive and seemingly thorough means are used, as is attested by comparatively recent occurrences at the Ontelaunee Dam, at Reading, Pa.

As stated, although the writer, like Mr. Dore, considers the observations and data obtained as the results of pumping in pits and test wells of considerable value in determining the average effective permeable quality of the over-burden as a whole, where this factor is desired, he does not consider the average permeability of an over-burden (which is to become the foundation material of a dam) of any great value. It may even be disastrously misleading, if considered alone, in determining the safety of the dam from "piping" or other phenomena of undermining, because calculations of velocities of flow of under-seepage from average permeability will result, ordinarily, in a very re-assuring figure for velocity. The maximum rate of flow through the coarsest stratum, lense, or vein of material existing at the site, will indicate the possible danger from piping and if determinable will be vastly greater and considerably less re-assuring. Such conditions will seldom be discovered by pumping tests or by any known method of sampling, except from test pits, which are usually much too far apart for this purpose. Let it be assumed that the vein in question has no actual direct connection to the river or watercourse, or to the ground surface under the proposed reservoir by any of a large variety of possible means. Then, under full reservoir conditions, all other materials of the valley floor or flood plain are acting, or will act, as a vast filter admitting water to the vein rather freely and in considerable quantities, producing a high velocity. Should the vein lie relatively close to the surface of the reservoir floor, or have connections close to it, the so-called piping distance necessary to protect against failure is reduced from a comfortably long distance greater than the base width of the dam, to substantially no more than the vertical depth of the over-burden above the seam. In most cases the distance is reduced to the thickness of a fairly thin stratum of comparatively impervious material, which cannot usually be proved to be continuous or without fault or opening. It is immaterial, of course, whether the pervious stratum is of coarse gravel or whether it is an open and uncorrected seam or solution channel in the underlying bed-rock. In either case the relative conducting capacity is so great under the increased head of the reservoir as to warrant deducting practically the entire width of the base of the dam from any calculation of safety from "piping."

Whether or not one or more sink-holes form in the bottom of the reservoir, the area of the filter bed will usually be such that water will be supplied to any open vein or stratum with so little loss of head that although the "blanket" below the dam is thick and comparatively impermeable, the hydraulic gradient of the seeping water, under the conditions, is likely to be well above the ground surface at one or more points. In some cases a pressure can be calculated readily to exist, which is sufficient actually to lift the overlying blanket, particularly if only one of several permeable veins should chance to end, or be substantially reduced in carrying capacity near the down-stream toe of the embankment. However, some spring or other natural "seep" will probably be present to encourage escape and to start "piping" action. The phenomenon of flotation of the finer particles, of course, will render the thickness and tightness of the down-stream blanket of little value, whereas the gravel seam or solution channel in the rock (the chances being that there will be a number of one or the other), will usually have ample unfilled voids to accommodate the displacement of the relatively small quantity of the "fines" from the up-stream blanket necessary to begin the action of piping at that end. The next step is the formation of sink-holes, and then, possibly, disaster.

The foregoing may seem a pessimistic view, but that such results are very possible is attested by the comparatively recent partial failure of the Ontelaunee Dam at Reading, Pa., previously referred to and recently described in engineering periodicals. The writer has often had opportunity to verify suspicion by excavating in foundations, and finding seams of large, perfectly clean gravel located in apparently quite impervious fine sand and silt, which seams were producing water copiously. He also has discovered numerous partly filled and water-bearing solution channels in limestone where few were indicated by the customary spacing of exploratory drill holes. Some "chimneys" were actually found leading into the latter, and subsequent pumping from the seams proved the presence of other "chimneys."

Fortunately, in the case of most important projects, some means of investigating the possible under-flow has been undertaken, and although not usually conducted with the thoroughness of that described by Mr. Dore, has resulted in a cut-off wall or other positive element being incorporated in the structure as a safeguard. Such action is seldom too conservative; in effect, it corresponds to the usual safety factor against uplift, sliding, etc., for concrete dams. The relative proportion of the cost of safety against under-failure of earth dams to total cost seldom, if ever, approaches that of the usual practice in utilizing three or four times the theoretically necessary material in wood, steel, or concrete structures.

Dam failures of recent years are believed to indicate that, through continued success, the profession is in danger of becoming somewhat too optimistic in regard to factors of safety, particularly in foundations with respect to the elements herein discussed.

The determination of permeabilities by laboratory analyses of bore samples is of value only in the case of the finer and more cohesive materials, as the result depends entirely upon whether a representative actual sample is furnished to the laboratory. Wash-boring samples are practically valueless. Samples of fine cohesive materials may be taken from the ends of drill casings by various methods, which will be suitable for mechanical analysis, but seldom in natural unconsolidated, or unbroken, state for direct permeability tests. True samples of the coarser veins, accidentally encountered, cannot be taken by any means or tool known to the writer, without the probability of mixing in it at least 10% or 15% of the finer strata above or below it, which admixture will control the "effective size" and render worse than valueless any calculations made from it. Such calculations may lead to serious errors in resulting decisions, always tending to the unsafe side.

This admixture of fines is almost certain to occur in sampling where thin strata of very open materials are encountered, even in open test pits, and particularly with running ground-water, unless a competent observer is present in the pit at all times. to take true samples. Samples from the bottoms of casings, below ground-water level, or where wash-boring methods are used, are always subject to question. In all cases where pumping has been resorted to in a casing or pit, samples of coarser materials may have fines washed into or out of them before the casing or pit has quite reached such a stratum. Furthermore, it is extremely unlikely that the coarsest stratum or other opening has been discovered unless boreholes or test pits are prohibitive in number (at least as close as 10-ft centers over the entire area under investigation) and more unlikely that it has been identified and a true sample taken.

It is noted that Mr. Dore dissents from the use of the coarsest particle of the finest 10% of the sample or the term, "effective size," in making permeability computations, by means of the Slichter formula. After discarding certain extreme variations, such as 5% and 26%, however, he finds that the permeability is controlled in most cases by sizes within the fairly close range of from 8% to 15%, with a reasonable mean of all results at 12%, which he uses in connection with the Slichter formula. The writer agrees with his conclusion that all such methods are subject to errors of from 50% to 100%, and, therefore, such a small difference in practice in the selection of the controlling size seems unimportant. For that reason no close decision as to the need of a cut-off wall should ever be decided in the negative, on the basis of pumping tests or permeability calculations from samples, but a factor should be used, on the side of safety, considerably greater than the possible error inherent in the methods of investigation.

The belief and hope that the methods developed and used at Quabbin may find application elsewhere, has been expressed by Mr. Dore, and his paper adds greatly to the practical application of the meager methods and tools that are available for predicting under-seepage in order to determine the necessity of cut-off provisions. Where, as at Quabbin, pumping was done on such a scale, and for a period long enough so that the ground-water was considerably lowered along the axis of the dam from one abutment to the other, many valuable data and determinations are possible, particularly as to general permeability. However, high productivity in the caissons used as wells would seem to leave little doubt as to the need for cut-off provisions, as without such provisions it would seem probable that under full reservoir head the leakage would be at least as much as, or several times, the total rate of pumping.

In his investigations, Mr. Dore represented the variation of the ground-water table by means of contours. The use of ground-water contours is worthy and capable of wide general application to the study of sub-strata with respect to permeability and the writer has used this tool with satisfactory results at several locations in both glacial and alluvial soils. In general, the method involved a system or grid of a greater or lesser number of observation wells, consisting of auger holes, perforated pipes, supplemented by such test pits, existing dug wells, or ponds, depending on which were available With the latter, of course, care should be exercised to determine that the free water surface does not represent a perched water-table, as frequently happens in the case of ponds, ditches, and even rivers at certain locations. Periodic readings of the elevation of the water in such wells are taken very conveniently and accurately with a 50-ft or a 100-ft steel tape (with 0.01-ft graduations) on a compact reel, and fitted with a cupped lead weight.

By taking observations of all wells and plotting ground-water contour maps at 1-day, 5-day, or 10-day intervals, during variations in the water-table, much may be deduced in regard to the direction of flow of the underground water. Occasionally, different elevations will be registered in pairs of pipes driven to different depths, where the ground-water is moving rapidly in some stratum. The variation in the water-table may be due to the natural rise and fall of the adjacent reservoir or river; or it may be produced by "gunning" the river by means of discharge from the outlet valves or spillway of some dam, or by pumping from various pits or wells. The time lag and the response of the wells to fluctuations in river or reservoir level, will give a general indication of the relative general permeability of the materials. It is very enlightening to plat time graphs of a group of wells, superimposed upon a time graph of the river-gage readings.

Certain general information may be derived from the displacement of the ground-water table in a given time, over a given area, due to a rise or fall of the river; but usually a calculation from the amount of upward displacement of the water-table is of no value on account of the impracticability of calculating the quantity of ground-water escaping from the area in question. Valleys in the water-table contour maps will often indicate this condition and the direction of escape.

The ground-water contour maps would be closely analogous to contour maps of the free water surface of a flowing river were it not for the varying resistance of the materials through which the ground-water is moving. A steep slope or a nearly flat slope in the water-table may indicate various rates of flow, depending upon the characteristics of the materials. The difficulties in determining the characteristics of the more open materials by sampling has been discussed. For a complete and dependable determination of conditions, therefore, it is necessary to have a means for determining directly the actual rate of flow in the soil at definite locations and depths. Such a means was happily suggested to the writer by chancing upon a reference to the electrical ground-water timing apparatus devised by Professor Charles Slichter.[19] Such an apparatus was constructed for use at one dam site, with the exception that the timing and recording were done by attaching a commutator to the shaft of a recording ammeter of the type used on power-house switchboards.

The configurations of the water-table contours will usually indicate valleys where, presumably, the rate of underground flow is greatest. In the first two such locations selected, in a terrain previously considered quite impervious, rates of flow approximating 400 ft per day were observed. Sal ammoniac was used as the electrolyte, and the direction of flow checked that assumed from the contours. At one place it was at right angles to the river and at the other location approximately parallel to the river, both being caused by the same rise in the river level. The direction was corroborated further by the fact that the only electrode affected in each case being brass, was found upon removal to have been discolored to a deep purple, whereas the other electrodes were still bright. The discoloration was easily removed by sand paper after each test. Calculating backward from the measured rate of flow indicated an effective size of the particular gravel vein approximately ten times larger than that obtained from the coarsest samples recovered from 24-in. casings driven to rock at several places relatively near-by.

With a few determinations made in critical or representative locations, and with the slope of the water-table accurately determined by precise measurements in the wells used with the Slichter apparatus, and roughly checked from the contour map, a much more complete understanding of conditions is had than may be determined by any other method known to the writer. If the hydraulic gradient found between the test wells differs materially from the general slope taken from the contour map, the latter should be disregarded in calculating the flow constant.

Remembering that even where the materials do not differ greatly in gradation (approximately 90% of the total water movement will be found to occur in the one coarsest layer), the pipes for the Slichter test may be perforated for only 2 ft or 3 ft at the bottom and driven directly to this stratum when its location is known. Where the locations of suspected coarse strata are unknown, the pipes may be perforated for the entire depth of the over-burden if desired, to permit a maximum flow rate being recorded on the first trial. On the other hand, 2 ft or 3 ft may be perforated, and the pervious strata located, by surveying the entire depth of over-burden by successive tests, driving the pipes deeper each time by a distance equal to the length of perforations. The pipes may be driven by hand for shallow depths, but where the over-burden is deep or gravelly, a drill rig of the usual type will be found to expedite the work greatly.

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[19] *Ground-Water Paper No. 140*, U. S. Geological Survey.

If successive tests are to be made at the same set-up of pipes, it will usually be found necessary to allow from one to several days between tests for the sal ammoniac or salt from the previous test to become dissipated or washed away.

This direct method of measuring the flow through the actual materials in place, at sites selected by means of the ground-water maps, seems far superior to calculations from soil samples. It is not particularly necessary to compute the effective size of grains from the Slichter test, as this apparatus provides a direct measurement of rate of flow under a known hydraulic gradient, and the possibility of piping under increased gradients may be calculated directly from it.

A word of warning may be in order in regard to the use of this equipment in locations where solution cavities or other water passages or faults are possible in the underlying bed-rock. Assuming that the pipes have been properly cleaned, and unless the materials are entirely clay or fine silt, should the apparatus fail to register a positive test after a reasonable time interval, it is entirely possible that the flow may be straight downward or in an inclined direction beneath the receiving electrodes. The ground-water map, if constructed from a sufficient number of test wells, should be examined carefully for the presence of a depression or pot-hole, marked by a closed contour over a small area, which may not have been recognized in the interpolation of the contours. This condition would indicate a "chimney" in the rock, into which, under certain conditions, the ground-water may be flowing. It is often desirable to drive additional test wells to assist in solving peculiar conditions.

To avoid disappointment, and to attain satisfactory results in the use of driven perforated pipes for observing ground-water fluctuations and for use with the Slichter apparatus, the following suggestions are offered. The writer has found that the commercial type of screened well-points are not sturdy enough to stand the hard driving necessary in deep, sandy, or gravelly over-burden. The most satisfactory and economical points were devised by blowing four rows of holes at 90° in extra heavy pipe, the holes being spaced 4 to 6 in. in the rows, with the oxy-acetylene torch, and pointing and welding the ends of the pipes in the same manner, or by employing a blacksmith. The pipes should be 2 in. or larger, as desired or required.

After driving, the pipes are frequently found to be plugged with sand or silt forced through the holes, even if the water level may seem to stand at approximately the correct height. The pipes should be washed by inserting a $\frac{1}{2}$-in. or $\frac{3}{4}$-in. pipe used as a water-jet, rather than by attempting to blow or jet them clear with air. After the air pressure has been turned off the inrushing water usually plugs the pipes again with silt and sand, and serious error and lost records result in assuming that they are clean. The writer has seen pipes stand inactive for days after several attempts to clean them with air, and later register freely after washing them with a water-jet. The pipe lengths should be measured as driven, and the sounding lead used to ascertain whether they are clear for the full length after the completion of driving. After driving and cleaning, if necessary, each pipe should be tested by pouring a bucket or two of water into it and observing with sounding lead whether the water level subsequently subsides at a reasonable rate. Only if the perforations are in contact with very dense silt or clay will the water level remain stationary. When tested with the bucket, all stationary or sluggish pipe should be regarded with suspicion.

In driving, all pipes should be threaded so that, with application of considerable force, the ends can be made to abut firmly in the couplings. Ordinarily, extra heavy pipe should be used, with recessed couplings.

Driven pipes are seldom exactly plumb and may deviate several feet. For this reason, pipes driven for the Slichter apparatus should be spaced well apart to reduce the percentage of possible error in spacing between the perforated parts, and the corresponding error in the apparent timing of passage of the electrolyte. A spacing of 20 ft to 25 ft from the salted well to the receiving wells is desirable, although this will require more time for the observations, and additional receiving pipes and electrodes may also be required.

Considerable difficulty and annoyance will be avoided if the perforated basket for lowering the dry sal ammoniac, the insulators for the electrodes, and sounding leads used on steel tapes are all made sufficiently small to avoid catching on the burrs on the ends of the pipes in the couplings.

Clothes-line ropes and electrical wires should be very securely attached to the electrodes, and the wires protected from injury, at such connections, by electrician's tape. Measuring marks should be established on the ropes or wires to ascertain that the electrodes reach the bottom of the well. The electrodes should be lowered slowly and carefully to place, and thereafter should not be pumped up and down as the pumping action of the insulators has been found to draw sand into the pipes, which usually means jammed and lost electrodes and a useless pipe.

Apparently, for the purpose of being able to utilize a recording instrument, the Slichter apparatus is arranged for the use of dry cells, the measured variations in the induced current, when connected to the several circuits, furnishing the desired information as to the passage and arrival of the electrolyte. If the automatic-recording feature is not desired or essential, the process may be reversed and the varying resistance in the several circuits measured at regular intervals by means of an electrical "megger" or ground tester. At one time the writer utilized this principle with fair success between observation wells spaced approximately on 100-ft centers but without electrodes, having previously charged an up-stream well with common salt. The "megger" has certain advantages over the recording ammeter since difficulties are frequently experienced with the latter in poor commutator contacts and in adjusting the number of dry cells so that the record will stay on the scale of the paper. Since the resistance is very low between the electrodes and their respective pipes after the electrolyte has entered the pipes, it is probable that two or three dry cells and a small, comparatively insensitive ammeter, carried from pipe to pipe, would give satisfactory results, and much wiring and labor in setting up would be obviated. With this method, after the electrolyte has definitely reached one of the electrodes, the observations could be taken more frequently if desired, and observations on the other wells omitted, which is not practicable where the commutator clock is used, and the observations timed to 10-min or 15-min intervals, since there are eight different circuits in the regular set-up.

Where a grid or system of observation pipes has been driven for the study of a site, all pipes possible should be preserved in order to continue the program of periodic readings throughout the filling stage of the reservoir and for some time thereafter. Should it not be convenient (as in the case of a rolled fill) to extend certain of the pipes up through the fill to the crest of the dam, it is believed to be a justifiable expense, particularly where a cut-off has been constructed, subsequently, to drive a sufficient number of pipes through the completed fill into the more pervious strata of the foundation, closely up stream and down stream of the axis, in order to observe the hydraulic gradient at different locations after the filling of the reservoir, and to check the efficiency of the cut-off provisions.

In addition to the Slichter apparatus, valuable data have sometimes been obtained by the introduction of fluorescein dye into the perforated test walls and timing its arrival in test pits and excavations in which pumping is being done. In one case a rate of flow of approximately 200 ft in 2 hr was observed on a moderate gradient, in a limestone seam, with the probability that the greater part of the 2 hr was consumed by the fluorescein finding its way from the well into the chimney leading into the seam.

Stanley M. Dore,[20] M. Am. Soc. C. E. (by letter).—The discussion by Mr. Snow calls attention to the "more or less blind groping" that occurs in the investigation of the permeable qualities of an over-burden which depends solely upon analyses of bore-hole samples. The writer agrees that the pumping of ground-water from one of many measuring wells is decidedly, and by far, the superior means of estimating the permeability, and should preferably be used wherever such means are available. However, in many cases, the determination might be made after the bore-hole casings have been pulled. For such cases the usefulness of results obtained by analyses of bore-hole samples depends upon the number of samples available and the manner in which the volume of the over-burden has been investigated with bore holes. To take samples from a relatively small number of bore holes for the determination of the permeability of an over-burden as a whole is practically useless, as the result obtained may be correct or it may be absurd, and in such a case the groping is decidedly blind. However, with a large number of samples from an adequate number of well-placed bore holes, the law of averages operates to reduce the guesswork and materially to increase the probability that the result is a reasonable one.

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For free-draining glacial materials, similar to those at the sites of the Quabbin Dam and Dike, it was estimated that about 90% of the water in the pores was obtained quickly from storage by pumping. Probably, having been under water for years, the pores of these materials were entirely filled with water. Part of the remaining 10% would trickle slowly down to the lowered ground-water surface, and part would be held in the corners of the pores and would never travel toward the pumps.

The division of the quantity pumped into that obtained from depletion of storage, that obtained from infiltration within the circle of influence, and that obtained from flow into the circle from without, can not be precisely made for any site for all periods of time, as it depends upon variable conditions, such as rainfall, temperature, run-off, vegetation, etc. The quantity flowing underground into the circle from without is usually relatively very small; and, for a valley-shaped over-burden, it is the quantity that ordinarily flows down the valley underground. For instance, for the relatively pervious over-burden at the Quabbin Dike such quantity was estimated to be about 100 gal per min. The quantity flowing on the surface into the circle from without depends on local conditions, and the quantity that reaches the areas affected by the pumping operations depends upon the slope and water-tightness of the stream bed. At Quabbin Dike nearly all the surface drainage up stream (drainage area, 1.4 sq miles), flowed indirectly into the circle of influence, sank into the ground, and was pumped. Practically, then, the water pumped is nearly all infiltration within the circle of influence and depletion of storage. The quantity of pumpage remaining constant, the quantity depleted from storage, depends directly upon the infiltration, which, in turn, is large or small as affected by recent rains or other conditions. At the Quabbin Dike the percentage of the total pumpage coming from depletion of storage varied generally from about 10 to 90.

Mr. Snow expressed a desire that the determinations of permeability might be used as means of estimating the yield of ground-water wells. It was the writer's intention that such would be the case although he presented the investigations in connection with unwatering and seepage problems. In both cases the permeability of the over-burden would be estimated in a like manner, the data in case of a water-supply development being obtained, if possible, by test pumping from one bore hole and observing the drop in water level in others.

Once this average permeability is obtained, circles of influence can be estimated, and the ground-water maximum draw-down curves plotted in profile for wells at various depths. The maximum continuous yield of any well, after the ground-water curve is drawn down to its maximum and there is no further reduction of storage available, depends upon the infiltration portion of the pumpage, which in turn depends upon local conditions of rainfall and run-off and of surface and sub-surface flow into the circle of influence. This part of the yield as a continuous water supply is comparable to the prime power of a hydro-electric development, and in case of an extended and continuous excessive demand, the yield of the well will be limited to this quantity. However, the demand upon a well as a water supply is seldom constant and continuous. The volume of the pores between this maximum draw-down curve and the original ground-water surface, in such a case, can be utilized as a storage reservoir, to be filled in wet periods and drawn upon in dry periods.

Thus, the underground supply can be treated in a manner similar to the surface supply, the infiltration being the source and the volume of pores above the maximum draw-down curve being the storage reservoir, upon which certain demands can be made, limited by the capacity of the pumps and the permeability of the over-burden. For estimating the maximum draw-down curve and the rate at which water can be drawn from the over-burden for other positions of the draw-down curve, the same formulas for pumpage as were described in the paper apply when the permeability is known.

After the local factors affecting the infiltration, the volume of underground storage, and the rates of pumpage permissible at various depths have been determined, the probable capacity of any well to meet certain demands can then be estimated. The spacing of wells is most advantageous if the circles of influence do not interfere, but, in such cases, although more complicated, the yields may be similarly estimated.

Mr. Gongwer evidently feels that, although the methods described in the paper are useful in obtaining the average permeability of an earth over-burden, such a value does not necessarily indicate the lack of a need for a cut-off in case this result is a relatively small one, as the average permeability is not indicative of what the largest permeability of some of the materials may be. Of course, if the total quantity of percolation indicated by the average permeability determinations is large enough to warrant the construction of a cut-off (which was the case at the Quabbin Dams), the distribution of this percolation throughout the over-burden is not important, except in cases where the cut-off provisions considered are partial or not positive. The writer agrees with Mr. Gongwer that, for the partial or non-positive cut-off provisions, and for the case in which the total quantity of percolation in itself is not large enough to warrant a cut-off, the final decision on omitting the complete cut-off provisions should not be made until investigations and studies indicate that there is no serious concentration of the total percolation (although small) in one or more veins, channels, or strata, which might jeopardize the safety of the dam through "piping." For such cases, the presence of a concentrated flow condition would be definitely indicated in a pumping program with an adequate number of observation wells, because the loss of head in the wells through or near the coarse veins or strata would be comparatively much less than elsewhere at the site where the wells were in the tighter materials. The writer, however, wishes to thank Mr. Gongwer for describing methods used in determining the location and size of concentrated flows in over-burdens. These constitute a valuable addition to the paper as they can be utilized, if desired, in connection with a pumping program or with bore-hole investigations for locating and measuring concentrated flow conditions or for demonstrating the lack of any such flow.

Mr. Gongwer indicates that it is practically impossible to obtain satisfactory bore-hole samples that contain the proper quantity and quality of coarse materials to indicate the existence of the coarser veins and strata in the over-burden. For certain types of soils, such as alluvial deposits, the writer agrees that this is the case, but in many soils, such as random glacial deposits, reasonably good, so-called, dry samples are obtainable from bore holes which are fairly representative of the materials from which they are taken, the coarse materials usually occurring in thick veins, pockets, lenses, and strata. If the number of samples taken is large enough to be indicative, the chances of appreciable distortion in the estimate are not large, although the writer agrees that, as in the case of the pumping program, the possible existence of any coarse "piping" channel should be investigated before definite conclusions are reached regarding the omission of the cut-off.

The writer notes particularly that Mr. Gongwer concurs with him in the opinion that in over-burdens the coarser veins or pockets are usually interconnected, because the study of permeable qualities of an over-burden, especially from bore-hole samples, cannot be made reasonably, if it is assumed that the coarser portions are segregated and insulated from each other by the finer parts.

It is believed that, although the pumping arrangements at Quabbin are fairly ideal, because of the caisson pumping intakes, it would not be difficult to obtain a good pumping program elsewhere using well-points and pumps installed in the bottom of small pipe wells sunk to the desired depths at suitable locations.

Mr. Gongwer's comparison of the expense of cut-off provisions with the cost of the materials used to obtain the factor of safety in structural work is particularly well made; but sufficient and proper investigations in some cases will reveal that considerable saving can be made instead of spending money for cut-off insurance, and, in other cases, will eliminate the practice too often followed of deciding not to build a cut-off, risking the unknown consequences.

The point that the writer wishes to emphasize is not that the 12% size was a reasonable means to be used in making permeability computations by means of the Slichter formula for materials at Quabbin Dams, but that it might be some size other than the 10% size commonly used for the materials found at some other location, say, the 2%, the 5% or the 15% size; and that in using the common method the permeability results might be distorted so seriously as to be extremely misleading if not useless.

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ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

BY HAROLD CONKLING,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOSEPH JACOBS, W. D. FAUCETTE AND J. E. WILLOUGHBY, R. E. SAVAGE, H. J. F. GOURLEY, O. J. BALDWIN, G. E. P. SMITH, DAVID G. THOMPSON, WELLS A. HUTCHINS, JOHN E. FIELD, GEORGE S. KNAPP, CHANDLER DAVIS, L. STANDISH HALL, AND HAROLD CONKLING.

SYNOPSIS

Wells may be drilled in almost all valley areas of the United States with the expectation of securing sufficient water for household purposes and stock watering. Such supplies are of first importance in pioneer and farming communities, but are not of commercial importance. Neither legislative nor Court control of them has often been attempted. Usually it is only when supplies sufficient for irrigation, municipal, and industrial uses exist that engineering and law are involved. This paper deals only with the larger supplies.

A large percentage of the major cities of the United States is supplied with water produced from wells and probably most of the smaller ones are similarly supplied. Irrigation from wells exclusively increased 55.1% and, from wells as a supplemental supply, 177.5% in the 1919–1929 decade, whereas irrigation from surface streams decreased 6.3 per cent. This growing use of underground water has brought about much study of underground supplies throughout the United States and is causing changes in underground-water law. An important question that arises is whether the underground supplies should be placed under administrative control and the nature of legislative enactments creating such control.

NOTE.—Published in April, 1936, *Proceedings*.

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This paper opens a discussion of the laws of underground waters, their consistency with those of surface waters, the possibility of enacting statutes for their administrative control, the form of such statutes, and the drawbacks and benefits of such control. The paper is not a treatise on underground-water law. This subject has been treated many times by minds much better qualified for the task than that of the writer. It is rather at attempt to reach a practical solution of a problem which, unlike most engineering problems, does not rest on physical laws alone, but on the laws of physics as interpreted by legal minds, and also on the fundamental law of property as applied to water. The foundation combines the legal with the material. Were this paper concerned with the design of a bridge the writer would not think of introducing into it the foundation concepts of elementary mechanics, but since it treats of the use of water, it is deemed necessary and advisable to outline the growth and development of water law so that the foundation for the conclusions will be apparent.

For those who may wish to go further into the subject, reference is made to Kinney on "Irrigation and Water Rights"; Wiel on "Water Rights in the Western States"; "Corpus Juris"; "American Law Reports"; and the great body of decisions of the Federal and State Courts.

INTRODUCTION

The usual physical situations in which ground-water of commercial importance is found, are as follows: (1) Definite underground channels; (2) percolating through basins; (3) underflow which may support a surface stream; (4) artesian areas or basins; and (5) alluvium or glacial till supplied by (*a*) irrigation; and (*b*) rainfall. These different physical situations are not clearly delimited and merge one into another. Underground supplies derived wholly from surface rainfall on valley floors are found in humid areas, but rarely, if at all, in arid areas. No essential difference exists between deep percolation from irrigation and that from rainfall. A description of typical instances would clarify the subject more than general descriptions.

DEFINITE UNDERGROUND CHANNELS

If the underflow of a stream passing through bottom-lands several miles wide, bounded by bluffs, hills, or mountains, is regarded as a definite underground stream, it may be said that such streams, usable for irrigation, often occur in the West. The attitude of the Courts is not entirely clear as to the status of this supplementary flow, but in one case the Court of Arizona has defined it, definitely, as an underground stream. In most such streams in Arizona there is not perennial surface flow. The law covering such waters generally is the same as the law of surface waters, but nevertheless the subject is treated herein because such occurrences of underground water are important.

PERCOLATION THROUGH BASINS

The San Gabriel Basin[3] lies immediately east of Los Angeles, Calif. In it are large areas devoted to citrus groves and walnut orchards and many towns and cities, including such famous municipalities as Long Beach and Pasadena. The basin consists of a mountain drainage area of 294 sq miles, an upper valley of 278 sq miles, called San Gabriel Valley, and a lower valley, known as the Coastal Plain, supplied by several streams of which the San Gabriel River is the largest. Of the entire area of San Gabriel Valley, 151 sq miles are using water for irrigation or for domestic and municipal purposes, and the Coastal Plain is likewise well developed. These are structural valleys filled with detritus from the mountains and surrounding hills.

A range of hills separates San Gabriel Valley and the Coastal Plain and through this barrier the San Gabriel River has cut a pass about two miles wide. Another range of hills almost faces the ocean and through this obstruction the stream has also cut two passes several miles wide. The alluvial plain below these hills faces directly on the ocean and is a true coastal plain as distinguished from the area above the lower range of hills which, although known as the Coastal Plain, is really a structural valley.

The summers are rainless and most of the precipitation occurs in the four months, December to March, inclusive. The annual rainfall in the San Gabriel Valley varies from about 26 in. near the mountains to 17 in. at the lower or southern side of the valley. In the Coastal Plain it is about 12 in. Rainfall is heavy at times and, although most of it is absorbed in the valleys, the steep impervious mountain slopes rising to elevations of about 7 000 ft and as high as 10 000 ft above sea level, shed the rain rapidly and cause large flashy floods. In such floods all the streams cross the pervious valley fill and flow into the ocean, but during most of the year the flows from the mountains are entirely absorbed if not used close to the mountains.

The stream beds in the valley are dry sandy washes during most of the year, but at the gap between the San Gabriel Valley and the Coastal Plain and, again, at the gap in the hills near the ocean, the underflow comes to the surface, creating a perennial stream of rising water, much larger at the upper gap than at the lower one.

The average slope of the surface of the San Gabriel Valley from the mountains to the outlet is about 50 ft to the mile and that of the Coastal Plain is about 12 ft to the mile. The water-table is 200 to 300 ft, and even 400 ft, below the surface in places near the mountains; it is at the surface through the outlet and is again 30 to 50 ft beneath the surface immediately below the gap. A few miles below the hills separating the San Gabriel Valley and the Coastal Plain artesian conditions begin and the piezometric level is above the surface between this point and the lower range of hills. Down stream from the latter point the water-table is again below the surface. The small summer flows from the mountains and the rising water are the only gravity supplies available for irrigation. All other supplies are from wells which, in general, have good yields.

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[3] "San Gabriel Investigations", by Harold Conkling, *Bulletin No. 7*, Div. of Water Rights, Dept. of Public Works, State of California.

The climate is subject to long cyclic fluctuations of rainfall. In 1934, the average water-table in the San Gabriel Valley was about 60 ft lower than in 1916, when the highest was recorded. In small basins partly cut off from the main area by underground formations the recessions have been much larger, whereas in the Coastal Plain and near the gap in the San Gabriel Valley they have been smaller. The area using water has increased from 32 000 acres in 1904 to 97 000 acres in 1934. The recession of the water-table is due partly to the increased draft to supply this demand, but mainly to lack of rainfall during the period, 1916 to 1934, and cumulative overdraft is not indicated. Likewise, rising water at the outlet from the San Gabriel Valley has decreased from an estimated average of 164 cu ft per sec in 1916 to 38 cu ft per sec in 1934. Opportunity for percolation from the San Gabriel River is probably two to three times as much with the lowered water-table of 1934 as it was with the conditions of 1916, because of the greater distance the stream travels over an area beneath which the water-table is far below the surface. Opportunity for percolation of rainfall on the valley floor to the water-table is likewise greater than it was in 1916.

The particularly noteworthy points to bear in mind in a discussion of an underground-water law, is the decrease in rising water, the recession in the water-table, and the steadily increased use of water during the period when the water-table was dropping.

The characteristics of the San Gabriel Basin are typical of most of the coastal valleys of California south of San Francisco, and of the valleys or basins in which ground-water is found throughout the Great Basin which embraces parts of California, Oregon, Idaho, Nevada, Utah, and Wyoming. The major differences between Great Basin conditions and the San Gabriel Basin and others in California is the greater aridity in the Great Basin. As a consequence run-off is less, stream channels are often not cut through barriers down to the level now necessary to discharge water, and the water reaching the water-table is disposed of at the lower side of the basins by evaporation from lakes either ephemeral or constant, playas, and moist areas, instead of by the surface streams of rising water.

UNDERFLOW WHICH MAY SUPPORT A SURFACE STREAM

The South Platte River has its source in South Park, Colo., on the eastern slope of the Continental Divide. After reaching the plain it flows northward through Denver, Colo., not far from the toe of the mountain and is joined by numerous tributaries, the last of importance being the Cache La Poudre. Then it flows eastward into Nebraska, is joined by the North Platte, and empties its waters into the Missouri River, near Omaha, Nebr.

Most of the area irrigated in Colorado from the South Platte is near the mountains in the high plains which begin their rise at the Missouri River and slope gradually upward to an elevation of 5 000 or 6 000 ft at the mountain toe where the precipitation is about 12 in. This rainfall increases, with distance eastward, to 30 in. at Omaha, occurring mainly in the growing season. Irrigation is practiced west of the 98th Meridian. The crops are practically the same throughout the irrigated area, consisting of grain, alfalfa, and a hoed crop which is corn in the eastern part, giving way to sugar-beets toward the west.

The South Platte River furnishes a supply for 1 233 000 irrigated acres in Colorado and 45 000 acres in Nebraska, a total of 1 268 000 acres. Of the area irrigated in Colorado about 8 000 acres are supplied from pumped wells. Of the 457 wells used for irrigation, 294 are in minor tributary valleys and only 4 in the valley of the main river. There are three wells in the valley in Nebraska.

Below the Cache La Poudre influx the river flows in a broad valley, generally ten miles or more wide, but sometimes narrowing to about two miles. The bottom-lands are only slightly above the water surface of the stream. The valley has been carved through the plain and the bottom refilled with porous detritus carried by the stream. The uplands are fertile, and their topography is suitable for irrigation, but as they rise from 150 to 350 ft above the river, it is not feasible either to pump for them or to reach them by gravity canals below the point where the river turns east. Wells drilled in the uplands do not reach sufficient porous water-bearing material to furnish more than domestic or stock supplies. The wells in the valley generally find porous aquifers from which irrigation supplies can be produced. Although reservoir development for impounding the floods of the South Platte System is exceptionally large, considerable water escapes into the more humid eastern area in flood times. More pumping in the river bottom would provide storage space into which the floods would percolate and greater development could be made. At the same time, supplies to existing gravity diversions below would be reduced and the people dependent on them would also be forced to pump to secure their quota of water.

The principal points to bear in mind in the foregoing discussion are the possibilities of greater development by pumping in the river valleys and the greater expense to secure a water supply, which would be incurred by all interests below the pumping plants.

ARTESIAN AREAS OR BASINS

The Roswell Basin[3] is situated in Southeastern New Mexico and is a narrow strip about 65 miles along the Pecos River, a tributary of the Rio Grande from the north. The original area of artesian flow was 663 sq miles, but by the winter of 1926–27 it had shrunk to 425 sq miles because of heavy draft. Under a considerably larger area to the west ground-water is under pressure but development is not economically feasible. The principal intake area lies still to the west and water for it is supplied by run-off from the Sacramento Mountains which are parallel to the river, the crest being about 80 miles distant. The streams from the mountains percolate into their beds in crossing this area, and the water finds its way to the artesian aquifers. This is the principal supply but there is an additional supply from deep percolation of rainfall on the intake area.

The Roswell Basin is not a structural basin, but is more nearly an artesian slope with the strata dipping from west to east. The permeable artesian strata consist of honeycombed and cavernous limestone, but moderate quantities of water are produced from sand strata and sandstone. Water is found also in the surface alluvium or valley fill derived from surface drainage, local precipitation, irrigation return, and upward leakage from the artesian strata. The Pecos River acts as a drain to the valley and its waters are diverted for irrigation in, and at points below, the artesian area.

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[3] "Geology and Ground Water Resources of the Roswell Artesian Basin, New Mexico", by Albert G. Fiedler and S. Spencer Nye, *Water Supply Paper 639*, U. S. Geological Survey.

The principal crops are cotton, alfalfa, orchard products, and corn. The annual precipitation is 12 to 14 in. in the agricultural area, and the climate may be classed as semi-arid. The annual average temperature is from 59° to 63°, depending on the altitude which is from 3 100 to 3 600 ft above sea level.

The draft on the artesian basin evidently has been greater than the recharge, as the continuous loss of head on the artesian wells and the shrinkage in area of artesian flow testify. The safe annual yield is thought to be somewhat greater than 165 000 acre-ft, but the annual draft is now (1936) about 200 000 acre-ft, of which more than 100 000 acre-ft are required for the irrigation of crops now supplied from artesian wells. Various non-beneficial natural disposals of water are in process which, together with leakage from wells, account for the remainder of the 200 000 acre-ft. The non-beneficial disposals are decreasing as the pressure lowers.

Originally, large springs flowed in the vicinity of Roswell, but these no longer exist as the water formerly issuing from them is now diverted underground to the artesian wells because of the depressed piezometric head. The area formerly irrigated by gravity by means of ditches is reduced to that which can be irrigated from "return from irrigation" and other minor sources. It is probable that other adjustments in the area irrigated by gravity from the river and tributaries will be made if the excess demands on the artesian basin continue.

The principal points for attention in the foregoing is the probable present overdraft, with consequent past and future decrease in surface flows, along a river formerly available for irrigation.

Alluvium Supplied by Deep Percolation from Irrigation

The Boise River has its source in the Sawtooth Range, in Idaho, and flows west into Snake River, a tributary of the Columbia River. At an early date, diversions were made on both sides of the river but principally on the south side. The normal flow in late summer is not sufficient to supply all water rights. In 1907, the United States Bureau of Reclamation started the Boise Project on the south side of the river paralleling older irrigated areas but at a higher elevation, and completed Arrowrock Reservoir about 1914. The rising water-table caused by deep percolation from irrigation on the Boise Project threatened ruin to some of the land on the Project and to a considerable area of the older irrigated lands along the lower river and on the south side of it. Surface drains to the river were installed and the land was thus protected. The discharge from the drains is used by rights diverting from the river below them during the irrigation season and water formerly allowed to flow down river from the head-waters is now diverted farther up stream by other rights. During the winter the drainage water wastes into the Snake River.

Some, but no great quantity, of underground water has been pumped for irrigation from this area of high water-table, but water shortages in recent years have forced the consideration of greater development of this source. If a sufficient quantity were pumped, the ground-water would be lowered and the waste from drains in the winter would decrease, or perhaps cease entirely. The rights down stream might not have a full supply from drains in such a case and, if so, would have to be supplied with discharge direct from the head-waters.

The principal point to observe in this situation is that a greater area could have been developed or a better supply to the present area could have been given if, instead of building surface drains, wells and pumping plants had been installed to accomplish the same thing (assuming the existence of the necessary porous aquifers). The cost to the farmer of using this water would have been greater, also, due largely to the subsidy on capital costs given in the financing of Federal irrigation projects, but the additional cost would have been spread over a large area. Under the appropriative doctrine if the proprietors of any land not now irrigated, or to be irrigated as a part of the Boise Project, should attempt to use this water, they might find the cost excessive because of the upset to the existing situation. As a supplemental supply to existing projects either over-lying the underground basin, or near-by, this water might prove very valuable, and might be secured without great legal difficulty.

Alluvium or Glacial Till Supplied by Rainfall.—In New York State, Long Island is marked by a range of hills from east to west forming the backbone of the Island and rising about 100 to 200 ft above sea level[4]. From these hills streams drain both north and south into the ocean. The surface soils are of glacial origin of particularly porous nature on the south side of the Island. The rainfall averages about 45 in. annually and, except when the ground is frozen, most of it is absorbed by the soil. Perhaps 40% or 50% reaches the water-table which, in most of the area, is below the root zone. The water-table slopes toward the sea and probably most of the underground water reaches the sea without coming to the surface. Near the stream channels, carved deeply by large ancient glacial streams, the water-table slopes toward the valleys and they act as drains. Other surface disposals are by evaporation from ponds and swamps where the water-table intercepts the surface.

As shown by the water development of the Borough of Brooklyn, N. Y., on the western end of Long Island, wells yield heavily. As irrigation is not practiced, except perhaps in a very limited way, there is little draft on the underground water except in the western end, to supply Brooklyn. If average annual deep percolation from rain averages about 20 in., as estimated, the yield from the approximate 500 sq miles of Southern Long Island would be very large and suited to the requirements of a large municipal center which requirement is the only one of magnitude for consumptive use in humid climates.

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[4] "Long Island Sources", Rept. of the Board of Water Supply of the City of New York, 1912.

The Long Island situation is of interest because, aside from the western end used by the Borough of Brooklyn, the underground water resources are practically undeveloped, although they have been thoroughly explored as a supply to New York City. This is an unusual situation since in most cases knowledge comes only after development.

Exploitation of the supply would lower the water-table and thus create additional storage space underground for the wet years. Movement toward the ocean would decrease and salt water from the ocean would move farther under the land as pressure was removed. The discharge of streams fed from ground-water would decrease and ponds and swampy areas would diminish in size. If, instead of the ocean, a stream flowed along the southern shore its flow would be decreased.

The area is typical of those in humid regions suitable for large development except that some are tributary to a stream of magnitude instead of to the ocean.

UNDERGROUND WATER IN GENERAL

Aside from the flood flows of a stream, its discharge is the result of precipitation on the tributary water-shed which has percolated below the root zone of vegetation and moved underground in the direction of the slope, to the stream (or other body of surface water). Underground flow is merely an evidence of water on its way to the surface. It may be used either directly from underground or after it has found a stream. It is the same water in either case, but in the one it has been brought to the surface by Nature, and in the other, by Man; but if Man had not interfered, Nature would have done the job at some lower elevation.

Any withdrawal of water from the aquifers causes a decrease in the water reaching the stream. It may be the floods that are decreased, inasmuch as the withdrawals will provide additional space into which the rainfall may percolate. On the other hand, the underflow may be decreased and thus the low water of the stream diminished. Water moves very slowly underground, its rate depending on slope and on porosity and thickness of the aquifers. If pumping withdrawal occurs at a sufficient distance from the surface outlet it may be that several rains, or even annual rainy seasons, may occur before the depressed water-table caused by pumping reaches the outlet; and, in such cases, sufficient replenishment may occur to restore the former underground supply completely. Such replenishment is unlikely in arid regions, but in humid regions the longer the period the more likely that the depletion will be made up by percolation which would not have occurred if the withdrawal had not been made. Likewise, the smaller the quantity of water pumped the more likely it is that it will be replenished. Pumping from the underflow of a surface stream would undoubtedly decrease the surface flow for a distance below the pumps, but might have no effect on it farther down stream.

Natural disposal of underground water must be in equilibrium with the recharge over a term of years, but the water-table constantly fluctuates with the wetness of the season, or cycle of seasons, because disposal is more uniform than recharge. For a time, perhaps many years after pumping draft begins, disposal continues as before, and the water-table in the general vicinity of the pumping lowers with comparative rapidity until natural disposal is decreased by the quantity of water pumped unless, as may occur (especially in humid regions), the draft is made good by additional recharge. If the new draft is not greater than natural disposal the water-table assumes equilibrium at this new lower level. If it is greater, pumping draft must eventually decrease to equal recharge and again equilibrium will be established. In any event the stream below would be decreased.

In arid regions, especially, evaporation may be so great and the supply so small that disposal is accomplished from wet areas in the lower levels of the valley, and no surface stream issues from it. In such cases the lowered water-table caused by pumping merely decreases the evaporation.

In considering the different physical situations heretofore described in which ground-water of commercial importance is found, it is discovered that there are no essential differences. In some cases (such as that of Long Island and closed valleys in arid regions where the disposal is, in the one case mainly into the sea and, in the other, into an evaporating area) pumping would produce no general effect on a stream system but would nevertheless upset local conditions by lowering the water-table. In humid regions, where water-courses are more frequent than in arid regions, the distance between the recharge area and the point of disposal may be small and the effect on stream flow felt more immediately. On the other hand, greater probability of additional recharge below the pumped area exists in humid regions. Each situation is a separate study which must be exhaustive and widespread to reach reliable conclusions.

Ordinarily, the exploitation of an underground source has proceeded without supervision or knowledge on the part of those using the water. Such knowledge comes usually after a series of lawsuits to enjoin exploitation, or when alarm is caused by the receding water-table. A notable exception is the survey of the underground supply of Long Island previously mentioned. Presumably similar surveys have been made by other large organizations in seeking a water supply. The United States Geological Survey has done, and is doing, much work throughout the United States on investigations largely in co-operation with States or local entities, but because of paucity of funds its work is naturally most intensive in those areas where exploitation has caused alarm. Probably State supervisory boards are doing similar work in many States. In California, the State Division of Water Resources is doing considerable investigational work, but as the use of underground water in California far exceeds that in all the other States of the Union its work is likewise being done in regions already overdrawn, or where alarm has been caused by lowered water-table.

EXTENT OF UNDERGROUND WATER USE

In this paper it is implicit that use of underground water for ordinary domestic or stock supplies is not considered. This draft is insignificant and has never, to any extent, been a matter of legislative or Court control. In humid regions (that is, in regions of the United States east of the 98th Meridian) underground water is exploited to the point where far-reaching effects are produced only to secure supplies for metropolitan areas and information as to the extent of such exploitation has not been found. West of the 98th Meridian the climate in the regions where agriculture is developed is arid, semi-arid, or sub-humid, and irrigation is required. It is in part of this region that the possibilities, proportionately to those of the East, are highly exploited, and in some cases over-exploited.

There are more statistical data in available form for the West than for the remainder of the United States, and these data are summarized herein in tabular form. Table 1 contains data on irrigation for the years 1919 and 1929 for each of the nineteen States in which it is practiced.

This information is selected from a *Bulletin* of the U. S. Bureau of the Census, published in 1930[5]. In addition, the *Bulletin* reports an item under "Supplements from Pumped Streams, Pumped Wells, and Flowing Wells." The total shown for pumped wells[6] is 293 026 acres, of which 287 136 acres are in California and the remainder scattered through the other States. The same information is not given for 1919 and, therefore, it is not considered in Table 1. If it were added, the total for California, irrigated partly from wells, would be 1 070 318 acres in 1928, whereas the total for other States would be only slightly affected.

The total area irrigated is given by the Census as 19 191 761 acres in 1919 and 19 547 544 acres in 1929. The area in Table 1 covering three different classes of irrigation is 17 745 182 and 18 189 425 acres, or 92.5 and 93.0% of the total, respectively. The remainder not covered by Table 1 is irrigated from springs, lakes, etc., and from supplementary well supplies not owned by the owners of the surface rights. Table 1 gives: (1) The area receiving its entire supply from streams; (2) the area receiving its entire supply from wells; and (3) the area receiving a supply from streams supplemented by wells. The total shows for the decade a decrease of 6.3% in the first class and increases of 55.1% and 177.5% in the second and third classes. When California is excluded the increase in irrigation from wells becomes much less impressive. The decrease in the area irrigated from streams exclusively is then 2.6%; the increase in the area irrigated from wells exclusively is only 18.0%; and the areas with a supplemental supply from wells shows an increase of only 20.0 instead of 177.5 per cent.

Even this increase indicates a vitality in the growth of the use of underground water which does not exist in the case of irrigation from streams, or gravity irrigation, as it is commonly called. The use of underground water has gone forward in spite of conditions adverse to agricultural development which existed during the decade. The causes for this increase are probably: (1) Pressure for new development in certain areas in spite of over-development of agriculture in the United States and even in the State in which the

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[5] "Irrigation of Agricultural Lands", U. S. Bureau of the Census, 1930, Tables 11 and 12.

[6] *Loc. cit.*, Table 37.

TABLE 1.—COMPARISON OF AREA (IN ACRES) IRRIGATED IN THE UNITED STATES

State	AREAS RECEIVING ENTIRE SUPPLIES FROM:				PERCENTAGE CHANGE FROM 1919 TO 1929		AREAS SUPPLIED FROM STREAMS WITH SUPPLEMENTAL WELL SUPPLY		Percentage change from 1919 to 1929
	Streams		Wells						
	1919	1929	1919	1929	Streams	Wells	1919	1929	
Arizona	196 453	170 797	41 810	106 002	—13.1	153.5	218 324	292 721	32.5
Arkansas	6 129	1 502	135 260	142 978	—75.5	5.7	250	....	—100.0
Colorado	3 050 964	3 169 559	14 390	15 929	3.9	10.7	84 138	17 656	—79.0
Idaho	2 384 010	2 029 016	1 545	5 569	—14.9	260.5	2 284	74 667	3 170.0
Kansas	32 137	56 412	13 285	11 651	75.5	—12.3	1 540	405	—73.7
Louisiana	271 152	259 001	155 575	175 787	—4.5	13.0	10 045	....	—100.0
Montana	1 550 827	1 487 751	351	1 064	—4.1	203.1	6 223	2 694	—56.6
Nebraska	437 532	503 653	546	23 452	15.1	....	115	70	—39.2
Nevada	470 179	395 236	1 171	3 426	—15.9	192.6	5 039	4 534	—10.0
New Mexico	434 368	436 955	52 295	58 115	0.6	11.1	2 026	1 015	—50.0
North Dakota	11 499	8 253	....	....	—28.2	.....	....	....	....
Oklahoma	2 710	675	125	63	—75.1	—49.6	....	....	....
Oregon	851 183	739 569	2 405	3 891	—13.1	61.8	305	3 322	1 010.0
South Dakota	93 360	65 916	130	528	—29.4	306.2	520	160	—69.2
Texas	495 870	699 146	44 466	62 624	41.0	40.8	499	850	90.5
Utah	1 116 130	1 040 577	12 394	19 655	— 6.8	58.6	662	3 520	431.0
Washington	471 145	450 067	20 665	20 995	—4.5	1.6	2 856	708	—75.3
Wyoming	1 157 121	1 183 252	166	320	2.3	92.8	400	137	—65.4
Sub-total *	13 032 769	12 697 337	496 576	586 775	—2.6	18.0	335 226	402 459	20.0
California	2 920 396	2 254 712	868 060	1 464 960	—22.8	68.8	92 152	783 182	641.0
Total	15 953 165	14 952 049	1 364 639	2 117 012	—6.3	55.1	427 378	1 185 641	177.5

* Exclusive of California.

new development is made; (2) failure of gravity supplies due to drought, or to additional use of the stream at higher elevations; (3) increased efficiency of pumps; and (4) lower power cost.

A clearer picture of the whole may be made by roughly grouping the arid States in accord with difference in economic, physical, and climatic conditions to determine in what general areas the increase has taken place and to lay the basis of an estimate for the future.

Group 1 consists of Louisiana, Arkansas, and Texas (southeastern part). Although Western Texas is arid, the eastern part, with all of Arkansas and Louisiana, is humid. Irrigation by pumping from wells is practiced for rice even in the most humid portions of each of these States, as this crop should be partly submerged after a certain stage of its growth. Rice is grown on leveled land on which water may be ponded by dikes surrounding or going through the fields. In the section of this group of States where rice is grown, the terrain is so flat that gravity irrigation is impracticable in most cases and recourse must be had to wells or to pumping from streams. The ground-water supply is not sufficient to maintain the present draft in certain of the areas. The acreage in rice depends on the rice market. Future increase in the acreage irrigated from wells may not be large as it is unlikely that there will be any considerable failure of stream supplies making it necessary to substitute wells.

Group 2 consists of Montana, Wyoming, North Dakota, and South Dakota. In all four States only 1 912 acres were irrigated from wells exclusively in 1929. Although this is 196% increase from the 647 acres thus irrigated in 1919, the total is insignificant. The climate of these States is not conducive to crops which can stand the comparatively large annual expenditure required for pumping from wells, and it is believed improbable that the total cost will ever be so large as to cause it to be a problem.

Group 3 consists of Colorado, Kansas, and Nebraska. Colorado east of the Rocky Mountains is an area of high plains with an eastern gradient which continues through Kansas and Nebraska to the Missouri and Mississippi Rivers. The eastern limit of irrigation is at about Longitude 98° W, west of which lies about three-fourths of the length of Nebraska and two-thirds the length of Kansas. The principal streams from the eastern slope of the Rockies in Colorado flow on through Nebraska and Kansas. Precipitation occurs principally in the growing season. Both temperature and precipitation increase to the east and south. Crops raised by irrigation are quite similar over the entire area, but corn—the hoed crop in the eastern part of the area—gives way to sugar-beets as one goes west. Such irrigation from wells as occurs is practiced entirely in the broad bottom-lands of the streams. It is not feasible for the bordering plains. The cost of securing water in the bottom-lands is small, as the water-table is high and other conditions are favorable. Physical conditions are favorable to an increase in the area watered from wells. Surface supplies from the principal stream are unreliable especially as the distance from the mountains increases and until the humid section is reached, but there should be ample underground water for any demand that is likely to be made upon it.

Group 4 consists of Idaho, Utah, and Nevada. Except as to Northern Idaho these States are arid. The climate in a considerable area of each State is conducive to high crop values. Underground water is found: (1) In isolated basins from which the water that enters them is dissipated without surface outflow; (2) in larger basins, more prolifically supplied, from which issue streams originating in the ground-water; and (3) along river bottoms and under irrigation projects. Physical conditions are favorable to an increase in the area supplied from wells, and in some basins of Utah the demand is said now to exceed replenishment. In 1919, the area thus supplied in this group was 15 000 acres and the increase by 1929 was 90 per cent.

Group 5 consists of New Mexico and Arizona. The southern part of these States is characterized by an intensely hot and arid climate. Irrigation from wells in New Mexico is largely in the great artesian areas. In Arizona, to a large extent it is along the Salt River and other tributaries of the Gila where irrigation has increased the ground-water supplies. Water is pumped, however, from the bottom-lands and valleys formed by the principal streams and from closed basins. The physical conditions are favorable to the extension of the area irrigated from wells.

Group 6 consists of Oregon and Washington. West of the Cascade Mountains irrigation is not practiced, but east of these mountains both States are arid. The climate decreases in aridity east from the Columbia River, in Washington. Some development of well irrigation exclusively is occurring in Eastern Oregon from basins, but in Washington little extension is being made.

Group 7 consists of California alone. Almost 75% of the total area in the United States irrigated from wells exclusively, and 66% of that which uses wells for a supplemental supply is found in California. To a limited extent the areas in which ground-water exists are found along the flood-plains of the

TABLE 2.—AREA IRRIGATED EXCLUSIVELY FROM WELLS, BY GROUPS OF STATES

Group No.	State	Remarks	Area, in Acres		Percentage increase
			1919	1929	
1	Louisiana Arkansas Texas	Humid climate; water used for rice growing.	335 301	381 389	13.7
2	Montana Wyoming North Dakota South Dakota	Cold climate; expensive pumping impossible.	647	1 912	196.0
3	Colorado Kansas Nebraska	Well suited to pumping from wells along streams. Much additional development possible.	28 221	51 032	80.9
4	Idaho Utah Nevada	Well suited to pumping in parts of this group. Considerable development possible.	15 110	28 650	89.6
5	New Mexico Arizona	Climate suitable to expensive development. Water naturally available is limited.	94 105	164 117	57.3
6	Oregon Washington	Climate favorable over great part, but ground-water conditions not suitable for large development.	23 070	24 886	8.0
7	California	Ground-water over exploited in parts and must be replenished by importation if present development maintained.	828 060	1 464 960	68.8

streams, but most of the ground-water supply is found in the coastal valleys and in the great central valley. These valleys are filled with detritus and are supplied by percolation from streams crossing them and rainfall on the valley floor. Some areas are now over-developed. It is to be expected that only a small percentage of increase will take place in the future in the area of the State irrigated from wells, because most of the development possible has occurred unless (as is proposed for some areas) water is imported for artificial recharge.

Table 2 shows the comparison of area in each group irrigated from wells exclusively in 1919 and 1929. It is in Groups 3, 4, and 5, and possibly in Group 6, that climate and physical conditions point to further development with possibilities of over-development and attendant legal conflicts, such as have occurred already in some States, and particularly in California.

WATER LAW IN GENERAL

Any legislative acts concerning underground water would presumably follow closely any existing legislation as to surface waters in the jurisdiction wherein the enactment is made, and it is desirable, therefore, to inquire into the law of surface waters in the various States before discussing that of underground water, in order to discover something of its nature. The law of underground waters, as now developed, is less than a century old, whereas the law of surface waters of the United States had its antecedents and development in the Roman Empire; hence for intelligent discussion reference must first be had to the law of surface waters. The civil law of the Romans is the foundation for the common law of England, which was adopted by each of the jurisdictions of the United States that came in after the original Thirteen Colonies where not incompatible with economic conditions prevailing in the new country. It appears probable, however, that the common law of England on water was taken from early American decisions which were derived from the Code Napoleon, which, in turn, was derived from the Roman law.

The doctrine of surface water originally made it the common property of the public, just as air and fish in the water—economically a "free good"; but because private ownership of land came to exist, it follows that the public is excluded from the use of the water except in the case of navigable streams and lakes, or the sea, and, consequently, the stream and the lake are the common property only of those who own lands bordering them. As a final outgrowth (in this country, at least) the common law doctrine of riparian rights permits each owner of riparian lands to use the natural flow of the stream on his riparian land in any reasonable way. In other words each riparian owner has a right correlative with, and equal to (proportionate to his holdings of riparian land), that of every other riparian owner. Riparian rights do not attach to all land in the water-shed, but only to the extent of the holding (which has never been severed) of lands bordering the stream, and even then not past the water-shed boundary. Priority of right does not exist; nor does the right exist to store water for use in a later season. Use must be made of the stream in its natural regimen if at all.

Water may be appropriated for use, however, on distant non-riparian lands in all jurisdictions. This is at the sufferance of the riparian owner where the riparian right is recognized and, if he insists, after compensation for any damage he may suffer if he makes his claim before the period of limitation forecloses it. If no damage is suffered no claim can be made, as the riparian right, similar to the appropriative right, is the right to use, but not to hold, unused water against others who may have use for it. The definition of the use for riparian rights varies with the jurisdiction and is probably very broad in some States. Appropriations could be made under the civil law of Rome and can also be made under the legal systems of all countries the laws of which derive from the civil law. Undoubtedly, this was the case also under all legal systems foreign to the Roman system, which were developed under civilizations where irrigation was necessary. Mexico does not seem to have recognized the riparian doctrine and from reading its water codes it is concluded that its officials exercised a free hand in granting appropriations. Most of the arid States of the United States have rejected the riparian doctrine and have substituted for it an appropriative procedure entirely, based on Mexican procedure probably, but adhering strictly to the doctrine of priority of right obtained with an appropriation.

In the arid States water is more important than land, since it is the limiting natural resource in development. Under the appropriative doctrine it is conceived that water is public property, but that the State owns it in trust for all the people, and it is not the property of only those who have access to it because of riparian holdings. These States have ruled that the right to use water should be exclusive instead of correlative. A water right is a property right and the exclusive appropriative doctrine is somewhat more akin to the law of real property than the riparian doctrine of water law. In the appropriative doctrine, as it now prevails in the States, he who is first in time is first in right and can insist on his full right as long as reasonably needed even if it takes all the water in the stream except that necessary for domestic and stock use in most cases. In sequence of priority all users take their full right as long as the water in the stream will yield it without infringing on prior rights.

Wyoming, Nevada, Utah, Colorado, Arizona, and New Mexico, which are wholly arid States, and Idaho, which is partly arid, abrogated the riparian doctrine and adopted the appropriative doctrine either by constitutional provision or by statute at an early date in their history. Montana by a Court decision uttered in 1921 appears to have joined this same category. In the States immediately east of the Rocky Mountains (that is, North Dakota, South Dakota, Nebraska, Kansas, Oklahoma, and Texas) irrigation is necessary or desirable only west of about the 98th Meridian; that is, in only about three-fourths or less of their area, and is, therefore, not so important as in the aforementioned States. Both the riparian and the appropriative doctrines exist in these States, although incompatible in theory; and yet, because the riparian right in the arid parts has been limited to reasonable use consonant to the definition of reasonable use in connection with appropriative rights, or has been limited in some other way, the conflict is not important. In Oregon, riparian rights as defined herein have been virtually abrogated and, in Washington, they still exist but are no longer of importance. In California, the basic doctrine is strongly riparian but an appropriative procedure has been superimposed on it. Riparian rights have been limited by constitutional amendment (1928) to that reasonably required for beneficial use comparable to beneficial use under appropriative rights. This was upheld and interpreted by the Court in 1935 (Peabody, *et al.* *v.* City of Vallejo, 2 Cal. 2d 351).

The appropriative doctrine lends itself to administrative control more fully than the riparian doctrine. In all the aforementioned States, where the most arid conditions prevail, an elaborate procedure as to surface waters has been set up for receiving filings for appropriation, granting permits and licenses, adjudicating water rights by stream systems, patrolling streams, regulating head-gates, and, in general, seeing that each water user gets the water to which he is entitled. In the less arid Western States the procedure is not so much elaborated.

One point in surface-water law, especially important to this discussion, needs to be considered: "Is the diverter from the stream protected in his means of diversion?" Under the common law doctrine the riparian owner has a right correlative with that of every other riparian owner to the use of surface water; he possesses nothing exclusive. If one suffers from water shortage all must suffer. Under the appropriative doctrine, the user of surface water has an exclusive right, which may be exercised when sufficient water is in the stream no matter what may happen to those having rights junior to him except that, under certain conditions, stock water and domestic supplies for the junior rights take precedence over every other kind of use by the senior rights.

Does this correlativity and this exclusiveness extend to the means of diversion? In other words, can the riparian diverter claim that an undue burden has been placed upon him if other up-stream riparians, after his works are completed, take from the stream sufficient to make his diversion works useless and to require him to incur additional expense if he wishes to continue to use the water? Furthermore, can the appropriator using surface waters claim estoppel on projects up stream which, while not depriving the stream of sufficient water for his right, cause him additional expense to divert?

It is patent that all degrees of deprivation might be incurred in such a case from the most minor to one in which the cost of changing diversion methods would be so expensive as to be complete confiscation of property. Actually, this latter would rarely be the case, and it would seem just that protection be given against such contingency. Most cases are not so serious but nevertheless any additional expense would be partial confiscation unless recompense were made. On the other hand the "hurdle" faced by the individual who proposes to divert at an up-stream point would indeed be high if he had to compensate all down-stream riparians whose diversion works would have to be reconstructed because of his act. Development would be seriously curtailed.

It would seem that a right to an exclusive method of diversion would be incompatible with the riparian doctrine. The riparian doctrine is such that any owner up stream or down stream can take a part of the supply of the original riparian diverter and deprive him of his use. As compared to this, the benefits of a certain method of diversion would be of comparatively small moment. As there are only a small number of riparian rights being exercised in the United States in locations where water is important, it is probable that the matter has never come to the attention of the Courts. At least, "Corpus Juris" does not list any cases exactly in point.

When it comes to appropriative rights it is found that two cases were decided at a comparatively early date—Shodde *v.* Twin Falls Land and Water Company (161 F. 43) Idaho, and Natoma W. and M. Co. *v.* Hancock (101 Cal. 42) California—which held that the appropriative right did not include a further right to insist on a certain means of diversion. In the first case, the force of the current turned the plaintiff's water-wheel, which, in revolving, lifted water in buckets attached to the wheel. The water in these buckets was dumped into a canal and reached the land of the plaintiff. The Court held that the plaintiff had appropriated water for irrigation, had made no appropriation for power, and denied injunction when a dam below caused backwater at this wheel. The Natoma case is exactly in point and declared flatly that the plaintiff must change his diversion system when up-stream takings interfered with his ability to obtain water by means of existing works. No other cases have been found.

Legislative enactments or "water codes" of the various States contain nothing on protection of means of diversion. It would seem from this fact, and from the foregoing cases, that short of absolute or a very great degree of confiscation, protection is not given to means of diversion. Protection of the water right seems to be regarded as important in the "water codes", but not protection of the other phases of the property right. Constitutional provisions regarding confiscation of property as interpreted by the Courts are the protection in such cases.

Comparison—Appropriative and Riparian Doctrines

Both the appropriative doctrine and the riparian doctrine have faults and both have advantages. The appropriative doctrine, particularly is an attempt to give the closest approximation possible of the law of real property; but land is fixed in position and stable in area, whereas running water is evanescent and its quantity varies from day to day, from season to season, from year to year, and from cycle to cycle. Manifestly, with such an unstable natural resource, no such security of possession can be given as is the case with land; but the appropriative doctrine attempts to approximate it as closely as the nature of flowing streams and vagaries of climate permit. In times of normal flow the difficulties are not such as to be serious, but in cycles of unusual drought the deficiencies of the doctrine become apparent and these defects result from this very attempt to give stability. A preferred class of first users exists and all junior appropriators can be deprived of even partial supplies (except domestic and stock necessities under certain limitations) to give a full supply to the preferred class. This result of the doctrine has been masked on many streams by the reservoirs constructed by the U. S. Reclamation Bureau which in many cases although originally built for the new projects of the Bureau, have been found large enough to supplement such of the prior rights as were deficient.

The riparian doctrine recognizes the variability of water supply and prorates it to the riparian owners; and thus, in times of deficiency, all riparian users suffer equally just as in humid areas all suffer during drought. Nevertheless, the riparian owners are a preferred class, as is the early appropriator. The base is somewhat broader, however, and the preference much more limited. Since it deals only with streams in their natural regimen, reservoir development is impossible under it and recourse must be had to the doctrine of appropriation. The riparian doctrine creates uncertainty as to water rights and encourages resistance to non-riparian diversions; but where limited to reasonable use (as rights under this doctrine now have been in all arid areas of America), the resistance is not serious.

As interpreted by the Courts neither doctrine has been entirely satisfactory for present complex conditions and there is a tendency toward modification. Compacts between States have been consummated, or are in process of consummation, which allocate certain shares of water in the stream system to the different States regardless of priority. The Colorado River Compact is a notable example. In California, the terms of the act creating the Water Project Authority which relates to the Central Valley Project, introduces another new phase in water law in that priority of an entire water-shed is given. In California, also, streams have been adjudicated by zones in which the relation of the water rights of one zone to those in another has not been observed. Such an adjudication is very similar to the Colorado River Compact.

In the water conflicts between States the Supreme Court of the United States is the only judicial body having power to assume jurisdiction no matter what doctrine of water rights prevails in them. The Court has assumed this power and has shown a tendency toward making an "equitable" allocation of the unused water where the appropriative doctrine is not in force in both or in all of the contesting States, leaving present uses untouched. It has also shown a tendency to continue to hold such situations under its control with the apparent expectation of revising its allocation as development proceeds.

It seems probable that whatever system now exists will continue to exist for the smaller stream systems, in all jurisdictions, but that construction of reservoirs and modification of interpretation will finally occur by which a more uniform and flexible distribution of the unstable water supply will be secured. For the larger stream systems, special laws (either legislative or judicial) framed to meet the situation, are probable. In other words, the two doctrines will continue to meet the requirements of minor matters, but modifications of both are being made for the large developments.

UNDERGROUND WATER LAW IN GENERAL

Only in comparatively recent times has underground water become important. The first case used as a precedent by the Courts of the United States was decided in England in 1843 and the controlling case in 1857. In these decisions, underground water was considered as a mineral which could be extracted by the owner of land at his will without recourse on the part of others who might be damaged by such extraction. All American Courts followed this rule for a time, but in 1862 the New Hampshire Court departed from it and promulgated the rule that the use of underground water must not be greater than reasonably necessary for the tract of land in which the water is produced. Under the former rule exportation of water could be made even if it damaged others dependent on the underground water either directly or indirectly, but in the newer rule such exportation was not allowed if damage resulted. This is termed the "doctrine of reasonable use", or the "American rule"; it is applied in a considerable number of the States in the humid sections of the United States whereas in the others the former doctrine of what may be termed "unreasonable use", or the English rule, still prevails. All Western jurisdictions, also, followed the English rule at first, but when greater knowledge of physical conditions came about through the use of underground water for irrigation, especially in California, the California Court (Katz *v.* Walkinshaw, 74 Pac. 766) introduced another doctrine which abrogated the English rule and expressly adopted a further and entirely logical development of the doctrine of reasonable use. This rule holds that each land owner overlying a basin has a right to the underground water co-equal and correlative with that of all other land owners overlying the same basin. In contradistinction to the doctrine of reasonable use this means that one land owner can not extract more than his proportion of the underground water even for use on land overlying the source when by so doing he encroaches on the right of another owner to do the same; nor can water be exported if injury to the overlying land owners results.

This places underground waters in California under the same doctrine as surface waters on riparian lands; that is, under the doctrine of correlative right. By later decisions, all the waters of a stream system, whether percolating and diffused underground; flowing underground in contact with a surface stream; flowing underground in a definite channel; or flowing as a surface stream or ponded in a lake, were treated in the same way, and the use of underground water was made correlative with the use of the surface stream.

The rule of underground water as promulgated in Katz *v.* Walkinshaw was adopted by the Courts in some Western jurisdictions in spite of the fundamental appropriative doctrine which exists in many of them. This rule is compatible with the riparian doctrine of surface streams, but is not compatible with the doctrine of exclusive appropriation of surface waters prevailing in the strictly arid States. Some other Western States still adhere to the English rule; thus, in many Western States, there are Court rulings which define the law for underground water either on the reasonable or the unreasonable use basis, and an entirely different legislative doctrine for the surface stream which may be fed by the underground basin, a condition that is certain to cause legal confusion where economic conditions permit the development of underground water.

A digest of Court decisions made in 1928 (55 A. L. R. 1390) lists thirty-one States (including the District of Columbia), mostly in the humid region, but including five in the arid region, in which former decisions are favorable to the common law rule of absolute ownership of underground waters. This doctrine, as heretofore stated, was first challenged in 1862 in New Hampshire, and the doctrine of reasonable use was introduced. Not until 1900 did any other State concur, but in that year the Court of New York adopted the same rule and, in 1909, the Court of New Jersey followed. Of the States listed in the foregoing digest a total of six had reversed themselves and eight have not recorded decisions since 1900. Twelve States not listed in the digest have adopted the doctrine of reasonable use (Neb. L. B. 12; 191-6N, 1933). It appears that the trend in humid States is toward the American rule, but that many jurisdictions still hold to the English rule.

The status of the law as to underground waters in the various Western States (there are statutes in many States prohibiting the waste of artesian water, but this phase is not considered in this paper), is as follows:

Arizona.—There are no statutes on underground water in Arizona. The Court has ruled that such water is not subject to appropriation (Maricopa County Municipal Water District *v.* Southwest Cotton Co., 1931, 4 P. 2d 369), and, at first, reserved decision as to whether the English rule or the correlative doctrine which originated in California should prevail; but in 1934 (Fouryan *v.* Curtis, 29 P. 2d. 722), it decided in favor of the American doctrine of reasonable use—that is, that the water belongs to the land under which it is found and could be taken to distant lands if injury did not result to another on the stream system. In the case of Pima Farms *v.* Proctor (245 P. 369, 1926), the underground water in a stream valley several miles wide was in question. No surface flow exists except in flood times. The Court ruled that this was a definite underground channel (the litigants stipulated at the outset that the water involved was the immediate underflow of the river), that the prior user of underground water had a vested right to the maintenance of the water level, and that subsequent users must deliver water to him at no greater cost than had been incurred prior to the new use. This is similar to declaring that a prior appropriator of surface waters had a vested right in the means of diversion; in fact, the Court states that this is the law, but cites no decisions to that effect.

California.—There are no statutes on underground water in this State, but it may be appropriated by taking on sufferance of the overlying land owners. These appropriations ripen into a right after five years of open taking and are so recognized by the Courts.

No exclusive rights are permitted in underground waters except the appropriations previously noted. Cities and other municipal organizations are regarded as appropriators even if they are located directly above the underground basin. All overlying lands, and lands riparian to a stream where the percolating waters feed a surface stream, have correlative and equal rights to the stream system whether water is on the surface or underground (that is, percolating). A diversion from a surface stream made prior to 1914 may be an appropriation even if on riparian land, and, as such, prescription obtains against an underground water user below, who may be supplied wholly or in part by percolation from the surface stream.

The law of waters in California, both surface and underground, is highly developed and rests on reasonable use and the correlative doctrine of equal rights.

Colorado.—Underground water is not mentioned in Colorado statutes as subject to appropriation. The Courts have held that it is subject to appropriation, however, and subject to the same regulation as surface water. Rights under appropriation are in order of priority of filing on the stream in question. The Courts seem to be tending to the rule that in Colorado all waters, whether surface or underground, are presumably tributary to a surface channel, and that their taking is thus subject to prior appropriation of the surface stream.

In Colorado the taking by an underground user is stopped if it decreases the surface flow available to the down-stream user even if the down-stream user could sink pumps to the underflow and get a full supply. This amounts to a guaranty of the method of diversion previously discussed.

The theory of underground water law in Colorado is consistent with that of surface streams, but the application varies in the aforementioned particular. The waters of an entire stream system are treated as one.

Idaho.—Underground water is not specifically mentioned in statutes in Idaho. The Court concurs in the doctrine of appropriation of underground waters and states that it may be by the procedure of the water code, or by taking (Silkey *v.* Trego, 5 P. 2d, 1049, 1931). The subsequent appropriator of underground water must not lower the water-table from which the prior appropriator pumped (Noli *v.* Stonen, 26 P. 1112, 1933). In other words, the prior appropriator is protected in his means of diversion when underground water is in question.

Kansas.—All underground waters in the northwest quarter of the State are, by statute, subject to appropriation. Disputes on underground water have not been before the Court to any great extent, but so far decisions appear to be based on the English rule.

Montana.—There are no statutory provisions for the appropriation of underground water in Montana. Conflicts involving underground water have not been frequent. A case decided in 1912 followed the English rule (Ryan *v.* Quinlan, 124 P. 512).

Nebraska.—There is no legislation concerning underground water in this State, but in 1933 the Court declared in favor of the doctrine of reasonable use (Olson *v.* City of Wahoo, 124 Neb. 802).

Nevada.—The statutes provide that all water within the State, whether above or below the ground surface, belongs to the public and may be appropriated for beneficial use as provided in the act and in no other way; but it specifically eliminates percolating water, the course and boundaries of which are incapable of determination. Use of underground water is not great and details of administration have not been established by State authorities. There have been no recent Court decisions on the matter.

New Mexico.—In 1927, a statute as to appropriation of underground waters was passed in New Mexico, but was declared unconstitutional in 1929 because of faulty title. In 1931, a new act, designed to satisfy the Court's objections to the first, was passed by the Legislature.

The statute applies to waters of underground streams, channels, artesian basins, reservoirs, or lakes having reasonably ascertainable boundaries; and declares them to be public waters and subject to appropriation. As a result of the definition, waters diffused and percolating toward a stream in the manner customary in humid countries, may not be included.

As a result of investigation the State Engineer has declared three basins as coming within the scope of the law. Two of these are basins in which the water is not under pressure, but in a condition such as that defined previously under the heading, "Percolation Through Basins." The other is the famous Roswell artesian basin previously mentioned.

The degree of co-ordination between procedure in the case of underground water and surface water in the same stream system is not apparent in the statute or the procedure outlined by rulings of the State Engineer. Apparently, the State Engineer proposes to accept filings only in those basins which have been examined and which have been declared to come within the purview of the statute. The Court has declared that adjudications should embrace both ground and surface water in one proceeding (El Paso and R. I. Ry. *v.* District Ct. 8 P. 2d. 1064, 1932).

North Dakota.—The statutes of North Dakota declare underground waters to be in the same ownership as the land on which they are found. No Court decisions have been made.

Oklahoma.—The statutes of this State make no mention of underground water and Court decisions on the question have not been found.

Oregon.—All waters, according to the Oregon statute of 1909, may be appropriated for beneficial use. In 1927, underground water east of the Cascades was declared subject to appropriation when it occurred in basins the boundaries of which could be defined with reasonable certainty. As finally amended in 1932 the statute as to underground waters still limits appropriation to the areas east of the Cascades and conforms to the law of New Mexico. Applicants for appropriations follow the general procedure outlined for surface waters. Development is proceeding east of the Cascades. There are no recent Court decisions as to the status of the underground water in Oregon.

South Dakota.—The statutes in force in South Dakota are "silent" on underground water. All Court decisions uphold the common law doctrine of absolute ownership and unreasonable use.

Texas.—There are no statutory enactments as to underground water in Texas. All Court decisions uphold the doctrine of absolute ownership of underground water.

Utah.—By a number of decisions, the Utah Court has held that owners of overlying land have co-equal and correlative rights in underground water (Katz *v.* Walkinshaw, *supra*) and also that such owner may export his *pro rata* share to distant points. The controlling case is Glover *v.* Utah Oil Refining Co.

(218 Pac. 955, 1923). In the case of Wrathall *v.* Johnson (1935), the Court —in a peculiarly divided opinion—held that the use of underground water is by appropriation even if on overlying land and is subject to the same restrictions as prevail for appropriations of surface water. This was a decision on demurrer and cannot be regarded as conclusive.

The Legislature of 1935 enacted a statute placing underground water in the same status as surface water; that is, as a right secured only by application for appropriation to the office of the State Engineer. This act follows a model recommended by the Association of Western State Engineers and is similar to the New Mexico statutes.

Washington.—All water in the State of Washington is declared to be subject to appropriation which would include underground water, but State officials consider that no authority is conferred on the State over such waters. One Court decision on underground water adopts the doctrine of reasonable use on overlying lands; another that underground water may not be taken to the injury of surface water diverters from the stream. The doctrine of the Courts is not well defined.

Wyoming.—No mention is made of underground water in the statutes of the State; nor are there recent Court decisions to clarify the law.

All Other States.—The only attempt at statutory control coming to the notice of the writer is in New York State where wells on Long Island drawing more than 100 000 gal per day are placed under the jurisdiction of the State Conservation Commission.

Summary.—Summing up the results of the foregoing examination, there are found to be four doctrines of law in the United States on which use of underground water is based: (1) Absolute ownership of water because of the ownership of the land beneath which the water is found, with no obligation to respect the rights of others, is herein termed the "doctrine of unreasonable use" or the English rule; (2) absolute ownership of water to the extent of reasonable use on the land beneath which the water is found, but with no right to export to distant land if by so doing damage is caused to another, is herein termed the "doctrine of reasonable use", or the American rule; (3) ownership, co-equal and correlative with that of every other land owner, of water lying over the basin, or riparian to a stream fed by water rising from the basin, is herein termed the California doctrine; and (4) entire lack of ownership on the part of the proprietor of the land, but ownership by the State instead—which allows use by appropriation under a procedure set by the State, or otherwise, and which is subject to prior rights of other users whether from the surface stream or from underground sources tributary to the stream—that is, the doctrine of prior appropriation. These differences are successive and cumulative impositions of control or, broadly speaking, the police power as found desirable because of the growing use of underground water, and as found possible because of increased knowledge of groundwater hydrology.

Unless analyzed these diverse doctrines would seem to entail endless confusion, but when it is remembered that there is also great diversity of climate in the United States and also great variation in the present stage of development and possibilities of future development, the probable confusion appears not to be very great as a whole although potentially bad enough in limited areas. The worst legal confusion could result from lack of consistency between surface-water law and underground water law in the same general region; but even where these laws are inconsistent, the climate may be such that costly development is impossible and, if so, the conflict may be more apparent than real.

ADMINISTRATION

Surface Streams.—From a stream of any extent, in arid regions, there are numerous diversions made for irrigation and it is obvious, in view of fluctuations of stream flow, that if use is large as compared to flow, chaos would result if these diversions were not controlled. In times when the flow is not sufficiently greater than the rights so that control is not needed, the administrative official allows to each diversion its right as determined by the law, Court decisions interpreting the law, Court decrees dealing with water rights in the stream, custom, and his own interpretation of the law. The decrees or the law not only determines the priority, but the amount of diversion. This administration is possible under both riparian and appropriative systems of law. Under the appropriative doctrine the administrator goes further and permits or denies new rights (if the law gives authority to deny), depending on his determination of the existence or non-existence of unappropriated water. In many States authority to deny is not given but control rests on the water master. So far as known no administrator denies a permit or in his policing of the stream closes a diversion because of difficulty which may be, or is, caused to a lower diverter as long as the water required by the right of the lower diverter will reach his point of diversion on the surface; but accompanying almost every surface stream is underflow which is legally a part of the stream system, and where this exists, it can be diverted by pumping so that even if the surface stream is all diverted at a point up stream, the lower user is not necessarily deprived of water. So far as known, however, administrators take no cognizance of this in their administration. This custom is based on Court decisions.

The Courts in all jurisdictions, in so far as can be found, have enjoined surface diversions which would have made it necessary for a prior and lower diverter of surface water to pump to secure water. It seems probable that this is the outgrowth of custom antedating the use of underground water, or it may be that a change from surface diversion to pumping diversion is unconsciously regarded as a rough dividing line between complete confiscation and endurable confiscation, discussed previously, in connection with surface water; or, it may be that, because the appropriation of surface water was made, it is regarded as necessary that surface water be preserved to the right. It is only recently that connection between surface water and underground water has been recognized legally. Formerly, the two were treated in separate and distinct doctrines of law, and still are, to a great extent. This matter has an important bearing on the development of underground water because it is probable that once control of the use of underground water is attempted, both classes of water will be brought under the system which prevails as to surface water in those situations where both are involved but most statutes so far enacted for control seem to ignore this probability.

Underground Water.—The discussion of State laws under the heading, "Underground Water Law in General", indicates that there has been little legislation on underground water and that administrative control has been attempted in few jurisdictions. Under the headings, "Percolation Through Basins", "Underflow Which May Support a Surface Stream", "Artesian Areas or Basins", "Alluvium Supplied by Deep Percolation from Irrigation", and "Alluvium or Glacial Till Supplied by Rainfall", several typical situations were described in which underground water is of commercial importance. Consider the situation in San Gabriel Valley. No administrative control of the use of underground water is exercised in California, and all underground water rights are correlative one with another and with rights in the surface stream of the system. Consider the present condition had the surface rights at the lower end of the valley been empowered to enjoin the use of underground water at up-stream points in order that the surface flow might be maintained, as such rights probably are empowered in all appropriative doctrine States, or will be if the two systems of law are welded. In 1912, 32 000 acres were irrigated in the valley at up-stream points, whereas, in 1934, there were 97 000 acres, but there is no indication of long-time deficiency in supply in spite of the increase. This increase could not have occurred had such power existed. Had this additional draft not taken place the water-table would now be higher than it is; the capacity of the underground basin above the water-table would have been smaller than it is at present; and the amount of percolation from floods would have been much less. In short, unless these paramount rights had been extinguished, development would now be much less than it is and the water now percolating in flood time into the underground reservoir would be wasting into the ocean. On the other hand, those having surface water rights would secure it at less cost than they now do, as they have been forced to resort to pumping to secure part of their supply. The result of the existing law has been a more complete development and the greater unit cost which is generally incurred by recent developers has been spread more or less evenly over all users.

Consider the Roswell Basin: The water which supplies the artesian area of the Basin, in part at least, flowed through, or out of, the region as sustained flow usable for irrigation; but surface users have now been deprived of this water. In part, there may be recharge now from floods which was formerly wasted. If the paramount consideration had been to preserve the surface flow to satisfy rights therein, the Basin, in all probability, could not have been developed as it now is, and the irrigated area served by the stream would be less unless surface reservoir capacity had been substituted. If this is true, development of the Basin has gone forward because of lack of administrative control.

Consider the South Platte: The control of underground water has existed through Court decree, and encroachment upon surface rights has been inhibited. Some, but no great percentage of, stream discharge is now wasted and most of this water could be captured were pumping allowed in the river bottom and were more acreage developed. Additional cost would be incurred by all users down stream from the location of pumping and the cost of new development would have been spread to some of the prior users; but to conserve this water by surface reservoirs would also cost money, which would be furnished by the direct beneficiaries, and perhaps construction of a reservoir would not be financially feasible for them.

The foregoing examples suffice to illustrate the results of administrative control under the concepts now existing. Clearly, development now exists which would not exist had control been in effect, yet, in many cases, in spite of this greater development, draft does not exceed supply. Whether this added development, with its attendant increased costs to early water users, is more desirable than a smaller development and lower costs is a question that need not be discussed in this paper, but where draft does exceed supply there can be little argument that the condition which exists is desirable.

Although control of surface waters is not simple, the administrator charged with the control of underground water finds himself involved in a vastly more complicated situation than he who administers only surface streams. He may be faced with the necessity of re-examining his concepts of what constitutes a water right. His decision must enter the field of economics from which he has been free in the administration of surface streams. Practically, no extensive use of underground water can be made in the West without diminishing the normal summer flow of surface streams below it where such flow exists and as these streams in most cases are appropriated the precedent of decisions will incline the Courts to protect the user of surface rights against depletion by underground-water users. If this is the case, administrative control merely means complete inhibition of pumping where surface streams are involved unless pumping occurs so far from the stream that the depressed water-table has no effect on the summer flow.

The Court of Idaho has already indicated by decisions, previously cited, that it considers that the first underground-water user has a right to compensation against later pumpers in the same basin, whose draft would lower the water-table. The Court of Arizona (Pima Farms *v.* Proctor, 245 P. 369) has adopted the same attitude as to water in a definite underground stream. Pumping cannot be done without lowering the water-table, and each additional draft causes additional lowering. Consequently, even without reference to surface rights, these rulings constitute absolute prohibition of the development of underground water in these jurisdictions, except in similar situations. Statutes embodying control by the water authority merely make it easier to accomplish the same result if the administrator either voluntarily adopts the same view, or is forced to do so by the Courts.

Administration of a surface stream involves only matter which is in plain sight on which, as compared to underground water, only a small amount of study is required. The problem is solved largely in the field and a set rule is followed if only priorities are considered. Administration of underground water involves gathering and studying data. The work is in the office, and decision is not so easy. The cost of obtaining sufficient information to provide a basis for intelligent decision is considerable and this cost would not be justified where economic conditions do not permit extensive use of underground water and, from the same consideration, administrative control would not be justified.

If funds are sufficient for study, administrative control should lead to greater knowledge of hydrology and the education of engineers and Courts so that water law can be modified logically. If such changes are to be constructive the powers of the administrator must not be too hampered, and his function must at least be quasi-judicial. The increased responsibility requires greater power than ordinarily lies in the water administrator's office. Above all, it requires power to deny applications.

Summary, Discussion, and Conclusions

1.—The use of underground water in the United States is increasing continually. In humid regions this use is principally for municipalities, but in arid regions the draft is for both municipalities and for irrigation. In the East it is only in the vicinity of very large or very numerous municipalities that the draft is sufficient to be of concern and in the West it is only where physical and climatic conditions are favorable. There has been very little use of underground water in the Northwestern States; and there is no conceivable prospect for increase except in the arid or semi-arid portion of those Northern States favored by the Pacific slope climate.

2.—An underground basin contains a body of water that is moving; but, at the same time, it is a reservoir because it is moving very slowly and it may take several years for a unit of water to pass through it. Withdrawal of water from the underground reservoir decreases the quantity that must be disposed of by Nature unless conditions are such that the quantity pumped is replenished by percolation from stream beds, or by deep percolation from rainfall, neither of which would have occurred had not underground space been created by pumping. This replenishment, if it occurs, does so as Nature sees fit and is not under the control of Man. If the underground water is tributary to a stream, the stream flow will be decreased by a draft on the underground basin. This decrease may be either in the period of low water or of high water, or both, and its time is non-controllable.

3.—In humid regions there are few statutory enactments as to rights to use water. The body of Court decisions rules. Both the English and American rules exist for underground water, but the trend is toward the American rule in which ownership of the water is not vested absolutely with ownership of the surface. This is not on the same basis as the riparian doctrine in surface stream flow, but satisfies the requirement in most cases, as does also the English rule, apparently, where it prevails.

4.—All the arid States of the Union have instituted more or less far-reaching administrative control of surface waters in order to obviate the chaotic conditions which generally result without such control. Such legislation was adopted by many very soon after Statehood was granted and, at that early period, the use of underground water was nil, or practically so. Probably, as a consequence of this condition, reference to underground water in the statutes of many of the States is lacking, inadequate, or must be inferred by the all-embracing wording of the statutes. In California, because of the clear-cut doctrine developed by the Court prior to the enactment of statutes regarding surface waters vesting the rights to underground water in overlying lands, reference to it was omitted from these statutes.

5.—In California, which is the only jurisdiction in the United States recognizing the riparian doctrine where the law of underground water has been fully harmonized with surface water law, each land owner having access to the usable water of the stream system, whether it is on the surface, or underground, has a right to a reasonable use of the water. If the owner whose land overlies an underground basin, in exerting his right, infringes upon a surface-water user's supply the latter has no recourse if the infringement is not excessive and he may be deprived of his surface right entirely if he can secure water by pumping. He is thus deprived only as the result of the exercise of another right equally as good, vested in all who have access.

On the other hand, in Colorado, which recognizes the appropriative doctrine instead of the riparian doctrine, and in which State the law of underground water appears to have been more fully correlated with the law of surface waters than in any other State except California, the owner whose land overlies a body of underground water is prohibited from using that water if such use would decrease the surface flow necessary to supply the appropriative rights of surface users.

The result of the legal doctrine recognized in California has been so far to give a free hand in the development of underground water. The result of the legal doctrine recognized in Colorado is practically complete prohibition of such development where rights to diversion from surface streams are involved, because the surface streams were used in most cases prior to the time that the use of underground water began.

6.—In the absence of definite statutory enactments the Courts have adopted diverse views as to the doctrine of law applying to underground water and in many jurisdictions even in the West the doctrine as enunciated by the Courts is not compatible with the doctrine of law prevailing for surface waters. There is a long line of decisions in the United States which vests with the owner of the surface the right to use water under his property although the right is not absolute but rather relative in most jurisdictions and the tendency is toward the relative doctrine. This right to use underground water seems a natural one and the Courts tend to promulgate it, but in a State where the theory of the appropriative doctrine prevails (State ownership of water), underground water should be regarded as belonging to the State if the law of underground water is to be compatible with that of surface water.

7.—It is notable that, in California and Colorado, the two States in which use of water for irrigation is largest (see Table 1), underground water and surface water are in the same system of law. In other words, the fact that they are part of the same stream system is recognized. It seems entirely probable that this same development of law will come about in other States whether or not statutes enacted for the control of underground water ignore the interdependence. It would be convenient in many cases if this did not occur, but to continue to ignore, legally, the interconnection while at the same time putting administrative control of underground water into effect would seem to be an impossibility. The following Conclusions assume that the two classes of water will come under the same system of law.

8.—The appropriative doctrine (Conclusion 6) rejects the right of each individual to pump commercial quantities of water from his own land, and makes it necessary to secure a permit from the State to do so. In any effective administration the permit would be granted only after a survey of the entire situation and, since the appropriative doctrine seeks to preserve to prior users the full measure of their water right, would lead to rejection if the investigation disclosed that use of underground water would cause decrease in the water to which they are adjudged to have a right provided the concept prevails, that a water right on a surface stream cannot be satisfied by pumping from underflow.

9.—No doubt legislation on underground water in States recognizing the appropriative doctrine would be a powerful influence in future Court decisions, but if the legislation should be contrary to the doctrine of underground-water rights laid down by the Courts in sufficient previous decisions to make it well grounded, it might be declared unconstitutional in certain jurisdictions inasmuch as the existence of previous decisions means that development exists under the doctrine of law adopted by the Court.

10.—It is improbable, but not impossible, that legislation on underground water—even in some States recognizing the appropriative doctrine—might recognize that a right to use such water exists in the ownership of the overlying land, in which case prior use in surface streams fed by underground water would not constitute a right which could be exercised against the users of such water. Much more probable legislation, however, would place all water, whether surfac or underground, on the same basis and under the control of the water authority.

11.—The water authorities of the arid States generally grant a permit to an applicant for the use of surface waters. This can be done because the streams are patrolled. If the project is constructed the diversion works can be closed when the surface flow is not sufficient for all prior rights; but when an underground supply feeds a surface stream it will be impractical to follow the same course on application for rights to pump from the underground basin because the evidence of the effect of the draft on underground water is so long delayed, and because the time at which the effect will occur is uncontrollable. Action by the authorities (if it is intelligent) must have behind it the knowledge gained by a thorough investigation and if, in those States where the law of underground water is, or becomes, compatible with the surface-water law, this leads to the conclusion that the use of underground water will reduce the supply available to surface-water users, a permit would be denied if the surface right is paramount. There is no half-way point. At present, practically all surface streams are fully used in the late summer and, on many of them, reservoirs have been constructed which impound all the flood flows in ordinary years.

12.—In isolated basins with no surface stream outlet the problem is less complicated by traditional concepts of what constitutes a water right. The average recharge and draft can be estimated with reasonable accuracy after thorough study. At first thought, it would seem that a simple solution would be to issue permits up to the amount of recharge as a greater draft which may be sustained for a period would only necessitate a future decrease. However, decrease in draft most often occurs because cost of pumping from a lowered water-table becomes too great for some users and not because the water has become physically unavailable. The recharge during a long period of years may be deficient due to vagaries of the climate. Even though pumping draft is less than the long-time average recharge the water-table will drop during such periods, and excessive pumping costs will occur so that some users quit. The result is different only in degree from that which would be the case with actual long-time overdraft. Obviously, the administrator is faced with consideration of water costs to determine the safe yield in such situations and this, instead of merely quantity of water available, may guide his decision.

13.—The administrator will also be faced with another situation not discussed in the body of this paper and pertinent only to the economic questions which are potentially present in the intensive use of almost all streams, but particularly so in the intensive use of underground water. The use and re-use of water increase the content of dissolved solids. Some of the salts held in suspension appear to be actively poisonous to plant growth whereas others merely decrease the growth by partial smothering. Still others destroy the soil and make it unworkable. Theoretically, the use of water from an underground basin can be developed to equal the average recharge, but this would result in a slowly cumulative concentration of solids possibly to the point at which all agricultural values would be destroyed. A certain percentage of the supply must be wasted to carry off the salts. This fact must be considered in any intelligent administrative control of underground water.

14.—The foregoing sections refer to natural underground supply. When water is diverted from a stream it passes into the possession of the appropriator. As more water than can be consumed is almost always diverted and placed on the land the surplus percolates to the water-table, and, unless pumped out, finally reaches the stream. The law differs between States; in some States the water which has percolated, is deemed the property of the diverter until it reaches a natural channel, and hence it can be pumped and used on additional land under his control without interference with other users down stream. In other States the diversion is deemed to be applicable for use on the land which it will serve directly. The use is deemed completed when the water in the surface conduit is exhausted, and deep percolation is regarded as belonging to the stream system. In such cases, it cannot be pumped without a permit, which would not be given in most cases for reasons previously stated.

15.—The decisions of the Idaho Court (Silkey *v.* Trego, 5 P. 2d, 1049, 1931; and Noli *v.* Stonen, 26 P., 1112, 1933) prohibited additional pumping from an underground basin because it lowered the water-table and thereby increased the pumping cost for a prior user. The decisions of the Arizona Court cited previously follow the same reasoning, but in this case the underground water is regarded as being in a definite channel, although several miles wide. As underground water cannot be used without lowering the water-table, this reasoning, if persisted in, means inhibition of the further development of underground water in these jurisdictions. The fact that considerable underground water is used in these States is no criterion of the future. This use has created a body of water users who will unite behind these decisions to enjoin additional use.

16.—In any State having the appropriative doctrine it should not be difficult to enact a statute that will clearly put underground water under the control of the water authority, or to frame rules and regulations in those States in which the status is already clear. If the doctrine of the statute is compatible with the appropriative doctrine it clearly means prohibition of further underground-water development when the water authority takes control of an underground basin which contributes to a surface stream for reasons before stated.

In States recognizing a different administration of water law than the appropriative doctrine, statutes regulating the use of underground water, if enacted, must rest on police power. In a State wherein the water law is as well developed as is the case in California, such a statute would probably extend only to prohibition in the case of overdraft and would not have the effect noted in the preceding paragraph.

17.—Lack of administrative control leads to the fullest development of underground water but, at the same time it leads to excess development. A statute creating control, such as those adopted by New Mexico, Oregon, and Utah, apparently permits the water authority to declare control at his option. This permits a thorough investigation of the situation prior to taking control and is a flexible arrangement which may guard against the evils of too much and too little control.

18.—Perhaps future development, particularly in cases where the regulation of underground water is involved, will bring sharply to the fore a trend toward modification of water laws applicable to smaller units than those involved in such agreements as the Colorado River Compact.

19.—To summarize: Administrative control of underground water, to be intelligent, requires much research and investigation, which, in turn, requires large legislative appropriations. When administrative control is initiated it may well result in the prohibition of underground water development in States following the appropriative doctrine, in most basins tributary to surface streams, and plunge the administrator into the question of economics in isolated basins. On the other hand, control will stop over-development and thus becomes desirable at some stage no matter what the legal doctrine of the jurisdiction. If the fullest development of the water resources of a State are desired, administrative control of underground water should not be adopted until overdraft threatens, and then only in particular areas. Conditions probably would never warrant administrative control in the arid States having a cool climate or in most of the humid States.

20.—All the foregoing comments lead to a re-statement of the paramount conclusion that, as control of underground water becomes necessary in the various jurisdictions, the complexities of the situation can be dealt with only by lodging greater power in the water authority than is now given in most jurisdictions. However, a step at a time may be the safest course.

ACKNOWLEDGMENTS

The writer expresses his appreciation of the authorities cited in this paper and acknowledges his debt in the preparation of the résumé of the law given herein. In addition, he expresses his appreciation of the corrections and criticisms offered by the members of the Committee of the Irrigation Division on the Conservation of Water.

DISCUSSION

JOSEPH JACOBS,[7] M. AM. SOC. C. E. (by letter).—This exceptionally able paper deals with the subject of the administrative control of underground water—a consideration of steadily increasing importance in the economic development, not only of the semi-arid sections of the West, but of the entire country. In his reference to irrigation in Oregon and Washington, the author inadvertantly states that "west of the Cascade Mountains irrigation is not practiced, but east of these mountains both States are arid." The writer desires to offer a correction of that statement.

As to the sections east of the Cascades it would be more accurate to state that they would, in general, be classed as semi-arid although there are considerable areas along the eastern border of Washington, and to some extent in Northeastern Oregon, that have sufficient rainfall for successful agriculture without irrigation. The status of irrigation west of the Cascades is portrayed in the following quotation from a paper prepared by the writer for the Pacific Northwest Regional Planning Commission and which appeared, in abstract, in a report published in 1936 by the National Resources Committee:

> "There has, as yet, been but limited irrigation development west of the Cascades. A relatively high annual precipitation, and a relatively high humidity, even in summer, have served to discourage such development. However, despite the rather substantial annual precipitation, its 3 inches or less of summer rainfall, when water is most needed by the crops, is less than that of many irrigated sections on the east side. The cost of west-side irrigation should, in general, be much less than that of the east side due to the shorter irrigation season, the less amount of water required, and the more readily available water supplies. Unquestionably, the agricultural output would be increased by such irrigation and unquestionably, too, the expense involved would, in many instances, be fully justified economically. About 12 000 acres are now irrigated in western Washington and 65 000 acres in western Oregon, of which about 10 000 acres are in the Willamette Valley. A recent study made by the Oregon State Planning Board, reports 1 140 000 acres of irrigable land in that valley. A more detailed investigation of the valley is now being made by the Corps of Engineers, United States Army. It is easily conceivable that west-side irrigation may ultimately amount to as much as 1 500 000 acres."

W. D. FAUCETTE[8] AND J. E. WILLOUGHBY,[9] MEMBERS, AM. SOC. C. E. (by letter).—In view of the prevailing lack of administrative control of the ground-waters and the reckless destruction of those waters by interests having other objects in view, this paper is particularly timely. There is now (1936) a determined effort, well-financed, to obtain the excavation of a sea-level ship canal by the Federal Government across the underground flow in the peninsula of Florida, which has aroused the apprehension of the many people whose farming and fruit-growing industry is dependent on that flow. The physical conditions in Florida make a résumé thereof an item of interest in connection with Mr. Conkling's paper.

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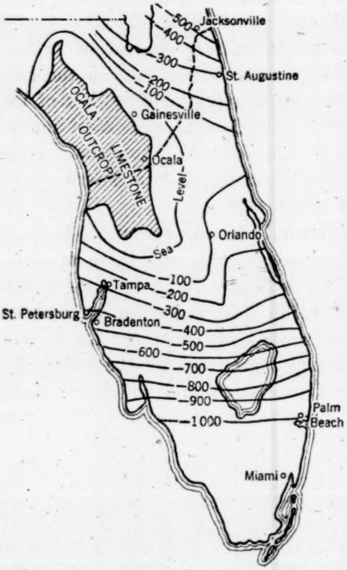

FIG. 1.—STRUCTURAL MAP WITH CONTOUR DRAWN ON TOP OF THE OCALA LIMESTONE IN FLORIDA.

The author discusses conditions that obtain in the arid West; in Florida, the rainfall is generous, and the geological relationships are different from any mentioned in his paper. Artesian water in Florida is not found in structural basins, but it moves outward and downward in a great dome of Ocala limestone. This limestone (which is a porous, soft, cavernous material through which ground-water moves freely) lies at or near the surface in the area from about Webster and Bushnell on the south, to Silver Springs on the east, and Gainesville, High Springs, and Mayo on the north, and extends to the Gulf of Mexico (see Fig. 1)[10]. The flanks of this limestone outcrop are covered with younger and often more impervious formations. The Ocala limestone outcrop area forms a great catchment basin. Copious rainfall thereon percolating outward and downward through this formation which underlies the entire State of Florida and is the most important water-bearing formation in the State, is the source of a large percentage of the ground-water supply available through deep wells and springs in the Peninsula.

A most important and pertinent question raised by the advocates for the conservation of the ground-waters of Florida is the effect of a 200-mile ditch cut 30 ft below sea level, across the catchment area that supplies the greater part of the artesian water of Florida. Would such a ditch so intercept, contaminate, and pollute the ground-waters of the Florida Peninsula as to destroy much of the beauty and charm of the great winter playground of the Nation, and, at the same time, ruin its groves, farms, and its market gardens? Would the canal bring about a situation whereby the residents of Florida could no longer depend upon ground-water supplies for industrial, municipal, agricultural, and domestic uses?

For several years, in co-operation with the U. S. Geological Survey, the Florida State Geological Survey has been making a study of the ground-waters of the State. Realizing the danger of promiscuous borings for artesian water and of contamination by drainage wells, the State Geologist has advocated a very careful supervision and conservation of ground-water supplies, and laws have been passed to that end. In spite of these recommendations, certain engineers and geologists have made superficial investigations and recommended the building of the canal, without giving due consideration to its effect upon ground-waters.

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[10] Twenty-First and Twenty-Second Annual Repts., Florida Geological Survey, Fig. 8, p. 50.

To the present time the weight of scientific evidence as to the effect of a canal on the geological problems involved, has been against, rather than for, the building of the canal. The geologists who have reported favorably on the canal admit the possibility of grave danger to such scenic features as Silver Springs and Blue Springs, but state that this danger may be minimized or entirely prevented by proper engineering precautions. However, lowering the water-table to 40 ft below the present level of Silver Springs and Blue Springs, and the lower courses of the Withlacoochee and Ocklawaha Rivers, will undoubtedly destroy these springs. That is what an effective drainage ditch will always do—drain. It is difficult to imagine any engineering precaution that can be taken to prevent it. The author points out that springs in the vicinity of Roswell, N. Mex., no longer exist, because the water formerly issuing from them has been diverted, and the area of artesian flow within that region has shrunk from 663 sq miles to 425 sq miles, because of depressed piezometric head.

In regard to the effect of salting artesian wells, the geologists favoring the canal frankly admit that they do not know what will be the result, if the canal is constructed. However, Dr. Herman Gunter, Florida State Geologist, specifically states that such salting is greatly to be feared, especially in that area where highly mineralized water is present at moderate depths. Any lowering of head due to excessive drainage in the Ocala catchment area may cause further encroachment of salting in those areas where there already exists a delicate balance between fresh water and salt water. This condition is already faced by some vegetable growers in the Sanford Section. Less than normal rainfall on the catchment basin over a period of several years has reduced the quantity of ground-water available in the Ocala formation, and this, together with the attendant necessity of heavy draft on flowing artesian wells for the irrigation of crops, has resulted in some wells in the Sanford District ceasing to flow, and the waters in others showing high salinity. The construction of the proposed canal could accentuate and make permanent these conditions, not only in the Sanford area but also in other important vegetable and fruit-producing districts of the State. Growers in the Sanford District, having experienced the disastrous effects on their crops of depleted ground-water supplies, have arisen *en masse* protesting the construction of the canal, fearing that it would completely destroy their livelihood. It is also true in certain areas, such as Sanford, that any diminution of head would necessitate the pumping of wells where they now flow freely. This would greatly increase the cost to the growers of water used for irrigation.

Geologists and engineers who have favored building the canal have definitely stated that the pursuit of agriculture and the growth of vegetation will not be affected; that the progress of agriculture and the growth of vegetation in general are only remotely related to the water-table. These are strong statements when it is remembered that three-fifths of the rainfall of Florida is in June, July, August, and September (which is the non-growing season, as Florida farmers produce their crops during the winter months for the Northern markets), and evaporation is so great that probably less than 20 of the 50 in. that fall as rain sinks into the soil. The control of water through irrigation and drainage is rapidly becoming a limiting factor in successful crop production in all areas of the United States. Areas in Florida that are not irrigated, and where the water-table is 60 to 100 ft below the surface, are adapted, only under exceptional conditions, to a highly developed agriculture. Florida growers are beginning to realize the necessity for more precise control of water in crop production if the State is to maintain its horticultural prominence. In that State, with the exception of the Everglades, irrigation is mostly from the underground water supply. Vast producing districts have been established in many parts of the Peninsula where perishable crops are grown under intensive methods based on irrigation from deep wells. This shows that Florida farmers realize that they cannot depend entirely on rainfall for the production of their crops. Like many areas in the West, noted by the author, Florida is using more and more artesian water for irrigation, and for municipal, industrial, and domestic purposes; and there is, therefore, a constantly increasing drain on underground supplies.

The statement has been made that although the water-table at the edge of the canal would be lowered to sea level, normal water-table slopes would be approached 10 to 15 miles from the canal. Unfortunately, the areas used in calculating these slopes were not taken in the Ocala limestone, but were in the Tampa and Hawthorne formations, which are younger and more impervious. As the Ocala material has a mean porosity of 30.6%, and as the canal would cross "the ramifying system of ground-water drains which has been developing over a period of millions of years", discharge from this area would be very rapid and the water-table slopes in such a formation much more gentle than those used in the calculations previously cited.

Over a large area in Central Florida the water-table stands between 40 and 50 ft above sea level. With more complete drainage established by cutting into the Ocala limestone, the water-table will fall to sea level over approximately the same area with the following results: (1) Decrease of head will cause salt water to rise in artesian wells approximately 40 ft for every foot the head is reduced; (2) lowering the water-table will remove supports from cavern roofs and walls, and there will be a progressive formation of sink holes from the canal, both to the north and south; and (3), the areas of high water-tables (the piezometric surface), both north and south of the canal, will be lowered, whereas high water-tables have a beneficial effect on temperature during cold waves.

Amplifying Result (2), perched lakes and swamps will be drained. It is interesting to note that many of the lakes of Florida may be of this character. A notable example of a lake of this type existed at Payne's Prairie, near Gainesville, Fla. Payne's Prarie has an area of 18 to 20 sq miles. In the summer of 1790, William Bartram visited this section and found a vast savanna that afforded pasturage to large herds of horses and cattle belonging to the Alachua tribe of Indians. When visited by James Pearce, in 1824, this basin was still dry land. About 1871, Alachua sink became clogged, and the body of water thus formed was known as Alachua Lake, which is reported to have been navigable for small steamers. This lake continued until the summer of 1891, when it was drained through a sink. Since that time the lake bed has remained dry land, with the exception of temporary overflows, and is known as Payne's Prairie, one of the best cattle-grazing areas in the State.

As the author points out, the use of ground-water in the United States as a whole is constantly increasing, and it is important that this valuable natural resource be conserved. Ground-water is the very life blood of Florida; and in view of this fact one can readily understand the concern which many localities in that State, south of the route of the proposed canal, have shown with regard to the conservation of their most important natural resource. It is the hope of the writers that the present agitation in Florida will lead to more effective administrative control, and to the development in the mind of engineers and of the public of a greater sense of responsibility to the future of preserving the ground-water resources throughout the Nation

R. E. SAVAGE,[11] ASSOC. M. AM. SOC. C. E. (by letter).—A complete review of a new and important subject is presented in this paper. Mr. Conkling has handled the legal background of water legislation and Court decisions in an able manner. The paper points out some of the implications of public control of ground-water, assuming that such control will be exercised by the States in a wise and inclusive manner, in places where there is an economic justification and need for such a program.

The extreme variation in topographical, geological, and climatic conditions in the United States present a wide basis for the application of remedial legislation and governmental control of ground-water. In addition, there is the differing laws, customs, and application of laws by Civil Courts, resulting in the building up of a complicated structure. The author has emphasized all these phases and has shown the need for future control of underground water through the growing use of it for beneficial purposes.

The increase in population and the decreasing water supplies have brought the need of conservation of water to the fore. There is an ever-increasing demand for large supplies of water of relatively high standards of quality for industrial purposes in the centers of population in the United States. Large metropolitan areas have striven heroically to meet this demand by the construction of aqueducts bearing the supplies of mountain water-sheds a great distance. It is evident, however, that the economic and physical limit of these distant supplies is in sight. In the past few years engineers have witnessed a heightened discussion of the climatic cycles and their relation to the quantity of run-off water available for storage, irrigation, and domestic use, and the studies presented have been of absorbing interest. There is an increasing efficiency and excellence in the design of pumping machinery and motor-power equipment for the utilization of ground-water. There should be a corresponding advance in regulation and protection and in making the existing ground-water supplies available.

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[11] Asst. Engr., California Railroad Comm., San Francisco, Calif.

The writer has been greatly impressed by the need of regulation, by law, of the loss of wells in basins subject to harmful encroachment from salt water from adjacent coast lines and inlets, and also the need of eliminating the waste from virgin artesian supplies due to careless and destructive methods. There is no doubt that much benefit would result from the co-ordinated control of ground-water by public authority. After reading the paper, one is more than impressed with the need of distinct legislation for the various phases of water-supply control, wisely and skilfully drafted.

Any student of ground-water resources realizes the necessity of collecting information concerning the geologic nature, the history, and the yield of wells. In the private development of ground-water much of this information is lost, and these data can be collected and made available by future regulation.

In this comment on the physical aspects the writer has endeavored to keep in mind the difficulties besetting the progress of any comprehensive program of public control of ground-water, because of the diversity of the legal treatment of property relations in the ownership of water in the various States and political subdivisions. Mr. Conkling has ably clarified this phase of the situation. As stated under the heading, "Conclusion", it is incomprehensible that the control should be taken from the State, due to complex and extensive legal framework that has been constructed in the past as a means of controlling the surface waters.

H. J. F. GOURLEY,[12] M. AM. SOC. C. E. (by letter).—The subject-matter of this paper is of great interest, if only as an indication of the variation in the practice in different parts of the United States, and forms a valuable record. To complete the picture, the following notes of the position in England are submitted.[13]

In 1859, the highest legal tribunal—the House of Lords—established the principle that the rights of riparian owners, in respect of water flowing in visible surface streams or in known and defined channels, do not apply to underground water which simply percolates into and through the ground and has neither certain course nor defined limits. Subsequent cases decided in 1881, 1886, and 1902, made it clear that a "defined" channel is one which is definitely contracted and bounded, although the actual course of a channel may be undefined by human knowledge, and, furthermore, that a defined subterranean channel must be "known" not by excavation but by reasonable inference from observations on the surface of the ground.

By these decisions an owner claiming riparian rights in the flow of water in a subterranean channel must prove the existence of such a channel without recourse to excavation, and as recently as 1932 this statement of the legal position was affirmed by the Court. The general position, therefore, is that there is no property right in underground water, and nothing to prevent the sinking of a well on land adjacent to the site of an existing well, to the possible detriment of that well.

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[12] Civ. Engr. (Binnie, Deacon & Gourley), Westminster, London, England.

[13] "Some Recent Developments in British Waterworks Practice", by H. J. F. Gourley, *Journal*, New England Water Works Assoc., December, 1936, p. 345.

As between neighboring industrial concerns using underground water, the principal effect of the existing law has been that when "Peter" robbed "Paul" by sinking to, and pumping from, a greater depth than "Paul", the latter's only remedy was to deepen his own well and pump from still greater depths, and the only people who gained were the well sinkers.

Until comparatively recently an industrial concern using more than 100 000 gal of water per day from an underground source was the exception; within the past decade, artificial silk and other specialized works have been established, and these, by drawing on underground resources to the extent of several million gallons per day in some cases, have so depleted (over-pumped) the area in which the works happen to be, as to affect adversely the yield of public wells upon which the community relies for all or part of its supply.

It will have been appreciated that a private individual or manufacturing concern has only to buy land in order to have the indisputable right to obtain such water from underground sources as the strata are capable of yielding.

When, however, a public authority or statutory water undertaker[14] contemplates the augmentation of its resources by an underground supply, the position is not so simple, since the authority of Parliament or of the Ministry of Health must be obtained first. Parliamentary plans are then deposited in both Houses (that is, the House of Lords and the House of Commons), in other specified places, and with Government departments. A Bill is prepared describing the "works" for which powers are sought, and among its other provisions is a so-called "protective clause." At the same time, by notices published in the newspapers, all persons or interests likely to be concerned are made aware of the proposals of the Bill, which is, in due course, the subject of inquiry, in turn, by a committee of each House. The promoters and the various opposing interests are represented by counsel and supported by engineers, and other technical witnesses, and, in the sequel, the Bill emerges as an Act upon receiving the Royal Assent, although, generally, with amendments put upon the promoters by one or other committee or inserted by agreement with opponents. It is by such an Act that a public authority is empowered to purchase land by condemnation, if agreement is not possible, and to sink a well or wells, and to build other structures, on it. For less important works and where there is no opposition, a simpler and less expensive procedure is available, for the Minister of Health has authority to grant a Provisional Order, which must subsequently be confirmed, if not then opposed, by Parliament.

For thirty years or more most private Acts involving the construction of wells have included the protective section previously mentioned. Under this section, the owner of any well or spring (and, sometimes, pond) within a specified radius, used at the date of the Act as a source of water supply, who can prove that his supply has been diminished as a consequence of the pumping at the authorized well, is entitled to compensation in water, or otherwise. The radius of protection varies with the geological formation and in special circumstances, but it is usually 1 mile for wells in chalk and 2 miles for wells in Triassic sandstone. It also sometimes happens that, having regard to representations made during the Parliamentary proceedings on behalf of opposing statutory water undertakers, a limit is put on the quantity that may be pumped in any day.

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[14] Throughout this discussion the term, "undertaker", means, according to the context, a municipal authority or a public utility company empowered by statutory authority to construct and operate works for supplying water within a specified area.

Thus, it will be seen that, whereas private individuals have been unfettered in their exploitation of underground water resources, those charged with the duty of affording supplies to the community have, by precedent and practice, been restricted as it were outside the general law; and until 1936 there was no case of reciprocal protection being given to the public authority and nothing in law to prevent an industrial concern, with the knowledge that an authorized well had proved the presence of a large supply, from sinking its well immediately adjacent to the authorized well and developing it so as seriously and adversely to affect the yield of the authorized well.

Partly to meet the growing needs of its area of supply and partly because one of its authorized wells had had its yield reduced from 1 500 000 to 700 000 gal per day in the course of eight years, the Wolverhampton Corporation sought Parliamentary powers this year (1936) to sink additional wells at some distance from the Borough. The reduction in yield was attributed to the pumping of increasingly large quantities (now about 2 000 000 gal per day) from the same Triassic formation by an industrial concern the works of which are within the Borough and less than a mile from the well; and evidence was given to the effect that, in the course of a few more years, the yield of that well would become so small as not to be worth pumping by the Corporation.

Although the Bill, which included the usual protective clause with a radius of 2 miles, was opposed, it emerged with a reciprocal clause by which, after the date of the Royal Assent, no land-owner within the same radius could sink a well or otherwise abstract underground water, except by consent of the Corporation, except for the domestic supply of persons on his land, for the watering of stock, or for other purposes of an agricultural nature connected with the product of that land. It was not the intention of the Corporation to impose undue restrictions on the normal development of the lands within the 2-mile radius, and the clause was definitely directed to preventing the incursion of large industrial works. The clause provided that on refusal by the Corporation of consent to sink a well, the land-owner may require the Corporation to afford a supply at cost price to the Corporation, although there is no obligation if by so doing there is likely to be interference with the Corporation's ability to maintain either the supply for domestic purposes within its statutory limits of supply, or with existing trade supplies; nor if the aggregate of such supplies, together with what water was necessary to compensate owners of existing sources within the radius of protection, exceeds the yield of the authorized well.

Having regard to the extent to which underground resources were being exploited without control, water-works undertakers for many years have been pressing for general legislation to control the development of underground sources, and advocating a system of licensing by which regard would be had to the limits of potential yield of particular underground basins; and during the past two or three years the Institution of Water Engineers of Great Britain, acting jointly with other associations representing water-works interests, has made representations to the Minister of Health which were supported by him in a memorandum placed before a Joint Committee of both Houses of Parliament which sat at intervals during 1935 and 1936. This Committee reported on July 29, 1936, and, after referring to the anomalous position of water supply undertakers as compared with private individuals (as previously described), states that "it would be more equitable and more in the public interest if property owners were treated in the same way as water undertakers in any project where the former may wish either to sink wells below a certain depth or extract more than a certain quantity of water from them except for their own agricultural or domestic purposes." It remains to be seen how far general legislation follows this recommendation; it is sufficient to observe that the Wolverhampton Act is in advance of the general law as it now stands, and since, in the absence of general legislation, Parliamentary Committees on Private Bills are more often than not guided by precedent, this Act will serve as an argument in favor of similar provisions in future Bills.

So far as regional considerations affect underground-water resources and supplies, the problem is by no means as simple and as capable of reasonably clear-cut limitations as in the case of surface supplies. To appraise with any reasonable and approximate degree of accuracy the limits of a particular underground basin, its rate of replenishment, its storage capacity, and its potential yield, is a difficult and often involved undertaking. In most cases it is only after years of exploitation that it becomes apparent, on an investigation and consideration of ground-water levels, on the character of the water yielded, or on a combination of these factors, that water is being extracted in excess of replenishment. Often, particularly in areas adjacent to the sea, it is too late to remedy a partial or complete spoilation of an otherwise potable supply which has resulted from over-pumping in such circumstances.

What has been stated herein, indicates that there is now a tendency in England toward what the author describes as the "American rule" and also to what may be considered as a measure of State control. In so complicated a matter, the step-by-step approach, as the author observes, is probably the safest line and one which will allow of re-orientation in the light of experience.

O. J. Baldwin,[15] Jun. Am. Soc. C. E. (by letter).—The able manner in which this paper has been presented is a subject for congratulation. The value of administrative control of underground waters is well emphasized and a careful study of the paper would be of great value to all concerned with the utilization and conservation of this natural resource.

There are other aspects of the use of underground water, not mentioned by Mr. Conkling, which are of major importance in many States, particularly in the humid sections in the Mid-Western and Eastern United States where irrigation is not an important water use. In this region the demands of business and industry upon underground water for air-conditioning and industrial use have placed drafts of such magnitude on small areas that static levels are being, and have been, seriously lowered. The continued exploitation of underground waters will result in conditions which, undoubtedly, will produce a fifth doctrine of law, or a modification of the fourth doctrine discussed by the author.

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[15] Chf. Engr., The Iowa State Planning Board, Ames, Iowa.

Specifically, the point that must be decided by the Courts, or written into future legislation, concerns the proposed use of the water. Shall the municipal or domestic consumption of underground waters be considered a more basic or fundamental use than those of industry (manufacturing), refrigeration, and air-conditioning? If so, what shall be the priority of use, and shall a prior claim on the underground waters be subject to condemnation proceedings to meet the needs of a more fundamental use, as is the case with surface waters in some States? Inasmuch as air-conditioning and refrigeration may be obtained by other, although more expensive, means it would appear that the development of sub-surface waters for these purposes should be definitely controlled by legislative action.

Legislative control of air-conditioning and cooling usage should not be construed as a prohibition of such usage. Where the cooling effect is obtained by circulation through a closed system, the water may be returned to the source, through recharge wells, without changing the mineral or bacterial content. This method of maintaining the static level has already been used successfully in some of the Eastern metropolitan areas.

The use of a recharge well at a point removed from the supply well is more expensive to the individual user than wasting the water into sanitary sewers. On the other hand, the overloading of the sanitary sewerage system and the sewage disposal plant of a municipality with waste water from air-conditioning plants will greatly increase the cost of operation and maintenance. Already, some cities have found legislative action necessary to force the operators of air-conditioning plants to cease overloading sanitary sewers by connecting to the storm sewers. It is not unlikely that a continued development of sub-surface waters (provided the supply is not exhausted) may lead to an overloading of storm sewers also. Even if the storm sewers prove adequate to carry the flow, the wasteful use of artesian water is not halted. Viewed in the light of conserving the underground waters, any reasonable increase in expense, incurred in returning the waste water from closed-circuit air-conditioning plants to the ground, can be justified.

The use of a recharge well presents certain problems aside from those of expense. Foremost of these are the questions of minimum spacing between wells to prevent recirculating the returned water, and the temperature rise to the return of the heated water. The interval between recharge and supply wells must be determined by studies of the individual cases, giving due weight to the porosity of the aquifer, the movements of the artesian stream, the location of adjacent wells, and other factors peculiar to the site.

A rise of only a few degrees in the temperature can materially lower the efficiency of a refrigerating or air-conditioning system. To offset the temperature rise, it seems entirely possible, from the practical aspects, to utilize a closed system with a recharge well both as a cooling system in the summer and a heating plant in the winter. With an underground water temperature of 50 to 60° it would be possible to use the cooling system as a heating plant probably 20% of the year. A temperature differential of only a few degrees would permit a material saving in fuel costs in heating fresh air brought into the building, whereas warehouses might be completely heated to a satisfactory temperature.

Temperature differences of 75° would be quite common for short periods in the Northern United States. Under such conditions it is possible that sufficient "cold" could be stored in the sub-surface formations to eliminate the temperature rise of the following summer. It is quite likely that an actual economy could be effected, over a yearly operation of individual heating and air-conditioning plants, that would more than offset the added expense of a closed system. Regardless of the comparative cost, however, a less wasteful usage of sub-surface water resources will be one of the conservation problems of major importance in the near future.

The need of wisely drafted legislation, based on a thorough study of the water resources, is most urgent. The Engineering Profession is perhaps as responsible as any one group for the uncontrolled exploitation of this natural resource. It is now its responsibility to encourage the conservation of artesian waters and to lend its support to obtaining legislative appropriations for studies of the many problems relating to sub-surface waters. The tempo of the present development calls for an immediate study of extensive proportions in order that control measures may be adopted before depletion of the underground supply occurs. The writer is in accord with the conclusions that the control of underground water should be retained in the State. The complex situations existing in the various States make legislative control sufficiently difficult even within the State. Only in special situations where the interest of more than one State is involved, should there be any attempt at regulation by higher authority.

G. E. P. SMITH,[16] M. AM. SOC. C. E. (by letter).—This very valuable paper is more comprehensive than its title indicates. It is a rather thorough treatment of the entire subject of ground-water law, but with special emphasis on the control of ground-water by the State. If the following discussion appears to be critical, it does not follow that the writer is unappreciative of the high value of the author's timely analysis of an intricate subject.

A great handicap in the development of ground-water law is that legislators and advocates in the Courts usually have little conception of the hydrology of ground-waters. Fortunately, this has led to an avoidance of the issues in many States, and questions which might have been settled wrongly are still open. Progress is being made in the understanding of ground-water regimen and hydrologic records are being accumulated, but it may be long before ground-water law becomes fully crystallized. Although uniformity in the seventeen Western States might be desirable, it can scarcely be hoped for; even in the use of terminology, the various State Courts are not in agreement at present. Hydrologists should have an active part in the development of the law, both statutory and judicial, and administrative control should be in the hands of engineers whose training includes hydrology.

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[16] Prof. of Irrig. Eng., Univ. of Arizona, Tucson, Ariz.

The most notable pioneering advance in recent years is the legislation in New Mexico, enacted in 1931, placing underground streams and bodies of ground-water the boundaries of which "can be reasonably ascertained" under the effective control of the State Engineer. The excellent administrative control that has been established has solved the difficult problem of a seriously decreasing water supply and has even made a significant further advance in the use of public funds, in part, in shutting off the loss of water from abandoned and leaky wells.

The author has described and contrasted the various systems of law relating to surface waters and ground-waters. The riparian doctrine, as modified and developed by the Courts in California, is so different from the original riparian theory introduced into this country from the English law that it ought to be given a new name for the sake of clarity. Among its disadvantages in an arid region should be mentioned that, in theory at least, it may lead to the spreading out of the water supply, in dry years, into a thin irrigation over too great an area, so that on no field is there a profitable crop. Furthermore, since all ditches will be wetted, the proportion of the water lost in the canal system will be high; and, if a real attempt were made by a State administrative body to prorate the waters from day to day through a drought period, the task would be found well-nigh impossible. It is understood that the irrigation interests in California have had to tolerate the riparian doctrine, but have not relished it.

In his comparisons of laws relating to ground-water the author has tended to minimize the disadvantages of the American and California systems, which are, first, the almost certain over-development of the supply to the injury of all the water users; second, the insecurity as to the future of a developed supply and the insecurity attending the 5-yr probationary period; and, third, the likelihood of ultimate scattered small patches of irrigated land instead of more compact areas. Under the appropriative doctrine a water user at least knows where he stands, assuming that the determination of the existing rights has been made by the State's administrative officer.

The writer questions the statement that the rule of law promulgated in the Katz *v.* Walkinshaw case was adopted in other States. The one exception was the Utah Oil Refining Company case, in which the Court adopted the correlative rule, but permitted export; and more recently Utah, by statute, has placed all ground-waters under the full appropriative rule.

In an Arizona case, involving a small seepage of water, the query was as to just what constitutes a spring. The Court added considerable dicta, and indicated a leaning toward the correlative rule, but such tendency should not be considered as adoption. When the issue arises directly in a case, the American or the California rule may possibly be followed. In a case in the State of Washington, where a hillside well had been drained by reason of a new railway cut, the chief issue was damages, and the references to the correlative doctrine were of little moment. Judges often wander afield in the preparation of decisions.

The author points out clearly that the doctrine of correlative rights is incompatible with the doctrine of priority of appropriation. By reason of that incompatibility the correlative rights theory should be rejected in arid States other than California, at least with regard to "commercial" supplies of water.

The author's analysis of the terminology of rules of law is timely. The term, correlative, has been used in decisions with several different meanings. He might well have emphasized that a number of States have a dual system of law—the American system for "diffused percolating waters" and the appropriative system for underground streams flowing in definite channels. The division is not along the same lines as that between non-commercial and commercial supplies. New Mexico, Arizona, Nevada, Oregon, and Washington have the dual system. A recent case in point is that of Evans *v.* City of Seattle (47 Pac. (2d) 984).

Normally, the best pumping supplies are located in the Quaternary valley fill. If they are controlled under the appropriative system of law, it may be assumed that domestic supplies will be made available under the rule of preferential rights. If meager supplies are obtainable from Tertiary formations (at least in Arizona), it may be best to leave them under the rule of reasonable use, the American rule; rarely, if ever, will the effect of such use on the Quaternary supplies be of any considerable importance.

The writer agrees that the extension of the right of priority to the means of diversion, requiring that the water level at the well of the prior appropriator be not disturbed, is absurd. For the rule respecting means of diversion of surface water, the case of Salt Lake City *v.* Gardner (114 Pac. 147) is a leading one and was followed in the case of Pima Farms Co. *v.* Proctor, cited by the author. In the few cases where this rule has been upheld, it is not unlikely that sooner or later the Courts will reverse themselves, and will adopt the rule of reasonableness; even now in those jurisdictions the rule is ignored in practice.

The future success of administrative control over ground-waters will depend to a large extent on the right to reject applications when development has reached a safe limit. It will be simpler to prevent drilling of wells than to exercise constant control over surplus wells. A private communication from the State Engineer indicates that the attitude in Wyoming as to rejections is that "if an individual wishes to proceed * * * he is the one who must bear the burden of costs and should satisfy himself as to the chance of securing water for his appropriation in accordance with its date of priority." It is understood that, in California, despite frequent strong opposition, the Division of Water Rights refuses applications to appropriate when there is no unappropriated water, "getting away with it when they can."

A State administrative office has all the data on vested and initiated rights and on stream flow and storage, and is in a better position to determine when a water supply is fully appropriated than any other agency. Surely an "individual" in an isolated location with limited finances cannot be expected to go into the question thoroughly; nor can a promotional company do it. It does not promote the public welfare when great sums are invested in irrigation works and in clearing land, building homes, and planting orchards, subdividing land, and luring settlers, only to find later that the water supply is inadequate or undependable. The attitude in Wyoming seems to be to evade the responsibility; this is lamentable because engineers most favorable to State administration of water supplies consider Wyoming's record of administration as the ideal one.

The author states that there are no statutes on underground water in Arizona. On the contrary, the State Water Code includes water "in definite underground channels, whether perennial or intermittent", and also springs, among the waters that belong to the public and are subject to appropriation. This statutory provision is far-reaching, and has been recognized in Court decisions. It remains now to establish just what constitutes a definite underground channel, and much thought is being given that problem. In the Southwest Cotton Company case (4 Pac. 2d, 369), the Court in its decision suggested a test which hydrologists agree is impractical. When another case comes before the Court, or by statute, both definition and test should be settled definitely.

The author states, also, that in the case of Pima Farms Co. *v.* Proctor (245 Pac. 369), the Court ruled that the ground-water beneath the inner valley of the Santa Cruz River is in a definite under-ground channel. No technical data or competent evidence on that point was presented to the Court. The litigants stipulated that it was a definite channel, and the Court stated, "it is assumed or conceded throughout that this body of water is a known independent subterranean stream." How the Court would "rule" on this point if the evidence were presented, cannot be known. The status of ground-water law in Arizona is one of uncertainty.

In no Western State has the final word been said; ground-water law is still in the making. There will be refinements, especially in border-line cases, and there will be some reversals. In the further development of the law and of administrative control of ground-waters, it should be kept in mind that the interests of the public are an important factor. On some points, property rights may be modified or may give way entirely. The public is interested primarily in having the limited water supplies fully utilized. Furthermore, in the future, it may be emphasized that, in cases of conflict, the most important uses must prevail over those less vital to the general welfare.

DAVID G. THOMPSON,[17] ESQ. (by letter).—In this paper Mr. Conkling discusses a problem of great importance in planning the efficient development of one of the country's valuable natural resources, namely, the underground water or, as it is commonly called, the ground-water. In some States almost all public and domestic supplies are derived from ground-water, and large industries and thousands of acres of irrigated land are dependent upon this source of water supply. There is abundant evidence that, in many areas, the supply of ground-water is not sufficient to meet the needs or demands of all who desire to use it.

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[17] Senior Geologist, Div. of Ground-Water, U. S. Geological Survey, Washington, D. C. Published with permission of the Director, U. S. Geological Survey.

The paper is written largely from the viewpoint of problems in the Western States where much water is used for irrigation. It is the aim of this discussion to raise for consideration certain viewpoints arising from a study of ground-water conditions in many parts of the United States, but most particularly in certain of the humid States. The writer has delved assiduously into the laws of many of the States and of numerous Court decisions relating to ground-waters, but he must emphasize the fact that his viewpoint is that of one not versed in the principles of law. His approach is primarily from the viewpoint of a ground-water hydrologist whose duty has been to advise in the development of the country's ground-water resources.

Mr. Conkling has stated that, in his paper, he has not considered the phase of control of ground-water relating to waste. He is concerned primarily with the orderly development of the available supply to the greatest possible extent without exceeding the safe yield of the source. Meinzer[18] has pointed out that the problem of legal control of artesian water relates: (1) To the proper construction and operation of wells to prevent waste; and (2) to the equitable distribution of the water. It is the writer's belief that laws relating to control of the use of ground-water should not avoid consideration of the phase of prevention of waste. As a matter of fact, a definite requirement in the statutes of most of the States that have sought to establish a property right in either surface or ground-water, makes such a right dependent upon a "reasonable use" of the water. As a result of its quantitative studies of ground-waters in many parts of the country the Division of Ground Water of the U. S. Geological Survey now defines the safe yield of an aquifer somewhat as follows: "The safe yield is the rate at which water may be drawn perennially from the formation * * * without depletion of the supply or impairment of its quality to such an extent that withdrawal at this rate is no longer economically feasible." In other words, safe yield involves not merely the elements of quantity of water, time, and cost of pumping, but also the element of quality of the water; for example, the controlling factor in the maximum use of water from beds along a sea coast may be the rate at which water can be withdrawn without drawing in salt water. Elsewhere, the usefulness of fresh water-bearing beds may be impaired by improper well construction in oil fields, or in other areas where salt water is present, by disposal of waste in industrial areas, or by pollution when wells are used for drainage. Any law seeking to bring about a reasonable development of ground-water must recognize these dangers and give authority to control them just as much as it gives authority to limit quantity of withdrawal. However, this discussion is confined to the phase of equitable distribution or limitation of the use of ground-water.

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[18] "Progress in the Control of Artesian Water Supplies", by O. E. Meinzer, *Engineering News-Record*, Vol. 113, pp. 167-169, 1934.

Under "Extent of Underground Water Use" Mr. Conkling has stated that "in humid regions (that is, in regions of the United States east of the 98th Meridian) underground water is exploited to the point where far-reaching effects are produced only to secure supplies for metropolitan areas." However, in Table 1, the two States following California, in second and third rank (namely, Louisiana and Arkansas), and a large part of the irrigated area in Texas, are in the humid part of the country..

It is doubtless true that the problem of the control of ground-water has been most urgent in the arid Western States. There is evidence, however, that increasing attention must be given to the problem in other States. In a most interesting discussion of problems of legal control in England, H. J. F. Gourley[19], M. Am. Soc. C. E., shows some of the legal difficulties encountered in an area that certainly is as humid as most of the United States. Within recent years the need for some kind of legal control has been recognized by State geologists and other officials in several Eastern and Central States, and legislation has been drafted in some States. In the last two or three years special impetus has been given to the consideration of projects for irrigation by ground-water in drought-affected areas. There have been numerous developments of ground-water for municipal supply and for private industrial use. Particularly has there been a great increase in the use of ground-water for air-conditioning, and this use is certain to increase. In all too few instances in the actual and proposed developments has adequate consideration been given to the safe yield of the source aquifers, and control of development becomes increasingly desirable. A few instances may be cited to supplement, for the Eastern States, Mr. Conkling's excellent summary of the present status of control in the Western States, and to show the further need for, and problems of, control.

Stated briefly, the writer's thesis is as follows: Much of the classification of ground-waters adopted in many Court decisions and by writers of legal textbooks is not consistent with scientific principles of ground-water hydrology. Efforts frequently have been made to distinguish between "percolating water" and water moving in "known and defined subterranean channels or watercourses", where geologic and hydrologic conditions do not warrant such distinctions. The rule adopted in some Courts to apportion the ground-water among land owners in accordance with the doctrine of correlative rights will not provide a universally satisfactory solution because in many localities the supply of ground-water available is inadequate to fulfill the reasonable needs of all land owners and, especially in industrial areas, there may be no relation between the actual reasonable needs of different individual land owners and the area of lands owned by them, respectively. If this doctrine were interpreted in such a way as to limit each user to no more than his *pro rata* share of the ground-water on a basis of area of land owned, in some localities such share would be so small that no ground-water development would be practicable; and this great economic resource would largely go unused. Therefore, if the ground-water resources are to be utilized most beneficially it is believed to be essential that a doctrine of appropriation and beneficial use sufficiently flexible to be adaptable to the conditions of each locality is the only practicable solution. It is realized that adoption of this doctrine in some States will encounter divergences of opinion and conflicts of interests among various groups which must be most carefully and sympathetically reconciled, but it is not believed that any insurmountable obstacles exist.

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[19] "English Law Does not Control Ground-Water Draft", by H. J. F. Gourley, *Water Works Engineering*, Vol. 89, pp. 1294–1297, September 30, 1936.

Attempts at, or Lack of, Control in Eastern States.—In the Grand Prairie region of Arkansas for about 20 yr from 100 000 to 160 000 acres of rice land has been irrigated with water from wells[20]. Since 1927, the water level in observation wells has dropped to lower and lower levels in successive years and the evidence seems indisputable that the safe yield of the beds has been exceeded. The area of land that is suitable for irrigation is much larger than that actually irrigated in any season, and conceivably at some future time, with conditions of price and other factors favorable to the rice farmer, there may be a great increase in the use of ground-water for irrigation in the Grand Prairie region.

Arkansas has no law providing for the control of ground-water, and so far as the writer knows, there have been no Court decisions upon the subject within the State. The available scientific facts disclose that limitation of the use of ground-water in the Grand Prairie region is desirable if not necessary to protect an important agricultural development. With conditions as they now exist in the region, undoubtedly it would be necessary to recognize the rights for water already developed, and if these rights exceed the safe yield of the aquifers, to require a proportionate decrease in use by present water users to bring the total consumption within the safe yield.

Within a radius of 25 miles of Houston, Tex., at least 80 000 000 gal of water per day is used for public supply and industrial purposes and irrigation. It is believed that the pumpage has not yet exceeded the safe yield of the aquifers, but there may be larger future development of ground-water, which (at least as far as mechanical problems are concerned) can be obtained from properly constructed wells. In this region, as in other areas in Texas, to protect the large investments already made, regulation of the use of ground-water is desirable.

There has been great lowering of artesian head in a large area around Chicago, Ill.[21]. Less severe drops have been reported from other parts of Illinois. It would seem that some control in this area would also be desirable.

Ground-water is used for irrigation in several areas of the Florida Peninsula and in certain areas some wells already yield salt water. There is reason to believe that any considerable increase in draft will cause further contamination. In an attempt to give some protection to the Gulf Coast area, the Florida Legislature in 1929 passed a law relating solely to Sanford, Manatee, and Charlotte Counties[22]. An effort has been made, but is as yet (1936) unsuccessful, to pass a law applying to the entire State. The present law defines waste from wells, sets up provisions for the prevention of waste, and violation of these provisions is a misdemeanor subject to the penalties prescribed by law. It also makes provision for inspection and supervision by the State Geologist of drilling operations in order to prevent leakage of water, known in some places to be salty, from one aquifer to another. The law further states that,

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[20] "Ground-Water Supplies for Irrigation in the Grand Prairie Region, Arkansas", U. S. Dept. of the Interior (Geological Survey) Press Memorandum, January 26, 1931, 21 pp. (mimeographed); also, observations continued since this report was issued.

[21] "The Artesian Waters of Northeastern Illinois", by Carl B. Anderson, *Bulletin 34*, Ill. State Geological Survey, pp. 93–95, Fig. 3, and Pl. IV, 1919.

[22] The text of the law is printed in "Ground-Water Resources of Sarasota County, Fla.," by V. T. Stringfield, Twenty-third and Twenty-fourth Annual Repts., Florida State Geological Survey, pp. 179–181, 1933. (This paper is also printed separately.)

> "Should the underground-water supply of any district become so depleted by heavy draft as to endanger the water supply of such district by saline contamination, the State Geologist, his assistants, or duly authorized representative, shall be empowered to regulate the draft upon all wells in such district so that the water resources can be protected and preserved."

This provision, however, relates only to the prevention of salt-water contamination. There is no provision of funds to enforce the law, nor for bond or other insurance that a well owner or driller will properly plug or otherwise control a leaking or flowing salt-water well. The prevention of leakage from one formation to another or of flow around the outside of wells has been difficult and costly, and laws should definitely require a bond by the driller or owner to assure proper protection of other well owners in areas where these difficulties are likely to be encountered.

In New Jersey there has been a partial control of ground-water by the State since 1907. This has been accomplished by laws which provide that new or additional developments of water, either surface or underground, for public supply, by municipalities or privately owned water supply companies, can not be made until the supervising commission (at present (1936), the State Water Policy Commission) has approved the plans for such development.

In the State of New York there is a control of public water supply systems exercised by the State Water Power and Control Commission, very similar to that in New Jersey. The power seems to be limited to an approval of plans and not to the granting of water rights. In 1933, the New York Legislature amended the law to provide that,

> "* * * no person, firm, or corporation shall hereafter install or operate any new or additional wells in the counties of Kings, Queens, Nassau and Suffolk to withdraw water from underground sources for manufacturing or industrial purposes or for use in manufacture or industry where the capacity of such well or wells singly or in the aggregate is in excess of one hundred thousand gallons a day without first obtaining the approval of the Commission. * * * The provisions of this act shall not apply to the use of water for agricultural purposes."

The four counties mentioned constitute all of Long Island. The first section of the law, after referring to reports by State and Federal agencies in regard to depletion of the underground waters of Long Island and the danger of encroachment of salt water, states that "this enactment is made in the exercise of the police power of the State and its purposes generally are to protect public health and public welfare in conserving the supply of water for domestic consumption."

Parenthetically, it may be noted, in contrast, that an Hawaiian Court expressed the opinion that the police powers of the Territory "cannot be deemed to justify, under the showing made in this case, the prohibition of the appellant's proposed well while at the same time permitting all existing wells to continue to be operated without diminution"; but it did recognize that the Territory could exercise the police power to the extent of regulating the exercise of "private rights to the end that the health and safety of the community may be preserved and that the right of co-owners may not be violated." (City Mill Co., Ltd. *v.* Honolulu Sewer and Water Commission, 30 Haw. 912.)

In both New Jersey and New York favorable action by the controlling commission apparently does not constitute a grant of any right to use the water, but merely an approval of plans, etc. Nevertheless, the laws in these States have had the effect of controlling to some extent the development of ground-water supplies in certain areas where there is danger of the safe yield being exceeded, either by actual refusal of approval, or by modifying provisions as to location of wells or quantity to be withdrawn.

In both New Jersey and New York, except Long Island, the law does not apply to diversions of water by industries or others using the water for purposes other than public supply systems. This distinction has brought about a peculiar situation in that upon showing that a proposed development for municipal supply may injure an existing private supply the controlling commission may refuse, and (at least in New Jersey) has refused, to approve the public development; but, on the other hand, after a municipal development has been completed, a private development may be made which may greatly reduce the capacity of the wells of the public system or perhaps even destroy its usefulness. A similar situation in England has been cited by Mr. Gourley[19].

Confusion in Classification of Ground-Waters.—There is considerable inconsistency or confusion in thinking concerning certain phases of the situation of control, or lack of control, of the use of ground-water as it now exists. These conditions are largely the cause of the present mixed, and to large extent unsatisfactory, status of legal control. This situation doubtless is due in part to the fact that the legal principles were established before there had been developed an adequate knowledge of the geologic and hydrologic conditions that govern the occurrence of ground-water; but even after these conditions have become better known, the hydrologists have, perhaps, not presented them clearly to the Courts and legislative bodies. It may be said that some progress is being made, but there is still room for more enlightment.

An outstanding confusion of ideas exists in the distinctions or classes into which underground waters have been divided from a legal viewpoint. In many cases the question arises as to whether the water under consideration is ground-water or part of a surface stream. From the viewpoint of many hydrologists this is a starting point of ambiguity and uncertainty. In his discussion of ground-water law, Mr. C. S. Kinney divides underground waters into three general classes, namely, "subterranean water courses or streams", "artesian waters", and "percolating waters."[20] The first class is subdivided into water flowing in "known and defined channels or water courses", or in "undefined and unknown" courses. Artesian waters apparently were originally placed in the class of "percolating waters", but as their characteristics became better known a separate class was made for them. Percolating waters have been subdivided into: Diffused percolations or percolating waters; percolating waters tributary to surface watercourses or other bodies of surface waters; percolating waters tributary to underground reservoirs or other bodies of underground waters; and, finally, seepage waters. Other subdivisions include "underflow dependent on surface streams", "percolating waters tributary to springs", and "percolating waters supplying surface wells" (as distinguished from waters supplying artesian wells).

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[20] "A Treatise on the Law of Irrigation and Water Rights and the Arid Region Doctrine of Appropriation of Water Rights", by C. S. Kinney, Bender-Moss Co., San Francisco, Calif., Second Edition, p. 2095, 1912.

There is no doubt that, at the mention of the term, "known and defined subterranean channels or water courses", most geologists visualize streams flowing in channels with definitely identifiable banks, such as are found principally in solutional channels in limestone caverns (for example, the Echo River in the Mammoth Cave of Kentucky); or of so-called tunnels in lava flows, or in smaller openings with definite walls such as crevices, rather than minute interstices in sand and gravel, even if the water-bearing beds are beneath a definitely recognizable surface stream. On the other hand, the geologist would consider the water flowing in alluvium beneath a stream bed to be percolation just as much as water moving in an artesian aquifer, or ground-water tributary to surface streams or underground reservoirs is percolating water. (The Standard Dictionary defines "percolate" as to cause, as a liquid, to pass through fine interstices; filter; strain. It then defines "percolation" as the act of percolating or passing through small interstices; infiltration. Furthermore, the Standard Dictionary defines "channel" as: (1) The bed of a long body of water, especially the hollowed course of a stream; (2) the deep part of a river, harbor, straight, or estuary where the current or tide is strongest; etc. It defines "watercourse" as: (*a*) A stream of water; river; brook, especially, in law, a stream usually flowing (although not necessarily running all the time) in a definite channel, having a bed and banks; (*b*) the course or channel of a stream of water. The definition in Webster's Unabridged Dictionary is in essentially similar language.)

Further difficulties arise when one attempts to discover the distinction, for example, between underflow, percolation, and artesian water. In many localities, where a stream flows over a considerable thickness of permeable alluvium confined in a rock-walled canyon, most, if not all, Court decisions recognize underflow as part of the surface stream. The question becomes more complicated where the stream flows across a wide alluvial plain of essentially homogeneous material, so that the stream is continuous with the water-table beneath the plain for many miles on either side of it, the stream being at about the lowest part of the water-table. Perhaps there would be no question as to the portion of the ground-water body directly beneath the stream, provided the course of the stream is fairly straight; but what would be the determination where the stream, instead of flowing in a reasonably straight line, meanders in large ox-bow curves several miles in diameter across a wide plain as does the Mississippi River, and other rivers in wide alluvial filled valleys? In this case the movement of water in the lowest part—the thalweg[24] of the water-table—may be expected to be in a rather straight line toward the mouth of the stream and not parallel to and beneath the meanders of the stream. Thus, although certain drops of water may at one place be directly beneath the stream bed and, therefore, might be called underflow, upon reaching and passing down stream beyond a sharp sideward meander of the stream, they apparently cease to be "underflow" in the sense of water flowing beneath a stream bed, and become "percolating water" which, in its journey to the sea, may reach a point several miles from the river before again passing under it.

In both the foregoing cases there is a question as to where boundaries shall be drawn to divide the underflow from the "percolating" water beneath the plains on either side of the river. In many localities this is hydrologically impossible because, at least in part, percolation laterally from the plains to the river maintains the underflow and at different seasonal periods or in different sections of the river system, this direction of movement may be reversed. In some localities it can be demonstrated both by studies of the geological history of the regions in question and by hydrological data, that the underflow is derived largely, if not wholly, from such lateral percolation, and not from the surface stream as is stated by Mr Kinney to be the legal presumption both under the common law doctrine and under the doctrine of appropriation[25]. In fact, the usual geological and hydrological conception of the relations between a permanent surface stream and the ground-water is that the stream comes into existence only after erosion has cut a channel deep enough to intersect the top of the zone of saturation or water-table. This view is in agreement with Mr. Conkling (see "Introduction") who uses the title of "underflow which may support a surface stream." The theory stated by Mr. Kinney that the stream supplies the underflow can be demonstrated completely only where the stream is influent; that is, where the water-table is some distance below the bottom of the stream and water percolates from it downward to the ground-water body. In this case it might be well argued that the person who diverted the surface water was taking water that would otherwise naturally supply the ground-water reservoir of percolating water.

Mr. Conkling has described certain conditions under the headings of "Definite Underground Channels" and "Percolation Through Basins." Presumably, these conditions comply with those described by Kinney as waters flowing in subterranean streams or watercourses and percolating waters, respectively. It will be noted that Mr. Conkling states ("Definite Underground Channels"): "If the underflow of a stream * * * is regarded as a definite underground stream, it may be said that such streams, usable for irrigation, often occur in the West" This suggests that there is some doubt in his mind as to what really constitutes a definite underground stream. Under the heading of "Percolation Through Basins", he describes the general hydrological features of the San Gabriel Basin. The writer does not know whether any Court decision has classified the movement of ground-water in the San Gabriel Basin as "percolation", but he believes that it is a proper classification according to the customary conception of that term as it is commonly used except perhaps for minor parts of the basin where certain geological conditions produce special conditions such as small artesian basins. This general classification is supported by maps of the water-table which show it to be continuous, with gradual slopes over a large area, and with no demarcation or indication of difference in water-table conditions beneath the wide alluvial wash channel of San Gabriel River and lands that extend for several miles on either side of the present channel[26].

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[24] "The Motions of Underground Waters", by C. S. Slichter, *Water Supply Paper 67*, U. S. Geological Survey, p. 32, 1902.

[25] "A Treatise on the Law of Irrigation and Water Rights and the Arid Region Doctrine of Appropriation of Water Rights", by C. S. Kinney, Bender-Moss Co., San Francisco, Calif., Second Edition, pp. 2111–2115, 1912.

San Gabriel Valley may be compared with San Fernando Valley. As far as the writer can determine from the statement of facts in the case of City of Los Angeles *v.* Hunter (156 Cal. 603), and from other information, the geologic and hydrologic conditions in the latter valley are essentially the same as in San Gabriel Valley in that the ground-water occurs in a vast body of alluvium surrounded by bed-rock hills, with a water-table continuous over practically the entire valley. Contours[27] of the water-table show that the ground-water for practically the entire valley is moving toward the locality where the Los Angeles River, which rises in the valley, passes through "narrows" with conditions essentially similar to the pass traversed by San Gabriel River between San Gabriel Valley and the Coastal Plain as described by Mr. Conkling; and yet, in the case of City of Los Angeles *v.* Hunter, the Court decided that water pumped from wells (some of which were 1 000 ft distant, but others 2 or 3 miles distant, from the Los Angeles River) was a part of the subterranean flow of that river and was not percolating water.

In another case relating to San Fernando Valley (City of Los Angeles *v.* Pomeroy (124 Cal. 597)) the difficulty in distinguishing between water in subterranean channels and percolating water is just as evident and the reasoning of the Court is confusing. In this case the decision of the Supreme Court quotes the instructions to the jury by the Superior Court. In Paragraph XVII of these instructions the Court stated certain physical conditions which, if they were considered to fit the case, should require the jury to decide the waters of certain tributary streams of the valley to be a part of Los Angeles River. In Paragraph XIX were other statements to show under what conditions the ground-water would not be in a definite watercourse, but would be percolating water. Both statements seem to fit the conditions in San Fernando Valley in so far as they go, and reading either of them alone one would conclude that the ground-water should properly be classified by that statement. In neither this case nor in Los Angeles *v.* Hunter (156 Cal. 603), does the writer find any discussion of the critical geological or hydrological factors that really would determine whether the subterranean channel or watercourse flowed in a definite, confined channel, with "beds, banks, or sides."[28] Such pertinent evidence should show either definite differences in the character of the water-bearing materials constituting the supposed subterranean channel of the river from the materials on either side that would be considered to constitute the banks; or it should show that the water-table or piezometric surface of the water in the materials constituting the banks and sides of the watercourse was discontinuous with the level of the surface water in the stream or in the water-table beneath it.

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[26] "Ground Waters and Irrigation Enterprises in the Foothill Belt, Southern California", by W. C. Mendenhall, *Water Supply Paper 219*, U. S. Geological Survey, Pl. VII and IX, 1908; also, "South Coastal Basin Investigation—Geology and Ground-Water Storage Capacity of Valley Fill", by Rollin Eckis, *Bulletin 45*, California Div. of Water Resources, Plates D and E, 1934.

[27] *Loc cit.*, Rollin Eckis, Plate E.

It would seem that, from the decision of Los Angeles *v.* Hunter and Los Angeles *v.* Pomeroy, by analogy, logically all the water in the vast underground reservoir of San Gabriel Valley, which eventually passes through the Whittier Narrows either as surface flow of San Gabriel River or underflow beneath it in a restricted channel, should be considered to be part of that stream. Mr. Conkling has pointed out ("Percolation Through Basins") how, with great increase in pumping from wells the quantity of "rising water" at the Narrows (that is presumably, the low-water flow of San Gabriel River) has greatly decreased; and this very fact suggests a relation between the body of "percolating water" and the river similar to that determined by the Courts for Los Angeles River in San Fernando Valley.

Since a large part of the firm flow of practically all streams of importance in the United States comes from the ground-water reservoir, from the foregoing reasoning it can also be argued that ground-water almost anywhere is a part of the nearest surface stream toward which the water-table slopes. The obvious consequence of this argument would be that riparian owners of surface waters might compel abandonment of many wells now diverting such percolating water. For most parts of the country this interpretation may seem absurd, but for certain regions, conceivably, it may have application, as, for example, along certain stretches of the Mohave River, in California, the Rio Grande in New Mexico and Texas, and the Platte River in Nebraska. On stretches along each of these rivers there is undoubtedly contribution of ground-water to the stream from lands some miles back from it.

In its length of more than 100 miles the Mohave River, in California, is alternately, here an effluent and there an influent stream, with stretches in which most of the time there is a surface stream and farther down stream no surface flow. Probably most of the flow comes from the San Bernardino Mountains, but undoubtedly some of the water moves ("percolates") toward the river beneath the bordering "mesas" or alluvial slopes that reach back to the mountains of the desert. In some, if not in all, of the stretches, both influent and effluent, the underflow might be considered to be part of the river. There is reliable evidence that, in a stretch between Hodge and Barstow, Calif., some of the underflow of the river—which is probably augmented by percolation from the surface flow when there is water in the river—leaves the valley of the Mohave River underground and moves northwestward beneath an area known as Hinkley Valley, past a surface divide and toward an undrained valley containing a "playa" known as Harper Dry Lake[29]. Apparently, at least in years of normal run-off, none of this ground-water that leaves the valley of the Mohave River, appears on the surface again, but is disposed of by evaporation from the soil and transpiration by plants. The question may now be asked: When it reaches Harper Valley, is this water still to be considered as part of the Mohave River although it is entirely outside the drainage basin of that river; although subsequently it does not reach any other stream; and, although it moves in a manner entirely similar to "percolating water" in numerous basins of the desert region? The answer might be different, depending upon whether a down-stream user along the Mohave River sought to enjoin a user in Harper Valley, or a Harper Valley user sought to enjoin an up-stream user along the Mohave River.

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[28] City of Los Angeles *v.* Pomeroy 124, Cal. 597, Paragraph XIX of charge to jury by Superior Court, quoted in the decision of the Supreme Court.

One more example may be cited to show the difficulties of attempting to set up definite distinctions between different types of occurrence. According to co-operative studies by the Nebraska State Geological Survey and the U. S. Geological Survey, between Chapman and Gothenburg, Nebr., about 30 000 acre-ft of water is pumped from wells on the flood-plain of the Platte River[30]. The writer does not know whether any Court decisions have defined this water in Nebraska, but under similar conditions in Colorado, it would certainly seem to be considered a component part of the Platte River The field studies have demonstrated beyond a doubt that a large quantity of water, estimated to be about 56 000 acre-ft per yr, percolates southward out of the Platte River Valley and beneath a high upland from a few to as many as 15 miles wide and re-appears in the tributaries of the Republican and Blue Rivers which flow at considerably lower altitudes[31]. The studies indicate that more water for irrigation can be taken from wells in the Platte River Valley as well as on the upland plains. If such development occurs to the extent that the flow of the Republican and Blue Rivers is seriously diminished, shall the ground-water beneath the upland (which normally would be considered to be "percolating water") and also the water used in the Platte River Valley, be considered to be part of the Republican and Blue Rivers? A case recently decided (Osterman *et al v.* Central Nebraska Public Power and Irrigation District, 268 N. W. 334), relates to the diversion of surface water from the Platte River into the drainage basins of the Blue and Republican Rivers. In this case the probability of underflow from the Platte River to the basins of the other rivers was recognized, but the ground-water conditions apparently were not specifically considered in the recent decision.

The foregoing examples have been cited primarily to point out inconsistencies in the reasoning by which certain principles of classification of ground-water have been set up in the course of a long period of litigation.

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[29] "The Mohave Desert Region, California", by David G. Thompson. *Water Supply Paper* 578, U. S. Geological Survey, pp. 428–429, 1929; also "Mohave River Investigation", by Harold Conkling, M. Am. Soc. C. E., *Bulletin* 47, California Div. of Water Resources, pp. 27–28, and contours of water-table on Pl. 5-B, 1934.

[30] "Ground-Water Resources of South-Central Nebraska with Special Reference to the Platte River Valley between Chapman and Gothenburg", by A. L. Lugn and L. K. Wenzel, U. S. Geological Survey. (Report released in manuscript form and waiting publication, mimeographed preliminary Interior Dept. Notice 106376, September 16, 1935.)

[31] For a cross-section showing this condition see "Water-Bearing Formations of Nebraska", by G. E. Condra and E. C. Reed, Nebraska Geological Survey *Paper No. 10*, p. 17, 1936.

Apparently, the complicated classification given by Kinney has been found necessary as case after case came up for decision in which the geological and hydrological conditions did not exactly fit the conditions of some previous case. He seems to have discovered the essential difficulty of the entire problem when he wrote, concerning the criteria for determining the classification of ground-waters[32]: "The difficulty has been not so much to determine the law as to rights which may be acquired in subterranean waters, where certain water has been determined to be of a certain character, as it has been to determine the character of the particular class to which the water belongs." This difficulty may be expected to become greater as still other cases, with peculiar but perfectly natural conditions, as in some of the areas suggested in the paper, come up for decision. The very existence of this difficulty raises a question as to whether it may be due to the fact that the classification that has developed in past cases, as outlined by Kinney, is not on a sound basis of geological and hydrological facts. All water beneath the surface of the ground, after all, is purely and simply ground-water, moving according to certain well recognized laws of physics. There seems to be no scientific reason why an elaborate and expanding classification of ground-waters should be necessary

One may wonder as to how the aforementioned confused state of affairs came about. It can be attributed to lack of knowledge of the geologic and hydrologic principles that govern the flow of water beneath the surface of the earth. It is not surprising, of course, that many facts now well known in regard to the occurrence of ground-water were not so widely understood when the early case relating to percolating water cited by Wiel[33] (Acton *v.* Blundell (12 Mees and W. 324)) was decided in 1843, although they were not wholly unknown even then. Some essential facts were known to a few scientific observers[34], but undoubtedly these ideas had not been very widely circulated. Unfortunately, even to-day not a few persons believe that subterranean water actually occurs in more or less distinct and open bodies with free surface similar to lakes and rivers that are seen above ground. The conception of "definite subterranean channels or watercourses" probably at least in part arose from this erroneous belief.

So far as percolating water is concerned the theory set up in early cases likewise rests on lack of knowledge of ground-water conditions. In Los Angeles *v.* Hunter (156 Cal. 603), the Court stated that percolating waters, in the common law sense of the term, were "vagrant, wandering drops moving by gravity in any and every direction along the line of least resistance." This judgment was in contrast to the Court's conception of water moving in a definite subterranean channel or watercourse of which it said,

> "These waters percolate, it is true, but only in the sense that they form a vast mass of water confined in a basin filled with detritus, always slowly moving downward to the outlet, in the effort, in conformity with physical law, to attain a uniform level."

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[32] "A Treatise on the Law of Irrigation and Water Rights and the Arid Region Doctrine of Appropriation of Water Rights", by C. S. Kinney, Second Edition, p. 2115, Bender-Moss Co., San Francisco, 1912.

[33] "Water Rights in the Western States", by S. C. Wiel, Second Edition, Vol. 2, p. 970, 1911.

[34] "History and Development of Ground-Water Hydrology", by O. E. Meinzer, *Journal*, Washington Acad. of Science, Vol. 24, pp. 14–16, 1934.

As a matter of fact, with the present knowledge of ground-water hydrology, it may be stated with considerable emphasis that practically all ground-water conforms with the requisite conditions set up in the last quotation because, until diverted by artificial means, the ground-water flow, in conformity with physical laws, at any given point is moving in a definite direction as a result of the interaction of geological and hydrological conditions, and, except for loss by transpiration and evaporation, nearly all ground-water is moving to maintain the flow of surface streams Furthermore, the direction and rate of movement can be determined within reasonable limitations of accuracy, although this may not everywhere be practical because of the cost involved.

If the foregoing arguments are tenable, namely, that from a logical continuation of the reasoning stated by the Court in such cases as Los Angeles *v.* Hunter and Los Angeles *v.* Pomeroy, it would seem to be a further logical conclusion that since nearly all ground-waters tend to maintain surface streams, they should be considered waters in subterranean channels, and thus practically all ground-waters should be subject to the laws that govern surface waters Under this reasoning, in States in which the doctrine of riparian rights now applies to surface water, this doctrine would logically be applied to ground-waters. In these States under a strict interpretation of the doctrine of riparian rights it would seem that the use of ground-water would be essentially prohibited unless all well owners were considered to have riparian rights equal to the riparian owners along the surface streams. At best, it would mean the application of the doctrine of correlative rights (see heading "Water Law in General"). On the other hand, in States in which the doctrine of appropriation is now applied to surface waters that doctrine would also be applied to all ground-waters, with consideration given to the question of the effect of their diversion on surface water rights already granted.

It may be stated that the application of the doctrine of appropriation to the control of ground-waters in certain areas in New Mexico is based on the scientific evidence of the movement of the water from its source to points of discharge in areas, which, although they may be of large extent, have boundaries that can be, and have been, definitely ascertained. To this extent there is recognized a similarity to "definite subterranean channels or watercourses." Thus, by recognizing definite boundaries and a definite direction of flow the doctrine of appropriation, which has been applied for many years to surface waters in New Mexico, has now been applied to ground-waters in areas in which, as pointed out by Mr. Conkling ("Underground Water Law in General"), the water occurs under conditions that formerly would have been considered to require a classification as "percolating water."

It is pertinent, perhaps, to state that the New Mexico law was first drawn up following an extensive study by the U. S. Geological Survey, of ground-water conditions in the Roswell Basin, to which the law was first applied[85]. This study showed that the safe yield of the artesian reservoir had been exceeded and that any increase in the use of ground-water would jeopardize the investments of prior users; and it recommended legislation to prohibit new developments of artesian water except as it can be shown that such developments will not injure present users of water. The information collected in the investigation furnished the necessary basis for the determination and declaration by the State Engineer that the Roswell Basin comes within the purview of the new law in so far as the water is in an "artesian basin * * * having reasonably ascertainable boundaries." It is worthy of note that the law has indeed accomplished its objective in the Roswell Basin, and this is perhaps the outstanding instance of limitation of use of ground-water by law in the United States.

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[85] "Report on Investigations of the Roswell Artesian Basin, Chaves and Eddy Counties, New Mexico", by A. G. Fiedler, Seventh Biennial Rept. of the State Engr. of New Mexico, pp. 21-60, 1926; also, "Geology and Ground-Water Resources of the Roswell Artesian Basin, New Mexico", by A. G. Fiedler and S. S. Nye, U. S. Geological Survey *Water Supply Paper 639*, 1933.

Doctrines of Correlative Rights and Appropriation.—The writer wishes to make himself clear that in what he has written thus far he is not suggesting what he believes should be a proper basis for the determination of water rights. He has merely endeavored to present what he believes must be logical conclusions if certain assumptions in legal decisions are accepted. At this point it is desirable to consider another phase of the problem, namely, the apportionment of water among land owners under the doctrine of correlative rights.

It is first necessary to consider just what is meant by the doctrine of correlative rights. Mr. Conkling states ("Underground Water Law in General") presumably referring to the doctrine of correlative rights, but not using the term specifically in the sentence quoted: "This rule holds that each land owner overlying a basin has a right to the underground water co-equal and correlative with that of all other land owners overlying the same basin." This interpretation is considered by some to require, in part, that if there is not sufficient water for unlimited use by all owners of land overlying water-bearing strata, there shall be a proportionate distribution among all. This view is stated, apparently as dictum of the Court—that is, mere discussion upon some collateral question of law not necessary to the decision of the case before the Court—and not as a part of the actual decision, in several cases, notably Katz *v.* Walkinshaw (141 Cal. 116 (especially p. 136)); and City Mill Co., Limited *v.* Honolulu Sewer and Water Commission *et al* (30 Haw. 912 (especially p. 922)).

The writer is aware that this view is not shared by others. Some appear to consider the doctrine of "correlative rights" as applied to ground-waters to be synonymous with the doctrine of "reasonable use." (See such expressions as the following in which the two phrases are used together, quoted from the annotation appended, Clinchfield Coal Corporation *v.* Compton (55 A. L. R. 1376): "a reasonable exercise of such right; or as said by the Court, the rights are correlative"; "* * * in Katz *v.* Walkinshaw, 1902 (141 Cal. 116, 64 L. R. A. 236, 99 Am. St. Rep. 35, 74 Poc. 766, 80 Poc. 633), the first case in which the rule of reasonable use applied to correlative rights is applied"; "the American rule, or rule of reasonable use, or as expressed by some Courts and writers, the rule of 'correlative rights'"; and, finally, in Horne *v.* Utah Oil Ref. Co. (59 Utah, 279, 31 A. L. R. 883, 202 Poc. 815), "the doctrine of correlative rights or reasonable use." On the other hand, in at least one case the Court recognized a distinction between, and defined, these two doctrines, separately. (See heading "Underground Water Law in General"; also, City Mill Co., Limited *v.* Honolulu Sewer and Water Commission *et al* (30 Haw. 912 (especially p. 922)). In this case a definite part of the doctrine of reasonable use was stated to be that the water shall be used on "the owner's land" (in cases in other jurisdictions limited to lands of the owner overlying the water-bearing strata). As a matter of fact, in many, if not all, cases where the doctrines of reasonable use or correlative rights (whatever the application of this term may be) have been considered, water was taken for sale by public supply systems or for other use on lands distant from those beneath which the water was found; and this has generally been held to be an unreasonable use (Clinchfield Coal Corp. *v.* Compton (55 A. L. R.), annotation p. 1404). The writer has failed to find a single case in which there was actually decided by the Court the question of division of water among owners of land overlying the water-bearing strata when the supply was insufficient for all desired use, without the collateral question of use on distant land. Therefore, it is probable that the meaning of the term, "correlative rights", as applied to such a condition of insufficient supply, is still an open question except as Courts subsequently may be influenced by the dicta in the aforementioned cases.

Next, there is to be considered the doctrine of correlative rights from the viewpoint of ground-water hydrology. Of the total quantity of ground-water that flows beneath the average parcel of land, only part of it originates on that land and much, if not most of it, originates on lands farther up the ground-water gradient. In the case of an artesian aquifer covered by an impervious bed beneath the entire parcel no water would be contributed from rainfall on that parcel, and it would all move in from lands beyond its boundaries, perhaps from an outcrop many miles away. In fact, an artesian aquifer, perhaps, is one of the best examples of a condition that, by stretching considerably the geological concept, might fit the description of a subterranean watercourse with definitely bounded walls.

The supply of ground-water available in any area is dependent upon the rainfall either in the immediate area, or in some other area, perhaps fairly remote, from which it moves to the area in question either in a surface stream, which loses water to the underground reservoir, or through some underground aquifer. In many localities, particularly in the arid and semi-arid States, ground-water is available in large quantity only where geologic conditions are such as to concentrate part of the precipitation or surface run-off from large areas, or areas of heavy precipitation, into ground-water reservoirs of comparatively small size, or where large reservoirs have been filled by infiltration from small areas continuing through long geologic periods. Under certain conditions the available supply is also limited by the permeability of the water aquifer; that is, the rate at which it can transmit water. As a consequence of these factors, there is a definite limit to the safe yield of water that can be taken perennially from wells in any locality. This limit has been reached and passed in several parts of the United States, although the supply may not yet be exhausted. In many other parts of the country this limit is far from being reached and, at present, gives no concern. In some areas it probably never will be reached. However, it seems necessary that the possibility of it being reached must be recognized in any effort to provide adequate control of ground-waters. This is particularly so where the possible use of ground-water might greatly exceed the safe yield of the water-bearing formations.

In Antelope Valley, California, for example, 200 000 acres or more is suitable for irrigation, but the annual contribution to the ground-water supply is of a magnitude of only about 50 000 acre-ft[36]. If this quantity were distributed over 200 00 acres it would be only 0.25 acre-ft per acre, which is not enough to do material good in irrigation. Obviously, if the water were divided among all lands proportionately under the doctrine of correlative rights, as suggested by the Court in Katz *v.* Walkinshaw, much of the land would be nearly worthless. Prior beneficial users would be insecure. Only by a reasonable application of the doctrine of appropriation can the maximum economic use of water be obtained in Antelope Valley.

In the Borough of Brooklyn, New York, a number of industrial concerns pump large quantities of water from wells on their property, and as a result there has been a serious drop in the water-table. However, if all industries of the Borough undertook to pump from wells, claiming a share of water under the doctrine of correlative rights as interpreted in Katz *v.* Walkinshaw, either salt water would quickly be drawn in, spoiling the water for many purposes, or, if the water was rationed in such a way as not to exceed the total safe yield, each owner's share would be so small that if no other supply were available many of the industrial concerns could not operate. Although the principle of correlative rights may be said to hold in the humid States, in many localities where much ground-water is used the actual situation is that there is no equitable distribution of water in proportion to ownership of land, but the water is available primarily only to those best able to pay for the deepest wells or the largest pumps.

Another important fact, which is not generally appreciated, is that, as stated by Mr. Conkling under "Administration: Underground Water", it is impossible to take water from any well either by natural flow from an artesian well in which the static head is above the surface, or by pumping from wells in which it is below the surface, without causing a drop in head, or static level, beneath the territory surrounding the well. Theoretically, this drop in head should extend ultimately to the outermost borders of the ground-water body under consideration[37]. The loss of head resulting from the withdrawal of water from several wells if within the cones of influence of each other, may be significant in amount over a large area, perhaps many square miles. Some loss of head can not be avoided even if the quantity of water withdrawn is only a small part of the total safe yield of the aquifer; and if a considerable part of the safe yield is to be obtained in some regions there must be a considerable loss of head. It should be distinctly understood, however, that loss of head does not necessarily mean that the permanency of a well owner's supply is endangered. These facts are of interest in connection with the situation in Idaho, where, as pointed out by Mr. Conkling ("Summary, Discussion, and Conclusions (Item 15")) a strict interpretation of the Court's reasoning in Silkey *v.* Trego, (5 P. (2d), 1049), would prohibit every one from using ground-water except the first appropriator.

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[36] "The Mohave Desert Region, California", by D. G. Thompson, U. S. Geological Survey *Water Supply Paper 578*, p. 322. 1929.

[37] In this connection see "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge in a Well Using Ground Water Storage", by C. V. Theis, *Transactions*, Am. Geophys. Union, Pt. 2, pp. 519–524, 1935.

Examples of Lack of Appreciation of Fundamental Hydrologic Principles.— The failure to appreciate these facts has resulted in some unfortunate situations. Several years ago the writer had an opportunity to read the transcript of evidence in a case in a Western State. It was a striking fact that in this testimony wells were said to have gone "dry" when the water had merely ceased to flow over the top of the casings at or near ground level. Apparently, it was not evident to counsel or expert witnesses for the defendant that the wells were, in fact, not dry, and that it would only be necessary to install pumps in the wells to obtain as much water as, or more than, was originally obtained by natural flow. The failure to understand this simple hydrological principle is almost unbelievable, but it is borne out by other evidence that in the same region until a few years ago both well drillers and water users did not understand other essential principles to be considered in the construction of wells and developments of ground-water.

In 1935, the Colorado Legislature passed a law (Chapter 80) which provided that "it shall be unlawful for any person, persons, or corporations to pump water from an artesian well for any purpose or purposes whatever except for domestic or manufacturing purposes." An artesian well was defined as a well "the waters of which, if properly cased, will flow continuously over the natural surface of the ground adjacent to such well at any season of the year." The Act was worded so that apparently it was intended to apply principally to a particular area in which there are numerous flowing wells used for irrigation. As written, the law obviously prohibits pumping, but does not prohibit the drilling of a well that may perchance flow. The framers of the law appear to have been unaware of, or to have overlooked, the fact that the damage they sought to prevent by prohibiting pumping can be brought about by drilling flowing wells of large capacity in parts of the artesian basin where the head above the ground surface is high; for, as stated previously, a cone of depression, or an area of influence and interference, is created when a well with artesian head above the ground surface is permitted to flow just as much as when a well with head below the surface is pumped.

According to the definition of artesian well in this law, a well that may be near the border of the area of flow and which does not flow at any season of the year is not an artesian well and not affected by the law. Its owner can put in a pump and operate it to his heart's content, creating a considerable drop in head in wells in the near-by "artesian" part of the basin. On the other hand, a man whose well is situated just inside the "artesian" area, so that it flows in some seasons of the year and not at others, or one whose well is still farther inside the "artesian" area, so that the well flows continually but in some seasons the flow is very small (perhaps only a few gallons per minute) can not install pumps to obtain water from his wells. Since the lowest head, and hence the period of low flow or non-flow, is generally in the height of the irrigation season, the owners of these wells are prohibited from pumping their wells to irrigate crops. It thus appears that if this law were applied strictly as worded it would in fact prohibit the use of ground-water by certain established users in a zone between the truly non-artesian area where water may be pumped from wells and the part of the artesian area where adequate supplies may be obtained by natural flow.

In connection with the situations just cited, it may be noted that in some cases it has been ruled that one riparian owner, in taking water from a surface stream, has so changed conditions at the intake or other control works of another riparian owner in such a way as to make such works useless, the latter owner does not have cause for redress, but must remodel his works at his own expense. By analogy it may be argued that when one well owner deepens his well or pumps it heavily in such a way as to lower the water level in another owner's well, or even dry it up, the latter must deepen his well at his own expense, provided the former is not taking more than his share of the water. This has, indeed, been the conclusion in cases of record relating to correlative rights. The result, as pointed out by Mr. Gourley[19], is a race to construct the deepest well, in which the weakest party financially, who may be the earliest user, "gets left."

Ground-water laws in some States have been written very poorly, in part, because of lack of understanding on the part of the writers as to principles of ground-water occurrence, construction of wells, etc. The poor wording of laws applies, for example, to sections defining artesian wells. In some laws, this covers only wells that flow. The quotation from the Colorado law, previously cited, is an example. A definition in another law includes not only wells "through which water is raised or carried to or above the surface of the ground by natural pressure or gravity", but also wells "through which water is or may be raised or carried to or above the surface of the ground by artificial means." This definition would include practically any well from which water is taken, whether by natural flow, by pump, or even by bucket; and the term, "artesian", which here is used very loosely, is superfluous. The really significant hydrologic principle involved in a scientific definition of artesian water and artesian wells is that the water is under artesian pressure beneath an impervious stratum so that it will rise above the level at which it is struck, regardless of whether the water flows at the surface or how many impervious layers may be between the aquifer and the level to which the water will rise[38]. A State legislature, in general, has the right to define terms as it wishes. Such action, however, does not help in developing scientific accuracy that is sought by technical workers and it can not change the natural laws and principles involved or, by mere edict, rectify the difficulties that the law is designed to meet. One is reminded of the law introduced in a State legislature a few years ago which, in order to simplify geometry for students and engineers, said that hereafter and forevermore the value of π, the factor used to designate the relation of the circumference of a circle to its diameter, shall be 3.00.

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[38] "Outline of Ground-Water Hydrology", by O. E. Meinzer, U. S. Geological Survey, *Water Supply Paper 494* (definitions of zone of saturation and water-table, pp. 21-22, and definition of artesian water, p. 39), 1923.

Conclusion.—The writer has discussed what he believes to be inconsistencies and misconceptions that have grown up both in written laws and in Court decisions relating to the control of underground waters. The need of control is being shown more and more, not only in the arid and semi-arid States but also in the humid States. A more definite control must come some time. The longer it is deferred the more complicated will the problem become. It is desirable therefore, that action be no longer delayed.

What shall the method of procedure be? It is the writer's opinion (and this is shared by his colleagues in the Division of Ground Water of the Geological Survey who have given serious thought to the problem), that all lines of argument lead to the conclusion that the only feasible and reasonable method of control of the use of ground-water for any purpose can be under the doctrine of appropriation, with the necessary control exercised by some State official. The recognition of the doctrine of appropriation as applied to ground-waters "whether flowing in definite channels, or standing in lakes or basins, or percolating through the soil, in areas of which the boundaries may be ascertained" is the first of several recommendations in regard to the control of ground-water, submitted by an advisory committee of the National Land Use Planning Committee[39]. As indicated by Mr. Conkling's statement ("Underground Water Law in General: Utah") in regard to the 1935 law of Utah, in a draft of a proposed "uniform underground water law for Western States", a committee of the Association of Western State Engineers has also applied the principle of appropriation for beneficial use[40]. (The draft of this law in many respects is patterned after the New Mexico law).

The Committee recognizes that the principle is not now applicable in States wherein the doctrines of riparian rights or correlative rights are recognized. The two organizations mentioned have concerned themselves primarily with conditions in the arid and semi-arid States, but it is the writer's belief that similar action can be recommended both for the humid States and those arid or semi-arid States in which the doctrines of riparian rights and correlative rights are now recognized; and definite steps should be taken to make possible the application of the doctrine of appropriation for beneficial use in these States.

In some States it may be necessary to apply the doctrine of appropriation with modification, for example to exempt, or to give special preference to the comparatively small use of ground-water for rural domestic use and watering stock. It may be desirable to provide control only in areas where there is large use of water, by permitting the formation of water districts; but control should be initiated at the earliest possible time in any area where it seems likely that at any time in the future the supply will be deficient. It may be desirable to provide some means for water districts to acquire, for the common good, established rights in some parts of a ground-water basin, which, if they were utilized, would prevent a full and efficient use of the available supply. Thus, to satisfy early rights to a small quantity of water on high lands might mean the prevention of pumping which would lower the water level beneath such lands.

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[39] Suggested principles of State legislation relating to the use of underground water, *Publication No. 3* (6 pp.) (mimeographed), prepared by Technical Advisory Committee on Reclamation, Drainage, and Irrigation Policies, W. W. McLaughlin, *Chairman*. The National Land Use Committee, organized in February, 1932, is not to be confused with the present National Resources Committee.

[40] *Proceedings*, Seventh Annual Conference, Assoc. of Western State Engrs. (1934), p. 45.

How may the principle of appropriation be introduced into States where it is not now recognized? This is a problem for the law-makers and the Courts to decide, of course, but it seems not inappropriate that the ground-water hydrologist give thought to the problem. Perhaps, in some States, it can be done by passing a law stating that all waters of the State, whether above or under the ground, are the property of the public, and, subject to all existing rights to the use thereof, hereafter shall be subject to appropriation for beneficial use under laws of the State existing or to be enacted. This method has been adopted in New Mexico, Utah, and Oregon. In some States it may be necessary to place such a declaration in the State Constitution by amendment. It may be difficult to arouse sufficient public interest to bring about the necessary legal enactment. However, it is not inconceivable that the Local Sections of the Society, the American Water Works Association, and similar organizations, could bring about the desired results, especially if campaigns of education were pushed during times of drought or other emergencies which emphasized the need of administrative control.

Even after the enabling declarations are obtained the problems of the control of ground-water will necessarily continue to be complicated. There will be the matter of determining rights established before the new principle became effective. All this will require much study of legal, hydrologic, and engineering problems, and funds for such study must be made available. Appropriation of funds raised by taxation perhaps will not be readily approved by the general public. However, inasmuch as the waters would be considered to be owned by the public, the developer using them by permission, it would not be unreasonable to require a fee in proportion to the quantity of water used. Since 1917 by law the New Jersey State Water Policy Commission and its predecessors have collected a fee of $1 per 1 000 000 gal for all water used by public supply systems in excess of certain initial allowances. This fee is very small, and it could be increased to several times as much without becoming an undue burden to the water user. The New Jersey Commission has used the money thus collected not only to carry on its work relating to current applications for approval of plans to divert water, but it has also undertaken studies as to the requirements of different parts of the State 10, 25, and 50 yr or more in the future, in order that it may be guided to a most efficient use of the water resources of the State. Some kind of fee has been collected for use of certain waters in other States, for example, in Ohio and Pennsylvania.

One other fact should be borne in mind, namely, that the movement of ground-waters is not limited by State boundaries, and sooner or later (in some States already) interstate problems must be considered. When these interstate problems are considered it further becomes evident that the general problems should be considered not from the standpoint of any one section of the country, but on a country-wide basis By this statement the writer does not refer to the possibility of any Federal control of waters, which is doubtless both undesirable and unlikely of attainment, but merely the desirability of having the laws of adjacent States sufficiently similar to simplify the solution of interstate problems relating to ground-waters. This conclusion is readily reached when it is realized that certain of the water-bearing beds that underlie such of the Great Plains States as Nebraska and Kansas, extend into Iowa and Missouri; other formations of the latter States extend into Wisconsin and Illinois; and so on from one State to another with related problems.

Some progress has been made in the problem of legal control of ground-water, but the situation is still far from satisfactory. Conditions are ripe for another forward step. Probably no single organization has in its membership as many persons who are in close touch with the variety of problems of the control of ground-waters as the Society. Some of its members are involved primarily in the development of ground-water for public supply, industrial use, or irrigation; others in problems of the contamination of waters or of drainage; and others in positions of State administration. Accordingly, the Society is in a position to take the first step in initiating discussions on this important problem. However, the discussions should be on a very broad basis and should not be restricted to problems of one section of the country. They should include not merely the consideration of control of ground-water from the standpoint of the problem of water supply for irrigation or for public use, but also from the standpoint of contamination of ground-water supplies by oil-field developments or salt-water encroachment, the effect of ground-water developments on navigation of rivers, of drainage projects and the construction of dams on ground-water developments and *vice versa.* The discussions should be participated in by a variety of specialists, representatives of those who use the water—the public supply systems, industrial concerns, irrigators—to get their viewpoint; the well drillers and engineers to determine construction problems to be considered; the ground-water hydrologists to show the controlling natural principles; and, the State officials who generally are best aware of the difficulties of administration now encountered and who will have the duty of administering the laws subsequently to be enacted. Experienced men in these various lines can be obtained from a number of companion associations such as the American Water Works Association, the Section of Hydrology of the American Geophysical Union, the American Public Health Association, the Society of Economic Geologists, the Geological Society of America, the Association of State Geologists, the American Association of Petroleum Geologists, the American Petroleum Institute, the Association of Western State Engineers, and the American Association of Water Well Drillers. The Legal Profession, also, must be consulted; but is it not pertinent to suggest that in this problem—concerning whch all who have practical contact with it agree that conditions are most unsatisfactory—when once the majority of the technologists agree, the position of the lawyer is to show not what the desired result shall be, but how it can be brought about? The writer has seen a statement (but can not now find definite reference to it), attributed to the late Chief Justice Marshall in which he indicated his belief that the Courts should be permitted to call more freely upon men of technical training entirely independent of the experts produced as witnesses in cases. In the case of Los Angeles *v.* Hunter (156 Cal. 603), Justice Henshaw stated: "For experts learned in geology to give their theories as to the manner in which Nature has created or developed a given physical condition is not an invasion of the domain of the law." From these statements it seems entirely fitting that the technologists take the initiative in endeavoring to solve the problem of the control of ground-water which is necessary to the most efficient use of this valuable resource.

Acknowledgments.—The writer wishes to acknowledge his indebtedness to his colleagues, Messrs. O. E. Meinzer, A. G. Fiedler, Walter N. White, and J. F. Deeds, of the U. S. Geological Survey, and to H. D. Padgett, of the U. S. Resettlement Administration, whose opinions have served to clarify the writer's viewpoint. Attention should also be called to a recent paper by Mr. S. C. Wiel[41], which deserves consideration in connection with the problems considered herein.

WELLS A. HUTCHINS,[42] ESQ. (by letter).—A scholarly discussion of ground-water law in the United States and of the physical situations to which it applies, is presented in this paper. Mr. Conkling's conclusions will merit study by those concerned with legislation pertaining to underground waters.

Status of Ground-Water Law in Several States.—Under the heading, "Underground Water Law in General", a statement is made as to the status of the law governing such waters in each of the seventeen Western States. Additional comments in the case of several States may be in point.

Arizona.—The water code provides that "the water of all sources, flowing * * * in definite underground channels" belongs to the public and is subject to appropriation. The Courts have held, as Mr. Conkling states, that waters percolating generally through the soil belong to the land-owner and that the rule of reasonable use applies, subterranean streams flowing in natural channels between well-defined banks being subject to appropriation under the same rule as surface streams.

The decision in Maricopa County Municipal Water Conservation District Number One *et al.* *v.* Southwest Cotton Company *et al.* (39 Ariz. 65, 4 Pac. (2d) 369), held that the presumption is that underground waters are percolating in their nature, and that one who asserts that they are not must prove his assertion by clear and convincing evidence; furthermore, that before an underground stream becomes subject to appropriation, there must be certainty of location as well as of existence of the stream. Whether surface or subterranean, a watercourse is held to be essentially a channel, consisting of a well-defined bed and banks, and a current of water which need not flow continuously. Hence, a prior appropriator from a surface stream, who asserts that his rights are being infringed by pumping plants in a valley traversed by the stream, must demonstrate this physical interference, according to the foregoing rule, "by clear and convincing evidence."

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[41] "Fifty Years of Water Law," by S. C. Wiel, *Harvard Law Review*, Vol. L, pp. 252–304, 1936.

[42] Irrig. Economist, Bureau of Agri. Eng., U. S. Dept. of Agriculture, Berkeley, Calif. Approved for publication by the Bureau of Agri. Eng., U. S. Dept. of Agriculture.

Nevada.—Important additions and amendments to the ground-water law were made in 1935. The State Engineer is to designate underground areas and sub-areas for administrative purposes. No administrative areas, however, have yet been designated.

Utah.—After a line of decisions supporting the correlative doctrine, the Supreme Court of Utah has now (1936) announced the application of the appropriation doctrine to at least some underground waters. The two cases in point are Wrathall *v.* Johnson *et al.* (86 Utah 50, 40 Pac. (2d) 755, January 2, 1935), and Justeson *v.* Olsen *et al.* (86 Utah, 158, 40 Pac. (2d) 802, January 10, 1935).

The Wrathall case went up on demurrer, and the Supreme Court held that the complaint stated a cause of action and that the demurrer should have been over-ruled; but the Justices were not in agreement as to why it should have been over-ruled. Two Justices stated that the doctrine of appropriation and beneficial use should be applied to underground percolating waters, and a third that a cause of action was stated under either doctrine, and that waters in an artesian basin should be distinguished from mere "percolating" or "diffused" waters in privately owned land, not connected with other waters. The other two Justices indicated that the complaint stated a cause of action under only the correlative doctrine.

A week later came the decision in the Justeson case, involving a conflict between owners of adjoining tracts, part of each being underlaid by a common artesian basin. Three Justices applied the appropriative doctrine to an "artesian basin", which is "nothing more than a body of water more or less compact, moving through the soils with more or less resistance." The concurring opinion distinguished such waters from "mere percolating or diffused waters." The two Justices who in the Wrathall case concurred in the results but in favor of the correlative doctrine, dissented in the Justeson case.

Shortly afterward the Legislature declared all waters in the State to be the property of the public, subject to existing rights, and provided complete machinery for appropriating underground waters. To the present time (November 2, 1936) the Supreme Court has not had occasion to construe these provisions. In the meantime, the State Engineer is actively administering the statute, twenty-six underground water areas having been defined to September 28, 1936.

In view of the facts: (*a*) That the recent decisions—all handed down prior to the comprehensive legislation—were rendered by a divided Court, with sharp and definite dissenting opinions; (*b*) that the prevailing opinions classified underground waters; and (*c*) that the legislation covering all underground waters has not yet been before the Supreme Court, it appears that further decisions should be awaited before stating, too conclusively, the law of Utah in regard to underground waters.

Wyoming.—So far as the writer is aware, the only decision of the Wyoming Supreme Court on underground waters is Hunt *v.* City of Laramie (26 Wyo. 160, 181 Pac. 137, June 2, 1919). This holds that, if developed artificially, percolating waters belong to the owner of the land on which they are developed.

Co-Ordination of Surface and Ground-Water Rights.—In discussing the administration of surface streams, the author states that only recently has the connection between surface water and underground water been recognized legally, and that the two are still, to a great extent, treated in separate and distinct doctrines of law. Doubtless, this situation has resulted from the fact that surface water is out in the open for all to see, its courses and boundaries readily ascertainable, and its quantity subject to reasonably accurate measurement; whereas, exactly the reverse is characteristic of underground water, although the technique of estimating ground-water supplies has been greatly developed and improved in recent years.

One of the cardinal principles of the doctrine of prior appropriation is that an appropriative right is entitled to protection on all sources of supply of the stream to which the right attaches. An attempted diversion of water from a tributary, at a time it is needed by prior appropriators on the main stream, will be enjoined; for it was recognized in early decisions that the unlimited acquisition of rights on tributaries would eventually deprive the main-stream prior appropriators of their water supply; and, yet, some of these States which follow the appropriation doctrine exclusively as to surface streams have applied the common-law doctrine to underground waters generally, regardless of the fact that such waters may be an important source of supply of surface streams to which prior rights have attached. Doubtless, these inconsistencies have resulted, in part, from the location of tracts over which the first controversies arose; from the failure of river appropriators to recognize the implications of a suit between owners of land overlying a basin constituting one of their sources of supply, and to intervene to protect their rights; and from the difficulties of proving the physical inter-connections of subterranean and surface waters.

Obviously, underground and surface waters cannot be entirely dissociated. The connection between surface streams and tributary or supporting underground waters is too marked. An aggregate flow of 100 cu ft per sec entering a river is no less tributary, physically, when it rises in numerous places through the bottom or seeps through the sides of the channel than when it spills over the bank. To intercept this flow of 100 cu ft per sec by pumping plants, if underground, or by dams, if a surface flow, reduces the river to the same extent; yet in a correlative-doctrine State, such as California, these methods of interception are subject to different rules of law. In States in which the doctrine of appropriation of all waters is carried to its logical conclusion, such as in Colorado, neither pumping nor surface diversions would be permitted to interfere with the passage of this 100 cu ft per sec while it is needed to satisfy earlier river priorities. The pumping diversion, however, would have its own priority to the same extent as would a surface interception if it antedated the river appropriation or became vested with a prescriptive right. All priorities, in other words, relate to the stream system—the river and all contributing sources. To divide the surface waters in an area embracing any part of that system according to one set of priorities, and the underground waters according to a different set, without regard to possible inter-relationships, would (if the development proceeded far enough), inevitably infringe sooner or later upon prior river rights and result in a hopeless confusion of water titles.

Co-ordination of rights to surface and underground waters, therefore, appears to be a condition precedent to the most complete utilization of water in areas in which physical interconnections exist. Under the doctrine of appropriation this will involve in a given region the separation of percolating waters tributary to surface streams from those not so tributary; adjudications of existing rights; allocation of the remainder of the water supply to further appropriations at diversion points consistent with those of established priorities; and extinguishment of some existing priorities, either through voluntary agreement or condemnation by public authority, in order that ground-water development may proceed, where the proposed development is sufficiently valuable to justify the expense.

Desirability of Administrative Control.—In summing up the results of his examination of State laws affecting the use of underground waters, the author lists four doctrines of law, three of which recognize ownership of water by the land owner and one by the State. Administrative control will be concerned with this fourth doctrine, under which the title to the ground-water is in the public or the State, subject to beneficial use by individual appropriators. There is a possible field for State control under the correlative doctrine likewise, in basins subject to overdrafts; but if experience with riparian rights on surface streams may be taken as a guide, adjustments of controversies over water in individual ownership will be left to the Courts

When a State is once committed to a doctrine of private ownership of water, change to a public ownership system is difficult. The change is much more feasible before great development has been made, than afterward; before vested rights and their protection offer serious resistance. The Nevada Courts for a period of thirteen years recognized the riparian doctrine as to surface streams, to a certain extent at least, and then abrogated it. Riparian rights embodied in Court decrees became vested, but the abrogation was sufficiently early in the State's development to avoid widespread complications. After a long line of decisions supporting the correlative doctrine of ground-waters, the Utah Court now favors the appropriation doctrine; but only a very small percentage of the irrigated area receives its supply from wells. The situation in California is very different; in that State nearly one-third of the total irrigated area secures water exclusively from wells. A movement now under way to effect a modification of the correlative doctrine in California is faced with difficult constitutional questions.

Undoubtedly, in various areas, development of ground-water exists that would not have existed had control been in effect. Nevertheless, a certain amount of this has taken place at the expense and to the injury of early appropriators. This may or may not be in the public interest in a given locality, depending upon local conditions. In parts of California—a correlative-doctrine State—early users of underground water have been forced to go deeper and deeper into the ground for their water supplies as a result of increasing later use by other owners of land overlying the same basin. Development has been great; but against this must be cited the great increase in cost of operation forced upon the early users, the failures on the part of those who could not stand the mounting costs, and the uncertainty as to available water supply if and when all those who are entitled to pump the water choose to exercise their rights. Under the correlative doctrine, as compared with the appropriation doctrine, those who install pumping plants have little protection in the maintenance of their water right; no matter how long they may use the water, its adequacy and availability may be impaired at any time by the installation of pumping plants by other correlative owners. In other words, a correlative right to pump in an area in which the tillable land exceeds the water supply simply means that when all exercise their rights no one will have enough; if some develop a supply adequate for their needs in the meantime, they will be deprived of at least part of it later.

Extreme application of the appropriation doctrine, on the other hand, is open sometimes to serious objections. For example, the first appropriator is entitled to have the water flow in the stream to his point of diversion when it will do so in quantities sufficient to be useful; and he is not to be deprived of this right in favor of a later appropriator higher up the stream on the ground that the water would be greater in quantity and, consequently, more useful up stream. Regardless of heavy losses in the stream bed, the earlier appropriator is entitled to the flow to the extent of his appropriation. To require 100 cu ft per sec to be released up stream to supply 5 cu ft per sec to the early priority down stream may appear unreasonable, but it is the latter's right. Analogous to this situation would be the prevention of any later development of underground water that would lower the water level at the earliest user's well below the point from which he can afford to pump. The earliest user in each case would have protection under the strict application of the appropriation doctrine. This is a property right, important to the individual. The application in such case is objectionable from the standpoint of the public interest, for it requires an allotment of natural resources to one individual that is capable of serving many to greater advantage.

The appropriation doctrine has proved better suited to the Western States than the riparian doctrine. Nevertheless, the last word in its development has not necessarily been spoken. The author indicates that there is a tendency toward modification; undoubtedly, there is room for modification to include economic use, and for laying more stress upon the element of public benefit.

Granted that the appropriation doctrine as to surface waters, in substantially its present form, will be retained, and that interest in bringing ground-waters under State control on an appropriative basis has grown in recent years, the key to the future development of ground-waters tributary to a stream system in a strict appropriation State may lie in some form of public or co-operative acquisition and extinguishment of prior rights which interfere with development, compensation to be made to the holders of the rights thus extinguished. The same reasoning applies to extinguishing prior ground-water rights along the rim of a basin, which if not extinguished may prevent the use of a much greater water supply drawn from storage in the interior of the basin. The prospective ground-water use must justify itself however; and if the user can not afford to pay just compensation, the necessity for any such exchange of water right is questionable unless some other public benefit is involved. In the case of surface-water appropriations, the working out of an equitable basis for an adjustment should not present great difficulties. In the case of ground-water it is more complicated, of course, both physically and economically.

Difficulties of Administration.—The author indicates some of the difficulties that face the administrator charged with control of ground-water. It is a complicated proceeding, of course, and it is appreciated that precise determinations may be difficult or impossible in a given case, or so costly as to be prohibitive. Nevertheless, in view of the increasing knowledge and mastery of ground-water hydrology, there seems little reason to doubt that a careful, scientific investigation will yield substantial justice to the claimants.

Administration of underground waters will be more expensive than that of surface streams, due to the cost of necessary investigations. That, however, is an element to be considered in any proposed development of ground-waters, just as is the high cost of pumping from underground, as compared with the cost of gravity diversions from surface, streams.

The fact that the administrator must enter the field of economics, from which he has been free in the administration of surface streams, may be an unwelcome departure to the individual, but is not an insuperable obstacle. Methods have been developed for determining costs and returns in farming operations, costs of water, and capacity of lands to pay for irrigation development. The administrator will adapt existing methods in reaching his decisions as to issuance of permits and regulation of pumping drafts, rather than develop an entirely new technique.

JOHN E. FIELD,[43] M. AM. SOC. C. E. (by letter).—Dealing as it does, with a very complicated subject, this paper is unusually comprehensive and clear. For simplicity, it would be well to separate the discussion into two major parts: First, where riparian rights obtain—geographically east of the 98th Meridian; and, second, where the appropriative doctrine rules (that is, west of that line). The westerly area should be subdivided into the several States, and the one which offers the least departure from the strictly appropriative doctrine and in which, through judicial decision, the rules are becoming fixed, should be discussed first. With a code for such a State as a basis, modifications in conformity with laws and customs can be made and applied to other States.

Colorado seems to be the State where riparian rights intrude the least, and where the laws and practice have reached a fair degree of stability. The practice in Colorado is based largely on judicial decisions and many of the statutory laws have followed rather than preceded the decisions of the Court. Until 1879, the practice was "go as you please" with only the Constitution as a guide, which provided that "the right to divert water for beneficial purposes shall never be denied" and "the first in time is the first in right." In 1879, a law was passed providing for "proving up" on claims of the irrigator, of recording and defining the rights, and for "adjudication." State-wide adjudications were had in the period from 1883 to 1886. The right to transfer water from one canal to another was recognized and although it sometimes led to injustice to the individual, nevertheless, it encouraged economy of use and caused the transfer of water from the poorer land to the more productive. These transfers were decidedly in the public interest, and, in part, they account for the rapid development in Colorado. On the other hand, State governments that tried to tie the water to the land and to avoid individual injustices, lagged in their development. A statute prescribing the method of effecting the transfer was passed in 1899, which the State Supreme Court declared was the expression of the existing law and merely made definite the procedure to be followed.

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[43] Cons. Engr., Denver, Colo.

In the matter of laws, regulations, and administration of underground waters, as the author indicates (see "Underground Water Law in General: Colorado"), the Colorado Courts have applied the same regulations as for surface water, but the time has now arrived when the method of administering and recording underground water rights should be embodied in statutory law, based on practice, custom, and the decisions of the Courts. This orderly procedure which has characterized the evolution in water laws in Colorado, will result in an equitable and practical expression of pre-existing fact. It is for these reasons that the writer uses Colorado as the example to clarify his theories and arguments.

As it passes from the mountains to the plains, the South Platte River derives less than one-half its supply from the South Park area (see "Underflow Which May Support a Surface Stream.") It passes through Denver about ten miles east of the foot-hills and at the bend near Greeley, is 30 miles from the mountains and enters Nebraska close to the northeast corner of Colorado. The minimum precipitation on the eastern slope (about 12 in.) is found about 50 miles east of the mountains; it increases to the west to 15 in. at the foot-hills, and increases easterly to about 16 in. at the Kansas-Colorado line. Due to the rapid fall in the river (about 2 000 ft in 200 miles), existing canals could be extended to cover the greater portion of the area east of the mountains and between the Wyoming line on the north to the South Platte-Arkansas divide on the south, and water could be carried on to land in the Arkansas water-shed or, following the divide, enter Kansas at the point of highest elevation in that State. In other words, practical gravity canals can reach lands far in excess of the water supply and if the present supply could be increased (through trans-mountain diversions) by 50%, only moderate extensions of existing canals would be required to use the increase. This possibility explains the absence of pumping. Pumping to provide storage space has not been proposed heretofore, for the South

Platte Valley, as it is water and not reservoir space that is needed; and, furthermore, diversion dams now raise the underflow into many canals quite as effectively as pumps could. Below these dams, during the period of ordinary flow, the river bed is dry and would remain dry for the remainder of its course, except for the invisible inflow of natural and return seepage water, which amounts to about 1 000 cu ft per sec, or 5 cu ft per sec per mile of river.

Consequently, the writer must differ with Mr. Conkling in his facts and conclusions as stated under the headings, "Underflow Which May Support a Surface Stream" and "Administration: Underground Water", as far as they refer to conditions on the South Platte.

Numerous pumping plants on the mesa lands are used to supplement the canal supply. These wells are below the canals and their supply is from seepage water on its way, underground, to the watercourses. The right to pump has not been seriously questioned due in part to the difficulty of proving that the pumped water would reach the stream at a time when needed by a plaintiff. The damage in reality is inconsequential to any one canal. One-half the seepage water reaches the river in the non-irrigation season and the other half is diverted and demanded by one canal to-day and another to-morrow. All are probably affected at one time or another, each in its turn, depending on the stage of the river flow and the dates of priority. If all factors were determinable, canals that are remote and in other districts, may be the ones damaged at some particular time. A small quantity of the return seepage is intercepted by the pumps, but which canal is damaged, and to what extent, is impossible of even approximate determination. The only benefit that pumping would accomplish would be to furnish, from the underflow, some water to meet the emergency needs of canals. Numerous pumps have been installed in the five years (1932 to 1937), for such emergency purposes and the "live and let live" attitude of canal administrators who have been adversely affected has permitted their use, because each canal administrator suspects that he too will sometime wish to do the same thing. Contrary to the author's contention, the South Platte does not offer an example of the wisdom or possibilities of pumping; nor will pumping create reservoir capacity that may be filled by water now escaping.

Under "Artesian Areas or Basins", the author does not mention the great artesian wells of the San Luis Valley, near the head-waters of the Rio Grande. Slichter gave their number as about 6 000, and the aggregate supply as about 300 000 acre-ft. After more than forty years, they are still active and such as show a decreased discharge have probably become choked, the casing has failed, or the water is escaping before reaching the surface.

Table 1 seems to be in serious error in the columns, "Areas Supplied from Streams with Supplemental Well Supply", for in the San Luis Valley there is an area of more than 200 000 acres receiving water from wells, in addition to their canal supply. The area is sub-irrigated, and so this item of double source of supply escaped the eye of the field agent taking the census. In Table 2, remarks as to Colorado should read: "Pumping for emergency purposes only, is advisable." An increase in water supply will be necessary before much further development is possible. Otherwise, only through economy of use, can any further development of consequences be expected east of the mountains in Colorado. The error mentioned lays the accuracy of Table 1 open to suspicion. In any case the data pertaining to agriculture in 1919, are eccentric (that is, out of balance), especially when considering trends. They should be used with great caution as the stimulus to agriculture due to the World War makes them unreliable.

The question, "Is the diverter from a stream protected in his means of diversion?" (see "Water Law in General") is answered by a decided "No". It is quite enough that the early appropriator enjoys at all times an exclusive 100% right (when it is in the river) and, although he is entitled to the maintenance of conditions as of the date of his appropriations, to have it apply to the means of diversion would lead to intolerable conditions, contrary to the public interest. For example, when, by reason of diversion of the water above, the first appropriator finds his head-gate too high and the water too low for him to divert his full right, it would be wasteful to send sufficient water down to him to raise its surface to the old gage height; and, therefore, the law provides that he may move his head-gate up stream, build a diversion dam, or put in a pump. Assuming that it is Canal No. 2 which causes the low flow, to compel Canal No. 2 to pay for the change might well impose such a burden on that canal as to prevent its construction. Canal No. 1 could, by reason of its preference right, better afford to assume the burden. The author states that (see "Water Law in General") "the hurdle, faced by the individual who proposes to divert at an up-stream point would indeed be high, if he had to compensate all down-stream appropriators." Although there are no statutes in Colorado which protect the appropriator in his means of diversion there are statutes which permit him to take such measures as will enable him to secure his full appropriations and which by implication denies him the right to insist on a *status quo* as to his means or instrumentality of diversion.

In the case of overflow on meadows, when the river falls so low by reason of diversions above, that the meadows are no longer overflowed, the law permits the construction of canals, with a date of priority as of the date of its first use as a meadow.

In a recent decision of the Supreme Court of Colorado, the Court reversed itself and ruled that reservoirs and canals were on the same basis—and that the first in time, was the first in right. Here, it is quite definitely indicated that the instrumentality employed to accomplish beneficial use, in no way affects the right of use. Therefore, pumps have no different standing from gravity canals, reservoirs, pipe lines, or carrying the water in a bucket. Such being the case, it seems that in Colorado a practical plan of administration alone remains to be evolved and that at this time, the Administrator, the State Engineer, and Water Commissioner, should be given definite rules and clear instructions under which they will perform their administrative duties in handling underground water. The statute should require that filings be made as in the case of canals, that all wells to be used, for other than household and stock purposes, must be drilled under the direction of the State Geologist who would preserve a careful log of the well and make a study and report on the geology, the replenishment, and the source of supply. This report would be filed with the State Engineer, who, in turn, would make a tentative determination of the duty of water, and at the beginning and end of each irrigation season would check the height of the water-table. The statute should require the installation of a meter, a reading of which would be taken by the Water Commissioner as often as the State Engineer deemed advisable, or the latter could require the installation of a self-recording meter. The quantity of water pumped would be limited by the tentative duty of water as fixed by the State Engineer and the acreage irrigated, and the quantity pumped each year would be recorded and would form the basis for later adjudication.

Adjudication should be initiated by the State Engineer when, in his judgment, it was advisable, or upon the petition of the water users. In this adjudication, the Court would fix the depth below which the water could not be drawn, based on the records gathered and compiled by the State Engineer or by any competent investigator. The appropriator should be limited in the quantity of water he could pump in any one year; but the time or the rate of flow should be at the discretion of the user. The State Administrators would be concerned principally in the gross acre-feet pumped per year. The tentative duty of water would be changed from time to time until a fixed duty could be determined on the basis of the records. At first, the tentative duty might well be placed at 1.25 acre-ft per yr, which approximates the consumptive use for Colorado, a use over and above the rainfall. After the adjudications were had, the water officials would act thereunder, shut down such pumps as had reached the quantity allowed by the decree, or when the water-table approached its minimum allowable elevation. In periods of extra large supply, or of drought, the pumps with the later decrees would be allowed to run or would be closed down. After a period of years, the replenishment possibilities would be determined quite closely and the junior decrees would, in practice, become void. The writer does not anticipate that there would be an under-development; if this should occur, there would be almost no call for administrative control.

If pumps should be installed in the bottom-lands (land a few feet above the stream), they would be allowed to operate in the order of their priority; that is, after the demands of prior rights had been satisfied. Pumps more remote from the river should be allowed to operate until holders of earlier priorities make complaint, in which case, the State Engineer would investigate and render his decision, which would be binding until altered by the Courts.

In time, the necessity of classifying the land would probably arise, the entire underground supply in some areas would be reserved for domestic and stock uses, and, in others (as in river bottoms) all water pumped would be considered as a diversion from the river supply. In the matter of preference uses, the laws now provide that domestic use is superior to agricultural use, but by the Court's decision this means that the water used for irrigation may be condemned and converted to domestic use.

Summary, Discussion, and Conclusions.—In addition to what the author states in Item 2 of his "Conclusions", it should be added, that the damage occasioned by the decrease of supply on the South Platte is inconsequential, as it is shared by all priorities, each in its turn. Referring to Item 4, the fundamentals of Colorado laws were adopted before Statehood (in fact, coincident with the creation of the Territory in 1861). The same is true of most of the other arid States.

The practical operation of Colorado laws and decisions relative to underground water (see Item 5), may have prevented the development of pumping to some extent, but they have prevented infringement on existing rights, have protected developed areas, and have prevented unwise and "wild-cat" enterprises. The writer, with an intimate knowledge of the State for more than fifty years, is unable to visualize a materially greater development by the use of pumps than has occurred without them; and, on the other hand, he can readily visualize interminable legal controversies and a less development had the California law prevailed. In fact, the physical conditions in Colorado and in California in the matter of underground waters are so different that no comparison can be made; nor can conclusions be drawn with profit.

Much research and investigation are necessary for accurate and intelligent control (see Item 19), but unless the development is sudden and complete, the data can be gathered over a period of years, and as a part of the ordinary administration duties. The cost will be borne largely by the appropriator, just as it is now borne by canals in the construction of weirs, registers, and co-operative measurements. The administrative control and the gathering of data should not wait until overdraft threatens, but to be of greatest value, must be gathered during the years of development, and must be available when overdraft threatens and adjudications are asked.

In conclusion, the paramount consideration in such matters as the paper touches, is the public interest; the greatest economical development possible, even at the sacrifice or infringement of individual or property rights, should be the objective. Old laws must be abrogated and new ones enacted to meet physical and economic conditions. In the arid States, the riparian doctrine was ruthlessly set aside, or so modified as to meet the necessities of the people. Most of the early Court decisions were based on the necessities, and on common sense rather than on precedent, or the common law; and it is to be hoped that the Courts will continue to recognize paramount necessities as controlling, rather than adhere to the outworn laws and outworn decisions.

The State of Colorado has been very fortunate in having a Court willing to reverse itself and willing to consider facts and the public interest in making its decisions; and, in Wyoming, the Court, recognizing the benefits of the transfer of water, "hedged" on the "water-attached-to-land" constitutional provision and held that this provision did not apply to ditches built before the Territory became a State. It is axiomatic that water should be used where the user can pay the most for it.

In a mining camp, in Colorado, a "district" was formed, about 1859, and rules and regulations were adopted. In conclusion this document reads: "In matters not covered by the above rules, the laws of the United States shall apply." The writer's theory also is, that "when existing laws and decisions do not conflict with the public interest, they shall prevail."

George S. Knapp,[44] M. Am. Soc. C. E. (by letter).—Studies such as those made by Mr. Conkling in the preparation of this paper should aid materially in an understanding of ground-water administrative problems, and in a region as young, relatively, as that of the Western States, where ground-water laws are still in a formative state, the paper should be a valuable guide in the shaping of legislation for administrative control.

The paper shows the great diversity among the various States in laws governing rights to the use of ground-water. In their scope they vary from those vesting absolute and unrestricted ownership to ground-water in the ownership of the land under which it is found, to those which apply the appropriative doctrine so strictly that, in some States, a land owner may be barred from using any of the water under his land for irrigation because owners of other land have been permitted to appropriate the entire supply.

The object of such laws and administrative regulations is to permit the orderly development of ground-water supplies and, at the same time, to prevent over-development of the supply and injury to water users. Some effort has been made in recent years to secure the enactment of uniform ground-water laws for irrigation in the Western States. Uniformity of State laws is desirable in so far as it can be attained. With respect to ground-water under land no longer in public ownership, the process of changing the law is difficult, particularly if it is attempted to change from some form of the riparian or ownership doctrine to the appropriative doctrine. The process of separating the ownership of the water from that of the land by legislative enactment would undoubtedly be held by the Courts as the taking of property without due process of law; that is, without condemnation and compensation.

In some States both surface water and ground-water are under the same rule of law. In others, it is not. A number of States attempt to apply the rule of priority of right to ground-water in the same manner as it is applied to surface water. In this connection, it should be noted, perhaps, that the appropriative doctrine does not apply to ground-water, as among various appropriators, with the same effect, and in the same manner, as it does to surface streams. The available quantity of ground-water does not fluctuate rapidly from day to day as surface water does. Even if it moves in well-defined channels, ground-water constitutes essentially a reservoir and not a stream. Its surface is drawn down slowly with use, and the supply is replenished slowly. It is only the earliest appropriators from surface streams who enjoy their full measure of water in all years. The remainder receive their appropriation on a fewer number of days, depending on the relative priority of their appropriations. The last appropriators may be entitled to water only during the few days from time to time when the stream may be at flood-stage. In applying the appropriative doctrine to ground-water it is necessary to determine the quantity which may be withdrawn each year, and from year to year, without depletion of the reservoir. That group of users, whose aggregate appropriations come within this quantity, becomes, together, a preferred group which receives a full supply each year, whereas the latter, would-be appropriators, are denied any part of the water supply.

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[44] Chf. Engr., Div. of Water Resources of Kansas, Topeka, Kans.

There has been a great increase in the acreage irrigated from wells in the Arkansas and Platte River Valleys in Colorado, Kansas, and Nebraska, since the 1929 census data were secured. Much of it has occurred during the five years of drought experienced in this region in 1932–37. Some of these recently constructed pumping plants are used for the irrigation of new land. Others are used as a supplemental supply for land hitherto "dry-farmed", or for land under canal systems, either pumping into the canals, or directly on to the land to supplement deficient surface supplies. All these pumping plants are situated in the river valleys. As Mr. Conkling states, no extensive use of this ground-water can be made without diminishing the flow of the streams to users below. If the rule of priority of right had been enforced in Colorado, these pumping plants, being junior to canals down stream, as to dates of construction, would not have been permitted to operate when the lower canals were short of water. The immediate effect of the operation of these pumping plants would be the lowering of the water-table in the valley rather than a reduction of stream flow, since the water-table would necessarily have to be lowered before stream flow could be affected. At the time, therefore, they virtually drew on a reserve water supply during a period of drought. To permit them to continue to operate regularly from year to year, however, will result in an increased irrigated acreage, or an increased use of water at such points to the injury of surface-water users down stream.

Ground-water supplies constitute a reservoir that can be held in reserve for long periods without the evaporation losses sustained in the storage of surface supplies. The value of these underground supplies in supplementing deficient stream flow during the recent drought suggests the possible wisdom, as a matter of public policy, of holding them in reserve in certain localities to be used only in times of drought, but that could not be done legally under either the appropriative or the absolute ownership doctrines. It would require an entirely new rule of law.

Ground-water laws are still in the making throughout the West. As they develop it scarcely seems probable that they will grow in uniformity among the States as much as they will grow to meet the different needs and the different conditions of use in the various States.

Chandler Davis,[45] M. Am. Soc. C. E. (by letter).—The engineer's mission is to establish the mean number of gallons of water that may safely be pumped from the underground reservoir; the exploitation of the source should be controlled by the Federal Government or by the State Government, and the pumping should be limited to the quantity determined by the engineer; thus, there need be no fear of overdrawing on the sources of supply and causing alarm as Mr. Conkling states happened in California, in areas exploited prior to the organization of the State Division of Water Resources.

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[45] Cons. Engr., New York, N. Y.

Mr. Conkling is correct in advocating that the exploitation of an underground water source be preceded by a thorough investigation, that pumping stations be erected only under license and supervision of some branch of Government, and that the pumping be limited to the findings of the engineer. To emphasize these views, the writer wishes to comment briefly on the status of underground storage in Long Island.

After the recession of the ice of the last glacial period, what is known to-day under the name of Long Island, New York, covered a much larger area than it does now, the south shore extending more than 100 miles farther out to sea. When the subsidence occurred the clay deposited by the melting ice settled below the surface of the Atlantic and is found at various depths, and in various positions (horizontal layers, vertical, and others at all angles). Soon afterward, the sea began to pile sand on the shore front eventually forming the present Island, with its highest dune or backbone nearer the north shore, so that the slope to the north is quite steep, whereas the land slopes gently to the south. Similarly, the ground-water slopes toward the Atlantic very gradually, the highest point of the water-table being south of the highest point of the land. Streams and ponds are found where land contours cut the water-table, their size varying with the rainfall; that is, they shrink after a period of drought and grow larger after a very heavy rainfall period. A perceptible lag is observed due to time required for the rain (which is absorbed by the ground) to reach the water-table. There is one interesting exception, the high lying Lake Ronkonkoma, which does not seem to have any connection with the remainder of the underground water of the Island and seems to follow rules of its own.

The Borough of Brooklyn received its water supply from a line of wells driven along the south shore of the Island. Between 1900 and 1905, a group of farmers sued the City for diverting the water from beneath the farms situated along the line of, and in the vicinity of, the aqueduct. At first the City was not very successful in defending these suits; but, later, its engineers completed their studies of the situation and were able to prove that the farmers had suffered no damage, with the exception of such litigants whose farms lay directly alongside the aqueduct and within the cone influence of the pumps when operating.

A study of the tree growth within the area of the farms under litigation, as well as elsewhere in the United States, where rainfall and underground-water data were obtainable, made by the writer, established the fact that such growth was not affected by the pumping, and that as the water-table lay so far below the surface throughout the area under litigation, vegetation depended on rainfall for its growth.

There is an important question to be solved: How much of the rainfall, which is absorbed by the ground and finally reaches the ground-water, may be pumped safely without damage to the underground reservoir? This is especially important in cases such as Long Island, as the fresh water holds back the salt water, and if the latter is permitted to encroach on the former the volume of fresh water available will be reduced by that quantity irretrievably, and this is the real problem which confronts the engineer.

Mr. Conkling is to be congratulated on his very valuable and timely paper.

L. STANDISH HALL,[46] ASSOC. M. AM. SOC. C. E. (by letter).—A thorough treatment of the physical and legal aspects of the control of underground water is presented in this paper. The many and varied possibilities with reference to the physical conditions are adequately expressed in the sentence (see heading, "Underground Water in General"), "each situation is a separate study which must be exhaustive and widespread to reach reliable conclusions". The only generality that can be expressed is that all surface, vadose, and ground-waters were derived primarily from rainfall. (Vadose waters[47] are the suspended sub-surface waters found in the zone of aeration between the land surface and the water-table.)

The author states that ground-water cannot be developed without reducing the flow of the adjacent streams. This statement is true, in general, because it was implied that the streams in question were perennial, and that the development of surface-water diversions was possible. Such streams in semi-arid regions usually rest on the water-table and, to some extent, are effluent, or they may be both influent and effluent along various stretches of the channel. A ground-water development in these areas will naturally result in a lowering of the water-table, but if the pumping is not excessive a balance will be reached ultimately subject to the normal fluctuation of ground-water levels resulting from the natural variation in rainfall. This balance is reached from the increased seepage or percolation from the stream induced by the steeper gradient from the water level in the channel to that in the adjacent ground-water body. If the streams were originally influent, a change to effluent will result.

A natural limit to the process of increasing percolation from ground-water development is reached if the ground-water is lowered below the stream bed at points adjacent to the channel. When this condition occurs, no further lowering of the water-table will increase percolation under a given flow. The stream then would have reached the natural condition existing in ground-water adjacent to intermittent and ephemeral streams, where stream channels are above the water-table at all times. Under such conditions, percolation depends on the volume and duration of flow, and ground-water withdrawals exert no influence on the recharge.

If surface-water irrigation has developed in a basin prior to the use of the underground water, it does not appear possible, on a perennial stream, to permit a subsequent pumping development under administrative control. If the surface-water use occurs on an intermittent or an ephemeral stream, the ground-water development would not conflict with the surface-water use. However, if surface-water use on the latter type of streams succeeded ground-water development, there would be a conflict of rights, since diversion of surface flow would reduce the quantity that would percolate.

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[46] Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

[47] "Outline of Ground Water Hydrology", by Oscar E. Meinzer, *Water Supply Paper No. 494*, U. S. Geological Survey, p. 22.

Even a subsequent ground-water development on a perennial stream subject to appropriation, must occur up stream from the point of diversion to be of damage since (as is noted by the author) excess water is always carried from the stream with surface irrigation and frequently results in a rising water-table in the lower part of the irrigated area. Ground-water development down stream from the surface irrigation would frequently be a benefit and, in fact, is sometimes resorted to under a canal system to prevent water-logging of the lower areas.

The economics of the situation may also enter into the extent that development of ground-waters may progress unhindered. Sufficient lowering of the water-table eventually would result in excessive costs so that some users would be forced to abandon their development. Here, again, priority might establish the justice of such a method of survival depending on whether or not the lands forced out of cultivation were first in the ground-water field but, because of their location, were the first to feel the effects of mounting water costs.

The variety of conflicts that may arise in connection with ground-water uses and surface-water diversions would lead to the careful consideration of the author's conclusion (Item 19) that where fullest development of water resources are desired, "administrative control of underground water should not be adopted until over-draft threatens, and then only in particular areas." Furthermore, "a step at a time may be the safest course."

HAROLD CONKLING,[48] M. AM. SOC. C. E. (by letter).—The able and instructive discussions of the subject-matter of this paper "bring home" the considerable thought which is being devoted to the subject by hydrological engineers. The paper and the discussions do not deal with a matter that has been finished, nor with a mathematical certainty, nor with the results of an experiment; rather they deal with what may be the difficulties in, and what may be the desirability of, administrative control of a natural resource, the limitation on the use of which is obscure and is enwrapped with questions of economics. In such discussions informed opinion naturally plays a large part.

The discussions have indicated at least a part of the errors of commission and omission in the original paper. The writer is grateful for this, and it is hoped that all readers will note them. Any errors in acreage irrigated by pumping are not at all material to the purpose of the paper. These data are from the Federal Census which is the only source available, as was found by the writer after attempting to obtain the same information from the various State offices. In the discussions the scope of the paper has been amplified to the benefit of all. It was hoped that both correction and amplification would be the outcome. "Consider," said Prah-hotep 5 000 yr ago, "How thou mayest be opposed by an expert in council. It is foolish to speak on every kind of work;" and said Tai Tung, "Were I to await perfection my book would never be finished."

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[48] Deputy State Engr., Sacramento, Calif.

Many lines of thought have been suggested in the discussion which, however tempting, would lead to an inordinately long closure, and because of this, specific mention is made of only a few of the factual discussions of conditions in specific areas.

That of Mr. Gourley gives light on conditions in England where the development of modern industry is bringing to the fore the desirability of the control of underground water use. Mr. Baldwin has also called attention to industrial developments in the Mid-Western United States which show that even there a problem now exists. The discussion of Professor Smith describes in detail conditions in Arizona and neighboring States. In this connection a *Bulletin* by Professor Smith[49] on ground-water law, published in 1937, is well worth reading. Mr. Thompson has supplemented and amplified the original paper by a broad and masterly outline of conditions in the eastern part of the United States and by an analysis of legal decisions in both the West and the East. A discussion of conditions in Colorado, by Mr. Field, is illuminating because of the intensive development of water supplies in the eastern part of Colorado. This latter discussion appears to support the writer's thesis that in a State having the appropriative doctrine, use of underground water would be legally inhibited by surface rights below the point of use as long as the concept prevails that surface flow is necessary to fill a right which, heretofore, has depended on surface flow. The successful pumping which has been instituted along the South Platte River in the recent years of drought, has been successful because "the live and let live attitude of canal administrators who have been adversely affected, has permitted their use."[50] In other words, development of ground-water succeeded, and the crops of the area were preserved in the recent stressful years because the law was not invoked. This is cogent commentary on the difficulties that arise in the complete application of the law of prior appropriation when ground-water is involved. The foregoing statement, however, seems somewhat contradictory to a subsequent statement in the same discussion to the effect that the prior appropriator is not protected in his means of diversion and may be forced to pump. Perhaps Mr. Field did not have in mind pumping from under-flow as an alternative means of diversion.

All who have discussed the paper unite as to the need of administrative control. To the writer this seems clear also, but he is not so definite in his belief that actual control is needed at an early stage of development. A ground-water basin in its original state contains large deposits of water which, in most cases, can be withdrawn without great harm except that the water-table is lowered and the permanent supply is thus rendered more expensive to secure. In this respect, it is not different from an ore body or an oil field. If these deposits of water are not exploited, the potential wealth of the country is not developed; and yet, if they are exploited, it means that some of those using the water during the period of exploitation must cease using it at some future period. In other words, these users have only a temporary right; but it seems to violate a traditional concept to speak of a temporary right in water. How valuable this exploitation may be in some cases is witnessed by development in Southern California and in the San Joaquin Valley of California where, with mining of water as a foundation and mining of oil as a supplement—in the fields side by side with the water basin—wealth has been produced which has enabled or is enabling the communities to construct huge aqueducts to import water from sources hundreds of miles distant to replace the mined water and to bring in supplies much greater than the permanent local supplies.

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[49] "Ground Water Law in Arizona and Neighboring States", by G. E. P. Smith, M. Am. Soc. C. E., *Bulletin No. 65*, Coll. of Agriculture, Tucson, Ariz.

[50] See p. 826.

An uncontrolled development may bring about the best development or it may not; but whatever the result the means of its achievement are costly. Administrative control at its best would bring about the best result in the least costly manner, but it cannot depart too widely from the natural trend of events. If a temporary over-development which mined the impounded water supplies of the past ages seemed best for the public it would be permitted and, therefore, permanent and temporary rights would exist. The same results would thus be achieved finally as are achieved in many cases by uncontrolled development; that is, temporary over-draft and final adjustment of draft to recharge, with resulting abandonment of part of the development. The difference in such case would be that the temporary rights would be designated in the case of control, whereas, with lack of control, such rights would be those of the economically marginal users instead of the last users.

The point is that the need for flexibility in administration is paramount. The principle of prior appropriation, if actually administered according to the doctrine, can be very inflexible. The doctrine of correlative rights can be very flexible. Either doctrine may lead to beneficial results or either may lead to harmful results. In contemplating possibilities, the writer confesses to a distinct distaste for "freezing" any situation within the narrow confines of administrative action and control unless the administrator can be given the right to accept or to reject the areas to be placed within his control, either by his own initiative or by action on a petition of those using water. As an alternative the administrator can be given a power he is not now given in any jurisdiction; namely, the exercise of judicial functions.

Mr. Thompson's discussion brings out the apparent necessity felt by the Courts to define classes of underground water so that each situation falls into a certain category as to which a precedent exists or as to which a new precedent can be set. Even by a hydrologist, the artificiality of such definitions can be appreciated only when he must examine them in detail. Recently, the writer was faced with the necessity of deciding, for purposes of determining jurisdiction, whether a certain body of water was a "definite underground stream," or otherwise. Any reasoning a hydrologist could bring to bear on the matter which led to the conclusion that this body of water is an underground stream, would also lead to the conclusion that almost any moving underground water is an underground stream, and *vice versa;* but the first-mentioned reasoning process seemed the soundest. The writer is heartily in accord with Mr. Thompson's axiom that "all water beneath the ground is * * * simply ground-water." The difficulty is that the definitions and the need for definitions have been due to lack of knowledge, with little appreciation of the broader phases of the matter.

To summarize: The numerous and able discussions have done much to clarify the subject. It is felt that control is desirable, but that it can be accomplished best through a step-by-step process whereby the results of each step can be considered before going further. The attempts at control in the various jurisdictions and in England have been outlined. Of course, they are first steps, but it can scarcely be concluded after reading these discussions that a uniform law is desirable. Contrast conditions in Colorado as described by Mr. Field, with conditions in California as typified by San Gabriel Valley; or contrast with these, conditions in Long Island as described by Mr. Davis. Nevertheless, the basic ideal would necessarily be the same no matter how diverse the form of the law; and by it questions of draft which would cause final retrogression would be encompassed whether the retrogression were caused by marine or other saline intrusion, by too great concentration of chemicals, or simply by drawing more water from the basin than the recharge permits.

If engineers feel that control is desirable they must formulate the kind of control. This matter has become, in some areas (and will become in others), a matter which ranks with other phases of life which have been given into the control of commissioners with judicial or quasi-judicial power. If the development of underground water brings such a situation about as to its control, it will naturally extend to surface water since the two are physically interdependent. It is suggested that such a body, combining in its personnel, both scientific knowledge and ability for investigational research, can most intelligently cope with the present problem, subject of course, to review by the Court.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1972

DYNAMIC DISTORTIONS IN STRUCTURES SUBJECTED TO SUDDEN EARTH SHOCK

By Harry A. Williams,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Arthur C. Ruge, H. M. Engle, Merit P. White, L. S. Jacobsen, and Harry A. Williams.

Synopsis

An inspection of earthquake records indicates that the ground may have practically harmonic motion for several successive swings after the initial shock. The amplitude and perhaps the period of the motion is then changed, sometimes quite radically. Some engineers feel that it is scarcely possible, under these conditions, for a structure with a period of vibration very close to that of the ground to attain critical distortions because of insufficient time for the amplitude to build up. This paper contains the results of model experiments on a single-mass system and a theoretical analysis of the behavior of an elevated water tank, all subjected to a sudden shock followed by harmonic vibrations. The results indicate that for single-mass structures in the near-resonance condition, critical displacements can be built up before the type of ground motion changes. There is as yet insufficient experimental evidence from which to draw similar conclusions for multi-mass structures.

Experimental Procedure[2]

The experimental investigation, which was made in the Vibration Laboratory at Stanford University, in California, dealt with the dynamic equivalent of a single-mass structure. A diagrammatic sketch of the model as installed on the shaking-table is shown in Fig. 1. The operation of the vibrating system was as follows: The table was given a sudden acceleration by dropping the pendulum against the bumper spring. When the stiff bumper spring recoiled,

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Note.—Published in May, 1936, *Proceedings*.

[1] Asst. Prof. of Civ. Eng., Stanford Univ., Stanford University, Calif.

[2] For a complete mathematical analysis see "Vibration Research at Stanford University", by L. S. Jacobsen, *Bulletin*, Seismological Soc. of America, Vol. 19, No. 1, March, 1929.

contact with the pendulum ceased, and the latter was withdrawn from action. The resulting motion of the table—the ground motion—was harmonic, the amplitude gradually decreasing because of friction.

The model consisted of a 4-in **I**-beam approximately 2 ft long, supported and guided on three balls and having a tension spring attached to each end. The model was constructed so that the amount of viscous damping (resistance to vibration which is directly proportional to velocity, such as that offered by air or fluid friction) could be varied and the effect on the model motion studied, by fastening a brass plate on the under side of the **I**-beam in such a way that the plate moved through a magnetic field set up by two electro-magnets. The damping that resulted from this arrangement was directly proportional to the relative velocity of the model with respect to the shaking-table. It was found that constant or sliding friction could be neglected in the experiment.

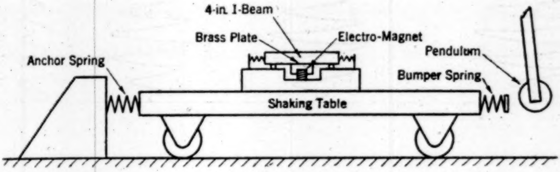


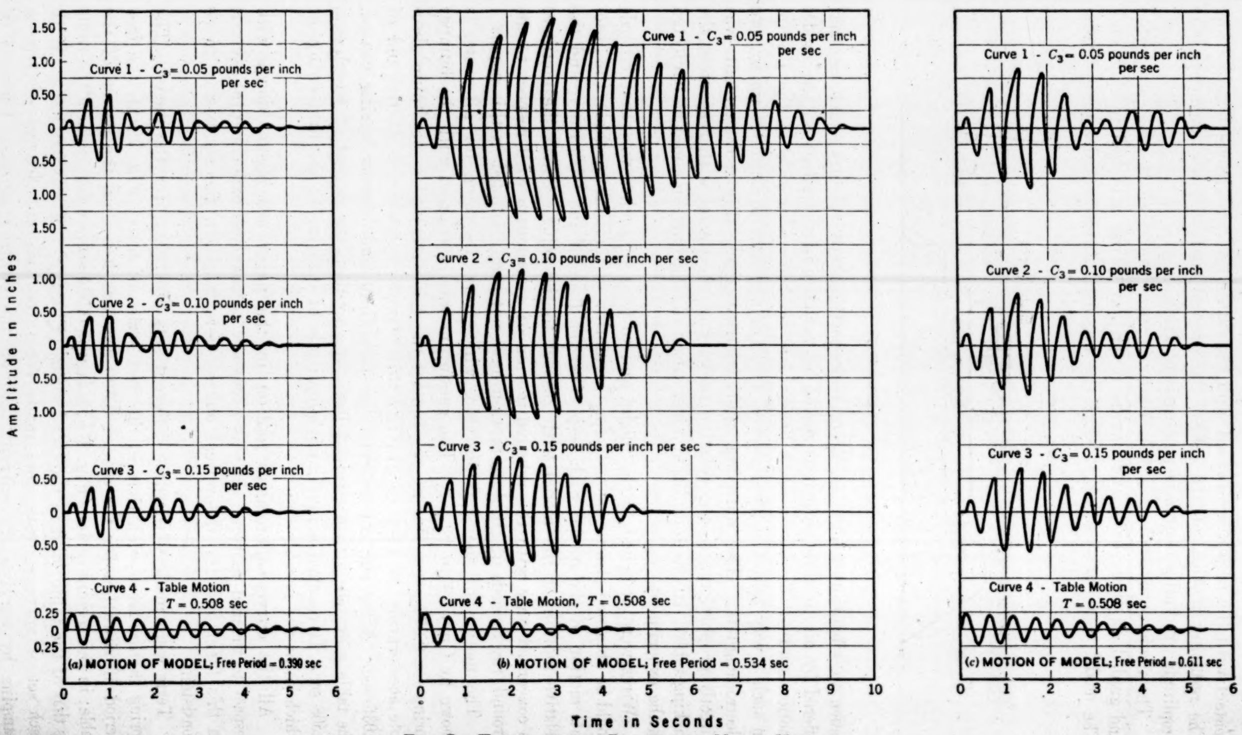
FIG. 1.

Wires were attached to the model and to pens in such a way that records could be obtained simultaneously for the motion of the shaking-table, the absolute motion of the model with respect to the floor of the laboratory, and the relative motion of the model with respect to the table. Since the engineer is concerned only with the relative motion of a structure with respect to the ground, the records of absolute motion have not been included in this paper.

The motion of the shaking-table, which was the same for all tests, is shown by Curve 4 in Fig. 2. The pendulum was in contact with the bumper spring for 0.077 sec. During the impact the table received a maximum absolute acceleration of 14% of gravity, this maximum occurring at the end of 0.035 sec. After contact between the pendulum and bumper spring ceased, the table vibrated with a damped harmonic motion, the period of which was 0.508 sec. It reached a maximum amplitude of 0.23 in. on the first swing, at which point the acceleration was 8.5% of gravity.

All the curves of relative motion in the paper are upside down with respect to ground motion. It was necessary to take the experimental records in this form and the theoretical curves were made in the same way to be consistent.

Tests were made with three different sets of springs for the model. The spring factors were such that in the first series of tests (Fig. 2(*a*)), the free period of vibration for the model, 0.390 sec, was less than the period of the table; in the second series (Fig. 2(*b*)), it was 0.534 sec, or very nearly the same as that of the table; and in the third series (Fig. 2(*c*)), it was 0.611 sec. With each set of springs, tests were made for three different degrees of viscous damping by varying the current through the electro-magnets. Curve 1 of


(a) MOTION OF MODEL; Free Period = 0.390 sec

(b) MOTION OF MODEL; Free Period = 0.534 sec

(c) MOTION OF MODEL; Free Period = 0.611 sec

FIG. 2.—EXPERIMENTAL RECORDS OF MODEL MOTION.

Fig. 2(*b*), for instance, records the test in which the viscous damping constant, c_3, was quite small. It is increased from 0.05 to 0.10 and thence to 0.15 lb per in. per sec in the tests recorded in Curves 2 and 3, respectively, of Fig. 2(*b*).

Very good agreement was obtained between the experimental records and corresponding theoretical curves.

DISCUSSION AND ANALYSIS OF TEST RESULTS

When the pendulum strikes the bumper spring on the table, the latter is accelerated for a very short period—0.077 sec in the present case—after which the pendulum ceases to affect the natural free vibration of the table. The amplitudes of these free vibrations are reduced by friction until motion finally ceases. Only the first two or three complete oscillations of the table are of interest in this investigation. An examination of the records shown in Fig. 2 reveals that:

(1) When the free period of the model is somewhat less (Fig. 2 (*a*)), or somewhat greater (Fig. 2(*c*)), than that of the table, the amplitude is not large, and the maximum is attained in a very few cycles;

(2) When a near-resonance condition prevails (Fig. 2(*b*)), the amplitude is large, and the time required to reach the maximum is somewhat longer; and

(3) The effect of increasing the viscous damping factor, c_3, is to decrease the amplitude.

With reference to Item (2), it is important to notice that the amplitude built up in the first few cycles is large when compared to the maximum. The enveloping curves for these tests, plotted in Fig. 3, show that a large percentage of the maximum amplitude reached in any given test is attained very quickly.

In connection with Item (3), the effect of damping on the period of vibration is negligible. If the friction is small, the amplitude will build up somewhat more rapidly and will be greater eventually than if considerable friction is present. The curves of Fig. 3 show that friction has no appreciable effect during the first few cycles. Theoretically, this would be expected since the major part of the damping results from the presence of the exponential factor, $e^{-\frac{c}{2m}t}$, in the mathematical expression for displacement. When t is small, this factor differs but little from unity, and, therefore, the amplitude decreases slowly. Hence, the presence of a moderate amount of viscous friction in an actual structure may not prevent the attainment of dangerous amplitudes. A critical amplitude may be reached before friction has much effect.

BEHAVIOR OF MODEL FOR DIFFERENT PERIODS OF TABLE MOTION

Theoretical curves in Fig. 4 show the behavior of the model for three different table motions. In Fig. 4(*a*), the period of the table is greater than that of the model; in Fig. 4(*b*), it is the same as in Curve 3 of Fig. 2(*b*), which is the near-resonance condition; in Fig. 4(*c*), the table period is smaller than that of the model.[3]

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[3] The mathematical expressions used in plotting the curves in Fig. 4 are similar to those developed by Professor L. S. Jacobsen, in his paper, "Vibration Research at Stanford University", *Bulletin*, Seismological Soc. of America, Vol. 19, No. 1, March, 1929.

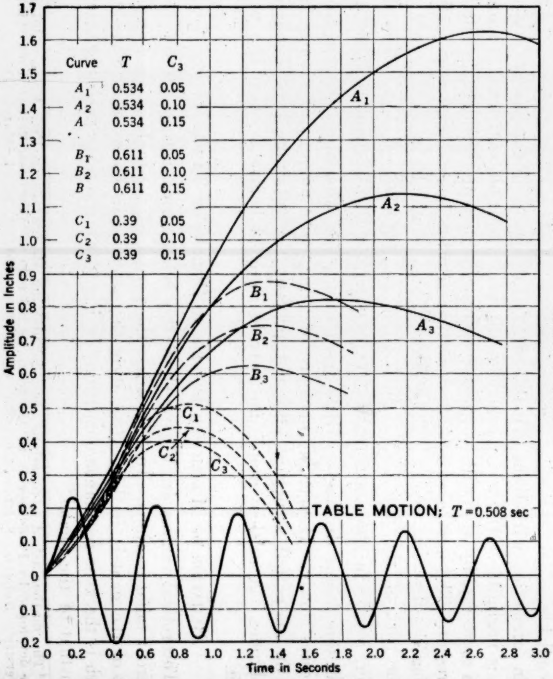

FIG. 3.—ENVELOPES OF MODEL MOTION FROM EXPERIMENTAL RECORDS.

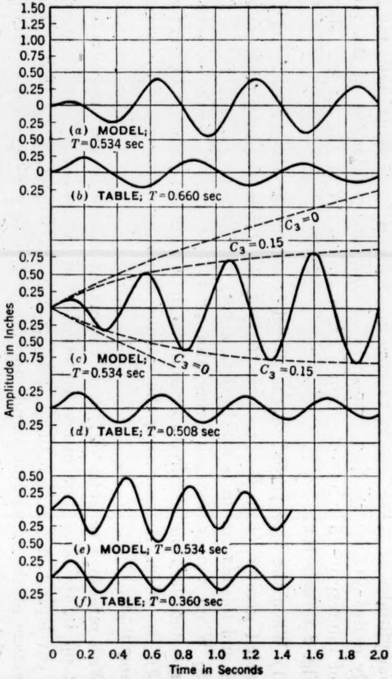

FIG. 4.—THEORETICAL BEHAVIOR OF MODEL FOR DIFFERENT PERIODS OF TABLE MOTION.

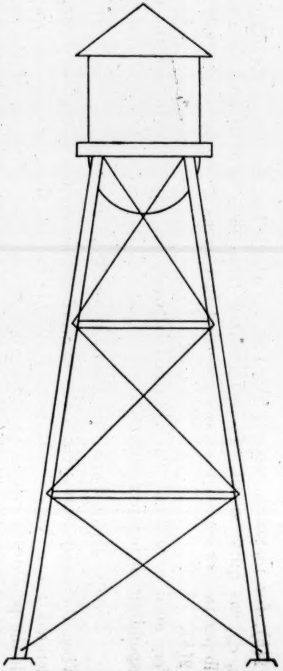

FIG. 5.—ELEVATED WATER TANK.

The result, in general, is the same as for the actual tests. As the system is removed from the near-resonance condition, the maximum amplitude is less than it is for that condition, and it is built up in a very few cycles.

It should be emphasized at this point that the distortion attained by the vibrating mass is directly proportional to the ground amplitude. Hence, it is entirely possible for an actual structure to be out of resonance with the ground and still be over-stressed because of the large amplitude of the ground motion.

AMPLITUDE VERSUS ACCELERATION

The preceding discussion has purposely been concerned with the amplitude of the motion rather than with the acceleration. In the final analysis, the important item is the stress created in a structure by the ground movement. If the structure does not distort with respect to the ground, obviously there is no stress. A large ground acceleration may cause large distortions and, hence, high stresses; but these stresses may not actually exist until the ground acceleration has decreased almost to zero. Moreover, the force causing the high stresses may bear no relation whatever to that fictitious force so frequently used, which equals mass times acceleration.

NOTATION

The following symbols of this paper conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials", compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932 (the numbered subscripts are consistent with those used in the reference previously cited[2]):

c_3 = viscous damping factor for the model, in pounds per inch per second.

F = frequency, in radians per second, of the elevated water tank $= \sqrt{\frac{\kappa}{m}}$; F_2 = frequency of the table or ground motion $= \sqrt{\frac{\kappa_2}{m_2}}$; F_3 = frequency of the model $= \sqrt{\frac{\kappa_3}{m_3}}$.

G = maximum amplitude of table or ground motion, in inches.

g = acceleration due to gravity, in inches per second^2.

m = the effective mass of the elevated water tank $= \frac{w}{g}$, in pounds-second^2 per inch; m_3 = mass of the model.

r = a subscript denoting "resonance condition."

T = period, in seconds.

t = time, in seconds.

V_0 = initial absolute velocity, in inches per second, when $t = 0$.

W = total weight.

Y = displacement with respect to the ground, in inches.

y = absolute deflection, deformation of spring, or amplitude of the center of gravity of a structure, in inches; y_0 = absolute displacement of the ground, in inches.

elastic constant, or spring factor for the elevated water tank; $_2$ = spring factor for the shaking-table; κ_3 = spring factor for the model, in pounds per inch.

THEORETICAL BEHAVIOR OF AN ELEVATED WATER TANK

The model experiments show that the amplitude of motion builds up rapidly but, since the model is not intended to show stress similitude, the experiments do not indicate at what point the distortion might become critical. The behavior of a 125-ft, 50 000-gal elevated water tank is investigated mathematically with this in mind. A sketch of the structure is shown in Fig. 5. For purposes of the investigation, certain assumptions are made:

(1) It is assumed that the tank is filled with water, but only 50% of this water acts as a rigid mass in affecting the motion of the structure. This admittedly is a rough approximation. The value was selected because, at the time the investigation was begun, it seemed consistent with available data. More recent studies, together with records[4] obtained by the United States Coast and Geodetic Survey, indicate that it is probably low and that the original discrepancies between computed and actual periods were due largely to other factors. If the expressions relating to the instantaneous shock properties of water in a tank, as given by Professors Hoskins and Jacobsen[5] are applied to a rectangular tank having the same cross-sectional area and containing the same volume of water as the tank in question, a value of 78% is obtained. Professor Hoskins also computed that for shock approximately 75% of the water would act as a rigid mass in a cylindrical flat-bottom tank, 19 ft in diameter and 17.5 ft high, the same dimensions as the cylindrical portion of the water tank. The effect of a considerable error in the assumed value of 50% is discussed subsequently.

(2) In order to simplify the problem, it is assumed that the gravitational surge of water from side to side in the tank does not materially affect the behavior of the structure during the first few cycles. Tests made by the U. S. Coast and Geodetic Survey indicate that the vibrating tank tower water system may be thought of as a coupled system and that, for certain characteristic depths of water in a tank, the resulting oscillations of the system are affected considerably by the water surges even during the first few seconds.[4] Experiments made at the Massachusetts Institute of Technology with an elevated tank tower model show a "cushioning effect" which has been attributed to water surge.[6] The latter tests indicate that to neglect water surge is probably on the side of safety for the particular tank tower used for illustration. However, experimental evidence does not yet warrant the same assumption being made for elevated tanks of all sizes, or with water at any depth.

(3) The total effective mass, which includes 50% of the water, the tank, and its appurtenances, and one-half the supporting tower structure, is assumed as being concentrated at the walk line. (It is higher than this for the static condition, but if only one-half the water is considered effective during motion, it would seem reasonable to lower the center of gravity somewhat, and such an assumption does not materially affect the result in any event.)

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[4] "Observed Vibrations of Steel Water Towers", by D. S. Carder, *Bulletin*, Seismological Soc. of America, Vol. 26, No. 1, January, 1936, p. 69.

[5] "Water Pressure in a Tank Caused by a Simulated Earthquake", *Bulletin*, Seismological Soc. of America, Vol. 24, No. 1, January, 1934.

[6] "Changed Elevated Tank Design Required for Safety Against Earthquakes", by A. L. Brown, *Engineering News-Record*, October 4, 1934, p. 424.

(4) The structure is assumed to rest on a rigid foundation. (Actually, the elasticity of the ground under the footings will lengthen the period of the structure.)

(5) Viscous damping and constant friction are neglected. (Records obtained by the U. S. Coast and Geodetic Survey from the motion of elevated water tanks indicate the presence of only a small amount of friction. Damping ratios given by Mr. Carder[4] show that friction decreases the amplitude of each succeeding cycle about 1%.)

(6) Initial tension in the diagonal rods is neglected. (The effect of this assumption is discussed subsequently.)

Under the foregoing assumptions, the mass, m, of the system is 663 lb-sec^2 per in., and the elastic constant or spring factor, $\kappa = 12\,300$ lb per in.[7] Hence, the free period of the structure is $T = 2\pi \sqrt{\frac{m}{\kappa}} = 1.45$ sec. Let it be assumed that for a number of cycles the ground has a harmonic motion in accordance with the expression:

$$y_0 = G \sin F_2\, t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let it also be assumed that this motion starts when the time $t = 0$; that is, the motion is assumed to start with a fictitious finite velocity from the zero displacement position. The differential equation for the absolute motion of the system then becomes:[8]

$$m \frac{d^2y}{dt^2} + \kappa\,(y - y_0) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

or, substituting for y_0:

$$m \frac{d^2y}{dt^2} + \kappa\, y = \kappa\, G \sin F_2\, t \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The solution of Equation (3) is:

$$y = \frac{\kappa\, G}{\kappa - F_2^2\, m} \left(\sin F_2\, t - F_2 \sqrt{\frac{m}{\kappa}} \sin \sqrt{\frac{\kappa}{m}}\; t\right) \ldots\ldots\ldots (4)$$

For the resonance condition, this expression can be differentiated with respect to F_2. In the limit, $\sqrt{\frac{\kappa}{m}} = F_2$, and the result is:

$$y_r = 0.5\, G\, (\sin F_2\, t - F_2\, t \cos F_2\, t) \ldots\ldots\ldots\ldots\ldots (5)$$

However, this is the absolute motion of the assumed center of gravity. The displacement with respect to the ground is:

$$Y_r = y_r - y_0 = -\,0.5\, G\, (\sin F_2\, t + F_2\, t \cos F_2\, t) \ldots\ldots\ldots (6)$$

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[7] For method of computing the elastic constant see "Computation of the Vibration Period of Steel Tank Towers", by R. S. McLean and W. W. Moore, Juniors, Am. Soc. C. E.. *Bulletin*, Seismological Soc. of America, Vol. 26, No. 1, January, 1936, p. 63.

[8] See "Vibration Problems in Engineering", by S. Timoshenko, p. 13.

In Equation (6) the term, sin $F_2\ t$, will be much smaller that $F_2\ t \cos F_2\ t$. Hence, for all practical purposes, the motion can be expressed as:

$$Y_r = -0.5\ G\ F_2\ t \cos F_2\ t \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The resulting motion is shown by Curve A in Fig. 6 for the case in which the maximum amplitude of the ground displacement is 1 in. (Displacements of this order, with periods of 1 sec to 1.5 sec occurred in the Long Beach (Calif.) earthquake of March 10, 1933.) If the latter is 0.5 in., the tower displacement will be one-half that shown in Fig. 6. The maximum allowable deflection of the top of the tower, if the elastic limit in the diagonals of the top panel is not to be exceeded, is approximately 2 in. This critical value will be exceeded near the end of the first cycle of the ground motion, if the ground amplitude is 1 in., and, at the end of one and one-half cycles, if the latter is 0.5 in. The time element is too short for a moderate amount of friction to be of much help.

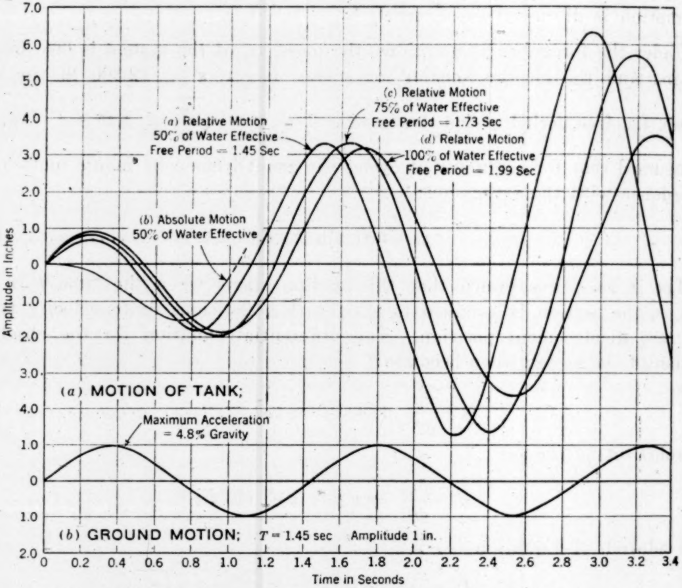

FIG. 6.—THEORETICAL BEHAVIOR OF ELEVATED TANK.

Admittedly, the foregoing quantitative results should be accepted with some reservations, since their accuracy is limited by the original assumptions. A study of the effect of possible errors in the assumptions, however, showed that the results are indicative of the probable behavior of the structure during the first few major swings of the ground. If 75% instead of 50% of the water acts as a rigid mass, the free period of the structure becomes 1.73 sec. When subjected to the ground motion shown in Fig. 6, the relative motion is as shown by Curve *C*. If 100% of the water is effective, the motion is shown by Curve *D*. The expression used in plotting the latter curves was obtained by subtracting Equation (1) from Equation (4).

The behavior of the structure was studied also for the condition in which considerable initial tension was present in all diagonal rods during the first few cycles of the ground motion. The elastic constant is increased about 70% in this case and the free period is reduced to 1.12 sec. If the ground has the same period as before (1.45 sec.), the tank motion curve is of the same type as Curve *A* in Fig. 6, but the cycles are shorter and the peak amplitudes are roughly 30% less during the first two cycles.

It should be apparent that a considerable change can be made in the free period of the vibrating system and yet not eliminate the possibility of critical distortions occurring during the first few cycles, especially if large ground amplitudes prevail. As pointed out in connection with Fig. 4, the period of the ground motion can be considerably larger or smaller than the free period of a given single-mass structure with relatively little effect on the amplitude during the first cycle, or so. This statement is further substantiated by experiments made at the Massachusetts Institute of Technology with an elevated tank tower model.[6]

If the initial tension in the diagonal rods is not large, the behavior of the structure is as follows:[4, 7] As the top of the tower deflects relative to the ground, a pair of rods in the top panel drop out of action first and the elastic constant is decreased; a further deflection, and a pair of rods in the second panel drop out of action, with a consequent additional decrease in the elastic constant. On the back-swing these changes are reversed. If a rod on one side drops out of action before the corresponding rod on the opposite side, the motion is further complicated by a twisting action, which might be serious.

The behavior of the structure after the yield point has been reached in the top panel rods is even more complicated. (During the following description, the reader will do well to bear in mind a typical stress-strain diagram for structural steel.) Thus, if initial tension is negligible, the elastic constant does not vary until a critical distortion is reached and the stress in the top panel rods exceeds the elastic limit. Then this constant is decreased to a rough average value which is about 8% of its former value and the structure becomes very flexible. If the ground motion continues with the same period and amplitude, the time required for the structure to reach the end of its outward swing under these conditions depends largely on its absolute velocity at the instant the critical distortion is reached.

As the structure starts on its back-swing, the rods again increase the elastic constant for a short time until they drop out of action because of permanent elongation. That is, during the unloading, stress is again approximately proportional to strain for a steel rod which has been permanently stretched. When the center of gravity of the structure crosses the position of static equilibrium, the unimpaired rods which resist motion in the other direction come into action. Under certain conditions, the same cycle could be repeated on that side. Once all top panel rods have been broken, the free period of the structure will be in excess of 5 sec.

There is no doubt but that the behavior of an elevated water tank beyond the critical distortion stage is an important phase of the problem. In the Long Beach earthquake of March, 1933, the top panel rods in many such tanks were badly stretched or entirely broken and yet the structures did not collapse. It is quite possible that many of the failures took place very soon after the start of the major shock and the fact that the elastic constant for such a structure no longer had a constant value but changed several times during a full swing enabled it to "weather" the remainder of the shock. Another view is that expressed by Mr. Carder.[4] He points out that, since the torsional period of most tank towers was about the same as the dominant period (0.3 sec) of the Long Beach earthquake, "it seems probable that much of the observed damage to these structures in the Long Beach region resulted from the circumstance that a large torsional motion was set up in resonance with the earth motion."

No attempt is made in this paper to plot theoretical curves for the behavior of the structure after the critical distortion has been exceeded, or to show the early behavior when there is initial tension in the diagonal rods, Any such theoretical computation should be verified by experiment.

Remarks on Behavior of Buildings

A single-story building can be expected to act as a single-mass system only if the walls are very light in comparison with the roof structure. In this case, its response to the initial shocks of an earthquake should parallel that of the model on the shaking-table. If the structure has heavy masonry walls, it usually cannot be considered a single-mass system and it is difficult to design a model that will adequately simulate its behavior.

Impact tests on a model of a multi-story building have been made by Professor Jacobsen. These tests, as yet unpublished, indicate that the distortions occurring during the first two cycles of the table motion are as large as any which occur later. Further experiment is needed, however.

Effect of Small Waves Before Major Shock

A number of waves of small amplitude usually precede the major shock. If the period of these waves is very short as compared to that of the structure the latter has very little motion, and the effect of the major shock will be practically the same as if the system had been at rest. If the preliminary waves are in resonance wih the structure for a short time, the structure will be "tuned in" and the first large swing might be disastrous.

Several theoretical curves have been plotted (see Fig. 7) which show the behavior of the elevated water tank for various starting conditions. The relative amplitude for the resonance condition can be expressed as:

$$X_r = \left(\frac{V_0}{F_2} - \frac{G}{2}\right) \sin F_2 t + \left(Y_0 - \frac{G}{2}\right) F_2 t \cos F_2 t \ldots\ldots (8)$$

in which Y_0 is the initial absolute displacement, in inches; and V_0 is the initial absolute velocity, in inches per second, at the instant when $t = 0$; that is, at the assumed starting point of the major ground displacements.

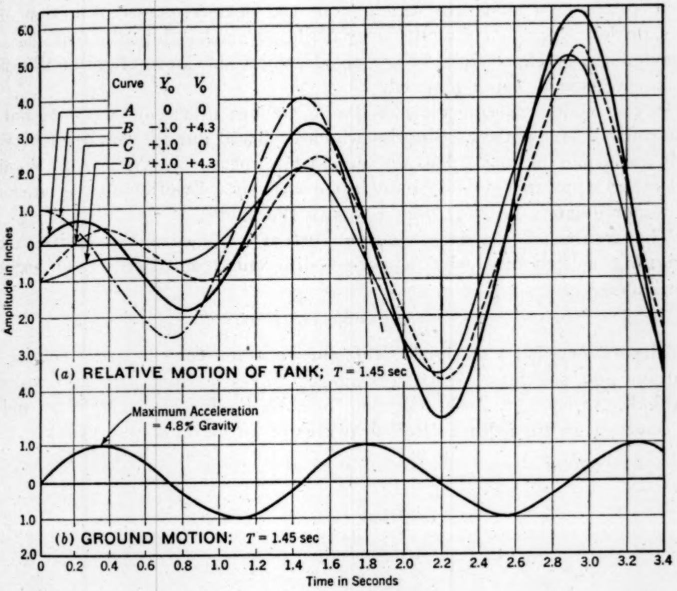

FIG. 7.—THEORETICAL BEHAVIOR OF ELEVATED TANK AT RESONANCE FOR VARIOUS STARTING CONDITIONS.

Referring to Fig. 7, Curve *A* is the same as Curve *A* in Fig. 6. The slope of this curve is not zero when $t = 0$, because of the assumption regarding ground motion. Note the slope of the curve of absolute motion in Fig. 6. It is assumed for the beginning of Curve *B* that the previous ground motion has been such that when t is assumed equal to zero, the tower has a displacement of 1 in. to the left and is moving at a velocity of 4.3 in. per sec to the right (the same direction as the ground motion on the first swing). The effect of the other starting conditions is indicated by Curves *C* and *D*.

Curves could be drawn for many other starting conditions. Some would be more and some less favorable than those shown. There is no consolation to be gained from the former because the latter are just as likely to occur. In any event, it is important to note that the rather severe starting conditions used in the example materially affected only the first one and one-half cycles. This further illustrates the fact that large ground motions and resonance are a dangerous combination at any time even if the condition prevails for only a few cycles.

CONCLUSIONS

The investigation indicates that if the ground is suddenly accelerated from rest or near rest, and then continues for a few cycles with a harmonic motion of constant period and amplitude, large displacements will be built up for a single-mass structure which is in the near-resonance condition. The tests made to date on a multi-story building model indicate that the same behavior can be expected of this type of structure. More study of this phase of the problem is being planned.

In the model experiments a moderate amount of viscous damping did not materially decrease the amplitudes which occurred during the first few cycles. This experimental result does not necessarily mean that friction in buildings and other structures should be neglected entirely. Friction in the latter may be proportionately greater than it was in the model.

The results of the investigation further emphasize the desirability of designing a structure with a period different from that of an expected earthquake.

ACKNOWLEDGMENTS

The writer wishes to thank Professor Jacobsen for the many helpful suggestions and assistance given during this investigation. He also wishes to thank L. A. Elsener, Assoc. M. Am. Soc. C. E., for his suggestions and for his courtesy in furnishing detail drawings of an elevated water tank.

DISCUSSION

Arthur C. Ruge,[9] Assoc. M. Am. Soc. C. E. (by letter).—Engineers who have been complacently designing "quake-proof" structures by the statical method will find much to disturb their peace of mind in Professor Williams' excellent paper. The importance of considering the dynamics of structures is made clear, and the author has experimentally demonstrated the validity of theoretical analyses in a manner that should satisfy the most skeptical.

The author's attitude toward interpreting the results obtained from experiments in simple harmonic motion is well taken. To carry the interpretation beyond the first one or two cycles of the assumed simple ground motion is meaningless from the engineering standpoint and, as the results show, is unnecessary in the case of elevated water tanks. The writer was led to substantially the same conclusions that Professor Williams has stated by means of experiments of quite a different character, in which the water motion in the tank was not neglected.

There is only one statement in the paper (see "Conclusions") which seems to the writer to be open to question: "The results of the investigation further emphasize the desirability of designing a structure with a period different from that of an expected earthquake."

Unintentionally, this statement is misleading because there is no known single period or narrow range of periods to be expected in an earthquake, and because the practical problems of avoiding even the known earthquake periods are quite difficult in the case of tall tank structures.

If, for the sake of argument, one assumes that there exists in an earthquake a motion having the simple form expressed by Equation (1), in which the amplitude, G, is 1 in., and the period, $\frac{2\pi}{F_2}$, is 1.45 sec (exactly in resonance with the tank), then Professor Williams has shown that critical stresses will be reached during the first cycle of the ground motion. Now, suppose it is decided to strengthen this structure in order to destroy the condition of resonance. Let the new stiffness be twice the original stiffness (which means the addition of 8 or 10 tons of steel to the framework); the new natural period of vibration will be $\frac{1.45}{\sqrt{2}}$, or 1.02 sec.

Fig. 8 shows the deflections of the stiffened structure as compared with the curve for the original structure in Fig. 7 (shown dotted in Fig. 8). It is seen that critical stresses are still reached during the first cycle of the ground motion. Hence, although the stiffening has limited the deflections, it has not prevented overstressing of the structure. Once the elastic limit has been exceeded, the subsequent behavior of the structure is largely a matter of chance and the calculated deflections become fictitious (that is, after $t = 1.25$ sec).

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[9] Research Assoc. in Seismology, Dept. of Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

Now, suppose the earthquake motion contains, somewhere in it, one wave of 1-sec period with a 1-in. amplitude, as well as one wave of 1.5-sec period with a 1-in. amplitude (both of which are entirely possible). Doubling the tower stiffness would then have absolutely no beneficial effect and, even if the stiffness of the original structure were to be quadrupled, critical stresses would still be reached, and the stiffening would not have achieved its purpose. Furthermore, since the wave periods and amplitudes cannot possibly be predicted in advance for an actual earthquake, it clearly depends purely upon chance whether a moderate amount of stiffening will increase or decrease the safety of a given tank structure.

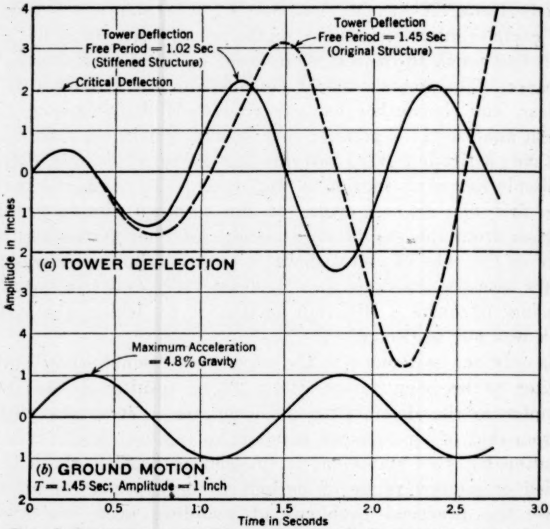

FIG. 8.—EFFECT OF DOUBLING THE TOWER STIFFNESS TO AVOID RESONANCE.

Measurements of actual tank-tower periods made[10] by the U. S. Coast and Geodetic Survey clearly illustrate how little is accomplished in avoiding known earthquake periods by moderate strengthening of structures in the 60 000 to 100 000-gal class. These excellent data should be of inestimable value in analyzing failures in future earthquakes.

The author rightly points out that consideration of the behavior of elevated water tanks beyond the critical distortion stage is of importance in attempting to explain many of the near-failures in the Long Beach, Calif., earthquake. In this phase of the problem chance again enters to a great extent. Even given some simple analytical method of attack, there is little justification for applying mathematical analysis very far beyond the yield point because of the strong probability of one side of the tower failing before the other side, which means almost certain collapse of the structure if the strong earthquake motions have not ceased by that time.

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[10] *Bulletin*, Seismological Soc. of America, January, 1936.

The Long Beach earthquake was indeed violent, but the most violent part was over in a few seconds. What might have happened to the many near-failures had there been a few more seconds of strong shaking can only be conjectured. There is reason to believe that the strong-motion phase of the San Francisco earthquake of 1906 was of longer duration than that of the Long Beach shock.

H. M. ENGLE,[11] ESQ. (by letter).—The Long Beach, Calif., earthquake of March 10, 1933, stimulated interest in the behavior of structures of various kinds. Elevated tank towers were adversely affected in many cases (although only three collapsed), and considerable attention has been focused on this particular type of structure. Due to the symmetry and structural simplicity of design, these towers lend themselves to model experiments and to theoretical attempts to explain what happened as well as to the determination of what the proper means of safeguard may be.

There has been a tendency on the part of those interested in shaking-table experiments and so-called "dynamic design" to consider the behavior of these structures as somewhat of a mystery, requiring complicated model study and theoretical experimentation. The writer has had intimate contact with the investigation, re-bracing, and repair of perhaps one hundred of such structures and also of a number of new towers. It is his belief: (1) That there is no particular mystery involved; (2) that a reasonable explanation of the damage can be made; and (3) that design to give assured safety against earthquakes is neither difficult nor economically impossible. Furthermore, no radical change in the proportions of towers, method of bracing, erection procedure, or past methods of design are necessary even if design in the past has been based on a static method of analysis. At present, there is no definite proof that the static method of analysis involves any more assumptions or uncertainties than the so-called dynamic method. A static analysis of damaged towers shows that they should have failed where failure occurred. There should be no particular mystery about a structure being damaged under action of forces of a magnitude it was never designed to resist. The writer does not mean to infer that all tall towers of standardized design were wrecked or fatally damaged. Considering that they were braced for wind loads only, their behavior was surprisingly good, and not such as to warrant the idea that the entire conception of their design is faulty for earthquake resistance. They are the result of long years of development and, contrary to some expressed opinions, are skillfully detailed and proportioned not only for economy, but also to utilize the metal involved to its fullest extent.

Following the Long Beach shock of March 10, 1933, the writer inspected in detail a large number of tank towers, some in the Long Beach area and many in other parts of California, which had never been subjected to a severe shock. It was found that the towers had usually been erected with the bracing rods rather loose. Conversation with the erection foremen brought out the fact that in some cases this was done purposely as it was considered desirable that the tower be somewhat limber. How this idea originated the writer was unable to discover. In some cases the towers were considerably out of line. It is probable that correction of this defect alone would have reduced, somewhat, the damage to such structures in the Long Beach area. It is easy to see that, where loose, the rods would pick up their loads with a jerk. The tank would surge around on the tower, with possible heavy impact. Furthermore, twisting of the tank on the tower would be possible. Proper rigging with rods having good initial tension would reduce these secondary effects and would give a better chance for the structure to develop its strength effectively.

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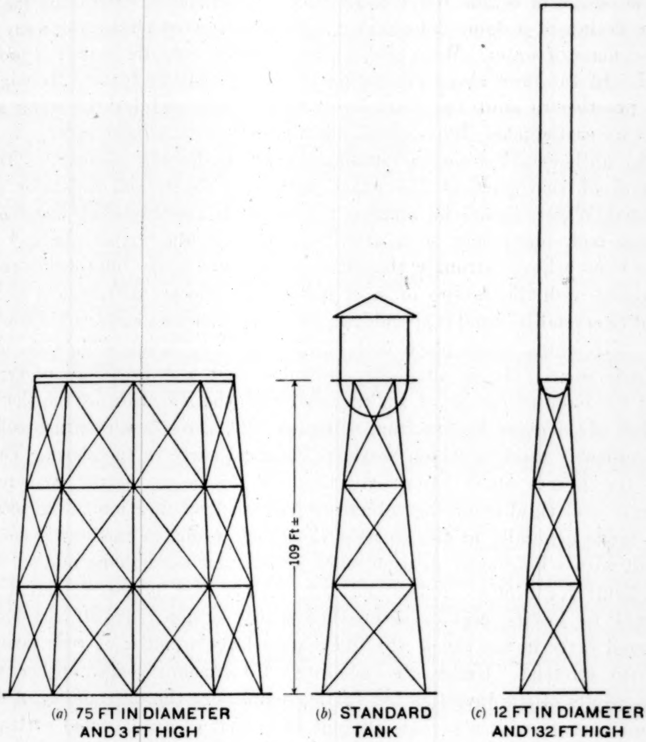
[11] Engr., Board of Fire Underwriters of the Pacific, San Francisco, Calif.

In the past most tanks have been built with hemi-spherical bottoms and a body about as high as, or somewhat higher than, the diameter. The tower legs are attached at the junction of the bottom and the cylindrical shell. The center of the mass of the tank and water is, therefore, a considerable distance above the column-to-tank connection. Under lateral movement the tank would tend to rotate in a vertical plane about the top of the tower. With wave action developed in the tank appreciable stresses could be developed in the tower and bracing. With loose rods this tendency to rotate on top of the tower might add additional impact. Tanks with ellipsoidal bottoms have this defect in a less degree, and the writer believes their more general use in earthquake zones would be desirable.

In regard to tank design and behavior a number of problems may be discussed. Consider three tanks, each with a capacity of 100 000 gal. Let one tank be 75 ft in diameter and 3 ft high, with a flat bottom (see Fig. 9(*a*)). It could probably be shaken indefinitely without any appreciable lateral forces due to weight of water being developed. Friction between tank and water would be small. No appreciable wave action could develop. It would be much like a popcorn shaker. Consider another tank 12 ft in diameter and 132 ft high (see Fig. 9(*c*)). Under shock probably the entire mass of water, or most of it, would have to be moved. Consider as a third tank the common standard design approximately 22 ft in diameter and 24 ft high (see Fig. 9(*b*)). The action of this tank would probably be intermediate. A certain proportion of the water would probably move with the tank, and appreciable wave action could develop. A strictly theoretical analysis of one of these towers to resist earthquake is probably impossible, being a complicated problem in wave action combined with various inertia effects, all produced by a chaotic earth motion impossible, as yet, to forecast. Perhaps Nature's shaking-table experiments on full-sized structures are the best guide. Actual disaster experience where properly interpreted is a fairly good foundation for basing future practice. One tower near Long Beach known to have been designed for a lateral force of 10% of the weight was undamaged in the shock of March 10, 1933. A static method of analysis was used with the lateral force applied at the center of gravity of the tank.

Proponents of dynamic design have laid considerable emphasis on the natural period of structures, generally assuming that there are certain dangerous and predominate natural ground periods which should be avoided. A period of from 1 sec to 1.5 sec has often been considered to be particularly dangerous and one to be avoided. As far as the writer can discover there is no certain basis for this belief. It is not concurred in by all outstanding seismologists. Periods from a fraction of a second to several minutes have been recorded. For instance, both the Long Beach and Helena (Mont.) shocks had natural periods of only a fraction of a second. It would appear that the range of natural periods of structures is such that it is nearly impossible to dodge the possibility of a comparable natural ground period.



(*a*) 75 FT IN DIAMETER AND 3 FT HIGH (*b*) STANDARD TANK (*c*) 12 FT IN DIAMETER AND 132 FT HIGH

FIG. 9.

Resonance is often dwelt on in connection with behavior of structures. The danger from resonance has probably been exaggerated. The chaotic and changing nature of the earth motion alone takes away some of the danger. It is probable that the combination of tank and water has no set natural period of vibration. The movement of the water in the tank probably has a tendency to break up the natural period of the tank and water as measured under the slight movement produced by moderate wind pressure. It is also a proved fact, and one consistent with structural design, that as a structure is damaged or loosened in an earthquake its natural period changes. Few structures are put together so that some loosening might not occur in a severe earthquake even if the resulting damage was minor. Any change in a structure's natural period during an earthquake would have a tendency to break up resonance.

The design of a tank tower to resist an earthquake is essentially the same as the design of a dam. The tank is alternately moved into and away from a fluid mass of water. What part of the water is actually moved is problematical. In the tank also wave action is a complicating factor. It might be good practice to omit the roofs from these tanks and let the water splash out in an earthquake. Such splashing might act as a safety valve.

The author makes several statements which deserve comment. Toward the end of that part of the paper entitled "Theoretical Behavior of an Elevated Water Tank" he states: "It is quite possible that many of the failures took place very soon after the start of the major shock * * *". The writer believes strongly that such action was quite improbable and not consistent with the failure of steel tension members. Although a steel rod might be crystallized and snap suddenly, in most instances such would not be the case. Stretching takes place first and it takes an appreciable time to jerk the rods apart. It is altogether probable that the breakage occurred at about the close of the heavy shaking and that the cessation of the shock was all that saved many towers from collapse. The three towers which collapsed were probably those in which breakage occurred early in the shock. The fact that the towers stood after extensive rod breakage speaks well for the strength and rigidity of the column-to-tank shell connection. The idea that rods broke generally at the start of the shock seems to the writer so inconsistent with a knowledge of usual steel failure and earthquake motion, that it is difficult for him to understand how it was ever proposed, except as an attempt to justify certain assumed dynamic action. If all the breakage occurred early in the shock why did such a large number of rods take "time out" to stretch? Under the heading "Conclusions", the author states: "The results of the investigation further emphasize the desirability of designing a structure with a period different from that of an expected earthquake." Who is going to forecast what type the expected earthquake will be, or how much energy will be released, or exactly how the varying types of soil in a district will respond? Who will guarantee that the period of the grand motion will be 0.2 sec, or 1.5 sec? Who will guarantee what time of year the shock may occur and whether the soil in a district is saturated and lubricated from long rains, or well drained after a long dry period? It seems to the writer that when some one can foretell what type the expected earthquake will be, that assured earthquake prediction will be achieved. Some competent seismologists doubt whether such exact prediction will ever be possible. It certainly is not possible now.

Many of the measurements of natural periods of vibration of structures are of no practical value whatever unless closely correlated with the known features of structural design. There has been ample disaster experience to show that, as yet, no practical substitute has been evolved for ample and assured strength and unity of structure.

MERIT P. WHITE,[12] JUN. AM. SOC. C. E. (by letter).—To engineers of certain regions the question of earthquake effects is most important. Although Professor Williams' paper does not pretend to furnish rules for the earthquake-resistant design of structures, nevertheless it does offer, to a certain extent, something which all engineers should possess before undertaking any such design, namely, an understanding of what happens to a structure subjected to an earthquake shock. The following facts (which are either stated in this paper or may be easily verified) seem especially important:

(1) Moderate viscous friction has no great effect in reducing the maximum deflections occurring in the early part of the motion (the effect of solid friction was not investigated);

(2) Amplitudes are built up very rapidly in the first stages of the motion;

(3) The region of dangerous frequencies (causing large deflections) is somewhat wider for suddenly applied simple harmonic motion than for established simple harmonic motion. (Fig. 10 shows the calculated relation between the maximum deflections of any one-story structure during one cycle or several cycles of suddenly applied simple harmonic ground motion ($Y_0 \sin F\, t$); also, for comparison, it shows the maximum deflections due to established simple harmonic motion, and $\frac{F}{F_2}$ (in distorted scale, horizontally), the ratio of the ground-motion frequency to the natural frequency of the structure);

(4) An earthquake acts not as a force, but as an irresistible motion; furthermore, the stresses produced in a structure by an earthquake are dependent not only on the ground motion (or on the maximum ground acceleration) but on the dynamical properties of the structure as well; and,

(5) It can be shown that for a given amplitude and number of cycles of suddenly applied simple harmonic ground motion the maximum deflection of a structure such as the one considered herein depends only on the so-called resonance ratio, $\frac{F}{F_2}$ (see Fig. 10).

From Item (5) it follows that an attempt to strengthen such a structure will affect its maximum deflections (and, therefore, its maximum stresses) only indirectly, through changing the stiffness and thus the ratio, $\frac{F}{F_2}$. This change is quite likely to increase deflections instead of decreasing them. An increase will result if $\frac{F}{F_2}$ approaches unity.

In view of the care which was probably taken in constructing the shaking-table and elevated tank model used in the author's experiments, it is not surprising that the model results should agree so well with those obtained from theory. It would be very surprising to the writer if this were not the case.

In discussing the effect of small waves preceding a major shock the author makes a relatively simple matter unnecessarily complicated. Whatever the preliminary waves and whatever the major shock, the effect of each alone as a function of time may be computed. Then, these deflections can be simply superimposed to give the actual deflection due to the combination. In any case the preliminary waves (ceasing at the beginning of the major shock) will cause an oscillation of frequency, F_2, and of amplitude, D_0, the latter (in the notation of the paper) being expressed as:

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[12] Asst. Engr., SCS, Iowa Inst. for Hydr. Research, Iowa Univ., Iowa City, Iowa.

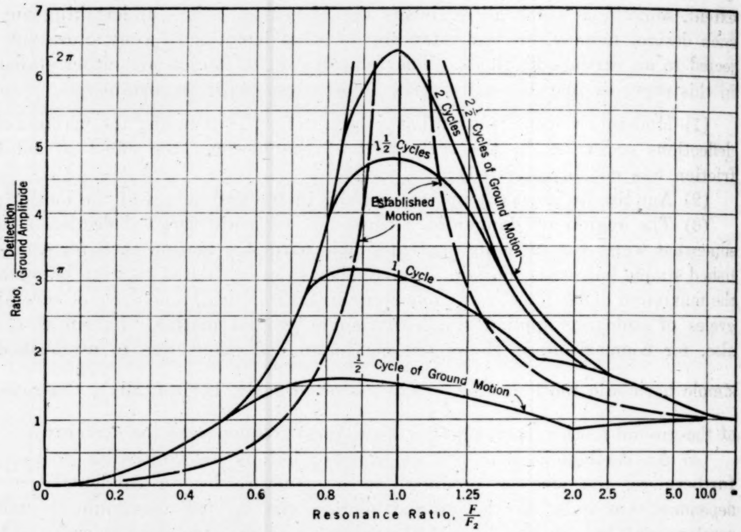

FIG. 10.—MAXIMUM DEFLECTIONS OF A SYSTEM OF ONE DEGREE OF FREEDOM, SUBJECTED TO SUDDENLY APPLIED SIMPLE HARMONIC DISPLACEMENTS AND DEFLECTIONS OCCURRING IN THE CASE OF ESTABLISHED SIMPLE HARMONIC MOTION.

$$D_0 = \sqrt{Y^2_0 + \left(\frac{V_0}{F_2}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

This oscillation can increase or decrease the maximum amplitudes which would result from the major shock alone by amounts not greater than D_0. What the effect will be, will depend on the phase difference between maxima of the motion due to the preliminary shock and those due to the major shock.

L. S. JACOBSEN,[13] ESQ. (by letter).—If a vibrating system with one degree of freedom experiences a transitory ground motion of constant period, but of variable amplitude, the dynamic distortions depend mainly on the ratio of the period of the ground to the one of the system. This fact has been demonstrated conclusively by Professor Williams. The paper shows by model experiments that for the case of near resonance large dynamic distortions of the system are produced even by a few oscillations of the ground.

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[13] Prof. of Mech. Eng., Stanford Univ., Stanford University, Calif.

The primary object of this discussion is to emphasize the absolute necessity of considering the dynamic properties of structures if a rational understanding of the problem of earthquake resistant design is to be evolved in the future. The secondary object of the discussion is to supply information concerning the dynamic distortions of a multiple-mass system as exemplified by a model of a 17-story building.

It is true that, at present, very few engineers can afford to make searching studies of the dynamic properties of the structures they design. Time limitations demand that so-called "rules of thumb" must be used for assuming the magnitudes of the dynamic loads. Because of this insufficient and imperfect knowledge of ground motions that occur during earthquakes, it is obvious that designers are not justified in making any specific assumptions in regard to the periods, durations, and intensities of the composite motions. Perhaps definite probable periods exist in particular localities, or perhaps the entire question of probable ground motions is so vague that no one can afford to risk making assumptions concerning the specific motions. Whatever their attitudes are toward the ground-motion question, they are forced to admit that forward and backward, as well as up and down, motions occur. This fact alone means that dynamic loadings varying with time must be sustained by the structures.

Professor Williams attempts to show the effects of a few specific ground motions in a horizontal direction on a single mass structure. It is of importance to note that the ground motion must be specific; otherwise, an elementary analysis is impossible. Moreover, if the ground motion were not specific and simple, it would not be feasible to extend and interpret the analysis by principles of superposition to more complicated types of ground motions. It is true, however, that when the results of the simple system and simple ground motion are applied to actual problems, the quantitative aspect of the analysis is lost and only the qualitative remains. It is precisely here that engineering judgment is necessary.

How is sound engineering judgment concerning seismic loadings to be formed? Naturally, one must admit that actual field evidence has given, and still gives, the first and indisputable clue as to what "stands up" and what "falls down", or what is seriously damaged. Under no circumstances can one afford to minimize the value of the expensive lessons given by "Nature's own laboratory." It is well known, however, that a group of engineers visiting the stricken region after an earthquake can, and does, find abundant evidence for supporting many different individual points of view. This is especially so if the damage is considerable so that the wreckage is largely in the form of heaps of bricks, concrete, tiles, and steel. Too many observers of earthquake damage lack something which is of extreme importance for the formation of sound engineering judgment, namely, a "pattern of characteristic behavior" of simple systems subjected to simple, although perhaps improbable, types of ground motion. Before it is possible to estimate how a complicated phenomenon has occurred one must have not only a picture, but also a quantitative picture of the simple phenomenon. Professor Williams has attempted to give such a "pattern of behavior", for which he deserves credit.

Since the customary practice of engineers is to design structures to resist fictitious, horizontal, static loadings expressed in percentages of the static, vertical loads, or in terms of horizontal, constant accelerations of the ground, it has occurred to the writer that a slight re-arrangement of the author's results may show more clearly the relation of the truly dynamic loadings to the fictitious, static loadings. It must be remembered that, in Professor Williams' experiments, the dynamic loadings have been inferred from the actually observed dynamic distortions; they are real loads, and, therefore, can not be explained away by the all too frequently heard remarks that such dynamic loads will not have time to become real loads and have the same effect as static loads.

Results obtained from Fig. 3 of the paper have been re-arranged as shown in Table 1, with the following supporting data:

(1) The properties of the model are: Weight, 25.5 lb; spring factors, 17.3, 9.2, and 7.1 lb per in.; friction factors, 0.05, 0.10, and 0.15 lb per in. per sec; and free vibration periods, 0.390, 0.534, and 0.611 sec.

(2) For ground motion, or shaking-table motion: The period is 0.505 sec; the duration of motion is about 5.5 sec; the maximum impact acceleration is about 14% of gravity; the maximum harmonic acceleration is about 8.5% of gravity; the maximum displacement is about 0.25 in.; and, the constant friction reduces the amplitude of the table by approximately 0.022 in, per cycle.

TABLE 1.—RE-ARRANGEMENT OF RESULTS OBTAINED FROM FIG. 3 OF THE PAPER

	BELOW RESONANCE			NEAR RESONANCE			ABOVE RESONANCE		
Ratio of model period to ground period	0.77			1.06			1.21		
Ratio of consecutive, free, damped amplitudes of model corresponding to the three values of damping used by Professor Williams	1.16	1.34	1.55	1.22	1.49	1.83	1.26	1.58	1.99
Ratio of maximum dynamic force on model to weight of model	0.38	0.30	0.27	0.58	0.41	0.29	0.24	0.20	0.17
Number of cycles of ground motion when the maximum dynamic force occurs	1.8	1.7	1.6	5.1	4.3	3.9	2.8	2.6	2.4
Ratio of maximum dynamic force to a constant fictitious force of same value as the maximum harmonic acceleration of the ground, 0.085 g times mass of the model	4.5	3.5	3.2	6.8	4.8	3.4	2.8	2.4	2.0

For the three cases investigated Table 1 shows that the ratios of the maximum dynamic forces, acting on the model as a result of the motion of the ground, are greatly in excess of the fictitious static forces which are equal to the mass of the model times the maximum, harmonic acceleration of 8.5% of gravity. In other words, the inherent vibrational properties of the model for the three cases are such that, with the minimum friction used, a

TABLE 2.—SHOWING MASS AND ELASTIC PROPERTIES OF THE SEVENTEEN-STORY MODEL (FIG. 11), TOGETHER WITH RATIOS OF EXPERIMENTAL MAXIMUM DYNAMIC SHEARS AS COMPUTED FROM THE RECORDS OF FIG. 3.

Floor No. (1)	Weight of floor, in pounds (2)	Rigidity of floor, in pounds per inch (3)	SHEAR RATIOS: $\frac{\text{Impact}}{0.32\,g}$ (4)	SHEAR RATIOS: $\frac{\text{Maximum dynamic}}{0.20\,g}$ (5)
17	3.24			
		13.6	0.19	4.95
16	2.55			
		14.6	0.31	4.15
15	2.48			
		19.2	0.29	2.90
14	2.48			
		20.5	0.22	2.25
13	2.65			
		17.6	0.38	3.00
12	2.65			
		18.6	0.62	2.80
11	2.65			
		19.7	0.41	1.70
10	2.65			
		20.8	0.25	1.40
9	2.65			
		21.8	0.31	0.65
8	2.65			
		22.8	0.60	0.90
7	2.65			
		23.8	0.34	0.55
6	2.65			
		24.9	0.28	0.55
5	2.65			
		25.9	0.28	0.80
4	2.65			
		26.8	0.38	0.90
3	2.65			
		27.8	0.31	0.90
2	2.65			
		28.8	0.39	1.15
1	2.65	∞		

force of 4.5 times the fictitious, static force results for the first case, or the "below resonance" case, whereas a force of 6.8 times the fictitious, static force is experienced by the model in the second or "near resonance" case; and, finally, a force of 2.8 times the fictitious, static force is in action for the third or "above resonance" case. If one notes that the number of cycles of ground motion necessary to establish the distortions corresponding to the foregoing dynamic forces are 1.8, 5.1, and 2.8, respectively, one is forced to conclude that the first case (or the "below resonance" case) assumes a sinister aspect since in an actual earthquake the probability of encountering a constant periodic motion lasting for 1.8 cycles is quite high. For the three cases, with the maximum friction used, the force ratios are 3.2, 3.4, and 2.0, whereas the numbers of cycles necessary for producing the distortions corresponding to the forces are 1.6, 3.9, and 2.4.

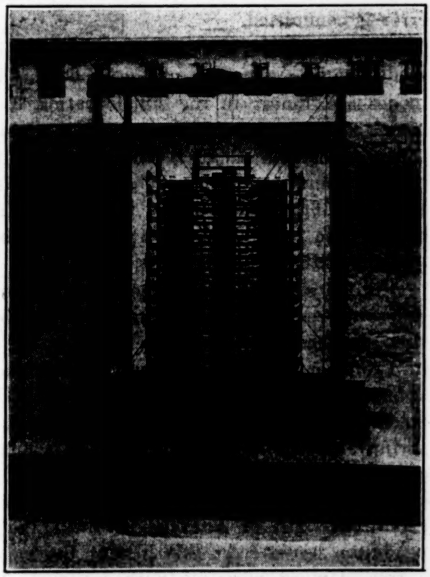

FIG. 11.—MODEL OF 17-STORY BUILDING.

Furthermore, in this case, the "below resonance" condition seems to be rather dangerous.

The first three columns of Table 2 give information concerning the mass and elastic shear properties of the 17-story model shown in Fig. 11. The model, built in 1931, was tested in 1932–33. Since the construction of this model allows only distortions due to shear and neglects those due to flexure of the building as a whole, the experiments have never been published before. It can be shown, however, that in the case of a rather stubby building, flexure of the building as a whole may be neglected, so that if the following model experiments are restricted to stubby buildings no serious error is committed.

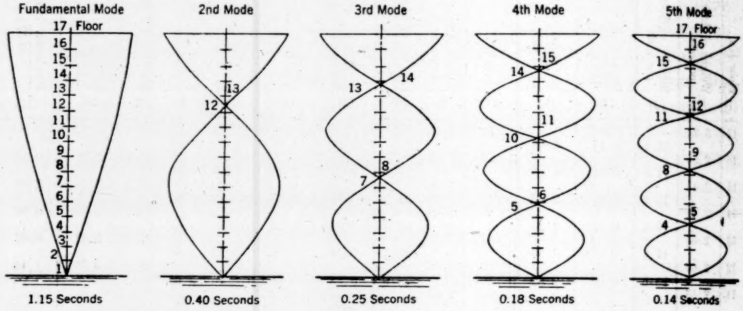

FIG. 12.

Fig. 12 shows the calculated displacement diagrams of the first five modes of free vibration of the model. The free vibration periods of these modes are almost in the simple ratios, 3, 5, 7, and 9, corresponding to a model with constant mass and shear rigidity distribution. The maximum shear distortions occur at the nodal points. Thus, for the second mode, the maximum shears occur between the 1st and 2d floor and between the 12th and 13th floor, whereas for the fifth mode, the maximum shears occur between the 1st and 2d, the 4th and 5th, the 7th and 8th, the 11th and 12th, and the 14th and 15th floors.

Fig. 13 shows sixteen time records of the dynamic forces acting between the seventeen floors of the model when the ground motion is given by the time displacement records below it. The ground motion is generated, as in Professor Williams' experiments, by the impact of a pendulum against a bumper spring on the shaking-table. During the impact interval (that is, the time during which the pendulum is in contact with the bumper spring), the ground motion may be thought of as being composed of two component motions, the primary motion of 0.15-sec period, lasting for about one-half cycle and the secondary motion of 0.46-sec period, lasting for about one-sixth of a cycle, or until the instant when contact with the pendulum ceases. At this instant the motion of the shaking-table becomes a free, damped vibration with a period of 0.47 sec, and this motion lasts for several cycles until friction in the shaking-table makes it cease. Nearly all the sixteen records in Fig. 13 give evidence of the existence of the two ground periods, 0.15, and 0.47 sec, in the dynamic force fluctuations.

The reasons for the pronounced effects are that the natural period of 0.14 sec of the model's fifth mode of vibration is in "near resonance" with the period of 0.15 sec of the primary impact motion of the shaking-table. In spite of the fact that this ground motion exists for only one-half cycle, its effect is observable throughout the records. Moreover, the secondary ground motion of 0.46-sec period, or the free vibration ground motion of 0.47-sec period, is also in "near resonance" with the model's second mode period of 0.40 sec. The effect of this "near resonance" is a building up of the dynamic forces with time, especially at the nodal points; that is, between the 1st and 2d, and between the 12th and 13th floors. Almost no dynamic forces of this period occur at the anti-nodal point between the 7th and 9th floors.

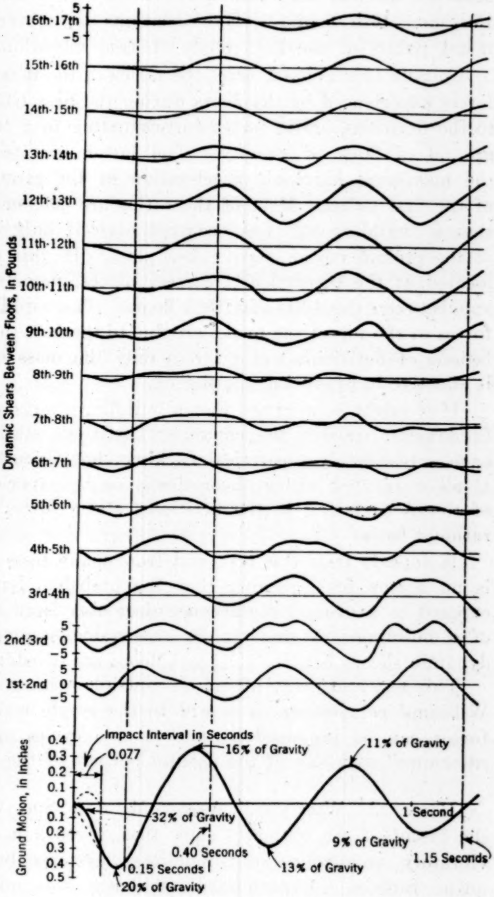

FIG. 13.

Columns (4) and (5), Table 2, give the measured, dynamic, maximum forces between adjacent floors divided by the fictitious, static forces corresponding to an assumed constant acceleration of the model. Column (4) relates to the maximum loads experienced by the floors during the first 0.15 sec of the ground motion. They are compared to the fictitious, static loads corresponding to a constant ground acceleration, equal to the maximum impact acceleration of 32% of gravity.

These ratios are all less than unity, signifying that the full effects of such a rapid ground motion are not established in the model. It should be noted, however, that the effects are maxima between the 3d and 4th, between the 7th and 8th, between the 11th and 12th, and between the 15th and 16th floors. These locations agree very well with those of the nodal points of the fifth mode of free vibration of the model.

Column (5), Table 2, gives the ratios of the maximum, "built-up", dynamic forces experienced by the floors during the first 1.15 sec of the ground motion to the fictitious, static forces corresponding to a constant acceleration of the ground of 20% of gravity. The latter gravity acceleration is equal to the maximum harmonic acceleration of the ground occurring at the peak of the first swing. Most of the ratios are greater than unity for the reason that a "building up" has occurred even if only about two complete cycles of the ground motion have taken place. In this case, also, the maxima are located at the theoretically correct places, namely, between the 1st and 2d and between the 12th and 13th floors. The rapidly increasing ratios of the forces at the top of the model are due to the fact that for the customary calculations of fictitious, static forces only the mass of the model above a floor is effective in producing the shears.

If it might be assumed that a building always were to vibrate solely in its fundamental modes, the customary fictitious static force calculations would have a reasonably theoretical backing; but since such an assumption is not at all warranted, either theoretically or by experience, the customary "rules of thumb" relating to the fictitious, static force assumptions are without a rational basis.

It appears from this test, and from many tests yet unpublished, that there is no reason for assuming that the probable dynamic shear in a building exposed to a ground disturbance decreases from the top toward the bottom of a building. A much more reasonable assumption is that the probable dynamic shears between adjacent floors are constant throughout the building.

This test and many others on multiple-mass systems corroborate Professor Williams' conclusions in regard to the single mass system. Large dynamic forces can be produced by one or two cycles of ground motion in "near resonance" with one of the natural periods of the multiple-mass system.

Harry A. Williams,[14] Assoc. M. Am. Soc. C. E. (by letter).—It was the intent of the writer to show that, given a near resonance condition, the vibratory amplitudes of single-mass systems build up to critical values quite rapidly. Experimental evidence was offered in support of the mathematical theory. The theory was then simplified and applied to an elevated water tank. This was not done with the idea that all future elevated tanks or other single-mass structures should be designed only in accordance with the simplified equation presented by the writer. It was offered rather as another approach by which an engineer might check his designs and guide his judgment.

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[14] Asst. Prof. of Civ. Eng., Stanford Univ., Stanford University, Calif.

Discussions were presented by Messrs. Ruge, Engle, White, and Jacobsen. Professor Ruge further substantiated the writer's contentions. His comments on the writer's conclusion regarding the desirability of designing a structure with a period different from that of an expected earthquake are well taken. The writer's original statement was made with the hope that in the distant future the type of local ground motions might be reasonably predictable. Only a study of the records of many future earthquakes will answer this question. Meanwhile, it is doubtful whether an elevated water-tank structure can be designed along conventional lines and not be in resonance with any dangerous ground motions. Designing the structure so that the elastic constant varies with the amplitude might be a solution to the problem. Certainly the method of adding more steel to stiffen the structure is open to the criticism offered by Professor Ruge.

Mr. Engle presents the so-called "static" point of view and in so doing raises a number of questions with which the writer finds himself in disagreement. A few of these are as follows (the quotations are from Mr. Engle's discussion):

(1) "There has been a tendency on the part of those interested in shaking-table experiments and so-called 'dynamic design' to consider the behavior of these structures [elevated water tanks] as somewhat of a mystery, requiring complicated model study and theoretical experimentation." Knowing that the behavior of a vibrating structure cannot be explained by the static method, those interested in the dynamic viewpoint are inclined to approach such a problem with some humility and study the reactions of a structure to simplified motions rather than attempt to explain its actual behavior during an earthquake by reasoning *a priori* from theories having very little, if anything, to do with the problem. Mr. Engle states, furthermore, that "there is no particular mystery involved" in the behavior of these structures, and that "a reasonable explanation of the damage can be made." Yet he asserts in a later paragraph that the problem is extremely complicated.

(2) To quote further: "At present there is no definite proof that the static method of analysis involves any more assumptions or uncertainties than the so-called dynamic method." Obviously, it is necessary to make assumptions in applying the dynamic method to practical problems. The writer made a number of these assumptions in his paper when he investigated the behavior of the elevated water tank. He also showed how there could be considerable variation in many of the assumed conditions and yet the results would remain fairly indicative of the probable behavior of the structure during a short interval of an earthquake; and he warned the reader against drawing quantitative conclusions from such an investigation. In the static method, a fictitious force is applied to the center of gravity of the structure, this force being made equal to the mass of the structure times an arbitrarily assumed acceleration. Only by sheer coincidence, as emphasized in Professor Jacobsen's discussion, will it give stresses which are indicative of the actual stresses which might be

expected. It would be just as logical a procedure to design the tank structure for a 200-mile wind. In doing this, one would be candidly admitting that he is merely applying an empirical formula to his problem. The writer, of course, is not advocating that the wind-force method be substituted for the static method. The latter is at least a step in the right direction, and it is making designers "mass conscious."

(3) "A static analysis of damaged towers shows that they should have failed where failure occurred." No doubt this was true, yet it does not justify the static method, because this method only divulges the weak points in the structure. If a collapsed tower is replaced with a structure which is identical except that the weaker points are strengthened, another earthquake of no greater intensity than the first may easily wreck the new tower.

The structural engineer has always been trained to design for strength. If a structure failed because of the weakness of a particular member, that member was strengthened in the new structure. The mechanical engineer designed in the same way until he was confronted with the development of high-speed machinery. He then discovered that it was sometimes better—as in De Laval's steam turbines and cream separators—to make a shaft more flexible rather than stronger and stiffer in order to avoid resonance and large vibrations at operating speeds. It is true that in some cases an equally acceptable result could have been attained by using a heavier shaft, but this would have required very accurate workmanship and, consequently, would have materially increased the cost of the machine. In the same manner, certain structures can be made quite safe if they are stiffened sufficiently, but this is economical and practicable only in certain cases.

(4) During the Long Beach earthquake, the top panel rods in many tank structures were badly stretched or entirely broken. The writer suggested that possibly many of these failures occurred soon after the beginning of the major shock, because of a near-resonance condition. Mr. Engle strongly disagreed with this contention, mainly because such action was "not consistent with the failure of steel tension members" since "stretching takes place first and it takes an appreciable time to jerk the rods apart." The elementary theory of dynamics and strength of materials should make it apparent that the dynamic forces take time to stretch the rods of such a structure. The length of time taken depends on the velocity of the mass center of the structure at the instant the elastic limit is reached in the top panel rods. As soon as this critical deflection is reached the tower becomes a structure with radically different dynamic and elastic properties. It may not return to its original position for several seconds. It seems to verge on the miraculous that "the cessation of the shock was all that saved many towers [with badly stretched or broken rods] from collapse"—not to mention the chimney of the Seal Beach Power Plant with broken diagonals in its supporting steel structure. It appears improbable to the writer that each tower was sub-

jected to a motion which stretched or broke its top panel rods and then stopped just in time to prevent collapse. In evaluating the effect of slack rods picking up their loads with a "jerk," it should be remembered that a flexible tower may not necessarily have very high velocities engendered by the ground motion. Moreover, the nature of the "jerk" is in reality a gradually applied elastic force.

There are a number of other points presented by Mr. Engle that could be discussed. It is agreed that the behavior of elevated tanks in recent earthquakes has been surprisingly good. Only further study and earthquake experience, combined with a rational dynamic point of view, will divulge how much present design can be improved.

The writer does not believe that the dangers from resonance or near resonance have been exaggerated. Chaotic motion does alleviate some of the danger, but evidence is reasonably conclusive that large distortions do build up rapidly, at least for single-mass systems, and records show that the ground does follow a more or less regular motion for a few cycles at a time. Shaking-table experiments made at Leland Stanford University and at the Massachusetts Institute of Technology leave little doubt but that a vessel and a part of the liquid it contains act as a single-mass system for a few cycles. Wave action, although having some "cushioning effect" at once, is most noticeable later. Mr. Engle's remarks concerning resonance apply if one is thinking of the general behavior of the elevated-tank structure throughout the major part of a shock. They do not detract from the writer's contentions.

Mr. White's emphasis and elaboration of certain points of the paper are interesting. It is important to note, as he indicates, that the region of dangerous frequencies is somewhat wider in the case of suddenly applied motion. This point further emphasizes the difficulty of avoiding near resonance by making only nominal changes in a single-mass system subject to earth shock.

The writer is very much indebted to Professor Jacobsen, not only for his comments on the paper, but also for the addition of a very desirable discussion pertaining to buildings. The last line of Table 1 shows how much in error one might be if he were guided entirely by the static method. As Professor Jacobsen points out, the dynamic forces involved for the "below resonance" condition are of a large order, and they occur very shortly after the beginning of the ground motion. In comparing the "below resonance" and "near resonance" results, the reader may be interested in the data as it is arranged in Table 3 for the tests involving the least friction only. These results show that relatively large dynamic forces are set up also for the "near resonance" condition during the first few cycles even if the maximum does not occur for some time. It will be noted that the dynamic force equals 38% of the weight of the model at the end of 1.8 ground-motion cycles in the case of "below resonance" whereas it reaches 27% in approximately the same time for the "near resonance" model. If the models represented two conventionally designed structures, there would be a question, depending on the design, as to which structure was being subjected to the more critical unit stresses. The rigidity of one structure may be double that of another whereas its critical strength is only slightly greater. In other cases, the more rigid structure may be very much stronger.

TABLE 3.—DATA OBTAINED FROM FIGS. 2 AND 3

	BELOW RESONANCE		NEAR RESONANCE		ABOVE RESONANCE	
(*a*) Ratio of model period to ground period	0.77		1.06		1.21	
(*b*) Relative values of elastic constants	2.4		1.3		1.0	
(*c*) Number of cycles of ground motion when the dynamic force occurs	1.1	1.8	1.9	2.4	1.5	2.0
(*d*) Ratio of dynamic force on model to weight of model	0.31	0.38	0.27	0.40	0.19	0.23
(*e*) Ratio of dynamic force to a constant fictitious force based on maximum harmonic acceleration of the ground, 0.085 *g* times mass of the model	3.7	4.5	3.2	4.7	2.3	2.7

The results of experiments with a 17-story building model, which Professor Jacobsen presented as a part of his discussion, further emphasize the effects of resonance, these effects becoming quite important in one or two cycles of ground motion.

It is agreed, as Mr. Engle stated, that "actual disaster experience where properly interpreted is a fairly good foundation for basing future practice." The proper interpretation of the results of "Nature's shaking-table experiments" is of the utmost importance. Although it is entirely possible that at some future time those interested in the dynamic approach will conclude that there are too many variables in most practical problems to justify the use of mathematical expressions based on the more exact dynamic theory and that an empirical approach similar to the existing static method must be resorted to, that time has not yet arrived. When and if it does arrive, those applying the empirical method will find that a familiarity with the behavior of structures when subjected to very simple motions will be an invaluable aid.

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ANALYSIS OF VIERENDEEL TRUSSES

By Dana Young,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. L. J. Mensch, A. A. Eremin, Leon Blog, A. W. Fischer, L. C. Maugh, John E. Goldberg, Louis Baes, E. C. Ingalls and Ralph B. Peck, Harold E. Wessman, A. Floris, Kimball R. Garland, J. D. Gedo, and Dana Young.

Synopsis

A Vierendeel or quadrangular truss is a hyperstatic frame composed of a series of rectangular or trapezoidal panels without diagonal members. The end connections of all members are rigid and designed to take moment.

Although it is possible to analyze such trusses by any of the so-called "exact" methods, the labor involved is prohibitive in all except the simplest cases. The purpose of this paper is to present a practical and accurate method of determining the stresses in the Vierendeel type of trusses. This method is based upon the theory of virtual work and involves a re-arrangement of the standard equations to produce a workable solution.

The interesting questions of design, economics, and field of usefulness of such trusses are not considered, the scope of this paper being limited to stress analysis.

Historical

Vierendeel trusses have been used for bridges in Europe, chiefly in Belgium, since 1896. The first bridge of this type was built at Tervueren, Belgium, by Professor Arthur Vierendeel, of the University of Louvain. Professor Virendeel is the originator of this form of truss and has been its main sponsor. To date (1936), approximately one hundred bridges of this type have been built. The longest has a span of 274 ft and carries a double-track railway. Both steel and reinforced concrete have been used and, since 1932,

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Note.—Published in August, 1936, *Proceedings*.

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several arc-welded steel trusses of this type have been built, including the trusses for a swing bridge.[2]

In addition, various other structures using this system have been constructed. Eight wireless towers, each 960 ft high and only 13 ft square in plan, with bracing of the Vierendeel type, have been built in Belgium. Towers for elevated water tanks have been similarly braced.

In the United States, Vierendeel trusses have not been used for bridges, but a number of other structures have been built with this system of bracing. The towers of the old Kinzua Viaduct of the Erie Railroad Company were of that form.[3] Various reinforced concrete viaduct bents have been built utilizing this construction. The towers of the Waldo-Hancock Suspension Bridge in Maine are braced transversely as Vierendeel trusses.[4] A modified form of Vierendeel truss was used for the foundation of the Telephone Building[5], at Albany, N. Y.

Introduction to Stress Analysis

In the general type of Vierendeel truss, with n panels and $(n + 1)$ web members, there are $3(n + 1)$ unknowns. It is necessary to obtain an equal number of equations to solve the problem. The equations of statics furnish three relations and the remainder are found from the elastic properties of the truss.

Any of the standard methods of elastic analysis may be used. In books on structural theory methods of solution will be found for a few limited cases, using the principles of least work[6], virtual work[7], slope deflection[7], and moment distribution.[8] However, in all except the simplest cases, the application of these methods is laborious.

The aim of the writer is to describe a method of analysis which is less laborious to apply. It is based upon the principles of virtual work and consists in re-arranging the fundamental equations, and the order of their solution. For trusses with chords that are symmetrical about a longitudinal axis (such as parallel chord trusses and viaduct bents), the solution is exact; that is, it involves no assumptions except those that are inherent in the elastic theory. For other trusses it is necessary to make use of certain simplifying assumptions. In most cases the relative error thus introduced is negligible.

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[2] "Vierendeel Truss Bridges Popular in Belgium", by L. C. Rucquoi, M. Am. Soc. C. E., *Engineering News-Record*, July 25, 1935, p. 116.

[3] "The Kinzua Viaduct of the Erie Railroad Company", by C. R. Grimm, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVI (1901), pp. 21–77.

[4] "Building First Long Span Bridge in Maine", by D. B. Steinman, M. Am. Soc. C. E., and C. H. Gronquist, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, March 17, 1932, p. 386.

[5] "Foundation for Albany Telephone Building", by G. W. Glick, *Engineering News-Record*, November 27, 1930, p. 836.

[6] "Elastic Energy Theory", by J. A. Van den Broek, M. Am. Soc. C. E., John Wiley & Sons, New York, pp. 81–87.

[7] "Modern Framed Structures", Pt. II, by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, New York, 1926.

[8] "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., John Wiley & Sons, New York, pp. 236–238; also, see, "A Rapid and Concise Method of Analyzing Rigid Viaduct Bents", by L. C. Maugh, Assoc. M. Am. Soc. C. E., *Engineering News-Record*, March 14, 1935, p. 379.

The first part of the analysis follows, in general outline, the method developed by Professor Vierendeel.[9] However, in order to obtain greater accuracy, this analysis does not involve all the simplifying assumptions that Professor Vierendeel uses. As a result, the detailed treatment and final formulas are different.

Notation.—The letter symbols in this paper are introduced in the text as they occur and are summarized for reference in Appendix I. An effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering and Testing Materials"[10], compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

ANALYSIS OF TRUSSES WITH SYMMETRICAL CHORDS

By a truss with symmetrical chords is meant one in which the chords are symmetrical about a longitudinal axis, with the corresponding chord members having equal moments of inertia. This class includes parallel-chord trusses and viaduct bents with inclined posts.

For the present, deformations due to axial stress will be neglected. Subsequently, the analysis will be extended to include this effect. Fig. 1 represents a general form of such a truss. From the symmetry of the truss, it is seen that the bending moments and shears in the upper and lower chords will be equal. It also follows that the point of contraflexure of each vertical member will be at its mid-height.

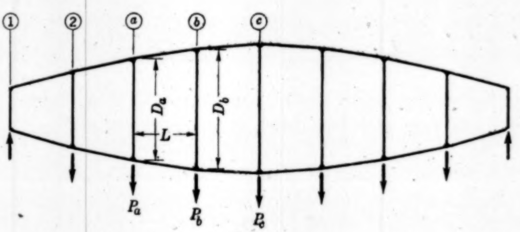

FIG. 1.

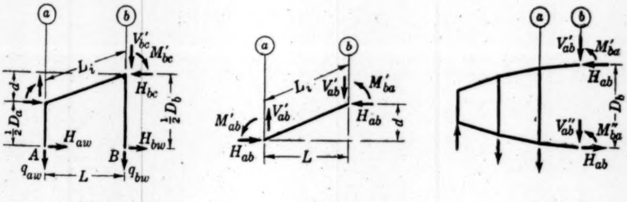

FIG. 2.

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[9] "Cours de Stabilité des Construction", Tome IV, by Arthur Vierendeel, Louvain, 1920, and succeeding editions. This text contains Professor Vierendeel's approximate method of solution and a bibliography on the subject. The references in the bibliography are mainly to European publications, few of which are available to engineers in the United States.

[10] A. S. A.—Z10*a*—1932.

Consider a typical panel, *ab*, removed from the truss and cut into two parts by a horizontal plane through the centers of the vertical members. Let Δ_x = horizontal deflection; L_i = the length of an inclined chord member; H_w = transverse shear in a web member; H_{ab} = horizontal component of axial stress in Chord Member *a-b*; D_a = depth of truss at Panel Point *a*, etc.; V'_{ab} = vertical shear in the upper chord between Panel Points *a* and *b*, etc.; M''_{bc} = bending moment at Point *b* in the lower chord member, *b-c* (that is, the member extending between Panel Points *b* and *c*), etc. The forces acting on the upper part are shown in Fig. 2(*a*). For this section the horizontal displacement of Point *B* with respect to Point *A* is:

$$E\,\Delta_x = \int \frac{M\;y}{I}\,ds = \frac{1}{I_b}\int_0^{0.5D_b} H_{bw}\,y^2\,dy + \frac{1}{I_C}\int_0^{L_i}\left[H_{bw}\left(\frac{D_b}{2} - y\right) - M'_{bc}\right.$$

$$\left. - (V'_{bc} + q_{bw})\,x + H_{bc}\,y\right]\left(\frac{D_b}{2} - y\right)dL_i + \frac{1}{I_a}\int_0^{0.5D_a} - H_{aw}\,y^2\,dy \;\ldots(1)$$

The values of the first and third integrals of Equation (1) are:

$$\int_0^{0.5D_b} H_{bw}\,y^2\,dy = \frac{H_{bw}\,y^3}{3}\Bigg]_0^{0.5D_b} = \frac{H_{bw}\,D_b^3}{24} \;\ldots\ldots\ldots\ldots(2a)$$

and,

$$\int_0^{0.5D_a} - H_{aw}\,y^2\,dy = \frac{H_{aw}\,y^3}{3}\Bigg]_0^{0.5D_a} = -\frac{H_{aw}\,D_a^3}{24} \;\ldots\ldots\ldots(2b)$$

The forces acting on the upper chord are shown in Fig. 2(*b*). Comparing these forces with the forces in Fig. 2(*a*), it is obvious that:

$$H_{ab} = H_{bc} - H_{bw} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3a)$$

$$V'_{ab} = V'_{bc} + q_{bw} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3b)$$

and,

$$M'_{ba} = \frac{H_{bw}\,D_b}{2} - M'_{bc} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3c)$$

Substitute Equations (3) in the second integral of Equation (1) and perform the integration. In the resulting expression substitute $d = \frac{1}{2}\,(D_b - D_a)$ and $H_{ab} = \sum_1^a H_w$, and this integral finally becomes:

$$\int_0^{L_i} (M'_{ba} - V'_{ab}\,x + H_{ab}\,y)\left(\frac{D_b}{2} - y\right)dL_i = \frac{M'_{ba}\,L_i}{4}\,(D_b + D_a)$$

$$- V'_{ab}\,\frac{L_i\,L}{12}\,(D_b + 2\,D_a) + \frac{L_i}{24}\,(D_b^2 + D_b\,D_a - 2\,D_a^2)\sum_1^a H_w \ldots(4)$$

The integral values in Equations (2*a*), (2*b*), and (4) substituted in Equation (1), give Equation (5) directly, as follows:

$$E\,\Delta_x = \frac{D_b^3\,H_{bw}}{24\,I_b} - \frac{D_a^3\,H_{aw}}{24\,I_a} + \frac{L_i\,M'_{ba}}{4\,I_C}\,(D_a + D_b) - \frac{L_i\,L\,V'_{ab}}{12\,I_C}\,(D_b + 2\,D_a)$$

$$+ \frac{L_i}{24\,I_C}\,(D_b^2 + D_b\,D_a - 2\,D_a^2)\sum_1^a H_w \;\ldots\ldots\ldots\ldots(5)$$

From symmetry, it is seen that $\Delta_x = 0$. Now, consider that part of the entire truss which is to the left of Panel Point b, as shown in Fig. 2(c). Taking moments about Panel Point b:

$$M_b - M'_{ba} - M''_{ba} - H_{ab}\,D_b = 0 \dots\dots\dots\dots\dots (6)$$

and, since $M'_{ba} = M''_{ba}$ and $H_{ab} = \sum_{1}^{a} H_w$, Equation (6) becomes:

$$M'_{ba} = \frac{1}{2}\,M_b - \frac{1}{2}\,D_b \sum_{1}^{a} H_w \dots\dots\dots\dots\dots (7)$$

It is obvious that:

$$V'_{ab} + V''_{ab} = 2\,V'_{ab} = V_{ab} \dots\dots\dots\dots\dots (8)$$

Substituting these values for M'_{ba} and V'_{ab} in Equation (5) and collecting terms:

$$E\,\Delta_x = 0 = \frac{D^3_b\,H_{bw}}{24\,I_b} - \frac{D^3_a\,H_{aw}}{24\,I_a} + \frac{L_1\,M_b}{8\,I_C}\,(D_a + D_b) - \frac{L_1\,L\,V_{ab}}{24\,I_C}\,(D_b + 2\,D_a)$$
$$- \frac{L_1}{12\,I_C}\,(D^2_b + D_b\,D_a + D^2_a) \sum_{1}^{a} H_w \dots\dots\dots\dots\dots (9)$$

Solving for H_{bw}:

$$H_{bw} = \frac{D^3_a\,I_b\,H_{aw}}{D^3_b\,I_a} + \frac{2\,L_1\,I_b}{D^3_b\,I_C}\,(D^2_b + D_b\,D_a + D^2_a) \sum_{1}^{a} H_w - \frac{3\,L_1\,I_b\,M_b}{D^3_b\,I_C}\,(D_a + D_b)$$
$$+ \frac{L_1\,L\,I_b\,V_{ab}}{D^3_b\,I_C}\,(D_b + 2\,D_a) \dots\dots\dots\dots\dots (10)$$

For parallel chords and constant, I, Equation (10) reduces to:

$$H_{bw} = H_{aw} + 6\,\frac{L}{D} \sum_{1}^{a} H_w - \frac{6\,L}{D^2}\,M_b + \frac{3\,L^2}{D^2}\,V_{ab} \dots\dots\dots (11)$$

Equation (10) is the general equation for trusses with symmetrical chords. All the coefficients involved are constants which depend only on the dimensions of the truss. Hence, Equation (10) is really of the simple form:

$$H_{bw} = C_1\,H_{aw} + C_2 \sum_{1}^{a} H_w + C_3\,M_b + C_4\,V_{ab} \dots\dots (12)$$

and since M_b and V_{ab} are constants which depend on the external loads, Equation (12) may be expressed in the simpler form:

$$H_{bw} = C_1\,H_{aw} + C_2 \sum_{1}^{a} H_w + C_5 \dots\dots\dots\dots\dots (13)$$

An expression of the form of Equation (13) may be written for each panel in the truss. Then, by successive substitutions, each value of H_w may be expressed in terms of the shear, H_{1w}, for the first vertical. Since the sum of all the H_w-values is equal to zero, it is a simple matter to solve for the value of H_{1w} and thence for the other values of H_w. The following two examples illustrate this method of solution.

Example 1.—Parallel Chord Truss.—Consider a four-panel, parallel chord truss with a height of 10 ft and a span of 40 ft, carrying 1 000 lb at each panel point, as shown in Fig. 3. The moment of inertia of all members is assumed to be the same. Substituting numerical values for this particular case in Equation (11):

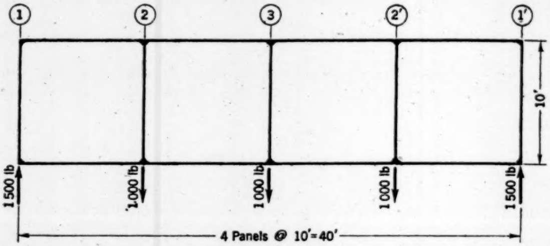

Fig. 3.

$$H_{bw} = H_{aw} + 6 \sum_{1}^{a} H_w - 0.6\, M_b + 3\, V_{ab} \dots\dots\dots\dots (14)$$

It should be noted that Equation (14) remains the same for any loading, since the coefficients depend only on the truss dimensions. The values for M_b and V_{ab} for this particular loading are, respectively: $M_2 = 15\,000$ ft-lb; $M_3 = 20\,000$ ft-lb; $V_{12} = 1\,500$ lb; and $V_{23} = 500$ lb. Substituting these values in Equation (14):

$$H_{1w} = H_{1w} \dots\dots\dots\dots\dots\dots\dots\dots (15a)$$

$$H_{2w} = H_{1w} + 6\, H_{1w} - 9\,000 + 4\,500 = 7\, H_{1w} - 4\,500 \dots\dots (15b)$$

and,

$$H_{3w} = H_{2w} + 6\,(H_{1w} + H_{2w}) - 12\,000 + 1\,500 \dots\dots\dots (15c)$$

Substituting Equation (15*b*) in Equation (15*c*):

$$H_{3w} = 55\, H_{1w} - 42\,000 \dots\dots\dots\dots\dots\dots (16)$$

From the symmetry of the loading in this particular case, it is obvious that $H_{3w} = 0$; hence:

$$H_{3w} = 55\, H_{1w} - 42\,000 = 0 \dots\dots\dots\dots\dots (17)$$

Solving for H_{1w}: $H_{1w} = \dfrac{42\,000}{55} = 763.6$ lb, and, thence, $H_{2w} = 7\ (763.6) - 4\,500 = 845$ lb.

Example 2.—Viaduct Bent.—As a second example, a bent of the Kinzua Viaduct will be studied. This particular structure has been analyzed by Mr. C. R. Grimm[8], using the method of least work, and by Messrs Johnson, Bryan, and Turneaure[11], using the general method of virtual work. Their solutions afford a check on the method presented herein.

Fig. 4 shows the dimensions of the bent and the lateral loads to be considered. The bases of the posts are assumed to be fixed. This is equivalent to assuming that the shear, H_{6w}, in the bottom strut is zero. Equation (10) applies in this problem. Evaluating the coefficients in this equation by substituting the numerical values of the truss dimensions, the following series of equations is obtained from Equation (10):

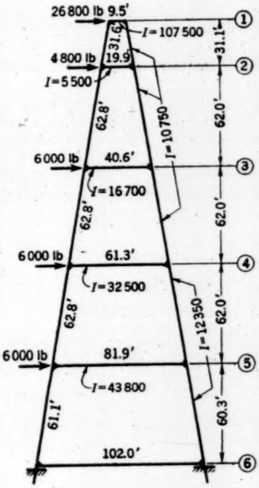

FIG. 4.

$$H_{1w} = H_{1w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(18}a\text{)}$$

$$H_{2w} = 0.00557\ H_{1w} + 2.77\ H_{1w} - 0.1810\ M_2 + 2.50\ V_{12} \ldots\ldots \text{(18}b\text{)}$$

$$H_{3w} = 0.358\ H_{2w} + 8.30\ (H_{1w} + H_{2w}) - 0.264\ M_3 + 7.26\ V_{23} \ldots\ldots \text{(18}c\text{)}$$

$$H_{4w} = 0.565\ H_{3w} + 13.02\ (H_{1w} + H_{2w} + H_{3w}) - 0.252\ M_4 + 7.29\ V_{34} \ldots \text{(18}d\text{)}$$

$$H_{5w} = 0.565\ H_{4w} + 12.52\ (H_{1w} + H_{2w} + H_{3w} + H_{4w}) - 0.1745\ M_5 + 5.14\ V_{45} \ldots\ldots\ldots \text{(18}e\text{)}$$

and,

$$H_{6w} = 0 = 1.043\ H_{5w} + 21.0\ (H_{1w} + H_{2w} + H_{3w} + H_{4w} + H_{5w}) - 0.227\ M_6 + 6.60\ V_{56} \ldots\ldots\ldots \text{(18}f\text{)}$$

Equations (18) are true for any loading. For the loading in Example 2, M_b and V_{ab} have the following values: M_2 = 835 000 ft-lb; M_3 = 2 798 000; M_4 = 5 128 000; M_5 = 7 830 000; M_6 = 10 825 000; V_{12} = 26 800 lb; V_{23} = 31 600; V_{34} = 37 600; V_{45} = 43 600; and V_{56} = 49 600. Substituting these numerical values in Equations (18):

$$H_{1w} = H_{1w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(19}a\text{)}$$

$$H_{2w} = 0.00557\ H_{1w} + 2.77\ H_{1w} - 84\,100 \ldots\ldots\ldots\ldots \text{(19}b\text{)}$$

$$H_{3w} = 0.358\ H_{2w} + 8.30\ (H_{1w} + H_{2w}) - 509\,000 \ldots\ldots\ldots \text{(19}c\text{)}$$

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[11] "Modern Framed Structures", Pt. II, by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, New York, 1926, pp. 353–354.

$$H_{4w} = 0.565\ H_{3w} + 13.02(H_{1w} + H_{2w} + H_{3w}) - 1\,018\,000 \tag{19d}$$

$$H_{5w} = 0.565\ H_{4w} + 12.52\ (H_{1w} + H_{2w} + H_{3w} + H_{4w}) - 1\,142\,000 \tag{19e}$$

and,

$$0 = 0.0497\ H_{5w} + 1.00\ (H_{1w} + H_{2w} + H_{3w} + H_{4w} + H_{5w}) - 101\,300 \tag{19f}$$

To this point, slide-rule calculations have been used; but now the computations must be carried to more places. This is not false accuracy. It is an arithmetical procedure which is necessary because of the subtractions that occur in solving the equations.

By successive substitutions, each H_w may be expressed in terms of H_{1w}; thus:

$$H_{1w} = H_{1w} \tag{20a}$$

$$H_{2w} = 2.776\ H_{1w} - 84\,100 \tag{20b}$$

$$H_{3w} = 32.33461\ H_{1w} - 1\,237\,138 \tag{20c}$$

$$H_{4w} = 488.4292\ H_{1w} - 18\,919\,500 \tag{20d}$$

$$H_{5w} = 6\,843.200\ H_{1w} - 265\,245\,500 \tag{20e}$$

and,

$$0 = 7\,707.8473\ H_{1w} - 298\,770\,300 \tag{20f}$$

Solving Equation (20*f*) to determine H_{1w}: $H_{1w} = \frac{298\,770\,300}{7707.847}$ $= 38\,761.83$. Substituting this value of H_{1w} in Equations (20*a*) to (20*e*) supplies the remaining values of H_w, thus: $H_{2w} = 23\,500$ lb; $H_{3w} = 16\,200$; $H_{4w} = 12\,900$; and, $H^5{}_w = 9\,400$.

The corresponding values, as found by Johnson, Bryan, and Turneaure, are: $H_{1w} = 38\,900$; $H_{2w} = 23\,400$; $H_{3w} = 16\,300$; $H_{4w} = 12\,400$; and $H_{5w} = 9\,900$. It is seen that these values are in close agreement with those computed by the writer. The differences are due to the slide-rule calculations of the coefficients in this Example.

ANALYSIS OF TRUSSES WITH INCLINED UPPER CHORDS

Trusses with inclined upper chords and horizontal lower chords are generally the most desirable type for bridges. The exact analysis of this form is more complicated than the case of the symmetrical chords. However, by making a few reasonable assumptions a satisfactory solution may be obtained. Fig. 5(*a*) represents a Vierendeel truss with an inclined upper chord. Consider a typical panel, *a–b*, removed from the truss and cut into two parts by sections through the top of each vertical. The forces acting on each part are shown in Fig. 5(*b*) and Fig. 5(*c*).

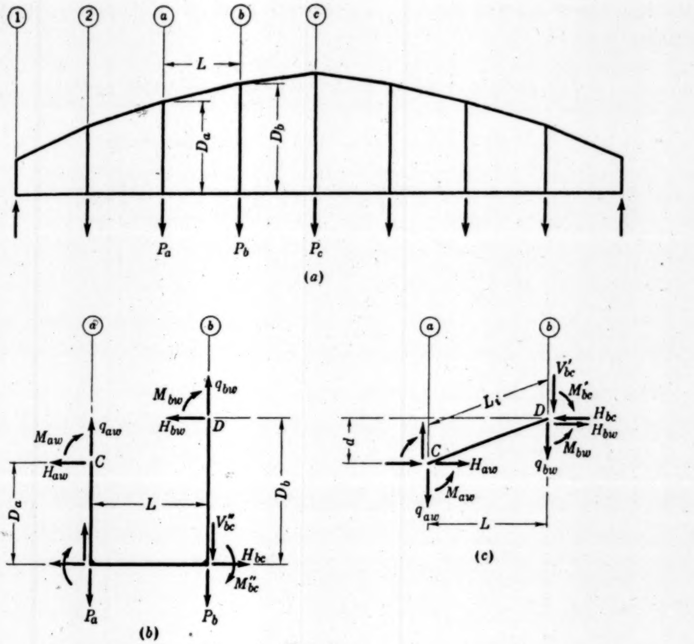

①
②
ⓐ
ⓑ
ⓒ
L
D_a
D_b
P_a
P_b
P_c
(a)
M_{bw}
q_{bw}
H_{bw}
D
M_{aw}
q_{aw}
H_{aw}
C
L
V''_{bc}
H_{bc}
M''_{bc}
(b)
V'_{bc}
M'_{bc}
L_i
H_{bc}
H_{bw}
M_{bw}
q_{bw}
d
H_{aw}
M_{aw}
q_{aw}
(c)

FIG. 5.

Considering the lower part, as shown in Fig. 5(*b*), the horizontal displacement of Point *D* with respect to Point *C* is:

$$E\,\Delta''_x = E\,\theta_C\,(D_b - D_a) + \int \frac{M\,y}{I}\,ds = E\,\theta_C\,(D_b - D_a)$$

$$+ \frac{1}{I_b}\int_0^{D_b} (M_{bw} - H_{bw}\,y)\,y\,dy + \frac{1}{I''_C}\int_0^{L} \Big[M_{bw} + M''_{bc} + (V''_{bc} + P_b$$

$$- q_{bw})\,x - H_{bw}\,D_b \Big]\,D_b\,dx + \frac{1}{I_a}\int_0^{D_a} \Big[H_{aw}\,(D_a - y) - M_{aw} \Big]\,(D_b - y)\,dy \,..(21)$$

The value of the first integral in Equation (21) is:

$$\int_0^{D_b} (M_{bw} - H_{bw}\,y)\,y\,dy = M_{bw}\,\frac{D^2_b}{2} - H_{bw}\,\frac{D^3_b}{3} \,\ldots\ldots\ldots(22)$$

From statics, it is evident that (see Fig. 5(*b*) and Fig. 6(*b*)):

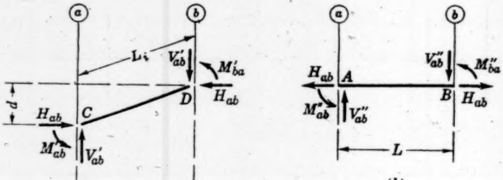
$$M''_{ba} = H_{bw}\,D_b - M_{bw} - M''_{bc} \ldots\ldots\ldots\ldots\ldots(23a)$$

and,

$$V''_{ab} = V''_{bc} + P_b - q_{bw} \ldots\ldots\ldots\ldots\ldots\ldots(23b)$$

Making these substitutions in the second integral of Equation (21) and integrating:

$$\int_0^L (V''_{ab}\, x - M''_{ba})\, D_b\, dx = V''_{ab}\, \frac{D_b\, L^2}{2} - M''_{ba}\, D_b\, L \quad \ldots\ldots(24)$$



(b)

FIG. 6.

The value of the third integral is:

$$\int_0^{D_a} [H_{aw}(D_a - y) - M_{aw}]\,(D_b - y)\, dy = H_{aw}\, \frac{D_a^2}{6}\,(3\, D_b - D_a) - M_{aw}\, \frac{D_a}{2}\,(2\, D_b - D_a) \quad \ldots\ldots(25)$$

Equations (22), (24), and (25), substituted in Equation (21) give:

$$E\, \Delta''_x = E\, \theta_C\,(D_b - D_a) - \frac{D_b^3}{3\, I_b}\, H_{bw} + \frac{D_b^2}{2\, I_b}\, M_{bw} + \frac{L^2\, D_b}{2\, I''_C}\, V''_{ab} - \frac{D_b\, L}{I''_C}\, M''_{ba} + \frac{D_a^2}{6\, I_a}\,(3\, D_b - D_a)\, H_{aw} - \frac{D_a}{2\, I_a}\,(2\, D_b - D_a)\, M_{aw} \quad \ldots\ldots(26)$$

The forces acting on the upper chord, as shown in Fig. 5(*c*), may be replaced by the equivalent system shown in Fig. 6(*a*). Then, for the upper chord, the horizontal displacement of Point D with respect to Point C is:

$$E\, \Delta'_x = E\, \theta_C\,(D_b - D_a) + \int \frac{M\, y}{I}\, ds = E\, \theta_C\,(D_b - D_a) + \frac{1}{I'_C} \int_0^{L_i} (V'_{ab}\, x - H_{ab}\, y - M'_{ba})\, y\, dL_i \quad \ldots\ldots(27)$$

From the geometry of the truss:

$$x = \frac{L}{d}\, y \quad \ldots\ldots(28a)$$

and,

$$d\, L_i = \frac{L_i}{d}\, dy \quad \ldots\ldots(28b)$$

Substituting Equations (28) in the integral of Equation (27), changing the limits of integration, and integrating:

$$\int_0^{L_i} (V'_{ab}\, x - H_{ab}\, y - M'_{ba})\, y\, dL_i = V'_{ab}\, \frac{L\, L_i\, d}{3} - H_{ab}\, \frac{L_i\, d^2}{3} - M'_{ba}\, \frac{L_i\, d}{2} \quad \ldots(29)$$

Substituting $d = D_b - D_a$ and $H_{ab} = \sum\limits_{1}^{a} H_w$, Equation (29) becomes:

$$V'_{ab}\,\frac{L\,L_1}{3}\,(D_b - D_a) - \sum_{1}^{a} H_w\,\frac{L_1}{3}\,(D_b - D_a)^2 - M'_{ba}\,\frac{L_1}{2}\,(D_b - D_a)\,.\,(30)$$

Equation (30) substituted in Equation (27) gives:

$$E\,\Delta'_x = E\,\theta_C\,(D_b - D_a) + \frac{L_1\,L}{3\,I'_C}\,(D_b - D_a)\,V'_{ab} - \frac{L_1}{2\,I'_C}\,(D_b - D_a)\,M'_{ba}$$

$$- \frac{L_1}{3\,I'_C}\,(D_b - D_a)^2 \sum_{1}^{a} H_w \ldots\ldots\ldots\ldots\ldots\ldots(31)$$

Equating Equations (26) and (31):

$$0 = -\frac{D^3_b}{3\,I_b}\,H_{bw} + \frac{D^2_a}{6\,I_a}\,(3\,D_b - D_a)\,H_{aw} + \frac{D^2_b}{2\,I_b}\,M_{bw} - \frac{D_a}{2\,I_a}\,(2\,D_b - D_a)\,M_{aw}$$

$$+ \frac{L^2\,D_b}{2\,I''_C}\,V''_{ab} - \frac{L\,L_1}{3\,I'_C}\,(D_b - D_a)\,V'_{ab} - \frac{D_b\,L}{I''_C}\,M''_{ba} + \frac{L_1}{2\,I'_C}\,(D_b - D_c)\,M'_{ba}$$

$$+ \frac{L_1}{3\,I'_C}\,(D_b - D_a)^2 \sum_{1}^{a} H_w \ldots\ldots\ldots\ldots\ldots\ldots(32)$$

Referring to Fig. 5(*b*), and Fig. 6(*b*) (which show the forces acting on the lower chord), the vertical deflection of Point D with respect to Point C is:

$$E\,\Delta''_y = -E\,\theta_C\,L + \frac{1}{I''_C}\int_0^L (M''_{ba} - V''_{ab}\,x)\,x\,dx$$

$$+ \frac{1}{I_a}\int_0^{D_a} (M_{aw} - H_{aw}\,y)\,L\,dy \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

Integrating Equation (33):

$$E\,\Delta''_y = -E\,\theta_C\,L + \frac{L^2}{2\,I''_C}\,M''_{ba} - \frac{L^3}{3\,I''_C}\,V''_{ab} + \frac{L\,D_a}{I_a}\,M_{aw} - \frac{L\,D^2_a}{2\,I_a}\,H_{aw}\,..(34)$$

Referring to Fig. 6(*a*), the vertical deflection of Point D with respect to Point C is:

$$E\,\Delta'_y = -E\,\theta_C\,L + \frac{1}{I'_C}\int_0^{L_1} (M'_{ba} - V'_{ab}\,x + H_{ab}\,y)\,x\,dL_1 \ldots(35)$$

Integrating Equation (35):

$$E\,\Delta'_y = -E\,\theta_C\,L + \frac{L\,L_1}{2\,I'_C}\,M'_{ba} - \frac{L^2\,L_1}{3\,I'_C}\,V'_{ab} + \frac{L\,L_1}{3\,I'_C}\,(D_b - D_a)\,H_{ab}\,..(36)$$

Equating Equations (34) and (36):

$$0 = -\frac{L\,D^2_a}{2\,I_a}\,H_{aw} + \frac{L\,D_a}{I_a}\,M_{aw} - \frac{L^3}{3\,I''_C}\,V''_{ab} + \frac{L^2\,L_1}{3\,I'_C}\,V'_{ab} + \frac{L^2}{2\,I''_C}\,M''_{ba}$$

$$- \frac{L\,L_1}{2\,I'_C}\,M'_{ba} - \frac{L\,L_1}{3\,I'_C}\,(D_b - D_a) \sum_{1}^{a} H_w \ldots\ldots\ldots\ldots\ldots(37)$$

Referring to Fig. 5(*b*), the angular displacement of Point D with respect to Point C is:

$$E\,\Delta''\,\theta = \int \frac{M}{I}\,ds = \frac{1}{I_b}\int_0^{D_b} (M_{bw} - H_{bw}\,y)\,dy + \frac{1}{I''_C}\int_0^{L} (V''_{ab}\,x - M''_{ba})\,dx + \frac{1}{I_a}\int_0^{D_a} (H_{aw}\,y - M_{aw})\,dy \;\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Integrating Equation (38):

$$E\,\Delta''\,\theta = \frac{D_b}{I_b}M_{bw} - \frac{D^2_b}{2\,I_b}H_{bw} + \frac{L^2}{2\,I''_C}V''_{ab} - \frac{L}{I''_C}M''_{ba} + \frac{D^2_a}{2I_a}H_{aw} - \frac{D_a}{I_a}M_{aw} \cdot (39)$$

Referring to Fig. 6(*a*), the angular displacement of Point D with respect to Point C is:

$$E\,\Delta'\,\theta = \int \frac{M}{I}\,ds = \frac{1}{I'_C}\int_0^{L_1} (V'_{ab}\,x - H_{ab}\,y - M'_{ba})\,dL_1 \;\ldots . (40)$$

Integrating Equation (40):

$$E\,\Delta'\,\theta = \frac{L\,L_1}{2\,I'_C}\,V'_{ab} - \frac{L_1}{2\,I'_C}\,(D_b - D_a)\,H_{ab} - \frac{L_1}{I'_C}\,M'_{ba} \;\ldots\ldots (41)$$

Equating Equations (39) and (41):

$$0 = -\frac{D^2_b}{2\,I_b}\,H_{bw} + \frac{D^2_a}{2\,I_a}\,H_{aw} + \frac{D_b}{I_b}\,M_{bw} - \frac{D_a}{I_a}\,M_{aw} + \frac{L^2}{2\,I''_C}\,V''_{ab} - \frac{L\,L_1}{2\,I'_C}\,V'_{ab} - \frac{L}{I''_C}\,M''_{ba} + \frac{L_1}{I'_C}\,M'_{ba} + \frac{L_1}{2\,I'_C}\,(D_b - D_a)\sum_1^{a} H_w \;\ldots\ldots (42)$$

Equations (32), (37), and (42) are the fundamental expressions for a typical panel. A set of these formulas could be written for each panel and the series solved simultaneously to find the unknowns. Such a procedure, although possible, would be too laborious for practical use. However, by transforming these fundamental equations, as described subsequently, a workable solution may be obtained.

Solution for the H_w-Values.—First, the terms involving M_{aw} and M_{bw} may be eliminated from Equations (32), (37), and (42). This can be accomplished by multiplying Equation (37) by $\frac{1}{2\,L}$ $(D_a - D_b)$; thus:

$$0 = -\left(\frac{D^3_a}{4} - \frac{D^2_a\,D_b}{4}\right)\frac{H_{aw}}{I_a} + \left(\frac{D^2_a}{2} - \frac{D_a\,D_b}{2}\right)\frac{M_{aw}}{I_a} - \left(\frac{L^2\,D_a}{6} - \frac{L^2\,D_b}{6}\right)\frac{V''_{ab}}{I''_C} + \left(\frac{L\,L_1\,D_a}{6} - \frac{L\,L_1\,D_b}{6}\right)\frac{V'_{ab}}{I'_C} + \left(\frac{L\,D_a}{4} - \frac{L\,D_b}{4}\right)\frac{M''_{ba}}{I''_C} - \left(\frac{L_1\,D_a}{4} - \frac{L_1\,D_b}{4}\right)\frac{M'_{ba}}{I'_C} - \frac{L_1}{6\,I'_C}\,(D_b - D_a)\,(D_a - D_b)\sum_1^{a} H_w \;\ldots\ldots\ldots\ldots\ldots (43)$$

Multiplying Equation (42) by $\frac{D_b}{2}$:

$$0 = -\frac{D^3_b}{4 I_b} H_{bw} + \frac{D_b D^2_a}{4 I_a} H_{aw} + \frac{D^2_b}{2 I_b} M_{bw} - \frac{D_a D_b}{2 I_a} M_{aw} + \frac{L^2 D_b}{4 I''_C} V''_{ab}$$

$$-\frac{L L_1 D_b}{4 I'_C} V'_{ab} - \frac{L D_b}{2 I''_C} M''_{ba} + \frac{L_1 D_b}{2 I'_C} M'_{ba} + \frac{L_1}{I'_C} (D_b - D_a) \frac{D_b}{4} \sum_1^a H_w \text{. (44)}$$

Subtracting the sum of Equation (43) and (44) from Equation (32) gives Equation (45), as follows:

$$0 = -\frac{D^3_b}{12 I_b} H_{bw} + \frac{D^3_a}{12 I_a} H_{aw} + \frac{L^2}{12 I''_C} (D_b + 2 D_a) V''_{ab} + \frac{L L_1}{12 I'_C} (D_b + 2 D_a) V'_{ab}$$

$$- \frac{L}{4 I''_C} (D_b + D_a) M''_{ba} - \frac{L_1}{4 I'_C} (D_b + D_a) M'_{ba}$$

$$+ \frac{L_1}{12 I'_C} (2 D^2_a - D^2_b - D_a D_b) \sum_1^a H_w \text{ .... (45)}$$

Equation (45) may be simplified by substituting various statical relations. For equilibrium of the upper chord (see Fig. 6(*a*)), it is seen that:

$$V'_{ab} L - H_{ab} d = M'_{ab} + M'_{ba} \text{ .... (46a)}$$

For equilibrium of the lower chord (see Fig. 6(*b*)), it is seen that:

$$V''_{ab} L = M''_{ab} + M''_{ba} \text{ .... (46b)}$$

Let $\frac{M'_{ab} + M'_{ba}}{M''_{ab} + M''_{ba}} = \alpha$ and note that $H_{ab} = \sum_1^a H_w$; then, dividing Equation (46*a*) by Equation (46*b*):

$$V'_{ab} = \frac{d}{L} \sum_1^a H_w + \alpha V''_{ab} \text{ .... (47)}$$

Substituting $V'_{ab} = V_{ab} - V''_{ab}$ and $V''_{ab} = V_{ab} - V'_{ab}$ in Equation (47):

$$V'_{ab} = \frac{\alpha V_{ab}}{1 + \alpha} + \frac{\frac{d}{L} \sum_1^a H_w}{1 + \alpha} \text{ .... (48a)}$$

and,

$$V''_{ab} = \frac{V_{ab}}{1 + \alpha} - \frac{\frac{d}{L} \sum_1^a H_w}{1 + \alpha} \text{ .... (48b)}$$

Now, consider that part of the truss to the left of Panel Point b, as shown in Fig. 7. For equilibrium it is obvious that:

$$M_b - M'_{ba} - M''_{ba} - H_{ab} D_b = 0 \quad \text{.............. (49)}$$

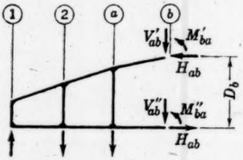


FIG. 7.

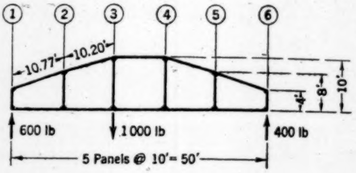


FIG. 8.

Let $\dfrac{M'_{ba}}{M''_{ba}} = \beta$ and note that $H_{ab} = \sum\limits_{1}^{a} H_w$; then, solving Equation (49), first for M'_{ba} and then for M''_{ba}:

$$M'_{ba} = \frac{\beta M_b}{1 + \beta} - \frac{\beta D_b \sum\limits_{1}^{a} H_w}{1 + \beta} \quad \text{.............. (50a)}$$

and,

$$M''_{ba} = \frac{M_b}{1 + \beta} - \frac{D_b \sum\limits_{1}^{a} H_w}{1 + \beta} \quad \text{.............. (50b)}$$

For reference purposes, designate the seven terms in Equation (45) by the letters, a to g, in order, from left to right. Substituting Equation (48b) in Term (c) gives:

$$\frac{L^2}{12 I''_C} (D_b + 2 D_a) V''_{ab} = \frac{L^2 (D_b + 2 D_a)}{12 I''_C (1 + \alpha)} V_{ab} - \frac{L (D_b + 2 D_a)(D_b - D_a)}{12 I''_C (1 + \alpha)} \sum_{1}^{a} H_w \quad \text{............ (51c)}$$

Substituting Equation (48a) in Term (d), gives:

$$\frac{L L_t (D_b + 2 D_a)}{12 I'_C} V'_{ab} = \frac{\alpha L L_t (D_b + 2 D_a)}{12 I'_C (1 + \alpha)} V_{ab} + \frac{L_t (D_b + 2 D_a)(D_b - D_a)}{12 I'_C (1 + \alpha)} \sum_{1}^{a} H_w \quad \text{........... (51d)}$$

Substituting Equation (50*b*) in Term (*e*) gives:

$$-\frac{L}{4\,I''_C}\,(D_b + D_a)\,M''_{ba} = -\frac{L\,(D_b + D_a)}{4\,I''_C\,(1+\beta)}\,M_b$$

$$+\frac{L\,D_b\,(D_b + D_a)}{4\,I''_C\,(1+\beta)}\sum_1^a H_w \quad \text{.................}(51e)$$

Substituting Equation (50*a*) in Term (*f*) gives:

$$-\frac{L_1}{4\,I'_C}\,(D_b + D_a)\,M'_{ba} = -\frac{\beta\,L_1\,(D_b + D_a)}{4\,I'_C\,(1+\beta)}\,M_b$$

$$+\frac{\beta\,L_1\,D_b\,(D_b + D_a)}{4\,I'_C\,(1+\beta)}\sum_1^a H_w \quad \text{................}(51f)$$

From Equations (51*c*) to (51*f*), collect the terms containing V_{ab}, to form Term (*h*):

$$\frac{L^2\,(D_b + 2\,D_a)}{12\,I''_C\,(1+\alpha)}\,V_{ab} + \frac{\alpha\,L\,L_1\,(D_b + 2\,D_a)}{12\,I'_C\,(1+\alpha)}\,V_{ab}$$

$$= \frac{L^2\,(D_b + 2\,D_a)}{12\,I''_C}\left[\frac{1}{1+\alpha} + \frac{\dfrac{L_1\,I''_C}{L\,I'_C}\,\alpha}{1+\alpha}\right]V_{ab}$$

$$= \frac{L^2\,(D_b + 2\,D_a)}{12\,I''_C}\left(\frac{1+\phi\,\alpha}{1+\alpha}\right)V_{ab} \quad \text{................}(51h)$$

in which ϕ = ratio of stiffness, top and bottom chords $= \dfrac{L_1\,I''_C}{L\,I'_C}$.

From the same group of terms, collect the ones containing M_b; thus, for Term (*i*):

$$-\frac{L}{4\,I''_C}\,\frac{(D_b + D_a)}{(1+\beta)}\,M_b - \frac{\beta\,L_1\,(D_b + D_a)}{4\,I'_C\,(1+\beta)}\,M_b$$

$$= -\frac{L}{4\,I''_C}\,(D_b + D_a)\left[\frac{1}{1+\beta} + \frac{\dfrac{L_1\,I''_C}{L\,I'_C}\,\beta}{1+\beta}\right]M_b$$

$$= -\frac{L}{4\,I''_C}\,(D_b + D_a)\left(\frac{1+\phi\,\beta}{1+\beta}\right)M_b \quad \text{..............}(51i)$$

From the same group of terms, collect the ones containing $\sum\limits_1^a H_w$, for Term (*j*):

$$-\frac{L\,(D^2_b + D_b\,D_a - 2\,D^2_a)}{12\,I''_C\,(1+\alpha)}\sum_1^a H_w + \frac{L_1\,(D^2_b + D_b\,D_a - 2\,D^2_a)}{12\,I'_C\,(1+\alpha)}\sum_1^a H_w$$

$$+\frac{L\,D_b\,(D_b + D_a)}{4\,I''_C\,(1+\beta)}\sum_1^a H_w + \frac{\beta\,L_1\,D_b\,(D_b + D_a)}{4\,I'_C\,(1+\beta)}\sum_1^a H_w$$

$$= \left[\frac{L\,(2\,D^2_a - D_b\,D_a - D^2_b)}{12\,I''_C}\left(\frac{1-\phi}{1+\alpha}\right) + \frac{L\,D_b\,(D_b + D_a)}{4\,I''_C}\left(\frac{1+\phi\,\beta}{1+\beta}\right)\right]\sum_1^a H_w.\,(51j)$$

Adding Term (j) to Term (g), Term (k) equals:

$$\text{Term }(j) + \frac{L_1}{12\,I'_c}\,(2\,D^2_a - D_a\,D_b - D^2_b)\sum_1^a H_w$$

$$= \text{Term }(j) + \frac{\phi\,L}{12\,I''_c}\,(2\,D^2_a - D_a\,D_b - D^2_b)\sum_1^a H_w$$

$$= \left[\frac{L\,(2\,D^2_a - D_a\,D_b - D^2_b)}{12\,I''_c}\left(\frac{1+\phi\,\alpha}{1+\alpha}\right)\right.$$

$$\left. + \frac{L\,D_b\,(D_b + D_a)}{4\,I''_c}\left(\frac{1+\phi\,\beta}{1+\beta}\right)\right]\sum_1^a H_w \qquad \text{(51}k\text{)}$$

By these transformations, Terms (c) + (d) + (e) + (f) + (g) have been reduced to the three terms, (h) + (i) + (k). With these changes, Equation (45) becomes:

$$0 = -\frac{D^3_b}{12\,I_b}\,H_{bw} + \frac{D^3_a}{12\,I_a}\,H_{aw} + \frac{L}{12\,I''_c}\left[(2\,D^2_a - D_a\,D_b - D^2_b)\left(\frac{1+\phi\,\alpha}{1+\alpha}\right)\right.$$

$$\left. + 3\,D_b\,(D_b + D_a)\left(\frac{1+\phi\,\beta}{1+\beta}\right)\right]\sum_1^a H_w - \frac{L}{4\,I''_c}\,(D_b + D_a)\left(\frac{1+\phi\,\beta}{1+\beta}\right)M_b$$

$$+ \frac{L^2}{12\,I''_c}\,(D_b + 2\,D_a)\left(\frac{1+\phi\,\alpha}{1+\alpha}\right)V_{ab} \qquad \text{(52)}$$

Transposing the term containing H_{bw} and dividing through by its coefficient gives:

$$H_{bw} = \frac{D^3_a}{D^3_b}\,\frac{I_b}{I_a}\,H_{aw} + \frac{L}{D^3_b}\,\frac{I_b}{I''_c}\left[3\,D_b\,(D_b + D_a)\left(\frac{1+\phi\,\beta}{1+\beta}\right)\right.$$

$$\left. + (2\,D^2_a - D_a\,D_b - D^2_b)\left(\frac{1+\phi\,\alpha}{1+\alpha}\right)\right]\sum_1^a H_w$$

$$- \frac{3\,L}{D^3_b}\,\frac{I_b}{I''_c}\,(D_b + D_a)\left(\frac{1+\phi\,\beta}{1+\beta}\right)M_b$$

$$+ \frac{L^2}{D^3_b}\,\frac{I_b}{I''_c}\,(D_b + 2\,D_a)\left(\frac{1+\phi\,\alpha}{1+\alpha}\right)V_{ab} \qquad \text{(53)}$$

Equation (53) is the fundamental expression for determining the H_w-values. No approximations have been made in its derivation, except to neglect deflections due to axial stresses. The effect of these deflections is considered subsequently. For the case of parallel chords and constant moment of inertia, Equation (53) reduces to Equation (11). It may be used for parallel-chord trusses with members having different moments of inertia.

After evaluating the coefficients of each term of Equation (53), it reduces to the simple form of Equation (13). In this case, however, the coefficients depend not only on the truss dimensions, but also upon α and β, which are ratios of the bending moments in the upper chord to those in the lower chord.

In order to evaluate the coefficients it is necessary to assume values for these ratios. It can be shown that, in most cases, choosing widely different values for these ratios effects the H_w-values only slightly. Hence, by assuming any reasonable values for α and β, a close approximation to the true values of H_w is obtained upon solving the equation.

For the particular case in which the moments of inertia are such that $\phi = \frac{L_1}{L} \frac{I''_C}{I'_C} = 1$, the factors involving α and β reduce to unity and the solution becomes exact. For values of ϕ close to unity, the factors involving α and β will be nearly equal to unity for any values of these ratios and, therefore, in such case, little error is introduced in assuming these ratios.

The following examples illustrate the method of solution and the effect on the H_w-values of assuming different magnitudes for α and β.

Example 3.—Consider a truss with dimensions and loads as shown in Fig. 8. For this example assume that the moments of inertia are of such values that $I_a = I_b = I''_C = I'_C \frac{L}{L_1}$. Then, for this case, ϕ will equal 1, and Equation (53) reduces to:

$$H_{bw} = \frac{D^3_a}{D^3_b} H_{aw} + \frac{2\,L}{D^3_b} (D^2_a + D_a\,D_b + D^2_b) \sum_1^a H_w - \frac{3\,L}{D^3_b} (D_b + D_a)\,M_b + \frac{L^2}{D^3_b} (D_b + 2\,D_a)\,V_{ab} \ldots\ldots (54)$$

Apply Equation (54) to each panel in the left half of the truss. Then, after evaluating the coefficients, the following series of equations is obtained:

$$H_{1w} = H_{1w} \ldots\ldots (55a)$$

$$H_{2w} = 0.125\,H_{1w} + 4.37\,H_{1w} - 0.702\,M_2 + 3.12\,V_{12} \ldots\ldots (55b)$$

$$H_{3w} = 0.512\,H_{2w} + 4.88\,(H_{1w} + H_{2w}) - 0.54\,M_3 + 2.60\,V_{23} \ldots (55c)$$

and,

$$H_{4w} = 1.00\,H_{3w} + 6.00\,(H_{1w} + H_{2w} + H_{3w}) - 0.60\,M_4 + 3.00\,V_{34} . (55d)$$

For the loading given: $M_2 = 6\,000$ ft-lb; $M_3 = 12\,000$ ft-lb; $M_4 = 8\,000$ ft-lb; $V_{12} = 600$ lb; $V_{23} = 600$ lb; and, $V_{34} = -400$ lb. Substituting these values in Equations (55) and then expressing each value of H_w in terms of H_{1w}:

$$H_{1w} = H_{1w} \ldots\ldots (56a)$$

$$H_{2w} = 4.495\,H_{1w} - 2\,340 \ldots\ldots (56b)$$

$$H_{3w} = 0.512\,H_{2w} + 4.88\,(H_{1w} + H_{2w}) - 4\,920 = 29.117\,H_{1w} - 17\,537 . (56c)$$

and,

$$H_{4w} = 1.00\,H_{3w} + 6.00\,(H_{1w} + H_{2w} + H_{3w}) - 6\,000$$

$$= 236.789\,H_{1w} - 142\,799 \ldots\ldots (56d)$$

Now consider the truss reversed in position, as shown in Fig. 9. The prime mark, ($'''$), will be used to designate terms for this position. Moments

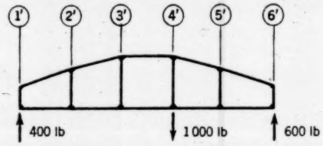


FIG. 9.

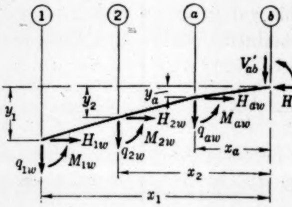


FIG. 10.

and shears for the left half of the truss are: $M'''_2 = 4\,000$ ft-lb; $M'''_3 = 8\,000$ ft-lb; $V'''_{12} = 400$ lb; and $V'''_{23} = 400$ lb. Substituting these values in Equations (55) and then expressing each value of H'''_w in terms of H'''_{1w}:

$$H'''_{1w} = H'''_{1w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57a)$$

$$H'''_{2w} = 4.495\ H'''_{1w} - 1\,560 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57b)$$

and,

$$H'''_{3w} = 0.512\ H'''_{2w} + 4.88\ (H'''_{1w} + H'''_{2w}) - 3\,280$$

$$= 29.117\ H'''_{1w} - 11\,692 \ldots\ldots\ldots\ldots\ldots\ldots (57c)$$

Since it is obvious from their identity that $H_{4w} = -H'''_{3w}$: $236.789\ H_{1w} - 142\,799 = -29.117\ H'''_{1w} + 11\,692$; and,

$$H'''_{1w} = -8.13233\ H_{1w} + 5\,305.87 \ldots\ldots\ldots\ldots\ldots (58)$$

For equilibrium:

$$H_{1w} + H_{2w} + H_{3w} + H_{4w} - H'''_{2w} - H'''_{1w} = 0 \ldots\ldots\ldots (59)$$

Substituting for each value of H_w in Equation (59), its value in terms of H_{1w}: $271.401\ H_{1w} - 162\,676 - 5.495\ H'''_{1w} + 1\,560 = 0$; $271\,401\ H_{1w} - 162\,676 - 5.495\ (-8.13233\ H_{1w} + 5\,305.87) + 1\,560 = 0$; and, $H_{1w} = 601.96$. Substituting this value for H_{1w} in Equations (56), (57), and (58): $H_{2w} = 366$ lb; $H_{3w} = -10$ lb; $H_{4w} = -262$ lb; $H_{5w} = -H'''_{2w} = -285$ lb; and, $H_{6w} = -H'''_{1w} = -411$ lb.

Example 4.—Consider the same truss as in Example 3, but assume that $I_a = I_b = I''_C = I'_C$. In this case, in order to evaluate the coefficients, it is necessary to assume values for α and β. To show how slightly these values affect the magnitudes of the H_w, it will be assumed that $\alpha = \beta = 2$. This is a highly improbable value, of course, and is chosen only for purposes of illustration. A more reasonable assumption would give more accurate results.

Referring to Equation (53), the factors, $\frac{1+\phi\,\alpha}{1+\alpha}$ and $\frac{1+\phi\,\beta}{1+\beta}$, become $\frac{1+\frac{10.77\times 2}{10}}{1+2} = 1.051$ for the first panel; $\frac{1+\frac{10.2\times 2}{10}}{1+2} = 1.013$ for the second panel; and $\frac{1+\frac{10\times 2}{10}}{1+2} = 1$ for the third panel. The coefficients for this example will equal those in Example 3 multiplied by these factors, except for the H_{aw}-term, which remains the same. Performing this multiplication yields the following series of equations:

$$H_{1w} = H_{1w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (60a)$$

$$H_{2w} = 0.125\ H_{1w} + 4.59\ H_{1w} - 0.738\ M_2 + 3.28\ V_{12} \ldots\ldots\ldots\ldots (60b)$$

$$H_{3w} = 0.512\ H_{2w} + 4.94\ (H_{1w} + H_{2w}) - 0.547\ M_3 + 2.63\ V_{23} \ldots\ldots (60c)$$

and,

$$H_{4w} = 1.00\ H_{3w} + 6.00\ (H_{1w} + H_{2w} + H_{3w}) - 0.600\ M_4 + 3.00\ V_{34} . (60d)$$

Substituting the values for the moments and shears and solving as was done in Example 3: $H_{1w} = 600$; $H_{2w} = 369$; $H_{3w} = -10$; $H_{4w} = -262$; $H_{5w} = -H'''_{2w} = -287$; and, $H_{6w} = H'''_{1w} = -409$. Comparing these values with those of Example 3, it is seen that the two assumptions give nearly the same results, even though α and β in this example were chosen far greater than their probable value.

DETERMINATION OF CHORD MOMENTS AND SHEARS

After the H_w-values have been determined from Equation (53), the next step is to solve for the M_w and q_w-values. This is done by transforming the fundamental equations, as follows: Multiplying Equation (37) by $\frac{3}{L}$ and adding the result to Equation (42), gives:

$$-\frac{D^2_b}{2\,I_b}\,H_{bw} - \frac{D^2_a}{I_a}\,H_{aw} + \frac{D_b}{I_b}\,M_{bw} + \frac{2\,D_a}{I_a}\,M_{aw} - \frac{L^2}{2\,I''_C}\,V''_{ab} + \frac{L\,L_1}{2\,I'_C}\,V'_{ab}$$
$$+ \frac{L}{2\,I''_C}\,M''_{ba} - \frac{L_1}{2\,I'_C}\,M'_{ba} - \frac{L_1}{2\,I'_C}\,(D_b - D_a)\sum_1^a H_w = 0 \ \ldots . (61)$$

Substituting the statical relations, $V''_{ab} = V_{ab} - V'_{ab}$ and $M''_{ba} = M_b - M'_{ba} - D_b \sum\limits_1^a H_w$, in Equation (61) and collecting terms:

$$-\frac{D^2_b}{2\,I_b}\,H_{bw} + \frac{D^2_a}{I_a}\,H_{aw} + \frac{D_b}{I_b}\,M_{bw} + \frac{2\,D_a}{I_a}\,M_{aw} - \frac{L^2}{2\,I''_C}\,V_{ab} + \frac{L^2}{2\,I''_C}\,(1+\phi)\,V'_{ab}$$
$$+\frac{L}{2\,I''_C}\,M_b - \frac{L}{2\,I''_C}\,(1+\phi)\,M'_{ba} + \left[\frac{L_1\,D_a}{2\,I''_C} - \frac{L\ D_b}{2\,I''_C}\,(1+\phi)\right]\sum_1^a H_w = 0 \ .. (62)$$

Fig. 10 shows the forces acting on the upper chord to the left of Panel Point *b*. For equilibrium:

$$M'_{ba} = -\sum_{1}^{a} M_w - \sum_{1}^{a} (q_w x) - \sum_{1}^{a} (H_w y) = -\sum_{1}^{a} M_w$$

$$+ \sum_{1}^{a} (V' L) - \sum_{1}^{a} (H_w y) \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

Equation (63) may be written:

$$M'_{ba} = -M_{aw} - \sum_{1}^{(a-1)} M_w + V'_{ab} L + \sum_{1}^{(a-1)} (V' L) - \sum_{1}^{a} (H_w y) \ \ldots (64)$$

From the geometry of the truss it is seen that $H_w y = H_w D_b - H_w D$. Substituting this value in Equation (64):

$$M'_{ba} = -M_{aw} - \sum_{1}^{(a-1)} M_w + V'_{ab} L + \sum_{1}^{(a-1)} (V' L)$$

$$-\sum_{1}^{a} (H_w D_b) + \sum_{1}^{a} (H_w D) \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

Substituting Equation (65) in Equation (62) and collecting terms:

$$\frac{D_b}{I_b} M_{bw} = -\left[\frac{2\,D_a}{I_a} + \frac{L}{2\,I''_C}(1+\phi)\right] M_{aw} - \frac{L}{2\,I''_C}(1+\phi)\sum_{1}^{a-1} M_w$$

$$+\frac{L^2}{2\,I''_C}(1+\phi)\sum_{1}^{a-1} V' + \frac{D^2_b}{2\,I_b} H_{bw} + \frac{D^2_a}{I_a} H_{aw} + \frac{L^2}{2\,I''_C} V_{ab} - \frac{L}{2\,I''_C} M_b$$

$$-\frac{L_t\,D_a}{2\,I'_C}\sum_{1}^{a} H_w + \frac{L}{2\,I''_C}(1+\phi)\sum_{1}^{a} (H_w D) \ \ldots\ldots\ldots (66)$$

For the first panel Equation (66) reduces to:

$$\frac{D_2}{I_b} M_{2w} = -\left[\frac{2\,D_1}{I_a} + \frac{L}{2I''_C}(1+\phi)\right] M_{1w} + \left[\frac{D^2_1}{I_a} + \frac{L\ D_1}{2\,I''_C}\right] H_{1w} + \frac{D^2_2}{2\,I_b} H_{2w} \ldots (67)$$

Substituting the same statical relations in Equation (37) and simplifying:

$$\frac{L^2}{6\,I''_C}(1+\phi)\,V'_{ab} = \left[\frac{D_a}{I_a} + \frac{L}{2\,I''_C}(1+\phi)\right] M_{aw} + \frac{L}{2\,I''_C}(1+\phi)\sum_{1}^{a-1} M_w$$

$$-\frac{L^2}{2\,I''_C}(1+\phi)\sum_{1}^{a-1} V' - \frac{D^2_a}{2\,I_a} H_{aw} - \frac{L^2}{3\,I''_C} V_{ab} + \frac{L}{2\,I''_C} M_b$$

$$+\frac{L_t}{6\,I'_C}(D_b + 2\,D_a)\sum_{1}^{a} H_w - \frac{L}{2\,I''_C}(1+\phi)\sum_{1}^{a} (H_w D) \ \ldots (68)$$

For the first panel Equation (68) reduces to:

$$\frac{L^2}{6\,I''_C}(1+\phi)\,V'_{12} = \left[\frac{D_1}{I_a} + \frac{L}{2\,I''_C}(1+\phi)\right] M_{1w}$$

$$-\left[\frac{D_1^2}{2\,I_a} - \frac{L_1}{6\,I'_C}(D_2 + 2\,D_1) + \frac{L\,D_1}{2\,I''_C}(1+\phi)\right] H_{1w} + \frac{L^2}{6\,I''_C}\,V_{12} \quad \ldots\ldots (69)$$

Equations (66), (67), (68), and (69) furnish the necessary relations for solving for each value of M_w and V'. Each value of q_w, of course, is determined directly from the V'-values.

Equations (66) to (69) appear somewhat involved but, after substituting numerical values for the truss dimensions and for the previously calculated H_w-values, they reduce to:

$$C_1\,M_{bw} = C_2\,M_{aw} + C_3 \sum_{1}^{a-1} M_w + C_4 \sum_{1}^{a-1} V' + C_5 \quad \ldots\ldots\ldots (70)$$

which corresponds to Equation (66);

$$C_6\;M_{2w} \;=\; C_7\;M_{1w} \;+\; C_8 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

which corresponds to Equation (67);

$$C_9\;V'_{ab} = C_{10}\,M_{aw} + C_{11} \sum_{1}^{a-1} M_w + C_{12} \sum_{1}^{a-1} V' + C_{13} \quad \ldots\ldots (72)$$

which corresponds to Equation (68); and,

$$C_{14}\;V'_{12} \;=\; C_{15}\;M_{1w} \;+\; C_{16} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (73)$$

which corresponds to Equation (69).

The general method of solution is to apply Equations (66) and (68) to each panel in turn. This gives a series of expressions which may be solved simultaneously to obtain the magnitude of each unknown function. It will be noted that the equations for any given panel involve the moments, M_w, and shears, V', for the preceding panels only. Hence, by starting with the equations for the end panel and substituting, successively, in the equations for the following panels, each M_w may be expressed in terms of M_{1w} and the entire series is then readily solved.

This method of solution is best explained by solving an example. Accordingly, Equations (66) to (69) will be applied to find the values of M_w and V' for the truss used in Example 3.

Example 5.—For this example, the moments and shears will be computed for the truss of Example 3, using the H_w-values obtained in that example. For this case it should be noted that $\phi = 1$ and that $\frac{L_1}{I'_C} = \frac{L}{I''_C}$. Then, for this particular problem, Equations (66) to (69) become:

For Equation (66):

$$D_b M_{bw} = -(2D_a + L) M_{aw} - L \sum_{1}^{a-1} M_w + L^2 \sum_{1}^{a-1} V' + \frac{D^2{}_b}{2} H_{bw}$$

$$+ D^2{}_a H_{aw} + \frac{L^2}{2} V_{ab} - \frac{L}{2} M_b - \frac{L D_a}{2} \sum_{1}^{a} H_w + L \sum_{1}^{a} (H_w D) \quad \text{..(74)}$$

For Equation (67):

$$D_2 M_{2w} = -(2D_1 + L) M_{1w} + \left(D^2{}_1 + \frac{L D_1}{2}\right) H_{1w} + \frac{D^2{}_2}{2} H_{2w} \quad \text{...(75)}$$

For Equation (68):

$$\frac{L^2}{3} V'_{ab} = (D_a + L) M_{aw} + L \sum_{1}^{a-1} M_w - L^2 \sum_{1}^{a-1} V' - \frac{D^2{}_a}{2} H_{aw} - \frac{L^2}{3} V_{ab}$$

$$+ \frac{L}{2} M_b + \frac{L}{6} (D_b + 2D_a) \sum_{1}^{a} H_w - L \sum_{1}^{a} (H_w D) \quad \text{......(76)}$$

and, for Equation (69):

$$\frac{L^2}{3} V'_{12} = (D_1 + L) M_{1w} - \left[\frac{D^2{}_1}{2} - \frac{L}{6}(D_2 + 2D_1) + L D_1\right] H_{1w} + \frac{L^2}{6} V_{12} . \text{(77)}$$

Applying Equations (75) and (77) to the first panel gives:

$$8M_{2w} = -(2 \times 4 + 10) M_{1w} + \left(4^2 + \frac{10 \times 4}{2}\right)(602) + \frac{8^2}{2}(366); \text{ or,}$$

$$M_{2w} = 4\,173 - 2.25\, M_{1w} \quad \text{..................(78)}$$

and, $\frac{10^2}{3} V'_{12} = (4 + 10) M_{1w} - \left[\frac{4^2}{2} - \frac{10}{6}(8 + 2 \times 4) + 10 \times 4\right](602)$

$+ \frac{10^2}{6}(600)$; or,

$$V'_{12} = 0.420\, M_{1w} - 86 \quad \text{....................(79)}$$

Applying Equations (74) and (76) to the second panel gives:

$10\, M_{3w} = -(2 \times 8 + 10) M_{2w} - 10\, M_{1w} + 10^2\, V'_{12} + \frac{10^2}{2}(-10)$

$+ 8^2 (366) + \frac{10^2}{2}(600) - \frac{10}{2}(12\,000) - \frac{10 \times 8}{2}(602 + 366) + 10\,(602 \times 4$

$+ 366 \times 8)$; or,

$$M_{3w} = -2.6\, M_{2w} - M_{1w} + 10\, V'_{12} + 759 \quad \text{..........(80)}$$

and, $\frac{10^2}{3} V'_{23} = (8 + 10)\ M_{2w} + 10\ M_{1w} - 10^2\ V'_{12} - \frac{8^2}{2} (366) - \frac{10^2}{3} (600)$ $+ \frac{10}{2} (12\,000) + \frac{10}{6} (10 + 2 \times 8)\ (602 + 366) - 10\ (602 \times 4 + 366 \times 8)$; or,

$$V'_{23} = 0.54\ M_{2w} + 0.3\ M_{1w} - 3\ V'_{12} + 508 \ldots\ldots\ldots\ldots (81)$$

Applying Equations (74) and (76) to the third panel gives: $10\ M_{4w} = -\ (2 \times 10 + 10)\ M_{3w} - 10\ (M_{2w} + M_{1w}) + 10^2\ (V'_{23} + V'_{12})$ $+ \frac{10^2}{2} (-262) + 10^2\ (-10) + \frac{10^2}{2} (-400) - \frac{10}{2} (8\,000) - \frac{10 \times 10}{2}$ $(602 + 366 - 10) + 10\ (602 \times 4 + 366 \times 8 - 10 \times 10)$; or,

$$M_{4w} = -\ 3\ M_{3w} - M_{2w} - M_{1w} + 10\ V'_{23} + 10\ V'_{12} - 6\,964 \ldots (82)$$

and, $\frac{10^2}{3} V'_{34} = (10 + 10)\ M_{3w} + 10\ (M_{2w} + M_{1w}) - 10^2\ (V'_{23} + V'_{12})$ $- \frac{10^2}{2} (-10) - \frac{10^2}{3} (-400) + \frac{10}{2} (8\,000) + \frac{10}{6} (602 + 366 - 10)\ (10 + 2 \times 10) - 10\ (602 \times 4 + 366 \times 8 - 10 \times 10)$; or,

$$V'_{34} = 0.6\ M_{3w} + 0.3\ M_{2w} + 0.3\ M_{1w} - 3\ V'_{23} - 3\ V'_{12} + 1\,481 \ldots (83)$$

Substituting Equations (78) and (79) in Equations (80) and (81):

$$M_{3w} = 9.05\ M_{1w} - 10\,960 \ldots\ldots\ldots\ldots\ldots\ldots (84a)$$

and,

$$V'_{23} = 3\,021 - 2.175\ M_{1w} \ldots\ldots\ldots\ldots\ldots\ldots (84b)$$

Substituting Equations (78), (79), and (84), in Equations (83) and (82):

$$M_{4w} = 51\,090 - 43.45\ M_{1w} \ldots\ldots\ldots\ldots\ldots\ldots (85a)$$

and,

$$V'_{34} = 10.32\ M_{1w} - 12\,648 \ldots\ldots\ldots\ldots\ldots\ldots (85b)$$

By means of these substitutions, all the values of M_w and V' are expressed in terms of M_{1w}.

Now, consider the truss turned end for end as shown in Fig. 9, and write a corresponding series of expressions, beginning with the first panel on the left, as before. Functions for the truss in this reversed position will be designated by the prime mark (′′). The coefficients for all terms will be the same as for the truss in its original position. The only difference in the calculations is that the values of H_w, M_b, and V_{ab}, of course, must be those for the reversed position. Because of this similarity, the detailed calculations are omitted in this case and only the final equations are given. Applying Equations (75) and (77) to the first panel, gives:

$$M'''_{2w} = 2\,990 - 2.25\, M'''_{1w} \quad \ldots\ldots\ldots\ldots (86a)$$

and,

$$(V'_{12})''' = 0.42\, M'''_{1w} - 63.04 \quad \ldots\ldots\ldots\ldots (86b)$$

Applying Equations (74) and (76) to the second panel:

$$M'''_{3w} = -\,2.6\, M'''_{2w} - M'''_{1w} + 10\,(V'_{12})''' + 2\,274 \ldots (87a)$$

and,

$$(V'_{23})''' = 0.54\, M'''_{2w} + 0.3\, M'''_{1w} - 3\,(V'_{12})''' + 245 \ldots\ldots (87b)$$

Applying Equations (74) and (76) to the third panel, gives:

$$M'''_{4w} = -\,3\, M'''_{3w} - M'''_{2w} - M'''_{1w} + 10\,(V'_{23})''' + 10\,(V'_{12})''' + 420.\,(88a)$$

and,

$$(V'_{34})''' = 0.6\, M'''_{3w} + 0.3\, M'''_{2w} + 0.3\, M'''_{1w} - 3\,(V'_{23})''' - 3\,(V'_{12})''' + 481.\,(88b)$$

Substituting Equations (86) in Equations (87):

$$M'''_{3w} = 9.05\, M'''_{1w} - 6\,130 \quad \ldots\ldots\ldots\ldots (89a)$$

and,

$$(V'_{23})''' = 2\,049 - 2.175\, M'''_{1w} \quad \ldots\ldots\ldots\ldots (89b)$$

Substituting Equations (86) and (89) in Equations (88):

$$M'''_{4w} = 35\,680 - 43.45\, M'''_{1w} \quad \ldots\ldots\ldots\ldots (90a)$$

and,

$$(V'_{34})''' = 10.32\, M'''_{1w} - 8\,258 \quad \ldots\ldots\ldots\ldots (90b)$$

It is obvious that $M_{3w} = -\,M'''_{4w}$ and that $M_{4w} = -\,M'''_{3w}$. Equating the values for these functions as given in Equations (84), (85), (89*a*), and (90*a*):

$$9.05\, M_{1w} - 10\,960 = -\,35\,680 + 43.45\, M'''_{1w} \quad \ldots\ldots\ldots (91a)$$

and,

$$51\,090 - 43.45\, M_{1w} = -\,9.05\, M'''_{1w} + 6\,130 \quad \ldots\ldots\ldots (91b)$$

Solving Equations (91*a*) and (91*b*) the following values are obtained: $M_{1w} = 1\,206$; and, $M'''_{1w} = 820$. The remaining moments and shears may now be found by substituting these values in Equations (78), (79), (84), (85), (86), and (89). This gives the following results: $M_{2w} = 1\,460$; $M_{3w} = -\,50$; $M_{4w} = -\,M'''_{3w} = -\,1\,290$; $M_{5w} = -\,M'''_{2w} = -\,1\,150$; $V'_{12} = 420$; $V'_{23} = 399$; $V'_{34} = -\,205$; $V'_{45} = -\,(V'_{23})''' = -\,265$; and, $V'_{56} = -\,(V'_{12})''' = -\,281$.

The q_w-values are determined directly from the V'-values, giving: $q_{1w} = -420$; $q_{2w} = 21$; $q_{3w} = 604$; $q_{4w} = 60$; $q_{5w} = 16$; and $q_{6w} = -281$.

This completes the analysis of the truss. With the values of H_w, M_w, and V' known, the stresses at all points in the truss may be calculated.

SPECIAL APPROXIMATIONS

Only general methods of solution have been illustrated in Examples 1 to 5. There are various places where short-cut methods of computation or, in special cases, further simplifications may be applied. Furthermore, the general equations may be solved by successive approximations.

For example, the values of H_w may be obtained by successive approximations, as follows: Equations (10) and (53) are of the form shown in Equation (13). Assume the location of the points of contraflexure in Panel bc and calculate the value of H_{bc}. Then, substituting the relation, $H_{bw} = H_{bc} - \sum_{1}^{a} H_w$, into Equation (13) an expression of the form, $H_{aw} = C' \sum_{1}^{a-1} H_w + C''$, is obtained. Starting with an end panel, an equation of this form may be set up and solved for each panel in turn, since $\sum_{1}^{a-1} H_w$ is known from the preceding calculations. The operations may then be repeated using for H_{bc} the results of the first approximation. Similarly, Equations (66) and (68) may be solved by successive approximations by assuming a value for M_{bw}. A good assumption for a first approximation is $M_{bw} = \frac{1}{2} D_b H_{bw}$.

Professor Vierendeel uses an approximate method for calculating the chord moments and shears[12] which is very rapid. Equations (48) and (50) are expressions for these functions. After determining the values of the H_w-values from Equation (53), the only unknowns in these expressions are α and β. By assuming values for these ratios, the desired moments and shears may be obtained quickly. Professor Vierendeel uses the assumption that $\alpha = \beta = \frac{I'_C}{I''_C} \frac{L}{L_4}$. It is obvious that this assumption greatly simplifies the work in comparison with the method used in Example 5. Experience in analyzing many trusses has shown that this approximation gives results which are sufficiently accurate for most designs. The only serious errors that occur as a result of using this method are in the values for the moments and shears in one or two panels at the ends of some trusses. For such cases a close approximation may be secured by combining the general method presented in this paper with Vierendeel's method; that is, use the former to obtain the moments and shears in the first two panels and the latter for the remainder of the truss.

EFFECT OF AXIAL STRESS

The effect of deformation due to axial stress in the chord members upon the H_w-values may be taken into account by the addition of two terms to Equation (53). Refer to Equation (21) which is the expression for the horizontal deflection in the lower part of a typical panel. The deflection due to direct stress in the lower chord is obviously $\frac{H_{ab}\,L}{A''\,E} = \frac{L\sum_{1}^{a} H_w}{A''\,E}$. The total deflection is obtained by adding this term to Equation (21).

---

[12] "Cours de Stabilité des Constructions", Tome IV, by Arthur Vierendeel, Louvain, 1920.

Similarly, for the upper chord, the horizontal deflection due to direct stress is $-\left(\frac{L\,H_{ab}}{A'\,E} + \frac{V'_{ab}\,d}{A'\,E}\right)\frac{L}{L_t}$. Substituting for V'_{ab} its value from Equation (48), these terms become:

$$-\left[\frac{L\sum_{1}^{a} H_w}{A'\,E} + \frac{\alpha\,(D_b - D_a)}{(1+\alpha)\,A'\,E}\,V_{ab} + \frac{(D_b - D_a)^2}{(1+\alpha)\,L\,A'\,E}\sum_{1}^{a} H_w\right]\frac{L}{L_t}$$

Adding these terms to Equation (27), the total horizontal deflection for the upper chord is obtained.

Adding these terms to Equations (21) and (27) and carrying them through all the operations to Equation (53), they become, in the final stage:

$$\frac{12\,I_b}{A'\,D^3_b}\left\{\sum_{1}^{a} H_w\left[\frac{L\,A'}{A''} + \frac{L^2}{L_t} + \frac{(D_b - D_a)^2}{L_t\,(1+\alpha)}\right] + V_{ab}\left[(D_b - D_a)\frac{L}{L_t}\left(\frac{\alpha}{1+\alpha}\right)\right]\right\}$$

By adding this expression to the right-hand side of Equation (53), the effect of direct stress is included in the solution. This is equivalent to slightly increasing the numerical values of the coefficients of the original terms, involving $\sum_{1}^{a} H_w$ and V_{ab}. In most cases, the change is small and may be neglected. In applying the foregoing term, it should be noted that I and A must be in the same units as D and L.

For trusses with symmetrical chords the effect of axial stresses may be included in a similar manner.

The deformation of the web members due to axial stress could be included in Equation (37), but this effect is generally so slight as to be negligible.

CONCLUSIONS

In summary, the following conclusions regarding this method of analyzing Vierendeel trusses and its limitations are noted:

(1) All the equations derived are based upon the principles of virtual work and are, therefore, "exact"; that is, they include no more assumptions or limitations than the basic elastic theory;

(2) The solution of the equations for trusses with symmetrical chords requires no assumptions and is, therefore, "exact";

(3) The solution of the equations for trusses with inclined upper chords involves an assumption of values for α and β, and, therefore, is approximate;

(4) The method of analysis presented in this paper, although not extremely rapid or simple, is reasonably so, and gives results that are sufficiently accurate for good design; and,

(5) A large part of the labor involved in the analysis is in the evaluation of the coefficients and setting up an equation for each panel. This part of the solution depends only upon the truss dimensions. Therefore, in investigating the effect of different loadings to obtain influence lines, this part of the analysis is the same for all cases, and so need be performed only once.

In any particular solution, the numerical effect of Assumption (3) upon the value of the coefficients is immediately evident and its effect upon the results may be estimated. In cases where the assumption has a large effect upon coefficient values, this method should be used with caution. In designing it is possible to assume such relative values for the moments of inertia of the members that the effect of the assumption is negligible or even nil.

ACKNOWLEDGMENTS

Grateful acknowledgment is due Professor Arthur Vierendeel for his kind assistance in investigating this form of truss and for furnishing much invaluable data regarding the bridges which he has designed.

APPENDIX I

NOTATION

The following symbols, defined where first introduced in the paper, are re-arranged herein for convenience of reference: In general, single primes denote upper chord values; double primes denote lower chord values; and triple primes denote values for the truss in a reversed position.

A = area; A' = cross-sectional area of an upper chord member; A'' = cross-sectional area of a lower chord member;

a = a subscript; Subscripts a, b, and c, denote three successive panel points (see Fig. 1), and are used to designate the location of the various functions; for example, V'_{ab} = vertical shear in the upper chord between Panel Points a and b; M''_{bc} = bending moment at Point b in the lower chord member, bc; that is, the member extending between Panel Points b and c; M_b = external moment at Point b; H_{aw} = shear in web member at Panel Point a;

b = a subscript (see a);

C = a constant; as a subscript to the moment of inertia, C denotes "chord member";

c = a subscript (see a);

D = depth of a truss at a given panel point (designated by subscript); length of a web member transverse to the longitudinal axis of a truss;

d = drop in chord between panel points; for trusses with only the upper chord inclined, $d = D_b - D_a$; for trusses with symmetrical chords, $d = \frac{1}{2}(D_b - D_a)$;

E = modulus of elasticity;

H = in general, a horizontal force; a horizontal component of axial compression or tension in a chord member; H_w = the total transverse shear in a web member; $\sum_{1}^{a} H_w$ = the sum of the H_w-values from Panel Point 1 to Panel Point a, inclusive; H_{aw} = the shear in a web member at Point a;

I = moment of inertia; I'_C and I''_C = moments of inertia of the upper and lower chord members, respectively; I_a and I_b = moments of inertia of the web members at Panel Points a and b, respectively;

i = a subscript denoting "inclined";

L = length of a panel; L_i = length of an inclined chord member;

M = external moment, considered positive when the external forces on that part of the truss to the left of the point of moments have a clockwise moment about that point. Internal moments and forces are considered positive when they act in the direction shown in the illustrations; M' and M'' = bending moments in the upper and lower chord members, respectively; M_{aw} = bending moment at Point C on the upper end of web member (at the panel point, a); M_{bw} = bending moment at Point D on the upper end of web member (at the panel point, b);

P = an external, concentrated load; P_a, P_b, and P_c, are, respectively, loads suspended from Panel Points a, b, and c;

q_w = total axial tension or compression in a web member;

V = total external shear; V'_{ab} = vertical shear in the upper chord between Panel Points a and b; V'' = vertical shear for the lower chord; the external shear, V, is considered positive when the resultant of the external forces on that part of the truss to the left of the point under consideration, acts upward. Internal moments and forces are considered positive when they act in the direction shown in the illustrations;

w = a subscript denoting "web";

x = a distance measured parallel to the X-axis;

y = a distance measured parallel to the Y-axis;

α = a measure of bending moment ratios $= \dfrac{M'_{ab} + M'_{ba}}{M''_{ab} + M''_{ba}}$;

β = a measure of bending moment ratios $= \dfrac{M'_{ba}}{M''_{ba}}$;

Δ = deflection; Δ_x = horizontal deflection; Δ_y = vertical deflection:

θ = angular deflection; θ_A = angular deflection at Point A;

ϕ = ratio of stiffness, top and bottom chords $= \dfrac{L_i I''_C}{L I'_C}$.

DISCUSSION

L. J. Mensch,[13] M. Am. Soc. C. E. (by letter).—From his splendid analysis of open-web trusses and bents Professor Vierendeel developed practical design formulas which relieve the structural engineer from the necessity of deep thinking when designing such structures. Why then has this practical method been ignored by American engineers, and by many European engineers? Professor Young's paper gives an answer to this question. The analysis is difficult to follow; there is a bewildering number of unknowns shown in Fig. 2, and in Fig. 5; in the absence of adequate explanations the directions of the unknown moments and shears are difficult to trace; and the reason for Δx being zero is rather difficult to find. The most serious obstacle to the proper understanding of the analysis is the omission of diagrams showing the deformation of the truss members under the assumed loading. The fact that Professor Young begins with a rather difficult case is another impediment.

The object of this discussion is to make it easy for the young engineer to grasp Professor Vierendeel's thoughtful conception by describing step by step how the simple open-web truss with symmetrical parallel chords deforms. Fig. 11(*a*) shows the typical deformation of every member of such a truss or bent, and the reader who has followed the latest discussions on wind-bracing bents will be surprised by the coincidence of the deformations. On account of the symmetry it is not difficult to see that the point of contraflexure of the web members must be in the center of the spans. As a rule, the points of contraflexure of the chords will not be in the center of the panels, but will be near the support or free ends, farther from the end than the center line of the panel, and will be found farther to the left in the panels near the center line of the truss or near a fixed end, as will be shown in detail subsequently.

Fig. 11(*b*) shows part of a half truss cut along the horizontal center line. In order to re-establish equilibrium it is necessary to apply, at each point of contraflexure of the web members, one-half the force acting at each panel point, this force being in the direction of the web member; and in order to prevent confusion in the signs of these forces, all forces, P, have been assumed to act in the same direction as in a wind-bracing bent. At the same points of contraflexure in the web members it is also necessary to apply the shear forces, H_1, H_2, H_a, etc., in a direction such that they will cause a curvature of the web member, as shown in Fig. 11(*a*).

By inspecting Fig. 11(*b*) one can readily see that all longitudinal forces and bending moments in each member may be found, after the statically unknown forces, H_1, H_2, etc., are known. The assumption that the longitudinal deformations of the chord members are so small that they may be neglected safely, leads peremptorily to the conclusion that the distance between the points of contraflexure of the web members is the same after, as before, the deformation of the truss.

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[13] Civ. Engr. and Constructor, Chicago, Ill.

At a section, k, an infinitesimal distance from Point a in Fig. 11(b), the left part of the truss or bent will exert the following forces and moments: A longitudinal compression $= H_1 + H_2 = \sum_{1}^{a-1} H$; a shear in an upward direction $= \frac{P_1}{2} + \frac{P_2}{2} = \frac{1}{2} \sum_{1}^{a-1} P$; a right-hand moment from $\frac{1}{2} \sum_{1}^{a-1} P$ (which in the author's notation is designated, $\frac{1}{2} M_a$); and a left-hand moment from $\sum_{1}^{a-1} H$, or $\frac{1}{2} D \sum_{1}^{a-1} H$.

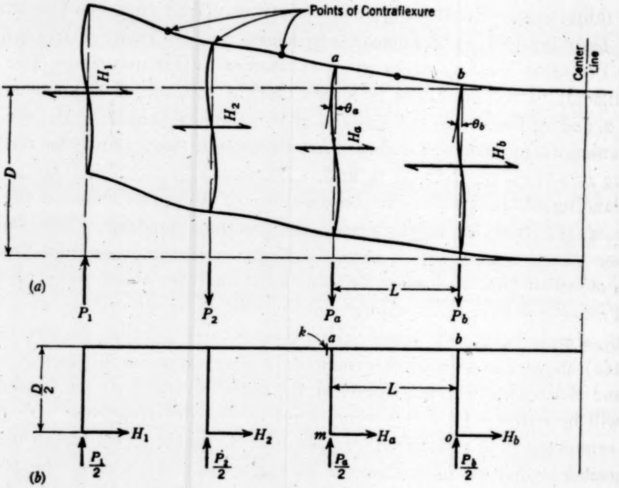
Points of Contraflexure
Center Line
a
b
H_1
H_2
θ_a
H_a
θ_b
H_b
D
(a)
L
P_1
P_2
P_a
P_b
k
a
b
$\frac{D}{2}$
L
H_1
H_2
m
H_a
o
H_b
$\frac{P_1}{2}$
$\frac{P_2}{2}$
$\frac{P_a}{2}$
$\frac{P_b}{2}$
(b)

FIG. 11

Fig. 12 shows an exaggerated picture of the deformed half-panel, a-b. If the deformation of the chords may be neglected, one may assume that m-o = m'-o'. The lines, b'-o'' and a'-m'', are drawn at right angles to the tangents at the chords at these points, and a'-m''' was drawn parallel to b'-o''.

Angles θ_a and θ_b are the rotations of the chord at a' and b', respectively, and $\theta_a - \theta_b$ is the change of angle of these rotations due to the moments acting in the chord, a-b, which may be found by the well-known formula:

$$\theta_a - \theta_b = \frac{1}{E\,I_c} \int_a^b M\,dx \ldots\ldots\ldots\ldots\ldots\ldots (92)$$

in which M is the moment acting at any point distant x from Point a.

The expression for M is found by adding to the moment acting at Section k: (a) The moment from the shear force acting at k, or $\frac{x}{2}\sum_{1}^{a-1} P$; ($b$) the moment from $\frac{1}{2}\,P_a$, or $\frac{1}{2}\,P_a\,x$; and (c) the moment from H_a, or $\frac{1}{2}\,D\,H_a$, which acts in the opposite direction. Moments (a) and (b) may be contracted to $\frac{x}{2}\sum_{1}^{a} P$, so that:

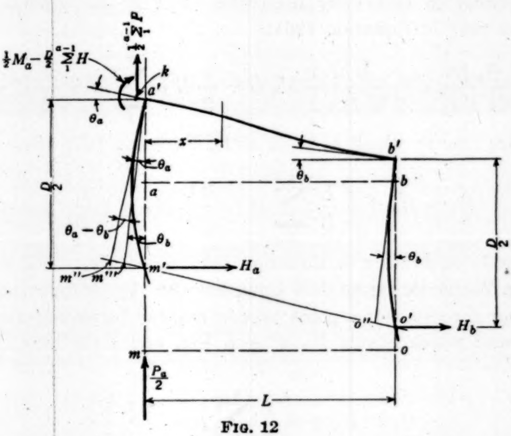
$\frac{1}{2}\sum_{1}^{a-1} P$
$\frac{1}{2}M_a - \frac{D}{2}\sum_{1}^{a-1} H$
k
a'
θ_a
x
θ_a
$\frac{D}{2}$
a
$\theta_a - \theta_b$
θ_b
m''
m'''
m'
H_a
b'
θ_b
b
$\frac{D}{2}$
θ_b
o''
o'
H_b
m
o
$\frac{P_a}{2}$
L

FIG. 12

$$M = \frac{1}{2}\,M_a - \frac{1}{2}\,D\sum_{1}^{a-1} H + \frac{x}{2}\sum_{1}^{a} P - \frac{1}{2}\,D\,H_a$$

$$= \frac{1}{2}\,M_a - \frac{1}{2}\,D\sum_{1}^{a} H + \frac{x}{2}\sum_{1}^{a} P \quad \ldots\ldots\ldots\ldots\ldots (93)$$

and,

$$\theta_A - \theta_B = \frac{1}{E\,I_c}\int_0^L \left(\frac{1}{2}\,M_a - \frac{1}{2}\sum_{1}^{a} H + \frac{x}{2}\sum_{1}^{a} P\right) dx \quad \ldots . (94)$$

In the author's notation, $\sum_{1}^{a} P = V_{ab}$, and,

$$\theta_A - \theta_B = \frac{1}{E\,I_c}\left(\frac{L}{2}\,M_a - \frac{D\,L}{2}\sum_{1}^{a} H + \frac{1}{2}\;V_{ab}\;\frac{L^2}{2}\right) \quad \ldots\ldots (95)$$

For the movements, o'-o'' and m'-m'', the author gives the correct values in Equations (2), and the elastic equation which enables one to find the indeterminate values of the horizontal shear forces is obtained by,

$$m'\text{-}m'' = m''\text{-}m''' + m'''\text{-}m' = \frac{1}{2} D\,(\theta_A - \theta_B) + o'\text{-}o'' \quad \ldots\ldots(96)$$

o'-o'' being equal to m'-m''' by inspection of Fig. 12. Substituting Equations (2) and (95) in Equation (96):

$$\frac{H_a D^3}{24\,E\,I_a} = \frac{H_b D^3}{24\,E\,I_b} + \frac{1}{2}\,\frac{D}{E\,I_c}\left(\frac{1}{2} M_a L - \frac{1}{2} D\,L \sum_1^a H + \frac{1}{2} L^2 \frac{1}{2} V_{ab}\right) \ldots(97)$$

In the rare case in which $I_a = I_b = I_c$, Equation (97) reduces to:

$$H_b = H_a + 6\,\frac{L}{D} \sum_1^a H - \frac{6\,L}{D^2}\,M_a - 3\,\frac{L^2}{D^2}\,V_{ab} \quad \ldots\ldots\ldots\ldots(98)$$

which is exactly equivalent to Equation (11), since $M_b = M_a + L\,V_{ab}$.

Professor Vierendeel simplified Equation (98) by introducing the moment of the exterior forces about a point exactly midway between a and b, which he called M^b_a, and which equals $M_a + \frac{1}{2}\,L\,V_{ab}$, and Equations (11) and (98) reduce to:

$$H_b = H_a + 6\,\frac{L}{D} \sum_1^a H - \frac{6\,L\,M^b_a}{D^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(99)$$

For the more practical case in which the moments of inertia of the various members are different as long as the I-value of the chords in each particular panel are alike due to the assumption of symmetry, the writer has used the equation:

$$\frac{I_a}{I_b}\,H_b = H_a + 6\,n \sum_1^a H - 6\,\frac{n}{D}\,M^b_a \quad \ldots\ldots\ldots\ldots(100)$$

which he derived from Equation (97) by introducing the relative stiffness of the web member to the chord in Panel ab:

$$n = \frac{I_a}{D} \div \frac{I_c}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(101)$$

The writer remembers that, when he first studied Professor Vierendeel's analysis, he had some difficulties with the statement that H is to be set to zero at the base of a fixed bent, even where no shear member exists. Fig. 12 shows that, in this particular case, Point o'' returns to Point o'; Point m''' returns to Point m'; and Equation (96) must be replaced by:

$$m'\text{-}m'' = \frac{1}{2}\,D\,\theta_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(102)$$

The first term is, again, $\dfrac{H_a D^3}{24\,E I_a}$, and the second term is given again by Equation (95). Therefore, it is clear that the term of H_b must really be set equal to zero in Equations (11), (97), (98), (99), etc., when it is used in the lowest panel of a fixed bent even where no web member exists.

Another difficulty encountered by students when they first try to apply this analysis occurs at the base when the ends are hinged; the solution is easy as the sum of all H-values must be equal to the reaction, which, in this particular case, is given by $\frac{M_b}{D}$. Often, there are solid walls as web members at the support of open web trusses. This case, also, is easily solved by omitting the first term in Equation (97), I_a being many hundred times larger than I_b.

American engineers should be vitally interested in Professor Vierendeel's analysis as it is the quickest method known for finding the moments and shears in the top and bottom stories of a wind-bracing bent, in which the points of contraflexure of the columns may not be near the middle of the story height or may be entirely absent, as shown in Fig. 11(a) in the panel near the center line of the truss. To illustrate, let Fig. 13 represent a six-story wind-bracing bent. A force, P, is acting at each floor and the assumption is made that all columns have the same length, L, and the same moment of inertia, I_c, and that all girders have the same moment of inertia, I_a. Equation (100) becomes,

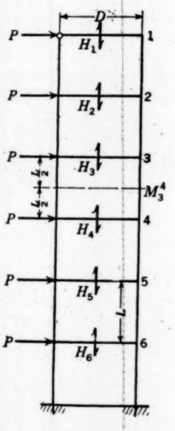

FIG. 13

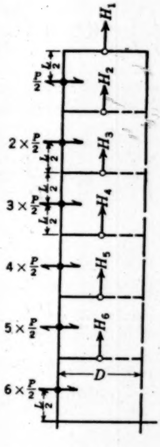

FIG. 14

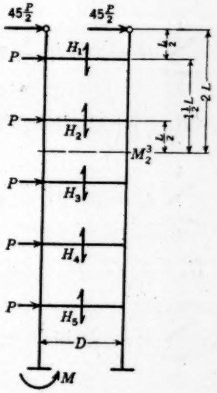

FIG. 15

$$H_b = H_a + 6\,n \sum_{1}^{6} H - 6\,\frac{n}{D}\,M^b{}_a \quad \text{..............} \quad (103)$$

When the relative stiffness of girders and columns is $n = \frac{1}{2}$, for example,

$$H_b = H_a + 3 \sum_{1}^{6} H - \frac{3}{D}\,M^b{}_a \quad \text{................} \quad (104)$$

The values of M^b_a for the various panels from the top down are easily found as $P\,L$ times 0.5, 2, 4.5, 8, 12.5, and 18, and one can write the elastic equations as follows:

$$H_2 = H_1 + 3\,H_1 - \frac{3\,P\,L}{2\,D} = 4\,H_1 - 1.5\,\frac{P\,L}{D}\,\ldots\ldots\ldots(105a)$$

and, $\sum\limits_1^2 H = 5\,H_1 - 1.5\,\dfrac{P\,L}{D}$;

$$H_3 = H_2 + 3\left(5\,H_1 - 1.5\,\frac{P\,L}{D}\right) - \frac{6\,P\,L}{D} = 19\,H_1 - 12\,\frac{P\,L}{D}\,.(105b)$$

and, $\sum\limits_1^3 H = 24\,H_1 - 13.5\,\dfrac{P\,L}{D}$;

$$H_4 = H_3 + 3\left(24\,H_1 - 13.5\frac{P\,L}{D}\right) - 3 \times 4.5\,\frac{P\,L}{D} = 91\,H_1 - 66\,\frac{P\,L}{D}\,.(105c)$$

and, $\sum\limits_1^4 H = 115\,H_1 - 79.5\,\dfrac{P\,L}{D}$;

$$H_5 = H_4 + 3\left(115\,H_1 - 79.5\,\frac{P\,L}{D}\right) - 3 \times 8\,\frac{P\,L}{D} = 436\,H_1 - 328.5\,\frac{P\,L}{D}\,.(105d)$$

and, $\sum\limits_1^5 H = 551\,H_1 - 408\,\dfrac{P\,L}{D}$;

$$H_6 = H_5 + 3\left(551\,H_1 - 408\frac{P\,L}{D}\right) - 3 \times 12.5\frac{P\,L}{D} = 2\,089\,H_1 - 1\,590\frac{P\,L}{D}\,.(105e)$$

and, $\sum\limits_1^6 H = 2\,640\,H_1 - 1\,998\,\dfrac{P\,L}{D}$; and,

$$H_7 = 0 = H_6 + 3\left(2\,640\,H_1 - 1\,998\,\frac{P\,L}{D}\right) - 3 \times 18\,\frac{P\,L}{D}$$

$$= 10\,009\,H_1 - 7\,638\,\frac{P\,L}{D}\,\ldots\ldots\ldots\ldots\ldots\ldots(105f)$$

Whence,

$$H_1 = 7\,638\,\frac{P\,L}{10\,009\,D} = 0.76311\,\frac{P\,L}{D}\,\ldots\ldots\ldots\ldots\ldots(106)$$

Substituting Equation (106) in Equations (105*a*) to (105*f*), the other indeterminate shear values, H_2, H_3, etc., may be found quickly.

Values of H $\left(\text{to be multiplied by } \dfrac{P\,L}{D}\right)$ in a six-story bent, for ratios of $n = \frac{1}{10}, \frac{1}{6}, \frac{1}{2}$, and 1, are given in Table 1.

TABLE 1.—VALUES OF H IN TERMS OF $\frac{PL}{D}$

Relative stiffness, n	H_1	H_2	H_3	H_4	H_5	H_6
$\frac{1}{10}$	1.282	1.75	2.38	2.91	3.08	2.42
$\frac{1}{6}$	1.09	1.68	2.45	3.17	3.56	3.01
$\frac{1}{2}$	0.76311	1.55	2.50	3.45	4.20	4.14
1	0.64545	1.52	2.50	3.48	4.34	4.20
Common theory	0.50	1.50	2.50	3.50	4.50	5.50

It will be interesting to compare the shear values found by Professor Vierendeel's analysis with those computed by the common theory which is based upon the assumption that the point of contraflexure of the columns is at the mid-story height. Fig. 14 illustrates this assumption and by taking moments about the juncture of column and girder the following statical equations may be written, from the top downward:

$$\frac{1}{2}P \times \frac{1}{2}L = H_1 \times \frac{1}{2}D;\ \text{or, } H_1 = \frac{1}{2}\,\frac{PL}{D}$$

$$\frac{1}{2}P \times \frac{1}{2}L + 2 \times \frac{1}{2}P \times \frac{1}{2}L = H_2 \times \frac{1}{2}D;\ \text{or, } H_2 = 1.5\,\frac{PL}{D}$$

$$2 \times \frac{1}{2}P \times \frac{1}{2}L + 3 \times \frac{1}{2}P \times \frac{1}{2}L = H_3 \times \frac{1}{2}D;\ \text{or, } H_3 = 2.5\,\frac{PL}{D}$$

$$3 \times \frac{1}{2}P \times \frac{1}{2}L + 4 \times \frac{1}{2}P \times \frac{1}{2}L = H_4 \times \frac{1}{2}D;\ \text{or, } H_4 = 3.5\,\frac{PL}{D}$$

$$4 \times \frac{1}{2}P \times \frac{1}{2}L + 5 \times \frac{1}{2}P \times \frac{1}{2}L = H_5 \times \frac{1}{2}D;\ \text{or, } H_5 = 4.5\,\frac{PL}{D}$$

and,

$$5 \times \frac{1}{2}P \times \frac{1}{2}L + 6 \times \frac{1}{2}P \times \frac{1}{2}L = H_6 \times \frac{1}{2}D;\ \text{or, } H_6 = 5.5\,\frac{PL}{D}$$

Comparing the values given in Table 1 it will be noted that the shear, H_1, for the top girder is entirely too small as derived by the common theory, whereas H_2 computed by the common theory nearly agrees with the values found by this analysis for $n = \frac{1}{2}$ and 1 and is only 16% at variance for $n = \frac{1}{10}$. For H_3, which is only two panels down from the top (a point of singularity), the agreement is quite remarkably close. The great variation of stiffness has scarcely any influence on the value of H_3 and it is quite permissible to compute it by the common theory. Another point of singularity is found at the bottom of the structure where the discrepancies in the shear values of H_6 are as great as for H_1 at the top. The irregularity at the bottom extends for three panels in the case of $n = \frac{1}{10}$ and $n = \frac{1}{6}$, and for two panels, or even only one panel, in the case of $n = \frac{1}{2}$ and $n = 1$.

It is interesting to compute the moment at the base of the columns for the case, say, of $n = \frac{1}{10}$. The sum of all H-values is found from Table 1 to be $13.83 \frac{P L}{D}$, whereas the moment of one-half the exterior forces about the base is 10.5 PL; that is, the moment at the base:

$$M = 10.5\,P\,L - \frac{1}{2}\,D \times 13.83\,\frac{P\,L}{D} = 3.59\,P\,L \,\ldots\ldots\ldots (107)$$

which is very much larger than the value of $6 \times \frac{1}{2}\,P \times \frac{1}{2}\,L$ found by inspection of Fig. 14 by using the common theory.

The shear at the base being only 3 P, it requires a leverage of more than the story height to produce a moment of 3.59 P L. Therefore, there is no point of contraflexure in the lowest story for this particular case ($n = \frac{1}{10}$).

From the foregoing considerations it follows that, in a tall building bent with many stories, the shears in the girders (except, possibly, two stories at the top and two to six stories at the bottom) may be computed by the common theory; that is, by assuming ideal hinges in the columns at their mid-story heights. For the computation of H_1 and H_2 the writer uses the following short-cut: Write the elastic equations for H_2 and H_3 by using Equation (103); then compute H_3 by the common theory; and equate the two values. For example, in the case of the six-story frame, shown in Fig. 13, for $n = \frac{1}{2}$, Equation (105*b*), $H_3 = 19\,H_1 - 12\,\frac{P\,L}{D}$, which must be equal to the value of $H_3 = 2.5\,\frac{P\,L}{D}$ found by the common theory as given in Table 1. Therefore, $2.5\,\frac{P\,L}{D} = 19\,H_1 - 12\,\frac{P\,L}{D}$; and $H_1 = 0.763\,\frac{P\,L}{D}$, which is nearly the same as the value given in Table 1.

For computing the girder shears in the lowest stories of tall bents (say, of 50 stories), the following procedure will save considerable time: At each point of contraflexure of the columns, just above the fifth story (see Fig. 15), apply one-half the wind load $\left(\text{namely, } 45\,\frac{P}{2}\right)$, and then write the elastic equations of the type of Equation (103). Assuming that $n = \frac{1}{10}$, Equation (103) becomes:

$$H_b = H_a + 0.6 \sum_{1}^{a} H - 0.6\,\frac{M^b_a}{D} \,\ldots\ldots\ldots\ldots\ldots (108)$$

and the values of M^b_a for the various stories from the fifth story down, are found as P L times 45.5, 92, 139.5, 188, and 237.5, respectively, or,

$$H_2 = H_1 + 0.6\,H_1 - 0.6 \times 45.5\,\frac{P\,L}{D} = 1.6\,H_1 - 27.3\,\frac{P\,L}{D} \,..(109a)$$

$$H_3 = H_2 + 0.6\left(2.6\,H_1 - 27.3\,\frac{P\,L}{D}\right) - 0.6 \times 92\,\frac{P\,L}{D}$$

$$= 3.16\,H_1 - 98.88\,\frac{P\,L}{D} \;\ldots\ldots\ldots\ldots\ldots\ldots (109b)$$

$$H_4 = H_3 + 0.6\left(5.76\,H_1 - 126.2\,\frac{P\,L}{D}\right) - 0.6 \times 139.5\,\frac{P\,L}{D}$$

$$= 6.616\,H_1 - 258.3\,\frac{P\,L}{D} \;\ldots\ldots\ldots\ldots\ldots\ldots (109c)$$

$$H_5 = H_4 + 0.6\left(12.376\,H_1 - 384.5\,\frac{P\,L}{D}\right) - 0.6 \times 188\,\frac{P\,L}{D}$$

$$= 14.04\,H_1 - 601.6\,\frac{P\,L}{D} \;\ldots\ldots\ldots\ldots\ldots\ldots (109d)$$

and,

$$0 = H_6 = H_5 + 0.6\left(26.42\,H_1 - 985.1\,\frac{P\,L}{D}\right) - 0.6 \times 237.5\,\frac{P\,L}{D}$$

$$= 29.9\,H_1 - 1\,335.2\,\frac{P\,L}{D} \;\ldots\ldots\ldots\ldots\ldots\ldots (109e)$$

Hence,

$$H_1 = 1\,335.2\,\frac{P\,L}{29.9\,D} = 44.6\,\frac{P\,L}{D} \ldots\ldots\ldots\ldots\ldots (110)$$

Equation (110) substituted in Equations (109) results in: $H_2 = 44.1\,\frac{P\,L}{D}$; $H_3 = 42.1\,\frac{P\,L}{D}$; $H_4 = 39\,\frac{P\,L}{D}$; and, $H_5 = 24.6\,\frac{P\,L}{D}$. By the common theory, the shear for the fifth floor girder is obtained by the equation, $H_1 \times \frac{1}{2}\,D = 22.5\,P \times \frac{1}{2}\,L + 23\,P \times \frac{1}{2}\,L$; or,

$$H_1 = 45.5\,\frac{P\,L}{D} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (111)$$

which varies only by 2% from the value in Equation (110).

The study of the two-column bent may be safely used in estimating the shear values in the girders of the upper two stories and those of the lowest two to six stories of a bent with three, four, or more columns. For this purpose an estimate of the column shears of the outside columns must be obtained by any valid method, such, for example, as the writer has indicated elsewhere.[14] Twice this shear must be considered as the load on the two-column bent and the girder shears may be computed for the irrgular panels at the top and bottom, as shown previously herein.

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[14] *Journal*, Am. Concrete Inst., February, 1932; see, also, p. 438.

The shears thus found in the girders will differ from those found by the common theory, and the shears in the corresponding girders of the inside bays may be changed in the same proportion, provided there is not too great a change in the value of the corresponding value of n. Some objections may be raised to the use of the formulas for a symmetrical two-column bent to the case of a bay cut from a regular wind-bracing bent where the two columns may not be of the same stiffness. The writer has studied, thoroughly, the two-column bent with columns of different stiffness (the unsymmetrical bent), and has found that, except for the top and bottom stories, the following relation holds:

$$\frac{X_1}{X_2} = \frac{(6 + n'')}{(6 + n')} \qquad \text{(112)}$$

in which X_1 is the shear at the left column; X_2 is the shear at the right column; n' is the relative stiffness between the girder and the left column; and n'' is the stiffness in relation to the right column. From Equation (112) one may observe at once that the shears in the columns will vary only a little when the values of n' and n'' are small, even where they differ from each other by as much as 100 per cent. For $n' = \frac{1}{8}$ and $n'' = \frac{1}{4}$, $\frac{X_1}{X_2} = 1.022$, from which it may be concluded that no great errors will be made in most cases of unsymmetrical frames when a corresponding symmetrical frame is analyzed first and the possible variation is studied later.

For end conditions the factor, 6, had better be replaced by 1 in Equation (112). If the author is correct in stating (see heading "Special Approximations") that Professor Vierendeel uses the assumption:

$$\alpha = \beta = \frac{I'_c L}{I''_c L_1} \qquad \text{(113)}$$

then the writer is convinced that Professor Vierendeel's students did not accept his analysis seriously or they would have analyzed many examples and would have found that Equation (113) "falls far from the mark"; although it does not seem to effect the results in Example 4 of the paper.

Professor Young's paper is an important contribution to American literature on structural engineering. However, it is no improvement on the 40-yr old method introduced by Professor Vierendeel. In fact, many of his practical simplifications have been omitted. The paper deserves the earnest discussion of the keenest talent in the United States because this analysis is susceptible of great simplification to the vast benefit of structural engineers, especially at this time when arithmetical methods are lauded as being preferable to a clear understanding and scientific processes.

Of course, Professor Young's analysis is not "exact" even in his own sense. He does not say whether clear spans or center-to-center spans should be used in the calculations and there is certainly a difference of at least 20% from this source alone, in most cases. There are considerable "roundings" at all the junctures of web members and chords, and they may change the results 5 to 10 per cent. If to this is added the serious error made in all textbook rules for the design of members subject to combined loading (which are often 50% in error), it is quite probable that an investigator who tests a model to destruction which has been designed by Professor Young's "exact" method may find it 100% too strong and will declare Professor Young's method a "safe" guide.

Fig. 16 shows a 100 000-gal sprinkling tank on a 80-ft tower, built in 1911 for the Chicago City Railway Company, Chicago, Ill., and Fig. 17 shows one of several open-bent structures (a coal-bin of 1 900-ton capacity, 142 ft high) built for the St. Louis Coke and Chemical Company, in 1919, at Granite City, Ill. The latter is probably the most unusual Vierendeel type of structure ever built. A large number of open-bent structures have been built in the United States.

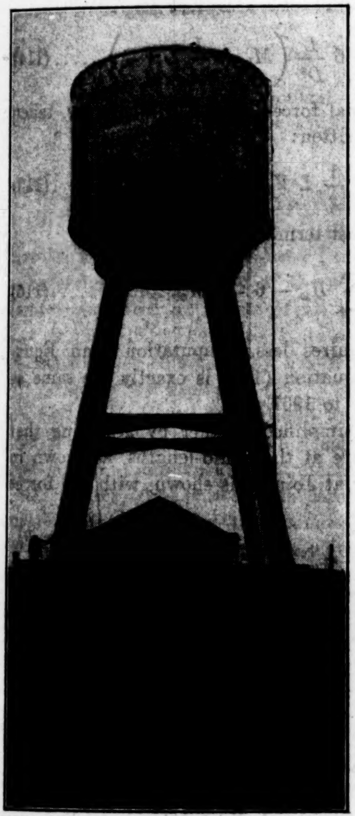

FIG. 16—VIEW OF 100 000-GALLON SPRINKLING TANK ON 80-FOOT TOWER, CHICAGO, ILL.

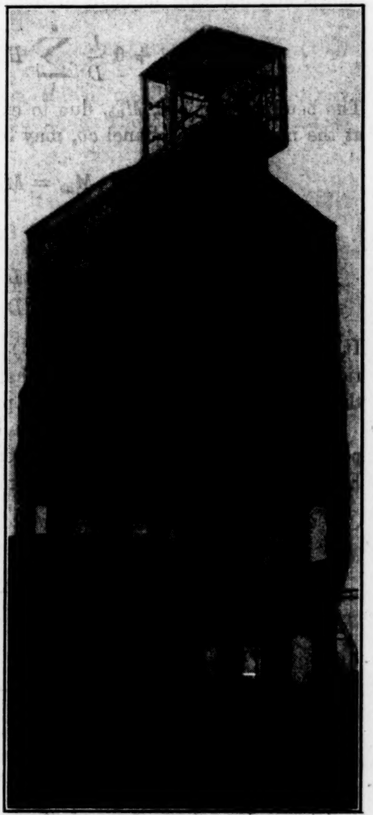

FIG. 17—VIEW SHOWING ONE OF SEVERAL OPEN-BENT STRUCTURES, COAL-BIN OF 1 900-TON CAPACITY, GRANITE CITY, ILL.

A. A. EREMIN,[15] ASSOC. M. AM. SOC. C. E. (by letter).—Although little has been written on this subject in English, the distribution of stresses in Vierendeel trusses has been studied extensively abroad. An exact computation of these stresses is a forbidding task and, therefore, various simplified methods of computation have been developed. The advantage of a Vierendeel truss is that, for all practical purposes, stresses can be computed with reasonable accuracy for assumptions as to distribution.

The method described by the author is not simple, although it can easily be simplified. For example, in the case of a Vierendeel truss with parallel chords and constant moment of inertia, Equation (11) may be written:

$$H_{bw} = H_{aw} + 6\,\frac{L}{D}\sum_{1}^{a} H_w - 6\,\frac{L}{D^2}\left(M_b + \frac{1}{2}\,L\,V_{ab}\right) \;\ldots\ldots (114)$$

The bending moment, M_{ab}, due to external forces carried by the truss, taken at the mid-length of Panel *ab*, may be written:

$$M_{ab} = M_b + \frac{1}{2}\,L\,V_{ab} \;\ldots\ldots\ldots\ldots\ldots\ldots (115)$$

Substituting Equation (115) in the last term of Equation (114):

$$H_{bw} = H_{aw} + 6\,\frac{L}{D}\sum_{1}^{a} H_w - 6\,\frac{L}{D^2}\,M_{ab} \;\ldots\ldots\ldots\ldots (116)$$

It is evident that Equation (116) requires less computation than Equation (11). It may also be noted that Equation (116) is exactly the same as that developed by Dr. F. Gebauer[16] prior to 1907.

Computation for this case may be simplified further by assuming that points of contraflexure in all members are at their mid-lengths, as shown in Fig. 18. In Fig. 19 the vertical member at Joint *b* is shown, with the forces acting at the points of contraflexure. Stresses indicated in these diagrams may be determined by statics. Using the author's notation, direct stresses in the chords are:

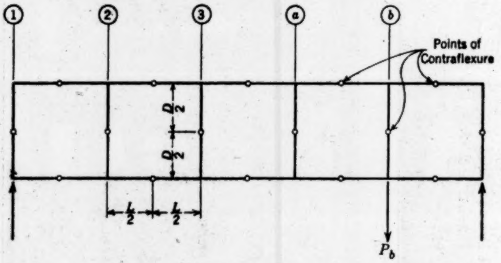

FIG. 18

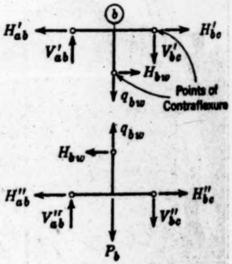

FIG. 19

$$H'_{ab} = -H''_{ab} = \frac{M_a + M_b}{2\,D} \;\ldots\ldots\ldots\ldots (117)$$

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[15] Assoc. Bridge Designing Engr., Div. of State Highways, Sacramento, Calif.

[16] *Beton und Eisen*, 1907, p. 252.

The shear stresses, V'_{ab}, at the points of contraflexure in the chords at Panel *ab* are:

$$V'_{ab} = V''_{ab} = \frac{1}{2} V_{ab} \dots\dots\dots\dots\dots\dots (118)$$

The shear stresses, H_{bw}, at the point of contraflexure in vertical members at Joint *b* are:

$$H_{bw} = H_{bc} + H_{ab} \dots\dots\dots\dots\dots\dots (119)$$

The direct stress, q_{bw}, in the vertical member at Joint *b* is,

$$q_{bw} = V_b - P_b \dots\dots\dots\dots\dots\dots (120)$$

The sum of the bending moments at the ends of members meeting at any joint is equal to zero; thus, for Joint *b* at the upper chord, an expression may be written (see Fig. 19):

$$V'_{ab} \frac{L}{2} + V'_{bc} \frac{L}{2} - H_{bw} \frac{D}{2} = 0 \dots\dots\dots\dots (121)$$

Equation (121) may also be used for checking the computations.

Referring to Example 1 of the paper, the shear stress at the end vertical member computed by means of Equation (117) is, $H_{1w} = 750$ lb; likewise, $H_{1w} = 1\,000$ lb. Therefore, it is evident that the shear stress at the end vertical member computed by means of Equation (117) differs slightly from that computed by Equation (11). The error increases toward mid-span, but will be less in Vierendeel trusses with a large number of panels. Furthermore, there is generally a greater factor of safety in verticals toward the mid-span, due to architectural requirements that vertical members be uniform.

If, in Vierendeel trusses with parallel chords, the moment of inertia of the upper chord is I_t, and the moment of inertia of the lower chord is I_b, the assumed distances of points of contraflexure in vertical members to top chords are:

$$a = D \frac{1}{1 + \frac{I_b}{I_t}} \dots\dots\dots\dots\dots\dots (122)$$

Equation (122) is similar to the formula for locating points of contraflexure developed by Professor R. Saliger.[17]

In the case of a Vierendeel truss with a curved top chord and straight bottom chord (see Fig. (5*a*)), a limitation of the author's method is that the true values of α and β can not be computed. In practice, the values of α and β differ slightly from unity. Furthermore, the probable error in the computation of the stresses (even with the help of modern mechanical calculating machines) is always greater than the difference between the true values of α and β and the values assumed. This is especially true in the case of Vierendeel trusses with a large number of panels.

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[17] **"Der Eisenbetonbau", 6 Aufl., Leipzig, 1933.**

Equation (54) is reasonably simple and may be used for the general cases in practice. However, the computation of the moments, M_w, and the shear stresses, V', may be simplified. Assume that the points of contraflexure are at the middle of the vertical members and that the shear stress, H_w, is applied at those points. Then, the maximum bending moment, M_{aw}, in the vertical member at Joint a, is,

$$M_{aw} = H_{aw} \frac{D_a}{2} \qquad \text{.......................(123)}$$

In Example 3 of the paper, the moments in the vertical members as computed by Equation (123) are: $M_{1w} = 601.96 \times 2 = 1\,203.92$; $M_{2w} = 366 \times 4 = 1\,464$; $M_{3w} = -10 \times 5 = -50$; $M_{4w} = -262 \times 4 = 1\,048$; and, $M_{5w} = -411 \times 2 = 822$.

The shear stress, V', may be computed from Equation (48a), assuming $\alpha = 1$; thus:

$$V'_{ab} = \frac{1}{2}\left(V_{ab} + \frac{d}{L}\sum_{1}^{a} H_w\right) \qquad \text{...............(124)}$$

Shear stresses computed by Equation (124) are: $V'_{12} = \frac{1}{2}\,(600 + 0.4 \times 600) = 420$; $V'_{23} = \frac{1}{2}\,[600 + 0.2 \times (600 - 369)] = 396.9$; $V'_{34} = -\frac{1}{2}\,(400) = -200$; $V'_{45} = -\frac{1}{2}\,[400 + 0.2\,(409 + 287)] = -269.6$; and $V'_{56} = -\frac{1}{2}\,[400 + 0.4\,(409)] = -281.8$.

It is evident that the moments and shear stresses determined by Equations (123) and (124), respectively, differ slightly from those computed by Equations (75) and (77) of the paper. The labor involved, however, is considerably reduced. The author is to be congratulated for his valuable and interesting contribution.

Leon Blog,[18] Assoc. M. Am. Soc. C. E. (by letter).—Formulas for analyzing several of the more familiar types of Vierendeel trusses are presented in this paper. As the largest part of the paper is devoted to the analysis and solution of problems dealing with the case of an inclined upper chord, the writer has confined his discussion to that phase.

Whether one will prefer to use the formulas presented by the author or the approximate equations derived by Professor Vierendeel, depends upon the exigencies of one's practice. Some engineers prefer to use formulas approximate to a known degree of accuracy and affording a speedy solution. They know full well that a liberal correction must be made to compensate for causes which make the theoretical stresses unattainable.

The main difference between the analyses of Professors Vierendeel and Young is that the former develops accurate formulas at the beginning of his study, demonstrates that they are too unwieldy for speedy solution, and, after making certain assumptions which carry conviction, evolves formulas which, although admittedly approximate, have a simple nomenclature, are easy to apply, and, above all, are direct. No "cut-and-try" process is required. Professor Young evolves a formula for horizontal shear in the vertical members which is theoretically exact according to the postulates stipulated; but it is difficult to apply because it is not simple. It is expressed in terms of unknowns. It can only be made simple by assuming certain convenient relations between the moments of inertia of the truss members, the fulfillment of which, in practice, would be purely fortuitous. How often will ϕ equal 1? When it does not, the various moments of inertia remain in Equation (53) and the fractions containing α and β must be evaluated before that formula can be solved. The difficulty of evaluating α is indicated subsequently under the heading "Comparison of Formulas."

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Using Professor Vierendeel's equations[19] the writer has made a check of the solution of Example 3 which the author solved by use of the formulas of the paper. The results of this solution are compared with that of the author in Fig. 20. All the forces are in pounds and the moments are in foot-pounds.

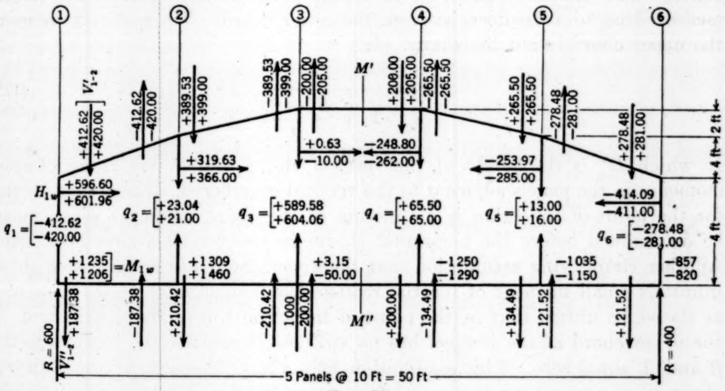
①
②
③
④
⑤
⑥
V'_{1-2}
+412.62
+420.00
−412.62
−420.00
+389.53
+399.00
−389.53
−399.00
−200.00
−205.00
M'
+208.00
+205.00
−265.50
−265.50
+265.50
+265.50
−278.48
−281.00
+278.48
+281.00
+0.63
−10.00
−248.80
−262.00
+319.63
+366.00
−253.97
−285.00
H_{1w}
+596.60
+601.96
−414.09
−411.00
$q_1 = \begin{bmatrix}-412.62\\-420.00\end{bmatrix}$
$q_2 = \begin{bmatrix}+23.04\\+21.00\end{bmatrix}$
$q_3 = \begin{bmatrix}+589.58\\+604.06\end{bmatrix}$
$q_4 = \begin{bmatrix}+65.50\\+65.00\end{bmatrix}$
$q_5 = \begin{bmatrix}+13.00\\+16.00\end{bmatrix}$
$q_6 = \begin{bmatrix}-278.48\\-281.00\end{bmatrix}$
+1235
+1206
$= M_{1w}$
+1309
+1460
+3.15
−50.00
−1250
−1290
−1035
−1150
−857
−820
R = 600
V''_{1-2}
+187.38
−187.38
+210.42
−210.42
1000
−200.00
M''
+200.00
−134.49
+134.49
−121.52
+121.52
R = 400
2 ft
4 ft
4 ft
5 Panels @ 10 Ft = 50 Ft

FIG. 20

Shear in the lower part of the vertical members balances the shears that are shown, and the writer's values are placed above, and to the left of, those determined by the author.

In the writer's opinion, the author did not depart essentially from the method of analysis used by Professor Vierendeel for the inclined upper chord truss. He arrived at different formulas by using the tool of least work to evaluate the relative displacements of the junctions of the verticals with the chords for two successive verticals. Professor Vierendeel based his method upon the linear and angular displacements of a point in the axis of an arch of flat curvature. He applied the formulas for these displacements to find the relative displacements of two successive verticals above a plane passed through them between the chords, and equated the displacements for the same points in the verticals below the plane. The result was an exact expression for the horizontal shear in any vertical which the author deemed to be too complex for easy and speedy solution. Making use of the relation which he had previously proved, that $\frac{M'}{M''} = \frac{I'\,dx}{I''\,ds}$, in which M' and I' are the internal bending moment and the moment of inertia at any section of the upper chord and M'' and I'' refer to a point in the lower chord in that same section; and since $\frac{dx}{ds}$ is the cosine of the angle of slope of the panel in which the section is taken, then, letting $\frac{I'\,dx}{I''\,ds} = \beta$, $M' = \beta\,M''$; that is, either chord moment is known when the other is known. This is the fundamental concept which underlies the derivations of formulas for the chord shears and horizontal shears through the verticals evolved by Professor Vierendeel. It is based upon the tenable assumption that the length of the verticals remains constant. This concept also underlies the location of the point of inflection of the vertical measured up to a distance, y, from the lower chord. When the curvature of the upper chord is not too sharp:

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[19] "Cours de Stabilite des Constructions", Tome IV, by Arthur Vierendeel, Louvain, 1920, Equation (16); Equation (3) p. 194; Equation (4) p. 183; and Equations (8) and (9), p. 188.

$$y = \frac{D}{1 + \frac{\beta + \beta_1}{2}} \qquad \text{(125)}$$

in which D[20] is the height of the vertical and β and β_1 are ratios of chord moments in the panels adjacent to the vertical considered. The true expression for the point of inflection is a function of the chord moments which cannot be determined before the horizontal shears in the verticals have been found. Another simplifying assumption that the upper chord has a finite area but an infinitely small moment of inertia, reduces the denominator of the value of y, as stated, to unity; that is, the point of inflection lies in the upper chord. If the upper chord at the vertical has no stiffness it can take no moment so that β and β_1 equal zero. This assumption may affect the value of the horizontal shear in a vertical as much as 5.5 per cent.[21] Based upon the simplifying assumptions made, and proceeding from the exact analysis for the horizontal shear in a vertical, Professor Vierendeel evolved the following formula:[22]

$$H_{r+1} = \frac{H_r\,D^2_r\,(3\,D_{r+1} - D_r)}{2\,D^3_r} + \frac{L\,(3\,D_r\,D_{r+1} + d^2)}{D^3_{r+1}} \sum_1^r H - \frac{3\,L\,(D_r + D_{r+1})\,M_{r+1}}{2\,D^3_{r+1}} + \frac{L^2\,(2\,D_r + D_{r+1})\,V_r}{2\,D^3_{r+1}} \qquad \text{(126)}$$

The subscript, r, denotes the left-hand panel point whereas $r + 1$ denotes the one immediately to the right; H denotes the horizontal shear in the vertical; D, its height; d, the difference in heights of two successive verticals; L, the panel length; and, M and V, the external bending moment and shear, respectively. Note the absence of all symbols except dimensions or forces in Equation (126) and the freedom from terms which themselves involve unknowns. Regardless of the value of H_r, $\sum_{1}^{r} H$, M_{r+1} or V_r, their coefficients are always the same and need be computed but once. Professor Vierendeel also reverses the position of the load in order to derive all the shears, as did the author.

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[20] "Cours de Stabilite des Constructions", Tome IV, by Arthur Vierendeel, Louvain, 1920, p. 186.

[21] *Loc. cit.*, p. 179.

[22] *Loc. cit.*, Equation (16), p. 194.

Comparison of Formulas.—Vierendeel wrote a formula[33] for horizontal shear in vertical members as follows:

$$H_{bw} = \frac{D^2_a\, H_{aw}}{2\, D^3_b}\,(3\, D_b - D_a) + \frac{L}{D^3_b}\left[(3\, D_b\, D_a) + (D_b - D_a)^2\right] \sum_{1}^{a} H_w$$

$$- \frac{3\, L\, (D_b + D_a)}{2\, D^3_b}\, M_b + \frac{L^2\, (D_b + 2\, D_a)}{2\, D^3_b}\, V_a \quad \ldots\ldots\ldots\ldots (127)$$

in which the notation is that of the paper. It is obvious that Equation (127) is more simple than Equation (53), involving fewer terms, and including only the linear dimensions of the truss and the known external bending moments and shears. All the assumptions are involved in the formula and, assumptions that $\phi = 1$, or that $I_a = I_b = I''_c = I'_c \dfrac{L}{L_t}$, such as the author makes to solve Equation (53), need not be made The difficulty about using Equation (53) is that H_{bw} involves unknown chord moments represented by α and β. These chord moments are expressed, in turn, in terms of themselves and of H_w as shown by reference to Equations (50*a*) and (50*b*). The ratio, β, is known from the properties of the truss, but H_w must still be found before the moments can be determined. This is a circuitous process. Professor Vierendeel suggested[33]:

$$M''_{ba} = - \frac{M_{ba}}{1 + \beta} + \frac{D_b \sum_{1}^{a} H_w}{1 + \beta} \quad \ldots\ldots\ldots\ldots\ldots (128)$$

It is to be noted that Equation (128) differs from Equation (50*b*) in the sign convention and that $\beta = \dfrac{M'_{ab}}{M''_{ab}}$.

The author's formulas for chord shears, Equation (48*a*) and Equation (48*b*), are complex because they involve the upper and lower chord moments of two adjacent panels which must first be found from Equation (53) and Equation (50*b*). Such moments need not be found when using the Vierendeel formulas[34],

$$V'_{ab} = \frac{V_{ab}}{1 + \beta} + \frac{\dfrac{d}{L} \sum_{1}^{a} H_w}{1 + \beta} \quad \ldots\ldots\ldots\ldots\ldots (129)$$

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[33] "Cours de Stabilite des Constructions", Tome IV, by Arthur Vierendeel, Louvain, 1920, Equation (9), p. 188.

[34] *Loc. cit.*, Equations (3) and (4), p. 183.

and,

$$V''_{ab} = \frac{V_{ab}}{1 + \beta} - \frac{\frac{d}{L} \sum_{1}^{a} H_w}{1 + \beta} \quad \text{................} (130)$$

because $\frac{M'_{ab}}{M''_{ab}} = \beta = \frac{I'\,dx}{I''\,ds}$, which is a known constant and, when substituted for the easily found ΣH_w, readily yields V'_{ab} and V''_{ab}.

Discussion of Results in Fig. 20.—Fig. 20 shows a comparison of all the values found by the author with those found by the Vierendeel formulas. In addition, the writer computed the shears in the lower chord by the Vierendeel formula, Equation (130). The sum of the shears in both chords in any panel must equal the external shear at that point and yields a check on the underlying evaluations.

The maximum shear variation in the upper chord occurs in Panel 3–4 and is 2.50% less than the value given by the author. The maximum significant horizontal shear variation in any vertical occurs in q_8 and is 12.50% less than the author's value. The maximum variation in direct stress in any vertical occurs in q_5 and is 18.70% less than the author's value. The maximum significant moment variation in any vertical occurs in q_2 and is 10.30 per cent.

The term, significant, is used to exclude the values for horizontal shear and bending moment in q_3. For shear, the writer obtained + 0.63 as against the author's − 10.00. These values are both small and of opposite signs. In any practical design, they would be allowed a factor of error because of the proximity to zero which might be due to the sensitivity of both methods to the underlying assumptions. For instance, in the author's solution of Examples 3 and 4, H_{3w} each time equals − 10 lb despite his different relations between the moments of inertia. For the moments in q_3, the writer's value is + 3.15 and that of the author, − 50.00 ft-lb. The ratio of these shears and moments is about 15.9 to 1. Except for the values for q_3, the agreement between the two solutions is close.

Under the heading, "Special Approximations", the author states that "Professor Vierendeel uses an approximate method for calculating the chord moments and shears[13] which is very rapid. Equations (48) and (50) are expressions for these functions." He should have made it clear that Equations (48) and (50) are his own expressions. The corresponding Vierendeel formulas are Equations (128), (129), and (130).

Referring to Conclusion (3) of the paper, the writer agrees that the solution for a truss with an inclined upper chord although proceeding from an exact formula, must, in the last analysis, be approximate. As to Conclusion (4), the formulas proposed cannot be as rapid as those of Professor Vierendeel because the simplifying assumptions required to be made with regard to moments of inertia, ϕ, and values of a to arrive at a simpler formula (Equation (54) from Equation (53)) will not, in general, correspond with the conditions of the problem. These difficulties are eliminated by Professor Vierendeel, who makes rational assumptions during the development of his final simple and direct formulas. The author's remarks in Conclusion (5) apply to the Vierendeel formulas, but the numerical effect of Assumption (3) is not immediately evident except for those happily selected cases, such as for $\phi = 1$, unless a solution is made by another method as a check. Appreciable errors can be made when the slopes of the upper chord vary considerably from panel to panel. Equation (53) contains too many unknowns for a rapid solution having any assured degree of accuracy. How much error is involved when ϕ does not $= 1$ and when α and β are assumed?

To those who are interested in the subject of Vierendeel trusses, the writer recommends reference to Professor Vierendeel's comprehensive treatment of trusses of various shapes and purposes as well as the design of panel joints.[13] An analysis of Vierendeel trusses by the combined Cross and Grinter methods was presented in an able paper before the International Association for Bridge and Structural Engineers.[25]

From the standpoint of rapid design, the writer does not consider the author's formulas as rapid, or as unfailingly accurate, as those presented by Professor Vierendeel.

A. W. FISCHER,[26] ESQ. (by letter).—The author has certainly contributed a valuable paper to the Engineering Profession on the analysis of the Vierendeel truss with either symmetrical or inclined upper chords. At first glance the theory seems rather long, but after it is studied and understood it is very simple (as are the final general equations) in application, if the theory from which they were developed is understood.

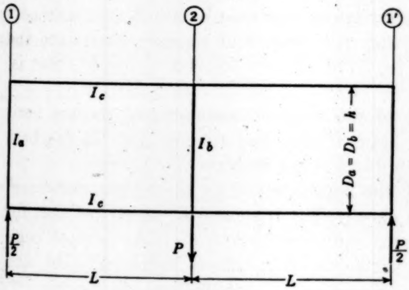

FIG. 21

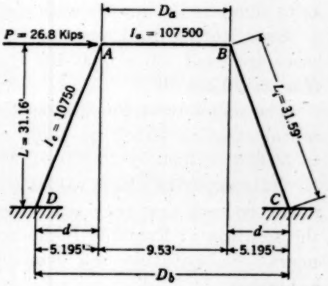

FIG. 22

Equation (10) is a general formula and solves symmetrical structures with either parallel or inclined chords in a short time.

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[25] Rept. of the International Assoc. for Bridge and Structural Engrs., Vol. 3, 1935, by L. C. Maugh, Assoc. M. Am. Soc. C. E.

[26] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

Example A.—For example, consider a symmetrical quadrangular bridge truss as shown in Fig. 21 which is a two-span Vierendeel truss, with parallel chords. Substituting in Equation (10):

$$H_{2w} = \frac{I_b\, H_{1w}}{I_a} + \frac{6\, L\, I_b\, H_{1w}}{h\, I_c} - \frac{6\, L\, I_b\, M_2}{h^2\, I_c} + \frac{3\, L^2\, I_b\, V_{12}}{h^2\, I_c} \quad \ldots (131)$$

From the symmetry of the loading in this particular case, it is obvious that $H_{2w} = 0$; $M_2 = 0.5\, PL$; and $V_{12} = 0.5\, P$. Substituting in Equation (131):

$$\frac{H_{1w}}{I_a} + \frac{6\, L\, H_{1w}}{h\, I_c} - \frac{3\, L^2\, P}{h^2\, I_c} + \frac{1.5\, L^2\, P}{h^2\, I_c} = 0 \quad \ldots\ldots\ldots\ldots (132)$$

Reducing Equation (132):

$$H_{1w} = \frac{1.5\, P\, \phi\, \dfrac{L}{h}}{1 + 6\, \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (133)$$

in which $\phi = \dfrac{I_a\, L}{I_c\, h}$.

The moment at the intersection of the end vertical and the top chord,

$$M = 0.5\, H_{1w}\, h = \frac{0.75\, P\, \phi\, L}{1 + 6\, \phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (134)$$

which is the same value as that given elsewhere.[27] The moments at the other joints can now readily be computed by statics.

Example B.—As another example the writer will use the top story of the bent analyzed as Example 2 in the paper and will assume the bases fixed, so as to form a symmetrical frame with inclined legs. In Equation (10) the change of the length of the various members is considered zero. Instead of using the dimensions given in Fig. 4, those shown in Fig. 22 will be used so as to compare the results with a similar frame analyzed by a different method.[28]

Substituting in Equation (10) and reducing, and noting that, since the bases are fixed: $0 = 0.005602\, H_{1w} + 2.770\, H_{1w} - 150\,900 - 66\,560$; that is, $H_{1w} = 30\,390$ lb.

The moment at the intersection of the inclined member and the top horizontal strut = 30 390 × 4.765 = 144.8 ft-kips and the moment at the base = 30 390 × 9.96 − 13 400 × 31.16 = − 114.8 ft-kips.

On comparing these values with the results given by the writer elsewhere[28] it can be seen that they agree very closely, but it seems to the writer that for the solution of Example *B* the general equations[29] are preferable to most engineers. Substituting the symbols shown in Fig. 22 in Equation (10) and reducing:

$$M_{AD} = - \frac{P\, D_a\, \phi_t\, L\, (3\, D_a + 4\, d)}{2\, \left\{ D^2_a\, (1 + 6\, \phi_t) + 12\, D_a\, \phi_t\, d + 8\, \phi_t\, d^2 \right\}} \quad \ldots\ldots\ldots (135)$$

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[27] "The Design of Steel Mill Buildings", by the late M. S. Ketchum, Hon. M. Am. Soc. C. E., Fourth Edition, Rewritten, Equation (56), p. 303.

[28] See pp. 362–364.

[29] See Equations (41) and (42), p. 364.

and,

$$M_{DA} = -\frac{P D_a L \left\{ D_a (1 + 3\phi_i) + 2\phi_i d \right\}}{2\left\{ D^2_a (1 + 6\phi_i) + 12 D_a \phi_i d + 8\phi_i d^2 \right\}} \quad \text{.........(136)}$$

in which $\phi_i = \dfrac{I_a L_i}{D_a I_c}$.

All calculations in Example *B* were carried to four significant figures. If they had been carried to ten significant figures the moments would have been 144 699 ft-lb and − 115 089 ft-lb, respectively. As the results differ by a very small amount the results to four significant figures are satisfactory and, therefore, a 20-in. slide-rule will give results that are reliable.

In Example *C* the truss shown in Fig. 23 is analyzed to show how the results by the author's analysis for a truss with an inclined upper chord compares with the results as calculated by relative deflections as proposed by Professor H. Yu.[30] The moments of inertia for all the members are equal.

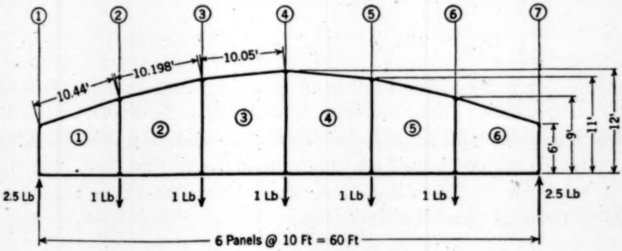

FIG. 23

The values of α and β are not known for an original design, but as the moments in the top and bottom chords have been determined by Professor Yu, these values have been used for determining the bending moment ratios, as shown in Table 2.

TABLE 2.—FACTORS FOR THE SOLUTION OF EXAMPLE *C*

Panel No.	VALUES OF:				
	α	β	ϕ	$\dfrac{1 + \phi\,\alpha}{1 + \alpha}$	$\dfrac{1 + \phi\,\beta}{1 + \beta}$
1............	0.9839	0.9862	1.044	1.022	1.022
2............	1.003	0.9954	1.0198	1.010	1.010
3............	1.011	1.0	1.005	1.003	1.003

Substituting the proper values from Fig. 23, and the proper factors from Table 2, in Equation (53), and reducing:

$$H_{1w} = H_{1w} \quad \text{......................(137a)}$$

$$H_{2w} = 0.2963\,H_{1w} + 4.795\,H_{1w} - 0.6309\,M_2 + 2.944\,V_{12} \quad \text{....(137b)}$$

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[30] "Stresses in Statically Indeterminate Structures", by Prof. H. Yu, National Wuhan Univ., Wuchang, Hupeh, China, Second Edition, 1935, pp. 486–494.

$$H_{3w} = 0.5477\,H_{2w} + 4.568\,(H_{1w} + H_{2w}) - 0.4553\,M_3 + 2.201\,V_{23}. \tag{137c}$$

and,

$$H_{4w} = 0.7703\,H_{3w} + 4.609\,(H_{1w} + H_{2w} + H_{3w}) - 0.4005\,M_4 + 1.973\,V_{34}. \tag{137d}$$

For the loading shown in Fig. 23: $M_2 = 25$ ft-lb; $M_3 = 40$ ft-lb; $M_4 = 45$ ft-lb; $V_{12} = 2.5$ lb; $V_{23} = 1.5$ lb; and, $V_{34} = 0.5$ lb. Substituting these values in Equations (137) and then expressing each value of H_w in terms of H_{1w}:

$$H_{1w} = H_{1w} \tag{138a}$$

$$H_{2w} = 5.0913\,H_{1w} - 8.4125 \tag{138b}$$

$$H_{3w} = 30.61356341\,H_{1w} - 57.94632625 \tag{138c}$$

and,

$$H_{4w} = 192.7543434\,H_{1w} - 367.5198853 \tag{138d}$$

From the symmetry of the loading in this particular example, it is obvious that $H_{4w} = 0$; hence, from Equation (138*d*), $H_{1w} = 1.90667$ lb and, from Equations (138*b*) and (138*c*), $H_{2w} = 1.29493$ lb and $H_{3w} = 0.423637$ lb.

After the H_w-values have been determined from Equation (53) the next step is to solve for the M_w-values for comparison. Applying Equations (67) and (69) to the first panel and reducing:

$$M_{2w} = 19.81 - 2.469\,M_{1w} \tag{139}$$

and,

$$V'_{12} = 0.4761\,M_{1w} - 1.171 \tag{140}$$

Applying Equations (66) and (68) to the second panel and reducing:

$$M_{3w} = -\,2.554\,M_{2w} - 0.9181\,M_{1w} + 9.181\,V'_{12} + 8.349 \tag{141}$$

and,

$$V'_{23} = 0.5674\,M_{2w} + 0.3\,M_{1w} - 3\,V'_{12} + 0.6575 \tag{142}$$

Applying Equation (66) to the third panel and reducing:

$$M_{4w} = -\,2.669\,M_{3w} - 0.8354\,M_{2w} - 0.8354\,M_{1w} + 8.354\,V'_{12} + 8.354\,V'_{23} + 5.907 \tag{143}$$

The value of V'_{34} is not required for evaluating M_{1w}, M_{2w}, and M_{3w}, and is, therefore, not given. Substituting Equations (139) and (140) in Equations (141) and (142):

$$M_{3w} = 9.759\,M_{1w} - 53.00 \tag{144a}$$

and,

$$V'_{23} = 15.41 - 2.529\,M_{1w} \tag{144b}$$

Substituting Equations (139), (140), and (144) in Equation (143):

$$M_{4w} = -41.969495\,M_{1w} + 237.953072 \ldots\ldots\ldots\ldots (145)$$

From the symmetry of the loading in this particular example, it is obvious that $M_{4w} = 0$; hence, from Equation (145), $M_{1w} = 5.6697$ ft-lb. The value determined by Professor Yu is 5.6695 ft-lb. Substituting the value of M_{1w} in Equation (139) gives $M_{2w} = 5.8115$ ft-lb; Professor Yu[30] found it to be 5.8115 ft-lb. Substituting the value of M_{1w} in Equation (144*a*) $M_{3w} = 2.3306$ ft-lb; Professor Yu reported 2.3298 ft-lb.

From the foregoing values it is seen that the author's general equations give results that agree with those found by other methods; and they should agree, because no approximations are assumed except that in all the analysis in this discussion the change in the length of the members is considered equal to zero.

The value of M_{3w} as given is not the absolute maximum because, if there is no unit load at Panel Point 2, but unit loads at Panel Points 3, 4, 5, and 6, then the value of M_{3w} will be larger and the value given by Professor Yu is 3.3355 ft-lb.

For the symmetrical loading assumed, $M_{4w} = 0$; but if unit loads are placed at Panel Points 2 and 3, M_{4w} no longer $= 0$, and the value given by Professor Yu is -2.1459 ft-lb.

If it is assumed that the H_w-values act at the center of the vertical members, the moments are as follows: $M_{1w} = 1.90667 \times 3 = 5.7200$ ft-lb; $M_{2w} = 1.29493 \times 4.5 = 5.8272$ ft-lb; and $M_{3w} = 0.423637 \times 5.5 = 2.3300$ ft-lb. The corresponding values given by Professor Yu are: 5.7122 ft-lb; 5.8398 ft-lb; and, 2.3316 ft-lb, respectively. In this case, again, the two different methods agree very closely.

To solve for the maximum moments in all the members of a Vierendeel truss is a rather lengthy process no matter what method is used, but it appears that the author's method is as short as any.

L. C. MAUGH,[31] ASSOC. M. AM. SOC. C. E. (by letter).—In the United States the use of Vierendeel trusses has been opposed consistently (in print at least) for such reasons as lack of rigidity and economy or for the difficulties that are involved in the analysis, design, and construction of such monolithic structures. In actual practice, however, structures of this type have been built or, more frequently, designed as a more standardized type, such as in the bowstring and open spandrel arch, when the Vierendeel truss arrangement would sometimes be the simpler solution. This simplicity is especially noticeable when welded or reinforced concrete construction is used. The present trend in the adoption of various structural types indicates that there will be an increasing use of the quadrangular panel system by those engineers who appreciate both its advantages and disadvantages (because it certainly has both). For this reason a general discussion of some of the methods that are applicable to the analysis of the quadrangular framework will be pertinent at this time.

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[31] Asst. Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

In the analysis of the Vierendeel truss, as in any hyperstatic frame, the engineer has the choice of two different methods of approach: One involving the strain energy of the system or its counterpart, internal and external work, and the other, the various deformation methods that use the displacement of the joints. In this paper the author has used the first method to obtain a general form for the three algebraic expressions that will provide a system of minimum strain energy, these equations being expressed in terms of the resultant internal forces acting on the vertical members. Ordinarily, there will be three of these unknown forces in each of the strain equations but, by ingenious algebraic arrangement, certain groups of equations are expressed in terms of the horizontal components of the vertical members as the only forces, and two variables, α and β, that represent the ratio between the unknown end moments of the top and bottom chords. This algebraic arrangement, which is a characteristic feature of the method of Professor Vierendeel and Professor Young, is probably as convenient and simple as any results that can be obtained by the use of energy or work theorems. In fact, the author has done exceptionally well in the development and application of this method of approach. By assuming various values of α and β, the horizontal components of stress, H, in the vertical members can be computed with sufficient accuracy; but what is true for the H and V-components, the writer believes, is not necessarily true for the bending moments in the chord members.

Small errors made in the calculation of the H and V-values (errors that can easily be made because of the necessity of carrying the computations out to many decimals, or due to the rather indeterminate quantities, α and β) may produce relatively large errors in the end moments acting on the chord members Thus, for example, in Fig. 8, the author obtained values of $H_{1w} = 601.96$ lb and $V_{12} = 420$ lb, which, if applied at the mid-point of the first vertical, gives a moment at the right end of the chord member equal to: $M_{21} = (420)\ (10) - (601.96)\ (6) = 588$ ft-lb. If an error of -3% is assumed in H_{1w} and of $+3\%$ in V'_{12}, the computed value of the chord moment will be: $M'_{21} = (1.03)\ (420)\ (10) - (0.97)\ (601.96)\ (6) = 823$ ft-lb, or an increase of 40 per cent.

In view of the fact that, for structures with top and bottom chords of different rigidity, the values of α and β are affected by this variation of the chord moments, the writer believes that it would be difficult to obtain accurate numerical results. When the top and bottom chords in the same panel have the same $\frac{I}{l}$-value, then $\phi = \alpha = \beta = 1$, and a direct solution can be made regardless of the inclination of the chord members.

For Vierendeel trusses or for any rigid frame in which the members have considerable variation in rigidity and in which the joints undergo relatively large linear displacements as well as rotation, the writer has frequently combined the use of deformation equations with auxiliary force systems to obtain a direct method of solution. With this method of approach, the various forms of successive approximations, such as the method of iteration or moment distribution, can be incorporated. Various forms of the deformation methods are in general use, but the advantage of introducing auxiliary force systems to control the motion of the structure does not seem to be so well known.

To illustrate the use of auxiliary forces, the truss shown in Fig. 8 will be subjected to various systems that will allow only one linear displacement at a time. Thus, in Fig. 24(a), there is one vertical displacement Δ_1, and in Fig. 24(b), a vertical displacement, Δ_2. These displacements will produce end moments as shown when no rotation of the joints is considered and when a value of $I = 40$ is used. The final moments that will be developed for each value of Δ when the necessary rotation of the joints is allowed can easily be determined by moment distribution, or by calculating the angular rotation, using the method of iteration. In general, there will be as many of these special problems as there are panels in the truss although advantage can be taken of symmetry, as in the truss used by the author only three problems need be solved.

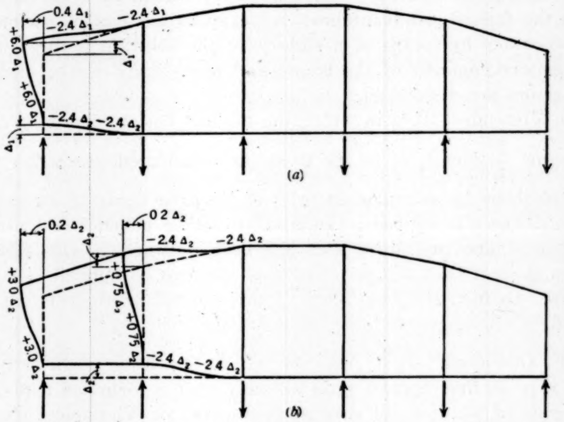

FIG. 24

After the values of the moments that are consistent with each value of Δ have been obtained, an equilibrium equation for the moments in each panel for any desired load can be written easily. For the truss considered, with $I = 40$, the writer obtained the following set of equilibrium equations:

$$-15.62\,\Delta_1 - 1.53\,\Delta_2 - 0.276\,\Delta_3 + 0.044\,\Delta_4 - 0.004\,\Delta_5 + 6\,000 = 0 \quad \text{..(146}a\text{)}$$

$$-1.65\,\Delta_1 - 7.64\,\Delta_2 + 1.64\,\Delta_3 - 0.261\,\Delta_4 + 0.042\,\Delta_5 + 6\,000 = 0 \quad \text{..(146}b\text{)}$$

$$-0.272\,\Delta_1 + 1.6\,\Delta_2 - 5.96\,\Delta_3 + 1.6\,\Delta_4 - 0.272\,\Delta_5 - 4\,000 = 0 \quad \text{..(146}c\text{)}$$

$$+0.042\,\Delta_1 - 0.261\,\Delta_2 + 1.64\,\Delta_3 - 7.64\,\Delta_4 - 1.65\,\Delta_5 - 4\,000 = 0 \quad \text{..(146}d\text{)}$$

and,

$$-0.004\,\Delta_1 + 0.044\,\Delta_2 - 0.276\,\Delta_3 - 1.53\,\Delta_4 - 15.62\,\Delta_5 - 4\,000 = 0 \quad \text{..(146}e\text{)}$$

Equations (146) satisfy all the requirements for quick convergence when solved by the method of iteration. The following values were obtained after performing the numerical calculations for five cycles: $\Delta_1 = 337$; $\Delta_2 = 583$; $\Delta_3 = -696$; $\Delta_4 = -652$; and $\Delta_5 = -180$. These relative displacements are, of course, E times the actual values. For different applied loads the only change in Equations (146) is in the constant terms.

The foregoing method of solution requires considerable numerical work, but it has the following advantages: All calculations can be performed with sufficient accuracy by means of a slide-rule; the solution is direct and accurate for any arrangement of the truss; and corrections can be made quickly whenever errors are discovered.

For the Vierendeel truss in which the top and bottom chords in each panel have the same $\frac{I}{l}$-factor, as in the truss just considered, the writer prefers to make the analysis by considering each panel as a separate structural unit. This method has already been explained in other papers[32] and need not be repeated here. The procedure that has been outlined in this discussion is much more general than the panel method and can be applied to many types of problems. In fact, the wide scope of this method of analysis is not always appreciated.

JOHN E. GOLDBERG,[33] JUN. AM. SOC. C. E. (by letter).—Slightly complex though it may at first appear, it is unlikely that a formular method for the exact analysis of the general case of open-panel or Vierendeel truss can be simplified to any greater extent than the fine method presented by Professor Young in his creditable paper Some simplification is possible, however, when cautious use is made of assumptions and approximations. Simplification is also possible in the analysis of special cases, although it may be necessary or desirable to abandon the general method of consistent deflections in favor of some special theory

Using a totally different method of attack, a special slope deflection method developed by the writer in 1933 for the analysis of symmetrical, parallel chord bents and trusses, a perfect check of Professor Young's Example 1, the frame shown in Fig. 3, was obtained. The basis of the writer's method is an exact and easily set-up working equation which gives the rotation, θ_n, of the nth panel points in terms of the rotations of the two adjacent panel points:

$$\theta_n \left(6 \frac{I_n}{D} + \frac{I'_{mn}}{L_{mn}} + \frac{I'_{no}}{L_{no}} \right) = \frac{V_{mn} L_{mn} + V_{no} L_{no}}{2} + \frac{I'_{mn}}{L_{mn}} \theta_m + \frac{I'_{no}}{L_{no}} \theta_o \ldots (147)$$

In actual use, an equation of this type is set up for each panel point along one chord, the resulting simultaneous equations then being solved by algebraic methods or by successive approximations. When successive approximations are used, the equations are reduced to the form:

$$\theta_n = A + B\,\theta_m + C\,\theta_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots (148)$$

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[32] "The Analysis of Vierendeel Trusses by Successive Approximations", Publications of the International Assoc. of Bridge and Structural Engrs., Vol. 3, (1935).

[33] With Dept. of Buildings, Chicago, Ill.

For Points 1 and 2 (Fig. 3) Equation (147) yields:

$$\theta_1 (6 + 1) = \frac{1.5 (10)}{2} + 1 (\theta_2) \ldots\ldots\ldots\ldots\ldots\ldots (149)$$

and,

$$\theta_2 (6 + 1 + 1) = \frac{1.5 (10) + 0.5 (10)}{2} + 1 (\theta_1) \ldots\ldots\ldots (150)$$

in which the shears are in kips. The simultaneous solution of Equations (149) and (150) (although the method of successive approximations using the form of Equation (148) would have been practically as easy) gives: $\theta_1 = 1.272$; and $\theta_2 = 1.409$. Final chord and cross-member moments are obtained by substituting the θ-values in the appropriate equations. For the moment at the end, *n*, of the chord member, *n-m*:

$$M_{nm} = \frac{V_{nm} L_{nm}}{4} + \frac{I'_{nm}}{L_{nm}} \frac{(\theta_m - \theta_n)}{2} \ldots\ldots\ldots\ldots (151)$$

and, for the moment at either end of the *n*th cross-member:

$$M_n = -3 \frac{I_n}{D} (\theta_n) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (152)$$

Thus, by Equation (152), the moment at each end of the first cross-member is $-3(1.272) = -3.816$ kip-ft, and at each end of the second cross-member, $-3(1.409) = -4.227$ kip-ft. Respective shears are, therefore, $\frac{2 (3.816)}{10} = 0.763$ kips, and $\frac{2 (4.227)}{10} = 0.845$ kips, which check exactly Professor Young's calculations.

LOUIS BAES,[34] ESQ. (by letter).—The author develops the equations of his problem very skilfully, but he states, under the heading "Conclusions", that his method is based upon the principles of virtual work. The writer does not discern anywhere in the paper that these principles are involved. On the other hand, Equations (1), (21), (27), (33), and (38), which are the fundamental formulas, are merely the Bresse formulas, relating to the deflection of the mean fiber of curved members, applied to the case of rigid frames (polygonal members).

Under the heading, "Introduction to Stress Analysis", the author states that his aim is to describe a method of analysis less laborious than others actually in use in the United States. In Belgium, where a great number of Vierendeel trusses have already been built, methods of analysis much simpler than that of the paper are being used. Among others, there is a method developed by Professor Vierendeel himself and another by Professors Keelhoff and Magnel. These methods are exact, in the usual sense of this word, for symmetrical trusses; they include simplifying assumptions for non-symmetrical trusses, which is also true for the author's method.

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[34] Prof. of Civ. Eng., Univ. of Brussels, Brussels, Belgium.

The simplifying assumptions in the paper involve the reasonable selection of the values of the two coefficients which are, in fact, the ratios between the bending moments taken by the upper chord and by the lower chord. The author rightly shows that the value reasonably assumed for these ratios only slightly affects the results. This is only true within the customary limits, but not in a general way. In fact, the author recalls that Professor Vierendeel uses the assumption that,

$$\alpha = \beta = \frac{I'_c L}{I''_c L_1} \qquad \text{.....................(153)}$$

and that this assumption greatly simplifies the work. The assumptions generally made in Belgium refer to the location of the point of contraflexure in the vertical members of the truss.

The methods by Professor Vierendeel and Professors Keelhoff and Magnel assume the location of these points of contraflexure to be known and, this location being given, make the truss statically determinate in cutting it into two great combs separated from each other at the points of contraflexure in each vertical. This leads them to one equation which is the equivalent of Equation (53), but instead of the long Equations (66) and (68), they obtain only simple formulas. This results from the fact that the author does not take advantage of the location of contraflexure points, except in the case of symmetrical trusses. Consequently, he finds himself confronted by three systems of equations of the type of Equations (53), (66), and (68)

These three systems are rather long and difficult to solve and require (as has been recognized in Belgium) either very long direct numerical computations, or the use of the method of successive approximations (see Example 4 or Example 5). In fact, the results are derived from the difference between two very great numbers which differ slightly, so that if one does not compute by successive approximations, one might obtain entirely erroneous numerical results. Thus, the method suggested by Professor Young seems to be longer than those developed by Professor Vierendeel or by Professors Keelhoff and Magnel.

The writer has proved[35] that the location of the points of contraflexure in the verticals can be found with great precision by a single formula, which has been checked by more than fifty photo-elastic tests conducted on fifteen models of non-symmetrical trusses. These tests cover a considerable field, extending beyond all the needs of bridge construction, and show the points of contraflexure in the verticals remarkably well. They reveal that these points are actual physical elements on which it is logical to base the method of analysis.

Finally, the writer has shown[35] that Kriso's method may be generalized to all cases. In this method, contrary to what has been done by Professor Vierendeel or by Professor Keelhoff, the system is made statically determinate by cutting a section in one of the chords in each panel. This generalized method leads to one equation for every group of two or three successive panels separated by two successive verticals. The other equations are elementary.

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[35] "La Poutre Vierendeel—Genéralisation de la methode de calcul par ouverture des mailles par sectionnement d'une des membrures—Controles par la photo-élasticité", par L. Baes, *l'Ossature Metallique* (Brussels, Belgium), No. 10, October, 1936.

The only long computation is in the solution of the system of these special equations, which are equal to the number of panels. This solution, however, does not involve any difficulty and the equations may well be compared to the three-moment equations of the continuous beams.[36] Each unknown is obtained by an expression in which all the terms have the same sign. This is very important in that it permits computation by slide-rule. On the other hand, it is impossible to make any error in sign. In fine, it must be noted that by this method the designers very easily, can determine the influence lines of all the variables in the problem. For the ordinary cases of bridges, influence lines show that computations may be limited to the case of one or two conditions of loading.

To the writer, this is the shortest method; it is "exact" for symmetrical trusses, and almost "exact" for the others since from experimental proofs[35] of this essential fact, the location of the points of contraflexure is known quite accurately.

E. C. INGALLS,[37] ESQ., AND RALPH B. PECK,[37a] JUN. AM. SOC. C. E. (by letter). —It is unusual to find a paper so carefully prepared that the derivations can be followed step by step without need for pencil and paper. The author is to be complimented for the excellent presentation of the development of his working equations.

The writers have used Professor Young's method to analyze the truss shown in Fig. 25. The relative moments of inertia assumed are indicated at the mid-point of each member. The quantities, α and β, were each assumed to unity. The symmetry of both the truss and the loading made the labor involved comparatively slight.

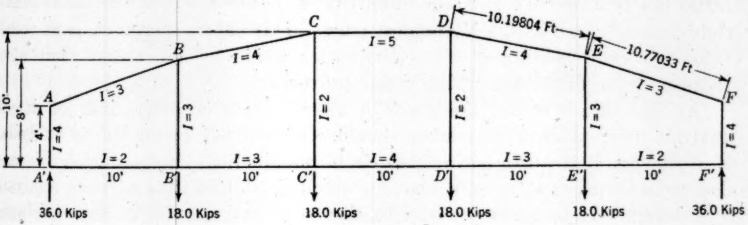

FIG. 25.

To obtain a more correct analysis, involving no assumption of α and β, the writers analyzed the truss using the slope-deflection equations. Although the solution of the simultaneous equations makes the labor great, so that the writers do not recommend the slope-deflection method as a practical means of solving this truss, the problem is more general than any which they have seen solved by slope deflection. It includes a modified form of the bent equation and certain relations between the R-values for each panel that would seem to justify its consideration as an alternate method of solution for the Vierendeel truss.

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[36] "Calcul d'une poutre élastique reposant, sur des appuis inégalement espaces", par M. Clapeyron, *Comptes Rendus* des l'Académié des Sciences, Paris, Tome 45, 1857, pp. 1076 à 1080.

[37] Troy, N. Y.

[37a] Detailer, Am. Bridge Co., Ambridge, Pa.

TABLE 2.—COMPARISON OF RESULTS BY SLOPE DEFLECTION AND BY THE YOUNG METHOD

Quantity	Slope deflection	Young	Quantity	Slope deflection	Young	Quantity	Slope deflection	Young	Quantity	Slope deflection	Young
H_{1w}	33.7	34.2	$M_{B'B}$ ...	−60.7	−58.4	M_{BC} ..	+16.0	+16.1	$M_{B'C'}$.	+17.9	+18.5
H_{2w}	15.5	15.1	$M_{CC'}$ ...	−13.7	−13.5	M_{CB} ..	+25.0	+24.7	$M_{C'B'}$.	−22.6	−22.2
H_{3w}	2.7	2.7	$M_{C'C}$ ..	−13.6	−13.4	M_{CD} ..	−11.4	−11.1	$M_{C'D'}$.	− 9.0	− 8.7
$M_{AA'}$ ...	−73.0	−75.4	M_{AB} ...	+73.0	+75.4	$M_{A'B'}$.	+61.8	+61.6	V'_1....	24.5	25.8
$M_{A'A}$ ...	−61.8	−61.6	M_{BA} ...	+47.6	+46.1	$M_{B'A'}$.	+42.9	+39.9	V'_2....	13.9	13.9
$M_{BB'}$ ...	−63.6	−62.2	.....	....	....	......	....	....	....	....	....

A detailed discussion of alternate methods is beyond the scope of discussion; but a comparison of results obtained by the slope-deflection method for the truss of Fig. 25, with those obtained by Professor Young's method, assuming $\alpha = \beta = 1$, is given in Table 2. The agreement is quite satisfactory.

HAROLD E. WESSMAN,[38] ASSOC. M. AM. SOC. C. E. (by letter).—There have been considerable additions in recent years to the literature of multi-story rigid frames, namely, the steel or reinforced concrete building skeleton. Little has been written in the United States, however, about the multi-panel rigid frame known as the Vierendeel truss. This paper, therefore, is of some value not only because of the contribution made by the author, but also because of the discussion which it has provoked.

As the author states, the paper is restricted to considerations of stress analysis only. Hence, discussion should confine itself primarily to analysis. Nevertheless, it is worth noting that, from the practical viewpoint, the Vierendeel truss is one of those structures in which it is difficult to separate analysis and design. It is simple enough to assume values for the relative stiffness of chords and web members in order to make an analysis of the shears, bending moments, and direct stresses; but when the designer proportions the various members on the basis of this analysis and in accordance with limiting unit stresses established by specification, and then makes a new analysis based on correct values of relative stiffness, he finds that the resulting shears, moments, and direct stresses may be radically different from those obtained in the first analysis. The normal procedure is to revise the design in accordance with the new values, but then, when the process of analysis is again repeated, considerable variations may again be found. In other words, the analysis is so sensitive to changes in sections, that it is almost impossible to obtain the final design in one or two trials, unless it is recognized that excess section must be used for some members.

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[38] Associate Prof. of Structural Eng. and Mechanics, Coll. of Eng., Univ. of Iowa, Iowa City, Iowa.

Here, again, the designer faces the questions: "From where does one start? Can one use the same limiting unit stresses in all members or must one 'waste' material in order to get a design that conforms to the analysis?" This particular problem is another one which emphasizes the need for research in preliminary design methods, research in which analysis and design are more closely correlated.

From the standpoint of elementary mechanics, the author's paper is of value in illustrating the statics of free body diagrams. From the standpoint of advanced structural analysis, the paper is of value in illustrating the application of virtual work equations to satisfy the condition that there must be no relative translation, horizontally or vertically, or any relative rotation at a cut section in a continuous structure. The writer feels, however, that the author's presentation could have been improved at certain points, although he realizes the limitations imposed by condensing a paper. For example, Equation (1) would be much clearer if the author had noted that the origin of co-ordinates for the first integral on the right side is at Point *B* (see Fig. 2(*a*)) and that the origin of co-ordinates for the second integral is at the upper right-hand corner. It is also worth noting that, although the displacement of Point *B* is found with respect to Point *A*, the absolute base of reference is the upper left-hand corner in Fig. 2(*a*). By fixing this corner and drawing all moment diagrams as if there were two cantilevered members, one extending down to Point *A* and the other extending to the right and then down to Point *B*, the basic equation may be easily visualized in terms of area moments.

The same criticism also applies to Professor Young's treatment of the unsymmetrical case. In Equation (21), origins of co-ordinates are not the same for each integral. In connection with this equation, it may be stated that students using area moments blindly are prone to omit the first term, $E\,\theta_c\,(D_b - D_a)$. In solving for the movement of Point *D* relative to Point *C*, the rotation of the end, *C*, when it is not on the same level as Point *D* must be considered, of course.

In the development of his equations the author emphasizes one method for the analysis of rigid frames. It is a method which is based fundamentally on setting up independent equations for all the unknowns and solving them simultaneously. Presumably, it is an "exact" method; nevertheless, in attacking the unsymmetrical case, certain approximations must be made in order to obtain a solution readily. These approximations, as the author indicates, are not necessary in the symmetrical case. It is evident from the work done by Professor Young and from the equations set up by various other investigators that the computations necessary in order to reach a final design by this method are over-burdening, especially when several analyses are required.

The writer much prefers another method for the solution of the multi-panel rigid frame. It is moment and shear distribution facilitated by superimposing arbitrary joint rotations in order to hasten convergence. There

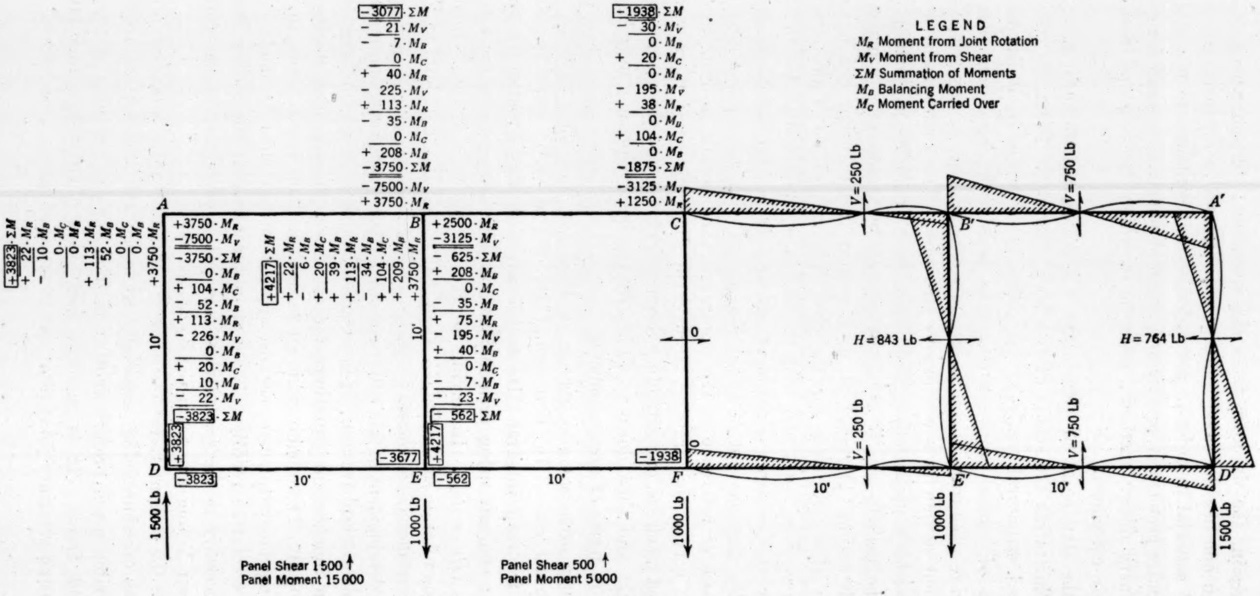
LEGEND
M_R Moment from Joint Rotation
M_V Moment from Shear
ΣM Summation of Moments
M_B Balancing Moment
M_C Moment Carried Over
A
B
C
D
E
F
A'
B'
D'
E'
+3823 · ΣM
+ 22 · M_R
− 10 · M_B
0 · M_C
0 · M_B
+ 113 · M_R
− 52 · M_B
0 · M_C
0 · M_B
+3750 · M_R
+3750 · M_R
−7500 · M_V
−3750 · ΣM
0 · M_B
+ 104 · M_C
− 52 · M_B
+ 113 · M_R
− 226 · M_V
0 · M_B
+ 20 · M_C
− 10 · M_B
− 22 · M_V
−3823 · ΣM
+3823
−3823
+4217 · ΣM
+ 22 · M_R
− 6 · M_B
+ 20 · M_C
+ 39 · M_B
+ 113 · M_R
− 34 · M_B
+ 104 · M_C
+ 209 · M_B
+3750 · M_R
−3077 · ΣM
− 21 · M_V
− 7 · M_B
0 · M_C
+ 40 · M_B
− 225 · M_V
+ 113 · M_R
− 35 · M_B
0 · M_C
+ 208 · M_B
−3750 · ΣM
−7500 · M_V
+3750 · M_R
−3677
+2500 · M_R
−3125 · M_V
− 625 · ΣM
+ 208 · M_B
0 · M_C
− 35 · M_B
+ 75 · M_R
− 195 · M_V
+ 40 · M_B
0 · M_C
− 7 · M_B
− 23 · M_V
− 562 · ΣM
+4217
−562
−1938 · ΣM
− 30 · M_V
0 · M_B
+ 20 · M_C
0 · M_B
− 195 · M_V
+ 38 · M_R
0 · M_B
+ 104 · M_C
0 · M_B
−1875 · ΣM
−3125 · M_V
+1250 · M_R
−1938
0
0
10'
10'
10'
10'
10'
10'
V = 250 Lb
V = 750 Lb
V = 250 Lb
V = 750 Lb
H = 843 Lb
H = 764 Lb
1 500 Lb
1 000 Lb
1 000 Lb
1 000 Lb
1 500 Lb
Panel Shear 1 500
Panel Moment 15 000
Panel Shear 500
Panel Moment 5 000

FIG. 26.

is nothing new[39] about the method, although it does not seem to be widely known, at least in the accurate sense.

The solution of the symmetrical case (see Example 1) by this method is shown in Fig. 26, in which, consistent with the notation of the paper, M_R = the moment from joint rotation; M_V = the moment from shear; M_B = the balancing moment; M_C = the moment carried over; and, ΣM = the summation of moments. Note the rapid convergence of values. For all practical purposes, the procedure could have been terminated after two cycles and the results would be sufficiently accurate. If the arbitrary joint rotations had not been superimposed upon the shear translations, however, the convergence would be much slower.

The process will not be explained in detail in this discussion. It is worth noting, however, that a sketch of panel distortions due to joint translations acts as a guide in indicating the sense of the arbitrary joint rotations. It is not necessary to impose equal rotations at all joints; it is not necessary to make all of them of the same sense; moreover, one does not need to know the actual rotation as long as the arbitrary moments at each end of each member due to rotation bear the proper ratio to one another. If the ratio is correct, geometrical continuity is preserved.

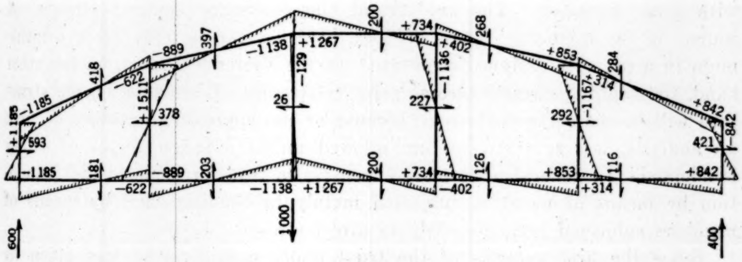

FIG. 27.

Fig. 27 shows the results from three cycles of computations when this process is applied to the author's unsymmetrical case (see Example 3). All calculations were made on a slide-rule.

This paper demonstrates to the writer that a slide-rule, combined with the proper method of attack, is a rather useful and universal instrument; it is much less expensive than a computing machine and its effect on the eyes not nearly so severe as a nine-place logarithmic table. Even if the author's results are obtained by the so-called "exact" solution of simultaneous equations, it must be kept in mind that his work also involved certain approximations. Moreover, when certain values depend upon small differences between two large values, it is evident that the large values must be determined with extreme precision to a great number of significant places in order to obviate errors in the final result. That is one serious drawback to methods such as that used by Professor Young.

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[39] "Continuous Frames of Reinforced Concrete", by Hardy Cross and N. D. Morgan, Members, Am. Soc. C. E., pp. 229–283; in particular, footnote at bottom of p. 283.

As long as procedure is scientific, however, the most important matter is not "what method was used", but rather, "how shall one interpret the results in the light of the actual design."

The alternate method cited by the writer[39] may also be used in the analysis of wind stresses in multiple-story buildings. It greatly facilitates convergence and obviates the need for extending a mathematical series to obtain the final answer.

A. Floris,[40] Esq. (by letter).—The classical methods used by the author in the analysis of the Vierendeel truss will undoubtedly divert the attention of engineers, somewhat, from the so-called arithmetical methods which have almost dominated the technical literature since the introduction by Hardy Cross, M. Am. Soc. C. E., of the moment distribution method.

The author sidesteps, wisely, the question of the merits of this type of structure. This question, however, is of great importance in practical problems involving safety and economy. Unfortunately, the Vierendeel truss is fundamentally inferior to trusses with diagonals, for the following obvious reasons. In the ordinary truss the primary direct stresses can be calculated with great accuracy. The analysis of the secondary bending stresses, of course, is less accurate. These stresses, however, can be reduced to a minimum in a properly designed structure. In the Vierendeel truss, on the other hand, the bending stresses become primary stresses. Consequently, the structure will be more expensive and, because of the approximate nature of such an analysis, less safe stresses are allowed unless a larger factor of safety is adopted. In general, it is more expensive to transfer loads to the foundation by means of members subjected mainly to bending than by means of members subjected preponderately to direct stress.

Since the first analysis of the truss under consideration was given by Professor Vierendeel[41], the problem has attracted, widely, the attention of theorists. This is obviously due to the desire to simplify the complicated, although not necessarily difficult, calculations. Theoretically, the analysis of statically indeterminate structures does not present essential difficulties. Any one familiar with such an analysis can write down the necessary equation with ease. However, the time consumed in the derivation of these expressions and their subsequent tedious numerical evaluation are serious obstacles to their use in practice.

Following the trend for greater simplicity and expediency the author gives an approximate (and, for practical purposes, sufficiently accurate) method of analyzing the Vierendeel truss. The writer does not intend to enter into a detailed discussion of this excellent paper. For a proper appreciation of the work done and the efforts made in this direction by the author and others, the available extensive literature on the subject should be studied carefully.[42]

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[40] Dipl.-Ing., Los Angeles, Calif.

[41] "Longerons en treillis et longerons à arcades", par Arthur Vierendeel, Paris, 1897.

Kimball R. Garland,[43] Esq. (by letter).—A series of equations is presented in this paper that should be of great aid in analyzing Vierendeel trusses, and the author deserves much credit for having presented the method and results in such complete form that they can be followed through and checked. Exact methods for solving this type of truss have been lacking in American literature, and a need for such methods (which this paper will help to supply) is growing. It is unfortunate that, in spite of the careful analysis which has been made of this type of truss, the solution is still extremely laborious. In the case of the truss with horizontal bottom chords and inclined top chords, even with the equations presented, and with the simple example which was solved in the text, the labor of obtaining a result is considerable. In the case of practical design, where many other complications must be considered, and several trial designs made, many American offices, in the writer's opinion, would consider the difficulty of solution as a weighty argument against its use.

Some years ago, the writer made some comparative designs for a building foundation which was intended to be rigid enough to resist the effects of unequal soil-bearing pressure, utilizing the entire basement story as a framework. The Vierendeel truss seemed to be the proper type of design for this purpose, so that the basement might be free from the obstruction of diagonal members. On this occasion, a set of equations, such as the author has presented, would have been of great value. Not having any literature immediately available on the subject, the writer made a number of solutions based on the method of moment distribution, and found that results could be obtained of sufficient accuracy for the purpose, although the work involved was rather laborious. One design which was studied had very large haunches at the top and bottom joints, and the horizontal members were also tapered between the haunches, for economy, being thinner near the middle of the panel where the bending was assumed to be least. This introduced a complication, in that the axis of all the horizontal members was bent in a vertical plane, so that the bending moment in these members was dependent in part on the axial stress in the chord, as well as on the shear which the panel carried. The process followed was to plot what was assumed to be the axis of each member, and determine its stiffness coefficient as if its axis were straight. Then, for the first trial analysis, it was assumed that this axis was straight, extending between the points where vertical and horizontal members intersected, and a solution was made by the method of moment distribution. The resulting axial stresses in the chords produced additional moments due to the eccentricity of the thrust, and a second moment distribution was made to correct for this. This cycle, in turn, changed the chord stresses, requiring a third correction, which seemed to have carried the process far enough. The proper method in this case would have been to substitute for the bent axis of each chord a straight axis, located so that a thrust or a tension acting along it would produce no angular displacement in either end of the member; or, in other words, through the center of gravity of the values of $y \frac{ds}{EI}$ for that member. After this is done, one analysis by moment distribution should suffice to determine the true values. Once this shifting of the axis of the member is accomplished, the author's method might be applied, although this would require further investigation. In any case, it seems as if some kind of correction should be applied where trusses have large gussets.

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[42] "On the Theory of Trusses Without Diagonals", by G. P. Perederiy, Moscow, 1905 (in Russian); also "Reinforced Concrete Bridges", by G. P. Perederiy, St. Petersburg, 1912, p. 166 (in Russian); "Trusses Without Diagonals, Their Analysis and Application to Steel and Reinforced Concrete Structures", by J. Podolsky, Moscow, 1909 (in Russian); also, *Beton-Kalender*, 1929, Pt. I, 290; "Beitrag zur Berechnung von Vierendeelträgern", von A. Ostenfeld, *Beton und Eisen*, 1910, p. 30; also, "Die Deformationsmethode", von A. Ostenfeld, Berlin, 1926, p. 86; "Beitrag zur Theorie der Vierendeelschen Träger", von H. Marcus, *Armierter Beton*, 1910, p. 208; "Einflusslinien für die Berechnung paralleler Vierendeelträger", von W. St. Ritter von Balicki, Berlin, 1910; also, "Vierendeel-Träger mit Parallelen Gurtungen", von E. Reich, Wien, 1911; "Gesetzmässigkeiten in der Statik des Vierendeel-Trägers", von L. Freytag, München, 1911; "Der Pfostenträger mit ungleichen Gurtungen", von L. Mann; in Festschrift für H. Müller-Breslau, Leipzig, 1912; "Die Berechnung der Pfostenträger", von Otto Mohr, *Der Eisenbau*, 1912, p. 85; also, "Abhandlungen aus dem Gebiete der technischen Mechanik", von Otto Mohr, Berlin, 1914, p. 506; "On the Analysis of Trusses Without Diagonals", by J. Podolsky, *Engineer* (Kiev), 1913, No. 7 to 10 (in Russian); "Die Berechnung der Rahmenträger", von F. Engesser, Berlin, 1913; "Die Berechnung statisch unbestimmter Tragwerke nach der Methode des Viermomentensatzes", von Friedrich Bleich, Berlin, 1918, p. 125; "Die Berechnung der Rahmenträger", von Tschalyscheff, *Der Bauingenieur*, 1922, pp. 208 and 244; "Die statisch unbestimmten Systeme des Eisen-und Eisenbetonbaues", von Friedrich Hartmann, Berlin, 1922, p. 169; "Statik der Vierendeelträger", von K. Kriso, Berlin, 1922; "Calcul des Constructions Continues à Eléments Droits", par P. Thomas, Paris, 1925, p. 57; "Berechnung Statisch Unbestimmter Biegefester Stab-und Flächentragwerke", von Peter Pasternak, Zürich, 1927. Pt. I, p. 240; "Analyse statique des poutres Vierendeel", par B. Enyedy, Paris, 1928; "Die Statik des ebenen Tragwerkes", von Martin Grünig, Berlin, 1928.

[43] Newton, Mass.

One uncertainty which makes it unwise to strive for too great refinement in the stress analysis of Vierendeel trusses of concrete, is the lack of knowledge regarding the tension chord. Should the reinforcement alone be computed, where both top and bottom rods are in tension, or should some weight be given to the concrete, which must certainly carry some stress, and, therefore, must add to the stiffness of the member?

As a matter of curiosity, the writer made a solution of the truss which the author solved in Examples 3 and 5 by the method of moment distribution. He made the usual assumption that every joint is held from rotating, although it may be displaced horizontally or vertically. He assumed, furthermore, that Vertical No. 3 took no horizontal shear, but that the members to the left took up all the thrust to the left, and those to the right took all the thrust to the right, due to the distortion of the inclined chords. It was then possible to write the expressions for the moments and shears in the two left panels in two simultaneous equations, and solve for the values. The same could be done for the two panels at the right. The center panel, being rectangular, was solved by inspection. The method of moment distribution was then applied. The steps would be as follows: (1) Solve for unbalanced vertical shear (two sets of short simultaneous equations); (2) balance joints; (3) carry over moments; (4) distribute the unbalanced horizontal shear among web members; and, (5) find the unbalanced vertical shear in each panel, and solve again by simultaneous equations.

The writer found that the moments converged consistently by this method, and that in five repetitions the results checked fairly closely (about 1%) with the answers given in the problem. This method, of course, was laborious. After working through the solution given in the paper for the same problem by the general method, the writer thinks that probably, for only one condition of loading, and where an accuracy of 1% is sufficient, the moment-distribution method would be as rapid as the solution by the general equations. For complete analysis of all possible conditions of loading, the general solution would save time in the end, and would give the exact results.

J. D. GEDO,[44] M. AM. SOC. C. E. (by letter).—The novelty in Professor Young's solution is that equations similar to those of the theorem of three moments may be written for the column shears. Thus, before any other unknowns are solved, the horizontal stresses are obtained from Clapeyronic equations. This is a valuable contribution.

It is surprising, however, that he under-estimates the value of the basic formulas, Equations (32), (37), and (42), in that he states: "A set of these formulas could be written for each panel and the series solved simultaneously to find the unknowns. Such a procedure, although possible, would be too laborious for practical use." As a matter of fact, Professor Young does write the three equations for each panel. Were these 3 n-equations such that they contain the 3 n-unknowns, not only collectively but also individually, the solution would be impossible in all but the simplest cases. The success of his analysis is not due entirely to the manipulation of these valuable equations; the secret is that when integrating the product of the virtual stresses and real deformations the integrations do not cover the entire structure, they extend only to the panel. Professor Young does not make any effort to explain why this is permissible; nor does he emphasize the importance of this procedure. It is true, however, that he refers to the work of J. A. Van den Broek,* M. Am. Soc. C. E., which contains such explanations.

Professor Young is misled when he assumes that, by solving the series simultaneously, the procedure would be too laborious for practical use. The writer has computed many Vierendeel trusses with similar elastic equations, and he has always found that it is possible to compute the influence lines of 10-panel Vierendeel trusses with horizontal or inclined upper chords in two working days, provided the truss is symmetrical about a vertical axis. With Professor Young's contribution this time is shortened by two hours.

Statements with which the writer does not agree are: That Vierendeel trusses may be built of reinforced concrete and that the effect of axial deformation is so small that it may be neglected in most cases. The example in Table 3 was computed to "throw light" on these statements and also to compare the Vierendeel truss with the closely related pinned truss and riveted truss. In the diagram of Table 3, the moments are shown on the sagging side of the system axis. The frame was designed as a Vierendeel truss and

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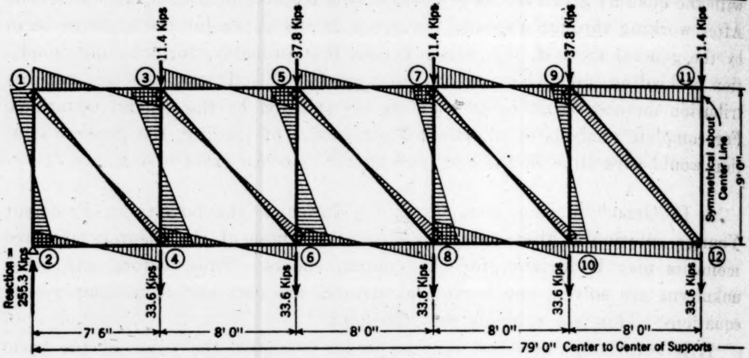
[44] Senior Designer, Park Dept., City of New York, New York, N. Y.

TABLE 3.—COMPARISON OF STRESSES IN VIERENDEEL, RIVETED, AND PIN-CONNECTED TRUSSES


①
②
③
④
⑤
⑥
⑦
⑧
⑨
⑩
⑪
⑫
11.4 Kips
37.8 Kips
11.4 Kips
37.8 Kips
11.4 Kips
33.6 Kips
33.6 Kips
33.6 Kips
33.6 Kips
33.6 Kips
Reaction = 255.3 Kips
Symmetrical about Center Line
9' 0"
7' 6"
8' 0"
8' 0"
8' 0"
8' 0"
79' 0" Center to Center of Supports

General			Bending Moment, in Foot-Kips			Direct Force, in Kips				Deflection, in Inches			
Member	Moment of inertia, in inches^4	Area, in inches^2	Vierendeel truss (direct forces neglected)	Vierendeel truss (direct forces considered)	Riveted truss	Vierendeel truss (direct forces neglected)	Vierendeel truss (direct forces considered)	Riveted truss	Pin-connected truss	Location	Vierendeel truss (direct forces considered)	Riveted truss	Pin-connected truss
1-2	2 798	64 36	476 6	464 6	155.7	127.7	125.9	207.7	255.3	..	......	......	......
1-3	2 798	64 36	476.6	464.6	164.1	105.9	104.3	173.1	212 8	..	......	......	......
1-4	797	21.76	......	......	8.4	......	......	—212.7	—332.3	..	......	......	......
2-1	......	......	476.5	474.4	167.6	......	......	......	......	2	0.00	0.00	0.00
2-4	2 798	64.36	476.5	474.4	167.6	—105.9	—104.3	—35.9	......	..	......	......	......
3-1	......	......	481.1	479.4	174.6	......	......	......	......	..	......	......	......
3-4	5 454	108.78	907 6	890.4	302.4	—11.2	—8.4	140.2	221.7	..	......	......	......
3-5	2 798	64.36	426.5	411.0	136.0	307.6	303.0	362.9	399.7	..	......	......	......
3-6	642	17.94	......	......	8.2	......	......	—182.4	—281.4	..	......	......	......
4-1	......	......	......	......	7.1	......	......	......	......	4	0.37	0.23	0.27
4-2	......	......	480.6	496.4	189.2	......	......	......	......	..	......	......	......
4-3	......	......	907 4	897.8	308.6	......	......	......	......	..	......	......	......
4-6	2 798	64.36	426 8	401.4	126.5	—307.6	—303.0	—241.0	—212.8	..	......	......	......
5-3	......	......	414 5	437.4	170.1	......	......	......	......	..	......	......	......
5-6	3 912	84.37	696.0	686.0	237.0	2.2	0.7	112.2	176.7	..	......	......	......
5-7	2 798	64 36	281.5	248.6	76.7	462.2	455.4	494.4	523.2	..	......	......	......
5-8	432	12.34	......	......	9.9	......	......	—117.0	—185.8	..	......	......	......
6-3	......	......	......	......	3.3	......	......	......	......	6	0.73	0.46	0.55
6-4	......	......	414.6	432.5	164.5	......	......	......	......	..	......	......	......
6-5	......	......	695.9	685.4	239.0	......	......	......	......	..	......	......	......
6-8	2 798	64.36	281.3	252.9	77.7	—462.2	—455.4	—415.7	—399.7	..	......	......	......
7-5	......	......	274.3	302.6	131.0	......	......	......	......	..	......	......	......
7-8	2 402	56 73	443.3	437.4	160.6	—11.1	—9.7	60.0	105.3	..	......	......	......
7-9	2 798	64 36	168.9	134.8	38.0	560.7	552.6	579 1	606.6	..	......	......	......
7-10	290	8.81	......	......	8.3	......	......	—72.8	—125.6	..	......	......	......
8-5	......	......	......	......	4.3	......	......	......	......	8	1.04	0.65	0.78
8-6	......	......	274.1	307.0	132.2	......	......	......	......	..	......	......	......
8-7	......	......	443 2	437 0	160.3	......	......	......	......	..	......	......	......
8-10	2 798	64.36	169.1	130.0	32.4	—560.7	—552.6	—530.0	—523.2	..	......	......	......
9-7	......	......	206.7	247.4	127.7	......	......	......	......	..	......	......	......
9-10	1 166	30.26	173.4	170.9	61.2	2.1	—0.1	37.5	60.3	..	......	......	......
9-11	2 798	64.36	33.3	76.5	58.5	599.2	590.7	609.4	626.6	..	......	......	......
9-12	290	8.81	......	......	8.0	......	......	—24.4	—30.1	..	......	......	......
10-7	......	......	......	......	2.3	......	......	......	......	10	1.27	0.78	0.97
10-8	......	......	206.5	239.0	122.4	......	......	......	......	..	......	......	......
10-9	......	......	173.4	172.0	62.7	......	......	......	......	..	......	......	......
10-12	2 798	64.36	33.1	67.0	57.4	—599.2	—590.7	—592.8	—606.6	..	......	......	......
11-9	......	......	122.9	155.5	78.6	......	......	......	......	..	......	......	......
11-12	290	8.81	......	......	......	—11.0	—8.4	6.4	11.4	..	......	......	......
12-9	......	......	......	......	1.5	......	......	......	......	12	1.35	0.83	1.01
12-10	......	......	123.5	163.0	74.4	......	......	......	......	..	......	......	......
12-11	......	......	......	......	......	......	......	......	......				

E = 30 000 000 lb per sq in.

the sections were retained in the case of the other trusses. Two analyses were made for the Vierendeel truss. In the second analysis the axial deformations were considered not only in the chords but also in the posts. Hence, the dissymmetry between top-chord and bottom-chord stresses. Comparing the two analyses it is evident that the effect of the direct forces on the moment distribution is far too great to be negligible. It is also evident that the truss in Table 3 can not be built of reinforced concrete. The direct tension in Member 10–12 is nearly 600 kips. This force alone would require about forty 1-in. square bars. The shear in Member 3–4 is about 200 kips. Allowing 100 lb per sq in. for shear, the area of the section would be 2 000 sq in. Thus, it is easy to see that reinforced concrete is not the proper material for a Vierendeel truss except, possibly, for very small ones.

Although the subject of this paper concerns only the stress analysis, the results can not be properly interpreted without comparing the Vierendeel truss with frames that resemble it. In Table 3 it is noticeable that the moments of the Vierendeel truss are much higher than the so-called "secondary stresses" of the riveted truss. The writer has not analyzed the riveted truss by the slope deflection method, the stresses were obtained by using the same reasonings as in the case of the Vierendeel truss. The direct forces are smallest in the Vierendeel truss, higher in the riveted truss, and highest in the pinned truss. Adding the diagonal material to the chords and posts, the sections thus increased may suffice for the higher stresses in the Vierendeel truss.

Furthermore, the stresses in the Vierendeel truss can be reduced by spacing the posts closer together near the supports and farther apart near the center. In other words, the ideal spacing for the posts is similar to that of the stirrups in a uniformly loaded reinforced concrete beam. In interpreting the relatively high deflections of the Vierendeel truss, it should be borne in mind that the dimensions of both the riveted and the pinned truss are greater than necessary, in the example. No doubt, many more comparative studies are needed to arrive at a definite conclusion. Nevertheless, it is certain that the Vierendeel truss has a great future. Professor Young's contribution, therefore, is doubly meritorious, although some of his minor statements are not quite exact.

Dana Young,[45] Assoc. M. Am. Soc. C. E. (by letter).—The discussions have revealed many interesting facts. Additional examples have been presented and checked, other methods of analysis have been recommended, and several "short-cut" procedures have been proposed.

Mr. Mensch has presented an instructive application of the method to a multi-storied bent. It may be noted that Equation (97) is simply a special case of Equation (53) and may be readily obtained from the latter. Table 1, which gives a comparison of *H*-values obtained by the general method with those from the approximate portal method, is helpful in showing the errors that may occur from using the latter method. Mr. Mensch has also given a "short-cut" method of applying the general equations to a specific problem. This is a good illustration of the simplifications that are practical in many cases, as was noted in the paper under the heading, "Special Approximations."

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[45] Asst. Prof. of Eng., Connecticut State Coll., Storrs, Conn.

Mr. Eremin has suggested that, for trusses with parallel chords and constant moment of inertia, it is sufficiently accurate to assume points of contraflexure in all members at their mid-lengths; that is, the portal method. However, it is well known that this method is often in serious error. This fact is clearly illustrated by Mr. Mensch in Table 1. Equation (122), which is given without proof, is readily derived from statics, using the assumption that $\frac{M'_{ba} + M'_{bc}}{M''_{ba} + M''_{bc}} = \frac{I'_c}{I''_c} = \frac{I_t}{I_b}$. Although this assumption is satisfactory in some cases, it may be shown that in trusses with large differences in the moments of inertia of the chords, the error will be considerable. The approximate formulas, Equation (123) and Equation (124), especially Equation (123), which Mr. Eremin proposes—are too inaccurate for general use. Although they give good results in the particular example given, they cannot always be relied upon to do so.

Mr. Blog has presented a summary of Professor Vierendeel's method of analysis and claims that this method is simpler and more accurate than that proposed in the paper. It is true that Equations (127) through (130), inclusive, are simpler than those in the paper, but that is because the former group is based upon several assumptions. Because of these assumptions, it is not true that the Vierendeel equations are as accurate as the others. The primary assumption in the Vierendeel method is that,

$$\frac{M'}{M''} = \frac{I'}{I''} \frac{dx}{ds} \qquad \text{.........................(154)}$$

Mr. Blog states that Professor Vierendeel proves that this relation is true. However, a careful check of this derivation[*] will show that it is based on the assumption that the vertical deflection of corresponding points on the upper and lower chords of a truss are equal throughout the length of the truss. This is not true except in special cases. The fact that Equation (154) is not exact may be readily shown by a numerical example in which the moments of inertia of the chord members are not equal. Such an example has been very conveniently solved by Messrs. Ingalls and Peck in their discussion. An inspection of their results, which are given in Table 2, shows this immediately. Another assumption which Professor Vierendeel makes, as stated by Mr. Blog, is that the upper chord has a finite area but an infinitely small moment of inertia. As a result Equations (127) to (130), inclusive, involve these assumptions and, therefore, must be used with caution.

Messrs. Fischer, Goldberg, and Ingalls and Peck have added materially to the paper by developing other examples and checking by other methods of analysis.

Several of the discussions have indicated other methods of analysis. Mr. Maugh has described an interesting method using deformation equations with auxiliary force systems, which may be solved either directly or by successive approximations. This discussion contains a numerical computation which purports to show that an error of -3% in H_{1w} and of $+3\%$ in V'_{12} might cause an error of 40% in M'_{n}. This example neglects the relation that exists between H_{1w} and V'_{12} which is given by Equations (77) and (79). From these equations, it is seen that, for the condition assumed by Mr. Maugh (that is, $M_{1w} = 2 H_{1w}$), the error in V'_{12} must be in the same direction as that in H_{1w} and, hence the error in M'_{n} will be much smaller than shown in his calculation. Professor Baes has referred to an ingenious method based upon the experimental determination of the point of contraflexure in each vertical member. This method appears to be very promising since experience in analysis tends to show that the position of the point of contraflexure in the vertical members varies only slightly for considerable differences in the moments of inertia of the truss members. Professor Baes objects to the statement that the fundamental equations are based upon the principle of virtual work. Unfortunately, it is true that the names of the various principles in structural mechanics are not standardized throughout the world. If this were done, the mass of current engineering literature would be considerably easier to follow. In this particular case, it is believed that the designation used is consistent with general practice in the United States.

Messrs. Wessman and Garland have applied the moment distribution method to the examples given. As these discussers have shown, although there are certain troublesome details in applying this method to a truss with inclined chords, the method is perfectly possible. In this connection, it is to be noted that the general equations given in the paper may also be solved by successive approximations, as was indicated, and will be found to converge very rapidly. Professor Wessman has raised the question as to whether the analysis of the Vierendeel truss is not so sensitive to changes in sections of members, that it is impossible to obtain a final design in one or two trials. Although such a condition is a possibility, it has been the writer's experience that no difficulty occurs in this direction in actual cases. The writer heartily agrees with Professor Wessman that analysis and design go hand-in hand" and must be considered together to insure satisfactory results, as in all good engineering.

Mr. Floris has stated that the Vierendeel truss is fundamentally inferior to trusses with diagonals. Although it is outside the scope of the paper to enter an extended argument as to the utility of this type of construction, it may be stated that the most usual form of Vierendeel truss for bridges is the bowstring type, which is very similar to a bowstring or tied arch, and possesses the advantages of this form of structure. In such cases, as mentioned by Mr. Maugh, the Vierendeel truss arrangement is often a simpler design, particularly with welded steel or reinforced concrete construction.

Mr. Gedo has presented an interesting and helpful discussion. His comparison of stresses in Vierendeel, riveted, and pin-connected trusses is very instructive. However, as mentioned previously, it would be more appropriate to compare the Vierendeel truss with a tied arch design, in order to understand best the possibilities of this form of construction. Mr. Gedo provides a refreshing contrast to some of the other discussers in that he advocates the direct solution of the general equation.

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SIMULTANEOUS EQUATIONS IN MECHANICS SOLVED BY ITERATION

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With Discussion by Messrs. Garrett B. Drummond, A. W. Fischer, M. B. Gamet, Marvin A. Gray, John E. Goldberg, A. Floris, and W. L. Schwalbe.

Synopsis

Iteration (meaning "repetition") stands for the process of successive approximations applied to the solution of simultaneous equations. A set of simultaneous equations may be characterized by a single equation such as, for instance, the well known equation of three moments for a continuous beam. Such an equation, with given boundary values, is easily solved by elimination or by determinants when the number of unknowns is small, say, eight or less, but requires less unwieldy methods when the number of unknowns is greater than eight. In this paper the method of iteration is applied to the equation of three moments, the equation of three angles, and the equation of five angles. All these equations occur in structural mechanics, the first two in the theory of continuous beams, and the third one in the theory of continuous frames.

The purpose in writing this paper is not to present a new method (all the subject-matter can be found in the references listed in Appendix I); but rather to aid the student in civil engineering in making a co-ordinated progress from the theory of beams in the elementary course in strength of materials to the theory of frames in the more advanced courses in structural civil engineering.

The derivations of formulas introduced are outlined separately in Appendix II and a list of symbols is arranged for convenience of reference in

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Note.—Published in August, 1936, *Proceedings*.

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Appendix III. In this notation an effort has been made to conform essentially with "Symbols for Mechanics, Structural Engineering, and Testing Materials"[3] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

EQUATION OF THREE MOMENTS

The equation of three moments, which relates the statically indeterminate moments at three consecutive supports of a continuous beam (supports fixed and modulus of elasticity of material constant), may be written in the form:

$$\frac{M_{x-1}}{K_x} + 2\, S_x\, M_x + \frac{M_{x+1}}{K_{x+1}} = U_x \;\dots\dots\dots\dots\dots\dots (1)$$

in which M_x = the bending moment at Support x; K = a stiffness ratio, $\frac{I}{L}$; S_x = a measure of stiffness $= \frac{1}{K_x} + \frac{1}{K_{x+1}}$; and U_x = a load factor in two adjacent spans, such that:

$$U_x = -\,\frac{6\, Q_{x-1}}{I_x\, L_x} - \frac{6\, Q_{x+1}}{I_{x+1}\, L_{x+1}} \;\dots\dots\dots\dots\dots\dots (2)$$

in which Q_{x-1} and Q_{x+1} are, respectively, the moments of the areas below the moment diagram in Spans L_x and L_{x+1} with respect to Supports $x - 1$ and $x + 1$. The span is regarded as simply supported under the given loads.

In order to solve Equation (1) by iteration, or successive approximations, the first approximation is computed for each support, x, as:

$$(M_x)_1 = \frac{U_x}{2\, S_x} \;\dots\dots\dots\dots\dots\dots (3)$$

Substituting this value back into Equation (1) the second approximation is found to be:

$$(M_x)_2 = (M_x)_1 - \frac{1}{2\, S_x}\left(\frac{(M_{x-1})_1}{K_x} + \frac{(M_{x+1})_1}{K_{x+1}}\right) \dots\dots\dots\dots (4)$$

and so on to the nth approximation ($n = 1, 2, \ldots$):

$$(M_x)_n = (M_x)_1 - \frac{1}{2\, S_x}\left[\frac{(M_{x-1})_{n-1}}{K_x} + \frac{(M_{x+1})_{n-1}}{K_{x+1}}\right] \dots\dots\dots\dots (5)$$

Added speed of convergence is obtained if corrected values are used as soon as they are obtained, for computing the next approximation. Thus, in working from left to right over the series of supports, Equation (5) may be written as:

$$(M_x)_n = (M_x)_1 - \frac{1}{2\, S_x}\left[\frac{(M_{x-1})_n}{K_x} + \frac{(M_{x+1})_{n-1}}{K_{x+1}}\right] \dots\dots\dots\dots (6)$$

That is, the nth approximation, $(M_{x-1})_n$, is used in computing $(M_x)_n$. The proof of convergence of Equations (5) and (6) to the solution of Equation (1) is given in Appendix II.

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[3] A. S. A.—*Z10a*—1932.

According to Equation (6) the first approximation, which is computed from the load factor, U_x, is repeatedly corrected by the factor, $\frac{1}{2\,S_x}\left[\frac{(M_{x-1})_n}{K_x} + \frac{(M_{x+1})_{n-1}}{K_{x+1}}\right]$, until the computed values remain stationary to any required degree of approximation. The correction factor is easily remembered as the sum of the products of the corrected moments on each side of a support and their corresponding $\frac{1}{K}$- values in the two adjacent spans, divided by twice the sum of the two $\frac{1}{K}$- values.

Case 1.—An example of a continuous beam over five supports with end moments zero, is shown in Fig. 1, including the computed values of the different quantities. To illustrate the use of Equation (6), the value, $(M_2)_2 = -\,2.84$, for $x = 2$, in Fig. 1, is computed as follows: The correction factor to the first approximation, $-\,3.22$, is $\left\{(-\,1.13 \times \frac{1}{2}) + (-\,0.47 \times \frac{2}{3})\right\} \times 0.428 = -\,0.38$. Then, $(M_2)_2 = -\,3.22 - (-\,0.38) = -2.84$.

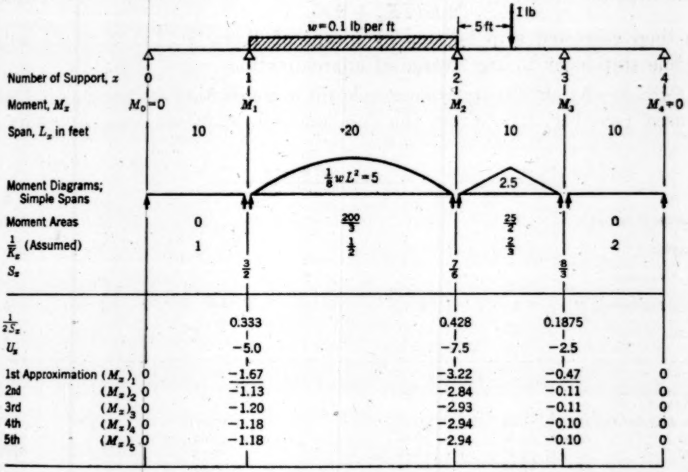

w = 0.1 lb per ft
5 ft
1 lb
Number of Support, x 0 1 2 3 4
Moment, M_x $M_0 = 0$ M_1 M_2 M_3 $M_4 = 0$
Span, L_x in feet 10 20 10 10
Moment Diagrams; Simple Spans
$\frac{1}{8}wL^2 = 5$
2.5
Moment Areas 0 $\frac{200}{3}$ $\frac{25}{2}$ 0
$\frac{1}{K_x}$ (Assumed) 1 $\frac{1}{2}$ $\frac{2}{3}$ 2
S_x $\frac{3}{2}$ $\frac{7}{6}$ $\frac{8}{3}$
$\frac{1}{2S_x}$ 0.333 0.428 0.1875
U_x −5.0 −7.5 −2.5
1st Approximation $(M_x)_1$ 0 −1.67 −3.22 −0.47 0
2nd $(M_x)_2$ 0 −1.13 −2.84 −0.11 0
3rd $(M_x)_3$ 0 −1.20 −2.93 −0.11 0
4th $(M_x)_4$ 0 −1.18 −2.94 −0.10 0
5th $(M_x)_5$ 0 −1.18 −2.94 −0.10 0

FIG. 1.

As a check, the values of the three moments, computed by elimination from the three simultaneous equations which are found when Equation (1) is applied three times, are: $M_1 = -\,1.18$; $M_2 = -\,2.93$; and $M_3 = -\,0.10$.

EQUATION OF THREE ANGLES

The equation of three angles expresses the relation between the angles of rotation at three successive supports of a continuous beam. The equation is:

$$K_x\,\theta_{x-1} + 2\,(K_x + K_{x+1})\,\theta_x + K_{x+1}\,\theta_{x+1} = \frac{\Delta\,M'_x}{2\,E}\;\ldots\ldots(7)$$

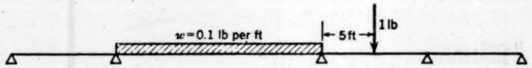
in which θ_x = angle of rotation of the beam at Support x (θ is plus (+) when clockwise); $\Delta M'_x$ = load factor = $(M'_x)_L - (M'_x)_R$; $(M'_x)_L$ = fixed-end moment at left of support, x, for the span, L_x, with given loads; $(M'_x)_R$ = fixed-end moment at right of support, x, for the span, L_{x+1}, with given loads; and E = modulus of elasticity of material.

The solution of Equation (7) by iteration proceeds in the same manner as that outlined for the equation of three moments. The approximating equation is written as:

$$(\theta_x)_n = (\theta_x)_1 - \frac{[K_x\,(\theta_{x-1})_n + K_{n+1}\,(\theta_{x+1})_{n-1}]}{2\,(K_x + K_{x+1})} \qquad \text{.............(8)}$$

in which $(\theta_x)_n$ is the nth approximation ($n = 1, 2, \ldots$), and the first approximation is $(\theta_x)_1 = \dfrac{\Delta M'_x}{2\,E\,(2\,K_x + 2\,K_{x+1})}$. The following approximations are then corrected step by step by means of Equation (8) until the values become stationary to any degree of approximation.

Case 2.—As an illustrative example the same continuous beam as in Case 1 is used (see Fig. 2). From the fundamental equations (see Appendix II, Equations (24) and (25)), the boundary values, θ_0 and θ_4, are obtained under the assumption of no restraints at the ends, thus: $\theta_0 = -\dfrac{\theta_1}{2}$; and $\theta_4 = -\dfrac{\theta_3}{2}$.



	0		1		2		3		4
Number of Support, x	0		1		2		3		4
Rotation, θ_x	$\theta_0 = -\frac{\theta_1}{2}$		θ_1		θ_2		θ_3		$\theta_4 = -\frac{\theta_3}{2}$
K_x (Assumed)		1		2		$\frac{3}{2}$		$\frac{1}{2}$	
Fixed-End Moment, M'_x	0		$0, -\frac{10}{3}$		$-\frac{10}{3}, -\frac{5}{4}$		$-\frac{5}{4}, 0$		0
$\Delta M'_x$	0		$+\frac{10}{3}$		$-\frac{25}{12}$		$-\frac{5}{4}$		0
$\frac{1}{2(K_x+K_{x+1})}$			$\frac{1}{6}$		$\frac{1}{7}$		$\frac{1}{4}$		
1st Approximation $2E(\theta_x)_1$	−0.28		$+\underline{0.555}$		$-\underline{0.2976}$		$-\underline{0.3125}$		$+\underline{0.15}$
2nd $2E(\theta_x)_2$	−0.35		+0.70		−0.44		−0.11		+0.05
3rd $2E(\theta_x)_3$	−0.38		+0.76		−0.48		−0.13		+0.065
4th $2E(\theta_x)_4$	−0.39		+0.78		−0.49		−0.13		+0.065
5th $2E(\theta_x)_5$	−0.395		+0.790		−0.494		−0.136		+0.068
6th $2E(\theta_x)_6$	−0.393		+0.786		−0.493		−0.1361		+0.0681

FIG. 2.

Computed values are included in Fig. 2. For example, the value, $2\,E\,(\theta_1)_2 = +\,0.70$, is computed as follows: The correction factor is $\frac{1}{6}\left\{(-\,0.28 \times 1) + (-\,0.30 \times 2)\right\} = -\,0.15$; and then, $2\,E\,(\theta_1)_2 = +\,\underline{0.55} - (-\,0.15) = +\,0.70$. Some labor of computation is saved if the corrections are made roughly at first, say, to one decimal place; then, when the values become about stationary, to the second decimal place, etc. From Equations (24) and (25) in Appendix II, the moments at the supports are computed as, $M_1 = -\,1.18$; $M_2 = -\,2.93$; and $M_3 = -\,0.10$.

EQUATION OF FIVE ANGLES; NO SIDE-SWAY

The equation of five angles is an expression of the equilibrium of a rigid joint in a continuous frame, relating the angle of rotation of the joint to the four neighboring angles and the loads on both vertical and horizontal members. For a frame with no side-sway (joints fixed in space), the equation is:

$$K_{x,\,y}\ \theta_{x-1,\,y} + K_{y,\,x}\ \theta_{y-1,\,x} + 2\,\Sigma K\ \theta_{x,\,y} + K_{x+1,\,y}\ \theta_{x+1,\,y} + K_{y+1,\,x}\ \theta_{y+1,\,x} = U_{x,\,y} \qquad (9)$$

in which $\theta_{x,\,y}$ = the angle of rotation at Joint x, y (see Fig. 9(*a*), Appendix II); $K_{x,\,y}$ = stiffness ratio for girder span; $K_{y,\,x}$ = stiffness ratio for column span; $\Sigma K = K_{x,\,y} + K_{x+1,\,y} + K_{y,\,x} + K_{y+1,\,x}$, $U_{x,\,y}$ = load factor $= \frac{1}{2E}\ (\Delta M'_{x,\,y} + \Delta M'_{y,\,x})$; $\Delta M'_{x,\,y}$ = difference, at Joint x, y, between the fixed-end moments of the girders with vertical loads, $[\Delta M'_{x,\,y} = (M'_{x,\,y})_L - (M'_{x,\,y})_R]$; and, $\Delta M'_{y,\,x}$ = the difference, at Joint x, y, between the fixed-end moments of the columns with horizontal loads, $[\Delta M'_{y,\,x} = (\Delta M'_{y,\,x})_L - (\Delta M'_{y,\,x})_R]$.

The approximating equation is written as:

$$(\theta_{x,\,y})_n = (\theta_{x,\,y})_1 - \frac{1}{2\,\Sigma K}\ [K_{x,\,y}\ (\theta_{x-1,\,y})_n + K_{x+1,\,y}\ (\theta_{x+1,\,y})_n + K_{y,\,x}\ (\theta_{y-1,\,x})_{n-1} + K_{y+1,\,x}\ (\theta_{y+1,\,x})_{n-1}] \qquad (10)$$

in which $(\theta_{x,\,y})_n$ = the *n*th approximation; and $(\theta_{x,\,y})_1$, the first approximation, $= \frac{U_{x,y}}{2\,\Sigma K}$.

The value of θ in Equation (10) that bears the index, n, depends on the order in which the correction factors (the quantity in parenthesis) are computed. The order is immaterial of course; any two of the four θ-values may be given the index, n. As written, Equation (10) indicates that the order of procedure is from left to right and from top to bottom. It expresses the fact that each value of $\theta_{x,\,y}$ is corrected, successively, by the sum of the products of the four neighboring θ-values and their K-factors, divided by twice the sum of the four K-values that meet at Point (x, y). The correction is always made from the first approximation, $(\theta_{x,\,y})_1$, which is first computed for each joint from the load factor. The proof of convergence for the procedure is indicated in Appendix II.

Case 3.—Fig. 3 shows a frame, two stories high (3)[a]. The values of the angles at the lower ends of the lower columns are assumed to be zero. For this example, $U_{x,\,y} = \frac{\Delta M'_{x,y}}{2\,E}$; and $\Delta\ M'_{y,x} = 0$, since there are no horizontal loads. To illustrate the application of Equation (10) the following values for $2\,E\,\theta_{0,2}$ and $2\,E\,\theta_{1,2}$ are computed:

---

[a] Numerals in parentheses refer to Appendix I.

$$2\,E\,(\theta_{0,2})_1 = \frac{U_{x,y}}{2\,\Sigma K} = \frac{\Delta\,M'_{0,2}}{2\,(\frac{1}{6}+\frac{1}{3})} = \frac{[0-(-3)]}{1} = +\,\underline{3.00};$$

$$2\,E\,(\theta_{0,2})_2 = +\,\underline{3.00} - \frac{1}{1}\left\{(-1.80 \times \tfrac{1}{3}) + (0.00 \times \tfrac{1}{6})\right\} = +\,3.60;$$

and,

$$(\theta_{1,2})_2 = -\,\underline{1.80} - \frac{1}{\frac{5}{3}}\left\{(+\,3.60 \times \tfrac{1}{3}) + (\underline{1.50} \times \tfrac{1}{6}) + (\tfrac{1}{3} \times \underline{1.80})\right\} = -\,3.03$$

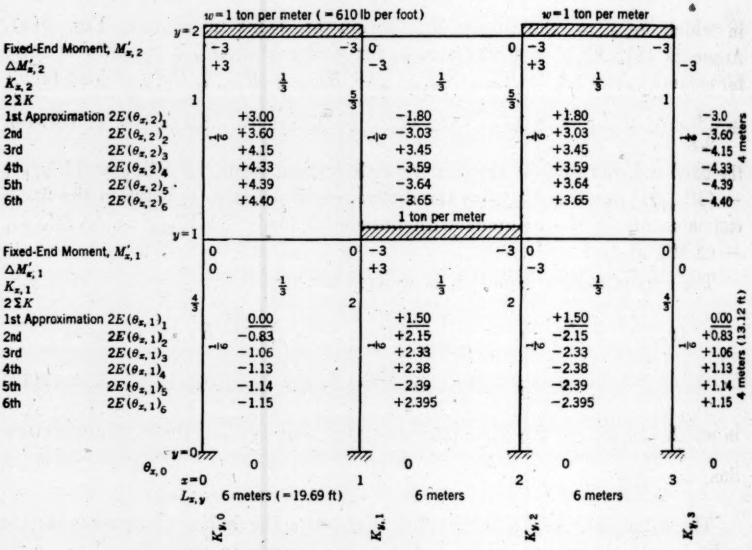

w = 1 ton per meter (= 610 lb per foot)
w = 1 ton per meter
y = 2
Fixed-End Moment, $M'_{x,2}$
$\Delta M'_{x,2}$
$K_{x,2}$
$2\Sigma K$
1st Approximation $2E(\theta_{x,2})_1$
2nd $2E(\theta_{x,2})_2$
3rd $2E(\theta_{x,2})_3$
4th $2E(\theta_{x,2})_4$
5th $2E(\theta_{x,2})_5$
6th $2E(\theta_{x,2})_6$
−3
+3
1
+3.00
+3.60
+4.15
+4.33
+4.39
+4.40
1/3
−3
5/3
0
−3
−1.80
−3.03
+3.45
−3.59
−3.64
−3.65
1/3
0
5/3
−3
+3
+1.80
+3.03
+3.45
+3.59
+3.64
+3.65
1/3
−3
−3
1
−3.0
−3.60
−4.15
−4.33
−4.39
−4.40
4 meters
1 ton per meter
y = 1
Fixed-End Moment, $M'_{x,1}$
$\Delta M'_{x,1}$
$K_{x,1}$
$2\Sigma K$
1st Approximation $2E(\theta_{x,1})_1$
2nd $2E(\theta_{x,1})_2$
3rd $2E(\theta_{x,1})_3$
4th $2E(\theta_{x,1})_4$
5th $2E(\theta_{x,1})_5$
6th $2E(\theta_{x,1})_6$
0
0
4/3
0.00
−0.83
−1.06
−1.13
−1.14
−1.15
1/3
0
2
−3
+3
+1.50
+2.15
+2.33
+2.38
−2.39
+2.395
1/3
−3
2
0
−3
+1.50
−2.15
−2.33
−2.38
−2.39
−2.395
1/3
0
0
4/3
0.00
+0.83
+1.06
+1.13
+1.14
+1.15
4 meters (13.12 ft)
y = 0
$\theta_{x,0}$
0
0
0
0
x = 0
1
2
3
$L_{x,y}$
6 meters (= 19.69 ft)
6 meters
6 meters
$K_{y,0}$
$K_{y,1}$
$K_{y,2}$
$K_{y,3}$

FIG. 3.—SUCCESSIVE APPROXIMATIONS FOR $2E\theta$

The procedure is continued until the values become stationary. On account of symmetry in this example computations for only one-half the joints need be made. The results shown in Fig. 3 check the values determined by elimination.

EQUATION OF FIVE ANGLES; SIDE-SWAY

For a continuous frame acted upon by horizontal loads, the rigid joints may undergo a horizontal displacement, or side-sway. The equation of five angles for this case is:

$$K\,(\theta) = U_{x,\,y} \qquad (11)$$

in which,

$$K\,(\theta) = K_{x,\,y}\,\theta_{x-1,\,y} + K_{y,\,x}\,\theta_{y-1,\,x} + 2(\Sigma\,K)\,\theta_{x,\,y} + K_{x+1,\,y}\,\theta_{x+1,\,y} + K_{y+1,\,x}\,\theta_{y+1,\,x} \qquad (12)$$

and,

$$U_{x,y} = \frac{\Delta M'_{y,x}}{2E} + 3\,K_{y,x}\,(\alpha_{y,x})_L + 3\,K_{y+1,x}\,(\alpha_{y+1,x})_L \ldots\ldots (13)$$

$\Sigma K =$ sum of four K-values of members meeting at a joint; and $\Delta M'_{y,x}$ = difference in fixed-end moments for the column ends meeting at Joint x, y; and $(\alpha_{y,x})_L$ = angle of column rotation due to the horizontal displacement of the upper end, y, relative to the lower end, $y - 1$, of the column, $L_{y,x}$. The subscript, L, indicates that the angle is to the left of a joint when the observer is to the right of the column (Fig. 10(a), Appendix II).

For convenience in computing Angles θ and α, the former is written $\theta = \theta_b + \theta_s$. If θ_b represents the angle of rotation due to the bending action alone of the horizontal loads on the columns (side-sway being zero), and θ_s is the rotation due to the side-sway alone, Equation (11) may be replaced by:

$$K\,(\theta_b) = \frac{\Delta M'_{y,x}}{2E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and,

$$K\,(\theta_s) = 3\,K_{y,x}\,(\alpha_{y,x})_L + 3\,K_{y+1,x}\,(\alpha_{y+1,x})_L \ldots\ldots\ldots (15)$$

Since the angles, α, as well as the angles, θ_s, in Equation (15) are unknown, it is necessary to write an additional equation of equilibrium for the horizontal forces above each floor level; thus:

$$\sum_{y-1}^{y=k}\sum_{x=0}^{x=m} P_{y,x} - \sum_{x=0}^{x=m} (R_{y-1,x})_{o,R} + \sum_{x=0}^{x=m} \frac{(M'_{y-1,x})_R - (M'_{y,x})_L}{L_{y,x}}$$

$$= -\sum_{x=0}^{x=m} \frac{6\,E\,K_{y,x}}{L_{y,x}}\,[\theta_{x,y} + \theta_{y-1,x} - 2\,(\alpha_{y,x})_L] \ldots\ldots\ldots\ldots (16)$$

in which k = number of floor levels minus one; $P_{y,x}$ = horizontal load on the column, $L_{y,x}$; m = number of columns minus one; $(R_{y-1,x})_{o,R}$ = horizontal reaction at joint on the level, $y - 1$, due to given loads on the columns, with end moments equal to zero; and M' = fixed-end moment, either to the right or to the left of a joint, as indicated by the index, R, or L (Fig. 11, Appendix II).

Case 4.—As an example of the application of Equations (14), (15), and (16) to a frame with side-sway, the frame in Case 3 is subjected to horizontal loads, w, equal to 2 tons per m (1 220 lb per ft; see Fig. 4). The first part of the problem is the solution of Equation (14). This is accomplished as outlined in Case 3 for Equation (9). The fixed-end moments, $M'_{y,x}$, are $-\frac{1}{12}\,w\,L^2 = -\frac{8}{3}$ and the first approximation is, $2\,E\,(\theta_b)_1 = \frac{\Delta M'_{y,x}}{2\,\Sigma K}$. In Fig. 4 the first approximations are underlined and the succeeding ones are tabulated below the first.

For the solution of Equation (15), subject to the condition expressed by Equation (16), it is assumed that the angle of side-sway, α, is the same for each column in any given story. Since, for this frame, $K_{y,x} = K_{y+1,x} = \frac{1}{6}$ for all columns, Equation (15) may be written:

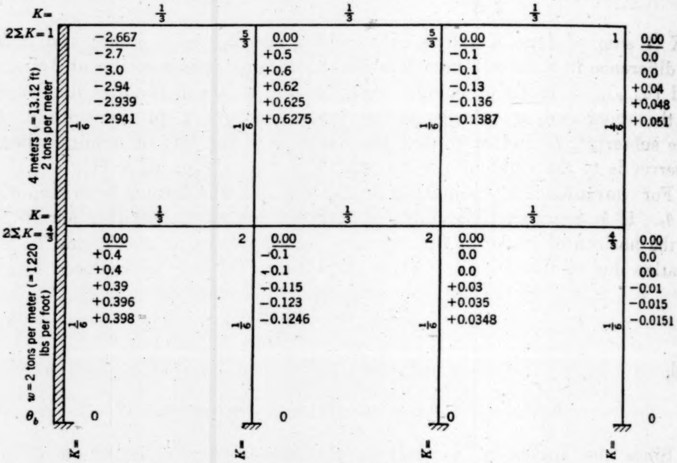

K=
2ΣK=1
1/3
1/3
1/3
−2.667
−2.7
−3.0
−2.94
−2.939
−2.941
5/3
0.00
+0.5
+0.6
+0.62
+0.625
+0.6275
5/3
0.00
−0.1
−0.1
−0.13
−0.136
−0.1387
1
0.00
0.0
0.0
+0.04
+0.048
+0.051
4 meters (=13.12 ft)
2 tons per meter
K=
2ΣK=4/3
1/3
1/3
1/3
0.00
+0.4
+0.4
+0.39
+0.396
+0.398
2
0.00
−0.1
−0.1
−0.115
−0.123
−0.1246
2
0.00
0.0
0.0
+0.03
+0.035
+0.0348
4/3
0.00
0.0
0.0
−0.01
−0.015
−0.0151
1/6
1/6
1/6
1/6
1/6
1/6
1/6
1/6
w=2 tons per meter (=1220 lbs per foot)
θ_b
0
0
0
0
K=
K=
K=
K=

FIG. 4.—SUCCESSIVE APPROXIMATIONS FOR $2E\theta_b$

$$K\,(\theta_s) = \frac{1}{2}\,(\alpha_{y-1})_L + \frac{1}{2}\,(\alpha_y)_L \ldots\ldots\ldots\ldots\ldots (17)$$

The joint rotation, θ_s, may be further divided into: $(\theta_s)_{y-1}$, the joint rotation corresponding to a side-sway $(\alpha_{y-1})_L$; and $(\theta_s)_y$, the joint rotation corresponding to a side-sway, $(\alpha_y)_L$. Consequently, for $y = 1, 2$ and $(\alpha_0)_L = 0$, Equation (17) may be replaced by:

$$K\,(\theta_s)_{y-1} = \frac{1}{2}\,(\alpha_1)_L = \frac{1}{2}\,\alpha_1 \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

and,

$$K\,(\theta_s)_y = \frac{1}{2}\,(\alpha_1)_L + \frac{1}{2}\,(\alpha_2)_L = \frac{1}{2}\,\alpha_1 + \frac{1}{2}\,\alpha_2 \ldots\ldots\ldots\ldots (19)$$

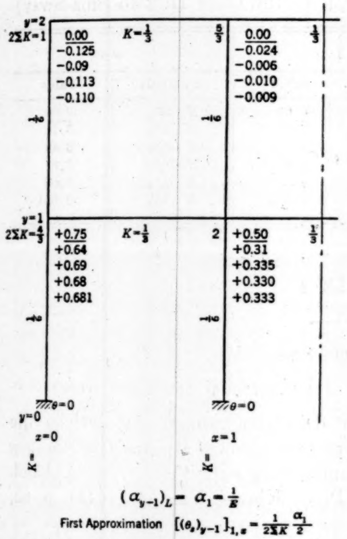
Equations (18) and (19) are solved by iteration, as outlined in Case 3, first for the values, $\alpha_1 = \alpha_2 = \dfrac{1.00}{E}$. The results are shown in Figs. 5 and 6. These values, for $\alpha = 1$, are then used in solving Equation (16), which, for Case 4, reduces to:

$$\alpha_1 = \frac{1}{8}\,(\theta_{0,1} + \theta_{1,1} + \theta_{2,1} + \theta_{3,1}) + \frac{6}{E} \ldots\ldots\ldots\ldots (20)$$

at the first story; and,

$$a_2 = \frac{1}{8}\left(\theta_{0,\,1} + \theta_{1,\,1} + \theta_{2,\,1} + \theta_{3,\,1} + \theta_{0,\,2} + \theta_{1,\,2} + \theta_{2,\,2} + \theta_{3,\,2}\right) + \frac{2}{E} \;.\;(21)$$

at the second story.

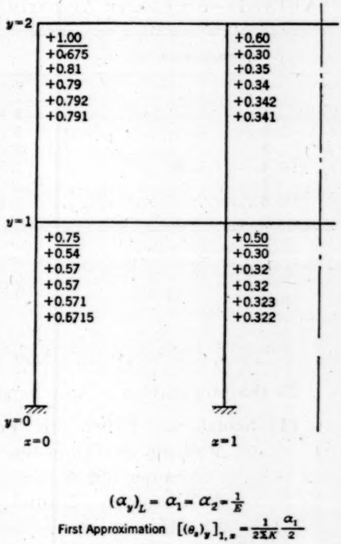

y=2
2ΣK=1
0.00
−0.125
−0.09
−0.113
−0.110
K=1/3
5/3
0.00
−0.024
−0.006
−0.010
−0.009
1/3
1/2
1/2
y=1
2ΣK=4/3
+0.75
+0.64
+0.69
+0.68
+0.681
K=1/3
2
+0.50
+0.31
+0.335
+0.330
+0.333
1/3
1/2
1/2
θ=0
θ=0
y=0
x=0
x=1
K=
K=

$(\alpha_{y-1})_L = \alpha_1 = \frac{1}{E}$

First Approximation $[(\theta_s)_{y-1}]_{1,\,x} = \frac{1}{2\Sigma K}\frac{\alpha_1}{2}$

$[(\theta_s)_{y-1}]_{2,\,x} = 0$

FIG. 5.—VALUES OF $2E(\theta_s)_{y-1}$


y=2
+1.00
+0.675
+0.81
+0.79
+0.792
+0.791
+0.60
+0.30
+0.35
+0.34
+0.342
+0.341
y=1
+0.75
+0.54
+0.57
+0.57
+0.571
+0.5715
+0.50
+0.30
+0.32
+0.32
+0.323
+0.322
y=0
x=0
x=1

$(\alpha_y)_L = \alpha_1 = \alpha_2 = \frac{1}{E}$

First Approximation $[(\theta_s)_y]_{1,\,x} = \frac{1}{2\Sigma K}\frac{\alpha_1}{2}$

$[(\theta_s)_y]_{2,\,x} = \frac{1}{2\Sigma K}\frac{\alpha_2}{2}$

FIG. 6.—VALUES OF $2E(\theta_s)_y$

In solving Equations (20) and (21) the first approximation for the a-values is obtained by placing all the θ-values equal to zero or $(a_1)_1 = \frac{6}{E}$ and $(a_2)_1 = \frac{2}{E}$. These values are then corrected by multiplying the values of $(\theta_s)_{y-1}$ and $(\theta_s)_y$ for unit rotation, a, by the first approximation for a and using these new values of $(\theta_s)_{y-1}$ and $(\theta_s)_y$, together with the θ_b-values (see Fig. 4), for a second approximation of a according to Equations (20) and (21), etc. To illustrate, the second approximations are:

$(a_1)_2 = \frac{1}{8}\left[\,(0.681 + 0.333)\,\frac{6}{E} + (0.571 + 0.322)\,\frac{2}{E} + (0.398 - 0.125 + 0.035 - 0.015)\,\frac{1}{2\,E}\,\right] + \frac{6}{E} = \frac{7.002}{E}$; and, $(a_2)_2 = \frac{1}{8}\Big[\,(0.681 + 0.333 - 0.110$

$$- 0.009)\ \frac{7.002}{E} + (0.571 + 0.322 + 0.791 + 0.341)\ \frac{2}{E} + (0.398 - 0.125$$

$$+ 0.035 - 0.015 - 2.941 + 0.628 - 0.139 + 0.051)\ \frac{1}{2\,E}\Bigg] + \frac{2}{E} = \frac{3.16}{E}.$$

Successive values of α up to the sixth approximation are shown in Table 1.

TABLE 1.—SUCCESSIVE APPROXIMATIONS FOR α (ROTATION DUE TO SIDE-SWAY)

Approximation No.	$y = 1$		$y = 2$	
	Symbols	Value	Symbols	Value
1	$E\ (\alpha_1)_1$	6.00	$E\ (\alpha_2)_1$	2.00
2	$E\ (\alpha_1)_2$	7.00	$E\ (\alpha_2)_2$	3.16
3	$E\ (\alpha_1)_3$	7.26	$E\ (\alpha_2)_3$	3.48
4	$E\ (\alpha_1)_4$	7.33	$E\ (\alpha_2)_4$	3.57
5	$E\ (\alpha_1)_5$	7.35	$E\ (\alpha_2)_5$	3.60
6	$E\ (\alpha_1)_6$	7.353	$E\ (\alpha_2)_6$	3.601

APPENDIX I

LIST OF REFERENCES

In the preparation of this paper some of the authors consulted were:

(1) **Seidel, L.** Ueber ein Verfahren die Gleichungen auf welche die Methode der kleinsten Quadrate führt, sowie lineäre Gleichungen überhaupt, durch successive Annäherung aufzulösen, Abh. d. Akad. d. Wissensch., Munich, Math.-Phys. Klasse, Vol. 11 (1874), p. 83, Pt. III.

(2) **Runge, C.** Praxis der Gleichungen, 1900. (Iteration procedure, p. 81.)

(3) **Gehler, W.** Der Rahmen, Third Edition, 1925. (Derivation of equation of three angles, p. 111; derivation of equation of five angles, p. 169.)

(4) **Mises, R. V. and Pollaczek-Geiringer, H.** Praktische Verfahren der Gleichungsauflösung, *Zeitschrift für Angew. Math. u. Mech.*, Vol. 9 (1929), p. 58.

(5) **Cross, H.** Analysis of Continuous Frames by Distributing Fixed-End Moments, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

(6) **Scarborough, J. B.** Numerical Mathematical Analysis, 1930. (Method of iteration, p. 184.)

(7) **Timoshenko, S.** Strength of Materials, Vol. 1 (1930). (Derivation and origin of equation of three moments, p. 226.)

(8) **Goldberg, J. E.** Wind Stresses by Slope Deflection and Converging Approximations. *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 962.

(9) **Seely, F. B.** Resistance of Materials, Second Edition 1935. (Theorem of three moments, p. 57.)

APPENDIX II

Mathematical Derivations

The fundamental equations relating moments and slopes for a single simply supported span (Fig. 7) are:

$$\theta_A = (\theta_A)_0 + \frac{M_A L}{3 E I} + \frac{M_B L}{6 E I} \qquad (22)$$

and,

$$\theta_B = -(\theta_B)_0 - \frac{M_B L}{3 E I} - \frac{M_A L}{6 E I} \qquad (23)$$

or,

$$M_A = 2 E K (2 \theta_A + \theta_B) + M'_A \qquad (24)$$

and,

$$M_B = -2 E K (2 \theta_B + \theta_A) + M'_B \qquad (25)$$

in which the subscript, 0, denotes that the moments at the ends are zero. Rotations are positive when clockwise; and moments are positive as shown in Fig. 7.

Equation of Three Moments.—The equation for three moments is found by applying Equations (22) and (23) to two adjacent spans in a continuous beam and making $(\theta_x)_L = (\theta_x)_R = \theta_x$ (see Fig. 8), in which $(\theta_x)_L$ is the angle of rotation at the left of Support x, and $(\theta_x)_R$ is the angle of rotation at the right of Support x. In a continuous beam these two rotations are equal. Expressing the angles, θ, in terms of the area-moments, Q_{x-1}, and Q_{x+1}, the equation of three moments becomes (7) (see Equation (1)):

$$\frac{M_{x-1}}{K_x} + 2 M_x \left(\frac{1}{K_x} + \frac{1}{K_{x+1}}\right) + \frac{M_{x+1}}{K_{x+1}} = -\frac{6 Q_{x-1}}{I_x L_x} - \frac{6 Q_{x+1}}{I_{x+1} L_{x+1}} \qquad (26)$$

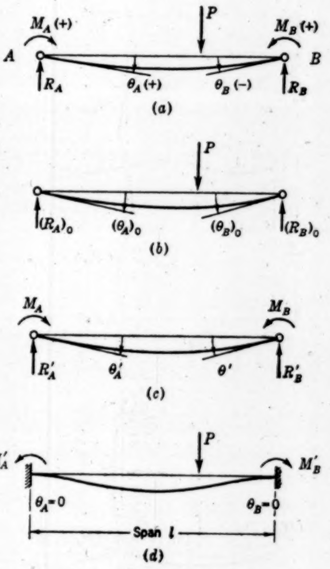


FIG. 7.

Equation of Three Angles.—The equation of three angles is obtained in a manner similar to Equation (26) by applying Equations (24) and (25) to two adjacent spans (Fig. 8) and equating the moments on either side of Support x; that is,

$$(M_x)_L = (M_x)_R = M_x \qquad (27)$$

in which $(M_x)_L$ is the moment at x in Member L_x to the left of x and $(M_x)_R$ is the moment at x in Member L_{x+1} to the right of x. Then (3) (see Equation (7));

$$K_x\ \theta_{x-1} + 2\ (K_x + K_{x+1})\ \theta_x + K_{x+1}\ \theta_{x+1} = \frac{\Delta\ M'_x}{2\ E}\ \ldots\ldots\ldots(28)$$

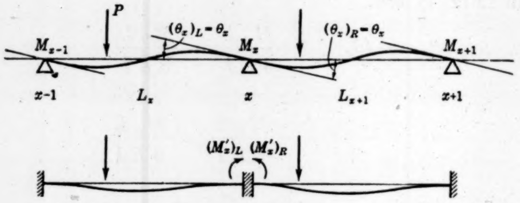
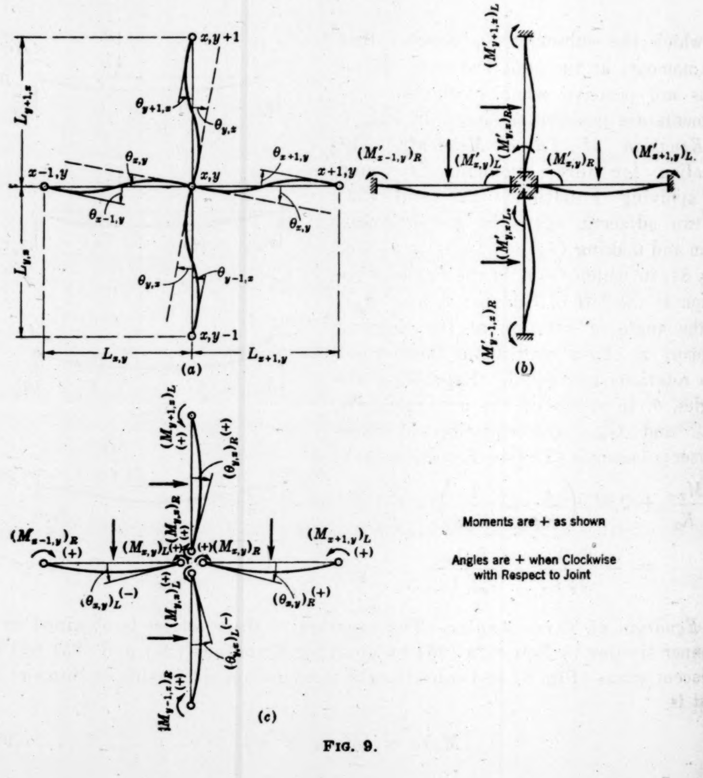

P
$(\theta_x)_L = \theta_x$
$(\theta_x)_R = \theta_x$
M_{x-1}
M_x
M_{x+1}
x-1
L_x
x
L_{x+1}
x+1
$(M'_x)_L$
$(M'_x)_R$

FIG. 8

in which $\Delta M'_x = (M'_x)_L - (M'_x)_R$ = the difference in fixed-end moments at x (see Fig. 8).


$x,y+1$
$L_{y+1,x}$
$\theta_{y+1,x}$
$\theta_{y,x}$
$\theta_{x,y}$
$\theta_{x+1,y}$
$x-1,y$
x,y
$x+1,y$
$\theta_{x-1,y}$
$\theta_{x,y}$
$L_{y,x}$
$\theta_{y,x}$
$\theta_{y-1,x}$
$x,y-1$
$L_{x,y}$
$L_{x+1,y}$
(a)
$(M'_{y+1,x})_L$
$(M'_{x-1,y})_R$
$(M'_{x,y})_L$
$(M'_{y,x})_R$
$(M'_{x,y})_R$
$(M'_{x+1,y})_L$
$(M'_{y,x})_L$
$(M'_{y-1,x})_R$
(b)
$(M_{y+1,x})_L$
(+)
$(\theta_{y,x})_R$ (+)
$(M_{y,x})_R$
(+)
$(M_{x-1,y})_R$
(+)
$(M_{x,y})_L$ (+)
(+) $(M_{x,y})_R$
$(M_{x+1,y})_L$
(+)
$(\theta_{x,y})_L$ (−)
$(M_{y,x})_L$ (+)
$(\theta_{x,y})_R$ (+)
$(\theta_{y,x})_L$ (−)
$(M_{y-1,x})_R$
(+)
(c)

Moments are + as shown

Angles are + when Clockwise with Respect to Joint

FIG. 9.

Equation of Five Angles: No Side-Sway.—Fig. (9*a*) shows Joint (x, y) rotated through a positive angle, $\theta_{x,y}$. Fig. 9(*b*) shows the fixed-end moments under the given loads. If Equations (24) and (25) are applied to the four spans joining at (x, y) (Fig. 9(*c*)), the following equations are obtained:

In Span $L_{x,y}$:

$$(M_{x,y})_L = -2\,E\,K_{x,y}\,[2\,(\theta_{x,y})_L + (\theta_{x+1,y})_R] + (M'_{x,y})_L \ldots (29)$$

In Span $L_{x+1,y}$:

$$(M_{x,y})_R = 2\,E\,K_{x+1,y}\,[2\,(\theta_{x,y})_R + (\theta_{x+1,y})_L] + (M'_{x,y})_R \ldots (30)$$

In Span $L_{y,x}$:

$$(M_{y,x})_L = -2\,E\,K_{y,x}\,[2\,(\theta_{y,x})_L + (\theta_{y-1,x})_R] + (M'_{y,x})_L \ldots (31)$$

and, in Span $L_{y+1,x}$:

$$(M_{y,x})_R = 2\,E\,K_{y+1,x}\,[2\,(\theta_{y,x})_R + (\theta_{y+1,x})_L] + (M'_{y,x})_R \ldots (32)$$

Substituting Equations (29) to (32) in the equilibrium equation,

$$(M_{x,y})_R + (M_{y,x})_R = (M_{x,y})_L + (M_{y,x})_L \ldots\ldots\ldots\ldots (33)$$

and, making use of the equality of angles at Joint x, y,

$$(\theta_{x,y})_R = (\theta_{x,y})_L = (\theta_{y,x})_L = (\theta_{y,x})_R = \theta_{x,y} \ldots\ldots\ldots (34)$$

the five-angle equation (3) (see Equation (9)), is obtained; thus:

$$K_{x,y}\,\theta_{x-1,y} + K_{y,x}\,\theta_{y-1,x} + 2\,\theta_{x,y}\,(K_{x,y} + K_{x+1,y} + K_{y,x} + K_{y+1,x})$$
$$+ K_{x+1,y}\,\theta_{x+1,y} + K_{y+1,x}\,\theta_{y+1,x} = \frac{1}{2\,E}\,[\Delta M'_{x,y} + \Delta M'_{y,x}] \ldots\ldots (35)$$

in which (see Fig. 9(*b*)): $\Delta M'_{x,y} = (M'_{x,y})_L - (M'_{x,y})_R$; and $\Delta M'_{y,x} = (M'_{y,x})_L - (M'_{y,x})_R$.

Equation of Five Angles: With Side-Sway.—It is assumed that no vertical loads act on the girders. Fig. 10(*a*) shows the horizontal displacement of Joints $x, y + 1$ and $x, y - 1$, relative to Joint x, y. The angles of displacement are $(\alpha_{y,x})_R$ to the right of Joint x, y, and $(\alpha_{y,x})_L$ to the left of Joint x, y. The other angles involved are shown in Fig. 10. Applying Equations (24) and (25) to the members in Fig. 10(*c*), the following equations are obtained:

In Span $L_{x,y}$:

$$(M_{x,y})_L = -2\,E\,K_{x,y}\,[2\,(\theta_{x,y})_L + (\theta_{x-1,y})_R] \ldots\ldots\ldots (36)$$

In Span $L_{x+1,\,y}$:

$$(M_{x,\,y})_R = 2\;E\;K_{x+1,\,y}\;[2\;(\theta_{x,\,y})_R + (\theta_{x+1,y})_L] \ldots\ldots\ldots (37)$$

In Span $L_{y,\,x}$:

$$(M_{y,\,x})_L = -\,2\;E\;K_{y,\,x}\;[2\;(\omega_{y,\,x})_L + (\omega_{y-1,x})_R] + (M'_{y,\,x})_L \ldots (38)$$

and, in Span $L_{y+1,\,x}$:

$$(M_{y,x})_R = 2\;E\;K_{y+1,x}\;[\;2\;(\omega_{y,\,x})_R + (\omega_{y+1,\,x})_L] + M'_{y,\,x})_R \ldots (39)$$

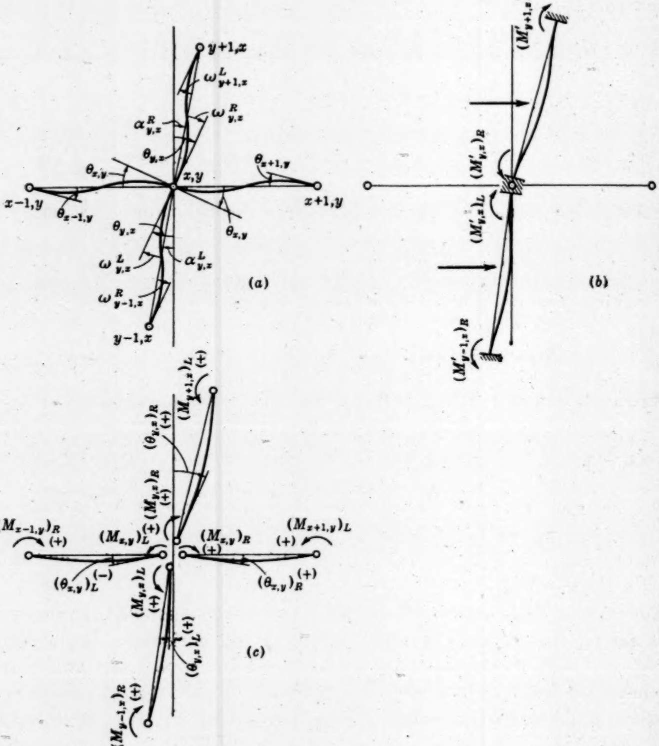

y+1,x
$\omega^L_{y+1,x}$
$\omega^R_{y,x}$
$\alpha^R_{y,x}$
$\theta_{y,x}$
$\theta_{x,y}$
x,y
$\theta_{x+1,y}$
x−1,y
$\theta_{x-1,y}$
x+1,y
$\theta_{y,x}$
$\theta_{x,y}$
$\omega^L_{y,x}$
$\alpha^L_{y,x}$
$\omega^R_{y-1,x}$
y−1,x
(a)
$(M'_{y+1,x})_L$
$(M'_{y,x})_R$
$(M'_{y,x})_L$
$(M'_{y-1,x})_R$
(b)
$(M_{y+1,x})_L$ (+)
$(\theta_{y,x})_R$ (+)
$(M_{y,x})_R$ (+)
$(M_{x-1,y})_R$ (+)
$(M_{x,y})_L$ (+)
$(M_{x,y})_R$ (+)
$(M_{x+1,y})_L$ (+)
$(\theta_{x,y})_L^{(-)}$
$(\theta_{x,y})_R^{(+)}$
$(M_{y,x})_L$ (+)
$(\theta_{y,x})_L$ (+)
$(M_{y-1,x})_R$ (+)
(c)

FIG. 10

By substituting Equations (36) to (39) into Equation (33), and making use of the relations, $(\omega_{y,\,x})_R = (\theta_{y,\,x})_R - (\alpha_{y,\,x})_R$; $(\omega_{y,\,x})_L = (\theta_{y,\,x})_L - (\alpha_{y,\,x})_L$ (see Fig. 10(*a*)); and, $(\theta_{x,\,y})_L = (\theta_{x,\,y})_R = (\theta_{y,\,x})_L = (\theta_{y,\,x})_R$

$= \theta_{x,y}$, the equation of five angles, for horizontal loads, with side-sway, is obtained; thus (see Equation (11)):

$$K_{x,y}\,\theta_{x-1,y} + K_{x+1,y}\,\theta_{x+1,y} + 2\,\theta_{x,y}\,(K_{x,y} + K_{y,x} + K_{x+1,y} + K_{y+1,x})$$

$$+ K_{y,x}\,\theta_{y-1,x} + K_{y+1,x}\,\theta_{y+1,x} = \frac{\Delta M'_{y,x}}{2\,E} + 3\,K_{y,x}\,(a_{y,x})_L + 3\,K_{y+1,x}\,(a_{y+1,x})_L \text{.} \quad (40)$$

Equation (16) expressing the equilibrium of horizontal forces for the part of the frame above a horizontal section through any floor level is derived as follows: The fundamental relations between moments and reactions (see Fig. 7) are:

$$R_A = (R_A)_0 + \frac{M_B - M_A}{L} \dots\dots\dots\dots\dots\dots (41)$$

and,

$$R_B = (R_B)_0 + \frac{M_A - M_B}{L} \dots\dots\dots\dots\dots\dots (42)$$

If Equation (41) is applied to a single column (see Fig. 11(*a*)):

$$(R_{y-1,x})_R = (R_{y-1,x})_{0,x} + \frac{(M_{y,x})_L - (M_{y-1,x})_R}{L_{y,x}} \dots\dots\dots (43)$$

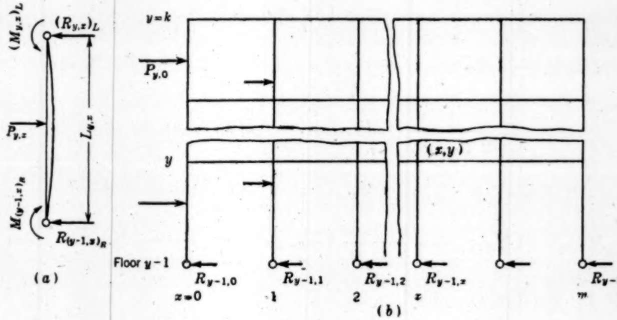


FIG. 11.

From Equations (38) and (39):

$$(M_{y,x})_L - (M_{y-1,x})_R = -\,6\,E\,K_{y,x}\,[\theta_{x,y} + \theta_{y-1,x} - 2\,(a_{y,x})_L]$$

$$-\,[(M'_{y-1,x})_R - (M'_{y,x})_L] \dots\dots\dots\dots\dots\dots (44)$$

Since the horizontal loads and reactions are in equilibrium (see Fig. 11(*b*)):

$$\sum_{x=0}^{x=m}\sum_{y=1}^{y=k} P_{y,x} = \sum_{x=0}^{x=m} (R_{y-1,x})_R \dots\dots\dots\dots\dots\dots (45)$$

Finally, substituting Equations (43) and (44) into Equation (45) (see Equation (16)):

$$\sum_{x=0}^{x=m} \sum_{y-1}^{y=k} P_{y,x} = \sum_{x=0}^{x=m} (R_{y-1,x})_{o,R} - \sum_{x=0}^{x=m} \frac{6\,E\,K_{y,x}}{L_{y,x}} \left[\theta_{x,y} + \theta_{y-1,x} - 2\,(\alpha_{y,x})_L\right]$$

$$- \sum_{x=0}^{x=m} \frac{(M'_{y-1,x})_R - (M'_{y,x})_L}{L_{y,x}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

Convergence Proof.—Equation of Three Moments.—That the sequence, $(M_x)_n$ (Equation (5)), is convergent for increasing integral values of n is demonstrated if it can be shown that the sequence of differences between successive approximations converges to zero. It is assumed that the coefficients, $\frac{1}{K}$, are positive and that $S_x = \frac{1}{K_x} + \frac{1}{K_{x+1}}$ does not equal zero; in other words, the $\frac{1}{K}$ - values are not zero for any two adjacent spans in a continuous beam. The nth approximation is expressed by Equation (5) of the paper and the $(n-1)$th is:

$$(M_x)_{n-1} = (M_x)_1 - \frac{1}{2\,S_x} \left[\frac{(M_{x-1})_{n-2}}{K_x} + \frac{(M_{x+1})_{n-2}}{K_{x+1}}\right] \quad \ldots\ldots (47)$$

The difference between Equations (5) and (47) is:

$$(M_x)_n - (M_x)_{n-1} = -\frac{1}{2\,K_x\,S_x}\left[(M_{x-1})_{n-1} - (M_{x-1})_{n-2}\right]$$

$$-\frac{1}{2\,K_{x+1}\,S_x}\left[(M_{x+1})_{n-1} - (M_{x+1})_{n-2}\right] \quad \ldots\ldots\ldots\ldots\ldots (48)$$

For $n = 2$, since $(M_{x-1})_o = (M_{x+1})_o = 0$, the difference is:

$$(M_x)_2 - (M_x)_1 = -\frac{1}{2\,K_x S_x}(M_{x-1})_1 - \frac{1}{2\,K_{x+1}\,S_x}(M_{x+1})_1 \quad \ldots\ldots (49)$$

Let M be the largest absolute value among all the moments in the first approximations, including boundary values. Then, the difference, $(M_x)_2 - (M_x)_1$, lies between $+\,0.5\,M$ and $-\,0.5\,M$. Writing the difference for $n = 3$:

$$(M_x)_3 - (M_x)_2 = -\frac{1}{2\,S_x\,K_x}\left[(M_{x-1})_2 - (M_{x-1})_1\right]$$

$$-\frac{1}{2\,S_x K_{x+1}}\left[(M_{x+1})_2 - (M_{x+1})_1\right] \quad \ldots\ldots\ldots\ldots\ldots (50)$$

it is seen that this difference lies between $+\,0.25\,M$ and $-\,0.25\,M$. In general, the difference expressed by Equation (48) lies between $\frac{+\,M}{2^{n-1}}$ and $\frac{-\,M}{2^{n-1}}$.

Hence, this difference approaches zero for increasing values of n. To show that $(M_x)_n$ approaches the solution, M_x, expressed by Equation (1), the difference is written in the form $\left(\text{remembering that } (M_x)_1 = \frac{U_x}{2\,S_x}\right)$:

$$(M_x)_n - (M_x)_{n-1} = -\frac{1}{2\,S_x}\left[\frac{(M_{x-1})_{n-1}}{K_x} + 2\,(M_x)_{n-1}\,S_x + \frac{(M_{x+1})_{n-1}}{K_{x+1}} - U_x\right] \text{ . (51)}$$

Since $(M_x)_n - (M_x)_{n-1}$ approaches zero, the right-hand expression approaches zero; hence, $(M_x)_{n-1}$ satisfies the original Equation (1) for the condition that n becomes infinite.

Convergence is easily shown also for Equation (6) when $(M_{x-1})_n$ is used in computation instead of $(M_{x-1})_{n-1}$; thus:

$$(M_x)_n - (M_x)_{n-1} = -\frac{1}{2\,K_x\,S_x}\left[(M_{x-1})_n - (M_{x-1})_{n-1}\right]$$
$$-\frac{1}{2\,K_{x+1}\,S_x}\left[(M_{x+1})_{n-1} - (M_{x+1})_{n-2}\right] \ldots\ldots\ldots\ldots (52)$$

and, for $n = 2$:

$$(M_x)_2 - (M_x)_1 = -\frac{1}{2\,K_x\,S_x}\left[(M_{x-1})_2 - M_{x-1})_1\right] - \frac{1}{2\,K_{x+1}\,S_x}(M_{x+1})_1 . (53)$$

By making use of the upper and lower bounds, as stated for Equation (48) (that is, $(M_x)_2 - (M_x)_1$ is less than $+ 0.5\,M$ and greater than $- 0.5\,M$), it is also true that $(M_{x-1})_2 - (M_{x-1})_1$ is less than $+ M$ and greater than $- M$. Hence, for Equation (53), $(M_{x-1})_2 - (M_{x-1})_1$ is less than $+ 0.5\,M$ and greater than $- 0.5\,M$; and, in general, $(M_x)_n - (M_x)_{n-1}$ is less than $\frac{+M}{2^{n-1}}$ and greater than $\frac{-M}{2^{n-1}}$, as before.

For the equation of five angles the proof of convergence is easily obtained by generalizing the proof as given for the equation of three moments; in the equation of five angles the value, ΣK, must not be equal to, or less than, zero. A proof of convergence based upon the minimum of a positive definite quadratic form associated with a set of simultaneous equations is given in the paper (4).

APPENDIX III

NOTATION

The following letter symbols, introduced in the paper, are arranged herein, for convenience of reference and for the guidance of discussers:

b = a subscript denoting "bending".

I = rectangular moment of inertia; I_x = moment of inertia in Span L_x, etc.

K = a stiffness ratio, $\frac{I}{L}$; ΣK = a sum of four stiffness ratios $= K_{x,y} + K_{x+1,y} + K_{y,x} + K_{y+1,x}$ at Joint (x, y).

k = number of floor-levels minus one.

L = length; L_x = span length between Supports $x - 1$ and x, etc.; as a subscript, L, denotes left and as a subscript to α, it indicates that the angle is to the left of a joint when the observer is to the right of a column.

M = moment of force; M_x = bending moment at Support x, etc.; M' = fixed-end moment; $\Delta M'_x$ = a local factor = a difference, $(M'_x)_L - (M'_x)_R$, between fixed-end moments at the left and right of Support x.

m = number of columns minus one.

n = a limiting number.

P = a horizontal force applied at any story; P_{yx} = horizontal force between floor levels, $y - 1$ and y.

Q = statical moments of area about a given axis; Q_{x-1} *and* Q_{x+1} are, respectively, the moments of the areas below the moment diagram in Spans L_x and L_{x+1} with respect to Supports $x - 1$ and $x + 1$.

R = a subscript denoting "right side".

S = a sum of two stiffness ratios; $S_x = \frac{1}{K_x} + \frac{1}{K_{x+1}}$.

s = a subscript denoting "side-sway".

U = a load factor in two adjacent spans.

α = angle of rotation of one end of one member; $(\alpha_{y,x})_L$ = angle of column rotation due to the horizontal displacement of the upper end, y, with respect to the lower end, $y - 1$, of the column, $L_{y,x}$; the subscript, L, applied to α indicates that the angle is to the left of a joint when the observer is to the right of a column.

Δ = a difference; $\Delta M'_x$ = a load factor = difference, $(M'_x)_L - (M'_x)_R$, between fixed-end moments.

θ = rotation of a joint; θ_b = the rotation due entirely to the bending action of horizontal loads on the columns; θ_s = the rotation due entirely to side-sway.

ω = an angle $= \theta - \alpha$.

DISCUSSION

Garrett B. Drummond,[4] Esq. (by letter).—A method of practical utility for the solution of simultaneous equations, is demonstrated in this paper. The steps for solving n simultaneous equations containing n unknown quantities, by the method of successive approximations, can be summarized as follows:

(1) Choose one equation in which the coefficient of one unknown is quite large in comparison with the coefficients of the other unknowns;

(2) Let all the unknowns, except that with the largest coefficient, assume the value zero;

(3) Solve for a first approximate value of this unknown;

(4) Choose another equation in which the coefficient of one of the unknowns, other than that chosen in Step (1) is quite large in comparison with the coefficients of the remaining unknowns;

(5) Substitute the first approximate value of the unknown already determined, allowing the remaining $n - 2$ unknowns to assume the value zero;

(6) Solve for a first approximate value of the second unknown;

(7) Continue in this manner, using first approximate values of each unknown, until first approximate values are determined for all unknowns; and,

(8) Repeat Steps (1) to (7) until the desired degree of approximation is reached, or until the difference between successive approximate values becomes negligible.

A very simple demonstration of this process can be given by solving the three-moment equations of the beam shown in Fig. 12. The equations are:

$$15\ M_2 + 54\ M_3 = -\ 132\,480 \quad\quad (54)$$

and,

$$50\ M_2 + 15\ M_3 = -\ 104\,820 \quad\quad (55)$$

From Equation (54): $54\ M_3 = -\ 132\,480$; and, $M_3 = -\ 2\,446$, which is the first approximate value of M_3. Substituting this value into Equation (55): $50\ M_2 + (15)\ (-\ 2\,446) = -\ 104\,480$; or (simplifying and solving for M_2): $M_2 = -\ 1\,356$, which is the first approximate value of M_2.

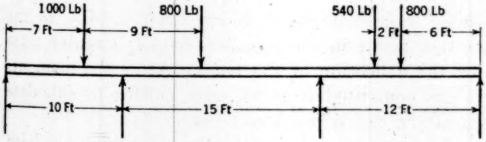


FIG. 12

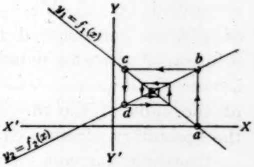


FIG. 13

Returning to Equation (54) with this value: $(15)\ (-\ 1\,356) + 54\ M_3 = -\ 132\,480$; and solving, $M_3 = -\ 2\,077$, which is the second approximate value of M_3.

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[4] With U. S. Engr. Dept., Memphis, Tenn.

Continuing in this manner, the second approximate value of M_3 is found to be $-1\,473$; the third approximate value of M_2 is found to be $-2\,041$; and the third approximate value of M_3 is found to be $-1\,484$. These third approximate values are identical with the exact values determined from the ordinary simultaneous solutions of the two equations.

This apparently trivial problem was selected because it is possible to show, graphically, the precise nature of the steps in the approximate solution of two equations. Consider the two functions, $y_1 = f_1(x)$ and $y_2 = f_2(x)$, as shown in Fig. 13. The process was to begin at Point *a*, where $y = 0$ in Equation (54). With the corresponding value of x, the next step is to find the value of y in Equation (55)—in other words, proceed to Point *b*, Fig. 13, with this value of y; the third step is to find the value of x in Equation (54)—that is, move to Point *c*, Fig. 13. Continuing in this manner, with the successive values found, from the graphical standpoint, one merely follows the path shown in Fig. 13, which approaches closer and closer each time to the intersection of the graphs of the two equations, or to the numerical values of the simultaneous solutions of the equations.

Of course, from its practical use, the purpose of this method of solution is to lighten the labor of solving a large number of simultaneous equations, such as are encountered in indeterminate structures. This raises the question of the desirability of "exact" solutions. Just what is meant by "exact" solutions?

Setting up the equations, say, for the solution of a frame by the method of slope deflection, is it not true that certain assumptions are made regarding the action of forces at joints, and is the assumption not also made that the moment of inertia of the various members are constant throughout their cross-section? Then, it appears to be stretching the point to insist that the equations resulting from these assumptions be solved for "exact" values. Approximate values are sufficient, and in arriving at such values the method explained by Professor Schwalbe is approximate and entirely applicable.

A. W. FISCHER,[5] ESQ. (by letter).—A rather unique method for the solution of simultaneous equations is presented in this paper. The simple examples are especially valuable, but the method is just as simple for solving equations of a more complicated nature. For example, it lends itself readily to the solution of the end moments that occur in the members of a triangular truss having rigid joints, caused by the deflection of the truss. After the moments at the ends of the members are computed it is an easy matter to calculate the secondary stresses caused at the end of each member.

Consider, for comparison, the Pratt truss shown in Fig. 14, which has been analyzed elsewhere[6] by Sophus Thompson, Assoc. M. Am. Soc. C. E., and Ralph W. Cutler, Jun. Am. Soc. C. E. Adding all the clockwise fixed-end moments and subtracting the counter-clockwise fixed-end moments at each joint and dividing them by $2 \Sigma K$ for all the members entering the joint the first approximation of $2 E \theta$ is determined for the various joints. For Joint 1, the sum of all the clockwise fixed-end moments[6] = 4 768 + 2 290 = 7 058, and $2 \Sigma K = 2\,(9.15 + 3.80) = 25.90$. From this, the first approximation of $2 E \theta$ at Joint 1 $= \dfrac{7\,058}{25.90} = 272.5$. For Joint 6 the sum of all the fixed-end clockwise moments minus all the counter-clockwise fixed-end moments $= 6\,440 - 6\,440 = 0$, and $2 \Sigma K = 2\,(12.43 + 0.774 + 12.43) = 51.268$. From this value the first approximation of $2 E \theta$ at Joint 6 = 0. All the first

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[5] Care, Pennsylvania Sugar Co., Philadelphia, Pa.

[6] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), pp. 108–110.

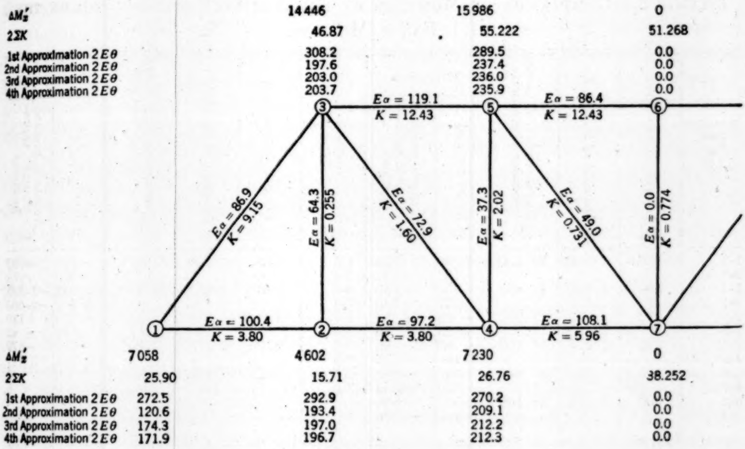
ΔM'₂
2ΣK
1st Approximation 2Eθ
2nd Approximation 2Eθ
3rd Approximation 2Eθ
4th Approximation 2Eθ
14446
46.87
308.2
197.6
203.0
203.7
15986
55.222
289.5
237.4
236.0
235.9
0
51.268
0.0
0.0
0.0
0.0
Eα = 119.1
K = 12.43
Eα = 86.4
K = 12.43
3
5
6
Eα = 86.9
K = 9.15
Eα = 64.3
K = 0.255
Eα = 72.9
K = 1.60
Eα = 37.3
K = 2.02
Eα = 49.0
K = 0.731
Eα = 0.0
K = 0.774
1
2
4
7
Eα = 100.4
K = 3.80
Eα = 97.2
K = 3.80
Eα = 108.1
K = 5 96
ΔM'₂
2ΣK
1st Approximation 2Eθ
2nd Approximation 2Eθ
3rd Approximation 2Eθ
4th Approximation 2Eθ
7058
25.90
272.5
120.6
174.3
171.9
4602
15.71
292.9
193.4
197.0
196.7
7230
26.76
270.2
209.1
212.2
212.3
0
38.252
0.0
0.0
0.0
0.0

FIG. 14

approximate $2 E \theta$-values are shown in Fig. 14. The second approximate value of $2 E \theta$ for Joint 1 $= 272.5 - \dfrac{1}{25.90}\,(9.15 \times 308.2 + 3.80 \times 292.9) = 120.6$. The second approximate value of $2 E \theta$ for Joint 2 $= 292.9 - \dfrac{1}{15.71} \times (3.80 \times 120.6 + 0.255 \times 308.2 + 3.8 \times 270.2) = 193.4$. Proceeding in this manner from joint to joint, in the order numbered in Fig. 14, all the second approximate values of $2 E \theta$ are calculated and are shown in the diagram.

The third approximate value of $2 E \theta$ at Joint 1 $= 272.5 - \dfrac{1}{25.90}\,(9.15 \times 197.6 + 3.80 \times 193.4) = 174.3$. Thus, all the third approximate values of $2 E \theta$ are calculated (see Fig. 14). The fourth approximate values are then determined and as these values of $2 E \theta$ at all the joints agree fairly closely with the third approximate values, there is no need to continue the operations. Values of $E \alpha$ can be computed by drawing a Williot diagram, then, applying the well known slope-deflection formula:

$$M_{AB} = 2 E K_{AB}\,(2 \theta_A + \theta_B - 3\,\alpha_{AB}) \ldots\ldots\ldots\ldots\ldots (56)$$

the moments at the end of all the members can be solved. Equation (56) will give the moment at End *A* for any member, *A-B*. In this case, the sign convention is the same as that recommended by W. M. Wilson, F. E. Richart, and Camillo Weiss, Members, Am. Soc. C. E.[7] The moments at the ends of all the members can also be solved by the formula:

$$M_{AB} = 2\,E\,K_{AB}\,(2\,w_A + w_B) \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

TABLE 2.—COMPARISON OF MOMENTS BY THE AUTHOR'S METHOD AND BY THE EXACT METHOD

Member (see Fig. 14) (1)	Length, *L*, in inches (2)	Moment of inertia, *I* (3)	Displacement, *D*, times modulus of elasticity (4)	MOMENTS		Member (see Fig. 14) (1)	Length, *L*, in inches (2)	Moment of inertia, *I* (3)	Displacement, *D*, times modulus of elasticity (4)	MOMENTS	
				Exact method (5)	Author's method (fourth approximation) (6)					Exact method (5)	Author's method (fourth approximation) (6)
1-2.....	320	1 218	32 130	+230	−235	4-5....	372	750	13 870	−875	+882
2-1.....	.....	.....	....	+185	−141	5-4....	.....	.....	....	−924	+930
1-3.....	490.7	4 490	42 630	−230	+239	4-7....	320	1 907	34 600	+1 342	−1 335
3-1.....	.....	.....	....	−527	+530	7-4....	.....	.....	....	+2 602	−2 600
2-3.....	372	95	23 900	− 53	+ 54	5-7....	490.7	358	24 040	−130	+130
3-2.....	.....	.....	....	− 55	+ 56	7-5....	.....	.....	....	+42	− 42
2-4.....	320	1 218	31 100	− 80	+ 86	5-6....	320	3 978	27 650	+562	−579
4-2.....	.....	.....	....	−141	+145	6-5....	.....	.....	....	+3 544	−3 511
3-4.....	490.7	805	35 780[9]	−307	+292	6-7....	372	288	0	0	0
4-3.....	.....	.....	....	−320	+305	7-6....	.....	.....	....	0	0
3-5.....	320	3 978	38 100	+900	−886						
5-3.....	.....	.....	....	+495	−486						

From the values given in Fig. 14, all the moments at the end of each member can be calculated and the results are given in Table 2. (In their Williot diagram[8], Messrs. Thompson and Cutler give a value of the displacement of 38 470 for Member 3–4. For a fixed-end moment of 700 as shown[9] the correct value would be 35 780, which is the value used by the writer.)

For purposes of comparison the values of the end moments by an exact method[10] is also given in Table 2 and on comparing these values with those computed by Professor Schwalbe it is seen that the moments by the author's method agree very closely. The signs shown are different for each member for the two methods, due to a different sign convention assumed. Furthermore, it should be made clear that, in Column (4), Table 2, the values of *D* times *E* are displacements (at right angles to the axis of the member) of one end relative to the opposite end.

From the foregoing it seems that the author's method can be used to solve for the end moments of all the members in a triangular truss with rigid joints in a very short and simple manner. The analysis is such that the results with a 20-in. slide-rule are reliable.

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[7] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method", *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[8] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 52 and Table 17.

[9] *Loc. cit.*, Fig. 53.

[10] "Modern Framed Structures" by Messrs. Johnson, Bryan, and Turneaure, p. 440.

M. B. GAMET,[11] JUN. AM. SOC. C. E. (by letter).—An interesting terminology to apply to methods of solution for, and types of, equations is developed in this paper. The writer wishes to commend highly the use of the term, "iteration", as a substitute for "successive approximation."

The writer wishes to make some very important additions to the author's "List of References" in Appendix I of the paper. These additions[12] represent the pioneer work done in the United States with this so-called "iteration" procedure in the solution of simultaneous equations of the type discussed by the author.

It will be noted that in Appendix I the author has listed no references to any of the numerous American developments in this field which occurred prior to 1930. All the references which he cites refer to German sources prior to 1930, whereas the method was well developed in the United States by that time.

Another important change in terminology is the use of the expressions, the "Equation of Three Angles", the "Equation of Five Angles: No Side-Sway", and the "Equation of Five Angles with Side-Sway", as a series of substitutes for the term, "slope-deflection" equations. Evidently, this terminology originates with the author's reference to the work of W. Gehler (3)[a] published in Germany in 1925.

There seems to be a continual need to remind recent students of the use of angular joint rotations as unknowns in rigid-frame problems, of the facts in the history of the development of this useful method of approach. As early as 1880, Manderla made use of angular rotations of joints in a rigid frame structure to determine the secondary-stress bending moments in a bridge truss. Soon after this (1893) Mohr used a similar approach which was identical with that which the slope-deflection method makes to the "general" problem of the rigid frame. Before 1915, only the isolated problem of secondary stresses had been approached in this manner.

In 1915, G. A. Maney, M. Am. Soc. C. E. published the first general statement of the slope-deflection method in this country[13]. In this treatment, not only are secondary stresses solved, but wind stresses and all types of frames with members transversely loaded as well. The idea of fixed-beam moments was there first developed, although at the time it was not called by that name. This work led immediately to the important publications originating at the University of Illinois.

In 1914, Professor Maney and W. M. Wilson, M. Am. Soc. C. E., developed, and in June, 1915, published[14] the solution of wind stresses in a twenty-story office building by the slope-deflection method. This solution was by the exact simultaneous method. In 1918, Professor Wilson and F. E. Richart, and C. Weiss, Members, Am. Soc. C. E., published a bulletin[15] giving general formulas for a large number of types of structures by the slope-deflection method. These two publications, following Professor Maney's publication and using his methods, gave the "angular unknowns" approach to rigid frame and continuous beam solutions for bending moment distribution the great popularity it has had in the United States.

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[11] Instr. in Civ. Eng., School of Eng., Northwestern Univ., Evanston, Ill.

[12] "Statically Indeterminate Stresses", by John I. Parcel, and George A. Maney, Members, Am. Soc. C. E., Wiley & Sons, 1936, Fig. 142 and footnote, p. 276; "Investigation of Secondary Stresses in the Kenova Bridge," by John I. Parcel and George A. Maney, Members, Am. Soc. C. E., *Studies in Engineering, No. 4,* Univ. of Minnesota, 1922; and by G. A. Maney, M. Am. Soc. C. E., *Studies in Engineering, No. 1,* Univ. of Minnesota, 1915, p. 17.

[13] *Studies in Engineering, No. 1,* Univ. of Minnesota, 1915.

[14] "Wind Stresses in Steel Frames of Office Buildings", by William M. Wilson, and George A. Maney, *Bulletin No. 80,* Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1915.

It should be emphasized that this is a typically American development and that the "general" use of the method of "angle unknowns" has lagged considerably in Europe. (This statement excepts only the problem of secondary stresses which was mentioned by Professor Maney[14].) The European approach to the general case of the rigid frame by the "unknown angles" method is outlined as follows: In May, 1914, Axel Bendixsen[16] published, in Denmark, a monograph which gives the first general statement of the slope-deflection relation for any type of loading. This publication appears to have been generally overlooked even in Europe until about 1923 when Professor A. Ostenfeld amplified it somewhat. In 1923, Professor Ostenfeld published[17] an amplification of Bendixsen's work, the presentation covering all cases for deformations rather than stresses.

An unusual development has followed Professor Maney's general statement of the use of angles as unknowns. Numerous ideas such as moment distribution, and the balancing of angle changes, have followed in its wake and they have all been strictly American variations of the "slope-deflection" theme.

Although this general idea of treating angles as unknowns was first presented in the United States by Professor Maney in 1915, it has since almost completely replaced the "least work" and "virtual work" approach to the problem. In Europe, these latter methods are still much used, in spite of their greater mathematically cumbersome nature.

In the following development some of the important equations and methods referred to by the author may be traced.

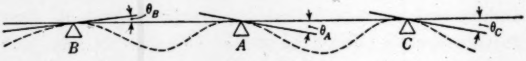

FIG. 15

Item (1).—In Fig. 15,

$$M_{AB} = \pm M_{F(AB)} + 2E K_{AB} (3 R_{AB} - 2 \theta_A - \theta_B) \dots\dots\dots (58)$$

is the general form of the slope-deflection equation stated by Professor Maney (13) in 1913 If two bending moments are equal and opposite in sign, as occur at each support of a continuous beam: $M_{AB} + M_{AC} = 0$; and, when

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[15] "Analyses of Statically Indeterminate Structures by the Slope-Deflection Method", by William M. Wilson, F. E. Richart, and Camillo Weiss, *Bulletin No. 2*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1918.

[16] "Die Methode der Alpha-Gleichungen zur Berechnung von Rahmenkonstructionen", von Axel Bendixsen, May, 1914.

[17] "Die Deformationsmethode", by A. Ostenfeld, *Der Bauingenieur*, January 31, 1923.

$R = 0$,

$$M_{AB} = \pm M_{F(AB)} - 2EK_{AB}\,(2\theta_A + \theta_B) \ldots\ldots\ldots\ldots (59)$$

and,

$$M_{AC} = \pm M_{F(AC)} - 2EK_{AC}\,(2\theta_A + \theta_C) \ldots\ldots\ldots\ldots (60)$$

Therefore,

$$K_{AB}\,\theta_B + 2\,(K_{AB} + K_{AC})\,\theta_A + 2\,K_{AC}\,\theta_C = \frac{\pm M_{F(AC)} \pm M_{F(AB)}}{2\,E} \ldots (61)$$

It will be seen that Equation (61) is identical with Equation (28) of the paper and is merely the slope-deflection "joint" equation for continuous beams. This is the equation which Gehler (3) called the "equation of three angles" in 1925, and has led to the author's use of this terminology. Similarly, the "equation of five angles" is merely a slope-deflection joint equation for the typical case of a rigid frame with four members framing into a rigid joint, instead of two, as in the preceding case of a continuous beam.

Item (2).—When the factor, $2E$, is cancelled and θ_B and θ_C are assumed to be zero temporarily, then Equation (61) becomes: $\theta_A = \dfrac{\pm M_{F(AC)} \pm M_{F(AB)}}{2\,(K_{AC} + K_{AB})}$; and, in general, $\theta_A = \dfrac{\Sigma M_{F(A)}}{2\,\Sigma K_A}$ if there are two or more members at the joint, and $\Sigma M_{F(A)} =$ the algebraic sum of M_F-values at Joint A whereas $\Sigma K_A =$ the numerical sum of the K-values $\left(K = \dfrac{I}{L}\right)$ around Joint A.

Similarly, $\theta_B = \dfrac{\Sigma M_{F(B)}}{2\,\Sigma K_B}$. If these expressions for θ_A and θ_B are substituted in Equation (61):

$$M_{AB} = \pm M_{F(AB)} - \frac{\Sigma M_{F(A)}}{\Sigma K_A} - \frac{1}{2}\left(\frac{\Sigma M_{F(B)}}{\Sigma K_B}\right) \ldots\ldots\ldots (62)$$

Equation (62) is simply the "moment distribution" equation in which $\dfrac{\Sigma M_{F(A)}}{\Sigma K_A}$ is the "distribution" factor and $\dfrac{1}{2}\left(\dfrac{\Sigma M_{F(B)}}{\Sigma K_B}\right)$ is the "carry-over" factor. By this variant of the slope-deflection method, the "iteration" method is applied through this "carry-over" factor until error is negligible.

Conclusion.—It is important to remember in connection with the entire general plan of solution by means of "angle unknowns", that the M_F-value, or the "fixed-beam" moment is the essential starting point, and that until the slope-deflection method was evolved in 1913, no one had previously used this approach. In connection with the problem of secondary stress, this conception was not involved.

Why do all equations of the type discussed by the author invariably converge when the so-called "iteration" process or "converging approximations" is used? The answer is found to be a purely physical reason which will only be confounded by any mathematical approach. If Equation (59) is examined it reveals that the angle change, θ_A, at the joint in question, has twice as much influence on the final moment, M_{AB}, as the angle change, θ_B, at the far end from the joint in question. Furthermore, the last term of Equation (62), or the "carry-over" factor, reveals the same thing. When an error at any joint adjacent to the one for which the angle or moment is desired is always at least twice the error caused at the joint in question, rapid convergence by halving of the error at each iteration is assured.

The simple physical facts make convergence, or solution of equations by iteration, possible. If the quantitative importance of angle changes at each end of a member were nearly the same in affecting the moments at both ends, then "successive convergence" or "iteration" would have no place as a method of solution for moments in continuous-beam and rigid-frame members.

MARVIN A. GRAY,[18] JUN. AM. SOC. C. E. (by letter).—Equation (1) of this paper is a conception of the theorem of three moments which, in itself, requires additional explanation for teaching; in a typically popular book of applied mechanics the theorem of three moments is given nine times (probably many more times in other texts and reference books) as:

$$M_1 L_1 + 2 M_2 (L_1 + L_2) + M_3 L_2 = -\frac{1}{4} w_1 L^3_1 - \frac{1}{4} w_2 L^3_2 \quad \ldots\ldots(63)$$

The symbol, Q, is used in only one special problem, whereas there are forty to fifty problems in which the last mentioned form (Equation (63)) is used; and U_x is not mentioned in the entire book. It might be of interest to note that this particular problem of moments, in which Q was used, is not even mentioned in this paper. The development of the author's expression of the three-moment theorem comes from the following algebraic maneuvers:

$$M_1 L_1 + 2 M_2 (L_1 + L_2) + M_3 L_2 = -\frac{1}{4} w_1 L^3_1 - \frac{1}{4} w_2 L^3_2 \quad \ldots(64a)$$

and,

$$M_1 \frac{L_1}{I_1} + 2 M_2 \left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_3 \frac{L_2}{I_2} = -\frac{1}{4} w_1 \frac{L^3_1}{I_1} - \frac{1}{4} w_2 \frac{L^3_2}{I_2} \quad \ldots(64b)$$

Then,

$$\frac{M_1}{K_1} + 2 M_2 \left(\frac{1}{K_1} + \frac{1}{K_2}\right) + \frac{M^3}{K_3} = -\frac{1}{4} w_1 \frac{L^2_1}{K_1} - \frac{1}{4} w_2 \frac{L^2_2}{K_3} \quad \ldots(64c)$$

with the addition of symbols for symbols and changing from common notation to that suggested by the author, the reader will arrive at Equation (1)—and the student will be confused.

Equation (8) is also of interest because it too adds to the student's problem. In school before the theorem of three moments is taught, the problem is solved by the calculus. Then, because this does not seem to be sufficient explanation, it is again derived by the moment-area method; and again by several variations, or very slight modifications, of moment areas. However, there is a general formula for finding moments, rotations, and movement of beams; this equation (derived from a knowledge of moment areas) is known as the slope deflection equation: $M_{AB} = M_{F\text{-}AB} - \frac{2\,E\,I}{L}\,(2\,\theta_A + \theta_B)$, or,

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[18] Chicago, Ill.

$$M_{AB} = +\,M_{F-AB} - \frac{2\,E\,I}{L}\,(2\,\theta_A + \theta_B - 3\,R) \quad \text{.........}(65)$$

Using standard notation (as in the three-moment theorem), the writer obtains the two equations:

$$M_{2-1} = -\,M_{F-2-1} - K_1\,(2\,\theta_2 + \theta_1) \quad \text{..............}(66a)$$

and,

$$M_{2-3} = M_{F-2-3} - K_2\,(2\,\theta_2 + \theta_3) \quad \text{................}(66b)$$

but $-\,M_{2\text{-}1} = M_2 = M_{2\text{-}3}$; so that:

$$+\,M_{F-2-1} + K_1\,(2\,\theta_2 + \theta_1) = M_{F-2-3} - K_2\,(2\,\theta_2 + \theta_3) \quad \text{.......}(67)$$

Solving Equation (67) by elementary algebra, the writer derives an equation for a given θ-value:

$$\theta_2 = \frac{M_{F-2-1} - M_{F-2-3} - (K_1\,\theta_1 + K_2\,\theta_3)}{2\,(K_1 + K_2)} \quad \text{............}(68)$$

which is as exact as any elastic beam theory used. If it does nothing else, this method at least omits many new and varied symbols that are confusing not only to the student but to the practicing engineer.

The writer omits the author's applications of slope deflection as presented in his paper, and calls attention to Equations (22), (23), (24), and (25). By using the slope deflection equation,

$$M_{AB} = M_{F-AB} - K\,(2\,\theta_A + \theta_B) \quad \text{................}(69)$$

and correcting for the author's conception of signs from the simpler convention of slope deflection, the writer obtains from Fig. 7:

$$\theta_A = (\theta_A)_0 + \frac{M_A\,L}{3\,E\,I} + \frac{M_B\,L}{6\,E\,I} \quad \text{................}(70a)$$

$$\theta_B = -\,(\theta_B)_0 - \frac{M_B\,L}{3\,E\,I} - \frac{M_A\,L}{6\,E\,I} \quad \text{...............}(70b)$$

$$M_{AB} = M'_A + 2\,E\,K\,(2\,\theta_A + \theta_B) \quad \text{................}(70c)$$

and,

$$M_{BA} = M'_B - 2\,E\,K\,(2\,\theta_B + \theta_A) \quad \text{................}(70d)$$

A little time spent in following the procedure (or algebra) necessary to arrive at the formulas mentioned, should convince the student or engineer that no "take off" on moment areas or variation of the slope deflection method could be as simple and practical as the slope deflection method itself. Certainly, the writer has proved his point that, fundamentally, this paper is an expression of slope deflection confused by new symbols; the same slope deflection, that is given in the paper (in general, Equation (65)), was originally presented[19] by G. A. Maney, M. Am. Soc. C. E.

The author would create a boon for students if he would take slope deflection and use it with its accepted convention in his teaching practice. The writer has shown that the background for slope deflection, and probably its derivation, is already taught; it seems that the use of Equation (65) with algebra or arithmetic (as is common in engineering) brings a widening of the structural field and applied mechanics in the accurate commercial solution of problems. In aeronautics alone slope deflection should find many uses because of its accuracy and speed; it also lends itself easily to rapidly converging approximations; and with the coming of aeronautical materials into the every-day structural field, the method is indispensable to the Engineering Profession. The author has started on the solution of a very important and useful problem, however, which should end in a symposium on the slope deflection method with its wider use in to-day's engineering, for students and for reference.

JOHN E. GOLDBERG,[20] JUN. AM. SOC. C. E. (by letter).—Discussion of Professor Schwalbe's paper affords an opportunity to emphasize several significant facts in the development of modern American slope-deflection practice:

(1) As early as 1913, Allston Dana, M. Am. Soc. C. E., used the method of iteration to obtain a slope deflection analysis of secondary stresses in the Kenova Bridge.

(2) In 1915, G. A. Maney, M. Am. Soc. C. E., proposed[21] the present general form of the slope deflection equation and demonstrated its application to a wide variety of rigid frame problems. Previous to this time, the slope deflection method existed solely as a means of analyzing secondary stresses in trusses. Professor Maney introduced the fixed-beam moment factor which makes possible the analysis of stresses in loaded frames and which forms the basis of the present simple and popular methods for the analysis of rigid or continuous frames. With the exception of the secondary stress problem, solution of the simultaneous equations was effected by algebraic methods.

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[19] *Bulletin No. 1, Eng. Studies*, Univ. of Minnesota, March, 1915.

[20] With Dept. of Buildings, Chicago, Ill.

[21] See "Secondary Stresses and Other Problems in Rigid Frames", by G. A. Maney, *Engineering Studies No. 1*, Univ. of Minnesota, 1915; see, also, "Wind Stresses in the Steel Frames of Office Buildings", by W. M. Wilson and G. A. Maney, *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, 1915. (See footnote at bottom of page 1, *Bulletin No. 80*, on credit for the theory.)

(3) In 1931, the writer presented[22] a practicable method, based upon slope deflection and successive approximations, for the analysis of rigid building frames and similar structures under the action of gravity loads, largely developed by Professor Maney directly from the original slope deflection equations.

(4) In 1933, several years after its development, the writer presented[23] the method of wind-stress analysis of slope deflection and converging approximations which he developed directly from the basic theory.

A. FLORIS,[24] ESQ. (by letter).—This illuminating paper will undoubtedly bring to the attention of engineers in practice the great importance of iteration in the analysis of statically indeterminate structures. This method of solving simultaneous equations of statics can be applied equally well to other problems than those treated by the author.

The use of converging approximations for determining the values of the end moments of beams and columns in the moment distribution method is, in reality, an iteration process. Furthermore, the treatment, by this method, of problems involving side-sway yields simultaneous equations the number of which is equal to the number of stories. These equations can also be solved by iteration[25].

The author treats the problem of side-sway in rigid frames by applying the equations of five angles. In these equations the unknown angles are determined by iteration. The writer's practice[25] in using the moment distribution method in case of side-sway is, first, to determine the coefficients of proportionality in the equilibrium equations by iteration. The values of the moments are then found by multiplying the distributed moments by these coefficients. In every case of lateral forces or unsymmetrical loads there are obtained simultaneous equations in which the coefficients of the diagonal falling from left to right are considerably greater than the remaining coefficients. In this case, the calculated values approach the true values by repeated approximations. In other words, the equations converge. Contemporary contributions to this important criterion of convergence are: Runge, (2)[3], and Mises and Pollaczek-Geiringer (4) whose work has been cited by the author, and Helmut Wittmeyer[26].

W. L. SCHWALBE,[27] ESQ. (by letter).—The procedure for solving *n* simultaneous equations of a general nature by successive approximations has been outlined by Mr. Drummond. The iteration procedure in the paper is more useful for those special systems that can be expressed by means of single equations, the single equation stating precisely what the iteration procedure must be for the solution of the n equations.

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[22] "Vertical Load Analysis of Rigid Building Frames", by John E. Goldberg, *Engineering News-Record*, November 12, 1931.

[23] "Wind Stresses by Slope Deflection and Converging Approximations", by John E. Goldberg, *Transactions*, Am. Soc. C. E. Vol. 99 (1934), p. 962.

[24] Dipl.-Ing., Los Angeles, Calif.

[25] "Analysis by Moment Distribution Aided Through Use of Iteration", by A. Floris, *Engineering News-Record*, June 25, 1936, p. 922.

[26] "Ueber die Lösung von linearen Gleichungs-systemen durch Iteration", von Helmut Wittmeyer, *Zeitschrift für angewandte Mathematik und Mechanik*, October 1936, p. 301.

[27] Asst. Prof. of Theoretical and Applied Mechanics, Univ. of Illinois, Urbana, Ill.

The example given by Mr. Fischer is a useful one for making a direct comparison of values computed by his exact method, by the procedure in the paper, and by the moment-distribution method according to Messrs. Thompson and Cutler.[6]

The development of "slope-deflection" methods in the United States proceeded essentially as pointed out by Messrs. Gamet and Goldberg. An additional reference may be cited, namely, the work of F. E. Richart, M. Am. Soc. C. E.[28]

In Germany, formulas of the type of Equations (22) and (23), Appendix II, were used by F. Lippich,[29] in the derivation of the three-moment equation. His equations included terms due to settlement of supports. F. Grashof[30] used formulas of the type of Equations (24) and (25) in deriving the three-moment equation for level supports. In his equations the load factor was not expressed in terms of fixed-end moments.

The writer would extend Mr. Gray's suggestion for a "symposium" to include not only "slope deflection," but all the theories of statically indeterminate analysis, with a glossary of symbols and terminology and a bibliography. In this connection the brief review and history, by H. M. Westergaard, M. Am. Soc. C. E.,[31] of the four general theories would serve as a basis.

The purpose of the paper (as stated in the "Synopsis") was not to present a new theory of statically indeterminate beams and frames, but to demonstrate the method of iteration in the solution of certain types of sets of simultaneous equations which occur in the application of theories already in existence. Consequently, the list of references is not a "who's who" in any particular theory.

It seemed desirable to extend the terminology which was begun by the introduction of the now well-established term, "three moment," to other equations of a similar character; hence, the use of the terms, "three angle" and "five angle" (3)[3]. To call equations of this type "slope deflection" equations, is a misnomer since "slope deflection" formulas are originally of the type of Equations (22) and (23). The latter are only a part of the story of statically indeterminate analysis. Virtual work or continuity and equilibrium conditions are also necessary to determine the final relationships.

The rapidly growing literature in the field of structural analysis gives one the impression of "schools of thought." Perhaps an increasing number of such esoteric sects is necessary for the growth of a subject but, surely, for a beginner the learning of fundamental principles underlying any later ritual is most important.

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[28] "Statistically Indeterminate Stresses in Stiff Framed Structures," by F. E. Richart, presented to the Univ. of Illinois, 1915, in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

[29] "Theorie des Kontinuirlichen Trägers Konstanter Querschnitte," *Allgemeine Bauzeitung*, Vol. 36 (1871), p. 104.

[30] *Zeitschrift*, Vereines Deutscher Ingenieure, Vol. 3 (1859), p. 155.

[31] "One Hundred Fifty Years Advance in Structural Analysis," *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 226.

The importance of iteration in analysis has been emphasized by Mr. Floris. This procedure is especially useful when the statically indeterminate unknowns are selected in such a way that the set of simultaneous equations can be represented by a single equation in the form of a difference equation or recursion formula. Any unknown is thus expressed in terms of its neighbors. With known boundary conditions and convergence, the unknowns can then be computed by starting with any set of arbitrary values for the unknowns and correcting each value step by step by means of the recursion formula until all the values become fixed.

In the theory of elasticity, a differential equation can always be replaced by a difference equation which represents a set of algebraic equations. According to R. D. Carmichael[32]: "A differential equation, with or without boundary conditions, may be realized in an infinite number of ways as the limiting form of an algebraic system, so that there is always room for choice in setting up the system, and in fact need for care that it shall be done in a convenient way." The idea thus expressed may well serve as a guide for approximation methods in many problems for which integral solutions are impossible. The algebraic equations may be regarded as defining a statically indeterminate net or framework, the difference equation serving as a bridge from one field to the other.

Iteration as a method for the determination of roots of algebraic equations had its beginnings in 1674.[33] The earliest applications to the solution of problems in elasticity were probably those of C. Runge[34] and L. F. Richardson,[35] Since then the use of the method has been considerable.

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[32] "Algebraic Guides to Transcendental Problems," *Bulletin*, Am. Math. Soc., Vol. 28 (1922), p. 184.

[33] "The Calculus of Observations." by Whittaker and Robinson, 1924.

[34] "Ueber eine Methode die partielle Differentialgleichung, $\Delta U =$ constant, numerisch zu integrieren," *Zeitschrift für Math. u. Physik*, Vol. 56 (1908), p. 225.

[35] "The Approximate Arithmetical Solutions by Finite Differences of Physical Problems Involving Differential Equations, with an Application to Stresses in a Masonry Dam," *Philosophical Transactions*, London, Vol. 210A (1910), p. 307.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1975

SIMPLIFIED METHOD OF DETERMINING TRUE BEARINGS OF A LINE

By Philip L. Inch,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Earl F. Church, Paul E. Wylie, James B. Goodwin, C. H. Swick, Philip Kissam, George D. Whitmore, O. H. Chilton, Chalmers C. Schrontz, Frank M. Johnson, Walter H. Starkweather, C. I. Day, F. L. McRee, F. J. Duarte, Leonard C. Jordan, J. C. Pinney, R. L. Vaughn, John C. Penn, Robert H. Merrill, Crosby J. Wilkin, C. S. Jarvis, and Philip L. Inch.

Synopsis

A simplified method of computing a direct observation of the sun, for azimuth, is offered in this paper. It involves reference to a comprehensive table[1a] specially prepared for this purpose. The objects of the table are:

(1) To place the determination of a meridian within the attainment of engineers, unprepared, by reason of the pressure of their usual work;

(2) To obviate textbook reference, at the time a meridian is necessary;

(3) To displace the inaccuracy of compass meridians in field conditions where the compass is affected by local attraction;

(4) To reduce the time necessaary to compute a direct solar observation; and,

(5) To reduce trigonometric formulas to simple arithmetic.

Theory

Problems involving the determinations of true bearings may be solved by applying the formula,[2]

$$\cos Z = \frac{\sin \delta}{\cos h \cos \phi} - \tan h \tan \phi \dots\dots\dots\dots\dots (1)$$

Note.—Published in September, 1936, *Proceedings*.

[1] U. S. Cadastral Engr., Public Survey Office, Reno, Nev.

[1a] See Table 9 of the closing discussion, p. 1012.

[2] "Surveying Theory and Practise", by R. E. Davis, F. S. Foote, and W. H. Rayner, Members, Am. Soc. C. E., McGraw-Hill Book Co., 1928, p. 425.

TABLE 1.—VALUES OF FACTORS A AND B, FOR A GIVEN ALTITUDE OF THE SUN, AT A GIVEN LATITUDE ON THE EARTH

Vertical angle, h	Symbol	LATITUDE, $\phi = 37°$			LATITUDE, $\phi = 38°$			LATITUDE, $\phi = 39°$			LATITUDE, $\phi = 40°$			LATITUDE, $\phi = 41°$			LATITUDE, $\phi = 42°$		
		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′	
(1)	(2)	(3)	h (4)	ϕ (5)	(6)	h (7)	ϕ (8)	(9)	h (10)	ϕ (11)	(12)	h (13)	ϕ (14)	(15)	h (16)	ϕ (17)	(18)	h (19)	ϕ (20)
21°.......	A	1.34085	15.5	30.1	1.35893	15.7	31.7	1.37793	15.9	33.3	1.39790	16.1	35.0	1.41889	16.3	36.8	1.44097	16.6	38.7
	B	0.28866	25.3	17.7	0.29929	26.3	18.2	0.31020	27.3	18.7	0.32143	28.3	19.3	0.33300	29.3	19.9	0.34492	30.3	20.5
22°.......	A	1.35012	16.3	30.3	1.36832	16.6	31.9	1.38745	16.8	33.5	1.40756	17.0	35.2	1.42869	17.3	37.1	1.45093	17.6	39.0
	B	0.30388	25.7	18.7	0.31507	26.7	19.2	0.32656	27.6	19.7	0.33838	28.7	20.3	0.35056	29.7	20.9	0.36310	30.7	21.6
23°.......	A	1.35992	17.3	30.6	1.37826	17.5	32.1	1.39752	17.8	33.8	1.41778	18.0	35.5	1.43907	18.3	37.3	1.46146	18.6	39.3
	B	0.31932	26.1	19.6	0.33106	27.2	20.2	0.34314	28.0	20.7	0.35557	29.1	21.3	0.36836	30.1	22.0	0.38154	31.2	22.7
24°.......	A	1.37028	18.3	30.8	1.38875	18.6	32.4	1.40817	18.8	34.0	1.42858	19.1	35.7	1.45002	19.4	37.6	1.47259	19.7	39.6
	B	0.33497	26.5	20.6	0.34730	27.5	21.1	0.35996	28.5	21.7	0.37300	29.4	22.4	0.38642	30.6	23.1	0.40025	31.7	23.8
25°.......	A	1.38122	19.2	31.0	1.39989	19.4	32.6	1.41946	19.7	34.4	1.44004	20.0	36.0	1.46166	20.3	37.9	1.48440	20.6	39.9
	B	0.35087	26.9	21.5	0.36378	27.9	22.1	0.37705	28.9	22.8	0.39070	30.0	23.4	0.40476	31.1	24.1	0.41924	32.2	24.9
26°.......	A	1.39277	20.3	31.3	1.41154	20.6	32.9	1.43128	20.9	34.6	1.45202	21.2	36.3	1.47382	21.5	38.2	1.49676	21.8	40.2
	B	0.36702	27.4	22.5	0.38054	28.4	23.1	0.39441	29.5	23.8	0.40869	30.5	24.5	0.42340	31.6	25.3	0.43855	32.8	26.1
27°.......	A	1.40495	21.4	31.6	1.42389	21.7	33.2	1.44380	22.0	34.9	1.46472	22.3	36.7	1.48671	22.6	38.6	1.50985	23.0	40.6
	B	0.38347	27.9	23.5	0.39758	28.9	24.2	0.41208	30.0	24.9	0.42700	31.1	25.6	0.44236	32.2	26.4	0.45820	33.3	27.2
28°.......	A	1.41777	22.5	31.9	1.43689	22.6	33.5	1.45697	23.1	35.2	1.47809	23.4	37.0	1.50028	23.8	38.9	1.52363	24.2	41.0
	B	0.40020	28.4	24.6	0.41493	29.5	25.2	0.43006	30.5	26.0	0.44563	31.6	26.7	0.46166	32.8	27.5	0.47818	34.0	28.4
29°.......	A	1.43126	23.7	32.2	1.45055	24.0	33.8	1.47083	24.4	35.5	1.49215	24.7	37.3	1.51455	25.1	39.3	1.53812	25.5	41.4
	B	0.41724	29.0	25.6	0.43260	30.0	26.3	0.44838	31.1	27.1	0.46461	32.2	27.9	0.48133	33.4	28.7	0.49855	34.6	29.7
30°.......	A	1.44547	24.9	32.5	1.46496	25.2	34.1	1.48544	25.4	35.9	1.50697	26.0	37.7	1.52959	26.4	39.7	1.55340	26.8	41.8
	B	0.43461	29.5	26.7	0.45061	30.6	27.4	0.46704	31.8	28.1	0.48395	32.9	29.0	0.50136	34.1	29.9	0.51931	35.3	30.9
31°.......	A	1.46041	26.2	32.8	1.48010	26.5	34.5	1.50079	26.9	36.3	1.52254	27.3	38.1	1.54540	27.7	40.1	1.56945	28.2	42.2
	B	0.45233	30.1	27.8	0.46898	31.3	28.5	0.48609	32.4	29.3	0.50369	33.6	30.2	0.52181	34.8	31.1	0.54048	36.1	32.1
32°.......	A	1.47612	27.5	33.2	1.49602	27.5	34.9	1.51694	28.2	36.6	1.53892	28.7	38.5	1.56203	29.1	40.5	1.58634	29.5	42.7
	B	0.47043	30.8	28.9	0.48775	32.0	29.7	0.50554	33.1	30.5	0.52384	34.3	31.4	0.54269	35.6	32.4	0.56211	36.9	33.4
33°.......	A	1.49261	28.9	33.6	1.51274	29.3	35.2	1.53388	29.7	37.1	1.55611	30.2	39.0	1.57948	30.6	41.0	1.60406	31.1	43.1
	B	0.48893	31.2	30.0	0.50693	32.7	30.8	0.52542	33.9	31.7	0.54444	35.1	32.7	0.56403	36.4	33.7	0.58422	37.7	34.7
34°.......	A	1.50997	30.2	33.9	1.53032	30.7	35.7	1.55172	31.1	37.5	1.57421	31.5	39.4	1.59784	32.0	41.5	1.62271	32.5	43.6
	B	0.50786	32.3	31.2	0.52656	33.5	32.0	0.54576	34.7	32.9	0.56552	35.9	33.9	0.58587	37.2	34.9	0.60683	38.6	36.1
35°.......	A	1.52811	32.0	34.3	1.54871	32.5	36.1	1.57036	32.9	37.9	1.59312	33.4	39.9	1.61704	33.9	42.0	1.64221	34.4	44.1
	B	0.52723	33.1	32.4	0.54664	34.3	33.2	0.56657	35.6	34.2	0.58708	36.9	35.2	0.60821	38.2	36.3	0.62997	39.5	37.5

in which Z = the azimuth of the sun; δ = the sun's declination; h = its altitude; and ϕ = the latitude of the point of observation. Let $\frac{1}{\cos h \cos \phi} = A$; and, $\tan h \tan \phi = B$; then, Equation (1) becomes, simply,

$$\cos Z = A \sin \delta - B \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The writer has developed a complete table[1a], of which Table 1 is a very condensed form, intended only for the solution of the illustrative example in the paper.

FIELD PROCEDURE

The field party will need a transit with a full vertical circle, an ephemeris of the sun (obtainable from the Superintendent of Public Documents, United States Printing Office, or from most dealers in engineering instruments), a table of sines and cosines, and the latitude of the point of observation, as scaled from an accurate map.

Example 1.—To find the true bearing of Line XY (Fig. 1): (1) Set the transit firmly at Point X; (2) set the horizontal graduated plates at 0°; (3) level the instrument with extreme care; (4) sight on Point Y; (5) clamp the lower plate; and (6) loosen the upper plate. Using a solar prism or colored glass as a sunshade on the telescope, place the vertical and center horizontal cross-hairs, tangent, respectively, to the right and lower limbs of the sun as shown in Fig. 2(a). The observations are recorded as shown in Table 2, Item No. 1. Reverse the instrument, place the vertical and center horizontal cross-hairs tangent to the sun's left and upper limbs, respectively, and, as before, record the time, the vertical angle, and the horizontal angle, as shown in Table 2, Item No. 2. The mean of the recorded quantities (Item No. 3, Table 2) gives the time when the center of the of the sun has a certain elevation above the earth's horizon and is at a certain horizontal angular distance from Line XY.

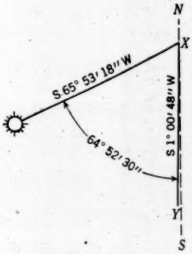

FIG. 1.

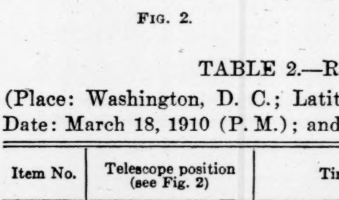

FIG. 2.

TABLE 2.—RECORD OF OBSERVATIONS

(Place: Washington, D. C.; Latitude: 38° 53′ 40″ N; Eastern Time Belt; Date: March 18, 1910 (P. M.); and Point Y (Fig. 1) to the Left of the Sun.)

Item No.	Telescope position (see Fig. 2)	Time	Vertical angle	Horizontal angle
1.....	Direct	3 hr 53 min 05 sec	25° 20′ 00″	65° 00′ 00″
2.....	Reversed	3 hr 54 min 55 sec	25° 31′ 00″	64° 45′ 00″
3.....	Mean	3 hr 54 min 00 sec	25° 25′ 30″	64° 52′ 30″

Concisely, the computations may be arranged as follows:

Substitution Factors =	*A*	*B*
$h = 25°$; $\phi = 38°$ (see Table 1)	1.39989	0.36378
$+h =$ 0° 25′ 30″	$495 = 25\frac{1}{2} \times 19.4$	$712 = 25\frac{1}{2} \times 27.9$
$+\phi =$ 0° 53′ 40″	$1750 = 53\frac{2}{3} \times 32.6$	$1186 = 53\frac{2}{3} \times 22.1$
Total	1.42234	0.38276
$\sin \delta$ =	× 0.01811	......
$A \sin \delta$ =	0.025759	......
Natural cosine, bearing of the sun.................. =	0.38276 − 0.02576	= 0.40852
Bearing of the sun......... =	S 65° 53′ 18″ W	
Horizontal angle (see Table 2) =	64° 52′ 30″	
Bearing of Line *XY* (see Fig. 2).................. =	S 1° 00′ 48″ W	

The foregoing computations may be further explained as follows: From Table 1 select the values of *A* and *B* corresponding to the even degree of vertical angle and latitude, refining both for the additional minutes. Since the altitude used for computing Table 1 is obtained by applying the average refraction correction for the corresponding vertical angle, the computer is relieved of the necessity of applying the refraction correction and the parallax.

The ephemeris gives the declination of the sun, either north or south, at noon in Greenwich, England, and also gives an hourly change, which is either additive or subtractive. This declination at noon is the declination of the sun at:

7: 00 A. M. within the Eastern Time Belt
6: 00 A. M. within the Central Time Belt
5: 00 A. M. within the Mountain Time Belt
4: 00 A. M. within the Pacific Time Belt

Thus, in Example 1, the sun's declination, δ, is found to be 1° 02′ 16″ S; and the natural sine of $\delta = 0.01811$. Multiply Factor *A* by $\sin \delta$ to obtain 0.02576, and this value, added to $B = 0.38276$, yields 0.40852, which is the natural cosine of the bearing of the sun. If the declination of the sun is north, subtract $A \sin \delta$ and *B*, the smaller from the larger. If $A \sin \delta$ is the larger, the bearing of the sun will be from the north (as, northeast and northwest); and, if *B* is the larger, the bearing of the sun is from the south (as, southeast and southwest). If the declination, δ, is south, add $A \sin \delta$ and *B*, and the bearing of the sun is from the south (as, southeast and southwest).

Finally, to the bearing of the sun, apply the horizontal angle to obtain the bearing of Line *XY*, S 1° 00′ 48″ W, allowing a probable error of 1′.

CONCLUSION

The writer disclaims all intention of introducing a "new method" to the Engineering Profession; his intention is simply to reduce to the commonest wording the intellectual niceties of an astronomical observation. The method of observation described herein is an old one; yet arranged more nearly in keeping with the elementary design of the paper. Any suggestions for the improvement of Table 1, compatible with its intent and purpose, will be appreciated.

DISCUSSION

Earl F. Church,[a] Assoc. M. Am. Soc. C. E. (by letter).—Modern revolutionary developments in the science of surveying, in instruments used, in methods followed, and in results required, are making heavy demands on the technical training of surveyors. Practical astronomy is one phase of the subject in which proficiency would seem imperative. Certainly every surveyor should understand thoroughly the astronomic triangle (sometimes called the *PZS*-triangle) and its applications; the full significance of its six elements; which of the six can be found in an ephemeris; which of them can be observed; the spherical trigonometry methods of calculating any missing elements when three are known; and the practical use of the six elements in geodetic astronomy. In fact, these very points, together with the various time transformations, actually constitute the basis for a working knowledge of practical astronomy, which should be one of the working tools of every surveyor. Any work, such as the preparation of Mr. Inch's paper, in so far as it assists engineers in fulfilling the demands of modern surveying practice, is to be highly commended.

Free use of astronomic methods for azimuth determination, is certainly to be encouraged. Doubtless, it is fitting that the author should recommend the altitude method of making the observation, for the computations are simpler than those for the hour-angle method. If the altitude method of observing is used, there are three methods of computation to be considered: (1) The method described by Mr. Inch, utilizing the special table (expanded Table 1); (2) logarithmic computation of the desired azimuth, using a different form of the astronomic triangle formula better adapted to logarithmic calculations than that shown in the paper; and (3) computation of the desired azimuth by means of the same formula shown in the paper, using a calculating machine, and without using Table 1. The writer proposes to discuss these three methods.

(1).—Computation of Azimuth as Recommended by the Author.—The surveyor would require a table of angular functions, a solar ephemeris, and the data presented by Table 1. The computation itself is shown in the paper and is not reproduced here.

(2).—Computation of Azimuth by Logarithmic Calculation of the Astronomic Triangle.—The surveyor would require a logarithmic table and a solar ephemeris, including a table of refraction and parallax corrections, but not the auxiliary table given in the paper. The formula used is:

$$\sin\frac{1}{2}Z = \sqrt{\frac{\sin\frac{1}{2}(z+\phi-\delta)\cos\frac{1}{2}(z+\phi+\delta)}{\cos\phi\sin z}} \quad \ldots\ldots\ldots(3)$$

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[a] Associate Prof. of Photogrammetry, Coll. of Applied Science, Univ. of Syracuse, Syracuse, N. Y.

(in which, in addition to the notation of the paper, z = zenith distance), and the computation follows:

$$\begin{aligned}
\text{Observed } h &= 25^\circ\ 25'\ 30''\\
\text{Refraction} &= -0^\circ\ 2'\ 01''\\
\text{Parallax} &= +0^\circ\ 0'\ 08''\\
\hline
h &= 25^\circ\ 23'\ 37''\\
z &= 64^\circ\ 36'\ 23''\\
\phi &= 38^\circ\ 53'\ 40''\\
\hline
z+\phi &= 103^\circ\ 30'\ 03''\\
\delta &= -1^\circ\ 02'\ 16''\\
\hline
z+\phi-\delta &= 104^\circ\ 32'\ 19''\\
z+\phi+\delta &= 102^\circ\ 27'\ 47''\\
\tfrac{1}{2}(z+\phi-\delta) &= 52^\circ\ 16'\ 10''\\
\tfrac{1}{2}(z+\phi+\delta) &= 51^\circ\ 13'\ 54''
\end{aligned}$$

$$\begin{aligned}
\log\sin\tfrac{1}{2}(z+\phi-\delta) &= 9.89812\\
\log\cos\tfrac{1}{2}(z+\phi+\delta) &= 9.79670\\
\text{colog}\cos\phi &= 0.10885\\
\text{colog}\sin z &= 0.04413\\
\hline
2)\ & 9.84780\\
\hline
\log\sin\tfrac{1}{2}Z &= 9.92390\\
\tfrac{1}{2}Z &= 57^\circ\ 03'\ 45''\\
Z &= 114^\circ\ 07'\ 30''\\
& \text{S } 65^\circ\ 52'\ 30''\text{W}\\
& 64^\circ\ 52'\ 30''\\
\hline
& \text{S } 1^\circ\ 00'\ 00''\text{W}
\end{aligned}$$

(3).—Computation of Azimuth by Solving the Astronomic Triangle by Natural Functions and a Calculating Machine.—The surveyor would require a table of natural functions of angles, a solar ephemeris, and a calculating machine, but not the special table (expanded Table 1).[1a]

Equation (1) is used for this case and the entire computation is as follows:

$$\begin{aligned}
\text{Observed } h &= 25^\circ\ 25'\ 30''\\
\text{Refraction} &= -0^\circ\ 2'\ 01''\\
\text{Parallax} &= +0^\circ\ 0'\ 08''\\
\hline
h &= 25^\circ\ 23'\ 37''\\
\sin\delta &= -0.01811\\
\cos h &= 0.90341\\
\cos\phi &= 0.77830\\
\tan h &= 0.47470\\
\tan\phi &= -0.80674
\end{aligned}$$

$$\begin{aligned}
\phi &= 38^\circ\ 53'\ 40''\\
\delta &= -1^\circ\ 02'\ 16''\\
\frac{\sin\delta}{\cos h\cos\phi} &= -0.02576\\
\tan h\tan\phi &= 0.38296\\
\hline
\cos Z &= -0.40872\\
Z &= 65^\circ\ 52'\ 32''\\
& 64^\circ\ 52'\ 30''\\
\hline
& \text{S } 1^\circ\ 00'\ 02''\text{W}
\end{aligned}$$

It will be noted that by far the simplest calculation is the third, by means of natural functions with a calculating machine, but without the special table given in the paper. It will also be noted that the logarithmic calculation without Table 1 is really no longer or more difficult than the computation shown in the paper by means of the special auxiliary table.

The conclusions, therefore, are obvious: (1) The altitude method of determining azimuths by solar observations is simple, and its use is to be encouraged; (2) the field procedure described by the author is entirely adequate; (3) the quickest and most convenient method of computation, however, is by means of the formula, Equation (1), using natural functions and a calculating machine, but without the special table; and (4) further development of Table 1 for the purpose of facilitating the computation is scarcely worth while, because the calculating machine method, without an auxiliary table, is more accurate, and is easier.

Paul E. Wylie,[4] M. Am. Soc. C. E. (by letter).—A valuable short method of solving the astronomical triangle for azimuth is presented in this paper. It would be interesting if Mr. Inch would add data concerning the effect upon the accuracy of the result due to his assumption that *A* and *B* each vary in direct proportion throughout 1° of altitude.

Modern methods of navigation are based upon the solution of this same astronomical triangle, and much ingenuity has been expended upon labor-saving tabulations for the purpose. The navigator needs the azimuth of the sun (or other heavenly body) not only for the determination of compass error, but also in order to lay down upon the chart, in the proper direction, the line of position on which his ship is located. He does not, however, require azimuth within an error which is less than the error of plotting; consequently, most navigators' tables are not adapted to the use of the engineer.

To this rule there is at least one exception. A table[5] compiled by Lieut. Arthur A. Ageton, U. S. N., involves no knowledge more complicated than the addition or subtraction of two numbers. There is no interpolation and no multiplication. One addition and one subtraction of tabular values suffice to determine the azimuth, with a maximum error of less than a half minute of arc.

In the writer's opinion, Lieut. Ageton's method is as rapid in use as that of the author, and it probably presents even less opportunity for error. Moreover, it is already available at any agency of the Hydrographic Office at (to quote the statute) "the cost of printing and paper." An example, adapted from "Problem III", in Lieut. Ageton's 50-page book, follows: On December 17, 1934, at Latitude 20° 10′ N.; Longitude 163° 33′ W., the altitude of the sun (corrected) was observed to be 43° 36.0′ at G. C. T. 21 hr. 45 min. 26 sec. Find the sun's azimuth.

From the almanac, determine the local hour angle and the declination of the sun in the usual manner. Opposite each half minute of arc in the tables, two columns are found, marked *A* and *B*. Using the tables as noted:

Local hour angle	=	(arc) 16° 13.4′ E.	*A*	55 376	(using nearest tabular value)
Declination	=	23° 21.9′ S.	*B*	3 716	
				59 092	
Altitude	=	43° 36.0′	*B*	14 076	
Azimuth = 159° 13.5′, corresponding to			*A*	45 016	

The simple underlying theory is given in the publication, and need not be repeated herein. The Ageton method involves an accurate knowledge of the time, and a working familiarity with the co-ordinates of the celestial sphere. In this respect, it is inferior to the author's method, and is disqualified for ordinary or occasional use, but its virtues make it well worth the attention of surveyors who must determine astronomical azimuths habitually.

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[4] Structural Engr.-Builder, Los Angeles, Calif.

[5] "Dead Reckoning Altitude and Azimuth Table", by Lt. Arthur A. Ageton, U. S. N., Hydrographic Office, *Publication No. 211*, U. S. Navy Dept., Washington, D. C.

JAMES B. GOODWIN,[6] M. AM. SOC. C. E. (by letter).—Suggestions have been solicited for making Table 1 serve its purpose more efficiently, and the only criticism that the writer would think applicable is that, perhaps, it might be more convenient if the sines of the angles of the sun's declination were embodied in an additional column, together with an accompanying column of differences for each minute. The column of altitudes (Column (1)) would then be combined to include the declinations. It would then be necessary to refer to only one source for altitude, latitude, and declination factors.

Equation (1) may be written in another form, namely:

$$\cos Z = \frac{1}{\cos h \cos \phi} (\sin \delta - \sin h \sin \phi) \ldots\ldots\ldots\ldots (4)$$

where $\frac{1}{\cos h \cos \phi}$ is as defined in the paper and $\sin h \sin \phi$ replaces $\tan h \tan \phi$. However, it is questionable whether this is any improvement on the author's procedure.

It would appear that, in the selection of the declination, standard time has been used for the 75th Meridian. In some possible conditions the use of standard time might lead to appreciable errors in establishing azimuths, especially at or near the limits of standard time belts. As an illustration, assume two traverse lines on either side of the boundary between two standard time belts or zones, run generally northerly and relatively close to the division line. One would have azimuths derived from a time 1 hr different from the other, whereas the same declination for the sun should practically apply, if close to the boundary.

Since the sine of the sun's declination is multiplied by $\frac{1}{\cos h \cos \phi}$, the effect on the resulting azimuth is apparent. It would seem more nearly precise to consider the longitude of the place of observation in determining the time for the sun's declination referred to Greenwich.

In the illustration as given, there would be a difference in time corresponding to 2° of longitude, assuming Washington to be on the 77th Meridian. It would be of interest to know why, in determining azimuths, preference appears to be given to the method outlined as against observations on Polaris.

C. H. SWICK,[7] ESQ. (by letter).—Simplification of engineering methods and computations is always desirable, but this is especially true when astronomical measurements are involved. Ordinarily, an engineer has little need to start or check his surveys by the use of astronomical determinations and, therefore, is likely to lack familiarity with astronomical methods and computations. The method proposed by Mr. Inch is quite simple to apply and, therefore, will probably prove useful in certain types of surveys. The factors which he proposes to obtain from the table are rather easily computed for the individual observations, and it is questionable whether the double interpolations can be made from the table any more readily and quickly than the factors could be computed directly. The table has the advantage, however, that gross errors in the computations are probably less likely to occur when it is used.

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[6] Toronto, Ont., Canada.

[7] Chf., Section of Gravity and Astronomy, U. S. Coast and Geodetic Survey, Washington, D. C.

The form of table proposed by Mr. Inch is compact and easily followed. As published, the interpolation interval is large, and this results in some inaccuracy especially in those parts of the table where the tabulated differences change rapidly, but the table would become rather unwieldy to compute and publish if the interval were decreased to 10 min or even to 30 min.

PHILIP KISSAM,[8] ASSOC. M. AM. SOC. C. E. (by letter).—There are many advantages inherent in the determination of azimuth by sun observations. The method is chiefly applicable to route surveys or traverses of this type where neither established azimuth control is available nor high precision necessary. Under average conditions an azimuth determined by this method will be within 0° 2′ of the correct value, depending on the latitude and time of year. The observations require 10 min, or less, time in the field and about 20 or 30 min for computation. They can be made during the regular program of observations and should be made once every clear day. Polaris observations usually are made more successfully at night than in the late afternoon when Polaris is visible during working hours, and the preparation necessary for night observations, including the transportation of the field party to the site, usually requires an hour or more, is inconvenient, and often interferes with the progress of the next day's traverse work.

Since the sun method is so useful and can be made so frequently, it is important that the computations should be made with least difficulty. The value of the paper by Mr. Inch is the saving of time for computation. It will be interesting to compare the two usual methods of computation with that of the paper: Method (*a*) involves a formula for use with a computing machine, as follows:

$$\cos Z = \frac{\sin \delta - \sin \phi \sin h}{\cos \phi \cos h} \quad \dots\dots\dots\dots\dots\dots (5)$$

and Method (*b*) involves a formula for use with a table of logarithms, as follows:

$$\cot^2 \frac{Z}{2} = \frac{\sin (s - \phi) \sin (s - h)}{\cos s - \cos [s - (90 - \delta)]} \quad \dots\dots\dots\dots (6)$$

in which, in addition to the notation of the paper.

$$s = \frac{\phi + h + (90 - \delta)}{2} \quad \dots\dots\dots\dots\dots\dots (7)$$

The steps required for computation are shown in Table 3. It will be noted that Method (*b*) requires several additions and subtractions of angles (9) which require time and may cause blunders. The Inch method requires fewer operations, and the references to the table are more convenient.

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[8] Associate Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

The author mentions the necessity of a solar ephemeris for computation of the sun's declination. It might be well to note that the best type of ephemeris to use is one that gives the declination at civil time (as in the Nautical Almanac). It is necessary to have an ephemeris for the proper year. If a permanent declination table is desired, the reader is referred to the report entitled "Azimuth Determination", by E. F. Coddington, M. Am. Soc. C. E.[9] Tables in this *Bulletin* extend through the year 2000, but the necessary computation is slightly more laborious.

TABLE 3.—COMPARISON OF METHODS OF COMPUTATION

Steps required	Method (*a*)	Method (*b*)	Inch method
References to a table	6	5	4
Interpolations	6	5	6
Multiplications or divisions	3	0	1
Other minor steps	0	9	3

The "Field Procedure" suggested by Mr. Inch is not the only practical one available. An excellent method is described under the title, "Topographic Instruction of the U. S. Geological Survey."[10] The chief feature of this method is in bisecting the sun's disk rather than in bringing the cross-hairs tangent to the disk. Ten pointings are recommended instead of two. The field time necessary for the ten pointings is about 6 min so that the average of the ten pointings can be used without danger of errors due to the curved path of the sun. Both the aforementioned *Bulletins* recommend using a white card behind the eye-piece on which the sun's image appears with the cross-hairs visible across the image. This method is rapid, as the observer does not have to place his eye at the eye-piece, and easy to use as no special equipment is necessary. The eye-piece must be focussed for this method, but this is balanced by the time required to attach a colored glass or prism.

(There is a difference of opinion as to the relative accuracy of pointing the instrument at the edge of the sun's disk rather than bisecting it. One of the faults of a sun observation is the difficulty of pointing a moving object. Although the sun's disk is large, the writer believes that bisection is more accurate. It is very difficult indeed to bring the cross-hairs to the edge of the disk without bringing them into the disk. Any effort to bring a cross-hair to the edge of a target is fraught with difficulties, whereas the eye can bisect with surprising accuracy as long as the entire image is visible without shifting the eyeball. An experimental determination of the comparative accuracies might well be made. It is also noted that special cross-hair arrangements are available from many instrument makers in which intersecting wires form a square slightly smaller than the sun's disk and for which claims are made of greater accuracy of pointing.)

In connection with the field procedure described in the paper, Step (3), namely, "level the instrument with extreme care", it might be well to recommend leveling with the bubble tube attached to the telescope if such a bubble is available. No amount of reversals will eliminate the errors caused by the vertical axis not being exactly vertical.

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[9] *Bulletin No. 79*, Eng. Experiment Station, Ohio State Univ., Columbus, Ohio, September, 1930.

[10] *Bulletin 788-C*, U. S. Geological Survey.

GEORGE D. WHITMORE,[11] M. AM. SOC. C. E. (by letter).—Unquestionably, the method presented by Mr. Inch for computing solar azimuth observations is a time-saver. Actual tests indicate that by his method the sun's azimuth can be found in about one-half the time required for solving the formula by use of trigonometric functions. It must be realized, however, and prospective users should be so warned in the explanatory text, that the azimuth found by this short-cut method may be incorrect by as much as 1′ in azimuth, solely because the interpolated values for Factors *A* and *B* may be incorrect. This is because the tables give the *A* and *B* factors only for each degree of altitude and each degree of latitude. For practical reasons, the intermediate values must be based on straight-line interpolation, whereas actual intermediate values would follow a curved line.

Taking the author's example for illustration, the azimuth obtained by solving the spherical trigonometry formula gives a result about 30″ lower than the one he secures by using the *A* and *B* tables. Other experiments showed that, under certain conditions and combinations, the azimuth secured by using the tables of this form could be in error by as much as 60″.

In Example 1, Factor *B* is taken for 38° of latitude, the actual latitude being 38° 50′ 40″. The author finds the value for Factor *B* by interpolating between 38° and 39°, taking the tabulated difference of 22.1 for 38° of latitude, multiplying this value by 53′ 40″, and adding the result to Factor *B* for Latitude 38°. (A closer result would have been secured, incidentally, had he found *B* by interpolating "downward" from 39°, the actual latitude being only 6′ 40″ less than 39°, compared with 53′ 30″ plus from 38°.) The point, however, is that the difference in the fifth place of logarithms for Factor *B* for 1′ at 38° is 22.1, and for 1′ at 39° is 22.7. It is apparent that this difference of 0.6 will create rather sizable differences in the values for Factors *A* and *B*, when the latitude is about midway between the even degrees.

Readers should be warned also that Factor *B* is perhaps a more critical value than Factor *A*. They will both be subject to some error, but whatever the error might have been in Factor *A*, as taken from the tables, its effect is considerably reduced when *A* is multiplied by sin δ, which may be any quantity between 0.0 and 0.4. Factor *B* is used in the formula, however, just as it is taken from the tables, and is not reduced in any manner. The explanation should include a warning also that the computed result will vary in accuracy with the seasons, since the greater the declination and the larger its sine, the less the error in Factor *A* is reduced. It also appears that a south declination is likely to give a greater error than a north declination, since the two factors, *A* and *B*, are always added when the declination is south.

Inexperienced computers should be warned as to the declination tables to be used. Some of the solar ephemeris tables published by instrument manufacturers, supposedly to be used in connection with solar attachments, give the declination as including refraction corrections. The author's table gives the values for *A* and *B* already corrected for refraction; hence declination tables used in connection with this method should never include refraction corrections.

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[11] Chf. of Surveys, Maps and Surveys Div., T.V.A., Chattanooga, Tenn.

The tables might be more conveniently arranged by using one full page for each degree of latitude, and giving *A* and *B* values for altitudes from about 10 to 49°. Under this arrangement, the page might be divided into four columns, each column covering a range of 10° of altitude. This arrangement might also prove to be of advantage to a survey party wishing to compute solar azimuths in the field, as only one or two pages of the tables would be required for any one locality, thus eliminating the need for carrying a full set of tables for all latitudes. For the benefit of inexperienced computers, it might be well to explain that the "Differences for 1′" are in units of the fifth decimal place.

Except for the questions raised herein, it is believed that the five objects stated in the "Synopsis" will be accomplished, except possibly Item (2), which is to "obviate textbook reference." The tables and the necessary explanations suggested by the author will in themselves constitute a textbook, and will have to be present always when solar azimuth observations are to be computed. It is suggested also that the principal uses of the method may be by computers who are engaged in calculating many solar observations as a routine operation, rather than by the engineer making an occasional solar observation and computation. Such an engineer may not be aware of the existence of these tables, or may not have them in his library, and the few extra minutes required to develop the regular formula is not so important where only an occasional observation is required. On the other hand, the saving in time for a computer calculating many observations, day after day, may be important. If this is to be the principal use, then it might be well to expand the table, showing the factors for, say, every 10′ or 20′ of altitude, and similarly for latitude. Such expansion of the tables would eliminate also the previous comments on inaccuracy in the interpolated values for the *A* and *B* factors.

O. H. CHILTON,[12] ESQ. (by letter).—Many engineers, to whom the intricacies of field astronomy are a mystery, will welcome this paper and the simplification of the work of finding the azimuth of a line which it enables. For the very reason that it is addressed to those who are not fully acquainted with the subject, it seems all the more desirable to draw attention to some points which, as they stand at present, are likely to cause confusion.

Consideration of the formulas of the paper shows that the exact time of observation does not enter into the computations. The only purpose for which the time is observed is that of obtaining the correct declination of the sun from the ephemeris.

The limitations of Table 1 in its present form with somewhat wide intervals of 1°, will result in no great accuracy in the azimuth obtained.

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[12] Asst., Dept. of Civ. Eng., Imperial Coll. of Science and Technology, London Univ., London, England.

The author suggests a probable error of 1′. Accepting some such degree of accuracy, it can be easily shown that even at the more unfavorable seasons of the year, when the sun's declination is changing most rapidly, if the time of observation is known to the nearest 10 min, the present purpose will be adequately served. Thus, it would be quite sufficient for the observer to look at his watch at the beginning and at the end of the observations, and to take the mean of the times, or even to record merely one time, to the nearest minute, when reversing the instrument between pointings at the sun. The recording and averaging of times to the nearest 5 sec in Table 2 is not necessary and is misleading.

In the instructions for the field work there should be a clear statement that observations for azimuth should not be made on the sun except when it is favorably situated in the heavens. It can be shown that the effects, on the azimuth obtained, of errors in the assumed values of the latitude and the declination, are least when the sun is on the prime vertical—that is, due east and west. However, for the present purpose, it would suffice to limit observations to those periods when the sun is more than 3 hr off the meridian, that is, to mornings before 9:00 A. M., and afternoons after 3:00 P. M.—always having regard to the possible incidence of summer (daylight saving) time and any large variation between the local civil time and the true sun time at the place of observation.

The engineer will require some indication of the accuracy requisite in the value of the latitude of the place of observation, which he is instructed to obtain from "an accurate map." A map may be accurate, but the scale may be too small for some purposes. By differentiation of the fundamental equation relating the azimuth to the observed and assumed quantities it can be shown that:

$$\delta Z'' = -\delta c'' \frac{\cot t}{\sin c} \dots\dots\dots\dots\dots\dots\dots (8)$$

in which Z = azimuth; $c = 90° - \phi$; and t = hour angle.

Assuming that azimuth observations will be made by this method only when the sun is more than 3 hr off the meridian, and that the paper is addressed to engineers working in the United States between limits of latitude of, say, 49° and 30° N, it can easily be computed that for $\delta Z = 60''$, δc has a mean value of about 45″. There will be other sources of error in the computation, although possibly operating to balance each other. However, allowing, say, one-third of the error to this source (say, 15″ in the latitude, which is about 1 500 ft on the ground) a map of not less than 0.5 in. to 1 mile is indicated.

It is unfortunate that the author has chosen an example solved as long ago as 1910, and has included in the text a statement which was true for that year, and indeed until 1924. The American Ephemeris, in common with the ephemerides of other countries such as the British "Nautical Almanac", now gives the elements of the sun's position, such as the declination, for each day "for 0 hours Greenwich Civil Time which is 12 hours before Greenwich Mean Noon of same date." A mean hourly variation of declination is no longer tabulated, but finite differences between each of the daily values are given. As it stands at present the text would cause much confusion to an engineer, uninstructed in these matters, who was attempting to apply the method.

With regard to a possible extension of Table 1 the proposed range of values of the vertical angle and of the latitude, is not stated. In Table 1 as printed, an altitude as low as 21° is given. This might be open to question on account of the magnitude of the refraction correction involved. A lower limit of 25° is suggested. If it is intended to compute the table for latitudes north of the United States, the unsuitability of the method to very high latitudes will doubtless be borne in mind.

Until the limits of latitude for which the table is to be computed are known, it is not possible to comment finally on its form, but even for the part of the table printed, there is doubt as to its suitability. The writer has compared the methods of computation of the worked example, and the azimuth so obtained appears to differ by 45″ from that given by Table 1. Taking the author's standard of accuracy, "a probable error of 1′", a discrepancy of 45″ arising solely from the method of interpolation from the table is too great to be acceptable when there are other sources of error all operating to effect the final accuracy of the determination, such as errors in the assumed latitude of the place, sun's declination, and the observed angles.

Examination of the part of the table available shows that the method of linear interpolation by first differences becomes still more inadequate as the values of altitude and latitude increase. The worked example is taken from the most favorable part of the table. As it stands it could, of course, be made to serve its purpose if interpolation by second differences is resorted to; but that is scarcely likely to appeal to the engineer in the field, and indeed a direct computation by formula adapted to logarithmic working would be almost as rapid. If the form of the table is to be retained with interpolation direct by first differences, then it does appear that the intervals of 1° in both altitude and latitude may have to be broken down to 30′, or perhaps 20′, for the higher values.

CHALMERS C. SCHRONTZ,[13] M. AM. SOC. C. E. (by letter).—Equation (1) of this paper can be solved mechanically by the solar compass or by a solar attachment to the transit when the apparent altitude and declination of the sun are employed instead of their true values, as used in the method by direct observation. There are no means by which the correction for direct refraction in altitude can be made in the use of the solar attachment. A compensating correction must be made which can be applied to the declination arc of the solar attachment.

The declination correction is applied to an angle normal to the plane of the equator, whereas the direct refraction is applied to an angle normal to the horizon at the place of observation. Consequently, the declination correction for refraction is an increment of an angle in a system of polar co-ordinates corresponding to an increment of an angle in another system of polar co-ordinates, the two systems of which are coincident in the vertical plane passing through the pole of one, and the zenith of the other, system at the place of observation. Therefore, the refraction correction for declination is mathematically comparable with the direct refraction for altitude.

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[13] Associate Engr., U. S. A. Engrs., Jacksonville, Fla.

This relation of polar co-ordinates has made it possible to solve this problem mechanically by the use of the solar attachment and the same solution may be made mathematically from a direct observation of the sun.

The table for corrections to be applied to the true declination in order to determine the apparent declination of the sun for the hour and latitude of the place always accompanies the solar ephemeris. In either case, whether the solution of the spherical triangle is to be made from the true or from the apparent altitude of the sun, the declination as given in the ephemeris for noon at Greenwich must be corrected for the hourly change from Greenwich noon to the time of observation. The apparent declination is then obtained by the addition or subtraction of the refraction correction for declination corresponding to the time and latitude of the place of observation. When the apparent declination is used, these two corrections for hourly change and refraction are the only ones required, no correction being necessary for the observed altitude of the sun.

In the writer's experience this problem can be solved most satisfactorily by the use of a table of logarithmic and natural trigonometric functions, together with a solar ephemeris (furnished gratis by the instrument manufacturers), and a map from which the longitude and latitude of the place can be determined. In the tables of logarithmic functions the cosines and tangents of the latitudes occur on the same page and on the same line and also the same functions, for the vertical angles, which greatly facilitates the selection of the values of functions. Examples of the solution of the formula based upon the true and apparent altitude and declination follow.

Example 2.—Solution based upon apparent altitude and declination of the sun:

Date: April 30, 1936.
Place: Latitude, 40° North; longitude, 105° West.
Declination sun, Greenwich, noon, N 14° 48.3′.
Hourly change, + 0.76′ (declination increasing).
Time of observation: 9:00 A. M. (local mean time).
Altitude of sun's center, 43° 31′ 30″. (No correction to be applied to observed altitude.)

Apparent declination of sun at time of observation:

Longitude, 105° West, at 15° per hr is 7 hr from Greenwich, noon, to place of observation, noon.
Time of observation, 9:00 A. M., is 3 hr before noon at place of observation.
∴ Time of observation is $7 - 3 = 4$ hr after noon at Greenwich.

Declination, Greenwich, noon	N. 14° 48′ 18″
Hourly change, 4 × 0.76′	0° 3′ 2″
Declination at time of observation	N. 14° 51′ 20″
Refraction for third hour (from noon), Latitude 40° N.	+ 0° 0′ 40″
Apparent declination for time and place	N. 14° 52′ 00″

Solution:

$$\phi = 40^\circ;\ h = 43^\circ\ 31'\ 30'';\ \text{and},\ \delta = +\ 14^\circ\ 52'\ 00''$$

Logarithms:

cos ϕ	9.884254	tan ϕ	9.923713
cos h	9.860382	tan h	9.977630
log	9.744636	log	9.901443
Co-log	10.255364		
sin δ	9.409207		
log	9.664571		

Natural functions (corresponding to foregoing logarithms):

(+) 0.461924 (−) 0.796971
(−) 0.796971
cos Z = (−) 0.335047; and, $Z = 70^\circ\ 25'\ 49''$

Example 3.—Computation based upon true altitude and declination of the sun:

Observed altitude, sun's center	43° 31′ 30″
Correction for refraction and earth's parallax, 0.9′	− 0° 0′ 54″
True altitude	43° 30′ 26″
Sun's true declination for time of observation (as in Example 2)	14° 51′ 20″

Solution:

$$\phi = 40^\circ;\ h = 43^\circ\ 30'\ 26'';\ \text{and},\ \delta = 14^\circ\ 51'\ 20''$$

Logarithms:

cos ϕ	9.884254	tan ϕ	9.923813
cos h	9.860510	tan h	9.977360
log	9.744764	log	9.901173
Co-log	10.255236		
sin δ	9.408890		
log	9.664126		

Natural functions (corresponding to foregoing logarithms):

$$\begin{aligned} & (+)\ 0.461451 \\ & (-)\ 0.796475 \\ \cos Z = & (-)\ 0.335024;\ \text{and},\ Z = 70^\circ\ 25'\ 35'' \end{aligned}$$

The difference in the two results, of 14″ of arc, is deemed negligible in view of the interpolation of the values in the tables of functions for seconds of arc and the tabulated difference in refraction for the two systems of polar co-ordinates.

The signs of the declination (+ for north and − for south) become important when the direction of the sun is close to the east or west point from the point of observation. If the result, cos Z, is +, the direction of the sun is to be referred to the north point, and when it is −, as in this case, the direction is to be referred to the south point.

It is to be noted that the cos ϕ, tan ϕ, and sin δ can be taken as constants for several observations taken within 15 or 20 min without appreciable error in the results. This provides ample time to make several observations in which only the altitude and azimuth of the sun will change materially. The arrangement of the solution lends itself to several computations simply by supplying the changed values of cos h and tan h. Consequently, the solution is thus expedited.

Field Operations.—The travel of the sun's disk when viewed through the telescope of the solar attachment is parallel with the two horizontal cross-hairs and when the sun's disk is brought between these parallel cross-hairs by the lower tangent screw of the transit, the transit telescope points to the true north or south, provided, of course, that the proper latitude and apparent declination have been used. In this position it will be some time before the sun's disk can be detected to move above or below the horizontal cross-hairs of the solar telescope, which is due only to the sun's change in declination. This allows plenty of time to make a precise setting of the cross-hairs of the solar telescope on the limbs of the sun, and, hence, a precise pointing of the transit telescope on the true meridian. When the proper declination is once set off for the time and latitude of the place, one single observation is sufficient to determine the meridian.

In a direct solar observation the sun does not travel parallel with either the horizontal or the vertical cross-hairs of the telescope, but in a diagonal direction across the field of vision. In the usual method of pointing the telescope for forenoon observations, the vertical and horizontal cross-hairs are brought simultaneously in contact with the right and upper limbs of the sun, recording the vertical and azimuth angles; then reversing the telescope and setting the cross-hairs in contact with the left and lower limbs of the sun and recording the vertical and azimuth angles. For afternoon observations, the lower and right, and upper and left, limbs are observed and recorded in the same way. The means of the vertical and azimuth angles for each pair of observations are taken as the apparent altitude and true azimuth of the sun's center for the mean time of the two observations.

It is evident that such observations cannot be made with the ease and precision attained in the setting with the solar attachment. The results obtained in a direct observation are never so uniform, satisfactory, or trustworthy as those obtained with the solar attachment, unless in the case of two or more direct solar observations. One single determination from direct observation is scarcely trustworthy.

The results of a direct observation are easier, quicker, more uniform, and more satisfactory when the observation is made upon the center and tangent to the lower limb of the sun by bisecting, with the vertical cross-hair, a small segment of the sun just before contact of the horizontal cross-hair with the lower limb of the sun. To the vertical angle thus obtained the sun's mean semi-diameter is added, 16′ of arc, to obtain the altitude of the sun's center. The greatest variation of the sun's mean semi-diameter is 18″ of arc, and may be neglected unless greater precision is required. The horizontal angle is the azimuth of the sun, and no correction to it is necessary. Five observations are usually taken alternatively in direct and reverse positions of the transit. Each observation is treated independently. At least two observations are used to determine the azimuth. When these agree within less than 1′ of arc the mean of the two results is accepted. If not, one or more of the remaining observations are used for azimuth determinations. Seldom more than two or three determinations are necessary to obtain the desired result. The apparent declination is determined for the mean time of all the observations and is used as a constant for all the observations, so that for each independent determination of azimuth of the sun there are three constant values that must be taken once only from the tables of trigonometric functions, namely, cos ϕ, tan ϕ, and sin δ.

In this procedure one erratic result occasionally may be obtained in the azimuth determination which will evidence an error either in the solution or in the observation and may be corrected or discarded. The error thus discovered is confined to one single observation, whereas in the use of the mean of two observations, the error is reduced one-half and usually is not discoverable.

By the use of these expedients and the arrangement for the solution of the formula for azimuth, a set of observationes and computations may be made in the field with little delay, and the only tables required are the regular trigonometric tables which are familiar to all engineers and surveyors. Additional tables seem scarcely necessary and do not materially simplify the solution of the formula for azimuth.

The factor, A, is the natural number corresponding to the co-logarithm of cos ϕ cos h; or if natural functions are used, it is the reciprocal of cos ϕ cos h in which, for either case, the correction for refraction and parallax has been applied to h.

The factor B, is the natural number corresponding to tan ϕ tan h, also in which the correction for refraction and parallax has been applied to h; but if the apparent altitude of the sun is used there is no correction to be applied to h and all the correction for refraction and parallax is applied only once to one value of the declination, which may be used for several azimuth determinations, provided not more than 10 to 20 min are consumed in making the observations.

It is not the writer's purpose to criticize or to discourage any such worthy purpose or effort to simplify the solution of the azimuth formula. It is only to call attention to the fact that the apparent altitude of the sun may be used in solving the azimuth formula, and by so doing the refraction correction is referred and applied to the declination, once and for all, for a single set of several observations, and that the solution of the formula is made easily and quickly by a simple arrangement and use of a logarithmic function already familiar to the engineer and the surveyor.

The writer is unaware of the use of the apparent altitude and declination having been suggested heretofore, or used, in the determination of the true meridian by direct solar observation. It has only been suggested to him by the fact that in the use of a solar transit the solution of this formula is accomplished mechanically. It would seem that the mathematical solution has been overlooked.

It has always been somewhat of a mystery why the solar attachment has not come into more general use. The attitude seems to be that its use is derogatory to the dignity of the engineer, or an admission of a lack of scientific technique and ability, and that the transit and its use is the acme of perfection. Except for those who are continuously employed in public land surveys, the expense of the solar attachment seems a burden. Its occasional use does not justify the expense. Many are not fully convinced of its accuracy and dependability. Its adjustments are thought to be difficult and can be made satisfactorily only by the instrument makers. Furthermore, it adds weight and makes the transit unbalanced, or otherwise interferes with the routine work of the transit. All these objections are largely imaginary, and a very little experience in its use will dissipate such objections.

FRANK M. JOHNSON,[14] M. AM. SOC. C. E. (by letter).—The method proposed in this paper has been studied carefully to ascertain whether it does, in fact, afford a saving in the number of steps ordinarily required in the use of trigonometric tables. The saving, if any, is not as great as it is made to appear by the author.

It seems better to state frankly that there are no so-called "short-cuts" in making direct observations on the sun for azimuth, or in the reduction of steps by the trigonometric process, if reliable results are to be secured. By "reliable" is meant the ascertainment of results that can be verified within the limits of the accuracy of the instrument used for the observation; this should be less than 1′, and with care may be kept within 15″. Observations and results should be such as may be verified by others if occasion arises, thus removing elements of doubt or approximation.

This paper is not a satisfactory substitute for the explanations on the subject which may be found in textbooks on surveying, and several official publications. It is unfortunate to create the impression that the textbook treatment of the subject has been made difficult; the method has been brought into general use, and the number of examples offered are designed to clear all questions of doubt if the engineer will make a fair attempt to grasp the several steps and make a few practice observations.

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[14] U. S. Superv. of Surveys, Dept. of the Interior, Denver, Colo.

The use of the tabulations in the paper does not dispense with the need for tables of natural sines and cosines, and it is not believed that their use materially lessens the time required to resolve the formula in the usual way. On the other hand, it is felt that the method adds materially to the uncertainties of the results in case of one or more errors in making the observations, or other discrepancies.

TABLE 4.—First and Second Differences for Factors *A* and *B*, Table 1

Vertical angle, h, in degrees (1)	Factor *A*			Factor *B*		
	Factor* (2)	First difference, Δ_1 (3)	Second difference, Δ_2 (4)	Factor† (5)	First difference, Δ_1 (6)	Second difference, Δ_2 (7)
21	1.34085			0.28866		
		927			1 522	
22	1.35012		53	0.30388		22
		980			1 544	
23	1.35992		56	0.31932		21
		1 036			1 565	
24	1.37028		58	0.33497		25
		1 094			1 590	
25	1.38122		61	0.35087		25
		1 155			1 615	
26	1.39277		63	0.36702		30
		1 218			1 645	
27	1.40495		64	0.38347		28
		1 282			1 673	
28	1.41777		67	0.40020		31
		1 349			1 704	
29	1.43126		72	0.41724		33
		1 421			1 737	
30	1.44547		73	0.43461		35
		1 494			1 772	
31	1.46041			0.45233		

* Values of Factor *A* from Column (3), Table 1. † Values of Factor *B* from Column (3), Table 1.

TABLE 5.—First and Second Differences for Factors *A* and *B*, Table 1

Latitude, ϕ, in degrees (1)	Factor *A*			Factor *B*		
	Factor* (2)	First difference, Δ_1 (3)	Second difference, Δ_2 (4)	Factor† (5)	First difference, Δ_1 (6)	Second difference, Δ_2 (7)
37	1.52811			0.52723		
		2 060			1 941	
38	1.54871		105	0.54664		52
		2 165			1 993	
39	1.57036		111	0.56657		58
		2 276			2 051	
40	1.59312		116	0.58708		62
		2 392			2 113	
41	1.61704		125	0.60821		63
		2 517			2 176	
42	1.64221			0.62997		

* Values of Factor *A* for a vertical angle, $h = 35°$, in Table 1. † Values of Factor *B* for a vertical angle, $h = 35°$, in Table 1.

On examination, it is believed that the interval of Table 1 is altogether too great, as demonstrated by Tables 4 and 5. Assuming that all these values have been carefully verified and are available as indicated, with an interval of 1° in latitude and 1° in vertical angle, it will be noted that a considerable number of pages similar to Table 4 will be required in order to provide for latitudes ranging from about 25 to 65°, and similar to Table 5 for vertical angles ranging from about 15 to 60 degrees. The third variable, that of the sun's declination, as well as the final result in azimuth angle, are provided for by the use of tables of natural sines and cosines.

The method of direct observation on the sun for azimuth is regarded in high favor by engineers, and has been used extensively for many years. Reliance upon compass meridians for all classes of public land surveys was discontinued in 1890. Federal surveying and mapping agencies have stressed the importance of exact methods so long that it seems out of place to suggest a practice that has the value of an approximation only.

An obvious improvement for the tabulation of values of Factors *A* and *B*, if the method is to be favored, is to make the intervals of the latitude and vertical angles not greater than 0° 10′, but this extends the tables to such a length that it is doubtful whether it would be worth while to go to that much trouble. The tabular differences will increase more rapidly in the higher latitudes, and as the vertical angles become greater, the results then become correspondingly more uncertain.

The example given by the author does not derive the value to the nearest even 1′, as the bearing of the line by trigonometric reduction is S 1° 00′ 02″ W (not S 1° 00′ 48″ W, as ascertained by the use of the factors, *A* and *B*).

The trigonometric steps are not many, as will be seen from the following:

Observed vertical angle			25° 25′ 30″
Refraction			− 0° 2′ 00″
Parallax			+ 0° 0′ 08″
True vertical angle			25° 23′ 38″
Latitude			38° 53′ 40″
Sun's declination, south			1° 02′ 16″
		log sin δ	8.257958
log cos ϕ	9.891149		
log cos h	9.955871		9.847020
		log	8.410938
(Sun, south declination)		nat (−)	0.02576
(−) log tan ϕ	9.906733		
log tan h	9.676423		
log	9.583156		
nat (−)	0.38296		0.38296
Sun's azimuth, nat cos, Z			0.40872
True bearing of sun			S 65° 52′ 32″W
Angle from sun to flag			64° 52′ 30″
Bearing of line			S 1° 00′ 02″ W

Walter H. Starkweather,[15] M. Am. Soc. C. E. (by letter).—As arranged by Mr. Inch, with double increments for 1′ of altitude and 1′ of latitude, Table 1 is quite ingenious and should prove welcome to field engineers who have occasion to determine azimuths of lines, since it shortens the necessary computation considerably. One operation is also eliminated completely by having the factors in the table corrected for refraction in altitude.

There are several points which might prove to be confusing, however, to any one who had not used this formula or one evolved from it. For many years Equation (1) and formulas evolved from it have been used by field engineers and in the technical press with all the variables represented by letters of the English alphabet; but in most textbooks and handbooks, with a few exceptions, Greek letters have been used for some of the variables. The formula appears less formidable if stated as follows:

$$\cos Z = \frac{\sin d}{\cos h \cos L} - \tan h \tan L \ldots\ldots\ldots\ldots\ldots (9)$$

in which Z = the angle between the sun and the local meridian; d = the true declination of the sun; h = the altitude of the sun corrected for refraction; and L = the latitude of the observer.

TABLE 6.—Notes for Computing the Bearings of a Line

d	$-1°\ 02.5'$	......
h	$25°\ 25.5'$	......
L	$38°\ 53.5'$	......
Factor A	For 25° h and 38° L	1.3999
Correction h'	$25.5' \times 1.9$	48
Correction L'	$53.5' \times 3.3$	177
A	$\frac{1}{\cos h \cos L}$	1.4226
Factor B	For 25° h and 38° h	0.3638
Correction h'	$25.5' \times 2.8$	71
Correction L'	$53.5' \times 2.2$	118
B	$\tan h \tan L$	−0.3827
$\sin d$	....	−0.0182
$A \sin d$	....	−0.0259
$\cos Z$	$A \sin d - B$	−0.4086
Z	$65°\ 53'.1$	
Horizontal angle	$64°\ 52'.5$	......
Azimuth	$1°\ 01'$	......
Bearing	S 1° 01′ W	......

The writer wishes to present a plea for using d for declination and L for latitude, believing that some confusion will thereby be eliminated. As d and L are the initials of the words, declination and latitude, they would not be likely to cause the uncertainty as to what they represent as would the Greek letters, δ, ϕ, θ, and λ.

As the readings of the vertical angles usually cannot be taken closer than the nearest minute of arc with an ordinary transit, refining any of the computations to a point closer than 0.5′ of arc, or farther than four decimal places in the computations, is not justified, as the results within the required limits will be obtained without these refinements. For the same reason, the time is not needed to seconds, as the nearest 15′ to 20′ of the correct time will define the declination of the sun sufficiently close.

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[15] Civ. Engr.; Technical Asst. Engr. to Chf. Civ. Engr., U. S. Coast Guard, Washington, D. C.

In Table 6 the computation is made from the values used for Example 1 of the paper, arranged for convenient placing on the page of a standard field notebook, and is carried only to the limits suggested herein. The results are within less than 0.5′ of arc of the values calculated to seconds of arc with five-place computations. This is believed to be sufficiently close, as authorities are apparently agreed that the accuracy of the results by this method is limited to one-half the least reading on the horizontal or vertical arc, in making the observation, and the author writes "allowing a probable error of 1′," but the azimuth has been computed to seconds.

C. I. DAY,[16] ASSOC. M. AM. SOC. C. E. (by letter).—The avowed purpose of this method is to improve the accuracy of true bearing determinations, now largely dependent upon the compass needle. Many county surveyors and others who have not been fortunate enough to receive the necssary mathematical training avoid direct readings on the sun because of the complicated appearance of the formulas. This has been somewhat remedied by the various solar attachments that can be added to the transit, but which are too expensive for only occasional use.

The formula (Equation (1)) from which the author's tables (illustrated in part by Table 1) have been derived does not lend itself to rapid mathematical solution, particularly when logarithms are used. Therefore, the tables are faster to use and can be used by any surveyor who is not familiar with logarithms. The $\tan^2 \frac{1}{2} A$ formula lends itself to solution by use of logarithms in about the same time as the tables, unless a computing machine is used in making the cumbersome multiplications.

Some surveyors are always somewhat uncertain as to how to apply the refraction correction to altitude observations and for them the tables suggested by the author are a boon.

In comparing the tables against the formulas, the accuracy seems to be within that claimed and certainly is considerably better than can be gained with a compass. Mr. Inch is to be commended highly for his efforts.

F. L. MCREE,[17] ESQ. (by letter).—Some mention should be made of the accuracy of the values obtained from the table prepared by Mr. Inch. In working with Table 1 the writer was surprised to discover how little error is introduced by the use of an average parallax and refraction correction. The principal error seems to be due to straight-line interpolation being used between tabular values. This error, fortunately, is partly compensating for values of Z (measured from the north) greater than 90 degrees. An examination of the following examples will show that when $A \sin \delta$ is negative and is added to B, although the individual values of $A \sin \delta$ and B, as computed from Table 1, differ from the exact values, the result does not differ materially from the exact value for cos Z. However, when Z is less than 90° (when $A \sin \delta$ is positive), this compensation does not exist, and larger errors may appear. Unfortunately, the range of Table 1 does not allow investigations for smaller values of Z. The writer would like to raise this question with the author: Do larger errors occur for interpolated values from the tables when Z (measured from the north) becomes small?

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[16] Pres., W. & L. E. Gurley, Troy, N. Y.

[17] Associate Prof. of Civ. Eng., Texas Technological Coll., Lubbock, Tex.

To illustrate the writer's viewpoint with respect to the accuracy of the results obtained from the tables the following examples are given.

Example 4.—Latitude = 35° 32.0′ (North); observed h = 35° 36.1′; declination = N 23° 24.0′; and, temperature = 50° C.

By the exact formula:

$$\begin{array}{rl}
\text{Observed } h & = 35^\circ\ 36.1' \\
\text{Parallax and refraction correction} & = \ \ 0^\circ\ \ \ 1.1' \\
\hline
h & = 35^\circ\ 35.0'
\end{array}$$

$$\cos Z = \frac{\sin 23^\circ\ 24.0'}{\cos 35^\circ\ 35.0' \quad \cos 37^\circ\ 32.0'} - \tan 35^\circ\ 35.0' \tan 37^\circ\ 32.0'$$

$$\cos Z = \frac{0.39715}{0.81327 \times 0.79300} - 0.71549 \times 0.76825$$

$$\cos Z = 0.61581 - 0.54968 = 0.06613$$

$$Z = 86^\circ\ 12.5'$$

By the author's tables:

$$\begin{array}{lrlr}
 & \text{Factor } A & & \text{Factor } B \\
h = 35^\circ;\ \phi = 37^\circ & 1.52811 & & 0.52723 \\
36.1 \times 32.0 & = 0.01155 & 36.1 \times 33.1 & = 0.01195 \\
32.0 \times 34.3 & = 0.01098 & 32.0 \times 32.4 & = 0.01037 \\
\hline
 & 1.55064 & & 0.54955 \\
\sin \delta & = 0.39715 & & \\
\hline
A \sin \delta & = 0.61584 & &
\end{array}$$

$$\cos Z = 0.61586 - 0.54955 = 0.06629$$

$$Z = 86^\circ\ 11.9'$$

Example 5.—Latitude = 40° 30.0′ (North); observed h = 21° 32.4′; declination = S 22° 30.0′; and, temperature = 0° C.

By the exact formula:

$$\begin{array}{rl}
\text{Observed } h & = 21^\circ\ 32.4' \\
\text{Parallax and refraction correction} & = \ \ 0^\circ\ \ \ 2.4' \\
\hline
h & = 21^\circ\ 30.0'
\end{array}$$

$$\cos Z = \frac{\sin\,(-22^\circ\ 30.0')}{\cos 21^\circ\ 30.0'\ \cos 40^\circ\ 30.0'} - \tan 21^\circ\ 30.0' \tan 40^\circ\ 30.0'$$

$$\cos Z = \frac{-0.38268}{0.93042 \times 0.76041} - 0.39391 \times 0.85408$$

$$\cos Z = -0.54089 - 0.33643 = -0.87732$$

$$Z = 151^\circ\ 19.2' \text{ (measured from the north)}$$

By the author's tables:

	Factor A		Factor B
$h = 21^\circ;\ \phi = 40^\circ$	1.39790		0.32143
32.4×16.1	$= 0.00522$	32.4×28.3	$= 0.00917$
30.0×35.0	$= 0.01050$	30.0×19.3	$= 0.00579$
A	$= 1.41362$		$B = 0.33639$
$\sin \delta$	$= -0.38268$		
$A \sin \delta$	$= -0.54096$		

$$\cos Z = -0.54096 - 0.33639 = -0.87735$$

$$Z = 151^\circ\ 19.4'$$

Example 4 gives a variation from the exact value as obtained from the formula of 0.6′. If Table 1 had been based on exact values of h, and if corrections had been made for parallax and refraction, the values of Z for this particular example would still be 86° 11.9′.

In both examples, it is noted that, from Table 1, that $A \sin \delta$ is larger and B smaller than the like expression in the exact formula. When $A \sin \delta$ is negative and is added to B the discrepancies compensate partly so that $\cos Z$ is not materially in error.

Examples 4 and 5 show that the proposed tables cannot be used where extreme accuracy is desired, in which case a more precise instrument than the transit should be used. It has been the writer's experience that the ordinary observer does well to obtain results accurate to the nearest minute with a transit. There are many instances in which azimuths or bearings to the nearest minute are sufficiently accurate and Table 1 seems to be well within this limit of accuracy.

The writer would like to see the complete table published. Due to his own labors in that direction he realizes the enormous amount of labor involved in its preparation, and that the author deserves considerable credit for his work.

F. J. DUARTE,[18] ESQ. (by letter).—A more complete table than Table 1 could be computed if the corrections for mean refraction and parallax were omitted. It would then be sufficient to compute another subsidiary table, for every degree of altitude of the mean refraction less the altitude parallax. With this auxiliary table the observed altitude angle would be corrected before entering Table 1.

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[18] Director del Observatorio de Caracas, Caracas, Venezuela.

The differences for 1′, both for altitude and latitude, have been computed on Table 1, dividing the first differences by 60 (chord interpolation). It would perhaps be more convenient to compute the derivatives (tangent interpolation), as follows:

$$f'(x) = \text{unit variation} = \frac{\Delta_1 - \frac{1}{2}\Delta_2 + \frac{1}{3}\Delta_3}{60}$$

The method described is easy to apply and precise enough for the orientation of a topographic map. The novelty of the method consists in Table 1 designed to facilitate the solution of the formula. This work, however, could be accomplished more rapidly by the use of a table of natural circular functions and a calculating machine, than with Table 1.

LEONARD C. JORDAN,[19] M. AM. SOC. C. E. (by letter).—For his effort to simplify and reduce the labor of field computations, Mr. Inch is to be commended. A transitman cannot always wait until evening to compute his bearings by candlelight in the office tent, but, sometimes, must take a solar observation, compute the results (perhaps on a windswept mountain side), and proceed to run lines in accordance with predetermined bearings. Any method that aids in making quick and accurate field computations will be welcomed by those men who find it suitable to their needs.

Attention should be called to the particular necessity for frequent correction of bearings on surveys that extend over considerable east and west distances. On such surveys, bearings very soon become incorrect, the errors increasing with the latitude of the location.

A base line, run in a straight line, 90° from any meridian, and from any latitude, will cross the equator when it is one-fourth of the distance around the world. Obviously, for a line north of the equator, the true bearing will be slightly southward, within the course of a few miles.

In sighting the sun, the writer has found it most convenient to focus the instrument on a distant object and, with his back to the sun, direct the telescope by means of its own shadow until it is nearly in the correct position. Then, the standard method of focusing the sun's image upon a page of field notebook held at the eye-piece (with the bisecting cross-hairs plainly visible) gives accurate results. Five independent observations, computed individually, generally are found to be in agreement. The most convenient transits for these observations are those which have additional hairs at 45° and at equal distances from the intersection of cross-hairs. These diagonal hairs are placed so that they cut thin segments from the sun's image. The slightest error in direction will cause one segment to be conspicuously longer than the opposite one. Looking through the instrument, no matter how shielded, should be avoided since the observer is almost certain to look past the telescope and directly into the sun. His work during the succeeding few minutes would be unreliable.

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[19] Cons. and Designing Engr., New Rochelle, N. Y.

J. C. Pinney,[20] M. Am. Soc. C. E. (by letter).—The determination of the true meridian by direct solar observation is a practical necessity in ordinary surveying work, and any method which will reduce the time necessary for computing the field observations without sacrificing accuracy is worthy of consideration. Several formulas have been developed for computing the azimuth, Z, of the sun from the spherical triangle. The four which the writer recalls give the solution in terms of $\cos Z$, $\text{vers } Z$, $\cot \frac{Z}{2}$, and $\sin \frac{Z}{2}$, and from these terms an equation can be developed for other trigonometric functions. With the ordinary instruments and methods used by the land surveyor, about 1′ in azimuth is the limit in accuracy, and computations carried beyond 0.1′ are futile.

Mr. Inch has presented a method for solving the cosine formula arithmetically in a much shorter time than a regular arithmetic solution would take without the use of a computing machine. This is accomplished by presenting the major computations pre-worked in the form of Table 1. The writer's main criticism of this table is that a straight-line interpolation is used for determining the constants, A and B, between whole degrees. For certain values of the altitude and latitude, this may lead to appreciable errors.

The cosine formula, Equation (1), is usually assumed as being adaptable to natural functions rather than to logarithms. The writer, however, believes that when properly arranged it is very susceptible to logarithms, and he presents herewith the solution of Example 1 by logarithmic computations of both the cosine and versine formulas, comparing the number of operations involved in these formulas and in Mr. Inch's solution. For convenience of reference the versine formula is given:

$$\text{vers } Z = \left\{ \sin\left[90 - (h + \phi) \right] + \sin \delta \right\} \sec h \sec \phi \ldots\ldots(10)$$

Example 6.—Using Equation (1), with north as the zero azimuth, and correcting the observed h for refraction and parallax:

$$\begin{array}{rrl}
\text{colog cos } 0.044128 & \ldots h = & 25^\circ\, 23.'6 \ldots \text{log tan } 9.676408 \\
\text{colog cos } 0.108851 & \ldots \phi = & 38^\circ\, 53.'7 \ldots \text{log tan } 9.906741 \\
\text{log sin } 8.258190n & \ldots \delta = & -1^\circ\, 02.'3 \\
\hline
\text{log } A \quad 8.411169n & A = & -0.02578 \\
 & B = & 0.38296 \qquad\qquad 9.583149 \\
\hline
 & A\text{–}B = & -0.40874 \\
 & = & \cos 245^\circ\, 52.'5 = \text{S } 65^\circ\, 52.'5 \text{ W}
\end{array}$$

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[20] Asst. Cadastral Engr., U. S. Biological Survey, Des Moines, Iowa.

Example 7.—Using Equation (10) with south as zero azimuth, and correcting h as in Example 6:[22]

$$\begin{array}{lrrcrl}
 & & h & = & 25^\circ\ 23.'6 & \text{.. log sec } 0.044128 \\
 & & \phi & = & 38^\circ\ 53.'7 & \text{.. log sec } 0.108851 \\
\hline
 & & \text{sum} & = & 64^\circ\ 17.'3 & \\
\text{nat sin} & 0.43384 & \text{....co-sum} & = & 25^\circ\ 42.'7 & \\
\text{nat sin} & -0.01812 & \text{....}\ \delta & = & -1^\circ\ 02.'3 & \\
\hline
\text{Algebraic sum} & 0.41572 & \text{..........................} & & & \text{log } 9.618801 \\
 & & & & & \text{log vers } 9.771780 \\
 & & & & Z = 65^\circ\ 52.'5 = & \text{S } 65^\circ\ 52.'5\text{ W}
\end{array}$$

Although the most rapid method for obtaining the final result depends largely upon the computer and the method of procedure with which he is best acquainted, it may be worth while, nevertheless, to compare these three solutions in respect to the number and kind of operations required in each. These operations may be classified as: Arithmetic multiplication and division, arithmetic addition and subtraction, and references to tables. Of these, multiplication and division are to be particularly avoided as tedious and, therefore, subject to errors.

TABLE 7.—COMPARISON OF WORK INVOLVED IN SOLUTIONS

Method	Example 6	Example 7	Inch method
Addition and subtraction	0	0	5
Multiplication and division	4	5	3
Table reference	7	7	2

The comparison (adding one more addition and one more table reference to the cosine and versine methods than is shown in Examples 6 and 7 because Table 1 includes refraction correction) is shown in Table 7. It is assumed for the cosine formula (Example 6) that the cosine and tangent combined will constitute only one table reference as the logarithmic sines, cosines, tangents, and cotangents are contained in a single table.

As compared with the method given by Mr. Inch the writer believes either of the other two methods are preferable, because of the five arithmetic multiplication and division operations alone. The cosine method seems to have a very slight advantage over the versine method only if the computer is able to write down the colog cosine directly from his reading of the log cosine. Even in this case there is a fertile field for error. This same objection is also true for the versine method in case the computer has no table of log secants (= colog cosines). If the computer is equipped with a handbook[21] containing consecutive, five-place tables of log secants, natural sines, logarithms of numbers, and log versines, the versine method is considerably shorter.

The foregoing discussion applies primarily to conditions where the computations are to be made in the field, or under circumstances where a computing machine is not available.

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[22] This solution is taken from "Azimuth" by the late George L. Hosmer, M. Am. Soc. C. E., and the arrangement is taken from Form Bi–1096 of the U. S. Biological Survey.

R. L. VAUGHN,[22] M. AM. SOC. C. E. (by letter).—Of all methods of obtaining a true bearing that by observation of Polaris at elongation is the most simple. It has several disadvantages, among which may be mentioned: (1) In high latitudes it is difficult to observe, and even more difficult to obtain, a desirable degree of accuracy; and (2) in low latitudes, even on clear nights, the star is likely to be obscured by atmospheric haze, the necessary arrangements for illuminating the cross-hairs, the verniers, and the station sighted on entail more or less bother. A fairly accurate determination of the latitude is essential. Lastly, the time of elongation is more than likely to be at a most inconvenient hour. Accurate local time is not indispensable.

The "any hour method" of observation on Polaris is subject to all but one of the foregoing objections, and furthermore requires accurate local time (not standard time). It does possess the advantage that the observation may be made at any time when Polaris is visible.

Methods of obtaining true azimuths or bearings by observation on the sun, of course, are by no means new. To an engineer not particularly versed in field astronomy and who has only occasional need for making an azimuth determination, the method may appear involved and laborious. Such, however, is not the case. Whenever a determination accurate to the nearest minute will suffice, sun observations will be found to be greatly superior to observations on Polaris both as regards speed and as regards convenience.

A number of formulas are available, one of which is given by the author in Equation (1). This formula is well adapted to computations where natural functions and a calculating machine are used. For computations using logarithmic functions a somewhat more convenient formula is:

$$\cot \tfrac{1}{2} A = \sqrt{\sec S, \ \sec (S - P), \ \sin (S - h), \ \sin (S - \phi)} \ldots\ldots (11)$$

in which A is the angle between the true south and the sun, measured clockwise if the observation was made in the afternoon; counter-clockwise, if the observation was in the morning; P is the north polar distance of the sun (90° plus a south declination, or minus a north declination); h is the altitude, of the sun, corrected for parallax and refraction; ϕ is the latitude of the station from which the observation is taken; and,

$$S = \tfrac{1}{2} (P + h + \phi) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Whenever S is less than P or h or ϕ, merely subtract the smaller angle from the larger and proceed without regard to the negative algebraic sign.

During the early years of its existence, regulations of the Bureau of Public Lands of the Insular Government of the Philippine Islands required that an astronomical determination of azimuth should be made as a part of each land survey. The solar method was preferred on account of reliability and convenience. The standard observation consisted of eight pointings on the sun, as shown in Table 8, Sets I and IV being made with the telescope erect and Sets II and III with the telescope reversed, and the A vernier being read in all cases. A fairly rapid instrumentman could set up an instrument and take the entire observation in a period of 20 min, or less; a fairly proficient computer could perform all the calculations in an additional 20 min. It cannot be said that the determination of a true bearing was an unduly burdensome task.

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[22] Cons. Engr., San Francisco, Calif.

The true purpose of the time of observation seems to be misunderstood by some surveyors. Accurate local time is not required for a solar observation. The only purpose for which time is used is to obtain the sun's declination at the time of observation, which requires that the Greenwich time of the observation be known within an accuracy of about 5 min. Within the area of the United States the observer's watch will almost certainly be keeping standard time, and in order to obtain Greenwich mean time it is merely necessary to add 5, 6, 7, or 8 hr to the observed time, depending on which standard time is carried. If the observation is made in some other part of the world, it will be necessary to ascertain in some manner the relation between watch time and Greenwich time, but in no case is the longitude of the place of observation needed. In some cases it will be found that the particular Ephemeris which happens to be available gives the sun's declination for Greenwich mean noon. In this case the calculation of the declination for the time of observation is simple and obvious. It may be found that the declination is given for Greenwich apparent noon, in which case the apparent time of observation may be found by adding or subtracting the equation of time (also given in the Ephemeris) to the mean time.

Solar attachments, and sextant telescopes, contain special arrangements of wires by the use of which the sun may be centered. Ordinary transits are not so equipped and it is necessary to make observations in pairs and use averages, as is indicated in Table 8. This method assumes that the path of the sun is a straight line between the two positions in each set. Such is not the case, but no appreciable error is introduced providing the observations are taken with reasonable speed. However, the telescope should not be reversed between the two pointings which constitute any one "set."

Two methods are available by means of which the necessary readings may be taken. In one method the sun's image and the pattern of the cross-hairs are cast upon a card held about 6 in. from the eye-piece. There will be no difficulty in focusing so as to obtain a sharp image of the sun, but the eye-piece must be especially set to obtain a sharp image of the cross-hairs. With some transits there will be no difficulty about focusing the eye-piece. With other instruments, however, it will be found impossible to obtain a sharp image of the cross-hairs within the limit of motion of the eye-piece. In such instances, the small screw which limits the motion of the eye-piece may be removed, after which the latter may be drawn outward a sufficient distance to give the required sharp image of the wires. Fairly good results may be obtained by this method in the hands of an experienced observer. However, at the exact instant of tangency of any wire with the edge of the sun, the image of that wire is not visible on the card and in order to get really accurate results the observer must be a good guesser.

TABLE 8.—AZIMUTH COMPUTATION FROM SOLAR
(Latitude, 37° 21′ 00″ N; Longitude, 122° 54′

Description (1)	Set I (Direct) Horizontal angles (2)	Set I (Direct) Vertical angles (3)
Mean	140°–03′	31°–28′–30″
Parallax and refraction		0°– 1′–20″
True altitude, h		31°–27′–10″
Local (average) time of observation	9:00 A.M.	
Longitude (hour angle)	8:00 A.M.	
Greenwich (average) time of observation	5:00 P.M.	
Interval from Greenwich noon	5:00 P.M.	
Corrected declination	S 11°–52′–24″	
±90°	± 90°– 0′– 0″	
North polar distance, P	101°–52′–24″	
Altitude, h	31°–27′–10″	
Latitude, ϕ	37°–21′– 0″	
$2S$	170°–40′–34″	
S	85°–20′–17″	
North polar distance, P	101°–52′–24″	
$S - P$	16°–32′–07″	
S	85°–20′–17″	
Altitude, h	31°–27′–10″	
$S - h$	85°–53′–07″	
S	85°–20′–17″	
Latitude, ϕ	37°–21′– 0″	
$S - \phi$	47°–59′–17″	
log sec, S	1.09 008	
log sec, $S - P$	0.01 834	
log sin, $S - h$	9.90 732	
log sin, $S - \phi$	9.87 099	
log cot^2, $(0.5A)$	0.88 673	
log cot, $(0.5A)$	0.44 336	
$0.5A$	19°–48′–45″	
A	39°–37′–30″	
Azimuth of sun (North = 0°–0′–0″)*	140°–22′–30″	
Angle from sun to mark†	140°–03′– 0″	
Azimuth, station to mark†, Set I	0°–19′–30″	
Azimuth, Set II	0°–19′–00″	
Azimuth, Set III	0°–19′–30″	
Azimuth, Set IV	0°–18′–30″	
Total; divide by four	0°–76′–30″	
Average azimuth, station to mark†	0°–19′–00″	
Bearing, station to mark†	N 0°–19′–00″ E	

* In the afternoon, the azimuth of the sun is A; in the forenoon, it is 360° A (from south point).

† For example, "East corner of the chimney, gymnasium of the P. Brunett, Jr., High School.

A much better method of observation is by the use of a diagonal eye-piece. Most of the better instruments are fitted to receive this simple and inexpensive attachment which consists, essentially, of a mirror and a colored-glass shade. When using the diagonal eye-piece it is not necessary to change the focusing of the wires, and the latter will be visible whenever they are in any

Observations; Solution of Equation (11)

00″ W; Standard Time Meridian, 120° W)

Set II (Reversed)		Set III (Reversed)		Set IV (Direct)	
Horizontal angles (4)	Vertical angles (5)	Horizontal angles (6)	Vertical angles (7)	Horizontal angles (8)	Vertical angles (9)
320°-38′-30″	31°-46′-30″ 0°- 1′-20″ 31°-45′-10″	321°-16′-30″	32°-06′-0″ 0°- 1′-20″ 32°-04′-40″	142°-13′-30″	32°-33′- 0″ 0°- 1′-20″ 32°-31′-40″
9:00 A.M.		9:00 A.M.		9;00 A.M.	
....		....		....	
S 11°-52′-24″ +90°- 0′- 0″		S 11°-52′-24″ +90°- 0′-00″		S 11°-52′-24″ +90°- 0′- 0″	
101°-52′-24″ 31°-45′-10″ 37°-21′-00″		101°-52′-24″ 32°-04′-40″ 37°-21′-00″		101°-52′-24″ 32°-31′-40″ 37°-21′-00″	
170°-58′-34″		171°-18′-04″		171°-45′-04″	
85°-29′-17″ 101°-52′-24″		85°-39′-02″ 101°-52′-24″		85°-52′-32″ 101°-52-24″	
16°-23′-07″		16°-13′-22″		15°-59′-52″	
85°-29′-17″ 31°-45′-10″		85°-39′-02″ 32°-04′-40″		85°-52′-32″ 32°-31′-40″	
53°-44′-07″		53°-34′-22″		53°-20′-52″	
85°-29′-17″ 37°-21′-00″		85°-39′-02″ 37°-21′-00″		85°-52′-32″ 37°-21′-00″	
48°-08′-17″		48°-18′-02″		48°-31′-32″	
1.10 423 0.01 800 9.90 649 9.87 201		1.12 005 0.01 765 9.90 559 9.87 311		1.14 307 0.01 716 9.90 442 9.87 462	
0.90 073 0.45 036		0.91 630 0.45 815		0.93 927 0.46 964	
19°-31′-15″ 39°-02′-30″		19°-12′-00″ 38°-24′-00″		18°-44′-00″ 37°-28′-00″	
140°-57′-30″ 140°-38′-30″		141°-36′-00″ 141°-16′-30″		142°-32′-00″ 142°-13′-30″	
0°-19′-00″		0°-19′-30″		0°-18′-30″	

position near the edge of the sun. It will be found necessary to remove the attachment each time the instrument is transited.

Observed altitudes of the sun must be corrected for refraction. The correction is always subtractive, and is usually obtained from a table of average refraction values. Theoretically, there should be a correction for parallax because the point of observation is on the surface of the earth, not at its center. Parallax is not of great importance in a solar observation, although it is a serious factor in lunar observations. However, both refraction and parallax are a function of the altitude and may be combined quite simply. Many standard texts give tables of combined refraction and parallax, and the correction should be taken from such a table rather than from one giving refraction only. In this connection it is well to emphasize the fact that the author has incorporated the parallax and refraction correction in his table and when using it the observed altitudes should not be corrected. In fact, attention should be invited to this circumstance in the heading to the tables.

Properly used, the solar method is capable of excellent results. The writer does not approve of reversing the instrument between the two observations constituting a pair, as indicated by the author in Fig. 2; nor should reliance be placed on a single pair of observations. The method used by the Bureau of Lands was found to be quite satisfactory. The results of Sets I and IV, Table 8, should agree within 1 min, as should the results of Sets II and III. The discrepancy between the former and the latter may be considerably wider, reflecting the effect of lack of instrumental adjustment. Taking the average of all four sets will eliminate the effect of such instrumental errors and, of course, will minimize accidental errors of observation.

One instrumental error that cannot be eliminated by the method of reversal, is the effect of error in leveling the horizontal plates. Any error in the determination of the latitude will be reflected to an appreciable extent in the results. However, error from this source may be eliminated by taking one complete series of observations in the morning and another in the afternoon.

The method of dealing with horizontal angles shown by the author in Fig. 1 is correct; but with the sun in a different relative position, it might cause confusion because of the necessity of measuring angles in a counterclockwise direction. Adherence to the following rule will avoid all difficulty: Set the horizontal plates so that they read 0° 00′ when pointing along the line the direction of which is to be determined. Read all horizontal angles to the sun in a clockwise direction, from 0° to 360°. Compute the azimuth of the sun (not its bearing). Subtract the observed horizontal angle from the computed azimuth of the sun, adding 360° to the latter if necessary. The remainder is the desired azimuth of the line.

Table 1 of the paper certainly represents an impressive mass of labor, and the publication of the complete table would be a service to the profession which should be encouraged. It entails double interpolation: One to reach the values for the actual latitude, and another to reach the actual altitude. Interpolation by second differences must be resorted to in order to achieve an accuracy in the computations commensurate with the precision of the method. Experience has shown that it is almost impossible to publish such a table for the first time without some errors. The possibility of such errors should be guarded against when using the table.

The formulas given in this discussion apply equally well to any other heavenly body. The Nautical Almanac gives the necessary data concerning the planets and a selected list of fixed stars. The only difficulty is to identify the particular body under observation. Star charts are available by means of which the fixed stars may be identified. Planets are most easily identified by computing the approximate elevation and time of transit across the local meridian. In some locations and under some atmospheric conditions, it may be desirable to determine azimuth by observation on some object other than the sun or Polaris. In case a planet is observed, which is closer to the earth than the sun, the parallax correction should be given due consideration. Observations on the moon are not likely to give very good results, and this body is best avoided.

JOHN C. PENN,[23] M. AM. SOC. C. E. (by letter).—The most favorable time for observations on the sun is when it is on or near the prime vertical; that is, when it is east or west, or nearly so. The azimuth of the sun is then ± 90° (0° is assumed to be north). Any formula that solves the azimuth by means of its "cosine" necessitates entering a logarithmic table or tables of natural functions where the interpolation is most difficult and most inaccurate. In the United States, most work will be done in the field when the sun's declination is north (that is, in the time between March 21 and September 21), and with a lower limit for altitude of 25° (because of uncertainty in refraction correction below that figure), azimuth will fall between 80° and 100°, and slide-rules, tables, and any other arrangements for using $\cos Z$ will not be accurate.

Any astronomical method for finding azimuth requires the nautical almanac, an ephemeris, or a similar table. The choice of the year 1910 in the example in Mr. Inch's paper is unfortunate. The astronomical day, like the civil day, has begun at midnight ever since 1925. The American Nautical Almanac now (1937) contains tables for refraction, parallax, etc.

The solar ephemerides, as published by instrument companies for use with solar attachments, contain tables for refractions, but care must be taken in not using these refraction corrections for the setting of the attachment. Refraction corrections involve the latitude of the place and cannot be applied as corrections for an observed altitude.

There are several other formulas available, and surveyors are not necessarily limited to observations on the sun for the determination of meridian. Navigators use,

$$\cos \frac{Z}{2} = \sqrt{\cos S \cos (S - P) \sec \phi \sec h} \;\ldots\ldots\ldots\ldots (13)$$

in which Z = the azimuth of a heavenly body (star, planet or sun); $S = \frac{1}{2}(h + \phi + P)$ in which h = corrected altitude; ϕ = latitude; and P = polar distance (90° ± declination). For those not familiar with "secants," this formula may be written:

$$\cos \frac{Z}{2} = \sqrt{\frac{\cos S \cos (S - P)}{\cos \phi \cos h}} \;\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Equation (14) solves for azimuth in terms of $\frac{Z}{2}$ and involves logarithmic tables in the vicinity of 45° where the values for cosine mean something.

It is a pity that the present American Nautical Almanac gives the azimuth of Polaris for various hour angles of Polaris only to the nearest tenth of a degree. Older almanacs and the American Ephemeris give azimuths of

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[23] Director of Civ. Eng., Armour Inst. of Technology, Chicago, Ill.

Polaris to 0.1′. If the American Nautical Almanac gave the same consideration to surveyors as it now does to aviators, the necessity for tables and slide-rules would be entirely obviated and, incidentally, there would be no need for an accurate map for the determination of latitude, as both azimuth and latitude can be determined by an observation on Polaris with very little arithmetic.

If the table suggested by Mr. Inch is to be of any value to a surveyor it must be well understood that the surveyor goes into the field with the following equipment:

(*A*) The table itself, extended to allow for better interpolation;

(*B*) Instructions for use of this table to include: (1) A bold statement that altitudes need not be corrected for parallax and refraction; (2) a similarly strong statement that if the declination of the sun is north, $A \sin a$ and B are subtracted (the smaller from the larger); (3) if $A \sin a$ is the larger, the bearing of the sun will be from the north, and if B is the larger, the bearing is from the south; and (4) if the declination is negative, the bearing is from the south;

(*C*) A table of natural sines;

(*D*) An almanac for the year;

(*E*) A watch correct within a few minutes; and,

(*F*) A quadrangle sheet of the U. S. Geological Survey, or a similar map for the determination of latitude. (Perhaps this map could be left in the office.

Some of these items are facetious, of course. A little knowledge of trigonometry will eliminate Items *B* (2), *B* (3), and *B* (4), but to one who is not "on his toes" in the mathematics of surveying and the bearing should prove to be 90° ± 20′, these very rules will save many an anxious moment.

The writer would rather carry an almanac, a log table, with the value of cos 0.5 Z neatly printed inside the cover, and a small pad of paper, for meridian determinations during the day. The solution of Example 1 of the paper, by Equation (13) is as follows: Refraction plus parallax correction is 0° 1′ 54″; corrected altitude equals 25° 23′ 36″.

$$\begin{array}{rcll}
P &=& 91^\circ\ 02'\ 16'' & \\
h &=& 25^\circ\ 23'\ 36''\ \log\sec & 0.04413 \\
\phi &=& 38^\circ\ 53'\ 40''\ \log\sec & 0.10885 \\
2 &\big\lfloor& \underline{155^\circ\ 19'\ 32''} & \\
S &=& 77^\circ\ 39'\ 46''\ \log\cos & 9.32974 \\
S - P &=& 13^\circ\ 22'\ 30''\ \log\cos & \underline{9.98835}\ \text{(always positive)} \\
 & & & 2\,\lfloor 19.47107
\end{array}$$

$\log\cos 0.5\,Z = 9.73554 = 57^\circ\ 3'\ 4''$; and $Z = 114^\circ\ 6'\ 8''$.

$$\begin{array}{rl}
\text{Bearing of line} = 180^\circ - (114^\circ\ 6'\ 8'') = & \text{S } 65^\circ\ 53'\ 52''\ \text{W} \\
\text{Given} = & \underline{\text{S } 64^\circ\ 52'\ 30''\ \text{W}} \\
 & 1^\circ\ \ 1'\ 22'' \\
\text{By table} = & \underline{1^\circ\ \ 0'\ 48''} \\
\text{Difference} = & 0^\circ\ \ 0'\ 34''
\end{array}$$

In the determination of azimuth by a time-altitude of the sun, regardless of the method that may be used, some knowledge of astronomy is absolutely necessary. Without a calculating machine, and in the field, logarithms must take preference over natural functions. All methods so far proposed require either one or the other. A table so involved and so limited as all tables must be, can never take the place of the foregoing simple computation.

Robert H. Merrill,[24] M. Am. Soc. C. E. (by letter).—By way of revising "tables in their most convenient form, as an Appendix", with due allowance for second differences, Fig. 3 shows (in the four margins) the four rates per minute of arc that are involved. In plotting the two columns under each degree of latitude from Table 1, many points fail to fall precisely on line. To secure uniformly spreading curves, these columns (to be eliminated by a graph) should be recalculated accurately to an additional decimal place.

For comparison the rates used in the typical example given in the paper are tabulated in Column (2) of the following example; the rates at the terminal values, $h = 25°\ 25'\ 30''$ and $\phi = 38°\ 53'\ 40''$ in Column (3) are also not necessary. Only at the mid-points of the interpolation (namely, at $h = 25°\ 13'$ and $\phi = 38°\ 27'$ are the true values found which are listed in Column (4).

Rate of:	In: (1)	From: (2)	To: (3)	Mean: (4)		Minutes added to $h=25°$ and $\phi=38°$ (5)		A 1.39989 (6)	B 0.36378 (7)
A	h	19.4	20.2	19.8	×	25.5	=	505	...
A	ϕ	32.6	34.3	33.5	×	53.7	=	1 798	...
B	h	27.9	29.1	28.5	×	25.5	=	...	726
B	ϕ	22.1	23.2	22.6	×	53.7	=	...	1 212
								1.42292	0.38316

$d = -\ 1°\,02'\,16''$ nat sin $= \times\ 0.01811 = 0.02580$

$Z = 65°\,51'\,38''$ nat cos $= 0.40896$

Instead of $Z = 65°\,53'\,18''$ (of the paper) $= 0.40852$
and since $Z = 65°\,52'\,30''$ by more precise calculation, the range in using Table 1 is + or − more than 30″.

The late James B. Davis, M. Am. Soc. C. E., explained forty years ago "the mystery as to why the solar attachment has not come into more general use" by emphasizing the delicacy of adjustments, the extra weight, and the cost, which is quite out of proportion to its infrequent use. About twenty-three years ago a "mechanical navigator"[25], costing $2 400 was placed on the market. It solved, mechanically, the general spherical triangles, but was heavy and bulky, with seven axes measuring altitude, latitude, hour angle, azimuth, declination, and 90° on 6-in. disks with verniers that were precise to $\frac{1}{1200}$ in. Obviously, great mechanical skill was attained in adjusting all parts so perfectly that the last setting gave results accurate to 1′ or 2′ of arc. With a relatively cheap 10-in. straight slide-rule, or a 7-in. circular slide-rule, one may attain similar accuracy in solving the same problems.

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[24] Cons. Engr. (Spooner & Merrill), Grand Rapids, Mich.

[25] *Engineering News*, Vol. 71, p. 180.

For greater precision it is easy to recompute by the rather clumsy but adequate formula, using both naturals and logarithms of standard field tables thus, to solve the formula, $\cos Z = \dfrac{\sin d - \sin h \sin \phi}{\cos h \cos \phi}$:

Quantity	Angle	nat sin	log sin	log cos
d (–)	1° 02′ 16″	–0.01811	....	....
h	25° 23′ 37″	....	9.632290	9.955872
ϕ	38° 53′ 40″	....	9.797882	9.891149
$\sin h \sin \phi$	....	0.26926	9.430172	....
$\cos h \cos \phi$	....	....	....	9.847021
Numerator	....	–0.28737	....	9.458441
Z	65° 52′ 32″	....	....	9.611420

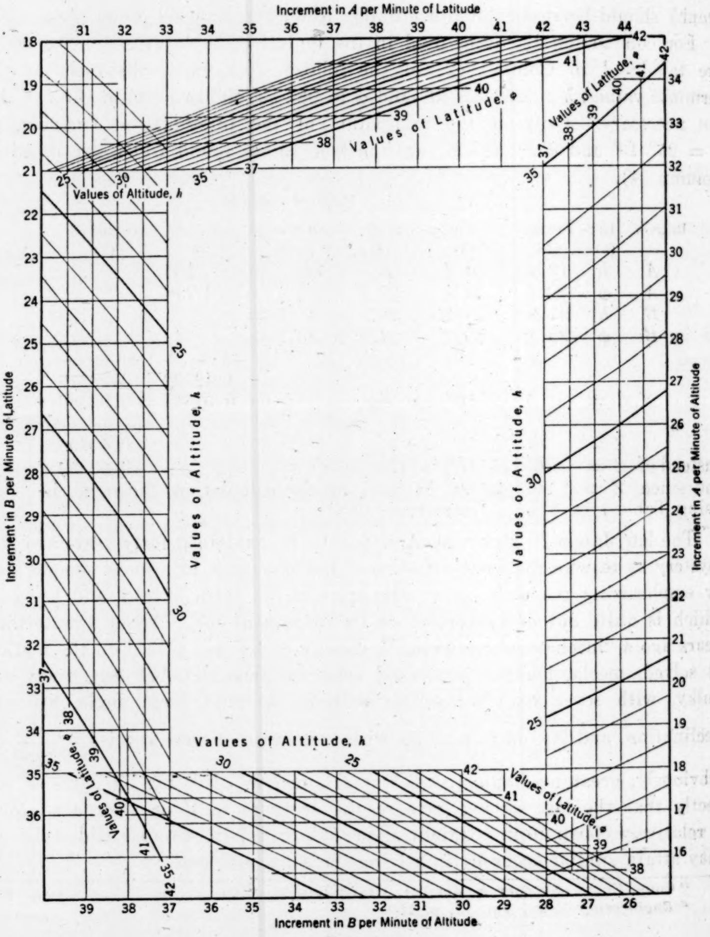

FIG. 8.

The writer favors the sun-disk bisections for observations on the sun as the easiest and the one developing all the accuracy of which an ordinary 1′ transit is capable. In the absence of colored-glass shades and a prismatic eye-piece, it is often handy to use a carpenter's 24-in. folding rule to hold the focusing card. With a spring clip near the 9-in. mark, clamp the 6-in. to 12-in. leg of the rule to the left front standard of the transit. Flex the rule at the 12-in. and the 18-in. joints so that the 21-in. mark falls on the line with the telescope about 3 in. below the eye-piece, and clip on the card at that point.

Although it is not necessary to use the uniform time intervals it is easy to check as the observations progress, according to the following schedule:

	Watch Time:					
Hours	Minutes	Seconds	*Z*	*dZ*	*h*	*dh*
4	03	00	276° 03′		31° 10′	
				0° 04′		0° 13′
4	04	00	276° 07′		30° 57′	
				0° 05′		0° 14′
4	05	00	276° 12′		30° 43′	
				0° 04′		0° 13′
4	06	00	276° 16′		30° 30′	
				0° 04′		0° 15′
4	07	00	276° 20′		30° 15′	

The uniformity of the increments per unit of time, shows at a glance that the middle pointing of the five was a fair mean value in spite of the erratic intervals. It is customary, however, to require the series of pointings to be continued until five consecutive ones show equal increments to the nearest 1′. Note also that, if well chosen, the more rapid motion in *dh* lessens the probable error in computed azimuth.

CROSBY J. WILKIN,[28] ASSOC. M. AM. SOC. C. E. (by letter).—Textbooks, slide-rules, tables, etc., are of little avail in obtaining the correct true bearings of a line if the data used in the formula are not right. The writer has yet to see a typical problem fully solved by the use of commonplace data. In his work, he would prefer to use true azimuths in all property surveys but he is somewhat hesitant to do so without considerably more experience to gain complete confidence in his computations; and he can imagine that surveyors with less education would be more non-plussed. This is probably the main reason why most surveys are based on a magnetic north, which is unreliable for re-running surveys, large or small, particularly in the absence of sufficient monuments.

The following solution was made on a survey in the summer of 1936. An engineer's transit was used in the field work and the writer obtained the data without assistance. The sun's image was focused on a white card wired to the telescope and the cross-hairs were made tangent to the image.

For the calculations, the Ephemeris of the Sun and Polaris, obtained from the U. S. Government Printing Office, was used, together with a calculating machine. The method is a composite of various other methods none of which was complete in itself for the tables at hand.

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[28] Land Surv., Gardiner, N. Y.

Field Work: Date, July 22, 1936; long., W 74°-08.4′; lat., 41°-38.1′; watch slow 9 sec.; temperature, 75° F:

Observation	Time A.M.	Horizontal angle	Vertical angle
Direct	8:54:35	115° -42′	44° -46′
Direct	8:56:04	116° -00′	45° -02′
Direct	8:56:55	116° -12′	45° -12′
Direct	8:57:42	116° -22′	45° -20′
Reversed	9:01:10	296° -19′	46° -31′
Reversed	9:02:35	296° -38′	46° -46′
Reversed	9:03:28	296° -49′	46° -55′
Reversed	9:04:28	297° -02′	47° -05′
Average	8:59:37.1	116° -23′.0	45° -57′.1

Computations:

Correction	+9. watch slow	— 0′.8 parallax and refraction
Corrected,	8:59:46.1 A.M.	45° -56′.3

Standard time of observation	8:59:46.1 A.M.	
Difference in time from standard time meridian, 75° — (74° -08′.4) = 0° -51′.6; or 0.86 × 4 = 3.44′ = 4 min for a degree of longitude	+3:26.4	
Local mean time of observation	9:03:12.5	
Equation of time	—6:17.3	
Apparent time of observation	8:56:55.2	= 3:03:04.8 before noon
Longitude reduced to time (74° -08′.4) ÷ 15 = or difference in time from Greenwich noon and place of observation.	4:56:33.6	
Time of observation before noon	—3:03:04.8	
Time elapsed since Greenwich noon	1:53:28.8	= 1.892 hr
Apparent declination at Greenwich noon	N + 20° —16.1′	
Difference for 1 hr = 0.50′		
Difference for difference in time, 0.50 × 1.892 =		— 0.95
Declination for place of observation	20° -15.15′	

$$\cos Z = \frac{\sin d - \sin h \sin l}{\cos h \cos l} = \frac{-0.13127}{0.51976} = 0.25276, \text{ or } Z = -75° \text{-}22'$$

from south point, in which:

$d = 20°\text{-}15.15'$ $h = 45°\text{-}56.3'$ $l = 41°\text{-}38.1'$

Horizontal angle based on magnetic needle	116° -23′
True azimuth of sun 180° — 75° 22′	104° -38′
Declination of needle	11° -45′ W.

The writer has no criticism to offer of the method proposed by Mr. Inch, for he gladly adopts simplification of any process in solving a problem which is as accurate as the usual method. However, the addition of a set of tables to the Ephemeris of the Sun and Polaris might only confuse the computer further. It seems to the writer that engineers had better continue using the customary method, eliminating any steps in the solution compatible with the precision required in the answer.

The difficulty with the usual solution is that, apparently, no one has endeavored to publish a method using the ephemeris only. The formula introduced in the foregoing example is easy to use with any calculating device, or even by long hand.

C. S. Jarvis,[27] M. Am. Soc. C. E. (by letter).—A simplified method of determining azimuth by direct observation of the sun has been ably presented by the author, but simpler methods of determining true bearings of a line deserve consideration and extended use.

It is possible to determine the true meridian within an error of 1′ or 2′ consistently day after day, by using methods similar to the one described by the author, differing only in details, with or without the solar attachment for the ordinary engineer's transit; but it happens too often that the uncertainties or errors of bearing from such methods are considerably larger. No doubt this fact guided the standard practice of the General Land Office in requiring that the bearings of land-subdivisional lines be checked occasionally by observations on Polaris, and that the records of such tests and results be incorporated in the official field notes of public land surveys.

In view of the accessibility of correct standard time to within a second or two, through telegraphy and radio broadcasts, it appears that the transit of the sun's center across the meridian at apparent noon should be included. True solar time, derived from standard time by corrections for longitude and for the equation of time as published in a solar ephemeris, might be as much as 45 min slower or faster than standard time. An error of 4 sec in time would account for an error of 1′ of arc.

As the sun's semi-diameter varies from 15′45″ on July 1 to 16′18″ on January 1, and the time of its passing the meridian varies from 64 sec to 71 sec, as obtained from a solar ephemeris, it is quite feasible to observe the western limb of the sun at the corresponding time before apparent noon, and the eastern limb at the same interval after noon, but with the telescope reversed. These two lines should coincide on the meridian, except as affected by instrumental errors. A bisector of the small angle between them, therefore, should be the true meridian, obtained by the simplest method available, but susceptible to considerable errors if the time is not known accurately. However, by observing the sun's passage over the highest point of its orbit, together with its position a few minutes before and after noon (that is, at equal altitudes) both the apparent solar time and the meridian may be determined approximately, perhaps within 5′ as to bearing of the line.

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[27] Hydr. Engr., SCS, Washington, D. C.

Caution should be exercised to avoid injury to the observer's eyes through exposure to direct rays of the sun. Colored glasses of suitable density, or a colored translucent shield to be placed over the object glass of the telescope, may afford the required protection. However, after using such expedients and having to devise other methods when necessary, the writer has abandoned the telescopic sighting except for the purpose of bringing the sun's image into the field of the telescope. Thereafter, the observer's eyes are directed away from the sun in order to adjust the cross-wires, through movement of the tangent screws, to the edge of the solar disk as projected on a sheet of white paper, held perpendicular to the telescopic line of sight, and a few inches from the eye-piece. The cross-wires appear in magnified dimensions if the eye-piece is moved sufficiently from its normal position; that is, if thrown out of focus, and the paper is held far enough from the telescope. With such a magnified image of the sun and the cross-wires, either the bisecting or the tangential position may be readily attained by slow motion, vertically and horizontally.

Of all methods used by the writer for determining a meridian, the hour-angle observation of Polaris proved to be the most satisfactory. Utilizing twilight for illuminating the cross-hairs of the telescope, morning or evening, or both, the direct and reversed readings on the star and on the traverse line may be completed with very little time devoted to instrumental work and final calculations. Moreover, by occupying a station on the traverse line and sighting to one of the flags or marks, the observer is enabled to obtain the data for determining the bearings without the use of a flagman or rodman. As compared with observations at either culmination or elongation of Polaris, often involving tedious hours of waiting during the night, the hour-angle method has many advantages.

At the time of the vernal equinox (about March 21) the right ascension or time of meridian passage of Polaris at upper culmination is 1 hr 38 min by both sidereal and solar time, and the former gains nearly 4 min per day on the latter so as to accommodate one more day each year. It is evident, therefore, that on April 21 the upper culmination will occur 2 hr earlier, or at 11:38 A. M., and the lower culmination at 5:36 P. M., so that a twilight observation at about an hour later (assuming the latitude as 40° N), would define a line bearing 21′ east of north, and the altitude of the star would be about 39 degrees. An error in time amounting to 3 min would account for an error of 1′ in azimuth at or near culmination; or for the entire hour extending equally before and after elongation, the change of bearing would not exceed 1′, the actual azimuths at 30-min intervals being 80′.1, 80′.7, and 79′.8, beginning with 5 hr 30 min as the hour angle.

When the atmosphere is exceptionally clear and free from haze, dust, and smoke, it is quite possible to locate Polaris through a transit telescope a few minutes before sunset or after sunrise, and thus to complete the observation and calculations within daylight. In order to bring the star within the field of the telescope, it is necessary to set off an angle on the vertical circle to elevate the telescope equal to the latitude of the station plus or minus the correction for position of Polaris above or below the pole; or the cosine of the hour angle times 62′. Then, by setting the horizontal circle at approximately the correct position for the star, and using the tangent screw to search back and forth, the image should be made to streak part way across the field; and after centering it, the observation is soon completed.

PHILIP L. INCH,[28] ASSOC. M. AM. SOC. C. E. (by letter).—The intent and purpose of Table 1 was to enable an engineer to obtain a meridian, within a limit of 1′ of accuracy in azimuth, with the least inconvenience or preparatory study, ever bearing in mind that thousands of surveyors are not equipped educationally to cope with trigonometric formulas, logarithms, etc.; nor are computing machines generally available. Although the discussion has revealed little on the subject of the form of the table, the writer believes that the methods discussed have enriched the general knowledge of the subject.

The writer approves the suggestion to eliminate the use of Greek symbols, however familiar they may be; he agrees that less confusion will result.

As to star observations: To observe the upper or lower culmination of Polaris, or to determine a meridian by an hour-angle observation requires accuracies in time readings that are too great for those who are working to an accuracy of 1′. Furthermore, both these methods and elongation require night work unless an instrument is specially equipped for daylight observations.

The determination of a meridian by the equal altitude method is orthodox; but, again, the necessary preparation and the time consumed fits it inadequately to the needs of the field engineer.

The computation of the fifth decimal place of the table has received some comment. It will be noted that, in the tables of natural sines and cosines, a 1′ variation in either appears as from 1 to 3 in the fourth decimal place. In the determination of the fifth place, an even number for a 5 in the sixth place was maintained.

The discussion has included the advisability of computing the table of 1′ differences in A and B upon the arc, instead of one-sixtieth of the difference between two arc-determined, even degrees. Intensive investigation has demonstrated that such a refinement would add little or no increase in the general accuracy of the table. Before any one of the basic calculations was accepted, from two to six observations on the arc were computed and comparisons made. An experimental table was also constructed and comparisons were made of solar observations in latitudes as high as 60° north; the declinations used ranged from the highest to the lowest. The computation by table was found to be well within the required limits. For reference, the entire table has been included herein as Table 9. This form includes values of A and B for vertical angles, h, from 15° to 54°, inclusive; and latitude, ϕ, from 31° N to 48° N.

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[28] U. S. Cadastral Engr., Public Survey Office, Reno. Nev.

TABLE 9.—VALUES OF FACTORS *A* AND *B*, FOR A GIVEN ALTITUDE OF THE SUN, AT A GIVEN LATITUDE ON THE EARTH
(Including corrections for parallax and refraction)

Vertical angle, *h*	Symbol	LATITUDE, $\phi = 31°$			LATITUDE, $\phi = 32°$			LATITUDE, $\phi = 33°$			LATITUDE, $\phi = 34°$			LATITUDE, $\phi = 35°$			LATITUDE, $\phi = 36°$		
		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′	
			Vert. angle, *h*	ϕ		Vert. angle, *h*	ϕ		Vert. angle, *h*	ϕ		Vert. angle, *h*	ϕ		Vert. angle, *h*	ϕ		Vert. angle, *h*	ϕ
15°.......	*A*	1.20746	9.8	21.7	1.22045	9.9	22.7	1.23409	10.0	23.9	1.24843	10.1	25.1	1.26351	10.2	26.4	1.27932	10.4	27.7
	B	0.16036	18.9	10.7	0.16677	19.6	10.9	0.17331	20.4	11.2	0.18001	21.2	11.4	0.18687	22.0	11.7	0.19390	22.8	12.1
16°.......	*A*	1.21331	10.3	21.8	1.22637	10.6	22.9	1.24008	10.7	24.0	1.25448	10.8	25.3	1.26964	11.0	26.5	1.28553	11.1	27.9
	B	0.17168	19.1	11.4	0.17854	19.9	11.7	0.18556	20.6	12.0	0.19273	21.4	12.2	0.20007	22.2	12.5	0.20759	23.1	12.9
17°.......	*A*	1.21961	11.2	21.9	1.23273	11.3	23.0	1.24651	11.5	24.1	1.26098	11.6	25.4	1.27622	11.7	26.6	1.29219	11.9	28.0
	B	0.18313	19.3	12.2	0.19045	20.0	12.5	0.19793	20.8	13.1	0.20558	21.6	13.1	0.21341	22.5	13.4	0.22143	23.3	13.7
18°.......	*A*	1.22634	12.0	22.0	1.23953	12.1	23.1	1.25338	12.3	24.3	1.26794	12.4	25.5	1.28326	12.5	26.8	1.29932	12.7	28.2
	B	0.19468	19.5	13.0	0.20246	20.3	13.3	0.21042	21.0	13.6	0.21855	21.8	13.9	0.22688	22.7	14.2	0.23541	23.5	14.6
19°.......	*A*	1.23352	12.8	22.1	1.24679	12.9	23.2	1.26073	13.0	24.4	1.27537	13.2	25.7	1.29078	13.4	26.9	1.30694	13.5	28.3
	B	0.20637	19.7	13.7	0.21461	20.5	14.1	0.22304	21.3	14.4	0.23166	22.1	14.7	0.24049	23.0	15.1	0.24953	23.8	15.5
20°.......	*A*	1.24118	13.5	22.3	1.25453	13.7	23.4	1.26855	13.8	24.6	1.28329	14.0	25.8	1.29879	14.1	27.1	1.31504	14.3	28.5
	B	0.21820	20.0	14.5	0.22692	20.8	14.9	0.23583	21.6	15.0	0.24494	22.4	15.6	0.25427	23.3	16.0	0.26384	24.1	16.3
21°.......	*A*	1.24929	14.4	22.4	1.26272	14.6	23.5	1.27684	14.7	24.7	1.29167	14.9	26.0	1.30727	15.1	27.3	1.32363	15.3	28.7
	B	0.23017	20.2	15.3	0.23937	21.0	15.7	0.24877	21.9	16.0	0.25838	22.7	16.4	0.26823	23.6	16.8	0.27832	24.5	17.2
22°.......	*A*	1.25792	15.2	22.6	1.27145	15.3	23.7	1.28566	15.6	24.9	1.30060	15.7	26.2	1.31631	15.9	27.5	1.33278	16.1	28.9
	B	0.24231	20.5	16.1	0.25199	21.3	16.5	0.26189	22.2	16.9	0.27201	23.0	17.3	0.28237	23.9	17.7	0.29299	24.8	18.2
23°.......	*A*	1.26705	16.1	22.7	1.28068	16.3	23.9	1.29500	16.4	25.1	1.31004	16.6	26.4	1.32586	16.8	27.7	1.34246	17.0	29.1
	B	0.25461	20.8	17.0	0.26479	21.6	17.3	0.27519	22.5	17.7	0.28582	23.4	18.2	0.29671	24.3	18.6	0.30787	25.2	19.1
24°.......	*A*	1.27670	17.1	22.9	1.29044	17.3	24.0	1.30486	17.5	25.1	1.32002	17.7	26.6	1.33596	17.9	27.9	1.35268	18.1	29.3
	B	0.26709	21.1	17.8	0.27777	22.0	18.2	0.28868	22.8	18.6	0.29983	23.7	19.1	0.31126	24.6	19.5	0.32296	25.6	20.0
25°.......	*A*	1.28694	17.9	23.1	1.30079	18.0	24.2	1.31533	18.2	25.5	1.33061	18.5	26.8	1.34668	18.7	28.1	1.36353	18.9	29.6
	B	0.27977	21.5	18.6	0.29095	22.3	19.1	0.30238	23.2	19.5	0.31407	24.1	19.9	0.32603	25.0	20.4	0.33829	26.0	21.0
26°.......	*A*	1.29765	18.9	23.3	1.31161	19.1	24.4	1.32627	19.4	25.7	1.34168	19.6	27.0	1.35789	19.8	28.3	1.37488	20.1	29.8
	B	0.29265	21.9	19.5	0.30435	22.7	19.9	0.31630	23.6	20.4	0.32853	24.5	20.9	0.34104	25.6	21.4	0.35387	26.4	21.9
27°.......	*A*	1.30901	19.9	23.5	1.32309	20.1	24.7	1.33788	20.4	25.9	1.35342	20.6	27.2	1.36976	20.8	28.6	1.38691	21.1	30.1
	B	0.30577	22.2	20.4	0.31798	23.1	20.8	0.33047	24.0	21.3	0.34324	25.0	21.8	0.35632	25.9	22.3	0.36972	26.9	22.9
28°.......	*A*	1.32095	21.0	23.7	1.33516	21.2	24.9	1.35009	21.4	26.1	1.36577	21.7	27.5	1.38226	21.9	28.9	1.39957	22.2	30.3
	B	0.31910	22.7	21.3	0.33186	23.6	21.7	0.34489	24.5	22.2	0.35822	25.4	22.8	0.37187	26.4	23.3	0.38585	27.4	23.9
29°.......	*A*	1.33352	22.0	23.9	1.34786	22.3	25.1	1.36293	22.4	26.4	1.37876	22.8	27.8	1.39541	23.1	29.1	1.41288	23.4	30.6
	B	0.33270	23.1	22.2	0.34599	24.0	22.7	0.35958	25.0	23.2	0.37348	25.9	23.7	0.38771	26.9	24.3	0.40229	27.9	24.9
30°.......	*A*	1.34676	23.2	24.2	1.36125	23.4	25.0	1.37646	23.7	26.7	1.39245	24.0	28.0	1.40927	24.3	29.4	1.42691	24.6	30.9
	B	0.34655	23.6	23.1	0.36039	24.5	23.6	0.37455	25.5	24.1	0.38902	26.5	24.7	0.40385	27.5	25.3	0.41903	28.5	26.0
31°.......	*A*	1.36068	24.4	24.4	1.37531	24.7	25.6	1.39069	24.9	26.9	1.40684	25.2	28.3	1.42383	25.5	29.7	1.44165	25.9	31.3
	B	0.36068	24.1	24.0	0.37509	25.1	24.6	0.38982	26.0	25.1	0.40489	27.0	25.7	0.42032	28.0	26.3	0.43612	29.1	27.00

32°	*A*	1.37532	25.6	24.7	1.39011	25.9	25.9	1.40565	26.2	27.2	1.42198	26.5	28.6	1.43915	26.8	30.0	1.45717	27.1	31.6
	B	0.37511	24.6	25.0	0.39010	25.6	25.5	0.40542	26.6	26.1	0.42109	27.6	26.7	0.43713	28.6	27.4	0.45357	29.7	28.1
33°	*A*	1.39068	27.0	24.9	1.40564	27.2	26.0	1.42135	27.6	27.5	1.43786	27.9	29.0	1.45523	28.2	30.4	1.47345	28.6	31.9
	B	0.38986	25.2	26.0	0.40544	26.2	26.5	0.42136	27.2	27.2	0.43765	28.2	27.8	0.45432	29.3	28.5	0.47141	30.4	29.2
34°	*A*	1.40685	28.2	25.2	1.42198	28.5	26.5	1.43788	28.8	27.8	1.45458	29.1	29.3	1.47215	29.5	30.7	1.49058	29.9	32.3
	B	0.40496	25.7	27.0	0.42114	26.8	27.6	0.43768	27.8	28.2	0.45459	28.9	28.9	0.47191	30.0	29.6	0.48966	31.1	30.3
35°	*A*	1.42376	29.8	25.5	1.43907	30.2	26.8	1.45516	30.5	28.1	1.47206	30.9	29.6	1.48984	31.2	31.1	1.50849	31.6	32.7
	B	0.42040	26.4	28.0	0.43720	27.4	28.6	0.45437	28.5	29.3	0.47193	29.6	30.0	0.48991	30.7	30.7	0.50833	31.9	31.5
36°	*A*	1.44166	31.3	25.9	1.45717	31.6	27.2	1.47346	31.9	28.5	1.49057	32.3	30.0	1.50858	32.7	31.5	1.52746	33.1	33.1
	B	0.43622	27.1	29.1	0.45366	28.1	29.7	0.47147	29.3	30.4	0.48969	30.4	31.1	0.50835	31.5	31.9	0.52747	32.7	32.7
37°	*A*	1.46041	32.8	26.2	1.47612	33.2	27.5	1.49262	33.5	28.9	1.50996	33.9	30.4	1.52820	34.3	31.9	1.54732	34.8	33.6
	B	0.45246	27.8	30.1	0.47054	28.9	30.8	0.48902	30.0	31.5	0.50792	31.2	32.3	0.52727	32.4	33.1	0.54710	33.6	33.9
38°	*A*	1.48009	34.5	26.5	1.49601	34.9	27.9	1.51274	35.3	29.3	1.53031	35.7	30.8	1.54879	36.1	32.3	1.56818	36.5	34.0
	B	0.46913	28.5	31.2	0.48787	29.7	31.9	0.50703	30.9	32.7	0.52663	32.0	33.5	0.54670	33.3	34.3	0.56725	34.5	35.2
39°	*A*	1.50079	36.4	26.9	1.51693	36.8	28.3	1.53389	37.2	29.7	1.55170	37.6	31.3	1.57045	38.1	32.8	1.59010	38.6	34.5
	B	0.48625	29.4	32.4	0.50568	30.6	33.1	0.52554	31.8	33.9	0.54585	33.0	34.7	0.56665	34.2	35.5	0.58796	35.5	36.4
40°	*A*	1.52262	38.0	27.3	1.53899	38.4	28.7	1.55620	38.8	30.1	1.57427	39.3	31.7	1.59329	39.8	33.2	1.61323	40.3	35.0
	B	0.50388	30.2	33.6	0.52401	31.4	34.3	0.54459	32.7	35.1	0.56564	33.9	35.9	0.58719	35.2	36.8	0.60927	36.6	37.8
41°	*A*	1.54541	40.1	27.7	1.56204	40.5	29.1	1.57950	41.0	30.6	1.59784	41.4	32.2	1.61714	41.9	33.7	1.63738	42.5	35.5
	B	0.52202	31.3	34.8	0.54287	32.4	35.5	0.56419	33.7	36.4	0.58600	35.0	37.2	0.60833	36.3	38.1	0.63120	37.7	39.1
42°	*A*	1.56946	42.2	28.1	1.58634	42.7	29.6	1.60407	43.1	31.1	1.62270	43.6	32.7	1.64230	44.2	34.2	1.66286	44.7	36.1
	B	0.54071	32.2	36.0	0.56232	33.5	36.8	0.58440	34.8	37.7	0.60699	36.1	38.4	0.63012	37.5	39.5	0.65381	38.9	40.5
43°	*A*	1.59477	44.2	28.6	1.61193	44.9	30.0	1.62994	45.4	31.6	1.64888	45.9	33.2	1.66879	46.5	34.8	1.68968	47.0	36.6
	B	0.56001	33.2	37.3	0.58239	34.6	38.1	0.60526	35.9	39.0	0.62866	37.3	39.9	0.65261	38.7	40.9	0.67715	40.2	42.0
44°	*A*	1.62141	46.7	29.1	1.63885	47.3	30.5	1.65717	47.8	32.1	1.67641	48.3	33.8	1.69666	48.9	35.4	1.71790	49.5	37.3
	B	0.57995	34.4	38.6	0.60312	35.7	39.5	0.62681	37.1	40.4	0.65104	38.6	41.3	0.67584	40.0	42.4	0.70126	41.5	43.5
45°	*A*	1.64945	49.3	29.6	1.66720	49.8	31.1	1.68583	50.4	32.6	1.70541	51.0	34.3	1.72601	51.6	36.0	1.74762	52.2	37.9
	B	0.60056	36.6	40.0	0.62456	37.0	40.9	0.64909	38.5	41.8	0.67417	40.0	42.8	0.69986	41.5	43.9	0.72618	43.0	45.0
46°	*A*	1.67901	52.0	30.1	1.69708	52.5	31.6	1.71604	53.1	33.2	1.73598	53.7	35.0	1.75695	54.4	36.7	1.77894	55.1	38.6
	B	0.62191	36.9	41.4	0.64676	38.4	42.3	0.67216	39.9	43.3	0.69814	41.4	44.3	0.72474	43.0	45.4	0.75199	44.6	46.6
47°	*A*	1.71019	54.8	30.7	1.72858	55.4	32.2	1.74790	56.0	33.9	1.76821	56.7	35.6	1.78957	57.3	37.3	1.81197	58.1	39.3
	B	0.64404	38.3	42.7	0.66978	39.8	43.8	0.69608	41.4	44.9	0.72299	43.0	45.9	0.75053	44.6	47.1	0.77876	46.3	48.3
48°	*A*	1.74306	57.9	31.3	1.76181	58.5	32.8	1.78151	59.1	34.5	1.80220	59.9	36.3	1.82397	60.6	38.1	1.84680	61.3	40.0
	B	0.66701	39.7	44.4	0.69367	41.4	45.4	0.72091	43.0	46.4	0.74877	44.7	47.6	0.77730	46.4	48.7	0.80653	48.2	49.7
49°	*A*	1.77779	61.0	31.9	1.79692	61.9	33.5	1.81700	62.6	35.2	1.83811	63.3	37.0	1.86031	64.3	38.8	1.88359	64.8	40.9
	B	0.69090	41.5	46.0	0.71851	43.1	47.0	0.74673	44.8	48.1	0.77559	46.5	49.2	0.80514	48.3	50.5	0.83542	50.1	51.8
50°	*A*	1.81451	64.7	32.5	1.83403	65.4	34.2	1.85453	66.2	35.9	1.87607	68.6	37.8	1.89873	67.8	39.6	1.92249	68.6	41.7
	B	0.71577	43.2	47.7	0.74438	44.9	48.7	0.77361	46.7	49.8	0.80351	48.5	51.0	0.83413	50.3	52.3	0.86549	52.2	53.6
51°	*A*	1.85334	68.4	33.2	1.87328	69.3	34.9	1.89422	70.0	36.7	1.91622	70.9	38.6	1.93937	71.7	40.6	1.96364	72.6	42.6
	B	0.74170	45.1	49.4	0.77133	46.9	50.5	0.80163	48.7	51.6	0.83261	50.6	52.9	0.86433	52.6	54.2	0.89683	54.6	55.6
52°	*A*	1.89446	72.6	34.0	1.91484	73.4	35.7	1.93624	74.2	37.5	1.95873	75.1	39.4	1.98239	76.0	41.4	2.00720	77.0	43.5
	B	0.76876	47.2	51.2	0.79948	49.0	52.3	0.83087	51.0	53.5	0.86299	52.9	54.8	0.89587	55.0	56.2	0.92956	57.0	57.6
53°	*A*	1.93803	77.1	34.8	1.95888	77.9	36.5	1.98077	78.8	38.4	2.00378	79.7	40.3	2.02798	80.7	42.3	2.05337	81.7	44.5
	B	0.79705	49.3	53.1	0.82890	51.4	54.4	0.86146	53.4	55.5	0.89475	55.5	56.8	0.92884	57.6	58.2	0.96377	59.8	59.7
54°	*A*	1.98430	82.0	35.6	2.00564	82.9	39.0	2.02806	83.8	39.4	2.05162	84.8	41.6	2.07640	85.8	43.3	2.10239	86.9	45.6
	B	0.82670	51.9	55.1	0.85973	54.0	56.8	0.89350	56.1	57.6	0.92803	58.3	58.9	0.96339	60.5	60.4	0.99962	62.8	61.9

TABLE 9.—(*Continued.*)

Vertical angle, h	Symbol	LATITUDE, $\phi = 37°$			LATITUDE, $\phi = 38°$			LATITUDE, $\phi = 39°$			LATITUDE, $\phi = 40°$			LATITUDE, $\phi = 41°$			LATITUDE, $\phi = 42°$		
		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′	
			Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ
15°.......	*A*	1.29596	10.5	29.2	1.31344	10.6	30.6	1.33180	10.6	32.2	1.35110	10.9	33.8	1.37139	11.1	35.6	1.39273	11.3	37.4
	B	0.20111	23.7	12.3	0.20851	24.6	12.7	0.21611	25.5	13.1	0.22394	26.4	13.4	0.23200	27.3	13.8	0.24030	28.3	14.3
16°.......	*A*	1.30225	11.3	29.3	1.31981	11.4	30.8	1.33826	11.6	32.3	1.35765	11.7	34.0	1.37803	11.9	35.8	1.39948	12.1	37.6
	B	0.21531	23.9	13.2	0.22324	24.8	13.6	0.23138	25.7	14.0	0.23976	26.6	14.4	0.24838	27.6	14.8	0.25727	28.6	15.3
17°.......	*A*	1.30900	12 0	29.4	1.32665	12.2	30.9	1.34519	12.4	32.5	1.36469	12.6	34.2	1.38518	12.7	35.9	1.40674	12.9	37.8
	B	0.22967	24.2	14.1	0.23812	25.1	14.5	0.24680	26.0	14.9	0.25574	26.9	15.3	0.26494	27.9	15.8	0.27442	28.9	16.3
18°.......	*A*	1.31622	12.9	29.6	1.33397	13.0	31.1	1.35262	13.2	32.7	1.37222	13.4	34.3	1.39282	13.6	36.0	1.41450	13.8	38.0
	B	0.24416	24.4	15.0	0.25315	25.3	15.4	0.26238	26.2	15.8	0.27188	27.2	16.3	0.28166	28.2	16.8	0.29174	29.2	17.4
19°.......	*A*	1.32394	13.7	29.8	1.34179	13.9	31.3	1.36054	14.1	32.9	1.38026	14.3	34.6	1.40099	14.5	36.3	1.42279	14.7	38.2
	B	0.25881	24.7	15.9	0.26833	25.7	16.3	0.27812	26.6	16.8	0.28819	27.5	17.3	0.29856	28.5	17.8	0.30924	29.6	18.4
20°.......	*A*	1.33215	14.5	29.9	1.35011	14.5	31.5	1.36898	14.9	33.1	1.38883	15.1	34.8	1.40968	15.4	36.6	1.43162	15.6	38.5
	B	0.27364	25.0	16.8	0.28372	26.0	17.2	0.29406	26.9	17.8	0.30471	27.9	18.3	0.31567	28.9	18.6	0.32697	29.9	19.5
21°.......	*A*	1.34085	15.5	30.1	1.35893	15.7	31.7	1.37793	15.9	33.3	1.39790	16.1	35.0	1.41889	16.3	36.8	1.44097	16.6	38.7
	B	0.28866	25.3	17.7	0.29929	26.3	18.2	0.31020	27.3	18.7	0.32143	28.3	19.3	0.33300	29.3	19.9	0.34492	30.3	20.5
22°.......	*A*	1.35012	16.3	30.3	1.36832	16.6	31.9	1.38745	16.8	33.5	1.40756	17.0	35.2	1.42869	17.3	37.1	1.45093	17.6	39.0
	B	0.30388	25.7	18.7	0.31507	26.7	19.2	0.32656	27.6	19.7	0.33838	28.7	20.3	0.35056	29.7	20.9	0.36310	30.7	21.6
23°.......	*A*	1.35992	17.3	30.6	1.37826	17.5	32.1	1.39752	17.8	33.8	1.41778	18.0	35.5	1.43907	18.3	37.3	1.46146	18.6	39.3
	B	0 31932	26.1	19.6	0.33106	27.2	20.2	0.34314	28.0	20.7	0.35557	29.1	21.3	0.36836	30.1	22.0	0.38154	31.2	22.7
24°.......	*A*	1.37028	18.3	30.8	1.38875	18.6	32.4	1.40817	18.8	34.0	1.42858	19.1	35.7	1.45002	19.4	37.6	1.47259	19.7	39.6
	B	0.33497	26.5	20.6	0.34730	27.5	21.1	0.35996	28.5	21.7	0.37300	29.4	22.4	0.38642	30.6	23.1	0.40025	31.7	23.8
25°.......	*A*	1.38127	19.2	31.0	1.39989	19.4	32.6	1.41946	19.7	34.4	1.44004	20.0	36.0	1.46166	20.3	37.9	1.48440	20.6	39.9
	B	0.35087	26.9	21.5	0.36378	27.9	22.1	0.37705	28.9	22.8	0.39070	30.0	23.4	0.40476	31.1	24.1	0.41924	32.2	24.9
26°.......	*A*	1.39277	20.3	31.3	1.41154	20.6	32.9	1.43128	20.9	34.6	1.45202	21.2	36.3	1.47382	21.5	38.2	1.49676	21.8	40.2
	B	0.36702	27.4	22.5	0.38054	28.4	23.1	0.39441	29.5	23.8	0.40869	30.5	24.5	0.42340	31.6	25.3	0.43855	32.8	26.1
27°.......	*A*	1.40495	21.4	31.6	1.42389	21.7	33.2	1.44380	22.0	34.9	1.46472	22.3	36.7	1.48671	22.6	38.6	1.50985	23.0	40.6
	B	0.38347	27.9	23.5	0.39758	28.9	24.2	0.41208	30.0	24.9	0.42700	31.1	25.6	0.44236	32.2	26.4	0.45820	33.3	27.2
28°.......	*A*	1.41777	22.5	31.9	1.43689	22.6	33.5	1.45697	23.1	35.2	1.47809	23.4	37.0	1.50028	23.8	38.9	1.52363	24.2	41.0
	B	0.40020	28.4	24.6	0.41493	29.5	25.2	0.43006	30.5	26.0	0.44563	31.6	26.7	0.46166	32.8	27.5	0.47818	34.0	28.4
29°.......	*A*	1.43126	23.7	32.2	1.45055	24.0	33.8	1.47083	24.4	35.5	1.49215	24.7	37.3	1.51455	25.1	39.3	1.53812	25.5	41.4
	B	0.41724	29.0	25.6	0.43260	30.0	26.3	0.44838	31.1	27.1	0.46461	32.2	27.9	0.48133	33.4	28.7	0.49855	34.6	29.7
30°.......	*A*	1.44547	24.9	32.5	1.46496	25.2	34.1	1.48544	25.4	35.9	1.50697	26.0	37.7	1.52959	26.4	39.7	1.55340	26.8	41.8
	B	0.43461	29.5	26.7	0.45061	30.6	27.4	0.46704	31.8	28.1	0.48395	32.9	29.0	0.50136	34.1	29.9	0.51931	35.3	30.9
31°.......	*A*	1.46041	26.2	32.8	1.48010	26.5	34.5	1.50079	26.9	36.3	1.52254	27.3	38.1	1.54540	27.7	40.1	1.56945	28.2	42.2
	B	0.45233	30.1	27.8	0.46898	31.3	28.5	0.48609	32.4	29.3	0.50369	33.6	30.2	0.52181	34.8	31.1	0.54048	36.1	32.1

32°.......	*A*	1.47612	27.5	33.2	1.49602	27.5	34.9	1.51694	28.2	36.6	1.53892	28.7	38.5	1.56203	29.1	40.5	1.58634	29.5	42.6
	B	0.47043	30.8	28.9	0.48775	32.0	29.7	0.50554	33.1	30.5	0.52384	34.3	31.4	0.54269	35.6	32.4	0.56211	36.9	33.4
33°.......	*A*	1.49261	28.9	33.6	1.51274	29.3	35.2	1.53388	29.7	37.1	1.55611	30.2	39.0	1.57948	30.6	41.0	1.60406	31.1	43.1
	B	0.48893	31.2	30.0	0.50693	32.7	30.8	0.52542	33.9	31.7	0.54444	35.1	32.7	0.56403	36.4	33.7	0.58422	37.7	34.7
34°.......	*A*	1.50997	30.2	33.9	1.53032	30.7	35.7	1.55172	31.1	37.5	1.57421	31.5	39.4	1.59784	32.0	41.5	1.62271	32.5	43.6
	B	0.50786	32.3	31.2	0.52656	33.5	32.0	0.54576	34.7	32.9	0.56552	35.9	33.9	0.58587	37.2	34.9	0.60683	38.6	36.1
35°.......	*A*	1.52811	32.0	34.3	1.54871	32.5	36.1	1.57036	32.9	37.9	1.59312	33.4	39.9	1.61704	33.9	42.0	1.64221	34.4	44.1
	B	0.52723	33.1	32.4	0.54664	34.3	33.2	0.56657	35.6	34.2	0.58708	36.9	35.2	0.60821	38.2	36.3	0.62997	39.5	37.5
36°.......	*A*	1.54733	33.5	34.8	1.56819	34.0	36.5	1.59011	34.5	38.4	1.61316	34.9	40.4	1.63738	35.5	42.5	1.66286	36.1	44.7
	B	0.54708	33.9	33.6	0.56722	35.2	34.5	0.58790	36.5	35.5	0.60919	37.8	36.5	0.63110	39.2	37.7	0.65369	40.6	38.9
37°.......	*A*	1.56745	35.2	35.2	1.58858	35.7	37.0	1.61079	36.2	38.9	1.63414	36.7	40.9	1.65867	37.3	43.0	1.68449	37.8	45.3
	B	0.56744	34.5	34.8	0.58833	36.1	35.8	0.60978	37.4	36.8	0.63186	38.8	37.9	0.65459	40.2	39.1	0.67802	41.6	40.3
38°.......	*A*	1.58858	37.0	35.7	1.60999	37.5	37.5	1.63250	38.2	39.3	1.65616	38.6	41.5	1.68103	39.2	43.6	1.70719	39.7	45.9
	B	0.58834	35.8	36.1	0.61000	37.1	37.1	0.63224	38.5	38.2	0.65515	39.9	39.3	0.67871	41.3	40.5	0.70300	42.8	41.8
39°.......	*A*	1.61079	39.1	36.2	1.63251	39.6	38.0	1.65533	40.1	40.0	1.67932	40.7	42.0	1.70453	41.3	44.1	1.73101	42.1	46.6
	B	0.60982	36.8	37.4	0.63227	38.2	38.4	0.65532	39.6	39.6	0.67905	41.0	40.7	0.70348	42.5	42.0	0.72866	44.1	43.3
40°.......	*A*	1.63422	40.8	36.7	1.65625	41.3	38.6	1.67940	41.9	40.4	1.70374	42.5	42.6	1.72932	43.2	44.9	1.75624	43.8	47.2
	B	0.63192	37.9	38.8	0.65518	39.3	39.8	0.67907	40.8	41.0	0.70366	42.2	42.2	0.72898	43.7	43.5	0.75507	45.3	44.9
41°.......	*A*	1.65868	43.1	37.3	1.68105	43.6	39.2	1.70455	44.2	41.2	1.72925	44.9	43.3	1.75521	45.5	45.5	1.78253	46.2	47.9
	B	0.65467	39.1	40.2	0.67877	40.5	41.3	0.70352	42.0	42.5	0.72899	43.5	43.7	0.75522	45.0	45.1	0.78225	46.7	46.7
42°.......	*A*	1.68449	45.3	38.0	1.70720	45.9	40.0	1.73107	46.2	41.8	1.75616	47.2	44.1	1.78252	47.9	46.2	1.81026	48.7	48.5
	B	0.67812	40.3	41.3	0.70308	41.8	42.7	0.72872	43.4	44.0	0.75511	44.9	45.3	0.78227	46.6	46.7	0.81027	48.2	48.2
43°.......	*A*	1.71166	47.7	38.5	1.73474	48.3	40.4	1.75899	49.0	42.5	1.78448	49.7	44.7	1.81128	50.4	47.0	1.83946	51.2	49.5
	B	0.70232	41.7	43.1	0.72818	43.2	44.3	0.75473	44.8	45.6	0.78206	46.4	46.9	0.81020	48.1	48.3	0.83919	49.8	49.9
44°.......	*A*	1.74025	50.2	39.1	1.76371	50.7	41.1	1.78836	51.6	43.2	1.81428	52.3	45.6	1.84153	53.1	47.8	1.87018	53.9	50.3
	B	0.72733	43.1	44.6	0.75410	44.7	45.8	0.78160	46.3	47.2	0.80990	48.0	48.6	0.83904	49.7	50.1	0.86907	51.5	51.7
45°.......	*A*	1.77035	52.9	39.8	1.79422	53.6	41.8	1.81930	54.3	44.0	1.84567	55.1	46.2	1.87338	56.0	48.6	1.90253	56.8	51.1
	B	0.75317	44.6	46.2	0.78090	46.3	47.5	0.80938	48.0	48.8	0.83868	49.7	50.3	0.86886	51.5	51.8	0.89995	53.3	53.5
46°.......	*A*	1.80208	55.8	40.5	1.82637	56.5	42.6	1.85190	57.3	44.8	1.87875	58.1	47.0	1.90695	59.0	49.5	1.93663	59.9	52.1
	B	0.77995	46.3	47.9	0.80866	48.0	49.2	0.83815	49.7	50.6	0.86849	51.5	52.1	0.89974	53.4	53.7	0.93194	55.3	55.4
47°.......	*A*	1.83554	58.8	41.2	1.86028	59.6	43.4	1.88629	60.4	45.6	1.91363	61.3	47.9	1.94236	62.2	50.4	1.97259	63.2	53.0
	B	0.80771	48.0	49.6	0.83744	49.8	50.9	0.86798	51.6	52.4	0.89941	53.5	53.9	0.93177	55.4	55.6	0.96511	57.4	57.4
48°.......	*A*	1.87082	62.1	42.0	1.89604	63.1	44.2	1.92255	63.8	46.4	1.95041	64.8	48.8	1.97970	65.7	51.4	2.01051	66.8	54.0
	B	0.83652	49.9	51.2	0.86731	51.8	52.7	0.89894	53.7	54.2	0.93148	55.6	55.9	0.96500	57.6	57.6	0.99953	59.7	59.4
49°.......	*A*	1.90810	65.7	42.9	1.93382	64.9	45.1	1.96085	67.5	47.4	1.98928	68.5	49.8	2.01914	69.5	52.4	2.05057	70.6	55.1
	B	0.86648	52.0	53.2	0.89837	53.9	54.6	0.93113	55.9	56.2	0.96485	57.9	57.9	0.99956	60.0	59.6	1.03533	62.1	61.6
50°.......	*A*	1.94750	69.5	43.8	1.97376	70.4	46.0	2.00135	71.4	48.3	2.03036	72.4	50.8	2.06084	73.5	53.5	2.09291	74.7	56.3
	B	0.89767	54.2	55.1	0.93071	56.2	56.6	0.96465	58.2	58.2	0.99958	60.3	59.9	1.03554	62.5	61.8	1.07260	64.7	63.8
51°.......	*A*	1.98919	73.5	44.7	2.01600	74.5	47.0	2.04419	75.6	49.4	2.07381	76.7	51.9	2.10495	77.8	54.6	2.13771	79.0	57.5
	B	0.93017	56.6	57.1	0.96442	58.7	58.6	0.99958	60.8	60.3	1.03578	63.0	62.1	1.07304	65.3	64.0	1.11144	67.6	66.1
52°.......	*A*	2.03331	78.0	45.7	2.06072	79.0	48.0	2.08953	80.1	50.5	2.11982	81.3	53.0	2.15164	82.5	55.8	2.18513	83.8	58.7
	B	0.96411	59.2	59.2	0.99961	61.3	60.8	1.03606	63.6	62.5	1.07357	65.9	64.4	1.11220	68.2	66.3	1.15200	70.7	68.5
53°.......	*A*	2.08008	82.8	46.7	2.10812	83.9	49.1	2.13759	85.1	51.6	2.16857	86.3	54.3	2.20113	87.6	57.1	2.23539	89.0	60.1
	B	0.99960	62.6	61.3	1.03640	64.3	62.9	1.07419	66.6	64.8	1.11308	69.0	66.8	1.15313	71.5	68.8	1.19440	74.0	71.0
54°.......	*A*	2.12974	87.9	47.9	2.15845	89.1	50.3	2.18862	90.4	52.9	2.22035	91.9	55.6	2.25368	93.1	58.5	2.28876	94.6	61.5
	B	1.03678	65.0	63.6	1.07495	67.1	65.3	1.11414	69.9	67.2	1.15448	72.4	69.2	1.19602	75.0	71.3	1.23882	77.8	73.7

TABLE 9.—(*Continued*)

Vertical angle, h	Symbol	LATITUDE, $\phi = 43°$			LATITUDE, $\phi = 44°$			LATITUDE, $\phi = 45°$			LATITUDE, $\phi = 46°$			LATITUDE, $\phi = 47°$			LATITUDE, $\phi = 48°$		
		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′		Factor	Differences for 1′	
			Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ		Vert. angle, h	ϕ
15°.......	*A*	1.41519	11.4	39.4	1.43882	11.6	41.6	1.46371	11.9	43.7	1.48994	12.1	46.1	1.51760	12.3	48.6	1.54679	12.5	51.4
	B	0.24887	29.3	14.8	0.25772	30.3	15.2	0.26688	31.4	15.8	0.27637	32.4	16.4	0.28620	33.7	17.0	0.29640	34.9	17.7
16°.......	*A*	1.42205	12.3	39.6	1.44579	12.5	41.8	1.47084	12.7	43.9	1.49717	13.0	46.3	1.52496	13.2	48.9	1.55429	13.4	51.6
	B	0.26645	29.6	15.8	0.27593	30.6	16.3	0.28573	31.7	16.8	0.29582	33.0	17.6	0.30641	34.0	18.2	0.31734	35.2	18.9
17°.......	*A*	1.42942	13.2	39.8	1.45329	13.4	41.9	1.47844	13.6	44.3	1.50500	13.7	46.5	1.53287	14.1	49.1	1.56235	14.4	51.9
	B	0.28421	29.9	16.9	0.29432	31.0	17.4	0.30478	32.0	18.0	0.31561	33.2	18.7	0.32683	34.4	19.4	0.33849	35.6	20.2
18°.......	*A*	1.43731	14.0	40.0	1.46131	14.3	42.2	1.48660	14.5	44.4	1.51324	17.8	46.8	1.54133	15.0	49.5	1.57100	15.3	52.1
	B	0.30215	30.2	17.9	0.31289	31.3	18.5	0.32401	32.4	19.2	0.33552	33.6	19.9	0.34746	34.8	20.6	0.35985	36.0	21.5
19°.......	*A*	1.44573	15.0	40.2	1.46987	15.2	42.4	1.49530	15.4	47.7	1.52391	12.7	47.1	1.55035	16.0	49.7	1.58017	16.4	52.5
	B	0.32027	30.6	19.0	0.33167	31.7	19.6	0.34346	32.8	22.0	0.35566	34.0	21.1	0.36832	35.2	21.9	0.38145	36.4	22.8
20°.......	*A*	1.45470	15.9	40.5	1.47899	16.1	42.6	1.50457	16.4	44.9	1.53154	16.7	47.4	1.55996	17.0	50.0	1.58999	17.3	52.7
	B	0.33864	30.9	20.1	0.35068	32.1	20.8	0.36314	33.2	21.5	0.37604	34.4	22.3	0.38942	35.6	23.3	0.40331	36.9	24.0
21°.......	*A*	1.46421	16.8	40.7	1.48865	17.1	43.0	1.51443	17.8	45.2	1.54157	17.7	47.7	1.57018	18.0	50.3	1.60038	18.4	53.1
	B	0.35722	31.4	21.2	0.36993	32.5	21.7	0.38307	33.7	22.7	0.39668	34.9	23.5	0.41079	36.1	24.4	0.42544	36.9	25.5
22°.......	*A*	1.47432	17.9	41.0	1.49894	18.1	43.2	1.52487	18.4	45.6	1.55220	17.7	48.0	1.58100	19.1	50.7	1.61142	19.5	53.5
	B	0.37606	31.8	22.3	0.38943	33.0	23.0	0.40326	34.1	23.9	0.41759	34.9	24.8	0.43245	36.6	25.7	0.44787	37.9	26.7
23°.......	*A*	1.48503	18.9	41.3	1.50982	19.2	43.5	1.53594	19.5	45.9	1.56347	19.8	48.4	1.59249	20.2	51.2	1.62311	20.6	53.9
	B	0.39516	32.3	23.4	0.40921	33.4	24.2	0.42374	34.5	25.1	0.43880	35.7	26.0	0.45441	36.9	27.0	0.47061	38.3	28.1
24°.......	*A*	1.49634	20.0	41.6	1.52132	20.3	43.8	1.54764	20.6	46.2	1.57537	21.0	48.8	1.60462	21.4	51.4	1.63548	21.8	54.3
	B	0.41452	32.8	24.6	0.42927	34.0	25.2	0.44442	35.3	26.3	0.46021	36.6	27.3	0.47658	37.9	28.3	0.49358	39.3	29.4
25°.......	*A*	1.50834	20.9	42.0	1.53353	21.3	44.1	1.56000	21.7	46.7	1.58800	22.1	49.1	1.61744	22.5	51.8	1.64854	23.0	54.8
	B	0.43420	33.3	25.7	0.44964	34.6	26.6	0.46562	35.8	27.6	0.48217	37.0	28.6	0.49932	38.3	29.7	0.51713	39.7	30.8
26°.......	*A*	1.52089	22.2	42.3	1.54629	22.6	44.6	1.57304	22.9	47.0	1.60124	23.3	49.5	1.63096	23.8	52.3	1.66232	24.2	55.2
	B	0.45419	33.9	26.9	0.47035	35.1	27.9	0.48707	36.4	28.8	0.50437	37.6	29.9	0.52231	39.0	31.1	0.54094	40.4	32.3
27°.......	*A*	1.53420	23.3	42.7	1.55982	23.7	45.0	1.58679	24.1	47.4	1.61523	24.6	50.0	1.64521	25.0	52.7	1.67685	25.5	55.7
	B	0.47454	34.5	28.1	0.49142	35.7	29.1	0.50888	37.0	30.1	0.52696	38.3	31.2	0.54570	39.7	32.5	0.56517	41.1	33.7
28°.......	*A*	1.54820	24.6	43.1	1.57405	25.0	45.4	1.60128	25.4	47.9	1.63000	25.8	50.4	1.66023	26.4	53.2	1.69216	26.9	56.2
	B	0.49524	35.2	29.4	0.51286	36.4	30.4	0.53108	37.7	30.5	0.54995	39.0	32.6	0.56952	40.4	33.8	0.58983	41.8	35.2
29°.......	*A*	1.56293	25.9	43.5	1.58902	26.3	45.9	1.61653	26.7	48.3	1.64550	27.2	50.9	1.67604	27.7	53.7	1.70827	28.2	56.7
	B	0.51634	35.8	30.6	0.53470	37.1	31.7	0.55370	38.4	32.8	0.57337	39.8	34.0	0.59376	41.2	35.3	0.61494	42.7	36.7
30°.......	*A*	1.57845	27.0	43.9	1.60480	27.7	46.3	1.63257	28.1	48.8	1.66182	28.6	51.4	1.69267	29.2	54.3	1.72522	29.7	57.3
	B	0.53783	36.6	31.9	0.55696	37.8	33.0	0.57675	39.2	34.2	0.59724	40.6	35.4	0.61848	42.1	36.8	0.64055	43.5	38.2
31°.......	*A*	1.59476	28.6	44.4	1.62139	29.0	46.8	1.64944	29.6	49.3	1.67900	30.1	52.0	1.71017	30.7	54.8	1.74306	31.2	57.9
	B	0.55976	37.3	33.2	0.57967	38.6	34.3	0.60027	40.0	35.6	0.62160	41.5	36.9	0.64371	42.9	88.3	0.66667	44.5	39.8

32°	*A*	1.61192	30.4	44.9	1.63883	30.5	47.3	1.66718	31.1	49.8	1.69706	31.6	52.5	1.72856	32.1	55.4	1.76180	32.8	58.8
	B	0.58216	38.2	34.5	0.60287	39.5	35.7	0.62430	40.9	37.0	0.64648	42.4	38.3	0.66948	43.9	39.8	0.69336	45.4	41.4
33°	*A*	1.62993	31.6	45.4	1.65714	32.1	47.8	1.68582	32.7	50.4	1.71604	33.2	53.1	1.74789	33.9	56.0	1.78150	34.5	59.2
	B	0.60506	39.0	35.9	0.62658	40.4	37.1	0.64884	41.9	38.4	0.67190	43.3	39.8	0.69580	44.9	41.4	0.72061	46.8	43.0
34°	*A*	1.64888	33.0	45.9	1.67641	33.6	48.4	1.70542	34.3	50.9	1.73598	34.9	53.7	1.76820	35.6	56.7	1.80220	36.3	59.9
	B	0.62848	40.0	37.3	0.65084	41.3	38.5	0.67396	42.8	39.9	0.69790	44.4	41.4	0.72273	46.0	43.0	0.74851	47.6	44.7
35°	*A*	1.66869	35.0	46.4	1.69655	35.6	49.1	1.72600	35.0	51.6	1.75693	36.7	54.4	1.78954	37.4	57.4	1.82395	38.1	60.6
	B	0.65245	40.9	38.7	0.67565	42.5	40.0	0.69967	43.9	41.4	0.72453	45.5	43.0	0.75030	47.1	44.6	0.77706	48.8	46.4
36°	*A*	1.68968	36.6	47.0	1.71789	37.2	49.6	1.74762	37.2	52.2	1.77893	37.9	55.1	1.81196	38.6	58.1	1.84680	39.3	61.3
	B	0.67701	42.0	40.1	0.70109	43.5	41.5	0.72600	45.0	43.0	0.75180	46.6	44.6	0.77854	48.3	46.3	0.80631	50.0	48.1
37°	*A*	1.71165	38.5	47.6	1.74023	39.1	49.8	1.76993	40.4	52.9	1.80164	41.2	55.8	1.83509	41.9	58.8	1.87038	42.6	62.1
	B	0.70221	43.1	41.6	0.72718	44.7	43.1	0.75302	46.2	44.6	0.77978	47.9	46.2	0.80752	49.6	48.0	0.83632	51.3	49.9
38°	*A*	1.73472	40.5	48.3	1.76368	41.1	50.9	1.79420	41.8	53.6	1.82637	42.5	56.5	1.86026	43.2	59.6	1.89603	44.2	63.0
	B	0.72807	44.3	43.2	0.75397	45.9	44.7	0.78075	47.4	46.2	0.80849	49.5	48.0	0.83726	51.0	49.8	0.86712	52.8	51.7
39°	*A*	1.75897	42.7	49.0	1.78834	43.4	51.6	1.81929	44.0	54.3	1.85189	44.8	57.3	1.88627	45.8	60.5	1.92254	46.5	63.9
	B	0.75465	45.6	44.7	0.78149	47.2	46.3	0.80924	48.9	48.0	0.83821	50.3	49.4	0.86783	52.3	51.6	0.89878	54.3	53.6
40°	*A*	1.78456	44.5	49.7	1.81435	43.5	52.2	1.84566	46.2	55.1	1.87874	47.0	58.3	1.91374	47.7	61.1	1.95041	48.8	64.8
	B	0.78200	46.9	46.4	0.80982	48.6	48.0	0.83859	50.3	49.7	0.86838	52.1	51.4	0.89921	54.1	53.6	0.93135	55.9	55.6
41°	*A*	1.81127	47.0	50.1	1.84132	47.8	53.1	1.87338	48.6	56.0	1.90695	49.5	59.0	1.94235	50.4	62.3	1.97970	51.4	65.7
	B	0.81015	48.7	48.0	0.83897	50.1	49.7	0.86878	51.9	51.4	0.89964	53.7	53.4	0.93165	55.6	55.4	0.96488	57.6	57.6
42°	*A*	1.83946	49.5	51.2	1.87017	50.3	53.9	1.90248	51.1	56.8	1.93659	52.1	59.9	1.97255	53.0	63.2	2.01048	54.0	66.8
	B	0.83917	49.9	49.8	0.86902	51.7	51.4	0.89990	53.5	53.3	0.93186	55.4	55.2	0.96500	57.4	57.4	0.99942	59.4	59.6
43°	*A*	1.86913	52.1	52.0	1.90033	52.9	54.7	1.93317	53.6	58.5	1.96783	54.2	60.9	2.00438	55.8	64.2	2.04290	56.8	67.9
	B	0.86912	51.6	51.5	0.90004	53.4	53.3	0.93200	55.3	55.6	0.96511	56.5	57.2	0.99945	59.3	59.4	1.03507	61.5	61.8
44°	*A*	1.90034	54.8	52.7	1.93207	55.7	55.6	1.96545	56.8	58.7	2.00068	57.8	62.0	2.03786	58.8	65.2	2.07700	60.0	69.9
	B	0.90007	53.3	53.4	0.93208	55.2	55.3	0.96518	57.2	57.2	0.99948	59.4	59.2	1.03502	61.3	61.6	1.07195	63.5	64.0
45°	*A*	1.93321	57.8	53.8	1.96549	58.7	56.7	1.99950	59.7	58.8	2.03536	60.8	63.0	2.07314	61.9	66.4	2.11300	63.1	70.2
	B	0.93205	55.2	55.3	0.96521	57.2	57.2	0.99950	59.2	59.2	1.03500	61.3	61.4	1.07182	63.5	63.7	1.11005	65.8	66.2
46°	*A*	1.96786	60.1	54.8	2.00072	61.9	57.7	2.03532	62.9	60.8	2.07181	64.0	64.1	2.11029	65.2	67.6	2.15085	66.3	71.4
	B	0.96519	57.3	57.2	0.99952	59.3	59.2	1.03502	61.4	61.3	1.07180	63.6	63.5	1.10992	65.8	66.0	1.14950	67.8	68.6
47°	*A*	2.00440	64.2	55.8	2.03786	65.3	58.7	2.07305	66.5	62.0	2.11024	67.7	65.3	2.14940	69.0	68.9	2.19075	70.2	72.8
	B	0.99954	59.4	59.3	1.03509	61.5	61.3	1.07187	63.7	63.5	1.10995	65.6	65.8	1.14942	68.4	68.0	1.19042	70.8	71.1
48°	*A*	2.04293	67.8	56.9	2.07704	69.0	58.8	2.11295	70.1	63.2	1.15085	71.3	66.6	2.19080	72.7	70.2	2.23290	74.1	74.2
	B	1.03519	61.8	61.4	1.07201	64.0	63.5	1.11010	66.2	65.7	1.14953	68.6	68.2	1.19042	71.1	70.8	1.23289	73.6	73.6
49°	*A*	2.08363	71.7	58.0	2.11842	72.9	61.0	2.15500	74.1	64.4	2.19365	75.5	67.9	2.23440	76.8	71.6	2.27737	78.3	75.6
	B	1.07227	64.3	63.6	1.11041	66.6	65.7	1.14984	69.0	68.1	1.19069	71.4	70.6	1.23306	74.0	73.3	1.27703	76.6	76.2
50°	*A*	2.12666	75.9	59.2	2.16217	77.1	62.2	2.19947	78.4	65.8	2.23895	79.7	69.2	2.28050	81.3	73.1	2.32433	82.9	77.2
	B	1.11086	67.1	65.9	1.15038	69.4	68.1	1.19124	71.9	70.5	1.23356	74.5	73.1	1.27744	77.1	75.9	1.32300	79.8	79.0
51°	*A*	2.17218	80.3	60.5	2.20845	81.7	63.5	2.24653	83.1	67.1	2.28679	84.6	70.8	2.32926	83.2	74.7	2.37406	87.8	78.9
	B	1.15109	70.0	68.3	1.19204	72.5	70.6	1.23437	75.1	73.1	1.27824	77.7	75.8	1.32370	80.5	78.7	1.37091	83.4	81.8
52°	*A*	2.22037	85.1	61.6	2.25744	86.5	64.9	2.29637	88.0	68.6	2.33756	89.5	72.4	2.38100	91.9	76.2	2.42672	93.0	80.6
	B	1.19309	73.2	70.7	1.23553	75.8	73.2	1.27944	78.5	75.7	1.32488	81.2	78.3	1.37200	84.2	81.6	1.42097	87.1	84.8
53°	*A*	2.27144	90.4	63.2	2.30936	91.9	66.3	2.34916	93.4	70.2	2.39128	95.2	74.0	2.43567	97.0	78.0	2.48250	98.8	82.5
	B	1.23701	76.7	73.3	1.28101	79.4	75.9	1.32652	82.2	78.5	1.37362	85.2	81.1	1.42250	88.2	84.6	1.47323	91.3	87.4
54°	*A*	2.32566	96.1	64.7	2.36450	97.7	67.9	2.40522	99.3	72.0	2.44839	101.0	75.8	2.49384	106.8	79.9	2.54178	104.9	84.5
	B	1.28302	80.5	76.1	1.32865	83.4	78.9	1.37584	86.3	81.3	1.42473	89.3	84.5	1.47540	92.5	88.4	1.52803	95.8	91.2

In the use of Table 9, the slide-rule is commended. The necessary multiplications are made quickly, with an advantage both in the assurance of accuracy and in the time consumed.

The solar attachment has been introduced into the discussion. The writer is of the opinion that the initial cost is not so prohibitive as the continuing cost of adjustment and maintaining the adjustment.

The suggestion that sun altitudes between 10° and 49° be used has its inconsistencies. The writer agrees that 49° is high enough, considering the factor of safety. When it is at an altitude of 10° the sun is moving vertically at too rapid a rate to insure a good observation. Moreover, the refraction is large and variable. Above 49°, although the refraction element is nearing its minimum, the sun is moving horizontally at a comparative rate as great as it is moving vertically at minimum altitudes. Furthermore, the least error in the horizontal reading appears directly in the bearing that is sought.

The ideal time for sun observations is about three hours either before or after apparent noon. Experiments, however, have demonstrated that everything depends upon the accuracy with which the observation is taken and how nearly the personal equation can be eliminated. At certain seasons of the year excellent observations can be obtained as early or as late as five hours before or after apparent noon. The freedom from personal inaccuracies is the paramount issue, not the time nor the computation by any particular accepted method.

A suggestion for solar observation is as follows: Reduce the use of the tangent screws to a minimum. Hold a limb of the sun tangent to one hair of the telescope with the tangent screw and allow the sun to drift into tangency with the other selected hair. Personally, the writer holds the faster moving limb of the sun tangent with the tangent screw. A little practice and one's judgment of the sun's movement can be placed within a 2-sec interim. In an observation in which all four quadrants are used, the perfect co-ordination of two hands and an eye at eight different times is rare.

In the discussion, reference is made to the additional hairs that can be placed in a telescope to facilitate a solar observation. They enable the sun to be centered approximately at the intersection of the horizontal and vertical hairs. This arrangement gives excellent results; but few instruments are equipped with this accessory.

From the viewpoint of a field engineer, the most needed addition to the field procedure is some check on the accuracy of a solar observation in the field, and at the time of observation. Fig 4 will suggest a method of observation that will afford an excellent check upon the accuracy of each complete observation, by comparison with the others in any group. Observe the sun in this manner, recording the time, the horizontal angle, and the vertical angle. Combine the observations, respectively; thus: $\frac{1 + 8}{2}$, $\frac{2 + 7}{2}$, $\frac{3 + 6}{2}$, and $\frac{4 + 5}{2}$.

In theory, and in theory alone, the mean time, the mean horizontal angle, and the mean vertical angle of each complete set will be equal; one's judgment, together with the degree of accuracy required, should be one's best guide as to the rejection or acceptance of any set. One good observation is better than the mean of one good one and three poor ones. Observations taken 1 min apart, with 2 min between the series of direct and reversed readings, give very satisfactory results.

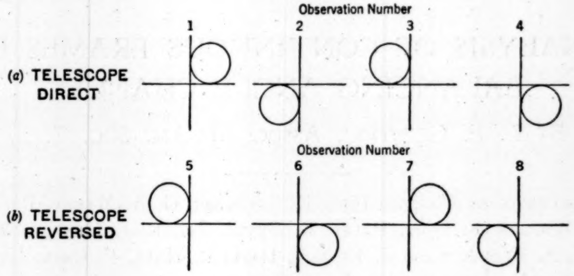

FIG. 4.

The ease of computing a solar observation by means of simple formulas, logarithms, and computing machines is apparent. The writer approves of their use, but only in skilled hands. The selection of a solar observation taken in 1910, as an example, has been commented upon; it was taken from the Manual of Instructions for the Survey of Public Lands of the United States, 1930 (page 112); it represents a criterion.

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ANALYSIS OF CONTINUOUS FRAMES BY BALANCING ANGLE CHANGES

BY L. E. GRINTER[1], ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOHN E. GOLDBERG, G. A. MANEY, PAUL ANDERSEN, WILLIAM F. LUCE, RALPH E. BYRNE, JR., L. J. MENSCH, MARVIN A. GRAY, A. FLORIS, FRED J. BENSON, DAVID B. HALL, P. CRAIG LIVESAY AND WILLIAM M. SIMPSON, AND L. E. GRINTER.

SYNOPSIS

The interest that has been shown by the profession in the method of balancing fixed-end moments (introduced by Hardy Cross, M. Am. Soc. C. E.[2]) as an alternative to the classic methods of analyzing continuous frames, has suggested the publication of a second alternative process based upon the conception of balancing angle changes to obtain joint rotations. The procedure of balancing angle changes is shown to parallel the procedure used in balancing and distributing fixed-end moments. No claim is made that the suggested method surpasses the method of moment distribution either in simplicity or in a reduction of time consumed by the analysis. In fact, it will take somewhat longer to arrive at the final end moments. However, the additional time consumed may be far more than equally repaid by the fact that the method is practically self-checking. Furthermore, upon completion of the analysis, one has at hand all necessary data for sketching, rapidly and accurately, the deflected structure, a procedure that experience has proved to be a most valuable tool both in analysis and in design. The method of balancing angle changes is particularly useful for studying secondary stresses in trusses and for investigating the effect of elastic distortion or of slip in the riveted or welded connections of a continuous steel frame. Approximate influence lines for live load studies can be obtained very rapidly thereby. The pro-

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NOTE.—Published in September, 1936, *Proceedings*.

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[2] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

cedure of balancing angle changes is susceptible to expression in many forms. It is thought, however, that discussion will be used primarily for presenting evidence tending to test the theory proposed herein.

INTRODUCTION

Slope and Deflection.—The method of balancing angle changes should appeal to engineers who are most familiar with the slope-deflection equations because it makes use of the same physical variables of end slope, joint rotation, and relative deflection, at the same time possessing the other advantages which are so apparent in the method of moment distribution. There are other published methods[3] in which joint rotations have been obtained by series convergence, but these methods seem to involve relatively tedious procedures. The present method was developed by the writer in 1927. A few of the many special applications that have been found useful will be presented herein, but the basic method is essentially in its original form.

The well-known slope-deflection equations make it possible for one to compute the end moments in any unloaded member of a continuous frame from known values of end slopes and relative end deflections. If the member is loaded, the slope-deflection moments become the changes in the end moments from an originally fixed-end condition. Hence, the analysis of a continuous frame can be made upon the basis of a treatment in which joint rotations and relative deflections are chosen as the redundants. Many advantages have been claimed for the use of these functions as the redundants, most of which have not been justified by experience, but there is one which can not be disputed. Joint rotations and deflections are the equivalents of the physical action of the structure and the emphasis of these factors lends a desirable pictorial clearness to the method, provided this picture is not obscured by the mechanical details of the procedure.

Successive Rotation.—The method of balancing fixed-end moments starts with all members restrained against end rotation. The alternative procedure to be developed will start with all horizontal members acting as simply supported beams and all columns as pin-ended. Then, by successively balancing the angle changes at the joints until continuity is fully restored, one will obtain values of the true joint rotations from which the deflected structure can be sketched and the joint moments determined. Naturally, joint translation will complicate the analysis somewhat. The discussion that follows will present the method for use in continuous-beam analyses, after which its application to frames in which the joints translate will be considered briefly. A thorough understanding of a previous paper by the writer[4], will assist the reader in his interpretation of this paper.

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[3] "Wind Stresses by Slope-Deflection and Converging Approximations", by J. E. Goldberg, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 962; also, "Rigid Frame Analysis by the Criterion Series of Converging Angular Approximations", by S. M. Cotten, M. Am. Soc. C. E. (Mimeographed notes under date of 1930); and "Statically Indeterminate Stresses," by J. I. Parcel and G. A. Maney, Members, Am. Soc. C. E., Fig. 142, John Wiley & Sons, New York.

[4] "Wind Stress Analysis Simplified", by L. E. Grinter, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 610.

DEFINITION OF TERMS

A few new terms will be needed and for clearness they will be defined at once:

(1) By "angle change" (ϕ-value) will be meant the change in slope produced at the end of a member either by the loads or by an applied end moment; for example, certain easily determinable "angle changes", ϕ, occur at the ends of a simple beam when it is loaded.

(2) By "joint rotation" (θ-value) is meant the angular twist of a joint in a loaded frame when absolute continuity exists between all members meeting at the joint.

(3) By "joint rotation factor" is meant the ratio between the joint rotation and the angle change existing at the end of a connecting member when the angular discontinuity between that member and the joint is removed by applied moments. It is assumed that the far ends of all members are pin-connected when the joint rotation factor is computed. Evidently, there is a joint rotation factor associated with each member meeting at a joint.

(4) By "carry-over factor" is meant the angle change at the far end of a simply supported or pin-ended member when the near end is rotated through an angle change of unity.

PROCEDURE OF BALANCING ANGLE CHANGES

The condensed statement of the procedure of balancing angle changes that follows will be helpful to the reader and much clarified, if it is used for continual reference while the illustrative examples are being studied.

Angle Change and Joint Rotation.—With all joints of the frame held fixed against all possible translations and with the ends of all members either simply supported or pin-connected, one "permits" the members to deflect under the loads and computes the values of the end angle changes. Any joint is selected and the end angle change existing at that joint in one of its members is eliminated by twisting that particular member and the joint in opposite directions until continuity is restored. The recorded value of the joint rotation is the angle change that existed at the end of the member selected, after being multiplied by the joint rotation factor for that member.

Carry-Over Angles.—All members meeting at this joint have now been rotated through determinable end angles. The end angular rotation is the joint rotation for all members other than the one operated upon. For that particular member, the end angular rotation is the difference between the joint rotation and the end angle change caused by the loads. Since all these members were considered to be simply supported at their far ends, one must record an angle change (not a joint rotation) at the far end of each member equal to the angular rotation at the near end, when multiplied by the carry-over factor. Then, one proceeds from member to member and from joint to joint balancing the angles at each joint as many times as necessary, which means to continue the procedure until the carry-over angles are negligibly small. Continuity has then been established. Under normal conditions, it will be necessary to balance each joint from three to four times.

SIMPLE BEAM ANGLE CHANGES

The starting point in the analysis is to obtain the angle changes or the slopes at the ends of the loaded spans. This problem is solved in all textbooks on the strength of materials. For rapid calculation one should use the conjugate-beam method[6]. It scarcely seems necessary to present this treatment and instead the angle changes are given for a few simple cases in Fig. 1.

$$\phi_A = -\phi_B = \frac{w L^3}{24 E I}$$

$$\phi_A = -\phi_B = \frac{P L^2}{16 E I}$$

$$\phi_A = +\frac{P a b}{2 E I L}\left(\frac{2 b}{3} + \frac{a}{3}\right)$$

$$\phi_B = -\frac{P a b}{2 E I L}\left(\frac{2 a}{3} + \frac{b}{3}\right)$$

$$\phi_A = +\frac{M L}{3 E I}$$

$$\phi_B = -\frac{M L}{6 E I}$$

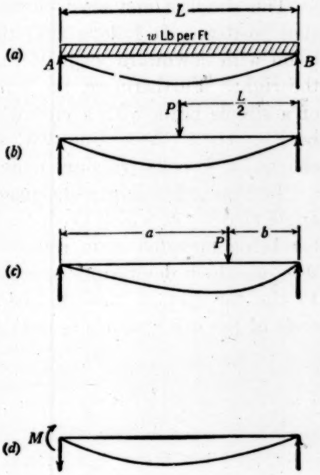

FIG. 1.—END ANGLE CHANGES FOR SIMPLE BEAMS.

JOINT ROTATION AND CARRY-OVER FACTORS

Stiffness of Joint and Member.—The stiffness of a beam may be defined as the end moment necessary to produce a unit end rotation and, therefore, it is proportional to the ratio, $\frac{I}{L}$, which is commonly designated as K. If an angle change, ϕ_1, occurring at the end of a member which frames into a joint, is to be eliminated and continuity is to be restored by twisting the member and the joint in opposite directions (by equal end moments), it is apparent from the definition of stiffness that the joint must be rotated through an angle, θ_1, equal to the angle, ϕ_1, times the stiffness ratio, $\frac{K_1}{\Sigma K}$. Here, as elsewhere, K_1 is the stiffness factor for the member having the end slope, ϕ_1, and ΣK is the sum of the stiffness factors for all members meeting at the joint. Hence, $\frac{K_1}{\Sigma K}$ is the "joint rotation factor" for Member No. 1 at the joint under consideration. In this paper, the far ends of all members are considered to be pin-connected or simply supported.

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[6] *Transactions*, Am. Soc. C. E., Vol. 98 (1927), p. 1.

Usual Carry-Over Factor.—From Fig. 1 it is observed that the end angle changes in a simple beam loaded with a single end moment are in the ratio of 2 to 1. This represents the condition of a member when one end is being rotated into continuity. Hence, the carry-over factor for angle change is 0.5 whenever the member is prismatic or of uniform section.

SIGN CONVENTION

The simplest and most obvious sign convention is to call clockwise rotation positive. It follows that the end angle changes in a simply supported beam with downward loads will always be positive at the left and negative at the right. Furthermore, the same signs of angle changes apply for the case of a simple beam with a clockwise moment applied to the left end, as shown in Fig. 1(*d*). Accordingly, it follows that the carry-over factor for angle change is of negative sign, namely, − 0.5.

The sign of a joint rotation, θ, necessarily is the same as that of the end angle change, ϕ, which produces it. Note, however, that the end of the member being operated upon rotates in the opposite direction through an angle, $\theta - \phi$, which determines both the value and the sign of the carry-over angle to the far end of this member. All of the carry-over angles to the far ends of the other members meeting at the joint are simply $-0.5\ \theta$.

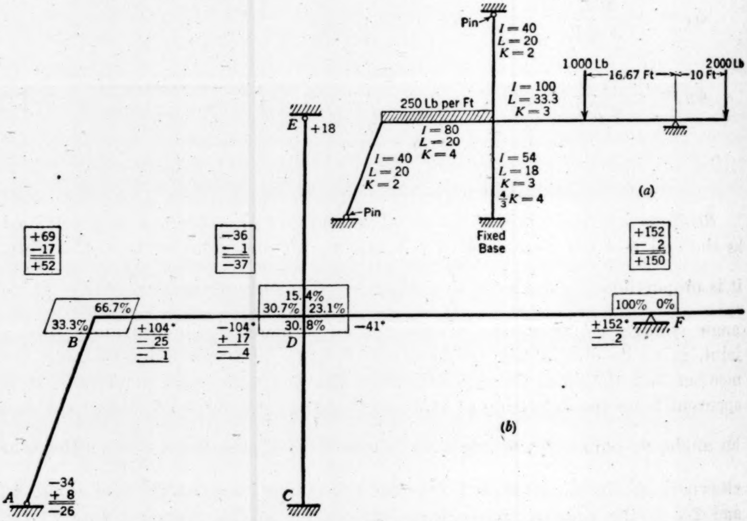

Pin
I = 40
L = 20
K = 2
1000 Lb
2000 Lb
16.67 Ft
10 Ft
I = 100
L = 33.3
K = 3
250 Lb per Ft
I = 80
L = 20
K = 4
I = 40
L = 20
K = 2
I = 54
L = 18
K = 3
4/3 K = 4
Pin
Fixed Base
(a)
E
+18
+69
−17
+52
−36
− 1
−37
+152
− 2
+150
15.4%
30.7%
23.1%
30.8%
66.7%
33.3%
B
+104*
− 25
− 1
−104*
+ 17
− 4
D
−41*
100% 0%
+152*
− 2
F
(b)
A
−34
+ 8
−26
C

FIG. 2.—BALANCING ANGLE CHANGES FOR A CONTINUOUS FRAME. (* = RELATIVE SIMPLE BEAM ANGLE CHANGES IN LOADED SPANS; ϕ - VALUES.)

ILLUSTRATIVE EXAMPLE

The Frame.—Fig. 2 shows the application of the method of balancing angle changes to a continuous frame with prismatic members when all joints are fixed against translation. This academic example was chosen to illustrate, quite simply, the general procedure. Fig. 2(*a*) shows the loaded frame and gives the properties of the members. The end angle changes of the loaded spans, when the joints are pin-connected, are marked with an asterisk in Fig. 2(*b*) which further illustrates the balancing of these angle changes to obtain the joint rotations. The end angle changes in Span *BD* are those of a simple beam carrying a uniform vertical load. The angle changes in the span, *DF*, are the resultant end slopes of the simple beam with a concentrated center load of 1 000 lb and an applied moment at the right-hand end of 20 000 ft-lb. From these joint rotations, the deflected structure can be sketched and the end moments calculated by use of the standard slope-deflection equations. Only the balancing of angle changes to obtain joint rotations will be discussed, since the remainder of the procedure should be clear to any one who is familiar with the ordinary theories of indeterminate structures.

Balancing the Angle Changes.—The balancing procedure was started (Fig. 2(*b*)) at Joint *B*, although any other unbalanced joint might have been selected. The angle change of + 104 at End *B* of Member *BD* produced a joint rotation of 66.7% of + 104 or + 69. This is simply the angle change times the joint rotation factor, or $\phi \left(\frac{K_1}{\Sigma K}\right)$. Joint rotation factors $\left(\frac{K}{\Sigma K}\text{-values}\right)$ are expressed as percentage factors and enclosed in boxes at the joints for convenient reference. End *B* of Member *BA* has been rotated through an angle of + 69 which gives rise to a carry-over angle of − 34 to Joint *A*. The carry-over factor is − 0.5 and only whole numbers are recorded in this example. End *B* of Member *BD* has been rotated backward from + 104 to + 69, or through an angle of − 35. The carry-over angle is + 17 to End *D*.

Two angle changes occur at Joint *D* which was selected as the second joint to be balanced; that is, − 41 in Member *DF* and − 87 in Member *DB* obtained as the algebraic sum of − 104 and + 17. The joint rotation is 23.1% of − 41 plus 30.7% of − 87, or a total of − 36. The carry-over angle to *E*, therefore, is + 18. End *D* of Member *DF* has rotated positively from − 41 to − 36, or through an angle of + 5 which produces a carry-over angle of − 2 to End *F*. End *D* of Member *DB* has rotated positively from − 87 to − 36, or through an angle of + 51. The carry-over angle to *B* is − 25. There is no carry-over angle to End *C* of Member *DC* which is a fixed end. To allow for the added resistance of this member to end rotation at *D*, its *K*-value or stiffness factor was increased $33\frac{1}{3}\%$ when joint rotation factors were being determined. One will observe this relationship from Fig. 3.

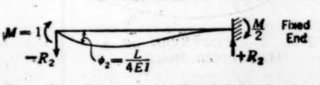


FIG. 3.—EFFECT OF END RESTRAINT UPON STIFFNESS.

The Cantilever End.—The third joint selected for balancing was *F*, where the entire resistance to rotation occurs in Member *FD* which has a joint rotation factor of 100 per cent. Hence, each value of ϕ is simply recorded as a joint rotation, θ, and there is no carry-over angle to Joint *D*. In other words, there is no external resistance to rotation at Joint *F* which means that it can be treated like Joints A and E where there is no distinction made between values of ϕ and θ. The fourth and fifth joints balanced were B and D which involved merely a repetition of the procedure that has already been described. The repetition was necessary in order to balance out the carry-over angles mentioned previously.

ACCURACY IN BALANCING ANGLES

In Fig. 2 the balancing process was stopped when the carry-over angles became less than unity. This is the proper procedure. Greater accuracy could be obtained if desired by introducing an extra significant figure into the original angle changes or ϕ-values. A device frequently found useful is to double or triple the first three significant figures of the original ϕ-values and then balance joints until the carry-over angles are less than unity. This device can be used to improve the accuracy of the analysis without adding a fourth significant figure to the original ϕ-values. Of course, the final joint rotations must be reduced in the same ratio that the original angle changes were increased. Accuracy was sacrificed for clearness in Fig. 2.

CHECKING THE FINAL MOMENTS

The simplest procedure for checking the analysis is to apply the equation $\Sigma M = 0$ to each joint after final moments are determined from the θ-values by use of the slope-deflection equations. Since the balancing is performed on angles rather than moments and since fixed-end moments are introduced through the slope-deflection equations, any error in the computation of an original angle change, in the balancing procedure, or in the application of the slope-deflection equations, will be detected by this simple check. Of course, compensating errors might occur, but this slight possibility for error exists in all methods. A sketch of the deflected structure will help to locate such unusual errors.

HAUNCHED BEAMS AND CURVED MEMBERS

Basic Terms Redefined.—A variation of cross-section or a curved axis affects the resistance of a member to end rotation and makes necessary a careful calculation of end angle changes, joint rotation factors, and carry-over factors. End angle changes can be computed conveniently by use of the conjugate-beam method. Since the possible variations of loading are infinite, it would be unreasonable to attempt to formularize this procedure. Joint rotation factors and carry-over factors can be obtained quite simply from the angle changes that occur at the ends of the simply supported beam loaded with a unit moment at one end (see Fig. 4). The carry-over factor for angle

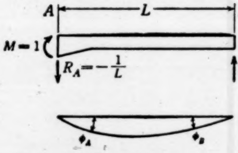


FIG. 4.—STIFFNESS OF A HAUNCHED BEAM.

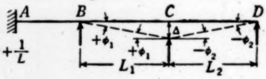


FIG. 5.—SETTLEMENT OF A SUPPORT.

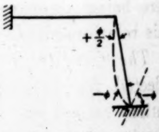


FIG. 6.—ROTATION OF A FOOTING.

change from Point A to Point B is $\frac{\phi_B}{\phi_A}$ and has a negative sign as in a prismatic beam. Stiffness is defined rigorously as the moment necessary to produce a unit end rotation, and evidently is $\frac{1}{\phi_A}$. The factor, K, has been used for prismatic beams to define relative stiffness. Hence, since $\phi_A = \frac{L}{3\,E\,I} = \frac{1}{3\,E\,K}$, or since $\frac{1}{\phi_A} = 3\,E\,K$ for such a beam, it is permissible and possibly desirable to omit the factor, $3\,E$, in making calculations for a haunched beam. Accordingly, one can define K as $\frac{1}{3\,E\,\phi_A}$ for the haunched beam and then the definition for the joint rotation factor remains unchanged as $\frac{K}{\Sigma K}$. Curved members can be handled by this procedure without re-defining any terms. The carry-over factor for angle change may reverse its sign for an arched member.

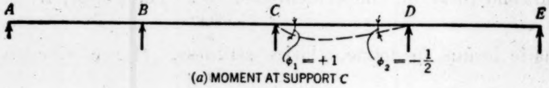
SETTLEMENT OF SUPPORTS AND OTHER IMPRESSED DISTORTIONS

The effect of a settling support is readily introduced into the calculations. Fig. 5 makes it apparent that a settlement of Δ at Support C is represented by angle changes of $\phi_1 = +\frac{\Delta}{L_1}$ at the two ends of Span BC and $\phi_2 = -\frac{\Delta}{L_2}$ at each end of Span CD. One balances these angle changes and computes the moments in exactly the same manner as for angle changes produced by loads. There are no fixed-end moments to be considered when applying the slope-deflection equations. Temperature change of a continuous frame gives rise to linear distortions represented by end angle changes in a similar manner. The unplanned rotation of a footing could be represented by two angle changes, as shown in Fig. 6.

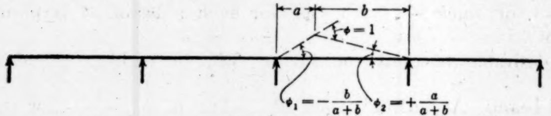
INFLUENCE LINES FOR CONTINUOUS BEAMS

The Deflected Load Line.—The process of balancing angle changes offers a highly pictorial method of determining influence lines for continuous beams. It is well known that the influence line for any function, such as moment or shear, can be obtained as the deflected load line produced by an impressed unit distortion in the nature of the stress function; that is, a unit rotation must be impressed upon the structure to obtain an influence line for moment and a unit shearing distortion must be used to obtain an influence line for shear. The corresponding end angle changes are illustrated in Fig. 7. Fig. 7(*a*) shows the angle changes that will give rise to an influence line for M_C, the moment over Support C. In balancing angles, one maintains the discontinuity, $\phi = 1$, at Joint C by balancing only carry-over angles at Joint C.

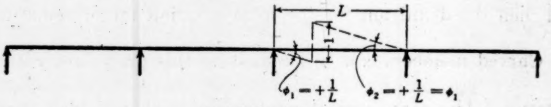
An influence line for moment between supports of a continuous beam would be obtained as illustrated by Fig. 7(b). In this case the end angle changes, $\phi_1 = -\dfrac{b}{a+b}$ and $\phi_2 = +\dfrac{a}{a+b}$, must be balanced out and the internal, unit, angular discontinuity will be maintained automatically by the standard procedure of balancing angles. Similar configurations for obtaining influence lines for shear between supports and at a support, are illustrated by Fig. 7(c) and Fig. 7(d). Observe that there are no angular discontinuities to be removed at the sections of vertical displacement.



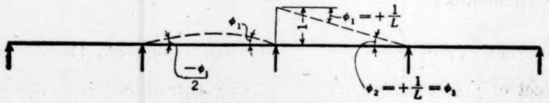
(a) MOMENT AT SUPPORT C



(b) MOMENT IN SPAN CD



(c) SHEAR IN SPAN CD



(d) SHEAR AT END C OF SPAN CD

FIG. 7.—ANGLE CHANGES FOR PRODUCING INFLUENCE LINES FOR A CONTINUOUS BEAM

Approximate Influence Lines.—The procedure, as described for continuous beams, is quite as applicable to all types of continuous frames in which joints do not translate. Hence, the method may be particularly useful as a tool for sketching an influence line rapidly and for obtaining the approximate load divides; that is, balance the angle changes shown in Fig. 7 without making any attempt at great accuracy, and then, from the approximate values of the joint rotations, sketch the shape of the deflected structure, which is the influence line desired. Approximate influence lines are frequently quite as useful as exact ones. However, the exact influence lines can also be obtained rapidly by this method.

THE PROBLEM OF JOINT TRANSLATION

Known or Estimated Deflections.—Rigid frame structures usually have certain joints that are free to translate as permitted by the elastic distortions of the structure. Evidently, the analysis of such a structure would be simplified by a previous knowledge of the deflections. From the known deflections, it would be possible to write a corresponding set of angle changes, balance these angle changes to obtain the joint rotations, and then compute the corresponding end moments. When exact deflections cannot be known beforehand, it usually is possible to estimate the deflections approximately and to base the analysis upon them, later introducing correction factors if necessary. The procedure of basing an analysis upon estimated deflections has been presented by the writer in a paper on wind stress analysis.[4]

Successive Distortion.—If deflections cannot be estimated with sufficient accuracy, one can always revert to the device of obtaining the shears in all stories or panels from an applied distortion (represented by equal end angle changes) produced, successively, in each story or panel. Then, by combining these configurations to produce the desired shears, one has obtained the true deflected structure and the corresponding moments are precise. This process is more exact and consumes more time than is justified in wind stress analysis, but it is useful in the study of open web trusses where it furnishes the data required for a study of partial live loadings.

SECONDARY STRESS ANALYSIS

Bar Rotations.—The analysis of secondary stresses in bridge trusses forms a perfect illustration of the use of predetermined deflections for computing indeterminate moments. Since the deflections are produced primarily by the changes in lengths of the truss members and only to a slight extent by the secondary moments, they are readily calculable. However, only the bar rotations are needed to start the balancing process and these can be obtained without the formality of drawing a Williot-Mohr diagram. The angle changes of the triangles of a truss can be expressed by the following relations which were derived by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., in his book entitled "Steel Mill Buildings":

$$E\,(\Delta\,A) = (s_{bc} - s_{ab})\cot B - (s_{ca} - s_{bc})\cot C \;\ldots\ldots\ldots(1)$$

$$E\,(\Delta\,B) = (s_{ca} - s_{bc})\cot C - (s_{ab} - s_{ca})\cot A \;\ldots\ldots\ldots(2)$$

and,

$$E\,(\Delta\,C) = (s_{ab} - s_{ca})\cot A - (s_{bc} - s_{ab})\cot B \;\ldots\ldots\ldots(3)$$

In Equations (1), (2), and (3), $\Delta\,A$ represents the change in Angle A caused by the stresses, s_{ab}, s_{ca}, and s_{bc}, in the correspondingly lettered bars of the truss of Fig. 8. Accordingly, if it is assumed that Bar *a-c* remains horizontal, $\Delta\,A$ becomes the rotation of Bar *a-b*. Similarly, Bar *b-c* rotates through an angle, $\Delta\,A + \Delta\,B$, Bar *b-d* rotates through an angle, $\Delta\,A + \Delta\,B - \Delta\,D$, etc. These bar rotations are relative to a zero rotation for Bar *a-c*. The true rotations relative to a line through the supports, or *a-i*, are obtained by subtracting from each bar rotation relative to Bar *a-c* the true rotation of Bar *a-c*, which,

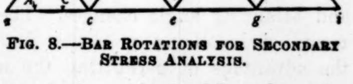


FIG. 8.—BAR ROTATIONS FOR SECONDARY STRESS ANALYSIS.

for Fig. 8, can be shown to be the average relative bar rotation for all lower chord members.

Secondary Moments.—Bar rotations are treated as end angle changes and balanced by the usual procedure to obtain joint rotations. Then, the secondary moments follow from the slope-deflection equation,

$$M_{AB} = 2EK\,(2\,\theta_A + \theta_B - 3\,\rho) \qquad \text{(4)}$$

in which θ represents a joint rotation and ρ is the bar rotation.

BENT ANALYSIS

Shear Adjustment by Proportion.—A simple bent will be used to illustrate the general method of adjusting shears by proportion. Such a bent is characterized by having a single degree of lateral freedom — its side lurch. Hence, if moments and shears are known for any given side-sway, those corresponding to any other side-sway can be obtained by multiplying the known moments and shears by the ratio of lateral deflections. It follows that the balancing procedure can be started by imposing upon the bent an unresisted side lurch represented by equal angle changes at the top and bottom of each column. Then, by balancing angle changes, one obtains the joint rotations from which the column shears, V_H, are determined by the relation:

$$V_H = \frac{6\,E\,K}{h}\,(\theta_A + \theta_B - 2\,\rho) \qquad \text{(5)}$$

The ratio of actual horizontal shear to total calculated horizontal shear is then to be used as a multiplication factor for adjusting lateral deflection, joint rotations, and column and girder moments. The "actual" horizontal shear may be caused by lateral loads or by an artificial restraint introduced through the balancing process when one is studying the effect of dissymmetry.

ANALYSIS INVOLVING JOINT SLIP

Riveted connections undergo elastic distortion and slip, both of which permit angular discontinuities to occur at the joints. For the purpose of outlining a method of analysis, it is unnecessary to distinguish between elastic and plastic joint deformation, although it is evident that these effects would have to be isolated and studied separately to investigate the action of moving loads.

Balancing Moments and Angles.—Test data on joint slip are available in a paper[6] by J. Charles Rathbun, M. Am. Soc. C. E. The cases to be discussed will be chosen so that these data are applicable. The method of analysis given herein will involve the double conception of balancing moments and balancing angle changes. The procedure can be developed entirely in terms of balancing moments, but the combination suggested herein has the advantage of permitting the use of standard stiffness factors and the unchanged carry-over factor of 0.5 for a prismatic member. The method of balancing angle changes can also be used without the aid of balancing moments but less advantageously.

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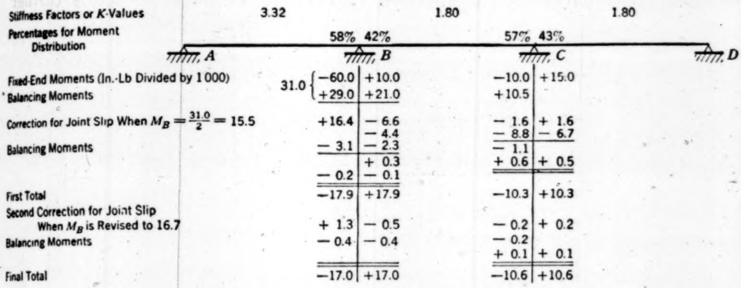
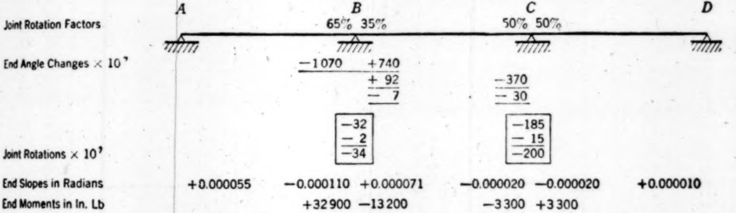
[6] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 524.

Continuous Beams with One Elastic Connection.—The first example presented (Fig. 9) is one solved by Professor Rathbun[7]. Joint B is assumed to be an elastic connection having a modulus of rotation of 4.2×10^8 in-lb per unit (radian) rotation in Span BC and 2.9×10^8 in-lb per unit rotation in Span BA.


Stiffness Factors or K-Values
3.32
1.80
1.80
Percentages for Moment Distribution
58% 42%
57% 43%
A
B
C
D
Fixed-End Moments (In.-Lb Divided by 1000)
Balancing Moments
31.0
−60.0 | +10.0
+29.0 | +21.0
−10.0 | +15.0
+10.5
Correction for Joint Slip When $M_B = \frac{31.0}{2} = 15.5$
+16.4 | − 6.6
− 4.4
− 1.6 | + 1.6
− 8.8 | − 6.7
Balancing Moments
− 3.1 | − 2.3
+ 0.3
− 0.2 | − 0.1
− 1.1
+ 0.6 | + 0.5
First Total
−17.9 | +17.9
−10.3 | +10.3
Second Correction for Joint Slip When M_B is Revised to 16.7
+ 1.3 | − 0.5
− 0.2 | + 0.2
Balancing Moments
− 0.4 | − 0.4
− 0.2
+ 0.1 | + 0.1
Final Total
−17.0 | +17.0
−10.6 | +10.6

(*a*) BALANCING MOMENTS WITH CORRECTIONS OBTAINED FROM (*b*)


A
B
C
D
Joint Rotation Factors
65% 35%
50% 50%
End Angle Changes × 10^7
−1070
+740
+ 92
− 7
−370
− 30
−32
− 2
−34
−185
− 15
−200
Joint Rotations × 10^7
End Slopes in Radians
+0.000055
−0.000110
+0.000071
−0.000020
−0.000020
+0.000010
End Moments in In. Lb
+32900
−13200
−3300
+3300

(*b*) BALANCING ANGLE CHANGES TO OBTAIN JOINT ROTATIONS

FIG. 9.—ANALYSIS OF CONTINUOUS BEAM WITH ELASTIC CONNECTION AT JOINT *B*.

Fixed-end moments are computed and the moments at Joint B are balanced once to obtain the first approximation of 31 000 in-lb for M_B. This moment will be reduced considerably by joint deformation and as a starting point it was assumed that the final moment at Joint B would be reduced 50%, to 15 500 in-lb. The moment at Joint B gives rise to angle changes from joint deformation of $\phi_{BA} = \frac{31\,000}{290\,000\,000} = 0.000107$; and, $\phi_{BC} = \frac{31\,000}{420\,000\,000} = 0.000074$. These angle changes must be maintained as angular discontinuities and, accordingly, they are not balanced. Nevertheless, there is a carry-over angle of $-0.5\ \phi_{BC}$ to Joint C which is to be balanced as in previous examples. (See Fig. 9(*b*).)

End moments are calculated from end rotations by the slope-deflection equations and are introduced as fixed-end moments. Joint moments are then completely balanced, and the joint moment at B is found to be 17 900 in-lb as compared to the estimated moment of 15 500 in-lb. The true moment lies between these values and was then estimated at 16 700 in-lb. Correction moments can be obtained in this simple example by proportion since there is only one elastic joint in the structure. The revised value of M_B is 17 000 in-lb, which is close enough to the assumed value of 16 700 in-lb to make a further

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[7] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 561, Fig. 33.

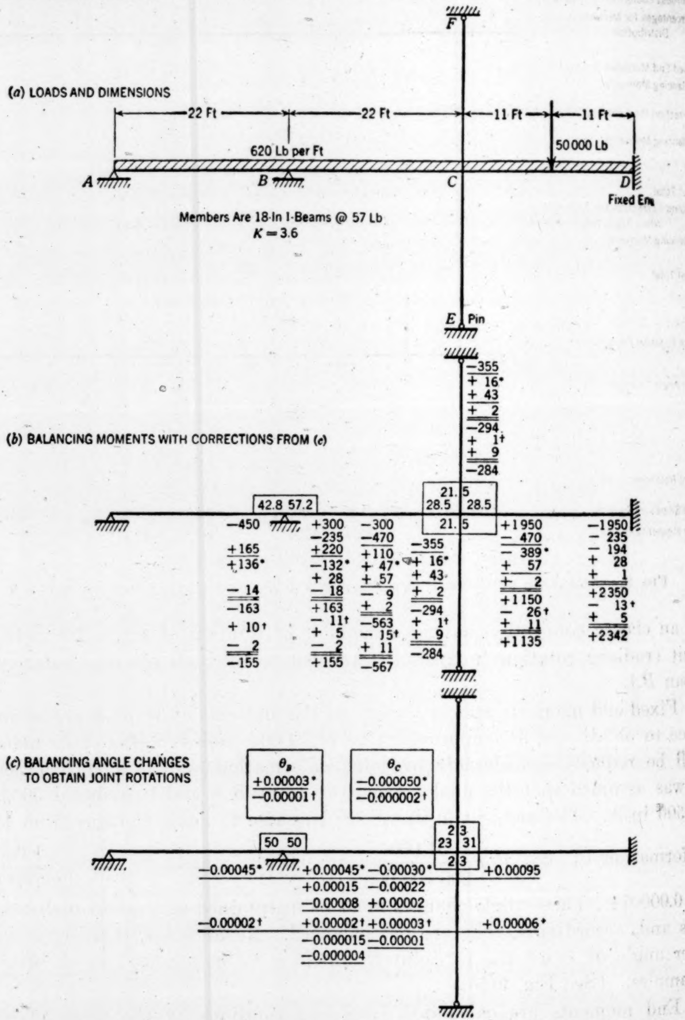

(a) LOADS AND DIMENSIONS
22 Ft
22 Ft
11 Ft
11 Ft
620 Lb per Ft
50 000 Lb
A
B
C
D
F
Fixed End
Members Are 18-In I-Beams @ 57 Lb
K = 3.6
E
Pin
(b) BALANCING MOMENTS WITH CORRECTIONS FROM (c)
42.8 57.2
21.5
28.5 28.5
21.5
−355
+ 16*
+ 43
+ 2
−294
+ 1†
+ 9
−284
−450
+165
+136*
− 14
−163
+ 10†
− 2
−155
+300
−235
+220
−132*
+ 28
− 18
+163
− 11†
+ 5
− 2
+155
−300
−470
+110
+ 47*
+ 57
− 9
+ 2
−563
− 15†
+ 11
−567
−355
+ 16*
+ 43
+ 2
−294
+ 1†
+ 9
−284
+1950
− 470
− 389*
+ 57
+ 2
+1150
− 26†
+ 11
+1135
−1950
− 235
− 194
+ 28
+ 1
+2350
− 13†
+ 5
+2342
(c) BALANCING ANGLE CHANGES TO OBTAIN JOINT ROTATIONS
θB
+0.00003*
−0.00001†
θC
−0.000050*
−0.000002†
50 50
23
23 31
23
−0.00045*
−0.00002†
+0.00045*
+0.00015
−0.00008
+0.00002†
−0.000015
−0.000004
−0.00030*
−0.00022
+0.00002
+0.00003†
−0.00001
+0.00095
+0.00006*

FIG. 10.—ANALYSIS OF A FRAME WHERE JOINT SLIP IS INVOLVED. (*CORRECTIONS FOR JOINT SLIP ARE BASED ON ASSUMED MOMENTS OF:—$M_{BA} = M_{BC} = +150$ INCH-KIPS; $M_{CB} = -600$ INCH-KIPS; AND $M_{CD} = 1\,100$ INCH-KIPS. † REVISION OF CORRECTIONS FOR JOINT SLIP WHEN:—$M_{BA} = M_{BC} = 155$ INCH-KIPS; $M_{CB} = -570$ INCH-KIPS; AND $M_{CD} = 1\,140$ INCH-KIPS.)

correction unnecessary. These final moments check the values computed by the writer by Professor Rathbun's procedure.

Continuous Frame with Two Deformable Connections.—The second example, shown in Fig. 10, represents a more complicated analysis of a frame having two joints where both elastic deformation and plastic joint slip occur. The modulus of rotation for the connections at Joints B and C are as shown by Fig. 11. It is of importance to note that it is not permissible in this case to simplify the analysis by the assumption of a straight-line relationship between joint slip and end moment.

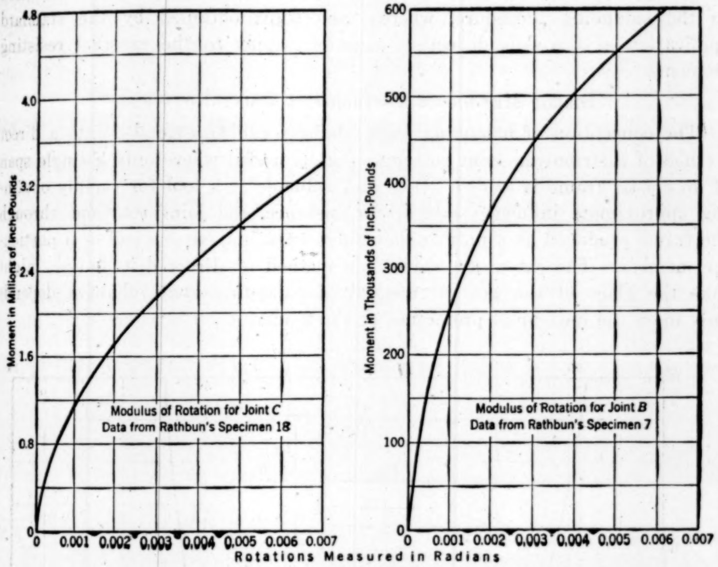


FIG. 11.—MODULI OF ROTATION FOR RIVETED JOINTS OF FRAME OF FIG. 10.

The procedure is started in Fig. 10(b) by a single balance of fixed-end moments at Joints C and B. From these approximate moments, an estimate of final moments is made. Care should be taken in making this estimate because a close approximation of the final moments will remove the need for making additional corrections. From the estimated moments and the data of Fig. 11, one obtains approximate values for joint slip. These angular discontinuities give rise to carry-over angles which are balanced in Fig. 10(c). From the computed values of end rotation, one obtains end moments which are shown followed by an asterisk in Fig. 10(b). When these moments had been balanced out, the end moments were compared with the estimated values and found to be in reasonable agreement. However, to illustrate the procedure of introducing corrections, an improved estimate of final moments was made, and the changes in these moments were used to obtain correction angles from Fig. 11, which also are balanced in Fig. 10(c). The correction moments obtained from these correction angles are introduced into Fig. 10(b) and balanced to obtain final moments. Second corrections to moments and angles are marked with a dagger in Fig. 10(b) and Fig. 10(c).

Sign Reversal.—The foregoing procedure will not be found easy to understand until one is quite familiar with the methods of balancing moments and balancing angle changes. Possibly a confusing point is the reversal of signs of the moments as obtained by the slope-deflection equations. This is necessary because of the fact that the signs of internal moments are used in the balancing procedure, whereas the signs obtained by the standard application of the slope-deflection equations apply to the external resisting moments.

DIRECT METHOD OF DISTRIBUTING JOINT ROTATIONS

The conception of balancing angle changes can be extended into a direct method of distributing joint rotations that is useful where only a single span of an elastic frame is loaded. It is also valuable as a tool for rapidly obtaining approximate influence values; for instance, the joint rotations through the frame produced by a unit angle change (ϕ-value) at one end of a particular member. The extent to which this method of direct distribution might take the place of the general method of balancing angle changes depends only upon the individual preference of the reader.

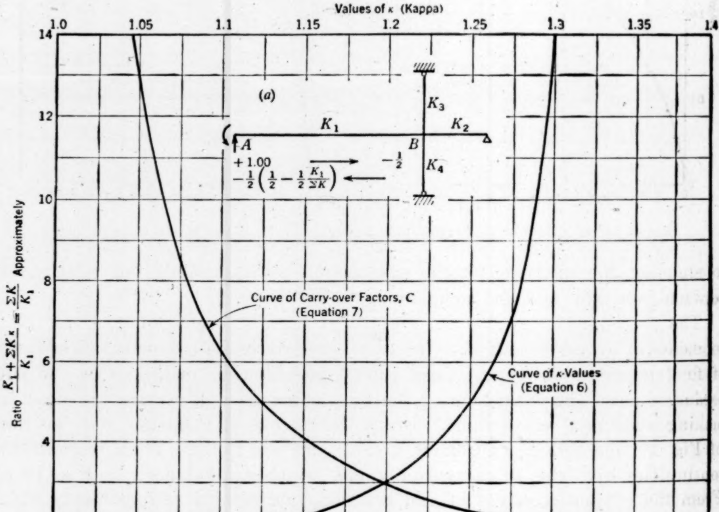


FIG. 12.—CARRY-OVER FACTORS AND K-VALUES FOR DIRECT DISTRIBUTION OF JOINT ROTATIONS.

Stiffness Dependent Upon End Restraint.—Fig. 12(*a*) shows the resultant θ-values as produced by a unit ϕ-value at End A of Member AB which is restrained but not fixed at Joint B. Since stiffness is defined as resistance to rotation, the stiffness of the member in terms of rotation at End A is increased by the restraint at Joint B. The stiffness at a joint usually is expressed as $\Sigma K = K_1 + K_2 + K_3 + K_4$. If κ is defined as the ratio of the stiffness of the restrained member to its stiffness when the far end, B, is simply supported, then $K \kappa$ becomes the stiffness factor for a restrained member exactly comparable to the K-value for an unrestrained member. From the small sketch, Fig. 12(*a*), it will be evident that $\theta_A = 0.75 + 0.25 \dfrac{K_1}{\Sigma K}$ and that $\theta_B = -0.5 \dfrac{K_1}{\Sigma K}$. Accordingly,

$$\kappa = -\frac{1}{\theta_A} = \frac{1}{0.75 + 0.25 \dfrac{K_1}{\Sigma K}} = \frac{4}{3 + \dfrac{K_1}{\Sigma K}} \quad \dots\dots\dots\dots (6)$$

The carry-over factor for joint rotation is,

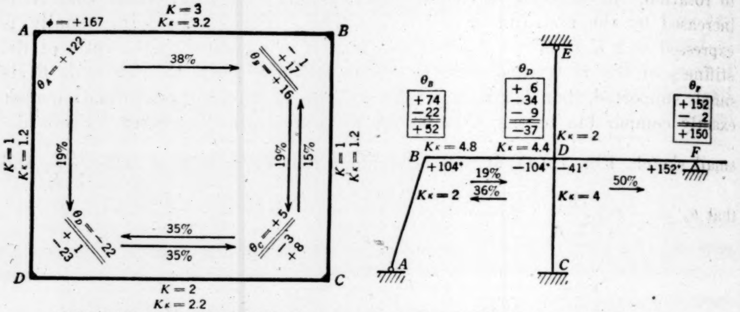
$$C = -\frac{\theta_B}{\theta_A} = -\frac{0.5 \dfrac{K_1}{\Sigma K}}{0.75 + 0.25 \dfrac{K_1}{\Sigma K}} = -\frac{2}{1 + 3 \dfrac{\Sigma K}{K_1}} = -\frac{\kappa}{2} \frac{K_1}{\Sigma K} \ . (7)$$

When the far ends are restrained, substitute in the foregoing equations, $\dfrac{K_1}{\Sigma K \kappa}$ for $\dfrac{K_1}{\Sigma K}$, in which $\Sigma K \kappa = K_2 \kappa_2 + K_3 \kappa_3 + K_4 \kappa_4$.

The curves of Fig. 12 show the plotted values of κ and C for a wide variation of the ratios, $\dfrac{\Sigma K}{K_1}$. Actually, when the far ends of the restraining members are not simply supported, one should substitute $\dfrac{K_1 + \Sigma K \kappa}{K_1}$ for $\dfrac{\Sigma K}{K_1}$, but this refinement is unnecessary when approximate θ-values are satisfactory.

Examples of Direct Distribution.—The use of the curves of Fig. 12 is illustrated by the two examples of Fig. 13. The elastic frame in Fig. 13(*b*) was analyzed previously in Fig. 2. The ϕ-value of + 104 at End B gives rise to the joint rotation of + 74 which, in turn, produces the joint rotation of + 6 at Joint D. The remainder of the θ-values are produced by balancing Joint D and carrying over the proper rotations to Joint B and Joint F.

A box culvert is analyzed in Fig. 13(*a*) for the effect of a single ϕ-value of + 167 at Corner A. The joint rotations become influence values from which the effect of any symmetrical loading could be obtained. The calculations are self-explanatory except perhaps for the use of a single κ-value for each vertical member. Actually, there are slightly different κ-values for the two ends of the vertical member, but this difference is small enough to be neglected. The corresponding effect upon the carry-over factor was of greater importance, and, accordingly, two carry-over factors were recorded for the vertical member.


A
B
C
D
$\phi = +167$
$K = 3$
$K_\kappa = 3.2$
$\theta_A = +122$
38%
$\theta_B = +16$
$+17$
-1
$K = 1$
$K_\kappa = 1.2$
19%
19%
15%
$\theta_D = -22$
$+1$
-23
35%
35%
$\theta_C = +5$
-3
$+8$
$K = 2$
$K_\kappa = 2.2$
E
θ_B
+74
−22
+52
θ_D
+6
−34
−9
−37
θ_F
+152
−2
+150
$K_\kappa = 4.8$
$K_\kappa = 4.4$
$K_\kappa = 2$
F
+104*
19%
36%
−104*
−41*
50%
+152*
$K_\kappa = 2$
$K_\kappa = 4$

(a) INFLUENCE VALUES FOR CLOSED FRAME

(b) ANALYSIS OF CONTINUOUS FRAME IN FIG. 2

FIG. 13.—DIRECT DISTRIBUTION OF JOINT ROTATIONS. (*RELATIVE ϕ-VALUES.)

CONCLUSION

The procedure of balancing angle changes has been found to offer a parallel method to that of balancing moments. Either tool can be used to analyze continuous beams, rigid frames, open-web trusses, tall building frames, continuous arches on elastic piers, and other continuous framed structures. In certain cases they can be used to advantage to supplement each other, as in the analysis of a frame undergoing joint slip. The procedure of balancing angle changes possesses the inherent disadvantage that final moments are computed from the differences between angular rotations. Hence, angular rotations must be computed with greater accuracy than joint moments by the method of moment distribution. However, the method of balancing angle changes possesses two important advantages of its own: (1) It is essentially self-checking; and (2) it offers one the clearest possible picture of the physical action of the structure. These attributes seem sufficient to justify its use by those designers who are accustomed to thinking in terms of joint rotations and deflections.

DISCUSSION

JOHN E. GOLDBERG,[8] JUN. AM. SOC. C. E. (by letter).—The method of analyzing continuous frames by balancing end rotations presented by Professor Grinter is similar in many respects to the simple slope-deflection method largely developed by G. A. Maney, M. Am. Soc. C. E., from his basic slope-deflection equations and presented[9] by the writer in 1931. Both methods consist, essentially, of determining the angular positions at which the various joints of the continuous structure are in equilibrium, or balanced, and then substituting these values of the angular rotations in the basic slope-deflection equation to obtain the final or actual moments at the ends of the various members. Both methods may be extended, by the application of well-known and well-understood principles, to the solution of a wide variety of rigid-frame problems. The advantages which Professor Grinter cites for his method are at least as applicable to the earlier slope-deflection method.

The slope-deflection method seems, to the writer, to have certain additional inherent advantages; for example, the starting point of the slope-deflection method is the fixed-beam moments which are again used to obtain the final or actual moments after the various rotations have been determined. This itself may be construed as a twofold advantage: First, a complete series of calculations (the simple beam end-angles) is avoided; and, second, the fixed-beam moments are themselves better known and more easily remembered for the various standard loadings than the simple beam end-angles. Another advantage lies in the fact that, throughout the entire slope-deflection analysis, the concept of continuity is firmly maintained, a fact that adds greatly to the clarity and physical significance of the method, besides being inherently more direct. In Professor Grinter's analysis, on the other hand, this continuity is first completely destroyed and then successive parts of it are re-established. Another inherent advantage of slope-deflection methods lies in the fact that there are frames the analysis of which by methods based upon successive approximations, corrections, or balancing, may be inadvisable, impractical, or even impossible.

Various types of simple frames, for example, may be analyzed by slope deflection by the solution of a limited number of equations, in many cases by only one equation. Simple bridges, culverts, and many other similar frames fall into this category of frames which may be analyzed with actually less effort and greater reliability by direct methods. The frame shown in Fig. 2, although chosen by the author for its classical values, is nevertheless an illustration of a structure that could be solved very neatly by these direct slope-deflection methods. One who is adept and familiar with the slope-deflection theory could very easily set up the two equations which would solve the frame. Solution of these equations would give, with absolute certainty, the joint rotations from which the final or actual moments may be calculated. There are also frames which, by their very proportions, can not be solved successfully by methods based upon successive approximations or corrections. For the analysis of these frames there always remains the alternative method of solving the slope-deflection equations by algebra.

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[8] Chicago, Ill.

[9] "Vertical Load Analysis of Rigid Building Frames", by John E. Goldberg, *Engineering News-Record*, November, 12, 1931; see, also, "Simplified Methods for the Analysis of Multiple Joint Rigid Frames", by George A. Maney and John E. Goldberg, *Northwestern Univ. Bulletin*, Vol. XXXIII, No. 7, October 17, 1932.

Professor Grinter, it seems, has touched upon the matter of bent analysis too lightly. He implies that if the shear for any assumed side-sway at a particular story is known, the actual shear in the various members may be calculated by a single, simple, proportionate adjustment. Unfortunately, the analysis is not so simple, except in a very few mathematically perfect cases, chief among which is the single-story bent, itself not a very complex problem. Equations (4) and (5), the slope-deflection equations, show that the shear in a member is as much a function of the joint rotations at the ends of the member as of the relative joint translation. These rotations, in turn, are affected just as much by the shears above and below the story under consideration as they are by the shears in that particular story, etc. Consequently, except in these few perfect cases, this simple adjustment is itself only an approximation.

Professor Grinter has referred to the method of analyzing wind stresses by slope-deflection and converging approximations, which the writer developed from basic slope-deflection theory and presented[10] in 1933, as being based upon series convergence. Actually, the method consists simply of obtaining, successively, closer or more nearly correct values of the various rotations in accordance with simple and easily understood principles, and thus to label the method with an abstract mathematical term is confusing to the student and to the reader. Furthermore, he has referred to the method as being "relatively tedious" without, however, stating to what it is relative. Nevertheless, after an exhaustive survey of methods of wind stress analysis, the results of which are contained in its latest report[11], Sub-Committee No. 31, Committee on Steel of the Structural Division of the Society, on Wind-Bracing in Steel Buildings points out that among the desirable characteristics of the writer's method are speed, accuracy, simplicity, and the clarity of the physical concept maintained throughout the analysis. Professor Grinter's present appreciation of the slope-deflection equation and of the importance and significance of the physical concept of joint rotations is definitely complimentary to the slope-deflection method and to those who have consistently sponsored and developed its use.

The frame of Fig. 2 is analyzed herein by slope deflection to illustrate the general methods of attack. One familiar with the slope-deflection theory can set up, practically by inspection, the joint equations which express the simple physical fact that ΣM (in terms of fixed-beam moments and joint rotations) equals zero. For the frame of Fig 2:

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[10] "Wind Stresses by Slope-Deflection and Converging Approximations", by John E. Goldberg, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 962–975.

[11] *Proceedings*, Am. Soc. C. E., March, 1936, p. 397.

$$\text{Joint } B\text{: } 11.0\ \theta_B + 4.0\ \theta_D = 8.333$$

and,

$$\text{Joint } D\text{: } 4.0\ \theta_B + 21.5\ \theta_D = -\ 12.0833$$

From which, by simple algebra:

$$\theta_D = \frac{11.0\ (-12.0833) - 4.0\ (8.333)}{11.0\ (21.5) - 4.0\ (4.0)} = -0.754$$

and,

$$\theta_B = 0.758 + 0.274 = 1.032$$

Substitution of these values in the basic slope-deflection equation,

$$M_{AB} = M_{F\text{-}AB} - K\ (2\,\theta_A + \theta_B) \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

gives the final end moments; that is, $M_{DB} = -\ 8.333\ -4.0\ (-\ 1.508 + 1.032) = 6.429$; and, similarly, for the other end moments. The moments are in kip-feet; and, K-values are used in only a relative sense. As a result of these facts, the joint rotations in the foregoing analysis are approximately one-fiftieth of the rotations indicated by Professor Grinter.

When direct analysis of a frame is so simple, there is scant justification for the use of methods based on successive balancings or corrections, or even for the use of methods based on semi-empirical formulas. However, as an illustration of an alternative slope-deflection method, particularly applicable to frames in which the number of joints precludes the use of algebraic methods, the analysis of the frame of Fig. 2 by converging approximations is also given: The respective joint equations are, for simplicity, reduced and solved for the respective values of θ:

$$\text{Joint } D\text{: } \theta_D = -\ 0.562 - 0.186\ \theta_B$$

and,

$$\text{Joint } B\text{: } \theta_B = 0.758 - 0.364\ \theta_D$$

For the purpose of obtaining a first approximation, θ_B is taken to be equal to the constant term in its own equation; that is, 0.758. Then, $\theta_D = -\ 0.562 - 0.186\ (0.758) = -\ 0.703$. From which, $\theta_B = 0.758 - 0.364\ (-\ 0.703) = 1.014$.

For a second approximation: $\theta_D = -\,0.562 - 0.186\ (1.104) = -\,0.751$; and, $\theta_B = 0.758 - 0.364\ (-0.751) = 1.031$. For all practical purposes, this second approximation is sufficiently accurate. However, a third approximation gives: $\theta_D = -\ 0.562\ - 0.186\ ((1.031) = -\ 0.754$; and, $\theta_B = 0.758 - 0.364\ (-\ 0.754) = 1.032$.

By comparison of these results it is clear that convergence is now complete. Substitution of these final θ-values in the respective basic slope-deflection equations gives the final end moments for the various members. It is clear that no appreciable inaccuracy would have resulted if the θ-values obtained as a second approximation were used to calculate the final end moments.

G. A. MANEY,[12] M. AM. SOC. C. E. (by letter).—In comparing the method used in his paper with the moment distribution method, the author comes to two conclusions as follows: (1) It is essentially self-checking; and (2) it offers the clearest possible picture of the physical action of the structure. The writer agrees heartily with these conclusions and submits four short solutions in tabular form, which are made by using the slope-deflection method first proposed by him in the United States in 1915. These solutions (which explain themselves because of the tabular forms used), together with the past

TABLE 1.—COMPLETE SOLUTION OF PROBLEMS IN FIG. 2 AND FIG. 9

Line No	(a) END MOMENTS IN FIG. 2				
	Symbol (1)	Coefficients of: θ_B (2)	Coefficients of: θ_D (3)	Fixed-beam moments, M_F (4)	Value of final moment, in foot-kips (5)
COMPLETE CHECK COMPUTATIONS FOR FINAL END MOMENTS					
1	M_{BA}	−3	……..	……..	−3.09
2	M_{BD}	−8	−4	+8.33	+3.09
3	M_{DB}	……..	−8	−8.33	−6.36
4	M_{DE}	……..	−3	……..	+2.25
5	M_{DF}	……..	−4.5	−3.75	−0.38
6	M_{DC}	……..	−6	……..	+4.50
7	M_{CB}	……..	−3	……..	+2.25
…	M_{ED}	……..	……..	……..	+20.00
CONVERGING SOLUTION FOR VALUES OF $2E\theta$					
	EQ for:				
8	J_B	+11	+4	+8.33†	……..
9	J_D	+4	+21.5	−12.08†	……..
10*	…….	+1.0000	0.364	……..	……..
11*	…….	+0.186	1.000	……..	……..
12*	…….	+0.758	−0.566	……..	……..
13*	…….	+0.205	−0.141	……..	……..
14*	…….	+0.051	−0.038	……..	……..
15*	…….	+0.014	−0.009	……..	……..
16*	…….	+0.003	−0.002	……..	……..
17	$2E\theta$	+1.031	−0.756	……..	……..

(b) END MOMENTS IN FIG. 9, INCLUDING EFFECT OF ANGULAR DISCONTINUITIES				
Symbol (6)	Coefficients of: $-\theta_B$ (7)	Coefficients of: $-\theta_C$ (8)	Fixed-beam moments M_F (9)	Corrections due to the angles, ϕ (10)
MOMENT EQUATIONS				
M_{BA}	−4.98	……..	−60.0	−0.996 M_{BA}
M_{BC}	−3.60	−1.80	+10.0	−0.448 M_{BC}
M_{CB}	−1.80	−3.60	−10.0	−0.249 M_{BC}
M_{CD}	……..	−2.70	+15.0	……….
FINAL MOMENT EQUATIONS				
M_{BA}	−2.49	……..	−30.0	−17.1‡
M_{BC}	−2.40	−1.20	+6.7	+17.1‡
M_{CB}	……..	−3.30	−11.7	−10.7‡
M_{CD}	−1.20	−2.70	+15.0	+10.7‡
CONVERGING SOLUTION FOR VALUES OF $2E\theta$				
EQ for:				
J_B	+4.89	+1.20	−23.3	……….
J_C	+1.20	+6.00	+3.3	……….
*	+1.00	+0.246	……	……….
*	+0.20	+1.00	……	……….
*	−4.76	+0.55	……	……….
*	−0.14	+0.95	……	……….
*	−0.23	+0.03	……	……….
*	−0.01	+0.05	……	……….
$2E\theta$	−5.14	+5.18	……	……….
……	……..	……..	……	……….
……	……..	……..	……	……….

* Balancing angle changes. † ΣM_F. ‡ Values of final moment, in inch-kips.

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[12] Prof. of Structural Eng., School of Eng., Northwestern Univ., Evanston, Ill.

history of slope-deflection and of converging approximate solutions, lead to the following comments:

(1) "Balancing angles changes" is merely a new group of words to describe what has been done for years in the solution of slope-deflection equations by converging approximations of the θ-values.[13]

TABLE 2.—SOLUTION OF PROBLEM IN FIG. 10

Symbols	(a) ASSUMING RIGID JOINT			(b) ASSUMING FINAL ANGLE SLIPS IN THE JOINTS AND DETERMINING THE REDUCTION IN MOMENTS THUS CAUSED					
	Coefficients of:		Fixed-beam moments, M_F (3)	Coefficients of:		Fixed-beam moments, M_F (6)	Correction due to yielding angles (7)	Value of final moment (8)	Percentage reduction in moment (9)
	θ_B (1)	θ_C (2)		θ_B (4)	θ_C (5)				
COMPLETE CHECK COMPUTATIONS FOR MOMENTS									
M_{BA}	−5.4	....	−450	−5.4	....	−450	+147	−164†	41
M_{BC}	−7.2	−3.6	+300	−7.2	−3.6	+300	−127	+164†	41
M_{CB}	−3.6	−7.2	−300	−3.6	−7.2	−300	+40	−556†	19
M_{CF}	....	−5.4	....	....	−5.4	....	....	−292†	23
M_{CD}	....	−7.2	+1 950	....	−7.2	+1 950	−421	+1 140†	21
M_{CE}	....	−5.4	....	....	−5.4	....	....	−292†	23
M_{DC}	....	−3.6	−1 950	....	−3.6	−1 950	−210	−2 355†	−7
EXACT SOLUTION FOR VALUES OF $2E\theta$									
EQ for: J_B	+12.6	+3.6	−150*	+12.6	+3.6	−130*	....	....	....
J_C	+3.6	+25.2	+1 650*	+3.6	+25.2	−1 269*	....	....	....
....	−3.6	−1.04	+43	−3.6	−1.04	+37	....	....	....
M_F for:	....	+24.16	+1 693	....	+24.16	+1 306	....	....	....
θ_C	....	....	+70	....	....	+54.0	....	....	....
θ_B	....	....	−32	....	....	−25.7	....	....	....

* ΣM_F. † Inch-kips.

(2) In the problems of Tables 1 and 2 values of $2E\theta$ were determined by the usual method of "balancing angles" or "converging slope values", instead of the θ-values themselves. This has always been done merely as a convenience in calculation, since the quantities resulting are easier to handle. In all slope-deflection solutions made in recent years in which the number of unknowns is large, this method of "balancing angles" or "converging end slope values" has been used, but the convergence or balancing has started from the zero end angle case in which M_F is the fixed-end bending moment.

(3) In Table 1(*b*) a solution is given for the case of elastic angle slip at the joints, which has the advantage of making a trial solution unnecessary. Note the close check on values of final moments. In another problem checked by the writer (see Table 2), the case of both elastic and plastic joints is

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[13] For an example, see "Statically Indeterminate Stresses", by John I. Parcel and G. A. Maney, Members, Am. Soc. C. E., Fig. 142, opposite p. 276, John Wiley and Sons, 1936.

treated. The "corrections due to yielding angle" (see Table 2(*b*) where this correction is − 127) herein used can only be,

$$\Delta M_{BC} = -2\,E\,K_{BC}\,[2\,\phi_{BC} + \phi_{CB}] \qquad \text{(9)}$$

using the slope-deflection formula in which ϕ-values are angular discontinuities. When they are elastic, ϕ_{BC} becomes $\frac{M_{BC}}{R_{BC}}$ and $\phi_{CB} = \frac{M_{CB}}{R_{CB}}$ (in which the *R*-values are the moduli of rotation) which are ratios of bending moment to corresponding angular rotation of the end of the member, in radians. In Table 2(*b*), this correction becomes $-2\,E\,K\left(\frac{2\,M_{BC}}{R_{BC}}\right)$ for M_{BC}, or $-0.498\,M_{BC}$.

(4) Particular attention is called to the case of "plastic" discontinuity of end angles as included in the author's problem in Fig. 10 and in the writer's corresponding solution in Table 2(*b*). The percentage change in end moments due to lack of joint rigidity is listed in Column (9), Table 2(*b*). This percentage varies from a decrease of 41% to an increase of 7%, and emphasizes the need for a solution as given in Table 1(*b*) which eliminates guesswork regarding the final moment values.

(5) If there is a "plastic" feature of the angular discontinuity, why should not all end moments closely approach zero finally? A plastic condition *per se* means "give" under continued load with a corresponding steady increase of ϕ-values with time. If this simple beam condition is approached, why indulge in rigid-frame analysis for frames with "flowing" or "plastic" joints?

Tables 1 and 2 may be clarified by examining Table 1(*a*), as follows:

(*a*) Line 2 indicates that $-8\,\theta_B - 4\,\theta_D + 8.33 = M_{BD}$, which is a specific statement of the general equation applicable to this case. The general form would be stated:

$$M_{BD} = \pm M_{FBD} - K_{BD}\,(2\,\theta_B + \theta_D) \qquad \text{(10)}$$

in which, $\pm M_{FBD} = +8.33$ ft-kips; $2\,K_{BD} = 8$; $K_{BD} = 4$; and θ_B and θ_D are multiples of the angular rotation of Joints *B* and *D* when equilibrium is reached.

(*b*) Line 8 shows that $\Sigma\,M_B = 0$; or that the net rotational tendency of all moments acting on Joint *B* is zero when equilibrium is reached. The expressions for M_{BA} in Line 1 and for M_{BD} in Line 2 are added to obtain the joint equation of Line 8, which is: $11\,\theta_B + 4\,\theta_D = +8.33$. Likewise, equations of Lines 3, 4, 5, and 6, are added to obtain the equation (Line 9) for Joint *D*.

(*c*) Lines 10, 11, and 12, indicate that $1.000\,\theta_B + 0.364\,\theta_D = 0.758$ and that $0.186\,\theta_B + 1.000\,\theta_D = -0.566$. Lines 13, 14, 15, and 16 are the balancing steps for the angle values ($0.364\,\theta_D$ for θ_B, and $0.186\,\theta_B$ for θ_D) at the far end, converging until the corrections approach zero.

(*d*) Line 17 represents final values of $2\,E\,\theta$, or additions of five preceding quantities, which are substituted in the original moment expressions (Lines 1 to 7, inclusive) and values such as $-$ 3.00 ft-kips are obtained for all member ends, such as End B of Member BA (M_{BA}). It will be noted that the sum of all moments around a joint must be equal to zero, a desirable check on the numerical accuracy of the work.

PAUL ANDERSEN,[34] ASSOC. M. AM. SOC. C. E. (by letter).—In many respects Professor Grinter's method will prove a more flexible tool to the designing engineer than the Cross method. It is true that the latter has the advantage over the former in that it makes use of moments throughout its procedure and leads directly to the actual bending moments; but this disadvantage of the Grinter method is more than compensated for by the facts that it is self-checking and that the actual work involved in establishing continuity has been reduced materially. Thus, in using the Cross method, it is necessary to balance the moments at each end of a member, whereas by balancing angle changes, only one set of values is needed at each joint.

The procedure used in solving the example in Fig. 2 seems incorrect; Joint B is balanced and the carry-over angle from B is added to the angle changes of Joint D and included in the first process of balancing Joint D. The proper procedure is to balance all joints first and then to carry over the angle changes, and repeat.

Side-Sway.—Instead of obtaining the corrections for side-sway by imposing equal angle changes at the top and bottom of each column, the writer suggests applying, at the column tops, any set of angle changes inversely proportional to the column heights, and distributing these changes throughout the structure. It is readily seen that, by equating horizontal deflections before and after balancing the angle changes, the moment at the top of a column is:

$$M = \frac{Vh}{2} \times \frac{\phi - \theta}{\phi - \dfrac{3}{4}\,\theta} \quad \text{..(11)}$$

in which ϕ = an arbitrary angle change of the top of the column; θ = the resulting joint rotation; V = column shear, proportional to $\frac{K}{h^2}$; and h = column height.

This procedure is applied to the two-legged bent

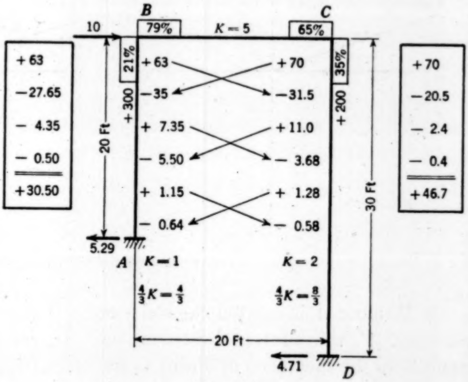

FIG. 14.

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[34] Balboa Heights, Canal Zone.

shown in Fig. 14 in which angle changes of 300 and 200 are applied to the column tops, resulting in joint rotations of 30.5 and 46.7, respectively, after continuity has been established. The final moments (see Fig. 14) are,

$$M_B = \frac{1}{2} \times 5.29 \times 20 \times \frac{300 - 30.5}{300 - \frac{3}{4}\,30.5} = 51.4 \text{ ft-kips}$$

and,

$$M_C = \frac{1}{2} \times 4.71 \times 30 \times \frac{200 - 46.7}{200 - \frac{3}{4}\,46.7} = 65.6 \text{ ft-kips}$$

WILLIAM F. LUCE,[15] JUN. AM. SOC. C. E. (by letter).—A valuable tool for the analysis of continuous frames is contained in this paper. Computations by the method introduced involve only simple arithmetic with no simultaneous equations, and can be made, in many cases, in a fraction of the time required by the so-called "exact" methods, such as the slope-deflection method, the method of least work, etc. Furthermore, both the development and the application of the method involve concepts familiar to the structural engineer.

The presentation by Professor Grinter has been so complete that further explanation is scarcely necessary. The following development, however, may help some readers to visualize the balancing process and the determination of the joint rotation factor. In Fig. 15(*a*), the members, *A*, *B*, *C*, and *D*, have stiffness ratios $\left(\frac{I}{L} = K\right)$ of K_A, K_B, K_C, and K_D, respectively.

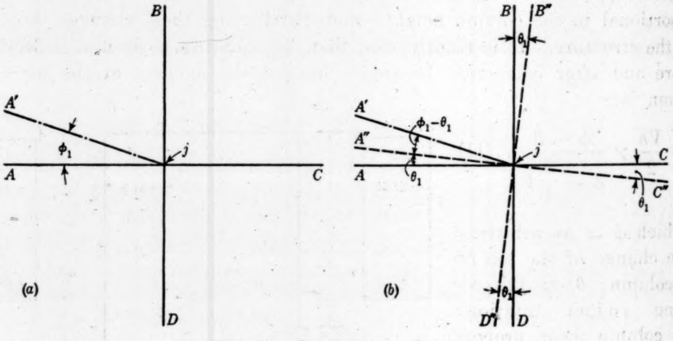

FIG. 15.—BALANCING A JOINT.

If Member *A* is rotated clockwise about Point *j* through an angle, ϕ_1, to the position, *A′*, the joint is discontinuous. In restoring continuity, by moments applied to the members, at Point *j*, Members *B*, *C*, and *D* will be rotated clockwise through an angle, θ_1, to the new positions, *B″*, *C″*, and *D″*, whereas

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[15] Univ. Fellow in Structural Eng., Univ. of Michigan, Ann Arbor, Mich.

Member A will be twisted counter-clockwise through an angle $(\phi_1 - \theta_1)$ to the position, A''. The internal moments balance at Joint j:

$$M_A = M_B + M_C + M_D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

and, since the moment in a member depends upon the product of its stiffness and the angle through which it rotates:

$$(\phi_1 - \theta_1)\, K_A = \theta_1\, K_B + \theta_1\, K_C + \theta_1\, K_D \quad \ldots\ldots\ldots\ldots (13)$$

or,

$$\theta_1 = \phi_1 \frac{K_A}{K_A + K_B + K_C + K_D} = \phi_1 \frac{K_A}{\Sigma K} \quad \ldots\ldots\ldots\ldots (14)$$

Factor $\frac{K_A}{\Sigma K}$ is termed the joint rotation factor for Member A at Joint j. The development is equally simple if two or more members are given an original angular discontinuity.

It is noted from the paper that members having their ends free to rotate are the norm in this method, and that the stiffness values for fixed-end members are corrected by the factor, $\frac{4}{3}$. This is the reverse of the procedure used in the method of balancing moments.

The writer has solved problems on secondary stresses in trusses, wind-stress analysis of building frames, and joint slip, by the method of balancing angle changes. The structure investigated for secondary stresses was a symmetrical Pratt truss of eight panels. Angle changes were computed by Dean Ketchum's formulas as suggested by the author. The balancing required four to five cycles per joint. The end moments found in this manner balanced within 1% at every joint, providing a check on the work. As a final check, the secondary stresses were calculated by the method of balancing moments, using deflections taken from a Williot-Mohr diagram. The variation of stress values as obtained by the two methods was small in most members, and the maximum difference was 400 lb per sq in. It was observed, however, that the balancing process in either case should be started with comparatively large numbers if the final results are to be accurate.

The first wind-stress problem investigated was that of a 6-story building[18], the analysis being made by estimating the shape of the deflected structure as outlined by the author elsewhere[4]. The final moments obtained, requiring only one balancing process, checked those calculated by balancing moments within 1% to 2 per cent. The latter method, however, seems to be definitely better suited to the solution of wind-stress problems. Only four cycles of balancing moments were required to give the same accuracy as six or seven cycles of balancing angle changes, and, of course, did not require the use of the slope-deflection equations at the end of the process. The analysis of the lower stories of another building frame[17], also indicated that the value of the check embodied in the method of balancing angle changes is decreased because of the introduction of different criterion ratios in adjacent stories. The method of balancing moments has the further advantage in the analysis of frames in which joint translations occur, such as wind-stressed buildings, that fixed-end moments may be added during the balancing process to offset joint restraints. A similar procedure for the method of balancing angle changes is not apparent.

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[18] Structural Engineer's Handbook, by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., 1924 Edition, p. 102.

The solution of joint slip problems and the plotting of influence lines for continuous beams can be performed very successfully by balancing angle changes.

Ralph E. Byrne, Jr.,[18] Jun. Am. Soc. C. E. (by letter).—Another method for the analysis of continuous structures is presented in this paper. It is a close parallel to the method of moment distribution, but inasmuch as the quantities dealt with are rotations and displacements rather than moments, the procedure must necessarily differ. Some of the most important differences between the two methods occur in the definitions of some of the terms involved. In the moment-distribution method, stiffness is defined as the moment required to produce unit rotation at one end of a member, the other end being fixed; the carry-over factor is defined as the ratio of the moment at the fixed end to the moment at the end being rotated. In the method of balancing angle changes, stiffness is defined as the moment required to produce unit rotation at one end of a member, the other end being hinged; and the carry-over factor is defined as the ratio of the angle change at the hinged end to the angle change at the end being rotated. Likewise, the method by which applied loads are brought into the analysis is different for the two methods In the former method, fixed-end moments are used, these being the moments which would occur at the ends of the loaded member if the ends were fully restrained. In the latter method, the quantities which are analogous to the fixed-end moments are the angles of rotation of the two ends of the member under the applied loads, the ends being hinged. These quantities are easily calculated for use in either method as long as the members are prismatic; the calculation of the quantities becomes a major task when haunching occurs.

Various tables are available giving the values of stiffness, carry-over factors, and fixed-end moments for various types of haunched beams and various loadings, for use in the method of moment distribution. The writer knows of no similar tables giving directly the analogous quantities for use in the method of balancing angle changes. However, simple relationships exist between the various quantities which greatly simplify the calculation of the quantities required for the latter method, those required for the former method being known.

The following notation will be used in establishing these relationships: S and K will denote stiffness, and r and C will denote the carry-over factor, as defined, respectively, for the method of moment distribution and the method of balancing angle changes; F will be used to denote the fixed-end moment, and ϕ, the angle change at the end of the simply supported member, due to applied loads; and the subscripts, A and B, applied to these quantities will distinguish them for the two ends of the member. In this connection, it should be noted that r_A and C_A denote the carry-over factor to the end, A.

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[17] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 664.

[18] Structural Designer, Pacific Elec. Ry., Los Angeles, Calif.

Referring to the member shown in Fig. 16: First, consider the member fully restrained at End B and a moment applied to End A to produce unit rotation of End A (Fig. 16(a)); this moment is defined as S_A. Next, release End B, the moment at B now becoming zero (Fig. 16(c)). This is accomplished by applying a moment of $-S_A\, r_B$ to the end, B (Fig. 16(b)). Adding the results of Fig. 16(a) and Fig. 16(b) the results of Fig. 16 (c) are obtained. The angle of rotation of End A is unity, and the moment at End B is zero; hence, by definition, the moment at End A is the stiffness at End A as defined for the method of balancing angle changes. Expressing this relationship as an equation,

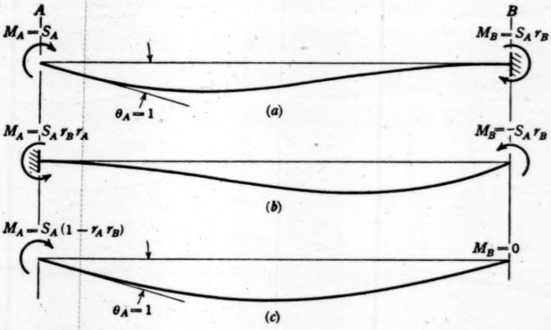

FIG. 16.

$$K_A = S_A\,(1 - r_A\, r_B) \qquad (15)$$

Consider the member shown in Fig 17. In Fig. 17(a), a moment, M, is applied at End A, End B being simply supported. The moment diagram is shown in Fig. 17(b). Assuming the origin of co-ordinates to be at End B, the following equations may be written directly, using the two area-moment propositions:

$$\theta_A = \frac{1}{L}\int_0^L \frac{m_x\, x\, dx}{E\, I} = \frac{M}{L^2}\int_0^L \frac{x^2\, dx}{E\, I} \qquad (16a)$$

and,

$$\theta_B = \frac{1}{L}\int_0^L \frac{m_x\,(L - x)\, dx}{E\, I} = \frac{M}{L}\int_0^L \frac{x\, dx}{E\, I} - \frac{M}{L^2}\int_0^L \frac{x^2\, dx}{E\, I} \qquad (16b)$$

The carry-over factor, C_B, as defined for the method of balancing angle changes, may be expressed, therefore, as follows:

$$C_B = \frac{\theta_B}{\theta_A} = \frac{L\int_0^L \frac{x\,dx}{E\,I} - \int_0^L \frac{x^2\,dx}{E\,I}}{\int_0^L \frac{x^2\,dx}{E\,I}} \qquad \text{(17)}$$

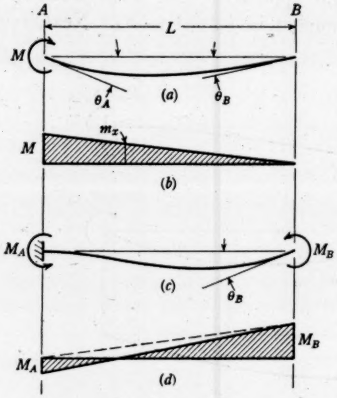
A
B
L
M
θ_A
(a)
θ_B
m_x
M
(b)
M_A
M_B
(c)
θ_B
M_B
M_A
(d)

FIG. 17.

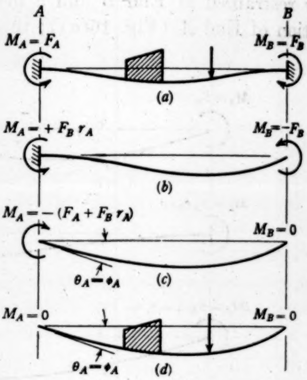
A
B
$M_A = F_A$
$M_B = F_B$
(a)
$M_A = +F_B r_A$
$M_B = -F_B$
(b)
$M_A = -(F_A + F_B r_A)$
$M_B = 0$
$\theta_A = \phi_A$
(c)
$M_A = 0$
$M_B = 0$
$\theta_A = \phi_A$
(d)

FIG. 18.

Next, consider the same member, as shown in Fig. 17(*c*), fixed at End *A* and a moment, M_B, applied to End *B*. The moment diagram is shown in Fig. 17(*d*). Again, assuming the origin of co-ordinates at End *B*, the following equations may be written directly, using the two area-moment propositions:

$$\frac{M_B}{L}\int_0^L \frac{(L-x)\,dx}{E\,I} - \frac{M_A}{L}\int_0^L \frac{x\,dx}{E\,I} = \theta_B \qquad \text{(18a)}$$

and,

$$\frac{M_B}{L}\int_0^L \frac{(L-x)\,x\,dx}{E\,I} - \frac{M_A}{L}\int_0^L \frac{x^2\,dx}{E\,I} = 0 \qquad \text{(18b)}$$

Equations (18) may be solved for M_A and M_B, obtaining,

$$M_A = \theta_B \frac{L\int_0^L \frac{x\,dx}{E\,I} - \int_0^L \frac{x^2\,dx}{E\,I}}{\int_0^L \frac{dx}{E\,I}\int_0^L \frac{x^2\,dx}{E\,I} - \left[\int_0^L \frac{x\,dx}{E\,I}\right]^2} \qquad \text{(19a)}$$

and,

$$M_B = \theta_B \frac{\int_0^L \frac{x^2\,dx}{E\,I}}{\int_0^L \frac{dx}{E\,I}\int_0^L \frac{x^2\,dx}{E\,I} - \left[\int_0^L \frac{x\,dx}{E\,I}\right]^2} \qquad \text{(19b)}$$

The carry-over factor, r_A, to End A as defined for the method of moment distribution, may be expressed as follows:

$$r_A = \frac{M_A}{M_B} = \frac{L\int_0^L \frac{x\,dx}{EI} - \int_0^L \frac{x^2\,dx}{EI}}{\int_0^L \frac{x^2\,dx}{EI}} \text{.............(20)}$$

Comparing Equations (17) and (20), it is seen that,

$$C_B = r_A \text{..........................(21)}$$

Similarly, it could be shown that $C_A = r_B$.

Consider a member under any system of transverse loading fixed at both ends, as shown in Fig. 18(*a*). The fixed-end moments are F_A and F_B. Release End B, by adding to Joint B a moment equal to $-F_B$, resulting in a moment at End A equal to $+F_B\,r_A$ (Fig. 18(*b*)). By adding, the condition is obtained in which the beam is fully restrained at End A, and hinged at End B; for this condition the fixed-end moment at End A is seen to be $F_A + r_A F_B$. Next, release End A by applying to that end (Fig. 18(*c*)) a moment equal to $-(F_A + r_A F_B)$. It was shown (see Equation (15), that a moment equal to $S_A\,(1 - r_A\,r_B)$ applied at End A produced unit rotation at A when End B was hinged. Therefore, the angle change produced at End A when a moment of $F_A + r_A F_B$ is applied to End A must equal the ratio, $\frac{F_A + r_A\,F_B}{S_A\,(1 - r_A\,r_B)}$. This angle change has been defined as ϕ_A for the method of balancing angle changes; hence,

$$\phi_A = \frac{F_A + r_A\,F_B}{S_A\,(1 - r_A\,r_B)} = \frac{F_A + r_A\,F_B}{K_A} \text{.............(22a)}$$

and, similarly,

$$\phi_B = \frac{F_B + r_B\,F_A}{S_B\,(1 - r_A\,r_B)} = \frac{F_B + r_B\,F_A}{K_B} \text{.............(22b)}$$

Using Equations (15), (21), and (22), and any tables giving values of S, r, and F, for use with the method of moment distribution, the corresponding quantities, K, C, and ϕ, may be computed. The extra step entailed by these computations may be considered an advantage of the method of moment distribution over the method of balancing angle changes; however, for any case in which tables are not available, it will be somewhat easier to calculate K, C, and ϕ, than the corresponding S, r, and F.

The method of analysis introduced in this paper has both advantages and disadvantages when compared with other available methods. The author has enumerated some of them; others might be mentioned. Inasmuch as the choice of method must depend largely on the designer's preference, it should be to the advantage of each individual designer to list and weigh the various advantages and disadvantages of available methods for himself. In this way, he will be better enabled to judge the most advantageous method to use in any particular case.

L. J. Mensch,[10] M. Am. Soc. C. E. (by letter).—The analysis of highly hyperstatic structures has not been taught in engineering schools or textbooks in such a practical manner that it can be used by busy engineers. They have avoided such structures, but when compelled to use them, have made safe guesses as to the stresses and, as a rule, have not produced economical designs.

Structures only one degree indeterminate are comparatively easy to solve, although requiring considerably more effort than ordinary beam construction and even such cases are avoided by many. Structures indeterminate in the second degree give twice as much trouble; and, three-degree indeterminacy necessitates much more mathematical skill and can only be designed by experts in this line.

Arched bridges are at least three times indeterminate when no hinges are used, evolve the expenditure of sizeable sums in construction, and have been analyzed regularly in the last twenty years by the classical theoretical methods. A continuous frame may be many times indeterminate, may evolve the expenditure of only moderate sums, and no busy engineer can afford the time for a complete theoretical analysis as taught in standard literature.

There is a crying demand for shorter and reasonably correct methods of design and Professor Grinter has tried to supply this deficiency by some novel procedures. The writer considers this and similar methods a retrograde step in structural engineering. The author makes the bold and gratituitous assumption that no side-sway may occur, or that it can easily be considered afterward. In the writer's opinion the analysis for side-sway is a much more difficult task than the analysis for vertical deformation only, and special short-cuts must be found before these methods are of much value to the profession.

Consider, for example, the case of a railroad trestle with expansion joints over every third or fourth column. Side-sway may occur there. What prevents side-sway in a school building with classrooms on both sides of a comparatively narrow corridor?

The first example in this paper is a flagrant case of disregard of the fundamental condition of equilibrium, namely, that the sum of the forces acting in any of three principal directions must be zero. They are not zero in a horizontal direction by 25%, as shown by the analysis. Would any engineer be satisfied with a solution in which the sum of the computed vertical reactions would differ from the acting vertical loads by 25 per cent? By taking this unwarrantable liberty with statics, the author claims that he has found a new and revolutionary short-cut in the analysis of indeterminate structures.

There are cases in which this and related methods may be used. Symmetrical structures with symmetrical loading have no side-sway. If the side-sway is in such a direction that the structure leans against a rigid body, and temperature and shrinkage may be neglected, side-sway has no chance to act. In skeleton constructions, with many bents, the side-sway may be very small. There is no side-sway in the classical problem of Clapeyron for continuous girders.

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[10] Civ. Engr. and Constructor, Chicago, Ill.

In all other cases these methods are only crude guesses at solutions and require more brain work than the classical methods, provided St. Venant's principle is used, namely, that members in a continuous frame which are several spans or stories distant from the girder which is being analyzed have only a very small influence on the necessary elastic equations, as the following demonstration will show: Let AB be a girder under a load forming part of a frame, as shown in Fig. 19; let K, K'_A, K''_A, * * *, be the ratio, $\frac{I}{L}$ of the various members; let M_A, M_B be the negative moments acting in $A\,B$, at the ends, A and B, according to the classical sign convention; and, let α_A, and α_B, be the rotations of the joints, A and B, due to the load and the restraint of the adjoining and further members. When these joints are assumed to be rigid, it is clear that this rotation must be equal to the slope of the elastic line of all members meeting at any joint.

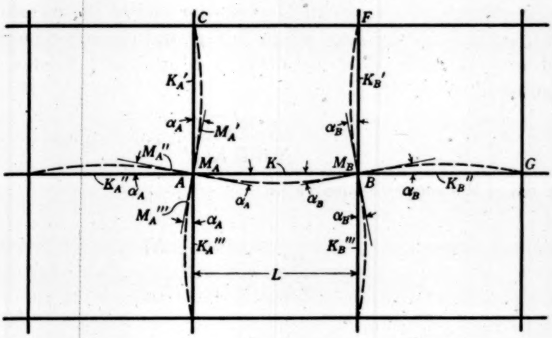

FIG. 19.

It is known, and may be found in almost any elementary textbook, that,

$$\alpha_A = \phi_A - \frac{(2\,M_A + M_B)\,L}{6\,E\,I} \quad \text{................} (23a)$$

and,

$$\alpha_B = \phi_B - \frac{(2\,M_B + M_A)\,L}{6\,E\,I} \quad \text{..................} (23b)$$

For a uniform load the author gives $\phi_A = \phi_B = \frac{W\,L^2}{24\,E\,I}$; and for a concentrated load in the center, $\phi_A = \phi_B = \frac{P\,L^2}{16\,E\,I}$.

If α_A and α_B are known, the unknown moments, M_A and M_B, can be obtained from Equations (23). The problem is reduced, to as close an estimate of the values of α_A and α_B as possible, from the deformation of the members adjacent to the joints, A and B, and the load on AB.

It is known from mechanics that for a member, AC, for example,

$$\alpha_A = \frac{M'_A}{m'\,E\,K'_A} \qquad \text{(24)}$$

in which m' is a factor depending on the restraint at the end, C. When C is hinged, $m' = 3$; when fixed, $m' = 4$; when AC is deformed so that the elastic line has a point of contraflexure just at mid-length of the member, $m' = 6$; and when AC is deformed so that the elastic line is symmetrical about the mid-point, $m' = 2$. In a previous discussion on this subject[20] the writer has shown that the final results for the unknown bending moments, M_A and M_B, do not greatly differ whatever the restraint is at the far end of the members adjacent to the joint, A, and for the case where this restraint cannot be readily estimated and may be assumed to lie between hinged and fixed conditions, a practical guess is:

$$\alpha_A = \frac{M'_A}{3.6\,E\,K'_A} \qquad \text{(25a)}$$

In rare cases the more precise value,

$$\alpha_A = \frac{M'_A}{E\,K'_A}\;\frac{1 + \frac{4}{3}N_C}{4\,(1 + N_C)} \qquad \text{(25b)}$$

may be used. One can write:

$$\alpha_A = \frac{M'_A}{m'\,E\,K'_A} = \frac{M''_A}{m''\,E\,K''_A}$$

$$= \frac{M'''_A}{m'''\,E\,K'''_A} = \frac{M'_A + M''_A + M'''_A}{\Sigma\, m\,E\,K_A} = \frac{M_A}{\Sigma\, m\,E\,K_A} \qquad \text{(26a)}$$

and, similarly,

$$\alpha_B = \frac{M_B}{\Sigma\, m\,E\,K_B} \qquad \text{(26b)}$$

Let,

$$N_A = \frac{3\,K}{\Sigma\, m\,K_A} \qquad \text{(27a)}$$

and,

$$N_B = \frac{3\,K}{\Sigma\, m\,K_B} \qquad \text{(27b)}$$

then, by combining Equations (26) with Equations (23) one easily obtains for a uniform load:

$$M_A = \frac{w\,L^2}{4}\;\frac{1 + 2\,N_B}{4\,(1 + N_A)\,(1 + N_B) - 1} \qquad \text{(28a)}$$

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[20] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1682.

and,

$$M_B = \frac{w\,L^2}{4}\;\frac{1 + 2\,N_A}{4\,(1 + N_A)\,(1 + N_B) - 1}\;\ldots\ldots\ldots\ldots(28b)$$

For a concentrated load in the center, $\frac{w\,L^2}{4}$ must be replaced by $\frac{3\,P\,L}{8}$.

Once the moments, M_A and M_B, are computed they can be distributed to the adjoining members, AC, for example, as:

$$M'_A = M_A\,\frac{m'\,K'_A}{\Sigma\,m\,K_A}\;\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

The moment, M'_A, produces a moment at the far end of AC, depending on the value of m', as follows: When $m' = 2, 3, 3.6, 4, 6$, or $4\,\frac{(1 + N_C)}{1 + \frac{4}{3}\,N_C}$,

$$M_{CA} = M'_A\left(-1, 0, \frac{1}{3}, \frac{1}{2}, 1, \frac{1}{2 + 2\,N_C}\right)\ldots\ldots\ldots\ldots(30)$$

The moment, M_{CA}, can again be distributed to the adjoining members meeting at C, if found desirable; but this is generally unnecessary. In most cases it is near enough to compute the maximum positive moment in AB as the center moment by,

$$M = \frac{w\,L^2}{8} - \frac{M_A + M_B}{2}\;\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

If it is desired to compute the maximum negative moment in AB at B, one must assume also that the girder, BG, is loaded, and compute from Equations (28) the negative left end moment, and then by distribution according to stiffness, as shown in Equation (29), the moment, M_{BA}, which has to be added to the moment, M_B, previously found.

This is the classical method of moment distribution, except that the distant spans have so little influence, in most cases, on the elastic equations of the particular member, which is being analyzed, that they may be neglected in practice. This holds true whether Castigliano's theorem, the Maxwell equations, or the principle of virtual velocities are used. These are only different names for the identical elastic equations.

Experts in the analysis of hyperstatic structures of the continuous-frame type have rarely used the latter principles and have found that the moment-area principle, in combination with the rotation of the joints, offers a number of short-cuts, not inherent in the foregoing methods, as may be found in the excellent works of Professor E. Winkler[21], the works of W. Ritter[22], W. Gehler[23], A. Strassner[24], etc.

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[21] "Vorträge über Brückenbau", G. Gerold, Vienna, 1875.

[22] "Graphische Statik", III, Zuric, 1900.

[23] "Der Rahmen", Wm. Ernst & Sohn, Berlin, 1913.

[24] "Neuere Methoden", Berlin, 1916.

By the method herein presented by the writer an attempt will be made to analyze the structure shown in Fig. 2. All the K-values are shown in that diagram and $m = 3$, except for DC where $m = 4$.

From Equations (27), for Beam BD: $N_B = \dfrac{3 \times 4}{3 \times 2} = 2$; and $N_D = \dfrac{3 \times 4}{3 \times 2 + 3 \times 3 + 4 \times 3} = 0.445$; and, from Equations (28): $M_B = 0.0289\, w L^2$; and $M_D = 0.0765\, w L^2$.

It is clear that the moment, M_{BA}, at B is also $0.0289\, w\, L^2$; and the horizontal reaction at $A = 0.0289\, w\, L$. The author did not give the inclination of the member, BA, as implying no importance in his opinion, and one may assume the member to be vertical. The moment, M_D, must be distributed in the proportion, $3 \times 2 : 3 \times 3 : 4 \times 3$, into the members, DE, DF, and DC, respectively, and these moments are easily found to be 0.017, 0.0255, and $0.034\, w\, L^2$, which lead to reactions of $0.017\, w\, L$, $0.0153\, w\, L$, and $0.0567\, w\, L$, at Points E, F, and C, respectively.

The horizontal reactions at Points A and E are acting to the right and their sum is $(0.0289 + 0.017)\, w\, L$, whereas the horizontal reaction at Point C is acting to the left and is only $0.0567\, w\, L$—or the opposing reactions to the latter are more than 24% greater, an impossible condition. Sidesway will occur in the girders to the left so that the moment, M_B, is increased about 20% and the moment, M_D, decreased a similar amount.

The girder, DF, may be analyzed as follows: The stiffness of the member, DF, beyond the hinge, F, being zero, as there is no restraint to the right, $N_F = \infty$. From Equations (28),

$$M_D = \frac{3 P L}{8} \, \frac{1 + 2 N_F}{4 (1 + N_D)(1 + N_F) - 1} \quad \text{............(32)}$$

Dividing the numerator and the denominator by the infinite value of N_F:

$$M_D = \frac{3 P L}{8} \, \frac{2}{4 (1 + N_D)} = \frac{3 P L}{16 (1 + N_D)} \quad \text{............(33)}$$

in which, $N_D = \dfrac{3 \times 3}{3 \times 2 + 4 \times 3 + 3.6 \times 4} = 0.278$; and $M_D = 0.148\; P\, L = 4\,930$ ft-lb. This negative moment will be diminished by the cantilever moment at F of 20 000 ft-lb, as follows:

From Equation (25*b*),

$$m = \frac{4 (1 + N_C)}{1 + \frac{4}{3} N_C} = \frac{4 (1 + 0.278)}{1 + \frac{4}{3} \times 0.278} = 3.73$$

From Equation (30)

$$M_{DF} = \frac{20\,000}{2 + 2 \times 0.278} = 7\,830 \text{ ft-lb.}$$

which is a positive contribution to the moment in *DF* at *D*, producing there a positive moment of 7 830 − 4 930 = 2 900 ft-lb; which must be distributed according to the stiffness ratios of 3 × 2: 4 × 3: 3.6 × 4 to the members, *DE*, *DC*, and *DB*, respectively. These additional moments will decrease somewhat the sum of the horizontal reactions found for the uniform load on *BD* alone, but will still leave a serious difference.

The writer claims that Equations (28) are so simple that they can be used by busy engineers and will give reasonably correct results in a few minutes without deep thinking and less likelihood of mistakes than the new methods, such as that proposed in this paper. He would advise the author to improve his scholarly paper by showing more practical methods of estimating the influence of side-sway, which is of greater importance at present than the computation of the primary moments. There are plenty of methods for finding primary moments in engineering literature, whereas side-sway has generally been overlooked, or, if treated at all, unbalancing of the vertical reactions has often resulted.

MARVIN A. GRAY,[25] JUN. AM. SOC. C. E. (by letter).—The purpose of the following discussion is to test the theory of this paper in all its technical phases. Throughout, Professor Grinter uses such terms as "slope" and "deflection", apparently without realizing that it is, in fact, "slope deflection", because he uses ϕ in special cases instead of the more generally known symbol, θ, that is used in connection with slope deflection. He presents some values of ϕ derived by the conjugate-beam method (a variation of moment areas) and each of these ϕ-values needs a separate derivation. This type of method is meant to displace slope deflection, which needs only one proof or derivation and the single set of unknowns (θ-values) instead of θ-values and ϕ-values.

To show how closely this paper follows the slope deflection method the writer herein develops the ϕ-values by using the slope deflection equation,

$$M_{AB} = M_{F\text{-}AB} - \frac{2\,E\,I}{L}\,(2\,\theta_A + \theta_B) \;\ldots\ldots\ldots\ldots\ldots (34)$$

in a simple beam. For a concentrated load at mid-span, $-\theta_B = \theta_A$; so that (with $M_{AB} = 0$) Equation (34) yields for θ_A (or for ϕ_A and $-\phi_B$): $0 = \frac{P\,L}{8} - \frac{2\,E\,I}{L}\,(2\,\theta_A + \theta_B)$; or,

$$\theta_A = \frac{P\,L^2}{16\,E\,I} = -\,\theta_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

For a uniform load on the entire span, θ_A (or ϕ_A and $-\phi_B$) equals $\theta_A = \frac{w\,L^2}{12}\,\frac{L}{2\,E\,I} = -\,\theta_B$, that is:

$$\theta_A = \frac{w\,L^3}{24\,E\,I} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

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[25] Chicago, Ill.

For a concentrated load not at mid-span, in a similar manner, Equation (34) yields, for θ_A (or ϕ_A);

$$\theta_A = \frac{P\,a\,b^2}{L^2}\,\frac{L}{2\,E\,I}\left(\frac{2}{3} + \frac{a}{3\,b}\right) = \frac{P\,a\,b}{2\,E\,I\,L}\left(\frac{2\,b}{3} + \frac{a}{3}\right) \ldots (37a)$$

and, for θ_B (or ϕ_B):

$$\theta_B = -\,\frac{P\,a^2\,b}{L^2}\,\frac{L}{2\,E\,I}\left(\frac{2}{3} + \frac{b}{3\,a}\right) = -\,\frac{P\,a\,b}{2\,E\,I\,L}\left(\frac{2\,a}{3} + \frac{b}{3}\right) \ldots (37b)$$

For an induced moment, M, at End A, with $\theta_A + 2\theta_B = 0$; and $\theta_A = -\,2\,\theta_B$, Equation (34) yields, for θ_A (or ϕ_A):

$$\theta_A = \frac{L}{2\,E\,I}\,\frac{2M}{3} = \frac{M\,L}{3\,E\,I} \ldots\ldots\ldots\ldots\ldots\ldots (38a)$$

and, for θ_B (or ϕ_B):

$$\theta_B = -\,\frac{\theta_A}{2} = -\,\frac{M}{6\,E\,I} \ldots\ldots\ldots\ldots\ldots\ldots (38b)$$

and if the result appears with the wrong sign, the author's sign reversal explains that the author has taken reaction instead of action. The writer has now proved definitely that the ϕ-value is a special value or solution obtained directly by slope deflection from the value of θ.

Having stated his thesis, the author next proceeds through part of the paper to give a proof for the moment distribution method, and succeeds in demonstrating that it is really based on, or derived from, slope deflection; as, for example: "Accordingly, it follows that the carry-over factor [θ_A or θ_B] for angle change is of negative sign, namely, −0.5". * * * "All of the carry-over angles to the far ends of the other members meeting at the joint are simply -0.5θ", etc. The balancing process of the slope deflection method (which is a part of all similar "methods") is based on the following equations:

$$M_{AB} = M_{F\text{-}AB} - \left(\frac{2\,E\,I}{L}\right)(2\,\theta_A + \theta_B) \ldots\ldots\ldots\ldots (39a)$$

and,

$$M_{BA} = M_{F\text{-}BA} - K\,(2\,\theta_B + \theta_A) \ldots\ldots\ldots\ldots (39b)$$

The balancing part of Equation (39a) is $-\left(\dfrac{2\,E\,I}{L}\right)(2\,\theta_A + \theta_B)$; and that of Equation (39b) is $-\,K(2\theta_B + \theta_A)$. If, momentarily, the direct effects of M_{AB} and $M_{F\text{-}AB}$, are disregarded, the following result is obtained due to M_{BA}: $0 = 0 - K\,(2\,\theta_A + \theta_B)$. Then, $\theta_A = -\,0.5\,\theta_B$, or,

$$K\theta_A = -\,K\,0.5\,\theta_B \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

In the moment distribution, $K\ (2\ \theta_A)$ is solved for directly and $K(2\ \theta_B)$ in Equation (39*b*) is $-\ 0.5\ (K\ 2\ \theta_A)$, as in Equation (40). In other words by slope deflection, the writer has proved in a truly direct method, that an angle at one end, *A*, of a beam induces an angle (or its equivalent moment) at the other end, *B*, opposite in sign and one-half its value; or θ_A induces a value, $-\ 0.5\ \ \theta_A$ (or $-\ 0.5\ K\ \theta_A$), at Point (*B*), the far end of the beam; and, conversely, Angle θ_B, from an external moment, produces a rotation (or a moment), $-0.5\theta_B$ (or $-0.5\ K\ \theta_B$), at End *A*, the rear end of the beam, which is what the author terms the rotation "balancing angle". Expressed in terms of moment distribution multiplying by *K* ($K\ \theta$ is a moment), a moment induced at one end creates a moment (or angle) at the other end equal to $-\ 0.5$ of the original induced moment (or angle), the correction or carry-over factor that is used in the moment distribution method. Thus are the foregoing quotations from the paper explained; and the contention that moment distribution (of which the method of balancing angle changes is a branch) is a variation of slope deflection by changing symbols, is proved conclusively.

The subject of continuity at joints is by far the most important phase of rigid frames as far as practice is concerned, because without this knowledge, all the theory pertaining to it does little good. However, if in a homogeneous beam it works on the basis of plastic flow as the author infers, the entire beam would deform until it acts like a cable. It is more probable that one of two conditions may exist: (1) The member may be free to move due to loose rivets, or there may be "play" in the connection; or (2), rivets or the connection may yield because they are overstressed until the ends finally hold or balance the load. The first case seems illogical, and failure of the material is very probable in the second case; but in either case a definite θ-value can be given for the movement, with a solution by slope deflection.

The publication of this paper affords an opportunity to clarify the relation between rigid frame methods (solutions) and their basis, the slope deflection method. The author is correct in emphasizing the superior virtue of angle movement, or the balancing of angle changes; such advantage has been shown many times in papers on slope deflection.[26] The adaptation of the standard Slope Deflection Equation[27] constitutes the most accurate, as well as the fastest, method having a theoretical proof based on angle movement. Slope deflection and its method of solution by converging approximations will be of great value in the new fields of structural analysis, such as airplanes, airships, light-weight trains, and high-strength large bridges.

A. FLORIS,[28] ESQ. (by letter).—The analysis presented in this paper is a complicated variation of the well known and widely used simple method of moment distribution.

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[26] See, for example, *Engineering News-Record*, Vol. 107, No. 20, p. 770, November 12, 1934.

[27] "Engineering Studies," *Bulletin No. 1*, Univ. of Minnesota, March, 1915.

[28] Dipl.-Ing., Los Angeles, Calif.

As a base system the author takes a beam that is freely supported or free to rotate at the ends. This is one of the limiting cases. The other limiting case, the fully restrained beam, used in the moment distribution method[3] could not be utilized as a base system in the author's analysis, because the tangent to the elastic curve at the supports being horizontal, there are no angles to distribute.

However, the advantage of the choice of the angle changes instead of the fixed end moments is debatable. In general, engineers are not concerned with angle changes of bars framing into a joint. It is true that in the slope-deflection method these changes are introduced in the analysis, but this is done for the purpose of reducing the degree of statical indeterminacy of the structure. The procedure of balancing the statically indeterminate quantities is the same in the author's method as in the moment-distribution method. Hence, there is no need to use the less convenient angles, if the required moments can be found more directly by moment distribution.

In the author's analysis, the angle changes at a joint must be distributed to all members framing into it, in such a way, that, by changing the slopes, the continuity of the structure is restored. By doing this, however, moments are induced at the ends of these bars. In the moment distribution method the moments produce the angle changes. In the author's analysis the angle changes produce the moments. Consequently, the angle changes at the ends of bars are inversely, and not directly, proportional to the K-values, as the author states. A simple analysis will prove it. In spite of this discrepancy, the results obtained are correct because of the fact that the distribution is made by taking percentages of the stiffness of the bars framing into a joint.

With the use of the base system in the moment distribution as well as in the angle distribution methods, the continuity of the structure is destroyed. To restore the continuity by unlocking or restraining the joints, the moments or angle changes must be distributed to the members framing into a joint in proportion to their capacity to resist such moments or angle changes. This is the balancing process which, in turn, influences the opposite end of the bars under consideration. This influence upon the opposite end of the bars is the carry-over process. The moment distribution method is based upon these two simple steps.

The author's explanations regarding the application of his method to side-sway produced by lateral or unsymmetrical loading are extremely vague. In this case, the freely rotating bar at both ends cannot be used as a base system throughout the structure. The insertion of hinges in all joints necessitated by the choice of this beam produces a system that does not possess lateral stability. Perhaps the frames with columns fixed at the base, connected to beams that are hinged at the ends, can be chosen as a base system. The interesting properties of these frames, which are stable against lateral forces, have been discussed by the writer elsewhere.[29]

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[29] "Types of Rigid Frames for Existing Buildings to Resist Shocks", by A. Floris, *Southwest Builder and Contractor*, July 13, 1934, p. 22.

Fred J. Benson,[30] Jun. Am. Soc. C. E. (by letter).—The method of analyzing continuous frames by balancing angle changes, presented by Professor Grinter, is a valuable alternative to the method of balancing fixed-end moments. Professor Grinter discusses briefly the use of the device of successive distortions in applying his method to structures having more than one story or one panel. This important application seems to deserve more attention. The following analysis will serve to illustrate and clarify the points emphasized in the paper.

The two-story bent shown in Fig. 20 has two degrees of freedom of horizontal movement. Hence, two successive distortions must be produced in order to determine the moments produced by the lateral loads. In order to produce the first distortion it is assumed that Joints *E*, *F*, and *G* are fixed in position and that the remaining joints are pin-connected. An unresisted horizontal movement of Joints *B* and *D* is produced in the direction of the load, P_2. This movement produces equal angle changes at the top and bottom of each column affected. Such angle changes are then balanced by Professor Grinter's method. The shear in each column is computed from the balanced angle changes and the produced horizontal movements by Equation (5). The total shear in each story is determined by the summation of the column shears. These shears may be designated as V_{1a} and V_{2a}. In order to produce the second distortion it is assumed that Joints *A*, *B*, and *C* are fixed in position and that the remaining joints are pin-connected. An unresisted horizontal movement of Joints *E* and *F* is produced in the direction of the load, P_1. Again, the angle changes produced by this movement are balanced and the story shears, V_{1b} and V_{2b}, are obtained.

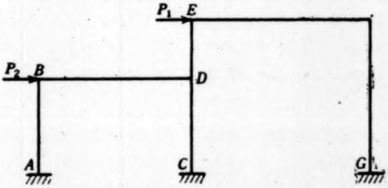

Fig. 20.—Two-Story Bent with Lateral Loads

The two distortions must be combined in such a manner that the true story shears are obtained. Therefore, it is necessary to determine the factor by which the balanced angle changes and produced horizontal movements for each distortion must be multiplied to obtain the true story shears. If x represents the factor for the first distortion, and y, the factor for the second distortion, the following equations may be written:

$$x\,V_{1a} + y\,V_{1b} = P_1 + P_2 \qquad (41a)$$

and,

$$x\,V_{2a} + y\,V_{2b} = P_1 \qquad (41b)$$

Equations (41) are solved simultaneously for x and y. The true rotation of any joint is found by multiplying the balanced angle change for each distortion by the corresponding factor and then adding these two corrected angle changes. The true horizontal movement of any joint is found in the same manner by applying the factors to the produced horizontal movements. The movement in any member at any joint is then found by using the slope-deflection equations. Since the summation of moments at any joint must equal zero, it is quite evident that the method is self-checking.

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[30] Hot Springs, N. Mex.

The rapid use of either the methods of balancing fixed-end moments or angle changes requires considerable practice. In general, the final moments can be obtained more rapidly by balancing fixed-end moments. However, for structures containing a number of joints, the fact that the method of balancing angle changes is self-checking is of considerable importance since it offers a rapid method of checking the solution without repeating the balancing process.

DAVID B. HALL,[31] ASSOC. M. AM. SOC. C. E. (by letter).—In the section of this paper entitled "Haunched Beams and Curved Members," the author calls attention to the fact that end-angle changes, and other factors, must be specially calculated for such members. The following observations may assist in the calculation of the angle changes due to loads.

A haunched beam with a unit moment at the left end is shown in Fig. 4 (Point *A*), and below it is shown the deflection curve. This diagram is the influence line for the angle change at Point *A* due to loads anywhere on the span, because, by the theorem of reciprocal deflections, the deflection at any point due to a unit moment at the end is identical to the rotation at the end due to a unit load at that point.

If the method of elastic weights is used to compute this curve, it will be found expedient to use a double summation process, adding up elastic weights to obtain shears between successive points, and adding up these shears, times length of divisions, to obtain moments. If this plan is regarded as a subterfuge, the identical operations may be performed, treating the elastic weights as angular changes (which they are); adding them to obtain slopes, beginning with ϕ_A as the slope at the end; and summing up the products of slopes and division lengths, to obtain the deflections.

P. CRAIG LIVESAY,[32] ESQ., AND WILLIAM M. SIMPSON,[32] ESQ. (by letter).—In opening his paper Professor Grinter has stated that the method of balancing angle changes has been presented as an additional tool to be used by the engineer wherever it may be found superior to other methods or an aid in checking a special problem. The relative merits of balancing angle changes as compared with other methods will depend upon the problem considered and the preference of the engineer who is making use of the method, which, again, will depend upon his familiarity with the various methods.

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The writers have used the method of balancing angle changes to solve a variety of problems most of which have been checked by the method of balancing moments, or by the column analogy, in order to prove the self-checking features of the method. No difficulty has been experienced in obtaining results to any desired degree of accuracy, and no problem has been encountered to which this method could not be adopted. The types of problems solved include the following: Secondary stresses: bent analysis, involving side-sway; joint slip; wind-stress analysis in building frames; influence lines for continuous beams; and settlement of supports.

Because of the briefness of the treatment of side-sway by Professor Grinter, it was thought that an illustration of two special cases of joint translation would be useful.

The first example chosen to illustrate the method of handling side-sway is a slant-leg bent with an applied horizontal load. The analysis by balancing angle changes is shown in Fig. 21. An unresisted side-sway similar to the one shown by the dotted lines has been assumed. (*E* has been omitted throughout, since it will cancel as may be shown by setting up the equations for the angle changes and the shears.) The angle changes were assumed as convenient values taken in proportion to the angles for a side-sway of 1 in. The small triangle above Joint *C* shows movements for a 1-in. deflection. These angles were then balanced by following the procedure outlined by Professor Grinter (under the heading, "Procedure of Balancing Angle Changes"). All joint rotations due to this balance are enclosed in rectangles near the joints. After the balancing of angles was

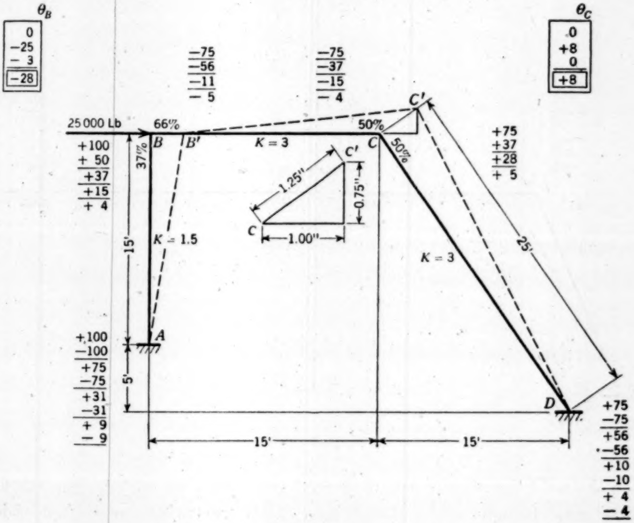

FIG. 21

completed, the horizontal shear in the vertical leg and the horizontal components of the shear and the direct stress in the slant leg were computed. The sum of these horizontal forces was found to 379 lb. The correction ratio was $\frac{25\,000}{379}$, or 65.9. The true joint rotations in the structure due to the horizontal loading were then found by multiplying the enclosed joint rotations by this ratio. The true ρ-values were found, similarly. This bent analysis illustrates the method of using a correction factor, as explained by Professor Grinter (see heading, "Bent Analysis").

The second example (Fig. 22) is an analysis of the frame presented by the author in Fig. 2, with the exception that the fixed support, *F*, has been replaced by a nest of rollers. Points *A* and *C* were assumed to lie in the same horizontal plane. This example illustrates the method of side-sway correction for an unsymmetrical structure carrying loads which are not in the direction of free movement. The author's example permitted no side-sway because of the fixed support at *F* and thus no correction for side-sway was necessary.

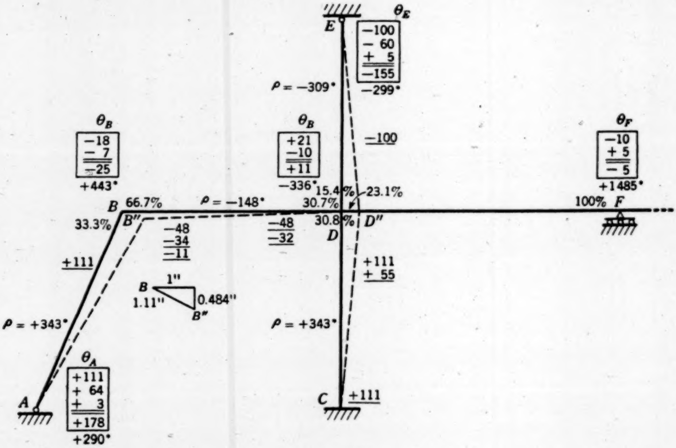

FIG. 22

When Joint *F* in Fig. 2 was replaced by a nest of rollers, the unbalanced restraint was found to be 1 044 lb. This restraint prevented side-sway which would have moved Joint *D* toward Joint *F*.

An unresisted side-sway was then imposed upon the pin-connected frame, as indicated by the dotted lines in Fig. 22. A rotation of − 100 was assumed in the member, *DE*. The geometrical deformation of the frame determined the rotations of the other members. The member rotations used in balancing angles were $\rho_{AB} = 111$, $\rho_{BD} = -48$, $\rho_{CD} = 111$, and $\rho_{DE} = -100$. All joint rotations due to this balance are enclosed in rectangles near the joints. The unbalanced horizontal force in the structure was found to be 338 lb. The modulus of elasticity has been neglected.

The joint rotations and the ρ-values of the assumed side-sway were multiplied by the ratio, $\frac{1044}{338}$. These corrected values were then added to the joint rotations indicated in Fig. 2 (multiplying the author's values by 10 before adding). Each final joint rotation and ρ-value is marked with an asterisk in Fig. 22. This completed the analysis for side-sway. In applying the slope-deflection equations to these values, the modulus of elasticity should be omitted.

When applying this method to particular problems many advantages and disadvantages, as compared to other methods, may be found. Regardless of the type of problem to which this method is applied there is one improvement over other methods. This advantage is its self-checking feature. There is a chance for compensating errors, of course, but that chance is very slight. In addition, the method has been very useful to the writers in giving them a clearer picture of the physical deformation of a structure under any given set of conditions.

L. E. Grinter,[33] Assoc. M. Am. Soc. C. E. (by letter).—The unusually clear and useful discussions have undoubtedly added much to the clarification of the paper on balancing angle changes. As anticipated by the opening and closing paragraphs of the paper, the method has interested those who were either familiar with the slope-deflection method of analysis or who had used the method of balancing moments. It is not to be expected that others would find the method either immediately convenient or attractive, since they would not only be faced with a new and uniquely disturbing procedure but probably with a new terminology as well. However, many persons are being exposed regularly to contact with modern methods that are now in use by some one in nearly every office so that a gradual evolution is in progress.

It is interesting to observe the different opinions expressed with reference to the similarity between balancing moments and balancing angle changes Mr. Floris, Mr. Byrne, and Mr. Mensch are seemingly of the opinion that the two methods are practically one and the same. On the other hand, Mr. Gray and Professor Maney seem positive that the method of balancing angle changes is really the old slope-deflection method in a new dress. However, Mr. Goldberg answers adequately Professor Maney's contention that "'balancing angle changes' is merely a new group of words to describe what has been done for years in the solution of the slope-deflection equations," when he points out that "throughout the entire slope-deflection analysis the concept of continuity is firmly maintained," while in balancing angle changes "continuity is first destroyed and then * * * re-established." The resemblance that Professor Maney noticed was for a special case of secondary stress analysis to which discussion the writer devoted only two sentences of the paper.

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Several discussers, notably Mr. Floris, Mr. Goldberg, and Mr. Mensch had some difficulty in analyzing for side lurch by balancing angle changes. Mr. Floris will find that Mr. Andersen has solved this problem most excellently without the use of an "unresisted side lurch" whereas Messrs. Livesay and Simpson and also Mr. Benson apply, effectively and brilliantly, the conception of an "unresisted side lurch." The interesting problem of a two-level bent, analyzed accurately by Mr. Benson, will clarify the misconceptions of Mr. Goldberg that the method would necessarily become an approximation for a two-story frame. Evidently, the work involved in each of the examples presented by the discussers is much less than by any of the classical methods. The final moments can always be obtained to any degree of accuracy desired.

Mr. Andersen questions the writer's procedure of "carrying-over" from one joint before "balancing" at adjacent joints, as in Fig. 2. Experience with the several methods of successive corrections has shown that the procedure used in Fig. 2 produces the fastest possible convergence. The number of terms involved in Fig. 14 could probably be reduced considerably without the sacrifice of accuracy by adopting the balancing procedure from Fig. 2.

Mr. Gray stated in his opening sentence that the purpose of his discussion would be "to test the theory of this paper in all its technical phases." Since his discussion does not mention the use of balancing angle changes for any of the many purposes considered in the paper, it appears that this purpose was not carried to a conclusion. His discussion is composed mainly of a carefully presented demonstration that the ϕ-values and the carry-over factors can be derived from the slope-deflection equations. Naturally, this is true, but it is also true that these primary factors can be found directly by area moments, by virtual work, or by any of the known methods of deflection and slope computation. Mr. Hall's excellent discussion points out other procedures. It is worth noting that the slope-deflection equations are themselves derived by area-moments in most textbooks.

Mr. Luce has tested the method of balancing angle changes on a sufficiently wide variety of structures to speak with assurance as to its general application. The difficulty that he has found with reference to its use in wind-stress analysis has probably occurred to others; namely, that criterion ratios are based upon shears rather than angles and, therefore, a correction ϕ-value or ρ-value in a story is not easily determined. This difficulty can be overcome, however, by a study of the influence of an unresisted side lurch in a typical story upon adjacent stories when joint continuity is restored. This study will involve a correlation of ϕ-values and θ-values with story shears. An investigation of these relationships has been performed successfully under the writer's direction.

The thoughtful discussion of Mr. Byrne was pointed mainly toward the need for obtaining ϕ-values, stiffness factors, and carry-over factors for starting the process of balancing. The integral signs in his discussion give it a rather formidable appearance, particularly when it is compared with the paper in which the ideal of mathematical simplicity was paramount. However, Mr. Byrne does show conclusively how factors for moment and angle distribution are related. In answer to Mr. Byrne's questioning statement that he knows of no tables giving directly the quantities used for balancing angle changes, the writer submits Table 3 which was prepared during a study that he made in 1925. The source of these data is now unknown, although it is recalled that they represented only a slight revision of standard tables. They supply values of Factors A and B for substitution in the formulas for symmetrically haunched beams:

$$\theta = \frac{L\,A}{1\,000\,E\,I} \qquad \text{(42a)}$$

and,

$$\beta = \frac{L\,B}{1\,000\,E\,I} \qquad \text{(42b)}$$

in which θ and β, respectively, are the rotations at the ends of a beam which is simply supported, symmetrically haunched, and loaded with a unit moment at one end.

Mr. Hall may also be interested in Table 3, since the constants obtained from this table are for haunched beams to which his excellent discussion refers. Mr. Hall has contributed the very brilliant and useful suggestion that the deflected curve for the effect of a unit end moment is an influence line for end angle change. The writer would add that the angle change caused by the unit moment is an inverse measure of stiffness from which distribution factors are obtained and that the ratio of the angle change at the far end to the angle change at the near end is the carry-over factor for angle distribution. Hence, one loading (applied unit end moment) and a single analysis will give rise to all data for balancing angle changes. It is suggested that the haunched beam is best analyzed for end slope and elastic deflections by area moments preferably modified by the conjugate-beam conception. Table 3 gives the end angle changes for a wide range of haunched beams.

Mr. Mensch states that the effect of side lurch was overlooked in Fig. 5 of the paper. The fact that side lurch was not discussed was not an oversight but was premeditated and to this end Support F was shown by the standard convention of a non-movable support. If lateral movement had been permitted, the support, F, would have been shown by the standard convention for a rocker or roller nest. These conventions do not ordinarily receive explanation in a technical paper. Hence, it seems that Mr. Mensch erred in making his statement that this example is a "flagrant case of disregard of the fundamental conditions of equilibrium" about as seriously as he over-stated the writer's actual claims when he wrote "the author claims that he has found a new and revolutionary short-cut in the analysis of indeterminate structures." These very modest "claims" are stated in the concluding paragraph of the paper as; (1) The method is self-checking; and (2) it offers the clearest possible picture of the physical action of the structure.

The discussion of Messrs. Livesay and Simpson shows how the introduction of a roller nest at Support F would change the calculations.

TABLE 3.—FACTORS FOR SYMMETRICALLY HAUNCHED BEAMS, EQUATIONS (42)

Values of $n^* = \frac{a}{b}$		FACTORS A AND B, FOR THE FOLLOWING VALUES OF $m\dagger = \frac{I}{I_o}$:												
		1.00	0.60	0.30	0.20	0.15	0.12	0.10	0.08	0.06	0.05	0.04	0.03	0.02
(a) STRAIGHT HAUNCHES														
0.5														
	A	333	251	173	140	121	109	99	89	78	71	64	56	47
	B	167	141	106	91	82	76	71	65	59	55	50	45	39
0.4														
	A	333	264	198	171	154	144	136	127	117	112	106	99	91
	B	167	147	125	115	108	104	100	96	92	89	86	83	78
0.35														
	A	333	271	212	187	172	162	155	147	138	133	128	122	114
	B	167	151	134	126	120	117	114	111	107	105	103	100	96
0.30														
	A	333	278	226	204	191	182	175	168	160	156	151	145	139
	B	167	154	142	135	131	129	126	124	121	120	118	116	113
0.25														
	A	333	286	241	222	210	203	197	191	184	180	176	171	165
	B	167	158	149	144	141	139	138	136	134	133	132	130	128
0.20														
	A	333	294	257	241	231	225	220	215	209	206	202	198	197
	B	167	161	155	152	150	149	147	146	145	144	143	142	141
0.15														
	A	333	303	274	261	254	249	245	241	237	234	231	228	224
	B	167	163	160	158	157	156	155	155	154	154	153	152	152
(b) PARABOLIC HAUNCHES														
0.5														
	A	333	273	213	186	170	158	150	141	130	123	116	108	102
	B	167	150	131	122	115	110	107	102	97	94	91	86	80
0.4														
	A	333	283	233	210	196	187	180	172	162	156	151	143	134
	B	167	156	143	136	132	128	126	123	120	117	115	112	107
0.35														
	A	333	289	243	223	210	202	195	188	179	175	169	162	154
	B	167	158	148	143	139	136	135	132	129	128	126	123	120
0.30														
	A	333	295	254	236	225	218	212	205	198	193	188	183	175
	B	167	160	152	148	146	144	142	140	139	137	135	134	131
0.25														
	A	333	300	266	250	241	234	229	224	217	213	208	204	198
	B	167	162	156	154	152	150	149	148	147	146	144	143	141
0.20														
	A	333	306	278	265	257	252	248	243	237	234	230	227	221
	B	167	164	160	158	157	156	155	154	153	153	152	151	150
0.15														
	A	333	312	290	281	274	270	267	264	259	257	254	250	246
	B	167	165	163	161	161	160	160	159	159	158	158	157	157

* n = the ratio of the length of the fillet, a, to the span length, L. † m = the ratio of the moment of inertia, I, at the center of the span, to the moment of inertia, I_s, at the support.

Mr. Benson, Mr. Andersen, Professor Maney, and others have generously emphasized the self-checking features of the method. That the method is definitely self-checking is its greatest single advantage over methods that deal with moments or stresses alone. Others have mentioned the fact that the method gives a clear physical picture of the action of the structure corresponding with the various mathematical steps in the analytical process. Some would conclude, apparently, that this same advantage exists in all slope and deflection analyses, but this is most decidedly not true. No one would attempt to visualize any connection between the elastic action of the structure and the steps performed in solving a group of simultaneous equations either directly or by successive routine corrections. In decided contrast, it will be noted that every step in the process of balancing angle changes has a physical significance that is easily visualized.

Professor Maney and Mr. Gray seem to have misunderstood the use of the word, "plastic." The writer's use of the word was consistently that denoting "inelastic." The profession will make no mistake that elasticity means straight-line variation of stress with strain whereas plastic or inelastic deformation denotes permanent or non-recoverable deformation. Such a fantastic idea as "time yield" or "continued flow" in a structural steel joint which would practically eliminate end restraint is a proved fallacy that formed no part in the choice of the word, "plastic." This fallacy formed the design basis for the steel building frames of two decades ago.

In closing this discussion, the writer wishes to express his appreciation of the time and effort which the discussers gave to the clarification of the subject matter of the paper. This also seems an appropriate time to express the conviction that the expanding group of modern methods of analysis based upon successive corrections which emphasize physical action under load will soon complete the displacement of the long entrenched classical methods dependent upon the solution of groups of simultaneous equations.

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THE MODERN EXPRESS HIGHWAY

By Charles M. Noble,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Fred Lavis, Joseph Barnett, G. E. Hawthorn, John F. Fairchild, Leslie R. Schureman, C. H. Purcell, Elmer R. Haile, Jr., H. W. Giffin, T. T. Wiley, F. L. McRee, Theron M. Ripley, W. W. Crosby, Richard S. Kirby, Harold M. Lewis, George Conrad Diehl, William E. Rudolph, A. C. Dennis, J. C. Carpenter, Henry B. Alvord, Robert Eugene Hiles, Chandler Davis, R. A. Moyer, and Charles M. Noble.

Synopsis

The purpose of the paper is to present for criticism and discussion a process of reasoning and analysis in highway design rather than to set forth definite rules and data; to suggest necessary test and research projects; to emphasize that design should provide positive safety at the speeds of which vehicles are now capable, or may be capable, in the near future; and to suggest an immediate raising of design standards for trunk highways.

The paper presents only an outline of a broad subject and is intended principally to direct attention in a positive manner to safety in highway design.

Introduction

The enormous loss of life and property on American streets and highways is a compelling reason for pausing to take cognizance of location and design practice and of the manner in which it should be improved to meet the increased speed of the present and future motor car. No other engineering structure has such a death record as the highway. Table 1 is a compilation of the accident record in the United States for 1935. Certainly many more accidents are chargeable to the highway itself than statistics indicate. The failure to attribute accidents to faulty road design is due to several causes principal of which is the subtlety of the problem. Police

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Note.—Published in September, 1936, *Proceedings*.

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officials are not always trained to analyze the basic causes of accidents properly, and there is a definite temptation to blame the driver because such a course is more concrete and along lines of least resistance.

TABLE 1.—TYPES OF ACCIDENTS RESULTING IN PERSONS KILLED AND INJURED IN 1935

Collision with	ACCIDENTS		PERSONS KILLED		PERSONS INJURED	
	No.	Percentage of total	No.	Percentage of total	No.	Percentage of total
Pedestrian	297 610	36.0	16 030	44.4	276 640	30.9
Automobile	374 490	45.3	8 900	24.6	450 320	50.3
Horse-drawn vehicle	4 960	0.6	140	0.4	5 370	0.6
Railroad train	4 960	0.6	1 440	4.0	4 480	0.5
Street car	13 230	1.6	310	0.9	11 640	1.3
Other vehicle	8 270	1.0	250	0.7	8 060	0.9
Fixed object	53 730	6.5	4 080	11.3	64 460	7.2
Bicycle	19 840	2.4	580	1.6	17 910	2.0
Non-collision	47 120	5.7	4.290	11.9	53 720	6.0
Miscellaneous	2 480	0.3	80	0.2	2 680	0.3
Total	826 690	100.0	36 100	100.0	895 280	100.0

The motor-car designer has set a pace which the highway designer has not anticipated properly. New automobile models appear each year, but new highways to replace outmoded highways cannot be produced so quickly. This imposes on the designer the task of producing a highway suitable for the car of ten or fifteen years in the future. So far, in general, this has not been done. The result is a staggering economic loss in life, property, and capital expenditure. To-day (1936) there is a large road mileage, less than ten years old, which is hopelessly obsolete so far as the safe and efficient operation of the present motor car is concerned. As an example, in 1935 several manufacturers of motor vehicles (some cars in the medium-price class) advertised that their cars would maintain a sustained speed of 100 miles per hr. The roads to accommodate these cars are not now in existence.

The rise in the accident and death rate is recognized to be due to increased speeds. The vehicle itself is far safer than formerly, but the higher speeds result in more serious accidents, involving more cars with a consequent greater loss of life. At high speed there is less time to correct an error of judgment, and it must be recognized that this time element is the new factor of importance which has entered the problem. Therefore, basic safety must be designed into the highway, taking this factor into account in such manner that the motorist is allowed sufficient time to cope with driving hazards. The reckless and unsafe driver must be considered. He affects the life and property of the innocent safe driver and cannot be disregarded with a shrug as formerly. In the past emphasis in design has been based on highway economics founded on vehicle operating costs and time losses. Broad highway economics founded on loss of life and property, as well as operating costs, has not received much attention. The injuries and the loss of life and property on American highways are issues squarely facing the highway engineer. It is his duty to design the highway so that the traveling public is safeguarded.

FUNDAMENTAL CONCEPTS

A consciousness of driver psychology is gradually emerging although a knowledge of its "laws" is only vague and half-formed. More knowledge is necessary, but, in the meantime, there should be a consciousness of its existence, and such facts as are now known should be applied to design. For instance, in many cases, it is possible to design a highway so that the motorist is led unconsciously to choose the safe act rather than that which is unsafe.

The trend of the times indicates the desirability of designing the highway on the basis of a constant rate of vehicle speed between large terminal points. It is evident that the decision as to the design speed should be arrived at only after very thorough study of the country to be traversed and that it should be made personally by the principal officer in authority. In the interest of safety it is also evident that the motorist must be notified on any change in the design speed. From present indications it would appear not unreasonable to design for a speed of 100 miles per hr in country not too mountainous. This will result in trunk highways having better alignment and curvature than the high-speed passenger railways. Generally, the present railway maximum speed is approximately 80 miles per hr. The drivers of trains are trained, picked men, familiar with their runs, riding on fixed rails, with definite orders and under close supervision. The reverse is true on the highways.

At every step in design the question should be asked: Is this safe under all driving conditions—darkness, snow, ice, rain, fog, ice and snow on the pavement, snow plowed on to the shoulders? Every conceivable accident hazard must be analyzed. Always there should be the consciousness of the number of feet per second a vehicle travels at any given speed, the length of headlight beams, the chances of visibility, all combined with the distance required to bring a vehicle to a stop.

THE DUAL HIGHWAY

The dual highway appears to be the only type of design developed so far to accommodate successfully the present high speeds with safety. A suggested minimum design is shown in Fig. 1. To provide increased speed with safety, wider lanes will be necessary. It is suggested that two 12-ft lanes in each direction may be sufficient if truck traffic is relatively light.

In the case of heavy truck traffic two lanes in each direction may be constructed in the unused center portion of the dual highway, but without any physical connection with the high-speed lanes. Access to the truck lanes may be provided at intervals by the use of "fly-under" crossings beneath (or over) the high-speed lanes. It does not appear likely that trucks will travel much in excess of 70 miles per hr and, therefore, the standards of design for these center lanes, except for ruling grades, need not be as high as for the outer lanes. A most effective method of eliminating time losses is to bypass all towns and cities between terminals, in addition to separating all highway and railway grade crossings. A safety factor is to prevent frontage of any type on the highway and to exclude farm and local road entry. It appears reasonable to restrict access to approximately 10-mile intervals.

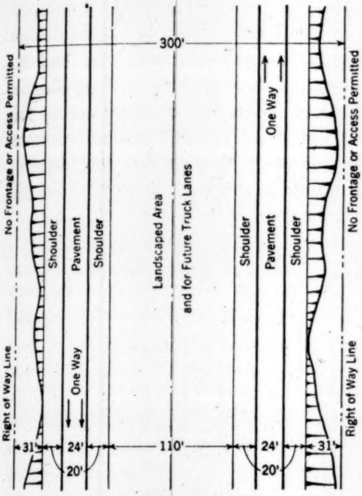
300'
No Frontage or Access Permitted
Right of Way Line
Shoulder
Pavement
Shoulder
One Way
Landscaped Area
and for Future Truck Lanes
Shoulder
Pavement
Shoulder
One Way
No Frontage or Access Permitted
Right of Way Line
31'
24'
20'
110'
24'
20'
31'

FIG. 1.—SUGGESTED MINIMUM DUAL HIGHWAY DESIGN

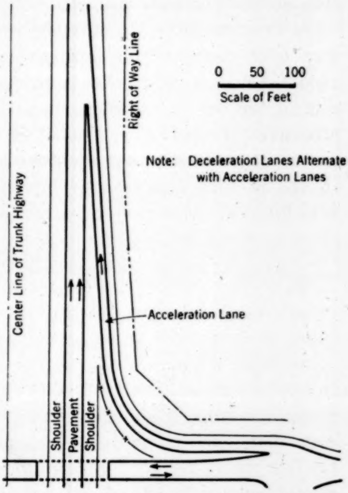
0 50 100
Scale of Feet
Right of Way Line
Note: Deceleration Lanes Alternate with Acceleration Lanes
Center Line of Trunk Highway
Acceleration Lane
Shoulder
Pavement
Shoulder

FIG. 2.—ONE QUADRANT OF SUGGESTED ACCESS ROAD DESIGN; OTHER THREE QUADRANTS ARE SIMILAR

Fig. 2 illustrates a simple type of design for rural access which has much to recommend it from a safety standpoint. The acceleration and deceleration lanes are the feature of the design. They provide full visibility on the trunk highway over a period of time sufficient to enable the motorist to adjust himself to the entrance and exit of other vehicles.

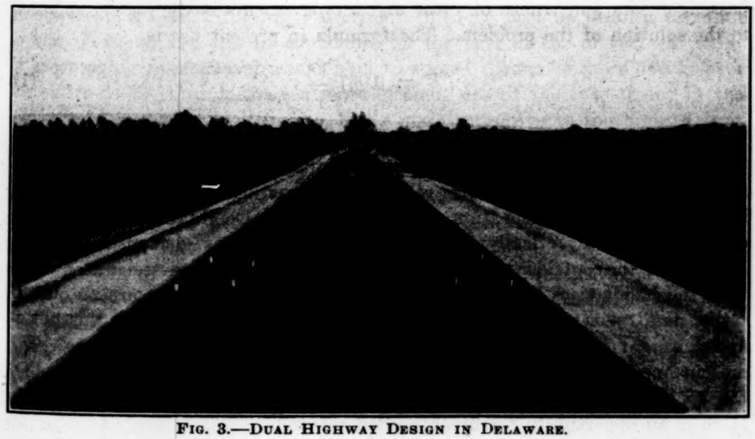

FIG. 3.—DUAL HIGHWAY DESIGN IN DELAWARE.

In the interest of economy the dual highway may be constructed in stages, but upon a full-width right of way. Two lanes may be constructed on one side and used as a two-way road until such time as funds become available for the complete project.

It is unnecessary to enumerate the points of superiority of the dual highway over conventional multiple-lane design to any one who has had the good fortune to drive over a properly designed dual highway. The elimination of the passing problem alone is sufficient reason for the adoption of this principle. Esthetically, also, it is capable of great possibilities and development. It is certain that the dual highway deserves very serious consideration in the present and future planning of trunk highways. Fig. 3 shows the dual highway as at present developed in Delaware.

DETAILS OF DESIGN

In order to define more clearly the writer's views with respect to highway design it will be more illustrative if details of design are discussed separately under certain more or less specific sub-divisions.

Superelevation.—With present and probable future high speeds it seems untenable not to bank (superelevate) curves. In many sections of the United States banking is standard practice, but in some States superelevation has been abandoned on multiple-lane projects on the plea of drainage difficulties. There appears no good reason why effective means of meeting the situation cannot be developed. A proper study of drainage and suitable design can eliminate nearly all the accumulated water which would normally flow transversely across the pavement. Certainly the motorist has a right to expect the protection of superelevation around curves. Negotiating at high speed a curve with "reverse" banking when the surface is covered with ice or snow is far from a pleasant or safe experience.

As far as the writer is aware the proper rate of banking for any given speed-radius combination is unknown. Comprehensive tests using modern passenger cars and trucks on clear and icy pavements is the logical approach to the solution of the problem. The formula in present use is,

$$E = 0.067 \frac{V^2}{r} \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

in which E equals superelevation, in feet per foot of width; V equals velocity, in miles per hour; and r equals radius, in feet. Equation (1) yields rates somewhat higher than are required for practical driving conditions. It is based on the superelevation maintaining equal weight on the inside and outside tires. Practically, this is not necessary, as demonstrated from experience the outside tires may have considerably more weight on them than the inside tires without danger. The point is that there are no quantitative data upon which conclusions can be based as to how much this unbalanced weight may be for safe, comfortable operation around curves. The present method of determining the superelevation is to use Equation (1), but to assume some arbitrary speed lower than the actual speed expected. Here is an opportunity for much needed research.

Curves.—From an analytical standpoint the minimum radius will depend on the maximum permissible rate of banking and the speed assumed. For example, suppose it is desired to design for a speed of 100 miles per hr, assuming 1 in. per ft of width as the maximum rate of super-elevation and, further, that Equation (1) will bank the curve safely if V is taken as 60 miles per hr. Substituting in the formula, a minimum radius of 2 900 ft is indicated. This method of determining the minimum radius is superior to the present practice of arbitrarily setting up a standard minimum radius based upon judgment only.

It is doubtful whether spiral curves will be necessary. The vehicle is free to spiral the slight amount necessary within the wide lane in order to accommodate itself to the large radii required by present speeds. Opinion is not of much value in solving this problem, however. Practical road tests are necessary before design practice can be established definitely.

Careful thought and designing are necessary in providing the transition between the normal crown and the banked section. It does not seem reasonable to adopt a standard length and treatment regardless of the radius and rate of banking. Certainly the treatment should be such that there is no tendency for the vehicle to be "thrown in" toward the center of the curve, or to swerve out. The ideal is for the design to be such that the vehicle may enter and leave the curve smoothly—almost automatically.

The distance between reversed curves depends on the time required for the driver of a vehicle to "come out" of one curve and prepare to enter the next one. As an illustration, if a 7-sec period and a speed of 100 miles per hr are assumed, the minimum distance between reversed curves will be approximately 1 000 ft. The writer is unaware of any observations upon which to base any conclusion as to the proper reaction time between reversed curves. This time interval may have some relation to the radii of the curves.

Grades.—The realization is growing that long smooth grades and vertical curves are essential for confident and safe operation at high speeds. Rolling grades create a hazard for night driving. Shadows cast by headlights result in lack of vision and loss of confidence on the part of the driver. A policy of encouraging night travel would tend to spread the traffic more evenly over the entire 24 hr, thus making a more efficient use of the investment in the highway. Long, sustained profiles, being more attractive to the night driver, may prove more economical than rolling grades when considered in the light of broad economics.

At a speed of 100 miles per hr a car travels 146 ft per sec. It would appear reasonable to provide uniform grade conditions for a period of at least 10 sec. If this speed is assumed in the design, a minimum length of profile tangent of 1 500 ft is indicated. The lengths of vertical curves should be such that, within the limit of vision of the headlights, shadows will not be cast and the normal length of the head-light beam will not be reduced. This will result in much longer vertical curves than those now in use. In general, definite data from which the proper length of vertical curves may be set are not available. Tests or optical computations will be necessary before definite values can be presented.

Present thought appears to favor good alignment in preference to low maximum grades, but it would seem desirable to limit maximum grades to about 5% in order to provide reasonable safety in descent on icy pavements. If commercial traffic is present in sufficiently large volume ruling grades may need to be limited in proportion to the economic importance of this class of traffic. It may be enlightening to study the operating efficiency and speed of various classes of trucks, loaded in accordance with current commercial practice, upon different rates and lengths of grades and allowing for motors not operating at "brand new" efficiency. It is well known that practically all trucks overload beyond the rated capacity of the vehicle. At present, manufacturers present data covering speeds and operating efficiency for different rates and lengths of grades, based on rated capacity and new motors. A test program simulating actual commercial conditions, in co-operation with motor manufacturers, may change present conceptions somewhat as far as economic grade design is concerned, as applied to trucks.

Sight Distance.—The proper approach to this problem, if the dual highway principle is utilized, would appear to be related to the distance in which a vehicle can be brought to a stop. This is dependent on the braking power and the interval of time required for the operator to start the application of the brakes after he has observed a danger. This latter interval may be termed the lag. The result of computations based on the laws of motion and using the 1931 rules of the New Jersey Vehicle Commission for stopping distances indicates a deceleration factor of 17.4 ft per sec per sec for four-wheel brakes. This is slightly higher than the factor deduced (approximately 16 ft per sec per sec) from the figures of the National Bureau of Standards[2]. The formula, based on well-known principles of motion, is:

$$s = \frac{1.075\,V^2}{17.4} \quad \text{.........................(2)}$$

Equation (2) will give the distance, s, required to bring a vehicle equipped with four-wheel brakes to a stop, neglecting the lag. The lag has been observed by various authorities as ranging from 0.75 sec to 1.5 sec. for an emergency stop. In connection with the deceleration factor of 17.4, tests conducted by a New York, N. Y., Taxicab Company indicate that a deceleration of about 19.0 ft per sec per sec will throw the occupants of a car out of the seats. This suggests a limit to a further increase in braking power.

Applying Equation (2), using V equals 100 miles per hr, a distance of 617 ft will be required to bring a car to a stop (on level grade) after the brakes are applied. Using a lag of 1.5 sec, a sight distance of 836 ft is required (617 + 219), at a speed of 100 miles per hr. The sight distances calculated, using the factor, $d = 17.4$, should be considered the minimum.

Fig. 4 shows the stopping distances for different speeds and rates of grades. Let $S =$ grade, in feet per foot ($-$ = down grade and $+$ = up grade);

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[2] Manual on Uniform Control Devices for Street and Highways, National Bureau of Standards, Appendix *C*, dated March 4, 1935.

m = mass; and, d = the deceleration of the car, in feet per second per second; then, since total work = kinetic energy + potential energy:

$$m\ d\ s = \frac{m}{2}\left(\frac{5\,280}{3\,600}\,V\right)^2 + 32.2\,m\,(-S)\,s \ldots\ldots\ldots\ldots(3)$$

For properly adjusted four-wheel brakes, using the New Jersey State traffic rules for 1931 a value of d equal to 17.4 is deduced from which it is possible to derive an expression for s in terms of S, as follows:

$$s = \frac{1.075\ V^2}{d + 32.2\ S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

If the dual highway principle is not used and the design consists of a two-lane, two-way highway, passing distance becomes the controlling element. For this case a sight distance of more than 2 000 ft may be required for reasonably safe passing. Even this distance does not take into consideration the speed of an approaching car. The passing menace is one of the serious problems facing the highway designer.

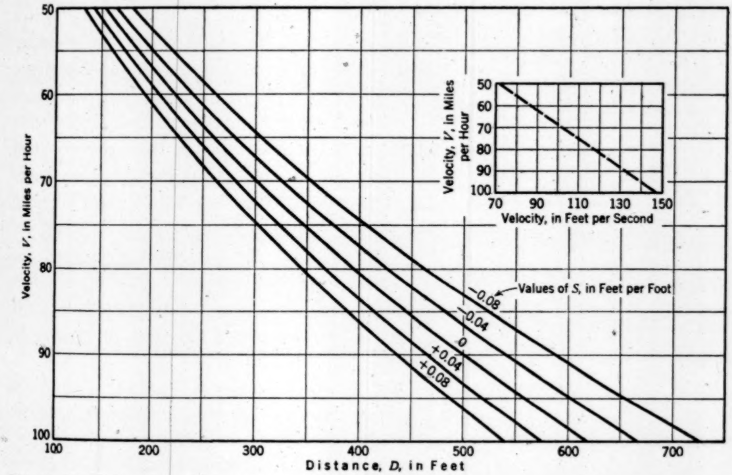

FIG. 4.—DISTANCE REQUIRED TO BRING A MOTOR CAR EQUIPPED WITH 4-WHEEL BRAKES TO A STOP AFTER BRAKES ARE APPLIED.

Shoulders.—If the shoulder is looked upon as a factor of safety a healthier attitude toward its cost will develop. In case of accident or mishap it may prove as important as the pavement itself. Many minor accidents develop into major accidents when cars swerve on to the shoulder and strike some fixed object, or over-turn. In many cases it is necessary for a car to dodge out on to the shoulder to miss a sudden accident ahead. For these reasons it is apparent the shoulders should be wide (preferably not less than 20 ft), level with the pavement, paved suitably to accommodate vehicles safely at high speed, and cleared of all obstructions. Ditches are recognized hazards and are being eliminated in present modern designs, but obstructions, such as poles, trees, warning signs, head-walls, light standards, etc., are not so generally recognized as sources of danger. The writer has often wondered why public officials permit private utility companies to locate poles within a few feet of pavements instead of along the right-of-way lines.

Curbing is another element of danger which has not been generally recognized. It is recommended that it be used very sparingly and only in places where it will prove to be a safety device instead of a hazard. It is well to keep in mind that the essential function of a curb is to protect pedestrians on sidewalks, to form drainage channels in urban areas, and as a first line of warning and defense on bridges, etc.

Signs.—The setting of warning signs as fixed objects near the roadway was probably derived from railway practice. Since the motor vehicle is not confined to fixed rails, signs set in this manner constitute a hazard.

Signs may either be suspended from overhead cables or mounted on flexible standards. To be effective they should be illuminated at night in such manner that the motorist is not blinded and so that objectionable shadows are not cast. The ideal would appear to be a luminous type visible for a sufficient distance, but with a limited light source. Reflectors are not always effective under certain conditions and, of necessity, must be set in a position to intercept head-light beams. In such a position the reflector usually becomes a menace.

A criticism frequently heard is that direction signs are not placed sufficiently in advance to permit the motorist time to make a turn properly and safely. This safe distance will be increasingly important because of advancing speeds. Braking power and feet traveled per second are the factors. Duplication of direction signs will also prove helpful.

Guard-Rail.—Probably no other single highway structure has been the cause of so much loss of life and property as the guard-rail. Because it is a hazard at best it is desirable that as little of it be placed as possible. For fills of moderate height it would appear logical, therefore, to substitute extra wide shoulders and flat side slopes. This would eliminate many miles of railing. In locations where the guard-rail is essential, only rail of such design that damage to the vehicle and injury to the occupant are avoided, should be installed.

The enlightening paper by Searcy B. Slack, M. Am. Soc. C. E.,[8] covering fundamentals of guard-rail design, points the way toward more rational design.

Lighting.—Practically all authorities agree on the present inadequacy and unsuitability of highway illumination. With increasing speeds and night travel this inadequacy assumes serious proportions. The lighting of to-day creates bright areas around which are pools of blackness difficult for the eye of the driver to penetrate. Indeed, some types of units defuse the rays laterally and upward leaving the pavement and an area some distance above the pavement in semi-shadow when compared with the brightness of the light itself. The effect on the motorist is to create a series of blind spots. Rain, snow, sleet, and fog increase this hazard. In this connection roadside stands with their brilliant lighting are very troublesome. Some of these stations produce a blind interval of several seconds. Proper legislation could eliminate this latter hazard. If highways are to be illuminated it is apparent that a radical departure from present methods will be necessary.

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[8] *Engineering News-Record*, June 9, 1932.

Pavements.—Present thought in pavement design appears concentrated on providing only sufficient strength, with a small factor of safety, to prevent actual failure. It may be that too little thought is being given to providing sufficient thickness and steel reinforcement to perpetuate the good riding qualities originally built into the pavement. The natural forces of weather, temperature, frost, and plastic flow in concrete tend to distort the riding surface. It is established that profileometer readings increase each successive year. Thus, after five or six years, the riding qualities are greatly impaired. The public is as interested in smoothness during the entire life of the pavement as in the first two or three years of its life. At present speeds, such roughness may not be unsafe, but at the speeds which may be common in the near future, it may be vital. A thick slab heavily reinforced top and bottom, with transverse joints less than 100 ft apart, may be the answer. Such a slab will be relatively stiffer and will have less deflection at transverse joints, and thus tend to lessen present troubles at the joints. A test program would do much to throw light on the problem.

At present, joints are receiving much deserved attention. They will be increasingly important in the future. Noise caused by the joint may be equally as important as smoothness. At a speed of 100 miles per hr with joints on 50-ft centers, a vehicle will pass over three transverse joints per second. If the joint causes noise it may prove more annoying and conducive to driver fatigue than the "clicking" of rail joints on the railways.

Terminals and Interchange Facilities in Urban Areas.—In such areas speeds may have to be limited to approximately 40 miles per hr in the interest of safety. Because of the specialized nature of the problem, the scope of this paper does not permit a detailed discussion. Each location presents individual design problems and must be treated on its merits. In the writer's opinion the simplest possible layout is best. A complicated design that confuses the motorist may be more conducive to lost time than a less mechanically perfect design that is simpler. The motoring public objects to "getting lost." The present tendency to eliminate all left turns on the secondary street may be unnecessary in most cases and may lead to undue complication. The decision as to the use of a design with no left turns depends on the peak-hour volume of traffic using the ramp and the street to be entered. If traffic on the street to be entered is light and not moving at high speeds, left turns may be permitted with safety. If the street to be entered is equipped with traffic lights the criterion will be that no more traffic shall accumulate in the ramp than will clear during a light interval. Observations show that at least ten cars in two lanes, will clear out easily during a 1-min. light interval. This is equal to not less than 300 cars per hr.

The principles of the traffic circle, the grade separation with ramps, and the clover leaf are well understood. The traffic-circle design is based on the volume of traffic as related to the diameter and the number of entrance streets, to the end that free weaving action may occur. The charge that traffic circles are dangerous is true if the circle is inserted in a high-speed highway where speeds are greater than 40 miles per hr. At speeds of about 40 miles per hr, a circle can be made safe with proper lighting, adequate illumined over-head warning signs placed sufficiently in advance, and by eliminating curbs and obstructions within and adjacent to the circle.

Center ramps, which require the main traffic to by-pass around them, are exceedingly dangerous. Heads of all turn-out ramps should be designed so that traffic does not tend to run into them. Curbs, if any, should be low and walls should be started sufficiently back from the nose to allow the motorist time to avoid crashing into them. Obstructions (signs, reflectors, light standards, etc.) placed on the nose are an added hazard. Proper lighting in the form of spot-lights and advance, over-head, illumined warning and direction signs will be of assistance in making ramp heads safer. Many of them have been designed in improper positions, and then loaded with signs and reflectors. Such installations have resulted in many serious accidents with accompanying loss of life and property. No part of the design deserves a more careful accident analysis than ramp heads. It is not necessary for the motorist to enter the ramp at breakneck speed; nor is it necessary to use curves of large radius. The ramp should be a definite turnout, a subsidiary of the highway, and should be so designed. If properly "signed" far enough in advance, the motorist will find it even if it is off to one side of the highway.

It is well to repeat that simplicity of layout consistent with safety and traffic-handling capacity is sound policy when designing terminal facilities.

Conclusion

Most of the highways designed twelve or fifteen years ago (before about 1920) are obsolete as far as alignment, grades, and details are concerned. These shortcomings are the cause of untold suffering and loss of life as well as an enormous economic waste in property loss and capital outlay. Unless there is a radical revision upward in design standards, history may repeat itself as far as the trunk system is concerned. Motor-car design and the public will dictate the speeds which will obtain. The engineer will have little voice in the matter.

There are evidences in various parts of the country of an awakening to the principles outlined in this paper, but nothing really concrete has been initiated on a national scale. Germany, on the other hand, actually has under construction the first unit of a system designed for speeds as great as 115 miles per hr.

The highway engineer is facing a tremendous responsibility. He must have courage and vision. He controls the destiny of billions of dollars and tens of thousands of lives. Safety for the motorist must be the watchword.

Acknowledgments

Acknowledgment is freely given to the Delaware State Highway Department, Travelers Insurance Company, and Mr. E. B. Shrope for assistance and courtesy in supplying data and illustrations used in this paper.

DISCUSSION

Fred Lavis,[4] M. Am. Soc. C. E. (by letter).—There can be little question that the facts and opinions expressed by the author should receive careful attention from engineers and others responsible for future design and construction of main trunk-line highways. This paper is timely and important. However, it raises a question that is fundamental. Mr. Noble seems to assume that it is the duty of the State (and by that is meant any governmental agency) to provide ample facilities for the operators of motor vehicles driven at high speeds on highways. It is a serious question, however, how far one should, or could, go in the expenditure of public monies for this purpose.

There is no doubt that future trunk-line highways, or future reconstructions of such highways, should be designed with reasonable consideration for future requirements, and at least one of these requirements is the operation of motor vehicles, trucks, buses, and private cars at high speeds.

Since the days of Wellington at the end of the Nineteenth Century, wherever proper engineering methods have been used, railway design has envisaged and provided for future traffic. In 1930 the writer[5] pointed out the desirability of applying Wellington's formulas, modified to suit the conditions, to the design of highways. Mr. Noble has amplified this suggestion to include safety of operation as well as the economic operating factors cited by the writer.

The fact must be borne in mind, however, that, although the highway accident record is bad and involves a considerable economic loss, and although it is in many ways a real disgrace, the development of high-speed operation imposes on the State a much more serious duty than that of providing a safe roadway—namely, the duty of carefully selecting, testing, and supervising the operators of those vehicles which are driven at high speeds. There is a further duty in the periodic inspection and testing of the vehicles themselves. Simply to provide speedways and let any one and every one use them who meets certain very minimum requirements—people whose responsibility is not frequently checked and verified—is really criminal and far more a subject of criticism than any charge that can be directed against the structure of the highways even as they exist to-day. Perhaps, some day the Utopia may arrive, in which all vehicles and drivers not meeting certain requirements will be limited automatically to certain comparatively low speeds.

An interesting commentary on this phase of highway safety is the statement of the Commissioner of Motor Vehicles of the State of New York of the revocation or suspension of licenses for the two-week period ended September 19, 1936. For the State the total was 733 and among the causes listed were: Driving while intoxicated; leaving scene of accident; failure to pay registration fee; false statements on application; reckless driving; speeding; etc.

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[4] Cons. Engr., New York, N. Y.

[5] "Highways as Elements of Transportation", by Fred Lavis, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

The fact that 733 licenses or registration certificates were suspended or revoked in only two weeks in one State indicates the extreme importance of the human element and it is, of course, almost certain that many other drivers than those who were discovered and penalized are not competent to drive at all and certainly not at high speeds. It will be noted also that no mention is made in these lists of the physical condition of the vehicles being driven.

There is also another phase to be considered in connection with the author's proposals for roads with large radius curvature, comparatively smooth profiles with light rates of gradient, and smooth rigid pavements. From the standpoint of safety, the effect on the driver of this monotonously even surface should be taken into consideration. There is some evidence that it may tend to dull the senses and produce a state, at least, of semi-somnolence which does not develop in driving over roads on which vigilance is obviously necessary. It may be admitted that the tendency of the future is likely to be toward the construction of highways of the very high type suggested by Mr. Noble but this argument, in turn, brings the subject back to the greater need of testing, licensing, and supervising the operators of the vehicles using the modern highway, and the vehicles themselves.

One statement is made in the paper in regard to pavements to which, perhaps, more particular attention may be drawn. Considerable has been said and written about the effect of heavy motor vehicles on highway costs, with the fairly general assumption that there is some more or less definite relation between the two and that the cost of highways is increased in some more or less direct ratio to the weight of the vehicles using it. More definite, perhaps, is the assumption that pavements (as also, to some extent, the alignment gradients, and width of roads) are controlled by the requirements of heavy motor trucks.

In December, 1934, the writer pointed out[6] some of the fallacies of this assumption and it is of interest, therefore, to note the author's statement that safe operation of modern private cars requires such widths and thicknesses of pavements that they will be amply sufficient for the heaviest vehicles.

In his "Conclusion", Mr. Noble states that "the highway engineer is facing a tremendous responsibility. He must have courage and vision." This sense of responsibility, courage, and vision have not been lacking, and the engineers of the United States have been far in advance of those of Europe in designing and building various types of so-called super-highways. It must be borne in mind that much of the policy of the development of transportation in Europe is based on military needs and cannot be assumed to be caused entirely by anxiety to provide for the speeding of private cars, although this latter may develop as an incidental factor.

The construction of the Westchester County Park System, in New York, incorporating the most advanced thought in widths of pavements, elimination of grade crossings, etc., was begun in 1913. The New Jersey State Highway from the Holland Tunnel to Elizabeth, N. J. (Route 25), was envisaged in 1923 and construction was begun in 1924. Boston, Mass., Philadelphia, Pa.,

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[6] "Effect of Heavy Motor Vehicles on Highway Costs", National Research Council, 1934.

Chicago, Ill., and other centers have contributed ideas and proceeded with construction.

The real question is not the courage and vision of the engineer, it is the reasonable adaptation of construction to future needs without unduly taxing certain classes of people, or all the people, for the benefit of the few.

The question as to how far one should go in making the expenditures necessary for the extension of these super-highways, is one of statesmanship just as much as engineering. One suggestion offered by Mr. Noble may be emphasized and that is the desirability of advance planning of such highways and the acquisition of the necessary land, the restriction of encroachments, etc., so that when the proper time comes the actual construction will not be unduly burdensome.

It should be stated that the author has done a service in emphasizing some of the factors that should be given consideration if, as, and when, such highways are required and the funds for their construction are made available.

JOSEPH BARNETT,[7] M. AM. SOC. C. E. (by letter).—In reading this paper one is given the impression that "Express Highway", "High-Speed Highway", and "Dual Highway" are considered to be synonymous. They are not so necessarily.

An express highway is one which may be traveled without undue interruption between terminals or other designated control points, much as express vehicles in any other form of transportation. It need not be high speed, necessarily, but the speed is higher, generally, than is common on adjacent local highways. The Westchester County and Long Island Parkways, in New York, properly may be classified as express highways. They may not be traveled at high speeds as that term is understood now, but may be traveled without undue interruption, in comfort, and with a sense of the beauty of Nature not possible on high-speed highways.

Dual highways are composed of two one-way highways. They need not necessarily be high speed or express. All highways carrying more than 10 000 vehicles daily should be constructed as dual highways. Increased safety and only a small additional cost over that of four-lane two-way highways are sufficient justification. Expensive 300-ft rights of way are desirable, but far from necessary. The largest part of the possible additional safety is attained as soon as opposing traffic is separated physically, even if the separation is only 3 or 4 ft of curb. All additional effort and expenditure, such as wide landscaped areas between highways resulting in wide shoulders to aid in avoiding collisions, reduction of head-light interference, etc., are worth while but they are subject to the law of diminishing returns in providing additional safety.

It should be possible to travel at high speeds on all modern express highways. However, the fact that motor vehicles are being constructed to travel safely at 100 miles per hr is not in itself sufficient justification for the adoption of this speed for design. It is only one of the factors which should be considered in weighing the problem. Of great importance are the topography, which is a factor in the determination of cost, and economic justification. One of the most important transcontinental highways in the United States would be included by many in any program for developing modern express highways, and yet hundreds of miles of this highway, where not under the influence of local urban traffic, carry about 300 vehicles daily. A two-lane, two-way highway designed for a speed commensurate with the particular topography is all that is justified.

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[7] Senior Highway Design Engr., U. S. Bureau of Public Roads, Washington, D. C.

Where traffic justifies the construction of four one-way highways, two lanes in each direction for trucks and two lanes for passenger vehicles, many advantages accrue to locating the truck or slow traffic highways outside rather than inside the fast traffic highways. "Fly-under" crossings are expensive, especially in flat country and the entrances to the slow traffic highways are comparatively numerous. By placing the slow traffic highways on the outside, fast-moving passenger vehicles may use the slow-moving highways for short distances to enter and leave the dual highway. Fig. 2 would then be modified, as shown in Fig. 5.

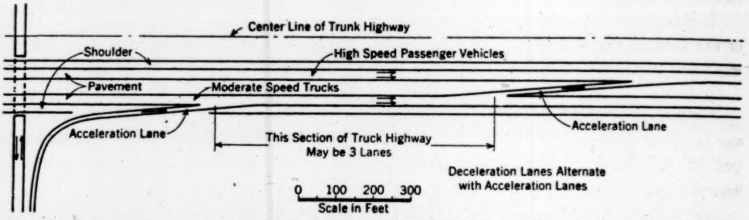
Center Line of Trunk Highway
Shoulder
High Speed Passenger Vehicles
Pavement
Moderate Speed Trucks
Acceleration Lane
Acceleration Lane
This Section of Truck Highway May be 3 Lanes
Deceleration Lanes Alternate with Acceleration Lanes
0 100 200 300
Scale in Feet

FIG. 5.—ACCESS PLAN FOR A FOUR-PAVEMENT TRUNK HIGHWAY. ONE QUADRANT IS SHOWN; OTHER QUADRANTS ARE SIMILAR.

Details of Design.—The complete formula for super-elevation (see Equation (1)) is,

$$E + F = 0.067 \frac{V^2}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which, in addition to the notation of the paper, F equals the side friction factor. On the assumption that there is an ample margin of safety against skidding when a vehicle rounds a curve at the minimum speed at which side pitch is noticed by driver or passenger, the U. S. Bureau of Public Roads has received reports on nearly 900 road tests to determine this speed. The results have not been analyzed thoroughly, but they lead to the conclusion that a side friction factor of 0.16 may be used with safety for vehicle speeds as great as 60 miles per hr. The results indicate lower factors for speeds greater than 60 miles per hr, but the combination of highway, vehicle, and driver capable of driving safely on curves at speeds greater than 60 miles per hr is rare, and few observations at higher speeds were received.

It is suggested that the minimum radius of curves be determined by the use of Equation (5); for example, for a speed of 60 miles per hr, a super-elevation of $1\frac{1}{4}$ in. per ft, and a safe side friction factor of 0.16, the limiting radius is 915 ft. For a speed of 100 miles per hr and an assumed safe side friction factor of 0.08, the limiting radius is 3 640 ft. This method seems superior to one in which side friction is ignored, and a lower velocity is assumed to compensate for it.

The writer cannot urge too strongly the adoption of transition curves. Mr. Noble states that the necessity for such curves is doubtful because the vehicle is free to make its own transition, necessary within the wide lane. It is not sufficient that highways be designed so that it is possible for vehicle operators to remain in their lanes. They must be encouraged to do so if safety is to be built into the highways. "Cutting corners" is one of the important factors contributing to the number of accidents on two-way highways which constitute the largest part of the highway system of the United States. Vehicle operators "cut corners" regardless of whether or not it is possible for them to keep to the traveled lane, chiefly because of a natural tendency to reduce the shock of the sudden application of centrifugal force. They are further encouraged to "cut corners" by the fact that the change from a crowned to a superelevated cross-section is placed on the tangent because of the lack of a transition curve. To keep from sliding down the inclined cross-section, the operator must turn his wheel slightly against the direction of the curve ahead, a most unnatural action, the necessity for which encourages him to head toward the inside of the curve. With transitions, superelevation increases with curvature, and no complicated methods of changing from crowned to superelevated cross-sections are required. Reverse curves need no special treatment and the ends of adjacent transitions may be a common station. With adequate tables the design and location of curves with transitions are not complicated and require no more time and effort than curves without transitions.

The required length of transition depends upon the rate at which the final constant centrifugal acceleration is approached. If a vehicle travels at a constant speed on a curve it is accelerating toward the center at the rate of $\frac{V^2_s}{r}$, in which V_s represents the speed, in feet per second. The total time required to traverse the transition curve is $\frac{L_s}{V_s}$, in which L_s represents the length of transition, in feet. The average rate at which the vehicle on the transition approaches this final constant centrifugal acceleration, therefore, is $\frac{V^2_s}{r} \div \frac{L_s}{V_s} = \frac{V^3_s}{r\,L_s}$. This average rate will vary for different drivers and numerous scientifically controlled tests are needed to determine the most desirable value. The few observations available indicate that a value of 2 for $\frac{V^3_s}{r\,L_s}$ is a fair average. Equating and changing to V, in miles per hour, results in the formula,

$$L_s = 1.58\,\frac{V^3}{r} \quad \text{.........................(6)}$$

For a speed of 100 miles per hr and a radius of 3 640 ft, the required length of transition is about 450 ft.

Regardless of the assumed design speed an alignment of curves with adequate transitions is greatly superior to an alignment of tangents and curves without transitions, in appearance and in ease and comfort of travel. Any one doubting this statement should operate a vehicle on a highway, such as the Mount Vernon Memorial Highway between Arlington and Mount Vernon, Va., on which all curves are constructed with transitions.

A deceleration of $d = 17.4$ ft per sec per sec is equivalent to a uniform rolling friction factor between wheels and pavement of 0.54. Although this is not too great for a motor vehicle on clean pavements even when wet, it is felt that in computing minimum sight distances for one-way highways a friction factor that would apply to most vehicles on pavements not altogether clean should be used. This friction factor is likely to be about 0.4. The general equation for braking distance based on a uniform rolling friction factor between wheels and pavement follows:

$$s = \frac{V_s^2}{2\,g\,(F \pm S)} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

Converting Equation (7) by substituting V to represent the velocity, in miles per hour, and 32.2 for g, results in the following:

$$s = \frac{0.0334\ V^2}{F \pm S} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

For an assumed uniform friction factor of 0.4, a vehicle at 100 miles per hr on a level highway would stop at a distance of 835 ft after the brakes are applied. What Mr. Noble terms "lag" is the sum of two reactions times, brake reaction time, and what may be termed "awake" reaction time. Brake reaction time has been tested widely and varies between 0.5 sec and 1 sec for most vehicle operators. Awake reaction time may be defined as the time it takes for a vehicle operator to come to the realization that the brakes must be applied. In this case, the operator must come to the realization that an object in his lane is stationary. It may take a few seconds to come to this realization if the object is a stationary automobile. Tests for awake reaction time are needed.

Conclusion.—The use of an assumed design speed of 100 miles per hr may be justified on many highways in many sections of the United States. Indeed, some engineers (notably those of the Oregon State Highway Department), are constructing highways designed for critical, as distinguished from assumed, design speeds as great as 100 miles per hr. The final determination to construct great systems of highways for this speed depends not on the vision and courage of the engineer, who is well endowed with these attributes, but on financial ability and justification.

However, to be safe, any highway on which the expected vehicle speeds are greater than 40 or 50 miles per hr should be designed with a refinement not thought necessary a decade ago. It is in this broad field that the principles outlined in Mr. Noble's excellent paper can be put to use. In many cases, little if any additional cost is involved. The principal requirements are more care in design and construction and the services of a high type of engineer, of which there is a plentiful supply.

G. E. HAWTHORN,[8] M. AM. SOC. C. E. (by letter).—The ever-increasing speed of automobiles presents a serious problem for the highway engineer. There does not seem to be any definite upper limit for speed which automobile designers will build into their vehicles of the future. At the present (1936) rate of increase, automobiles capable of speeds of 150 miles per hr may be expected within the next 10 yr. Mr. Noble, as well as many other highway engineers, fixes the maximum speed for which express highways should be designed at 100 miles per hr.

A decade ago the highway engineer had the problem of ever-increasing weights of vehicles. This problem has been solved, at least temporarily, by rigidly enforced laws limiting maximum size and weights of vehicles. This has been accomplished by co-operation between automobile designers and users with highway engineers.

The problem of speed is similar but more serious, in that loss of life and destruction of property, instead of destruction of pavement only, are involved. Unless some rigid upper limit of speed is established, either by mutual agreement between motor-car designers and highway engineers or by a rigidly enforced law, engineers will continue to design highways that will become obsolete as to safety features every decade. There is some question as to whether it is desirable to design main highways for speeds as high as 100 miles per hr just because some automobiles can travel that fast. Possibly 70 miles per hr would be more desirable; certainly, it would be much safer. The driver's control over his vehicle varies, other conditions being equal, with the square of the speed. The vehicle traveling at 100 miles per hr requires twice the distance of one traveling 70 miles per hr, and four times the distance of one traveling 50 miles per hr to be brought under control in an emergency. All possible safety features that can be built into both the automobile and the highway can in no way change these ratios. They hold also as to the violence of the impact resulting from any accident.

Minimum speeds on express highways are also important. Even on dual highways fast traffic may be "boxed" by one slow vehicle passing another, resulting in serious accidents.

The two main safety features that can be built into the highway itself are: (*a*) Proper superelevation on all necessary curves; and, (*b*) sight distances of sufficient length to allow vehicles operating at maximum speeds to be brought under control or stopped at all times in case of an emergency.

Safety Feature (*a*).—The superelevation used on curves can be considered from several angles. Equation (1) of the paper gives the theoretical superelevation desirable for vehicle operation on curves with the same safety as on tangents. Unfortunately, the maximum superelevation that can be used safely, if the road is to be used by slow traffic, is limited approximately to a 10% cross-grade, which allows a maximum superelevation of about 0.1 ft per ft of width. Using this value, the minimum radius of curve that could be used would be:

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[8] Asst. Prof. in Civ. Eng., Univ. of Washington, Seattle, Wash.

$$r = 0.67\ V^2 \ldots\ldots (9)$$

or for speeds at 100 miles per hr, $r_{\min.} = 6\,700$ ft. Automobiles can be operated on curves which are under-superelevated, however. Railroad engineers have determined that an under-superelevation of 3 in. in a gage width of 4.71 ft, may be used without discomfort to train passengers. Using this value for comfortable riding,

$$E = \frac{0.067\ V^2}{r} - 0.053 \ldots\ldots (10)$$

On curves where the superelevation is not enough, theoretically, the vehicle has a tendency to turn out and follow a curve of greater radius. This is an added hazard, particularly at high speeds. It can be shown that, if the deficiency in superelevation is equal to the coefficient of friction between tires and surface, vehicles will skid sidewise. At the skidding superelevation:

$$E = \frac{0.067\ V^2}{r} - f \ldots\ldots (11)$$

in which f is the coefficient of friction. Equation (11) is important only at high speeds or on slippery pavements.

Deficient superelevation may also cause overturning. It can be shown by the equilibrium of forces that the vehicle will overturn when,

$$E = \frac{0.067\ V^2}{r} - \frac{b}{2\,h} \ldots\ldots (12)$$

in which b = the width of the car from center to center of tires and h = the height of the center of gravity of the vehicle above the roadway.

Safety Factor (*b*).—The sight distance required for safety on highways depends on the distance the vehicle will travel while being brought under control or stopped. A fundamental formula for the required sight distance can be derived, using the relation of work and energy. Considering the kinetic energy of the rotating parts of the vehicle, its total kinetic energy is equal to $\frac{1.05\,W\,V^2}{2\,g}$, in which W = the weight. Then changing velocity from feet per second to miles per hour, $0.03511\ W\ V^2 = W\ f\ s + W\ S\ s$, and the required distance equals,

$$s = \frac{0.03511\ V^2}{f + S} \ldots\ldots (13)$$

in which S = the grade, in feet per foot (− = down grade and + = up grade). A term should be added to Equation (13) representing the distance traveled during the time used by the operator in applying the brakes. If s_a equals this time, in seconds,

$$s = \frac{0.03511\ V^2}{f + S} + 1.47\ s_a\ V \qquad (14)$$

For values of $f = 0.4$, $S = -0.05$ ft per ft, $V = 100$ miles per hr, and $s_a = 1$ sec.: $s = 1\,002 + 147 = 1\,149$ ft. The rate at which the vehicle decelerates varies with the value of the coefficient of friction, f, and $d = 32.2\ f$, in which d = the deceleration, in feet per second per second. Substituting d for f in Equation (14):

$$s = \frac{0.03511\ V^2}{\frac{d}{32.2} + S} + 1.47\ s_a\ V \qquad (15)$$

The first term of Equation (15) can be readily reduced to Equation (4) of the paper.

Sight distance and superelevation are built into the pavement and can not be changed once it is laid. The only unknown factor in these equations is the speed. Automobile designers and users, together with highway engineers, should agree on a reasonable maximum speed for which the main highways and automobiles are to be designed. Then, and only then, can highways be designed which are as safe as it is possible to build them. Speed will still claim its victims, but it will not be the fault of the highway.

Wide smooth shoulders free from poles and signs, flexible guard-rails placed at reasonable distances from the edge of the surface, and the limiting of grades are desirable, but they become really important only when the automobile is out of control. They can be replaced or improved as traffic demands change, at relatively small cost. Great improvement has been made along this line in existing highways in the United States during the last few years.

JOHN F. FAIRCHILD,[9] ASSOC. M. AM. SOC. C. E. (by letter).—A phase of this subject was omitted from the paper which, although it does not affect the construction of the highway itself, might affect its cost materially. For example, Fig. 1 shows a 300-ft right of way on the side lines of which appear the words "No Frontage or Access Permitted." In the paragraph following Fig. 1, Mr. Noble emphasizes this proposal by the statement that "* * * a safety factor is to prevent frontage of any type on the highway and to exclude farm and local road entry. It appears reasonable to restrict access to approximately 10-mile intervals."

If the express highway is in the location of an existing road, the construction of the highway under such restrictions would confiscate a very valuable right to the abutting properties for which compensation must be paid. If the express highway is located on an entirely new right of way, no doubt, it would cut some properties into separate parcels, so that it would be necessary to furnish under-crossings or over-crossings between the two parcels thus separated. This situation, of course, would vary in different localities, which fact should be taken into consideration in estimating the cost of the express highway.

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[9] Harrington Park, N. J.

LESLIE R. SCHUREMAN,[10] ASSOC. M. AM. SOC. C. E. (by letter).—The author renders a valuable service to the Highway Engineering Profession and to the general public in stimulating thought and discussion on what is probably the most pressing problem which confronts the highway engineer to-day —that of safety in highway design. However, any analysis of this problem cannot be complete without some consideration of the highway bridge which, although in itself a problem in design, must be included in any broad approach to the problem of determining and establishing standards of design for safe trunk highways. The accident record offers indisputable testimony in support of the fact that the highway bridge, as it exists to-day, is as hopelessly obsolete as the highway itself for the high-volume, high-speed traffic which must be provided for in the very near future.

The highway bridge engineer must develop design standards and details for a structure which will accommodate such traffic safely and efficiently. Its realization necessarily depends upon the co-operation of the highway engineer in providing straight-line tangents and continuity of grade and alignment at the site. Serious thought must be given to the roadway width of the structure. Obstruction to travel on the shoulder area constitutes a definite traffic hazard which can, and should, be eliminated. Pavement widening or flaring on approaches has proved to be a valuable safety feature. Approach slabs designed to span the back-fill area eliminate the startling end-of-deck bumps so common on present-day bridges. Balustrades must be structurally adequate to resist complete failure from collision and, preferably, should be provided with a first line of defense in the form of a curb. Poorly designed balustrades are much too frequently a contributing factor in serious accidents. Center piers on under-passes are a highly fertile source of motor-car disaster and, unquestionably, should not be used except in locations where the underpassing highway is divided by a center strip of ample width. Low points in the highway profile frequently occur at stream crossings. Adequate deck drainage eliminates the hazard of a wet or icy pavement condition.

Quite aside from purely utilitarian considerations, the structure should undoubtedly possess true grace and beauty. The question of æsthetics has become increasingly important with the growing need for grade-crossing eliminations. The intelligent selection of materials and design of proportions rather than detailed embellishment and ornamentation contribute much to an æsthetically pleasing result. The æsthetic treatment of any structure constitutes a problem in itself and should certainly be given considerably more time and thought than, in most cases, it has received in the past.

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[10] Asst. Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

C. H. PURCELL,[11] ASSOC. M. AM. SOC. C. E. (by letter).—The line of thought developed throughout this paper is well worth serious consideration. The next stage in highway design will be along the lines of greater safety, although perhaps not to the extent that speeds of 100 to 115 miles per hr will be common. It is quite feasible to eliminate a considerable percentage of accidents now classed, and rightly so, as "fault of driver." This can be done simply by the expenditure of sufficient sums of money. The engineering knowledge and experience are available now and can be applied whenever the work can be financed. The width of lane, radius of curvature, rate of superelevation, etc., for any maximum speed can be determined at a cost that will be insignificant in comparison to the total cost of improvement required. It is very doubtful indeed if all "fault-of-driver" accidents can be eliminated, and certainly not by engineering effort.

Records indicate that approximately one-half of all motor vehicle accidents in California occur in cities. A fair percentage occur, also, on secondary roads. Construction of super-trunk line roads, therefore, can eliminate much less than one-half the problem. In fact, such construction might easily increase the accident rate in cities and rural areas through emphasis of the high-speed complex.

On the basis of records for the first six months of 1936, accidents involving two or more vehicles fall into three groups of vehicle types:

Type of accident	Percentage of total
Passenger *vs.* passenger	69.70
Passenger *vs.* freight	21.67
Freight *vs.* freight	2.67
All others	5.96

Grouped in reference to the course being pursued, these accidents appear in the order of frequency, as follows:

Type of accident	Percentage of total
Approaching on the same road	41
Overtaking on the same road	31
Paths intersecting, but on the same road	16
Paths intersecting, but traveling different roads	12

From the foregoing, it appears that the separation of freight and passenger traffic would eliminate nearly 22% of accidents involving vehicles. The construction of divided lanes and the separation of grades of intersecting roads would reduce accidents by 53 per cent. Construction of pedestrian lanes and cross-overs would greatly reduce such accidents. To the extent that improvements of this kind were successful, the "fault-of-driver" type of accidents would be reduced. The extent of increase of accidents of the "overtaking", "side-swipe", etc., types which might result from increase in speed thus made possible, is entirely a matter of conjecture.

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[11] Chf. Engr., San Francisco-Oakland Bay Bridge; State Highway Engr., Sacramento, Calif.

Accidents in which single vehicles are involved are grouped, as follows:

Type of accident	Percentage of total
Collision	35
Non-collision	46
Pedestrian	19

The statement, "drove off the road", accounts for nearly 25% of all single car accidents. What this percentage might be for vehicles traveling at speeds of 150 ft per sec, controlled by a driver with a reaction time of 0.5 to 1.5 sec, is problematical. Certainly, accidents which might result would be spectacular, and perhaps more likely to involve vehicles in a lane 100 or even 200 ft to the left than the farm house or other fixed object an equal distance from the right of way.

In connection with consideration of design speeds, it is interesting to note the detail secured during a survey of actual road speeds at about sixteen locations, conducted by the Maintenance Department of the California Division of Highways. The speed of approximately 22 000 vehicles was recorded during daylight and of 11 000 at night. Fourteen of the locations were such that any desired speed could be maintained. The average speeds were, as follows:

Vehicle	Average Speeds, in Miles per Hour	
	In daylight	At night
Passenger	47	45
Trucks only	34	34
Buses only	50	50

Of the daylight traffic, 94.09% traveled at a rate of 55 miles per hr, or less, and, during darkness, 94.49% traveled at, or less than, that rate.

Before entering on a program looking toward ultimate development of highways adequate for speed of 100 miles per hr, there are other factors to be considered.

Are public officials prepared to admit, for example, that they are unable to control the careless, the reckless, and the incompetent drivers at present legal speeds? Are the present methods of control of traffic by "stop and go" and other mandatory signals and reasonable regulations a total failure in controlling traffic? Are they certain enough as to what the effect would be if construction were advanced to permit of higher speeds? Are they at all certain that the majority of drivers desire to travel at higher speeds than present roads permit? Have the people reached a stage where they are prepared to submit to the degree of regulation which German authorities, for example, can enforce in the operation of their express roads? What will be the effect of further improvements in airplane operation and construction of machines which can compete, in convenience and first cost, with the automobile?

There have been a number of highway projects constructed along the lines which Mr. Noble has in mind, and others will be developed which, from an experimental point of view, will provide information on which to base design standards.

The problem at present appears to be more politic and sociological than engineering. Every highway engineer should be alive to the situation, not only from a professional point of view but from that of the citizen as well, in order to serve the need in the best possible manner.

Elmer R. Haile,[12] Jr., Jun. Am. Soc. C. E. (by letter).—Since more and more monies are being expended for the renovation of obsolete highways, this paper comes at an opportune moment. It succeeds admirably in outlining the subject of building safety into highway design, and in suggesting desirable researches and tests. This discussion presents the results of experiments pertaining to various design details, refutes the theory that the design of highways for high speeds will lower the accident rate, and suggests an alternate method for lowering the accident rate.

The Design Speed.—The design speed of 100 miles per hr appears to be unnecessarily high. It may be true that automobiles can be built to cruise at 100 or even 120 miles per hr, but that fact in itself does not justify the design of highways for those speeds, because the average driver will never travel that fast. Conceivably, a few drivers might develop high speeds, but their numbers would be far too small to warrant the additional expenditure involved.

Studies indicate that the intelligence of the human race has not increased measurably in the last 100 or 1 000 yr. Hence, it is erroneous to assume that the ability of Man to reach quick decisions or that his tendency to make mistakes will change appreciably in the near future. To quote from the paper, "the present railway maximum speed is approximately 80 miles per hr. The drivers of trains are trained, picked men, familiar with their runs, * * *, with definite orders and under close supervision." Even so, there are instances of trainmen disobeying orders and signals. If men with only average judgment, slight supervision, little training, and perhaps no knowledge of road conditions were allowed to take over the operation of trains, the number of accidents would "sky-rocket." On the highways the personnel is exactly as described in the preceding sentence, with the additional formidable handicap that their vehicles do not run on fixed rails. The path of each car is controlled by the driver and, therefore, may be subject to every possible human reaction. Whereupon it may be reasonably deduced that highway speeds ranging close to the railway maximum of 80 miles per hr are out of the question for the average driver.

A speed of, say, 60 miles per hr might be safe, but even in this case it would be necessary to bar incompetent drivers from the road. In practice, this proves to be almost impossible. There are always those drivers who pass the most rigid of examinations and then proceed to be reckless, or at least lax, in concentrating their attention on driving; not to mention those who become incompetent due to ennui and fatigue, and intoxication.

All of which leads to one conviction, which is, distasteful though it may be to many persons, that the solution is to fix a maximum speed limit. Granted that there are some drivers who know what a safe speed is, and can safely "hit 90", yet for every such driver there are a dozen who can not judge whether or not they are proceeding at a safe speed until an emergency arises. Likely as not it is an innocent party who pays with his life for the driver's poor judgment. The proposed speed limit must be enforced, in sharp contrast with most present-day limits. Of the several means of enforcement, schooling is undoubtedly the best in the long run, supplemented with police patroling and, in some cases, mechanical speed control.

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[12] Junior Highway Engr., U. S. Bureau of Public Roads, Gettysburg, Pa.

After exhaustive tests and observations are made and the maximum safe speed is determined and agreed upon generally, then the minimum design standards for that limit may be established. Lower speed limits will be necessary for secondary roads, and for trunk roads in congested areas and mountainous country. Of course, there is no harm in building a highway safe for speeds greater than the speed limit; such construction increases the factor of safety. The point which the writer wishes to emphasize is that the construction of 100 miles-per-hr highways will not in itself serve to reduce the number of fatal accidents if cars are permitted to attain 100 miles-per-hr speeds. There is a certain speed above which the average driver can not go without appreciable risk of a fatal accident, no matter if the road is theoretically safe for 120 miles per hr. For example, statistics from several sources indicate that the majority of present-day accidents occur in the country, during the day, on dry pavements, and where the road is straight. These accidents are inexplicable; they must be charged to the frailties of Man.

The standard of 100 miles per hr has another vitally important disadvantage, namely, the high cost, not only for the right of way and area of pavement, but particularly for the elimination of grade crossings and points of access, and, except in flat terrain, for the grading. Only a few of the heavily traveled arterial routes can qualify as being sufficiently important to justify such expenditure.

The writer is not prepared to suggest a maximum value for the design speed, but he does conclude that it need not be very much higher than one-half the design speed suggested by Mr. Noble.

Accidents.—Referring to Table 1 in the paper, it is noted that more than one-third of all accidents involve pedestrians. These accidents cannot be reduced by improvement in road design, except by the construction of sidewalks, which, incidentally, must have a surface equally as good as, or better than, the roadway along which they are built or they will not be used by those they are intended to serve. About one-half the pedestrian accidents are due to the carelessness of the pedestrian. Education, particularly in the schools, is reducing that cause of accidents. The other half of the accidents that are blamed on the driver cannot be reduced by building high-speed roads.

Again, referring to Table 1, it is noted that less than 7% of the accidents involve collision with fixed objects. About one-half of these accidents occur on curves, and one-half, on straight sections. Therefore, an improvement in radii of curves will not materially reduce the accidents due to vehicles leaving the roadway, and it may increase them by encouraging higher speeds on curves.

Again, it is noted that about 45% of the accidents involve collision with other automobiles. Of these collisions, about 11% occur at intersections, 9%

are "head-on" collisions, and the remaining 25% occur with vehicles going in the same direction. The 11% part may be reduced by the proper development of intersections. The majority of all intersections, even in congested areas, will probably remain at grade for economic reasons. Much study is necessary to determine some type of intersection that will not be inherently hazardous. For example, islands or circles inserted in high-speed roads may be very dangerous. Higher speeds than those prevailing at present will certainly increase the number of accidents at grade intersections, regardless of their treatment, because of the unavoidable crossing of traffic. The 9% part—"head-on" collisions—may be reduced by the use of 20 or 22-ft roads instead of the usual 16 or 18 ft, by providing ample sight distances, by widening and easing of curves, and, in the few cases where four-lane construction is economically justified, by the use of divided roadways. However, since the great majority of trunk highways (about 95%) will remain two-lane, two-way roads, speeds higher than the present average will increase "head-on" collisions more than the improved design will reduce them. The 25% portion of all accidents can be reduced in about the same manner as the 9% just mentioned. In one instance, statistics show that accidents involving vehicles proceeding in the same direction are fewer on divided highways than on two-lane, two-way highways. yet, in another instance, the accident rate on the new divided highway is greater than the rate on the old two-lane, two-way highway, due possibly to the higher speeds obtained on the improved road.

To summarize, an analysis of the various types of accidents indicates that improvements in highway design will not serve to lower the accident rate as long as correspondingly higher speeds are permitted to develop, thus demonstrating the futility of trying to design roads for the maximum speeds of which modern cars are capable.

The Dual Highway.—The dual highway is undoubtedly indicated wherever a two-lane highway is inadequate in capacity. In at least one instance, the concrete slabs of one-half of a new four-lane highway have been jacked up and moved transversely so as to leave a neutral area between cars traveling in opposite directions.

The writer has observed a serious disadvantage in having a width of pavement in excess of approximately 22 ft, especially for a two-lane, two-way highway. Many drivers use the road as a three-lane highway, despite the narrow clearances, making it even more dangerous than the 30-ft, three-lane design.

Superelevation.—Past practice has been to superelevate according to rule of thumb, or not at all. Consequently, the driver never knows what to expect when he encounters a curve. The greatest danger lies in having a series of curves superelevated for a certain speed followed by a curve superelevated for only one-half that speed, as would be the case if the first-mentioned curves were banked to the limit, and the following curve was very much sharper, but not banked any higher, inasmuch as the limit had already been reached.

The writer has conducted road tests in an effort to determine the proper rate of superelevation for various combinations of speeds and radii. Modifying Equation (1) so as to take into account the transverse frictional force that is developed whenever the speed of the vehicle differs from the speed at which the gravity force due to the superelevation exactly balances the centrifugal force:

$$F + E = 0.067 \frac{V^2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which, in addition to the notation of the paper, F is the coefficient of friction between tires and road surface.

Observing the speeds at which drivers begin to encounter difficulty in keeping their cars under control on curves of known radii and rates of superelevation, substituting these values in Equation (16), and solving for F, values are obtained ranging from about 0.05, at which practically no centrifugal force is felt, to 0.50, at which a severe centrifugal force is felt, throwing passengers against the sides of the car, and resulting in considerable transverse creeping or skidding toward the outside edge of the curve. Upon plotting the coefficient, F, for numerous values of V, a wide band of points results. A curve drawn slightly below the average of these points is reproduced in Fig. 6, and represents the judgment of the more conservative drivers as to the allowable values of F for safe driving.

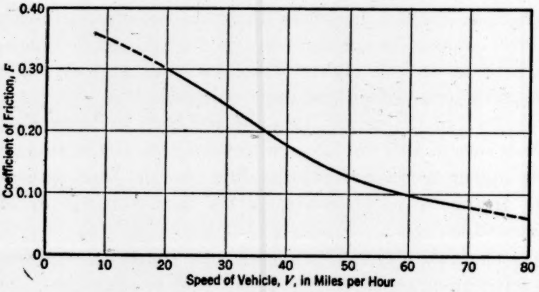

FIG. 6.—ALLOWABLE VALUES OF F FOR SAFE DRIVING ON CURVES.

An interesting feature of the graph is the fact that drivers do not hesitate to develop high values of F when traveling at slow speeds around sharp curves. However, a value of 0.30 for F still allows some factor of safety, for skidding does not occur under normal conditions until F reaches about 0.50. This margin need not be large at low speeds when the car can be slowed down within a few feet. At the other end of the graph, it is noted that much lower values of F are obtained, resulting in larger factors of safety. This is desirable at high speeds when the results of skidding may be disastrous, and when a considerable distance is required for reducing speed in case the car should begin to skid, as might happen if the pavement were uneven, or slippery with mud or grease.

Table 2 gives the maximum allowable curvature for various maximum speeds, the values of F being taken from Fig. 6, and the maximum value of E being assumed arbitrarily at 0.10 ft per ft of width. In sections where ice conditions prevail over a considerable part of the year, a lower value, say, 0.07, may be advisable, in which case the maximum allowable degree of curve would be somewhat less.

Since, in general, the average speed of vehicles will not be as high as the maximum safe speed, all curves flatter than the limiting curve should be superelevated for the average speed, using Equation (1), taking care, however, that the use of a low value of E resulting from that computation will not cause a transverse friction factor, F, greater than the allowable for the maximum safe speed as computed with Equation (16).

TABLE 2.—MINIMUM RADII FOR VARIOUS DESIGN SPEEDS, USING EQUATION (16) AND FIG. 6, AND ASSUMING $E = 0.10$

Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)	Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)	Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)	Maximum safe speed, V (1)	Maximum coefficient of friction, F (2)	Minimum radius of curve, r, in feet (3)
20	0.30	67	35	0.21	265	50	0.13	728	65	0.09	1,490
25	0.27	113	40	0.18	383	55	0.11	965	70	0.08	1,824
30	0.24	177	45	0.15	543	60	0.10	1 206	..	....	.....

Spirals.—The writer begs to differ with Mr. Noble's opinion that spiral curves are unnecessary. To illustrate, assuming curves of 2 900-ft radius with 1 000 ft of tangent between reversals, as suggested in the paper, and introducing spirals of 1 000-ft length (which will consume all the tangent), computations show that the offset required for inserting the spiral between the tangent and circular curve is 14.35 ft. This transition cannot be accomplished by a car within a 12-ft lane when spirals are omitted; the driver is forced to make his own spiral rather short in order to stay within the lane. Such an abrupt spiral requires special concentration and judgment, which is foreign to the ideal stated in the paper that vehicles should enter the curve smoothly, without tendency to be thrown in or out—almost automatically. When the spirals absorb all the tangent, as in the preceding illustration, there is no break in curvature—the driver is turning his steering wheel at a constant rate between the curves; but with a piece of tangent between the curves, the driver must adjust his car to run on the tangent for a few seconds, and then adjust his car to the next curve.

Contrary to general belief, the highway with the longest possible tangents is not the ideal to be sought. A flowing alignment is much more pleasing and restful to the traveler than monotonous "straightaways" followed by abrupt (although under the maximum degree) curves. This suggests the use of very long spirals between tangents and circular curves. The ideal treatment of a sharp curve, especially if it is sharper than the standard, is to introduce a spiral (or perhaps a series of other curves) of gradually increasing curvature for a sufficient distance ahead of the curve to enable the driver to release his throttle automatically and reduce his speed enough to make braking unnecessary. In addition, sharp curves should be signed to indicate their safe travel speeds, for the benefit of that great number of drivers who are not gifted with sufficiently accurate judgment. The foregoing treatment is far safer than the usual method, which is to design a road for high speed until a certain under-pass or topographical feature is encountered and then promptly to forget the standards.

In going around a curve, whenever a vehicle develops a lateral coefficient of friction, designated by F, the body tilts with respect to a line normal to the axle. Upon entering the curve, it is advisable the develop the factor, F, gradually; otherwise, there will be discomfort due to sudden tilting. Since F varies directly with the degree of curve, a transition with uniformly increasing curvature would allow a uniform increase in F. A spiral curve has the property of uniformly increasing curvature; therefore, it is used for the purpose.

If C is the rate of change of acceleration toward the center of the curve, L_s is the length of spiral, in feet, V_s is the velocity of the vehicle, in feet per second, r is the radius of the circular curve, in feet, and, if the curve is not superelevated:

$$C = \frac{V_s^3}{L_s\, r} \quad \text{.........................(17)}$$

Tests conducted by the writer indicate that: (1) A value of C less than 3 ft per sec per sec per sec does not cause a noticeable lurch of the vehicle; (2) a value greater than 6 does cause a noticeable lurch; and (3) a value of C as great as 10 ft per sec per sec per sec causes a bad lurch. Substituting 3 for C in Equation (17), and solving for L_s:

$$L_s = 0.333\, \frac{V_s^3}{r} \quad \text{.......................(18)}$$

Changing to miles per hour, and substituting $\frac{F + E}{0.067}$ for $\frac{V^2}{r}$:

$$L_s = 15.7\, V\, (F + E) \text{..................(19)}$$

On an unbanked curve, $E = 0$ and Equation (19) reduces to:

$$L_s = 15.7\, V\, F \text{.......................(20)}$$

On a banked curve, the part of the centrifugal force represented by E is balanced out by the gravity force due to the banking, and does not operate to tilt the vehicle with respect to a line normal to the axle; therefore, Equation (20) holds good for both banked and unbanked curves.

This formula gives the minimum length of spiral required for any combination of maximum safe speed, radius of curve, and superelevation. Upon applying the formula in practice, however, it develops that the length of spiral as computed is too short because of other considerations. If, for example, $V = 60$; $E = 0.10$; and $r = 1\,206$; whereupon, $F = 0.10$, it is found that $L = 94$ ft. As shown subsequently, the minimum length of transition from an unbanked section to a banked section, when $V = 60$ and $E = 0.10$, is

240 ft, with a recommended length of twice that amount. In all cases, the spiral should have the same length as the superelevation transition in order to provide uniformly increasing curvature simultaneously with uniformly increasing superelevation Therefore, Equation (20) may be discarded as superfluous.

Superelevation Transitions.—As a vehicle proceeds over a superelevation transition, it rotates about a longitudinal axis. This rotation, or angular velocity, or change per second of the rate of superelevation, may be expressed by $\frac{V_s}{L_s}(E_2 - E_1)$, in which V_s is the rectilinear velocity, in feet per second; $E_2 - E_1$ is the difference in rates of superelevation at the ends of the transition; and L_s is the length of the transition, in feet.

In the case where the superelevation increases uniformly from E_1 to E_2, the angular velocity jumps from zero to a certain value at the beginning of the transition, maintains this value throughout the length, L_s, and then drops back to zero at the end of the transition. This causes the vehicle to lurch at each end of the transition, because the change in angular velocity is very abrupt.

The change per second of angular velocity, or angular acceleration, may be expressed by $\frac{V_s}{L_v} \times \frac{V}{L_s}(E_2 - E_1)$, in which L_v is the length of the vertical curve at each end of the straight-line transition. The length of this vertical curve, or "rounding", may be as great as L_s, in which case there will be no tangent left, or it may be any length shorter than L_s. The writer has used $L_v = \frac{L_s}{2}$, although he cannot prove that that proportion is the best. Substituting $\frac{L_s}{2}$ for L_v, the expression for angular acceleration becomes, $\frac{2\,V_s^2\,(E_2 - E_1)}{L_s^2}$.

Experiments made by the writer show that an angular acceleration greater than 0.06 per sec per sec per sec produces an uncomfortable lurch, but that one-half that value, or, say, 0.027, is satisfactory. Substituting that value for the angular acceleration and solving for L_s:

$$L_s = V_s \sqrt{\frac{2\,(E_2 - E_1)}{0.027}} \qquad (21)$$

which, changing to miles per hour, reduces to:

$$L_s = 12.62\,V \sqrt{E_2 - E_1} \qquad (22)$$

If, for example, the maximum safe speed is 60 miles per hr and the change in superelevation is 0.10, the required length of transition is found to be 240 ft, with 120-ft vertical curves. At other maximum safe speeds, with the same difference in superelevation, the required lengths are in direct proportion. At a reversal of two sharp curves, if $V = 60$ and $E_2 - E_1 = 0.20$, the length, L_s is found to be 340 ft. This is almost one-third less than the 480 ft, which would be used if it were assumed that the length of transition should be proportional to the change in superelevation. This is an advantage in locations where the tangent distance is limited, which is probable where the curves are sharp enough to require the maximum allowable superelevation. On the other hand, if the tangent is not limited, and if the introduction of a longer transition is feasible, the writer suggests that double the values of L_s given by Equation (22) be used. When the transition is quite long, the rounding at its ends may be an unnecessary refinement, in which case a straight-line transition may be used.

Grades.—Safety on icy pavements does not justify limiting grades to 5%, assuming there are no other considerations making such a limit desirable. The extra cost of grading or the disadvantages of the more tortuous alignment necessary to provide a 5% grade may far exceed the accumulated cost of spreading sand or cinders over the icy pavement from time to time during the winter months.

If there is relatively little truck traffic, short lengths of 8% grade on tangent are not objectionable, but long grades should not exceed 7% on tangent, and should be compensated on curves. A gently rolling profile with very few, if any, tangents produces a more pleasing appearance than a series of tangents connected with comparatively short vertical curves. At summits, vertical curves should be flat enough to provide a sight distance equal to the distance required to stop a vehicle. If the design is a two-lane, two-way road, then passing would not be permitted on summits, but would be provided for at frequent intervals by sections of road allowing much longer sight distances on horizontal and vertical curves.

The normal length of head-light beam will not be affected if vertical curves of the foregoing standard are used, because, especially on a black road, the lights are of little value more than 300 ft ahead of the car, or, say, 400 ft with the brightest lights allowed by present regulations. At that distance, the edge of the road cannot be distinguished. An object, such as a cow, in the middle of the road is not recognized as an obstacle until the car is within 400 ft of it.

Sight Distance.—Tests conducted by the writer show that a wet bituminous surface may have a coefficient of friction as low as 0.54, and that when traveling at high speeds, it is dangerous to apply the brakes hard enough to develop all the coefficient of friction, especially on a curve. Using 0.40 as a safe maximum, and allowing 1.5 sec for the average driver to re-act in an emergency, the formula for stopping distance (on level grade) becomes,

$$s = 0.0835\ V^2 + 2.2\ V \quad \ldots\ldots\ldots\ldots (23)$$

Sight distances, both on vertical and on horizontal curves, should be not less than the stopping distance, as computed, using the safe maximum speed in Equation (23).

Conclusion.—The writer endorses the remainder of the paper. Classifying accidents according to three types of contributing factors—faulty cars, roads, and drivers—only 5% of the accidents are found to be the result of faulty cars; about 20% result from faulty road construction, and the remaining three-fourths result from faulty drivers. It is true that there are often poor features of road design involved, but the drivers show bad judgment in not allowing for those poor features. In support of the foregoing, statistics show that, in any one year, about 75% of all drivers have no accidents, and that 5% of all drivers have about 30% of the accidents. Several commercial firms, employing more than 1 000 drivers, have succeeded in reducing their accident rate to one-fifth, by eliminating drivers who were frequent repeaters.

A study of railroad grade-crossing accidents provides some insight into the actions of the average driver. In several million observations, about 10% of all drivers were classified as not exercising reasonable care in crossing tracks. When drivers survive an accident, they often claim that they never saw a train on that particular crossing before, and, further, claim they neither heard nor saw the train, although facts prove that it was impossible for a normal person not to have done so. Perhaps that is why many drivers overtake and pass cars at summits of hills, because they have never been caught by a car coming toward them on that particular hill! No wonder there are so many collisions on the highways if some persons drive "blind" and "deaf", and if 10% do not exercise reasonable care in driving!

The writer recommends that about 5% of drivers, the more prolific accident-makers, be deprived of the privilege of using the highways. Then, with the improvement of highways by designing for a safe, reasonable speed, and the positive regulation of traffic to conform to that speed, the accident rate should be reduced to a point well below one-fifth the present rate. This can be done, as proved by Evanston, Ill., that cut its accident rate to only 7% of its former rate within a few years.

It is to be hoped that there soon will be a nation-wide standardization and enforcement of traffic rules and licensing of drivers, and a standardization of features of road design.

H. W. GIFFIN,[13] ASSOC. M. AM. SOC. C. E. (by letter).—Some interesting questions as to the future of highway systems in the United States is raised in this paper. The subject deserves the attention of highway designers, administrators, and the general public. Before consideration of details of design three questions of a fundamental nature arise: (1) Is there a present need and demand for express highways as described in the paper? (2) Can they be operated safely? (3) Is there economic justification for them? Discussion of some of the phases of the present highway situation may throw some light on the future. Highway safety is much in the public mind and certainly every one concerned with the administration and design of highways should give serious thought to it in contemplating the highways of the future. A favorite method of foretelling the future is to examine the past for trends and to project these trends forward. Trends have a way of continuing, but many of them decrease in pace and some, finally, come to an end. Each trend should be examined carefully for those factors responsible for its growth and the conditions favoring its continuance.

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One of the conspicuous trends of the past is the higher average speed of automobile operation. Four-wheel brakes, low centers of gravity, better tires, better metals in improved power plants, and straighter, wider, and better surfaced highways have supported this trend. Will it continue and, if so, how far? Will it reach 100 miles per hr as an average, or for large numbers of vehicles, on the main roads? To-day, occasional vehicles operate at that speed at a few places. Is this an indication of future average conditions? On the other hand, what tends to arrest this trend?

Perhaps the increase in the accident rate which is now arousing the public will act as a damper. Can accidents be attributed to speed? Perhaps not altogether, but few will deny that speed is a contributing cause. Of ten accidents picked at random, how many would have been avoided by a reasonable reduction in speed? The effect of a reduction in speed is to give a greater margin of time and distance to avoid trouble. As most accidents occur within 2 or 3 sec, an extra second means much in deceleration or in steering clear of an object. Furthermore, the seriousness of many accidents is a function of the speed at contact.

It is maintained by some that the combination of speed and bad driving, and not speed in itself, is the cause of accidents. Most drivers over-estimate their ability to drive safely at the higher speeds. The formation of situations likely to result in accidents is of very frequent occurrence. Often, considerable skill is necessary to avoid contact. Should not good driving prevent the formation of such situations? One might define good driving as always having the vehicle under that degree of control which permits a sufficient margin of time and distance to take appropriate action to avoid contact. By this definition how many drivers would be classed as good? Even the best of drivers have lapses of inattention and absent-mindedness, and are saved from accident by fortunate circumstances. Any interested observer can testify to the many instances of bad driving he has seen. How many drivers never drive faster than is warranted by the range of their lights? What is the proportion of good and bad driving on high-speed highways? Should not something be done about the bad driving before average speeds increase?

Perhaps in the future there will be some method of eliminating the bad drivers and educating the others into being good drivers. How fast, then, can a good driver drive? The manufacturer constantly is improving the "roadability" of the automobile so that the individual can keep the vehicle on its course more easily. No one can foretell what the limit may be. Probably, however, there will always be a need of sudden stops as long as the operation of automobiles remains subject to the whims and judgment of individuals. As far as can be foretold now, the rate of deceleration of the vehicle must always depend on friction. The surfaces of roads, brakes, and tires may be improved in this respect, but it does not appear that any great improvement can be made in this direction. Certainly, the rate of vehicle deceleration cannot go beyond the possible rate of deceleration of the human body without injury.

With all the engineer can do to build safety into the road he cannot build good driving by individuals into it. That must always remain a matter of individual responsibility. Whether or not the engineer improves alignment, widens pavement and shoulders, separates grades by bridges, separates opposing lines of traffic, and solves the problem of posting adequate and proper signs, there will always be individuals who will drive carelessly and too fast for the conditions. There are some hazards, furthermore, which the engineer cannot remove, such as fog, sleet, and blinding snowstorms. It lies within the ability of the highway designer to design reasonably safe roads for the use of prudent drivers. Is it possible or desirable to try to go beyond this ideal? Many modern roads have all the features of safe design for reasonably prudent drivers, and yet the accident toll on them continues at an alarming rate. It seems that the removal of each hazard adds an increment of speed.

From an economic standpoint the justification of the construction of express highways must rest on their capacity to carry a very large number of vehicles and to save for them something worth the cost of the highway. Either distance, time, annoyance, operating cost, or dangerous conditions must be decreased. Distance often can be reduced by re-alignment, but for the main lines of travel it is doubtful whether this reduction would amount to more than 5% were all the main roads rebuilt on new alignment. Saving in time would depend on the saving in distance, elimination of stops, and a higher rate of speeed. The operating cost of automobiles is a function of speed, the increase being very rapid beyond 40 miles per hr. Improvements in the vehicle of the future will undoubtedly effect greater savings in operating cost, but the relationship of increased cost for high speeds will probably hold. Hazard on the main roads is almost entirely a function of speed and of the volume of traffic.

There is another economic aspect to the subject of express highways. There are now extensive systems of transportation by road, rail, water, and air in the United States. Each system is capable of producing a certain kind of service with the greatest advantage. Some overlapping of service is inevitable, and even essential, so that each system will render the best possible service in its field. The modern highway has already encroached upon the field formerly regarded as belonging to the railroad, forcing the railroad companies to re-adjust the service offered to the public. This development can be regarded as healthy as far as it has gone to-day and perhaps it can go still further without harm; but it seems probable that the principal railroads of the future will furnish a service not economically furnished by the highways. Is there need and economic justification of providing complete duplication? Are there not many highways yet to be built before there is necessity for further rapid encroachment on the railroad field? The railroads are adjusting their service to provide safe and comfortable means of travel for long distance at high speed. For speeds beyond those which the railroads of the future will provide there are airplane systems. Is the highway, with its individualistic mode of operation, an ideal system for providing all kinds of transportation service?

As better highways are built, the older roads suffer by comparison in the public mind; but all roads not conforming to the highest standards of design should not be thought obsolete. Their useful life is not nearly ended because they must be an important part of the highway system for many years. Day after day many of them are providing a service commensurate with the needs, and producing a return commensurate with the investment in them. Of course, many others can be rebuilt economically and, when rebuilt, should be changed to provide better service; but there are many which, although they are not so efficient as new ones would be, cannot be re-designed for a long time. Nor are they generally regarded as dangerous, inasmuch as they are driven at slower and more economical speeds by those who have become accustomed to them. An examination of the accident records of these roads does not indicate that they should be discarded.

There is one accident for approximately 200 000 miles of driving. If one could witness 200 000 miles of driving by one person—or better, 50 miles by 4 000 persons—over average streets and roads, one would see a wide range of speeds, great difference in the skilful handling of motor vehicles, and all shades of driving between good and bad; and yet only one accident would occur involving injury to driver, passenger, or pedestrian; 3 999 out of 4 000 would come through a 50-mile trip safely. What is a reasonable conclusion to draw from this fact?

Although the automobile manufacturer has done a good job in production, both from an economic standpoint and from that of convenience and safety, modern automobiles are now equipped with power plants far beyond the needs or even desires of the average motorist. This excess power is seldom used; it is an economic waste and probably a contributing cause for much bad driving. It has been said that excess power is very convenient in an emergency. On the other hand, the mere possession of it often invites the creation of such an emergency. The lack of it would force a reduction in high speed and, by and large, would be far safer. It is emergencies of this nature which require exceptionally good driving by others on the road. Is the fact that the manufacturer is providing something not needed and not in the public interest a valid reason for building roads to keep pace with it? If permitted, the manufacturer of the future will probably be able to supply power plants capable of speeds of 200 or even 300 miles per hr. Imperfections of the mechanical machine are gradually being overcome; but the human machine is the limiting factor in the attainment of highway safety.

The problem of the future highway, then, is related to the capacity of individuals to drive safely, to their economic needs, and to the standard of living of the motorist on which the cost of improvement must rest. There is much that is not clear, much that is unknown, and many questions to be answered before the specification for the future highway can be written. In the meantime, the highway engineer will continue to do what he can to interpret the immediate needs of the large majority of motorists, to build well in accordance with the funds provided, and to try to eliminate road hazards which conceivably are contributing causes of accidents. What lies beyond must await future developments.

T. T. Wiley,[14] Jun. Am. Soc. C. E. (by letter).—In presenting this paper the author has afforded a timely opportunity for discussing some of the factors that affect the design of the modern highway. The design of many highways and highway structures under construction to-day is scarcely adequate for safe driving at modern-vehicle road speeds and, if the present progress in motor-vehicle design is continued, the roads being built to-day will be obsolete almost before they are completed. A discussion of superelevation and curves seems especially pertinent, because these are features of design that have a direct bearing on safety and that have not been given the attention and study that they deserve.

It is true that Equation (1) yields rates of superelevation that are higher than required for safe and comfortable riding. However, the practice of using a speed lower than the design speed in this equation is not satisfactory. A more scientific attack on this problem can be made by considering the forces that prevent side-slip outward when the speed of travel is such that the resultant of the mass of the vehicle and centrifugal force is not normal to the pavement. When these frictional forces are considered, Equation (1) becomes,

$$E = 0.067 \frac{V^2}{r} - f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which f is the average coefficient of friction developed between the tires and the pavement in order to prevent side-slip.

Values of f varying from 0.15 to 0.20 have been used, but investigations have indicated that they are too high for comfortable riding and safe driving. The writer conducted a brief series of tests to determine the safe speed on curves, using Equation (24) and the definition that the safe speed was "the minimum speed at which the driver or passenger feels a side pitch outward." Two cars with a driver and one or two passengers were used for this investigation. One car was equipped with knee action and the other with transverse springs.

Test runs were made in both directions on seventeen curves, and the speeds at which side pitch became noticeable were determined Additional runs were made at speeds calculated from values of $f = 0.15$ and $f = 0.10$. Table 3 gives the curve data, test speeds, and values of f.

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TABLE 3.—TESTS TO DETERMINE SAFE SPEED ON CURVES

Curve No.	Degree	Superelevation, in feet per foot	Speed for side pitch, in miles per hour	Coefficient of friction, f, for side pitch	VELOCITY, V, IN MILES PER HOUR		Curve No.	Degree	Superelevation, in feet per foot	Speed for side pitch, in miles per hour	Coefficient of friction, f, for side pitch	VELOCITY, V, IN MILES PER HOUR	
					For $f = 0.15$	For $f = 0.10$						For $f = 0.15$	For $f = 0.10$
1	11	0.05	35	0.11	39	34	9	3	0.058	60	0.07	77	67
2	17	0.062	30	0.12	33	28	10	5	0.092	50	0.06	64	57
3	11	0.075	30	0.04	42	37	11	5.5	0.067	45	0.06	59	51
4	10	0.046	30	0.06	41	35	12	4	0.042	50	0.08	64	55
5	6	0.025	35	0.06	50	42	13	24	0.083	25	0.10	29	25
6	5	0.017	35	0.07	53	45	14	5.5	0.083	45	0.05	60	53
7	9	0.042	33	0.07	41	37	15	5.5	0.083	45	0.05	60	53
8	3	0.000	50	0.09	65	53	16	13	0.054	30	0.06	36	32
							17	12	0.062	30	0.07	39	34

The following conclusions were obtained as the unanimous opinion of all persons who participated in the tests:

(1) The speeds at which side pitch first becomes noticeable are slower than necessary for comfort or safety;

(2) The speeds which require a friction coefficient of 0.15 create very definite side pitches of such magnitude as to be uncomfortable, require undue effort in steering, and often border on being dangerous, particularly when meeting traffic; and,

(3) The speeds which develop a friction coefficient of 0.10 create definite side pitches that are reasonably comfortable, require appreciable but not excessive steering effort, are not dangerous, and are fast and reasonable speeds in every case.

In the author's example for determining the minimum radius by using Equation (1), the design speed was 100 miles per hr and the rate of superelevation was 1 in. per ft of width. Then, the assumption was made that it would be satisfactory to design for 60 miles per hr and the minimum radius was calculated as 2 900 ft. The danger in making such an assumption is shown by taking the same example and solving for the radius, using the true design speed of 100 miles per hr and Equation (24) in which f is taken as 0.1. From this formula, the calculated minimum radius becomes 3 700 ft. The difference between this radius and the one obtained by using the methods now in common use certainly indicates the need for more scientific design of curves, particularly for high speeds.

Table 3 shows a considerable range in the values of f for speeds which first create side-pitch. There are several reasons for this, one of which is the personal element which existed in noting side-pitch. However, certain physical characteristics of the curves also cause these differences. Such items as grade, length of run-off, and roughness of the pavement, undoubtedly affect the results of such tests and indicate that these items must be considered in design. This leads directly to the author's discussion of the transition between normal crown and superelevation, and his statement that "it is doubtful whether spiral curves will be necessary."

If the design is "to be such that the vehicle may enter and leave the curve smoothly—almost automatically", the spiral will be necessary on all superelevated curves. Although engineers know that the spiral provides a uniform rate of change in degree of curve, many of them fail to realize that its principal advantage is that it also provides a uniform rate of change in the superelevation so that the superelevation at any point on the spiral or curve is in exact agreement with the degree of curve at that point.

There is no need for presenting the theory of the spiral because several useful spirals have been developed. However, railroad engineers have learned considerable about curves and spirals, and their knowledge can, and should, be applied to highway design; for example, they have found that superelevation cannot be introduced across the track gage at a rate greater than 1¼ in. per sec without creating an uncomfortable riding condition. Therefore, superelevation on a highway should not be introduced at a rate exceeding 0.02 ft per ft per sec, and the length of spiral on a curve with a superelevation of 1 in. per ft should approximate the distance traveled in 4 sec at the design speed. On this basis, curves designed for a speed of 100 miles per hr, with superelevation of 1 in. per ft, should have a spiral approximately 600 ft long.

Highway engineers are loath to accept the spiral as a necessary feature of design, partly because they do not realize the ease with which it can be calculated and applied. The writer was taught a type of spiral which requires about 10 min of calculation in preparing the field notes and a negligible amount of excess time in staking out the spiral and curve. Resort to tables is unnecessary.

The author states that "failure to attribute accidents to faulty road design is due to several causes principal of which is the subtlety of the problem." This is probably true especially with respect to accidents at curves, which too often are listed as being caused by driving too fast for conditions.

An automobile entering a curve is subjected to new forces and, at modern fast speeds, these forces often change in magnitude and direction very rapidly. Superelevation for curves is usually introduced on the tangent over a distance termed the "run-off." A vehicle traveling over the run-off is subjected to a lateral force which must be overcome by turning the steering wheel in a direction opposite to that of the curve. Otherwise, the vehicle will move toward the inside of the curve, thus crowding the center line or pavement edge with occasional disastrous results.

The instant the car enters the curve, forces acting on the car change instantaneously, requiring a very rapid and usually complete reversal in the direction of steering. The changes may be so great that the steering wheel cannot be turned rapidly enough and the car is thrown partly or wholly out of control and an accident results. The subtlety of the problem now enters, for the accident report states "driving too fast for conditions." Is the mistake in the driving or in the conditions? The common assumption is that it is the driving. Engineers familiar with the spiral will claim that the conditions are at fault. There is absolutely no reason why the driver should have to attempt to overcome the sudden change in forces because a properly designed curve will be spiraled and a sudden change of any appreciable magnitude in the forces which must be overcome by steering will never occur.

The writer has expressed the opinion that Equation (24) gives satisfactory results in computing curve requirements. This formula was not advocated because it is better than Equation (1), but because it is better than Equation (1) as ordinarily used. In other words, Equation (24) is a logical way to take into consideration the degree of overbalancing that can be tolerated in rounding a curve, whereas the attempt to do this by substituting a speed less than the design speed in Equation (1) is not based upon scientific analysis and is often unsatisfactory. Equation (24) should afford a desirable "minimum standard", but if highways are to be designed for the greatest safety, why not adopt the best available, which is Equation (1) using the true design speed rather than a guess at a "safe" speed? Similarly, spiraled curves are certain to be safer than non-spiraled ones. Use the spiral.

Incidentally, this contention is applicable to many other design standards. Too often a "minimum standard" becomes "the" standard, so that, in many instances, standard curves of 1 500-ft radius have been built where flatter curves would have been just as economical and certainly more desirable. The highway engineer must remember that a minimum standard is the worst that he is permitted to use, and that good design requires the use of something better than the minimum allowable if it is reasonably practical. To use minimum standards when better can be obtained is usually "penny wise and accident foolish."

The writer cannot agree with the author's general contention that signs constitute a hazard. Wooden posts, 4 in. square, furnish excellent mounting for signs and, if struck by a car moving at a fair rate of speed, they snap off without causing appreciable damage to the car. The types of signs now in use would be useless because of lack of visibility, if suspended from overhead cables. Flexible standards of sufficient rigidity to furnish adequate support for standard signs subjected to wind pressure, vandalism, and other buffetings would be just as dangerous as wooden posts.

A properly designed modern highway should require few signs. If the design speed is to be placed at a limit, such as 100 miles per hr, the "standard" signs now advocated will be wofully inadequate.

The author is to be commended for emphasizing the fact that the highways being built to-day are scarcely abreast of the car of to-day, not to mention the car of the future. The highway engineer must exercise his imagination to visualize the conditions that will exist in the years to come, and base his work upon the conviction that the best design is scarcely good enough.

F. L. McREE,[15] ESQ. (by letter).—Considerable food for reflection and thought is presented in the paper by Mr. Noble. There is no doubt that steps should be taken to reduce the appalling death rate on highways in the United States. However, it is a moot question whether or not increased safety in design will bring about the desired result. Since a large percentage of the automobile accidents are caused by a small percentage of the drivers, can engineers produce a design that will be safe for these few inherently reckless drivers?

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[15] Associate Prof. of Civ. Eng., Texas Technological Coll., Lubbock, Tex.

The trend is, and will continue to be, toward faster motor-vehicle speeds, unless legal steps are taken to prohibit the increase. Motor-vehicle manufacturers have increased the speeds of their vehicles from year to year because the people have demanded it. As the author points out, there are few existing highways that can accommodate these speeds. John W. Wheeler, M. Am. Soc. C. E., has made[16] the statement that some highways are antiquated before they are built—a very broad statement, but which, nevertheless, may be true.

The author's idea of a dual highway appears to be good, but will the people submit to the cost of the suggested wide right of way? It seems to the writer that the width could be reduced considerably and still serve the purpose. It is doubtful whether truck traffic will need two lanes in each direction for many years to come. Furthermore, why should the people have their money tied up in an extensive investment to provide for truck lines of the future?

Restricting rural entries to 10-mile intervals makes it very difficult for the rural resident who may have to travel 10 miles to visit his neighbor who lives directly across the highway. Rural entries are undoubtedly a source of danger, but it seems unreasonable to restrict these entries to such long intervals. The people living in rural communities help pay for the highways, and, in return, expect to make use of them. The only basis on which rural entries could be restricted would be to provide parallel slow-speed roads which would increase costs and still impose a hardship on rural residents.

It is very necessary that curves be properly superelevated. The flat or insufficiently superelevated curve becomes a source of great danger with present high motor-vehicle speeds. The frictional resistance between tires and pavement surfaces should be given serious consideration in determining the superelevation. Investigations conducted at the Iowa Engineering Experiment Station[17] indicate that a working coefficient of 0.30 can be used safely, even for wet pavements. This will permit much higher speeds than the theoretical for which the curve is superelevated.

Equation (1) can be modified to take account of the frictional resistance. From mechanics, it can be shown that,

$$\tan\,(\theta + \phi) = 0.067\,\frac{V^2}{r} \quad \text{.................... (25)}$$

or, in terms of the degree of curve, D,

$$E = \tan\,(\theta + \phi) = \frac{DV^2}{85\,800} \quad \text{.................. (26)}$$

in which θ = the angle of superelevation; $\phi = \tan^{-1} f$; and f = the coefficient of friction. If the coefficient is used as 0.3, ϕ becomes 16° 42′. As an example, suppose that the minimum superelevation is required for a 3° curve and a speed of 100 miles per hr: $\tan(\theta + \phi) = \frac{3 \times (100)^2}{85\,800} = 0.35$; and $\theta + \phi = 19° 17'$. For $f = 0.30$, $\phi = 16° 42'$; and, therefore, $\theta = 2° 35'$; $\tan 2° 35' = 0.045$; and, by Equation (26), $E = 0.045$ ft per ft.

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[16] *Civil Engineering*, June, 1935.

[17] *Bulletin 120*, Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa, August, 1934.

A solution of the author's problem of finding the sharpest curve for a maximum superelevation of 1 in. per ft by the foregoing would be: $\theta + \phi = 4° 46' + 16° 42' = 21° 28'$; $D = \frac{85\,800 \times 0.393}{(100)^2} = 3° 22'$; or, $r = 1\,700$ ft.

In any consideration of superelevation of curves, minimum speeds should be considered. When ice forms on a pavement the coefficient of friction sometimes becomes very small and vehicles tend to slip toward the inside of the curve. This is particularly true for slow-moving, heavily loaded trucks. These considerations should limit the maximum superelevation to be used.

The minimum speed on superelevated curves is given by Equation (26) solving for V.

Assuming a coefficient of friction equal to 0.10, the minimum speed for the example noted would be zero since the friction angle is 5° 43′; that is, $\theta - \phi$ is negative, showing that, theoretically, a negative speed could occur without skidding.

Fig. 7 shows the minimum superelevation for a speed of 100 miles per hr and a coefficient of friction of 0.30, and also the maximum for a speed of 10 miles per hr and a coefficient of friction of 0.10, 10 miles per hr being assumed as the minimum speed likely to occur. It is not reasonable to assume that the curves would be built flat up to 2° 30′, therefore, the suggested superelevation is indicated. Diagrams similar to Fig. 7 can be drawn for any coefficient of friction that is assumed to be reasonable.[17] Table 4 shows values of the maximum speed for $f = 0.30$; maximum and minimum speeds for $f = 0.10$; and the theoretical speed for which the curve is superelevated ($f = 0$), all based on the suggested values of superelevation.

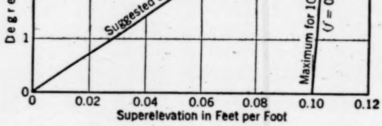

FIG. 7.—MINIMUM SUPERELEVATION FOR A SPEED OF 100 MILES PER HOUR; MAXIMUM SUPERELEVATION FOR A SPEED OF 10 MILES PER HOUR; AND THE SUGGESTED SUPERELEVATION.

The writer cannot agree with Mr. Noble that spiral curves are unnecessary, or that a motor vehicle has sufficient room within its lane to form its own spiral. For a motor vehicle to enter or leave a curve smoothly the curve should be spiraled and the spiral should be of sufficient length to accommodate the maximum speed for which the highway is designed.

TABLE 4.—MAXIMUM AND MINIMUM SPEEDS, AND THE THEORETICAL SPEED FOR WHICH THE CURVE IS SUPERELEVATED

Curvature, D, in degrees (1)	Superelevation, E, in feet (2)	VELOCITIES, V, IN MILES PER HOUR FOR:				Curvature, D, in degrees (1)	Superelevation, E, in feet (2)	VELOCITIES, V, IN MILES PER HOUR FOR:			
		$f = 0$ (3)	$f \leqq 0.30$ (4)	$f \leqq 0.10$ (5)	$f \geqq 0.10$ (6)			$f = 0$ (3)	$f \leqq 0.30$ (4)	$f \leqq 0.10$ (5)	$f \geqq 0.10$ (6)
1	0.028	49	169	105	0	4	0.10	46	95	66	0
2	0.057	49	125	82	0	5	0.10	41	84	59	0
3	0.086	49	106	72	0	6	.....	..	...	...	.

There is considerable disagreement among highway engineers as to the proper length of spiral. Past practice has been to use spirals that are too short. The American Railway Engineering Association has found by experiment that the maximum rate at which the tilt of superelevation can be attained without discomfort to passengers is about 2 in. per sec. There is no valid reason why the same rule will not apply to highway curves. Using 58 in. as the average width of automobile tread and E as the superelevation, in inches per inch of width, the total tilt would be 58 E in. The minimum time in which this tilt should be attained is $\frac{58\,E}{2} = 29\,E$ sec which gives the minimum length of spiral in terms of the velocity, V_s, in feet per second, as $L_s = 29\,E\,V_s$. Changing the velocity, V, to miles per hour the expression becomes,

$$L = 42.5\,E\,V \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which E = the superelevation, in feet per foot.

Another method proposed for determining the minimum length of spiral is based on the maximum rate at which a motor vehicle may change its radial acceleration without discomfort and danger. Experiments conducted at the Iowa Engineering Experiment Station[17] indicate that this value is from 2 to 3 ft per sec per sec per sec, with 2 ft as the recommended value. The engineers of the State Highway Department of Oregon use a value of 3 ft per sec per sec per sec for their spirals in planning their highways for a speed of 100 miles per hr. The radial acceleration varies from zero at the beginning of the spiral ($r = \infty$) to $\frac{V^2}{r}$ at the end. The time in which this acceleration is attained is $\frac{L}{V}$ sec. The rate of change of acceleration, then, will be:

$$d = \frac{V_s^2}{r} \div \frac{L}{V_s} = \frac{V_s^3}{L_s\,r} \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

Using a value of 3 ft per sec per sec per sec for this rate of change, changing V_s to miles per hour, and replacing r by $\frac{5\,730}{D}$:

$$L_s = \frac{DV^3}{5\,450} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

The writer favors using the expression that gives the greatest length of spiral.

Some engineers follow the practice of spiraling the center line of the pavement and offsetting to obtain the line for the pavement forms. It is much easier, and more accurate, to run separate spirals for each edge of the pavement. With widened pavements separate spirals should always be run on account of the longer length of the inside spiral. A good method is to compute the minimum length of spiral from one of the foregoing expressions, use this length to the nearest 10 ft for the outside edge of the pavement (unless the run-off of the superelevation requires a longer spiral), and then compute the length for the inside edge. This inside length is fixed by the length of the outside spiral and the widening.

Referring to the preceding statement that a motor vehicle does not have room to form its own spiral within its lane, it can be shown that the shift or offset, p, for a spiral is given by the following formula:

$$p = \frac{D^3\,V^6}{408 \times 10^{10}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

The derivation is based on the A.R.E.A. spiral expressions and Equation (29). Using a 3° curve and a speed of 100 miles per hr, the offset is $p = 6.6$ ft. If a value of 2 ft per sec^3 for the maximum rate of change of radial acceleration had been used in developing Equation (29), the spiral offset for this degree of curve and velocity would be 15 ft. Obviously, a motor vehicle cannot form its own spiral within a 10 or 12-ft lane when the offset is so large.

It appears that there is a tendency among highway engineers to avoid the spiraling of highway pavements because of the supposedly complicated computations involved. The A.R.E.A. spiral can be adapted easily to highway pavements, and the writer cannot agree that the computations involved are too complicated. Even if they were complicated the engineer should be willing to do the necessary work when increased safety is the goal.

THERON M. RIPLEY,[18] M. AM. SOC. C. E. (by letter).—The "express highway" is given as the principal antidote for accidents and fatalities and in this development the highway engineer is implied to be the contributing cause of the present accident record due to improperly designed travel lanes. The experience of seventeen years in the design, construction, and maintenance of highways and the study of traffic compels the writer to take issue with the foregoing assumption. He has seen the 14-ft, the 16-ft, and the 18-ft, two-way pavement, each in turn, abandoned for the 20-ft lane with all the attendant improvements in grade, alignment, and crossing elimination. The self-propelled vehicle has changed from an oddity and a toy to the commonplace and a necessity, and the dirt road to a paved street; yes, for many hundred miles, to a boulevard.

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[18] Cons. Engr., Buffalo, N. Y. Mr. Ripley died on November 30, 1936.

The discussion of the highway traffic situation, apparently, has arrived at that stage which can be represented by the man suddenly awakened by an unusual sound: when he is so awakened his first impression is one of bewilderment and he attributes the sound to one of great magnitude and danger. This is a natural reflex from the first law of Nature—self-preservation. In May, 1934, under the title, "The Balance Wheel", the president of a large corporation attempted to keep the feet of his organization on the ground by writing a few truths which are as applicable to the present highway situation as they then were to the petroleum industry. He said in part:

> "This is a time when mental indigestion is epidemic. The seats of learning have become sounding boards for intellectual jazz, and the Doctor Cooks of economics are having a field day.
>
> "The statement that two and two are four lacks intrigue or interest, and arguments to the effect that, in this modern time, two and two make five are the sex appeal of current economics. It is a day of intellectual emancipation, and it should not be turned into a period of mental anarchy.
>
> "These observations apply only to current discussions on religion, morals, politics, business, art, and science. Otherwise, things are normal.
>
> "The balance wheel has its uses even though it is not spectacular."

Under "Introduction" the first subject mentioned by the author is "the enormous loss of life and property on American streets and highways" and, included in this paragraph, is a table (Table 1) showing the number of persons injured, the number killed, and the total number of accidents in 1935. For the convenience of the reader the original table[19] is reproduced as Table 5 for the record.

After stating that "the reckless and unsafe driver must be considered", the author finishes his "Introduction" with the statement that "the injuries and the loss of life and property on American highways are issues squarely facing the highway engineer. It is his duty to design the highways so that the traveling public is safeguarded." A study of the accident record, perchance, may show where the engineer has failed.

As Table 5(*a*) shows that 79.0% of all accidents, 87.2% of all fatalities, and 78.6% of injuries happen "going straight", it seems reasonable to assume that doing away with all curvature is not the answer to the prevention of trouble. If the engineer is to blame he must look further into the causes.

Table 5(*b*) is a breakdown of the items in Table 1 of the paper. Of Table 5(*b*) the compilers[19] wrote:

> "Although total deaths last year [1935] exceeded fatalities of the year before by around 300, mistakes of drivers figured in nearly 1 500 more deaths than in 1934. That means the time for dilly-dallying about what to do to restrain careless drivers has long since passed. Drivers who will not act safely must be forced to change their conduct. Else they must be kept off the road."

The greatest number of fatalities, under one heading, is seen to be "Pedestrians"; these are segregated in Table 5(*c*). It is believed to be a fair assumption that all the items in Table 5, from Items Nos. 23 to 38, inclusive, can be assumed as wholly within urban communities. If this assumption is used for argument, then it is found that 12 020 fatalities occur in cities and villages and 3 030 fatalities in rural communities, with 980 fatalities to be distributed. If the 980 fatalities are divided between the two known localities on the basis of the assumed known amounts, the 16 030 total will be divided into 12 805 urban and 3 225 rural. It is self-evident that rights of way of 300 ft, or more, in width cannot be secured in urban centers; therefore, in order to save 80% of the disappearing pedestrians the engineer must look to some other savior than the width of the highway.

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[19] See "Live and Let Live", Travelers Insurance Co., Hartford, Conn., p. 4.

TABLE 5.—CONDITIONS RESULTING IN PERSONS KILLED AND INJURED IN 1935

Item No.	Description	ACCIDENTS		PERSONS KILLED		PERSONS INJURED	
		Number	Percentage of total	Number	Percentage of total	Number	Percentage of total
	(*a*) EFFECT OF THE DIRECTION CARS WERE TRAVELING						
	Vehicles:						
1	Going straight	653 090	79.0	31 480	87.2	703 690	78.6
2	Turning right	20 670	2.5	610	1.7	23 280	2.6
3	Turning left	56 220	6.8	1 120	3.1	63 560	7.1
4	Backing	12 400	1.5	330	0.9	13 430	1.5
5	Skidding	29 760	3.6	1 260	3.5	31 330	3.5
6	Parked, or standing still	23 970	2.9	610	1.7	25 070	2.8
7	Slowing down or stopping	26 450	3.2	400	1.1	30 440	3.4
8	Miscellaneous	4 130	0.5	290	0.8	4 480	0.5
9	Totals	826 690	100.0	36 100	100.0	895 280	100.0
	(*b*) EFFECT OF HASTY, NEEDLESS, AND CARELESS ACTION OF DRIVERS						
	Driver:						
10	Exceeding speed limit	121 460	22.8	7 240	30.7	161 550	22.9
11	On wrong side of road	85 770	16.1	3 940	16.7	111 460	15.8
12	Did not have the right of way	135 840	25.5	3 580	15.2	191 880	27.2
13	Cutting in	17 580	3.3	420	1.8	23 980	3.4
14	Passing standing car	2 130	0.4	70	0.4	2 820	0.4
15	Failed to signal or signaled improperly	27 700	5.2	260	1.1	35 980	5.1
16	Passed another car on a curve or hill	8 520	1.6	400	1.7	11 290	1.6
17	Passed another car on the wrong side	2 130	0.4	50	0.2	2 820	0.4
18	Absent; car ran away	3 200	0.6	280	1.2	4 230	0.6
19	Drove off the roadway	55 940	10.5	3 390	14.4	64 190	9.1
20	Drove recklessly	51 670	9.7	3 020	12.8	67 020	9.5
21	Miscellaneous	20 780	3.9	920	3.9	28 220	4.0
22	Totals	532 720	100.0	23 570	100.0	705 440	100.0
	(*c*) EFFECT OF THE ACTION OF PEDESTRIANS						
	Pedestrian Crossing at Intersection						
23	With signal	10 990	3.8	210	1.3	10 780	3.9
24	Against signal	36 200	12.4	1 070	6.7	35 130	12.7
25	No signal	37 280	12.7	1 870	11.7	35 410	12.8
26	Diagonally	5 350	1.8	370	2.3	4 980	1.8
	Pedestrian:						
27	Crossing between intersections	78 100	26.7	4 550	28.4	73 550	26.6
28	Waiting for, or stepping on or off, street car	3 210	1.1	80	0.5	3 130	1.1
29	Standing on safety aisle	1 210	0.4	100	0.6	1 110	0.4
30	Getting on or off another vehicle	3 210	1.1	260	1.6	2 950	1.11
31	Playing in street	45 850	15.7	1 600	10.0	44 250	16.0
32	At work in roadway	6 220	2.1	450	2.8	5 770	2.1
33	Riding or "hitch-hiking" on vehicle	3 920	1.4	320	2.0	3 600	1.3
34	Coming from behind parked car	34 340	11.7	1 140	7.1	33 200	12.0
35	Walking on rural highway	14 650	5.0	3 030	18.9	11 620	4.2
36	Not on the roadway	5 920	2.0	340	2.1	5 580	2.0
37	Miscellaneous	6 220	2.1	640	4.0	5 580	2.0
38	Totals	292 670	100.0	16 030	100.0	276 640	100.0

TABLE 5.—(*Continued*)

Item No.	Description	Accidents		Persons Killed		Persons Injured	
		Number	Percentage of total	Number	Percentage of total	Number	Percentage of total
	(*d*) Effect of Prevailing Weather Conditions						
39	Clear weather	699 710	84.6	28 600	85.6	671 110	84.6
40	Fog	15 940	1.9	870	2.6	15 070	1.9
41	Rain	92 160	11.2	3 310	9.9	88 850	11.2
42	Snow	18 880	2.3	630	1.9	18 250	2.3
43	Totals	826 690	100.0	33 410	100.0	793 280	100.0
	(*e*) Effect of Road Surface Condition						
	Road Surface:						
44	Dry	619 930	75.0	25 860	77.4	594 070	74.9
45	Wet	136 830	16.6	5 240	15.7	131 590	16.6
46	Snowy	23 970	2.9	770	2.3	23 200	2.9
47	Icy	45 960	5.5	1 540	4.6	44 420	5.6
48	Totals	826 690	100.0	33 410	100.0	793 280	100.0
	(*f*) Effect of Light Conditions						
49	Daylight	479 650	58.0	14 000	41.9	465 650	58.7
50	Dusk	30 100	3.6	1 540	4.6	28 560	3.6
51	Dark	316 940	38.4	17 870	53.5	299 070	37.7
52	Totals	826 690	100.0	33 410	100.0	793 280	100.0

Considering the remaining 20% of walkers: It is possible that the construction of sidewalks in special localities might be of greater value than widening the right of way or separating the vehicle movements. Pedestrians will not walk over rough ground, through wet grass, or in the mud in greater numbers simply because a change has been made in the pavement or highway limits.

Assuming the statements in the preceding paragraphs to be true as to facts and probable action, the fatalities chargeable to the highways can be reduced by 16 030, to 20 070; but Table 5(*b*) shows 23 570 fatalities due to "Hasty, Needless, and Careless Action of Drivers" and, therefore, some of these quantities overlap. It may be only a coincidence that this overlapping is almost equal to the number "Walking on Rural Highway" (Item No. 35, Table 5(*c*)).

Considering now the matter of pavement condition, attention is called to Table 5(*d*) and Table 5(*e*), which show that 84.6% of all accidents happen on clear days; that 75.0% happen on dry surfaces; that 85.6% of fatalities happen on the former and 77.4% happen under the latter.

When the matter of design is considered in relation to fog, snow, and ice, and their effect upon traffic, it is seen that these "Weather Conditions" account for 14.4% of the fatalities and these "Road Conditions" for 6.9 per cent. The handling of ice and snow conditions upon the highway is, primarily, a function of maintenance. When the designing engineer has provided the necessary room for the proper functioning of the snow-removal equipment and personnel he will have done his duty. Ice and packed snow on the pavement must be looked after by the maintenance organization.

Artificial lighting of improved highways has been discussed for a number of years, but the question of cost and the inadequacy of the lighting equipment available have been deterrents in the adoption of a general policy. An idea of the influence of light upon fatalities and injuries can be secured from Table 5(*f*), which shows that "Dusk" and "Dark" claim 58.1% of the fatalities and 41.3% of the injured. Lighting, properly done, is not merely the placing of electric lamps along the roadside; the writer has seen more than one installation of that character which was more of a hazard than a help. The new lamps in the experimental installation near Schenectady, N. Y., will help to solve this problem.

The foregoing remarks, born of Mr. Noble's paper, may seem to exonerate the designing engineer, but such is not the intention. The engineer has his work to do. The writer desires to impress upon the reader that the principal reason for automotive mortality is the operator of the machine and, in a lesser degree, the pedestrian or his guardian; and that the reduction of accidents is to be accomplished, primarily, by education and law enforcement.

To design for a speed of 100 miles per hr, as suggested by the author, would require a major change in the designing trend. Dr. F. C. Stanley, Chief Engineer for the Raybestos Company, of Bridgeport, Conn., has published a number of papers and graphs relative to the stopping distances required by automotive vehicles under ideal conditions. His studies have resulted in the adoption of an empirical formula which shows that 444 ft is the distance to stop a car traveling at 100 miles per hr—this with a dry pavement and a car in perfect condition, and the operator with a reaction time of 0.75 sec. A speed of 45 miles per hr, which is now recognized as legal in some States, would require, under the same conditions, 90 ft in which to stop.

As the whistling snorer refuses to admit that he keeps the family awake, so, the driver of an automobile refuses to admit his responsibility for the highway slaughter.

Recently the technical and lay press has been writing about "friction" as being a major cause of highway accidents. Vehicle friction, or highway friction, or traffic friction may be good or bad slogans. The writer has never seen or heard brain friction mentioned and yet it may have a legitimate place in the discussion.

As an example of public reaction, to curb highway traffic damage, the State of New York voted $300 000 000 for the elimination of railroad-highway grade crossings. The record then showed that 4.5% of fatalities and 0.5% of all accidents happened at such crossings. Much of this money has been spent and many crossings eliminated but the slaughter continues, the motorist is as discourteous as ever, and traffic rules and laws are broken as of yore.

What the speed of highway traffic will be ten or twenty years hence no one knows and few are competent to make an intelligent guess. What is known is that a nap at the wheel is a free pass to eternity; that alcohol in the stomach and gas in the tank is a combination that can make shambles of the public highway; and that halfwits, nitwits, and morons are at the throttle of machines of 60 hp and more, driving from somewhere to nowhere to do nothing when they get there and, in their joyously mad ride, they are a fatal menace to all other traffic on the highway.

Looking back to the water-bound macadam days of New York State and the time when a motor vehicle was a "red devil" which scared all the horses, and most of the drivers; and tracing the changes through the years to this day, it seems that both the civil and the mechanical engineer have done a most commendable job. Neither has reached perfection, but one has only to see what has been done adjacent to the larger cities in New York and other States, particularly in New Jersey, to realize that the highway designer is awake to the situation. To find the sleepers one must look in other directions.

The most potent cause of accidents, as well as the most difficult to control, is the driver. Fool-proof highways can not be built; therefore, a sincere effort must be made to keep the fools away from the steering wheel.

W. W. CROSBY,[20] M. AM. SOC. C. E. (by letter).—This frank and forceful paper is a subject for congratulation; the cogent statements contained in it are most opportune. The author points out some fundamental defects of present highway designs. These latter are the direct reflections of engineering ideals, imagination, and effort, and he rightly implies that "safety" should be a feature of the results. This latter point the writer has been trying to emphasize for some time[21].

Too often, however, the engineer is restricted by higher authority (such, for instance, as his Highway Commission or Department of Public Works) from developing advanced or new ideas, or from attaining effective results along progressive lines. Even Governors have prevented highway authorities from attempting novelties; and legislatures have usually restricted the authority and activities of highway departments. An illustration of the latter may be found in many States where laws limit the width of public rights of way for roads to 100 ft, or similarly inadequate widths, and compel the highway officials to produce results within that insufficient strip.

Safety can be enhanced by better road design, but room for that must be provided for engineers to use. The provision of this "elbow room"[21], or right of way, generally depends more on the electorate, the legislatures, and the authorities than on the engineers. It is the duty of the engineer of course, to force his higher authorities toward—and even hard against—their limits when his technical knowledge and experience convince him that this should be done; but the extension or removal of those limits is in most cases the responsibility of others rather than that of the engineer.

The author's statement as to "broad highway economics" (see "Introduction") is mild. Until recently the preponderance of discussion on the subject has been along regrettably narrow lines. The writer welcomes the author's assertion because he has tried on various occasions to combat the narrower views expressed[21].

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[20] Cons. Engr., Coronado, Calif.

[21] *Roads and Streets*, March, 1926, p. 154, and March, 1936, p. 48; also, "Notes on Highway Location", by W. W. Crosby, Chapter V *et seq.*, Gillette Pub. Co.; and the *Transactions* of the Society.

Under "Fundamental Concepts", the author's brief statement that the highway design should be such that "the motorist is led unconsciously to choose the safe act rather than that which is unsafe" deserves emphasis. The writer has previously expressed himself on this subject[23] and wishes here to applaud the author's statement. Similarly, Mr. Noble's endorsement of the "Dual Highway" has been supported by the writer[24] who, however, offers herein some discussion or emphasis of the details given[25].

If the central "110-ft reservation" (Fig. 1) is intended to be used later for the through ways (and each "pavement-one-way" for "service roadways"), the width (110 ft) may be insufficient. Otherwise, it is probably three to four times as much as is desirable, although the total width of right of way should be preserved.

The "Acceleration Lane" (Fig. 2) may be a questionable provision in many cases. The writer has learned from experience that dangers exist for a driver swerving from a closely parallel lane into a swift traffic current in the same direction. There is inevitably a "blind spot" in the left rear of a driver. It would be much better to bring the traffic from the side in on a track much more nearly normal to the through traffic so that the approach of the latter may be included in the usual front and side vision of the entering driver, even at the sacrifice of speed momentarily by the latter. The tangent of the curve on the turn to the "trunk highway" should go off the first half of that curve as drawn, making the angle with the trunk highway in no case less than 30°, and the "acceleration lane" should be abandoned as such.

Under "Details of Design", the writer agrees with Mr. Noble as to the need for longer vertical curves; but he believes that, for a maximum grade limit, the length of the grade should enter the consideration, together with the actual minimum rate of rise. With the improvements in cars, brakes, and tires, is not an increase of acceptable maxima for rates of grades now justifiable in many cases?

The writer wishes to add whatever emphasis he may be able to contribute to the author's statement following Equation (4), that "the passing menace is one of the most serious problems facing the highway designer."

Although in the past the writer has expressed himself quite fully as to "Signs"[26], he wishes now to applaud the author's remarks as to the need for signs "in advance" (see heading, "Signs"). He also agrees with Mr. Noble as to "obstructions, such as poles", etc., and as to "guard-rails". The writer would point out that, where guard-rails are unavoidable, their main objective should be to assist the driver to regain control of his vehicle on the highway.

In the "lighting" of streets the considerations are different in some respects from those on open roads. In both cases the glare of opposing headlights is a large factor in creating dangers. The dual highway removes this factor,

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[22] *Roads and Streets*, October, 1934, p. 362.

[23] *Loc cit.*, March, 1926, and March, 1936.

[24] *Loc. cit.*, November, 1936.

[25] *Transactions*, Am. Soc. C. E., Vol. 100 (1935) pp. 1052 *et seq.*, and p. 1085.

[26] "Notes on Highway Location", by W. W. Crosby, Gillette Pub. Co., Chapter VI and p. 72.

and greatly simplifies the problem, if, in many cases, it does not eliminate it. "Roadside stand lighting" would probably be unobjectionable if the stands were back where they belong and where they would be on proper widths of highway right of way.

The author's remarks on "Terminals and Interchange Facilities" are extremely important. Their essence seems to be that these provisions (and many others in the design) should be made so that the driver will go automatically where he should, without delay or hesitation that may create danger for himself or others. "What looks right may be wrong, but what looks wrong cannot be right".[27]

The writer heartily endorses the first two paragraphs of Mr. Noble's "Conclusion." A few States might be credited with actually expressing some of the principles mentioned. The State of Washington is reported to have adopted the "dual highway" for all its East and West State routes. New Jersey seems to be establishing the ideal, and possibly there are other States that should be mentioned. Italy set the example which Germany is developing. Great Britain has provided legal authority for it.[28]

In the third paragraph of "Conclusion" the author seems to go a bit too far when he states that the highway engineer "controls the destiny of billions of dollars and tens of thousands of lives." The engineer is fated to be loaded with the responsibility, of course, but actually the final authority or "control" is beyond him many times. Prompt general recognition of this fact should result in an earlier realization of desirable ideals.

The author's reference (see heading, "Conclusion") to the obsolescence of highways in the United States deserves emphasis and perhaps elaboration. The writer, therefore, ventures to append the following at this opportunity. In these days, "three things are certain: Death, Taxes, and Obsolescence." Obsolescence is a greater and more formidable menace on roads than on any comparable line of approach. It seems that the highway authorities to whom the public has the right to look for relief are actually doing the most to bring about the enormous penalty of antiquated, inefficient, and obsolescent public roads.

Generally, it may be stated that public road authorities are spending hundreds of millions of dollars annually "slicking up" a type of highway, which, when "perfected" will not only be inadequate for its purposes, but which also will *per se* hasten its own condemnation as being antiquated, effete, inefficient, out of date, and as requiring a substitute.

With a few notable exceptions, State and National highway authorities seem to cling to the ideal of the single-roadway type, and seem to be making no provision for its transformation into a better one even if they actually are conscious of its imminent obsolescence.

The opportunity for escape from the impasse ahead is so often neglected that one is almost forced to believe that they are not even looking ahead. Merely securing the extra width of right of way for the double-roadway type (while it is undeveloped and, in many cases, of no considerable value), would postpone the obsolescence indefinitely, beside securing concomitant benefits of immeasurable value. Instead, the practice seems to persist (even across waste lands) in taking only a 100-ft right of way, or even less, when 300 ft or 500 ft might just as well have been had. Later widening of the right of way is then almost always impracticable.

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[27] "Notes on Highway Location", by W. W. Crosby, Gillette Pub. Co., pp. 78, 130–132.

[28] *Roads and Streets*, October, 1936, p. 33.

It is probably too early to forecast accurately all the effects on highway design that may come from the future of "trailers." Their use is estimated as 100 000 in 1936 and this number is increasing rapidly. It is certain that they will hasten the arrival of the "Pairway"[28], and, hence, the need for width in which to build and to operate it.

According to a recent article[29], expenditures of $15 000 000 000 since the turn of the century, in the United States, have yielded a highway system unfit for modern motor traffic. The statement may be a bit extreme, but there is no doubt that the enormous expenditures made and annually being continued are rapidly bringing this country to that position when the highway authorities will have to retrace their steps, or seek a new course, at an immense cost of money and lives.

Highway engineers should urge the authorities that rank above them to adopt a new ideal for the public road. They should visualize divided roadways instead of the single broad pavement as something to work toward more or less gradually. They should urge the provision, as promptly as may be, of ample rights of way for that purpose; and, as engineers, they should do their utmost to have the safety built into, and the absolescence built out of, the highway.

RICHARD S. KIRBY,[30] M. AM. SOC. C. E. (by letter).—It is high time that automobile manufacturers and highway builders interchanged views. The former, in urging the public to invest in cars capable of speeds as great as, and even more than, 100 miles per hr are lacking in a sense of social responsibility and are blinking at certain obvious physical and mental limitations of the average human machine. Perhaps the latter are following where they should lead. It is obvious that there is a need for better designed highways, on which driving will be safer and less nerve-wracking; but why urge that public money be spent so that some Zelik Forlansky, who should be piloting a road roller, or some Percey Asteroid Vandervell, III, who should be attending his classes at college, can tear along over the country at rates 50% greater than that of the speediest limited express trains? What, in particular, is gained for society as a whole? The millions would far better be spent on careful selection of drivers, special training for those who obviously need it, elimination of the unfit, and swift, impartial, and certain punishment for flagrant offenders.

All of which is in no sense a condemnation of Mr. Noble's excellent presentation of a pressing technical problem. Suppose, however, that, ten or twenty years hence, engineers have developed highways which in their perfection of design are even beyond Mr. Noble's fondest dreams. If there still persists the present *laissez faire* attitude toward those persons who are allowed to drive cars on these superways the taxpayers' good money will have been spent to little purpose, for the killings and the maiming will be going on in ever-increasing volume. The country does indeed need safer highways for rational use; but persons who think they must travel at 2 miles per min will never be satisfied with any highways that engineers may build, because such individuals require three-dimensional space instead, and the good Lord has provided plenty of such space already.

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[29] *Fortune Magazine*, August, 1936.

[30] Prof., Eng. Drawing, School of Eng., Yale Univ., New Haven, Conn.

HAROLD M. LEWIS,[31] M. AM. SOC. C. E. (by letter).—The development of express highways has taken place within the last ten years. Only within the last few years has it been recognized that the dual highway, recommended by Mr. Noble, which provides separate roadways for each direction of traffic, should be made a feature of all major express routes. The safety of a highway would thus be tremendously increased and operation at higher speeds made possible.

The first comprehensive system of express highways for a metropolitan area was that presented in the Regional Plan of New York and Its Environs in 1929. This included about 253 miles of routes in the New York-New Jersey Metropolitan Area, only 3½ miles of which had then been developed as express routes. This system was supplemented by a separate system of parkways, most of which would also provide express movement, but all of which would be limited to passenger vehicles. The express highway system was designed for both passenger and commercial vehicles.

Express highways were needed to relieve the interference from cross-traffic on intersecting routes and to provide short cuts between population centers and by-passes around intensively built-up districts. In a few cases they have been developed as viaducts, or on embankments, where there could be no interference from the uses of abutting property, but where they have been built along existing grades, the permanency of free movement has been overlooked. An example is U. S. Route No. 1, between Newark and Trenton, N. J., which was reconstructed over an entirely new right of way and hailed as the first comprehensive express highway. For the first few months after it was opened to use, it was adequate for through traffic which traveled over it, but it soon became lined with gasoline stations, refreshment stands, and other commercial concessions so that parts of it are now little better than the old Lincoln Highway which it supplanted. It is recognized that a new and better route will be required to relieve it.

Mr. Noble has described a type of route which will be a great advance in highway design. He mentions that "a safety factor is to prevent frontage of any type on the highway and to exclude farm and local road entry. It appears reasonable to restrict access to approximately 10-mile intervals." The writer would like to emphasize the importance of this safety factor and believes that it is an essential feature if permanent efficiency is to be maintained.

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[31] Cons. Engr.; Engr. and Planning Consultant, Regional Plan Assoc., Inc., New York, N. Y.

A highway built with State or Federal funds without local assessment can logically be designed primarily for non-local uses. Why not make it entirely for through traffic and exclude the right of any abutting property to local access, except at those points designated by the agency building the highway and incorporated into its design?

The "freeway" or "limited way" is the term that has been used to designate such a highway, on which general traffic can have some of the advantages which passenger cars now enjoy on the modern parkway. The word, "freeway", was coined several years ago by Edward M. Bassett, Attorney, of New York City. Other countries have made notable advances in constructing modern highways of this type, notably the toll roads built by private corporations in Italy under Government authorization and with Government assistance, and the "Reichautobahnen" under construction in Germany as part of a national highway system.

Thus far, freeway construction has made little progress in the United States due to the necessity of new legislation authorizing public highway agencies to lay out and build highways to which abutting owners will not have the right of access.

Definition.—Probably the briefest definition of a freeway is as follows[33]: "A highway to which there is no vehicular access from abutting properties." In the report from which this definition is quoted a freeway was defined as one type of express highway.

A more complete definition of a freeway is obtained by summing up the various qualities which such a route should possess, as follows. (1) For fast-moving vehicles, long haul as against local traffic; (2) for mixed traffic, including passenger cars, buses, and motor trucks, but excluding horse-drawn vehicles; (3) free from crossings at grade, and left-hand turns, unless in isolated cases in rural areas where the construction necessary to eliminate them would not be justified; (4) providing no right of access to abutting property, except at points provided by the agency designing and constructing the freeway; (5) abutting service facilities, such as gasoline and parking areas, to be provided only by, or under the control of, the public agency building the highway at sites incorporated in the design of the freeway; (6) infrequent connections to other highways; and (7) low grades and easy curvature.

The freeway system should form a part of a more comprehensive system of express routes containing two other types of express facilities. One of these facilities would be a system of parkways, restricted to passenger vehicles and providing attractive, landscaped routes with grade separations at important intersections, expanded at places to provide recreational parks. The other would be express routes composed of highways also serving abutting property, but where freedom of movement is facilitated by occasional grade separations, side roadways for local traffic, or other special treatment.

Design of Freeway.—The general design of a freeway is well indicated by the foregoing Items (1) to (7) which list the qualities it should possess. Although the need of freeways is more acute in the closely developed parts of metropolitan areas, it is more feasible to establish them in suburban and rural areas. Accordingly, the widths of right of way to be provided will vary with the character of the territory. In congested areas of high land values, compromises with existing conditions will have to be made.

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[33] Rept. of the Committee of the City Planning Division, Am. Soc. C. E., on Street Thoroughfares Manual, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1049.

In general, the design should call for three lanes of traffic in each direction, and dual roadways separated by a central unpaved area. Two lanes in each direction will often be sufficient for initial construction and, in some cases, for final development. Border strips should be provided along each side.

The Regional Plan Association of New York City recommends that outside lanes and lanes next to the central unpaved strips be 13 ft in width. Lanes not bordering on unpaved shoulders or central strips could be 11 ft wide. A minimum width of 26 ft, including curbing, is recommended for the central strip. Within this width a screen planting of shrubs and trees will eliminate the glare of head-lights at night from drivers going in an opposite direction. A minimum width of 160 ft would thus provide an adequate right of way for a freeway with an ultimate capacity of six lanes, three in each direction.

In rural areas a width of 400 to 500 ft is advisable, but this might very well vary in accordance with the topography and the ownership of the land through which the freeway passes. In this case a minimum border strip of 50 ft is desirable which might well be supplemented by the prohibition of billboards and signs within 200 ft or more of the right of way. Through rural areas where land is under cultivation a narrower right of way, supplemented by an easement over adjacent land, will reduce initial and maintenance costs and give the motorist a more interesting and informative view of the

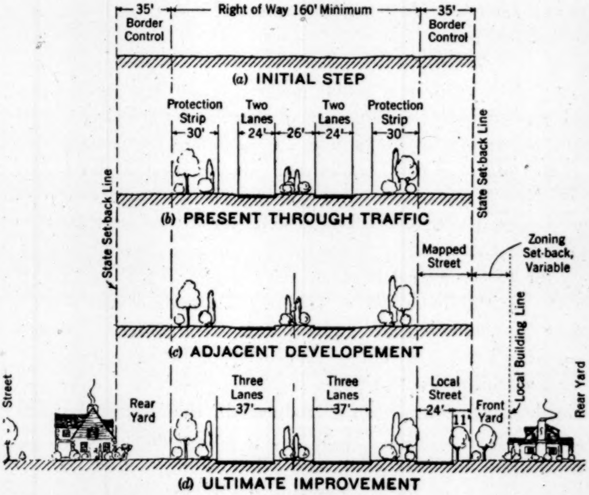
35' Border Control
Right of Way 160' Minimum
35' Border Control
(a) INITIAL STEP
Protection Strip 30'
Two Lanes 24'
26'
Two Lanes 24'
Protection Strip 30'
State Set-back Line
State Set-back Line
(b) PRESENT THROUGH TRAFFIC
Mapped Street
Zoning Set-back, Variable
Local Building Line
(c) ADJACENT DEVELOPEMENT
Street
Rear Yard
Three Lanes 37'
Three Lanes 37'
Local Street 24'
11'
Front Yard
Rear Yard
(d) ULTIMATE IMPROVEMENT

FIG. 8.—ADAPTATION OF ROAD CAPACITY TO TRAFFIC VOLUME AND LAND USE

country side. As much of the motor travel in the United States is of a recreational nature, many of the parkway features, such as attractively landscaped overlooks at scenic points, picnic areas, and bridle paths, might be incorporated advantageously in freeway design in rural areas.

A series of cross-sections, prepared by the Regional Plan Association and indicating the progressive development of a freeway in suburban areas contiguous to large population centers, is shown in Fig. 8. This calls for a minimum right of way of 160 ft and a 35-ft set-back line to be established by the State (see Fig. 8(*a*)), assuming that the freeway would be constructed by the State Highway Department. Early in the development (Fig. 8 (*b*)), an economical pavement is installed and trees are planted in their final location. As transition from an open area to a more intense use occurs (Fig. 8(*c*)), local circulation can be provided in "control borders." Local building lines should be established at this time. Ultimately, when future traffic justifies it, the roadway can be expanded to three lanes in each direction without disturbing the planting. A more effective separation is possible between traffic and buildings when abutting property is developed as shown on the right side of Fig. 8(*d*).

In urban areas marginal border strips might be cut down to 4 ft and the central strip to 8 ft. With these extreme measures, which should be applied only where local conditions make them necessary, a six-lane freeway could be constructed on a 90-ft right of way, or a four-lane freeway on a 64-ft width. In certain sections the elevated highway or vehicular tunnel must be resorted to for short distances.

Proposed Legislation.—Although there is general agreement as to the need of freeways and legislation to enable governmental departments to build them, there has been considerable difference of opinion regarding the details of such legislation. The only freeways now in existence in the United States are some short sections of general traffic roads in the Westchester County Parkway System, in New York State, and the Norris Freeway constructed by the Tennessee Valley Authority. The Westchester routes were built by the expedient of placing them within parts of the County Park System, turning the roadways over to the jurisdiction of the Division of Highways of the State Department of Public Works. These roadways were constructed with State funds supplemented by Federal aid. The Norris Freeway was constructed by the Tennessee Valley Authority, a Federal agency, without any special State enabling legislation.

Freeway legislation has been introduced in the States of California, Connecticut, and Massachusetts. It has been under consideration by the State Planning Boards in Maryland and New Jersey, and the Division of State Planning in New York. It seems likely that some of these States will soon provide themselves with such legislation.

Application to New York Region.—Fig. 9 shows a study prepared by the Regional Plan Association for the application of the freeway principle to a complete system of expressways for the New York-New Jersey-Connecticut Metropolitan Area. It shows a much more extensive system of general traffic express routes than was contemplated in the express highway system proposed in the Graphic Regional Plan in 1929, already cited.

In 1933, a report[33] on the preceding four years of progress in the regional development of New York and its environs showed progress on 179 miles of express highways, or 72% of the new mileage proposed in the original plan, which had looked forward to the year 1965. This tremendous growth in the adoption of the express-highway principle justifies an expansion of the original proposals. Fig. 9 might be described as an ultimate plan of expressways for

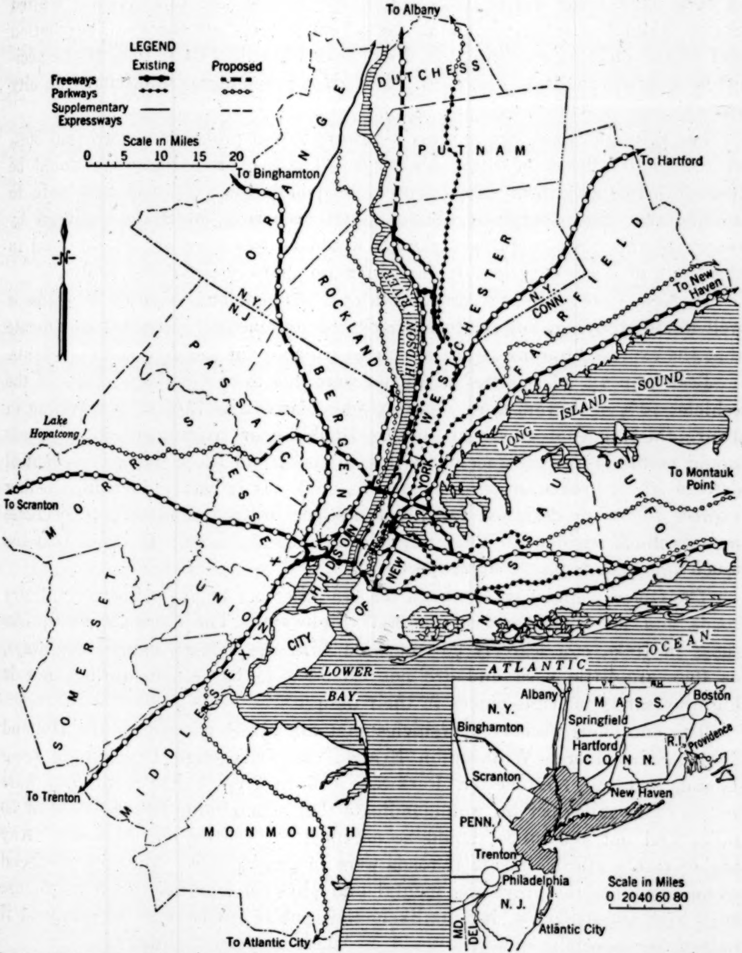
LEGEND
Existing
Proposed
Freeways
Parkways
Supplementary Expressways
Scale in Miles
0 5 10 15 20
To Albany
To Binghamton
To Hartford
To New Haven
To Montauk Point
To Scranton
To Trenton
To Atlantic City
Lake Hopatcong
DUTCHESS
PUTNAM
ORANGE
ROCKLAND
WESTCHESTER
FAIRFIELD
N.Y.
N.J.
N.Y.
CONN.
HUDSON RIVER
PASSAIC
MORRIS
ESSEX
BERGEN
HUDSON
NEW YORK
CITY OF NEW YORK
UNION
SOMERSET
MIDDLESEX
MONMOUTH
NASSAU
SUFFOLK
LONG ISLAND SOUND
LOWER BAY
ATLANTIC OCEAN
Albany
N. Y.
MASS.
Boston
Springfield
Binghamton
Hartford
R.I.
Providence
CONN.
Scranton
New Haven
PENN.
Trenton
Philadelphia
N. J.
Scale in Miles
0 20 40 60 80
MD.
DEL.
Atlantic City

FIG. 9.—STUDY OF RADIAL FREEWAYS FOR NEW YORK AND ITS ENVIRONS, SHOWING THEIR PLACE IN AN ULTIMATE EXPRESSWAY SYSTEM

the Region looking even further ahead than the published Regional Plan. Its advantage would be that any routes constructed in conformity with it would be permanently useful for the expeditious movement of vehicles along attractive rights of way and the past sad experience of the clogging of State highways with local traffic to and from roadside development will be eliminated along new routes.

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[33] "From Plan to Reality", Regional Plan Assoc., 1933.

GEORGE CONRAD DIEHL,[34] M. AM. SOC. C. E. (by letter).—Traffic accidents in New York State during 1935 were about 10% of the total for the United States, which is also approximately New York State's percentage of population and of automobile ownership. In New York State less than 10% of the accidents occurred on State highways. The great preponderance occurred on city streets.

The mileage of modern express highways would probably never equal 30% of the total mileage of State highways. These express highways would be located principally near large centers of population. It would be safe to assume that the number of accidents not occurring on State highways by reason of the use of express highways would be less than 3% of the total in the State of New York and far less in more sparsely populated States.

In New York State, the prevention of 1 800 accidents with 60 fatalities is well worth studying, but the foregoing brief statement is intended to indicate that the express highway is not a general panacea for motor-vehicle accidents.

Less than 4% of reported accidents were due to mechanical defects of the vehicle; 10% of drivers were violating speed laws; and 75% were traveling in a straight line (not on curves, etc.). Doubling or tripling allowable speeds would certainly increase accidents due to speed and those due to mechanical defects (tires, brakes, steering apparatus, etc.). It is rather doubtful whether express highways designed for 100 miles per hr. would materially decrease motor-vehicle accidents. They might even increase them. It seems that too much emphasis is placed upon speed.

The German highways designed for 115 miles per hr are primarily military roads and not "peace or commercial" highways. The German roads, like those in Italy, can scarcely be compared with the modern express highways, as they are little used and are not intended to be self-sustaining; nor is highway safety a prime object in their design.

The enormous number of vehicles that are safely carried by the Holland Tunnel, the George Washington Bridge, the Triborough Bridge, the Long Island, and the Westchester, Parkways (all in New York) with maximum speeds of 40 miles per hr, would indicate that a maximum hourly speed of 50 miles and not 100 miles should be assumed for the express highway. Any higher rate would be unsafe if roads were in general use. Although a special examination might prove the mental and physical stamina necessary to use such high speeds safely, it is unlikely that public funds could be obtained if only those especially qualified could use these modern highways.

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[34] Cons. Engr., New York, N. Y.

Assuming that the vehicles could be made mechanically safe, the individual operator is certainly limited as to speed, no matter how well the engineer designed the vehicle and highway. Some limit must be fixed in the interest of public safety, and the rate should be less in the case of long, high-speed buses, of trucks, and of cars with trailers, and of vehicles of unusual length.

Often the fact is overlooked that increasing speed beyond certain limits decreases the capacity of the highways. This decrease is due to the extra spacing required for safe operation. Vehicles traveling at 50 miles per hr, according to Fig. 4, can be operated safely at intervals of 150 ft, but at a speed of 100 miles per hr, the spacing should be 600 ft, four times the spacing at twice the speed, reducing the capacity one-half. Only one-half the number of vehicles can be carried safely. The only remaining advantage is speed. Much of this advantage is lost when speed is reduced to minutes saved. This saving will be comparatively small, as sections of such modern express highways, generally considered, cannot be justified economically for extensive mileages.

The various forms of transportation—air, rail, and highway—should be co-ordinated so that all three can be operated successfully. If greater average speeds than 50 miles per hr are desired, rail service could safely be developed from 50 to 100 miles per hr and air service much in excess of 100 miles per hr.

Expensive public improvements must be justified technically, economically, and socially. The express highway is a necessary transportation utility, but it involves unusually large expenditure. Fortunately, the mileage of such highways is very small when compared with the total.

Toll highways have not been popular in the past. In very few cases are there sound arguments in favor of the toll method of financing. If the toll method is not adopted, then the public, as well as appropriating bodies, must be convinced that the cost of express highways is justified and that there will remain in the available highway funds sufficient sums to care for all lesser classes of highways. Doubtless, the public will also insist that the work on all classes be carried on proportionately and contemporaneously.

Modern express highways must be justified by the traffic and probably a safe method of financing would require the users of such highways to pay from present registration or gasoline taxes a considerable part of the total cost. The organized motorist objects to any increase in motor-vehicle taxation, through tolls, or otherwise.

Every express highway which can be financed soundly should be built, but by the same reasoning it would be gross extravagance to construct such highways where less expensive roads would suffice.

If proposed speeds are slowed down the beautiful parkways with picturesque landscaping near the largest cities can often serve as modern express passenger highways, but their mileage should be limited, and recourse should be had to those rules of economy which properly apply to parks and parkways, as well as to business highways.

It would seem that definite general rules could be adopted for the design and construction of express highways for a period of, say, thirty years. This would permit a sufficient lapse of time to care for amortization of bonds or borrowings. Technical, economical, and social justification could be computed on the basis of thirty years, or on such other period of time as might be more equitable and desirable Proponents of each form of transportation could then develop and solve their own problems without unfair overlapping or competition.

Support for the large expenditure for express highways from legislators and those who plan the budget, whose home localities are not greatly affected by these great arteries, will be dependent upon a general plan which will provide for the equitably apportioned construction of all classes of highways within the sums available.

For at least twenty years the writer has urged the importance of "highway classification" and of "traffic distribution." All the many hundreds of thousands of miles of public highways and streets never can, or should, be designed for equal capacities, loadings, or speeds.

The individuals composing budgeting or appropriating bodies are prone to believe that the highways in their home localities are deserving of the highest and costliest types.

Formerly, the writer had suggested that the highways outside of cities be divided into four classes. Names which would be more or less descriptive to legislators were used: (1) High-cost roads; (2) low-cost roads; (3) little used roads; and (4) remote roads. It was endeavored to work out an economic justification by assuming that the actual payments of registration fees and gasoline taxes should equal one-half the road costs, including construction, maintenance, and amortization.

On this theory: (*a*) High-cost roads were assumed to carry as many as 600 000 vehicles annually; (*b*) low-cost roads were assumed to carry from 200 000 to 600 000 vehicles annually; (*c*) little used roads were assumed to carry 30 000 to 200 000 vehicles annually; and (*d*) remote roads were assumed to carry less than 30 000 vehicles annually.

The weight and length of vehicles to be permitted on high-cost roads were to be the full legal limit. On low-cost roads the maximum allowable capacity was 3½-ton trucks and less in the case of school buses. Long trailer truck trains or long, high-speed buses were to be excluded (Incidentally, this restriction only excludes 1% of the total number of vehicles permitted on high-cost roads.) Little used roads are limited the same as low-cost roads, except that trucks must not exceed 2½ tons. Remote roads are limited to 2-ton trucks and no trailers or buses (except smaller vehicles such as school buses) are permitted on them.

The maximum speeds varied from 35 to 60 miles per hr, with special restrictions on sharp curves of the lower types. Stage construction was planned so that one type could be changed to the next higher type with a minimum of unnecessary expenditure. A more complete description of these types cannot be made within the limits of a discussion of this paper.

The first class, namely, "high-cost roads", should be again subdivided, or at least a new class should be added; that is, the modern express highway. This latter highway could not be justified probably at less than 2 000 000 vehicles annually, and if clover-leaf intersections were adopted generally, a larger number of vehicles would be required. Very few miles of highways carry 10 000 vehicles daily and then only near large centers of population. Traffic congestion is local.

The mileage of high-cost roads is approximately only 10% of the total. The lower, less expensive types comprised 90% of the total. Formerly, traffic censuses were inadequate in number and scope to calculate the allowable mileage and justifiable costs of the several classes of highways. During the past few years, through Federal participation, there has been a great addition to traffic data and the more populous States have sufficient data to develop a fairly accurate general plan for road classifications and economic planning.

The writer's experience in the enforcement of road rules and restrictions convinces him that the traffic of the various classes could be practically, and readily, controlled. Closely allied to the subject of road classifications is the matter of traffic distribution on city streets as well as on suburban and rural roads On the most crowded Saturdays, Sundays, and holidays, there are, in the vicinity of many large cities, mile after mile of comparatively empty roads used by few vehicles, but open to many. Pleasure seekers use the busy roads because they are not directed to idle roads. Even when one's destination is fixed, little used parallel roads are available for the passenger car and lighter trucks. The average tourist could readily be routed to arrive at or to leave on, busy city streets at other than rush hours with peak loads.

Many city streets permit of easy and undisturbed travel when greatly accepted routes are slow and difficult. For instance, it is possible to drive from mid-town New York City to the Westchester County line by several routes at the same hour of day, one route taking 35 min, another, 1 hr and 15 min, and others at intermediate periods. On the busiest road not only is ½ hr wasted, but the traffic is still further cluttered. A continuing study and control can govern traffic by properly routing drivers who go daily between fixed destinations.

Traffic distribution is an important item in planning city terminals and within certain of the most congested areas, involves the question of compulsory mass transportation as contrasted with the use of these areas by the individual passenger car.

Obviously, the average taxpayer cannot analyze the traffic problem or study its intricate details, and yet he will re-act unfavorably to the enormous cost per mile of the express highway unless provision is made for road classification and traffic distribution. In these days of socially planned legislation one cannot disregard the lonesome vehicles on "forgotten highways" leading into hundreds of smaller cities and serving many thousands of villages and townships. Space does not permit a further discussion of traffic distribution, but much more should be said. It must be apparent that much travel could be taken from the express highway and the busiest high-cost roads.

The statement often made that most highways designed twelve or fifteen years ago are obsolete is true only of some of the crowded main highways near large centers. Literally, thousands of miles of the cheaper types are well designed for the purpose intended.

Such statements give an erroneous impression to laymen and to members of budgeting and appropriating bodies. As a matter of fact, it might better be said that the fine work of highway engineers during the past ten or twelve years has resulted in highways fully adequate for the next fifteen or twenty years. These excellent results should be emphasized so that highway engineers will be highly regarded by the general public, and their recommendations estimated at their true worth.

William E. Rudolph,[35] M. Am. Soc. C. E. (by letter).—Although there is much of interest and pleasure to be gained from reading this paper, the writer considers that its basis is altogether erroneous. Mr. Noble introduces his subject with a compilation of accidents in the United States during 1935 (Table 1). Having been away from his native land for nearly three years, the writer was not a little surprised to learn that accidents accounted for injury or death to nearly a million persons annually in the United States. Then, upon turning the page, he was still further surprised to find that the author was proposing high-speed highways to accommodate faster cars, in order to "safeguard the traveling public."

Is Mr. Noble "looking in the right direction"? Year after year the manufacturer has been producing cars, each in its turn safer than the previous model, without a doubt, and also faster. Year after year the engineer has been building safer and faster highways, until Mr. Noble now suggests designing for 100 miles per hr.; and yet, with safer cars and safer highways, engineers are confronted with Mr. Noble's tabulation of a single year's terrible toll of accidents. Are they working in a vicious circle?

Safer cars, equipped with safer tires which will blow out gently if they must blow out, will probably present no difficult problem to the super-enlightened automobile manufacturer of the present day. Safer highways can be designed and constructed by the engineer, as outlined in Mr. Noble's excellent presentation of his subject; but the safer driver, the person who is not subject to ordinary human weaknesses, such as fatigue, mental lapse, or a strong cocktail—what is being done about him? Would it not seem in order to correct such factors concurrently with the physical factors, even if it became necessary to establish special schools for instructing the driver regarding the laws of motion, and the proper management of the immense amount of power at his command when he starts the motor of his car? Later, perhaps, with drivers who would depend upon good judgment rather than upon mere good luck for avoiding accidents, there might be a real need for higher-speed highways and the inevitable higher-speed cars; but they should not be projected upon the basis of a tabulation showing an annual total of nearly a million accidents.

It might be that engineers in the United States are taking their cue in this matter from high-speed highways recently constructed in Europe. This is ill-advised—for Europe requires such highways because of the inevitable war she is facing, whereas conditions in the United States are quite different (or at least it is to be hoped they are).

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[35] Chf. Engr., Mauricio Hochschild & Co., Ltd., Potosi, Bolivia.

In Bolivia, Nature has imposed great obstacles for the road builder: Highways must wind in spectacular fashion in and out of rugged canyons, over steep mountain passes; at places there is scarcely width to pass another car. Many roads run through river bottoms, where there is no passage during the rainy reason; others become extremely difficult to travel when the clay surface becomes wet, and hard roads are almost unknown. Despite vertical cliffs and other hazards, however, the accident factor in Bolivia is indeed low. Moreover its citizens work and produce and are happy, although they do not live so fast. Road conditions are constantly being improved, but, fortunately, they have not reached the point, thus far, which permits high speeds that might occasion risks quite out of proportion to the useful ends to be gained by such speeds.

A. C. DENNIS,[36] M. AM. SOC. C. E. (by letter).—Studied in conjunction with a paper by Fred Lavis, M. Am. Soc. C. E., published in 1930[37], the paper by Mr. Noble constitutes an important advance toward the establishment of correct standards for trunk highways. The technical questions of design standards and economic location can safely be left to the decision of engineers, and the broader question of organization and financing may also be properly the concern of engineers even if the ultimate decision is the responsibility of others. If there is the possibility that certain trunk highways may be constructed, with no addition of taxes to the general public, to provide the users of these highways a safer and faster route free of interruptions at no additional cost to the user, then an engineer's self-interest as well as his duty as a good citizen should lead him to study and discuss this possibility and, if found to be sound, to work for its fulfillment.

The existing highways are, in effect, State toll roads, the toll being paid at the gas station as a State sales tax. These highways have generally been built with local use and convenience as the main objective. Practically all of them are of inadequate capacity for trunk-line traffic and are subject to various traffic interruptions. The result is a condition similar to that of a railway system consisting of many branch lines, but no main line.

Trunk highways are interstate structures beyond the province of the several States, but within the powers of the Federal Government to finance, construct, own, and operate. It appears that a major section of Federal trunk highway of normal cost, carrying about 7 500 vehicles daily on eight traffic lanes, between three or more emergency or parking lanes, may be self-liquidating in that a Federal gas tax collected along the highway at a rate not exceeding that of the local tax, together with revenue from concessions for furnishing service to vehicles and persons, will provide cost of administration, policing, maintenance, and other operating items, as well as interest on the investment with perhaps something toward retirement of investment.

The organization suggested previous to operation, is that a Chief Engineer shall report, possibly, to the Secretary of The Interior, with authority (as far as is practicable for a public servant) similar to that of a Chief Engineer of a railway during location and construction. He should select his staff and be allowed to pay salaries sufficient to secure proper assistants. The Highway Engineer of each State traversed should be a member of his consulting staff.

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[36] Gen. Supt., Dravo Corporation, Kerhonkson, N. Y.

[37] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1020.

The first duty of the Chief Engineer should be to select the general route for the highway, establish standards, and make a preliminary estimate of cost and an estimate of immediate and future revenue to determine whether the proposed section is self-liquidating, or whether it will become so in the future. When a proposed section appears to be self-liquidating and when authority has been given to proceed, the Chief Engineer should make location surveys, design structures, acquire rights of way and other necessary lands, prepare specifications, let unit price contracts in sections of several hundred miles each, and supervise the construction.

When a major section of the highway is ready for operation, it should be administered by a Manager reporting, perhaps, to the Secretary of The Interior. He should be responsible for the maintenance of the highway, the leasing and regulation of concessions, and the enforcement of operating regulations for vehicles and drivers, with the support of Federal Police and Magistrates.

The near future will probably be an unusually expensive period for highway construction, but some dense traffic sections may be economically sound even at excessive cost, whereas the plans for others, not at present self-liquidating, may contain useful knowledge to have available when the next depression again reduces construction costs.

J. C. CARPENTER,[38] M. AM. SOC. C. E. (by letter).—This timely paper covers a subject that deserves most serious consideration by the Engineering Profession. Perhaps the title, "The Modern Express Highway", was selected as a lure to induce some engineers to give thought to the subject of highway safety, who might not be interested in a paper on "The Necessity for Consideration of Safety in Highway Design."

The immediate answer to the problem of highway safety is not the "express highway." Engineers must consider safety in all their designs and cannot wait until funds are available and traffic warrants the construction of divided-lane highways on the large mileage that is now (1937) and will soon be under construction. Mr. Noble's excellent paper covers many features of design that are used in the development of all highway plans and his suggestions are generally applicable to single-lane, as well as to dual-lane, highways. The dual-lane highway is undoubtedly superior to the two-way single-lane, and should be used in every instance where the traffic volume and funds available will allow its use. Unless it is plainly obvious that this design is demanded, it will take more than vision or courage to persuade those in authority that the required extraordinary expenditure is justified. Perhaps this paper will provide the data needed by some engineers to convince the authorities that more dual-lane highways are necessary.

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[38] Senior Highway Engr., U. S. Bureau of Public Roads, Fort Worth, Tex.

As soon as the planning movement, now rapidly gathering headway, has advanced to a point where the results are available, the designing engineers will have reliable and convincing information that will allow them to select, and promote with confidence, the designs that are properly applicable to the conditions dictated by traffic, economic development, and a program based on a correct knowledge of future available funds. The planning program is now in its preliminary stages and the most difficult phases are just ahead. The collection of data is a comparatively simple and easy task, and presents no difficulties not previously surmounted by the engineer, but the co-ordination of these data, analysis, and utilization, to prepare a satisfactory plan that will be so perfect that its general features will be permanent, is an unexplored field and will demand the services of men of broad experience, who have the vision and the courage to look ahead and support their ideas with clearly established reasons. Highway engineers have "followed the old road" and built "within the money" for so long that it will be difficult to break new trails, but those who are privileged to work on this phase of the planning procedure have a large responsibility; and the skill, courage, and good judgment they show, will determine the value of the results to the public and all future generations. One important point will undoubtedly be developed, namely, the funds now collected from highway users are not sufficient to carry on the necessary construction and maintenance operations, including building and maintaining safety in the highways, and funds collected for highway uses should not be diverted for any other purpose.

The safety problem involves many features outside of engineering design and practice. The National Safety Council's three "E's" necessary for safety, are "enlightened engineering", "sagacious education", and "judicious law enforcement." The writer's estimate of the relative endeavor necessary on these three general phases of the safety problem is, engineering, 5%; education, 10%; and enforcement, 85 per cent. Reasons for this estimated rating will be given in the following discussion. This low rating for engineering does not mean to imply that engineers can sit complacently and let the public do the educating and enforcement, but rather that the engineer, with his specialized knowledge and training, while concentrating on the improvement in the engineering phases of the problem, should assume leadership in developing ways and means of providing effective programs for education and enforcement. Education can best be developed through the regular school curricula and through courses in safety required as a penalty for law violation. A regularly scheduled and complete course in all phases of safety, with emphasis on highway safety and practical demonstrations and teaching, should be included as an eighth-grade subject in all the schools of the country. Drivers' licenses should not be issued to children until they have completed this course and passed a State examination and the enforcement should fit in with this policy. Children of eighth-grade age are just beginning to drive, and development of this policy and legal recognition through the school system and enforcement authorities, will do more to bring about a citizenship thoroughly "safety minded" than any other means of education. Of the adults, the law breakers are the offenders who need to be educated and the majority of them are of the type who will not listen to good advice given as newspaper publicity or through other commonly accepted channels. If they were required to take a carefully planned course in safety before being allowed to drive and to pass a strict examination before having their license returned to them there is no doubt that a large number of violators would be converted into careful drivers.

Enforcement, of course, is a police function, but engineers in private practice and public position can, at every opportunity, stimulate popular support of intelligent enforcement of existing laws and encourage the enactment of necessary additional legislation conforming to the principles outlined in the Uniform Acts of the National Conference on Street and Highway Safety, and can insist on the appropriation of sufficient funds to insure thorough and complete enforcement of all traffic laws.

Highway safety is as much an engineering problem as industrial safety, and engineers can devise methods of control that will be as effective as those so successfully used by the railways and other industries. Careful study of information provided by such organizations as the National Safety Council, the American Automobile Association, and other similar agencies will equip the engineer to lead in community and State safety movements.

The statistics quoted in Mr. Noble's paper emphasize the loss of life and injuries due to traffic accidents. Although it may be that more accidents should be charged to highway design, it is believed that the percentage of the total will not be changed very much. When the figures given are analyzed, it is possible to conclude that the percentage chargeable to design is not as large as might be thought. National Safety Council's "Accident Facts" shows that of the total of 36 100 deaths in 1935, 11 800 were in cities of more than 10 000 population, and 24 300 were on rural highways, which are those under consideration. Of these totals, 35% of the casualties were pedestrians. Although improvement in the design may, accidentally, reduce the pedestrian deaths, it is logical to assume that the majority of them were caused by lack of care on the part of the pedestrian and excessive speed on the part of the driver. A closed right of way on the express highway recommended by Mr. Noble, might keep some of the pedestrians where they belong, but there are other complications, legal and otherwise, which would prevent the strict exclusion of the free and independent public from the public highways of the United States. After deducting 35% for pedestrians, there remains a total of 15 795 deaths which may be more or less remotely charged to defects in design.

The data in Table 6 have been obtained on three highways adjacent to Fort Worth, Tex., covering accidents during the fiscal year ending August 31. 1936. U. S. Highway No. 80 across Palo Pinto County was built in 1924, on what were considered modern standards at that time, but it has many sharp curves, steep grades, and a rough surface 16 to 18 ft wide, with more crown than is necessary. U. S. Highway No. 80, across Parker County, was completed in 1922 as a surface-treated gravel road about 16 ft wide, and approximately one-half of it was reconstructed to a higher standard and wider surface in 1927. State Highway No. 34, from Azle to Jacksboro, Tex., was built in 1934 on excellent alignment, the location having been selected from an aerial survey map, with low degree curves and much better sight distances than on the other two roads; it is surfaced with a bituminous top, 22 ft wide, built to a good section, and generally is in excellent condition. The three sections are about the same length, traverse the same type of terrain, and, except for their age, the surfaces are similar in character. Traffic is heavier in volume on the Parker County Section than on the other two roads, but the difference is not enough to affect the results materially.

TABLE 6.—ANALYSIS OF ACCIDENTS ON EXPRESS HIGHWAYS IN TEXAS

(a) GENERAL				(b) CAUSES OF ACCIDENTS				(c) TYPES OF ACCIDENTS			
Description	U. S. HIGHWAY No. 80		STATE HIGHWAY No. 34	Description	U. S. HIGHWAY No. 80		STATE HIGHWAY No. 34	Description	U. S. HIGHWAY No. 80		STATE HIGHWAY No. 34
	Palo Pinto County	Parker County	Azle to Jacksboro, Tex.		Palo Pinto County	Parker County	Azle to Jacksboro, Tex.		Palo Pinto County	Parker County	Azle to Jacksboro, Tex.
When constructed	1924	1922 to 1927	1934	Drunken drivers	4	...	2	Head-on collision	4	4	8
				Driver asleep	...	6	7	Overtaking car	3	15	15
				Blinding lights	...	...	4	Collision with:			
Total accidents	15	45	64	Speed	10	15	16	Standing car	6	6	2
Total killed	2	10	10	Car defects	...	3	12	Fixed object	6	6	7
Total injured	26	48	49	Miscellaneous	1	21	23	Miscellaneous	2	20	34

Table 6 indicates that there are more accidents on the roads of more modern design than on the older types. It is not logical to conclude that one should return to the older design, but one should analyze more carefully all accidents and determine exactly what is the cause; and if design is responsible the engineer must determine what changes are necessary to correct it. It would also seem that the modern design is likely to induce sleep and that about 25% of the accidents are caused by overtaking other automobiles. A change to the dual type will not prevent overtaking accidents, nor will it stop drivers from going to sleep. On all the roads, speed is responsible for a large percentage of the accidents; it is responsible for two-thirds of the total on the oldest road.

It is obvious that a greater volume of accident reports is needed and more complete detailed information on the reports that are made. It is difficult to get the entire "picture", for it is impossible to follow every car and to be on hand to obtain accurate and complete details of each accident. Any information furnished by a driver involved in an accident is usually prejudiced. The fact remains, however, that engineers need considerably more data than are now available, if they are to design intelligently. Congress, in 1936, appropriated $75 000 for use in investigating conditions and formulating legislation on highway accidents. The Bureau of Public Roads, in co-operation with the Highway Research Council, has appointed an advisory committee to study the causes of highway accidents and conditions contributing to them. These studies will be made in the States that have kept records of accidents and will consist of an analysis of the accident records of a large number of drivers. Publication of the results of this study and wide distribution among engineers will undoubtedly stimulate the more universal collection of accident data; bring about uniformity in the procedure; make available much valuable information for use in design; and result in the enactment of corrective legislation.

This and other recent papers on this subject[39] set a speed of 100 to 120 miles per hr for the design of highways. Such a high rate is out of all proportion to the capabilities of the present-day driver. Perhaps if there were a uniform drivers' License Law, administered in such a way as to classify all drivers in accord with their skill and dependability, one might allow a select few to pilot automobiles at this rate of speed. Highways designed for this rate of speed will be perfectly suited for the lower speeds that are required, and will provide a factor of safety against obsolescence; but encouraging high speeds will increase accidents, as is emphasized by National Safety Council's "Accident Facts", which shows that at speeds of 20 miles per hr., one accident in 61 is fatal, whereas if the speed is 50 miles per hr., or more, there is 1 death to each 11 accidents. It is believed that practically all the travel over the rural highways of the country can be executed without exceeding 50 miles per hr. and without serious loss of time and money due to this conservative rate. The deaths due to accidents, property damage, and other supplementary losses can undoubtedly be reduced materially if this maximum speed is adopted and the achievement of this control is as much an engineering function as the reduction in degree of curvature, or the determination of the correct rate for superelevation. This same source indicates that in the fatal accidents nine out of ten cars were going straight ahead when the accident happened, thus relieving curvature of the responsibility for a large proportion of the fatal accidents, and indicating that speed was probably at fault.

The dual highway is undoubtedly the correct design where traffic and planning data indicate the location to be of such a permanent nature that the extraordinary expenditure is justified. In a large percentage of the area to be served by an extension of the present highway service the construction of additional routes will better serve the territory than the development of one route to carry a large volume. To illustrate this point, the Fort Worth-Dallas area covers about ten miles in a north and south direction, the centers of the two cities being about thirty-two miles apart. The construction of four 2-lane highways will relieve traffic congestion on the one main-line route now heavily crowded, and these four routes will not cost much more than the dual type on one route. One or two of these routes can be planned for future development as a dual highway, in the manner suggested by Mr. Noble.

The formula for superelevation of surfaces on curves should include a factor for side friction. A superelevation of 1½ in. is about the maximum that can be used. All curves should be superelevated and it would seem to be a simple process to compute the rate between zero for a tangent, and 1¼ in. for a 6° curve, using the formulas, including the friction coefficient. There should be very little justification for curves sharper than 6° on any highway that will develop later as a location for a dual highway. If sharper curvature is demanded by the topography it would seem that another route should be investigated. In all highway location the smallest reasonable degree of curvature should be used. It is quite common practice to use a higher degree than necessary, even in new country, due to the fact that the use of a low degree of curvature has not been considered necessary. Many 4° to 6° curves have been used where a 1° curve would fit better, and some 1° curves have been placed where a 10′ or a 30′ curve is easily possible.

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[39] See, for example, "Highway Design Applied to the State System", by R. H. Baldock, *Civil Engineering*, October, 1936, p. 643.

Spirals should be used for all changes of direction. The spiral is the natural curve for fast traffic and will tend to keep the cars in the proper lane without cutting corners and, hence, will be a safety measure.

There should be no objection to a reasonable degree of undulation in grade line, provided the sight distance is not reduced. A break of this kind will hide the headlights of an approaching car and will be a welcome relief in some cases. An undulating grade line is almost always less expensive in construction cost, will allow shorter culverts, lower fills which may be sloped so as to avoid the construction of guard-rails, and thus be safer and less expensive to maintain. Grades should be fully compensated for curvature. Where dual highways are justified it would seem that a maximum of 5% grade should be demanded, but on the major part of the important highways of the country where the 2-way lanes are to be used, economy would appear to dictate a 7% maximum, unless ice is a factor to be met. In a similar line of reasoning, it seems that a sight distance of 700 ft is a reasonable minimum for the large percentage of the highways but, of course, in all cases, the longest reasonable obtainable distance should be used.

Except in very rough topography, the roadway section should provide a gradually changing slope away from the traveled way, eliminating the defined shoulder point, and providing the maximum of travelable surface on the right of way. All unnecessary obstructions should be removed, but signs are not as much of an obstruction as Mr. Noble indicates. More signs are hit by gunners than by automobiles, and, when judiciously used, signs are a necessary safety provision. It is impracticable to suspend signs in a windy country. The recently developed center-line reflector button has much merit as a safety device on 2-way traffic lanes.

Guard-rail should be used only where absolutely necessary. For fills less than 10 ft in depth, it is generally more economical to flatten slopes than to use guard-rail, and for deeper fills the additional safety will often justify extra expenditure to eliminate the use of the rail.

There is no immediate possibility of lighting any considerable mileage of rural highways, but with the development of rural electrification the prospect is encouraging. Where conditions demand a dual-lane highway it is logical to assume that lighting will be included.

Mr. Noble suggests the possibility of increasing the thickness of the pavement to provide smoothness. It is the writer's opinion that more detailed study of the supporting sub-grade and provision for adequate control of the moisture content (and, consequently, more uniform supporting power) are the most important phases of design for present study.

Henry B. Alvord,[40] Assoc. M. Am. Soc. C. E. (by letter).—If for no other reason than that it brings to a focus the relation of considerations of safety to the design of a highway, this paper is a timely one. Safety should be a dominant principle in the design of through, or express, highways as it already has been accepted in the design of other structures, such as bridges, buildings, and dams.

In order that this problem may be further particularized the writer presents herewith, a tentative set of suggestions in outline form which would apply to the design of an express highway. The attempt is to present a complete yet concise summary that will include all features necessary in a safe design. The writer may have omitted some points or given incorrect values in some instances; but if ideas such as these could be formulated and studied, they might help toward an agreement as to what is, and what is not, essential in such a program.

Competent engineers consider the following items to be necessary features of a safely designed through-traffic artery or express highway:

(1) Maximum allowable speed, 100 miles per hr;

(2) Two roadways of three lanes each, separated by a center strip having a width of 20 ft and a 20° sloping curb, each roadway to consist of two 12-ft lanes, the surface of the outside 20-ft parking lane to be of rough texture;

(3) Minimum radius of horizontal curve, 3 000 ft;

(4) Transition (spiral) curves;

(5) Horizontal curves to be superelevated for speed of 60 miles per hr;

(6) Maximum grade, 5%;

(7) Minimum vertical curve length = 160 times the percentage change in rate of grade. (This corresponds to a tangent offset of $\frac{5}{16}$ ft, 100 ft from any point of tangency);

(8) Minimum profile tangent, 1 500 ft;

(9) Minimum sight distance, 800 ft;

(10) Minimum horizontal tangent, 1 000 ft;

(11) Overpasses for all important cross-traffic. (No crossings of traffic at grade);

(12) Restriction of all side entrances to a minimum spacing of 5 miles. (No break in center strip at side entrances nor within 1 000 ft of a side entrance, and all breaks in center strip protected by a deceleration and acceleration lane of 500 ft);

(13) Acceleration and deceleration lanes at all side entrances (minimum length, 500 ft);

(14) White stripes separating lanes to control passing;

(15) Adequate lighting to provide clear visibility of all objects in roadway for a maximum distance of 2 000 ft, such lighting to be entirely in addition to, and independent of, automobile headlights;

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[40] Prof., Civ. Eng., and in Chg. of Dept., Northeastern Univ., Boston, Mass.

(16) Surface textures to provide at least a coefficient of friction of 0.5; smoothness of profile to be consistent with operation at 100 miles per hr;

(17) Surface of extreme right-hand lane of extra rough texture and to be used for emergency stops only; all other lanes to have specified minimum speed limits;

(18) No entrance or egress permitted from roadways to and from adjacent land; all use of marginal land for commercial purposes to be eliminated by the non-access feature herein contained;

(19) Utility wires to be placed in conduits or on poles adjacent to right-of-way boundary lines;

(20) Directional signs restricted to a few words, placed sufficiently in advance of the point to which they refer, and to be of such size of type as to be easily legible by an observer passing at a rate of 100 miles per hr; and,

(21) Any installation of traffic lights of any type at any place in this express highway is clearly incompatible with proper design as indicated heretofore.

Any set of numerical requirements such as Items (1) to (21) probably will not be consistent for use in the entire United States. At least three classifications are necessary: One for a climate where frost and ice never cause a serious problem; one where ice and snow are common during the winter months; and one where the topography is exceptionally rough.

The Civil Engineering Profession is charged with the technical responsibility inherent in highway design. It is evident that the demands of highway traffic have grown (like "Topsy") out of all bounds. These demands have swamped the highway designing engineer. Either his recommendations often have been inadequate, or his design has been emasculated by his superiors under the guise of economy, falsely so-called.

If a structural engineer allows a design of a building to pass his desk when he knows it is unsafe, he is not commended for the economy of the design. Safety properly takes precedence over economy. Safety principles accepted in building design apply equally to highway design, namely, safe capacity and safe use. If this premise is correct, the conclusion is obvious—an inadequate design of a highway from the standpoint of safety is unprofessional.

Robert Eugene Hiles,[a] Assoc. M. Am. Soc. C. E. (by letter).—In the paper by Mr. Noble, attention is called to the distinctive need for an inventory of the present worth of the modern express highways, as regards the purposes for which they were built, and the additional services that they are now rendering. He has suggested tests, research projects, and methods of handling the present and future requirements of heavy traffic conveniently and safely.

Briefly, the writer proposes that the modern express highway should be built and operated in a manner similar to the present modern express railroads, which have most certainly handled their large volume of freight and passenger traffic safely and conveniently for the public welfare.

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[a] Res. Engr. Insp., P.W.A., Little Rock, Ark.

In introducing this discussion, it might be interesting to call attention to the astonishing fact that during the same year (1935) that Mr. Noble used to show that there were 36 100 deaths due to automobile accidents, there was not a single death of a paid customer due to accident on the railroads. No other test than that should be needed, therefore, to prove that the system of turning 50 000 000 vehicles, more or less, loose on an open highway with just any one operating and guiding them at any rate of speed, perhaps to 120 miles per hr, proved to be wrong in the 36 100 cases in 1935. It would also be just as wrong to put a steering wheel on a modern stream-line train and turn it loose on the open highway at 100 miles per hr. The modern highways handle much more traffic than the railroads; so why not control them as rigidly as the railroads? No recommendations are made herein to handle local traffic, except that which is permitted entrance at established express terminals after complying with and passing certain rigid regulations and inspections for road-worthiness, safety, etc. The construction of these express highways and facilities would require the building of new roadways where they could not follow the alignment and otherwise utilize the present highways. The adequacy of the present system of handling the express highway will certainly and surely be tested in time of war when man power will be scarce to guide all the machines of war besides all the vehicles for domestic and military transportation. Finally, the payment for this proposed project would not only be made from tolls charged on vehicles using the express highway and rents charged for concessions in the terminals, etc., but also by reason of the savings accrued from reducing the present type of highway pavement to that actually required per lane width of the proposed express highway track.

Express-Highway Tracks.—Why not put the modern express vehicles on tracks or in guides so that the personal element of guiding will be eliminated from the many duties of the driver? The design of these tracks should be effective and of sufficient strength to guide the standard design trucks and truck trains without the aid of a driver in the vehicle and should give due consideration to the following factors: Installation, replacement, and adaptability to present highway slabs and bridges; adaptability of present vehicles to a standard express vehicle and track gage; selection of materials for guides; hazards of the elements; alignment and right-of-way clearances; automatic control devices with particular attention to the photo-electric cells and other safety devices for warnings and control under all hazardous conditions and locations along the right of way; design of track appurtenances, such as switches, frogs, cross-overs, sidings, passing tracks, and terminal facilities; and, finally, the adoption of a guide that will permit the use of the present type and design of automobiles, so that with slight delay local traffic, upon reaching an express terminal, may enter and be permitted the better facilities of safety and convenience afforded by the modern express highway system.

Physical Characteristics.—As shown by the record of accidents in Chicago, Ill., where certain sections of highway were increased from normal to 100 ft in roadway widths, the number of accidents increased instead of decreased. Apparently, then, this is not the solution, but rather the separation or division into lanes by mechanical means.

The proposed double-track express highway would handle, safely and conveniently (if properly regulated), as much as the present highways handle in perhaps four lanes. Passing tracks and sidings would be required, however, for cases of emergencies and convenience at various intervals along the system.

Any speed could be maintained either by law or by governors on the vehicles which could be checked at the terminals from previous tickets showing previous records of tests, "roadability" performance, accidents with rating as to frequency and seriousness, etc. By maintaining certain legal speeds the necessity and hazards of passing vehicles are eliminated.

Truck and passenger trains controlled by the express highway system, or privately owned passenger and freight train and single vehicles, could be operated over the same system.

Design.—Photo-electric cells, lights, neon tubing, and other control equipment could be placed on the front and rear of all vehicles and at certain necessary points along the right of way to maintain certain desired distances between vehicles and otherwise to control the speed and operation of the moving vehicles.

Certain familiar design assumptions (such as the increase in load or eccentric loading to trusses and girders, due to the shifting about of the design load from one curb to another, in order to produce probable maximum design conditions on the present type of bridge floor) would be eliminated by the proposed track system and in many structures this would effect a material saving in cost. Furthermore, the entire road-bed would not have to be paved. This advantage, however, would depend on the type of tracks or guides adopted.

Control.—Vehicles that failed to meet the requirements of speed, safety, and "roadability", while using the express highway, would be fined or otherwise penalized or disbarred in accordance with the severity and frequency of the offense. The routing and train, or vehicle, movement would be controlled by express dispatchers. The design and construction, supervision, inspection, collection of tolls, rentals, fees, fines, etc., operation, and maintenance would be under the direction of the present State highway commissions.

Payment.—The basis for payment in the form of tolls or express highway charges would be in accordance with the weight per mile hauled over the roads, payable and collected in advance at the express highway terminals. As noted under the heading, "Control", rentals, or leases for concessions at the express terminals, together with fines and penalties collected, would apply on operation and maintenance costs. Any saving by reason of design, in favor of the proposed modern express highway system should be credited toward payment. However, the greatest saving anticipated by this system is not financial, but is the saving of human lives, which at the present rate is a loss of 100 per day; and, the ultimate goal of the modern express highway should be the almost perfect record of the modern express railroads.

Conclusion.—The recommendations and suggestions contained herein are submitted for consideration, as suggested by Mr. Noble, for: Design, construction, operation, test, and research, to determine a safe and convenient modern express highway; and, perhaps, to prove that this can be accomplished by eliminating, as much as possible, the personal element from the control and operation of modern express vehicles on the highways.

CHANDLER DAVIS,[42] M. AM. SOC. C. E. (by letter).—The modern express highway, as described by Mr. Noble, is undoubtedly the highway of the future. Whether it will be feasible to permit driving at 100 miles per hr is very doubtful.

For 1935, Table 1 records 826 690 accidents of all kinds on American streets and highways, of which 374 490 were collisions between automobiles (that is, 45.3% of the total number), resulting in killing 24.6% and injuring 50.3% of the total reported for the year—this large and appalling list in spite of the numerous laws and ordinances.

Suggestions for minimizing such accidents and making the high-speed roads safe is treated by the author, who states that a car traveling at 100 miles per hr requires 617 ft in which to stop, provided the car is fitted with four-wheel brakes. To this value is added 219 ft for a lag of 1.5 sec for an emergency stop; that is, a total of 836 ft is required if a car should be required to stop suddenly. Naturally, each vehicle on the road will require the same space for a factor of safety and, consequently, each mile of road is limited to seven cars, if a speed of 100 miles per hr is to be maintained. This seems an absurd conclusion. Where can one find the data which will prove that, for safety, the road spaces given by Equation (2) are correct and conform to the proper use of highways. The United States Army has made a thorough study of the problem of moving supplies by truck and in developing rules of the road for motorized trains have reached conclusions, which seem to be based on Equation (2), or on some similar formula. It must be borne in mind, however, that the military units do not, and are not permitted to, exceed a speed which has been worked out as safe, although at times in case of an emergency the highest possible speeds are permissible; the safe rate for moving long columns (military) has been limited to 12 to 18 miles per hr.

The Army unit is the motor company, consisting of fifty-four trucks. Each vehicle is allowed a road space of 30 yd, which permits a maximum safe speed of 40 miles per hr. This is approximately the maximum speed found to be permissible on present-day highways, although, generally, it is exceeded. The ordinary speed for the Army is fixed at 20 miles per hr, although light tanks, carried on their trucks, are moved at 40 miles per hr when required. At night, it may be necessary to march without lights and naturally under such conditions (even when the cars may use their lights) the speed is considerably reduced. As 50 yd is left free between each company, it will be seen that one company requires 1 mile of road when moving; or at speeds as great as 40 miles per hr, with vehicles maintaining the proper distances, each mile is limited to fifty-four cars.

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[42] Cons. Engr., New York, N. Y.

Columns of motor-driven vehicles can be moved safely, day and night (the latter in case of an emergency only) at 40 miles per hr, provided all road crossings and junctions are properly controlled. Cars should only be allowed to enter the road under the direction of a traffic control; therefore, it seems that the approaches to the express highway should be similarly controlled. This would call for as few approaches as possible between terminals, and one for each lane so that there would be no crossing of cars at any time.

Undoubtedly, the perfect highway will be constructed eventually; but the writer believes that the high-speed commercial roadway is a long way from realization.

R. A. MOYER,[43] ASSOC. M. AM. SOC. C. E. (by letter).—Many controversial questions are involved in the proposals offered by the author; however, the most important question that remains unanswered is that of deciding upon the proper speed or speeds for which the highways of the future are to be designed and for which existing highways are to be re-designed and improved.

The writer discussed this subject at some length in 1936 and 1937,[44] including a recital of the more general aspects of the problems involved. In analyzing Mr. Noble's paper, the writer wishes to add further interpretations of the results of his extensive research on the subject of speed *versus* safety, as it applies more specifically in this case to the problem of designing express highways for 100 miles per hr recommended by Mr. Noble as a suitable design speed.

There seems to be general agreement among highway engineers to-day that many highways in the United States are in immediate need of modernization and that the greatest single factor which has been responsible for this situation is speed. Highway engineers are by no means in agreement in regard to the methods to be adopted to solve the many problems created by wide variations in speed and especially by travel at high rates of speed. The author leaves the impression that the best solution to the speed and highway accident problem lies in the design and construction of express highways for speeds of 100 miles per hr. After more than five years of intensive study of the many factors related to speed, the writer is convinced that safety on highways can not possibly be assured by following such a course and that the surest way to increase accidents and fatalities is to encourage high-speed travel by building highways permitting speeds of 100 miles per hr. Not only is travel at such speeds unsafe on the basis of the physical laws which govern motor-vehicle operation and which are inviolate, but the difficulties which confront the driver at such speeds constitute an even greater threat to his safety. Furthermore, the extra costs of providing suitable highways for travel at this speed, and especially the excessively high operating costs, definitely indicate that such travel is uneconomical and impractical.

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[43] Assoc. Prof. of Highway Eng., Iowa State Coll., Ames, Iowa.

[44] "Speed *versus* Safety on Straightaways", *Civil Engineering*, November, 1936; and "Speed *versus* Safety on Curves", *Civil Engineering*, February, 1937.

The author may defend his stand on the basis that if the highway is designed for a speed of 100 miles per hr, it will be safe for speeds less than that. Now, it is the writer's opinion, that this attitude is contrary to the first fundamental concept of good design offered by the author when he states that "it is possible to design a highway so that the motorist is led unconsciously to choose the safe act rather than that which is unsafe," and he further points out "the desirability of designing the highway on the basis of a constant rate of vehicle speed between large terminal points." With these concepts or principles of design the writer concurs. However, few persons driving cars at 100 miles per hr on a highway of the type suggested by the author could ever be expected to have the ease of mind or feeling of safeness commonly experienced when driving at the moderate speed of 50 miles per hr on a dual highway or even on a two-lane highway designed to present-day standards to accommodate such a speed. The tenseness and feeling of unsafeness commonly experienced at speeds greater than 60 or 70 miles per hr on the best of present-day highways is born out of the knowledge that there is a certain minimum time limit to any driver's reactions to meet certain emergencies and errors of judgment below which it is impossible to go. In that split second so much can happen over which the driver has no control that only by taking the control of the car almost completely out of his hands (as has been done in the safest rail operations) can safe operation be made possible. Not only is there a fixed limit to reaction time but, what is more important, the forces with which the driver has to contend at a speed of 100 miles per hr have been increased many times, compared to the forces encountered at present-day average speeds of 40 to 50 miles per hr. The time required to bring the car under control has likewise been increased; and the distance traveled during this interval extends far beyond the zone of safety within which control of the car may reasonably be assumed to be possible. Only a revolutionary change in the steering, braking, and driving of cars can correct this situation. Accordingly, even if the driver were led unconsciously to choose the safe act on a highway designed for 100 miles per hr, his chances of following his choice when driving at this speed would all too frequently be very small, indeed, to prevent the accident.

A valuable feature of this paper is that it has directed attention to the most vital factor in modern highway design—the speed factor—and furthermore, it has stimulated discussions on definite design details which the author contends should provide positive safety at a definite speed. However, to establish the design speed arbitrarily at 100 miles per hr, or at the maximum speed at which the cars are now, or may soon be, capable of, is contrary to another basic principle in design mentioned by the author in which he states that to-day emphasis should be based on broad highway economics founded on loss of life and property as well as on operating costs.

In the 1936 report of the National Safety Council on "Accident Facts," statistics are given which indicate that the severity of accidents increases with an increase in speed. Thus, at speeds of less than 20 miles per hr, 1 accident out of 61 is fatal; at speeds between 20 and 29 miles per hr, 1 accident out of 42 is fatal; at speeds between 30 and 39 miles per hr, 1 accident out of 35 is fatal; at speeds between 40 and 49 miles per hr 1 accident out of 25 is fatal; and at speeds greater than 50 miles per hr, 1 accident in 11 is fatal. Statistics of railroad-crossing accidents, involving cars and trains, have consistently shown that one accident in two or three is fatal. Tremendous forces are involved in accidents involving high fatality rates. It should be noted that the forces developed at high rates of speed vary as the square of the speed and that the aforementioned statistics indicate a certain amount of correlation in this respect.

Not only is the severity of accidents increased with an increase in speed, but the chances for accidents increase with an increase in speed. This was revealed in 1936 in a report,[45] by Charles J. Tilden, M. Am. Soc. C. E., in which the accident records of owners of cars observed traveling at high and low speeds were analyzed. It was found that in this instance "the high-speed drivers who have accidents have more of them, so that they account for 45 per cent more accidents than do the low speed drivers."

In regard to the economics of express highways, Mr. R. E. Toms summarized the highway cost phase of it very ably in a paper presented at the 1937 Annual Meeting of the American Association of State Highway Officials.[46] He stated that,

> " * * * however desirable they [express highways] may be, they are not possible except on a very limited mileage of our State Highway Systems. * * * Assuming that it would be desirable to improve 5 per cent [about 16 000 miles] of the State highway mileage with four or more traffic lanes with opposing traffic separated, grades at intersecting highways separated, border roads to eliminate unrestricted access from abutting property, and sidewalks for pedestrians where needed, the expense involved in this undertaking alone would amount to approximately four billion dollars. This is just about the amount that has been devoted to highway improvement in this country during the last 20 years through the program of Federal aid to the States matched with State funds, plus the amounts of Federal funds appropriated for emergency construction which did not require matching by the States. When these figures are considered, we must admit that in so far as we can visualize the future at this time from 95 to 97 per cent of the State highway mileage in this country may never progress in improvement beyond a two-lane highway."

Although Mr. Tom's estimate of the cost of constructing express highways may have been high, since it calls for an average initial cost of about $250 000 per mile, this figure is not unreasonable when stated in terms of the cost per vehicle-mile on a highway carrying 4 000 or more vehicles per day. The annual cost for such a highway, including maintenance costs, snow removal, depreciation, interest charges, and charges for policing, would probably be about $15 000 per mile and, for 4 000 vehicles per day, this would represent a tax cost of about 1 ct per vehicle-mile. The present average tax cost for highways in the United States, based on the license fee and gas tax, is about $\frac{1}{3}$ ct to $\frac{1}{2}$ ct per vehicle-mile. It would be quite possible for the Federal Government to undertake such a project and to assume full control of this system of highways. A Federal license would be required to operate on the highway. Tolls might be collected to finance the cost of construction and operation. The highway would be constructed, maintained, operated, and policed by the Federal Government.

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[45] "Motor Vehicle Speeds on Connecticut Highways", pub. by Committee on Transportation, Yale Univ., 1936.

[46] "Highway Safety Exemplified by Properly Designed and Constructed Highways", by R. E. Toms, *American Highways*, January, 1937.

Although the highway cost may not be unreasonably high where the traffic volume is sufficiently great to justify the construction of such a highway, the motor-vehicle operating costs are so much higher at 100 miles per hr than at the present-day average speeds of 40 and 50 miles per hr, as to make it very doubtful whether traffic would operate at such speeds for any considerable distance; and, certainly, it would be absurd to build highways for such a speed if only a small percentage of the traffic would travel at the designed speed, or if drivers were to operate at such a speed for only a few minutes at a time. Tests conducted by the writer indicate that fuel costs at 70 miles per hr are almost double the costs at 40 miles per hr, and at 100 miles per hr there is every indication that the costs for the same car are about three times the costs at 40 miles per hr. Oil costs seem to follow a similar trend, or if oil costs are reduced at the higher speeds, the engine costs are increased, due to increased wear on cylinders and pistons. Tire wear follows the square law very closely and the tire costs at 100 miles per hr are most likely to be about six times as great as at 40 miles per hr. Engine breakdowns are still very common in the Indianapolis, Ind., races where an average speed of slightly more than 100 miles per hr is maintained for only 500 miles. One can be quite certain that engine repair costs and car maintenance costs would be greater at 100 miles per hr than at 40 miles per hr. A conservative estimate of the increased operating costs for the same car due to speed is that the cost of operating at 70 miles per hr is 2 cts per mile more than at 40 miles per hr and at 100 miles per hr it may easily be 4 or 5 cts per mile more than at 40 miles per hr. Such a cost of high-speed travel can not be disregarded when proposals are made for spending billions of dollars to build express highways. It would be a sad commentary on engineering judgment to find that after such highways were built, traffic would not be willing to pay the tax to use them—the toll-free State highways with their lower speed limits carrying most of the through traffic.

In his discussion of details of design, the author states that there is great need for research before satisfactory design details can be developed. It has been gratifying to note that this paper has provided the means for bringing together many discussions in which considerable experimental data and many excellent theoretical analyses of the design problems are presented for the first time. Tests conducted by the writer[47] have provided much basic information that is needed in the solution of this problem. Although present knowledge is far from complete, it is the writer's conviction that the basic facts are now known, and it is not so much additional information that is needed as it is the proper use and interpretation of the information at hand.

The most important element in the design of an important highway to-day is the speed factor. This is not a matter to be decided upon by an individual, from experience based on local conditions, but rather it is an element of design which should be as rigidly controlled as are the weights, lengths, widths, and heights of vehicles. Speed may quite properly be thought of as a "fourth dimension" which controls curvature, superelevation, road width, and sight distance. In fact, practically every detail of design is wholly or in part controlled by the speed factor. For important highways, it is desirable, therefore, to standardize or control speed for the same reason that weights and lengths of vehicles are standardized. The establishment of such a standard is not an easy matter.

In this discussion, the writer has touched on only a few of the more important factors which should be considered before speed standards are decided upon. A complete analysis of the speed problem and of establishing design speed standards for various classes of highways is not within the scope of this discussion.

It is virtually impossible to design highways to permit safe travel on ice-covered surfaces for the high-speed travel, which the author suggests will be a controlling factor in determining the superelevation on curves. Tests clearly indicate that there simply is not sufficient friction available between rubber tires and ice to make it possible to develop a speed of 100 miles per hr on a level surface, not to mention the difficulties of steering or of stopping the car at that speed. The writer recommends that superelevation be used on all curves to permit easy steering and that the maximum rate of superelevation of 0.1 ft per ft be used on the sharpest curves, combined with a maximum value of f equal to 0.1 where this frictional force is necessary to counteract centrifugal force not taken care of by the superelevation. The safe speed on ice is almost never greater than 30 miles per hr on straightaways where traffic is separated and it certainly does not seem possible that curves could be designed to permit higher speeds with safety on ice by any known device commonly used in highway design to-day. The writer wishes also to state most emphatically that spiral curves are necessary and may very properly be used on all curves.

In this paper, the author has aroused the imagination of his readers. The paper has been timely because this country now appears to be approaching the end of what has been aptly called a "highway construction holiday." A new era of road building seems to lie immediately ahead. The modern car is so far ahead of the highways on which it must now travel that mere improvising will not suffice. A new conception of a modern highway is needed. Men of vision and of wisdom are needed, who can weigh all the variables involved, who can foresee the direction in which true progress lies, and who can formulate the broad plans upon which the highway systems of the future are to be built. The magnitude of the improvements under consideration are worthy of the most thorough and painstaking analysis and it is for this reason that this paper should prove to be a really worthwhile contribution to the literature on the subject and a benefit to all the people in this country who, in the last analysis, will profit most by such studies.

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[47] *Bulletin 120*, Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa, August, 1934.

CHARLES M. NOBLE,[48] ASSOC. M. AM. SOC. C. E. (by letter).—The response to the paper, and the frank expression of opinion, is gratifying. The discussion has been directed as much along sociological, psychological, and philosophical lines as along a train of thought devoted to technical design. Of the three elements which constitute the broad background of highway safety (namely, the operator and his control, the vehicle, and the highway) the paper treated only the latter, on the theory that the other two were, in a technical sense, out of the province of the designing engineer; but, since the discussion has broken free of the original limits, it may be in order to continue in the spirit of the discussion and begin at once with the non-technical phases before turning to the more technical features.

The writer wishes at once to concur with all that has been said respecting the desirability and advantage to be gained from a rigid restriction of the privilege of driving a motor car on the public streets and highways, as well as the adequate testing and training of the operator. There is no doubt that a very large percentage of accidents can be prevented in this manner.

The deprivation of one's license to drive, from the reckless and unfit, would relieve the highway at once of one of the gravest hazards, while at the same time it would have a sobering effect on the remaining drivers. Nothing in the paper could be construed as advocating looseness or laxness in this direction and, for some years, the writer has been keenly aware of the advisability of rigid control and policing. However, such control is more in the province of the administrator, the police official, the politician, and the engineer, acting in the broad capacity of his profession. Essentially the problem of the designing engineer is to design to comply with conditions as he finds them, or as he can reasonably predict they will be in the future. Perhaps a short review of elementary basic engineering design principles may not be out of order, as there appears to be a tendency to overlook some of the fundamentals which general engineering practice has evolved.

In designing a bridge, for example, the first task of the designer is to discover the magnitude of the loads which the bridge may be required to support, the physical properties of the materials of construction, the proper factor of safety to apply to the materials, the determination of the stresses of all kinds which will occur in all parts of the structure, and, finally, the proper proportioning of each member, based on the transmission of stresses to it. The completed design represents a logical and consistent process of thought, which combines a close attention to detail with a clear picture of the functioning of the structure as a whole. It is axiomatic that success in engineering design is dependent on the correct assumption of the loads to be carried, the proper selection of the materials of construction, and the correct stress to apply to these materials.

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Suppose bridge engineers designed structures, using loads less than the actual known loads, because they felt the actual loads were unreasonably high, that loads of such magnitude should be prohibited, and because they hoped legislation might be passed some time in the future to curtail the loading! This seems to be somewhat the attitude of a number of highway designers. There appears to be a reluctance to acknowledge the loads which the highway is called upon to support.

Traffic density and speed are the loads. Alignment, grades, superelevation, sight distance, shoulders, signs, lighting, and a multitude of details which add to vehicle safety, are the materials. Loads have been assumed that were too small, and the materials have been stressed up to and beyond the ultimate strength. This is not in accordance with established engineering practice.

A brief review of recent highway engineering history may be of assistance in clarifying past design measures and in illustrating that design practice is, at present, pausing at the cross-roads, while consideration is being given to the direction it shall pursue. The twenty years since about 1917 represent the pioneering period in highway design as related to the motor car. At the beginning of the period no road system, as at present defined, was in existence, and the motor industry was still in its infancy. The pressing need was to bring into existence, from a standing start, a connected road system embracing the entire United States. The task was colossal. Cheapness of original cost and speed in location and construction were the goal.

At that time, the property owner contributed the bulk of the road money, and he expected a road dollar to go a long way, geographically. The results may be compared in a rough sense with the earlier development of the railways, which, although not of a thoroughly permanent nature, were, nevertheless of vast benefit to the nation. America was "pulled out of the mud" in the incredibly short time of twenty years, thus making possible the almost magical development of the motor car and, even if the exigencies of the period had permitted of more leisurely and permanent construction, engineers had nothing on which to base a forecast of future requirements, for no one really dreamed of a stock car of moderate price which could travel with ease hour after hour at 70 or 80 miles per hr. Engineers were literally pioneering in a new form of transportation, and, consequently, one must not be too critical of their performance.

To-day, the situation is quite different. The pioneering period has definitely closed, and the highway and motor industries are entering upon an era of early maturity. The motor car owner is contributing the major portion of the monies for highway improvement, and consequently his safety is entitled to careful consideration; in addition, there is ample "case history" from which to make an intelligent forecast of future needs. From now on the engineer must produce a finished and polished product, perfected as to detail and equipment, and, in addition to expeditious travel service, an adequate margin of operating safety must be provided.

There has been a misconception on the part of many readers. It has been assumed that the writer is a proponent of speed, and that he advocates an improvement in design for the purpose of providing speedways to encourage the motorist to drive at high speeds. Nothing could have been further from the writer's mind. He does feel, however, that a great number of motorists always have driven at a higher rate of speed than road conditions warranted, and that they will continue to do so until road conditions are improved to a state more in accordance with the speed habits of the nation. As far as the writer personally, is concerned, he would be content never to exceed 50 miles per hr, but the problem the engineer is facing cannot be solved by injecting personal opinion into design practice. Engineers are designing an engineering structure, and engineering principles and methods must be applied—principles and methods which have evolved from the design of other engineering structures.

There has been considerable discussion and adverse criticism by many commentators respecting the writer's suggestion that design be based on a speed of 100 miles per hr. It must be recognized at once, however, that a design for express routes, based on this speed, is a very effective method of assuring a proper factor of operating safety for more moderate speeds, while at the same time providing a reasonably sure form of insurance against obsolescence. As subsequently demontrated the adoption of such a design speed does not involve as much difference in actual design standards as might be supposed in comparison with a design based on a sustained speed of 60 miles per hr. Be this as it may, the embarrassing question may quite reasonably be asked: Is there anything in the past history of the development of the motor car which justifies a confidence that top driving speeds have been reached? It should be remembered that the motor industry is highly competitive and therefore, will continue, in all probability, to strive to give the public the kind of vehicle which it demands.

As far as the speed situation is concerned, present design practice appears to be in a somewhat unfortunate position. If the road was so crooked or so congested that even moderate speed was physically out of the question, it would be reasonably safe as far as serious accidents are concerned; or, if the road was designed uniformly for a speed greater than any one cared to drive, then it would be safe from a physical standpoint at all the operating speeds to which it might be subjected. Present design, however, is neither the one nor the other. It is open enough to encourage speeding, but not open enough to justify such speeding, and thus highway designers are guilty of encouraging the motorist to operate at speeds not justified by the actual design. It should be emphasized that safe design consists not only of good alignment, superelevation, and grades, but upon a skillful combination of all the components outlined in the paper, so that a consistent and ordered whole is produced. The writer earnestly feels that engineers should face the facts squarely, should make an intelligent effort to forecast the future, and should then proceed in an ordered and logical manner to design along engineering principles, adopting a proper factor of safety.

The key to the widespread advance in sustained road speeds appears to be related more to the general improvement of the vehicle rather than specifically to increased horse-power in the motor. As long ago as 1920 there were a number of makes of American stock cars which were capable of speeds of 75 and 80 miles per hr; but, except for the most reckless and adventurous, there were few that even approached these speeds, because driving one of those cars at such high speeds was distinctly a nerve-wracking experience which only a thrill seeker would endure. The human nervous system could not stand continuous high-speed operation in such cars, and, consequently, continuous operation seldom exceeded 40 miles per hr. Thus, highway engineers had no experience or any other basis for predicting a rapid advance in road speeds. Along in the middle 1920's, when the manufacturers had mastered the production of a fully dependable motor, they turned their attention to other features of the vehicle. Improvements in the body, steering, tires, brakes, reduction in vibration, redistribution of weight, shock absorbers, rubber mountings, etc., pyramided in rapid succession, until to-day the car glides over the highway with swiftness and ease, eradicating all sense of speed values, with complete lack of nerve tension and fatigue for the operator.

In a word, all sense of speed has been destroyed. This, then, is the major cause of the general increase in sustained road speeds, and coupled with greater power plant in the medium priced and cheapest vehicles, it spreads the power of high speed among the great mass, and among all classes of people. The destruction of the sense of speed is the primary reason why many conservative motorists operate at higher speeds than conditions warrant. This is particularly true where conditions require speeds in the lower brackets.

A recent advertisement of a car "priced just above the lowest" listed, among others, the following: "No vibration or road rumble; new ease of steering; no 'wind wander' even in a gale; no 'edging off' in ruts or gravel; new sound-proofing; so silent inside you can talk in a whisper; 'city ride' on any kind of road; * * *" Incidentally, this car is equipped with 93 hp. There can be little doubt that manufacturers will soon reduce the cost of high speed. In view of the highly competitive nature of the industry, do highway engineers really believe advancement in automotive design will cease abruptly?

Mr. Purcell gives the results of observations in California showing that 94% of all cars traveled at or less than 55 miles per hr. Assuming the observations were taken in 1936, it should be remembered that more than 60% of these cars were three years old, or older. In other words, 60% were not newer than the 1933 model, and many of the most effective improvements in vehicle design which particularly deaden the sense of speed, have occurred since that year.

Since comparisons are inevitable, suppose for a moment, the essential difference between improvement in railway line and improvement in motor ways is examined. The improvement in track is entirely economic, using the definition in the conventional sense. Such improvement is actuated by a desire to lower the cost of hauling freight or to shorten passenger-train time. Safety is not an issue, because enginemen are given a definite speed order to apply to each section of track, and they are perfectly familiar with their runs. Because these operators are employees who may be disciplined for infraction of orders, obedience is quite general. Thus, from a safety standpoint, bad track conditions can be overcome.

The highway situation is quite different. The motorist, by and large, is not an employee, and he purchases a vehicle from an industry which is not responsible for the design, construction, or maintenance of the track upon which it operates. As long as he can escape the vigilance of law enforcement officers he can do, and does, about as he pleases. He usually feels he is clever if he continues to "get away" with breaking the motor laws. Often he is unfamiliar with road conditions, and more often is operating at a greater speed than conditions require. He has no "slow orders," and, therefore, any sudden change in road conditions which requires a slow speed introduces a hazard which, sooner or later, results in an accident. Moreover, the average motorist does not have the skill and judgment which an engineman possesses. Thus, an improvement in highway standards is a step forward in safety, and consistency in design is the very essence of safe design.

What, then, is there in this situation to assure the highway designer that he will ever catch up with automotive design; that obsolescence will not continue its costly toll? As far as the vehicle is concerned, there appears to be no immediate limit; possibly fantastic speeds may be possible, but a limit there must be, and that limit is more likely to be human flesh than steel and rubber. The human nervous system has a limit, and it has been stated that this limit is possibly 100 miles per hr. This, then, is the basis for the suggestion in the paper that 100 miles per hr may not be an unreasonable maximum design speed.

The purpose of the paper was to show that the present standard of first-class road design is inadequate for high speeds. The destructive forces at high speed are so terrific that an entirely new conception of trunk highways will be necessary to cope with the situation. This thought is expressed very aptly by Professor Moyer. The writer is not sure, given unlimited resources and money, whether engineers could produce, to-day, an inherently safe design. They may not know enough at this moment about safe design. Nevertheless, a sincere and intelligent effort must be made to provide the motorist with as much protection as knowledge and skill will permit, at the same time continuing an intensive attack on the problem in order that the profession may the more surely advance its knowledge and technique.

In general, the discussion by the commentators may be classified into two more or less distinct groups: (1) That the present standards of design are properly and efficiently performing their functions, and that the highway system, as at present constituted, is as perfect as the motorist may reasonably expect; and (2) that further advancement in the art of highway engineering, particularly as related to safety of operation, is distinctly indicated if the motorist is to be provided with the type of service he should reasonably expect.

It may be well to express to proponents in the first group that no engineering art should remain stationary—that none of the various branches of engineering is remaining stationary When an art remains stationary it becomes mere dogma. The writer feels sure no one really wishes to suggest such a fate for highway engineering. Indeed, it is clear that this branch of engineering has just emerged from the pioneering period and there is a vista ahead of mature but vigorous growth and development.

Specifically, the following suggestions have been offered in lieu of improved design standards, or as reasons against raising such standards:

(1) That vigorous control of the driver should be maintained and the unfit and the accident repeater be ruthlessly ruled off the road;

(2) That vehicle speed be arbitrarily controlled by a mechanical device;

(3) That a large class should not be taxed to provide speedways for the few;

(4) That the cost of adequate highways is too large and is not economically justified, and that the percentage of accidents caused by faulty highways is not great enough to justify an improvement in present standards; and,

(5) That it is questionable whether it is the duty of the State to provide expeditious service.

It may "clear the air" somewhat if Suggestions (1) and (5) are discussed in somewhat greater detail.

Control of Driver.—For twenty years the problem of control (Item (1)) has been discussed but very little has been accomplished. A few years ago it was thought that a system of licensing drivers would be a remedy, but such action has been found to be only mildly palliative. The difficulty is that the great mass of public opinion is opposed to rigid control measures; politicians will not pass statutes, nor will officials ruthlessly enforce legislation, which they know is not solidly supported by the mass of the people. The Eighteenth Amendment is an example.

As a group, highway engineers would welcome the elimination of all unfit and dangerous drivers, particularly those who prove to be accident repeaters. Granted such a procedure is most desirable and that it would be helpful, it would by no means solve the problem under discussion. It would not materially reduce those accidents caused by imperfections in the highway itself. Many careful and competent drivers have serious accidents. Should sober and substantial citizens be required to risk their lives day by day when it is possible to remove many of the hazards connected with motoring by designing safety into the highway?

Unfortunately, a continued discussion on the subject of control is almost academic. Does any one really believe that rigid and drastic laws will be passed and enforced in the near future? Without wishing to appear cynical, the writer believes the answer is definitely, "No!" Only by the slow process of education can real progress be made, and for this reason it is doubtful whether the goal can be reached within the next twenty years. By that time nearly 800 000 human beings will have paid with their lives, if the present death rate continues, and 25 000 000 will have been injured.

Mechanical Control Devices.—During the past twenty years mechanical speed governors (Item (2)) have been seriously proposed several times; but the public would have none of it. It is one thing for a private company operating a number of motor vehicles to apply governors to their cars but quite another to get a law on the statute books requiring all citizens to apply governors. An employee of a company is quite helpless to prevent the use of such a device, but the electorate have a way of showing their disapproval. In addition, there are practical difficulties.

Would such a law be a Federal statute, or would each of the forty-eight States have to pass legislation specifying an identical speed limit? Would motorists from other States be compelled to stop at State lines and purchase a governor in order to pass through States having such laws? The injection of these conditions into the motor problem would undoubtedly disrupt motor transportation. Finally, is there any one who can point to a strong, unified movement among the great mass of citizens for an enactment of mechanical speed legislation?

Taxation.—The writer can recall hearing, twenty years ago, the same complaint from owners of horses and buggies to the effect that the taxpayers were having to spend good money to create speedways for the motorists (see Item (3)), but does not recall hearing any motorists in recent years complaining about roads being constructed too well. Twenty years ago most of the road funds were being contributed by the property owner. To-day, most of the road money is being contributed by the highway user. As Mr. Dennis states, the American highway is, in effect, a toll road, the user paying a sales tax. For instance, in 1935, the State and Federal oil and gas taxes and license fees amounted to $958 753 000, the manufacturers' and personal property tax to $350 000 000, making a total of $1 308 753 000. If the motorist, who pays the bills, does not object to the expenditure of sufficient money to modernize the highways, there is no valid reason why the engineer should urge him to accept an article which does not provide him with reasonable safety facilities. To the writer's knowledge, there is no unified and concerted movement among the various associations of motorists for the curtailment of highway construction.

Economical Considerations.—It is often argued (see Item (4)) that it costs too much money to do a first-class job in constructing highways. That may have been true during the pioneering period when the vital need was to create a road system; but now that that era of stop-gap construction is past, engineers can proceed to replace their "log cabins" with brick and stone. It must be realized that adequate highways will cost a great deal of money, and it would seem the better policy to design and construct these highways with the thought of their being permanent, rather than to have to repeat the performance twenty years hence, thus wasting the sums now being spent. A cheap article is not always a bargain.

It is somewhat inexplicable that engineers should take the stand that adequate highways should not be designed and constructed because of the large cost involved, when it is remembered that considerable sums are diverted annually from motor vehicle funds to other than highway uses. A more commendable effort would appear to be to educate the public to the fact that the need of highway expenditure is not over, but in reality only begun, and that the highway industry needs every dollar collected from the motorist to remove hazards from the present highway system and to construct new highways in accordance with advanced technique, to the end that the public will receive more expeditious road service, will derive more pleasure from motoring, and will not be subjected to as many chances of being killed or injured. Thus, the engineer would be performing a notable public service, at the same time assuring himself, and his fellow engineers, of a continued opportunity to make use of his technical talents.

It is further contended that the percentage of accidents caused by faulty highway design is not sufficient to justify the expense of safe design. This is quite a cold-blooded argument when the terrific mental and physical suffering occasioned by traffic accidents is remembered. To make matters worse, the argument may not be altogether true. As stated in the paper, accidents occasion an economic loss from a purely financial point of view. The writer is not aware of any accurate appraisal of this loss. The Travelers Insurance Company, of Hartford, Conn., estimates that it amounts to from $2 000 000 000 to $2 500 000 000 each year. W. V. Buck, M. Am. Soc. C. E., states[49] that statistics indicate that 30% of all accidents may be traced to some fault in road location and design.

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[49] *Engineering News-Record,* October 29, 1936, p. 612, *and Civil Engineering,* January, 1937, p. 1.

If these figures and assumptions are correct, then faulty highway design is costing the American public from $600 000 000 to $750 000 000 each year; and to these figures must be added the economic loss due to obsolescence. It is evident that more study is necessary in order that more nearly accurate figures of the economic loss due to accidents may be made available.

In this connection it may be pertinent to mention railway-highway grade-crossing elimination. Acording to Table 1, only 0.6% of all accidents were due to this cause. If the economic loss from this type of accident is assumed to be at the same rate as for all accidents, then, assuming the foregoing estimate, the annual economic loss due to railway grade crossings only amounts to from $12 000 000 to $15 000 000 for the entire United States. Yet, as Mr. Ripley states, New York State recently voted $300 000 000 for grade-crossing elimination, and there is a strong movement all over the country for rapid elimination of all railway grade crossings.

Studied from a cold-blooded standpoint, this work has no economic justification; but is there any considerable body of engineers who condemn grade-crossing elimination? If engineers are willing to support such work, why are they not willing to support the elimination of other road hazards? Mainly, it is because of the subtlety of the problem. The actual causes of other types of accidents are not as evident even to engineers.

The public is supporting grade-crossing elimination heavily because it can readily visualize a railway crossing as a hazard. If the public really understood the mortality due to other highway hazards, they would without doubt approve the expenditure of the necessary funds to provide for their systematic elimination. More people are killed in motor accidents in the United States in one month than have perished in all the floods in the entire country during the past ten years. Therefore, it would appear wise for engineers to advise the public of the relation between highway design and the accident rate, rather than to remain indifferent until stung into action by public censure.

Responsibility.—Mr. Lavis questions whether it is the duty of the State to provide the motorist with expeditious road service (see Item (5)). This is a pertinent question and deserves careful consideration. As practiced in the United States, the State has, in general, assumed as a duty any function for which there is a persistent and widespread demand among the electorate. It may be assumed, therefore, that if the public demands expeditious service, the State will accept the responsibility within the limits of available highway revenues. The continued reconstruction of outmoded highways is an expression of an acceptance of that duty. Frequently, the relief from congestion in urban areas constitutes a provision of expeditious service, and highway engineers are in a position to know to what lengths the State is willing to go to provide such service.

From the foregoing discussion, it is rather evident that designers are facing (and probably will continue to face for some time) operating conditions over which they have little control. Thus, an open mind, continued thought, compilation and analysis of data, and continued technical progress, based on well-founded principles, is clearly indicated in order that that happy combination of safe design and over-all economy may be achieved.

Rights of Way.—Of all the features under discussion, the alignment and width of the right of way is by far the most pressing and important, not only from a financial point of view, but also because of its vital influence on vehicle safety. The ability of the average citizen to look into and prepare for the future is generally conceded to be rather poor. His thoughts are chiefly concerned with the present, and usually he is not prepared to make any sacrifices or provide monies to care for the future unless he can be shown conclusively the benefits that will accrue. The dissemination of this gospel and education is in the province of the engineer, for he is generally thought to have the ability to look into the future, to forecast the coming needs, and to evaluate, properly, the extent to which the present generation should prepare for the problems of the next. The water supply, flood control, electrical, and sewerage engineer has demonstrated his proficiency in this field, and has educated the public to respect his judgment. The right-of-way situation presents an opportunity to the highway engineer for public service.

Messrs. Lavis and Crosby have indicated the desirability of initially acquiring ample rights of way with proper alignment. The importance of the subject justifies an expansion of the discussion in order to focus attention sharply upon this vital question.

The right of way is the only element of the highway that is permanent. This axiom was current when the writer entered the highway field in 1914, yet it was heeded to so small an extent that to-day the inadequacy of rights of way, which is the most difficult and expensive error to correct, is acute in many States. Road improvement brought an increase in land values and physical improvement adjacent to the right of way which has made widening and straightening prohibitive in many cases. The critical street situation in cities is a manifestation of the ultimate results of inadequate width. As stated previously, lack of vision during the pioneering stage in securing adequate rights of way should not be criticized without a realization of the problems of the time. To-day, the situation is very different.

Because of its very nature, the trunk highway will be located to as large an extent as practicable upon new rights of way as far removed from developed areas as possible, and, in general, it will be purchased by the acre rather than by the square foot. Therefore, its cost will be less at the time of original improvement than at any period in the future. It can be purchased on the basis of the owner retaining (with restrictions), the mineral, timber, and cultivation rights of the unused part, thus reserving to him a large portion of the material benefits for many years, while, at the same time, he would be relieved of paying taxes on such lands. This procedure will reduce the cost of acquisition, and, at the same time, lessen the opposition of the public and the land owner.

In the discussion by the commentators, frequent mention has been made of the excessive cost of wide rights of way. The cost of anything is relative. Bearing in mind that the life of such land is relatively infinite, the yearly cost is consequently very small; but, for fiscal reasons, it may be advisable to liquidate it within a period of 50 yr, or possibly 100 yr. In Europe, there are rights of way in use to-day which have existed for more than 2 000 yr. The right of way is the very foundation of the pyramid of stage construction.

The initial construction (located off center), may cost less than the land, and in undeveloped regions its standards may be quite low (except for the alignment which is determined by the right of way), until traffic justifies further improvement, but the ultimate construction may cost $200 000 per mile. (In the East, highways have already been constructed through open, easy country which cost $600 000 per mile, exclusive of land.) If the right of way proves inadequate, the previous heavy investment in physical construction may be rendered impotent because when a highway is relocated to a new right of way, all improvements in the form of grading, sub-drainage, stabilization of sub-grade and slopes, drainage, bridges, and landscaping are lost. It is a type of financial failure. If the right of way is of proper alignment and of sufficient width, each step in advancing the standards will make full use of all previous effort and expenditure, thus conserving the people's investment and producing, step by step, a more perfect and up-to-date element of transportation.

It is difficult for those who reside in rural or undeveloped regions to visualize the throttling effect of growing population and development upon inadequate rights of way. Many troubles to-day are caused by the failure of previous generations to realize the possibilities of the future. It was difficult for them to visualize the effects of growth. Many wholly undeveloped regions through which right of way may be purchased at cheap prices, may suffer 100 yr from now if a niggardly right-of-way policy is pursued to-day.

An element of cost not sufficiently stressed is the adverse effect upon, and economic loss sustained by, farmers and property owners each time a highway is relocated. Many farms and parcels of property have been cut up into unusable or uneconomic pieces. Areas containing abandoned pavements usually are not reclaimed and put back into any form of production, and an abandoned right of way seldom finds its way back on the tax books. Certainly highway authorities should not subject land owners to this form of inconvenience and loss more than is necessary. Permanent right of way, of adequate width, would solve the difficulty and tend to stabilize land values, particularly when the freeway principle is used.

The present period appears most opportune to advance the cause of wide rights of way, in view of the continued statements from high governmental sources relative to retiring large tracts of land on the theory that more land is in use than is economically justified, and highway engineers should be quick to seize the opportunity of informing the public of the lasting benefits which will accrue to the nation from an enlightened right-of-way policy.

Classification.—Mr. Diehl brings up the very pertinent question of highway classification. The subject is not only important but pressing. The road must not only be classified according to present and future traffic trends, but it is quite as vital for it to be classified on the basis of safe operating speed. Fortunately, the groundwork[60] on which to base a classification is being laid by forty States in co-operation with the U. S. Bureau of Public Roads. Whether full and proper use will be made of these transport surveys depends on the initiative and unselfish public spirit of officials and engineers. For the first time, it will be possible to make a real and searching analysis of the social and economic structure of the highway system as a whole.

National Highways.—Without doubt, when the local and regional analysis is completed, thinking will be directed toward the possibility of a national system of highways, designed, constructed and maintained by the Federal Government. This is indicated by Mr. Dennis, and such a system may not be as fantastic as would appear at first thought. Undoubtedly, it would prove a real national asset not only for the every-day needs of the nation, but also as a vital unit in the national defense system. As Mr. Dennis intimates, a national system may be likened to the trunk lines of the railways, with the present system acting as the branch or feeder lines.

If it is assumed that 10 000 miles would constitute a reasonably complete system, it could be constructed for approximately $3 000 000 000, and if, as Mr Diehl suggests, it is developed within a period of 30 yr, the annual appropriation required ($100 000 000), is only a moderate portion of the taxes which the Federal Government at present imposes on motor-car owners and the motor industry. In view of the $15 000 000 000 quoted by Colonel Crosby[29] as having been spent on the existing highway system in the United States during the past 36 yr, $3 000 000 000 does not appear out of proportion.

In connection with the more technical features of the paper, the writer wishes to express his appreciation to those who brought the latest results of research and thinking before the members of the Society. At the time of the preparation of the paper (spring of 1935), the results of certain very illuminating and clarifying tests,[47] by R. A. Moyer, Assoc. M. Am. Soc. C. E., which have since been widely disseminated, had not come to the writer's attention, and he is grateful indeed for the addition of the material.

To simplify matters, and for convenience, it may prove desirable in covering the discussion, to follow the same order as used in the paper.

Dual Highway.—There is further opportunity for service in establishing design standards of the dual highway, because development has reached the interesting stage in which several variations of type are being constructed and subjected to the actual test of traffic. Further analysis of the various classes of accidents occurring on each type, and a thoroughgoing scrutiny of the comparative accident data, may clarify the situation and indicate the proper design to be used for certain particular conditions. Since the discussion did not record current design variations, or any analysis of the philosophy behind the use of particular designs, the writer takes the liberty of attempting to analyze the situation in order to define the issue more clearly in the hope that further thinking will be stimulated. The subject is so new, and there is so little supporting data, that the remarks should be considered only as a personal expression of opinion, and, therefore, fallible.

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[60] *Civil Engineering*, March, 1937, p. 178.

It is granted that a space separating opposing traffic is essential for trunk-line highways carrying any appreciable volume of traffic; but the width and treatment of this space is a very unsettled matter. There are cases when it is physically and financially limited to a width of a few feet, and other cases where it may easily be 150 ft or more. It is obvious that as the width increases, the chances of opposing accidents diminish. For the narrower widths it is clearly necessary to use some kind of curb to prevent impatient drivers from using the space for passing in case of traffic congestion. The treatment in use varies from straight-sided curbs, 6 to 11 in. high, to numerous types of sloping curbs.

When the width is only 3 or 4 ft, the vertical (batter of 6 on 1) curb may be the lesser evil, but it must be recognized that certain "elbow room" has been forfeited. The operator no longer has the chance to dodge sidewise the few feet necessary to avoid a side-swipe accident when he has been crowded or "pinched" by other vehicles. For widths of from 7 to 20 ft, some kind of sloping curb is indicated, the design to be such that it can be mounted at high speed without danger of loss of control. Considerable experimentation is being practiced in New Jersey in an effort to find a solution for this problem, and it is hoped that tests will be made, using different makes of cars operated at various speeds, in order to establish a design which will permit mounting without throwing the car out of control. A curb of improper design may cause a more serious type of accident than one produced by a side-swipe.

Particular attention must be paid to the material between curbs. In wet weather, and during the spring break-up, unstabilized grass plots are very dangerous, the ground being so soft that vehicles are thrown out of control, if by any chance they are forced into the space while traveling at even moderate speeds.

For widths from 20 to 30 ft, a narrow mall or island may be placed in the middle, using smooth stabilized shoulders adjacent to and level with the pavement; and for widths greater than 30 ft, level shoulders with grass and landscaping between is indicated.

It is axiomatic that no form of fixed obstruction, such as poles, light standards, trees, etc., be placed in the dividing safety space. Head-light glare can be eliminated by planting a type of evergreen, the trunk of which will not exceed 2 or 3 in., and such a screen will also tend to act as a cushion for cars out of control.

As a motorist, the writer cannot but be tremendously impressed with the general design developed in Delaware; the operator has a feeling of tranquillity and utter safety not experienced in any of the narrow center-width and curbed designs. Passing is no problem because the motorist keeps to the right without supervision, and the wide, smooth shoulders provide a factor of safety as well as a space for parked vehicles.

It is the writer's belief that the width separating opposing traffic is a function of safe allowable speed. As the width decreases, the speed should decrease. Where it is necessary to reduce the width for financial reasons, the additional cost to the State of policing to insure reduced speed, as well as an appraisal of time losses, should be added to the financial analysis. (It should be remembered that the predominating reason for the safety record in the Holland Tunnel is the presence of police officers stationed at frequent intervals to enforce the regulations and prevent motorists from crossing from one lane to the other.) Where restricted widths are required, it may be necessary for the designing engineer to arrange with the State Police to provide sufficient officers to enforce compliance, rigidly, at all hours, with the safe speed established by the engineer. The public will not reduce speed unless an adequate number of officers are present to compel obedience.

Thus far there are no available data to support the idea that safe speed is a function of the safety island width or from which to deduce recommendation of the safe speed for particular widths and it is hoped that research will be instituted in order to establish basic principles.

Mr. Carpenter states that adoption of the dual type will not prevent overtaking accidents. That may be true, but it will greatly reduce them. On the ordinary two-way highway the presence of an oncoming car causes many such accidents, because the passing car is often forced to "crowd" the car being passed in an effort to avoid a head-on collision. This condition is entirely absent on a properly designed dual highway, and thus passing accidents should be reduced to a very small percentage.

Mr. Lewis is to be congratulated on his able presentation of the subject as related to urban regions. The discussion of the freeway is most opportune, and highway engineers should read and digest all he has written with great care. The mention of the throttling effect of commercial development on the transportation efficiency of U. S. Route 1 in New Jersey is pertinent, and particular attention is directed to his statement respecting the reasonableness of barring access from property not directly assessed for the cost of highway improvements. The restriction of access should be no more serious than conditions imposed by the private railway lines traversing the United States.

In connection with Fig. 8, the writer wishes to inject a word of caution respecting the planting of trees adjacent to motor ways carrying high-speed traffic. He feels that no trees, other than small (2 in. to 3 in.

in diameter) ornamental trees, should be planted within a line 50 ft from the shoulder of the highway. The car is not forcibly confined to a track, and often gets out of control. Many lives have been sacrificed on trees, and, therefore, these hazards should not be planted within range of the destructive effects of a motor car.

Colonel Crosby raises the interesting question as to the effect of the "blind spot" in rear vision on safely entering the main highway from the acceleration lane. There is no reason why manufacturers cannot solve the "blind spot" problem by the proper use of rear vision mirrors or periscopes, but since they have not, design practice must admit this flaw. The matter at once resolves itself into a choice between two evils.

The right-angle intersection is dangerous because vehicles wishing to turn usually approach at a higher rate of speed than practicable for the turn, thus causing the vehicle to "swing wide," usually across the first lane into the second. The entering car usually appears without warning, swings into the main road, effectively blocking both lanes, and, not having any great velocity in the direction of the main-road traffic, speed differentials are great. The only recourse of the motorist on the main road is to swing out on to the shoulder, but often the condition occurs so quickly that he has crashed into the entering car before he realizes the situation.

In contrast, the acceleration lane places the entering car in full view of the motorist on the main road for a period of several seconds, thus giving him sufficient time to move over into the left-hand lane. There is no temptation for the entering car to crowd over into the left-hand lane, and the speed differential of the two cars will not be great. The angle of approach is shown so small in Fig. 2 that cars on the main road will begin to appear in present-design rear-view mirrors before the entering car actually begins to encroach in the right-hand lane. Access lanes coming in at more of an angle than shown, will exclude rear-view mirror vision and, at the same time, any angle short of 90° will not provide full visibility for the motorist looking back through the side window.

The access road condition shown in Fig. 2 is probably no more hazardous than an ordinary turnout for passing; certainly it is less hazardous than weaving and passing to right and left as practiced on present multiple-lane highways. This short statement is not intended to refute Colonel Crosby's well-warranted criticism, but only to bring out the comparative hazards of the situation.

Mr. Barnett suggests that the truck lanes be placed on the outside, with the high-speed lanes on the inside, in order to eliminate the expense of "fly-under" crossings. The condition is illustrated in Fig. 5 as contrasted with the original suggestion shown in Fig. 2.

The fundamental philosophy of the paper is that the highway should be designed for basic operating safety. The design shown in Fig. 5 requires the passenger vehicle to merge with the right-hand lane of truck traffic, pass across to the second lane, leave the second truck lane, and lastly merge with the passenger vehicles. This doubles the exposure to accident, particularly in passing across the truck lanes. The public is quite aware of the hazards entailed in maneuvering around trucks because of their speed, length, and size.

Access to inside truck lanes can be provided at an average approximate expense of $100 000 even in flat country (complete with four separate single lanes of traffic). Because the existing system of highways will carry all short-haul traffic and act as feeders to the express routes, truck access need not be spaced at intervals closer than 20 miles. Thus, such "fly-under" (under or over) connections will only add approximately $5 000 per mile to the cost, which does not appear out of proportion when the entire cost per mile of express highways is considered.

Superelevation and Curves.—The friction factor, f, has been added by several commentators in order to complete Equation (1). There is a difference of opinion among the commentators, however, as to the proper value of f to use, the discussion centering on the effect of centrifugal force acting on the driver and passengers. The problem in reality is divided into two parts by climatic conditions. In regions subjected to ice and snow, the low coefficient of tires on ice will govern; in warm climates, the effect of centrifugal force on the driver will govern. Because of the seriousness of side-skid on a curve at high speed, the writer feels that design should be conservative and that some factor of safety be provided in addition to that subsequently treated under the heading, "Factors of Safety."

Tests[47] indicate that skidding on ice occurs at friction factors ranging from 0.05 to 0.13. The writer has side-skidded on a curve at a friction factor of 0.04. It is recommended, therefore, that a friction factor of $f = 0.03$ be adopted for icy conditions up to the limiting speed that a car can operate on ice on a tangent. Superelevating a curve for icy conditions, for a speed greater than that at which a vehicle can operate on ice on a tangent, is unnecessary. Where ice is not a problem, centrifugal force on the driver will govern. Several recommendations have been made by the discussers, but the writer's tests confirm the recommendations of Mr. Wiley, namely, $f = 0.10$. The writer's experience coincides with that of Professor Moyer and Mr. Wiley, that a certain amount of skill not possessed by all drivers, is required to drive around a curve at a speed which develops a friction factor as great as 0.15.

In utilizing the foregoing recommended values, the radii of curves become so large that discomfort and possibly a hazard is experienced driving around them at the lower-speed ranges if the superelevation is a great as 1.25 in. per ft. It is recommended, therefore, that the transverse bank or slope be limited to 5%, provided, of course, the radii are expanded the proper amount based on the foregoing assumption. To illustrate, Fig. 10(a) shows the speed-radius relation for one cross-slope, 5%, and the design curve is based on icy conditions governing only up to a speed of 60 miles per hr, on the theory that operation at greater speeds on ice, on a tangent, is impracticable. For speeds greater than 60 miles per hr, it is based on friction factors varying (approximately) uniformly from a value of 0.10 to 0.08. The intermediate curve shows the speed-radius relation when the centrifugal force is exactly balanced by the superelevation. The lower curve, showing minimum speeds at which it is comfortable to operate, is based on the observation that it is fairly comfortable to operate for a limited period on a pavement on a tangent which is crowned 2 in. in 10 ft., and this amount of "unbalance" toward the inside of the curve is assumed as permissible. Fig. 10(*b*) is similar except that it is based on a superelevation of 8%, which is approximately 1 in. per ft. A study of these diagrams is instructive, especially as related to speeds below the "balanced" curve.

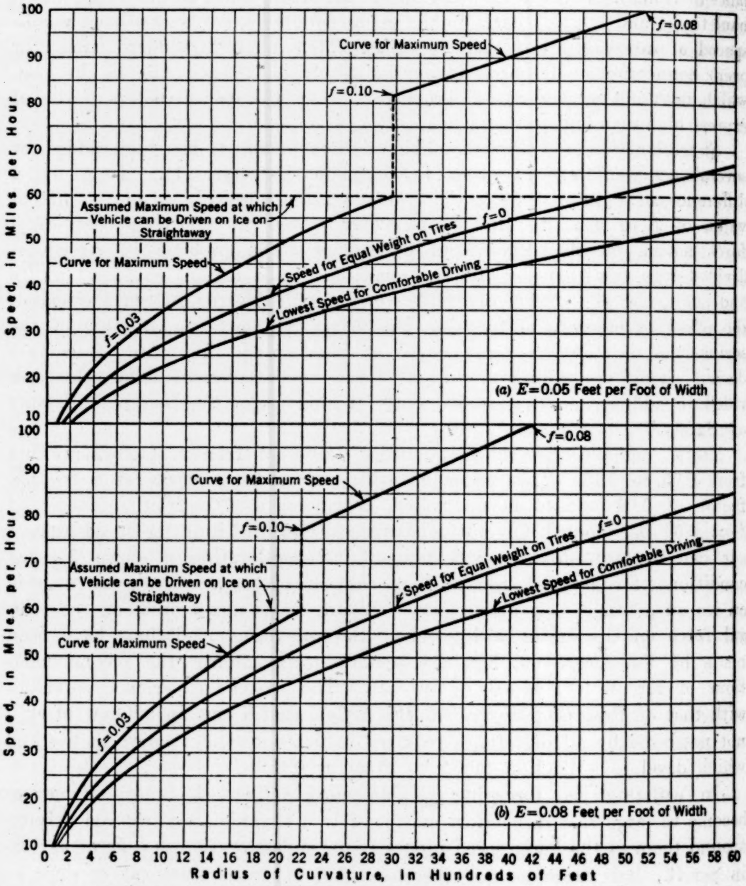

FIG. 10.—RELATION BETWEEN SPEED AND RADIUS OF CURVATURE IN REGIONS SUBJECT TO ICE AND SNOW

Before leaving this phase of curves, it will be illuminating to quote from a paper[51] by Professor Moyer, as follows:

> "A significant feature of the slippage tests is the magnitude of the slippage as the speed of the car is increased. The trend indicates that the slippage at speeds of 80 miles an hour or more is so large on curves sharper than 3 deg [$r = 1\,910$ ft] that the most skillful driver will have difficulty in steering the car and holding it within a 10-ft traffic lane."

Transition Spirals.—The writer wishes at once to state that the necessity for transition spirals where high speed is concerned, has been conclusively demonstrated by the discussers. There appears to be no agreement, however, as to the proper length to use. In actual practice the rate of change of centrifugal acceleration ceases to be a criterion because superelevation cancels centrifugal force for a given superelevation-speed-radius combination. The factors affecting length, then, reduce to the time interval required for the operator to turn the wheel and the permissible rate of rotation. No one has commented on the time interval and it is hoped that engineers will continue to study this problem.

The profession is indebted to Mr. Haile for presenting his analysis of the principles of rotation as related to superelevation transitions. Such analyses are a step in the right direction, and it is suggested that investigation be continued to establish well substantiated values for the length, L_v, and the assumption that 0.027 represents a correct value for the expression, $\frac{2\,V^2_2\,(E_2 - E_1)}{L^2_2}$. The correctness of the result is dependent on the proper assumption of these values.

The writer is not so sure that railway-highway conditions are strictly analogous as related to superelevation transitions. The wheel-base of railway coaches is so great that a car will twist if the rate of change of superelevation is too rapid. The wheel-flange striking the outer rail is also a factor. Those conditions are not present in the highway problem. He also wishes to question the general assumption that very long spirals are desirable. It is possible that spirals longer than required to turn the steering wheel properly will require a nicety of "balance" and judgment in steering not possessed by all drivers. Thus, there may be a tendency for the car to "wobble" in the lane, due to the necessity of changing the steering wheel constantly, and developing different slippage angles. It is suggested, therefore, that additional data be accumulated for the purpose of establishing design practice.

Vertical Curves.—It is disappointing that no contribution was submitted on the problem of the influence of the head-light beam on the lengths of vertical curves. Mr. Haile states that the normal length of the head-light beam will not be affected if vertical curves provide a sight distance at summits equal to the distance required to stop the car. This becomes a reality if stopping distance is based on icy conditions. The general problem is divided into two parts, at sags and at summits.

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[51] *Civil Engineering,* February, 1937, p. 114.

At sags, the factors are, length of head-light beam, height of the lamp above the pavement, the vertical angle which the upper beam makes with the horizontal axis of the lamp, and the algebraic difference of the grades. With known factors, a graphical solution is simple and may be applied during the process of establishing profile grades. A mathematical solution is simple, but it requires solving simple equations for two cases in order to determine the limiting curve and is not as rapid, therefore, as the graphical method. The vertical curves required for this condition are not particularly long, but some designers may have to revise their ideas of vertical curve lengths somewhat. All motorists are aware of the experience at sags of having the head-light beam cut off by the ascending grade ahead, and seeing the light advancing up the pavement until the car reaches a point on the curve where the normal length of beam is restored.

At summits, the rounding curve intercepts the light rays and the down-curving pavement beyond this tangent point is in darkness, the head-lights piercing only space, giving the impression of a black wall ahead or that the car is about to plunge into an abyss. A mathematical analysis involving the differential calculus produces formulas (two cases), which yield startling figures. For instance, with an algebraic difference in the grades of 8% and a head-light beam of 400 ft, a vertical curve 2 100 ft long is indicated. A vertical curve of more than 3 000 ft is required, if a sight distance (during daylight), of 1 330 ft is desired, which illustrates the correctness of Mr. Haile's observation.

In view of the lack of corroborative evidence, it is hoped that tests and observations will be conducted for the purpose of establishing practical design procedure. Space does not permit, nor is it proper to present at this time, the development of the formulas mentioned. Mr. Haile states that passing would not be permitted at the summits of two-way highways. The writer knows of no positive method of preventing motorists from passing on summits, unless a police officer is stationed on every summit.

Grades.—Many commentators question the value of limiting maximum grades for the purpose of providing reasonable safety in descent on icy pavements. If traffic is so light that not more than two or three cars are present in any mile, safety in descent then affects the individual car and that car only will be involved in a skidding accident, but where the traffic is heavy, with cars immediately adjacent to each other, a skid promptly involves other cars, with the result that from two to seven cars become snarled together, often with disasterous results. For instance, the Operation Department of the Port of New York Authority experiences difficulty as soon as ice forms on the short 4% grade of the New York approach to the George Washington Bridge, in spite of the fact that cinders are applied almost immediately. It has also been found necessary to close the Pulaski Skyway on Route 25, New Jersey, at certain times when ice has formed. The maximum grades on this structure are 3.5 per cent. Ice can form in a few minutes, and a maintenance crew with many miles to cover will find it difficut to spread cinders promptly. Consideration should also be given to the limiting friction factor of ice, which varies from 0.05 to 0.13.

It is true that, if locating engineers are looking for an "easy way out," the adoption of 8% and 10% grades will lessen their difficulties and shorten the time required for the location. This attitude is somewhat reminiscent of the early days of railway location. However, after the railways had passed from the pioneering period and became engaged in the reconstruction of their lines, the attitude disappeared. There are cases on record of the location work taking a considerably longer period of time to complete than the actual construction. Indeed, studies for some railway lines have extended over a period of ten years. Do highway engineers consider railway location more important than highway location, when the direct relation of alignment and grades upon vehicle safety is considered?

There should be a realization that the location of modern trunk highway routes is a very important matter, and all possibilities should be investigated in order that the best obtainable route and individual location be secured. If the physical conditions of a trunk route are of such a nature as to require 8% grades, that should be sufficient cause for its rejection, if it is in a climate subject to ice and snow.

Sight Distance.—Messrs. Barnett and Haile are quite correct in stating that minimum sight distance should not be computed based on 17.4 ft per sec per sec, because of the high friction developed between tires and road surface in order to decelerate at that rate This point should be particularly emphasized. Both suggest a friction value of 0.40, which is equal to a deceleration of 12.88 ft per sec per sec. This is greatly superior to the value given in the paper, and its use is recommended for climates not subject to ice and snow; consequently, Fig. 4 should be disregarded.

In regions subject to ice formation, the low friction coefficient of tires, on ice, should govern up to the maximum speed possible on such a surface. Because of the comparatively long distances resulting, and because the seriousness of an accident from this cause (within the lengths prescribed), is less than from insufficient superelevation, a friction value of 0.10 has been assumed. A new diagram, Fig. 11, showing stopping distances, has been prepared to be used instead of Fig. 4. The writer realizes that the diagram may be criticized on the basis of showing distances that are too short to care properly for icy conditions in view of tests[47] which show coefficients on ice as low as 0.05.

The question may also be asked whether there is any real justification for designing on the basis of icy conditions in localities where ice may not occur more than a few days during each year. The answer to this question may be found in the well-stablished practice of other branches of engineering. A building, for example, may not have to withstand the wind loads for which it is designed more than once in 10 yr, or flood-control dams, levees, and spillways may not be called upon to withstand full flood height for which these works are designed, more than once in

20 yr; yet engineers feel it is sound policy to design for the maximum condition. In this connection it may be stated that highway engineers apparently are more flood conscious than accident conscious, as it is quite usual to construct highways above the highest recorded flood level.

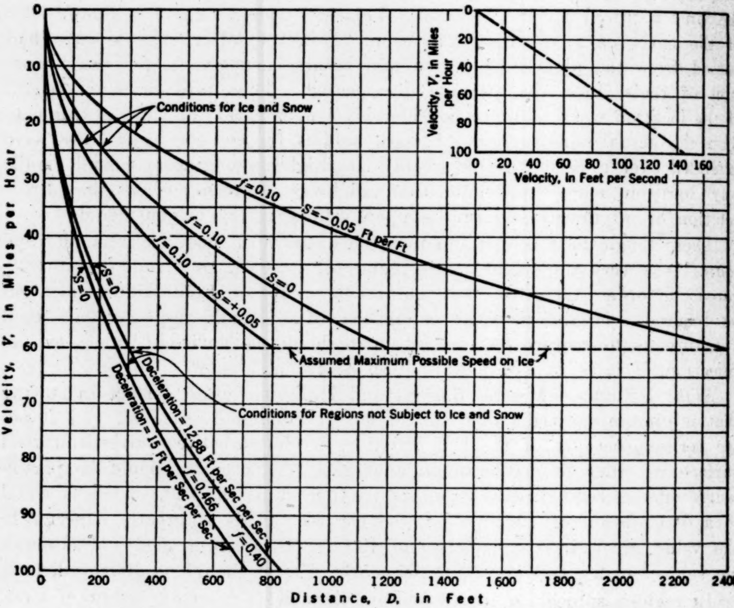

FIG. 11.—DISTANCE REQUIRED TO BRING A MOTOR CAR EQUIPPED WITH FOUR-WHEEL BRAKES TO A STOP AFTER BRAKES ARE APPLIED

It is to be noted from Fig. 11 that a speed of 60 miles per hr under icy conditions, requires a greater stopping distance than a speed of 100 miles per hr, with dry and wet surfaces. For a speed of 60 miles per hr on ice, a sight distance of 1 332 ft (1 200 ft + 132 ft) is required on level grade for a one-way (dual) highway. In order to obtain this sight distance on a curve (with a central angle greater than 14°), and assuming that all obstructions are cleared for a distance of 40 ft from the edge of the pavement, a curve with a radius greater than 5 300 ft, will have to be adopted. In other words, if all trees, cut banks, or other obstructions are kept uniformly 40 ft from the edge of the pavement, it will be necessary to adopt a curve of not less than 5 300-ft radius in order to obtain a sight distance of 1 332 ft.

Thus, in general, it is evident that if the design is based on icy conditions, sight-distance requirements will compel the use of curves of sufficient radii to care for speeds of 100 miles per hr.

Safety on Tangents.—There has been an implied assumption by some discussers that a tangent should be considered as a criterion of safety, merely because it is straight. Straightness alone does not assure safety. The mere fact that there are more miles of tangent than curves is not the only reason that more accidents occur on tangents than on curves. There are sound reasons for these accidents. Often a fairly long tangent is encountered after a series of sharp curves. This is an invitation to the motorist to speed, and frequently other operating conditions not connected with straightness are such that there is no justification for an increase in speed.

A tangent with no separation between opposing traffic, narrow soft shoulders, deep ditches, heavy crown, steep grade, and with poles, trees, head walls, bridge walls, and guard rails near the pavement, is surely more dangerous than a properly designed curve. Such a tangent, however, is in direct contradiction to the spirit of the paper. An analysis of accidents which can occur on a tangent of this nature may be enumerated as follows: Head-on collision; rear-end collision, with car standing on the pavement; side-swipe due to passing; forced off road by passing car and overturning in ditch; car sufficiently out of control due to soft shoulder to collide with tree, utility pole, guard fence, or other obstruction; car skidding with the aforementioned results; and a blow-out or other mechanical failure, resulting in all the foregoing types of accidents.

With separated roadways, wide smooth stabilized shoulders free of all obstructions, adequate sight distance, smooth profiles, and reasonable grades, some types of these accidents would be eliminated entirely, and the seriousness of a large number of accidents would be minimized. Sufficient time, space, and "elbow room" would permit a vehicle to recover from a minor mishap in time to prevent disaster.

It is not contended that a properly designed curve of sufficient radius, adequate superelevation, with well designed spirals, is more dangerous than any other part of the road structure. The philosophy of the paper is that hazards should be removed from the highway whether they occur on a tangent, on a curve, in a tunnel, on a bridge, in urban regions, or in the open countryside.

Statistics.—Several discussers presented detailed accident statistics and have deduced certain conclusions therefrom. Statistics may prove misleading unless thoroughly analyzed by a competent statistician. With no attempt to treat a subject with which the writer is unfamiliar, it may be pertinent, however, to point out certain of the more obvious pitfalls connected with traffic-accident statistics. In analyzing such records, the relative values that must be assigned to certain figures should be borne clearly in mind. For instance, unless it is realized that less vehicle miles are operated at night than during the day, the erroneous conclusion might be reached that it is less hazardous to motor at night than during the day, when it is noted in Table 5 (*f*) that 58% of all accidents occur during daylight, whereas only 42% occur during dusk and darkness. To particularize further, it must be realized that there are less vehicle miles operated during rain, fog, ice, and snow, that certain regions in the United States have little or no rain, certain sections have no ice, and, furthermore, that there are fewer hours of rain than dry weather, less hours of snow and ice on the pavement, and still less hours of actual snowfall and fog. As previously noted, there are more miles of tangent than curves.

When the statistics are properly adjusted to represent the true relation on a basis of comparison of hours-mileage-volume, a clearer picture of the actual exposure to accident for given conditions will result. There is little doubt that it can be shown that the exposure to accident is increased by rain, fog, snow, and ice, and that these are definite factors which should enter into the formula for safe highway design.

Capacity.—Mr. Diehl states correctly that the safe spacing of cars should be the distance required to bring the vehicle to a stop (braking distance + lag distance). He further states that because of this, the capacity of a highway is unduly reduced at the higher speeds. This is true, but not to the extent which some investigators have assumed, since their assumption does not coincide with the driving habits of most motorists. Without condoning or in any way recommending this dangerous practice, it must be realized that operators usually keep a distance back from the car ahead only sufficient to care for the space covered during the lag period. The motorist relies on being able to decelerate at the same rate as the car in front. If a lag period of 1.5 sec is assumed, this distance approaches the braking distance at the lower speeds, but is only a fraction of the distance at the higher speeds. This becomes very striking if the total stopping distance is computed for icy conditions.

Factors of Safety.—A careful study of Figs. 10 and 11, shows that the factor of safety for sight distances and curves is either zero or very low if the design for the actual expected speed is based on these diagrams. As Professor Hawthorn states, "the driver's control over his vehicle varies, other conditions being equal, with the square of the speed." In a rough sense, this relationishp can be used as a measure of factors of safety. In other words, if Figs. 10 and 11 are used in the sense that values derived therefrom represent the ultimate strength of a material, then established engineering practice requires that some factor of safety be applied. If the "ultimate strength" is assumed as 100 miles per hr, then a factor of safety of 1.5 is provided at a speed of 80 miles per hr, 2 at 70 miles per hr, and somewhat less than 3 at 60 miles per hr. It should be remembered that a factor of safety (against ultimate strength) of 3 is in established use for such a uniform and consistent material as structural steel. Fully realizing that such a method is not strictly scientific as applied to highways, and yet it does form some measurement of safety.

If, then, it is desired really to design a highway for safe operation for a speed of 60 miles per hr, curves, spirals, superelevation, and sight distance (except where icy conditions require greater values), should be designed on the basis of a speed of 100 miles per hr. This will give a measure of safety in a longitudinal direction.

Safety in a lateral direction is less easily measured. It is disappointing that the discussion did not contain any data and suggestions from which a start could be made to appraise factors of safety in a lateral direction. For instance, if the pavement was flanked by smooth, stabilized shoulders of infinite width, with all obstructions removed, complete lateral safety would be provided for many types of "out-of-control" skidding and "driver-asleep" accidents. Conversely, if the shoulder were only 1 ft wide, the accident hazard would be greatly increased. Manifestly, it is utterly impossible to provide shoulders of infinite width, but there must be some measure of the factor of safety for any given speed-shoulder-width combination. This problem is not simple, and, therefore, presents a challenge.

In this connection, certain engineers have assumed that once a vehicle is out of control, the responsibility of the highway designer ceases. If this assumption were logical, then from force of logic the conclusion would be reached that the operator should always drive perfectly, in which case there would be no highway problem. Since it is physically impossible for the motorist to drive perfectly, and since the results of an accident are the same from whatever cause, the designer does have a responsibility and, therefore, should make a reasonable allowance for the case in which the car is out of control.

For example, as Mr. Lavis suggests, somnolence is a factor which should be considered. Any one may be guilty of it, and a certain amount of protection should be provided. One-way roadways, with sufficient width between, furnish a certain degree of protection to the left of the motorist; the shoulder furnishes protection to the right. The slight additional roughness of the shoulder will awaken most drowsy drivers, and if given space and time, the car can regain the pavement without mishap. The same conditions apply to inattentive operators.

The statements of Messrs. Carpenter and Wiley should be emphasized, that the minimum standard often becomes "the" standard, and is used when better standards could be applied with little, if any, addition in cost. Many times the designer is so engrossed in solving structural or mathematical problems that he becomes "blind" to the essential issue of designing for basic operating safety, and, consequently, unwittingly inserts unnecessary hazards to operation. Often it is no more expensive to design safely, but there are many times when safe design requires courage and conviction on the part of the designer. Professor Alvord has performed a service in emphasizing that under-design is unprofessional. It should be repeated that consistency in design is the very soul of safe design.

The highway should be considered as a unit, and uniformity and consistency should be a feature of the results, whether the route be in the city, in a tunnel, on a bridge, or in the open countryside, to the end that it efficiently and safely serves its function as an element of transportation.

Application to Broad Field.—Mr. Carpenter is acute and is quite correct in his assumption that the writer's interest in safety also extends beyond the limits of the express highway. It covers the entire highway field, including farm roads and city streets, not only as related to new construction, but also to the removal of hazards from the existing system.

Design in this broad field must be approached with new vision, and enlightened standards must be established, based on a definite design speed, to the end that operating safety is the basic philosophy. As stated in the paper, the selection of the design speed for any particular class of road is one of the most important decisions connected with the design. The future possibilities of use and traffic volume must be determined accurately in order that adequate right of way and proper alignment be obtained. With such provisions, the initial design speed for remote roads may be set less than the ultimate as far as profile, graded width, type of pavement, and details are concerned, if economy is paramount. With such a procedure, speed standards may be improved from time to time without appreciable loss of investment. If the speed standard is less than the legal limit, the permissible rate of speed should be posted at frequent intervals.

There is a tremendous opportunity to perform a humanitarian service in the removal of hazards from the existing system of highways. The magnitude of the task is apparent when it is remembered that there are more than 3 000 000 miles of rural roads in the United States. There is crying need for a hazard survey to locate and classify hazards according to some rating; and a program with appropriations in each State and each political sub-division of each State for the continued and consistent removal of hazards. The removal of such death-traps is quite certain to yield greater dividends in lives saved than the complete removal of all railway grade crossings.

In order to provide a criterion of operating safety, it is suggested that the entire existing system be classified on the basis of safe allowable speeds. Thus, for example, it may be established that a given section between towns, or other natural division points, may permit vehicles to operate safely at a speed of, say, 35 miles per hr, except for a few particular locations where the safe speed may drop to a lower figure, say, for illustration, 20 miles per hr. These places, then, are of particular hazard, and every effort should be made to reconstruct them to the 35-mile standard at the earliest possible date. Except as related to the elapsed time of travel, it is inconsequential what the safe speed may be—10 miles per hr, or 60 miles per hr—as long as consistency is maintained. It is the unexpected hazard that will continue to take the lives of many conservative motorists on the existing system. The safe speed should be posted as a speed limit on all roads which are of a lower standard than the legal limit, and a gradual change in operating conditions should be provided when passing from one speed classification to another.

The writer realizes that the foregoing attitude is, in part, somewhat contradictory to the spirit expressed in the paper, but the paper applied to new construction, particularly as related to the trunk system. It is manifestly impossible to reconstruct all roads to high-speed standards. A program such as that outlined herein will do much to relieve the accident situation within a relatively short period, certainly before any large volume of new construction can be completed. These roads will always be feeders to the trunk lines.

Expensive Highways not Necessarily Safe.—Certain engineers appear rather complacent with respect to present and past highway-design thinking, and seem to feel that further mental and physical effort is not only unnecessary but somewhat puerile. In the interest of safety, and for that purpose only, it may prove educational if a few instances of expensive projects, which fall short of the philosophy of the paper, are given. Their identity is unimportant since no reflection on the designers is due or intended, and the illustrations are given solely to show that the engineer has not yet brought his thinking up to date as related to safety—he is not yet thoroughly safety conscious.

An express highway, several miles long, was constructed, which cost nearly $5 000 000 per mile, without right-of-way cost. It carries a daily traffic of approximately 45 000 passenger automobiles, trucks not being permitted; yet it has center ramps, the light standards are placed in the center safety island, and although the general posted speed is 35 miles per hr, in several places the posted speed is 20 miles per hr, because of sudden sharp curves and center ramps.

An elevated structure several miles long was constructed at a cost of more than $5 500 000 per mile. There is no physical separation of opposing traffic; it has center ramps, and the roadway has an odd lane. The economic justification for this structure, to a great extent, was based on its carrying a large volume of trucks. Present traffic is approximately 85 000 vehicles daily. During the first six months of operation the accident situation became so serious that trucks were excluded from the structure in an effort to reduce the accident toll.

This structure is a small part of a much larger project which cost an average of more than $1 000 000 per mile. The greater part of the route is of multiple-lane type, four to six lanes wide, constructed at grade in the usual manner, except that all railway and important highway crossings are eliminated by grade separation. When completed, it was considered an outstanding example of modern highway design. Within a year after being opened to traffic, the newspapers and public christened it "Death Highway." Three-car and four-car accidents were common, and six-car and seven-car accidents not uncommon. In the last two or three years, a large mileage has been reconstructed and made over into a divided way, with an island separating opposite direction traffic, at an additional cost of approximately $65 000 per mile. It is reported that accidents have decreased where the roadways have been separated.

These projects were designed by some of the most competent and alert highway engineers in the country, who were using the most modern and up-to-date design methods in vogue at the time. When it is considered that the work was performed within the past decade, the swiftness with which highway design thought is developing becomes apparent. It is also apparent from the unfortunate experience noted that the public is awakening to the situation, and unless the engineer becomes alive to the problem, he is quite likely to find himself in an uncomfortable position from which he may not be able to extricate himself without loss of prestige.

Conclusion.—There has been an assumption on the part of some commentators that the highway engineer is unduly restricted by laymen in the form of commissioners and politicians, and, therefore, of necessity must continue to design "within the money" in accordance with the ideas of other than technical men. This is true to a certain extent, even at the present time, and it indicates that engineers should rise to the occasion and make an intelligent effort to bring the true facts before governing bodies. It is also true that the layman will only value the engineer and his judgment as highly as the engineer values and has confidence in his own competence and judgment. There was a time when engineers designed bridges "within the money" because they were brow-beaten into it against their better judgment. A series of failures convinced the governing bodies that they did not know as much about the forces of Nature as engineers. As a consequence, it is very rare for a bridge to be badly under-designed in this day. The highway is failing to-day, but its failure is not as spectacular and obvious as a bridge failure.

It has been the writer's observation that governing authorities are willing to listen to, and act on, concerted engineering opinion when they are convinced of the hazards involved in disregarding it; but when the engineers are uncertain and not in agreement among themselves as to the proper method of solving a given problem, the governing bodies must not be blamed too much if they take a financially conservative, as well as a politically expedient, attitude.

Consequently, it is of the utmost importance for engineers to bring their ideas more in harmony with each other, and having achieved solidarity, to go forth and spread the gospel of highway safety in relation to design. In the last analysis the engineer will be held responsible by the public for safe highway design. The writer reiterates his conviction that the engineer must have vision and courage.

The paper has been broadened and enriched by the discussion, and, in closing, it is desired to express again appreciation to all the discussers who so generously gave their time and effort to the subject.

In addition to those noted previously, acknowledgment is freely given to the Automobile Manufacturers' Association, and Mr. C. George Krueger, for assistance and courtesy in supplying data and illustrations.

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Paper No. 1978

THE ENGINEER, AN EMPLOYEE-EMPLOYER

ADDRESS AT THE ANNUAL CONVENTION, DETROIT, MICHIGAN, JULY 21, 1937

By Louis C. Hill,[1] President, Am. Soc. C. E.

It is not uncommon to-day to read in a newspaper a glowing account of some outstanding engineering work and in the same issue of that paper to be confronted with the news that engineers are organizing into labor unions. Thus, we have public recognition that this is the age of the engineer, that the material progress of the world to-day is dependent upon his achievements; and, yet, engineers are resorting to collective bargaining because of a lack of economic recognition of the individual engineer.

In 1930, John F. Coleman, Hon. M. and Past-President, Am. Soc. C. E., opened his address[2] before the Annual Convention of the American Society of Civil Engineers, with the statement:

> "For years there was a constant cry to the effect that the engineer did not receive the recognition which he deserved. Lately, the same cry is heard although less frequently. In the past there was much to justify such complaint, and even now there is some excuse for it.
>
> "It seems probable, however, that the engineer himself is in great degree responsible for such a state of affairs in that he has been until recent times almost inarticulate in the councils of men; * * *."

During the seven years since the Society was so challenged, this country has passed through a period of severe depression out of which arose the most ambitious program of public works the world has ever known. Engineers by the thousands and tens of thousands have been employed in the design and on the construction of such works, all of which have been publicized to the extent that the names of many projects have become household words.

Yet, whenever engineers get together, at meetings of the Society or in other groups, at social gatherings or in their homes, an inevitable subject of conversation is the lack of professional recognition of the engineer by the public. Do these men mean professional recognition in the sense of public acknowledgment of the importance of engineering work in general, or do they mean something more material which affects them as individuals? I believe that little of the discussion so prevalent to-day has its origin in any failure of the world at large to appreciate the Engineering Profession in the abstract. Rather, I think it is the economic status of the engineer, his material and mundane reward, which is of consuming interest.

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[1] Cons. Engr. (Quinton, Code & Hill-Leeds & Barnard), Los Angeles, Calif.

[2] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1344.

The basic reason for this situation was suggested by the late Harrison P. Eddy, Past-President, Am. Soc. C. E., in his address[3] before the Annual Convention of the Society in 1934, when he defined Engineering as a true profession, although of a different character from those of Law and Medicine. In that address he brought out that lawyers and doctors in large measure are independent professional practitioners, whereas engineers are generally employees. The validity of his distinction is evident from the fact that when one is sick the personal services of a doctor of medicine are sought, and the services of a particular lawyer are engaged when one is faced with legal difficulty. On the other hand, the engineer is employed only rarely by another individual. Only a very few persons have any appreciation of the function of the engineer as an individual, even though every citizen of this country, whose reading goes beyond the tabloids and the sport sheets, must be conscious of the tremendous part which the Engineering Profession plays in affairs of the modern world.

The engineer has little more association with those who make daily use of his works than the men who produce the materials which go into his works have with him. Even the engineer in private practice meets professionally only a few laymen. From the nature of his employment, he deals generally with corporations, both public and private; where he has one client, a lawyer may have twenty and a doctor a hundred. Most engineers, therefore, by the very conditions which make their work possible, are substantially barred from individual personal contacts with the public.

To have a basis for action and at the same time to be frank with ourselves, we must accept the condition that engineers comprise fundamentally an employee group in which the world at large has the greatest confidence, but regarding whom the public has little individual concern. Going a little further with our introspective analysis, we must consider the significance of the unique fact that ours is the only profession in modern society which both works for itself and employs itself. It is because of this anomaly that each engineer is in great measure responsible for the status of his profession, both abstractly and concretely.

The current roster reveals that approximately 42% of the entire membership of the Society are in the public employ. Almost half of these are employees of the Federal Government; the others work for various State and county governments, municipalities, and other political subdivisions. About 6% of the members are connected with colleges and universities, many of which are State institutions. In round numbers, then, one-half the members of the American Society of Civil Engineers in all grades are employed in some capacity by governmental agencies. Probably a greater proportion of the civil engineers not members of this Society are likewise employed.

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[3] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1383.

About two-thirds of the other half of the membership are employees of private corporations, a few in high executive positions, the great mass in subordinate or employee positions. Little more than 10% of the total membership are evidently in private practice as consulting engineers or as principals and associates of engineering firms. The unclassified remainder includes those retired, or temporarily unemployed, and those engaged in special pursuits.

This predominance of the employee status in Society membership is even more significant when the occupational distribution of the members in the various grades is considered. Substantially 34% of those in the grade of Member and 42% of the Associate Members are employed by some governmental agency; nearly 60% of the young men in the Society, that is, those in the grade of Junior, are likewise governmental employees. If to these be added the hundreds who are on the faculties of institutions supported by taxation, it is apparent that many more than one-third of the Members, almost one-half of the Associate Members, and nearly two-thirds of the Juniors are public employees.

This greater proportion of young engineers who are employees of governmental agencies may be due in part to the fact that private practice is largely closed to them, but more likely it is due to changing conditions. Comparable statistics of twenty years ago probably would have shown that more than one-half the younger engineers in the Society were in the employ of private firms and corporations. Such a conclusion follows from analysis of the membership distribution of those now in the employ of private corporations: in that, roughly one-third the Corporate Members of the Society are corporation employees, as compared to only about one-quarter of the Juniors. It is evident, therefore, that any consideration of the economic status of the engineer must take into account the evident trend toward governmental employment.

Anomalous as it may seem, a large proportion of the engineers who are employees are at the same time employers of engineers. Particularly is this true of those in responsible charge of work, even though few engineers may be employers in the sense that they have authority to engage, to retain, or to dismiss subordinates according to their individual ideas of policy. In a very large measure, however, all of us who are Corporate Members of the Society are responsible for the work done by our subordinates and their assistants, and rarely will changes in their positions or salaries be made by our superiors or employers except on our recommendation.

In general, therefore, the engineer in responsible charge of work occupies the dual position of an employer of engineers while he himself is an employee of a public or a private corporation. To the same extent that he is responsible to his superiors for the execution of engineering work done by his subordinates, the engineer in charge is responsible to his subordinates for any recognition or lack of recognition given them by his superiors.

Much of this is not new to us. Other Presidents of the Society have pointed out that the responsibility is our own, and in papers read at the Annual Convention at Portland, Ore., in 1936, attention was called to conditions affecting the status of the engineer. Thus, it may well be asked: What has been done to advance the Engineering Profession in the eyes of the public and what has been done to improve the social and economic status of the individual members of the profession?

As to the former question, great progress has been made: Engineering works are news to-day and, in the abstract, the world at large has conceded to our profession the recognition we deserve. In all this the American Society of Civil Engineers has played a major part. Also, as a Society, we aided many engineers in obtaining employment during the depression, and we were effective in establishing salary scales for engineers on emergency relief works which were at least as high as the wages paid to skilled labor. Again, as a Society, we are seeking to establish standards for the proper compensation of engineers in the employ of public and private corporations.

However, so definite have become the demands for more effective action and more tangible results that quasi-technical organizations have been formed which have as their primary objective the improvement of the economic status of the engineer. Guilds and other associations are in existence for similar purposes, which make no pretense of being technical in character, and in some localities the younger engineers and engineering aides have gone so far as to join militant labor organizations.

It should be obvious that this trend toward trade unionism, if permitted to continue, will destroy whatever standing the engineer has obtained as a member of an acknowledged profession. Hence, it is imperative that we consider what line of action is available to us other than the direct action of collective bargaining.

It is evident to me that the answer lies in recognition of the responsibility of each engineer to his subordinates. I wish to emphasize that point: It means that most of the members of this Society must assume, actively and individually, the obligations of the dual function of employee-employer. Such a duty goes far beyond the administration of the work of engineers subordinate to him; it involves real concern for the working conditions of those subordinates and for the monetary and other rewards they receive for the work which they do.

With full appreciation of the altruism, ideals, and ethics of our profession, we must admit that the desire for money, with the power and security that money gives, is the controlling motive of modern civilization. Rightly or wrongly, the world to-day measures its recognition of the work of the individual by a money standard. Such being the situation, it is desirable and proper that each engineer should further the interests of every engineer subordinate to him by emphasizing to his superior or employer the importance of the engineering problem, the accuracy of execution, the responsibility, and the integrity of purpose that characterize the work of his subordinates. Just as often as he does so, he will take a positive step toward improvement of the status of his profession and at the same time promote his own well-being. On the contrary, each time such an engineer understates the worth of engineering services, he does positive injury to the profession and to himself.

In this latter category falls the consulting engineer who obtains work on the basis of its cheapness, and thus is required to hold down the rates of pay to his subordinates. The engineering executive of a governmental agency who promotes the use of his personnel by political subdivisions at costs to them which cannot be met by engineers in private practice likewise does a direct injury to the Engineering Profession. Similarly, each time that the chief engineer of a corporation reports to the executive officer that he has saved money by employing engineering talent at the market price for skilled labor, he belittles his profession, and he brings nearer the day when he himself becomes only the foreman of such labor, an honorable but not a professional position.

Such practices were challenged seven years ago by Past-President Coleman when he told the Society that the standing of the engineer was his own responsibility and that advancement must come from within the profession.[2] Instead of accepting this responsibility as an individual, the engineer generally has shifted it to some organization with the hope that something would be done. When he did so, and failed to sell to his superiors the value of the services of his subordinates, he failed also in his obligation to the profession. Improvement in the social and economic status of the members of the Engineering Profession can come as a result of our own individual efforts, if every engineer will acknowledge the responsibility of the employee-employer relation which is peculiar to the Engineering Profession, if also he will accept the obligation of this relationship, and, finally, if he will make that obligation the basis of his own professional action.

Failure to accept that obligation will jeopardize the professional standing of engineers and engineering, because any further trend toward trade unionism among engineers will destroy public confidence in us as members of a true profession. By our own efforts we can correct the conditions which already have forced many engineers to put aside the standards of the profession and accept those of a trade instead; but effective action may not be postponed and the responsibility rests directly upon the individual members of the American Society of Civil Engineers.

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STRUCTURAL APPLICATION OF STEEL AND LIGHT-WEIGHT ALLOYS
A SYMPOSIUM

PAGE

FOREWORD

By Jonathan Jones, M. Am. Soc. C. E. 1181

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FOREWORD

By Jonathan Jones[1], M. Am. Soc. C. E.

The automobile of to-day has been made possible—not merely made cheap, but actually made possible—by new types of steels, new treatments of steels, new knowledge regarding the properties of steels and how to control them, and, also, in some degree, by a corresponding development of the light structural metals. To-day's airplane also is the product of this research into, and development of, new metallic alloys. Where can be seen to-day, the airplane cloth so essential to the aviation of the World War? In the last few years, a new development has begun toward light-weight railway rolling stock based, again, on the newest metallic alloys.

In view of such rapid developments in various industries, of which the writer has mentioned three, it is incumbent upon any metal-using industry (upon none more than upon the fabricators of bridge and building frameworks and similar structures) to inquire what new economies, what new possibilities, may be anticipated in the structural field from the newer knowledge and the newer structural materials.

For the first quarter of the Twentieth Century this field was substantially stationary. Almost without exception the framed structures of that period were constructed of open-hearth steels of virtually a single simple class. The designers of a few monumental structures turned to nickel, manganese, or silicon for help, but these exceptional incidents made no difference in the average, or even in fairly impressive, bridge spans, and meant literally nothing to thousands of structural designers.

It would seem to-day as if that stationary condition is broken not to be resumed, or certainly not until after a long period of experimentation and development. New structural steels and light structural alloys are being pressed upon the structural engineer, not always, perhaps, with a very clear understanding of what he needs. Any survey of the situation, therefore, can be as of to-day only, but nevertheless it is unfortunate that neither as of yesterday nor of to-day has any such survey been undertaken, and the results arranged so that the average structural engineer may appreciate what is, and what is not, being done with new structural metals, and what may, and what may not, be "around the corner."

To fill this gap, and to perform this service, is the function of this Symposium. The specialists who have contributed papers are known to be both informed and informative; and they discuss questions such as the following:

(1) What does the newest laboratory knowledge reveal as to the important qualities of materials, of how to distinguish the best from the less suitable, of how to define, and attain in modern designs, that elusive function, the factor of safety?

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(2) What new tools for examination of stress conditions should structural engineers take seriously into account?

(3) What will variations in alloy additions, or in manufacturing practice, do to produce desirable qualities in structural steels, and at what cost?

(4) What degree of corrosion resistance is obtainable in materials subject to structural use, and can the price be paid?

(5) What alloys lighter than steel have properties that interest the structural engineer, and what seem to be the economic possibilities of their use?

(6) What has been done thus far, and what promises to-day to be done, in the actual utilization of these special structural metals?

(7) What is the objective in further development of new structural alloys, and for what can a premium be justifiably paid?

These several questions embrace one topic, that of the special structural metals of to-day and their structural use.

Note.—In order that the information imparted in the papers of this Symposium may be of the greatest possible usefulness to Civil Engineers it has been necessary to identify several of the alloys under the trade designations by which they are most commonly known. A consistent effort has been made, however, to avoid advocating special interests, with the understanding that the discussers, likewise, will confine their comments to the intended scope defined in each paper, as bearing on general classes of alloys rather than on individual proprietary types within those groups.

The term, "kips", to denote "kilo-pounds" or "thousands of pounds", has been used by each author because of the opportunity of thus arranging the tables in a more compact and convenient form; and an effort has been made to avoid conflicts in the introduction of the few algebraic symbols involved. The papers are not mathematical as a rule and only a few algebraic symbols have been required. These conform essentially to the American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[2].

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[2] A. S. A.—Z10*a*—1932.

STRUCTURAL APPLICATION OF STEEL AND LIGHT-WEIGHT ALLOYS

A SYMPOSIUM

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MODERN STRESS THEORIES

BY A. V. KARPOV[3], M. AM. SOC. C. E.

SYNOPSIS

Structural design is a field that has not been strictly defined. Commonly, it is assumed to include only stationary structures of a fundamental character. A broader definition would be more in line with the present scientific engineering attitude. The same basic principles that are used in the design of non-moving structures—as, for instance, bridge trusses—are applied in the design of railroad rolling stock, automobile frames, trusses of the lighter-than-air ships, or structural members of airplanes, etc. The inter-relation and reciprocal dependence of these widely different fields of engineering endeavor is growing in importance, extending structural design into fields that commonly are assigned to mechanical engineering.

Due to the experimental and theoretical investigation made during the last few decades and prompted mostly by the needs of automobile and airplane designers, engineering science has undergone a considerable change, not unlike the spectacular changes in physical and chemical sciences. These changes are influencing mechanical designs profoundly, but in so far as structural designers are concerned, there are diverging views concerning the necessity of changes in design conceptions and practice.

For quite some time the structural engineer avoided the issue by assuming that the new developments were confined to problems in mechanical engineering, and, in particular, to its most modern branches, aeronautical and automobile engineering. The rapid advance in other engineering branches necessarily must be reflected in structural engineering.

ENGINEERING DESIGNS AS PROBABILITY PROBLEMS

Every proposed engineering structure presents a probability problem that can be stated as follows: What is the probability that the design decided upon will result in a most suitable structure during the entire period of its assumed useful life?

In its broad sense, an engineering design is an attempt to solve this probability problem by determining the future suitability of an engineering structure, considering the many governing viewpoints. In a structural design the safety of the structure is the most important consideration; economy, utility, and durability are the other viewpoints that are most often considered.

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The determination of its suitability is based on the prediction of the probable future behavior of an engineering design. The difference between the predicted and actual behavior is an indication of the state of engineering knowledge at the time the design was made. This indication is of a very general nature since it encompasses not only the assumption on which the purely engineering aspects of the design were based, but also the general assumptions made, before the structure was built, concerning the future service conditions.

That every engineering structure must function during a certain period of time was always realized. The structures of ancient times in which only materials were used as they occurred in their natural state—as, for instance, natural stones—were very little influenced by aging. Such structures stood for indefinite periods of time.

At present, radically different materials are used which are much more effectively utilized and which have the tendency to change their properties either with time or with stress application or with both. As a general rule, the fact that a structure is adequate and safe under initial service conditions does not assure its future adequacy and safety. Fast-running engines in which numerous and rapid variations of stress are taking part were probably the first that called the attention of engineers, to the fact that a design that was entirely satisfactory during the initial load application may fail after a comparatively short service, due entirely to the changes in the stress-bearing capacity of the material.

The ultimate purpose of every engineering structure is to receive external and gravitational forces that must be distributed, transformed, and transmitted to some outside medium. The purpose of an engineering design is to predict these forces and the influence they will have on the structure during the entire period of its life.

CHANGES IN ENGINEERING ATTITUDE AS TO MATERIALS AND DESIGN METHODS AND ASSUMPTIONS

Present and, probably still more, future engineering progress depends very largely upon the changed engineering attitude as to the materials to be used in engineering designs. Instead of simply utilizing the existing easily obtainable materials, new materials are being developed which are not so easily obtainable and, consequently, are more expensive, but which, for one or a number of reasons, are preferable, or may be even more economical, notwithstanding their higher cost.

In most applications the advantageous and economical use of such materials requires a better utilization than is customary with less expensive materials. The improvement in utilization is attained by a better understanding of the properties of the materials and closer evaluation of the future behavior of the structure.

In the past, for instance, it was considered good engineering to build machinery that was able to carry a much higher load than was specified, reflecting the insufficient engineering knowledge with respect to the requirements and actual performances. With reference to bridges, even at present, the claim is often made that a particular bridge is well designed because it carries loads that are much heavier than were originally specified.

The structural designer realizes only gradually the up-to-date viewpoint of the aeronautical, mechanical, or electrical engineer, that good engineering requires a design that fits the conditions. That an over-ample design is poor engineering as well as an inadequate one. In that connection one of the interesting recent changes in engineering ideas is the attitude toward weight. A few years ago the weight was considered not only as a favorable factor in engineering designs, but as a criterion of quality—the heavier the better. Consequently, no particular efforts were made to reduce the weight. At present, the attempts can be followed through all fields of engineering development to create more economical or better suitable design by the elimination of useless weight: The use, if possible, of arch dams instead of gravity dams; the substitution of artificial materials in housing, which are either lighter or which can be used in less quantities; the substitution of high strength or light-weight alloys in metal structures; the introduction of welding; the lightening of bridge floors; the reduction of weight of automobiles and airplanes; and the use of weightless agencies in the numerous applications of electric energy.

THE EXTENT OF PRESENT THEORETICAL KNOWLEDGE

The present electronic theory is based on the assumption that the electric and magnetic forces of the electrons are ultimately responsible for the continuity of each physical body. The different orientation of these forces is responsible for the properties of the different materials and in particular for their stress-carrying capacities. The fundamental approach to the stress problems would be to consider these electric and magnetic forces. At the present inadequate state of knowledge that approach is impractical. Even the less fundamental relations between the individual particles or crystals of the material have not been sufficiently studied and rationalized to be practically applicable.

The present engineering approach necessarily must be simplified, which is done by the introduction of the theory of stress. This theory should make it possible to determine the distribution of forces applied to the structure and their transformation into strains and stresses. It should provide a method by which the behavior of the structure may be predicted if the applied external forces or loading are known or can be assumed with sufficient accuracy. In the engineering sense of the word, stresses are changes in the internal forces that are holding these particles together. Stresses introduced in any physical body are manifested in strains and resulting deformations and deflections. The mathematical expression of the engineering theory of stress is given in a set of differential equations. The mathematical solution of these equations should result in a set of functions that can be solved for each point of the body, giving, for each set of conditions, the corresponding position of each point and the stress in each desired direction. As a general rule, the mathematical functions are applicable only within the range of continuity and cannot be extended beyond a discontinuity.

The surface of each structural element will be a discontinuity, regardless of whether it is exposed to the atmosphere or to a liquid, or is a jointing or connection area of two structural elements or the border area between materials differing in their properties. In the engineering sense that would mean that each set of functions will determine the conditions within the body of a continuous structural element, and also at its surfaces or discontinuity areas, the latter being of particular importance. The conditions at such surfaces are referred to as "boundary conditions".

Present mathematical knowledge is limited and no general solution is available of the differential equations set up by introducing the theory of stress. Consequently, the solution of each structural problem resolves itself into finding the partial or approximate solutions of the general equations which will solve, with more or less accuracy, the particular stress problem.

All the approximate methods are based on a number of more or less reliable assumptions. Besides the traditional assumption that the material follows Hooke's law nearly all methods are based on the assumption of linear stress distribution throughout the thickness of the body. In many cases, such an assumption may be reasonable at a certain distance from the boundaries, but as a general rule it violates, grossly, the actual conditions at close proximity to the boundaries. As a consequence these methods, in general, do not satisfy the boundary conditions, although they may permit a reasonably close determination of conditions within the thickness of a uniform body.

At the boundaries, however, the difference between the theoretically determined, and the actual, conditions may be appreciable, particularly at places where a sharp change in the boundary conditions occurs. Such change may be due either to an abrupt change of cross-section or to local application of concentrated forces.

The theory of stress gives two major criteria for the determination of the suitability of a design—the deflection and the stress. The deflection is an integration of the differential strains over the entire structure or over an entire structural element. Consequently, if the average conditions determined by the use of an approximate method are close to the actual average conditions, there may be a very satisfactory agreement between the deflections determined by the use of such a method and the actual deflections, notwithstanding the discrepancies at the boundaries.

In so far as the stress is concerned, the suitability of a structure should not be judged by the average but by the maximum stress that may occur at any point, and that maximum, as a general rule, will occur at the boundary. The value of such governing maximum stress cannot be determined with reasonable accuracy by the use of the traditional approximate methods.

PROPERTIES OF METALS AND ALLOYS OF IMPORTANCE IN STRUCTURAL DESIGNS

The ideal material, which is the basis of conventional designs, not only has perfect elastic properties, obeying Hooke's law through the entire range of stress, and exhibiting a linear stress distribution through the thickness of the body, but it is also unaffected by time, retaining the same properties after an indefinite length of service. No actual material has such properties and in up-to-date structural design the behavior of the material must be considered during the first, as well as the intermediate and last, load application for the entire expected life of the structure. The possible deterioration of the material with time, the creep, and the fatigue properties, must be investigated before an engineering judgment can be formed as to the suitability of the particular material for a particular application.

The stress-resisting capacity of a metal or alloy is the fundamental property that makes possible its use in any engineering structure. The suitability of different metals and alloys is judged by comparing their ultimate strength, elastic limit and elongation, fatigue, creep, impact properties, weight, and price.

Metals and Alloys Used.—The pure metals are very unsatisfactory in so far as their stress-resisting capacity is concerned. No pure metals are used in structural designs; the carbon steels are the oldest and most important alloys. The modern steel alloys, the aluminum and magnesium alloys, are recent additions to the structural field.

From the seructural designer's viewpoint an ideal alloy should possess high stress-carrying capacity at light weight and low cost and, at the same time, should have a high degree of permanence or a high degree of corrosion resistance. A series of alloys having these qualities and of different moduli of elasticity would make it possible to use the most suitable alloy for each design.

No alloy or alloys are available at present that would meet all the requirements satisfactorily. The ultimate goal of metallurgical research is to find compositions and fabrication methods that will improve the stress-resisting as well as the other desirable properties. Such an ideal alloy is not even in sight, and structural engineers will be compelled, for a long time to come, to use numerous alloys, each one satisfying a part of the requirements among which the cost will always be important.

In so far as the modulus of elasticity is concerned, it seems that, at present, metallurgical science does not propose any methods by which that important property can be varied sufficiently.

Ultimate Strength.—Ultimate strength is measured by the stress at which the material fails during a single gradually increasing load application, based on the original cross-section of the specimen and using an arbitrary size and shape of specimen. One of the complications in the application of present stress theories, lies in the fact that the ultimate strength depends not only on the kind of material, but also on the kind of stress. The same material will show a different ultimate strength in tension, compression, bending, or shear.

Considering the simplest case, the theoretical maximum tensile strength of a material reflects the degree of adhesion between its molecules.

The general assumption is that this adhesion is due to the molecular or internal forces that hold the molecules of the material together. If the modern electronic theories are accepted, it is only logical to assume that there is a particular arrangement of all electrical and magnetic forces that will result in the maximum internal force holding the particles together and, consequently, will represent the conditions of the absolute ultimate tensile strength that cannot be exceeded by any material under any conditions. Then the ultimate tensile strength obtained in any material under a particular set of conditions could be expressed as a percentage of the absolute tensile strength. This percentage will be low for unsatisfactory arrangements of the internal forces and will increase if the internal forces are better orientated with reference to the tensile stress applied. No information is available as to what such absolute ultimate strength may be, but it is reasonable to assume that it is not even approached in any of the commercially used structural materials.

The present state of knowledge is insufficient to determine how a better orientation of the internal forces may be obtained, but practical experience shows that such improvements are possible even if an explanation for them is not apparent. It is not even clear whether the increase in the specific gravity of the material has any favorable influence on the orientation of the internal forces.

The most important improvement in the orientation of the internal forces, resulting in the increase of ultimate strength, is obtained by cold working suitable alloys. At the same time such cold working not only does not increase, but very often decreases, the density of the material. This fact could be taken as an important indication that the proper orientation of internal forces has no direct relation to the specific gravity, but depends on some factors unknown.

The general speculation permissible under these assumptions is that it should be possible by the use of means, unknown at present, to produce alloys of the same degree of orientation of internal forces irrespective of their specific gravities. Practically, that would mean that it should be possible to produce light alloys of very high ultimate tensile strength.

Materials identical in so far as the orientation of the electric and magnetic forces is concerned, should be identical in all other respects. There are numerous ways in which the orientation of these forces can be changed in metals, resulting in an unlimited variety of metals and alloys; but practically nothing is known as to the character of these re-orientations. The following classification of the known methods of re-orientation of the internal forces may be made:

I.—Re-orientation involving change in chemical composition:

(*A*) Changes brought about by the addition or substitution of different alloying elements, resulting in alloys of a different chemical composition.

(*B*) Changes in composition of the integral parts of the alloys, resulting in alloys of the same general chemical composition, but of different properties.

(*C*) Changes in surface conditions, known as chemical corrosion.

II.—Re-orientation not involving any change in chemical composition:

- (*A*) Changes in grain structure caused by cold working or heat treatment.
- (*B*) Changes in the internal equilibrium conditions caused by the introduction of internal stresses.
- (*C*) Changes in surface conditions caused by mechanical erosion.
- (*D*) Changes in surface conditions caused by repeated application of stress.

Factors that do not influence the stress-resisting capacity directly, but are customarily included, may be classified as follows:

III.—Factors reflecting insufficient theoretical knowledge:

- (*A*) Stress concentrations that cannot be determined exactly.
- (*B*) Unknown distribution of stress throughout the thickness of the specimen resulting in the introduction of a new variable—the size of the specimen.

IV.—Factors reflecting inadequate manufacturing procedure:

- (*A*) Non-uniformity of material.
- (*B*) Unsatisfactory surface conditions.

The chemical composition is naturally the major factor. Nothing is known concerning the fundamental relation between chemical composition and strength of the various materials and in particular metals. Much has been learned during the last few decades about the influence of comparatively small quantities of some alloying elements on the strength of commercial alloys. Nevertheless, the development of new alloys is very much a "cut and try" proposition. The present metallurgical knowledge of the fundamental factors influencing the strength of metals is rather limited. In general, every commercially used metal can be made stronger by adding proper alloying ingredients. Even the impurities that are unavoidable in any commercial process may change the strength of metals to a large extent.

On the other hand, in most cases, the increase of strength indicated by increased ultimate strength that is due to alloying, is obtained with a simultaneous decrease in ductility, indicated by decreased elongation. Since, for practical purposes, a metal of high strength and high ductility is desirable, it is necessary to arrive at some compromise as to the most favorable combination of strength and ductility. For alloys to be used as castings it is possible to utilize materials of very low ductility; metals that must be changed to a prescribed shape by forging , stamping, extrusion, or rolling necessarily must have a much higher ductility, at least at the working temperature.

Some of the alloys are susceptible to heat treatment and change their properties if heated to a certain temperature and cooled in a definite way. The wide influence that changes in chemical composition may have on alloys with different percentages of alloying elements is demonstrated in Fig. 1, which shows the total percentage of alloying elements and the ultimate tensile strength of a number of non-commercial and commercial aluminum alloys. It embraces the highest values of the ultimate tensile strength obtained.

In drawing this diagram, no attention was paid to the method by which the highest ultimate strength was obtained, such as heat treatment, cold working, etc. The greatest tensile strength obtained for each alloy was the

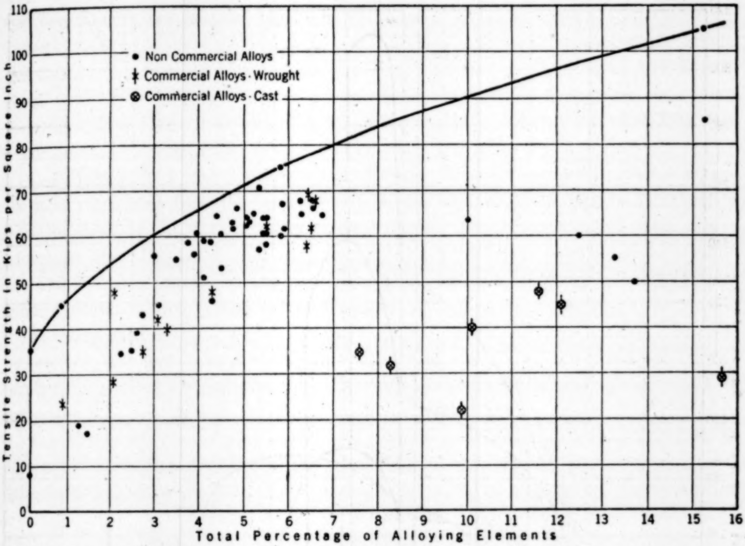


FIG. 1.—ULTIMATE TENSILE STRENGTH OF ALUMINUM ALLOYS.

information included in Fig. 1. It is reasonable to assume that every point of the enveloping curve can be filled in, with future progress in the study of the different possible alloys.

The increase in the percentage of alloying elements increases the ultimate tensile strength, but may decrease the ductility, and in many cases the corrosion resistance, to such an extent that the resulting alloys often have no commercial use. Increasing the number of alloying elements may improve these conditions, resulting in increased stress-resisting capacity without a detrimental influence on other desired properties, resulting in commercial alloys.

The large number of points representing non-commercial alloys indicates that a large number of factors needs to be considered in order to find whether a particular alloy will prove to be useful.

No information is available that would indicate the maximum strength that may be reached by the alloying of different base metals.

Stress-Strain Curves, Modulus of Elasticity, Yield Point.—Although very little is known about the ways in which the different chemical ingredients act in the alloy, the final results are reflected in stress-strain curves, particularly if these curves are obtained by the use of sufficiently sensitive instruments.

A number of such curves are shown in Fig. 2, which is drawn in such way that for the first part of the stress-strain curves, up to 0.01 in. per in., or 1% of strain, a large scale of strains is used, the remainder of the diagram is

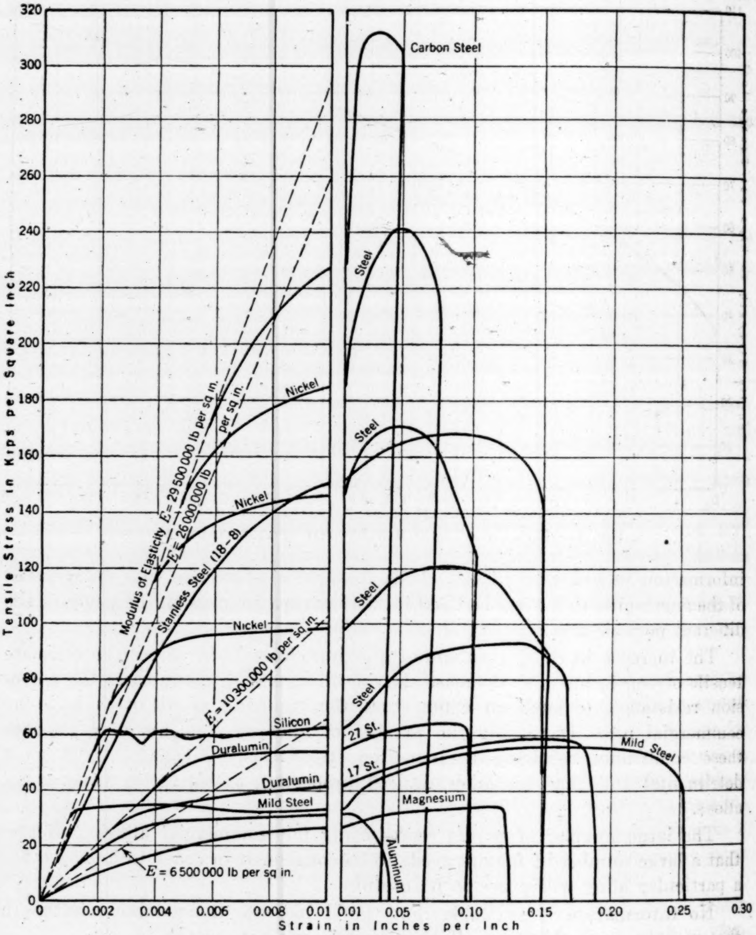


FIG. 2.—COMPARATIVE STRESS-STRAIN CURVES

drawn, using a strain scale one-twentieth of the size of the original one. This diagram, based on publications of V. D. I. (Verein Deutscher Ingenieur), and unpublished tests made by R. L. Templin, M. Am. Soc. C. E., shows the three types of commercial alloys that are used or may be used in structural designs, the steel, aluminum, and magnesium alloys. A study of Fig. 2 reveals a few interesting facts:

1.—Notwithstanding the very large variation of the ultimate strength and elongation, only four different moduli of elasticity are available, one between 29 000 and 30 000 kips per sq in. for low alloy steels, the other between 25 000 and 26 000 kips per sq in. for high alloy, stainless, steels, the third about 10 300 kips per sq in. for aluminum alloys, and the fourth about 6 500 kips per sq in. for magnesium alloys. In structural materials as commercially used at present, no intermediate moduli of elasticity are available.

2.—Yield points may be studied in Fig. 2. The low-strength steel alloys—the mild steel and the silicon steel—have pronounced yield points at which a definite break of the strain-stress curve occurs. The high-strength steel alloys as well as the aluminum and magnesium alloys have no definite yield points, the stress-strain curve changing from the straight part to a curved part gradually and steadily without any break.

The importance of the yield point for mild steel and particularly for silicon steel is obvious. Any stressing of these alloys above the yield point will result in considerable deformation that must take place before the increase of resistance will occur that may stop such deformation.

For alloys which do not have yield points it is customary nevertheless to designate as yield point the stress at which an arbitrarily chosen permanent deformation occurs. For non-ferrous alloys, 0.2% of permanent deformation is widely used. For ferrous alloys a permanent deformation between 0.1% to 0.5% is usually chosen.

Considering the properties of the alloy and its applications, these arbitrary yield-point values are rather meaningless, but they have some practical significance in limiting the acceptable design deformations.

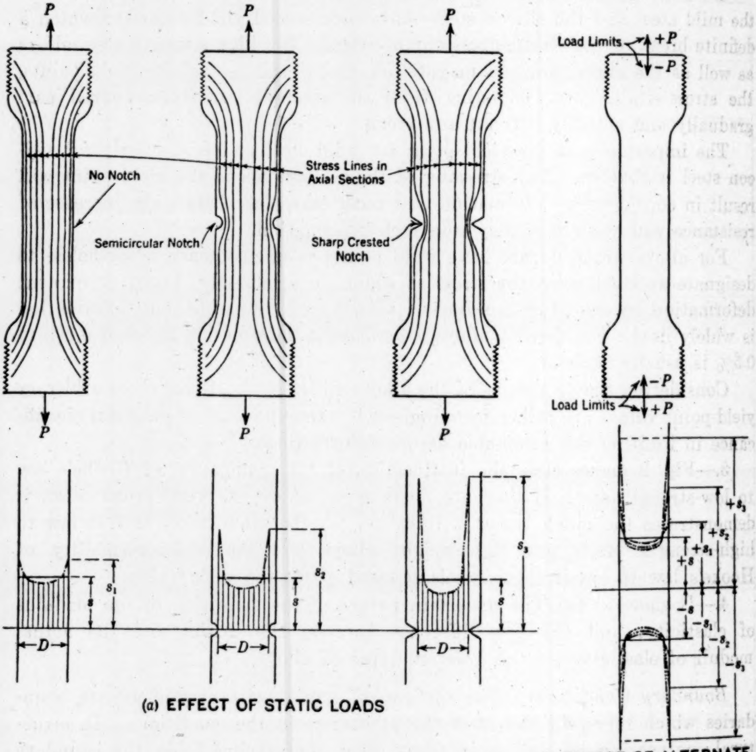
3.—Fig. 2 makes clear the justification of the application of Hooke's law to low-strength steels if they are not stressed above the yield point; also, it demonstrates the much lesser justification for the application of this law to high-strength steels and light-weight alloys and the non-applicability of Hooke's law to low-strength steels stressed above the yield point.

4.—It shows: (*a*) The uncertain nature of the definition of the modulus of elasticity; and (*b*) the difference between the initial and the actual moduli of elasticity for the different types of alloys.

Boundary Conditions.—The surfaces of any structural element are boundaries which represent the most abrupt change in the conditions. In structural engineering, as well as in many other engineering fields, the boundary conditions may become of considerable importance. They must be considered in any investigation of the stress-carrying capacity of alloys and should be reflected in the mathematical treatment of the stress problems. The present progress in engineering design, as well as in the utilization of alloys, depends to a large extent on the increased attention paid to the boundary problems.

That corrosion, or the disintegration, of metals and alloys starts at the boundary or surface and gradually penetrates the body of the material was realized, of course, a long time ago, and the metallic surfaces were protected by the use of different paints, or, more recently, by facing the less stable base metal with a more stable alloy or alloying element.

Even now the influence of the boundaries on the distribution of stress within the body of the metal is not always realized. For instance, the traditionally assumed linear distribution of stress does not represent the conditions close to the boundaries. During the last few decades considerable advance has been made in realizing the importance of the boundary or surface conditions in the fatigue phenomena. Without attempting to go into the complicated stress relation among the individual crystals or even more minute particles of the material, but keeping within the customary engineering conception of stress, a simple illustration may be of interest. The stress conditions of a circular bar subjected to pure tension or compression are shown in Fig. 3(*a*) and Fig. 3(*b*) for static and varying loads, respectively.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P
P
Load Limits
+P
-P
No Notch
Stress Lines in Axial Sections
Semicircular Notch
Sharp Crested Notch
-P
+P
Load Limits
P
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s
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D
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+s₃
+s₂
+s₁
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-s
-s₁
-s₂
-s₃

(*a*) EFFECT OF STATIC LOADS

(*b*) EFFECT OF ALTERNATE COMPRESSION AND TENSION

FIG. 3.—STRESS DISTRIBUTION IN CYLINDRICAL SPECIMENS.

The two plain bars have the same diameter, D, and the notched bars have the same diameter, D, as the plain bars, at the bottom of the notch. The static load, P, to which the bars in Fig. 3(a) are subjected, is the same. The varying load, P, to which the bar in Fig. 3(b) is subjected, alternates between $\pm P$, the numerical value of P being the same as in the case of the bars subjected to static loads.

Under the traditional assumption of linear distribution of stress, a tensile stress, s, will be developed at the middle section of the plain bar (Fig. 3(a)) and at the section taken at the bottom of the notches at the bars. This stress will be equal for each of these three bars and will be distributed uniformly over the entire available cross-section, as shown by solid lines in Fig. 3.

The actual stress in the plain bar (Fig. 3(a)) under static load probably will be greater at the surface and less at the middle as shown by the broken-line curve, the ratio between the maximum actual stress, s_1, and the traditionally assumed stress, s, being usually referred to as the stress concentration factor. The actual stress in the bar with the circular notch will be greater at the bottom of the notch and somewhat less at the middle, as compared with the plain bar, and as shown by the broken-line curve, the maximum stress being s_2. Finally, the actual stress in the bar with the sharp notch will be still greater at the bottom of the notch and somewhat less at the middle, and as shown by the broken-line curve, the maximum stress being s_3.

A much better visualization of the conditions may be obtained by the use of the stress fields or stress lines analogous to the fields and lines used in the study of electrical phenomena. Instead of visualizing the stress at one cross-section only, it is possible to visualize the flow of stress. In Fig. 3(a) such stress lines are drawn for an axial section of each cylinder. At each point the direction of the stress is a tangent to the stress line. The intensity of the stress is shown by the distance between the stress lines. The influence of the differently shaped notches may be visualized by the study of these diagrams.

Introducing the stress concentration factors, c_1, c_2, and c_3, the maximum stresses, s_1, s_2, and s_3, may be expressed as $s_1 = c_1 s$; $s_2 = c_2 s$; and $s_3 = c_3 s$. The values of c_1, c_2, and c_3, and, consequently, the values of the maximum stresses, s_1, s_2, and s_3, depend on the properties of the material, particularly on the shape of its strain-stress curve. As long as the material follows Hooke's law at the low range of stress, the strain-stress relation is expressed by the straight part of the curve. Under these conditions the concentration factors reach their highest value. When the material deviates from the straight part of the curve, increased deformation or plastic flow takes place. This tends to redistribute the material, tending to decrease the peak value of the stress and, consequently, to decrease the concentration factor.

Alloys having straight-line stress-strain curves and definite yield points will develop the maximum stress concentrations with increase of load until the maximum stress reaches the yield point, after which the stress concentration factor will decrease rapidly.

Alloys that gradually diverge from the straight part of the stress-strain curve will develop lower stress concentration factors at the lower range of stress.

Assuming, again, linear stress distribution, the traditionally evaluated stress in Fig. 3(*b*), under the varying load, will fluctuate between plus and minus s; the numerical value of s again being the same as in the bars subjected to static load, and being uniform over the entire cross-section. The actual stress in the bar will depend on a number of conditions. If the force, *P*, and, consequently, the actual maximum stress, s_1, is small, the stress in the bar will fluctuate between the limits shown by broken-line curves, corresponding to the actual stress of the bar in Fig. 3(*a*), without producing any changes in the material. Consequently, the reversing load may be applied an indefinite number of times, without producing a failure. If the stress, s_1, is great enough, the actual stress of the plain circular bar (Fig. 3(*b*)) will fluctuate between limits that will change gradually. At the first load application, the stress limits will be the same as before. If the reversing load is applied a number of times, incipient cracks will be formed at the surface of the bar, an increase of the surface stress will follow, and the stress limits will be approximately the limits of the bar with the circular notch (Fig. 3(*a*)), shown by the broken-line curve, with the maximum stress, s_2.

In this case, again, the phenomenon may stop if s_2 is low enough and the reversing load may be applied an indefinite number of times without producing a failure; but if s_2 is great enough the repeated application of the reversing load will cause the incipient cracks to change their shapes, increasing the surface stress until it corresponds to the conditions of the bar with the sharp notch in Fig 3(*a*), with the maximum stress, s_3, the notch being chosen so as to produce the highest possible value of c_3.

These conditions will be somewhat complicated due to the fact that the circular bar will contract or expand, not only parallel to the axis or in the direction in which the forces are applied, but also along the diameter or at right angles to the direction of the forces. This complication, however, in the simple case assumed, will only be of secondary importance.

If the stress, s_3, developed under such conditions is low enough, no more changes in the conditions of the incipient cracks will occur, and the specimen may be subjected to an indefinite number of load reversals without failure.

If the stress, s_3, is high enough, a permanent and continuous change in the crack conditions will be produced and the failure will occur after a number of load reversals during which the incipient crack will grow and attain larger dimensions, causing an increase of stress due to the reduction of the effective cross-section. It may take a considerable number of load reversals before the incipient cracks will come to such a condition that they start to grow; but after that the final growth is usually accomplished during a small number of load applications, and the failure takes place rapidly.

Finally, the reversing load, *P*, applied to the specimen may be so great that although the stress, s_1, will be lower than the ultimate strength, the stress, s_3, exceeds it. Under such conditions the failure will occur after a limited number of load applications as soon as the growing maximum stress approaches the ultimate strength of the material.

This description of the fatigue phenomena may not be exact in every case, but should give an understanding of the underlying conditions as known at present.

The properties of metals and alloys of importance in engineering structures may be divided into two classes: The first are the properties that depend mainly on the internal deep-seated conditions; and the second those that depend upon the surface conditions. Modulus of elasticity, shape of the stress-strain curve, ultimate strength under static load, creep, and single impact properties, all depend on the internal conditions and are affected very little by the surface conditions. Corrosion resistance, fatigue strength, and fatigue strength under repeated impact, are properties that depend fundamentally on the internal conditions, but may be influenced to a very large degree by the surface or boundary conditions.

Fatigue Properties.—Fatigue strength is determined by the formation of incipient cracks on the surface and by the extension of such cracks. If the formation and extension of these cracks can be retarded, the fatigue strength will be increased, and *vice versa.* The stresses developed at the surfaces are the major factors in the formation of the cracks. It is not the mean stress or the assumed stress that is of importance, but the actual maximum stress that may occur.

The extension of cracks depends on the load or stress fluctuation. A static load may develop minute cracks, but unless the maximum stress is close to the ultimate strength of the material such cracks will not grow. The variation in stress will change the conditions of the cracks. Small variations of stress from a constant mean value will make the cracks spread only if the mean value is high. Larger stress variations will make the cracks spread even at a smaller mean value until the complete reversal of stress may result in their extension and spread, although the mean value is zero.

These conditions may be summarized on a diagram with a base line of 45°, such as that shown in Fig. 4, in which the mean value of the stress is plotted on the 45° base line, using either the scale on the ordinate or the abscissa axes. The stress variations are plotted vertically as ordinates using the mean stress value as the zero point. This diagram, based on the publications of V. D. I., extends from the zero mean value which will be the origin of the co-ordinate system and to the value of the ultimate strength plotted at the 45° base line.

To draw such a diagram the fatigue strengths must be determined on a large number of identical specimens of a particular alloy under different conditions of load variations, but for a stress of the same kind, either tension or compression.

For practical purposes a diagram such as Fig. 4 should not be used above the true or assumed yield-point value, resulting in that part of the diagram shown by solid lines.

Number of Load Applications.—The fatigue strength of a material is determined by the testing of a sufficiently large number of identical specimens. Each specimen is tested in a similar manner but at a different stress range.

If, for instance, the fatigue strength at the complete reversal of stress is to be determined, the specimens are tested under conditions of complete stress reversal, starting with a number of specimens that are tested at high stress.

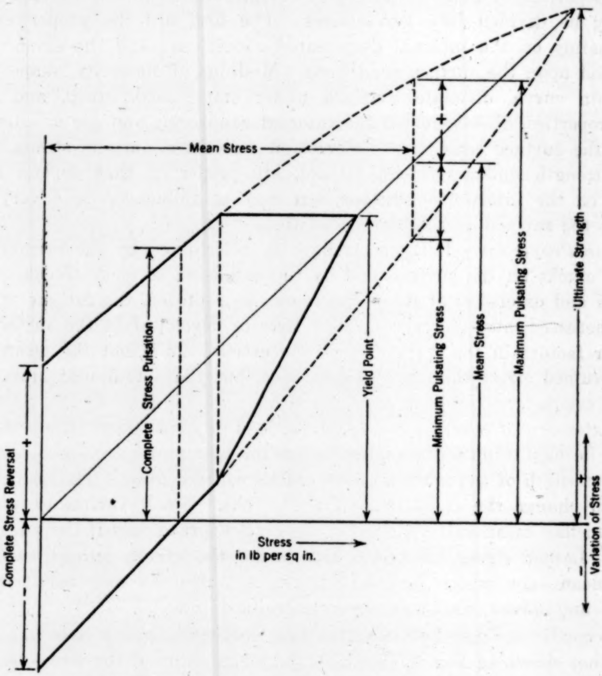

Mean Stress
Complete Stress Reversal
+
−
Complete Stress Pulsation
Yield Point
Minimum Pulsating Stress
+
Mean Stress
Maximum Pulsating Stress
Ultimate Strength
Stress in lb per sq in.
+
Variation of Stress in lb per sq in.
−

FIG. 4.—FATIGUE DIAGRAM; 45 DEGREE BASE LINE.

The testing is continued, each succeeding specimen or set of specimens being tested at a lower stress, so that the result represents a series of gradually decreasing values of stress to which the specimens were subjected. For each specimen two values are obtained, the stress at which the specimen was tested and the number of stress reversals at which it failed. The stress is determined by the use of traditional methods, disregarding any possible stress concentration.

In order to obtain complete fatigue data, tests must be made and diagrams plotted for complete and partial stress reversals and for complete and partial stress pulsations.

The accepted term, "endurance limit", can be defined as the fatigue stress under complete stress reversal below which metal will withstand, without failure, an indefinitely large number of cycles of stress. It may also be defined as the stress value at which the stress-cycle curve changes to a line, straight and parallel to the abscissas. For partial stress reversals and for pulsating stress the term, "fatigue strength", would seem more applicable.

There is a wide difference in the shape of the stress-cycle curve for different materials. For some materials the curve has a very pronounced "knee" and the endurance limit is a very definite term, but for other materials the curves have no definite "knee" and, for some materials, the stress-cycle curve does not change to a line straight and parallel to the abscissas.

To determine the properties of the different alloys it is necessary to run the fatigue tests either until a definite endurance limit or fatigue strength is established or until the proof is obtained that no definite value can be obtained.

The term "endurance limit", is often misused. For practical purposes, particularly in structural problems, data may be utilized which represent results of tests obtained at a lower number of load applications; such data should not be referred to as "endurance limit", but as "fatigue strength", and it would seem necessary in each case to supply a clear indication of the limiting number of load applications at which the data were obtained. The term, "endurance limit", should not be applied to cases of partial stress reversals or stress pulsations.

Surface Conditions.—Fatigue strength may be influenced to a large extent by changes in the surface conditions. The methods by which such changes may be obtained can be divided into a number of groups. The most radical is the group involving the change in the chemical composition of the boundary layer of the metal or alloy. If such a change results in a surface which prevents the development of cracks, or which prevents the minute cracks from spreading, an increase in fatigue strength may be obtained. The nitriding of steel changes the chemical composition of the surface, resulting in a considerable increase in the fatigue strength.

Changing the surface conditions without changing the chemical composition covers a group of methods of primary importance. First in this group is the case in which changes occur automatically during the repeated application of the fluctuating load. The changes produce what is known as the "strain-hardening effect." It seems that if, during the load application, stress concentrations occur, that bring the peak of the stress above the yield point for materials that have a yield point, plastic deformations are caused, thus changing the properties of the material that may retard or even stop the growth of the incipient cracks. For materials that do not have a yield point, the same condition occurs if the stress is high enough to produce a sufficient deformation.

Similar results may be produced by cold working the surface by use of rollers or by similar means. The increase in the fatigue strength in this case may be attributed partly to the compacting of the surface particles and partly to the pre-stressing of the surface which, if properly applied, may reduce the stress peaks.

The next group of methods involves the finish of the surfaces. Machined and highly polished specimens have the highest fatigue strength in this group. The elimination or reduction in size of the incipient minute cracks is the probable explanation. Fatigue strength is reduced if the surfaces are only machined but not polished, and it is still more reduced for rough, non-machined surfaces. The notching of the surface, or injury by sharp cuts, and, finally (for mill-rolled shapes), using the material with the mill scale left on, will result in the lowest fatigue strength. The presence of cracks and high stress concentrations are the most probable explanations.

The final and lowest in fatigue strength is the group in which the surfaces are covered with liquids. It would seem that if liquids of high viscosity are pressed into the incipient cracks, or liquids of low viscosity flow freely into them, they act as wedges during the closing of the crack, increasing considerably the stress concentrations and reducing the fatigue strength. The chemical reactions of some of the liquids cause a further lowering of the fatigue strength by increasing the sizes of the incipient cracks or deepening them.

Finally, a very considerable lowering of the fatigue strength and the destruction of the metal, submerged for instance in water, may occur due to cavitation and resulting corrosion, when the material is subjected to a very large number of very rapid blows due to the continuous and rapid forming and collapse of vacuum bubbles in the surface layer of the water adjoining the metal.

If similar metals or alloys are compared, those that have a lower ultimate strength can be improved as a general rule to a larger degree; and, on the contrary, high-strength alloys are more sensitive in so far as the lowering of the fatigue strength is concerned. Thus, the fatigue strength of a low-strength carbon steel can be improved by nitriding to a larger degree than the fatigue strength of high-strength steel; but if compared with machined and polished surfaces, a notch or surface injury will lower the fatigue strength of high-strength alloys to a larger degree as compared with similar low-strength alloys.

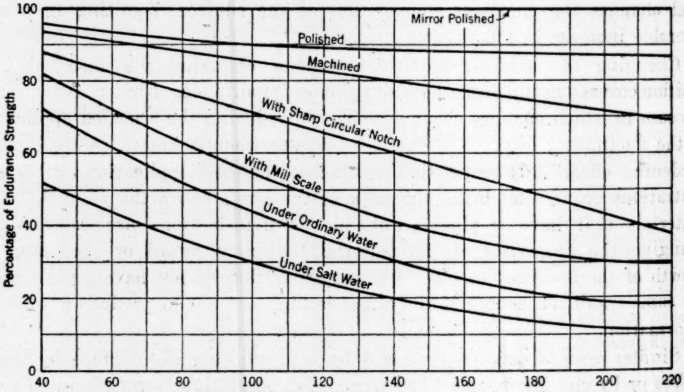
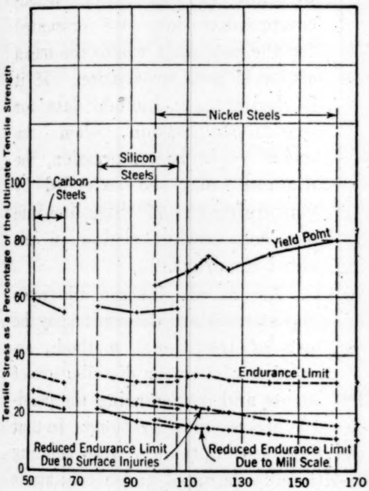


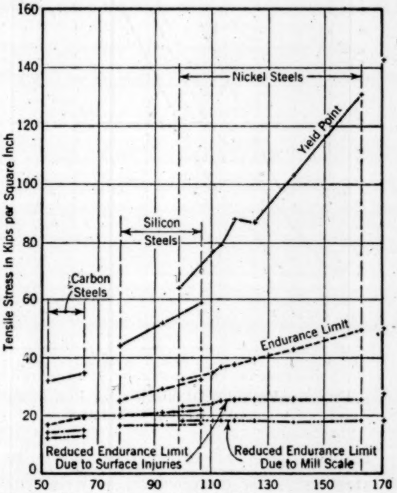
FIG. 5.—REDUCTION OF ENDURANCE STRENGTH OF STEEL ALLOYS DUE TO SURFACE CONDITIONS.

In so far as steel alloys are concerned, Fig. 5, based on V. D. I. publications, illustrates this conditions. The diagram is based on fatigue strength obtained on specimens in tension-compression and bending. The highly polished or mirror-polished specimens are given the 100% rating.

Reduction factors are to be applied, depending on the ultimate strength of the particular steel alloy and its surface conditions. They must be applied not to the mean value of stress but to the plus-minus stress fluctuation. If a combination of factors is involved the lowest values shown on Fig. 5 should be used. As usual, the values for sharp circular notches do not take into consideration the stress concentrations and give a good idea as to what the reduction in fatigue strength may be if the surface is injured by a blow leaving a sharp indentation.

The influence which that reduction of fatigue strength may have under conditions of complete stress reversal is demonstrated in Fig. 6, based on the data shown in Fig. 5, for different steel alloys. This diagram shows the yield point, the endurance limit for highly polished specimens, the reduced endurance limit due to surface injuries using the data for sharp circular notches, and the reduced endurance limit that is obtained if rolled material is tested with the mill scale left on. All these data are expressed in percentages of the ultimate tensile strength. Finally, the diagram, Fig. 7, shows that under the same conditions of a complete load reversal the high-grade steels have endurance limits somewhat proportional to their higher ultimate strength, only for highly polished specimens. For specimens with injured surfaces or with the mill scale left on, there is only a very slight increase of the endurance limit.





Ultimate Tensile Strength in Kips per Square Inch

FIG. 6. FIG. 7.

Complete Fatigue Diagrams.—The fatigue strength of a material, tested under identical conditions in so far as the properties of the alloy and its surface conditions are concerned, depends on the kind of applied stress.

Therefore, it becomes necessary to test the materials separately for three kinds of stresses—tension-compression, bending, and shear. Fig. 8, based on V. D. I. publications, shows a complete fatigue diagram of a silicon steel alloy on which the fatigue strength for all three kinds of stresses are shown. The characteristics of this alloy were: Carbon, 0.25%; silicon, 0.2%; manganese, 0.6%; ultimate strength in tension, 64 kips per sq in.; and, elongation, 0.20% in 4 in. In Fig. 8 the tension-compression data are presented for the case only where the mean stress is zero or tension. If it is desirable to include data for tension-compression when the mean stress is compression, the diagram may be extended by continuing the 45° base line into the compression region in the third quadrant.

As in all fatigue diagrams the stresses are determined by the use of traditional methods, assuming the linear distribution of stress and disregarding the possible stress concentrations. In that respect it is of interest to compare the tension-compression and bending stresses. The comparison between the traditionally assumed and actual stress distribution is shown on Fig. 3(*a*) and Fig. 9 for the tension-compression and bending, respectively. By comparing these two diagrams it can be seen that in the case of tension-compression the actual stress is higher, and in that of bending lower, than the assumed stress. If corrections due to non-linear distribution of stress were applied to the data in Fig. 8, the fatigue strength in bending would be lowered and that in tension-compression would be raised, bringing both of them together. The large difference between the shear and tension-compression fatigue would be made still larger.

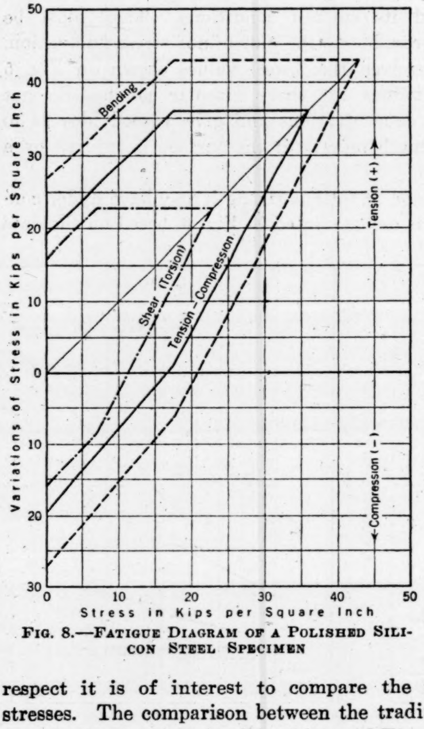


FIG. 8.—FATIGUE DIAGRAM OF A POLISHED SILICON STEEL SPECIMEN

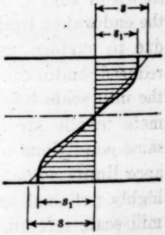


FIG. 9.—DISTRIBUTION OF STRESS IN A RECTANGULAR BAR SUBJECTED TO BENDING.

Comparative Fatigue Diagrams of Different Alloys.—The fatigue strengths of a number of alloys, determined on highly polished specimens under tension-compression stress, are shown in Fig. 10(*a*), based on unpublished tests by R. L. Templin, M. Am. Soc. C. E., and on V. D. I.

publications. The alloys were chosen to represent high-strength alloys of their respective classes.

The ultimate tensile strength and elongation of these alloys are given in Table 1. Although it gives the absolute fatigue strength values, Fig. 10(*a*) does not offer a proper basis for comparison, since the weights of the materials are not considered. Fig. 10(*b*) is based on the same data, but the weight is considered by assuming the steel as the material of unit weight, and multiplying the fatigue stress for duralumin by the steel-duralumin weight ratio, which makes possible a direct comparison of the different alloys on the strength-weight basis.

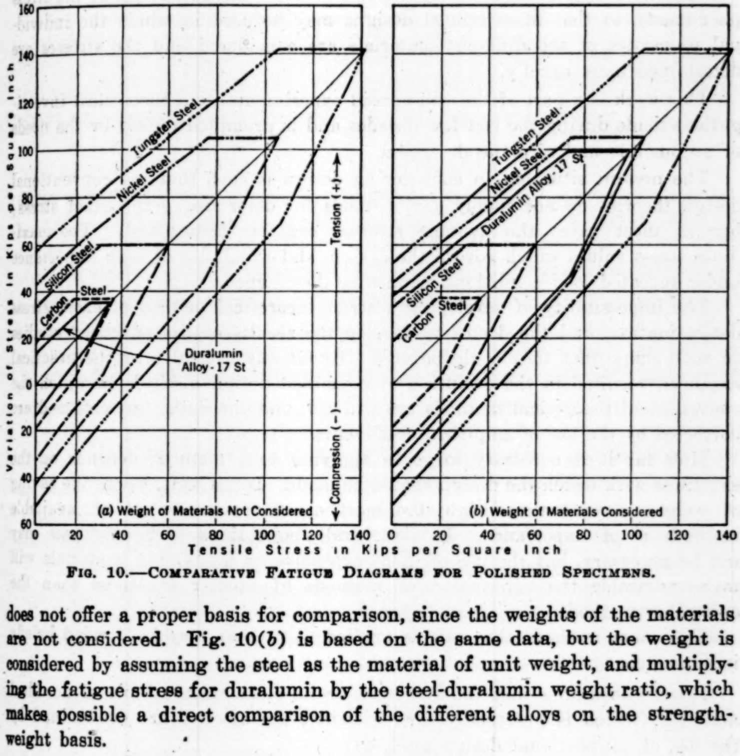


FIG. 10.—COMPARATIVE FATIGUE DIAGRAMS FOR POLISHED SPECIMENS.

TABLE 1.—CHARACTERISTICS OF ALLOYS IN FIG. 10.

Alloy	Ultimate tensile strength, in kips per square inch	Percentage elongation, in 4 inches
Duralumin	59	19
Carbon	64	20
Silicon	107	10
Nickel	139	10
Tungsten	170	10

IMPROVED DESIGN METHODS

Determination of Stress.—The assumption that the materials follow Hooke's law does not take care of the individual properties of the different materials, but bases all designs on the use of non-existing ideal materials, with perfect elastic properties. The assumption of linear stress distribution in many cases precludes the proper determination of stress, particularly at the boundaries.

Considerable experimental and theoretical work was done in the past and is being done at present in order to create a better understanding of the stress phenomena, so that more refined designs may be used in which the individual properties of the different materials are considered and the stresses are determined more exactly.

This work is a part of the widespread experimental and theoretical investigations made during the last few decades and is prompted mostly by the needs of automobile and airplane designers.

The present situation in engineering design is such that the conventional design theories do not supply any methods for determining the exact stress; but in most cases the average stresses can be determined. The maximum stress values which govern the design, and which are of such importance under repeated load conditions, cannot be determined.

The impossibility of developing a strict theoretical method of exact stress determination, and the desire to develop the results of the advanced studies in such shape that they could be used without any difficulty by the practical engineer, resulted in the evolution of combined design methods in which the conventional theoretical methods are utilized, and the data thus obtained are corrected by the use of empirical coefficients.

How far it is necessary to go in applying such methods depends on the exactness with which the design should be made. In aircraft, where the saving of weight governs the design, the most exact application of all available methods is of importance. In structural applications such exactness may not be necessary, but the use of more expensive or less known materials will make advisable the application of methods of greater exactness than the customary methods.

The exact methods that can be applied at present may be divided briefly in four steps:

First.—Assuming that the structure or its elements are built of an ideal material (which is practically non-existent), the stresses are determined by the use of conventional design methods;

Second.—The conventional stresses thus determined, are corrected to bring about close agreement between the design and correct stresses in the ideal material;

Third.—Stresses determined for the ideal material are corrected again, to take care of the substitution of materials actually used for the ideal material, so that a close agreement is assured between the design and actual stresses; and,

Fourth.—The finally determined stresses for the materials that are actually used are compared with the stress-resisting capacity of these materials under the expected loading conditions.

The traditional structural designs are based on the use of the first step only. The fourth step is seldom fulfilled; since the stress-resisting capacity of the materials is not correlated with the expected loading conditions, in particular, the fatigue properties are usually neglected.

The inclusion of the second and fourth steps in the design will insure much greater precision. All four steps probably represent the most exact design possible at present.

Stability.—Stability problems are of considerable interest in the more refined designs. In the past the tendency has been to design structures so that no stability failures were possible, the failure by overstressing the material occurring under smaller loads than those at which the condition of instability could be reached. In other words, the attempt was to keep the safety factor with reference to stability greater than the safety factor with reference to stress failures.

An economical design not only must be balanced in so far as the stress safety factors are concerned, but it should also be balanced in so far as the stability safety factor is concerned. Theoretically, there are no reasons why a structure or its elements should have different safety factors with reference to stress and to stability. Practically, the stress-carrying capacities of alloys are better understood than the conditions of instability, and the tendency of the designer is to use higher safety factors with respect to stability as compared to stress, with the exception of some aircraft designs.

No matter what the material, and how stiff the structure or each of its elements may be, the strains and resulting deformations and deflections will appear every time forces are applied, the only difference being in the amount of deformations and deflections. These factors will depend on the modulus of elasticity of the material used and the stresses developed.

The mathematical treatment of the stability problem is even less definite than that of other stress problems. The fact that the change from stable to unstable conditions in most cases occurs during a very small change in the loading conditions, makes the exact mathematical solution a problem of such refinement that it cannot be obtained exactly by the use of present mathematical methods.

Practically nothing is known about the question as to whether, and, if so, how, the local stress concentrations and repeated load applications influence the stability. In comparing the application of different alloys, the stability may become an important factor.

If higher stressed, stronger alloys of the same specific gravity and modulus of elasticity are substituted for low-strength alloys, the thickness of the members is reduced, the deformations and deflections are increased, and, consequently, the stability safety factor is lowered.

If alloys, equally stressed and of the same strength but of lower specific gravity and lower moduli of elasticity, are substituted, no direct conclusion may be drawn. The comparative stability will depend on the details of design with respect to the distribution of the additional volume of the alloy of lower specific gravity.

APPLICATION TO ACTUAL DESIGNS

Necessarily, the refinements of engineering design must start with a more exact prediction of the future loading conditions. Not only should the maximum force to be applied be known but, for varying forces, the amount of variation and the probable frequency of different load application must also be known. All these conditions have an important bearing on the behavior of the structure.

The relation between the shop and field labor costs and between the cost of material and labor are changing factors the importance of which is considerably accentuated since more expensive materials and different jointing methods are put at the disposal of the structural engineer.

Considering the different designs, the fatigue and stress concentration phenomena are of particular interest. The load that a metal structure carries produces stresses that may vary from a perfectly static to a completely reversing stress. There are very few metal structures the members of which are subject to perfectly static stress. Even the steel skeletons of buildings are subjected to a constantly varying stress, due not only to the wind action but also to temperature variations. As a general rule, these stress variations are small as compared with the mean stress. On the other hand, the number of structural elements that are subject to complete stress reversal is also comparatively small. The vast number of structural elements are subjected to varying stresses of different degrees of pulsation or reversal.

The number of stress pulsations or reversals may be considerable. The most outstanding instances are structures that are subject to vibrations in which the number of stress pulsations or reversals may be comparable to the number of stress reversals to which parts of fast running engines are subjected. Most of the structures are subjected to fewer stress pulsations or reversal.

These considerations would indicate that stress concentration and fatigue phenomena must be taken into account in structural design.

Joints.—The necessity to develop expensive high-speed internal combustion engines forced the mechanical engineer to take into consideration and to begin a study of stress concentration and fatigue phenomena. Joints, being the most expensive and most sensitive part of a structural design, are at present forcing the structural engineer to follow the same path.

Riveted joints are necessarily producing stress concentrations. If two similar riveted joints are compared, the traditionally assumed strength of which is the same, it would seem that the joint in which a small number of large rivets is used will have higher stress concentration than one in which there is a larger number of small rivets. All other conditions being equal the joint with a larger number of smaller rivets should have the higher fatigue strength.

The next conclusion would be that a continuous welded joint should have a greater fatigue strength than a riveted joint, which may not always be the case. The local heat application will result in residual stresses that may increase the peak stress. The different properties of the base material and of the welded material, and the possible changes in the properties of the base material adoining the weld may produce internal boundaries, resulting in high stress concentration similar to those produced by abrupt changes in shape. The stress concentrations thus produced, and the changes in the properties of the base material, may result in a considerably lower fatigue strength of the joint.

Safety Factors and Working Stress.—The adequacy of an engineering structure is expressed in terms of the safety factor, which is supposed to represent the ratio between the actual and the ultimate conditions, under which the structure will fail. The safety of a design is not determined by an average safety factor, but by the lowest safety factor at the danger spot of the weakest element. It may be influenced by the fact that in any redundant construction, the members can partly unload this burden upon other parts of the structure, thus making the actual safety factor higher than the minimum safety factor of a single element. The factor is to be considered first in strength, represented by the stress safety factor, and second in stability, represented by the load safety factor. In the consideration of any given safety factor of a structure the most important item is the reliability of its determination. If crude design methods are used, the actual safety factor in many places or instances may be much lower than the theoretical safety factor. If more refined design methods are used, the actual and theoretical safety factors will be in closer accord.

In practically all cases a structure designed by the use of conventional methods, with a high safety factor, will be less safe than one designed by the use of more refined methods and using a smaller safety factor. The conventional safety factor idea is that the safety factor for a particular design under given loading conditions is a constant coefficient; but actually the safety factor varies.

In all engineering structures the factor of safety decreases as time goes on. In some instances it may initially increase for some time, if the stress-bearing capacity of the material increases, until it reaches its maximum, and it will then decrease. Some of the alloys as well as concrete are typical in that respect. In most structures the initial safety factor is the largest and after the structure goes into service the safety factor decreases gradually. Taking into consideration only the decreasing stress-carrying capacity with repeated loadings, the initial or maximum safety factor in many cases may be twice as high as that which will obtain after a definite time period. The difference will be still greater if the changes in material—its corrosion or deterioration—are considered.

If these conditions are realized it should be clear that a simple statement that the structure is designed with a certain safety factor is not sufficient. The safety factor should be qualified. In structural designs in which the fatigue properties and the stress concentrations are neglected, the design safety factor represents a theoretical initial safety factor, which actually is not attained even at the initial period.

The safety factor specified in modern aviation designs in which the fatigue properties and stress concentrations are considered, represents the actual safety factor which the structure will have at the end of its useful life, the initial safety factor being higher. The selection of the proper safety factor requires considerable engineering judgment. Too high a factor will result not only in an uneconomical structure, but will make it difficult (as, for instance, in long span bridges) or even impossible (as in aviation), to design a structure that could fulfill its functions. The danger of too low a safety factor is obvious. In practical applications the safety factors vary between 1.25 and 4.0 and often are even higher. It seems only reasonable to accept lower safety factors for more refined designs and to accept high factors for less refined designs in which only the initial theoretical safety factor is considered.

Region of Permissible Stress Variations.—The data given in a 45° diagram may be applied to a design if the proper reductions are applied. Fig. 11 shows the method of applying reductions. It is based on data given in the previous diagrams on tests made on highly polished small-sized specimens.

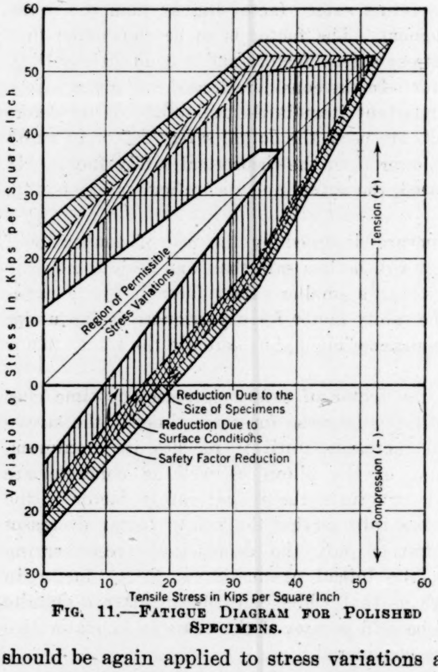


FIG. 11.—FATIGUE DIAGRAM FOR POLISHED SPECIMENS.

Part of the reductions must be applied to the stress variations. The first reduction applied takes care of the difference between the size of the testing specimen and the actual structural elements, and the possible difference in their properties. A small reduction in stress variations should take care of the usual difference in sizes. An increased reduction would be necessary if there is a difference in properties of the test specimens and the material actually used.

The next reduction shown in Fig. 11 should take care of the difference in surface conditions of the test specimens and the actual element. This reduction should be again applied to stress variations and should be taken from a diagram similar to the one shown on Fig. 5, giving, for instance, the reduction between highly polished and machined specimens.

The final reduction will be the safety factor reduction, which should be applied to the maximum value of stress left after the previous reductions were applied. The area finally left, shown in Fig. 11, will give the region of permissible stress variations, which should govern the design.

CONCLUSION

During the last few decades the theoretical bases of Structural Engineering have been very much standardized and have changed slowly. Structural engineers used ordinary mild steel and there was no particular need of refinement in material or in methods of design. At present, there seems to be a gradual change in conditions, making advisable a more critical attitude.

Conventional designs neglect stress concentrations, fatigue, and creep phenomena, and assume perfectly elastic properties of the materials. Such designs are idealized, presuming a mode of load application and jointing (which in the majority of cases may not be approached), and the use of non-existing ideal materials.

It is only natural that the development of engineering science should have started with the simplified, idealized conditions, and that at present the trend is toward evaluating the adjustments that are necessary in order to force the idealized and the actual conditions into a better agreement.

A clear visualization of these conditions will make possible the determination of the degree of refinement in the theoretical design that is practical and reasonable. There is a widespread belief that better designs are obtained if the conventional theories are applied and extended to cover a number of possible secondary influences.

Considerable time may be spent and very elaborate theoretical values obtained, which may represent a very refined extension of the conventional theories, but in most cases such efforts do not disclose the behavior of the actual structure that is not subjected to idealized loadings and is not made of an idealized material.

The conventional theories being only an approximation, the further extension of such approximate theories will result, in many cases, in larger discrepancies between the evaluated and actual behavior of the structure. More refined designs will involve additional engineering work, and it is desirable at least to indicate the methods by use of which the additional time invested in a design will bring returns in assuring a closer agreement between the evaluated and actual conditions. This purpose may be stated as the designing of structures in which the evaluated and the actual safety factors are identical.

The paper records an attempt to outline broadly the factors that should be taken into account. In the following papers of the Symposium some of the points mentioned in this paper are developed more authoritatively and in more detail. It is believed that at present the developments are too fluid to be stated in terms of definite rules and recommendations, although there is a possibility that the static conditions of the past will not return for many years. In that event, the structural engineer must change his attitude in line with that of the more advanced branches of engineering.

TESTS OF ENGINEERING STRUCTURES AND THEIR MODELS

BY R. L. TEMPLIN[4], M. AM. SOC. C. E.

SYNOPSIS

The scope of this paper is limited to tests of structural members, involving the determination of deflections, stresses, and general behavior under given loads, of engineering structures and their models. Following mention of the purposes and types of the tests made, is a discussion of the details of the methods used. Consideration is given to similarity conditions, model materials, testing apparatus, and testing technique. Reference is made to specific examples of tests of actual structures, and of small-sized and over-sized models. The results from an over-sized model example are given, illustrating the various purposes mentioned.

From consideration of the factors affecting structural tests of engineering structures and their models, and the results obtained from such tests, it is concluded that the purposes enumerated can be fulfilled if proper consideration is given to actual departures from strict similarity conditions.

TYPES OF ENGINEERING STRUCTURES AND TESTS TO BE CONSIDERED

Within recent years there is evidence of an increase in the number of engineering structures that are tested, either during or after construction, or both, under definitely imposed conditions of loading. Although these tests have been made with some differences in purpose, in general, they have been conducted with the intention of checking the design assumptions, postulates, and calculations. Examples of such tests may be found in the publications of the Society, particularly in the *Transactions* for the past seventeen years (1919–1936). Tests of the kind referred to, frequently afford a very satisfactory accelerated method for revealing the adequacy or inadequacy of the structures in lieu of the more generally used tests of time and service.

The types of structures that may be tested include essentially all those made of engineering materials with apparently no serious limitations being imposed by size, use, or location. The scope of this paper, however, will be restricted to structural tests involving determination of deflections, stresses, and general behavior under given loads.

The designs of most modern engineering structures are based on: (1) Some given over-all dimensions; (2) certain physical properties of the materials involved; (3) assumed conditions of loading; (4) analyses of stresses, deflections, and stability; (5) empirical rules; (6) "engineering judgment" with

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[4] Chf. Engr. of Tests, Aluminum Co. of America, New Kensington, Pa.

"suitable" allowances for safety; and usually with (7) consideration of the economics involved.

When an engineering structure performs the expected duties throughout or beyond its estimated life, it is generally considered a successful engineering project. Most engineering structures have successfully fulfilled their designers' intentions, but this does not mean, necessarily, that the actual behavior of the structures under service conditions, or the magnitude of the stresses throughout the various parts of the structures, have been in close agreement with the assumed or calculated values. Although many engineers are familiar with the assumptions involved in the design of a given structure, there seems to be a lack of data to show quantitatively the discrepancies between the actual behavior either of the structure as a whole or of its various parts and the predetermined theoretical values. The quantitative effects of the errors involved in the usual assumptions made concerning isotropy and homogeneity of materials, Hooke's law, continuity, and fixity are often difficult to evaluate.

In order that a better understanding of these factors, as well as of many other factors involved in the behavior of some engineering structures, either under arbitrarily imposed or actual service conditions, may be had, tests have been made on actual structures or on their models. In the majority of cases the results obtained from such tests have indicated quite definitely that this field of endeavor might be considerably enlarged with consequent advantages to all concerned Among the various advantages that might be anticipated, would be more efficient, safer, and, probably, more economical structures.

Purpose of Tests.—Tests of actual structures are usually made for any or all of the following purposes: (1) Checking the analysis against the actual behavior of the structure under known load conditions; (2) checking the actual behavior of the structure under service conditions; (3) providing data to be used as a basis for changes in design rules; and (4) checking the efficacy of any alterations made in an existing structure.

Tests of models of engineering structures may be made for any or all of the following purposes: (5) To check analysis by actual measurement of the behavior of the model under known load conditions; (6) to provide data to be used as a basis for changes in design rules; (7) to check the efficacy of any proposed alterations in a given construction; (8) to supplement theoretical analysis by experimental analysis; (9) to avoid theoretical analysis by experimental analysis; (10) to provide an easier, quicker, and usually a less expensive means of obtaining the desired information, than would be the case if the actual structure were tested; and (11) to provide a means for studying the behavior of a design under loading conditions not possible with a full-sized structure.

Types of Tests.—Types of mechanical tests which may be conducted with the foregoing purposes in mind, may be subdivided into two general classes: (*a*) Static; and (*b*) dynamic.

Under static tests may be included the determination of strains (and, from these, stresses) and deflections, at critical points or throughout the structure, resulting from known conditions of loading.

Dynamic tests include those involving the measurement of strains (and, hence, stresses) and deflections resulting from the impact of moving loads, as well as the determination of the natural vibration frequencies of various parts of the structure, or of the structure as a whole.

In both types of tests it should be emphasized that the stress and deflection values obtained, represent changes resulting from the conditions imposed, and although corrected for "no-load" or "dead-load" or "at-rest" conditions, should not be designated as "true" or "absolute" stress or deflection. Even a limited understanding of the strains induced in materials, especially metals, during production, and in the actual structure by fabrication and erection, would mitigate seriously against the use of such appellations unless heterodox definitions of their meaning are to be accepted.

TEST METHODS

Size of Models and Materials Required.—Considerable discussion has appeared in technical literature concerning the laws of similarity to be followed when using models to analyze the behavior of engineering structures. The papers by B. F. Groat[5], M. Am. Soc. C. E., and the discussions pertaining to them, have emphasized the theoretical requirements for models which are intended to comply with strict similarity conditions. In these same discussions it has been shown that one may make some departures from the ideal conditions and yet obtain worth while results. The writer is quite in agreement with the latter idea after a number of years of experience in the use of models to assist in analyzing engineering structures. Although it is recognized that extensive tests of actual-sized structures are preferable in many instances, for obvious reasons, they are often impracticable and, in some instances, impossible. Supplementary partial tests on actual structures frequently afford a satisfactory final check on both theoretical and experimental analytical studies, as, for example, those made on the towers of the George Washington Bridge[6].

Models of engineering structures are generally thought of as being smaller than their prototypes. This need not always be the case, however. In some instances over-sized models can be, and have been, used to advantage. An example of tests on an over-sized model will be cited subsequently. Irrespective of whether the model size chosen is larger or smaller than the prototype, conditions of testing, in most cases, can be controlled to better advantage than in the case of tests of actual structures.

Models Made of the Same Material as the Prototype.—As indicated by Mr. Groat[5] an engineering model should be one which is scaled down, or up, in such a way that the dimensional and time requirements explained in his paper are fulfilled. These are theoretically ideal conditions which are very difficult to comply with in actual practice. Consider, for instance, a model of a truss in which structural members are fastened together by means of gusset-plates. If the model is a photographic reproduction of the prototype, the cross-sectional areas of all the members will be changed in the proportion of the square of the scale ratio, but the moments of inertia of the members will be changed in a different proportion. The stiffness factors of the members—that is, the moment-of-inertia-to-length ratios of the various members—however, will be changed to still another scale. The stiffness of the gusset-plates will be changed to a scale different from that of the cross-sectional areas. Consequently, when the various members are considered individually, the degree of end restraint will not be the same as in the prototype, and the behavior of the various members of the model, under combined axial loads and bending, will be different from that of the corresponding members of the prototype.

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[5] "Ice Diversion, Hydraulic Models, and Hydraulic Similarity", *Transactions* Am. Soc. C. E., Vol. LXXXII (1918), p. 1138, and "Theory of Similarity and Models", *Loc. cit.*, Vol. 96 (1932), p. 273.

[6] *Transactions*, Am. Soc. C. E., Vol. 97 (1933), p. 181.

In the laboratories of the Aluminum Company of America, at New Kensington, Pa., the procedure of testing models of structures has been to consider the model itself as a structure and to make a combined analytical and experimental study of it as such, to determine its behavior under various types of loading. By so doing, all the factors introduced are treated automatically in the light of their proper magnitude, so that when the prototype is being studied, the various elements entering into the behavior of the structure are not considered by simply applying a scale factor to the results of the model tests, but the various factors and considerations are included in a detailed analysis. In joining various members to make an assembled structure, it is impossible to reproduce, in a small or in a large model, the same conditions that are normally obtained in the actual construction of its prototype. Therefore, the effect of connections cannot be determined reliably from small or large models unless all the factors entering into the determination are known. This is especially true in connection with loads which normally will produce fairly high stresses in the prototype and may cause a local yielding of the material, whereas, in a small model of the structure, in which the stresses are scaled down automatically, this local yielding does not occur.

In many models which depend for load upon their own dead weight, or on the weight contained within them, it follows inevitably that if a direct scale reproduction of the prototype is made, the resultant stresses in the various parts of the model structure will be changed also, to the same scale ratio. This procedure makes any measure of strength obtained from models of this type very ineffective, unless analytical work involving all the variable factors is carried on and used as a basis for interpreting test data.

In many structures the ultimate safe load that can be supported depends upon the stability of the structure as a whole, or of some of its parts. In general, the stress at which a particular member becomes unstable, is a function of the proportions of its parts; as, for instance, the stress at which a thin outstanding plate buckles is directly proportional to the modulus times the square of the ratio of thickness to the outstanding width. Thus, the stress at which an outstanding flange would become unstable would be the same for a prototype or a model, with the result that if the stresses in the model are changed as the scale ratio, a false sense of stability is obtained.

Models Made of Materials Different from Those of the Prototype.—It may be desirable to make models of an engineering structure from a material different from that of the prototype, so as to:

(1) Correct for the stresses that actually occur in the model; that is, the ultimate strength of the material for the model should be changed to the same proportion of the actual stress under test as the ultimate strength of the prototype bears to its actual stress in service.

(2) Correct for the stability; that is, to have a material with a modulus sufficiently low, so that the stress at which the various component parts will become unstable bears the same relationship to the stresses in the model, as the stress at which the prototype becomes unstable bears to the actual stresses in it.

(3) Correct for the different relative distribution of direct stresses, bending, and shear, by having either auxiliary loads or a composite model made up of two materials so that resistance to shear and bending of the model will be the same as that in the prototype.

(4) Correct for the relative proportion of dead and live weight in the model and in the prototype. In this event the specific gravity of the model should be comparable to the specific gravity of the prototype.

(5) Provide the same distribution of stress in two-dimensional or three-dimensional models. This requires the Poisson ratios for the materials in the model and the prototype to be equal.

In order to interpret model studies, Factors (1) to (5) must be considered in connection with the theoretical similarity studies indicated in Mr. Groat's papers previously cited, because, if any one of the conditions enumerated is violated, it may be that a direct comparison cannot be made between the model and the structure. Since it is practically impossible to fulfill all these conditions simultaneously, it becomes necessary to supplement model studies with adequate theoretical analyses in order to take into account the effects of the various factors.

In selecting the size of model to be used, consideration should be given to the size of the testing apparatus available and the magnitude of the strains and deformations to be measured. Even if specially designed apparatus must be used, for any given tests, limitations will be imposed, which must be recognized if the desired results are anticipated.

Testing Apparatus.—In the testing of actual structures or models it is customary to apply either static or dynamic loads, or both. In the case of static loads applied to actual structures, dead weight of one form or another is frequently used. Almost any form of available material, of suitable density, may be used for such loading, but care should always be taken to insure that the load is applied to the structure in as close agreement as possible with the assumed conditions of application, or in accordance with service conditions (preferably both), if close agreement between calculated and actual values is anticipated. The method sometimes used for loading both actual structures and models, is a cause of a considerable number of the discrepancies observed. There are numerous methods, other than dead weight, for applying loads to the structure being tested; for example, calibrated hydraulic jacks or springs can often be used to advantage. In the case of laboratory tests, some arrangement such as that used in the Materials Testing Laboratory of the University of Illinois[7] is worthy of comment. A part (23½ ft by 120 ft), of the concrete floor of the laboratory is heavily reinforced and provided with suitable inserts at regular intervals, into which may be screwed tension bolts that furnish the necessary reactions for any structure to be tested. When the structure is restrained by this method it may be loaded with dead weights and levers, or hydraulic jacks or springs with load-indicating devices.

When the structure to be tested can be put in a testing machine, the loading problems are usually very much simplified, although under such condition restraints may be applied to the structure under test, that are not in strict accordance with the assumed or desired loading conditions. The torsional and bending resistances offered by testing machines in the test of columns, using any type of heads except those substantially free from friction, is an example. The spherical heads usually provided with commercial testing machines are not substantially free from friction while under load. Furthermore, considerable lateral restraint is imparted to beams by testing machines when using the testing procedure generally followed, and this lateral restraint may appreciably affect the test values obtained.

Changes in the strains occurring in the various parts of a structure may be determined by using any one or more of a variety of strain-gages. Although a number of such instruments of different sizes and types are available commercially, special designs are evolved, occasionally, to meet particular requirements. The types used may be classified as mechanical, electrical, optical, acoustical, or some combination of the four. Some of the instruments are intended to be applied manually to the structure while making an observation, and then removed. Others are attached permanently for the duration of the test. Both indicating and recording (intermittent and continuous) types are available. Some can be read locally only, whereas others permit of distant observation.

The instruments to be selected for field use, in most instances, should be more rugged than those which may be used in the laboratory. Increased ruggedness of design, unfortunately is frequently accomplished at a sacrifice of sensitivity or accuracy, or both. Temperature effects are often negligible under laboratory conditions of test, but must be considered carefully in the field; thus, certain parts of the better field types of strain-gages are made of invar. Even so, such types of instruments should be checked with suitable standard bars to correct for residual temperature effects as well as for incidental maladjustments. An example of the consideration which should be given to a selection of strain-gages may be found in the report of the Committee on Arch Dam Investigation[8].

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[7] Described in pamphlet issued by Univ. of Illinois for distribution at the dedication of the Laboratory, May 2, 1930.

[8] *Proceedings*, Am. Soc. C. E., Vol. LIV, Pt. 3, May, 1928, p. 64.

Regardless of the type of strain-gages selected, considerable attention should be given to the details of the methods of applying the instruments to the gage lines, throughout which the changes in strain are to be measured. For those strain-gages which require holes at the ends of the gage length, variations in the size and condition of the edges of the holes used affect both the accuracy and the sensitivity of certain instruments more than others, depending on the mechanical design of the instrument. When clamps or machine screws are used for attaching the instruments, appreciable errors may occur because of an indeterminate degree of restraint in the apparatus. The method of attachment sometimes used with some types of tensometers, or strain-gages, are examples.

The sensitivity, accuracy, and total range of available strain-gages vary appreciably, and, therefore, deserve careful consideration from the viewpoint of what is desired in any given test. Just because a strain-gage is sensitive to ± 0.00001 in. per in., it should not be inferred that the instrument is accurate to the same degree. Quite frequently the limit of accuracy will be found to range from three to thirty times this value. The total range of an instrument may become a limiting factor when attempting to measure large strains. Some of the instruments, notably those of the optical type, have very limited total ranges or give values for large deformations which may include appreciable errors that would be negligible in the case of small deformations. There are a number of other details which should be given consideration in the application of strain-gages to a structure, which will be dependent on the particular instrument used, as well as on the test conditions. Only a few of the more important factors involved have been mentioned, for the purpose of emphasizing the need for attention to such details if the best results are desired.

Measurements of the deflections of structures, or of their component parts, require first a definition of the desired quantities with respect to some given datum planes or bench-marks. Under test conditions, the reference planes or marks should remain fixed or, if they do move, both the magnitude and direction of such movement should be known. Deflections of a beam, for example, are best determined by reference to points on the neutral axis directly over the centers of the supports, and deflections of a pin-connected truss by reference to centers of the end pins at the supports. Deflections of a concrete arch dam, however, might be construed as including deflections of the arch proper, the movement of abutments and of that part of the canyon affected by the load on the structure, with reference to fixed points beyond the influence of the structure and its load. There are excellent opportunities for both confusion and errors, in attempting to interpret given deflection values for structures, where there is evidence of lack of appreciation of the requirements for proper reference planes or points

The instruments for deflection measurements are so varied in character and type as to preclude any discussion other than mere mention of a few of those more commonly used: Scales, micrometer screws, dial-gages, tapes, plumb-lines, levels, theodolites, and combinations of some of these devices. Likewise, their sensitivity and accuracy vary over wide ranges. It will suffice for the purposes of this paper, to emphasize the necessity for selecting suitable deflection-measuring apparatus with regard to the character of the values to be determined and the desired accuracy, rather than let the choice depend entirely on availability of equipment. Deflection measurements, when properly made and correlated with known loading conditions, may, and generally do, give reliable data on the behavior of a structure and serve as a valuable check on the theoretical or experimental analysis.

All the measuring devices used in testing structures should be calibrated by comparison with acceptable standards, preferably those furnished by the National Bureau of Standards, U. S. Department of Commerce, Washington, D. C. Many types of measuring instruments, such as strain-gages, change with use because of wear in the moving parts or accidental misuse. In such instances, additional calibrations should be made at intervals, depending on the specific instrument and the usage it has received. Calibration devices suitable for checking load, strain, and deflection-measuring apparatus, are available, and can be certified as to accuracy by the Bureau of Standards, and then used by any one possessing them, for calibrating the instruments for which they are suited. The degree of accuracy of measurements, suggested by so much attention to calibration of apparatus, may not be in harmony with some of the tests of structures heretofore made; but in conformance with the idea of attempting to enhance, materially, the value of experimental analyses of structures, it is believed that more attention should be given to details of calibration of the instruments used.

Testing Technique.—In the testing of structures, the errors arising from the personal equations involved in the manipulation of the instruments used, must be considered, in addition to those already mentioned. The magnitudes of these errors usually become less, at a diminishing rate, as the experience of the observer increases. Some data exemplifying the magnitude of the personal errors, involved in strain-gage measurements, are shown in Fig. 12. The ordinates of these curves represent the differences in the lengths of the gage lines determined from two readings, and the abscissas represent the percentage of the total number of gage lines on which differences in readings did not exceed values indicated by ordinates. The data were selected at random from a large mass of similar data available. Fig. 12(*a*) shows a comparison of data obtained by two different observers using the same 2-in. strain-gage, on gage lines located on horizontal, vertical, and oblique surfaces. The data shown represent measurements on 536 gage lines by Observer *A* and on 569 gage lines by Observer *B*. In Fig. 12(*b*), the group of readings by Observer *B* has been broken down with respect to position in order to show how that factor affects the personal equation involved. Fig. 12(*c*) shows a comparison of results obtained by two different observers, one using a 2-in. strain-gage and the other a 10-in. strain-gage of a different type. The data given are from measurements taken on 1 050 gage lines by Observer *C* and on 310 gage lines by Observer *D*.

Observed values obtained during the tests of structures, must be corrected frequently or adjusted to compensate for changes occurring in the test conditions, such as rise or fall in temperature. Just how to make corrections

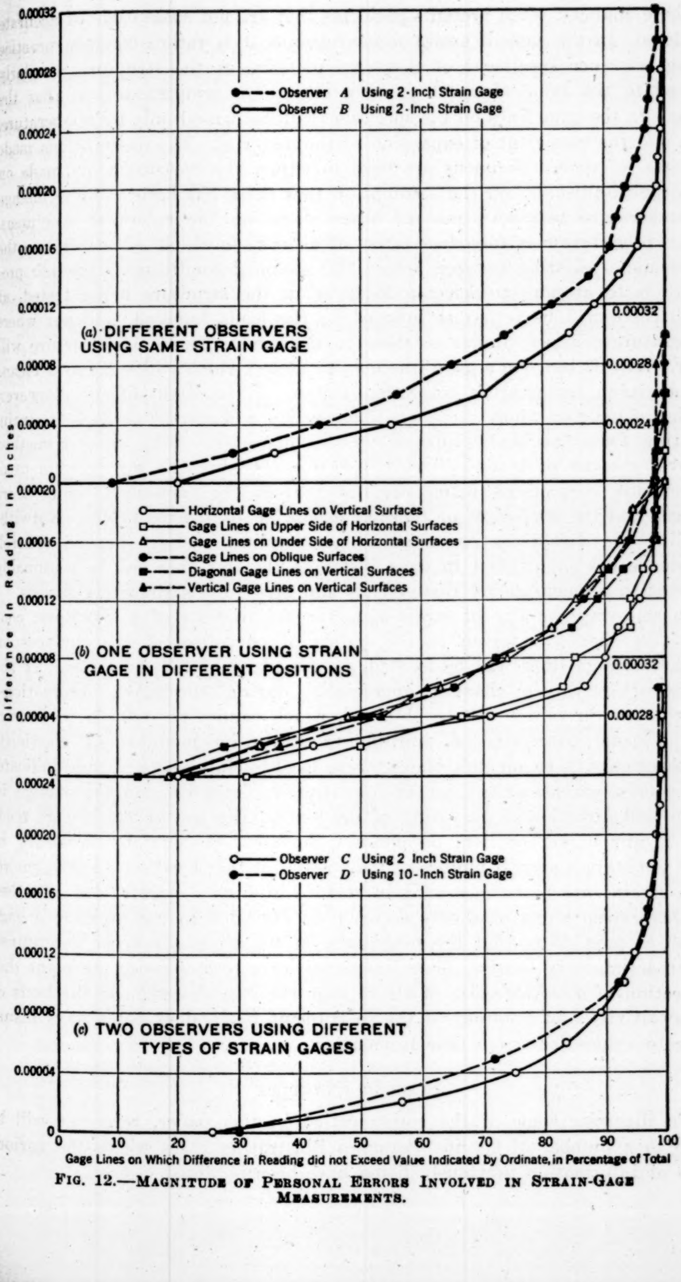

FIG. 12.—MAGNITUDE OF PERSONAL ERRORS INVOLVED IN STRAIN-GAGE MEASUREMENTS.

for these changes, often presents problems that are not susceptible of accurate solutions. In the case of strain measurements, it is rather common practice to use reference specimens of the same material as the structure, similarly exposed to the same elements causing change in temperature, so that the changes in the gage lines on the specimens will be caused only by temperature, time, and the coefficient of expansion of the material. The observations made on these reference specimens are used to adjust the measurements made on the structure, usually on the assumption that there has been a linear change in temperature between times of observations on the reference specimens. Such a procedure may introduce errors of magnitude which will depend on the discrepancies existing between actual and assumed conditions. Another procedure is to attempt to select a location on the structure being tested, at which there will be no stress induced by the loads imposed, and yet where temperature changes similar to those in the remainder of the structure will occur. At this location a gage line may be placed which will serve as a means for obtaining temperature correction values. It is often difficult, however, to select a location where there would be no doubt about the absence of strain, resulting from the loads imposed on the structure. Still another method involves the use of invar reference bars for checking the instruments used, temperature corrections being calculated from the measured temperature changes and the coefficient of thermal expansion of the materials. Notwithstanding the difficulties presented by temperature changes beyond control, experience has shown that in most instances the test data can be reasonably adjusted to compensate for these effects and give quite satisfactory results.

In selecting the size of strain-gage for use in tests of a structure, consideration should be given to the radius of curvature of the surfaces on which the gage lines will be located, in the plane of the gage line. If the radius of curvature changes appreciably during successive observations, apparent strain values will be obtained, which cannot be reduced to stresses by the usual procedure of multiplying strain by modulus of elasticity. Measurements taken on thin plates where local buckling occurs, may indicate, erroneously, exceptionally high or low stresses, depending on the change in radius and direction of curvature of the plate. The use of strain-gage readings at any given location, on opposite surfaces, is generally necessary in order to obtain a correct picture of the stresses at that location. Furthermore, when strains are to be measured in regions of high concentration of stress, or where steep stress gradients occur, it is desirable to use as short a gage length as possible so that the magnitude of the stresses may be determined within satisfactory limits. These instances serve to emphasize the point that a selection of a suitable size of strain-gage (or gage length), on the basis of the sensitivity and accuracy of the instrument itself, does not always insure accurate values for stress measurements.

Specific Examples

To illustrate some of the points raised in this paper, reference will be made to a number of specific examples illustrating cases where the various types of information previously listed have been obtained.

Actual Structures.—The tests made on the Hell Gate Bridge[9] comprise an example of those made for the purpose of checking analysis by measurements on the actual structure. The field tests referred to in the reports of the Special Committee on Stresses in Railroad Track[10] and of the Special Committee on Impact in Highway Bridges[11]; and the tests made on steel railway bridges[12] are examples of checking the actual behavior of structures under service conditions. The tests made on Stevenson Creek Dam[13] were instituted for the purpose of providing new experimental knowledge that would assist in better design rules for such structures. Extensive strain and deflection measurements, made on the Santeetlah pipe line[14] afforded a very satisfactory check on the efficacy of alterations made in the design of stiffener rings and supports after partial collapse of the structure during its initial use.

Model Tests.—Tests made on the model of Boulder Dam[15] are an example of those made to check the analysis by actual measurements of the behavior of a model under given load conditions. A series of models of pipe-line designs were made in connection with the investigation of the Santeetlah pipe line[14] for the purpose of providing data to be used as a basis for changes in the design rules.

One of the purposes of the tests by Wilbur M. Wilson[16], M. Am. Soc. C. E., on multiple-span concrete arches with and without decks, was to check the efficacy of variations in a given type of construction. The model tests in connection with the George Washington Bridge towers[17], were made to supplement theoretical analysis by experimental analysis. The model tests of multiple-span arches conducted by George E. Beggs[18], M. Am. Soc. C. E., offered a means of avoiding theoretical analysis by experimental analysis. Substantially all the tests on model dams are examples of efforts made to provide easier, quicker, and usually less expensive means of obtaining the desired information than would be the case if the actual structures were tested. Model tests made for the purpose of predicting the behavior of a falling dam[19] afford a good example wherein the use of models was a means for studying the behavior of a design under loading conditions not possible with the full-sized structure.

Numerous examples of models made from materials different from those used in their prototypes, may be found upon reference to the technical literature. A single example only, has been selected to exemplify each of the five reasons for using different materials, previously cited. Because aluminum models were used in the tests made in connection with the investigation of the Santeetlah pipe line[14], it was necessary to correct for the stresses that actually occurred in the model. The series of model tests conducted by H. E. Saunders and D. F. Windenburg for determining the strength of thin-walled structures[30] included corrections in the loads applied in order to give the same effective stability in the models as in the prototype. The model tests[31] by the late A. H. Beyer, M. Am. Soc. C. E., and Mr. A. G. Solakian, wherein photo-elastic methods were used to analyze stresses in composite materials, involved a choice of materials to correct for the different relative distributions of stresses, by using composite models of aluminum and bakelite. A special material was developed and used in the model tests of Calderwood Dam[32] for the purpose of correcting for the relative proportion of dead and live load weight in the model and in the prototype. The material used for a model of Boulder Dam[33] was selected, among other reasons, because it had a Poisson's ratio substantially the same as the concrete which would be used in the prototype.

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[9] *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1040.

[10] *Loc. cit.*, p. 1191; and, also, Vol. LXXXIII (1919–1920), p. 1409.

[11] *Loc. cit.*, Vol. 95 (1931), p. 1089.

[12] *Bulletin*, A. R. E. A., Vol. 37, No. 380, October, 1935.

[13] *Proceedings*, Am. Soc. C. E., May, 1928, Pt. 3.

[14] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 154.

[15] *Engineering News-Record*, April 7, 1932, Vol. 108, No. 14, p. 494.

[16] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 424.

[17] *Loc. cit.*, Vol. 97 (1933), p. 179.

[18] *Loc. cit.*, Vol. 88 (1925), p. 1208.

[19] *Civil Engineering*, July, 1932, Vol. 11, No. 7, p. 415.

Over-Sized Models Tests.—In each of the foregoing examples of model tests the size of the model was smaller than its prototype. In certain instances, however, more satisfactory results can be obtained by the use of over-sized models. The results from such experimental tests, made on what may be considered an over-sized model of a filleted, re-entrant, 90° corner, will be given as an example of such tests. The reason for using an over-sized model in this particular case was that with such device it was physically possible to obtain the stress distribution with available strain-measuring equipment, whereas, the usual full-sized re-entrant corners are so small as to make strain measurements quite difficult and unsatisfactory. Fig. 13 is a view showing the method used in testing a series of specimens cut from a plate $1\frac{1}{16}$ in. thick. The material used for these specimens was a high-strength aluminum alloy (17S-T)[34] plate (Young's modulus = 10 300 000 lb per sq in.; and the proportional limit = 20 000 lb per sq in.). The specimens were loaded in a testing machine having seven different capacity ranges, vary-

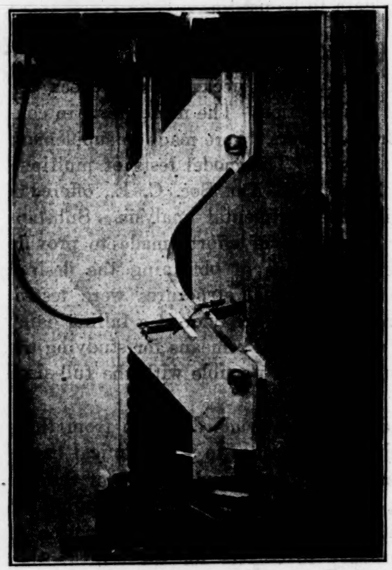

FIG. 13.—VIEW SHOWING METHOD USED IN TESTING 90 DEGREE, RE-ENTRANT, ANGLE SPECIMENS.

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[30] *Journal of Applied Mechanics*, A. S. M. E., December 15, 1932, Vol. 54, No. 23, p. 253.

[31] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1196.

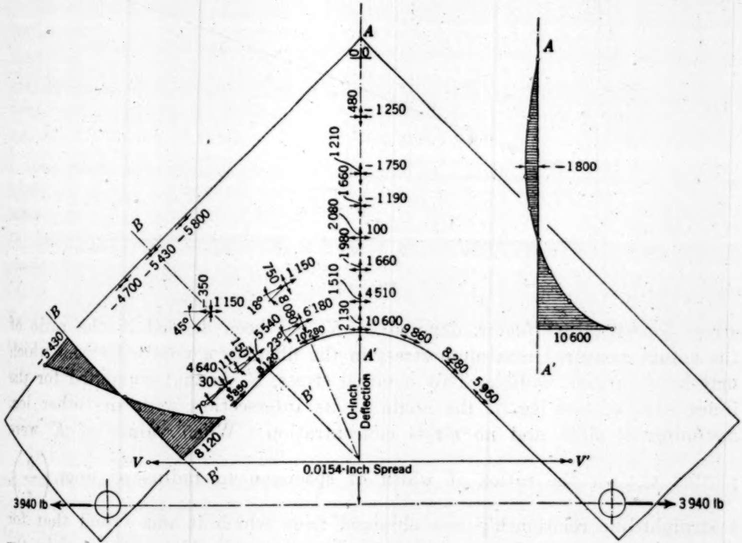
[32] *Loc. cit.*, Vol. 100 (1935), p. 185.

[33] *Loc. cit.*, p. 240.

[34] The symbols in parenthesis denote the trade designation by which the alloy is commonly recognized.

ing from 400 to 40 000 lb, but in this particular series of tests the range chosen was the lowest that would give the desired load. In all cases the load could be determined within ± 1 per cent. The machine was previously calibrated in accordance with Tentative Methods of Verification of Testing Machines of the American Society for Testing Materials[25].

Two tensometers, having 0.5-in. gage lengths and magnification ratios of 2000:1, were used to determine the strains. The instruments were placed in corresponding positions on opposite faces of the specimen, and the readings were averaged to take care of any variations that occurred throughout the thickness of the specimen. The magnitude of the load applied, in any instance, was that which would give a readily observable maximum strain value but was insufficient to distort the specimen permanently. To insure against permanent distortion and partly to provide measurements of the actual distortion under load, the distance from the line of action of the force to the center of the face of the fillet was measured in each test, using the taut wire shown in Fig. 13, and a suitable scale. In addition, the spread of the angle was measured by means of a 10-in. strain-gage, on a gage line parallel to the direction of load application. The tensometers and the 10-in. strain-gages were previously calibrated, using a device described elsewhere[26], and found to be correct within ± 0.000015 in. per in.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10 280
23°
5 540
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5 850
7°
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8 120
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B'
0-Inch Deflection
V
V'
0.0154-Inch Spread
3 940 lb
3 940 lb

FIG. 14.—DISTRIBUTION AND MAGNITUDE OF STRESSES IN RE-ENTRANT 90-DEGREE ANGLE SPECIMEN HAVING 3-INCH RADIUS FILLET

Observations were made under no-load and full-load conditions throughout most of the tests. Special tests, however, were made for the purpose of demonstrating that the load-strain relationship on any given gage line, throughout the range of loads used, remained a linear one, thus making it necessary to determine two points only (no-load and full-load), to define the complete load and strain relationship at any point. The special tests also showed that the distortions of the specimens varied directly with the load.

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[25] *Proceedings*, A. S. T. M., Vol. 35, 1935, Pt. I, p. 1303.

[26] *Loc. cit.*, Vol. 28, 1928, Pt. II, p. 714.

Fig. 14 shows the magnitude and direction of the stresses obtained from one of the test runs made. The small arrows are on the locations of the various gage lines, with diverging arrow-heads indicating tension along the gage lines and converging arrow-heads indicating compression. Stresses, in pounds per square inch, along the gage lines, are indicated by the numbers directly adjacent to each line. The directions and magnitudes of the stresses at points away from the edges of the specimen were obtained by measuring strains on rosettes[27]. The magnitudes and directions of the stresses at such points were determined by the dyadic circle method.[27]

With these data available on a series of angles of different proportions, a stress concentration factor was determined, as shown in Fig. 15. This stress concentration factor, denoted by K, has been defined as the ratio of the actual measured maximum stress in the fillet, to a nominal stress which may be computed readily. This nominal stress, S_1, is that computed for the inner edge of one leg of the angle at its intersection with the other leg, assuming no fillet and no stress concentration. When values of K were plotted against the ratios of width of specimen to radius of fillet, $\left(\frac{b}{r}\right)$, a straight-line relationship was obtained from which it was found that for the specimens tested, the stress concentration factor could be expressed by the equation,

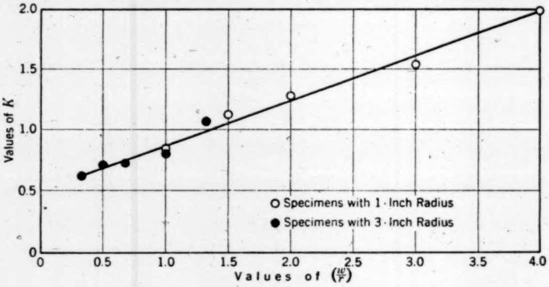

FIG. 15.

$$K = 0.5 + 0.37\left(\frac{b}{r}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For other loading conditions different stress concentration factors would be obtained.

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[27] *Journal of Research,* National Bureau of Standards, Vol. 10, May, 1933, Paper No. 559, and Vol. 15, December, 1933, Paper RP851.

An experimental study of this same type of problem has been made by means of photo-elastic methods[28]. In this connection, it is interesting to note that with a slight modification of the procedure and technique of strain measurement on over-sized models, it would be possible to extend the stress concentration studies to the case of local bending in the outstanding legs of structural shapes, whereas the photo-elastic methods are definitely limited, at present, to studies in a single plane.

Over-sized model tests were also found very useful in evaluating the strains occurring in multi-stranded cables under conditions of vibration[29]. In this particular instance the experimental analysis was supplemented by theoretical analysis and the two correlated very closely indeed.

CONCLUSIONS

From a consideration of the factors affecting structural tests of engineering structures and their models and the results obtained in many instances, from such tests, it may be concluded that:

(1) Tests of actual structures can be made, which will yield satisfactory results when the purposes of such tests are: (*a*) Checking an analysis by observing the actual behavior of the structure under known load conditions; (*b*) checking the actual behavior of the structure under service conditions; (*c*) providing data to be used as a basis for changes in design rules; and (*d*) checking the efficacy of any alterations made in an existing structure.

(2) Tests of models can be made which will give satisfactory results when the purposes of the tests are: (*a*) To check an analysis by actual measurement of the behavior of the model under known load conditions; (*b*) to provide data to be used as a basis for changes in design rules; (*c*) to check the efficacy of any proposed alterations in a given construction; (*d*) to supplement theoretical analysis by experimental analysis; (*e*) to avoid theoretical analysis by resorting to experimental analysis; (*f*) to provide an easier, quicker, and usually less expensive means of obtaining the desired information, than would be the case if the actual structure were tested; and (*g*) to provide a means for studying the behavior of a design under loading conditions not possible with a full-sized structure.

In the case of tests of models the results mentioned in Conclusion (2) may be anticipated in most instances, provided the actual departures from strict similarity conditions are suitably taken into consideration when interpreting test observations

ACKNOWLEDGMENTS

The writer desires to acknowledge the assistance given in the preparation of this paper, by R. G. Sturm, and E. C. Hartmann, Associate Members, Am. Soc. C. E., and by Mr. M. B. Moore, who made the measurements on the over-sized models of filleted corners.

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[28] "Factors of Stress Concentration", by M. M. Frocht, *Journal of Applied Mechanics*, A. S. M. E., Vol. 2, June, 1935.

[29] "Vibration of Over-Head Transmission Lines", by R. A. Monroe and R. L. Templin, Members, Am. Soc. C. E., ***Proceedings***, A. I. E. E., 1932.

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PHOTO-ELASTIC DETERMINATION OF STRESS

BY J. H. A. BRAHTZ,[30] ESQ.

SYNOPSIS

The purpose of this paper is to present an up-to-date picture of photo-elastic analysis, to discuss its applications and limitations, to show the relationship that it bears to other methods of stress investigations, and particularly to bring out the relationship that it should, and must, bear to the mechanical and structural designer and his problems. This is a timely subject, since the ever-growing acceptance of indeterminate structures, and the more expensive alloys makes it imperative to the designer that he avail himself of every possible means of refined stress analysis. Since this paper deals with the subject of photo-elasticity, there is naturally considerable space devoted to standard methods, model materials, and technique.

BRIEF HISTORY AND NATURE OF PHOTO-ELASTIC EXPERIMENTATION

Photo-elasticity had its inception more than a century ago, when Sir David Brewster, in 1816, demonstrated that plane-polarized light, on traversing a flat stressed transparent specimen, underwent some peculiar change in its optical properties, such that when viewed through another polarizing unit, colored bands appeared. These bands—isochromatic fringes—he assumed to be a measure of the internal strains. It is now known that these fringes are directly related to the state of stress in the model. The fringe order is, in fact, proportional to the maximum shears throughout the model without regard to their directions, which is equivalent to stating that these fringes (identified as Order 0, 1, 2, 3, etc.), are directly proportional to the difference of the principal stresses.

Just as engineers in the past have generally been slow in applying new mathematical tools, it was not until the beginning of the Twentieth Century that practical application was made of this physical phenomenon. An excellent treatise on this subject, covering a large number of recurrent practical problems, has been written by Messrs. E. G. Coker and L. N. G. Filon[31] who, following closely the pioneer work of Mr. S. P. Thompson[32], have advanced the art and science of photo-elasticity probably more than any other experimenters, and have co-ordinated their results with parallel mathematical analyses based on the mathematical theory of elasticity.

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[30] Director, Photo-Elastic Laboratory, U. S. Bureau of Reclamation, Denver, Colo.

[31] "A Treatise on Photo-Elasticity", Univ. Press, Cambridge, England, 1931.

[32] "Note on the Application of Polarized Light to Determine the Condition of a Body Under Stress", by S. P. Thompson and E. G. Coker, British Assoc. Repts., 1909.

THEORETICAL QUANTITATIVE LIMITATIONS AND PRACTICAL QUALITATIVE EXTENSIONS OF THE METHOD

Undoubtedly there are a large number of engineers who are only slightly acquainted with the subject of photo-elasticity, some of whom have very little faith in it as a useful tool. Probably there are others, on the other hand, who believe that it is a scientific achievement representing the last word in stress analysis, and that any problem, regardless of its nature and difficulty, can be put through this seemingly magic process and the answer will appear for them in black and white on a photographic plate. These concepts represent the extremes, and a little clarification of some of its limitations might be in order so as to establish a clear idea of just what the method can do and what it cannot do.

First, the photo-elastic method is a two-dimensional analysis, and the scope of problems must come within the classification of either plane stress or plane strain. For example, a buttress in a dam of the Ambursen type, or the multiple arch, represents a case of plane stress, whereas an imaginary 1-ft slice taken from the center of a long gravity dam is a case of plane strain.

Being an experimental method, coming from the laboratory, the reliability of photo-elasticity analysis is affected by experimental errors and the personal element of the experimenter, the degree to which the operator simulates model conditions to those in the prototype, and the care with which he manipulates his apparatus, will govern the probable error of his results. It is not always a simple matter to simulate conditions directly, or to keep the probable error as low as, say, 10 per cent. For instance, the maximum fringe order may be only four. The experimenter cannot read the fringe order in a photograph closer than one-fourth of a fringe and he may be in error as much as one-half a fringe, which is equivalent to an error of 12.5 per cent. Naturally, this error will be reduced to a minimum when the model is constructed of a highly sensitive material and is loaded close to its elastic limit. Nevertheless, to obtain proper similarity of model to prototype, the physical properties of the model material must be considered carefully. For example, if certain types of experiment are being conducted to simulate a steel structure, the model material should have an elastic modulus that is constant below the yield point. Now, it so happens that the materials that fit this requirement best are not the most sensitive, and those that are extremely sensitive do not behave elastically in a manner similar to steel; therefore, a compromise must be made in choosing a model material that strikes a mean in satisfying the requirements.

Another difficulty is the uncertainty as to just what forces, reactions, and effects are acting on a structure. The same difficulties arise in a mathematical analysis, in which case unknowns are assumed. It would be nice if these factors were always known, but they are not; for example, consider the restraint of the fixed ends of a beam. The fixation is probably never perfect, and no one knows the exact conditions. Numerous examples of this kind can be cited, but they are too well known. However, although these are uncertain conditions, the photo-elastic method can nearly always simulate conditions with more accuracy than any arbitrary assumption.

Corresponding to this indeterminacy of many boundary conditions, there is the difficulty of applying the true loads to models even when they are known exactly. Generally speaking, a concentrated load is much easier to apply than a distributed load, and a uniformly distributed normal load is easier to apply than a uniformly varying normal load or a loading involving shear forces. Furthermore, straight boundaries are easier to load than curved boundaries. These are mechanical difficulties, and the technique is constantly being improved to overcome them. Some new developments along these lines are cited subsequently.

There are some possibilities of expanding the scope of the photo-elastic method in order to deal with certain three-dimensional or composite structures (that is, structures containing materials of varying elastic properties), at least qualitatively. Some progress has already been made along these lines. In this connection mention should be made of some experiments by the late A. H. Beyer, M. Am. Soc. C. E., and Mr. A. G. Solakian, in which they tested simple bakelite beams that contained integral cast-aluminum rods to simulate concrete with steel reinforcement[33].

An experiment has been conducted by Howard G. Smits, Jun. Am. Soc. C. E., in which he attempted to measure the shrinkage stresses in a model of a concrete dam by stretching the base[34]. Although there may be some doubt as to the strict legitimacy of this experiment in simulating shrinkage stresses, nevertheless, as he suggests, it emphasizes the necessity of developing a cement that can be used to combine models of different elastic properties. For instance, in this experiment, the base could be compressed, the dam glued to the base, and then the compression removed. Then tensile stresses would develop in the dam and corresponding compressive stresses in the base. Furthermore, if such a glue were developed, it would open an entire new field of possibilities, such as testing the stresses in a dam resting on a base that is either softer or more rigid than the dam itself; it would also permit the analysis of such problems as a hole in a plate, with a rim of thicker material at the edge of the hole.

Methods, Apparatus, and Model Materials

There is little variation in the optical apparatus used by experimenters in obtaining isochromatic pictures. Essentially, the instrument consists of a monochromatic light source, a nicol polarizing prism, a large lens or concave reflector to create a field of parallel rays in which to place the model, another lens or reflector to focus the rays on another nicol prism, or analyzer, and a camera with a ground-glass back. The reflector type has the advantage of requiring a comparatively smaller space; it produces a large field of parallel rays at relatively low cost, and allows the experimenter to view the ground glass and operate the loading mechanism simultaneously. It is of interest to note that the reflectors are coated with aluminum, in place of the less durable silver, deposited by the method developed by Dr. John Strong, of the

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[33] "Photo-Elastic Analysis of Stresses in Composite Materials", *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 1196, 1207.

[34] "Photo-Elastic Determination of Shrinkage Stresses", *Transactions*, Am. Soc. C. E. Vol. 101 (1936), p. 927.

California Institute of Technology, at Pasadena, Calif., in connection with the 200-in. telescope[35].

In addition to giving the value of the difference of the principal stresses or maximum shear, at all points in a model, the polariscope (as it is commonly called) also readily adapts itself to giving the directions of principal stresses at all points, the observer utilizing the black isoclinic lines or lines of constant direction of principal stresses[35]. In many structures the only stresses that are required are those at the boundaries, more commonly referred to by engineers as the extreme fiber stresses. These stresses can be obtained directly from an isochromatic picture, if at a boundary the principal stress required is parallel to the surface, and the other is perpendicular to it; these boundary stresses are equal to zero if the structure is not loaded, or equal to the intensity of normal pressure when loaded normally. Since the normal stress and the magnitude of the difference given by the isochromatic picture are known, the extreme fiber stress is obtained in those cases either by adding or by subtracting the normal load from the measured difference of principal stresses. When the boundaries are loaded with inclined forces, the isoclinic lines must be used to evaluate the boundary stresses, but this is not difficult.

In order to use the photo-elastic method to determine the complete state of stress at interior points an auxiliary step is required in order to obtain the individual principal stresses. Since the difference of the principal stresses has already been found by means of the isochromatics, they could readily be separated by applying a little algebra at any point if one could, somehow, determine the sum of the principal stresses. This auxiliary step can be accomplished by several experimental methods. It is known that in a stressed model, the sum of the principal stresses is proportional to the change in thickness. This change is of an extremely small magnitude, but it can be measured. Coker has developed a very sensitive mechanical extensometer for this purpose, and R. W. Vose, Jun. Am. Soc. C. E., of the Massachusetts Institute of Technology, at Cambridge, Mass., has been successful in constructing a similar extensometer which measures the change in thickness by means of a small auxiliary interferometer[36], which is, of course, the most powerful method of measuring small dimensions. An interesting academic point in this connection is revealed by some experiments conducted by Professor M. M. Frocht, of the Carnegie Institute of Technology, at Pittsburgh, Pa., in which he constructed a test model from a plane-parallel optical flat, and used the two faces of the test pieces for an interferometer. In this manner he obtained interference fringes directly in the model, which again are a measure of the sum of the principal stresses. These bands he calls isopags or isopachic lines[37]. Experiments of this type require a knowledge of Young's elastic modulus and Poisson's ratio of the model material.

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[35] "Photo-Elastic Apparatus at the California Institute of Technology", by J. H. A. Brahtz. *The Review of Scientific Instruments*, Vol. 5, No. 2, February, 1934; see, also, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 1268–1277.

[36] "An Application of the Interferometer Strain-Gage in Photo-Elasticity", *Journal of Applied Mechanics*, A. S. M. E., September, 1935, pp. A-99, A-102.

[37] "On the Application of Interference Fringes to Stress Analysis", *Journal*, Franklin Inst., July, 1933, pp. 73–89.

A very practical method of obtaining the sum of the principal stresses is by means of the stretched rubber membrane (see Fig. 16)[20]. A horizontal rigid frame is constructed to the shape of the model and the vertical ordinates of the frame are proportional to the sum of the principal stresses of the boundaries, which are determined from the isochromatic photographs. A thin rubber sheet is then stretched uniformly over the frame forming a warped surface, every ordinate of which is proportional to the sum of the principal stresses, and these can be measured with an ordinary depth gage.

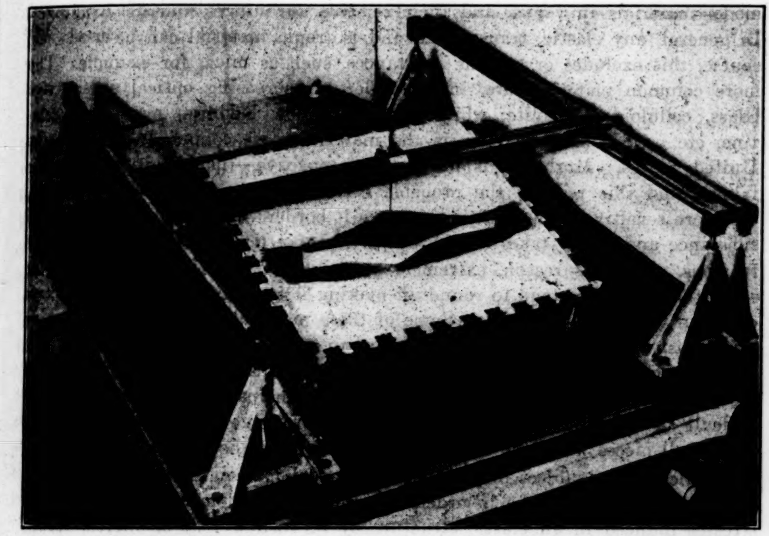

FIG. 16.—MEMBRANE PROFILOMETER (NOTE NEON GLOW LIGHT AND SWITCH IN BACKGROUND.)

Although all these methods will accomplish the desired results in completing the photo-elastic analysis at interior points, it is obvious that they have their respective advantages and disadvantages. The two extensometers require a quantitative knowledge of the elastic constants of the model material and depend on very accurate manipulation to reduce experimental errors. Furthermore, only one point on the model can be examined for a given position of the extensometer and each reading requires loading and unloading of the model; with a complicated loading mechanism this involves considerable labor. The only real objection to obtaining the isopachic lines is the cost of constructing the model from a relatively large plane-parallel optical flat. Perhaps the fastest method is the rubber membrane or soap film, but it fails wherever the slopes of the membrane become excessive, and the method depends on very accurate determination of the fringe orders at the boundaries of the isochromatic photographs, which is often very difficult due to edge effects.

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[20] Description of membrane analogy, by J. P. Den Hartog, *Zeitschrift Ange. Math. und Mech.*, Vol. II, 1931, p. 156, and *Journal*, Franklin Inst., July, 1933, p. 5.

Having briefly reviewed the standard methods of obtaining stresses in a loaded transparent model, it is interesting, next, to consider the various model materials in vogue, and their relative advantages and disadvantages. In general, any elastic, transparent, and isotropic material can be used. Of course, this excludes crystalline substances, such as mica, for example. The more common materials are, in the order of increasing optical sensitivity: Glass, celluloid, phenolite, white bakelite, yellow bakelite, marblette, gelatine, etc. Of course, bakelite is the material most commonly used in the United States. Marblette is much more sensitive—that is, it yields more fringes and this reduces the probable error in reading stresses. However, the more sensitive a material is the more it borders upon being a semi-liquid substance and partly takes on the properties of a liquid, particularly surface tensions. For this reason, extremely sensitive materials age rapidly in the sense that the edges tend to round off making it impossible to read the fringe order on the very boundaries. Some of these materials are easier to machine and polish into awkward model shapes than others. Marblette and gelatine, for instance, can be cast. Glass, which is frequently used for obtaining the isoclinic lines that give the directions of principal stresses, is the most difficult to machine and has the added disadvantage of cracking or flying into small pieces while the experimenter is applying the loads.

While on the subject of methods and model materials, a brief digression into body forces might be in order, covering the problem of determining the stresses induced in an elastic structure by its own weight or inertia forces. Naturally these forces cannot be used throughout the model by direct application of exterior forces, but must be accomplished by some other means. None of the materials mentioned previously, with the exception of gelatine, is sensitive enough to produce isochromatics from their own weight with the size and thickness of models ordinarily used. However, gravity forces can be magnified if the model is set in a centrifuge and rotated at a high angular velocity. Some interesting studies using this method have been made by Messrs. Philip B. Bucky, A. G. Solakian, and L. S. Baldin, of Columbia University[39], and independently by Fred L. Plummer, M. Am. Soc. C. E.[40]. A more satisfactory method has been suggested by Dr. M. A. Biot[41], in which he applies certain force distributions to the boundaries of the model, obtains the corresponding stresses, and then superposes these stresses to another system of theoretical stresses obtained from a simple gravitational stress function. The net result is the determination of stresses in the model due to gravity alone. This method has been used at the Laboratory of the United States Bureau of Reclamation, in Denver, Colo., and applied to the gravity stresses in the Grand Coulee Dam. The results have been very gratifying and there is no doubt that this method is more practical than the centrifugal method.

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[39] "Centrifugal Method of Model Testing", *Civil Engineering*, May, 1935, p. 287.

[40] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 1281–1282.

[41] "Distributed Gravity and Temperature Loading in Two-Dimensional Elasticity, Replaced by Boundary Pressures and Dislocations", *Journal of Applied Mechanics*, A. S. M. E., June, 1935.

There is another possibility of accomplishing this result by using thick gelatine models. A step in this direction has been made by R. R. Philippe, of the United States War Department, and Professor Plummer in connection with stresses in the foundations of earth dams[42].

From Model to Prototype

Without entering into too much detail, a brief explanation is included herein on the method of converting observed fringes into stresses in the structure. First, it is necessary to determine what the stresses are in the model. This is done with a simple test specimen cut from the same piece of stock as that from which the model is made. It is customary to make this test piece in the form of a simple beam subjected to pure bending, photograph it, and count the fringe order at the extreme fibers[43]. This same beam is analyzed mathematically and the stress computed at the same point. The stress is divided by the corresponding fringe order, resulting in the stress (the difference of principal stresses) in, say, pounds per square inch in the model per fringe. The two experiments of course, must be made under identical optical circumstances. Now, the law of similitude may be stated, as follows: The stresses in the model at all points will be to the stresses in the prototype at corresponding points as the normal boundary pressure on the model is to the normal boundary pressure at a corresponding point in the prototype.

Consider a simple example: A beam loaded uniformly with a pressure of 50 lb per sq in. on the edge. A bakelite model is constructed to any scale and loaded with, say, 5 lb per sq in., to obtain the isochromatic pattern. The material is calibrated and, perhaps, it is found that each fringe corresponds to a stress of 200 lb per sq in. in the model. Then, wherever there is a fringe order, 1, in the model, there will be a difference of principal stresses in the structure of 200 × 10 (the load factor), or 2 kips per sq in. If, for example, the fringe order at the center of the beam at the bottom fiber is Order 5, then the stress is 10 kips per sq in. in the prototype.

Comparison with Other Methods of Stress Measurements

It will be valuable to draw some comparisons between the photo-elastic method of stress analysis and two other experimental methods, namely, direct strain measurements on two-dimensional and three-dimensional models and the slab analogy. No mention will be made of the standard practice of testing models to failure, since this is in a class by itself, yielding no stresses, and is based on internal stresses beyond the yield point. In comparing the strain-measurement method with the photo-elastic method, several points should be mentioned. First, the strain-measurement method readily adapts

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[42] "Soil Mechanics Applied to Design of Dams", *Civil Engineering*, January, 1936, p. 25; see, also, p. 9.

[43] See, for example, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1269, Fig. 9.

itself also to three-dimensional models even with varying elastic properties. Because of its adaptability into three dimensions, the method stands alone; it dominates this field completely, and is the only known experimental method suitable for analyzing such a structure as a thick arch dam. Necessarily, it is a tedious expensive method both in execution and evaluation and calls for great skill. It allows elastic relationships between model and prototype to be simulated very closely because of the wide latitude of available materials. Quite often the applied loads produce strains that are scarcely detectable in certain regions and in every case the stresses obtained can represent only average stresses in the segment between gage points. Photo-elasticity yields the stress at a point.

Another method of obtaining experimentally the stresses in a two-dimensional structure is by the "slab analogy." This method is well adapted to obtaining the stress distribution, for example, in a gravity dam. A flat elastic slab made of rubber, for instance, is cut to the dimensions of the structure and warped by displacing and twisting the boundaries. The curvatures and twists can then be measured at any point within the boundaries of the model, and it can be shown that these curvatures are always proportional to the normal stresses in the structure in a direction perpendicular to that at which the curvature was measured, and that the shear stresses are proportional to the twists in the slab. Naturally, the slab must be set up in such a manner that the twists and curvatures at the boundaries are equivalent to the forces acting on the boundaries. The basis of this experiment lies in the fact that the differential equation of a warped elastic slab has the same form as the differential equation of compatibility of the Airy stress function. This is another one of those analogies that are being adopted by the Engineering Profession, and is in the class with the soap film analogy used in measuring torsional resistance of odd shapes, such as propeller blades and built-up sections. Other well known members of this family are the electric analogy and the aforementioned rubber membrane analogy also used to measure temperatures in the steady state, pressures, and equi-potential lines in highly viscous hydraulic flow that recurs in soil problems[44].

It may be said of the slab analogy that it is an excellent and reliable method because of the directness of the results; but, like the strain-gage method, it is costly and tedious, requiring a well developed technique. In its present state of development it is justified only in analyzing an expensive major structure.

Returning now to the photo-elastic method and without attempting in any way to depreciate the importance and value of the other methods, it can be said of photo-elasticity that it is an inexpensive method of analyzing difficult structures or elements of machines, discounting, of course, the high initial investment in optical equipment. Furthermore, it is a rapid method, especially when one is interested only in extreme fiber stresses. Moreover, it supplies, immediately, a rough picture of stress distribution in a structure and reveals concentrations wherever fringes are grouped close together. To emphasize these advantages further: Since the investigator immediately obtains a picture of the stress distribution in the model of a structure that is subject, say, to a number of different loading systems, he can, very rapidly, single out the critical cases. Furthermore, the shape of the model can be altered, reduced, or enlarged at weak points that are revealed in the photographs, and the process repeated. In other words, it furnishes a rapidly converging method of design, even when dealing with a structure that borders in the class which, off-hand, one might say is analyzed readily by the Begg's deformeter method, or the Cross method of moment distribution and column analogy. Naturally, the mathematical analysis should be made wherever possible, but now by photo-elasticity a good starting design and the critical loading are quite well isolated.

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[44] "The Flow Net and the Electric Analogy", by E. W. Lane, M. Am. Soc. C. E., F. B. Campbell, Jun. Am. Soc. C. E., and W. H. Price, *Civil Engineering*, October, 1934, p. 510.

PRACTICAL RELATION TO STRUCTURAL PROBLEMS

The true value that photo-elasticity must bear to structural and machine design is readily shown to those unfamiliar with the method by a few examples. It will be apparent that photo-elasticity has its own field, and completely dominates this field from a practical standpoint. Conversely, it is foolish to attempt to use it in those fields that are readily treated mathematically, such as standard beams, columns, tension members, etc., of lengths at least five or six times their lateral dimensions, except for academic purpose or as a check analysis. This includes all standard structural members. Where these ratios are shortened, where a structure is composed of short, thick, integral members, where it is otherwise complicated by holes, notches, reinforcements, etc.—in short, wherever it is obvious from the shape of the structure that it is indeterminate in the sense that the stress distribution is non-linear in character, and the principal stress trajectories assume a complicated curvilinear form—then the structural designer finds that his ordinary tools will not suffice and he must depend either upon his structural sense and previous experience, or perform a test to failure, or carry out a photo-elastic analysis. This type of problem is encountered continually in machine design. This does not mean that the photo-elastic method cannot, or should not, be extended to include fields susceptible to mathematical analysis for check or verification. As a matter of fact, the two overlap and, as has already been indicated, it is a rapid tool for finding critical loads and design sections for use in mathematical analyses. Furthermore, a check between the two goes a long way in removing that feeling of uncertainty which can hang like a nightmare over a designer who entertains a feeling, and rightly so, that perhaps he has made a few too many assumptions in dealing with a statically indeterminate problem without knowing whether or not they are on the side of safety.

Consider, for example, the subject of fillets, such as occur in elements of structures and machines. No one doubts their value, and yet an exact mathematical analysis would be extremely difficult. By testing a small bakelite model, however, a picture of the stress concentrations in the fillet can be obtained. It is possible also to see how they compare with stresses in the web, and the operator can begin with a large radius, cut back, and quickly try any number. Photo-elastic investigations have shown that a spiral fillet in some cases is more efficient than a circular one[45].

As a second example, consider the problem of the chain link of the type with two holes machined from a flat plate (see Fig. 17 and Fig. 18). A photo-elastic analysis can readily be used to compare a wide variety of conditions, such as various pin clearances, different ratios of hole to width of link, and various shapes of links. In this type of problem only the boundary stresses are important, and the designer can soon choose an economical and safe shape.

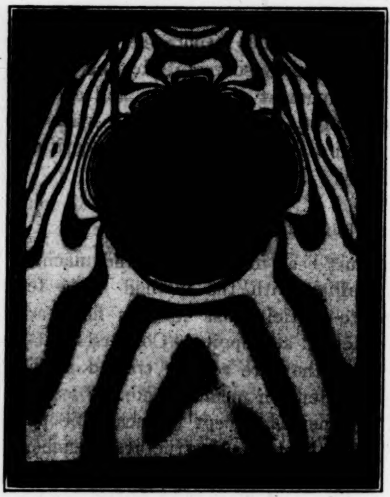

FIG. 17.—ISOCHROMATIC VIEW OF BAKELITE CHAIN LINK.

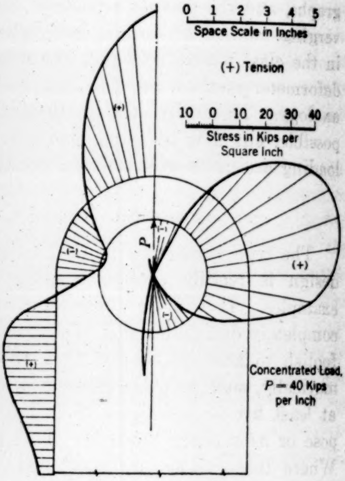

FIG. 18.—BOUNDARY STRESSES IN CHAIN LINK DERIVED FROM FIG. 17.

Fig. 19 illustrates an example in which the photo-elastic and mathematical fields overlap. and this experiment was conducted to find critical loading conditions and to check the mathematical analyses made for the critical cases. In this experiment two new loading devices were used[46]. The water pressure in the tunnels was simulated by air pressure applied through rubber inner tubes, restrained on the sides but bearing directly on the walls of the tunnels. The earth pressures were applied through a thick rubber sheet that fitted snugly over the model, and the loads were applied to the outside edges of the rubber.

Other interesting applications of the method are apparent if one side of a bakelite test piece is silvered, and if the light is sent through to the silvered face and back along the same path. For example, a piece of bakelite may be cemented on the surface of an actual structure, either two-dimensional or three-dimensional, and the changes observed in the induced isochromatics as the structure undergoes changes in loading; or the investigator may wish to measure the stresses in overlapping plates, such as occur in riveted connections. By silvering he can isolate the stress conditions in the two plates. Furthermore, he can study the behavior of viscous fluids under flow, by photographing the isochromatics that develop as ordinary salad oil is subjected to flow around obstacles set between parallel glass plates. Experiments of this type have been conducted successfully by Dr. Sadron, of the California Institute of Technology.

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[45] Discussion of Weibel's paper. entitled "Studies in Photo-Elastic Stress Determination", by A. G. Solakian, *Transactions*, A. S. M. E., August, 1934, pp. 652–655.

[46] "Photo-Elastic Analysis of Twin Concrete Conduit. Bull Lake Dam Outlet Works", by J. E. Soehrens, R. T. Cass, and J. E. Sower, Juniors, Am. Soc. C. E., *Civil Engineering*, September, 1936, p. 594.

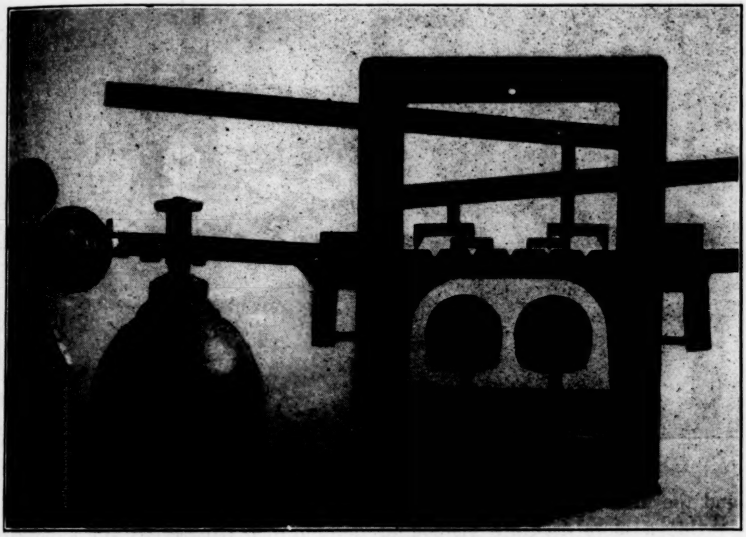

FIG. 19.—LOADING APPARATUS UTILIZED IN PHOTO-ELASTIC ANALYSIS OF DOUBLE-BARRELED CONDUIT FOR BULL LAKE DAM. (NOTE THAT PNEUMATIC PRESSURE IS USED TO SIMULATE INTERNAL HYDROSTATIC PRESSURE.)

THE INTERFEROMETER

Before concluding, a word should be said about the interferometer as a means of measuring individual principal stresses in a transparent model directly, thus eliminating the necessity of the auxiliary step of finding the sum of the principal stresses. It has been known for some time that this method can be applied, but in so far as the writer is aware it has never been utilized in the United States. One of these instruments has been designed and built at the Laboratory of the Reclamation Bureau, by J. E. Soehrens, Jun. Am. Soc. C. E. (see Fig. 20). Essentially, the method consists of splitting a pencil of light, one part of which traverses a loaded model at a point, vibrating in the plane of one of the principal stresses, and the other goes around the model. The two pencils are brought together again and interference fringes develop. The stress at the point in the model will be proportional to the relative displacement of these fringes as the load is applied. Next, the ray is made to vibrate in the plane of the other principal stress

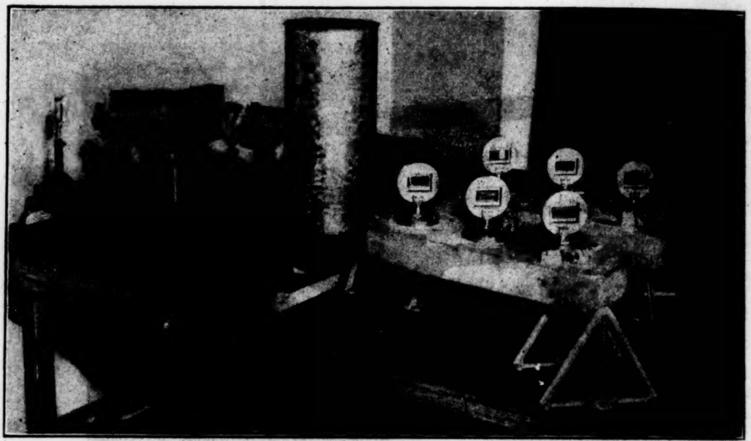

FIG. 20.—PHOTO-ELASTIC INTERFEROMETER.

and the measurements are repeated. It is believed that this apparatus will provide a more accurate method of determining stresses within a model; that is, stresses other than those at the boundaries particularly in cases in which the accuracy of a rubber membrane breaks down, as, for example, at a sharp corner or near the point of application of a load. This machine s much more delicate in its operation than the polariscope and must be designed in such a way that it is insensible to vibrations.

FUTURE POSSIBILITIES

At present, photo-elasticity has demonstrated its value and firmly established itself as an invaluable tool to the Engineering Profession. Its development has been somewhat erratic but positive, and has by no means reached its limit. New model materials and technique are being developed and will continue to be developed. The method is not a simple mechanical one, but a procedure that requires trained technicians. Some universities are now (1936) offering introductory courses in the subject to their undergraduates. As lighter, stronger, but more expensive, alloys assume their natural functions in construction, it will be more important than heretofore to have a complete knowledge of stresses. As these demands become more acute emphasis will be more and more toward photo-elasticity as a means of designing economical structures.

STRUCTURAL APPLICATION OF STEEL AND LIGHT-WEIGHT ALLOYS

A SYMPOSIUM

PAGE

II.—METALLURGICAL AND MANUFACTURING ASPECTS OF FERROUS AND LIGHT-WEIGHT ALLOYS OF INTEREST IN STRUCTURAL DESIGN AND FABRICATION:

Low-Alloy Structural Steels.
By E. C. Bain, Esq., and F. T. Llewellyn, M. Am. Soc. C. E. 1240

Stainless, High-Alloy Structural Steels.
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LOW-ALLOY STRUCTURAL STEELS

By E. C. Bain,[47] Esq., and F. T. Llewellyn,[48] M. Am. Soc. C. E.

Synopsis

In view of the fact that this Symposium is sponsored by a civil engineering society, the word, structural, in the title of this paper is to be construed as pointing with strong emphasis to framed static structures, such as bridges and buildings, almost to the exclusion of small moving structures and machinery. The other papers in Section II enumerate adequately the more restricted opportunities for over-all economy which are offered to the special high-strength steels in this restricted field of application. Information has been presented to show how the economical expenditure of the steel dollar may still call for the purchase of special steels that differ from common structural steel by having enhanced mechanical properties secured through the incorporation of alloying elements.

Introduction

In this inquiry, the writers begin with the assumption that the background of their subject has already been well drawn; that every one is aware that alloy steels may have a higher elastic range and a higher breaking load than the ordinary structural steels, but that the elastic modulus is the same. The consequences of these circumstances, as they affect design, are presented elsewhere in the Symposium, where the applications of those low-priced alloy steels are discussed. In this paper, the writers discuss the common alloying elements with respect to the direct and indirect influence their presence, both individually and concurrently, may have upon the properties of structural steels.

The writers first approached their task with a plan for a categorical enumeration or classification of the many so-called "grades" of low-alloy, structural steel that are now (1936) on the market, with their compositions and ordinary mechanical properties, as set forth by their makers. Since, however, such information has been given in a number of excellent compilations[49], the writers conclude that it may be more useful to treat the principles involved in designing alloy steels. Accordingly, two types of these steels have been chosen to illustrate these principles, and, as far as possible, the subject has been developed as a functional study. For convenience, however, a table of some of the current compositions offered to the trade and their reported mechanical properties is included.

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[47] Asst. to Vice-Pres., U. S. Steel Corporation, New York, N. Y.

[48] Research Engr., U. S. Steel Corporation, New York, N. Y.

[49] See, for example, *Proceedings*, Am. Soc. C. E., March, 1936, p. 361.

Although it will be convenient for subsequent reference to compare alloy compositions with the regular structural steel, designated "carbon steel", it may be well to forestall a false premise at the start. The vast majority of the structures within the scope of the present consideration are built of a steel which is by no means the pure metal iron or an alloy of iron and carbon alone; structural carbon steels carry about 0.5% of manganese, one of the important alloying elements. An inspection of hundreds of analyses indicates a probable most frequent composition about as shown in Table 2. Steel of such composition, as rolled, for example, in sections 0.50 in. to 0.75 in. thick, develops mechanical properties such as correspond, on the average, to approximately the tensile test data indicated in this table.

TABLE 2.—COMPARISON OF ORDINARY STRUCTURAL CARBON STEEL WITH PURE IRON

Description (1)	Ordinary, structural carbon steel (2)	Pure iron* (3)
Composition, in Percentages:		
Carbon	0.22	....
Manganese	0.45	....
Sulfur	0.035	....
Phosphorus	0.018	....
Silicon	0.05 or 0.20	....
Ultimate strength, in kips per square inch	60	39
Yield point stress, in kips per square inch	38	19
Percentage elongation in 2 in	45	48
Percentage elongation in 8 in	28	30
Percentage reduction in area	58	75

* Commercial purity.

It is interesting to compare the properties of the usual structural carbon steel with those of nearly pure iron (that is, iron of commercial purity) as indicated by Column (3) of Table 2. It is clear that the ubiquity of carbon in the metallurgy of iron and the availability and use of manganese, primarily as a specific or antidote for possible sulfur ills, have resulted in a considerable gain in desirable properties. One needs no marshaling of metallographic theory to convince himself of the general efficacy of these elements incorporated in steel, but it may be worth while, nevertheless, to digress further into this field of physical metallurgy.

Before having a "look" into the effects of the common alloying elements, it may be advisable to view the problem of stronger steels from the purely economical standpoint, but still through the eyes of the metallurgical engineer.

ECONOMIC FUNDAMENTALS OF ALLOYS

Members of the Society need scarcely be reminded that economy is one of the essentials of good engineering. In considering the economic fundamentals of alloys, or indeed of any materials, it is desirable to emphasize three facts. They need only be stated to be self-evident:

(1) The question as to the economy of a designated material becomes critical only when there is a choice of materials.

(2) If, as a result of its improved properties, the net cost of a substituted material is less than that of the material replaced, there is no economic problem to be solved. Manifestly, despite its higher cost per ton, the material in question would be used everywhere as far as it is available.

(3) A given material may be economical under certain conditions, and uneconomical under others.

It follows that the desirability of alloying steel with certain chemical elements, calculated to produce higher effective strength than ordinary grades, will vary according to the problem in question.

Having in mind these facts, some engineers have surmised that a simple chart could be prepared in which the higher cost per pound of each alloying ingredient, as a commodity, could be translated into final cost per ton per unit of resultant alloying content, and plotted against the corresponding improvement in given physical properties. A chart of this kind would be just what the civil engineer is seeking, but, unfortunately, those who operate steel mills, as well as Dame Nature, say that the matter is not quite as simple as that. Even if effects could be charted against percentage of alloying elements, comparative costs would not be indicated for the reason that rolling-mill costs vary greatly according to the quantities of a kind to be produced at a given time.

Quite apart from mill-operating conditions, however, such a chart is not feasible. As demonstrated subsequently, even the effect of individual alloying elements is involved, to say the least, and is not constant under all conditions. In many cases it depends on the inter-relation of other elements. For this reason, instead of one chart, one would require about as many charts as there are different types of steel, and these charts would be useless as aids in forecasting the properties of a new combination.

Most important of all, it must be emphasized that the improvement in certain properties which is afforded by many of the alloying elements is accompanied by a loss in other desirable properties. If Dame Nature and the economic cartographer attempt to join in holy wedlock, with metallurgy as sole officiating priest, the union cannot be blessed because the progeny would be as numerous and unmanageable as that of the "old woman who lived in a shoe."

Interpretation of Specimen Tensile Tests

It is well to reflect that the ordinary tensile tests, accompanied perhaps by compression tests, supply the engineer with some numbers which are not measures, exactly, of the precise properties which will be depended upon in the final structure, but are instead merely rough indicators of the possession of the needed properties. By way of example, a notch in a beam becomes a stress concentrator under conditions of bending, and failure may occur in a manner developing less than the expected load. On the other hand, in a carefully conducted tensile test, a bar in tension having a sharp circumferential notch may carry, without permanent deformation, a load in excess of the ultimate stress as ordinarily measured. A plate may show 30% or 40% elongation in the standard tensile test; and yet, when subjected to simultaneous tensional stresses at right angles, it may break with only a slight percentage of elongation as measured along any reasonable reference distance Nevertheless, the standard tensile test, interpreted in terms of experience, is a valuable guide and the writers would defend its general use.

The fact is that the gross loading as ultimately reflected in any small region of the metal may be considered as a force tending to rupture the metal, with a maximum in one certain direction, and a force tending to cause flow (that is, shear) likewise at a maximum, in another direction in a shearing plane. If the maximum shear component exceeds the elastic limit in shear and the metal has any capacity whatever for plastic flow, some deformation, without rupture, will occur. On the other hand, if the maximum disruptive component exceeds the fundamental tenacity or cohesion of the metal, a brittle break will occur. The former may occur first, followed by the latter, by virtue of a change either in the relative magnitude of the two components, or in the elastic limit in shear of the material during the deformation.

In an ideal tensile test in which a uni-axial tension is applied uniformly across the cross-section of a moderately long specimen, the numerical magnitude of the resulting disruptive stress is just twice that of the resulting shear stress. If, therefore, the elastic limit in shear of the specimen is more than one-half that of its cohesion, the tensile specimen will fail with a brittle rupture accompanied by no elongation; and yet, just such a material may possess considerable fundamental plasticity, as can well be shown by applying a torsion test, in which the resulting maximum disruptive stress is numerically equal to the maximum shear stress, on any small element in the highly stressed periphery of the specimen. Other tests, such as bending a notched bar, afford still less opportunity for any flow than does the tensile test, and one must conclude that the quality of realizable plasticity in a broad sense depends actually upon the ratio of the elastic limit in shear to the fundamental resistance to disruptive stress, for the second of which no known means of evaluation exists. This reasoning is introduced only to show that the tensile test supplies only a glimpse from one viewpoint of the mechanical properties of a steel. However, experience has demonstrated that good structures result from material with certain tensile test characteristics and may not result from others, and having no better practical test it is utilized with assurance as far as static loads are concerned.

With respect to structures as a whole, it is rare indeed that any general permanent deformation is tolerable, hence the interest really centers in the stress that can be borne within the elastic range of loading. For this reason increasing reference is made to the so-called "yield point" of the steel. Since the departure of the metal from strictly elastic behavior begins with almost undetectable gradations it is now customary to specify a very small but observable permanent elongation as a working definition of the elastic limit or yield strength; 0.1%, or 0.2% is usual and occasionally a proof stress corresponding to a permanent elongation of 0.01% is specified. The breaking load computed as a stress over the original section of the test has certain interest, to be sure, but of necessity plays a less important rôle in design than the yield strength.

The need for a degree of plasticity will not be detailed in this paper. The inescapable necessity for joints and the proper distribution of stress, otherwise highly localized and concentrated in and about such joints, can only be cared for by some capacity for flow. It is generally believed that 25% elongation in a test piece, 8 in. long, with certain cross-section limitations, is adequate. It should be added, however, that elongation should always be referred to a specimen which has a definite shape and ratio of length to cross-section; otherwise, comparative figures may be quite misleading. The reduction of cross-sectional area at the point of rupture is a factor of some complex significance not wholly clear even to the most erudite metallurgists and not of vital moment to the engineer.

It is not uncommon to measure the energy required to break a small, standard, notched bar by impact and to take this number of foot-pounds (or kilogram-meters) into account as a putative quality index of the steel. Considering that the most plastic and ductile metals known can be made to break in a brittle manner (low impact value) by introducing a sufficiently sharp and effective notch, and considering further the assiduous avoidance of notches in design, it might appear that such notched-bar impact tests would not yield broadly pertinent information. However, the steels under present consideration do not break in the brittle manner under the notches of customary sharpness at ordinary temperatures. If the break at very low temperatures should be brittle, and, therefore, should show low energy absorption (foot-pounds), the engineer should not reject the steel summarily, but instead, he should perhaps break a few specimens, at the same sub-zero temperature, which have been prepared with a more gentle notch and see that it is perhaps the notch, rather than the metal, which has been tested.

It happens that structures do not always bear a constant static load, but are subject to frequent loading and unloading, or even to reversal of load. Under such conditions a failure may occur, after a very large number of load-change cycles, at a specific stress lower than the yield point. Tests of the resistance to repeated stress effects (fatigue) have been devised, and the familiar "endurance limit" expresses this property usually as a maximum fully reversed stress which can be borne in a specimen during 10 000 000 cycles. For many materials this value is found to be well above 45% of the ultimate strength. However, when the stress changes do not entail reversal, but rather result from loadings and partial unloadings (surge loads), the endurance limit applicable approaches the yield point. It has been found that the character of the surface of the stressed member exerts a tremendous influence upon the practical fatigue resistance, and to make the tests represent, as nearly as possible, the metal itself rather than the surface condition, a polished test specimen is used. Accordingly, it would seem that for the civil engineer the yield point would be a more significant figure than the endurance limit, but that the endurance limit should be known and taken into account for the material under contemplation if stresses nearly or wholly reversed are to be encountered. Of probably greater significance is the possession of adequate plasticity, such as is measured, in part at least, by elongation and reduction of area in the tensile test.

In an exploratory manner, formability and other criteria of suitability may be judged by the data from conventional specimen tests, but it is to be expected that the final judgment will be based upon tests simulating operating conditions in practice.

The steels discussed in this paper were designed to retain adequate ductility and to provide an increment of yield strength. In practice, they have already achieved a remarkable degree of success. The design of machines (particularly automobiles) long ago opened up a need for steels that could first be endowed throughout large sections with the maximum properties previously securable by careful heat treatment upon only small sections of the higher carbon steels. Actually, it was soon evident that properties somewhat superior even to the best obtainable in small sections of carbon steel were securable in the alloy steels. This was comparatively easy because, by special heat treatment, tremendous improvements over the "as-rolled" properties are securable through the control of the microscopic structure; medium and high-carbon alloy steels with attendant high strength can be rendered tough and plastic as well.

The problem in the field of Civil Engineering is vastly more difficult. Not only is cost a serious deterrent from special heat treatment of structural steel, but the shapes themselves are scarcely amenable to the rapid cooling and reheating involved in such a method of enhancing properties. Since, in most cases, the steel perforce must be used in the as-rolled condition, and since the modern method of joining metals by the newer welding processes must be facilitated, the limitations are very strict. Not long ago it was a commonplace to hear in the metallurgical schools that alloying elements were of practically no value except where heat treatment was to be applied. The statement, never quite justifiable, is now known to be farther than ever from the truth, and to have accepted such a statement would have been to have missed the entire development under discussion in this paper.

MEANS OF STRENGTHENING STEEL

For the purposes of this survey, there are only three means of gaining strength in steel: (1) Cold working the members; (2) specially heat treating the members; and (3) incorporating alloying elements.

Cold Working.—Bridge wire is an outstanding example of this application, and the high tensile strength of cables is known to every one. Recently, cold-rolled strip, particularly a certain stainless steel, characteristically amenable to improvement by cold-work, and to spot-welding also, is becoming a very valuable material in movable structures, such as high-speed trains. One can scarcely visualize the extensive cold drafting of standard structural shapes.

Heat Treatment.—To the metallurgist, who realizes that the microscopic structure, and, hence, the properties of steel, depends upon the cooling schedule from an elevated temperature, the cooling of structural steel from the rolling temperature is, in itself, a heat treatment. Although this is proper, heat treatment, in current usage, generally means a subsequent controlled heating and cooling. Thus, for years (since 1914), heat-treated, carbon-steel, eye-bars have been utilized in bridges, and more than 17 000 of them are now (1936) in place. A 0.35% carbon, 0.60% manganese steel is used, having properties as rolled corresponding to a yield point of about 40 kips per sq in. and an ultimate strength of about 70 kips per sq in.; the ductility of the untreated steel is approximately represented by an elongation, in an 8-in. by 0.5-in. specimen, of about 23 per cent. After the closely controlled heat treatment of the eye-bar, properties are developed corresponding to an average yield point of 57 kips per sq in. and an ultimate strength of 88 kips per sq in. A full-sized tensile test bar with a cross-section, 12 in. by 2 in., and 18 ft long, exhibits a ductility so great that, in the full 18-ft test length, an elongation of nearly 2 ft occurs (10.5%), or in the 1 ft encompassing the point of rupture, 30%; the reduction of area is 46 per cent. A higher carbon steel containing 0.60% carbon, with 0.55% to 0.75% of manganese, is similarly heat-treated to an ultimate strength of about 119 kips per sq in. and a yield point of 82 kips per sq in. The elongation in 18 ft is about 8%, with a reduction of area of 23 per cent.

With such uniform regular sections as eye-bars, heat treatment has proved entirely practicable and economical, but for the usual shapes it is probably not "in the picture" at present; nor is it likely to become so for large welded structures, since any improvement derived from heat treatment would be nullified in and near the weld.

Use of Alloying Elements.—Turning next to the third and most readily applicable means of securing a general purpose steel, with a moderate increase in load-carrying capacity, it is to be noted that the metallurgist has at his disposal the elements—carbon, manganese, silicon, copper, phosphorus, chromium, nickel, vanadium and molybdenum—as well as some other less common elements, as possible addition agents in a structural steel. As far as small increments of equal weight are concerned, the elements are arranged about in the order of their cost, beginning with the cheapest. One should not be led to believe, however, that the order of merit is the same; nor, indeed, is there any basis available for any order of merit whatever unless one single property is under consideration at a time, and unless the proportions of the other elements are held entirely constant for the comparison. If one were to begin with commercial steel of the lowest carbon content and highest purity, and incorporate therein increasing quantities of the various elements, it is apparent at once that the Brinell hardness is increased in the rolled product about as shown in Fig. 21. These curves are drawn as the best probable average of scattered data and should not be regarded as being precise. It is intended to refer to wrought material, hot-rolled to a reasonable thickness, say, ½ to ¾ in., and cooled at a normal rate in air. The line for carbon in Fig. 21 refers to smaller specimens so as to reflect its effect upon hardenability. If larger sections are considered, the value at 1% carbon would be about 220 Brinell hardness. Hardness is not correlated perfectly either with yield point or with ultimate strength, but, in general, it increases with alloying elements in a manner such as to correlate somewhat with a value midway between the yield and the ultimate. Fig. 21

merely shows that the fundamental influence of alloys in iron is unique for each element; but such curves fall far short of telling the entire story. Added elements contrive to confer properties by several means, usually operating concurrently. The principal influences are, as follows: (*a*) Alteration of transformation characteristics (under-cooling and attendant fineness of microscopic structure); that is, "hardenability"; (*b*) solid solution hardness (and strength) increment in the iron itself; and (*c*) modification of the nature and the volume of the carbide constituent of the steel.

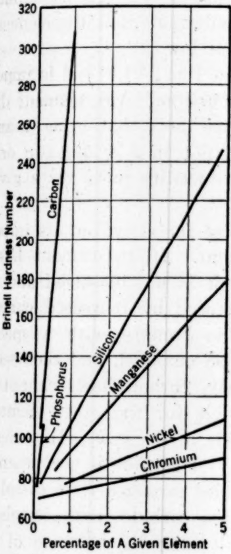

FIG. 21.—APPROXIMATE EFFECT OF SMALL ADDITIONS OF VARIOUS ELEMENTS TO NEARLY PURE IRON.

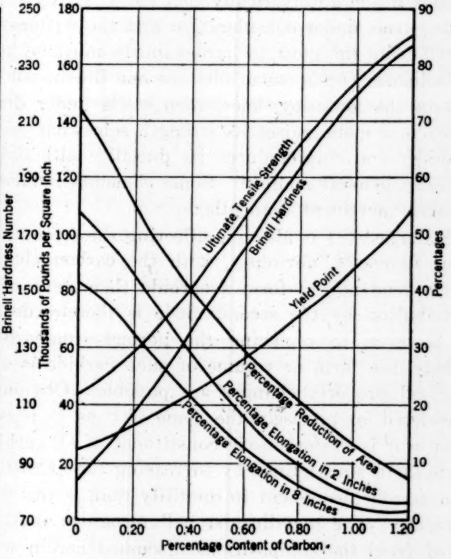

FIG. 22.—APPROXIMATE EFFECT OF CARBON CONTENT UPON THE TENSILE PROPERTIES OF STEEL, ASSUMING NORMAL MANGANESE CONTENT.

Another influence that almost warrants a separate listing has to do with the grain size in the heated condition; a fine grain generally connotes greater toughness.

Influence (*a*) is that which has to do with hardenability and, in the case of the higher carbon, higher alloy steels, such as some of the automotive steels—is of first importance. It is an influence which the element exerts by its presence in the steel at rolling temperature by determining the microstructure which finally develops upon cooling through the so-called critical temperature range at a given rate. It would operate to control the properties even if the alloying element were found to have vanished immediately after the steel had been transformed. Rapid cooling always produces finer distribution of carbide and, therefore, harder characteristics for any steel, and any of the alloying elements dissolved in the steel at rolling temperature acts in a way practically equivalent to a more rapid cooling, because these elements retard the reactions at the transformation, or "critical", temperature. Thus, it is customary to reduce the carbon or the alloy content for the thinner sections which automatically cool more rapidly. This influence must be mild in the steels under consideration and the writers will show that the elements contributing the most to hardenability must be kept low.

Influence (*b*) is essentially the one illustrated in Fig. 21, and is important in the low-alloy, low-carbon steels under discussion. Any element dissolved in a metal raises its strength somewhat (some more than others), and, generally speaking, reduces its ductility although not in a systematic or a perfectly general manner. Some elements reduce ductility more for a given strength increment than others.

Influence (*c*) is also one affecting the strength of the alloy, but its importance increases, obviously, with the carbon element. If an element has a strong tendency to form a carbide then some of it is not dissolved and its contribution by the second mode is thereby decreased by increased carbon. If, however, in replacing the normal iron-carbide lamellæ with a special carbide, the form or volume of such carbide is also changed, and then some practical property changes are possible. Obviously, carbon itself cannot be considered in precisely the same way as is possible for the other elements because it is the necessary constituent of all carbide in the steel. In general, strength in steel gained by increasing the quantity of carbide is accompanied by a greater decrement in ductility than is the added strength from dissolved elements. The metallurgist will recognize that the carbide in the steels as rolled from the compositions discussed herein will exist in the form of the minutest microscopic platelets or lamellæ, a form not nearly so conducive to ductility as the spheroidized form found in fully heat-treated steels. In the spheroidized condition, as may be surmised, the carbide is in the form of roughly spherical particles barely visible under the microscope. It will be understood that these features of microstructure are nothing that can be seen by the eye in the fractured surface, but are known only through application of the metallurgist's technique of examining a polished and etched section microscopically. Perhaps a philosophy of alloy steels can be developed by considering the elements, one at a time.

Characteristics of Individual Alloying Elements

Carbon.—Fig. 22 shows the approximate change in tensile-test properties created by the carbon content in steels of normal manganese (0.45%) and silicon content as rolled in sections of the order of 0.75 in. thick. As might be predicted from Fig. 21, carbon is the most powerful element, weight for weight, utilized in influencing the properties of steel; it is customary, however, to exclude carbon when referring to alloys or alloying elements. Its marked effect upon strength properties is gained at a considerable toll in ductility, and long experience indicates that at about 0.20% to 0.25% (with comparatively low content of other elements), a broadly applicable optimum is reached for structural carbon steels. At this content, a considerable range of other elements and of section is still permissible, and it happens also that the influence of carbon upon hardenability is not so great at this point as to preclude welding, bearing in mind the low content of other elements. Perhaps one may state that carbon contributes practically nothing by the second influence, that of solid solution, since at ordinary temperatures it is so nearly insoluble, and that the quantity of carbide as a microscopic constituent is proportional to the carbon content and, hence, a strong effect results in this (the third) manner. Furthermore, even with a minor content of other elements, carbon exerts a definite influence toward hardenability; only in its presence, and in proportion to its concentration, are other elements able to exert substantial influences toward hardenability.

Nevertheless, when need has arisen, engineers have not hesitated to use steels of higher carbon content. Because of the added requirement of wear resistance, rails carry about 0.75% of carbon; the eye-bars, already mentioned, are high in carbon since heat treatment eliminates a large part of the loss in ductility brought about by carbon. In cases wherein no welding is to be used and wherein joining problems are not complicated, carbon content higher than the usual can always be used with safety. However, where welding is to be utilized, the usual structural steel derives about all the strength that carbon alone can provide without excessively restricting other valuable properties.

Manganese.—After carbon and phosphorus, manganese is one of the most effective strengtheners of steel, rivaling chromium. Its effect may be greater than shown in Fig. 23 (*a*), which is an attempt on the part of the writers to correlate some scattered information. Manganese is an extraordinarily powerful hardening agent (Influence (*a*)), and it also strengthens greatly by solid solution. It has no particular carbide effect, although it divides itself between the iron solid solution and the carbide. It has the disadvantage, how-

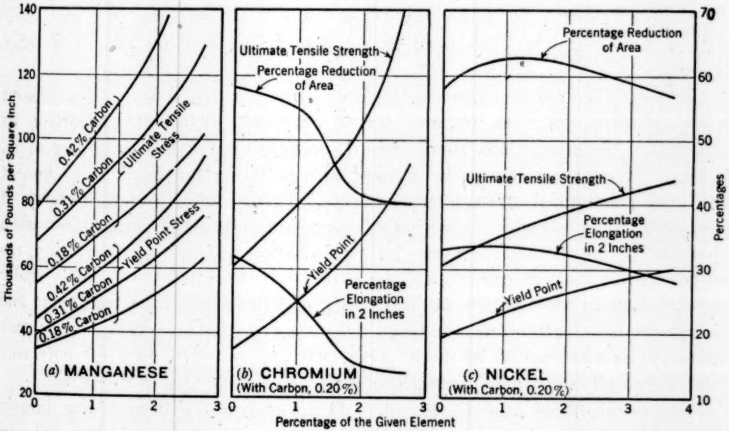

FIG. 23.—APPROXIMATE EFFECTS OF THREE DIFFERENT ALLOY ELEMENTS UPON THE TENSILE PROPERTIES OF CARBON STEEL. NORMAL MANGANESE IN CHROMIUM AND NICKEL CHARTS.

ever, of taking a considerable toll of ductility in the as-rolled condition, apparently through its solid-solution effect. In general, it should probably not be used much in excess of 1% to 1.3% in any non-heat-treated, general-purpose steel, and for the greatest ductility, perhaps 0.50% to 0.75% is sufficient. In a broad way, the over-all manganese effects are not unlike those of carbon; with very low-carbon steel, manganese is perhaps one-eighth as effective as carbon in developing strength, but in somewhat higher carbon steel it is certainly more like one-fourth or one-fifth as effective, weight for weight, because its hardenability effect enters. In order to preserve good ductility in the manganese steels the carbon must be decreased somewhat and more than the equivalent effective manganese is then to be added in order to gain strength; but even this substitution has strict limitations. In the ideal high-strength, general-purpose steel, other helpful elements should be called into use.

Silicon.—This element is found almost entirely in solution in the iron (ferrite), and its solid solution effect is strong. In small amounts (that is, as much as about 1%), it appears to reduce ductility only moderately, and some report that it has a curiously minor effect upon the reduction of area in the tensile test. It has a significant effect upon hardenability which is, however, small in comparison with that of manganese, carbon, or chromium, and in the presence of these elements its effect upon hardenability through its influence upon transformation rate must be taken into consideration. It is a valuable aid in the low-carbon steels in securing strength, and may be utilized safely up to 0.75% or, in some cases, nearly to 1.0%, depending upon the other alloy content. If it has any effect upon corrosion rate, it is not obviously adverse. Silicon has been rather less fully reported than the other elements.

Copper.—This element remains entirely dissolved in the iron when present in quantities as great as, perhaps, 0.50% or 0.60%, in which proportion its contribution to strength is small indeed, perhaps only 2 to 4 kips per sq in. In larger proportions it may be utilized as a precipitation hardening element with very substantial strengthening effects. In the lower range, however, it contributes to a valuable reduction of corrosion, a property again lost when the quantity is well into the range for precipitation hardening. Obviously, the age-hardening feature cannot well be utilized in the structural steels under consideration in view of the undesirability of subsequent heat treatment and the requirements attending welding. One may state, then, that copper may profitably be used up to at least 0.5%, with a valuable gain in corrosion resistance, a slight gain in strength, and no perceptible loss in ductility.

Phosphorus.—The traditional fear of phosphorus brittleness was largely dispelled by the classical work of Unger in 1918[80], who showed that phosphorus in quantities considerably greater than 0.10% contributed strength without undue loss of ductility. Indeed under some conditions an increment in strength could be provided by means of phosphorus with a penalty against ductility less than if the same increment were gained through a higher carbon content. In other words, a reduction of carbon content with a deliberate addition of phosphorus yields a steel in which strength is increased and ductility remains constant; it is then a suitable element for the structural steel under discussion. More recently the effect of phosphorus to reduce corrosion has been recognized, and since this is a factor of increasing importance, incidental advantage is taken of its quota of strengthening effect, while securing at the same time a marked improvement in corrosion resistance. At about 0.07% the strengthening effect of phosphorus is such as to necessitate a reduction of carbon content, but quantities as great as 0.15%, or even more, are still useful under some conditions.

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[80] "Effect of Phosphorus in Soft Acid and Basic Open-Hearth Steels", by John S. Unger, *Year Book*, Am. Iron and Steel Inst., 1918, p. 172.

Chromium.—This metal exerts its greatest influence through its contribution to hardenability, and large additions demand either a low carbon content or a heat treatment. In higher carbon steels, its strong carbide-forming tendency is outstanding, but this effect is of less consequence in steels carrying only 0.15%, or less, of carbon. Its straight, solid-solution effect in iron is extremely mild as may be seen in Fig. 21. This is particularly true in the case of the yield point which is scarcely changed by a small percentage of chromium in very low carbon steels or in extremely slowly cooled higher carbon steels. The ultimate strength, however, is more markedly affected. Fig. 23 (*b*) illustrates the approximate influence of chromium upon the properties of a 0.20% carbon structural steel as rolled to a 0.5 or 0.75-in. section. In steels used in the as-rolled condition, it acts to control the microscopic structure and to foster the stronger structure in thin sections. Like manganese, therefore, it may best be used with a reduced carbon content.

The effect of high percentages (as, for example, 11%, or more) of chromium to develop an inert surface is so marked in the corresponding stainless steels that its modest contribution to corrosion resistance in the lower chromium steels passed almost unnoticed until recently. As an auxiliary element to raise strength with little loss in ductility and to encourage a harder structure from the transformation, it is useful and may be employed safely in quantities as great as about 1% in the present class of steels whenever manganese is not already relied upon for most of the strength increment.

Nickel.—This element, like silicon, exerts little or no influence upon the quantity or character of the carbide formed in low carbon steel. It is somewhat more effective as a solid-solution hardener than chromium, but has rather less influence upon hardenability. Its effects are mild but positive, and strength gained by nickel is often accompanied by very little or no loss of ductility. The only disadvantage in its use is the relatively large quantity required to secure marked increase in strength. Its improvement of corrosion resistance is well known and compares with that of copper except that it is not so effective, weight for weight. Where it may be used in relatively large quantity, and where ductility is extremely important, it is particularly well suited. Fig. 23(*c*) shows the approximate effect of nickel upon the properties of a 0.20% carbon structural steel as rolled to 0.5-in. to 0.75-in. section.

Molybdenum.—As a carbide-forming element, molybdenum exhibits a grain-refining influence probably next to vanadium in intensity. Even with low carbon steels the molybdenum, as ordinarily used, is to be found chiefly in the carbide constituent where its effect is an indirect one through the grain-size effect mentioned previously. In this way, molybdenum may counteract some of the hardenability effects and promote a more advantageous distribution of the carbide than that of plain carbon steels. When it is dissolved at high temperature, however, as in the hotter zones of a weld, it exercises a hardening influence. In view of the cost of molybdenum, it is customary to use it very sparingly, in the alloys under consideration. The grain-size effect can be achieved, in large part, by other means, such as measured deoxidation utilizing aluminum in a special way. The outstanding value of molybdenum, revealed by its tendency to form persistent carbide globules, is manifest in the creep-resisting steels used at elevated temperature.

Vanadium.—The effect of vanadium, as ordinarily used, is essentially that of grain-size reduction, with its attendant control of hardenability and preservation of toughness. By this indirect influence it permits a more generous use of manganese, but does not enter the picture of low-priced structural steels to any great extent. It becomes a more important element in the higher carbon steels used in the heat-treated condition.

The foregoing comments show that the acquisition of high strength is easily securable by the addition to iron of practically any of the elements enumerated, but that in the case of the most effective ones the strength is secured at a considerable sacrifice in ductility. Carbon accomplishes its results through an increase in the non-ductile constituent carbide, whereas the other elements act more gently through solid solution effects and by a change in hardenability. To secure a greater strength with maximum preservation of ductility in the as-rolled condition, carbon can be used only sparingly, and even manganese and phosphorus must not be used in quantities contributing very greatly to strength.

COMPLEX STEELS: SIMULTANEOUS ADDITION OF SEVERAL ELEMENTS

A fortunate circumstance is that the concurrent use of moderate quantities of several elements appears to produce a more favorable combination of properties than would result from a single element in an amount sufficient to produce an equivalent strength increase. Roughly speaking, in the small quantities under discussion herein, the effects of the elements are sufficiently additive to permit rough first approximations of properties from composition, but even those who introduced the early formulas for predicting tensile test data from analyses recognized some inter-relation which rendered linear formulas unreliable.

Suppose, for example, that the objective is a steel of 55-kip yield strength. This is easily accomplished by increasing the carbon content to about 0.50% and holding manganese to 0.45%, but by so doing the elongation in 2 in. is reduced to about 20%, and less for 8 in., a figure regarded as inadequate for general purposes by many engineers; but such a steel will not yield a weld of desired properties in the metal adjacent to the weld because of undue hardenability, and the semi-fused zone may be still more susceptible to the loss of ductility. If one uses about 1.60% to 1.85% of manganese and about

0.20% of carbon, the 55-kip yield point may be achieved and the elongation in 2 in. will not be far below 25%; but, on occasion, the weld may still not be quite as free from undue hardening as might be desired. Further improvement can be secured by further reducing the carbon and adding silicon; or, finally, by using still less manganese and adding yet another element, a very attractive set of properties is securable. Thus, for example, in a chromium, manganese, silicon steel (Cromansil), as rolled in 0.5-in. to 0.75-in. section, with carbon, 0.12%; manganese, 1.0%; chromium 0.75%; and silicon 0.75%, one would expect to secure about such tensile properties as ultimate strength of 80 kips per sq in.; yield point, 55 kips per sq in.; elongation, 26% in 8 in., and reduction of area, 55 per cent. Slight improvement may result from a further moderate decrease in manganese and increase in chromium and silicon. Similar improvements are obtained by the use of nickel in rather higher proportions, say, 2% to 3%, and some corrosion resistance is secured simultaneously. Whatever combination is utilized the corrosion resistance seemingly may be improved by the incorporation of as much as 0.50% copper and by the use of 0.10%, or more, phosphorus. A steel with which the writers happen to be familiar, is constituted in accord with the foregoing line of reasoning, with particular attention to high-yield strength and corrosion resistance, and has an average analysis approximately, as follows:

Composition (Percentages):

Carbon (maximum)	0.10
Manganese	0.30
Phosphorus	0.10
Silicon	0.75
Chromium	1.0
Copper	0.35

Average Properties in Ordinary Sections:

Yield-point stress, in kips per square inch	55
Ultimate strength, in kips per square inch	72
Percentage elongation in 2 in.	25
Percentage reduction in area	60
Izod impact, in foot-pounds:	
At 70° F	60
At −40° F	40
Endurance limit, in kips per square inch	45

There are obviously many alloy combinations from which to choose in securing the mechanical properties set as the objective of the new structural steels. In selecting examples thus far the writers have only expressed their concept of a logical design of alloy-steel composition. It should be borne in mind that there are few, if any, sharp maxima in the ductility-strength combination when properties are regarded as a function of composition. Indeed the optimum composition ranges for the principal properties are probably broad ones, and the minor desiderata may have to be taken into account to enable a final choice to be made. An impression of the various alloy compositions offered to the trade may be gathered from an inspection of Table 3[51]. The writers are not responsible for certain apparent discrepancies that appear in Table 3.

TABLE 3.—COMPOSITION AND PROPERTIES OF SOME COMMERCIAL STRUCTURAL ALLOY STEELS

CHEMICAL ANALYSIS (PERCENTAGES)									MECHANICAL PROPERTIES						
											Percentage Elongation in:			Impact Test, in Foot-Pounds	
Carbon	Manganese	Silicon	Chromium	Copper	Nickel	Molybdenum	Phosphorus	Vanadium	Yield point stress, in kips per square inch	Ultimate tensile stress, in kips per square inch	2 inches	8 inches	Percentage reduction in area	Izod	Charpy
0 08	....	....	....	1.0	2 0	....	....	....	61	75	32	......	60	....	43
0 15	....	....	....	1.0	2.0	....	....	....	64	82	29	......	60	....	41
0 22	....	....	....	1.0	2.0	....	....	....	61	88	27	......	57	....	....
0.12	1.0	....	....	0.5	0.5	0.20	....	....	60	75	25	......	......	75	....
*	*	....	....	1.5	1.0	*	....	....							
0.30	1.0	....	....	0 5	0.5	0.20	....	....	70	90	18	......	50	....	....
*	*	....	....	1.5	1.0	*	....	....							
0.12	0.5	0.30	....	0 9	0 45	....	0.10	....	69	78	......	22	68	42	36
*	0.7	*	....	1.25	0.65	....	0.15	....							
0.12	0.15	†	....	0.3	0.3	0.05	0.05	....	47	67	28	......	......	....	....
*	0.90		....	0.8	0.8	0.25	0.15	....							
0.25	0.75	0 25	0.25	0.3	0.25	....	0.08	....	70	90	20	......	......	50	....
*	*	*	*	0.5	*	....	0.10	....	50	65	25	......	......	50	....
0.35	1.25	0.30	....	0.40	....	....	....	....	50	80	20	......	......	40	....
*	1.75	*	....	‡	....	....	....	....							
0.14	0.7	0.15	0.12	0.25	....	....	....	....	70	80	19	......	......	....	....
	0.9	0 20		0.30	....	....	....	....							
0.20	1.2	0.15	0.12	0.25	....	....	....	....	85	105	12	......	......	....	....
0.30	1 6	0.20		0.30	....	....	....	....							
0.08	0 6	0.50	0.25	0.40	0.25	§	....	....	50	65	20	......	......	....	....
0.30	0.9	*	*	0.60	*	....	....	....	72	90					
0.10	0.10	0.5	0.5	0.3	....	....	0.10	....	50	65	22	......	60	60	....
	0 30	1 0	1.5	0.5	....	....	0.20	....	60	75	27	......	......	40	....
0.35	1 25	0.10	....	0 25	....	0.40	....	0.20	55	90	20	......	......	....	....
*	1 70	0.30	....	*	....	*	....	*							
0.20	0.40	0 20	....	0.25	....	....	....	....	45	90	18	......	......	....	....
0.40	0.80	‡	....	*	....	....	....	....							

* = Maximum. † = Trace, ‡ = Minimum, § = Molybdenum present.

The writers are persuaded that the low-carbon content of most of these steels is an important feature and one that is worthy of careful consideration. Not only is high carbon unnecessary in view of its replacement by larger additions of alloying elements in securing high yield strength (say, 55 kips per sq in.), but ductility in the as-rolled condition in thinner gages depends upon its being kept low; but still more important is the manner of hardening during rapid cooling after welding, which often is not easily preventable. Some engineers regard about 0.15%, maximum, of carbon as highly desirable in the general case, although a higher percentage may be satisfactory in some instances. Clearly, the lower the carbon content, with physical properties constant, the less will be the opportunity for dissatisfaction in this respect.

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[51] "Low-Alloy, High-Yield Strength Structural Steels, An Extended Abstract", *Metals and Alloys*, March, 1936.

At the same time wherever welding is not used, or in cases amenable to stress-relief annealing following welding, the higher carbon steels are applicable, and they may possess still higher strength.

It is noteworthy that most of the low-carbon structural alloy steels manifest a relatively high endurance limit. Some of them are surprisingly high and others show an endurance limit of approximately 50% of the ultimate strength. Considering that comparatively few structures have to withstand full reversal of load, it is not believed that special consideration need be given this aspect of the question in the substitution of the alloy steels for ordinary structural steel.

Manufacturing Aspects

From the standpoint of the user, the manufacturing operations at the steel mill, involved in the production of the structural ferrous alloys covered by this paper, do not differ greatly from those used in the production of ordinary structural steel. There are differences, of course, in the manner in which the alloying elements are applied, in the furnace or in the ladle, in discard practice, and, in some cases, in the precise temperature at which the rolling process is finished, but it is believed that the civil engineer, who designs, fabricates, or erects structures in which the alloys in question are used, is not primarily concerned with these features. Indeed, operating practice differs in the several producing plants. Those who are interested may refer to the meritorious outline of mill practice in the production of the steels in question, which appeared in the Progress Report of Sub-Committee No. 2, Committee on Steel of the Structural Division, on Structural Alloy and Heat-Treated Steels[69].

Of greater interest to the civil engineer is the extent to which the special grades in various forms are readily obtainable from the steel mills. The fact that manufacturing operations are similar to those required for ordinary grades does not mean that the special grades are as readily obtainable; they are not as easy to manufacture in a wide variety of section for several reasons:

(1) At present, the demand for any given special grade is not as great as for ordinary grades; therefore, they are not rolled as frequently, or in as wide a variety of sections; and,

(2) The execution of orders for small tonnages of a given special grade must either await the accumulation of other similar orders, or the required items must be taken from stock. The probability of the existence of a stock of the desired grade, and in the desired form and length of section, is much less than in the case of ordinary grades.

For these reasons it is believed that prospective users should consult with the respective steel manufacturers as to the availability of a special grade of steel, for a designated structure, before they incorporate members of that grade into their design.

One interesting fact not always realized by users relates to the excess material produced. When an order for a given lot of special steel is taken, the steel mills regularly produce more than the quantity ordered so as to provide against possible rejections. If no material is rejected the excess is either scrapped or placed in stock. The latter practice explains the occasional offering by the manufacturers of odd lots of a special grade.

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[69] *Proceedings*, Am. Soc. C. E., March, 1936, p. 361.

SUMMARY

By incorporating moderate proportions of the common alloying elements, already used in other alloy steels, and at the same time reducing the carbon content, structural steels are now manufactured, which possess increased strength, particularly yield strength, with little or no diminution of plasticity or ductility. Weldability and formability are maintained by virtue of this altered composition. Under suitable conditions these gains in desirable properties may make possible an over-all economy in the use of alloy steel instead of the structural carbon steel in spite of its higher cost.

When advantage is taken of the superior properties to reduce the thickness of members greater attention must be paid to corrosion resistance, because corrosive attack does not reduce the effective metal section in proportion to thickness, but irrespective of section. Fortunately, alloying elements can be selected which confer a reduced corrosion rate to the steel, and this is a valuable property wherever a protective paint or other surface cannot be maintained continuously.

A number of alloy combinations are effective in achieving these results; manganese, nickel, chromium, and silicon are probably the most effective strengtheners, as ordinarily used, and copper, phosphorus, molybdenum, and vanadium are useful auxiliary elements. Copper, nickel, phosphorus, and chromium contribute important resistance to corrosion.

In these low-alloy steels, when the carbon content is less than about 0.15%, and when the high yield strength is derived by judicious combination of the other elements, no serious hardenability interferes with welding processes, the formability is more than adequate, and the product is not unduly sensitive to commercial variations in composition or gage. The content of carbon or the powerful hardening elements should be regulated by the needed ductility and the needed freedom from air-hardening characteristics.

ACKNOWLEDGMENTS

The writers are indebted to the Union Carbide and Carbon Company for the use of Fig. 23(*b*).

STAINLESS, HIGH-ALLOY STRUCTURAL STEELS

BY M. J. R. MORRIS[53], ESQ.

SYNOPSIS

In the realm of civil engineering the products discussed in this paper are new. The alloys generally used to-day are ferrous products of the mild structural steel type, which have definite physical qualities and stable surface characteristics. The Engineering Profession, therefore, has evolved a design within their limitations and, it may be stated, the curve of progress is now flattening out. Any hope of marked improvement in design involving this grade of steel does not appear possible.

Since about 1915 a class of steels has been produced in which the physical properties of the base (plain carbon) have been changed by the addition of alloys. This field of endeavor has been much capitalized in the structural steels for automotive engineering applications, but, otherwise, little use of these properties has been made by civil engineers; and, yet, this evolution in automotive practice accounted for the most important change in steel metallurgy within the last two decades, amounting in 1929 to 1 ton in 15 tons.

At the beginning of the World War a new chapter was opened, wherein the base plain carbon was alloyed with chrome, etc., to produce not only increased physical strength but also increased stability of surface. Unfortunately, this group has been termed stainless steels. They are not a single steel, but a large group, the base of which is approximately 12% chromium. This is the "corner stone" of the group which will be discussed in this paper.

INTRODUCTION

Steels designed under the specifications of the Society of Automotive Engineers have definite and desirable physical characteristics, but they do not possess the capacity to withstand various forms of corrosion; and it is this latter set of properties which enables this group to be used in lighter sections because of their better physical characteristics. It can thus be seen that one of the great drawbacks encountered in thinner sections of low-alloy, high tensile steels—surface deterioration—is not found among the high alloys, and now for the first time a group of steels is being developed which will have increasing fields of application.

Perhaps more than any other, the aircraft industry has exploited the use of stainless steels and of other high tensile alloys. This industry has been followed recently by the endeavors of the railroad companies. The Eads Bridge, at St. Louis, Mo., was a pioneer in raw products. It is a chromium

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[53] Chf. Metallurgical Engr., Central Alloy Div., Republic Steel Corp., Massillon, Ohio.

steel structure. Alloying 12% chromium with base plain carbon produces a new alloy, its ultimate tensile strength being capable of variation from 50 to 160 kips[64] per sq in. by heat treatment. At the same time, this alloy is highly resistant to atmospheric, and ordinary water, corrosion attacks (referred to base plain carbon life, it would be infinite).

This grade is in its simplest form, and can be further modified by increasing the chromium content with, or without, increasing the carbon. These properties are shown in Table 4. The chromium series are modified with small quantities of nickel (say, as much as 2.5%) to improve the physical properties and with molybdenum and silicon to increase the stability of the surface. These alloys are ferritic in structure and magnetic, and generally respond to heat treatment as a means of increasing their physical properties.

TABLE 4.—CHEMICAL AND PHYSICAL QUALITIES OF CHROMIUM AND CHROMIUM-NICKEL ALLOYS

Item No.	Alloy No.[f]	Carbon[a]	Chemical Analysis (Percentages)								Physical Properties	
			Chromium		Nickel		Silicon[b]		Other Elements		Yield Point Stress in Kips per Square Inch	
		(1)	From: (2)	To: (3)	From: (4	To: (5)	From: (6)	To: (7)	From: (8)	To: (9)	From: (10)	To[c]: (11)
1[g]..	12 Cr	0.12	12.0	14.0	....	....	......	......	......	......	40	170
2[g]..	12 Cr	0.12	12.0	15.0	....	....	(0.20)[j]	(0.40)[j]	0.45[l]	0.65[l]	40	150
3[h]..	18 Cr	0.12	16.0	18.0	....	....	......	......	......	......	40	100
4[i]..	28 Cr	0.20	23.0	30.0	....	....	......	......	......	......	....	45
5[h]..	18-8	0.20	17.0	19.0	7.0	9.5	......	......	......	......	35	225
6[h]..	18-8	0.20	17.0	19.0	7.0	9.5	......	......	0.15[m]	0.30[m]	45	130
7[i]..	18-8	0.20	17.0	19.0	7.0	9.5	2.0	3.0	......	......	..	40
8...	18-8	0.11	16.0	19.0	9.0	14.0	......	......	2.0[l]	4.0[l]	40	130
9[h]..	18-8	0.11	17.0	19.0	7.0	10.0	......	......	(4)[n]	(4)[n]	35	200
10[h]..	18-8	0.15	17.0	19.0	8.0	12.0	......	......	(6)[o]	(10)[o]	40	200
11[i]..	25-12	0.20	22.0	26.0	11.0	14.0	......	2.0[k]	......	......	....	40
12[i]..	25-20	0.25	24.0	26.0	19.0	21.0	......	[k]	......	......	....	40

TABLE 4—(*Continued*)

Item No.	Physical Properties (*Continued*)						Creep Strength for 1% Elongation per 10 000 Hours, in Kips per Square Inch, at							Modulus of elasticity, in pounds per square inch
	Tensile Strength, in Kips per Square Inch		Percentage Elongation in 2 Inches		Percentage Reduction in Area		900° C	1 000° C	1 100° C	1 200° C	1 300° C	1 400° C	1 500° C	
	From[c] (12)	To-[d] (13)	From:[c] (14)	To:[d] (15)	From[c] (16)	To:[d] (17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
1[g]	70	200	28	10	65	25	....	12	5.0	2.2	1.6	1.1	0.7	28 000 000
2[g]	70	175	25	10	60	30	....	....	....	....	....	....	....	28 000 000
3[h]	70	125	28	10	60	25	....	9	5.3	2.0	1.4	1.0	0.6	29 000 000
4[i]	......	75	28	....	60	....	....	....	3.5	1.7	0.8	0.2	....	29 000 000
5[h]	85	275	55	6	65	30	24	18	11.0	7.0	4.5	2.6	0.9	29 000 000
6[h]	100	160	55	20	60	40	....	..	....	....	....	....	....	29 000 000
7[i]	......	90	50	...	60	....	....	19	12.0	7.0	4.6	2.5	....	29 000 000
8	90	160	55	20	65	50	....	....	....	....	....	....	....	29 000 000
9[h]	85	250	55	5	65	30	....	....	14.0	8.0	6.0	3.0	....	29 000 000
10[h]	85	225	55	5	65	30	....	....	....	....	....	....	....	29 000 000
11[i]	......	90	55	....	65	....	....	....	....	....	....	....	....	29 000 000
12[i]	......	90	50	....	55	....	....	21	13.0	7.5	0.5	2.8	....	29 000 000

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[64] 1 kip = 1 000 lb.

TABLE 4—(*Continued*)

Item No.	Coefficient of Expansion at A° C		Magnetic properties	Welding properties	Heat treatment properties	Heat conditions,[e] in calories per cubic centimeter per degree Centigrade	Scaling Temperatures, in Degrees Centigrade		Brinell Hardness Number		Izod Impact, in Foot-Pounds	
	Values of A From 0° to: (26)	Coefficient, $\times 10^{-6}$ (27)	(28)	(29)	(30)	(31)	Continuous (32)	Intermittent (33)	From[c]: (34)	To:[d] (35)	From:[c] (36)	To:[d] (37)
1[g]	800	12.6	Yes	Poor[p]	Good	0.05	1 400	1 400	160	418	100	30
2[g]	800	12.6	Yes	Poor[p]	Fair[r]	0.05	1 400	1 400	160	340	70	20
3[h]	800	11.2	Yes	Poor[p]	None	0.045	1 500	1 600	160	255	70	1
4[i]	1 000	13.0	Yes	Poor[p]	None	0.04	2 000	2 100	160	...	30	1
5[h]	800	20.0	No	Good	None	0.035	1 600	1 400	156	418	110	10
6[h]	800	20.0	No	Fair[q]	None	0.035	1 400	1 400	179	350	80	10
7[i]	800	20.0	No	Good	None	0.035	1 700	1 500	156	...	80	..
8	800	20.0	No	Good	None	0.035	.....	.....	160	350	110	50
9[g]	800	20.0	No	Good	None	0.035	1 600	1 400	156	396	100	10
10[k]	800	20.0	No	Good	None	0.035	1 600	1 400	160	350	90	15
11[i]	1 000	20.2	No	Good	None	0.03	2 000	1 600	160	...	90	..
12[i]	1 000	22.5	No	Good	None	0.03	2 050	1 700	160	...	90	..

[a] Maximum allowable. [b] Except Item No. 2. [c] Annealed. [d] Heat-treated or cold-drawn. [e] Soft steel, 0.10 calories per cu cm per degree Centigrade. [f] Trade designation by which the alloy is commonly recognised. [g] Heat-treated to improve physical properties. [h] Cold-worked to improve physical properties. [i] These items will harden on cold-working but since they are used only for heat resistance they are annealed only. [j] Sulfur. [k] Maximum. [l] Molybdenum. [m] Selenium. [n] Titanium four times the carbon content. [o] Columbium six to ten times the carbon content. [p] Brittle welds. [q] May check. [r] Fair to good.

In the austenitic group of stainless steels the base is chromium plus nickel (approximately 18% chromium plus 8% nickel) with less than 0.20% carbon. These steels have low yield points, but high tensile strength, in the annealed condition; but they have an extremely high degree of ductility. They are incapable of gaining increased physical properties by heat treatment; they can be hardened by mechanical work only. The range of physical properties is large as shown in Table 4. This base alloy has been modified by molybdenum, columbium, titanium, silicon, and copper, each of which contributes some desirable modification, depending upon the different field of application.

Molybdenum is added within a range of 2 to 4% to increase its resistance to wet corrosion particularly of the sulfurous acid type. Columbium is added in quantities equal, roughly, to ten times the carbon content in order to affect the stability of the carbon at elevated temperatures. The effect is to prevent separation of carbon at the grain boundaries in the form of carbides, which separation is undesirable, as this condition is more susceptible to corrosion. Titanium is added for the same purpose as columbium and in quantities generally five times the carbon content. It was the first ingredient so used and may be supplanted by columbium. These end points are attained also by lowering the carbon content only, but the drawback is that grain refinement is not maintained at the same time; hence, the foregoing modification was developed.

Silicon is added to increase resistance to dry corrosion or scaling at elevated temperatures. There is no marked increase in strength from the addition of this alloy. Copper is added at the present time for the purpose of increasing resistance to wet corrosion, and possible definite data on its rôle will be forthcoming in the near future.

Physical Properties

The physical properties of the alloys are given in Table 4. One item that should be emphasized especially is the elastic limit of the mechanically worked austenitic type; the degree of mechanical working necessary to attain this value properly is in excess of 25 per cent. It is then noted that the elastic limit is indefinite and the curve is continuous. The evaluation of this characteristic can readily be seen to be widely divergent, depending upon the method utilized. This fact is shown in Fig. 24(*b*). The characteristics of types of stainless steel that are hardenable by heating are shown in Fig. 24(*a*). This is of the same type as in all the steels patterned after the specifications of the Automotive Society of America, but, as already mentioned, the austenitic steels can be hardened only by mechanical working. Therefore, referring to the basic plain carbon structure, a margin of superior physical properties is found which is several times the present base.

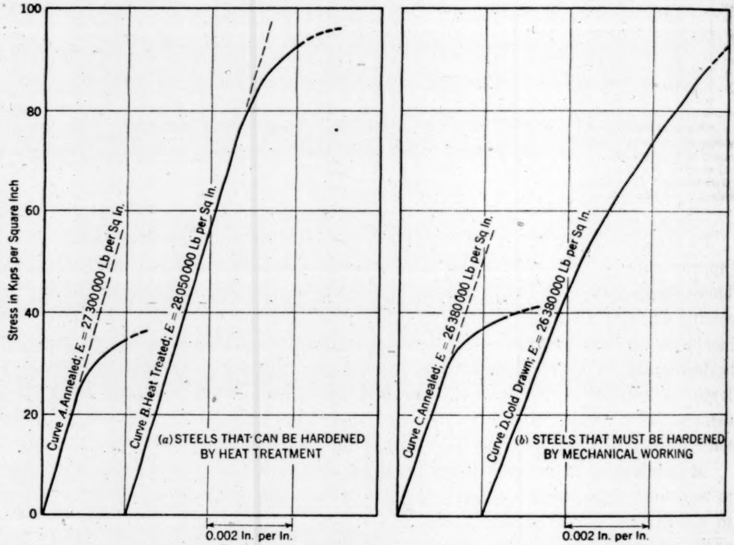

Fig. 24.

In addition, these austenitic steels are easily weldable by the modern electrical or gas-welding methods. They can also be formed, punched, and forged without difficulty, but it should not be forgotten that they are stronger steels and, therefore, will require more energy to work. It is safe to state that they are being handled successfully to-day.

It is now logical to discuss briefly the most important property of this group of steels; that is, their corrosion resistance. No one steel in this field has properties adaptable to every case and it is for this reason that the word, "stainless", has been unfortunate and, in fact, has hindered their proper development and use. It is just as improper to state that there is only one acid. Metallurgists know better to-day, and this important fact being appreciated is leading to more intelligent application and thorough understanding of these steels.

The civil engineer has various atmospheric conditions confronting him—from pure country air to sea air—and highly industrialized conditions. He encounters problems of diverse characteristics, from the practically non-corrosive to extremely corrosive types. These conditions can be analyzed much more readily to-day than in the past. Lastly, he meets a variety of soil conditions, which can be also measured much more intelligently to-day for their corrosion conditions. It can be seen, therefore, that knowing these conditions, the metallurgist and the engineer can design a more suitable steel product to serve a particular purpose.

Since about 1925, much has been done to stress the corrosion fatigue properties of steels. This emphasis has increased, definitely, the importance of obtaining the necessary information as to the medium of application. The surfaces must have a maximum degree of corrosion resistance.

MANUFACTURE OF STAINLESS STEELS

Because of their high alloy content, all the stainless steels discussed in this paper are made in an electric furnace of a maximum capacity of 20 tons, generally approximating 10 to 15 tons. The ingots are generally rolled, but the more special types are much stiffer and are forged from the ingots into blooms, which are then rolled.

Being much stiffer than the ordinary plain carbon steels or structural alloy steels, stainless steels are slower to absorb heat and have a narrower working range. The mills, therefore, are different from those used for manufacturing plain carbon steel. They are usually slower and more powerful. It is necessary to have very good temperature control and good fuel conditions. These steels demand a more complete understanding of the mechanical working operations; otherwise, no resemblance of success is obtained and much disappointment results. However, these are days of definite control of operations, and rule-of-thumb and good luck should not, and cannot, exist.

It has already been stated that the rolling requires extra strength, but that the annealing and cooling conditions must have definite control. It is a matter of general practice to handle straight chromium types as all ferritic and hardenable steels are handled—that is, they are cooled very slowly. In the austenitic types, however, the reverse is necessary in the case of nearly all sections in present use. When cooled slowly, the crystalline structure of these types is endangered and rendered more or less useless, but this condition is so easily tested that there is absolutely no excuse for a poor product being shipped from the steel mill or by the fabricator. Generally, it can be stated that sections containing less than $\frac{30}{1000}$ in. cool rapidly enough in air so that this harmful condition is avoided. The thicker sections are quenched to increase their rate of cooling. Hence, it is seen that the two annealing treatments are opposite.

The entire group of these alloys is complex and at the present time their methods of manufacture and production are truly in the evolution period or flux; it is encouraging to be able to state that rapid progress is being made. In a period of little more than 10 yr a product has been transformed from the jewelry classification to one of promising expanding industrial uses, and it is computed that the production for 1936 is well on the way to 100 000 tons.

APPLICATION OF STAINLESS STEELS IN CIVIL ENGINEERING

The application of stainless steels should be divided into fields of endeavor. It has been shown in recent years that, on certain jobs, definite use can be made of this group of steels, and all such applications have been major investments of the more permanent type. It would be better to divide the various uses into fields that have passed beyond the pioneer stage and describe the better known uses in each field.

Dam Construction.—A proper field for the application of stainless steels has been in the construction of dams, such as in gate-rollers, guides, seal strip, and screw mechanism for raising and lowering the gates. All this use applies to the gate structure and mechanism. It is conceivable that the larger gates could be made of structural steels overlaid with stainless steels and, although there are not yet a sufficient number of cases upon which to base sweeping conclusions, it is felt that such a suggestion would be accorded a high preference. These applications are seriously dependent upon freedom from corrosion in order to keep the mechanism free to operate, galling and rusting being fatal. The rollers and guides usually contain 12% chromium, heat-treated to a Brinell hardness number of about 350. The seal strip is austenitic, with low carbon content, of the type analyzed in Items Nos. 5 to 10, Table 4.

At present, expansion joints between concrete blocks in dam construction are being made of a thin gage strip of low carbon steel. Several of the projects in the Tennessee Valley Authority have this form of construction. It has been thought that, in rock-fill dams, the water face should be of thin stainless sheets, and it is reputed that one such application is being installed in the Severn River, in Great Britain, as an experiment. The ducts controlling the water to and from the locks should present a proper application of stainless steels as liners; this is being considered in some designs at present.

One of the largest uses of stainless steel in dam construction has been on the Aswan Dam, in Egypt, in which about 2 500 tons were used. The dam is 1.25 miles long, and was built in 1892. At the end of 1928, an International Commission inspected it and reported the necessity of further raising and strengthening it. Accordingly, it was decided to erect masonry buttresses between the sluices, but instead of these buttresses being fixed to the face of the dam, they were to be laid against a layer of stainless steel plates placed against the face. The buttress thus was to become a live load. These alloy plates, with a high chromium and nickel content, are 7 mm (0.280 in.) thick, and of varying sizes to suit the dimensions of the masonry buttresses in conjunction with which they are to be used.

More than 100 tons of 4 to 6% of chromium steel were used in the construction of a dam completed in 1935 at Hawk's Nest, W. Va. The steel was used as concrete reinforcing rods and represents one of the first applications of alloy steels to this type of work. The 4 to 6% chromium steel was selected because, by its use, a high tensile strength (180 kips per sq in.) could be obtained without special heat treatment and also because the chromium content renders these rods much more immune to the rusting that always occurs in such applications.

Great quantities of stainless steel were used for sluice-gates, rollers, etc. on the Arapuni Power Project, in New Zealand. This is a unique installation because of the great head of water. The gate is the most heavily loaded in the world, the total water pressure on the face being 2 000 long tons.

In bridge design, the rockers and rocker-plates have recently been made of stainless, heat-treated steel. Some experts maintain that the solid floors of bridges can be modified profitably by the use of stainless grating structures, which are claimed to be stronger, considerably lighter, more rust resistant, and more resistant to wear. One bridge has been so modified. This development received its first important impetus from experiences with the George Washington Bridge over the Hudson River, in New York, N. Y.

Stainless steel wire was used for auxiliary counterweight cables on the Buzzard's Bay Bridge, at Cape Cod, Massachusetts. This steel was the type analyzed in Items Nos. 5 to 10, Table 4, and the cables were 1$\frac{5}{8}$ in. in diameter. These cables are in a position not readily accessible for inspection and the use of stainless steel is an added safety factor.

Sewage Disposal Plants.—Stainless steel products should find definite application because of their anti-fouling properties. Heavy pipe lined with stainless steel has been used in large quantities in the City Sewage Plant, at Milwaukee, Wis., because of its low friction factor.

Intakes.—In water systems supplying cities, etc., the austenitic and straight chromium types of steel have been considered. These steels have been applied on intake screens, the type used depending on the design; and generally they have been the austenitic type. The filter screen in artesian well installations has disposed of considerable stainless steel. These screens are of several designs, either wire or sheets being used. Wells of this construction are very easily cleaned. In filter beds, thought has been given to utilizing perforated stainless steel for the floors, the perforation being of such size as to eliminate certain of the coarser gravels.

In 1935, a huge intake crib was completed in Lake Michigan off the foot of Chicago Avenue, Chicago, Ill. It covers the intake from which water for the municipal supply is drawn from the lake through a tunnel below the lake bottom. Older cribs were constructed of granite. A unique feature of this attractive new concrete and steel structure is the stainless steel ring that lines the shaft. It serves to protect the shaft and to resist the rust and erosion that could be expected to attack other forms of construction. It is 39 ft in diameter, 10 ft high, and weighs 3 700 lb.

Reinforced Concrete Structures.—In reinforced concrete structures several problems are presented which may be helped sometimes by the use of stainless alloy steels. When ordinary steels rust, the products of this rusting occupy a decidedly larger volume than that occupied by the non-rusted product from which it came. This produces expansion stresses which may result in cracking the concrete. If the superimposed thickness of the concrete is slight, the steel work is soon exposed; if the concrete covering is thick, this rate of rusting is much less. However, this thought has been embodied in the construction of one dam and, although the alloy was not the true stainless type of steel, its use was a pioneering effort which embodied the principles.

In constructing a new reservoir near Sheffield, England, the engineers decided to demolish an old weir on the River Derwent and to build a measuring channel instead. This channel is unique in that its masonry bed is covered with a stainless steel sheet, the purpose being to prevent the growth of moss and other vegetation that would choke the flow and induce errors in the measurements. This application of stainless steel, although unusual, is entirely practical and is indicative of the extent to which foreign engineers are utilizing its advantages.

Finally, large roofs demonstrate the first use of metals possessed of corrosion-resisting properties. Lead was an early form—then copper, brass, and other non-ferrous products. Necessarily, all this preliminary experience was with such metals as these because of the known attributes of steel and iron products. In these special applications, as ably demonstrated by the roofs of the Pennsylvania Railroad Station, in New York, N. Y., steel and iron rusted quickly and were not easily worked. Stainless steels could be used in such cases with a saving in weight of at least 10 per cent.

St. Paul's Cathedral, in London, England, is a splendid example of how masonry structures that are weakening from age may be strengthened by stainless steel. A few years ago Government inspectors discovered that the huge dome of this famous old cathedral was showing signs of deterioration. Not only was the life of the magnificent structure threatened, but the fractures in the dome constituted a serious danger to worshippers. Tests were made both with tie-rods of strong common alloy steels and with rods of special stainless chrome-nickel steel. The rough oxidizable surface of the common alloy steels adhered to the cement readily, which at first seemed to be an advantage. However, it required a maximum load of only 2.85 tons to withdraw the plain rods. Then bars of stainless steel were rolled and indented so that a series of flat rods was formed. To withdraw these bars from the hardened cement took a maximum pull of 18.88 tons. The result is that stainless steel tie-rods, 40 ft long, threaded at each end, and 4 in. in diameter, are now firmly embedded in the walls of St. Paul's Cathedral to tie the inner wall to the outer wall of the dome.

Irrigation.—Much stainless steel is being utilized in irrigation practice partly to resist corrosion. The Shanan Plant, on the Uhl River, in Punjab, India, is an excellent illustration of this development.

CONCLUSION

It would appear, therefore, that stainless steels of various composition, permit the selection of a type with attributes that will fill the demands of any given engineering problem. At present, it is not as simple to supply them as in the case of the plain carbon steels because the cost would be excessive. When the cost is secondary, or when a high first cost is justified by hazards to life, they can be applied. However, although the application of stainless steels to safeguard life is increasing, this is not the only field. In large structures, weight will be materially reduced by designing these alloys in conjunction with the low carbon types. As in the case of the railroad and the airplane, conventional designs will not be followed but types of structures will be evolved. This will enable a more widespread use of stainless steels. The problem now is to learn to use their valuable properties.

LIGHT-WEIGHT STRUCTURAL ALLOYS

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AND R. T. WOOD[57], ESQ.

SYNOPSIS

This paper is intended primarily to present those metallurgical principles and properties of the light-weight alloys which govern: (1) The selection of the most suitable composition, temper, or form of material; and (2) the selection of fabricating practices and degree of control required if the best results are to be obtained. As the paper embraces both aluminum and magnesium, and as their characteristics differ so widely, they will be dealt with separately.

PART I—ALUMINUM

HISTORY

Many aspects of the history of aluminum have been amply covered in the literature, especially in 1936, which marks the Fiftieth Anniversary of Hall's noteworthy discovery[58], an event which extracted this metal from the limited enjoyment of the few and made it available as a common article of commerce.

In spite of these previous writings, it is considered worth while to summarize briefly those pertinent events and developments that caused aluminum to become a structural engineering material. It may well be remembered that, for years, the position of this metal in the industrial "sun" was almost limited to essentially non-engineering applications, as, for example, the humble household cooking utensil. During those years, it was not deemed reasonable that such a light metal could do the world's "heavy work." It appears that it was not until aluminum alloys were used in the construction of aircraft that they were conceded the possibility of becoming engineering materials in any field.

In 1898, an Austrian, named Schwartz, constructed a rigid airship with the framework and outer covering of aluminum. Unfortunately, this ship was destroyed on its first landing and, hence, this effort should probably not be recorded as establishing the validity of aluminum alloys for engineering purposes. However, on July 2, 1900, Count von Zeppelin, after many difficulties, launched his first rigid airship. Gaged by the standards of that day the aluminum alloys, from which the framework of this ship was constructed, functioned satisfactorily. The composition of these early aluminum materials varied somewhat, but they were principally aluminum-zinc alloys. They were not heat treatable, but obtained their strength from the effects of the added alloying elements, coupled with cold working.

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[57] Chf. Metallurgist, Am. Magnesium Corp., Cleveland, Ohio.

[58] "Fifty Years of New Product Development", by Messrs. Frary and Edwards, *Chemical and Metallurgical Engineering*, February, 1936, p. 64.

In 1909, Alfred Wilm, of Germany, brought forth an aluminum alloy, since known as duralumin, which was infinitely superior to those formerly available. The superior combination of qualities of this alloy, such as high strength, workability, and resistance to corrosion, very quickly indicated its usefulness as an aircraft material. This alloy, and the process by which it is manufactured, is of particular historical interest for two reasons: (1) It marks the first noteworthy instance in which the strength of an aluminum alloy was increased materially by means of a heat-treating operation; and (2) in later years, the explanation of this behavior, originated by P. D. Merica[58] and his associates, revealed to the world a new metallurgical principle and one which not only profoundly affected later developments in the aluminum industry but also the metallurgical advance of many other metals.

Marked progress has been made during the past fifteen years in acquiring an understanding of what occurs within the structure of the metal during and subsequent to the heat-treating operation. This has made possible an intelligent selection and control of the alloying ingredients and manufacturing processes, resulting in an ever growing list of improved and more useful products.

Especially during the early years of this development period the principal urge for higher strength, greater uniformity of properties, increased resistance to corrosion, and greater diversity in the form and range of sizes came from the aircraft industry. This was not unnatural. Several conditions in the aircraft industry immediately following the cessation of the World War, which corresponded approximately with the commercial introduction of the heat-treatable aluminum alloys, all conspired to bring about an intensely active development on a co-operative basis between the aircraft and the aluminum industries. The aircraft industry was in its infancy; the application of the metal was of even more recent date. There are few engineering fields in which the demand for perfection of properties is necessarily so highly exacting, in which the materials of construction must be utilized so efficiently, and in which the factor of safety must, of necessity, be calculated to such a fine point. At the same time, the penalty of a failure is terrific.

The industry proceeded with the best materials then available, setting specifications that called for the most constant vigilance for their fulfillment, but pressed insistently for continued improvement and advance. The aluminum industry responded, especially in the United States, and there ensued a development which resulted in the commercial production of a variety of new aluminum products not hitherto available. The guaranteed minimum yield strength was increased more than 100% in less than twenty years. New products were produced, possessing such excellent resistance to corrosion that they could be used bare without any protective coating whatever and this, in sheets, as thin as 0.0095 in.

The object of relating this history is not primarily to praise the aircraft industry (although praise is due), but to indicate the standards of achievement sought during this development period. These same standards have governed the development of other aluminum alloys for other engineering applications.

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[58] "Heat Treatment of Duralumin", *Paper 347*, National Bureau of Standards, November 15, 1919.

The demands of other industries also contributed considerably during this period, and increasingly so. The railroads became actively interested in light alloy construction at least as early as 1923. Rightfully, they emphasized initial, as well as ultimate, cost; they stressed length of useful service far beyond that required for aircraft during that period. Aviation's first problem was to make something fly; and, next, to fly well and safely. Now it has reached the stage of adding "and economically."

Other industries came in for increased attention: The truck, the bus, the marine structure, construction equipment, building materials, sewage disposal equipment, and now the bridge industry.

Each field presented different problems, problems that differed as to technical characteristics of the material that were of governing importance, as to requirements dictated by design and feasible fabrication practices, and as to competitive costs. The result has been the development of a relatively large list of alloys and tempers, available in a very extended range of forms and sizes, some especially suited to one field, some particularly adapted to other fields. With the production of these new materials has come a substantial fund of knowledge of the metallurgical principles involved, an understanding of which is very helpful in the intelligent selection and use of these materials.

Important Metallurgical Principles of Structural Aluminum Alloys

Effect of Composition.—The more common metals used for alloying with aluminum are copper, silicon, manganese, magnesium, zinc, nickel, and chromium. Although occasionally only one of these alloys is added, frequently two or more are introduced, the quantity of each depending on the results desired. It is not possible to predict, definitely, the exact properties of a new composition, nor just how it will respond to various manufacturing processes; and yet certain general principles have been developed. The addition of any one of these elements results in increasing the ultimate strength, yield strength, hardness, shear, and fatigue or endurance values, usually at the expense of elongation, reduction of area, plasticity, and malleability. Magnesium is one exception. Although the strength is regularly increased (see Table 5), in conformity with the general principle, by the addition of

TABLE 5.—Effect of Addition of Magnesium Upon Mechanical Properties of Aluminum-Magnesium Alloys in the Annealed Temper

Magnesium content (percentages)	Tensile strength, in kips per square inch	Yield strength, in kips per square inch	Percentage elongation in 2 inches	Magnesium content percentages)	Tensile strength, in kips per square inch	Yield strength, in kips per square inch	Percentage elongation in 2 inches
0.75	15.00	6.00	30.0	5.30	38.70	17.50	27.0
2.50	26.76	10.70	25.7	6.00	42.00	18.50	29.0
3.00	28.90	11.65	26.7	8.00	46.00	21.00	32.0
4.25	35.50	15.25	26.0	10.00	52.00	24.00	32.0

more magnesium, the elongation at first decreases, but later recovers. The modulus of elasticity is only slightly affected by alloying. The greater the quantity of the alloying element, the greater in general will be the effect upon the several properties. The magnitude of the effects also varies with the particular alloying metal; that is, the addition of, say, 2% of copper will not affect the mechanical properties to the same degree as the addition of the same percentage of magnesium. Each alloying agent possesses its own quantitative influence.

The aluminum most resistant to corrosion is that of highest purity; in other words, the general effect of alloying is to lower this resistance. Again, the several metals vary in their effects. The alloys with copper and zinc are among the least resistant; the manganese and the magnesium alloys are on a par with commercially pure aluminum. The alloy consisting of 1.25% manganese (3S)[60], possesses a resistance to corrosion quite equal to that of commercially pure aluminum (2S). Likewise, the alloy (52S) consisting of 2.5% magnesium plus 0.25% chromium, and the alloy (53S) that consists of 1.25% magnesium, 0.7% silicon, and 0.25% chromium, and can be heat-treated, all possess a resistance to corrosion that places them in the same class as commercially pure aluminum. Corrosion resistance is one of the properties of these alloys that frequently dictate their selection.

Although the corrosion-resisting qualities of any particular new alloy cannot be definitely predicted in advance, many individuals and organizations throughout the world have concentrated on studying the specific behavior of present commercial and experimental compositions when subjected to a wide variety of laboratory and service conditions, with the result that there is available a wealth of useful information from which the prospective user can determine fairly well whether a particular alloy will meet his requirement satisfactorily.

Another effect of change in composition that may be of importance is that upon weldability. All the commercial aluminum alloys can be welded by the various methods—such as torch, electric spot, seam, butt, and arc—but the facility with which this can be accomplished and the properties of the resulting joint vary considerably, depending on the composition and, in some cases, on the welding method. For example, some magnesium-bearing alloys are less readily weldable by the torch than most other aluminum alloys, whereas they lend themselves especially well to joining by the electric spot and seam methods.

Welding means joining by fusion, regardless of the source of heat. Hence, during the act of welding, a portion of the parent metal becomes molten and, later, re-solidifies. Some compositions are rather tender or "hot short" at temperatures close to their melting points and cannot easily withstand the relatively high local contraction stresses set up during solidification and cooling. This is the case with some of the copper and magnesium-bearing alloys, whereas the silicon-bearing alloys behave differently. An alloy (43S) of 5%

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[60] The symbols in parentheses denote the trade designation by which the alloy is commonly recognized.

silicon possesses a relatively small solidification contraction, a low melting point, and a wide melting range.

Therefore, when a particular job is rather difficult to torch weld, because of the characteristics of the alloy or the complexity of the assembly, recourse is frequently had to a 5% silicon alloy (43S) for the welding wire. This is often the choice when the parts to be assembled must be held rigidly in a jig, a situation that prevents free movement of the parts to compensate for expansion upon heating and contraction upon cooling.

In any welded joint, there is a portion of metal that is in the form of a casting, unless the joint is of such a design as to permit the hammering, and thus the conversion, of the metal into its wrought state. Therefore, consideration must be given as to whether the properties of the particular alloy in the form of a casting will be satisfactory. Some compositions (99% aluminum) are as ductile and tough in the form of a casting as in their wrought state, whereas others have a relatively low elongation. Furthermore, the heat-treatable alloys, when in the form of a casting, do not respond to the heat-treating operation as quickly, nor to the same degree, as when in the wrought state. In the case of a torch weld, fusion takes place completely through the thickness of the parts being joined. On the other hand, electric spot and seam welding cause the melting of only a very small portion, and this area, in a properly made spot, does not proceed to the surface of the joint. This is illustrated in Fig. 25 and Fig. 26, which show in cross-

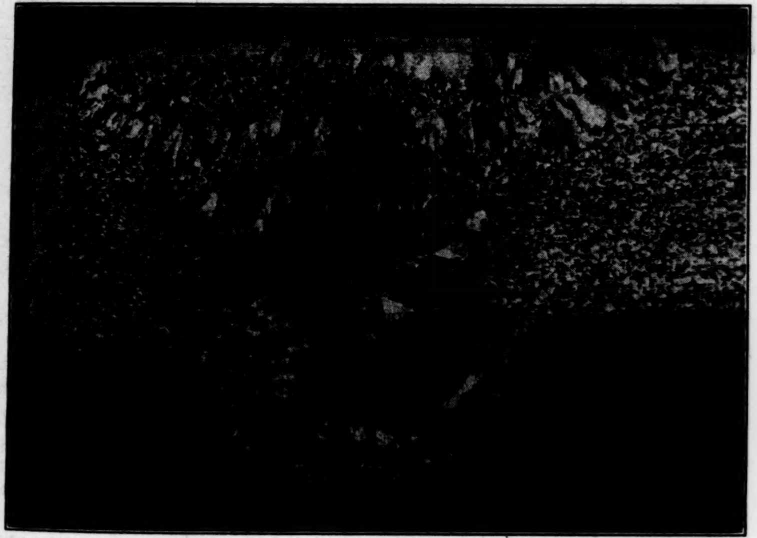

FIG. 25.—CROSS-SECTION OF BUTT TORCH WELD IN A 99% ALUMINUM SHEET. WELDING WIRE IS ALSO 99% ALUMINUM. (MAGNIFICATION, 7.5 TIMES.)

section a torch and a spot weld. The details regarding these matters have been well developed, and are available in current literature[m]. In Fig. 25, note that the metal in the welded portion has been melted completely through the thickness. In the case of the spot weld the area affected became molten during the process. In Fig. 26, note that this dark area has not proceeded to the surface of the joint.

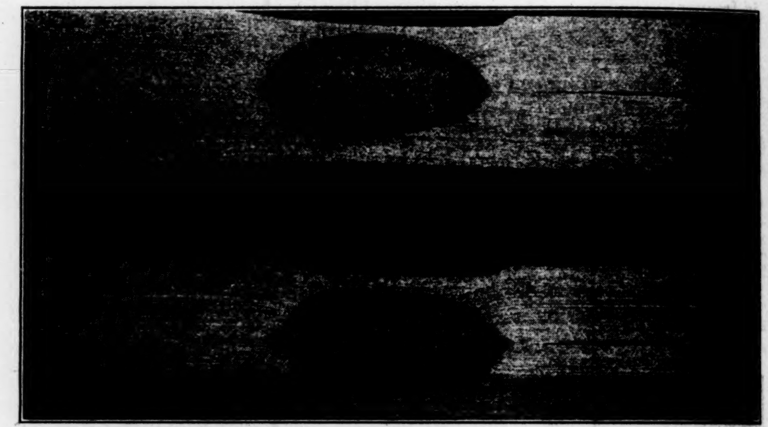

FIG. 26.—CROSS-SECTION THROUGH A LAP, ELECTRIC SPOT, WELD IN AN ALUMINUM-MANGANESE-CHROMIUM SHEET. (MAGNIFICATION, 10 TIMES.)

One of the major justifications for using aluminum in structural applications lies in its relatively low specific gravity in comparison with the other commercial metals. Although the specific gravity of the various aluminum alloys differs, this difference is so slight as seldom to constitute a factor in deciding whether to utilize one aluminum alloy rather than another. The greatest difference in this value among the present commercial aluminum alloys is about 3.5 per cent. Some of the alloys in which magnesium or silicon are the alloying agents are actually lighter than pure aluminum (see Table 6).

The coefficient of thermal expansion is also affected by the composition but here, again, the difference is so small as not to influence one's choice in favor of any particular alloy (except, perhaps, in some very special case), nor to argue against the selection of several different aluminum alloys in the same structure.

Other physical properties affected by composition are the electrical and thermal conductivities. These effects may be substantial. Typical values are shown in Columns (10) and (11), respectively, of Table 6.

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[m] "The Welding of Aluminum", by the Aluminum Co. of America; "The Aluminum Industry—Aluminum Products and Their Fabrication", by Messrs. Edwards, Frary, and Jeffries; and "Gas Welding Aluminum and Its Alloys", by G. O. Hoglund, *The Sheet Metal Worker,* February, 1936.

TABLE 6.—EFFECT OF COMPOSITION, COLD WORK, AND HEAT TREATMENT UPON SPECIFIC GRAVITY AND ELECTRICAL AND THERMAL CONDUCTIVITY

Item No. (see Table 7)	Description	Aluminum content (percentages)	ALLOYING ELEMENTS* (PERCENTAGES)					Specific gravity	Weight, in pounds per cubic inch	Electrical conductivity (percentages of international copper standard)	Thermal conductivity at 100° C (centimeter-gram-second units)
			Copper	Silicon	Manganese	Magnesium	All other alloying elements				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	Electrical conductor	99.5	....	....	....	....	....	....	....	61	....
1	Annealed	99.0†‡	....	....	....	....	....	2.71	0.098	59	0.53
	Hard rolled									57	0.51
	Annealed	98.75†	....	....	1.25	....	....	2.73	0.099	50	0.45
	Hard rolled									40	0.36
6	Annealed	95.0	4.0	....	0.5	0.5	....	2.79	0.101	45	0.40
	Heat treated									30	0.27
10	Annealed	93.85	4.5	0.8	0.8	....	0.05	2.80	0.101	50	0.45
	Heat treated									40	0.36
4	Annealed	97.25†	....	....	....	2.5	0.25	2.67	0.096	35	0.32
	Hard rolled									35	0.32
13	Annealed	97.80	....	0.7	....	1.25	0.25	2.69	0.097	45	0.40
	Heat treated									40	0.36

*Remainder, aluminum plus normal impurities. † "Common alloys". ‡ Minimum allowable.

Effect of Cold Working.—As with other common metals, cold working affects the mechanical properties of aluminum and all its alloys. All the various strength properties, stiffness, and endurance values are increased, and the plasticity, malleability, reduction in area, and elongation, are decreased. The modulus of elasticity is not affected.

For certain types of deforming operations, such as the cold rolling of sheets, in which the reduction in thickness is an accurate measure of the degree of cold working, a formula has been developed by R. L. Templin,[62] M. Am. Soc. C. E., which permits the prediction of the new tensile strength after the material has been reduced in thickness any specified amount. This formula is,

$$T = T_0 \, (1 + 0.9 \, R) \qquad (2)$$

in which T = the new tensile strength, in pounds per square inch; T_0 = the tensile strength of the properly annealed metal; and R = the reduction in thickness expressed as a decimal. Aluminum products which have undergone an 80% reduction of cross-sectional area by cold working, without intermediate annealing, are considered "hard", and the rate of strain hardening up to this point may be considered constant. Equation (2) applies to cold working up to the equivalent of $R = 0.80$. Mr. Templin[63] also shows the relationship between the tensile strength and the yield strength, and offers a formula to calculate the new elongation. As these latter formulas are rather complex, and as their use is based upon certain limitations, too lengthy to include in this paper, the reader is referred to the quoted reference. As a general rule the yield strength increases and the elongation decreases during the early stages of cold working at a much more rapid rate than the tensile strength increases. In annealed material, the yield strength is about one-third the ultimate tensile strength. With about 20% or more of cold working the yield strength is about 80 to 95% of the ultimate strength.

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[62] "The Aluminum Industry", Vol. II; "Aluminum Products and Their Fabrication", Edwards, Frary, and Jeffries, p. 406.

[63] *Loc. cit.*, p. 410.

However, most of the forming methods are complex, or deform various portions of the metal in different amounts; for example, in forming a simple right-angle bend, only that portion of the metal approximately included within the bending arc becomes worked, and that to a variable extent. The fibers along the inside surface will have been subjected to compression, whereas those on the outer surface will have been subjected to stretching, both of which types of deformation affect the metal in the general manner just indicated. No formula has yet been devised which measures the magnitude of such working. When forming a shape in a draw press, the metal is stretched in a longitudinal direction and is deformed under compression in a transverse direction with usually no appreciable change in thickness. Furthermore, as the metal slides over the radius of the die, it is first bent 90° and immediately thereafter it is re-straightened. Such a complex deformation introduces a large amount of working. No formula exists, which predicts the degree of the cold work nor, quantitatively, the values of the new set of properties. Although certain problems would be simplified if such formulas were available, their non-existence is not a particularly serious handicap.

It is the fabricator's task to fashion the metal into the shape and design as specified without fracturing it. He must depend on his own past experience and that of others in like or similar cases, and to "cut and try" methods. If a sample specimen fractures upon attempting to form a certain bend, a larger forming radius must be used, or a softer temper, and perhaps the shaped part will have to be hardened by heat treatment. The manner in which the working is performed may be important at times; for example, when forming a rivet head, it is preferable to accomplish the deformation by one squeeze or by a few sufficiently heavy blows, rather than by a great many lighter blows. Under the action of one continuous squeeze, the metal is quite uniformly worked throughout its mass. With many light blows, a much larger percentage of the energy is first absorbed by the surface layers before the centrally located portions have become deformed at all. If carried to the extreme, such surface hardening may be the cause of starting surface cracks before a satisfactory head will have been formed. The principle in this case is that cold working hardens the material and gradually depletes its capacity to withstand additional cold deformation. The rate of this effect varies among the metals. Tin and lead harden but very slowly; copper, iron, and aluminum harden much more rapidly. The alloys of aluminum (and this is generally true for the other metals) harden more rapidly than the pure metal; that is, with large total amounts of cold deformation a re-softening (annealing) might be necessary with the alloy, whereas this might not be required with the pure metal.

The engineer, on the other hand, is interested mainly in this matter of work hardness, in terms of the capacity of the finished structure to perform its intended purpose satisfactorily. This involves at least two questions: (1) Will the newly made structure possess adequate strength? and (2), will it retain that strength under the anticipated service conditions?

Because of lack of 100% accuracy of design methods, regardless of metal, influenced partly by the inability of any formula to take into consideration all the tolerances necessitated by commercial limitations, the answer to the first question is frequently determined by testing sample structures and accurately measuring the stresses induced in various members. With reference to the second question, it can be stated for aluminum and its alloys that the effects of cold work remain substantially constant unless some subsequent heating, sufficient in magnitude, should cause some change in the properties of the metal.

Cold work has little effect on the corrosion resistance of the aluminum alloys. As a matter of fact, there is some evidence that cold working may be slightly beneficial. Tests indicate that an alloy containing aluminum, copper, magnesium, and manganese (Item No. 4, Table 6) which has been heat treated and subsequently further reduced by cold working, is somewhat more resistant than the same alloy, heat treated but without the additional cold work. However, the magnitude of this difference is insufficient to give it commercial importance, other than to offer assurance as to the harmlessness of such cold work.

Cold work does not affect the weldability of any of the aluminum alloys. This would appear obvious as the heat, introduced by the welding operation, merely removes the effects of the cold working before the temperature of the metal has reached the point where welding can take place.

Cold work does not appreciably affect the specific gravity nor the thermal expansion. The electrical and thermal conductivity may be slightly reduced as indicated in Table 6.

Effect of Heating and Cooling.—When any aluminum alloy that has been merely cold worked is heated to its annealing temperature the effects of that cold work are removed immediately. This operation is called "'annealing." Details of proper annealing practice for the several alloys are well covered by existing literature and will not be repeated herein. This softening effect is of importance to the fabricator as occasionally he must have recourse to annealing in order to re-soften the metal and thus permit of further cold deformation. Any of the aluminum alloys may be cold worked, annealed, and further cold worked any number of cycles desired. If the annealing is performed correctly, the product will possess properties identical with those of the annealed temper.

It is preferable to heat the material rapidly as this is conducive to producing a fine crystal structure—a coarse structure being somewhat weaker. The more rapid the heating, the finer the metal structure. This is more critical with small amounts of cold work (equivalent to about 4% reduction in thickness) than with larger amounts.

In the annealing of the "common alloys" (that is, those not susceptible to increasing strength by a "heat-treating" operation), no particular precaution or care need be exercised against heating the metal somewhat above the required annealing temperature. Of course, heating the metal to, say, 800° F, when 650° F would have accomplished all that was desired, is unnecessary. It would be of value to have the furnace temperature substantially above the desired final metal temperature in order to re-crystallize quickly and thus obtain the finest crystal structure possible, but the load should be removed when the metal has reached the desired temperature. The feasibility of utilizing such a procedure depends on the particular furnace. Continuing to heat the metal beyond that point, and holding it there, is conducive to causing a coarser and, hence, less desirable crystal structure. Although no harm will result should the metal cool rapidly, such cooling is not generally of value. Something might be said in favor of permitting the mass to cool slowly on the grounds that the central portions of the load might not have reached the desired temperature, and such discrepancy would tend to be corrected by permitting a large mass to cool slowly.

With heat-treatable alloys, the principles are somewhat different. This can be better appreciated after covering the matter of heat treatment of aluminum alloys. No attempt will be made to describe in detail what occurs within the internal structure during and following the heat-treating operation. This also is well covered in the literature. Only a very brief résumé will be given herein.

The heat treatment of the aluminum alloys involves heating the metal to specified elevated temperatures, so as to put certain constituents in solution in amounts greater than their solubility at room temperature. Upon quenching the alloy, after such heating, it is in an unstable condition because of the excess of constituent in solution. This excess tends to precipitate, the effect being observable by an increase in strength, hardness, and reduction in general workability, but with no lowering of elongation values. In the case of certain alloys, such as the aluminum-copper-magnesium-manganese type, previously mentioned, this precipitation and self-hardening occur spontaneously at ordinary temperatures, very rapidly over the first few hours after quenching and gradually diminishing in intensity, substantially completing itself in about three or four days. The rate of this action may be retarded or completely postponed by placing the freshly quenched material in cold storage. Alloy 17S (Item No. 6, Table 7), when freshly quenched and placed in storage at 0°C, shows no precipitation after several days. Upon removing the material from cold storage and permitting it to warm again to ordinary temperatures, precipitation commences and continues in a normal manner. Advantage is taken of this delay in precipitation when it is desired to heat treat and quench a relatively large quantity of material at one time and maintain it in a soft and more workable condition for some days prior to forming the entire amount. Other compositions require a re-heating for several hours at temperatures approximating 300° F to cause this precipitation to occur to its maximum amount. The first heating and quenching cycle is termed "solution heat treatment," whereas the second type of heating is termed "precipitation heat treatment."

The foregoing indicates several principles of particular importance to the metal fabricator and the structural engineer. The highest strengths obtainable from these alloys result from heat treatment, although, of course, if the metal is subjected later to cold working the strengths may be increased still further. The fabricator frequently makes use of the fact that the metal is softer and more workable after the solution heat treatment, and before aging or precipitation has progressed to any substantial extent, by forming promptly after quenching. In certain more difficult cases, the metal may be formed in the annealed temper and the shape then heat treated.

Although each alloy has its most suitable heat treatment temperature, substantial solution of the constituent occurs below this optimum temperature and, in fact, the solution commences shortly above the annealing temperatures. Hence, when annealing the heat-treatable alloys, if fully annealed properties are desired, it is important to adhere as closely as possible to the recommended annealing temperature—namely, 650° F plus or minus 10 degrees. The rate of cooling is not important if these temperature limits have been observed. However, if these temperatures have been exceeded unintentionally, heat-treating effects may be avoided by slow cooling. As an extreme illustration, Alloy 17S (Item No. 6, Table 7), fully heat treated may be converted to the annealed condition by heating at 800° F for 2 hr and then cooling at a rate not exceeding 50° F per hr to less than 500° F. Occasionally, it is desired to produce some substantial softening, although without necessarily obtaining true annealed properties. In such an event, greater liberties may be taken with respect to temperatures and rate of cooling.

As the temperature of aluminum alloys is raised, the malleability and plasticity are markedly increased and the strength properties and resistance to deformation are decreased[64]. The metal fabricator may use this principle either to permit forming especially heavy sections, with less energy input than would be required by cold working, or to permit a severity of deformation that could not be accomplished cold without the metal fracturing. Even heating to as low as 400° F facilitates forming to a substantial extent. Occasionally, the material is heated to its proper heat-treating temperature and then, while still hot, pressed into the required shape, the cold dies producing substantial quenching effects[65].

There is not much difference in the resistance to corrosion between the several tempers of any common aluminum alloy. The word, "temper," in aluminum parlance, signifies any of the several metallurgical conditions in which wrought aluminum or its alloys is produced. For example, Item No. 2, Table 6 (containing 1.25% manganese, with the remainder of aluminum plus normal impurities) is produced in several "tempers:"

The soft temper is produced by annealing the alloy to remove the effects of cold working. The harder tempers are produced by strain hardening the alloy by varying amounts after it has been annealed. The alloy that has been cold worked the maximum amount practicable in commercial manufacturing operations, is said to be in the hard temper. By proper selection of the amount of strain hardening, the quarter-hard, half-hard, and three-quarter-hard tempers are produced, in which the tensile strengths are intermediate between those of the soft and the hard tempers in the manner indicated by the fractional designations. As these various tempers are merely the result of cold work, and as heating merely removes this cold-work effect, heating common alloys has no material effect on the resistance to corrosion. The facts are different with the heat-treatable alloys. In the case of Item No. 6, Table 6, the temper possessing the greatest resistance to corrosion is the heat-treated condition. Heating this material at temperatures much in excess of 212° F, for any substantial length of time, impairs its resistance. On the other hand, Item No. 13, Table 6, possesses an excellent and like corrosion resistance in any temper and, hence, heating any particular temper of this alloy produces no effect on its resistance. As a general rule, the heat-treatable alloys are in their most resistant state when they have been subjected to solution heat treatment. However, there are exceptions, as noted. Hence, when resistance to corrosion is of real importance, and it is intended to heat the material, the specific characteristics of the particular alloy should be ascertained and procedure governed accordingly. For example, the heat introduced by torch welding is sufficient, for the most part, to remove the heat-treatment effect, and so adversely affect the resistance to corrosion of a small area on either side of the weld in Item No. 6, Table 6 (heat-treated temper). On the other hand, in the case of Item No. 13, Table 6, no such harm results. The detailed facts are well known and are available either in present publications or from the producer of the alloy.

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[64] "Properties of Wrought Aluminum Alloys at Elevated Temperatures", by F. M. Howell and D. A. Paul, *Metals and Alloys,* October, 1935.

[65] U. S. Patent 1 751 500.

Whether or not the alloy may have been heated previously, has no effect upon its inherent weldability. However, in certain cases, a previous thermal treatment might increase the quantity of the oxide film on the surface of the metal and this, in turn, may affect the ease of welding. Upon heating Item No. 6, Table 6, to rather elevated temperatures, an oxide film is formed that interferes quite markedly with spot welding. For this reason, in order to spot weld or seam weld Item No. 6 (the heat-treated temper) the surface along the line of welding should be cleaned with emery cloth to remove this heavy oxide film. This is not necessary in the case of all compositions. This, again, is a matter of specific information which, however, is available.

Heating may affect certain other physical properties. When heating merely removes strain hardening, the effect upon electrical and thermal conductivity is just the reverse of introducing cold work, as previously mentioned. In the case of those alloys in which heating changes the degree of solution of certain constituents, the general rule is that, as the quantity of constituent put into solution increases, the electrical and thermal conductivity decreases. This is illustrated in Table 6. Heating and cooling have no appreciable effect on specific gravity.

AVAILABLE FORMS AND SPECIFIC PROPERTIES OF STRUCTURAL ALUMINUM ALLOYS

Wrought Products.—Table 7 shows the nominal composition and the mechanical properties of those wrought alloys of interest in structural fields.

TABLE 7.—NOMINAL COMPOSITION AND TYPICAL PROPERTIES OF WROUGHT-ALUMINUM ALLOYS

Item No.[a]	Alloy	Description	Aluminum content (percentages)	Alloying Elements (Percentages)								Unit Stresses, in Kips per Square Inch				Percentage elongation[h] in 2 in.	Brinell hardness number
				Copper	Silicon	Manganese	Magnesium	Chromium	Nickel	Zinc	All other alloying elements	Tensile strength	Yield strength	Shear strength	Endurance limit		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
		(a) COMMON ALLOYS															
1	2 S	Annealed Half hard[b] Hard rolled	99.0[e]	....	....	....	....	....	....	....	....	13 17 24	4 14 21	9.5 11.0 13.0	5.0 7.0 8.5	35 9 5	 32 44
2	3 S	Annealed Half hard[b] Hard rolled	98.75	....	....	1.25	....	....	....	....	....	16 21 29	5 18 25	11.0 14.0 16.0	7.0 9.0 10.0	30 8 4	28 40 55
3	4 S	Annealed Half hard[b] Hard rolled	97.75	....	....	1.25	1.0	....	....	....	....	26 35 42	10 31 38	16.0 19.0 23.0	14.0 15.0 16.0	20 5 3	45 65 80
4	52 S	Annealed Half hard[b] Hard rolled	97.25	....	....	....	2.5	0.25	....	....	....	29 37 41	14 29 36	18.0 21.0 24.0	17.0 19.0 20.5	25 10 7	45 67 85
		(b) HEAT-TREATABLE ALLOYS															
5	14 S	Heat treated[c]	93.70	4.4	0.80	0.75	0.35	....	....	....	....	68	53	45	15.0	12[i]	130
6	17 S	Annealed Heat treated	95.00	4.0	....	0.50	0.50	....	....	....	....	26 58	10 35	18 35	11.0 15.0	20 20	45 100
7	A 17 S	Heat treated[d]	97.20	2.5	....	....	0.30	....	....	....	....	43	24	25	13.5	24	70
8	24 S	Annealed Heat treated	93.80	4.2	....	0.50	1.50	....	....	....	....	26 65	10 43	18 40	14.0 14.5	20 20	42 105
9	25 S	Heat treated[c]	93.90	4.5	0.80	0.80	....	....	....	....	....	58	35	35	15.0	18[i]	100
10	27 S	Heat treated	93.90	4.5	0.80	0.80	....	....	....	....	[f]	60	50	37	13.0	9	115
11	51 S	Annealed W Heat treated	98.40	....	1.00	....	0.60	....	....	....	....	16 35 48	6 20 38	11 21 30	6.5 10.5 10.5	30 24 14	28 64 95
12	A 51 S	Heat treated[c]	98.15	....	1.00	....	0.60	0.25	....	....	....	48	38	32	10.5	14[i]	90
13	53 S	Annealed W Heat treated	97.80	....	0.70	....	1.25	0.25	....	....	....	16 33 38	7 20 32	11 22 26	7.5 10.0 11.0	25 22 14	26 65 80
14	Ni[d] A	Heat treated	97.50	0.5	....	....	0.50	....	1.0	....	0.5[g]	47	40	....	13.5	9	....
15	Ni[j] X	Heat treated[d]	95.75	1.0	....	....	0.75	....	1.0	1.0	0.5[g]	51	41	....	13.5	16	....

[a] See corresponding Item Numbers in Table 6. [b] Cold worked after its last annealing sufficient to raise the ultimate tensile strength to a point half way between the annealed and the full hard tempers. [c] Forgings only. [d] Extruded shapes. [e] Minimum allowable. [f] Traces. [g] Equal proportions of manganese, chromium, and molybdenum. [h] Except as noted these values are for $\frac{1}{16}$-in. sheets. [i] These values are for 0.505-in. round bars. [j] Known commercially as Nicral.

In some cases these compositions and heat treatments are protected by patents. Column (3), Table 7, indicates that the remaining material, other than the alloying elements, is aluminum plus normal impurities, such as iron and silicon. For the specimens listed, Young's modulus of elasticity is approximately 10 300 kips per sq in. Column (13), Table 7, contains the yield-strength stress that produces a permanent set of 0.2% of the initial gage length[66].

Referring to Column (16), Table 7, it is to be noted that the elongation will vary somewhat with the form and size of the specimen. The Brinell hardness test (Column (17), Table 7) involved a 500-kg load, exerted on a 10-mm ball. The values in Column (14) are single shear values, computed from double shear tests. The endurance limit (Column (15), Table 7) is based on a 500 000 000-cycle of complete reversed stress, using a standard form of machine and specimen[67].

There are well developed and standardized procedures for determining the mechanical properties and other characteristics of metals and there are well defined and accepted units of measurement by which these properties may be expressed[68]. On the other hand, in the case of resistance to corrosion, no such unit of measurement exists. This quality is evaluated by various empirical tests co-ordinated with actual field performance[69]. These observations merely indicate how one metal, or an alloy of a metal, behaves under certain corrosive conditions on a relative basis with another. Thus, the resistance to corrosion of a certain grade of aluminum cannot be given a rating of D, whereas another grade of another metal would be assigned a rating of E (in which the letters are supposed to possess definite numerical value). One can only state that the first is "somewhat superior" or "very much superior" or "inferior" to the other. After due experience of course, one can state, correctly, that Metal A will perform satisfactorily in a certain service, whereas Metal B will not. An attempt to list the relative resistance of the various products in tabular form would be misleading, as any correct rating should take into consideration the various forms in which the alloys are available and the service conditions applicable in any case. Although the resistance to corrosion of Item No. 9, Table 7, heat-treated, is not as great as that of Item No. 8, similarly treated, the former is utilized primarily as forgings, relatively thick in section, whereas the latter, which is not available as forgings, is normally used in thin sections, and usually under far more severely corrosive conditions. The resistance to corrosion of forgings made of Item No. 9, has proved quite adequate in the fields where they are adopted. The matter, therefore, can best be treated by discussion rather than by any table.

In general, it can be stated that aluminum alloys belong to the class which may properly be described as metals resistant to corrosion. Items Nos. 1, 2, 4, and 13, Tables 6 and 7; and Item No. 3, Table 7, in all tempers, possess superior resistance. In fact, except under the most severe service, such as the marine, they usually need not be painted for protection. Referring to Table 7, Items Nos. 5, 6, 7, 8, 10, and 14 (heat treated) and Item No. 11 (in all tempers), are next in order and substantially equal. Normally, they should be painted, although, in relatively heavy sections, such as would generally be used with forgings of Item No. 5 (heat treated), this might not be necessary. Next, would come Item No. 15, Item No. 9 (heat treated), and Items Nos. 6 and 8 (annealed). The most satisfactory way for the prospective user to take care of this subject is to consult with the manufacturer who is to supply the material.

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[66] Specification E8–33, Am. Soc. for Testing Materials.

[67] "Notes on Fatigue Tests on Rotating-Beam Testing Machines", Appendix to Rept. of Research Committee on Fatigue of Metals, *Proceedings*, A. S. T. M. Vol. 35, Pt. 1, 1935.

[68] "Standard Methods for Tension Testing of Metallic Materials", Am. Soc. for Testing Materials (E8–33).

[69] "Corrosion Resistance of Structural Aluminum", by E. H. Dix, Jr., *Proceedings*, Am. Soc. for Testing Materials, Vol. 33, Pt. II, 1933.

Aluminum alloys are available in the form of sheets, plates, wire, rods and bars, rolled and extruded shapes, tubing, rivets, and forgings, although not all the compositions listed in Table 7 are available in all these forms. Table 8 shows those forms that to-day are classed as "standard"; that is, are regularly produced in routine commercial production. Exceptions are made when warranted; for instance, although Item No. 6 (heat treated), is the only composition regularly carried in stock in the form of rolled shapes, Item No. 10 (heat treated), has been produced in rolled shapes when the quantities required, have justified it.

TABLE 8.—STANDARD COMMODITIES; WROUGHT ALLOYS

(Commodities marked with asterisk are standard).

Item No. (same as in Table 7)	Alloy	Sheets	Plates	Wires	Rod and bars	Rolled shapes	Extruded shapes	Tubing and pipe	Rivets	Forgings
1......	2 S	*	*	*	*	..	*	*	*	..
2......	3 S	*	*	*	*	..	*	*	*	..
3......	4 S	*	*	..	..	..	..	..	..	..
5......	14 S	..	..	..	..	..	..	..	..	*
6......	17 S	*	*	*	*	*	*	*	*	*
7......	A 17 S	..	..	..	..	..	..	..	*	..
8......	24 S	*	..	..	..	..	*	*	..	..
9......	25 S	..	..	..	..	..	..	..	..	*
10......	27 S	*	*	..	..	..	..	..	..	..
11......	51 S	*	*	*	*	..	*	*	..	..
12......	A 51 S	..	..	..	..	..	..	..	..	*
4......	52 S	*	*	*	*	..	..	*	..	..
13......	53 S	*	*	*	*	..	*	*	*	*
14......	Ni A	*	..	..	..	..	..	..	..	..
15......	Ni X	..	..	..	..	..	*	..	..	..

Plates are being produced as heavy as 2 000 lb. Although there are limitations as to sizes of certain alloys and tempers, they are available in widths of as much as 120 in. and lengths as determined by the 2 000-lb weight. Heat-treated rolled shapes in various sections and sizes up to 12-in. channels are produced in lengths as great as 85 ft. Heat-treated extruded shapes are available in lengths of about 50 ft.

Item No. 6, Table 8, (17S) is available in all the various forms, and because it presents an unusually favorable combination of properties, it has ranked as the most generally useful of the strong alloys.

The particular merits of Item No. 10 (heat treated) (27S-T) are its relatively high yield strength and the fact that, as this temper is produced by aging for some hours at approximately 300° F, neither the mechanical properties nor the corrosion resistance is impaired should the material be re-heated to this temperature, or even to slightly higher temperatures. This behavior is of significance because it permits the use of hot-driven steel rivets in joining heat-treated members composed of this alloy, without fear that the heat, thus introduced, will adversely affect the properties of the aluminum parts. The most noteworthy applications of Item No. 10 (heat treated) to date, comprise the rehabilitation of the floor of the Smithfield Street Bridge[10], Pittsburgh, Pa.; the floor of the Stratford Avenue Bridge, Bridgeport, Conn.; the side walk and railing of the Covington-Cincinnati Suspension Bridge[11], across the Ohio River; and the railings on the Laredo Bridge, in Texas. A thorough study has been made, contemplating the use of aluminum for the trusses and floor of the famous Brooklyn (N. Y.) Suspension Bridge[12].

The corrosion resistance of Item No. 13, Table 8, (53S) which is of a high order, is substantially the same in all tempers. It is for this reason that this alloy has found wide use in the construction of sewage disposal plants, in the form of extruded sections in the architectural industry for the fabrication of windows and frames, doors, building fronts, and various ornamental constructions. Increasingly, such items have been treated by a patented process, which applies a hard, weather-resistant, and corrosion-resistant coating and one that facilitates cleaning.

Cast Products.—Table 9 gives the composition of castings that are of widest interest to the structural engineer and shows their typical mechanical properties.

As will be noted, the heat-treated, aluminum-copper (Item No. 23, Table 9) and aluminum magnesium (Item No. 24), alloys are the strongest. Item No. 23, Table 9, has been used rather extensively since about 1920, whereas the alloy (Item No. 24) is a more recent development and still requires special handling in the foundry. The latter represents the highest combination of strength and shock resistance yet attained in aluminum castings and is especially desirable for large sections. It has given good service performance as parts of large shovel dippers. Fig. 27 shows the back of a 7-cu yd dipper, 6.83 ft long, 4.90 ft wide, and 3.0 ft deep, weighing 2 326 lb. Fig. 28 shows one hinge for a 32-cu yd dipper, 12.58 ft long, 5.25 ft wide, and 1.50 ft deep, weighing 2 242 lb. These castings are of Alloy 220 (Item No. 24, Table 9).

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[10] "Aluminum Enters Bridge Construction", by Charles M. Reppert, M. Am. Soc. C. E., *Engineering News-Record,* November 9, 1933; "Erecting an Aluminum Bridge Floor", by Henry D. Johnson, Jr., Assoc. M. Am. Soc. C. E., *Engineering News-Record,* November 23, 1933; and "Fabricating Structural Aluminum", by C. G. Schade, M. Am. Soc. C. E., *Engineering News-Record,* December 28, 1933.

[11] "The Use of Structural Aluminum in Bridges", by J. P. Growdon, M. Am. Soc. C. E., *Bulletin,* Associated State Eng. Societies, April, 1935.

[12] "Aluminum Trusses and Floor for Brooklyn Bridge", *Engineering News-Record,* April 18, 1935.

TABLE 9.—COMPOSITION AND MECHANICAL PROPERTIES OF ALUMINUM CASTING ALLOYS*

Item No.	Alloy (1)	Remarks (2)	Aluminum content † (percentage) (3)	Alloying Elements (Percentages)					Unit Stresses, in Kips per Square Inch				Percentage elongation in 2 in. (13)	Brinell hardness number (14)
				Copper (4)	Iron (5)	Silicon (6)	Magnesium (7)	Zinc (8)	Tensile strength (9)	Yield strength (10)	Shear strength (11)	Endurance limit (12)		
(a) Common Alloys														
16	43	As cast........	95.0	....	....	5.0	.....	....	19	9	15.0	6.5	4.0	40
17	47	As cast........	87.5	....	....	12.5	.....	....	26	11	18.0	6.0	8.0	50
18	112	As cast........	89.8	7.5	1.2	....	.....	1.5	22	14	20.0	7.5	2.0	65
19	212	As cast........	89.8	8.0	1.0	1.2	.....	....	22	14	20.0	8.5	2.0	70
20	214	As cast........	96.25	....	....	....	3.75	....	25	12	19.0	5.5	9.0	50
21	216	As cast........	94.00	....	....	....	6.00	....	27	16	23.5	.....	6.0	60
(b) Heat-Treated Alloys														
22	195	Heat treated...	96.0	4.0	....	....	.....	....	31	16	28.0	6.0	8.0	65
23	195‡	Heat treated...	96.0	4.0	....	....	.....	....	36	22	30.0	6.5	4.0	80
24	220	Heat treated...	90.00	....	....	....	10.00	....	44	26	33.5	7.5	13.0	75
25	355	Heat treated...	93.25	1.25	....	5.0	0.50	....	30	20	30.0	.....	4.0	60
26	355‡	Heat treated...	93.25	1.25	....	5.0	0.50	....	35	27	30.0	.....	2.0	80
27	356	Heat treatment	92.70	....	....	7.0	0.30	....	28	16	22.0	.....	6.0	55
28	356‡	Heat treatment	92.70	....	....	7.0	0.30	....	32	22	22.0	8.0	4.0	70

* In the case of some of these alloys, the composition or heat treatment, or both composition and heat treatment are patented.

† Plus normal impurities, such as iron and silicon.

‡ Different type of heat treatment.

This should be considered a special alloy for unusually severe service. The alloy (Item No. 20) is used where service conditions require the maximum resistance to corrosive attack. It is more difficult to cast into intricate, leak-proof castings than the aluminum-silicon alloys, but has higher mechanical properties than

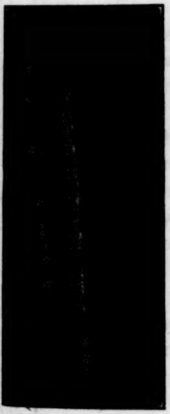

FIG. 27.—ONE HINGE FOR 32-CUBIC YARD DIPPER.

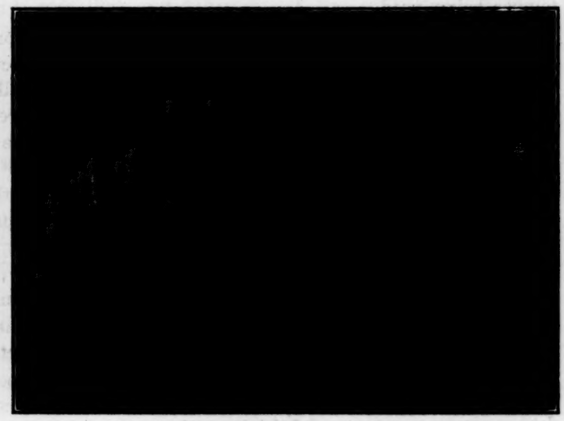

FIG. 28.—BACK FOR 7-CUBIC YARD DIPPER.

the alloy listed as Item No. 16, in Table 9, and is distinctly more resistant to corrosion. For this reason, it is utilized in the production of castings for use in sewage disposal plants, and for marine applications. The alloy (Item No. 21) is in a somewhat similar category, with equal resistance to corrosion, higher tensile and yield strengths, but with somewhat lower elongation.

The founding of aluminum alloys differs in many respects from that pertaining to other metals; in fact, each metal possesses its own foundry characteristics. It is important, therefore, when considering the detail design of any particular casting, and especially the pattern, to consult with the foundryman in order to permit of certain small modifications in design that might enable simplification of foundry problems, with consequent reduction in cost, that will insure the highest and greatest uniformity of properties.

The size and complexity of castings that can be produced varies so much with the particular instance that no general specifications can be given.

METALLURGICAL ASPECTS OF FABRICATING STRUCTURAL ALUMINUM ALLOYS

Aluminum alloys are almost always cut by mechanical means, such as shearing, sawing, or machining. Torch cutting is seldom recommended because of the high heat conductivity which makes this method impractical. Furthermore, the heat introduced would have adverse effects in certain instances, as previously discussed.

Aluminum alloys are formed by all the common methods, although they are usually done cold. They are occasionally formed hot for heavy sections and for forging, although consideration should be given as to whether the degree of heating has adversely affected strength or resistance to corrosion, and whether the shaped part should be re-heat treated.

Joints are fabricated by torch, electric spot, seam, arc, and butt welding, and by riveting and bolting. The choice of method depends upon the results desired and as influenced by the effect of heat upon the particular alloy or temper being used.

When using rivets larger than ⅜ in. in diameter, the present usual practice is to use hot steel rivets. Due to the high heat conductivity of aluminum, the heat thus introduced is rapidly dissipated with small chance of harmful results, especially when the rivets are driven in a scattered manner. In sizes smaller than ⅜ in., rivets are driven cold and, in the case of Alloy 17S (Item No. 6, Table 7), promptly after quenching before age hardening has progressed to any extent. Use is also made of the fact that this aging is arrested by placing the freshly quenched rivets in cold storage as previously mentioned. This process avoids the necessity of co-ordinating the heat treating and driving operations.

Torch welding requires the use of a flux, which should be removed later by a thorough washing to prevent a chemical attack of the metal.

Because of the high thermal and electrical conductivity of aluminum, the ordinary electric welder is not suitable. Suitable spot and seam-welding equipment has been developed, which can be relied upon, consistently, to reproduce spots, or a seam, of high quality. Although the quantity of heat introduced by the electric method is substantially less than that arising in torch welding, the effect of such heat should be considered. Arc welding is also being developed rapidly and is beginning to find commercial application. In this case, also, the amount of heating is substantially less than with torch welding, and harmful or annoying effects upon resistance to corrosion, or distortion, are greatly minimized. In general, torch welds of common alloys should be designed on the basis of the annealed temper. Although the heating does not fully anneal the heat-treated materials, there will be some softening and some over-aging in a small area on either side of the weld, thus reducing the strength and usually the resistance to corrosion. This effect can be corrected partly by a re-heat treatment. Usually, therefore, the strong alloys are not torch welded. The heating effects by the electric method are sufficiently less than by other methods so that it is coming into increasing use, especially in lighter work.

PART II.—MAGNESIUM

HISTORICAL

Although magnesium was isolated as an element as early as 1830, production of the metal on a commercial scale was not attempted until 1900 so that it can be truly called a product of the Twentieth Century. Historically, it was first produced about 1808 as a mercury amalgam by Sir Humphrey Davy who reduced the heated oxide with potassium vapor. In 1830, Bussy, the French chemist, obtained a fairly pure product by reducing magnesium chloride with potassium. In 1856, Deville and Caron used sodium for the reduction of the chloride followed by distillation in hydrogen, but the resulting metal was impure because of the presence of sodium and air reaction products. In 1863, Sonstadt developed a process in England based on the reduction of the anhydrous chloride by sodium, and a company was organized for small quantity production.

Commercial success was slow and the industry was of little importance until the electrolytic process was developed in Germany in the early part of the century. The development of fabricated products, such as sheets, castings, extrusions, and rods was undertaken and added considerably to the uses for the metal. The metal and its alloys were made and marketed under the name, Elektron, and, to-day, in England and on the Continent, this trade-mark is synonymous for magnesium alloys. Interest in the production of magnesium in the United States came in 1915 when the World War prevented importation from abroad.

GENERAL METALLURGICAL PRINCIPLES OF STRUCTURAL MAGNESIUM ALLOYS

Effect of Composition.—At present, most of the magnesium base alloys used in the United States, for structural purposes, are either ternary magnesium-aluminum-manganese or quaternary magnesium-aluminum-zinc-manganese alloys. The principal exception is the binary magnesium-manganese alloy which is used, preferably in the wrought condition, in applications where resistance to corrosion and ease of forming and welding are the chief requirements.

All the magnesium alloys now used for engineering purposes contain manganese. The binary magnesium-manganese alloys, usually containing 1.2 to 2.0% manganese, possess the greatest resistance to atmospheric and salt-water corrosion of any of the known magnesium base alloys, and manganese is added to the alloys containing aluminum, or aluminum and zinc, to enhance their resistance to corrosion. The manganese content of these alloys is rather definitely limited by the quantity of aluminum present and the possible variations are without influence on the more commonly determined physical properties. There is some evidence, however, that the addition of manganese to a wrought binary magnesium-aluminum alloy causes a marked increase in the proportional limit.

At present, aluminum is the most important element added to magnesium to produce structurally useful alloys. Alloys in commercial use in the United States contain aluminum in quantities varying from 3 to 13 per cent. Additions of aluminum in this range cause a marked increase in mechanical properties in both the cast and the wrought conditions.

Increasing the aluminum content of magnesium alloys causes changes in density, thermal conductivity, and electrical resistivity; but, in quantities up to 12% at least, the coefficient of thermal expansion is not greatly affected. The mean coefficient of expansion, α, for magnesium alloys from 0°C to any temperature, t, may be obtained from the following equation[illegible]:

$$\alpha = (25.07 + 0.00936\ t)\ 10^{-6} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The changes in density, thermal conductivity, electrical resistivity, tensile properties, and hardness brought about by the addition of increasing amounts of aluminum to magnesium are shown in Figs. 29 to 32, inclusive. In the sand-cast alloys, still other combinations of properties than those shown in the graphs may be obtained by aging heat-treated alloys containing more than about 8% aluminum.

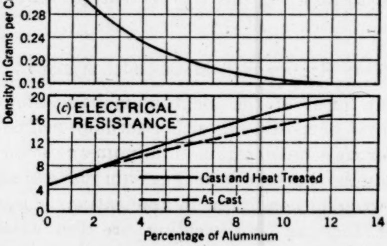

FIG. 29.

The shear strength and the compressive strength of magnesium-aluminum alloys tend to increase with the aluminum content in both the cast and the wrought conditions although this relation is more regular in the case of the wrought alloys. The shear strength of extruded pure magnesium is 16 kips per sq in. Additions of aluminum increase this value to a maximum of about 26 kips per sq in. in alloys containing 12% aluminum.

Additions of aluminum and zinc (either one or both) increase the endurance limit of magnesium alloys at least up to the point where the combined content of alloying elements amounts to 10 or 11%; beyond this amount, reliable data on endurance limit are lacking. Numerical data as to the endurance limits of magnesium alloys are given in Table 10.

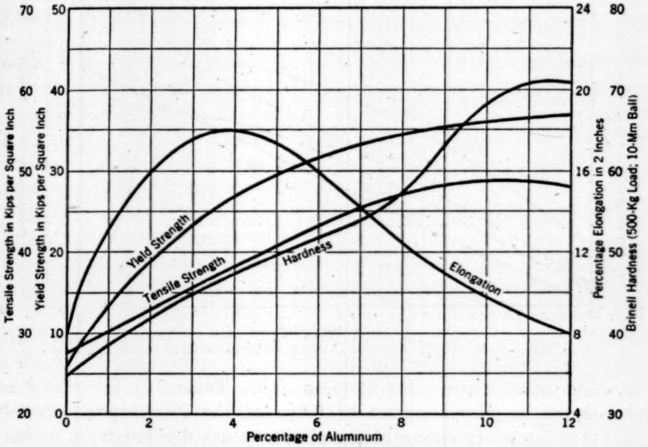

FIG. 30.—EXTRUDED ALLOYS.

The form in which these alloys are usually fabricated is as follows: Extrusions, Items Nos. 29, 30, 31, 32, and 33; sheets, Items Nos. 29 and 30; die castings, Item No. 38; sand and permanent mold castings, Items Nos. 34,

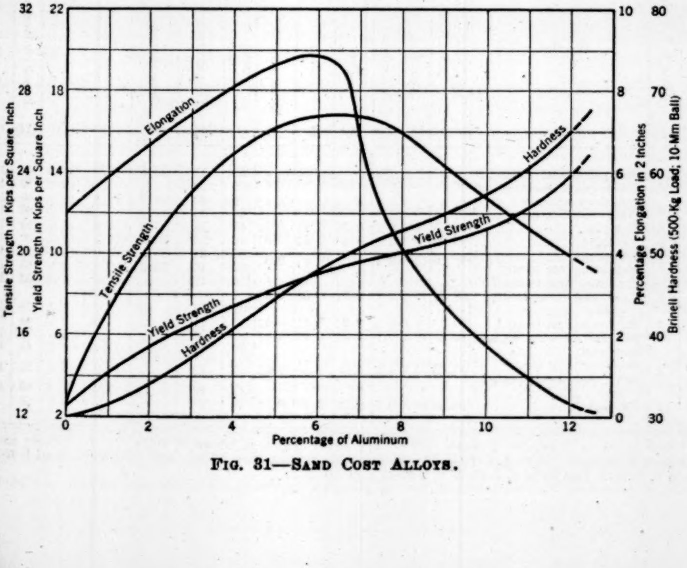

FIG. 31—SAND COST ALLOYS.

35, and 36; and, sand and semi-permanent mold castings, Item No. 37. Column (8), Table 10, gives the total allowable impurities, of which copper and nickel may not exceed the amount indicated in the starred footnote.

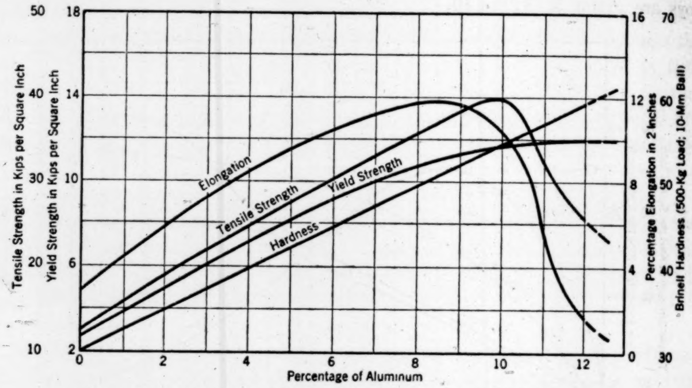

FIG. 32.—SAND CAST ALLOYS AFTER SOLUTION HEAT TREATMENT.

The yield point stress (see Column (10), Table 10) is defined as that which produces a permanent set of 0.2% of the initial gage length[66]. In Column (11), the shear strengths are single values derived from double shear

TABLE 10.—NOMINAL COMPOSITION AND TYPICAL MECHANICAL PROPERTIES OF MAGNESIUM ALLOYS

Item No.	Alloy, A. S. T. M.§ (1)	Description (2)	Magnesium content (percentages) (3)	Alloying Elements (Percentages): Aluminum (4)	Manganese (5)	Zinc (6)	Silicon (7)	Other elements* (8)	Unit Stresses, in Kips per Square Inch: Tensile strength (9)	Yield strength (10)	Shear strength (11)	Endurance limit (12)	Percentage elongation in 2 in. (13)	Brinell hardness number (14)	Specific gravity (15)
29	11	Soft sheet, heat treated †	97.9	....	1.5	.....	0.3‡	0.3‡	32	15	17.5	8.0	16.0	40	1.76
		Hard rolled sheet†							36	29	....	....	10.0	52	1.76
30	6	Soft sheet, annealed†	94.6	4.0	0.3	0.3‡	0.5‡	0.3‡	36	22	18.5	10.0	18.0	50	1.77
		Hard rolled sheet†							44	35	....	....	10.0	60	1.77
31	8	Extruded..................	91.75	6.5	0.2	0.75	0.5‡	0.3‡	43	33	21.0	15.0	16.0	55	1.80
32	9	Extruded..................	90.00	8.5	0.2	0.5	0.5‡	0.3‡	46	35	22.0	16.0	13.0	60	1.81
33	2	Extruded..................	89.10	10.0	0.1	.....	0.5‡	0.3‡	50	38	22.5	16.0	10.0	70	1.81
		Extruded, heat treated †.							56	40	25.5	16.0	3.5	85	1.81
34	2	Sand cast, heat treated† .	88.80	10.0	0.1	0.3‡	0.5‡	0.3‡	35	12	....	11.0	9.0	52	1.81
		Sand cast, heat treated, aged................							36	19	....	8.0	1.0	69	1.81
35	1	Sand cast, heat treated....	90.7	8.0	0.2	0.2‡	0.5‡	0.3‡	33	11	....	7.5	10.0	48	1.80
36	3	Sand cast, heat treated, aged	86.8	12.0	0.1	0.3‡	0.5‡	0.3‡	32	20	....	....	0.5	85	1.82
37	4	Sand cast..............	.....	.....	.....	.....	.....	.....	27	11	....	11.0	6.0	48	1.84
		Sand cast, heat treated...	90.00	6.0	0.2	3.0	0.5‡	0.3‡	35	12		11.0	10.0	50	1.84
		Sand cast, heat treated, aged................	.....	.....	.....	.....	.....	.....	37	19	....	9.0	4.0	69	1.84
38	12	Die cast...............	88.85	10.0	0.1	0.3‡	0.75	0.3‡	28	22	....	....	0.5	63	.....

* Such as copper (maximum allowable, 0.05%) and nickel (maximum allowable, 0.03%). † Wrought material. ‡Maximum allowable. §See A. S. T. M. Standards on Magnesium Base Alloys, 1936. Specifications B 92-36 T, B 93-36 T, B 80-36 T, B 94-34 T, B 90-36 T, B 91-36 T, B 107-36 T.

tests. The Brinell hardness number was based upon a 500-kg load on a 10-mm ball; and the endurance limits (Column (12), Table 10) were based on 500 000 000 cycles of stress. Composition has little influence on the modulus of elasticity. An average value suitable for design purposes is 6 500 kips per sq in.

The effect of aluminum concentration on the resistance to corrosion of magnesium-aluminum-manganese alloys depends to a considerable extent upon the microstructure. If the alloys are homogeneous that is, if all the aluminum is in solid solution, the corrosion resistance will decrease with increasing aluminum content, but if the alloys are heterogeneous, with considerable undissolved Mg_3Al_2 compound in the grain boundaries, the corrosion resistance will increase with aluminum content.

Torch welding of the magnesium-aluminum-manganese alloys becomes more difficult as the percentage of aluminum is increased. Alloys containing 4 to 5% of aluminum are welded without difficulty, but as the aluminum concentration is increased to more than 5%, hot shortness and cracking become gradually more serious. Torch welding of alloys containing more than about 8% aluminum is seldom recommended.

The alloys of magnesium used for structural purposes in the United States at present do not contain more than 3.5% zinc, and alloys with as much zinc as this are used only for casting. Zinc tends to make the alloys hot-short and for this reason the wrought alloys usually contain not more than 1% of this element. Like aluminum, zinc forms solid solutions with magnesium, and variations in zinc content are capable of producing changes in the mechanical and physical properties of the alloys.

The addition of zinc to binary alloys of magnesium and aluminum causes them to become more difficult to hot work and makes them less weldable. On the other hand, the addition of zinc usually improves the corrosion resistance and is slightly more effective than the equivalent percentage of aluminum in raising the tensile strength, yield strength, and hardness.

In the case of the magnesium-aluminum-zinc-manganese alloy used in the form of castings and containing a nominal 6% aluminum and 3% zinc, the zinc enters solid solution only after heat treatment and the changes in mechanical properties brought about by the zinc in this alloy are greater than can be obtained by adding an equivalent percentage of aluminum. In the sand cast condition, the 6% aluminum, 3% zinc alloy is stronger, more ductile, and tougher, and has a higher endurance limit than an alloy containing 9% aluminum. In the homogeneous heat-treated condition, the difference in mechanical properties is not marked, but the resistance to corrosion of the zinc-bearing alloy is very much superior to the 9% aluminum alloy. When these two alloys are artificially aged to enhance the yield strength and hardness, the beneficial influence of the zinc is again evidenced by greater strength, toughness, ductility, yield strength, and endurance limit combined with better resistance to corrosion than can be produced in the 9% aluminum alloy.

Although it is possible to weld alloys containing appreciable quantities of zinc, it becomes more difficult with increasing quantities. When the zinc content of wrought material is greater than 1%, or when the aluminum + zinc content is 9% or more, hot-shortness and cracking become so serious that welding is not usually recommended.

Effect of Cold Working.—Magnesium and all magnesium alloys work harden very rapidly and, in general, the rapidity of work hardening increases with the quantity of alloying material present. Pure magnesium and the binary magnesium-manganese alloy sheets may be cold rolled about 25% before failure occurs, but magnesium-aluminum-manganese alloy sheets containing 4% aluminum harden more rapidly and are seldom cold rolled more than 15 per cent. Magnesium alloys are so refractory toward cold deformation that practically all wrought forms are produced by hot working and, for the same reason, practically all forming and shaping operations on magnesium alloy sheets, plates, tubing, or structural shapes are performed at elevated temperatures in the range from 225° to 350° C (437° to 662° F).

In common with other metals, magnesium alloys with sufficient plasticity to withstand cold working may be strengthened and hardened by this process. The effect on the mechanical properties may be seen by referring to Table 10, which gives the properties of sheets in the soft and hard tempers. It has also been found that cold working extruded alloys slightly (1%) by stretching has a beneficial effect on their properties. The stretching operation increases the tensile and yield strengths without reducing the ductility.

Cold-worked sheets usually have a slightly lower resistance to corrosion than annealed sheets of the same composition.

Effect of Heating and Cooling.—A comprehensive discussion of the effect of heating and cooling on magnesium alloys is impossible in the limited space of this paper. The effect of heating varies with the composition of the alloy, its previous history, the temperature to which it is heated, and the length of time it is held at high temperature.

When cold-worked magnesium alloys are heated, they become re-crystallized and consequently soft, and they lose tensile strength, yield strength, and hardness. This type of heating is termed annealing and a common method of annealing cold-worked magnesium is to bring the mass of metal to 650° F and allow it to cool in air. Sudden cooling of annealed material does no harm, but would serve no useful purpose. Hot-worked magnesium alloys, such as extrusions and forgings, usually have the best mechanical properties in the "as worked" condition and annealing tends to reduce the tensile and yield strengths without increasing the ductility. Wrought material is usually formed at temperatures that will anneal the metal and, for this reason, the heating to forming temperature should be as rapid as possible and the time at temperature should be curtailed. Annealing of magnesium-alloy castings is without benefit and, in many instances, will result in lower properties.

Magnesium alloys are amenable to heat treatments of the same type as those used for aluminum alloys as described in Part I; that is, a supersaturated solid solution of the alloying element or elements is effected by heating to a relatively high temperature followed by rapid cooling. This produces a structure that is substantially homogeneous, with certain characteristic mechanical properties associated with such a structure. If the maximum obtainable yield strength and hardness are desired, the solution heat treatment is followed by heating the quenched (homogeneous) material for various times at lower temperatures, usually 12 to 20 hr at 300° to 350° F. This second heating or "artificial aging" is necessary because magnesium alloys do not age-harden at room temperature to any appreciable extent. The heat-treating temperature necessary to make them homogeneous depends upon the alloying constituent. Zinc may be put into solid solution at about 630° F, whereas 750° to 810° F is frequently used to effect the solution of the aluminum constituent.

As mentioned previously, wrought magnesium alloys generally possess their best mechanical properties in the as-worked condition and, for this reason, wrought alloys are seldom heat treated. In the case of magnesium alloy castings, however, heat treatment produces such a marked improvement in properties that the great majority of the castings supplied to the trade in the United States are in the heat-treated condition. The effect of heat treatment on the mechanical properties of cast magnesium alloys is shown in Table 10.

Heating and cooling have no effect on the welding characteristics of magnesium alloys nor on their specific gravity except that heat-treated and quenched alloys high in alloy content tend to "grow" very slightly if used at elevated temperatures. In common with aluminum alloys in which heating changes the degree of solid solution, the general rule is that, as the quantity of constituent put into solid solution increases, the electrical and thermal conductivity decreases.

The effect of temperatures encountered under service conditions must be taken into account when using magnesium alloys for engineering purposes. Although sub-normal temperatures as low as — 112° F have little or no effect on the mechanical properties, they undergo appreciable change at temperatures of approximately 300° F and more. Considerable data as to the effect of elevated temperatures on the properties of magnesium alloys are available and the one who is to supply the material should be consulted before magnesium is utilized in structures operating at elevated temperatures.

AVAILABLE FORMS AND SPECIFIC PROPERTIES OF STRUCTURAL MAGNESIUM ALLOYS

Wrought magnesium alloys are available as extrusions, sheets, plates, and forgings. Bars, rods, and structural shapes are produced by the extrusion process because of the readiness with which magnesium alloys may be worked in this manner. Rolled shapes and sections in high-strength magnesium alloys are not available at present. Because of the equipment and ingot sizes available, present extruded sections are limited in weight to about 300 lb and in size to sections corresponding to a standard 6-in. channel or I-beam. Rough press forgings are limited in weight to about 200 lb, whereas die forgings produced by hammering are limited to the forming capacity of an 18 000-lb hammer. Under present conditions obtaining in commercial fabricating plants, plates or sheets are limited in weight to about 80 lb, with a maximum width of 60 in. and a maximum length of 240 in.

Cast magnesium alloys are available as sand castings, permanent mould castings, and die castings. There are alloys readily adaptable for fabriction by all three methods, and the mechanical properties obtained compare favorably with aluminum alloy castings used for similar purposes. Particular attention is called to the high endurance limit of magnesium alloy castings as compared with most aluminum-base alloy castings. It should be mentioned, however, that sharp corners, notches, and other "stress raisers" seriously reduce the endurance limit of magnesium alloys, and this fact must be kept in mind when using such alloys in stressed structures.

Magnesium die castings and permanent mould castings are limited in size only by the die or mould equipment available. There is no inherent reason why magnesium castings of this type cannot be produced in the same sizes as die and permanent mould castings in other metals. Magnesium sand castings require a more specialized technique and oxidation inhibitors must be used in the moulding sand. In the present state of the art, sand castings with sections thicker than about 6 in., or which require more than about 1 500 lb of metal to pour would be difficult to produce. The composition of magnesium alloys is shown in Table 10.

METALLURGICAL ASPECTS OF FABRICATION OF STRUCTURAL MAGNESIUM ALLOYS

Magnesium alloys may be fabricated into structural units or assemblies by machining, cutting, forming, welding, or riveting. The precautions to be observed when machining or cutting consist mainly in keeping the tool sharp and free from chips to avoid overheating and consequent firing of the chips. Cutting must always be done mechanically as the high heat conductivity of the metal and the tendency of molten magnesium to burn precludes the successful use of torch-cutting. If it is necessary to use a liquid coolant (magnesium alloys are usually machined without lubricant or coolant), one should be selected that will not tarnish the finished work. Mineral oils are usually satisfactory, but rancid animal oils or acidic soluble oils should be avoided.

Severe cold-forming operations should be avoided as they are likely to cause invisible cracks which, later, will cause failure. Hot forming must be performed at temperatures below the eutectic temperature for all alloys containing aluminum and zinc (either or both). The binary magnesium-manganese alloy may be heated to 450° C (842° F), if necessary, but usually 260° to 350° C (500° to 662° F) is hot enough. The wrought magnesium-aluminum-manganese or magnesium-aluminum-zinc-manganese alloys may be readily formed in the range, 260° to 350° C (500° to 662° F), and the alteration in mechanical properties caused by this heating is not important unless the heating is prolonged.

Magnesium alloys not too high in alloy content may be joined by torch welding and the various types of electric resistance welding known as spot, seam, and butt welding. At or near the weld, the structure of wrought materials reverts to that of a casting with consequent lowering of properties. This condition can be greatly improved by hammering the finished weld at temperatures in the range, 290° to 370° C (550° to 700° F), and with certain types of mechanically operated electric butt welders, a cast structure at or near the weld is avoided for the most part. In order to prevent corrosion, welding flux must be thoroughly removed from torch welds, and seam or spot welds should be freed from copper particles picked up from electrode tips.

Because of the danger of electrolytic corrosion (magnesium is electro-positive to all common structural metals), rivets for magnesium alloy structures must be selected with care and, where possible, should be insulated with bitumastic paint. Magnesium alloy rivets cannot be used successfully because they would have a low shear strength and because even small sizes would have to be driven hot. Rivets of steel, copper, nickel, and copper-bearing aluminum alloys should be avoided. For joints not subject to high stresses, rivets of commercially pure aluminum (Item No. 1, Table 6) may be used, but for highly stressed parts, rivets of an aluminum base alloy containing 4 to 5% magnesium are recommended. As a further precaution against electrolytic attack, the contacting surfaces of riveted magnesium alloy sheets or extruded shapes should be painted before assembly. A synthetic resin varnish pigmented with $ZnCrO_4$ is recommended for this purpose.

When it is necessary to rivet magnesium alloy parts to other metals, electrolytic insulation must be provided. To prevent the possibility of cracks or notches that later will produce fatigue failures, rivet holes should be drilled instead of punched, the rivet size carefully selected to avoid overstressing the rivet hole during driving, and marring of the magnesium alloy part with riveting hammer should be avoided.

CONCLUSION

The past fifteen years have witnessed the recognition of aluminum as an engineering material in an ever-widening field of applications. This is due primarily to the relatively light weight of aluminum compared to that of other common metals. However, advantage could not be taken of this characteristic until alloys of high strength were developed. Especially in recent years, marked progress has been made in this direction. This has been particularly true in those fields, such as construction and transportation, in which the cost of moving the structure is a major item of operating expense. Although aluminum is a relatively new engineering material, the fund of information that has been developed regarding its properties and behavior, and the great variety of forms and sizes in which it has become available commercially, places it on an equal rank, in these respects, with the far older metals.

This paper indicates broadly the type and scope of the facts and materials now available.

Part II of this paper entitled, "Magnesium", shows that although that metal has not yet progressed as far as aluminum, it is following a somewhat similar path. As is the case with aluminum, so, also, the outstanding important characteristic of magnesium is its relative lightness. Marked improvement has been made in the properties of its alloys. Methods of fabrication have been developed so that, to-day, magnesium alloys are available in a great variety of cast and wrought products of known characteristics.

ACKNOWLEDGMENT

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CORROSION IN RELATION TO ENGINEERING STRUCTURES

BY JAMES ASTON[73], ESQ.

SYNOPSIS

The corrosion problem may be considered from two angles, each encompassing quite an extensive field, and worthy in itself of the space normally allotted to a technical paper. The one phase, dealing with the fundamental mechanism of corrosion, is necessarily theoretical, and of secondary interest to the practising engineer. The other phase, pertaining to the relative utility of metals in engineering structures, is undoubtedly of major interest to a gathering of civil engineers. This latter subdivision is quite complex, involving factors of experience, of economics, of conjecture, of controversy, and of personal opinion.

It is the intention of the writer to take the middle ground, in the belief that some knowledge of the fundamental mechanism of corrosion is a valuable background for a more adequate balancing of the many variables which are presented by the multiplicity of service conditions, and to interlink this discussion of the fundamentals with an appraisal of materials and protective measures which have a bearing upon prolonging the life of structures by combating the ravages of corrosion.

The seriousness of corrosion loss may only be guessed at on the dollar basis. One finds estimates running into hundreds of millions of dollars and even into the billions, as the annual toll. The resulting tax is enormous, and the tangible depreciation or the cost of abandoning structures is exceeded in a monetary sense by the more intangible effects of designing structures heavier than the requirements of working stresses, substituting more costly metals for others that would meet design conditions on a more economical basis, modifying design and fabrication, and applying paints and other protective coatings—all occasioned by the allowances necessary to offset or to minimize the effects of corrosion.

MECHANISM OF CORROSION

The electro-chemical theory is most generally accepted in explaining the mechanism of corrosion. Aside from the metal involved, the essential factors are moisture and oxygen. Water serves as the medium of electro-chemical reaction; it must be in contact with the metal, and its activity will be accentuated or decreased by salts or gases in solution, temperature, etc., somewhat in proportion to their effect in increasing or decreasing the electrical conductivity of the liquid. Oxygen, dissolved in the water, plays a dual rôle: (1) It serves to oxidize the products of reaction to the familiar form (in the case of iron) of rust; and (2) it is a depolarizer, neutralizing the effect of hydrogen formed as a product of the customary reactions, thus enabling a continuity of electro-chemical activity which otherwise would be suppressed.

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In the past, for iron in particular, undue emphasis was placed upon impurity or heterogeneity as an all-important factor in the inception and progression of corrosion. More recently, however, extraneous factors affecting the surface characteristics of the metal, and influencing the contact relationships with it of moisture and oxygen, have been recognized as playing an important rôle, at times, even to the extent that they overshadow certain of the internal characteristics of the metal. Rust may now be claimed to play a part in the progression of corrosion fully comparable to the other factors which formed the early basis of the electro-chemical theory. This later conception has served to amplify the theory, and to explain certain occurrences more rationally, notably pitting, which, at one time, were only vaguely understood.

All corrosion phenomena and effects revolve around the three essential factors: (*a*) The metal or metals concerned; (*b*) the moisture; and (*c*) the oxygen. The absence of any one of the factors will generally suppress the activity in its entirety, except in some special cases where a minor rate of progression may be observed. The rate, and more particularly the type, of corrosion resulting, will be markedly influenced by the relative balance in Factors (*a*), (*b*), and (*c*). Consequently, environment plays an important rôle in the problem of corrosion as related to structures.

TYPES OF SERVICE

Corrosion has most commonly been divided into three fields: Atmospheric, immersion, and soil. The metal may be considered common to all classes, the differentiation being primarily linked with the relationships between moisture and oxygen in the three types. Under atmospheric conditions there is a superabundance of oxygen. Moisture is the governing factor, with respect to its quantity, kind, and intermittent occurrence. Heavy precipitation naturally tends to increase corrosion. Contaminating gases of industrial atmospheres or the salt sprays of seaside conditions are aggravating influences. Of major significance is the interval between precipitations, and the opportunity thus afforded for the corrosion products to dry. Conditions favoring precipitation followed by relatively rapid drying will tend to spread the rust, rather than to concentrate the corrosion in pronounced pitting. Atmospheric corrosion is characterized by small, shallow pitting quite closely and uniformly distributed over the affected area. The result is a fairly uniform attack of the surface.

Immersion conditions represent an abundance of moisture and a varying, often deficient, supply of oxygen. The latter is a governing factor. True immersion conditions may be defined as those in which the metal surface is wetted continually with little or no drying of the corrosion products. The nature of the water may aggravate or retard the corrosion. Most salts in solution in moderate quantities accelerate the effect; but under certain conditions there may be film precipitations on the surface of the metal which are markedly beneficial. Temperature is an important factor, with the result that, other conditions being equal, the most aggravated effects are experienced under the conditions prevailing in hot-water supply systems, in boiler-feed waters, and in steam-condensate returns. It is important to bear in mind that the degree of corrosion is not in proportion to the purity of the water. Relatively pure, oxygenated waters are more to be reckoned with than contaminated waters which are foul with organic matter. The blanket statement may be made that immersion service presents the most variable and complex conditions in the entire corrosion problem. Its results are manifested as a rule in scattered pitting of a relatively large and deep character, with much of the surface showing little or no corrosion. The result will depend in a secondary manner upon the water characteristics and oxygen supply, with the primary influence quite largely dependent upon how continuously the metal surface was kept wet.

Soil corrosion may be considered as occupying a position intermediate between atmospheric and immersion service. It offers a great variety of conditions, such as those in which leachy salts affect the composition of the water, and those in which moisture is retained or drained from the metal surface. Inherently, the soil may be considered as a sponge, acting in a secondary capacity in relation to the nature, amount, and frequency of the moisture supply and its accompanying oxygen. The degree, and particularly the type, of corrosion will vary with the foregoing combination of influences. The corrosion may be similar to the uniform type characteristic of atmospheric conditions; it often shows the aggravated pitting typical of immersion service; and, not infrequently, it will display peculiarities which are akin to those present in the case of acid waters.

Factors Involved in Alleviation

With the foregoing exposition of fundamentals and mechanism as a background, it will be valuable to consider what may be done toward remedial measures which will be effective against the deterioration of engineering structures. Obviously, attention may be focused upon the metals involved, or upon the environment—that is, upon the prevailing moisture and atmospheric conditions.

With respect to environment, artificial measures are limited to special cases. In underground structures, better soil may be substituted in the contact zone and, particularly, benefit will result from proper attention to drainage and the elimination of moisture retentivity in this contact zone. Where liquid conveyance is a condition, as in pipe lines, benefit may result from an adjustment of water characteristics. A notable example is that of Coolgardie, Australia, where internal corrosion was reduced markedly in a large pipe line by reducing the dissolved oxygen content of the water by evacuation.

In numerous instances, moderate applications of lime have proved effective. On the other hand, certain water treatments, which have for their objective the extreme softening of water for domestic and similar uses, have been harmful from the corrosion viewpoint because they removed from the water the natural film protection which results from the raw water. Hardening treatment for extremely soft waters, elimination of bicarbonate content, neutralization of acidity, and the introduction of film-forming constituents, such as sodium silicate, have improved the water characteristics in the limited fields where such applications are practicable. De-aeration may be, and is being, resorted to in boiler-feed waters, in hot-water supplies, and in hot-water heating systems. In general, the efforts are confined to above normal temperature conditions, under which the corrosion is most aggravated and which enable the remedy to be applied most effectively.

Unfortunately, in dealing with structures of direct concern to the Structural Division of the Society, artificial alteration of the environment is limited to a narrow range of special cases. From a practical standpoint, one must cope with the natural surroundings as they exist, and must make allowances in design for such deterioration as may be indicated by experience for the particular environment prevailing. In some cases it may be logical to minimize deterioration by selecting suitable material and by protective metal surfaces. In general the battle with corrosion must be fought through the structure itself, rather than through its environment.

A most important phase of the problem is the selection of the metal. From the standpoint of the designing engineer, this must be primarily because of its fitness as a load-sustaining material, and, in most instances, corrosion, although it cannot be ignored, becomes a secondary feature.

In discussing the fitness of metals from the viewpoint of corrosion, one enters the most difficult field. Situations vary, metals vary, and there is no blanket rule by which any metal may be claimed to be best adapted to all conditions. This phase of the discussion is difficult, also, because it assumes a severely controversial and competitive aspect, in which personal experience and bias, and commercial associations will necessarily be provocative of argument. It will be the purpose of the writer to present a summary of opinion and of the status of various metals that are of interest to the structural engineer.

THE METALS OF INTEREST

For purposes of discussion the metals of interest may be grouped into the non-ferrous and ferrous divisions; with a further subdivision of the latter into the iron group, ordinary steels, and alloy steels.

In the non-ferrous group, this discussion is limited to copper and its chief alloys, brass and bronze, and to aluminum and its alloys. It may be rightly contended that copper is selected for structural purposes because of its noncorrodibility or because it is ornamental. In no case is its choice an acknowledgment of superior, or even equal, physical properties of strength, ductility, or unit weight obtained at a competitive cost figure in comparison with structural steel.

For the aluminum group of metals, the same general statements may be made, except for the important modification that the desired physical properties may be obtained with lessened weight of structure, and this may dictate the choice of material. As a specific example, the use of aluminum floorbeams in the Smithfield Street Bridge, in Pittsburgh, Pa., may be justified from the engineering viewpoint as prolonging the useful life of an existing superstructure by lessening the dead load and thereby coping with the necessity of taking care of the heavier live load of existing traffic. In a somewhat similar manner, the higher cost of aluminum window frames and other parts of the Empire State Building, in New York, N. Y., may be justified or even offset, because of a lessened load on a costly supporting structure.

For the usual corrosion conditions prevailing in the realm of structural engineering, the metals of the foregoing non-ferrous group may be claimed to have a high order of immunity. Special conditions or locations may be cited as exceptions, without defeating the generalization. In all cases, their use in structures is of a limited, specialized nature. Occasionally, their corrosion value is a dominant factor in their selection, but more often this desirable characteristic may be taken as an added feature in a choice made because of special considerations related to design.

Except for very special conditions and applications along the lines noted, the structural engineer will confine his selection of a metal to the ferrous group. Economic considerations, necessitating individual analysis and judgment, will determine the choice among the several types of iron or steel comprising the group as a whole. The presentation which follows is an attempt at a general appraisal of their features of merit or demerit.

It is logical that ordinary steel should be considered first since it is, and probably always will be, the dominant metal for structures. Furthermore, it serves as a well understood standard against which the alternative metals may be properly appraised.

STRUCTURAL STEEL

The "Age of Steel" began with the inception of the Bessemer process in 1855, and the open-hearth method of Siemens shortly thereafter. Since then tremendous progress has been made in a technical as well as in a manufacturing sense. One may claim that the customary type of structural steel represents the "bed-rock" standard in its all-around features of cost and physical properties, to meet the general need. It is an adaptable material in the hands of the metallurgist, who finds that, through adjustment of chemical composition, aided by heat treatment where desired, a wide range of controlled properties is obtainable. The customary metalloids—carbon, silicon, sulfur, phosphorus, and manganese—may be added or held to any practicable limits, even to a virtual approach to zero. Limits are set by the cost of the operation in comparison with the advantages accruing.

For the foregoing five elements, and in the range usually encountered in ordinary steels, one may summarize the following facts concerning corrosion.

Carbon is added, as a rule, for the purpose of conferring the desired tensile, elastic, and ductile properties. Although there is an apparent gain, from the corrosion viewpoint, as the carbon content is progressively lowered, the overwhelming advantage in fixing the percentage in accordance with the dictates of a standard fixed design relegates the corrosion factor into the background. Within the usual range, the influence of carbon upon the corrosion of the structure may be considered as relatively unimportant.

Silicon and manganese are the steel-makers' correctives, introduced to neutralize otherwise deleterious effects upon the physical properties. Although each may influence corrosion adversely as the quantity in the steel increases, the effect is no doubt small. In much of the literature and theory relating to corrosion, manganese has been given unwarranted prominence as a harmful agent. The structural engineer may rightly disregard these elements in the quantities customary in the steels that are of commercial interest, as serious factors in the corrosion of the structure.

Available evidence indicates that phosphorus has a retarding influence on corrosion, varying with conditions of service and probably being most pronounced in the case of atmospheric exposure. However, the well-known effect of phosphorus in promoting brittleness in steel will continue to dictate the low-phosphorus content of structural steel specifications. Design, and not corrosion, considerations will rule.

Sulfur is invariably recognized as being harmful from the corrosion viewpoint. It should be kept as low as possible with due recognition of the factors prevailing in any particular situation and of the added cost where abnormally low limits are demanded.

As a generalization, it may be stated that composition, within the specification limits set for plain structural steels, has little bearing upon its life from the corrosion angle. This generalization is subject to adverse modification, however, if steel is made so carelessly as to be characterized by segregation, non-uniformity, dirtiness, etc.

Cast and Wrought Iron

Before the introduction of steel, the iron group—namely, cast iron and wrought iron—formed the dominant materials available for structural uses. Cast iron has had an enviable record for corrosion service, especially for atmospheric and soil conditions. It is interesting to note that this is in spite of the fact that it is chemically the least pure and homogeneous of the ferrous metals. It is not reconcilable with a very widespread assumption that purity represents the goal in corrosion immunity; and it does lend support to the barrier principle of protection. In the case of cast iron, as corrosion proceeds, an increasing surface barrier of graphite flakes, aided perhaps by a tightly adherent layer of the corrosion products, serves to slow down, and to spread, the rusting effects. However, the brittleness and low tensile strength of cast iron, and its lack of desired fabricating qualities, will hold it to a very restricted and specialized place in modern engineering of structures.

One cannot impeach the integrity and experiences of those who believe that steel has failed to measure up, in corrosion service, to the standards set by wrought iron in the long history of its use. The Pillar of Delhi is a conspicuous example. England and Continental Europe contain many examples of a later date, whereas wrought iron dating from the Colonial period of United States history, such as the Bowling Green fence in New York, N. Y., furnishes factual evidence to support the contentions of wrought-iron adherents. Furthermore, one cannot dismiss lightly the contemporary records within the relatively brief period which defines the Age of Steel.

Wrought iron has a place in contemporary engineering; that is, in specialized applications where corrosion and shock or fatigue stresses are encountered. Limitations in manufacture have been handicaps, resulting in restriction of markets to a few specialized lines, such as pipe and bar iron; and cost problems have narrowed the zone of utilization. The manufacturing handicaps for wrought iron have been overcome, and one should expect an expansion in its use, both in quantity and in diversification of application. In the structural field, aside from possible fatigue considerations, wrought iron offers no advantage over steel, from a stress or fabrication viewpoint. It must find its place where experience indicates that it will give an economic return in the necessarily specialized fields of utilization. Superior resistance to corrosion will be the basis of selection.

Commercially pure iron, or ingot iron, is a contemporary metal of importance. It is interesting to note that its development is due to an acceptance of the principle that wrought iron had rust-resisting qualities superior to steel and to a belief that high purity of base metal was the fundamental feature contributing to this superiority. This subject is the center of much commercial controversy. The steel interests contend that nothing has been gained over what may be expected with customary grades of mild open-hearth steel. The wrought-iron advocate will not admit that open-hearth iron has proved an adequate substitute for his products; and quite rightly contends that it is not wrought iron. Although the two have in common a high base metal purity, the ingot iron lacks the physical incorporation of slag filaments which are characteristic of wrought iron.

ALLOY STEEL

In considering the alloy steels, one is in the fairyland of ferrous metallurgy. The field is relatively unexplored and seemingly unlimited in extent. Actually, however, in the light of present knowledge and requirements, the useful territory is somewhat restricted. In comparison with customary steels, the alloy additions must result in improved strength characteristics, in comparable manufacturing and fabrication qualities, and must have a reasonable cost. To-day, the field seems to be covered by additions of chromium, copper, manganese, molybdenum, nickel, silicon, and vanadium—together with carbon —singly, or in various combinations. Covering the low-alloy steels Mr. J. C. Whetzel[74] gives the general range of elements, as follows:

Carbon	0.10 to 0.40	Copper	0.01 to 1.40
Manganese	0.20 to 1.70	Nickel	0 to 3.5
Phosphorus	0.01 to 0.20	Chromium	0 to 12.0
Sulfur	0.05 maximum	Molybdenum	0 to 0.40
Silicon	0 to 1.0	Vanadium	0 to 0.20

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[74] *Proceedings*, Am. Iron and Steel Inst., May, 1935.

From the corrosion viewpoint, it appears that the carbon-silicon-manganese group has no particular merit and that the results of promise are in the combinations containing copper, nickel, and chromium.

Nickel steel (3.5% nickel) has been a standard, high-strength structural steel for many years. It has had a reliable performance record with high-strength characteristics, without the necessity of special heat treatment and radical deviations in manufacture and fabrication. The cost of the nickel addition has been the factor that has held this material to specialized applications in structures. The nickel content of the steel no doubt confers added corrosion resistance, but scarcely in proportion to the additional cost. Consequently, the use of nickel steel would not appear to be justified from the corrosion standpoint alone.

One of the most successful developments has been copper-bearing steel which has been successful because a small quantity of copper (0.30%) confers material benefit at low cost. Comprehensive tests by the American Society for Testing Materials indicate a substantial corrosion resistance under atmospheric conditions, particularly in industrial centers. A similar merit, however, cannot be claimed for immersion conditions. The added cost of this copper-bearing steel must justify its selection solely because of its corrosion characteristics; no benefit may be claimed because of its other physical properties in comparison with standard structural steels.

The results with higher copper additions (0.50% to 1.5%) are rather doubtful as far as structural applications are concerned. There is higher strength, and probably greater corrosion resistance, but the relationships of copper and iron in these higher ranges may lead to difficulties in manufacture and to unreliability in fabrication and in after service. It is not improbable that copper will play an important rôle in subsequent developments by recourse to other auxiliary alloying elements which will serve to shift the normal relationships of copper and iron, or to mask the otherwise detrimental effects. It may be that molybdenum has an influence of this character.

Low-Alloy Structural Steel

In the past few years there has been a marked impetus in the development and exploitation of new steels in the low-alloy group. Numbers of these alloys are on the market under commercial trade designations. The general types are well summarized by Whetzel[74] who gives the chemical composition, tensile and impact properties, and other characteristics. With a demand for lighter structures, the steel industry has endeavored to meet the competition of low-density metals by offering higher strength characteristics and consequent lessened section and weight. Quite naturally the greatest impetus has come through the transition which one observes in the transportation field. Promoters of the new low-alloy steels have the objectives of stepping into the territory so ably filled in the past by the use of nickel steel, but at a reduced cost. Manganese, silicon, and copper are the agencies presenting the greatest possibilities, because of the moderate required amounts and the low base cost of the alloy metals. Alone, or in various combinations, and often in association with nickel, chromium, molybdenum, and vanadium, one finds, to-day, the general list confronting the structural engineer.

As a matter of fact, the manganese and silicon structural steels have been used for a considerable period. From the corrosion viewpoint, they may be likened to ordinary steel and their selection should be made only because of benefit from a design basis.

In appraising the low-alloy steels from the standpoint of corrosion service, one quite commonly overlooked feature is worthy of emphasis. The basis of selection is almost invariably higher unit strength and consequent reduction of weight. In most structural shapes, this means little change in the over-all surface exposed to the elements and a reduction of section thickness in proportion to the increased strength of the metal. Consequently, if the strength is doubled, and the section thickness is reduced one-half, with a unit corrosion-merit value of twice that of ordinary steel, the life of the structure, as far as comparative corrosion is concerned, would be the same. In effect, therefore, in the selection of structural materials, the benefit to be derived, as far as it relates to corrosion, will be only to the extent that the corrosion-merit ratio exceeds the strength ratio for the metals under consideration.

In general, a proportionately high order of corrosion resistance in the low-alloy group does not appear to be attained unless the quantities of chromium or nickel, singly or in combination, are relatively high. In such a case the engineer is faced with a high-cost spread and the possibility of manufacturing and fabricating peculiarities. The writer believes that selection of metals in the low-alloy group would not be justified in engineering structures, as a major substitution for plain steels, if all other physical factors were the same, and corrosion was the only service consideration. On the contrary, where design characteristics of the several types dominate the selection, the equivalent corrosion characteristics may be considered as an additional premium obtained with certain of the materials; or the selection may be made by proper appraisal of all factors—strength, corrosion, and cost—peculiar to the competing metals. Obviously, with this criterion as the logical basis of choice, each structure becomes an entity unto itself—a problem for individual solution.

As illustrative of some considerations involved in any generalization regarding the low-alloy steels, one may cite an interesting alloy combination noted by Whetzel[74]. A chrome-copper-silicon-phosphorus addition gives to this steel the properties of the medium high-tensile group. It is claimed that the detrimental effects to be expected from abnormally high phosphorus (0.10% to 0.20%) are masked by the complex alloying relationships, and cold shortness is not induced in the metal. Graphs are given by Whetzel showing comparative corrosion in industrial atmosphere for two years, and the citation is made that copper-bearing steel has a resistance of two to three times that of ordinary steel, whereas for the chromium-copper-silicon-phosphorus combination the ratio is four to six times. If these comparisons are borne out by long-time service experiences, one would have a rational basis for the evaluation of the alloy for structural needs, with adequate balancing of all factors—strength, corrosion, and cost. Such evaluation at present would be unwise, since ultimate life cannot safely be predicted upon short-term, progress data. Furthermore, in the corrosion field, results vary markedly with conditions of service.

The greatest promise in the structural field is in adding alloys of low cost, in quantity and kind, which will give a positive corrosion merit warranting selection in competition with the usual grade of structural steel even if there should be little or no gain in the other physical properties.

One might almost claim that corrosion, for the conditions confronting the structural engineer, has been conquered by alloys of the 13.0% to 16.0% chromium, or the 18.0% chromium with 8.0% nickel classes—the high-alloy group. These steels have the ideal quality of forming thin self-healing surface films in usual environments which, in the last analysis, is the goal that the corrosion specialist dreams of achieving. However, aside from questions of fabrication and peculiarities of properties under certain conditions, stainless steels have very limited, specialized applications in structures because of a relatively high unit cost.

Surface Protection

Corrosion is a surface reaction, so that surface considerations or applications form a logical method of attacking the problem. Furthermore, in view of the fact that a ferrous metal of moderate cost is, and will remain, the dominant one for structures, and that rusting will occur in some degree, paint or equivalent surface applications are required in most installations, if only for æsthetic reasons. An admirable paper on the surface protection of steel has been presented by Mr. F. N. Speller[75]

Of the metallic coatings, interest centers upon hot-dip applications of zinc or tin. Of the two, galvanizing only need be given serious attention because of the high cost of tin. Although galvanizing forms a good protection for most exposure conditions, the cost is relatively high. Painting is usually a necessary accompaniment; and application of zinc is impracticable on the sizes and shapes used in large structures. In certain cases, such, for example, as bridge parts to which access for painting after erection is difficult, a zinc coating applied by hot-dipping, or by the metal-spray method, may be advantageous. Every one is familiar with the extensive use of galvanizing in building sheathing and in electrical transmission towers. In general, however, zinc and metallic coatings, as a whole, are of minor interest to the structural engineer.

As a protection against corrosion, the non-metallic coatings have merit only because they isolate the metal surface from contact with its environment —moisture in particular. The necessary qualifications are firm adherence to the underlying metal, a continued imperviousness, and resistance to disintegration during exposure. The metal surface is an important feature since dampness, grease, or rust affect adherence. Furthermore, the evidence is that iron or steel which corrodes to a dense, adherent rust is of decided advantage in the effectiveness of a paint coating, in contrast with results where the steel corrodes to a flocculent, loose type of rust. Paints containing pigments of the red lead, or of the chromate, types appear to have an inhibitive action on the corrosion of the steel base. To this extent they are of advantage in the contact zone; that is, in the priming coat.

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[75] *Transactions*, A. S. M. E., June, 1935.

Paints comprise the most important material available for the surface protection of structures. Coatings of greater thickness, such as bituminous or Portland cement mixtures, are more commonly applied in underground protection. In such cases, inaccessibility after erection demands a coating of reasonable permanence, since repeated renewal, as in painting, is practically out of the question. The fundamental features of protection are similar to those noted previously. In addition, the bituminous coatings should possess sufficient plasticity to yield under slight movement, and to resist cracking, and yet be of proper consistency to hold up under temperature influences and to resist deformation under earth pressures. For pipe lines, as an example, this has been satisfactorily accomplished by reinforcement with wrappings of burlap or asbestos embedded in the bitumen.

THE FUTURE

The metallurgist has advanced materially in his knowledge of the mechanism of corrosion. Likewise he has made progress in the improvement and development of materials for utilization by the engineer against its ravages. For this latter, more practical, accomplishment one may credit the eternal rivalry and struggle for supremacy among the several competing groups involved in the manufacture of materials for structural uses. One may look into the future with confidence of further progress which will be measured by: (*a*) Betterment of existing materials; (*b*) possible development of new types; and (*c*) reduction of cost spreads which will re-orient the economic basis of selection. After all, corrosion is an economic problem to an even greater degree than it is a technical one. There is no specific formula by which metals may be evaluated; results under one set of conditions are not necessarily transferable to another environment. Unfortunately, there is yet, no quick time test for appraising comparable to the tensile test which makes the diagnosis of physical properties reasonably satisfactory. Experience and good judgment remain as the best guide in selecting materials for corrosion service.

On the side of the structural engineer, the advances in knowledge and in materials open up a broader field for selection and economic placement. There are numerous places where special materials, even at higher cost, are justified from the viewpoint of corrosion alone. In bridges, there are inaccessable parts, such as connections and floor members; and, in buildings, harmful conditions may be encountered in foundations and where there is exposure to the atmosphere. Submerged structures (piers, etc.) present a serious problem, whereas the atmospheric conditions associated with marine and many industrial structures are conducive to accelerated deterioration. For these special cases it is good engineering to weigh all factors carefully and to select such metals or protective measures as experience and good judgment may indicate. However, in the broad field of structural engineering, and in spite of prospective development in the higher strength alloy steels, plain carbon steel will continue as the dominant material. The corrosion problem will remain as a serious handicap. The best defense will be "keep on painting."

STRUCTURAL APPLICATIONS OF STEEL AND LIGHT-WEIGHT ALLOYS

A SYMPOSIUM

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III.—STRUCTURAL APPLICATIONS OF SPECIAL STEELS:

Actual Applications of Special Structural Steels.
By V. D. Beard, M. Am. Soc. C. E. 1308

Evolution of High-Strength Steels Used in Structural Engineering.
By Leon S. Moisseiff, M. Am. Soc. C. E. 1333

ACTUAL APPLICATIONS OF SPECIAL STRUCTURAL STEELS

By V. D. Beard[76]. M. Am. Soc. C. E.

Synopsis

Within the meaning of the title selected for this paper it seems advisable first to preface the discussion by mentioning the steel adopted as the standard of reference, and then to give a list of materials to illustrate what are properly designated "Special Structural Steels":

(*a*) The standard selected as most suitable for the purpose is Carbon Steel A. S. T. M. Specifications A7 and A9. Comparisons of the special steels will be referred to this standard.

(*b*) Special structural steels, which draw their distinction from their physical properties, or their resistance to corrosion, or in some cases from both, afford a considerable variety. Representative members of this group are the following: High-carbon heat-treated steels; carbon steels containing copper; silicon steel; manganese steel; nickel steel; manganese vanadium steel; Cor-Ten; Cromansil; chromium steels; the stainless steels; and carbon steel, cold-drawn, for cables and wire rope.[77]

Bridges — Long Spans and Monumental Structures

Typical examples of structures in which special steel was used, are as follows.

San Francisco-Oakland Bay Bridge.—This great structure symbolizes an epoch in American bridge building. Approximately 4.5 miles of the 8.5-mile crossing consists of various types of bridge construction in which 200 000 tons of steel were used. The individual units, when viewed apart from the entire project, are notable bridges in their own right, and merit rather detailed description because of the extensive use of special structural steels. These

TABLE 11.—Physical Characteristics of Special Structural Steels

Description (1)	Unit Stresses, in Kips per Square Inch		
	Ultimate strength (2)	Yield point (3)	Working stress (4)
Medium carbon steel	62	37	28
Silicon steel	80	45	28
Nickel steel	85	55	34
Heat-treated eye-bars	80	50	34
Cable wire	220	150	82

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[76] Designing Engr., Am. Bridge Co., Pittsburgh, Pa.

[77] Progress Rept., Sub-Committee No. 2, Committee on Steel of the Structural Division, Am. Soc. C. E., on Structural Alloy and Heat-Treated Steels, *Proceedings*, Am. Soc. C. E., March, 1936, p. 361.

steels are classified as shown in Table 11 with regard to ultimate strength, yield point, and unit stresses.

The West Bay Crossing, from San Francisco, Calif., to Yerba Buena Island, is composed of two, nearly duplicate, double-deck, parallel wire cable, suspension bridges (see Table 12). The following tabulation will give an idea of the various types of steel used and their relative weights:

TABLE 12.—BASIC DATA, LONG-SPAN BRIDGES

Description	George Washington Bridge	San Francisco-Oakland Bay Bridge		Golden Gate Bridge	Ambassador Bridge	Delaware River Bridge	Bayonne Bridge
		West Bay Crossing	East Bay Crossing				
Type of structure	Suspension	Suspension	Cantilever	Suspension	Suspension	Suspension	Steel arch
Length of main span, in feet	3 500	2 310‡	1 400	4 200	1 850	1 750	1 675
Length of side spans, in feet	......	1 155	500	1 125	......	752	....
Weight of structural steel, in tons	......	84 700§	59 000	......	......	......	17 160
Cables:							
Diameter, in inches	36*	28‡	......	36.25‡	......	30	....
Number of galvanized wires	26 474	17 464	......	27 572	......	......	....
Diameter of each wire, in inches	...... 0.196	...... 0.195	....	0.192	....	...... 0.196	....
Maximum stress, in kips per square inch	82	82	......	82	......	72	....
Temper	Cold drawn	Cold drawn	......	Cold drawn	Cold drawn	Cold drawn	....
Ultimate strength, in kips per square inch	220†	......	......	220	215	220	....
Yield point stress, in kips per square inch	......	......	......	160	144	144	....

* Four each. † Average must be 225 kips per sq. in. ‡ Two each. § Total weight of two bridges.

Distribution	West Bay Crossing, San Francisco-Oakland Bay Bridge (Tons)	Distribution	West Bay Crossing, San Francisco-Oakland Bay Bridge (Tons)
Silicon steel in towers, cable bents, and center anchorage	18 500	Phosphor bronze	25
Heat-treated eye-bars and pins	350	Cable wire	18 700
Nickel steel	280	Cable castings	830
Silicon steel on suspended structure	24 000	2.25-in. suspender ropes	113
Manganese steel rivets	300		

The percentage of this weight that was of special steel may be noted by reference to Table 13(*a*). The East Bay Crossing, from Yerba Buena Island to Oakland, Calif., is composed of four 280-ft deck spans, one cantilever bridge with a 1 400-ft main span, and two 500-ft side spans, five 504-ft through spans on bents, fourteen 280-ft deck spans on steel bents and masonry piers, and ten 82-ft plate-girder spans. A comparison of this 1 400-ft cantilever with the Quebec and Firth of Forth Bridges is shown in Table 14. The special steels in this structure, their distribution in the various parts of the structure, and the percentage of the total tonnage, are given in Table 13(*b*).

TABLE 13.—DISTRIBUTION OF SPECIAL TYPES OF STEEL IN THE SAN FRANCISCO-OAKLAND BAY BRIDGE (WEIGHT, IN TONS)

Type	(a) West Bay Crossing					(b) East Bay Crossing					
	Towers 2, 3, 5, and 6	Pier No. 4	Stiffening trusses	Floor-beams and stringers	Total	Cantilever span	Towers and bents	500-foot span	Lattice truss spans	Girder spans	Total
Silicon	17 500	500	15 000	6 950	39 950	7 400	3 525	6 500	10 250	350	28 025
Nickel	......	280	......	.....	280	3 400	.....	.....	......	.....	3 400
Manganese rivets	......	.....	300	.....	300	25	.....	.....	......	.....	25
Phosphor bronze	......	.....	24	.....	24	.....	.....	.....	......	.....	......
Heat-treated steel	......	.....	......	.....	......	3 050	.....	1 200	......	.....	4 250
Percentage of total weight	79	62	82	52	......	77	64	62	52	43	64

Tables 15(*a*) and 15(*b*) show the chemical and physical properties of the special steels: For nickel (Table 15(*a*)) as averaged from the first ten melts; and for silicon (Table 15(*b*)), as averaged from thirty melts, ten from each of three mills. There are 370 tons of low-nickel chrome heat-treated pins in the cantilever structure which vary in size from 11½ in. to 24 in. in diameter and which have the chemical and physical properties shown in Table 15(*c*). The manganese rivets were furnished to the specifications shown in Table 15(*d*).

TABLE 14.—COMPARISON OF MATERIAL USED IN THESE CANTILEVER BRIDGES

Bridge	Span length, in feet	Total weight, in tons	Material Used (Percentages)			
			Nickel	Silicon	Heat-treated bars	Carbon
San Francisco–Oakland Bay	1 400	20 900	16	40	10	34
Firth of Forth	1 700	30 000	....	....	....	100*
Quebec	1 800	62 900	27	....	....	73

* Medium carbon steel.

Golden Gate Bridge.—The Golden Gate Bridge is the longest single suspension bridge in the world (see Table 12). It is a single-deck structure and is 90 ft wide from center to center of stiffening trusses, providing for a 60-ft roadway and two 11-ft sidewalks. The 700-ft towers are of the fixed-base, flexible type. The live load on the stringers was four 24-ton trucks and two 50-ton street cars abreast, with 50% impact for stringers and 25% for floor-beams.

Silicon steel was used for the top 200 ft of the towers and for the cross-strut below the roadway. The allowable unit stresses were 24 kips per sq in. on silicon steel and 18 kips per sq in. on carbon steel, except as follows (units are kips per square inch):

Carbon steel in stiffening trusses	21
Silicon steel in stiffening trusses	32
Silicon steel in lateral bracing	32
Silicon steel in flanges of floor-beams	22
Carbon steel in webs of floor-beams	16

There were 20 000 tons of silicon steel used on this bridge, and 2 500 tons of heat-treated eye-bars were used in the anchorages.

TABLE 15.—CHEMICAL COMPOSITION AND PHYSICAL CHARACTERISTICS OF SPECIAL STRUCTURAL STEELS

Item No.	Remarks	Chemical Composition (Percentages)								Unit Stresses, in Kips per Square Inch		Percentage elongation in 2 in.	Percentage reduction in area
	(1)	Carbon (2)	Manganese (3)	Phosphorus (4)	Sulfur (5)	Nickel (6)	Silicon (7)	Copper (8)	Chromium (9)	Yield point (10)	Ultimate strength (11)	(12)	(13)
	(a) NICKEL STEEL IN THE SAN FRANCISCO - OAKLAND BAY BRIDGE												
1	Average	0.29	0.62	0.015	0.023	3.40	0.21	0.28	.....	65.5	97.4	18.6	38.6
2	Minimum	0.25	0.50	....	.....	3.25	0.16	0.26	.....	59.5	90.6	16.2	30.5
3	Maximum	0.35	0.72	.....	.....	3.69	0.25	0.35	.....	72.6	104.6	21.2	47.3
4	Specified	0.40[p]	....	0.04[p]	0.05[p]	3.00[q]	.....	0.20[q]	.....	55.0[q]	90.0[q]	12[q]	30
	(b) SILICON STEEL IN THE SAN FRANCISCO - OAKLAND BAY BRIDGE												
5	Average	0.31	0.82	0.02	0.03	....	0.273	0.265	.....	53.9	89.9	22.8	46
6	Minimum	0.26	0.68	.....	.....	....	0.21	0.20	.....	46.4	81.0	19.	34.8
7	Maximum	0.35	0.95	.....	.....	....	0.34	0.33	.....	63.9	92.5	26.2	55.8
8	Specified	0.40[p]	1.00[p]	0.04[p]	0.05[p]	....	0.45[r]	0.20[q]	.....	45.0[q]	95.0[s]	14	35
	(c) LOW NICKEL, CHROMIUM, HEAT-TREATED PINS: EAST BAY CROSSING, SAN FRANCISCO - OAKLAND BAY BRIDGE												
9	Average, twelve heats	0.33	0.64	0.026	0.025	1.36	0.20	....	0.65	64.2	97.3	21	53.7
	(d) MANGANESE RIVETS IN THE SAN FRANCISCO - OAKLAND BAY BRIDGE												
10	Specified	0.30[p]	1.35[p]	0.04[p]	0.05[p]	....	0.25[p]	0.20[q]	.....	42	90[t]	....	45
	(e) SPECIFICATIONS FOR SPECIAL MANGANESE STEEL, BAYONNE (N. J.) BRIDGE												
11	Structural steel	0.40[p]	1.80[p]	.....	.....	....	0.30[u]	....	.....	55	90	....	....
12	Rivets	0.35[p]	1.80[p]	.....	.....	....	0.30[u]	....	.....	47	100[v]	....	....
	(f) HEAT-TREATED EYE-BARS IN THE HUEY P. LONG BRIDGE, AT NEW ORLEANS, LA.												
13	Average	0.35	0.63	.....	.....	....	0.08	....	.....	58.5	89.0	10[w]	....
14	Minimum	0.31	0.53	.....	.....	....	0.060	....	.....	52.7	82.1	7.6[w]	....
15	Maximum	0.39	0.71	.....	.....	....	0.112	....	.....	64.4	97.4	14.7[w]	....
16	Specified	0.30[q]	0.60[q]	.....	.....	....	.....	....	.....	50.0	80.0	8.0[w]	....
	(g) SPECIAL HEAT-TREATED EYE-BARS IN THE POINT PLEASANT BRIDGE (W. VA.)												
17	Average	....	....	.....	.....	....	.....	....	.....	81.0	114.5	6.8[w]	....
18	Minimum	....	....	.....	.....	....	.....	....	.....	78.3	111.6	4.1[w]	....
19	Maximum	....	....	.....	.....	....	.....	....	.....	85.7	119.6	7.7[w]	....
20	Specified	....	....	.....	.....	....	.....	....	.....	75.0	105.0	5.0[w]	....
	(h) TESTS OF STRUCTURAL STEEL FROM THE RICHMOND POWER STATION, PHILADELPHIA, PA.												
21	Minimum	0.31	0.68	.....	.....	....	0.15	....	.....	41.5	80.2	18.2	....
22	Maximum	0.39	0.90	.....	.....	....	0.33	....	.....	53.1	94.1	24.5	....
	(i) STANDARD MANGANESE BOLTS												
23	Specified	0.30[p]	1.35[q]	0.04	0.05	....	.....	....	.....	42	90[t]	[x]	....
	(j) GRILLS FOR THE KANAWHA POWER PLANT, HAWKS NEST, W. VA.												
	Range:												
24	From	0.15	1.10	.....	.....	....	0.70	....	0.40	60[q]	100[q]	18	....
25	To	0.35	1.50	.....	.....	....	1.00	....	0.70				
	(k) TOWERS OF THE PHILADELPHIA - CAMDEN BRIDGE												
26	Average	0.35	0.76	.....	.....	....	0.27	....	.....	....	....	....	....
	(l) TOWERS OF THE GOLDEN GATE BRIDGE												
27	Average	0.27	1.20	.....	.....	....	0.24	....	.....	....	....	....	....

[p] Maximum allowable. [q] Minimum allowable. [r] Minimum allowable value, 0.20 per cent. [s] 80 to 95 kips per sq in. [t] 75 to 90 kips per sq in. [u] From 0.10 to 0.30 per cent. [v] From 82 to 100 kips per sq in. [w] Percentage elongation in 18 ft. [x] For all items in this section, the percentage elongation in 8 in. $= \frac{1500}{\text{Ultimate strength}}$.

TABLE 15.—(*Continued*)

Item No.	Remarks	Chemical Composition (Percentages)								Unit Stresses in Kips per Square Inch		Percentage elongation in 8 in.	Percentage reduction in area
		Carbon	Manganese	Phosphorus	Sulfur	Nickel	Silicon	Copper	Chromium	Yield point	Ultimate strength		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	)	(10)	(11)	(12)	(13)
(*m*) Tests to Observe Difficulties of Fabricating High-Tensile Steel with Present Shop Equipment (See Table 17)													
	Brinell Hardness Nos.:												
28	228	0.45	2.00	.....	.....	....	.....	....	....	82	126	9.5	7.6
29	228	0.32	1.95	.....	.....	....	.....	....	....	75	109	16.0	42.4
30	228	0.32	1.95	.....	.....	....	.....	....	....	75	109	16.0	42.4
31	207	0.33	1.54	.....	.....	....	0.25	....	....	61	100	16.7	46.7
32	207	0.34	1.57	.....	.....	....	2.24	....	....	65	102	19.5	53.6
33	207	0.34	1.57	.....	.....	....	0.24	....	....	65	102	19.5	53.6
34	217	0.36	1.93	.....	.....	....	0.25	....	....	78	102	1.5	2.5
35	228	0.36	1.90	.....	.....	....	0.28	....	....	80	127	11.5	17.9
36	207	0.29	1.08	.....	.....	2.12	0.30	....	....	64	100	16.0	51.8
37	183	0.29	1.08	.....	.....	2.12	0.30	....	....	64	100	16.0	51.8
38	207	0.28	1.01	.....	.....	2.12	0.32	....	....	65	100	19.5	42.5
39	217	0.35	1.85	.....	.....	....	0.28	....	....	70	112	12.0	25.0
40	228	0.35	1.85	.....	.....	....	0.28	....	....	70	112	12.0	25.0
41	217	0.29	1.09	.....	.....	2.01	0.26	....	....	67	106	17.7	37.4
42	212	0.29	1.09	.....	.....	2.01	0.26	....	....	67	106	17.7	37.4
43	196	0.30	0.83	.....	.....	2.10	0.26	....	....	61	102	18.2	42.7
44	207	0.30	0.83	.....	.....	2.10	0.26	....	....	61	102	18.2	42.7
45	207	0.28	0.97	.....	.....	2.13	0.26	....	....	60	99	18.0	49.5
46	207	0.30	1.10	.....	.....	2.05	0.25	....	....	66	106	17.5	43.2
47	228	0.35	1.89	.....	.....	....	0.29	....	....	70	110	15.0	44.3
48	228	0.35	1.89	.....	.....	....	0.29	....	....	70	110	15.0	44.3
49	228	0.35	1.89	.....	.....	....	0.29	....	....	70	110	15.0	44.3
50	241	0.35	1.89	.....	.....	....	0.29	....	....	70	110	15.0	44.3
51	241	0.33	1.73	.....	.....	....	0.24	....	....	58	102	15.0	20.9
(*n*) U. S. Navy Specifications for Special Steel Suitable for Welding													
52	No. 48 S 5 e	0.18	1.45	.....	.....	....	0.25	....	Va. 0.08/0.18	50	80	....	....
(*o*) Tests on Silicon and Nickel Steel for the Delaware River Bridge (See Figs. 34 and 35)													
3	Sheared plates, all 5/8 in. thick	0.32	0.65	0.014	0.033	....	0.300	....	......	42.7 47.0 47.2 45.9 46.5 47.0	74.5 81.8 82.7 82.3 83.0 82.3	26.7 23.2 25.5 23.7 23.5 25.0	42.7 40.0 43.3 41.8 45.2 47.4
54	Angles: 8 by 8 by 3/4 8 by 8 by 1 8 by 8 by 1	0.35	0.75	0.023	0.040	....	0.208	....	......	46.7 48.4 45.0	81.4 82.5 79.3	22.2 22.5 25.0	40.1 33.1 38.3
55	Sheared plates: 5/8 in. thick	0.38	0.60	0.025	0.036	....	0.216	....	......	49.0 48.4	82.3 82.9	25.7 24.5	46.8 44.4
56	Universal mill plates: 5/8 in. thick	0.35	0.86	0.025	0.040	....	0.280	....	......	47.7 50.8	89.0 88.4	24.2 21.7	50.1 47.4
57	Universal mill plate: 5/8 in.	0.34	0.70	0.024	0.041	....	0.208	....	......	48.1	82.8	20.7	44.5
58	8 by 6 by 3/4-in. angles	0.39	0.72	0.016	0.026	3.28	0.052	....	......	72.8 80.2 75.2 80.1	121.0 119.0 121.0 121.2	13.7 8.2 14.7 12.7	30.1 29.2 33.2 43.0
59	Sheared plates: 13/16 in. thick 1/2 in. thick	0.31	0.57	0.013	0.027	3.23	0.080	....	......	59.5 56.9	101.0 92.0	20.5 21.5	43.5 50.4
60	Universal mill plates: 13/16 in. thick 7/16 in. thick	0.28	0.50	0.016	0.026	3.18	0.076	....	......	55.8 56.6	90.7 92.8	21.0 18.0	42.1 45.9
61	Sheared and Universal mill plates: 1 in. thick 3/4 in. thick 1 in. thick	0.30	0.55	0.016	0.034	3.28	0.068	....	......	56.0 55.6 55.6	95.3 95.2 97.4	21.7 20.5 18.2	39.7 41.3 39.8
62	Sheared plates: 13/16 in. thick 1/2 in. thick	0.28	0.51	0.013	0.033	3.28	0.060	....	......	56.3 54.1	89.2 86.0	20.7 22.0	43.3 45.5
63	....	....	....	.....	.....	....	.....	....	.....	....	....	....	....
64	8 by 8 by 5/8-in. angles	0.34	0.41	0.015	0.026	3.16	0.052	....	......	54.2 56.5	89.1 89.5	22.7 21.7	47.6 41.4

George Washington Bridge.—This well-known bridge over the Hudson River, in New York, N. Y., is a suspension bridge with a 3 500-ft center span. It is 106 ft wide, center to center of stiffening trusses[78]. Silicon steel was used for the chords of the stiffening trusses for the side spans, whereas nickel steel was used in the center span. Silicon steel was also used for the main columns of the towers and main material of the floor system, whereas carbon steel was used for the tower bracing and for details of the tower and floor.

The weight of steel on the bridge was divided as follows, in tons: Wire, 29 000; carbon steel, 31 000; silicon steel, 30 000; and nickel steel, 2 350. The towers contained 23 800 tons of silicon and 15 200 tons of carbon steel.

Ambassador Bridge.—This international bridge over the Detroit River at Detroit, Mich., is of the suspension type. The towers are 363 ft high. The 47-ft roadway carries five lanes of traffic, and there are two 8-ft sidewalks. The chords of the stiffening trusses are of silicon steel whereas the webs are of carbon steel for which unit stresses of 32 and 24 kips per sq in. were used. The cables were originally spun with heat-treated wire that had an ultimate strength of 220 kips per sq in. and a yield of 194 kips per sq in. This wire proved to be unsatisfactory when placed in the bridge, because of its inability to stand the bending stresses, and was replaced. The new wires were cold drawn (see Table 12), with the physical requirements of ultimate strength of 215 kips per sq in. and yield point of 144 kips per sq in., measured at an elongation of 75% in an original length of 10 in. A very extensive series of tests were made. Only 4 in 300 showed less than 215 kips per sq in., the lowest being 210 kips per sq in. A total of 45 yield-point tests met the specification requirements. Tension tests for permanent elongation indicated a proportional limit of 103 kips per sq in., which is well above design stresses. Creep tests showed very little difference between straight wires and those bent around sheaves. No wires broke over sheaves or at the point of tangency.

Delaware River Bridge.—This suspension bridge at Philadelphia, Pa., has a main span of 1 750 ft and side spans of 751 ft 8 in. The towers are 337.5 ft high. Three grades of steel were used in this bridge: Silicon steel for the main compression material of tower posts, web members, and lateral systems of stiffening trusses, and some of the heavy girders in the approaches; nickel steel for chords of stiffening trusses; and structural steel for the remainder. The two special steels were used to effect economy in weight (webs of stiffening trusses), or to secure great flexibility within the elastic limit of the material (in the towers, and still more so in the truss chords).

A unit stress of 24 kips per sq in. was used for the silicon steel in the towers for the sum of axial and bending stresses due to dead load, congested load, wind, and temperature. Including the secondary stresses, the allowable unit stress was 27 kips per sq in.

For the nickel steel (yield point, 55 kips per sq in.) in the chords of the stiffening trusses, a unit stress of 40 kips per sq in. in tension was used, and 35 kips per sq in. in compression, with the top chord working stress reduced 3 kips per sq in. on account of lateral forces; 32 kips per sq in. in tension was allowed for silicon steel in the web members and $32\,000 - 140\dfrac{L}{k}$ lb per sq in. in compression. For the laterals, the allowable unit stress for silicon in tension was 32 kips per sq in. and $30\,000 - 100\dfrac{L}{k}$ lb per sq in. in compression, but some value greater than 10 000 lb per sq in., of this was absorbed by participation in the chord stresses.

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[78] *Transactions*, Am. Soc. C. E., Vol. 97 (1933).

The Bayonne Bridge.—This bridge, which spans the Kill van Kull, is the longest arch span in the world. It has a span of 1 675 ft, a rise of 274 ft, and a width of 74 ft, center to center of trusses.

The total weight of steel in the arch is 17 160 tons, of which 4 000 tons is silicon and 4 700 tons is manganese steel. The 2 000 ft of viaduct approaches contained 4 150 tons of silicon and 350 tons of manganese steel. The allowable unit stresses on the three materials, in kips per square inch, were: For carbon, 20; for silicon, 27; and for manganese steel, 33. The special manganese steel for this bridge was specified as shown in Table 15(*e*).

Bridges—Movable Spans

The Cape Cod Canal Railroad Lift Span, at Buzzards Bay, Massachusetts, the longest lift span on record, is 544 ft, center to center of piers. The lift span is composed of silicon steel, except the chords in the end panels (see Table 16).

TABLE 16.—Basic Data—Movable Bridges of Medium Length

Description (1)	Buzzard's Bay Railroad Bridge across Cape Cod Canal (2)	Burlington to Bristol Bridge, across the Delaware River (3)	Railway bridge, at Boonville, Mo. (4)	Hackensack River crossing (5)	West Madison Street Bridge, Chicago, Ill. (6)	North Wabash Avenue Bridge, Chicago, Ill. (7)	South Harlem Avenue Bridge, Chicago, Ill. (8)	Outer Drive Bridge, Chicago, Ill. (9)
Type	Vertical lift	Vertical lift	Vertical lift	Vertical lift	Double-leaf bascule	Double-leaf bascule	Double-leaf deck trusses	Double-leaf deck trusses
Span length, in feet*	544	533.75	408.0	198	221.23‡	269	224	264
Weight of Steel, in Tons:								
Silicon	2 248	......	2 100	......	......	......	812	1 965
Carbon	2 144	......	......	......	......	......	......	......
Nickel	......	......	......	......	519	329	......	......
Width, in Feet:								
Center to center of trusses	27	......	20	45	......	......	......	......
Roadway	......	20	......	3†	38	60	56	38§

* Center to center of piers. † Tracks. ‡ Center to center of trunnions. § Two each.

The Burlington-Bristol Lift Span over the Delaware River is 533 ft 9 in. long. Light weight was obtained in this structure by the liberal use of silicon steel and a special light-weight floor. It was estimated that each pound added to the floor meant an additional cost of 12 cents in the trusses, towers, etc.

The 408-ft railway lift span, at Boonville, Mo., which carries the Missouri, Kansas, and Texas Railroad, was designed for E-70 loading.

The Hackensack River Crossing of the Delaware, Lackawanna, and Western Railroad, is a lift span 198 ft long. The maximum lift is 95 ft. To reduce the lifted load, an open deck was used, with silicon steel for the trusses and floor-beams.

The West Madison Street Bridge, in Chicago, Ill., is a double-leaf bascule. A total of 519 tons of nickel steel was used in the trusses and floor-beams.

The North Wabash Avenue Bridge, in Chicago, is a double-leaf trunnion bascule. In the trusses, 329 tons of nickel steel were used.

The South Harlem Avenue Bridge, in Chicago, is a double-leaf deck-riveted truss span, 224 ft, center to center of trunnions, and carries a 56-ft roadway and two 8-ft sidewalks. In the trusses, counterweight boxes, and two floor-beams, 812 tons of silicon steel were used.

The Outer Drive Bridge, in Chicago, is a double-leaf deck-riveted truss, 264 ft, center to center of trunnions, and carries two 38-ft roadways and two 14-ft sidewalks. The trusses and trunnion girders contain 1 965 tons of silicon steel.

RAILROAD BRIDGES

The Bessemer and Lake Erie Railroad High Bridge, over the Allegheny River, consists of two double-track continuous truss bridges. Silicon steel was used for the stringers, floor-beams, jacking girders, and for the main members of the trusses, except that the tension top chords over the piers were heat-treated eye-bars. The total weight of steel was 10 000 tons, of which 54% was silicon.

The three-span continuous truss bridge for the Chesapeake and Ohio Railroad, at Cincinnati, Ohio, is a double-track railroad bridge, 1 575 ft long. The center span is 675 ft and two side spans are 450 ft long. Silicon steel was used throughout at a unit stress of 24 kips per sq in. The resultant saving in steel weight was approximately 19% of the weight of a similar structure built of carbon steel using a unit stress of 18 kips per sq in. The decreased weight due to the use of silicon steel was particularly advantageous from an erection standpoint.

The railroad bridge across the Tanana River, for the Alaska Railroad, has a main span 700 ft long and 600 ft of plate-girder approaches. The live load was E-50. The item of freight was large and made the use of alloy steels attractive. Nickel-steel eye-bars were used on the 700-ft span, and silicon steel was used for all main members, the floor system, pins, and large detail parts. Silicon steel was also used for the flange angles of the 60-ft plate girders in the approaches.

The Atchison, Topeka and Santa Fé Railroad High Level Bridge, over the Illinois River, near Chillicothe, Ill., is a double-track structure, 1 696 ft long, consisting of a 400-ft and a 470-ft two-span continuous deck truss; a 440-ft through truss span; a 235-ft deck span; and two 68-ft plate-girder spans. The design live load was E-70. Silicon steel was used for all chords and main diagonal members, with carbon unit stresses increased 50% for tension and 40% for compression.

The Martinez-Benicia Bridge across Suisan Bay, in California, is a double-track railroad bridge, 5 600 ft long, built for the Southern Pacific Company. It consists of a 328-ft vertical lift span; seven through truss spans, 526 ft long; one deck span, 504 ft, and one deck span, 264 ft long; and a 780-ft approach viaduct. It was designed for E-90 loading, with unit stresses one-third higher than the standard. Silicon steel was used for the main members, except in the bottom chords where heat-treated eye-bars were used. The total weight of steel was 22 000 tons, of which 12 500 tons was silicon and 2 750 tons, heat-treated eye-bars.

The main river crossing of the Louisville and Nashville Railroad Bridge, over the Ohio River, at Henderson, Ky., consists of a double-track simple span, 670 ft long, and four double-track simple spans, 500 ft long. Silicon steel was used for the trusses, stringers, and floor-beams. The 670-ft simple span has a dead load of 14 kips per lin ft, and the total weight of steel in this span is 4 550 tons, of which 4 200 tons is silicon. The trusses are about 80% of the total weight of the span. The 500-ft simple spans weigh 2 500 tons each, of which 2 200 tons is silicon, and the trusses are about 67% of the total weight.

The Huey P. Long Bridge, over the Mississippi River, at New Orleans, La., is a combined double-track railroad and highway bridge (with one 18-ft roadway on each side). In the main spans and approaches 31 000 tons of silicon, steel was used. The 1 850-ft cantilever bridge and the five simple truss spans have a total weight of 20 000 tons, of which 11 000 tons are silicon steel and 1 400 tons, heat-treated eye-bars. The trusses, except minor members, are of silicon steel and heat-treated eye-bars. The railroad stringers and floor-beams are silicon, whereas the highway portion and the bracing are carbon. (For full-sized tests of heat-treated eye-bars for this bridge, see Table 15(*f*).)

HIGHWAY BRIDGES

The highway bridge over the Ohio River, at Louisville, Ky., consists of two 1 870-ft cantilever bridges, each composed of one anchor arm of 362 ft, one anchor arm of 500 ft, two 224-ft cantilever arms, and a 373-ft suspended span. It is 43 ft center to center of trusses and has two 6-ft sidewalks. The total weight of steel is 13 460 tons, of which 5 140 tons are silicon and 1 180 tons are heat-treated eye-bars.

A part of the Tri-Borough Bridge, at New York, N. Y., is a highway suspension bridge, having a 1 380-ft span and two 705-ft side spans. The 270-ft towers are of the fixed-base flexible type, having main columns of silicon steel with carbon-steel bracing. The stiffening truss chords are silicon steel, and the diagonals are carbon steel. The 96-ft floor-beams are of silicon. The total weight of silicon steel on the bridge is 15 000 tons. Each of the cables (20.63 in. in diameter) has 37 strands of 248 wires, 0.196 in. in diameter, and a total area of 277 sq in.

The Mount Hope (Providence, R. I.) Wire Cable Suspension Bridge has a 1 200-ft main span and two 504-ft side spans. The chords of the stiffening trusses are of silicon steel with working stresses of 34 kips per sq in. in tension and 28 kips per sq in. in compression. The material in the two 11-in. cables, is the same as for the Ambassador Bridge, previously cited.

The main structure of the cantilever highway bridge across Carquinez Strait, in California, is 3 350 ft long and consists of two 1 100-ft main spans, a 150-ft central tower, and two 500-ft anchor arms. Silicon steel was used for the main material of the towers, and for the compression members and the built tension members of the trusses. Heat-treated eye-bars having an ultimate strength of 80 kips per sq. in. and a minimum yield point stress of 50 kips per sq in., were used for the principal tension members. The total weight of steel in the main bridge is 11 400 tons, of which 5 400 tons is silicon and 1 370 tons, heat-treated eye-bars.

The eye-bar cable suspension bridge, at Florianopolis, Brazil, has a main span of 1 113 ft 9 in., and carries a 28-ft roadway. In this bridge a part of the cable, which is composed of heat-treated eye-bars, acts as the center part of the top chord of the stiffening truss. The heat-treated eye-bars were specified to have a minimum ultimate strength of 105 kips per sq in., a minimum yield point stress of 75 kips per sq. in., and an elongation of 5% in 18 ft.

The eye-bar suspension bridges over the Ohio River, at Point Pleasant and St. Marys, W. Va., have a 700-ft main span and two 380-ft side spans and carry a 22-ft roadway and one 5-ft sidewalk. These bridges are similar to the Florianopolis Bridge, except that in addition to the eye-bars replacing the twelve center panels (of twenty-eight panels) in the main span, they replace the first seven panels from the cable bent (of fifteen panels) in the side spans. The physical characteristics of the special heat-treated eye-bars in this bridge are given in Table 15(*g*).

The self-anchored, chain, suspension type of bridge was used on the 6th, 7th, and 9th Street Bridges over the Allegheny River, at Pittsburgh, Pa. The center span is 442 ft long, with two side spans of 221 ft. Each chain was made of eight and nine eye-bars alternating with a maximum section of 230 sq in. These eye-bars were heat-treated to a minimum yield point stress of 50 kips per sq in. and an ultimate strength of 80 kips per sq in.

The Waldo-Hancock Bridge, in Maine, is cited as an example of the twisted-wire strand cable, suspension bridge. It has a main span of 800 ft, and two side spans of 350 ft. Each $9\frac{5}{8}$-in. cable consists of 37 strands of $1\frac{3}{8}$-in. wire rope. The ultimate strength per strand is 216 kips and each strand was pre-stressed to one-half this amount. The modulus of elasticity, after pre-stressing, was 26 600 000 lb per sq. in.

BUILDINGS

Special steels have not been used extensively in tier buildings. In one of the buildings of Radio City, in New York, N. Y., silicon steel was used in a part of four columns in order to reduce the size.

The Tower Building for the Cleveland Terminal Company, in Cleveland, Ohio, is a 52-story building, being about 300 by 247 ft for 16 stories, and then a set-back tower, 489 ft high, has a base that is 98 by 98 ft. The total weight of steel was 16 850 tons, of which 4 610 tons was silicon.

The hangar built for the Goodyear Zeppelin Corporation at Akron, Ohio, is 325 ft wide by 1 184 ft long ("out-to-out" of doors) by 180 ft clear height, having eleven lattice arches spaced on 80-ft centers and two diagonal half arches at each end. Silicon steel was used for the main arch ribs only. Of the total weight of 7 210 tons, 628 tons is silicon.

In the Richmond Power Station, at Philadelphia, Pa., there are 10 000 tons of steel in the switch house, turbine hall, and boiler house, of which 8 000 tons is silicon. Silicon steel was used for all the principal members of the main building framework, except the framing of the turbine hall roof. Tests from sixty-five heats are listed in Table 15(*h*).

The United States Post Office Building, in Philadelphia, Pa., covers an area of 380 by 460 ft, and has five floors and a roof. The total weight of steel is 17 700 tons, of which 1 740 tons is silicon. Silicon steel was introduced in this building when two hundred and fifty-two 36-in. silicon beams without cover-plates were substituted for carbon beams with cover-plates.

The addition to the Post Office Building, at Washington, D. C., is an irregular-shaped structure (width, 230 to 120 ft; and length, 418 to 368 ft; height, six floors and roof). The total weight is 7 180 tons, of which 2 975 tons is silicon. Silicon steel was used for the heavy plate girders, the double girders, part of the column slabs, part of the floor-beams (usually 24 in. and deeper), and about 62% of the trusses and columns.

SPECIAL STRUCTURES IN WHICH TRANSPORTATION OR OTHER COSTS ARE FACTORS IN MAKING THE USE OF SPECIAL STEELS ECONOMICAL

One of the best examples is the steel derrick used in the erection of steel bridges. The additional cost of the special steel is more than compensated for in three ways: First, the saving in weight reduces transportation costs as the derrick is shipped to and from the tool house to the bridge sites; second, the larger capacity; and third, the saving in weight in the steel of the bridges due to smaller erection stresses.

Silicon steel derricks were designed for use on the San Francisco-Oakland Bay Bridge. With a 103-ft mast and an 85-ft boom, one of these derricks weighed about 32 tons and made lifts of 59 tons. A similar carbon steel guy derrick weighs about 34 tons and has a lifting capacity of 46 tons. For slightly less weight a silicon steel derrick will lift about 40% more load.

The traveling crane is another type of special structure in which saving in weight, due to the use of high tensile steels, more than pays for the additional cost, because power consumption is reduced. The writer knows of three mill types of cranes, 120-ft span, which have a 40-ton main hoist and a 15-ton auxiliary hoist. The main girders are riveted plate girders. A reduction in weight of 40 000 lb is due to the use of manganese steel. It was necessary to deepen the girders 1 ft (compensating for the higher unit stresses on a material having the same modulus of elasticity as the carbon steel) in order to keep the deflection the same as for the carbon-steel cranes. It is generally not economical to use special steel unless the crane span is more than 100 ft.

The single-track railroad viaduct of the Pittsburgh and West Virginia Railway Company near Banning, Pa., was being built on a new line, so that additional erection equipment could not be readily sent to the field. In this viaduct there were two 120-ft carbon steel plate girders, weighing 65 tons each, the weight and reach of which were beyond the capacity of the erection derrick. Permission was obtained to design the girders for the use of silicon steel. The silicon girders weighed 53 tons each and could be handled by the derrick.

Alloy steels are seldom used in mill building, but in the Open-Hearth Building for the National Tube Company, at McKeesport, Pa., some of the crane girders were 115 ft long. The type of erection rig used to construct mill buildings does not have the capacity to handle long heavy girders. The girders, in this case, were designed for silicon steel, thus reducing the weight to 93 tons which could be handled by two locomotive cranes.

Bridges in the United States have been growing longer and heavier, and with this growth have come increased stresses and larger field rivets. Because of the stiffer material and more thicknesses it is becoming increasingly difficult to draw connecting members tight for proper rivet driving. The erectors find it necessary to use pneumatic tools to tighten fitting-up bolts, which strip the thread or break the ordinary carbon-steel bolt. Manganese steel fitting-up bolts which are now used extensively, obviate this difficulty and can be utilized over and over again. Even the hand wrenches are being made of manganese steel. The specifications for this material are given in Table 15(*i*).

All bars forming grills for the tunnel intake for the New Kanawha Power Company, at Hawks Nest, W. Va., were special Cromansil steel welded to the main frame of carbon steel. It is of interest to note the latitude of the specifications (see Table 15(*j*).

A great variety of special forging steels and a considerable tonnage of stainless steel are used in the movable dams constructed for the Federal Government. Abrasion-resisting steel is also used in the structural steel work of chutes for the U. S. Coke and Coal Company.

Copper-bearing steel is coming into more general use on account of its corrosion-resisting properties. In some cases, particularly for the Louisville and Nashville Railroad Company, it is specified for steel in the floor of bridges and other parts likely to be affected by brine from refrigerator cars. In many cases, such as the San Francisco-Oakland Bay Bridge, it is specified for all parts of the structure. A minimum of 0.20% copper is usually specified.

Wire rope is used not only for main cables and hangers on suspension bridges, but also for hangers on several riveted bridges, such as the Bayonne, N. J., Bridge and the West End-North Side Bridge, in Pittsburgh, Pa. It is also used for counterweight and operating ropes on lift bridges.

It has recently become necessary, in certain work, to design and detail structural steel so that it can be transported by airplane. The usual requirements are that the weight of a piece shall not exceed 2 000 lb and that the piece can be loaded through a 3 by 7-ft door. The advantage of the saving in weight which is offered by the use of special steels, is apparent in such cases.

SHOP PRACTICE FOR SPECIAL STRUCTURAL STEELS

The fabrication of special steels has brought new problems to the shops because equipment was designed and powered for handling carbon steel. The equipment or capacity to fabricate the high tensile steels varies with each individual plant. The following discussion of shop practice is based on the typical or general equipment, and it must be realized that each plant may have some one tool with more power or capacity.

Cambered plates in heavy gages cannot be straightened with straightening rolls or in a press. Camber is removed from the large thick plates by planing the edges. The strength of high tensile steels is such that the usual fabricator is not equipped to straighten it as he would straighten rolled carbon steel. Efforts to cold-straighten heavy plates ($\frac{3}{4}$ in. and more) have produced badly cracked edges. One large fabricator prefers to divide plate thicknesses greater than $\frac{7}{8}$ in. into two thinner plates stitch-riveted together.

Some plates, $1\frac{1}{4}$ in. and more in thickness, cracked at the sheared ends when an attempt was made to flatten a slight shear bow by passing the plates through cold rolls. This bow was caused by the pressure of the movable shear blade on the outer ends of the plates where they were not supported by the fixed blade or die. The difficulty was overcome by planing $\frac{1}{4}$ in. from sheared ends and straightening the plates in the press. Subsequently, the bowing was avoided by turning the plates around so that in cutting both ends, the main body of the plate rested on the die of the shear.

As piping is deeper in silicon ingots than in carbon steel, angles should be ordered long enough for a preliminary shearing (or milling) at each end, in search of pipes.

It has been reported that the silicon plates for the towers of the Golden Gate Bridge were sub-punched with fewer punches broken than the plates for the towers of the Philadelphia-Camden Bridge. The records show that the average of the latter was as given in Table 15(*k*), whereas, the former, at the time the report was made, was as shown in Table 15(*l*).

A test has been devised which records on an indicator card the force of the punch, simultaneously with its penetration into the plate. Other tests have been made on power input required to drill and plane. Plates were used which had passed silicon steel acceptance tests and were of about the same strength. One analysis was: Carbon, 0.26%; manganese, 0.84%; and silicon, 0.27%; and the other was: Carbon, 0.35%; manganese, 0.84%; and silicon, 0.25 per cent. The former punched with 10% less maximum pressure than the latter. In drilling and planing, the difference was nearly the same, and in the same direction.

In 1929, a group of twenty-four types of high tensile steels in various structural shapes were studied to determine which of the different grades could be fabricated successfully with the present bridge shop tools. As described in Table 15(*m*), all these steels were in the manganese-silicon group and represented various combinations of these elements with carbon. In addition, there were a few sections containing about 2% nickel. The sections included ½-in. and 1-in. plates, 8 by 8 by ¾-angles, 12-in., 40.8 lb I-beams, and 14-in., 425-lb C-beams. These sections were subjected to various bridge shop operations which included punching, drilling, reaming, milling, planing, and coping, with the results noted in Table 17. Test coupons were cut from each item and tensile, bending, and hardness tests were made (see Table 15(*m*)).

TABLE 17.—FABRICATION OF HIGH STRENGTH STRUCTURAL STEELS WITH PRESENT SHOP TOOLS

Item No. (see Table 15 (*m*):	Punching	Drilling	Reaming	Rotary planing	Milling	Coping	Edge planing
28	*a, d*	*a*	....	....	....	....	....
29	*a, d*	*a*	....	....	....	....	....
30	*a, d*	*a*	....	....	....	....	....
31	*b*	*b*	*b*	*b*	....	....	....
32	*b*	*b*	*b*	*b*	....	....	....
33	*b*	*b*	*b*	*b*	....	....	....
34	*a d*	*c*	....	*a*	....	....	*a*
35	*a d*	*c*	....	*a*	....	....	*a*
36	*e*	*e*	*e*	*a*	....	....	....
37	*e*	*e*	*e*	*a*	....	....	....
38	*e*	*e*	*e*	*a*	....	....	....
39	*a*	*a*	....	....	*a*	....	....
40	*a*	*a*	....	....	*a*	....	....
41	*f*	*f*	....	....	*f*	....	....
42	*f*	*f*	....	....	*f*	....	....
43	*f*	*f*	....	....	*f*	....	....
44	*f*	*f*	....	....	*f*	....	....
45	*g*	....	....	....	....	*h*	....
46	*g*	....	....	....	....	*g*	....
47	*g, d*	....	....	....	....	*h*	....
48	*g, d*	....	....	....	....	*h*	....
49	*g, d*	....	....	....	....	*h*	....
50	*d*	....	....	....	....	*g*	....
51	....	*i*	....	*a*	....	....	....

[a] Considered impracticable; too difficult. [b] Satisfactory at speeds and feeds slightly less than those required for silicon steel. [c] Not quite as unsatisfactory as Item No. 28. [d] The shock to the punching machine was terrible. [e] Satisfactory at speeds less than Item No. 31. [f] Similar to Item No. 36. [g] Fair results. [h] Flanges split. [i] Practical at very low speeds.

In this general discussion, it should be recognized that two steels of the same ultimate strengths do not necessarily possess the same ease of fabrication, and that with the newer steels, current knowledge of this phase is quite incomplete.

Shearing.—Shears have been rated and powered for carbon steel, and it is not practical to reduce the speed. Consequently, with the higher tensile steels the capacity is correspondingly reduced. For silicon and manganese steels the shears are rated at approximately 75% capacity; for example, a large shear is rated to shear a carbon-steel plate, 1 in. thick by 100 in. wide. This same shear will safely shear a silicon plate, ¾ in. thick by 100 in. wide. The shear blades are not changed for the alloy steels, since the blades used for carbon steel are the hardest obtainable.

Punching.—The old "rule of thumb" for punching carbon steel was that the material should not be thicker than the size of the hole to be punched. With silicon steel and manganese steel, whether sub-punching or full-sized punching, and regardless of whether the holes are 13/16 in. in diameter, or larger.

the shop prefers a maximum thickness of $\frac{3}{4}$ in., whereas for nickel steel the preferred maximum is $\frac{1}{2}$ in. Heavier material than this puts too great a strain on the machines. When punching alloy steels, the speed of the punches is usually reduced about 25 per cent.

It is a recognized fact that, with variations in composition, alloy steels may vary in hardness, and that some melts punch more easily than others. Where the specifications permit full-size punching, the shop will try to punch silicon and manganese to a maximum thickness of $\frac{7}{8}$ in, and nickel steel to a maximum of $\frac{9}{16}$ in. If the metal is too hard, as shown by the breaking of the punches or the strain on the machine, the holes will be drilled.

Because of the excessive breaking of the punches due to the harder alloy steels they have been re-designed. Larger fillets are used where the section of the punch changes radically. The stress concentration at this point was accountable for 50% of the punch breakage.

It is desirable to maintain scrupulously, or if possible exceed, the edge distance (and rivet-center distances) ordinarily specified. With 1-in. rivets, no angles with legs less than 4 in. should be used in the design.

One of the items that causes additional punching costs is the special handling due to punching holes of two sizes in the same plate because sizes of shop and field rivets are not properly proportioned, or the amount of reaming varies for shop and field rivets.

TABLE 18.—DRILLING SPEEDS

Description	Peripheral Speed of Drill, in Feet per Minute				Speed of Reamers, in Revolutions per Minute									
					Reaming Sub-Punched Holes to Full Size					Spear Reaming Full-Sized Holes				
	Medium Open-Hearth Steel		Silicon Steel		Medium Open-Hearth Steel		Silicon Steel		Nickel steel	Medium Open-Hearth Steel		Silicon Steel		Nickel steel
	From:	To:	From:	To:	From:	To:	From:	To:		From:	To:	From:	To:	
Plant No. 1:														
Gantries......	50	70	45	60	170	260	150	240	.....	240	280	235	275	......
Multiple spindle.....	70	85	65	80	....	...	....	...	.....	.....	.....	.....	.....	......
Radials.......	50	60	45	55	160	270	150	250	.....	.....	.....	.....	.....	......
Plant No. 2:														
Medium open-hearth steel.	....	90	....	..	....	...	....	...	.....	.....	.....	.....	.....	......
Silicon steel...	70	75	....	..	....	...	....	...	.....	.....	.....	.....	.....	......
Nickel steel...	50	55	....	..	....	...	....	...	.....	.....	.....	.....	.....	......
Hi-cycle......	....	..	....	..	270		270		220	270		270		220
Hi-cycle	....	..	....	..	250		250		160	250		250		160

Drilling from the Solid.—With the harder alloy steels drilling from the solid is a matter of speed and feed. Thickness is not a factor unless the plates are stacked. The shop practice of sub-drilling, assembling the member, and reaming holes to size, or of assembling the member and drilling holes full sized, varies with the fabricating plants and also varies in an individual shop; it is determined by the types of work being fabricated.

The reduction in speed of the machines when drilling special steel can be seen from the report of plants as shown in Table 18.

The drilling of alloy steels requires more grinding of drills, which also entails the added cost of changing the drills more often. One plant reports that, in comparison with ordinary carbon steel, drills used in silicon and manganese steels must be ground somewhat more often, and those used in nickel steel, twice as often. One plant reports on nickel-steel bridge chords (in which the number of holes to be drilled per member averaged 539, with a thickness of $2\frac{5}{8}$ in.) that the best record on one chord required the drills to be changed fifty-five times, whereas on one chord the drills had to be changed ninety-seven times. For comparison purposes, they estimated that on a similar chord of carbon steel they would have required eight to ten drill changes.

The feed of drilling varies with the different alloy steels and even on one piece of angle or plate, and can only be stated as being less than for carbon steel.

Reaming.—Reaming of holes is more universally specified for alloy steels than for carbon steels. It will be noted from Table 18 (a report on the reduced speeds necessary with special steels) that the reduction is not as marked as noted for drilling.

Milling.—Planing speeds for silicon, manganese, and nickel steels are generally reduced about 25%, and the feeds are changed to suit the particular hardness of each metal. It is difficult to get a smooth finish on these alloy steels on account of the tool wear.

Chipping.—The increased difficulty of chipping silicon and manganese steels may be covered by stating that the speed is reduced about 25 per cent. Nickel steel is extremely difficult to chip under any conditions and depends on previous treatment, whether sheared or cold sawed; if cold sawed, it is almost impossible to chip, and may require pre-heating.

Flame Cutting.—Flame cutting of carbon steel of structural grades leaves a perfectly ductile edge for milling or chipping. If silicon steel is flame cut the edge is air-hardened, and it is difficult to edge-plane or mill. However, a milling cut of $\frac{5}{32}$ in. will remove all trace of damage. The portion affected by the flame varies in hardness and although the piece usually can be edge-planed without serious difficulty after a cutting torch is used near the surface to be planed, it is sometimes necessary to pre-heat it. Manganese and nickel steels offer more difficulty. If either is flame cut, it cannot be planed (without special treatment that is not practical).

Experiments with a special portable heat-treating and flame-cutting machine indicate the possibilities of flame cutting some of the alloy steels (the standard nickel, manganese, and silicon types), leaving an edge of sufficient ductility for further shop operations. The results of the bend tests and drift tests indicate that the process will meet any required specifications.

Bending.—In discussing alloy steels it is usually stated that the radius required to bend a plate cold depends upon the thickness of the plate, as expressed by:

$$r = C\,d \qquad (4)$$

in which r = the radius of bend; C = a coefficient; and d = depth, or thickness of the plate.

The writer has noted, in various papers, that, for plates with $d = \frac{3}{4}$ in., or less, $C = 2$ for Cor-Ten; 2.5 for Man-Ten; and 3.0 for Cromansil.

The radius required for bending a plate cold should vary with the kind of steel, the thickness of the plate, and the angle of bend. Table 19, which illustrates these variables gives the minimum radii required for bending silicon and nickel-steel plates. These radii are to be used when the bend is transverse to the plate. When the bend is longitudinal (that is, when the bend line is parallel to the direction of rolling the plate) these radii should be increased slightly.

TABLE 19.—MINIMUM RADII, r, IN INCHES, FOR BENDING PLATES AND ANGLES

Type of steel (1)	Plates				Angles; Bent Cold		
	Thickness of plate (2)	Bent Cold		Bent hot; bends greater than 90° (5)	3 by 3-in. to 4 by 4-in. (6)	6 by 6-in. (7)	8 by 8-in. (8)
		45° bend (3)	90° bend (4)				
Silicon	0.375	0.375	0.625	0.875	36.0	48.0	120.0
	0.625	0.625	1.5	1.75			
	0.875	0.875	2.625	2.875			
	1.125	1.125	3.625	4.0			
Nickel	0.375	0.5	0.75	1.0	42.0	54.0	120.0
	0.625	0.75	1.625	1.875			
	0.875	1.00	2.75	3.0			
	1.125	1.25	3.75	4.25			

Assembling.—The increased stiffness of the special steels makes it more difficult for the fitting-up bolts to draw the steel together into perfect contact, but this close contact is necessary for the driving of satisfactory rivets. The smallest fitting-up bolts which the shop should use are $\frac{3}{4}$ in. in diameter. This means that the minimum diameter of the holes should be $\frac{13}{16}$ in. Where sub-punched and reamed work is specified, the minimum shop rivet should be 1 in. in diameter. The increasing use of alloy steels and the difficulties of fitting up properly have necessitated the use of stronger fitting-up bolts, and the fabricating plants are beginning to make them of manganese steel. One plant now uses 100% manganese bolts for all bridge work.

Rivets.—The manufacture of carbon-steel rivets and their use in fabricated steel structures has become standardized. The need for a rivet of higher strength to match the special steels was recognized. Nickel-steel rivets with a nickel content of 3 to $3\frac{1}{2}$% have been used on a few structures, but have been so invariably unsatisfactory from every standpoint that they should not be specified for any structural steel work. However, tests have been made on rivets that meet Steel Specification No. 2115 of the Society of Automotive Engineers (carbon, 0.10 to 0.20%; manganese, 0.30 to 0.60%; silicon, 0.15 to 0.30%; and, nickel, 1.25 to 1.75%) which indicate that this grade is suitable for hot riveting and will develop a shearing strength comparable to that of other high-strength rivets in use to-day. Whereas silicon steel is admittedly not a forging steel and not satisfactory for rivet material, manganese steel has been found to be satisfactory both for manufacturing and driving. Shipbuilders have found that manganese steel makes a good high-strength riveting material. They have used it in large quantities and have had considerable research work done to establish the best chemical composition.

A large number of manganese rivets was used on the Bayonne Bridge (see Table 15(*e*), Item No. 2). The maximum rivet was 1¼ in. in diameter by 10¾ in. long. The rivets were tapered if the length was five or more times the nominal diameter. Later experiments indicated that this rivet material was a little too hard, and a more satisfactory material has been developed (see Table 15(*d*)). For the San Francisco-Oakland Bay Bridge the largest manganese rivets were 1¼ in. in diameter by 10½ in. long, 1⅛ in. in diameter by 9½ in. long, and 1 in. in diameter by 6¾ in. long. These rivets were tapered the same as for the Bayonne Bridge.

The driving technique for manganese rivets differs considerably from that for carbon rivets. The more important differences are: (1) Manganese rivets must be more carefully heated; (2) they are more easily burned than ordinary rivets; (3) the range of temperature is small between a proper driving heat and a burning heat; (4) more time is required in driving than for ordinary rivets; (5) the cost of driving is estimated at 10% greater; (6) it is necessary to burn off the heads before backing condemned rivets out; (7) it costs about twice as much per rivet to remove them; and (8) with the proper attention to heating and driving there will be no more cut-outs than for ordinary rivets. There is a need for a standard specification for high-strength structural rivets.

The action of riveted joints is a controversial subject and some criticism of manganese rivets has been offered because the tests show that they have an initial slip at lower values than carbon rivets. However, the point should be brought out that the factor of safety is not based on the initial slip value, but on the relation of the working stresses of the material and details, such as rivets, to the yield points, and upon the actions of these materials under laboratory tests and actual use, when stressed between these limits. Tests made for the Bayonne Bridge showed that carbon rivets averaged from 0.004 in. to 0.008 in. more slip than the manganese rivets. Other tests have shown a contrary result.

In order to maintain the factor of safety for the structure, the rivet factor must be maintained on an equal basis with the main material between the working stress and up to the yield point. Taking an average of all available tests, it seems fair to state that the slip for manganese and carbon rivets will be of approximately equal magnitude through the range of stress in which the designer is interested. At ultimate values the manganese rivets have a larger factor of safety than carbon rivets.

In 1906, tests of nickel and chromium steels were reported at the University of Illinois[79] with the following conclusions:

(*a*) The ultimate shearing strength of rivet joints depends on the shearing strength of rivet material, and this is influenced by the relative hardness of rivets and plates;

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[79] "Tests of Nickel-Steel Riveted Joints", *Bulletin*, Vol. 8, No. 26, Univ. of Illinois, Urbana, Ill.

(*b*) The ratio of the yield point stress in riveted joints to ultimate shearing strength of riveted joints is about the same as the ratio of the yield point stresses of plates in tension to the ultimate strength of the plate.

(*c*) In riveted joints designed on the basis of ultimate strength, the strength of the rivet material and of the plate material is of prime importance, and use of special steels of great strength may be of advantage.

(*d*) In riveted joints designed on the basis of the frictional hold of rivets there is little advantage in using rivets of special steel since joints with such rivets show about the same resistance to first noticeable slip as joints with ordinary carbon rivets; and,

(*e*) There was a noticeable slip of the joint, generally, at loads within the ordinary working shearing stress of rivets.

Tests by Commander E. L. Gayhart[80], U. S. N., were made on high-strength steels and rivets. He stated that joints fabricated with high tensile steel showed resistance to slip greatly inferior to that shown by medium steel—values so low that it appears doubtful whether any great part of the load could be considered as carried by frictional forces. In terms of single shear stress, according to Commander Gayhart, the stress in the rivet at which the first slip occurs is so much less than customary design shear stresses, and is so greatly affected by the number of rivets in the joint, that it does not appear desirable to design riveted joint construction on the basis of frictional resistance but rather by the commonly used method of proportioning on shearing and bearing.

As the designer depends on the bearing of the rivet on the plate to carry the stress, there is no advantage in using high-strength rivet steel in carbon plates; for example, on the Bayonne Bridge a bearing value was used for the manganese rivets, of 45 kips per sq in., on manganese plates; 40 kips per sq in. on silicon plates; and 30 kips per sq in. on carbon plates. As 30 kips per sq in. is also used on bearing carbon rivets on carbon plates, it is evident that the value of a manganese rivet and a carbon rivet in bearing on a carbon plate is the same.

Welding.—The scope of this paper does not permit the introduction of very much material on the subject of welding for to the writer's knowledge no structural steel bridges or buildings have been constructed in which the special steel was welded to carry stresses. However, considerable welding on special steels is being done in the plants. Pre-heating and annealing of large structural members is not practical; practically all the welding is done by the metallic arc.

Considerable confusion exists at present because there is no standard or definition stating what chemical or physical qualifications a steel must have in order to be classed as of weldable quality. The various steel companies list their special steels as those that can be welded, whereas the specifications of the American Society for Testing Materials do not list them as "steel suitable for fusion welding." On one hand the United States Navy

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[80] "An Investigation of the Behavior and of the Ultimate Strength of Riveted Joints Under Load", by Commander E. L. Gayhart, U. S. N., *Proceedings*, Soc. of Naval Archts. and Marine Engrs., Vol. 34 (1926), p. 55.

Department is quoted as being definitely against the use of the silicon-manganese types of steels for welded stress-carrying members; and yet U. S. Navy Specification No. 48S 5e specifies (under high tensile steel of weldable quality) a steel with the characteristics listed in Table 15(*n*).

Probably the most interesting point to be emphasized in the welding of special steels (for the purpose for which welding is used in structural shops) is that generally the ordinary soft steel, coated, welding rods are used rather than the special rods developed by various manufacturers. The technique required for the ordinary coated rods is much simpler, and the strength developed is more than ample for the fillet welds.

Tension tests were made on full butt welds in manganese-steel plates using several of the special welding rods and all the welds developed more strength than the parent metal. However, many of the special steels are subject to "air-hardening" or grain growth, so that if it is necessary to bend the steel, or if it is subject to bending stresses, the air-hardened surface must not be located on the tension side, unless the steel can be annealed, which is usually not practical for structural steel work.

Welding of special steels in the fabricating shops has generally been confined to: (1) Tack welding for assembling; (2) fillet welding fillers for permanent work; and (3) some welding of silicon slabs for shoes, with special supervision. Covering this type of welding, it is stated[77] that in the fabrication of the towers for the Golden Gate Bridge, it was desired to tack-weld the shaft angles and the web-plates so that the full thickness could be drilled from the solid steel simultaneously, with no subsequent separation before riveting. The question was raised as to whether this tack-welding would embrittle the silicon steel harmfully, which was of the composition listed in Table 15(*l*). Specimens, therefore, were tack-welded, broken apart, and bent; other specimens were drilled and bent until cracks appeared around the drilled holes. The broken weld edges would bend farther than the drilled holes before cracks appeared. Other plates were tack-welded on the edges, drilled, riveted, and bent. The rivets sheared before any cracks appeared in the welds. It was proved, therefore, that the tack-welding was not detrimental to the structure in comparison with the other acceptable processes of fabrications. This was not intended to, and did not, touch on the question of welding for transmission of stress.

Stress-strain data for the silicon steel used in the towers of the Golden Gate Bridge are listed in Table 20, the average chemical composition (previously discussed) being given in Table 15(*l*). Specifications for standard mill tests required that the yield point, or the drop of the beam, should be equal to, or greater than, 45 kips per sq in. The proportional limit was defined as the point on the stress-strain curve at which the rate of deformation is 50% greater than it is at the origin.

Sketches of the test coupons, and their standard position in plates and angles, are shown in Fig. 33. These specimens were made about 0.505 sq in. in diameter, except for material 0.5 in. thick, or less; in the latter case the

TABLE 20.—Check Tests on Silicon Steel in Towers, Golden Gate Bridge

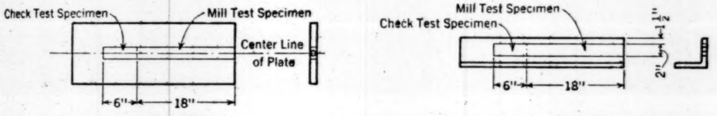
Test No.	Metal from top or bottom of ingot* (1)	Dimensions, in inches (2)	Unit Stresses, in Kips per Square Inch			Modulus of elasticity, in millions of pounds per square inch (6)
			Yield point (3)	Proportional limit (4)	Ultimate strength (5)	
(*a*) Sheared Plates (See Fig. 33 (*a*))						
1	Top	15/16 in. thick	53.5	48.5	94.4	28.9
1	Bottom	15/16 in. thick	48.5	48.5	82.0	29.5
2	Top	7/8 in. thick	55.0	55.0	90.3	29.2
2	Bottom	15/16 in. thick	50.0	50.0	82.1	28.2
3	Top	15/16 in. thick	51.3	51.3	89.9	27.4
3	Bottom	7/8 in. thick	47.5	44.3	87.6	26.5
4	Top	7/8 in. thick	52.2	50.9	93.0	28.7
4	Bottom	7/8 in. thick	52.0	50.0	91.5	30.0
5	Top	7/8 in. thick	45.0	41.2	87.0	28.4
5	Bottom	7/8 in. thick	45.0	41.2	85.6	30.4
6	Top	7/8 in. thick	50.0	48.8	87.4	29.9
6	Bottom	7/8 in. thick	47.2	44.8	87.8	29.7
7	Top	7/8 in. thick	57.5	53.7	97.1	30.2
7	Bottom	7/8 in. thick	52.2	51.6	85.0	28.8
8	Top	5/8 in. thick	51.3	51.3	88.2	29.5
8	Bottom	15/16 in. thick	42.5	38.0	81.6	30.0
9	Top	13/16 in. thick	43.8	35.6	87.1	29.6
9	Bottom	13/16 in. thick	42.6	37.5	83.9	28.0
10	Top	7/8 in. thick	51.3	42.5	95.1	28.4
10	Bottom	1/2 in. thick	52.0	52.0	84.9	29.2
11	Top	3/4 in. thick	47.2	44.8	83.4	28.3
11	Bottom	3/4 in. thick	46.0	46.0	83.7	30.1
(*b*) Universal Mill Plates (See Fig. 33 (*a*))						
12	Top	15/16 in. thick	46.0	42.6	83.4	29.3
12	Bottom	15/16 in. thick	46.0	43.3	84.8	29.4
13	Top	15/16 in. thick	49.7	46.9	88.6	29.3
13	Bottom	15/16 in. thick	48.5	47.2	80.3	30.1
14	Top	15/16 in. thick	43.8	35.0	86.4	28.1
14	Bottom	1/2 in. thick	47.9	47.3	82.7	28.3
15	Top	7/8 in. thick	47.1	43.6	84.3	30.2
15	Bottom	7/8 in. thick	45.8	45.8	86.0	30.2
16	Top	7/8 in. thick	48.5	43.5	91.3	30.2
16	Bottom	7/8 in. thick	47.1	42.6	85.3	29.8
17	Top	5/8 in. thick	50.0	48.7	88.2	30.3
17	Bottom	7/8 in. thick	47.2	43.4	88.4	30.9
18	Top	13/16 in. thick	51.0	41.2	93.4	31.2
18	Bottom	3/4 in. thick	(†)	39.2	88.8	30.1
19	Top	3/4 in. thick	50.0	45.0	93.0	28.8
19	Bottom	5/8 in. thick	46.4	42.8	81.6	27.5
20	Top	5/8 in. thick	50.0	48.8	92.0	27.6
20	Bottom	5/8 in. thick	47.5	45.1	85.3	26.9
21	Top	5/8 in. thick	51.5	42.7	93.9	29.6
21	Bottom	1/2 in. thick	50.8	41.2	87.6	31.8
22	Top	1/2 in. thick	50.0	43.3	88.5	25.9
22	Bottom	1/2 in. thick	50.0	41.0	86.8	26.7
(*c*) Angles (See Fig. 33 (*b*))						
23	Top	8 by 8 by 5/8	(†)	34.0	86.6	32.0
23	Bottom	8 by 8 by 7/8	42.5	36.6	82.8	28.1
24	Top	8 by 8 by 5/8	46.3	37.5	87.8	33.4
24	Bottom	8 by 8 by 7/8	48.5	48.1	87.4	29.1
25	Top	8 by 8 by 5/8	(†)	46.4	99.1	30.6
25	Bottom	8 by 8 by 5/8	(†)	45.0	96.5	31.4
26	Top	8 by 8 by 5/8	51.0	42.3	94.9	32.7
26	Bottom	8 by 8 by 5/8	46.0	40.2	84.1	28.7
27	Top	8 by 8 by 5/8	50.0	38.5	89.4	31.3
27	Bottom	8 by 8 by 5/8	48.9	35.1	89.1	30.8
28	Top	8 by 8 by 5/8	43.8	33.4	82.1	30.5
28	Bottom	8 by 8 by 5/8	45.0	30.9	82.1	31.3
29	Top	8 by 8 by 5/8	(†)	35.0	89.6	31.3
29	Bottom	8 by 8 by 7/8	47.5	47.1	85.8	29.1
30	Top	6 by 6 by 11/16	(†)	37.3	89.9	29.0
30	Bottom	6 by 6 by 11/16	(†)	36.3	84.8	32.4
31	Top	6 by 6 by 11/16	(†)	33.0	89.4	29.4
31	Bottom	6 by 6 by 11/16	(†)	36.1	88.7	29.6

TABLE 20.—(*Continued*)

Test No.	Metal from top or bottom of ingot* (1)	Dimensions, in inches (2)	Unit Stresses, in Kips per Square Inch			Modulus of elasticity, in millions of pounds per square inch (6)
			Yield point (3)	Proportional limit (4)	Ultimate strength (5)	
			(*d*) Flats (See Fig. 33 (*a*))			
32	Top	7 by 1/2	54.3	35.1	100.1	28.3
32	Bottom	7 by 1/2	53.8	51.5	93.8	28.9
33	....	5 1/2 by 7/8	54.7	53.5	91.8	30.5
34	....	7 by 3/4........	(†)	40.0	104.5	31.1
35	....	6 by 3/4........	57.5	56.6	93.0	29.3
36	....	6 by 3/4........	57.5	55.0	94.9	27.4
37	....	6 by 5/8	52.6	50.7	96.8	29.3
38	....	3 1/2 by 5/8	57.6	56.4	91.3	29.3
39	....	3 1/2 by 5/8	52.5	51.3	85.7	31.3
40	....	3 1/2 by 5/8	56.4	53.9	90.9	27.5
41	....	8 by 1/2	54.8	54.8	93.3	29.4

* Not necessarily from the same ingot in any test. † Indefinite yield point.

specimen was turned to the largest diameter possible. Stress-strain curves for a series of comparative tests of silicon and nickel steel used in the Delaware River Bridge are given in Fig. 34, the corresponding chemical and physical characteristics for this series being listed in Table 15(*o*). Fig. 35 illustrates the type of specimen tested in Table 15(*o*) and in Fig. 34. The letters, *C*, *B*, and *A*, indicate the location of coupons corresponding to Items Nos. 63, 64, and 65, respectively, in Fig. 34.



(*a*) PLATES (*b*) ANGLES

Fig. 33.

Heat-Treated Eye-Bars

Heat-treated eye-bars are made in two grades: (1) Grade HT—high tensile strength (Table 15(*f*)); and (2) Grade HT Special—special high tensile strength (Table 15(*g*)). When acceptance of eye-bars depends on the results of full-sized tests, the standard requirements are as shown in Table 21. When chemical and physical properties are specified, the following standards

TABLE 21.—Specifications Based on Full-Sized Tests

Description	Grade HT*	Grade HT Special†
Tensile strength, minimum, in kips per square inch................	80	105
Yield point, minimum, in kips per square inch.....................	50	75
Percentage elongation in 18 ft., minimum.........................	8	5

* See Table 15 (*f*). † See Table 15 (*g*).

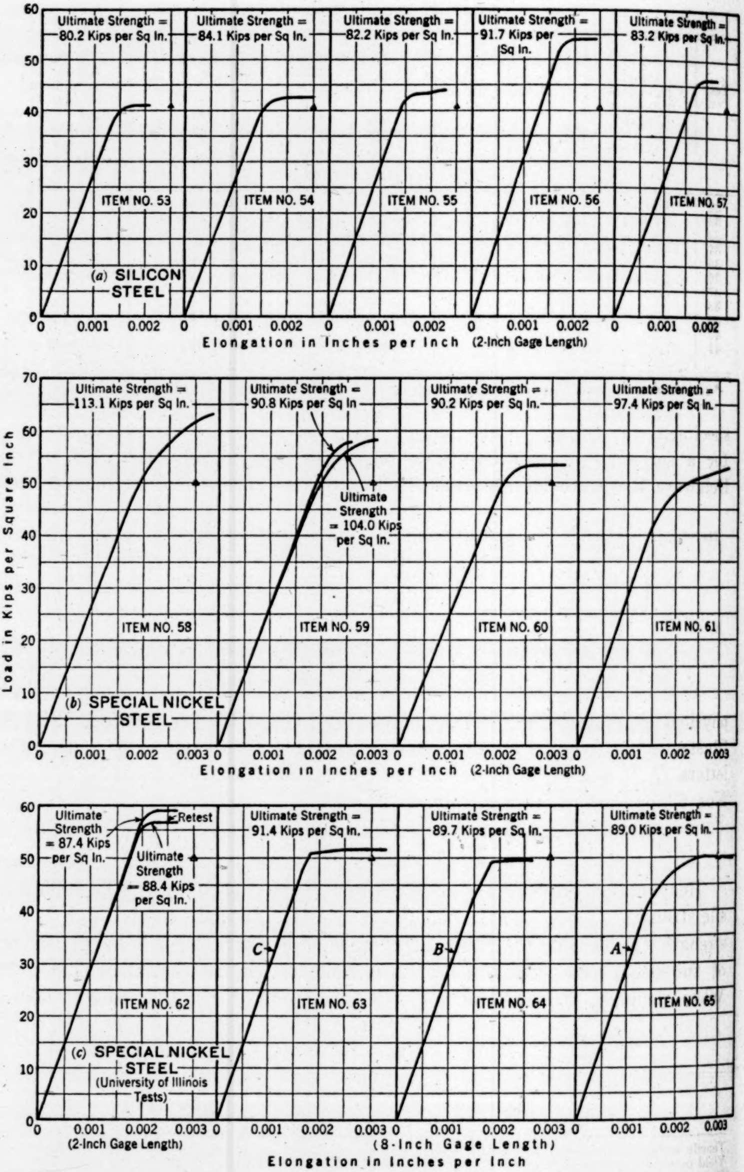
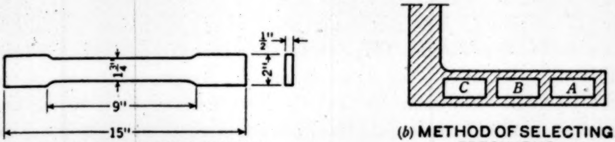


FIG. 34.—COMPARATIVE TESTS OF SILICON AND NICKEL STEEL COLUMNS, DELAWARE RIVER BRIDGE (SEE TABLE 15).

are satisfactory and will result in eye-bars that will meet full-sized test requirements:

	For high-tensile strength eye-bars
Percentage of carbon.......	0.30 to 0.40
Percentage of manganese....	0.50 to 0.75
Tensile strength, minimum, in kips per square inch....	70
Yield point, minimum......	0.5 of the actual tensile strength



(*a*) MACHINED SPECIMEN

(*b*) METHOD OF SELECTING SPECIMENS

FIG. 35.—STANDARD TEST SPECIMENS, DELAWARE RIVER BRIDGE.

All specifications for chemical and physical properties of specimen tests for the special high-tensile grade steel must be referred to the manufacturer of the eye-bars. The modulus of elasticity of all grades of eye-bars is approximately 29 000 000 lb per sq in.

Brinell tests for hardness are sometimes required on heat-treated bars. They are made to determine the uniformity of hardness throughout the length of the bar. These tests are for information only, and do not form a basis of acceptance.

Method of Manufacture.—The process of making heat-treated eye-bars is, briefly, as follows: The head on one end of a bar is formed by repeating the operations of heating, upsetting, and rolling, until the correct proportions are attained. A hole is punched in the head while hot, somewhat smaller than the finished bored hole, and the bar is allowed to cool. After cooling, the bar is sheared to the correct length, turned around, and a head is formed on the opposite end of the bar in the same manner as described previously.

This process insures a quantity of metal adequate to form the head completely. Excess metal is squeezed out between the dies on the two sides of the head and, later, is sheared off while hot. The rough edges on the sides of the head due to this shearing, are chipped smooth.

Heat-treatment of high-tensile and special high-tensile eye-bars consists of heating, quenching, and tempering. After removal from the tempering furnace, the eye-bars are cooled in the air. The bars are straightened, and the pin-holes in the two heads are bored simultaneously. When Brinell tests are made, they are made just prior to boring.

Full-Sized Tests.—Orders for eye-bars customarily include a small percentage (about 3) of full-sized test bars which are tested to destruction in a 2 000-ton hydraulic horizontal testing machine. Test bars are selected from the finished bars except that those from bars too long for the testing machine are selected from the full-length bars after the heads on one end have been formed. They are then cut, and the second head is formed to make the longest bars that can be tested.

The maximum distance, from center to center of pin-holes, with the ram of the testing machine at the beginning of the stroke, is 42 ft 6 in. The minimum is 3 ft 9 in. The length of stroke is 8 ft. The desired maximum length of the test bar is 35 ft.

Details of Eye-Bars.—High-tensile and special high-tensile heat-treated eye-bars are made in widths of 10 to 16 in. and in lengths, from center to center of pin-holes, of from 72 ft 7 in. (maximum) to 13 ft 0 in. (minimum). The excess thickness of head over body is about $\frac{1}{16}$ in. for Grade HT and $\frac{1}{8}$ in. for Grade HT Special. The minimum ratio of pin diameter to width of bar should be 0.875 for Grade HT, and 0.93 for Grade HT Special eye-bars. The pin clearance should be $\frac{1}{32}$ in. for Grade HT and $\frac{1}{50}$ in. for Grade HT Special. They can be made in sizes smaller than 10 in., but it is usually not economical to do so. Eye-bars more than 60 ft long are difficult to handle, and the cost of their manufacture is increased.

CONCLUSIONS

The use of special alloy steels has become increasingly prevalent, and is covering a wider range of structures as engineers are becoming more and more acquainted with its advantages.

ACKNOWLEDGMENTS

The material which forms the basis of this paper was collated from a number of articles in *Engineering News-Record* and various papers published by the American Society of Civil Engineers, especially the Progress Report of Sub-Committee No. 2, Committee on Steel, of the Structural Division on Structural Alloy and Heat-Treated Steels[81]. The writer is also indebted to Leon S. Moisseiff, Jonathan Jones, and C. F. Goodrich, Members, Am. Soc. C. E., for information in their files.

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[81] *Proceedings*, Am. Soc. C. E., March, 1936, p. 361.

EVOLUTION OF HIGH-STRENGTH STEELS USED IN STRUCTURAL ENGINEERING

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SYNOPSIS

The development which led to the use of higher strength steels for bridges in the United States is outlined in this paper.

The rapid increase of population by natural growth and immigration and the industrialization of the country demanded transportation facilities for freight and men. Heavy trains required bridges of great strength, and crowded cities demanded structures with many tracks and wide roads. The wealth of the country and the growth of automotive transportation called for comfort and speed and led to the building of large bridges of long spans. Engineers, by the use of higher strength steel, were able to construct many great structures. The reasons governing the design of most large American bridges afford an excellent illustration of the evolution of the use of higher strength steels, and are related in detail.

The practical advantages of higher strength steels are feasibility of construction and economy of cost. They are readily seen. The limitations of these steels are not quite so apparent. They are discussed at length from their technical and practical aspects.

INTRODUCTION

During Man's long struggle for existence with the forces of Nature he has acquired a mass of knowledge about phenomena around him and a number of ways of manipulating and forming the many materials among which he found himself. Out of the want of Man grew his knowledge of Nature and his ability to appraise and apply the strength of materials. Under the incessant pressure of his surroundings he acquired tools and skill in handling them and rules and numbers for Nature's behavior. In the course of time he learned that by applying these rules the same task could be done with less labor, at the same time, creating a better product; and thus labor began to be co-ordinated, rules pertaining to natural phenomena grew into science, and the planning and execution of work became engineering.

In the course of this evolution, engineering became the application of scientific principles to meet the practical needs of Man in the best way, with the least expense of labor and materials. It is based on necessity and economy, which are two hard masters to please. At times, one or the other prevails, but necessity is the more compelling.

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[82] Cons. Engr., New York, N. Y.

Transition from Wrought Iron to Steel

The meandering road of evolution is well illustrated by the changes that occurred during the transition from wrought iron to medium steel and from that to soft steel, in the United States and Central Europe.

Building Construction.—At about the beginning of the Twentieth Century steel came into use for frames and floors of multi-storied buildings. The growing demand for office buildings, hotels, and high-class apartment houses foretold the consumption of an enormous tonnage of steel for construction purposes. It became clear to the makers of steel that, although there would be a demand for large tonnage, there would also be a demand for a low-cost steel to compete with other building materials. It was realized that high strength and special qualities of steel were not required for this type of structure, which transmitted its loads directly to the foundations and which was not subject to impacts and reversals of stress.

It was important, from the point of view of the consumer, that the fabricated steel should be bought at a low price and that it be dependable; and from that of the producer, it was important that it should be manufactured in large tonnage without special precautions and manipulations and that it could be easily punched and fabricated. Above all, it was important that the fabricated product should be acceptable to the consumer and that there should be no rejections. This meant the avoidance of interruptions and the advantages of mass production. The demand, in the case of such relatively simple structures, was not for a steel of higher qualities developed like a race horse—highly nervous in behavior—to meet special requirements, but a dependable material that could stand abuse in fabrication—developed like a truck horse and possessing as little temperament. This was achieved by abandoning the medium steel that had prevailed before that period for bridges and adopting a soft steel of an average tensile strength of 60 kips per sq in., and a yield point, then known as "elastic limit," of one-half the tensile strength, as the standard material for engineering structures. To be more exact, the limitations for tensile strength were from 55 to 65 kips per sq in.

Steel Manufacture.—The demand at the beginning of the Twentieth Century for steel led to far-reaching re-organization of the steel manufacturing companies. A process of agglomeration set in; smaller mills were consolidated into large units, and in the process of co-ordination, many were abandoned in favor of enormous plants with capacity for large tonnage. Dispersed and competing fabricating companies were united in large groups which were more than able to compete with the remaining smaller independent organizations.

It cannot be regarded as a mere coincidence that while such re-organization took place in the United States, a similar process was effected at the same time in Central Europe, especially in Germany. It is still less accidental that, there also, after a systematic study of the advantages of soft steel, it was adopted as the standard material. By these changes and re-organizations the manufacturers were able to offer the market a uniform and dependable steel at a reasonable cost and a wide confidence in the product was created. In fact, in rare instances only did the consumer exercise his right of inspection at the mills and shops; the material was accepted on the Manufacturers' Certificate. Time has shown that, in general, the development was in the right direction and that the confidence in the product was justified.

Transportation.—The rapid growth of the United States and the great development of its industries brought with it enormous demands for transportation facilities for freight and men. One has but to consider that during the fifty years from 1880 to 1930 more than 27 000 000 people immigrated into the United States and that, in the same period, the total population of the country increased from 50 000 000 to 123 000 000, to realize the tremendous impetus given to transportation. The revenue of the railroads increased 64% in the first decade of the century and in the first two decades it increased three and one-half times. The growth of commerce and industry produced a demand for transporting the increased tonnage of freight and the greater number of passengers created by the new conditions. This demand was met by the double-tracking of existing railroads and by the use of longer trains and more heavily loaded cars, which required heavier locomotives with greater axle loads. By about 1900 the heaviest axle loading specified for railroad bridges was 43 000 lb; the weight of one engine and tender was 145 tons and the load following it 4.2 kips per lin ft. The 1935 Specifications for Steel Railway Bridges of the American Railway Engineering Association call for an axle loading of 72 kips per lin ft, followed by a uniform load of 7.2 kips per lin ft. The weight of one locomotive and tender is 256 tons. The increase in axle load is two-thirds and that in the engine, three-fourths. The comparison of these specified figures illustrates vividly the growth in the live load of modern railroad bridges.

Bridge Construction.—The double-tracking of railroads brought with it the building of new bridges for heavier loads. At the same time, the increase in revenue-producing freight and passenger traffic made it financially worth while, in some instances, to extend railroad lines and to bridge wider rivers, and, in other instances, to shorten routes and reduce grades. All these effects of the growth of railroads called for stronger and longer bridges.

The larger number of railroad bridges of the period were built of the new standard soft steel of a tensile strength of 55 to 65 kips per sq in. It was then known as the "railroad bridge grade" of steel. Whenever bridge engineers desired a higher grade for their bridges (largely because they felt that the severe strains and impacts to which the bridges were subjected justified a stronger steel) the mills rolled a special grade of a tensile strength of 60 to 70 kips per sq in., and a yield point of one-half that of the ultimate strength. The proximity of the tolerance limits of this steel to the standard steel being 5 kips apart, permitted overlapping and gave the mills a chance to use the higher melts of the standard steel for the special steel.

Demand for High-Strength Steel

Some bridge engineers, however, were not satisfied with the two grades of steel offered by the manufacturers and insisted on a higher grade of material for their bridges. They specified a steel of a tensile strength of 62 to 72 kips per sq in., and a minimum yield point of 35 kips per sq in. The steel industry looked with scant favor on specifications which called for material different from the standard product. The demands of the engineers were not always readily met and their path was not "strewn with roses." In spite of discouragement, individual engineers adhered to their demands, and a number of larger bridges were built of this higher strength steel. Finally, it became quite usual for engineers to ask for a special carbon steel for more important bridges.

It is quite in order that a structural engineer should demand for his particular structure a material which physically may be better suited for its task, provided he can obtain it from the manufacturers at a price that will justify it economically. It is also in order that the manufacturers should not wish to disorganize their regular procedure of production, both in the mills and in the shops, unless the demand is great enough to compensate them for it. As the demand for the higher quality of material becomes larger and steadier the manufacturers finally see the necessity and the reasonableness of acceding to the demands of the consumers, and they realize the advantage of adopting the modified material as their standard.

Specifications.—More recent changes in the economy of the commercial and industrial life of the country and, no doubt to some extent, the competition of reinforced concrete, have led to the consideration of a slightly stronger steel which will enable engineers to produce their structures more economically. The tendency has finally crystallized in the adoption of a specification by the American Society for Testing Materials (Standards Serial A7-34), which forms the basis of the Specifications for Steel Railway Bridges of the American Railway Engineering Association.

These specifications are a product of the co-operation of the consumers and the producers, the designing engineers, and the mills. The specified material is practically the medium steel of 1903. Its physical qualifications are: Tensile strength, 60 to 72 kips per sq in., and yield point, one-half the ultimate strength, but not less than 33 kips per sq in.

Economy.—It is understood that the manufacture of this steel will cause no increase in cost to the mills. A slight increase in the cost of fabrication in the shops may be expected, due to the greater wear and tear on tools and machinery. The same material will probably be used for all kinds of structures and will thus become the standard American structural steel.

Rapid Mass Transportation.—While the foregoing developments in railroad transportation were progressing and exerting their effect on railroad bridges, similar and still more important changes were modifying life in cities and affecting the character of city bridges. The extraordinarily rapid growth of population in cities due to immigration and the inflow from farm to city produced overcrowding and overflow into suburbs. The effect on the means of transportation is readily understood. The mechanism of commerce and industry is based on the co-ordination of many agencies, which can function properly only when the precision of time is observed. Many people must come to the city—to mills, shops, stores, and offices—and they must come there within a definite time on every working day. Moreover, the time is the same for each of them, within an hour or two. They also must return to their homes within other hours as closely set. Thus, the busy life of the city has produced the term, "rush hours." To transport this great and bustling mass of people, electric cars were running on busy streets and, later, entire trains were operated over elevated or subway structures, to achieve high-speed traffic uninterrupted by crossings. Larger and heavier cars were required and longer and longer trains were operated, attaining a length of ten cars in the Subway System of New York City. The higher speed of rapid-transit trains increased the danger of collisions and telescoping, and the underground operation of these trains made the adoption of all-metal, fire-proof cars imperative. All these requirements added weight to the cars. Thus, the weight of rapid-transit cars filled with passengers grew from 1 to 2 kips per lin ft of train.

In some cities the cars and trains had to be brought in over large streams requiring bridges with long spans. The bridges had to provide sufficient passage for cars and trains converging from many directions, and multiple tracks had to be provided on the bridges. At the same time, the increase in the wealth of the country justified the replacement of slow ferry traffic by long-span bridges. The growth of cities and the increased density of their population demanded provision for much heavier live loads than the old-time city bridge. It also demanded provision for the future growth of the frequency and continuity of these live loads.

Automotive Transportation.—During the twenty years, 1916 to 1936, a new factor in transportation began to exert a marked influence on the financing and building of bridges which, properly, should receive consideration. Favored by the increase of wealth due to the World War, the enormous development of automotive transportation of passengers, as well as of freight, brought with it new demands on bridges. Many capacious first-class highways were built throughout the United States and millions of automobiles are rolling over them at ever-increasing speeds. The new mode of travel by machine differs much from the old way by horse-drawn vehicle. It is not limited to immediate neighborhoods; its mileage is unbounded. It is swift and impatient; time has become essential. The automobilist, therefore, is willing to pay for his undelayed travel; and he does pay.

LONG-SPAN BRIDGES

Toll Bridges.—The economic aspect of bridges thus took a new turn because of automobile travel. Although toll bridges were built during a hundred years, all over the country, they were able to pay on the investment only because the structures were relatively small and cheap to build, and the low toll that the traffic could bear was sufficient to make the enterprise successful. Larger bridges could not be built on the expectations of income from tolls before the arrival of the automobile. The Brooklyn Bridge is a fair example. Gradually, the older toll bridges were being taken over by the State authorities and made free or replaced by modern bridges. All this was changed by automotive traffic. The unlimited traveling radius of the automobile and its high speed multiplied the number of vehicles on the highways and the higher standard of operating expenses made it possible and reasonable to charge substantial tolls.

The financial possibilities of building first-class bridges of longer spans were changed by the new factor completely. Where municipalities and Government agencies heretofore could build bridges only by borrowing funds, carrying and amortizing the bonds and maintaining the structures from general taxation, the possibility now offered itself to maintain the bridge, pay the interest, and amortize the bonds within a reasonable number of years from the income collected by tolls. The opportunity was quickly seen and taken. Practically all long-span highway bridges built in recent times are toll bridges. All the large highway bridges have been financed on the basis of toll income: The Delaware River, the Ambassador, the bridges of the Port of New York Authority, the New Orleans, Tri-Borough, Golden Gate, and San Francisco-Oakland Bay Bridges, and numerous less costly structures all over the country.

With this development new problems in the economic relations of bridges appeared. It was not enough to build a safe, substantial bridge of good appearance to accommodate present and future traffic but, first, the structure had to be designed not to cost more than the traffic would bear, and, for privately built bridges, a fair profit had to be earned. The full import and weight of economic design and construction made itself felt in the planning of toll bridges.

Highway Bridges for Motor Traffic.—With the development of highway bridges for automotive traffic, new conditions were created. Although the live load per lane of traffic on a bridge is lighter for automotive travel than for rapid-transit trains or trolley lines, the dead load of the bridge is greater. Automotive travel requires a floor with a continuous non-skidding surface, preferably fire-proof. Heavy automobile trucks have high axle concentrations which must be sustained at any point of the roadway by the pavement and its supports. The dead load of highway bridges has increased as compared with the live load. This shift of loads exerts an influence on the selection of the type of the most suitable and economical structure. Suspension bridges display their best advantages with high uniform loads and, therefore, have come to the foreground. Heavy dead load and relatively light live load also favor higher strength steels.

Economy of Materials.—The heavier live loads and the longer spans of bridges placed before bridge engineers the problem of technical performance. The question arose whether the available engineering materials were adequate and efficient to sustain the great forces developed in large bridges. Necessity demanded a material stronger than the available structural steel.

It is an essential characteristic of a bridge that it has a tendency to use stronger materials in its construction. It spans a horizontal distance by transferring the gravitational forces on the span to its abutments. The magnitude of the resisting forces increases with the weights on the span and with their distance from the bridge abutments. It is evident that the longer the span the heavier will be the bridge and the greater will be the forces required to uphold the equilibrium. To resist these forces acting through the material of the bridge, either more material (and, therefore, still more weight) is required, or a material of higher strength must be substituted which will resist the greater forces without adding to the weight of the structure. This states in different words what is known to all engineers, that with an increase of span a bridge becomes heavier at a rate greater than in a simple proportion. Thus, in the longest highway bridges, the ratio of dead load to live load becomes 5 to 1 in the George Washington Bridge and 5.25 to 1 in the Golden Gate Bridge.

Heavy and long-span bridges, therefore, will require excessively large members to meet the enormous forces developed in them. The sectional areas and weights of the members become difficult to fabricate, to connect, and to handle. The structure appears to outgrow its proportions, which are intimately tied not only to the bridge as completed but also to the machinery and capacity of the fabricating plants and the available transportation facilities. Finally, the cost of the structure becomes excessive and defeats its object economically. The application of suitable and stronger steels frequently makes the building of the bridge feasible and financially possible.

The aim of all bridge engineers, therefore, has always been to reduce the weight of the longer bridges by utilizing materials of higher strength. The history of bridge engineering shows that almost in all instances of longer bridges their designers have endeavored to obtain a material which, either by its strength or its dependability, would enable them to allow higher intensities of stress, and thus reduce the weight to be carried. Generally, this has been accomplished with more or less success.

Reasons for Adopting Higher Strength Steels

So far the general outline of the play of social and economic forces which has led to the demand for stronger steel has been discussed in general without direct application to built structures. Generalizations afford a bird's-eye view and give a historical panorama, but they do not supply the direct information and practical understanding which concrete examples offer. These will be brought out in the following paragraphs.

In relating the various reasons which have led engineers to adopt a higher strength steel for the several structures which will be discussed, the writer wishes to call attention to an observed phenomenon in the records of human investigations and history. In the meandering of human thought and in the constant play of forces, interests, and changing conditions, events and decisions evolve neither along smooth straight lines nor along continuous curves. Many a jolt is experienced and frequently the chain of events is interrupted. It is only by looking backward through a duration of time that people are able to perceive the general tendency and its flow and to describe it as sequences and logical action. Thus, the observer on the shore may see far away a smooth sea and a placidly moving ship.

Limitations to Progress.—The structural engineer in the past has had little choice in his search for the higher strength material. Laboratory tests, no matter how carefully made, can only indicate the quality of the product and establish its probable behavior when produced under mill conditions in large tonnage. Engineers have also learned, sometimes to their sorrow, that the mass product may develop failings and defects that cannot be anticipated in the laboratory samples, and even in small ingots. The engineer has often had to act in the rôle of stimulator and pioneer, and the manufacturer has brought out the new material with more or less enthusiasm and apprehension. It involves a risk in money and reputation to contract under specifications, which necessarily are rigid, to roll and fabricate the new steel. A higher price must necessarily be set to cover the cost of uncertainties and novelty to equipment and personnel.

Several instances may well illustrate the reasons which prompted the introduction of the use of nickel steel for bridges. At the beginning of the century two bridges over the East River—the Queensboro and the Manhattan—were being planned by the Department of Bridges of New York City.

Eye-Bars in the Queensboro Bridge.—The Queensboro Bridge is a double cantilever structure with two main spans of 984 and 1 182 ft. Originally, it was designed for two rapid-transit tracks, four trolley tracks, a roadway of 35.5 ft, and two sidewalks; after the letting of the contract two more rapid-transit tracks were added. In accordance with the loads then established, the specified congested live load was 16 kips per lin ft, and the average dead load, 35 kips per lin ft. The dead and live load stresses in the panels of the upper chord attain a maximum stress of 19 000 kips. Using the proportional unit stress which could be allowed on eye-bars of medium steel (20 kips per sq in.) about 950 sq in. of bars would be required. As built, the nickel-steel eye-bars have a greatest sectional area of 680 sq in., consisting of 20 bars, 16 by $2\frac{1}{8}$ in. An increase of 40% in sectional area would thus be required if the bars were made of carbon steel. As the largest eye-bars available were used, it meant a proportional increase in the number of bars. The increased weight of the tension members would add to the dead load and, in its turn, would require more bars. Structurally, it meant that the width of the chord and the length of the pins would have to be increased, which again would lead to further increase in dead load.

An increase of 40% in the number of bars would have required the widening of the upper chord in the panels of maximum stress from its present width of 90 in. to more than 120 in. The greater number of bars in the chord, in turn, would involve the widening of truss members to unwieldy proportions.

The engineers discussed various alloy steels with metallurgists and manufacturers, and, finally, the Carnegie Steel Company, in 1902, made a heat of steel containing 3.26% nickel, 0.21% carbon, and 0.53% manganese. From it 21 eye-bars were manufactured by the American Bridge Company in sizes of 6 by 1 in., 6 by 2 in., 8 by $1\frac{1}{2}$ in., and 10 by $1\frac{1}{4}$ in. Tests were made of specimens and of full-sized eye-bars, annealed and not annealed. The results of the tests indicated that a good structural material had been obtained which could be depended upon for bridge purposes and which, after further experimentation, could be specified to meet a yield point of 48 to 50 kips per sq in. and an ultimate strength of 100 kips per sq in.

In 1903 a further set of eight bars of basic open-hearth steel, containing 3.24% nickel, was made and tested. When tested, the full-sized eye-bars developed an elastic limit of 46 to 49 kips per sq. in., and an ultimate strength of 84 to 87 kips per sq. in. This gave assurance that a high-strength steel could be manufactured into eye-bars on which, for dead load and regular live load (8 kips per lin ft), a tensile unit stress of 30 kips per sq in., and for dead load and congested live load (16 kips per lin ft), a stress of 39 kips per sq in., could be allowed. During the construction of the bridge the bars were ultimately made to meet a minimum elastic limit of 48 kips per sq in., in full-sized tests, and an ultimate strength of 80 to 85 kips per sq in.; elongation in 18 ft, 9%; reduction of area, 40%; and bending, 180° around a pin of twice the thickness[83]. In this first application of nickel steel to bridge construction about 6 000 tons of eye-bars and pins were consumed.

The introduction of nickel as an alloy to the steel for the eye-bars of the Queensboro Bridge offers a good illustration of the causes which have led to the use of high-strength steels for bridges. A stronger steel was desired to hold the number of eye-bars and their section within manageable limits. The steel had to be as dependable and as workable as the standard bridge steel. A percentage of about 3.25 of nickel added to the steel supplied the necessary strength and quality, and this material was adopted for the eye-bars of the bridge. The type and general dimensions of the structure were not affected by the selection of the stronger steel.

Stiffening Trusses in the Manhattan Bridge.—Another illustration, but of a different character, of the use of high-strength steel is presented by the events which led to the use of nickel steel for the stiffening trusses of the Manhattan Bridge in New York City. It was planned at the same time as the Queensboro Bridge and was designed for four rapid-transit tracks, four trolley tracks, a 35.5-ft roadway, and two footwalks. For this capacity a congested live load of 16 kips per lin ft was adopted.

The dimensions of the bridge and its dead and live load created a problem for the engineers. The bridge had to clear the East River between pier lines and the length of the main span was determined by the location of the bridge and the width of the river between pier-head lines at 1 470 ft. A desire to present a well-balanced appearance made the side spans 725 ft. Before the development of the deflection theory of suspension bridges a reasonable design of stiffening trusses for a bridge with suspended side spans of similar dimensions, and a live load as heavy as two-thirds that of the dead load, would require trusses of great depth and extraordinary chord sections to come within allowable unit stresses and limited deflections as then computed. The introduction of the deflection theory led to a more correct determination of deflections and made it possible to design the bridge with relatively shallow trusses. Their depth of 24 ft was determined by the cross-section of the bridge, which has a double deck, and the required clearance for the rapid transit trains on the lower deck. It may be pointed out that this was the first suspension bridge of its kind and character designed in accordance with the deflection theory and built with shallow trusses.

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[83] "Nickel Steel Eye-Bars for Blackwells' Island Bridge" (later, Queensboro Bridge), by William R. Webster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 289.

The behavior of a suspension bridge is such that, given the general dimensions and the dead and live loads, its stiffness will be controlled more by these factors than by the stiffness of the suspended truss itself. This is explained by the fact that in the computations of the stiffening truss, the moment of inertia is not directly proportional to the deflection, and exerts a relatively small influence. However, it must follow such deflections as the system of forces on the cables and the elasticity of the latter will compel. In other words, the truss must bend in conformity with the cable deflections and it can only mitigate them. It is well known that in a beam of given depth deflecting to a given curve the stresses in the chords will be proportional to the degree of curvature. With the general dimensions of the Manhattan Bridge and its dead load a congested live load of 16 kips per lin ft will develop stresses of such intensity that the standard medium structural steel could not sustain them within its elastic limit. True enough, a uniform congested load moving on many lanes, either in unbroken or broken lengths, will rarely assume the positions which will cause maximum stresses in suspension bridges with side spans. In accordance with this argument the designers have given full consideration to the probability of maximum stresses in the stiffening trusses and have allowed high unit stresses on the selected material. The adopted material, however, had to have an elastic limit high enough at no time to cripple the trusses by exceeding that limit.

After the results obtained from the tests of nickel-steel eye-bars for the Queensboro Bridge, it was a matter of course for the engineers to turn to nickel steel as a material which can develop a sufficiently high elastic limit and ultimate strength for the moments and deflections of the stiffening trusses. Accordingly, the specifications for the nickel steel for the stiffening trusses of the Manhattan Bridge called for the following physical requirements:

Ultimate strength	85 to 95 kips per sq in.
Elastic limit, minimum........	55 kips per sq in.
Percentage of elongation in 8 in.	$\frac{1\,600 \text{ (minimum)}}{\text{Ultimate strength}}$
Percentage of reduction of area.	40 (minimum)

As shown by the results of the acceptance tests of all the heats rolled, the material furnished actually developed an average elastic limit of 55.9

to 59.8 kips per sq in., and an average ultimate strength of 87.2 to 90.8 kips per sq in. Having in mind the high intensity of loading assumed and the low probability of the positions of maximum stresses, a stress of 40 kips per sq in. in tension, properly reduced for compression, was allowed on this material. The permissible unit stress was thus 73% of the elastic limit of the steel. Altogether 8 000 tons of nickel steel were consumed in the Manhattan Bridge.

The recently proposed revised A. S. T. M. Specifications for Nickel Steel differ from those of the Manhattan Bridge in 1906 only by raising the tensile strength to 90 to 115 kips per sq in. and by lowering the reduction of area to 30 per cent.

In the case of the Manhattan Bridge the introduction of a stronger steel was due to a necessity which came from the character of the type of the structure and its dimensions and loads. If there were no steels of higher elastic limit available the bridge could not have been designed as it was. In this case the type and general dimensions of the structure were substantially affected by the selection of the stronger steel.

Nickel Steel in the Quebec Bridge.—In the construction of the present Quebec Bridge the Board of Engineers, which was created after the failure of the first bridge during its erection, make painstaking and careful studies of all phases of the work. It made extensive tests of carbon and nickel steel eye-bars and columns and, after careful considerations, it incorporated 16 000 tons of nickel steel in the structure. In the report[84] on the Quebec Bridge, the Board states: "Nickel steel was used wherever it would effect a saving in weight of the structure as a whole, and, by this means, notwithstanding its higher price, be a factor in reducing total cost."

Nickel Steel in the Delaware River Bridge.—The Delaware River Suspension Bridge, between Philadelphia, Pa., and Camden, N. J., was designed for a congested live load of 12 kips per lin ft. It has a span of 1 750 ft and a reinforced concrete roadway 57 ft wide; its design dead load is 26 kips per lin ft. The same considerations that led to the adoption of nickel steel for the stiffening trusses of the Manhattan Bridge led to the use of a high-strength steel for the trusses of the Delaware River Bridge. Realizing the need of high-strength steels in bridge building, the engineers of the Delaware River Bridge had in mind the possibility of opening the field for such steels, in order to encourage steel makers to produce various suitable high-strength steels and to foster competition. The specifications for the stiffening trusses, therefore, called for a special steel having a minimum yield point of 55 kips per sq in. and a minimum tensile strength of 90 kips per sq in., without specifying a definite alloy. It gave a free opportunity to steel manufacturers to offer an alloy to meet the physical requirements. The manufacturers finally furnished a nickel steel containing an average of 3.2% nickel.

One of the peculiarities of stiffening trusses in suspension bridges is that the shear is relatively low and that the web members are relatively light.

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[84] "The Quebec Bridge": Report of the Govt. Board of Engrs., 1919.

It was found that the web members in the trusses of the Delaware River Bridge, if made of nickel steel, would under-run the desirable minimum sections, and, consequently, they were made of silicon steel. This is the first instance in which a differentiation was made in members of a truss by using higher and lower steels. A total tonnage of 5 150 tons of nickel steel was consumed for the bridge.

The San Francisco-Oakland Bay Bridge.—A still more recent illustration of the use of nickel steel is furnished by the San Francisco-Oakland Bay Bridge in which the high-strength steel was used in two structures of the same bridge for different reasons. This project consists of a twin suspension bridge over the West Bay and of a cantilever structure and nineteen truss spans over the East Bay. It is a double-deck structure. The main span of the cantilever structure is 1 400 ft and the especially heavy dead load, due to the wide roadways, made it of major importance to hold the size and weight of the truss members within reasonable limits. To minimize the sizes of the stiff members 3 400 tons of nickel steel were consumed. The reasons for using stronger steel here were purely technical and not economical.

Still another instance in which a higher strength steel was used for technical reasons is furnished by the center anchorage of the suspension structure. The West Bay is about 10 000 ft wide and is crossed by twin suspension bridges, each having a main span of 2 310 ft. For a uniform fixed load over the entire length, the structure could have been built with three main spans and two side spans; but for a moving live load of 7 kips per lin ft, the bridge would be too flexible both for the towers and for the stiffening trusses, and would thus become impassible for traffic. To limit the deflections of the bridge a central anchorage pier was built which will restrain the motion of the cables by the stability of its weight. A simple solution was found to connect the cables to the pier and to each other by positive means. The cables of each of the bridges which meet at the central anchorage are displayed from their compacted cylindrical shape the same as at the land anchorages. The individual strands that constitute the cables are laid over thirty-seven anchor shoes, each of which is engaged by a pair of eye-bars similar to the usual suspension-bridge construction. Here, the similarity ceases. The eye-bars are not connected to other bars embedded in concrete, but at their free end they are connected by pins to vertical steel plates. In this way the plates are only intermediate links connecting the cable ends of the twin bridges. The cables are thus tied to each other continuously by bars and plates of steel. The steel tie-plates are made large enough (23 by 35 ft) to extend downward to provide for a pin connection to a steel structure anchored and embedded in the masonry of the central anchorage pier. The anchorage restrains the horizontal motion of the cables. When it is considered that the pull in the cables at the central anchorage may reach 36 400 kips per cable and that the unbalanced pull may attain 10 600 kips, the importance of the tie-plates—their size and their function—can be realized. Each cable connects to four plates by means of thirty-seven pins and the plates connect to the anchorage by a pin 27 in. in diameter. The geometrical dimensions are such that the distribution of stress in the plates is of the greatest importance. Although, in accordance with theoretical computations, the strength of silicon steel plates was sufficient for the purpose, the engineers felt that excess of strength in the steel was of special value in this case and that the well-known toughness of nickel steel would be much in place. The tie-plates, therefore, were made of nickel steel, requiring about 280 tons. In this instance, merely technical considerations and not economy were the determining factors. Not only was the higher tensile strength of the material considered but also its toughness.

Manganese Steel in the Bayonne Bridge.—In 1927 the engineers of the Port of New York Authority were planning a bridge over the Kill van Kull connecting Staten Island, New York, with the mainland at Bayonne, N. J. After much study it was decided to build an arch with a single span of 1675 ft between centers of bearings. The trusses were designed to act as three-hinged arches with the lower chords as continuous ribs from end hinge to end hinge. The lower chord was designed to sustain all the dead load and part of the live load, the upper chord and the web members acting merely as a participating and stiffening truss.

The greatest reaction on the pins is nearly 31 400 kips per truss. It was desirable to limit the thickness of the assembled plates and splices for riveting purposes but, above all, considerations of weight were governing. For every pound of weight added an additional pound of steel was required to sustain it; economy, therefore, decided the issue.

At that time, advanced engineering had already recognized the fact that the function of the various parts of a structure and that of their constituent members and details, demand in some parts greater strength and in others greater stiffness. It had realized that it is good design to utilize various grades of available steel and that, by the proper use of these various steels, a better and a more economical structure can be built. Modern design, therefore, will utilize steel of various strengths. Where the usual stresses must be met and where stiffness is of importance, the common carbon steel will be used; where greater forces must be sustained, medium strength steel such as silicon will be generally utilized. Finally, where exceptionally great forces are met and the members become excessively bulky, high-strength alloy steels will be introduced. This tendency was recognized systematically in the design and the proportioning of the various parts and members of the Bayonne Arch. The main material of the lower chord is high-strength alloy steel throughout; that of the top chord is silicon steel, except the three end panels which are of carbon steel; all the web members are of carbon steel, except some which were made of silicon steel because of erection stresses; all lateral bracing and sway-bracing are of carbon steel.

For the high-strength steel an alloy containing 3.25% nickel and having a minimum yield point of 55 kips per sq in. and a tensile strength of

90 kips per sq in., was specified. In the specifications it was stated, however, that, with the approval of the engineer, the contractor might substitute for the nickel steel another steel which would meet the physical requirements for nickel steel and in all other respects be equivalent in quality. The American Bridge Company, as the lowest bidder, was awarded the contract, and it offered as an equivalent substitute a carbon manganese steel at a reduction of $15 per ton below the price of nickel steel. The carbon manganese was to have the chemical and physical qualities indicated in Table 22.

TABLE 22.—CHARACTERISTICS OF MANGANESE STEEL, BAYONNE BRIDGE

Description	Specification, Port of New York Authority	Tests of actual product§
Chemical Properties (Percentages):		
Carbon	0.40*	0.33
Manganese	1.80†	1.63
Phosphorus	0.04†	....
Sulfur	0.05†	....
Silicon	0.10 to 0.30†	0.18
Physical Properties:		
Yield point stress, in kips per square inch	55‡	58.6
Tensile strength, in kips per square inch	90‡	101.6
Percentage elongation in 8 in	$\dfrac{1\,600\ddagger}{\text{Actual tensile strength}}$	19.5
Percentage reduction of area	30‡	42.6

* Maximum allowable; preferably not more than 0.35 per cent. † Maximum allowable. ‡ Minimum allowable. § 168 melts and 968 tests.

After a careful investigation, which included tests from a heat especially rolled for the purpose, and the adoption of precautionary provisions and more rigorous requirements regarding testing and inspection, the Port of New York Authority permitted the use of manganese steel in substitution for nickel steel. The total saving in price was about $80 000.

The average results obtained on the standard tests of manganese steel rolled for the chords of the Bayonne Arch are cited for comparison in Table 22. The elastic limit, being the Johnston point, was found to be 46 kips per sq in. None of the steel specimens was heat-treated, and the steel was used in the "as-rolled" condition.

A series of tests of large-sized columns was conducted by the Port of New York Authority jointly with the National Bureau of Standards; among them were two of the type of the George Washington Bridge and two of the type of the Bayonne Arch. The strongest column, of carbon manganese, developed an ultimate resistance of 9 653 kips. The ultimate compressive strength of the carbon manganese columns was from 58.6 to 62.3 kips per sq in.[85] Thus, the column strength was found to be the average yield-point strength of the material in the members. It should be added that carbon manganese steel rivets were used successfully for the field connections.

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[85] "Tests of Steel Tower Columns for the George Washington Bridge", by Stang and Whittemore, *Journal of Research*, National Bureau of Standards, Vol. 15, September, 1935, and paper in print.

The manufacture of manganese steel has shown that much care must be exercised in the production of the steel and the rolling of the plates and that the heavier plates are likely to encounter difficulties and to develop cracks. Tension tests of large specimens made by the contractor in connection with the Bayonne Arch gave poor results, and he finally decided not to use carbon manganese steel in the tension members of that bridge. In this instance, the reasons for using high-strength steel was due, as stated, to economical considerations. Substitution of carbon manganese steel for nickel steel was due to a saving in cost to the consumer. It is doubtful whether there was an actual saving in cost to the producer.

Summary.—The writer has discussed at length the application of high-strength steel to most of the large bridges in the United States and the reasons which prompted its use. The reason that nickel steel was used in most cases was simply that it was the material available and had been tried out first. Manufacturers were familiar with it and engineers knew it to be dependable and well behaving in finished structures. The reasons that prompted metallurgists and engineers to search for other high-strength steels suitable for structural purposes were those of economy. When used in the Queensboro and Manhattan Bridges in the first decade of the Twentieth Century, nickel steel cost about 3 cents per lb more than carbon steel. This differential in price has varied but little during the succeeding years. For the Delaware River Bridge, in 1924, the average of the bids was 2.25 cents per lb more for nickel steel than for carbon steel; for the Bayonne Arch, in 1928, it was 2.8 cents; and in the San Francisco-Oakland Bay Bridge, in 1933, it was 2.8 cents. The alloying nickel is practically controlled by a monopoly, and its cost to the mills has varied little. Additional cost of rolling and fabricating has kept the price relatively high. For the Harahan Bridge, a cantilever railroad bridge over the Mississippi River, at Memphis, Tenn., in 1914, 700 tons of Mayari steel was used in eye-bars and riveted members, with the evident intent to reduce the cost of high-strength steel and, apparently, good technical results were obtained. Mayari steel, however, has not been in the market for structural purposes in recent years.

HISTORY OF SILICON STEEL

Fortunately for the advance of bridge construction, a new steel was developed about 1916 which took its place in strength above structural carbon steel and below nickel or special steels. This steel, known as silicon steel, has a minimum yield point of 45 kips per sq in. and a tensile strength of 80 to 95 kips per sq in.

Notwithstanding its great popularity, silicon steel is fairly young. More than fifty years ago a silicon steel was proposed by English mills for ship construction. However, not before 1907 did silicon steel make its appearance in ship construction in plates for the S. S. *Mauretania.* Because of the wide use of this steel in recent times, the writer considers it important to discuss what is generally associated with the trade mark, "silicon steel," in European countries. The steel used in the *Mauretania* is said to have had the characteristics cited in Table 23.

TABLE 23.—EVOLUTION OF SILICON STEEL

Description	Silicon steel in the S. S. *Mauretania*, in 1907	Silicon steel in George Washington Bridge, in 1928-30
Chemical Analysis (Percentages):		
Carbon	0.27	0.35
Silicon	1.12	0.27
Manganese	0.72	0.78
Physical Properties:		
Yield-point stress, in kips per square inch	65	50.8
Tensile strength, in kips per square inch	92 to 105	88.8
Percentage elongation in 8 in	25 to 30	22
Percentage reduction of area	......	43

In later years, European manufacturers, and especially German mills, after the World War, had adopted a similar grade of steel, usually containing from 1.0 to 1.3% of silicon. This steel gave unsatisfactory results and ultimately was recalled. The greatest trouble was the high percentage of silicon as compared with the practice of American mills which limits it to 0.45 per cent. Because of the troubles experienced with the higher percentage, silicon steels in general acquired a bad reputation in Europe. In discussing steels with European engineers it will be well to explain to them the difference between European and American steel of the same name. As manufactured in the United States, silicon steel has a low percentage of silicon and a medium percentage of manganese. It could properly be named "Mansil" to indicate the ingredients which produce its characteristic qualities, or "Medium Manganese" steel.

It first made its appearance in bridge building in 1915 in the bridge across the Ohio River, at Metropolis, Ill. This bridge has a simple truss span of 720 ft and its long span and weight demanded a higher-strength steel. The demand was well met by silicon steel. So successful was the new material that to date about 350 000 tons of it has been consumed for bridges. It has been used for the towers of the Delaware River Bridge, Ambassador Bridge, Martinez-Benicia Bridge, George Washington Bridge, Golden Gate, New Orleans, Tri-Borough, and San Francisco-Oakland Bay Bridges. It has become a standard steel of to-day.

A fair idea of the quality of silicon steel that is being manufactured in such large tonnage can be had from the results of 895 melts and 1 884 tests of the silicon steel used in the towers of the George Washington Bridge, a total of 23 600 tons (see Table 23).

Economy.—The reason for the extensive use of silicon steel is its economy. Erected, in place, it costs 10 to 15% more than carbon steel. Its specified minimum yield point is 36% higher and a permissible unit stress of one-third higher is generally allowed for it. An elementary example will illustrate the economic relation of silicon steel to carbon steel. Assuming that 1 000 tons of carbon steel, in place, will cost \$100 000, the higher allowable unit stress, with practical allowances, will require only 800 tons of silicon steel at, say, \$112.50 per ton, amounting to \$90 000. A saving of \$10 000, or of 10% of the cost, will be realized.

The saving in cost shown in the foregoing example covers the lesser cost of the material in place. It does not take into account the saving in steel effected by virtue of its decreased total weight. Although it is of slight importance in buildings and minor bridges, the reduction in weight becomes a substantial factor in longer bridges where an additional pound of weight may require 3 lb of steel to sustain it. To this factor is due the saving sought by bridge engineers, and it is the determining cause for the selection of higher strength steel.

PRACTICAL ADVANTAGES AND LIMITATIONS OF HIGH-STRENGTH STEEL

From the point of view of the engineering of bridges and structures, a high-strength steel is an alloy that develops a substantially higher yield point and tensile strength than the so-called standard structural steel and is sufficiently ductile to be fabricated by the equipment and procedure in modern shops. It must also be ductile enough to overcome stress concentrations greater than the yield-point stress when they occur and to be capable of distributing the stress more or less over the entire section of the member without impairing its efficiency. The yield point is accepted as one of the measures of the quality of steel, not because it is a better index of quality than the limit of proportionality or the elastic limit, but simply because, for acceptance testing, it is much more feasible. Testing for commercial acceptance of material requires simplicity and speed. Both are obtained by the observation of the yield point or a conventional definition of it. The consumer, therefore, really buys the material on the basis of its yield point and its tensile strength as observed by testing procedures which are established conventionally to meet the exigencies of practical acceptance at the mills and shops. When the engineer specifies a high-strength steel he obtains a product with a higher yield point and higher tensile strength than is ordinarily used.

Criterion for Unit Stresses.—The object of the engineer in introducing a higher strength steel, whether for technical or for economical reasons, is to utilize its strength by applying to it higher permissible unit stresses. The question then arises here as to how much stronger is the new steel when in action in the structure than the common carbon steel. Can the "higher strength" of the new material be utilized by the designer of the structure in direct proportion to its yield point or tensile strength? If so, which of the two should guide the adoption of the unit stress, the yield point or the tensile strength? The engineer thus arrives at the fundamental considerations of what constitutes the resistance of steel when acting in a structure.

Engineering knowledge of the present time has found experimentally that a steel specimen subjected to a static load within its limit of proportionality will sustain the load indefinitely and when the load is released it will assume its original dimensions. Very little set and scarcely any creep due to time has been observed on specimens. The good behavior and long life of innumerable steel structures bear witness to these facts. Sufficient observations are available, furthermore, to justify the assumption that the variation of the intensity of a load applied in one direction only will not reduce the sustaining qualities of the material. Experiments have not yet established positively that the tensile strength of a specimen will not be reduced by applying a load and then removing it entirely a sufficient number of times to establish faith in the probability that such conditions would not be exceeded during the life of the structure. It has been established, however, that a reversal of stress within certain limits will cause the specimen to fail after many repetitions. The strength of the steel to resist an "infinite" number of stress reversals is known as fatigue strength and is lower than its static strength. Moreover, the fatigue strength depends on the tensile strength and not on the proportional limit or on the yield point. Taking the lowest values commonly set in specifications, the percentages of increase in yield point and tensile strength are indicated by reference to Table 24.

TABLE 24.—PERCENTAGE INCREASE IN YIELD POINT AND TENSILE STRENGTH

Description	Yield point stress	Tensile strength
Carbon steel to silicon steel	36	33
Carbon steel to special steel	67	50

Although the decrease in percentage of tensile strength is not much for the two varieties of steel in Table 24, attention is called to the fact because several steels recently have come into the market, the main characteristic of which is a higher ratio of yield point to tensile strength. When establishing the permissible unit stresses for the design of the structure, therefore, the engineer must consider the character of the loads to which it will be subjected and must choose the material best fitted for its function. For practically static loads the higher yield point will be the controlling influence, and for frequently reversed stresses the tensile strength will be the controlling influence.

While establishing permissible unit stresses the engineer should keep in mind that the proportional limit and the elastic limit are the limits that should not be exceeded in a structure and that the true margin of safety depends on them. Careful observations have shown that the elastic limit is lower than the yield point and sometimes substantially so. Generally, the elastic limit can be taken at eight-tenths of the yield-point stress and it may be substantially below it.

With the foregoing limitations in mind the engineer can take advantage of the higher strength qualities of the special steels. The recent Specifications for Steel Railway Bridges of the American Railway Engineering

Association call for the following permissible unit stresses in tension (in kips per square inch, net):

Carbon steel	18
Silicon steel	24
Nickel steel	30

It will be noted that the stress allowed for nickel steel is 1.67 times that of carbon steel, and that allowed for silicon steel is 1.33 times that of carbon steel. As a result, smaller sectional areas are required which will require less material. The reduced weight of steel required means less dead load for the structure to sustain and, in turn, reduces the steel tonnage. Where sections of carbon steel are too large, they will become proportionally smaller for the stronger steels and members will become easier to handle. Above all, when properly and judiciously applied, a saving in cost of the structure will be realized.

Limitations Imposed by Flexibility.—There are limitations, however, to the use of higher strength steels for structures which are inherent in all steels and are derived from their basic qualities. The modulus of elasticity of various steels is practically the same within narrow limits. All phenomena of the behavior of steel members and structures which are functions of the modulus of elasticity, therefore, will develop the same degree of deformation. The members of the structure composed of high-strength steel will shorten and elongate in inverse proportion to their sectional area, and the area being smaller, they will develop proportionately greater deformation. The structure as a whole, consisting of all the members, therefore, will be less stiff and will deflect more under load. This is not desirable. The more flexible a structure is the more secondary stresses it will develop in its stiff connections and if the deformation of the structure should become appreciable it will also develop additional stresses due to the distortion of the original geometrical pattern. Thus, very tall structures when exposed to high winds may sway sufficiently to produce additional stresses to an appreciable degree, so also may tall towers of long-span suspension bridges.

Limitations Imposed by Vibration.—Where the rapid application of loads produces vibrations, the intensity of these vibrations will be greater for the stronger steels because the intensity is inversely proportional to the moment of inertia of the structure and the latter value will be smaller for the stronger steel. Trains passing over railroad bridges at high speeds cause much vibration in the structure, and it is of serious importance rather to reduce than to augment these vibrations. Both the greater deformations and the higher vibrations which will be produced in structures built of stronger steels demand the careful thought of the designing engineer. In designing a structure it is important to consider seriously whether the advantage of the reduced cost effected by the use of the stronger steel is not outweighed by the disadvantage of the more flexible structure.

Buckling of Plates and Flanges.—Another phenomenon in the behavior of steel members in which the constancy of the modulus of elasticity plays an important rôle is that of the buckling of plates and outstanding flanges of steel members in compression. According to Euler's well-known theoretical study of the buckling strength of a deflected column under a compressive load, the column strength is directly proportional to the modulus of elasticity of the material and the moment of inertia of the column. The correctness of the Euler theory has been verified experimentally. The elastic stability or buckling strength of plates subjected to compression and shear has been derived from Euler's theory, and it has been established theoretically and verified experimentally that the critical unit stress at which buckling is imminent is directly proportional to the modulus of elasticity. A concise statement of the practical aspects of the problem has been presented[86] very ably by Otis E. Hovey, M. Am. Soc. C. E., who establishes a table of recommended ratios of the width of a plate to its thickness based on the yield point of the material used. For the purpose of showing that smaller ratios are permissible for higher yield points, three ratios will be quoted:

Yield point stress, in kips per square inch	Ratio of width to thickness
33	48.8
45	41.8
55	37.8

The foregoing data demonstrate vividly that a stronger steel is not proportionately more stable against buckling than common carbon steel and that when high-strength steel is applied it is sometimes penalized where wide plates are used in the structure.

The practical importance of the limitation presented by buckling resistance in design is well illustrated by the case of the towers of the Golden Gate Bridge and the San Francisco-Oakland Bay Bridge. The towers of the Golden Gate Bridge are 690 ft above the piers and each pier must sustain a vertical load of 61 500 tons. About 45 000 tons of carbon and silicon steel were used for both towers. The tower shafts are built as cellular structures with widths of 42 in. The general dimensions of the towers are such that in the lower portion the stability of the steel plates, and not high stresses, is controlling. Carbon steel, therefore, was used in the lower two-thirds of the shaft. In the upper one-third where the increased slimness of the shaft requires higher allowable working stresses and stronger steel, silicon was used. It was found that a number of silicon steel plates, located toward the outer faces, required thickening because the buckling resistance was not equivalent to the permissible unit stress. A number of single web-plates, therefore, were increased in thickness; a total of about 70 tons of steel was thus added.

Similar increases in thickness because of buckling strength were made in some web-plates in the towers of the San Francisco-Oakland Bay Bridge.

Details.—In the building of a steel member many auxiliary parts enter which are commonly known as "details." Designing engineers know that such auxiliary parts as lacing-bars, batten-plates, and stiffeners are not highly stressed and do not demand high-strength steel. In order to function well and to fulfill their purpose these parts should be rather stiff than strong. In other words, lacing-bars, batten-plates, and stiffeners should offer high resistance to deformation. To do this it is important that they have more section rather than that they be stronger per unit of area. These parts of high-strength members could very well be made of common carbon steel when they do not carry primary stresses and where there is enough tonnage to warrant the additional care required in sorting in the shop.

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[86] "Elastic Stability of Plates", by Otis E. Hovey, *Bulletin*, Am. Ry. Eng. Assoc., February, 1935.

Similar considerations hold true for most secondary members in bridge trusses. Hangers and parts in subdivided panels sustain concentrated panel loads only and, as a rule, they do not require heavy sections. Lighter members frequently under-run in size and should be made of carbon steel. The same is true for most lateral and sway-bracing in medium span bridges. Sway-bracing mostly functions to preserve the geometrical pattern of the structure and stiffness more than strength, is of importance. Lateral bracing requires heavy members where the lateral shear becomes high, which generally is in less than one-half the length of the bridge; in the lighter half, the least size of the member controls. If the tonnage of the steel is sufficient to warrant the use of two kinds of steel, carbon steel may be used in at least a part of the lateral system.

Adapting Steels to Parts of a Structure.—In the design of large structures engineers have recognized the value of differentiating between various members according to their functions and of using steels suitable for these functions. For example, the web members and all lateral and sway-bracing members of the Bayonne Arch are of carbon steel, whereas the lower chord is of carbon manganese steel and the top chord of silicon steel. The towers of the Delaware River Bridge are of silicon steel and their transverse bracing, as well as all diaphragms, etc., are of carbon steel; the chords of the stiffening trusses are of nickel steel; the webs, however, are of silicon steel and the lateral system, because of participation stresses, is also of silicon steel. In the towers of the George Washington Bridge the shafts are of silicon steel, the bracing, all secondary parts, and parts in which stiffness rather than strength was desirable, are of carbon steel. The method of differentiating between the various members of a structure in accordance with their function has become general practice on important structures where such a procedure is warranted.

PHYSICAL LIMITATIONS OF HIGHER STRENGTH STEELS

In considering various grades of steel for structural purposes the physical limitations of higher strength steels should be given attention because they will be reflected in the cost of the fabricated and erected steel.

Fabricating Problems.—All shaping and forming of metals is performed either by heating the material to a plastic state, or by exerting force on the cold metal. Within certain limits, structural steel is usually shaped by suitable devices at shop temperature. Cold-shaping is more convenient, speedier, and more economical. Practically all straightening of plates and bars is done in the cold state. Cold-shaping and straightening of steel are based on the principle of applying force through a motion of the forming device so that the elastic limit is exceeded and the metal enters the plastic state and, therefore, follows the shaping tools into the desired form. Without exceeding the elastic limit, the metal, of course, would spring back into its original form. The fabricator is well aware of the fact that the higher the elastic limit of the material, the more force will have to be exerted to pass it and the stronger the forming equipment will have to be. Shaping and straightening will be more difficult and tools will break more frequently. It will also require more time because the higher elastic limit steel will need to pass through the straightening rolls a greater number of times.

It should also be remembered that the stronger steels require a more careful procedure in the pouring and generally more ingots are discarded. Their rolling temperature has a more limited range, and some, if not all, of the higher steels require protection from cold blasts and drafts during the rolling process. The finer the material the more care it requires; the stronger the steel, the more difficult it is to form.

Distribution of Stress Among the Rivets in a Joint.—Another matter that deserves serious consideration is that of stress distribution in riveted joints. When a relatively wide plate is spliced to another plate by the common riveted connection, the question arises as to whether the stress is distributed uniformly through the rivets from one plate to the other. The phenomenon of splicing steel by a group of rivets is by no means simple. Like most phenomena encountered in practice it has been made simple by an assumption which, although not correct, makes it possible to approximate the facts roughly and to proceed with the task of splicing the members. The assumption is that all parts, the plates and the rivets, are rigid members and that a stress, uniformly or continuously distributed in a plate, will transmit its total force to the connecting rivets and these, when symmetrically arranged, will transmit the force to the second plate in the same symmetrical and uniform distribution. This assumption dates from the time when stresses were low and the elastic properties of metal were ignored. Engineers know that such uniform distribution does not prevail in the plates near the rivets; the originally distributed stresses meet their resistance in the rivets and rush to them to overcome it. Concentrations of stress are formed around the rivets which sometimes exceed the elastic limit. At that time a system of equilibrium establishes itself between the plates and the rivets by the mutual yielding of each; where the intensity of stress is too great for the elastic limit, it passes (for the time being) into the plastic state. The phenomenon of stress distribution through rivets still requires much study. Engineers are aware of the problem and, at present, are devoting their attention to it. One aspect of the problem, however, is quite clear: The higher the elastic limit of the material the greater may become the concentration of stresses and the less uniform will be the distribution of stress in the plates. In fact, too high an elastic limit may lead to such a high concentration of stress as to cause incipient failure. With the present knowledge of the phenomenon the prognosis may be made that the efficiency of riveted splices is not in direct proportion to the increase in the yield point of the material. The higher yield point, therefore, cannot be fully utilized in proportioning the connection.

What has been stated herein, concerning stress concentration on rivets, of course, holds generally true for all "hard points"; that is, for all places where the material encounters concentrated resistance. The same phenomenon is developed where the shape and area of the material change abruptly and the flow of stress is contracted and "choked." In such cases, the same as in riveted joints, it is desirable that the initial distributed stress be low and that the material should be able to adjust itself to the conditions. The development of photo-elastic studies in recent years has demonstrated, forcefully, the phenomenon of stress concentration.

The writer has dwelt at some length on the physical limitations of higher strength steels with the intent to emphasize the restrictions that may have to be imposed and to dampen the enthusiasm that the stronger material is likely to evoke.

Uses for High-Strength Steels

Modern economic, social, and technical development has brought people closer together in space and time than ever before in human history. The co-ordination of human efforts has brought together Man and mechanism. In the incessant endeavor to satisfy the ever-increasing demand for physical goods and to make the comforts of life accessible to the many, structures of great magnitude and capacity often become desirable as connecting links in production and distribution. Such structures and machines sustain heavier masses, move more freight, cross large rivers and wide bays, and lift up their spires into the skies. Because of their functions and of their magnitude they must sustain great forces, and materials of high strength are required for their construction.

Bridges with long spans require large members and, therefore, weigh more. Their stresses can be sustained by high-strength steel, with less material and at a lower cost. Tall buildings for special purposes, such as the Palace of Soviets, in Moscow, U. S. S. R., are subjected to high wind pressures and stronger steel will find its proper application for the skeletons of such structures. Derricks of large capacity and long-span cranes, and bridge members which are utilized during the erection of the structure and sustain great forces, may profitably be built of steel stronger than the common carbon steel. Wherever the demand for sustaining great forces will appear, necessity will compel the use of stronger steels independent of its economy. Where technical conditions of fabricating shops and facility of erection in the field will demand lighter sections and lighter members, higher strength steels will be applied. Their widest field of application, however, is in the reduced cost of the structure.

Engineering Economy.—In discussing the economy of large structures several interpretations and meanings of the word should be considered.

There is an economy inherent in the design of a structure which is part of the play of forces acting in the structure under load and of the arrangement of the sustaining members and their suitability to perform the functions which the behavior of the structure will demand. This economy is a measure of the mechanical fitness of the structure for its intended purpose. It is independent of the market price of the material and labor required to build it, and may well be designated as structural economy. Structural economy forms only a part of the engineering economy of a structure. The latter consists in utilizing for its various members such materials and shapes as will result in the lowest cost of the completed structure.

There is also another very important aspect of economy, namely, financial economy. In this economy enter considerations of the amount of capital invested in the project and the prospective life of the structure. It depends much on the prevailing rates of interest. It also is based on the economic organization of society in a given country. It is largely, however, through the development of engineering economy that higher strength steels will find their application and the field of stronger steels will be extended. The expansion will depend largely on the cost to the mills of the ingredient alloys which enter into the higher strength steels. It will depend on whether these alloys will be plentiful in the market or will be controlled by monopolies. It will also depend on the available processes of manufacturing and fabricating the stronger metals.

Improper Use of High-Strength Steels.—To realize engineering economy in structures fully, and to produce them at a lower cost, careful consideration must be given to the desired result in each case. Not only the first cost, but also the behavior of the structure under load, and its maintenance, should be weighed carefully.

A number of large bridges have been cited for which higher strength steels have been used, and an attempt has been made to give the reasons that have guided the engineers to adopt these steels. A few telling examples, out of many, will be brought to show instances in which higher strength steel has led to improper or uneconomical design.

In one instance an arch bridge of 140-ft span, of the plate-girder type, was designed. Its total steel weight was less than 200 tons, about one-half of which was of silicon steel. The arch ribs were made of 36 by $\frac{3}{8}$-in. webs and four flange angles, 6 by 6 by $\frac{1}{2}$ in. The ratio of the unsupported depth of plate to its thickness was 65. The web-plate was in compression and was manifestly too thin against buckling; it had to be thickened. Instead of using silicon steel, standard carbon steel should have been used. In general, it is not economical to use silicon steel for a short span of 140 ft. This case illustrates the physical and economical limitations in the use of silicon steel.

In another instance the designers evidently were aware of the buckling limitations of carbon steel and desired a compact section. They designed it, however, in an impractical manner. A highway bridge of the cantilever type was planned with a main span of about 800 ft. The chords were of silicon steel and of the enclosed box type with angles turned in. The outside dimensions were $18\frac{1}{8}$ by $24\frac{1}{2}$ in. The minimum section carrying stress was composed of one top cover, 18 by $\frac{5}{16}$ in., four angles, 4 by 4 by $\frac{5}{16}$ in., two web-plates, 24 by $\frac{7}{16}$ in., and one bottom cover, 18 by $\frac{3}{8}$ in. In the maximum sections the top cover-plates and the upper angles were thickened to $\frac{5}{8}$ in., the web-plates to $\frac{7}{8}$ in., two side-plates, 15 by $\frac{3}{8}$ in., were added, and the bottom angles were made 4 by 4 by $\frac{3}{4}$ in. For riveting purposes and for painting, manholes, 8 by 24 in., were provided in the bottom cover-plate. The clear inside dimensions were thus about 12 by 20 in. for the heavy section. It is very difficult, if not dangerous, at the large joints, to put a man in these chords to do field riveting. The small box-sections also prove to be expensive both in the shop and in the field. Notwithstanding the constricted section of the chord, the ratios of depth to thickness of the thinner web-plates are somewhat high for silicon steel. In this case larger dimensions of chords would have provided more accessibility for riveting and painting, and the use of carbon steel would have furnished well-designed and economic chord sections at a lower total cost.

A third instance of the uneconomical use of stronger steel consists in using silicon steel indiscriminately at random points in carbon-steel trusses.

In a fairly large bridge of the cantilever type there is a suspended span of 150 ft. All members in this span were designed of carbon steel except one bottom-chord section which was made of silicon steel. It was made of two web-plates, 20 by $\frac{1}{2}$ in., and four angles, 4 by 4 by $\frac{7}{16}$ in. By thickening the material to about $\frac{5}{8}$ in., this section could also have been made of carbon steel and the total cost of the section would have been less. In this case a critical review of the material used would have disclosed to the engineer at once that the introduction of silicon steel in one chord section was not good economy.

It may be well to point out that, although for a large tonnage fabricated silicon steel in place will command a differential of $12 per ton, for smaller tonnage the extra charge will be greater. Where the weight of each individual shape is less than 20 tons, the extra charge is $15 per ton. For plates smaller than 36 in., the differential is $10 and for plates larger than 36 in., it is $15. Silicon-steel bars of various sizes in quantities of less than 1 000 lb will cost $25 to $35 per ton extra. The higher extra charges for the smaller quantities are due to the care required to segregate and follow the small quantity of the special steel and to the difficulty of replacing from stock any items rejected during fabrication.

The foregoing examples of actual practice have been cited not in the spirit of criticism, but with the purpose of calling the attention of engineers to the possibility of errors of judgment and oversight and to establish warning signals.

CONCLUSION

The progress of the more civilized part of humanity has led to the growth and concentration of population in large cities. At the same time, it has created an enormous demand for rapid and economical transportation of large masses of freight and of great numbers of passengers. Among other efforts it has created a demand for bridges of large capacity over wide rivers. Engineers in their untiring search to meet the demands of growing communities and industry have successfully made use of a few higher strength steels which they have developed and found available. The field for the application of such steels appears to be extending and standardized specifications have been established for the use of manufacturers and engineers. The varieties of alloy steels of higher strength are by no means limited to the few which have actually been used in structures in the United States. At present, a number of high-strength steels have been developed and are available on the market. Many more will be produced to meet the demand. It is the task of the engineer to follow the development of materials and to utilize them to the best advantage. At the same time, it is incumbent on him to study the physical and economical limitations of the higher strength steels and to determine carefully in each case the best advantage that may be derived from their use.

Much effort is being made at present to increase the pay load on freight-carrying vehicles. While materials are being developed to reduce the dead weight of cars and trucks, the pay load, that is, the weight of the freight carried, will be increased and the tendency is strong to increase the total weight of the loaded vehicle. To be able to compete with freight transportation on highways, railroad officials aim to reduce the cost per ton carried. With the present tendency to higher cost of labor, the cost per ton carried must be held down by mechanical efficiency. The result will be heavier trains pulled by heavy locomotives, for which heavier bridges will be required. This is evidenced by the latest specifications for railway bridges. On the other hand, the growth of automobile traffic has made long-span bridges economically feasible. Engineering science, the art of metallurgy and fabricating technique, and erection skill will make it possible to build them economically in greater numbers. During the two decades, 1916 to 1936, the span of bridges has been more than doubled. Engineers have attained a span of 4 200 ft and will surpass it in the future. The use of stronger steels for structures has been intimately connected with the progress of bridge construction and will so continue.

STRUCTURAL APPLICATIONS OF STEEL AND LIGHT-WEIGHT ALLOYS

A SYMPOSIUM

PAGE

IV.—LIGHT-WEIGHT STRUCTURAL DESIGNS:

The Application of Stainless Steel in Light-Weight Construction.

By E. J. W. RAGSDALE, ESQ. 1360

Structural Application of Aluminum Alloys.

By E. C. HARTMANN, ASSOC. M. AM. SOC. C. E. 1369

Magnesium Alloys and Their Structural Application.

By A. W. WINSTON, ESQ. 1385

THE APPLICATION OF STAINLESS STEEL IN LIGHT-WEIGHT CONSTRUCTION

BY E. J. W. RAGSDALE[87], ESQ.

SYNOPSIS

In a broad sense, this paper covers the applicability to light construction of the two major types of stainless steel and then describes briefly the general problems of design, fabrication, and usages. Questions of plate stability, efficiency of spot-welded connections, and the economic phases of a relatively costly material are lightly touched upon. Although these seem to merit a more expansive treatment, the danger of such expansion is always that the paper may be thrown out of balance.

STAINLESS STEELS

The structural engineer need concern himself little about the metallurgy of the materials he uses. His purpose is to define requirement, and then to select the materials best suited to it. That selection is made on the basis of established properties, which become accepted values. The only concern is that these properties be not disturbed by continued use or by the processes of manufacture. So much appreciation of metallurgy must the structural engineer have.

When consideration, however, involves the innumerable alloys listed under the name of stainless steel, a mere appreciation of metallurgy no longer suffices. The structural engineer had then best invite the close co-operation of the metallurgist. No less than a dozen different alloys present themselves. Aside from a pronounced resistance to corrosion, they have little in common. Some owe their strength to heat treatment, others lose it thereby. Some are inherently stainless, whereas others are only stainless under certain conditions.

Fortunately, a structural application of stainless steel has already developed, and, through this, the choice of alloys becomes narrowed. Outstanding is a chrome-nickel alloy (18–8)[88], that has been described in detail by Mr. M. J. R. Morris, in this Symposium[89]. As a poor second is a straight chromium steel with from 12 to 17% chromium content. Requirement and the process of fabrication will determine the applicability of each. The first owes its high physical properties to cold working, and, by the same token, may lose them by welding. The other improves by heat treatment, which treatment automatically corrects the effects of any previous welding operation. Therefore, it becomes quickly apparent that the feasibility of heat treating a finished structure may determine the choice. Further considerations will be cost and strength. The cost of the chrome-nickel steel is about 50% greater than that of the chromium steel, and, at the same time, it is cold rolled to a 50% greater strength. An even greater advantage, however, lies in the susceptibility of chrome-nickel steel to good and cheap fabrication.

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[87] With Edward G. Budd Mfg. Co., Philadelphia, Pa.

[88] Symbols in parentheses denote the trade designation by which the alloy is commonly recognized.

[89] See Table 4, Items Nos. 5 to 10, inclusive.

Perhaps a very general distinction may be made by stating that the chrome-nickel steel is better suited to the requirement of light-weight construction, whereas the chrome iron offers a corrosion-resistant substitute for carbon steels in more conventional design.

DESIGN PROBLEMS

Light-weight construction has as its premise a more effective use of a more effective material. This indicates a wider spread of material, and, at the same time, a lesser material requirement. Both result in a reduction of the usual thicknesses, and herein lies a problem.

It is an old and convincing trick to illustrate structural stability by trying to stand a piece of paper on edge and then showing how this same sheet increases in load-carrying capacity as it is rolled into cylinders of smaller and smaller diameter. The limit is finally reached when the tube becomes so small as to lack column stiffness. A compromise position then results in a larger tube and a stiffened wall. This thin wall may be strengthened by structural members, by longitudinal corrugation, or by both. The ultimate effectiveness is reached when the column becomes a series of vertical members, mutually cross-braced. The wall has disappeared.

This question of wall or plate stability, however, has already been investigated, especially in connection with bridge construction. It becomes only more acute as the ultimate strength of metals is raised from 75 kips per sq in. to twice that value, and when thicknesses are measured in thousandths of an inch rather than in quarters of an inch. In this case, the question of relative flatness complicates an already complex mathematical treatment. The obvious solution becomes empirical and toward this end a curve on the so-called "flat pitch ratios", has been developed with which it is possible to compute the stability limit in compression of any closed section that is made up of a series of flat surfaces. The "flat pitch ratio", q, is the relation between the unsupported width of a flat surface and the total wall thickness; that is,

$$q = \frac{b}{d} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

Taking this value for the widest face of the section, Fig. 36 will show at what percentage of the ultimate strength of the metal, that particular face begins to undulate under compressive load. (The abscissas are plotted as ratios of ultimate compression stress, T_c, to ultimate tensile stress, T_t.)

Quite obviously, therefore, a metal of high tensile strength must be formed into sections that permit a stressing commensurate with its superior strength. An I-beam, for instance, may develop stresses up to the limit of mild steel, but if made of a steel having a working stress three times greater, the beam might collapse through instability long before that stress was realized. Therefore, the sections suitable for one metal cannot be extended arbitrarily for the use of another. Light-weight construction therefore, has brought into being a series of new and more efficient sections. These sections are largely of the closed-box type, or efficient open sections which are locally stabilized by stiffeners. Some are formed by rolling from a strip, whereas others are fabricated.

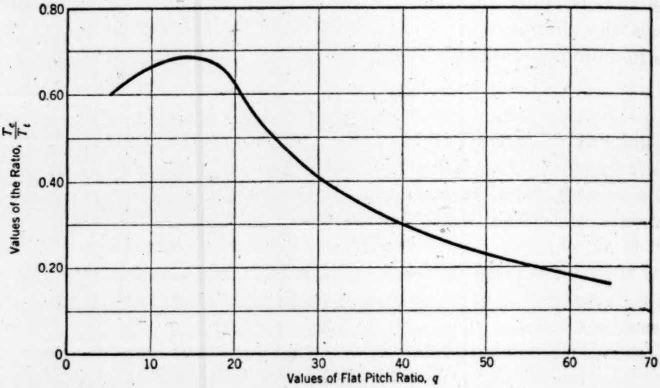

FIG. 36.

Fig. 37(a) and Fig. 37(b) are compression truss members, and Fig. 37(c) and Fig. 37(d) represent floor-beams. All of them (and, in fact, most sections in use) will develop at least 100 kips per sq in. on the compression side;

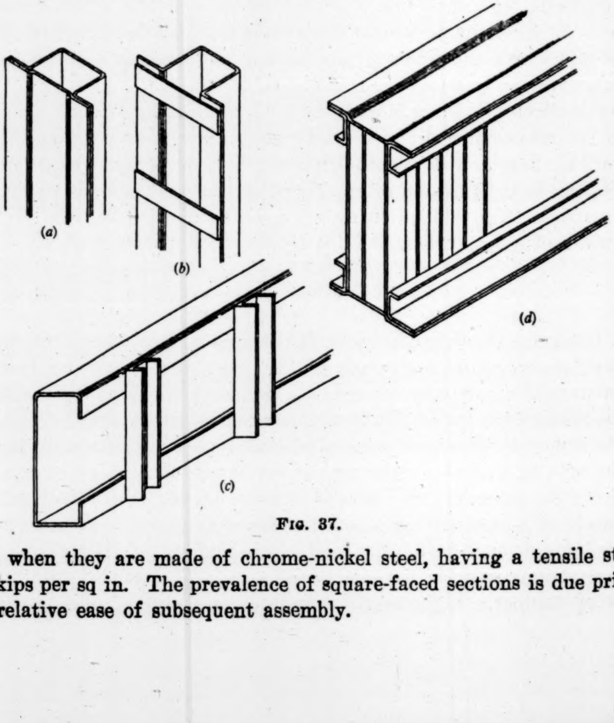

FIG. 37.

that is, when they are made of chrome-nickel steel, having a tensile strength of 150 kips per sq in. The prevalence of square-faced sections is due primarily to the relative ease of subsequent assembly.

Computed on a weight-to-strength ratio, such sections in material of high tensile strength, will be found to be from seven to ten times better than the conventional profiles of mild steel. One-half the advantage comes from the superior metal strength, and the other half from a more effective placement of that metal. Of course, this ratio relates to relatively small sections, such as 6-in. channels or I-beams, and becomes less apparent when compared to the efficient girders, such as obtain in bridge construction.

Efficient and strong as these individual sections may be, however, they are useful only to the extent that their connection, one with another, is equally efficient. Attachment becomes a problem, both as to area and avoidance of eccentricities. In fact, the same care and design exercised in large bridge structures must be interpreted into use for smaller and lighter ones of high-strength metal. Double gusset--plates, nested member ends, and efficient bonding, or joining, become necessary. These requirements again demand close attention to sequence of assembly and to accessibility of welding or riveting. The latter is complicated to the extent that the structure may be small or truly efficient. It is further aggravated often by lack of appreciation of shop procedure. A seemingly simple connection on paper, may baffle a Houdini in assembly.

Even when a type connection may be decided upon, there still remains, however, the question of providing sufficient bond. This bond may be by rivets, or by the fusion of faying surfaces as in welding. In either case, the function of the bond is always to resist straight shear. A definite practice has been established for this function in rivets and mild steel. It is not translatable immediately into high-strength material for the simple reason that rivets of correspondingly stronger physical properties cannot be applied. The requisite increase of shear is more aptly obtained by welding. This offers possibility of a 100% efficient joint. It involves, however, many new considerations of design and shop technique.

WELDING

Nothing is simpler than to calculate the strength of a welded connection, and few calculations can be more misleading. The difficulty lies in the assumptions. In the first place, one assumes that the weld is good, and, in the second, that the primary forces still govern. An improved welding technique has done much toward establishing welding certainty, but the relief of heat-induced stress concentrations in and about the welded area remains a problem, the solution of which differs for each individual case.

Although both chrome-nickel steel (18–8) and the chromium steel can be gas welded, arc welding is becoming the more favored of the two processes. It must be recognized, however, that either procedure is a severe heat operation and that these alloys are peculiarly responsive to such treatment. No responsible arc welding should be done, therefore, without subsequent heat treatment. This is not with a view to relieving stresses so much as to correct an impaired metallurgical condition in the weld and the adjacent metal. Since the chrome irons are usually improved by heat treatment, not only is the weld corrected, but the entire structure becomes benefited. Arc welding is strongly recommended for the straight chrome series, but the limitation of heat treatment also confines the practice to such structural members as may be susceptible to this treatment. These members are usually small and simple pieces, such as will not warp under the application of a high heat.

With chrome-nickel steel there are two difficulties attending arc welding. In cooling through the critical temperature gradient of from 1 200° to 1 600° F, carbides are precipitated. These carbides can only be returned to solution by heating to more than 2 000° and quenching. Then, again, chrome-nickel (18–8) owes its superior physical properties to cold working, and these properties are automatically reduced by either the heat of arc welding or the corrective re-heat treatment. Since this metal in the annealed state only shows an elastic limit of about 34 kips per sq in., its utility in highly stressed members is, therefore, defeated by arc-welded assemblies.

Thus far, the happiest solution for the joining of chrome-nickel steel appears to be found in a special system similar to that of ordinary spot welding. In principle, however, it differs by the establishment of such controls of current, time, and electrode pressure that the metal remains metallurgically unimpaired. This is made possible by the discovery that both carbide precipitation and the annealing of the metal adjacent to the zone of fusion are functions of the time of heat application. With a quick application of heat and a rapid cooling, neither of these undesirable phenomena appear. Welds may be made in as brief a time as one-half cycle, or $\frac{1}{120}$ sec. Rarely does the time of current dwell exceed ten cycles.

However, the process does not correct an inherently insusceptible condition, such as obtains in the chrome irons. In the latter case, regardless of the method of fusion, the bond is coarse-grained and brittle. It can only be corrected by subsequent heat treatment.

Chrome-nickel steel, on the other hand, is strangely susceptible to good welding. Its every property seemingly conspires toward this method of bonding. Recalling that in resistance welding the heat generated is expressed as $I^2R\,t$, the item, R, which is the electrical resistance imposed between the electrodes, assumes basic significance. With corrodible metals, R sums up the resistance of the body of the metal as well as that of the oxided surface. The latter not only varies widely with the extent of oxidation, but is usually many times greater than the resistance of the pure metal itself. Therefore, R is not a constant, and no matter how accurately controlled current and time may be, the resultant heat will vary. With stainless steel, the surface resistance is small and constant. Furthermore, the basic resistance is about six to eight times greater than that of mild steel. Accordingly, the current or time required for a given fusion is less.

Fusion initiates at the inner contacting surfaces of two or more sheets, and progresses outward in proportion to the total heat applied. Such is the heat-conducting property of the electrodes, however, that it can reach the outside surfaces only in the case of a badly burned weld. The shape and depth of the fused button are important. Although the diameter determines the area in shear, the penetration of the fusion into each adjoining sheet holds these sheets together much the same as the heads of a rivet. A shallow weld lacks tenacity; one that extends entirely through the sheets will invite corrosion. The control of weld shapes lies in the proper setting of the welding machine, which, in turn, is determined as a result of many tests with sectionalized and etched specimen welds.

Having established a proper welding practice, the next step is to insure a strict observance of such practice, because when welding becomes an element in structural procedure, the same standard of reliability is required of it as of the materials to be so bonded together. This seemingly ever-present problem in welding of any kind, has been solved in the special system herein cited, by use of a simple, but unique device for integrating the function, $I^2R\ t$, for each weld made. Any departure from the allowed tolerance rings a bell in the welder, and the result of each weld made or failed is recorded on a tape. Welds cannot be judged from the outside and, without the device mentioned, the operator is really working in the dark and beyond the scope of inspection.

On the other hand, how to make a weld is one thing; what it is worth is another; and therein is found again a remarkable property of chrome-nickel steel. The fused metal has been subjected to the very treatment recommended for annealing to a dead soft condition. Its tensile strength is 90 kips per sq in. More remarkable, however, is the fact that its shear strength is also 90 kips per sq in. This means that one weld has twice the strength of a rivet of like diameter, and since welds can also be placed closer together than rivets, a much more efficient joining results. Single lap-joints can easily show 85% efficiency, against 60% in good riveted connections. Just what stress distribution takes place in and about a welded joint, however, remains a neat problem to engage the ingenuity of some physicist.

When the conventional rivet hole is replaced by a metal softer than the surrounding sheet, and when this soft metal also assumes resisting capacity in tension with the sheet, it is difficult to visualize the proportion and paths of the lines of stress. However, the philosophy of welding is so engaging that one is likely to overlook the only practical consideration which makes this form of welding at all applicable to high-strength chrome-nickel steel (18–8). The point seems never before to have been emphasized; that is, that welding pressures can be used which are sufficient to draw the sheets closely together. In mild steel no such pressure is required because the steel is soft. Were such pressure necessary, the electrodes would break through the surface scale and, in thus reducing the resistance, would make for no weld. This phenomenon is known. With chrome-nickel steel, the electrode pressure is often more than 2 kips for a $\frac{1}{4}$-in. weld in two 16-gage sheets.

The overlapping of individual spots to form a gas-tight joint is known as seam welding. The same principle applies in this case, but pointed electrodes are usually replaced by rollers. So many considerations are involved, however, that this form of welding must be approached with a full appreciation of the difficulties. The problem is too specialized to be more than mentioned in this paper.

APPLICATIONS

Perhaps the first structural application of chrome-nickel steel was in connection with aircraft. The fact that it could be cold rolled to tensile strengths greater than 200 kips per sq in. and used in thin sheets without fear of corrosion, had a strong appeal. Some research has also been made in connection with bridges. This has been confined mostly to roadways and decorative effects. A much wider application awaits development. Weight reduction also has its place in this connection, not to mention the elimination of the high cost of painting in difficult and inaccessible places.

The development in marine structures has been far wider. It is generally known that warships have become weight conscious almost to an extreme limit and that deck-houses, masts, smoke-stacks, and bulkhead doors constitute only a beginning in the program. Several million pounds of chrome-nickel steel have already been used in this connection and the ultimate is not yet in sight. Nor is the interest confined to naval vessels; the merchant marine has also discovered that stainless steels have their definite uses. The installation of storm windows on the S. S. *Normandie*, for example, is but a modest beginning of the use to which the material can be applied economically and well. Add to the high strength and to the non-corrosive property of chrome-nickel steel a heat resistance almost as remarkable, and it becomes apparent that this type of stainless steel fits into the marine requirement as has no material of the past. Although the cost causes an initial hesitancy, this will be overcome as soon as ideas become adjusted to true economic worth. The present (1936) cost of chrome-nickel steel[80] is about 30 cents per lb. No wonder that structural engineers may hesitate, especially when alloyed steel for bridges is only a fraction more than 1 cent. Translate weight into motion, however, and metal costs become ridiculous. Dirigibles and some heavier-than-air craft cost as much as $20 per lb. Designers of automobile trucks and trailers consider that a pound saved and converted into pay load may be worth $1 per yr. Every extra pound in a Diesel-electric train carries 30 cents for additional power and equipment cost.

Most spectacular have been the developments in the application of chrome-nickel steel to railway cars, and, by the same token, the most publicized. Rarely have the results of research been more timely. With the railroads struggling under the weight of their own rolling stock rather than that of pay loads, and with a depression to emphasize the condition, an improvement was indicated—not just a slight betterment, but a radical one. It was typified by the first *Zephyr* train, the phenomenal performance of which has stimulated efforts not only along this line of construction, but in other directions as well. The field seems wide open for any material superior to mild steel. Ingenuity of design vies with ingenuity of cost argument, and the main objective is often lost from sight. This object in the case of the *Zephyr*, was to reduce the weight of a railway car to substantially one-third that of a heavy passenger coach, and to do so without sacrifice of strength or comfort. The three cars of the *Zephyr,* without the power plant, weighed 160 000 lb. Conventional railroad coaches weigh from 130 000 to 160 000 lb. So nearly was that part of the objective attained. Then, with a reduced mass, so were destructive forces also reduced. In a million miles of high-speed operation by these *Zephyr* type trains, and not without exposure to damage, there has been neither structural failure nor weakness. A more severe test of a comparatively new structural metal and a new method of fabrication can scarcely be imagined. It assumes a significance which should be more convincing than the listing of physical properties, or the discussion of welding merits.

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[80] See "Stainless High-Alloy Structural Steels", by M. J. R. Morris, Table 4, Items Nos. 5 to 10, inclusive, pp. 1258–1259.

Just what share of future railroad building will be enjoyed by stainless steel cannot, of course, be predicted. More certain seems the prediction of a prominent railroad consultant, to the effect that the last heavy railway car has been built. With this great industry becoming weight conscious, and with an inevitable building program ahead, the only limit to the part that can be played by stainless steel appears now to be one of facilities and organizations capable of handling it.

Stainless steel will open other lines of possible utility, as the knowledge of its uses is extended. An excellent book[n] published in 1935, contains a description not only of the properties and methods, but of the width of the fields already attempted. That this field is so sparsely covered reflects merely upon a lack of applied research; for instance, the structural application of chrome-nickel steel owes its development almost entirely to the efforts and enthusiasm of one man. The staff of engineers which he could afford to divert from his normal business in order to prosecute this development was pitifully small. The temptation to follow each new promising lead has been great, for they are promising—all of them. They require development, and this needs only time and enthusiasm, for the foundation work in stainless steels has now been done.

ACKNOWLEDGMENT

The writer is indebted to his employer, The Edward G. Budd Manufacturing Company, for the use of the basic data required for this paper and for permission to present Figs. 36 and 37.

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[n] "The Book of Stainless Steels", Edited by Ernest E. Thum, Am. Soc. of Metals, pub., 1935.

STRUCTURAL APPLICATION OF ALUMINUM ALLOYS

BY E. C. HARTMANN,[92] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

The aluminum alloy most frequently used for structural purposes is described in this paper from the standpoint of the designing engineer. The composition, properties, and forms available are given. Shop fabrication and workmanship are described to show that the practice differs very little from that for ordinary steel fabrication. The recommended design stresses for tension, compression, shear, and bearing are included, and the problem of designing to take advantage of the lighter weight is discussed. The paper is concluded with a brief study of the economics of the use of structural aluminum, indicating the extent of the extra costs and in what fields such extra costs may be justified.

CHARACTERISTICS OF STRUCTURAL ALUMINUM

The engineer interested in minimizing dead weight will find commercially available to-day more than a dozen aluminum alloys which have been used structurally. Each of these alloys, some wrought and some cast, is fitted for some particular field of structural work by virtue of its properties. The selection of the proper alloy to be used in any particular case, involves a number of problems outside the scope of this paper, and, therefore, in order to avoid confusion, the writer will confine himself principally to the wrought aluminum alloy which has been most commonly used in the structural field. This alloy (17S-T)[93] is of the duralumin type.[94]

It contains[95] nominally 95% of aluminum, 4% of copper, and 0.5% each of manganese and magnesium. It is a heat-treatable alloy and is never used structurally except in the heat-treated condition, when it develops the following typical properties:

Weight, in pounds per cubic foot.................	174
Ultimate tensile strength, in kips per square inch..	58
Yield strength, in kips per square inch............	35
Percentage elongation in 2 in....................	20
Ultimate shearing strength, in kips per square inch.	35
Shearing yield strength, in kips per square inch....	20

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[92] Research Engr., Aluminum Co. of America, New Kensington, Pa.

[93] The symbols in parentheses denote the trade designation by which the alloy is commonly recognized.

[94] Tentative Specifications B78-33T and B89-33T, Am. Soc. for Testing Materials.

[95] "Light-Weight Structural Alloys", by Messrs. Zay Jeffries, C. F. Nagel, Jr., and R. T. Wood, Table 6, Item No. 6, p. 1273, and Table 7, Item No. 6, p. 1279.

Modulus of elasticity in tension and compression, in pounds per square inch	10 300 000
Modulus of elasticity in shear, in pounds per square inch	3 800 000
Coefficient of expansion, in degrees per degree Fahrenheit	0.000012
Poisson's ratio	0.33

The yield strength is defined as the stress which produces a permanent set of 0.2% of the initial gage length[96] (see Fig. 38).

The guaranteed minimum properties for this material (in kips per square inch) are, as follows:

Description.	Ultimate tensile strength	Yield point strength.
Plates	55	32
Structural shapes	50	30

Fig. 38 is a typical tensile stress-strain curve for the material.

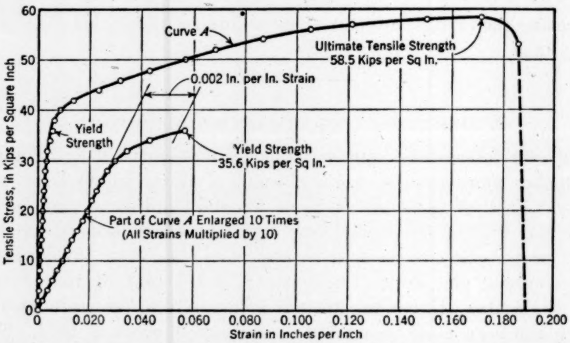

FIG. 38.—TYPICAL STRESS-STRAIN CURVE FOR STRUCTURAL ALUMINUM (DATA TAKEN FROM 0.5-INCH ROUND SPECIMEN.)

Structural aluminum is produced in the form of sheets, plates, structural shapes, rods, bars, rivets, tubing, and forgings. In the form of plates, widths as great as 120 in., thicknesses as much as 1 in., and lengths as great as 35 ft are available, with a weight limitation of 2 kips in any individual piece. In shapes, structural aluminum (17S-T)[96] is available in the form of angles as large as 6 by 6 in., channels as deep as 12 in., I-beams and H-beams as deep as 8 in., and Z-bars as deep as 5 in. Many of the structural shapes are available in lengths as great as 85 ft, but some of them have a length limitation of 35 ft.

FABRICATION OF STRUCTURAL ALUMINUM

Structural aluminum can be fabricated in the ordinary shop with practically no departure from commonly accepted good practice for other structural metals. The material can be cut by shearing or sawing, the use of the burning torch being prohibited, as it would be for any other heat-treated material. It can be sheared with equipment ordinarily used for steel. Both band and circular saws are used for sawing, and a lubricant or soluble cutting compound is recommended. High-speed metal-cutting band-saws, or heavy-duty wood band-saws with blade speeds of about 5 000 ft per min, are quite satisfactory. Circular saws with hollow ground blades are satisfactory for light work, but for heavy work the teeth should be swaged in order to provide clearance and prevent over-heating. Coarse-toothed blades should be used for both band and circular saws, and the teeth should have no top rake. With circular saws, a peripheral speed of 12 000 ft per min is recommended for the best results.

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[96] Specification E8-33, Am. Soc. for Testing Materials.

Structural aluminum can be worked cold to a limited extent. To obtain the best results in cold bending, it is necessary to have all tools in good condition, and to minimize friction between the tools and the metal that is being formed. The metal itself should be free from nicks and scratches, particularly along the edges, and the radii for all bends should be as large as practical.

For difficult forming the metal should be heated, but this should not be undertaken unless equipment for accurate temperature control is available. Heating the metal to a temperature of 300° F improves the cold-forming characteristics somewhat and does not seriously affect the properties of the finished piece, provided the metal is heated less than 30 min. For more difficult forming, however, it is usually necessary to heat the piece to the heat-treating temperature, 930° to 950° F. Such heating, of course, reduces the mechanical properties of the metal unless it is followed immediately by a suitable quench. In the ordinary heat treatment of the metal this quench is provided by immediate immersion in cold water, but since such a quench usually results in objectionable distortion, it is not generally utilized in the fabrication shop unless some means of straightening the finished piece is available. The more common method of providing a quench in the fabricating shop is to transfer the metal quickly from the heating medium to the forming die, which should be cold and of sufficient size to chill the metal thoroughly and suddenly during the forming operation. Since the metal is thoroughly supported and held in alignment by the die, the finished piece is not distorted.

It is clear from the foregoing that, in order that structural aluminum may be hot-formed successfully, it is necessary to have suitable heating equipment with accurate temperature control. Some fabricators of structural aluminum have found it desirable to install such equipment, but in other cases, where only a few pieces require forming, arrangements have been made to have such forming done at the plant of the producer of the metal prior to shipment. The engineer who appreciates the difficulties of forming and bending will make an effort, of course, to minimize the number of pieces that require such treatment, or he will try to limit such pieces to low stressed portions of the structure so that they may be made from one of the lower strength alloys having better forming characteristics than the particular structural aluminum alloy (17S-T), described in this paper.

Rivet holes in structural aluminum may be drilled, punched, or subpunched and reamed as desired, although drilled or reamed holes are preferable to punched holes. For reaming, the taper bridge reamer with spiral flutes is recommended.

Most aluminum alloy structures, particularly those involving the larger sizes of shapes and plates, are riveted with hot-driven steel rivets. The heating effect of the individual steel rivets is dissipated very quickly by the excellent thermal conductivity of the metal, so that the strength of the aluminum parts adjacent to the rivets is not affected adversely. Some fabricators have adopted the precaution of driving steel rivets at random, particularly in locations where they are closely spaced, to minimize the heating effect.

Aluminum alloy rivets have been used successfully in a number of structures, particularly in sizes smaller than $\frac{3}{8}$ in. which can be cold driven successfully. When it is necessary to use them in the larger sizes, the driving is done hot, and, again, it is necessary to have suitable heating equipment available. The hot-driving of aluminum rivets is very similar to the die-quench hot-forming operation previously described; that is, the rivets are inserted in the hole and driven immediately after removal from the heating medium so that a satisfactory quench is effected by the contact between the rivet and the relatively cold tools and metal around the hole. Some fabricators have successfully adopted the aircraft practice of driving cold rivets of structural aluminum (17S-T) in their semi-soft condition, immediately after heating and quenching, natural aging bringing the strength of the finished rivets up to maximum value over a period of three or four days. In general, however, the additional weight saved by the use of aluminum rivets has not been considered sufficiently important to cause them to be generally adopted instead of steel rivets.

The welding of structural aluminum, like that of most heat-treated materials, has not developed to the stage where it may be utilized to replace rivets. The heat of welding tends to anneal the metal adjacent to the welded area, greatly reducing the strength. When welding is necessary in aluminum construction, one of the lower strength alloys particularly suited for welding is used instead of the standard alloy treated in this paper (17S-T). This applies principally to tanks and light framework rather than to the larger structures.

Aluminum structures should be painted thoroughly, particularly when they are to be used in locations in which the corrosive conditions are severe or uncertain. The procedure generally recommended is to use a priming coat containing a substantial quantity of zinc chromate in a synthetic resin vehicle, applied to all surfaces of the structural parts and allowed to dry before assembly. All parts should be clean and dry, of course, before the application of the priming coat. Following the riveting operation, all rivet heads and adjacent parts should be touched up with the same primer. It is common practice to use at least two protective coatings of aluminum paint over the primer, and care should be taken to work the paint in well around the rivet heads.

No discussion of the fabrication of light-weight structures can be complete without proper consideration of workmanship. When aluminum alloys are utilized in a structure, it is generally for the specific purpose of reducing dead weight, and, therefore, one may logically assume that great care will be exercised in the design to produce a given result with the least possible material. Any such design deserves and should receive the most careful attention from the standpoint of workmanship. This means, of course, that: (1) Tools should be kept in first-class condition; (2) that holes should be round and no larger than necessary; (3) that edge distances should be carefully maintained and all edges should be well dressed to avoid notches and roughness; and (4) that all material should be handled in the shop work so that surfaces and edges are protected from dents and other injurious accidental defects. It is important that all re-entrant corners be provided with suitable radii to avoid stress concentration, and that rivets shall not be over-driven. In short, the fabricator should observe all the well-known rules of good structural metal practice as set forth in standard specifications in order that the excellence of the shop work may be commensurate with the care exercised in the design, and may contribute to the weight saving expected by the designer.

DESIGNING WITH STRUCTURAL ALUMINUM

In preparing the design of any structure the engineer, of course, must be familiar with the properties of the materials being used and the behavior of those materials under different stress conditions. Unfortunately, it is not always possible for the designing engineer to obtain this knowledge through first-hand experience in actual handling and testing of the materials, and, therefore, he must rely on other sources of information. Generally, he depends for his knowledge of the behavior of any material on some prepared set of specifications which limit the working stresses in the structure to values which are known to be safe on the basis of available test results, theoretical studies, and the experience of those engineers most familiar with the material.

In the case of aluminum alloys, particularly the standard product (17S-T) herein described, a wide variety of tests is available indicative of the structural behavior of the metal, as well as a considerable background of experience in its structural application. Although it is beyond the scope of this paper to go into detail regarding either of these interesting phases of the development of structural aluminum, the writer and his associates have drawn heavily on both in the preparation of the following design stresses. Theoretical studies, such as those by Professor S. Timoshenko[97], have also played an important part in the selection of suitable formulas.

These design stresses represent a factor of safety of at least two against permanent set and buckling, and a factor of safety of at least three against ultimate failure. In each case an attempt has been made to express the allowable stress in such a way that the designer has maximum freedom from arbitrary limitations so that he may exercise his ingenuity to the fullest extent in obtaining maximum weight savings within the limits of safety.

For the basic allowable unit stress on structural aluminum (17S-T) members in direct tension, or on the tension flanges of beams and girders (net section), 15 kips per sq in. is recommended. This value is one-half the guaranteed minimum yield strength for rolled structural shapes, and is less than one-third the guaranteed minimum tensile strength.

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[97] "Strength of Materials", by S. Timoshenko.

For compression on columns and other compression members having an effective slenderness ratio, $\frac{aL}{k}$, equal to or less than 81, an allowable stress (in pounds per square inch) is recommended in accordance with the following straight-line formula:

$$s_a = 15\,000 - 123 \frac{aL}{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In Equation (6), L is defined as the unsupported length of the member, in inches; k is the corresponding least radius of gyration, in inches; and a is a factor describing the end conditions of the member. For both ends fixed, a is equal to 0.5, and for both ends pinned, a is equal to 1.0. Since few if any structural members have completely fixed ends, the value of a should rarely be less than 0.6. In framed construction most compression members have partly fixed ends, and an a-factor should be selected between 0.6 and 1.0, depending upon the end conditions.

For compression on columns and other compression members having an effective slenderness ratio, $\frac{aL}{k}$, greater than 81, the allowable working stress should be in accordance with the following formula:

$$s_a = \frac{33\,000\,000}{\left(\frac{aL}{k}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Equations (6) and (7) for compression are plotted in Fig. 39, and it will be seen that the straight-line portion is tangent to the curved portion, the latter being simply the well-known Euler formula with a factor of safety of three applied. This combination of formulas has been found to represent

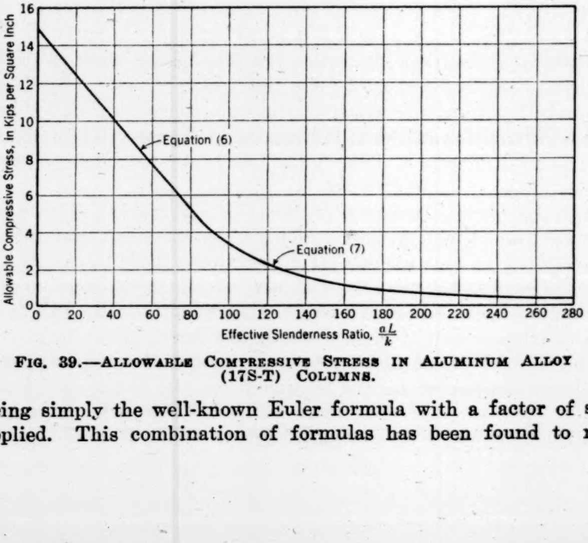

FIG. 39.—ALLOWABLE COMPRESSIVE STRESS IN ALUMINUM ALLOY (17S-T) COLUMNS.

fairly well the general trend of column tests on a large variety of structural aluminum compression members. It is used in the same manner as most other column formulas, inasmuch as it applies to the average stress on the gross cross-sectional area of the member at or near the center of the unsupported length. It will be noted that there is no rounding off of the column curve as it approaches $\frac{aL}{k} = 0$. In the writer's opinion it is much more logical to accomplish this effect by applying suitable restrictions to the allowable stresses on members which are likely to fail locally.

Flat plates or the legs, webs, and flanges of structural shapes tend to buckle locally when stressed in compression, particularly if such parts are fairly thin with respect to their unsupported widths. Because of the lower modulus of elasticity, aluminum alloys have less resistance to such buckling than steel, and, for this reason, it is more important, perhaps, that the allowable compressive stresses be defined more clearly than in the case of steel. In any event, it is far better to define the safe allowable stresses on such parts than arbitrarily to limit their dimensions because, as previously indicated, the designer should be allowed as much freedom as possible in working toward maximum weight savings.

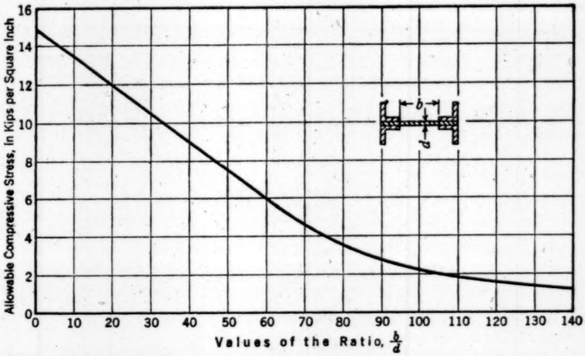

FIG. 40.—ALLOWABLE COMPRESSIVE STRESS ON ALUMINUM ALLOY PLATE SUPPORTED ON TWO EDGES

When flat plates buckle in compression, such buckling occurs in the form of local waves or wrinkles which are practically independent of the length of the member, provided such length is large with respect to the unsupported width of the plate. These local buckling failures in plates may be treated conveniently as local column failures of parts of the plate, using the column formula for the material, provided the proper "equivalent slenderness ratios" are used. From theoretical considerations and available test results the writer and his associates have evaluated these equivalent slenderness ratios for various cases as indicated subsequently.

For flat plates supported along two sides parallel to the direction of stress as in the case of the webs of H-shaped columns, or the compression cover-plates of double-web box girders, the equivalent slenderness ratio of the plate should be determined, as follows:

$$\frac{aL}{k} = 1.2\,\frac{b}{d} \qquad (8)$$

in which b is the unsupported width, and d, the thickness, both in inches. This equivalent slenderness ratio may be used in the column formulas, Equations (6) and (7), for determining the allowable compressive stress on such plates. The values thus determined are plotted in Fig. 40.

For flat projecting plates supported along one side parallel to the direction of stress, as in the case of the outstanding legs of single-angle, double-angle, **T**-shaped or star-shaped struts, the equivalent slenderness ratio of the projecting plate should be determined, as follows:

$$\frac{aL}{k} = 4.0\,\frac{b}{d} \qquad (9)$$

in which b is again the unsupported width and d the thickness (see Fig. 41, Case 1). Tests have shown that single-angle, double-angle, **T**-shaped and star-shaped struts have very little restraint against twisting if any outstanding leg begins to buckle, and the coefficient, 4.0, in Equation (9) is selected on this basis. For projecting plates or legs of angles built into heavier members, as in the case of the flanges of girders or **H**-shaped columns having relatively thick webs, the coefficient, 4.0, may be reduced to 3.0 because such members offer a much higher degree of restraint against twisting as the outstanding legs or flanges begin to buckle. For intermediate degrees of restraint the coefficient should be selected between the limiting values, 4 and 3, so as to be consistent with the degree of restraint assumed.

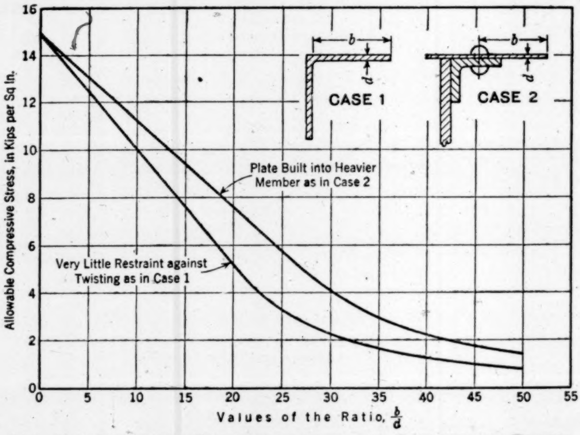

FIG. 41.—ALLOWABLE COMPRESSIVE STRESS ON OUTSTANDING PLATES AND LEGS, OF ANGLES OF ALUMINUM ALLOY

When flat plates supported on either one side or on two sides are used as component parts of columns (see Fig. 41, Case 2) they should be investigated for stability in accordance with the foregoing recommendations, independently of the stability of the column as a whole. If it is found that the allowable stress on the plate is less than that for the column as a whole, the former, of course, will control the design of the column, and *vice versa.* The use of equivalent slenderness ratios makes this comparison very simple because it is only necessary to determine whether the equivalent slenderness ratio of the parts is less or greater than the effective slenderness ratio for the column as a whole. In many cases it will be found economical to make the equivalent slenderness ratio of the parts equal to the effective slenderness ratio of the member as a whole, although this cannot be stated as an infallible guide.

For bearing on rivets or on tightly fitted pins and bolts, a working stress of 26 kips per sq in. on the projected area is recommended. Tests have shown that this value represents a factor of safety of at least two against measurable permanent distortion of the hole, and more than three against ultimate failure, provided the edge distance from the center of the hole in the direction of stress is at least twice the hole diameter. For smaller edge distances the foregoing allowable stress (26 kips per sq in.) should be reduced proportionately.

For shear on aluminum (17S-T) rivets or on tightly fitted aluminum (17S-T) pins or bolts, a design stress of 9 kips per sq in. is recommended. This same limiting shear stress applies to the gross section of the webs of beams and girders, provided the shear on the net area on such webs does not exceed 12 kips per sq in., and provided thin webs are adequately stiffened. For shear on steel rivets an allowable stress of 13 kips per sq in. is recommended.

For the webs of beams and girders the maximum shear stress, in pounds per square inch, at the center of any stiffened panel should not exceed:

$$\frac{12\,000\,000}{\left(\frac{h}{d}\right)^2}\left[1+\left(\frac{h}{l}\right)^2\right] \dots\dots\dots\dots \quad (10)$$

in which d is the web thickness; h is the clear depth of web between flanges; and l is the clear distance between stiffeners, all in inches. When it is desired to investigate intermediate points in large panels subjected to a varying shear along the length, l may be defined as twice the clear distance to the nearest stiffener. When no stiffeners are used, l becomes very large and the quantity in the brackets approaches unity. Equation (10) not only provides a means of checking girder webs for buckling, but also permits the spacing of stiffeners to be determined to suit any condition of varying shear. It should not be used beyond the limiting value of 9 kips per sq in. Fig. 42 is a set of curves representing Equation (10).

Members which in ordinary service are subject to stress reversal, tension to compression, should be proportioned so that the larger stress plus one-half the smaller stress, does not exceed the basic allowable stress of 15 kips per sq in. This precaution in design will guard against fatigue failures under all ordinary service conditions, because it provides for a million cycles of reversal with an allowance for "stress raisers", such as rivet holes. Of course, structures that are subject to a considerably larger number of reversals or that have sharp re-entrant corners, and other severe "stress raisers", should receive special treatment in design and cannot be classed with ordinary structures.

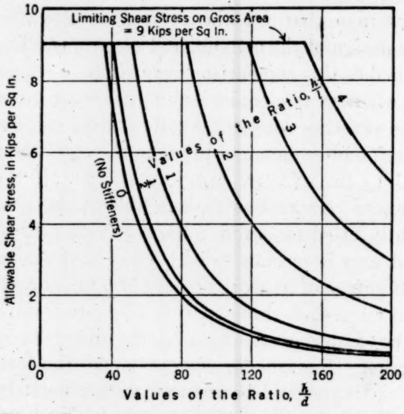

FIG. 42.—ALLOWABLE SHEAR STRESS ON ALUMINUM ALLOY (17S-T) GIRDER WEBS, EQUATION (10)

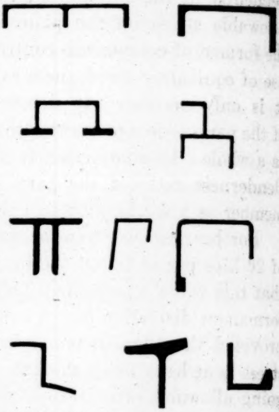

FIG. 43.—EXAMPLES OF ALUMINUM ALLOY SPECIAL SHAPES PRODUCED BY THE EXTRUSION PROCESS

The foregoing safe working stresses are intended to serve as a guide in the design of ordinary structures built of standard structural aluminum, in the same manner that the familiar design specifications for standard steel structures are used. The factors of safety are selected conservatively as indicated. In specialized fields of design these factors of safety may prove to be more conservative than necessary, in view of the exact knowledge of loads and other design conditions, and in such cases the engineer, of course, will use working stresses consistent with the nature of the problem at hand.

Impact allowances on aluminum alloy structures should be the same as those used for comparable steel structures. The entire question of dynamic loadings in ordinary structural design is still on a very unscientific basis, and the mere addition of an arbitrary percentage to the static live load stresses is only a crude attempt to provide a measure of safety against a little understood type of failure. Until a more scientific basis of designing for dynamic loadings comes into general use, there is no point in trying to make adjustments upward or downward in the conventional impact allowances to cover light weight, low modulus materials. Theoretically, of course, the relatively low modulus of elasticity makes aluminum alloys better suited to absorb energy within the elastic range, and this fact is readily demonstrated by tests of simple structural elements. A study of the results of tests and accidents involving more complex built-up structures, however, suggests that the behavior of such structures under dynamic loadings, either in the elastic range or beyond, may be more a function of design than of material, and that aluminum structures may be expected to withstand dynamic loadings at least as well as steel structures of comparable design.

In designing for light weight probably the most important single factor, is the ingenuity of the designer in selecting the best type of structure to carry the loads and meet the other controlling conditions. The designer and detailer must also be adept at arriving at the proper conclusion regarding such questions as the best distribution of metal in various members; when to substitute latticing for solid webs; when to use thin sections with many stiffeners as compared to thicker sections and fewer stiffeners; where to locate splices and connections; and when to use continuity to the best advantage. In all such matters the proper answers change from one structure to the next, depending upon the conditions of the design. The only trustworthy answer to any of these problems is obtained through a careful study of fairly complete comparative designs. Experience gained in the design of steel structures is not always a safe guide in determining how to arrive at the lightest weight in aluminum alloy structures.

The engineer designing in structural aluminum for the first time, is usually intrigued by the possibilities offered by the extrusion process for producing special shapes useful in structural design. In addition to the standard structural shapes, the user of structural aluminum has available an infinite variety of these special shapes, some of which are shown in Fig. 43, and many an aluminum structure owes at least part of its exceptional light weight and lower cost to the intelligent use of such shapes. On the other hand, it cannot be denied that the reverse may easily occur; that is, the novice in aluminum design may be over-enthusiastic about the possibility of extruded shapes, and burden a relatively small job with the cost of several extrusion dies where standard structural shapes might have served practically as well. It should not be difficult in any given case to determine the relative merits of special extrusions and standard shapes if the designer confers with the producer of the special shapes, so that if any extra costs are involved they can be evaluated properly early in the design.

It is a well-known fact that because of the lower modulus of elasticity, aluminum members deflect more than steel members of equal size under the same loading conditions. Occasionally, this fact has been a deterrent for engineers interested in the design of aluminum structures, and almost invariably the reason lies in the fact that the question of deflection is not faced squarely. There is a tendency on the part of some engineers to limit deflections entirely on the basis of their experience with existing steel structures rather than to make an effort to determine the allowable deflection, as an engineering problem. The writer fully appreciates that, in many instances, it is extremely difficult to determine just what the deflection requirements should be, but if minimum weight is to be attained in any structure it is essential for the designer to avoid all unnecessarily severe deflection restrictions.

In general, it is only live load deflections that need concern the designer because excessive dead load deflection can be corrected by proper camber. In addition, in light weight structures the dead load deflections are usually minimized by the weight savings. In members in which deflection becomes a controlling feature of design it is generally advantageous, of course, to increase the depth, thereby distributing the metal to the best advantage to resist deflections. A simple illustration may help to show the advantages of increased depth; for example, a 5-in. aluminum I-beam for a given span and load will deflect approximately three times as far as the same size of steel I-beam weighing three times as much. If the 5-in. aluminum beam is replaced by a 7-in. aluminum beam, however, the deflection will be reduced to the same as that of the steel beam, whereas the weight is increased to a value which is still only one-half that of the 5-in. steel beam.

Sometimes in striving for the lightest and best possible structure, the engineer will discover that other structural metals, such as the alloy steels, can be used to advantage in some part of what would otherwise be an all-aluminum structure. This is particularly true in the case of highly stressed tension members, shear pins, parts subjected to severe abrasion, shallow beams in which the depth cannot be increased readily, and special high-stressed castings. No special precautions are required in such composite construction, except that the possibility of temperature stresses due to differences in coefficient of expansion, should not be over-looked. Although the magnitude of such temperature stress rarely approaches one-half the design stresses, even where the different metals are riveted together over a considerable length, an attempt should be made to avoid such stresses wherever possible by arranging the different metals in such a way that temperature stresses are minimized. It should also be remembered that when a composite structure is loaded, the stresses in adjacent parts which are riveted securely together will be proportional to the moduli of elasticity. For example, if an aluminum cover-plate is riveted to a steel beam, it will carry only one-third its full share of the load and will not strengthen the beam nearly as much as it would an aluminum beam under the same conditions.

In using various metals in combination in a structure, care should be taken to avoid the possibility of galvanic action which might accelerate corrosion at the points of contact of the different metals, either by breaking the contact with an insulating material, or by excluding all moisture through the use of thoroughly protective paint coatings. The latter precaution is the only one necessary in most cases, and from the excellent condition of existing aluminum structures in which steel rivets have been used, one may readily deduce that this protection is entirely adequate.

The writer would like to emphasize the importance of giving proper attention to non-strength members and secondary strength members in a light-weight structure. Too often when an engineer is studying the possibility of weight saving he is inclined to overlook the fact that a fair percentage of the total weight of his proposed structure lies outside the main strength members. Careful attention given to such items as bracing, walk-ways, walk-way supports, hand-rails, closing sheet, etc., will often be rewarded by surprisingly large additional weight savings, and any structure in which weight saving is important certainly deserves this extra attention. Sometimes it will be found that a study of these structurally unimportant parts will lead to combining the functions of parts, thereby making possible the elimination of superfluous members and their connections. In any event, it is an important advantage of the light alloys that they permit weight savings in low stressed parts, which are very difficult to obtain in heavier metals that save weight only by virtue of their superior mechanical properties.

WEIGHT SAVING POSSIBILITIES

The question of when to use structural aluminum is an interesting one, which obviously cannot be treated adequately without going into a detailed and lengthy discussion. The writer will attempt only a brief review of the more important generalities. There are three questions to be answered: (1) What weight savings are possible? (2) What additional expense is incurred to obtain these weight savings? and (3) Can this extra expense be justified? Since structural aluminum weighs 35% as much as steel, a direct substitution, section for section, will lead to a weight saving of 65%, and this margin is often attained through exactly this procedure in actual practice. In most structures, however, the weight savings will range from 50% to 60%, because the designer finds it necessary, in certain members, to use slightly deeper or thicker sections than would be required for equivalent service in steel. The writer has made a study of the weight savings possible in various types of aluminum alloy structural members, tension, bending, and compression, assuming equal conditions of service, and finds that in the average structure in which dead load stresses are relatively small, theoretically, the weight of structural aluminum (17S-T) members should be about 45% that of the same members designed in structural steel for the same loads, a weight saving of 55 per cent.

In larger structures in which the dead load stresses become important in the design of the various members, the weight saved by the use of structural aluminum is greater due to an interesting pyramiding effect. When such a structure is made lighter, the decrease in weight causes a decrease in the dead load stresses which, in turn, permits an additional decrease in size of members, etc. Studies show that in these larger structures weight savings of 65% to 70%, and even greater, should be expected.

In discussing weight savings the use of percentages may lead to a certain degree of confusion. Throughout this paper the writer has used weight-saving percentages defined as follows:

$$\begin{matrix}\text{Percentage of}\\ \text{weight saved}\end{matrix} = 100\left(1 - \frac{\text{Total weight of aluminum members}}{\text{Total weight of equivalent steel members}}\right) \quad ..(11)$$

It does not apply to the over-all weight of structures, except in those instances in which the structural members themselves make up the entire weight of the structure; for example, consider the case of a dragline boom built entirely of structural aluminum and compared with a steel boom of exactly the same length and designed for the same service. The aluminum boom in this case might be expected to weigh 45% of the weight of the steel boom, provided both weights are for the structural parts only, exclusive of cables, sheaves, etc.

If the over-all weight of the dragline unit, including cables, machinery, and sheaves, is used in the comparison, the foregoing weight-saving percentage needs very radical revision. Other complicating factors in the interpretation of weight-saving percentages would be introduced into this same illustration if the aluminum boom was made longer than the steel boom to take advantage of the lower over-turning moment, or if only the outer two-thirds of the boom were constructed of aluminum instead of the entire boom. It is not the writer's purpose to enter into a discussion of these complicating factors, but it is necessary to emphasize that the weight-saving data introduced in this paper are intended to represent the differences in weight between any group of aluminum structural members and the same group of members designed in steel for equivalent service.

The price per pound of structural aluminum plates and shapes will average about fifteen times the corresponding price per pound of structural steel. Reduced to a volume basis instead of a pound basis the structural aluminum plates and shapes will cost about five times as much as those of structural steel. Therefore, comparing two structures having members of identical size throughout, one built of structural aluminum and the other of structural steel, the cost for material alone in the aluminum structure should be about five times that of the steel structure. For structures which in aluminum weigh 45% as much as the same structures designed in steel, the extra cost of the aluminum plates and shapes will be more nearly seven times that of the equivalent steel. If the extra cost is divided by the weight saved, the additional cost for material in the aluminum structures can be expressed conveniently in terms of extra cost per pound of weight saved. The writer has made a study along these lines, and has plotted the results in Fig. 44 which shows how the extra cost per pound of weight saved decreases as the percentage of weight saved increases, the most common range being from

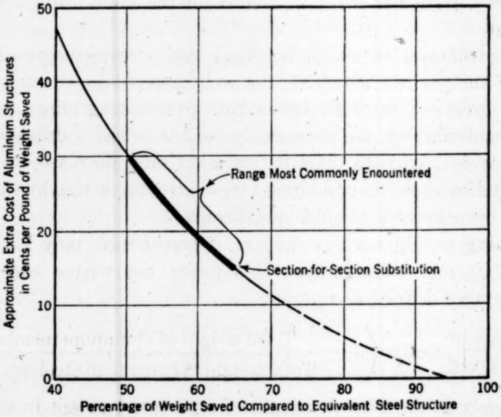

FIG. 44.—EXTRA COST OF STRUCTURAL ALUMINUM STRUCTURES COMPARED TO ORDINARY STEEL

15 cents to 30 cents extra per lb of weight saved. Of course, there are many factors that may operate to invalidate the values indicated on this curve, but the writer believes that they will be found fairly representative for the ordinary structures encountered to-day.

Although the extra cost values given in Fig. 44 are for material only, they should also represent, quite closely, the over-all difference in cost of fabricated structures because experience has shown that the cost of fabricating any aluminum structure is practically the same as that for fabricating a comparable steel structure. Of course, structural aluminum, because of its light weight, may effect slight savings through lower handling costs, but these savings are frequently offset by the fact that workmen may tend to handle the aluminum more carefully to avoid spoilage.

Since aluminum structures are more expensive than equivalent steel structures, the question of when to use aluminum becomes largely one of justifying the extra cost. Obviously, many structures are built, in which there is no possibility of justifying an extra expenditure of even 10 cents for each pound of weight saved, and for the present these structures are outside the field of structural aluminum. There are many structures, however, in which 15 to 30 cents extra cost per lb of weight saved (and even more) is easily justified. Most of such cases, of course, fall in the classification of moving structures where operating economies are effected. The aircraft and transportation fields have been, and will probably continue to be, the most fertile field for the use of structural aluminum, but engineers are rapidly finding other fields in which the light metals are making a place for themselves in spite of the cost. Such fields include dragline and crane booms, traveling cranes, mine-hoist cages, ship superstructures, etc. One interesting field of use of the light structural metals is in applications where the reduction of dead weight permits the useful life of an existing structure to be extended to accommodate changes in loading conditions not contemplated in the structure as originally designed. Examples are found in the replacement of heavy bridge floors with new light-weight floors, and the use of light-weight traveling cranes in buildings not sturdy enough to support a conventional crane of the required capacity.

As a specific example of the economics of the use of structural aluminum, the writer would like to cite the case of dragline booms, such as are used in levee construction. In this field, the weight saved by the use of aluminum is used most effectively to increase the length of the boom, and, hence, the operating range of the dragline unit. It has been found that a 150-ft steel boom, weighing 33 000 lb, can be replaced with a 175-ft composite aluminum-steel boom weighing about 23 000 lb, the weight saving being accomplished by using 12 000 lb of structural aluminum in the outermost 140 ft of the length. The resulting light-weight boom will handle the same size of bucket as the steel boom, with no greater overturning moment on the machine, and with no decrease in the swing speed. Because of the extra 25 ft in length there is less rehandling of material for the same capacity of bucket so that the speed of the work is increased about 10%, resulting in a saving of about 1 cent per cu yd of earth moved, and, thereby, increasing the earnings by about $1 000 per month per machine. The extra cost of the light-weight boom is thus defrayed in about five months, insuring an adequate return on the additional investment.

No discussion of the costs of aluminum alloy structures is complete without some mention of the scrap price of the metal, which in the case of fabricated structural shapes and plates is about 12 cents per lb at the present time. This high return value of the metal does not affect the extra first cost of structures, of course, and hence has no affect on the values shown in Fig. 44. It should not be overlooked in the general study of the economics of light-weight design, however, particularly on structures having relatively short periods of usefulness due to rapid obsolescence.

CONCLUSION

Structural aluminum is firmly established in those fields in which it is now being used successfully. Its future in other fields is a matter which lies almost entirely in the hands of designing engineers interested in producing structures which best serve their intended purposes. The writer has tried to indicate that the choice of structural aluminum is almost entirely an economic problem rather than a structural one; that is, structural aluminum may be utilized with confidence in any case where its cost can be justified.

There are many factors, of course, which may affect the future economic status of structural aluminum. Such factors include metallurgical or manufacturing changes which may effect the cost of the metal, the development of new alloys having better mechanical properties, or are otherwise better adapted for structural purposes, new developments in welding, or other fabrication methods, etc. Although some progress is being made at present along many such lines, the writer sees no immediate prospect for any radical change in the status of structural aluminum, and believes that its progress in the structural field in the near future will be simply a continuation of the steady, healthy growth which has marked the development to its present stage of usefulness.

ACKNOWLEDGMENTS

The writer is indebted to R. L. Templin, M. Am. Soc. C. E., and to R. G. Sturm and B. J. Fletcher, Assoc. Members, Am. Soc. C. E., for assistance and suggestions in the preparation of this paper.

MAGNESIUM ALLOYS AND THEIR STRUCTURAL APPLICATION

BY A. W. WINSTON,[98] ESQ.

SYNOPSIS

The combination of light weight with the improved mechanical properties of the newer magnesium alloys makes these materials of increasing interest to the civil engineer. Since 1933 or 1934, production costs have been reduced, and properties have improved to the point where the alloys are finding many successful applications formerly considered impractical for the material.

In this period a number of important applications have developed in the transportation field. As might be expected, because of the necessity for light weight, the airplane industry was the first to adopt these alloys in all types of service. Sand castings of magnesium alloys are used universally in airplane motor and starter castings, in landing wheels and brakes, and as brackets and fittings in the airplane structure. A number of truck and trailer bodies have been built of sheet and structural shapes, resulting in reduced weights and increased operating economy for their owners.

Although they are not used extensively as yet in structural engineering, these alloys give promise of development in this field of application. The purpose of this paper, therefore, is to present the characteristics of the standard magnesium alloys in which the civil engineer is most likely to be interested, in order that he may arrive at a proper appreciation of the possibilities in their use.

INTRODUCTION

In changing from one material to another it is very necessary that careful consideration be given to all the factors involved. The success of a design may depend upon the attention given to details which are automatically provided for by the engineer when working in steel, but which may escape attention when he is starting to use a new material.

The metallurgical aspects of magnesium alloys are presented in Part II of this Symposium. Not less important than a knowledge of these fundamental properties and characteristics is a practical knowledge of the performance of actual beams and columns, of correct shop and field methods of fabricating, and of the protective measures to be applied to insure long and satisfactory service.

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[98] The Dow Chemical Co., Midland, Mich.

The designer contemplating the use of magnesium has available a wide variety of alloys and forms which are adaptable to many specific uses. The manufacture of castings, structural shapes, sheet, and forgings, and the properties obtainable in these forms, have been described in detail in the paper, by Messrs. Zay Jeffries, C. F. Nagel, Jr., and R. T. Wood, entitled "Light-Weight Structural Alloys."[99] On comparing the properties of the standard, cast magnesium alloy with those for cast iron and the cast aluminum alloys (Table 25), it will be observed that the former compare exceptionally well on the basis of unit strength. Although not quite equivalent on a volume basis, the wrought alloys possess very favorable strength-weight ratios. Usually, it is possible to increase the sections slightly to secure the desired stiffness and strength without sacrificing the weight advantage of the magnesium alloys.

TABLE 25.—COMPARATIVE PROPERTIES OF MAGNESIUM AND OTHER STRUCTURAL ALLOYS

Item No.	Alloy and temper (1)	Specific gravity (2)	Weight, in pounds per cubic foot (3)	Tensile strength, in kips per square inch (4)	Yield strength, in kips per square inch (5)	Percentage elongation in 2 in. (6)	Modulus of elasticity, in kips per square inch (7)	Endurance limit, in kips per square inch (8)	Coefficient of thermal expansion, in inches per inch per degree Fahrenheit (9)	SPECIFIC STRESSES,† IN KIPS PER SQUARE INCH		
										Tensile strength (10)	Yield strength (11)	Endurance limit (12)
	(a) CAST METALS											
37*	Magnesium:											
	Heat-treated........	1.83	113	36	13	11	6 500	10.0	0.000016	19.7	7.1	5.5
	Heat-treated and aged	1.83	113	40	20	5	6 500	10.0	0.000016	21.8	10.9	5.5
39	Aluminum copper‡.....	2.83	178	22	14	2	10 000	7.5	0.000013	7.8	5.0	2.7
40	Aluminum Copper:											
	Heat-treated§.......	2.77	174	36	22	4	10 000	6.0	0.000013	13.0	7.9	2.2
41	Gray cast iron‖........	7.2	450	40	..	..	18 000	20.0	0.0000060	5.6	....	2.8
42	Cast steel; 0.30% annealed..............	7.86	490	76	42	..	29 000	33.0	0.0000066	9.7	5.3	4.2
	(b) WROUGHT METALS											
31*	Structural magnesium...	1.80	112	43	30	17	6 500	17.0	0.000016	23.9	16.7	9.5
43	Structural magnesium¶	1.80	112	45	35	12	6 500	....	0.000016	25.0	19.4	...
44	Duralumin............	2.79	174	60	36	20	10 000	15.0	0.000012	21.5	12.9	5.4
45	Low-alloy steel.........	7.85	490	80	60	22	29 000	40.0	0.0000066	10.2	6.8	5.1
46	Chrom-molybdenum steel	7.85	490	125	90	13	29 000	70.0	....	15.9	11.5	8.9

* See corresponding Item Numbers in Table 10 (see p. 1288). † Columns (4), (5), and (8), respectively, divided by Column (2). ‡ Alloy No. 12, Specification 30, Handbook, Soc. of Automotive Engrs., 1936 Edition. § Specification 38, Soc. of Automotive Engrs. ‖ Specification A 48-35 T, Class 40, Am. Soc. for Testing Materials. ¶ Alloy X, experimental alloy, heat-treated after extrusion.

DESIGN FACTORS FOR MAGNESIUM ALLOY STRUCTURES

In considering the more important properties of magnesium alloys and their effect upon structural design, reference should be made to Table 25 and Table 26 for details of the comparative properties discussed. The relative bending properties listed in Table 26 are based on the physical properties given in Table 27. The standard formulas for rectangular beams were used in making the calculations.

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[99] See p. 1267.

TABLE 26.—RELATIVE STRENGTH AND STIFFNESS IN BENDING OF SHEET METALS (WIDTHS CONSTANT)

Comparison	Material	Using Steel on a Basis of 100 for Comparison				Using Aluminum Alloy on a Basis of 100 for Comparison			
		Thickness 100	Strength 100	Stiffness 100	Weight 100	Thickness 100	Strength 100	Stiffness 100	Weight 100
Equal thickness	Aluminum alloy	100	58	34	36	.....	......	.....	......
	Magnesium alloy	100	50	22	23	100	86	65	64
Equal strength	Aluminum alloy	131	100	77	47	.....	......	.....	......
	Magnesium alloy	141	100	62	32	108	100	82	69
Equal stiffness	Aluminum alloy	143	118	100	51	.....	......	.....	......
	Magnesium alloy	165	136	100	38	115	114	100	74
Equal weight	Aluminum alloy	281	458	755	100	.....	......	.....	......
	Magnesium alloy	436	950	1 823	100	155	206	242	100

Specific Gravity.—The specific gravity of the present commercial magnesium alloys will average nearly 1.8, which may be compared to 2.8 for aluminum alloys and 7.85 for steel. The weights per cubic foot are, respectively, 112, 175, and 490 lb. For equal volumes the saving in weight by the use of magnesium alloy in place of aluminum would be 63 lb per cu ft, or 36%, and in the case of steel, the saving would be 378 lb per cu ft, or about 77 per cent. In general, not all this theoretical saving can be realized when allowance has been made for differences in strength and modulus of elasticity. With castings, however, these savings are attained frequently. An example of this is found in some large ventilating fans, 12 ft in diameter. When constructed of aluminum alloy, the castings weighed approximately 900 lb. The use of magnesium alloy castings made from the same patterns has lowered the casting weight to less than 600 lb, a net saving of more than 300 lb. The lighter fan has permitted the use of a smaller motor because of the lessened starting load and has reduced, considerably, the cost of the unit. Patterns used for aluminum or cast iron ordinarily may be used for magnesium alloys without modification other than adjustment for shrinkage and increase in the radii of fillets.

TABLE 27.—PHYSICAL PROPERTIES OF THE ALLOYS COMPARED IN TABLE 26.

Material	Specific gravity	Tensile yield strength, in kips per square inch	Young's modulus of elasticity, in kips per square inch
Steel	7.85	60	29 000
Aluminum alloy	2.79	35	10 000
Magnesium alloy	1.80	30	6 500

Thermal Expansion.—The coefficient of thermal expansion of magnesium alloys is generally accepted to be 0.000016 in. per in. per degree Fahrenheit. This value is slightly greater than that for aluminum alloys (0.000013) and considerably greater than that for steel (0.0000066), or that for cast iron

(0.000006). This difference becomes important when structures subject to temperature changes are being designed, in which magnesium alloy members may be rigidly connected to members of iron or steel; for example, a magnesium alloy floor-plate 10 ft long may be riveted to a steel framework made necessary by space limitations. If such a structure is subjected to a temperature rise of 100° F, the difference in expansion of the two materials will be almost $\frac{1}{8}$ in., an amount sufficient to cause partial shear of the rivets or serious and possibly permanent deformation of the structure. It is important, therefore, that the designer recognize this possibility and provide for it by the elimination of contributing causes. The use of shorter members, expansion joints, and care in the method of attachment will do much to make such composite structures practicable.

Modulus of Elasticity.—The generally accepted value for this constant is 6 500 000 lb per sq in. As this is somewhat lower than the modulus for aluminum (10 000 000), and considerably lower than that for steel (29 000 000), it is necessary to take this fact into consideration in re-designing structures for magnesium alloys.

To secure equal stiffness in bending, which usually is desired, it is necessary to increase the moment of inertia of the section. This may be accomplished with a relatively small increase in dimensions since the moment of inertia varies as the cube of the depth. By designing to equal stiffness, the yield strength will be increased over that of the steel or aluminum member being replaced, retaining very significant weight savings, as indicated in Table 26, and described in greater detail under the heading, "Beams."

The differences in moduli of elasticity must also be considered in designing composite structures to insure correct load-transfer conditions. Thus, if a magnesium alloy tension member is riveted to a steel section, both pieces should be adjusted in size to secure equal elongation between rivets when under operating load. Otherwise, the unequal elongation will throw an excessive load upon certain rivets, possibly resulting in their failure or in the elongation of the rivet holes.

Mechanical Properties.—An appreciation of the possibilities in the use of magnesium alloys is obtained by reference to Table 25 in which the comparative properties of some of the important structural metals are given. As previously noted, cast magnesium alloys have mechanical properties equal to those of the cast aluminum alloys or of cast iron on an equal volume basis. When compared on an equal weight basis, both the cast and wrought magnesium alloys are equal or superior to the other metals in the cast or wrought conditions.

Proportional Limit and Yield Strength.—When magnesium alloys are subjected to the usual tensile tests, there is a gradual breaking away from the modulus line, making determination of tangential proportional limits somewhat difficult. A suggested method is to consider the proportional limit as the stress where the stress-strain curve deviates 0.01% from the modulus line.

Similar to aluminum alloys but unlike steel, magnesium alloys do not exhibit sharp yield points. The yield strength is now accepted as the stress where the stress-strain curve deviates 0.2% from the modulus line. The usual value obtained for the proportional limit is from 50 to 60% of the yield strength.

A peculiar characteristic of wrought magnesium alloys, recognized by investigators for a number of years, is the low compressive yield strength compared to the tensile yield strength. This value, determined at 0.2% deviation from the modulus line as in the case of the tensile yield strength, is about 75% of the latter value. This ratio is subject to some variation and will be affected by mechanical working, composition, and heat treatment.

TABLE 28.—PROPERTIES OF EXTRUDED SECTIONS OF STRUCTURAL MAGNESIUM ALLOYS

Item No.*	Tensile strength, in kips per square inch (1)	Tensile yield strength, in kips per square inch (2)	Percentage elongation in 2 in. (3)	Compression yield strength, in kips per square inch (4)	Flexure yield strength, in kips per square inch (5)	Brinell hardness number (6)	Tensile strength, in kips per square inch (1)	Tensile yield strength, in kips per square inch (2)	Percentage elongation in 2 in. (3)	Compression yield strength, in kips per square inch (4)	Flexure yield strength, in kips per square inch (5)	Brinell hardness number (6)
	(*a*) 3 BY 4-INCH BY $\frac{1}{16}$-INCH ANGLES						(*c*) 2-INCH CHANNELS					
31	43.4	24.5	19.7	16.3	.....	58	41.5	21.5	19.5	15.3	.....	57
35	44.2	25.6	13.1	18.9	.....	67	46.0	29.4	13.1	18.4	.....	56
43	43.1	28.6	17.0	23.7	.....	69	44.5	30.0	15.0	27.4	29.5	69
	(*b*) 3-INCH I-BEAMS						(*d*) 4-INCH CHANNELS					
31	43.3	27.1	13.5	18.3	.....	....	42.0	24.2	18.0	18.5	.....	58
35	43.4	29.0	8.0	.....	.....	54	42.7	26.5	12.2	16.4	.....	55
43	45.2	29.6	12.5	22.0	24.1	64	44.3	31.0	9.8	24.3	23.0	72

* Item numbers corresponding to those in Table 10 (see p. 1288,) and Table 25 (see p. 1386)

In Table 28 are given the properties of the alloys in which wrought shapes are available, showing the progressive development, since 1934, in the direction of securing improved yield strength values. It has been noted that the methods presenting the best prospects for improvement along this line are those involving the heat treatment of wrought sections. The properties given in Table 28 are those obtained on structural sections. The yield strengths of small diameter bar stock as given in Table 25 are slightly higher than those of structural shapes, apparently due to more uniform conditions of flow through the extrusion die.

Fatigue Endurance Limit.—A noteworthy feature of magnesium alloys, particularly in the wrought condition, is the high fatigue endurance limit. Thus, it is not unusual to have an extruded magnesium alloy[100], of the same composition as that given in Item No. 31, Table 25, exhibit a fatigue endurance limit of 19 kips per sq in. with a proportional limit slightly less than this value, and a yield strength of about 33 kips per sq in. Endurance limit data probably will find increasing use in design as experience is obtained with actual service applications.

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[100] See p. 1288.

Bearing Strength of Sheet.—The bearing strength of magnesium alloy sheets is rather high compared to the tensile strength of the material, and in the case of a hard-rolled magnesium alloy (Item No. 30, Table 10)[100a] will run from 60 to 70 kips per sq in., whereas the annealed material will be from 55 to 65 kips per sq in. One-fourth of these ultimate values are considered to be safe design values.

Beams.—Magnesium alloy beams deflect appreciably with increasing load above the yield strength until failure finally occurs as stresses approach the ultimate tensile strength. This statement applies to simple beams, such as rectangular sections which fail by yielding of the material. The yield strength of a magnesium alloy beam is considered to be the point at which the outermost fibers are elongated 0.2% beyond the modulus line, and may be determined either by the use of strain-gages or by computing the deflection corresponding to this 0.2% permanent deformation of the outer fibers. This concept applies to the beams the same interpretation of yield strength as that used in the ordinary tensile or compression test on the material. Due to the reinforcing action of the material near the neutral axis, the flexural yield strength as determined in this manner will exceed the tensile or compressive yield strength of the material.

The most important factor that will interfere with the foregoing conclusion is that due to the form of the section. As the sections are made more slender, failure may occur through twisting, buckling, or flange crippling. In such cases, the yield strength of the beam may fall below the yield strength of the material in the outer fibers. Because of this fact, it is suggested that the design values be limited to one-half the flexural yield strengths as determined by actual tests of the sections under consideration. Examples of such tests are given in Table 28 for several typical sections.

Very few data have been obtained on the lateral buckling of compression flanges of beams in the relation of unsupported length to width of flange. Little information is available on the buckling of beam webs and on the crippling of compression flanges. Although most of the I-beam, channel, and plate girder tests conducted to date have failed by lateral buckling, the maximum stresses were about the same as those developed by similar sections which failed by compression yielding of the flanges.

Under the heading, "Modulus of Elasticity," attention was directed to the stiffness characteristics of magnesium alloy sections compared to those of aluminum alloy or steel sections. The relative effects of specific gravity, yield strength, and modulus of elasticity for these three materials on the design of rectangular beams have been computed and are given in Table 26. By reference to this table it will be observed that, on the basis of equal stiffness, the comparison is very favorable to magnesium alloys; for example, for equal stiffness, the magnesium alloy beam will be 65% thicker and 36% stronger than the corresponding steel beam, with a weight saving of 62 per cent. Compared to an aluminum alloy beam of equal stiffness, a magnesium alloy beam will be 15% thicker, 26% lighter, and 14% stronger.

The situation is slightly more complex when one wishes to substitute magnesium alloy structural sections for those of aluminum or steel. If the member to be replaced is a steel beam, it is generally desirable to choose a geometrically similar magnesium alloy section having a moment of inertia about four and one-half times as great as that of the steel beam. This rough rule will provide a member of the same stiffness, of considerably greater strength, and of less than one-half the weight.

[100a] See p. 1288.

In like manner, an aluminum section may be replaced by a geometrically similar magnesium section having a moment of inertia about one and two-thirds as great as that of the aluminum section, resulting in a member about 20% lighter, but having the same strength and stiffness.

If wood is to be replaced by magnesium alloy with equal stiffness, the metal section should have a moment of inertia at least one-fifth that of the wood.

The substitution of magnesium alloy beams for steel or aluminum members in composite structures should be made on the basis of equal stiffness, as better load transfer conditions can be maintained with an increase in the strength of the assembly.

Columns.—Magnesium alloy columns act much the same under loads and are as susceptible to design calculations as columns of other materials. The strength of such columns is dependent upon the following factors: (1) Compressive yield strength; (2) ultimate compressive strength; (3) modulus of elasticity; (4) size; (5) shape; (6) end conditions; (7) loading conditions; and (8) length. Perhaps the most important of these factors are the compressive yield strength and the modulus of elasticity.

The compressive yield strength is the governing factor for most short columns used in structural work. As discussed under the heading, "Proportional Limit and Yield Strength," the compressive yield strength of standard magnesium alloy sections has undergone considerable improvement since 1933. Recent determinations for this value during a series of column tests on a variety of sections of magnesium alloy (see Item No. 43, Table 25 and Table 28), have indicated a value of 24 kips per sq in. for the compressive yield strength. It was found that the results of these column tests were closely approximated by a modified Rankine-Ritter formula:

$$\frac{P}{A} = \frac{s_y}{1 + Q\left(\frac{L}{k}\right)^2} + \frac{80\,000}{\frac{L}{k}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which P = a load, in pounds; A = cross-sectional area, in square inches; s_y = compressive yield strength = 24 000 lb per sq in.; L = length of column, in inches; k = least radius of gyration; and Q = Ritter's constant:

$$Q = \frac{s_y}{C\,\pi^2 E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which C = a fixation coefficient and E = the modulus of elasticity = 6 500 000 lb per sq in. The results of the tests are plotted in Fig. 45. Although service experience is somewhat lacking in the application of these data, it is suggested that design values should not exceed one-half the unit strength given.

Because of the limited space available in this paper, it will be possible to give only a single example of the column characteristics of magnesium alloys compared to those of aluminum alloys or steel.

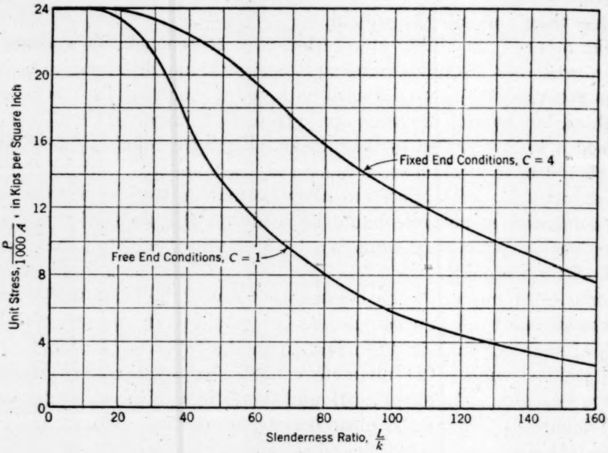

Fig. 45

Assume that a load of 20 000 lb is to be supported by a single fixed-end solid column of round cross-section and a length of 60 in. On computing the necessary diameters, using the published design values for steel and aluminum alloy and one-half the yield strength values for magnesium alloys, as given in Fig. 45, the results given in Table 29 were obtained.

TABLE 29.—COMPUTATION OF COLUMN DIAMETERS

Material	Diameter, in inches	Weight of column, in pounds	Percentage of steel weight saved
Steel	1.72	39.4	..
Aluminum alloy	1.89	17.0	57
Magnesium alloy*	2.10	13.7	65

* Alloy X; see Item No. 43, Table 25.

Some variation from these weight ratios will be experienced when other specific cases are estimated, due to changes in the relative permissible stresses, which will be dependent upon the shape of section, $\frac{L}{k}$-ratios, end conditions, eccentricity of loading, etc. In general, however, the order of weight difference will be similar to that given in Table 29.

FABRICATION METHODS

The successful application of magnesium alloys has been dependent largely upon the development of suitable fabrication methods. Among the important features of good shop practice are the results and conclusions of many years of experience in the production and use of a wide variety of articles made of magnesium alloys, including castings and built-up assemblies using structural shapes and sheets. As with any other structural material, careful attention to small details has been an important factor in the success of these applications.

Machining.—The machinability of the magnesium alloys is unexcelled by that of any other structural metal. A fine smooth finish is readily secured with no tendency to drag, tear, or chip out. Experience in the average machine shop has proved that practically all machine tools can be run at their maximum speeds with feeds to the full capacity of the machine. High-speed steel or tungsten carbide tools are generally recommended because of the long life between grindings.

Ordinarily, no cutting compound or lubricant is required as heavy cuts and feeds may be taken at high speed without excessive heating of cutting tools or work. A coolant is sometimes advisable on high-speed screw machine operations where a very fine finish is desired.

The power requirement in machining is less than that for any other metal. Comparative tests between magnesium and other alloys show that the power required will be about one-half that for aluminum alloys and brass, one-third that for cast iron, and approximately one-fifth that for steel.

Because of this excellent machinability, manufacturing costs frequently are reduced in spite of a larger cost for the rough casting or forging, particularly where the machining expense is a large proportion of the total cost. Machine production also is increased and, at the same time, operating and maintenance costs are lowered.

Forming.—Magnesium alloy sheet and shapes can be given a moderate amount of cold working, but when sharp bends are required it is necessary to heat the work and the tools. A temperature of 500 to 700° F is best for hot forming. The work may be heated locally with a torch, but a better method is to heat the part in an oven where the temperature can be controlled.

Because of the cold-hardening characteristic of the metal, spinnings, stampings, and drawings are limited to those of liberal bend radii in which only moderate deformation is required. In this case, also, heating of the stock and tools will permit more difficult operations. Tools should be clean, smooth, and well lubricated. Lard oil is generally satisfactory for the purpose.

Although the application of some of these processes may involve slight changes from present practice on other materials, most of these problems are capable of solution with a little expenditure of study and effort.

Riveting.—Rivets made of aluminum alloys, with properties given as Items Nos. 1, 2, and 6, in Table 6[101] (Alloys 2S, 3S, and 17S), may be used in magnesium alloy structures. The first two compositions are soft, are easily headed cold, and can be used where strength is not important. Item No. 6 of Table 6[101] (17S or AM55S), should be used where higher stresses are encountered. Alloy 17S has the higher strength, but must be heat treated just prior to driving. Alloy AM55S is especially recommended for use in magnesium alloy structures as the rivets may be driven cold and have the advantage of minimizing the possibility of galvanic corrosion under certain atmospheric conditions. The design of riveted joints, as with other metals, is based upon the shearing strength of the rivet and on the tensile and bearing strengths of the sheet. The use of steel rivets occasionally may be desirable in locations not subject to weathering where strength requirements or convenience in driving may make them more practical than aluminum alloy rivets. To prevent galvanic action, the steel rivets should be dipped in an insulating primer or compound before driving. One-fourth the bearing strength, or about 15 kips per sq in., may be taken as the safe design value for magnesium alloy (Item No. 30, Table 10).[102]

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[101] See p. 1273.

Welding.—Considerable success has been attained during the past few years in the development of welding processes for magnesium alloys. The methods generally in use at this time are gas welding and electric resistance welding. In oxy-acetylene welding a special flux must be used, while the filler rod should be of approximately the same composition as the part being welded. The presence of traces of flux left in the weld will tend to promote subsequent corrosion, and it is necessary, therefore, that considerable care be exercised during the welding operation to prevent such inclusions. After welding, the parts should be thoroughly cleaned in hot water and treated by immersion in the chrome-pickle solution (described under the heading, "Surface Treatment and Painting"), and then painted. Because of the difficulty in treating large structures, the acetylene welding process should be limited to articles and assemblies that can be cleaned adequately.

Electric spot welding has been used in a number of cases. It offers the advantage of speed and economy, and is applicable to the joining of sheet metal and extruded shapes. The following data give the shear strengths from tension tests of single spot welds joining strips of magnesium alloy sheet (Item No. 30, Table 10)[102]:

Single sheet thickness, in inches	Shear strength, in pounds per spot
$\frac{1}{32}$	500 to 600
$\frac{1}{16}$	900 to 1 000
$\frac{1}{8}$	1 600 to 1 900
$\frac{1}{4}$	1 900 to 2 200

Surface Treatment and Painting.—Under ordinary conditions of atmospheric exposure, magnesium alloys have proved remarkably stable over periods of years. The surface, particularly if polished or buffed, gradually tarnishes and becomes covered with a thin gray oxide film. Some powdering and roughening of the surface occurs in heavy industrial areas or in locations of continuously high humidity. This corrosion is a very slow process, however, being measured in terms of years, and is very much slower than the corresponding rusting of mild steel in the same atmosphere. In saline atmospheres along the sea coast, corrosion may become more serious and necessitate preventive measures.

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[102] See p. 1288.

Because of the difficulty in controlling the location and conditions under which an article will be used, it is always recommended that magnesium alloy assemblies or parts be given suitable paint protection. It has been demonstrated that, with proper care, paint systems on magnesium alloys will have a useful life in service as long and as satisfactory as on any other structural metal.

The surface treatment prior to painting is of great importance in the development of a protective paint system. The application of paint coats to the bare metal will result in unsatisfactory adhesion. A chemical treatment known as the chrome-pickle has been developed which, when applied to the metal, imparts to it definite corrosion inhibitive characteristics and a surface etch which promotes satisfactory mechanical bond. The chrome-pickle treatment consists of a dip for 30 sec at room temperature in a bath containing nitric acid and sodium bichromate. The yellowish irridescent coating formed by this treatment possesses good bond to the metal and in conjunction with suitable primers insures lasting adhesion of the paint system.

As a result of thousands of exposure tests, a number of paint schedules have been developed for a variety of service conditions, combining adequate protection with attractive decorative characteristics. A primer is extremely important and must be carefully selected for the particular service to be encountered. For general outdoor exposure, a primer recently developed by the United States Navy (Navy Specification P-27) has been very satisfactory, as it combines excellent adhesion with the desired inhibitive characteristic due to the zinc chromate pigment.

The last few years have seen the development of a number of very superior paint finishes compared to the old oil paints. They possess remarkably good weather resistance and imperviousness to moisture, due largely to the use of synthetic resins in their manufacture. Because of these qualities, they have proved to be very satisfactory for finishes over magnesium alloys. For ordinary atmospheric exposure, two coats of the pigmented enamels are generally satisfactory. For severe exposure conditions, it is recommended that three finish coats be applied, consisting of a varnish developed by the United States Navy (Navy Specification V-10*c*), containing 1½ lb per gal of aluminum powder.

In building assemblies from magnesium alloy structural shapes and sheets which will be exposed to the weather, care should be taken to avoid pockets that could entrap water. Enclosed areas should be provided with drainage and good ventilation, and should be given at least one coat of approved primer. Faying surfaces should be primed and allowed to dry before assembly. In locations where magnesium alloy surfaces will be in contact with wood or dissimilar metals, additional treatment with bituminous paint is recommended. A heavy sealing compound should be used if there is a possibility of water entering the joint.

Numerous structural magnesium parts and assemblies, such as motor castings, airplane wheels, and truck and trailer bodies, have proved that properly applied paint systems will give adequate protection. The use of magnesium alloys in structural applications now appears practicable (from the protection viewpoint) even in the more severe exposures along the sea coast.

CONCLUSION

The development of structural applications for magnesium alloys is dependent upon the economy possible through their use. It is a question of balancing initial, fabrication, and maintenance costs against the benefits of saving in weight. Castings of structural magnesium alloys already have demonstrated their ability to compete successfully with older metals and to-day are rendering satisfactory service in hundreds of applications. These castings are of excellent quality and in most cases the cost is very little, if any more than that of the heavier aluminum castings of the same quality.

With the introduction of stronger alloys and improved fabrication processes, the way is now open for the development of additional structural applications. A beginning has been made in the transportation industry, and it is logical to expect interest to continue in this field. On account of the relatively high cost compared to steel, and the lack of need for weight saving, it is improbable that the near future will see much use of magnesium alloys in stationary structures. On the other hand, the civil engineer is most likely to use them in reducing the weight of his equipment, in his crane booms, scaffolds, ladders, and other portable tools, to which magnesium alloys can contribute increased load capacity, strength, and convenience.

DISCUSSION

E. Mirabelli,[103] M. Am. Soc. C. E. (by letter).—In discussing safety factors and working stress, Mr. Karpov points out that the safety of a design in any redundant construction may be influenced by the fact that weak members can transfer part of their burden to other parts of the structure. In this manner the actual factor of safety is raised above that indicated by a comparison of computed stress with yield point or ultimate strength of material. In a simple beam there occurs a similar transfer of load from over-stressed outer fibers to under-stressed interior fibers, and the calculated stress does not give an exact indication of the margin available to the useful load-carrying capacity of the beam. After the maximum fiber stress has reached the yield point a certain amount of additional load may be applied without causing distress and, within definite limits, the beam deflection will continue nearly in direct proportion to the loading. Ultimately, the useful limit of the beam is reached, and deflection will increase rapidly with little or no increase in loading. The useful limit is affected to a large extent by the type of beam section and also to a small degree by the type of loading. These facts have been known for some time from results of tests and are used to some extent in the structural analysis of airplanes[104]. A process which would lead to a quantitative prediction of the behavior described herein would, to some degree, attain Mr. Karpov's apparent aim which is to diminish the gap between predicted and actual behavior. Such a process for beams of ordinary carbon steel may be based on an idealization of the stress-strain diagram into a series of straight lines, as shown in Fig. 46(*a*). The strain-hardening region to the right of $n_1 \epsilon$ is unimportant because failure takes place in the form of a rapid increase in the rate of vertical deflection before the outer fibers are strained sufficiently to reach this region. (This discussion refers to plastic failure only and not to elastic failure due to lack of lateral support.) The assumptions are made that through the entire range of stress, including the plastic range, a plane section before bending remains plane after bending[105] and also that the stress-strain relation is the same for tension and compression[106]. The stress distribution over a symmetrical beam section then will be as shown in Fig. 46(*b*), with a portion near the neutral axis in a state of elastic stress and the remainder of the section in a plastic state.

To illustrate the method of developing the deflection equations, the rectangular beam shown in Fig. 47 will be considered with a single concentrated load at mid-span of sufficient magnitude to produce a region of plastic stress. The limits of the plastic regions are defined horizontally by x and vertically by y. Using Fig. 46(b) and Fig. 46(c) and taking moments about the neutral axis, it can be shown that the moment of resistance is,

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[103] Asst. Prof., Structural Eng., Mass. Inst. Tech., Cambridge, Mass.

[104] "Airplane Structures", by Niles and Newell, John Wiley and Sons, N. Y., 1929, p. 239.

[105] "Elastizität und Festigkeit", by Bach-Bauman, Ninth Edition, Berlin, 1924, p. 289.

[106] "The Strength of I-Beams in Flexure", by H. F. Moore, *Bulletin No. 68*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

$$M = \frac{1}{6}\, s_1\, b\, h^2 \left(\frac{3}{2} - \frac{1}{2\, n^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

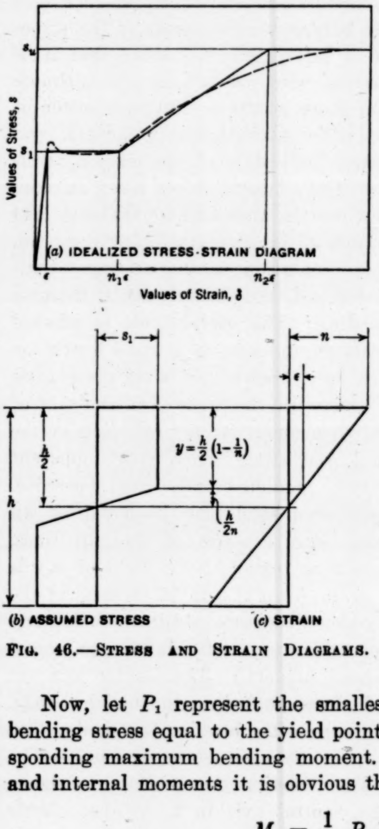
s_u
s_1
Values of Stress, s
(a) IDEALIZED STRESS-STRAIN DIAGRAM
ϵ
$n_1\epsilon$
$n_2\epsilon$
Values of Strain, δ
s_1
n
ϵ
$\frac{h}{2}$
h
$y = \frac{h}{2}\left(1 - \frac{1}{n}\right)$
$\frac{h}{2n}$
(b) ASSUMED STRESS
(c) STRAIN

FIG. 46.—STRESS AND STRAIN DIAGRAMS.

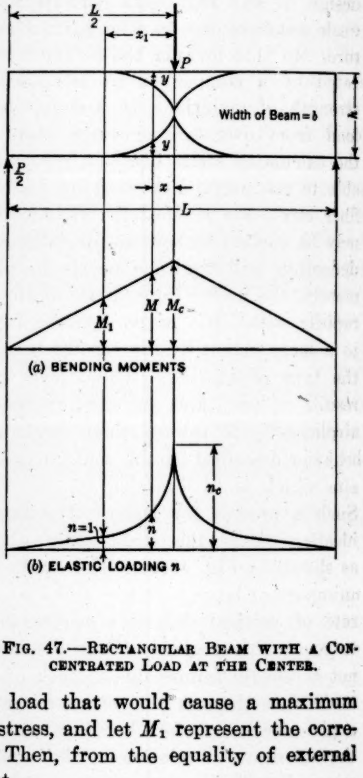
$\frac{L}{2}$
x_1
P
y
y
Width of Beam = b
h
$\frac{P}{2}$
x
L
M
M_c
M_1
(a) BENDING MOMENTS
n_c
$n=1$
n
(b) ELASTIC LOADING n

FIG. 47.—RECTANGULAR BEAM WITH A CONCENTRATED LOAD AT THE CENTER.

Now, let P_1 represent the smallest load that would cause a maximum bending stress equal to the yield point stress, and let M_1 represent the corresponding maximum bending moment. Then, from the equality of external and internal moments it is obvious that,

$$M_1 = \frac{1}{4}\, P_1\, L = \frac{1}{6}\, s_1\, b\, h^2 \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

At the section where $x = x_1$ (see Fig. 47), the bending moment is,

$$\frac{P}{2}\left(\frac{L}{2} - x_1\right) = \frac{1}{6}\, s_1\, b\, h^2 = \frac{1}{4}\, P_1\, L \ \ldots\ldots\ldots\ldots (16)$$

from which,

$$x_1 = \frac{L}{2}\left(1 - \frac{P_1}{P}\right) \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Equation (17) defines the horizontal limit of the plastic region along the outer fibers.

The strain along the outer fibers at any section which cuts the plastic region is measured by the value of the quantity, n, at that section. To obtain an expression for the strain factor, n, substitute Equation (15) in Equation (14) and equate internal and external moments, thus:

$$\frac{P}{2}\left(\frac{L}{2} - x\right) = \frac{1}{4} P_1 L \left(\frac{3}{2} - \frac{1}{2\,n^2}\right) \qquad \text{(18)}$$

When solved for n, Equation (18) gives,

$$n = \frac{1}{\sqrt{3 - 2\left(\frac{P}{P_1}\right)\left(1 - 2\,\frac{x}{L}\right)}} \qquad \text{(19)}$$

When $x = x_1$, it is obvious that $n = 1$, and, since in the purely elastic region the strain is proportional to the bending moment, it follows that in the region, $\frac{L}{2} > x > x_1$, the strain factor is given by the expression,

$$n = \frac{L - 2\,x}{L - 2\,x_1} \qquad \text{(20)}$$

The strain factor, n, may be used now to obtain deflections from the relation,

$$\delta = \int m \, d\theta \qquad \text{(21)}$$

in which m is the bending moment at any section due to a unit load applied at the section for which the deflection, δ, is desired, and $d\theta$ is the angular distortion in any element, dx. From Fig. 46(c) it is apparent that the angular distortion is,

$$d\theta = \frac{2\,n\,\epsilon}{h}\,dx \qquad \text{(22)}$$

If the deflection at mid-span is to be determined,

$$m = \frac{1}{2}\left(\frac{L}{2} - x\right) \qquad \text{(23)}$$

Substituting Equations (22) and (23) in Equation (21) and then performing the integration with the help of Equations (17), (19), and (20), and simplifying:

$$\delta_c = \frac{\epsilon\,L^2}{h}\left\{\left(\frac{P_1}{P}\right)^2\left[\frac{5}{6} - \left(\frac{3}{4} - \frac{1}{12\,n_c^2}\right)\frac{1}{n_c}\right]\right\} \qquad \text{(24)}$$

in which n_c is the strain factor at mid-span and may be calculated by use of Equation (19). If the quantity within the braces is given a substitution value, u, Equation (24) may be simplified thus:

$$\delta_c = \frac{u\,\epsilon\,L^2}{h} \qquad \text{(25)}$$

An alternative method for calculating deflections is by using quantities, $d\theta$, as "elastic weights." From Equation (22), it is seen that for a beam of uniform height the only variable is the quantity, n. This quantity may be used alone as an "elastic loading" if the results are corrected by multiplying by the constant, $2\frac{\epsilon_1}{h}$. The deflection at any point of the beam is the "bending moment" resulting from the application of the "elastic loading."

The foregoing method may be extended to beams with other types of cross-section. The loading-deflection curves shown in Fig. 48 were plotted from calculated deflections. The I-sections were assumed to have a ratio of flange thickness to depth of beam equal to 0.05 and a ratio of web thickness to width of flange equal to 0.10. These are average values for standard I-beams. The diamond-shaped section is a square with loading in the plane of a diagonal. In all calculations it was assumed that $n_1 = 12$ (see Fig. 46(a)) and that the slopes of the two inclined lines of the idealized diagram are in the ratio, 1:80. These are average values for medium carbon steel.

From the definition of P_1, it follows that the ordinate, $\frac{P}{P_1} = 1.0$ (Fig. 48), shows when yield point stress begins in the most stressed material. These curves indicate a yield point in flexure which differs from the yield point in direct stress and which varies widely with the shape of the beam section. These curves also indicate a possible exception to Mr. Karpov's statement that the stress-strain curves of Fig. 2 demonstrate the non-applicability of Hooke's law to low-strength steel stressed above the yield point. If Hooke's law may be interpreted as meaning that deflections are proportional to the forces which produce them, it is evident that sometimes the law is applicable with close approximation even when the steel is stressed above the yield point because the deflections continue very nearly in proportion to the loading for a considerable distance. The calculated curves are in good agreement with the published results of a number of experiments[107]. If the yield point in flexure is to be taken as the limit of the useful carrying capacity of a beam (that is, the point on which the factor of safety is to be based), it is evident that such a factor will vary with the shape of the beam. For example, using a working stress of 18 kips per sq in. for steel having a yield point of 36 kips per sq in., the factor of safety in flexure for the square section on edge is 4, for the rectangular section, 3, and for the typical I-section, 2.4. To obtain a uniform safety factor of 2.4, for example, the working stresses should be 30 kips per sq in. for the square section on edge, 22.5 kips per sq in. for the rectangular section, and 18 kips per sq in. for the typical I-section.

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[107] Test data may be found in the following publications: "Experiments on the Yield Point of Steel under Transverse Tests", by Sir A. B. W. Kennedy, *Engineering*, London, June, 1923, Vol. 115, p. 736: "The Strength of I-Beams in Flexure", by H. F. Moore, *Bulletin No. 68*, Eng. Experiment Station, Univ. of Illinois; "Strength of Light I-Beams", by the late Milo S. Ketchum, Hon. M. Am. Soc. C. E., and J. O. Draffin, M. Am. Soc. C. E., *Bulletin No. 241*, Eng. Experiment Station, Univ. of Illinois; "Beitrag zur Frage der tatsächlichen Tragfähigkeit einfacher und durchlaufender Balkenträger", von Maier-Leibnitz, *Die Bautechnik*, Berlin, 1928, Vol. 6, pp. 11, 27; and "Versuche mit eingespannten und einfachen Balken von I-Form", von Maier-Leibnitz, *Die Bautechnik*, Berlin, 1929, Vol. 7, p. 313.

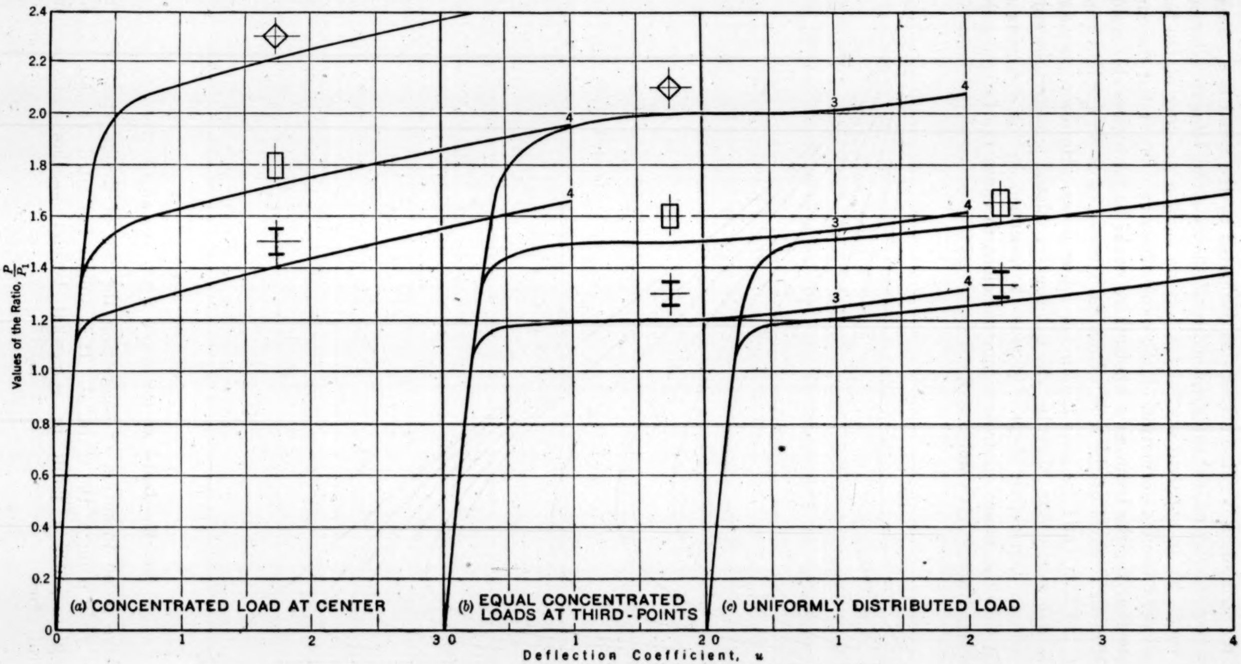

FIG. 48.—LOAD DEFLECTION CURVES (SEE EQUATIONS (24) AND (25)).

If the maximum bending stresses as computed by the usual beam formula should accidentally exceed the yield point of the material by a small amount, there would be an apparent over-stress, but no sudden yielding of the beam. A small permanent deformation of the outer fibers may occur but the curves indicate that for the beam as a whole there could be no appreciable residual deflection on removal of the load. Residual stresses of tension might remain in the top fibers and compression in the lower fibers. However, this condition is not worse than that in which beams or plates are straightened or bent cold in fabricating shops. It must be assumed in the foregoing discussion that the range of stress is not sufficient to cause the fatigue failure, which Mr. Karpov has emphasized.

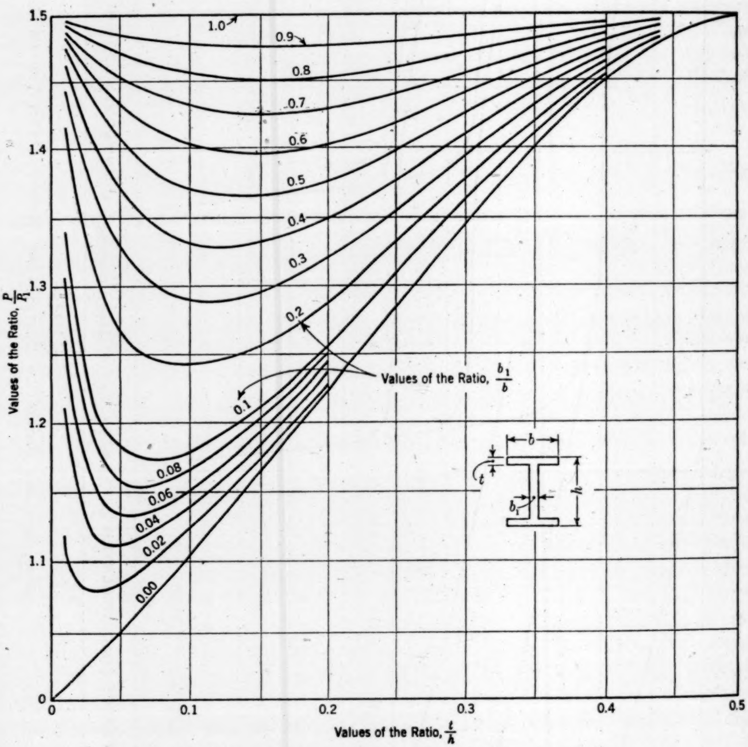

FIG. 49.—LOADS AT THE YIELD POINT FOR AN I-SECTION.

There is no intention of suggesting the use of design stresses as high as the yield point of the material. It is simply indicated that an occasional loading to such a point or somewhat beyond produces no permanent harmful effect.

There is still a question as to the effect of plastic flow or creep whereby long-continued, excessive loading may cause progressive increase in deflection. It appears reasonable to believe that plastic flow of an appreciable magnitude occurs only if the plastic region extends over a considerable part of the cross-section of the beam. Otherwise, the resistance of the elastically stressed portion of the section prevents further deformation.

The yield point in flexure for I-beams varies with the proportions of flanges and webs. The curves of Fig. 49 show the effect of variations in such proportions. It is interesting to note that keeping the flange thickness constant and increasing the web thickness always results in an increased ultimate strength, whereas keeping the web thickness constant and increasing the flange thickness does not always result in an increased ultimate strength.

The foregoing methods for simple beams may be applied to continuous beams with interesting results. For example, for the rectangular beam shown in Fig. 50, P_1 is one of a pair of equal loads which when applied at mid-span will start yield-point stress at a section over the middle support, and P is the corresponding load which will start yield-point stress at a section under the applied load. It is found that the loading may be increased from P_1 to 1.198 P_1, or about 20% without causing plastic stress anywhere except in the small shaded region over the middle support. With a group of four equal loads applied at the one-third points of the spans, the corresponding increase is nearly 50 per cent.

It would seem that any modern or future stress theories which aim at economical and efficient use of materials should recognize the effect of shape of section on the ultimate strength in flexure, and that some parts of a structure may be stressed to the yield point, and even beyond, without impairing its usefulness. Such recognition would tend toward the accomplishment of Mr. Karpov's purpose which is "* * * the designing of structures in which the evaluated and actual safety factors are identical."

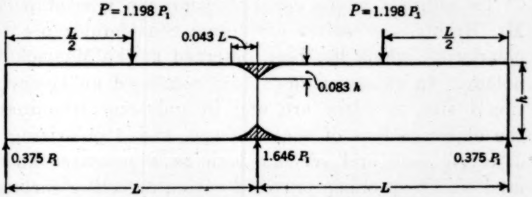

FIG. 50.—CONTINUOUS RECTANGULAR BEAM.

R. W. VOSE,[108] JUN. AM. SOC. C. E. (by letter).—The statement by Mr. Brahtz that "the photo-elastic method is a two-dimensional analysis", must be taken to apply not only to the general nature of the problem, but also to every individual point of interest in the structure to be investigated by the method. It has not been generally appreciated that in many cases which are nominally two-dimensional—such as the bearing of rollers, the distribution of stress in riveted gusset-plates, and the concentrations of stress in the vicinity of sharp corners—third-dimensional stresses arise in the flat model, due to surface distortions and to methods of load application in the actual model. These stresses inevitably balk the investigator, since they affect his tool—the light ray—in an indeterminate manner, and force him to avoid many localized regions of high stress which may be critical in certain types of design involving repeated stress.

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[108] Instructor, Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

The same limitation is encountered when an attempt is made to analyze the surface stresses in a three-dimensional structure by means of any of the "surface photo-elasticity" methods, of which Mr. Brahtz mentions one. A sheet of any of the photo-elastically active materials may be conveniently examined from one side only, if the other side is provided with a reflecting layer to return the polarized light to the analyzing apparatus. If the reflecting side of this sheet is attached to, or inserted in, a loaded model of any material, the sheet will partake of the strains in the model, and the stresses induced in it may then be determined photo-elastically and used as an indication of the stresses in the model. The method fails in the general case, however, due to the three-dimensional nature of the stresses induced in the surface sheet by the model, and it is found to give accurate results only in regions where the stress is not changing rapidly, in which case strain-gages might equally well be applied. Further limitations in this particular application come from the low stresses induced in the photo-elastic model due to their unusually low moduli of elasticity as compared with engineering materials, and from the present inability of the method to give other than the shearing stress at a point.

In addition to the usual transmission type of polariscope as described by Mr. Brahtz, the writer has found considerable use for a portable reflecting polariscope which has been designed at the Massachusetts Institute of Technology. In this instrument are combined an observing telescope of conventional size, mercury arc and incandescent illuminators projecting through the objective lens of the telescope, and a polarizing unit mounted over the objective lens and serving both as a polarizer and an analyzer. This is used with a specimen provided with a reflecting surface or a separate reflector, as mentioned previously, and owing to its small size may be carried to a model loaded in any suitable testing machine and moved from point to point around a model of any complexity. With its small size goes a correspondingly small field and low illumination, so that the instrument is not suited for projection work and must be used visually over small areas.

The same portable polariscope can be converted from the usual type of differential interferometer to an interferometer of the Fabry type with which the individual principal stresses can be determined. The principle of this interferometer and the equations for the solution of its indications are the same as those of the Mach-Zehnder interferometer which Mr. Brahtz describes, but instead of the expensive reflecting apparatus described by him the two surfaces of the model itself, polished to high reflecting power and parallelism, are substituted. This method promises to be an inexpensive supplement to other photo-elastic methods.

In connection with the determination of the individual principal stresses from the shear stress pattern furnished by the usual differential polariscope, it should be mentioned that each method has its own field of applicability. The point-to-point integration methods of Filon and of Neuber are dependent on having a number of free boundary points, or other points at which the complete stress conditions are accurately known; but on the other hand they are the only methods that can be applied to models which have flowed plastically. Such models are of importance in the fields of mechanical processing and soil mechanics where the possibilities of "photo-plasticity" have only been touched. On the other hand, the lateral extensometer methods, the Mach-Zehnder and Fabry interferometric methods, and the membrane methods all depend upon elasticity in the model.

In reviewing the past applications of photo-elasticity it seems that the field of mechanical engineering has been rather thoroughly covered by the present methods, and the designer in this field now has available sufficient data on stress concentrations and similar effects to enable him to handle, satisfactorily, most questions dealing purely with stress The overshadowing influence of fatigue, surface conditions, vibration, residual stresses, and many other factors, lessens the importance of accurate stress estimation. In contrast, the structural engineer is faced with problems in which stress calculations are of major importance in the design, and here it is that photo-elasticity may find increasing fields of usefulness in its present form.

RAYMOND H. HOBROCK,[100] ESQ. (by letter).—Considerable information on the mechanical and physical properties of the aluminum and magnesium alloys is presented in the paper by Messrs. Jeffries, Nagel, and Wood. The authors have wisely indicated quite a number of conditions for which it is best to consult the manufacturer of the metals before the selection of a particular alloy is made. It is the latter point which the writer intends to emphasize.

Civil engineers, in common with a majority of other engineers, are probably most familiar with the ferrous alloys and, through training and association, have become familiar with the interpretation of data on mechanical properties with the ferrous alloys in mind. The familiar mechanical properties for which values are usually given are the modulus of elasticity, the yield stress, the ultimate stress, the percentage elongation, percentage reduction of area, and, more recently, the endurance limit. If the engineer proceeds, by the same testing methods, to gain numerical values for these quantities for an aluminum alloy and for an iron-carbon alloy, the question arises, first, as to the legitimacy of a comparison of such numerical quantities as a basis for the determination of the general useful characteristics of the materials for specific applications. Secondly, one may inquire as to what other information is required for a reasonably complete analysis, providing a comparison of the measured numerical quantities of the mechanical properties are admitted as a legitimate first approximation for an estimation of usefulness.

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In order to gain some idea of the co-relation between the usual measured mechanical properties and the information which might be required by a civil engineer, it is important first to understand what the quantities determined for the mechanical properties actually measure, and then to examine whether or not the properties so measured are pertinent in the design of a successful structure.

Of the mechanical properties usually measured:

(1) The elongation, the reduction of area, and the contraction measure the capacity to flow; that is, they measure the plastic characteristics;

(2) The elastic limit and the yield stress give values at which the plastic characteristics begin; that is, they give numerical values for forces required for the material to deform or to flow some small amount fixed by general agreement;

(3) The ultimate strength is a specific stress value measured during the course of the plastic deformation; it measures the force required for plastic flow at the point where the flow presents the greatest resistance; and,

(4) The endurance limit is a value for a specific stress which may be applied any number of times in complete reversal without causing failure.

Perhaps the most striking fact about an examination of these mechanical properties is that, with the exception of the endurance limit, they are all associated with the plastic characteristics of the material. Since few structures are designed with the intention of causing the structure to undergo a plastic deformation, some doubt may arise as to the value of such quantities in the design of rigid structures. The value does lie chiefly in that these measured properties define a region that must surely be avoided. The yield stress or the elastic limit stands at the indefinite portal to this region —they reveal that at this point the material "flows a little"—how little is a matter that must be further investigated.

Nevertheless, the engineer has learned to make use of these quantities, and he learned them, perhaps, very largely in association with the ferrous alloys. He may express his doubt of the exact location of the beginning of the region in which the material "flows a little" by being on the safe side and assuming that that stress value is 20% too high; and he usually makes his structure two to ten times as strong as his best calculations (based upon the expected load and the yield stress of the material) indicate that it need be—he uses a safety factor. That safety factor can, and is, properly intended to compensate for more than accidental or uncontemplated excess loadings, or the imperfect state of current knowledge of stress analysis. It attempts to take into account a great many of the characteristics of the material that are unknown to the engineer or that are not quantitatively defined and subject to mathematical treatment.

With this in mind the first question may be expanded and restated as follows: If it is conceded that behavior characteristics are usually taken into account in structures of familiar ferrous alloys by the introduction of safety factors learned through much experience, are all these characteristics so inter-related in the case of the light structural alloys that the same safety factors might be used as in the ferrous alloys? If not, are there safety factors that will serve such purpose? Or do the light alloys have some characteristics quite different in relative importance from the same characteristics for a ferrous alloy that should receive special consideration in the design of structures?

It may be of value to examine a few of the conditions that are normally imposed upon a structure, or that might reasonably be expected to be imposed upon a structure during the course of its life, and then inquire into what behavioristic characteristics are indicated for determining the relative value of materials in respect to these requirements.

Anticipated Loadings.—Anticipated loadings of both a static and dynamic nature may be approached in the manner described by Mr. Karpov, making use of the best methods of computing stresses and based primarily upon the modulus of elasticity, the yield stress, and the fatigue properties.

Accidental Localized Overloads.—Such an accidental overload may result, for example, from the impact of an automobile or other vehicle upon a structural element of a bridge. Under such conditions it is valuable to know how the material of the structure, and, finally, how the structure itself, will re-act toward the redistribution of stresses. Will the structural element fail readily under an impact load? How sensitive is the structural material to boundary conditions, to stress concentrations, such as rivet holes, surface injuries, corrosion cracks, etc? Will the sudden application of a load cause failure at loads much less than if slowly applied? The answers are dependent upon the capacity of the material to undergo a plastic deformation quickly and, therefore, to redistribute the stresses. Some idea of the relative values of different materials in this respect is to be gained from impact testing, especially at various impact velocities. A more fundamental material property which perhaps serves as a measure of dynamic ductility is the damping capacity, that is, a measure of the capacity of the material to absorb energy.

Much work still needs to be done in the correlation and interpretation of values for damping capacities directly in terms useful to the design engineer. Indeed, the first need is to devise experimental means for measuring the damping capacity as a function of applied stress. At present, values for damping capacity are usually correlated with the maximum stress in the specimen only, except for very small stresses, as in the case of a free bar subjected to small longitudinal vibrations.

Relaxation or Creep.—If a sheet of metal contains a small hole at a distance of at least three times the diameter from the edge of the sheet, and if this sheet is subjected to tension forces, a stress concentration occurs on the circumference of the hole and on an axis of the sheet perpendicular to the direction of tension which is three times as large as the stress in the undisturbed part of the sheet. However, if the hole is filled tightly with a rivet, in such a manner that radial forces are exerted on the circumference of the hole, these stress concentrations may be eliminated. Suppose that a rivet is properly driven or squeezed in a hole and then exerts the radial force necessary to avoid the stress concentration, the question arises as to how long this condition can be maintained; that is, will the material relax after a certain length of time and relieve the radial forces? If it does, it will result in re-establishment of the stress concentrations. Obviously, information on relaxation and of creep, as well as the results of practical experiments, would be of value in this important case.

Properties at Elevated Temperatures.—These properties are of importance to bridges, buildings, and structures of this type in so far as they aid in predicting the behavior of the structures under unusual conditions as, for example, in the case of a local conflagration in a building. If the strength properties of a structural material decrease rapidly with increase in temperature, it may happen that a purely local fire will endanger the entire structure. It would seem that, because of their relatively low melting points, the aluminum alloys should show a much more rapid decrease in such strength properties with increasing temperature than a ferrous alloy. Furthermore, those aluminum alloys dependent upon precipitation-hardening effects for a goodly portion of their strength, might be expected to decline in strength rapidly at relatively low temperatures. On the other hand, the aluminum alloys possess a great advantage in their high heat conductivity, this being about 4.5 times as great as for low carbon steel at 100° C. It would be of considerable interest, however, to have comparative data on structures of the two materials designed for the same loading and subjected to identical conditions of local heating.

The Course of Destruction of a Structure.—One type of destruction, obviously, is started as a structure is built; that is, the natural deterioration. For a proper consideration of this factor, it is necessary to consider the corrosion properties in relation to the corrosive agents likely to act upon the material of the structure, the fatigue properties, the creep, the influence of the expected atmospheric temperature variations, and, in the case of precipitation-hardened materials, the influence of time itself. Since the acquisition of the highest strength properties in the case of some of the precipitation-hardened materials is a function of time after the solution heat treatment, one might expect that deterioration in properties might also be a function of time. This can be shown to be true in tests at elevated temperatures. From theoretical considerations one might expect a precipitation-hardened material to be thermodynamically more unstable at room temperatures than at elevated temperatures so that the tendency for a decrease in strength properties is greater at room temperatures. However, it is quite certain that the rate of approach to the condition of a thermodynamically lower energy level (and, consequently, to decreased strength properties) is much less at the room temperature. Information regarding strength properties of the light alloys after long times, under stress, and subject to the usual temperature variations of the atmosphere, would do much toward deciding whether or not this is a consideration of any practical importance.

In the event that a structure does fail by overloading, or a part of it fails in fatigue, and that this local failure leads to failure of the entire structure, how will it fail? Will it fail suddenly or will there be some warning, such as an apparent slow deformation of the structure? Obviously, some clue of this is to be gained by comparison of the plastic characteristics of the material—exactly those properties most often measured and listed.

Let no one gain the impression that all this desirable information is available about structural steel and is not available about the light alloys. This is certainly not the case; but engineers have used structural steel for a long time, and there is incorporated in the safety factors which they apply, some estimation of many of these unknown characteristics. The future will undoubtedly see a considerable use of the light alloys for civil engineering, and it has been the writer's intention to indicate that values for the ordinarily measured behavioristic characteristics, coupled with the usual safety factors—or perhaps with any one or two safety factors—will very likely not be sufficient to produce a good design.

At the present state of knowledge and in consideration of the relative unfamiliarity of structural engineers with the light alloys, the advice of Messrs. Jeffries, Nagel, and Wood to "consult the manufacturers" should be strongly emphasized.

WILLIAM F. CLAPP,[110] ESQ. (by letter).—The money spent in the effort to solve the various angles of the corrosion problem, with the ultimate object of producing superior metals, is staggering. Enormous numbers of long-time exposure tests are being conducted in many parts of the world under all conceivable conditions. On the other hand it seems that intensive studies of service records either have not been equally stressed, or the results have not been published or made available to the interested student. Therefore, it is to suggest that additional effort be made to analyze carefully the various factors that have resulted in the innumerable examples of exceptionally good and exceedingly poor service records of the various metals. Is it not within reason to anticipate that findings of value, comparable to those obtained from exposure tests, might result from the equally intensive laboratory studies of service records?

Although deeply interested in the various factors connected with the problems of corrosion, the writer's observations have been entirely confined to the study of a limited number of examples of metal deterioration, in which one or more of a large group of marine organisms has been designated as either directly responsible for, or as producing indirectly the conditions favorable for, accelerated corrosion. To read in recent engineering reports that the "marine borers" were responsible for excessive deterioration in certain steel structures in California; and in other reports that probably certain species of *Balanus* accomplished the same results by means of some powerful secretion, is certainly sufficient to arouse one's curiosity. Although the destructive powers of the marine borers should not be under-estimated, it is scarcely fair, when causes are difficult to find, nonchalantly to blame the "bugs." Some effort should be made either to prove or to remove the implication that marine organisms, particularly marine borers, are contributing factors of major importance. To prove them entirely innocent of this particular outrage is not as simple a matter as at first anticipated. Dr. F. N. Speller and others have indicated[111] some very insidious methods by means of which the marine organisms may after all contribute their share to metal corrosion. However, actual proof seems still to be lacking.

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[110] Cons. Biologist, Duxbury, Mass.

An accumulation of records of the biological, chemical, and electrical conditions surrounding outstanding examples of excellent and poor service, together with the analyses of the metal involved, would not be an appalling program of research, particularly when the annual losses due to corrosion, as described and tabulated by Mr. Aston and others, are taken into consideration.

J. C. HUNSAKER,[112] Esq. (by letter).—Three stimulating papers on the application of stainless steels, aluminum alloys, and magnesium alloys have been presented by Messrs. Ragsdale, Hartmann, and Winston, respectively. The three papers will be considered together by an engineer faced with the necessity for making a decision as to the materials for his design. He will appreciate Mr. Hartmann's paper best, perhaps, because it gives him practical design data rather than physical properties and generalities which, although significant, may not bear on his specific problems. Obviously, there are other criteria than strength, weight, and cost of the metal involved in structural or mechanical design. The engineer responsible for a decision must be assured of durability and maintenance, procurability in quantity without sacrifice of uniformity, shop and erection tooling and practice, and other factors which bear on over-all safety. There is less information offered concerning the latter factors, as applied to stainless steels and magnesium alloys.

The engineer, for example, may be interested in the possibility of light-weight floors for existing bridges, if the substitution of such a floor will prolong the life of the bridge; but he is loathe to abandon conventional materials. It is possible that such floors could be built of stainless steel with satisfactory weight, life, and durability, but the cost of material and fabrication would be high. Since neither rolled nor extruded sections are available, all beams, stringers, and secondary members must be built up. They could not be welded to the existing structure, in all probability, since torch-welding affects the stainless steel and shot-welding is out of the question on thick members of mild steel. It would be difficult to drill them in the field so that they could be riveted, and riveted-joint design of thin members is uncertain and complicated. Repairs to existing stainless steel structures would appear to be difficult. Moreover, for bridge loads, substantial sheets would be required instead of the light gages used in rail cars and deck houses. These sheets are well-nigh impossible to shear and are difficult to form as they have considerable "spring-back." If bent too sharply there seems to be an effect on the grain structure due to the excessive cold working, with the result that the corners corrode. From these considerations one might conclude that stainless steel, as utilized at present, would not be economical, although its durability and low maintenance costs tend to offset the higher costs of the material itself.

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[111] "Corrosion—Causes and Prevention", by F. N. Speller, *Transactions*, A. S. M. E., June, 1935.

[112] Prof. of Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

A real virtue of stainless steel lies in the indirect savings it makes possible for low-weight, high-speed means of transportation. The light-weight train operating at 90 miles per hr can do so on lighter rails and bridges than is possible with standard equipment. Duralumin or magnesium can be considered as equal to stainless steel from this point of view, although one may question both from the standpoint of fire hazard, particularly the magnesium alloys. The latter are also questionable from the standpoint of corrosion, especially in climates burdened with salt moisture or chemicals.

From what the writer has observed of the corrosion of duralumin in contact with steel in airplanes, he would question Mr. Hartmann's recommendation that steel rivets and pins be used for connections on structural aluminum. With adequate paint, corrosion can doubtless be prevented, but all one has to do is to look around at bridges these days to see how little paint they get. Such connections are questionable, particularly near the coast where the fogs are salt, or over railways where the blast from a locomotive blows sulfurous gases against the structure.

In buildings, the criterion for stiffness may be the plastered ceiling which has a definite deflection limit if cracks are to be avoided. It is true that aluminum beams can be made deeper so that they will have the required stiffness, and yet be lighter than steel; but when fireproofing is added, the total weight is as great as, or greater than, that of the steel. Little or nothing may be saved in the weight of the building. Magnesium or stainless beams would probably suffer from the same lack of stiffness; the one due to a low value of E, the other due to the use of deep, thin-walled elements carrying high unit stresses.

The engineer needs to know the limits of available standard elements of construction for stainless steel and magnesium. Stainless sheets seem to be limited in width because of the difficulty of rolling a hard metal to a constant thickness. The writer does not know what the limits are for the various gages; he assumes that magnesium is not available in long extruded sections having larger areas. Aluminum alloy was in this condition before 1926. Stainless steel can be formed through rolls so that long sections are available, but they are limited in width. It is difficult to know what can be done with tubes but the structural engineer considers this section an impossible one in any case, since he cannot fasten other members to it conveniently.

In general, the three materials—stainless steel, aluminum, and magnesium—are clearly of great interest both to the structural, and to the mechanical, engineer; but the specific applications will be approached with caution and step by step. It is fortunate that development of the three types of metal is in such competent hands, but it will require both experience and opportunity before engineers are able to proceed with confidence to the special application of each to uses in which it has a real advantage.

Horace C. Knerr,[113] Esq. (by letter).—A symposium on light-weight structural design seems incomplete without including that class of material which affords the lightest possible construction, namely, heat-treated alloy steel, in the range of 150 to 200 kips per sq in.

Considered in terms of load-carrying capacity for a given weight of material, metals may be compared on the basis of a strength-weight factor obtained by dividing the design strength value (that is, ultimate strength or yield strength), in kips per square inch, by specific gravity.[114]

Taking representative strength values for magnesium and aluminum alloys, respectively, and comparing them with steel on this basis:

$$\begin{array}{ccccccc} \text{Magnesium} & = & \text{Duralumin} & = & \text{Alloy Steel} & = & \text{Strength-Weight Factor} \\ \dfrac{35}{1.8} & = & \dfrac{55}{2.8} & = & \dfrac{150}{7.8} & = & 19 \end{array}$$

If the foregoing strength values are taken as ultimate, they are conservative for the metals indicated. If they are regarded as yield values, the factors for magnesium and duralumin are extreme, but that for steel can be exceeded. Steel, therefore, as a material, may equal or exceed in lightness the so-called light alloys.

Steel lends itself admirably to assembly by any of the welding processes—acetylene, electric arc, electric resistance, etc.—whereas the magnesium and aluminum alloys do not.

The autogenous welded joint is more efficient as regards material than the riveted joint and, therefore, it is lighter, thus contributing further to the lightness advantage of steel.

Comparison of the high-grade, heat-treatable alloy steels with the stainless steels, such as Alloy 18:8 (which derive their strength from cold work), shows certain advantages for the less costly material.

Stainless steels can be welded only by a proprietary process without sacrificing strength and corrosion resistance. They can be given high strength (150 kips per sq in., or more) only in the form of thin ribbons or strips and must be formed and fabricated in this hard, springy condition—a difficult feat.

Their necessarily thin structural sections and the relatively low elastic modulus of the material tend to result in flimsiness and excessive deflection under load.

Their costliness is somewhat offset by their corrosion resistance and attractive appearance, and, in some cases, by their non-magnetic character.

Heat-treatable alloy steels, such as chrome-nickel, chrome-vanadium, chrome-molybdenum, and others of moderate cost, can be used as forgings, rolled or pressed shapes, or, desirably, as seamless or welded tubing which possesses outstanding engineering advantages. These steels are easily worked, hot or cold, and some are better adapted to welding than others.

Parts may be heat-treated prior to assembly by means of bolts or rivets; or, units of moderate size, after assembly by welding, provided necessary precautions are taken to avoid distortion. The weld is strengthened by the heat treatment.

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[113] Cons. Metallurgical Engr.; Pres., Metlab Co., Philadelphia, Pa.

[114] "Material Selection on Strength-Weight Factors", by H. C. Knerr, *Automotive Industries*, April 19, 1923.

Magnesium, aluminum, and stainless steels have all been brought to their highest point of development in the construction of aircraft. The same is true of heat-treatable alloy steels. These steels, in tubular form, assembled by acetylene welding and then heat-treated, have been used successfully for many years in the most vital parts of aircraft, such as the landing gear, wing beams, and heavily stressed structural members where great strength, toughness, reliability, and resistance to shock and fatigue, combined with lightness, are essential. They are still unequalled by the more recently developed materials for these locations.

The three types of metal discussed in this Symposium are all more or less subject to proprietary interest. Their engineering and industrial applications have been promoted vigorously by their sponsors, chiefly in competition with the ordinary steels of low strength.

The heat-treatable alloy steels of high strength, being common property, have lacked this sponsorship and, consequently, have been rather neglected in the rush of new developments. They offer advantages of great importance in the field of light-weight construction, advantages to which the Engineering Profession may well give careful attention.

F. T. SISCO,[115] ESQ. (by letter).—Such a clear and comprehensive picture of the alloy steels and the light non-ferrous alloys has been presented in this Symposium that it seems impossible to add anything of interest on the properties and applications of these materials. There is one point, however, which was neglected by all the authors, which should be mentioned. That is, the rôle played by the research laboratories in developing these alloys so that the engineer will have readily available, at a reasonable cost, a material with certain distinctive properties or combination of properties. Although the writer will confine his remarks to alloy steel they are, perhaps, equally applicable to the non-ferrous alloys.

The history of alloy steels goes back scarcely fifty years, but in that short time splendid results have been attained and some remarkable materials have been produced. Considering the vastness of the field, it is quite astounding that the research laboratories have produced steels which meet almost every present industrial demand. The vastness of the alloy-steel field may be realized when it is remembered that of the alloys of iron with carbon and the eight common alloying elements about 50 000 000 combinations are possible.

Broadly, there are two kinds of research on alloy steels. It is difficult to give them names. They might be called scientific and practical; or if one wishes to be as cynical as a certain physicist, one might call them planned and "hit-or-miss", or "machine-gun" research. Under the research which the writer has tentatively termed "scientific" or "planned" is included work on constitutional diagrams and its ramifications. There are many outstanding examples of such work by American metallurgists. This type of research has as its object the addition of basic knowledge on alloy systems and may or may not have practical consequences. No metallurgist will deny, however, that it is important that the gaps in basic knowledge be filled as soon as possible, even if nothing practical results.

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[115] Metallurgist and Editor, Alloys of Iron Research, The Eng. Foundation, New York, N. Y.

Practical or "hit-or-miss" research is essentially carried on by using as a base material a commercial carbon steel or cast iron, and adding special alloys, or treating the material in special ways, to note what happens. For many years most of the research on alloy steels was conducted in this manner, and most of the common alloy steels are the result of such work. There is much justification for it, because the effect of combinations of alloying elements can rarely be predicted from the known effects of these alloys when used singly. Much of the research at the present time is a combination of planned and "hit-or-miss" methods.

One of the important essentials in beginning research work on alloy steels, whether it is to fill a gap in basic knowledge or to develop a new low-alloy steel, is for the metallurgist to be sure that he is not repeating work already done by some one else. Despite the fact that only a few hundred, or at most a few thousand, of the 50 000 000 possible combinations of alloys have been investigated, the literature on alloy steels is enormous. Reports of such research are scattered through thousands of journals and books in many languages. Reviewing this literature is time-consuming and, except where large libraries are available, practically impossible. Consequently it has frequently happened in the past that, after a metallurgist had completed his research, he found that similar work had been done previously by some one else.

To avoid such unnecessary duplication of effort "Alloys of Iron Research" was founded in 1929 by The Engineering Foundation. Its object is to review this vast literature, "weed out" the clearly erratic work, correlate and summarize the remainder, call attention to conflicts in data, and recommend research to fill gaps in basic knowledge. The results of this critical review of the world's research are correlated in a series of monographs which give to the metallurgist all the essential known data on alloy steels, thus enabling him to plan his research and development work with no loss of time and effort. They also give to the engineer an unbiased summary of the properties of all the alloy steels known to-day.

What will be the future of alloy steels is any one's guess. It has been said that the world is entering an alloy-steel age. No one knows now how many of the 50 000 000 possible alloy steels will be worthless commercially or at least no better than the ones available to-day. No one knows how many of them will have remarkable properties, so remarkable perhaps that a veritable revolution in construction may occur when they are developed. The most that can be said is that good alloy steels are available to-day (steels which are entirely satisfactory for many commercial applications) but that more research is needed to develop new steels, to improve the properties and reduce the cost of the older ones. That the iron and steel industry recognizes this is plain from the speed with which its laboratories are working, from the rapidly expanding literature reporting the results of research, from its splendid co-operation in the basic research to fill the gaps in the knowledge of alloy systems, and from the new alloy steels which are made available each year.

J. CHARLES RATHBUN,[116] M. AM. SOC. C. E., AND D. M. MACALPINE,[117] JUN. AM. SOC. C. E. (by letter).—The field of photo-elasticity has been covered by Mr. Brahtz in a most creditable manner. It does not, however, mention the multi-material use of the photo-elastic method. The idea has been suggested before, but the writers know of no attempt being made to use it. This type of model differs in several important respects from those described in the paper. At the College of the City of New York the photo-elasticity equipment is composed of a standard polariscope, with Nicol prisms and a carbon light source. Although this apparatus is equipped with a specially designed hydraulic loading device, it was considered more satisfactory to apply the loads as weights attached to the lower end of the models.

The advantage of this arrangement can be visualized best in the solution of the problem of determining the partition of a load among the rivets in a butt-joint connection. The model for this case consisted of three plates riveted together, the load being applied to the center plate and the reaction being furnished by the two outer plates.

An attempt was made to coat the inner surface of one of the outer plates with a reflecting material, with the intention of obtaining the patterns after the polarized light had passed through the same plate twice. The results were not satisfactory and the idea was abandoned. A mirror was then inserted between the plates to take the place of the reflecting coating. This appears to be unsatisfactory for several reasons, among them being the effect of the mirror on the stress distribution and also the effect of the distortion of the mirror when stress was applied to the model. Tin foil, aluminum foil, and silvered glass were all tried.

Models were then made which consisted of a central plate of marblette and two outer plates of celluloid, the idea being that, since marblette is much more sensitive than celluloid, the error due to the effect of the less sensitive material could be neglected in the first approximation to the solution. This error may possibly be evaluated and allowed for, if it is considered necessary, by using two sets of models alike in every respect except that different materials are used for the outer plates. There would then be two solutions of the same problem, each containing a set of errors. With the properties of the two materials, and the ratio between the errors known, the absolute value of the error might be computed or estimated, and the correction made.

The first model tested consisted of two strips of celluloid, 1 in. wide and 0.1 in. thick, riveted on either side to a strip of marblette 1 in. wide and 0.25 in. thick. Five rivets, 0.25 in. in diameter and spaced at 0.75-in. centers, were used, the rivets being in line. The value of two 0.1-in. plates of celluloid, when multiplied by the value of E (350 000 lb per sq in.) for celluloid, is approximately the same as that of one 0.25-in. plate when multiplied by the value of E (250 000 lb per sq in.) for marblette, so that the model represented three plates of approximately equal thickness riveted together. Furthermore, since the rivets were made of marblette, the plates had an effective thickness equal to the diameters of the rivets.

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[116] Assoc. Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

[117] Instructor, Coll. of the City of New York, School of Technology, New York, N. Y.

The problem of using rivets having a snug fit, but at the same time not introducing any initial stress in the sensitive marblette, was solved by using rivets having a loose fit and coating them with liquid marblette; that is, cementing the rivet into place with a very thin coat of marblette.

This design of model was used because both experimental and mathematical checks on the results were available. Although common practice assumes the partition of load as uniform between the rivets, it is easily shown that the outer rivets carry much more load than the inner ones. The partition obtained by the photo-elastic analysis of the multi-material model checked those obtained analytically within less than 2% in all cases and within less than 1% in most cases.

This experience has encouraged the writer to run a series of tests on rivet groups (not in a line) of five and six rivets when the load applied is eccentric to the grouping and also when the eccentricity is zero. That the results differ materially from those obtained using the assumption made in practice when designing the rivet groups, is quite obvious. It is hoped that a series of these investigations will yield very useful data. This same multi-materials method can be applied to the partition of the load among the rivets in the standard connection between beams and columns in building construction.

For the purpose of experimental study, models were made in which the three plates were of the same material. The resulting confused pattern could not be separated into the separate patterns of the inner and outer plates. This same tendency ensued when high loads were used with the multi-materials models. If the loads used were great enough to produce isochromatics and isoclinics in the celluloid, some confusion was found in interpreting the results. When an isoclinic from the celluloid crossed one from the marblette they destroyed each other, producing a light spot and not a dark spot. This confusion was avoided by using stresses so low that the effect on the celluloid was not visible whereas the isoclinics of the marblette were quite definite. A further advantage encountered in the use of low loads on marblette models is the reduction of creep to a negligible amount. The investigations found little evidence of creep under these loading conditions. This multi-material use of the photo-elastic method is worthy of further study.

FRED L. PLUMMER,[118] M. AM. SOC. C. E. (by letter).—The design of any part of a machine or structure, the size of which is determined by the stresses or deformations which may be imposed upon that part, represents a structural problem which deserves the attention of a competent structural engineer. Too many structural engineers think of fixed structures—buildings and bridges—as the only proper field for their activities. The design of the side frame or the truck of a railway car, the structural frame of an airplane or of a bus, and the hull and deck structures of boats are also structural problems. They sometimes present very difficult design problems which challenge the talents of the best engineers.

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[118] Cons. Engr.; Associate Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

Only a few years ago, many structures of this type were built by processes which placed definite limitations on the choice of sizes and shapes of the various parts. As a consequence, much of the analysis and design was reduced to an arbitrary "rule-of-thumb" or "cut-and-try" process. The introduction of high-strength and light-weight metals, together with the use of new fabrication processes, has caused such methods of design to become quite inadequate and has created a wide and comparatively new field of activity for the structural engineer. New problems are involved; however, structural engineers are better trained than any others to analyze and design these mobile structures properly. It will be unfortunate if through neglect the manufacturers of such structures and machines are forced to re-train men who are now trained primarily to design mechanisms and not structures.

The design of any structure involves first an attempt to predict what forces will act upon the completed structure and the conditions under which those forces will be active. It is then necessary to determine as accurately as possible (and the degree of precision is seldom even close to perfect) the stresses and distortions that will probably be created in the structure by the applied forces, whether those forces be applied loads or destructive agencies of entirely different character. Finally, the material must be selected, and the proper quantity and shape of that material must be determined, which will provide most efficiently a safe and useful service life for the projected structure. Included in this final step is the selection and design of proper connections so that the parts of the structure may be assembled and act as a unit. This problem, which is perhaps the most difficult of the group, frequently receives too little study.

High-strength steels and the light-weight alloys are usually selected for use in a given structure because of superior corrosion resistance properties or because the weight of the structure can be decreased by the use of such metals. This Symposium is devoted to a consideration of light-weight structures. Since these metals cost more than the materials now in common use, their selection can only be justified if the resulting structure is safer, if a longer service life can be expected, or if the extra cost can be balanced by lower maintenance and operation costs, or by greater income-producing possibilities.

The papers presented by Messrs. Ragsdale, Hartmann, and Winston have outlined the economical use of such metals in buildings, bridges, and erection equipment; excavating and materials-handling equipment, including cranes, trucks, etc.; in transportation equipment, including trains, buses, airplanes, and airships; on boats; and for a number of other miscellaneous uses.

The structural designer must re-study the fundamentals and not assume blindly that the principles followed in the analysis and design of structures built of materials now in common use must necessarily hold if similar structures are built of these newer metals.

In presenting these papers, the various authors of this Symposium have called attention to such factors as: (1) The non-linear distribution of unit stresses; (2) the effect of surface conditions on the strength of metal parts; (3) the effect of pulsating stresses "which may result in fatigue failures"; (4) the possible results of a small margin between yield point and ultimate strength, making less probable the relief of stress concentrations by the yielding of some of the over-stressed parts; (5) the difficulties of obtaining satisfactory joints of high-strength materials; (6) the problems created by the heat of welding if that method of fabrication is used; (7) the stress distribution between rivets in joints involving large plates and a great number of rivets; (8) the thin sections which frequently result when high-strength materials are used, give rise to a number of problems (corrosion may have a much greater relative effect; and failure by buckling becomes a greater possibility); and, (9) low moduli of elasticity and thin sections both contributed to greater distortions, more flexibility, and possible serious vibrations.

Not many years ago, few structural engineering organizations included on their staffs men capable of analyzing a highly redundant structure. It was common practice, in so far as possible, to design and construct all structures as simple non-continuous elements. Now, every large design staff includes a number of men trained in the analysis and design of statically indeterminate structures. Engineers are trained to meet and solve new and difficult situations, and they would be false indeed to the traditions of their profession if they were to allow the foregoing difficulties to prevent the use of these new metals. On the other hand, they must not use them without making a most careful study of the probable performance of the resulting structure as well as a thorough investigation of the economic factors involved.

The papers by Messrs. Ragsdale, Hartmann, and Winston contain a wealth of accurate design data. The authors have discussed in detail the proper treatment of many of the difficulties which accompany the use of these metals. Very briefly, they have indicated some of the economic factors that determine whether such metals should or should not be used. Each author might well amplify this part of his paper.

The increased use of these newer metals seems inevitable. Every structural engineer should study for himself the comparative design of at least one structure so that he can more fully appreciate the possibilities that lie in the development of these metals.

C. F. GOODRICH,[119] M. AM. SOC. C. E. (by letter).—All structural engineers who are faced with the problem of designing any large steel structure should read carefully the paper by Mr. Moisseiff because, although it contains no new technical formulas or data, it presents facts which, if studied and followed, will lead to better and more economical designs. There are two points, especially, which might well be more strongly emphasized:

(1) The very rapid development of the high-strength steels in the last decade and the contrastingly slow previous development; and,

(2) The note of caution against the promiscuous use of the various high-strength steels for structures, or for parts of structures, where ordinary standard carbon steels would have been more economical.

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[119] Chf. Engr., Am. Bridge Co., Pittsburgh, Pa.

Prior to 1927 there were only two high-strength, low-alloy, structural steels in use: Nickel steel and silicon steel. Since the introduction of the so-called manganese steel used in the Bayonne (N. J.) Arch, high-strength steels of various chemical content and heat treatment have appeared on the market. It took ten or fifteen years for silicon steel to come into such general use that it could be obtained from any rolling-mill or from stock, or could be fabricated in any shop and at a cost only $10 or $15 per ton greater than that of medium structural steel. The high-strength steels that have been produced since 1927 are far from standard at present. There is a tendency among many engineers to seize upon one or another of these newer steels, which exhibit desirable physical qualities, and apply them to structures, without much knowledge of their cost, simply because their use results in a lighter structure.

Mr. Moisseiff cautions against the use of high-strength steels in a small percentage of the members of a structure. The designer must realize that quantity is a large factor in the production cost of steel. Three or four members of high-strength steel in a structure, built generally of medium carbon steel, will cost far more per pound than this same steel in a structure built almost entirely of it.

The condition, pointed out by Mr. Moisseiff, which has existed in the last two decades (wherein the engineer is continually demanding a higher grade of steel from the producer) is a very healthy one. The laboratory, the mills, and the shops need this stimulus if the art of steel-making is to advance, but this growth must not be of the mushroom kind. The engineer must determine how his demands affect the producer: Whether they can be met at reasonable cost, or whether they will defeat the purpose for which they were made—economy and ultimate low cost to his client. Interruptions to continuous production, at the mills, of one grade of steel in order to introduce a special grade, cost money and the engineer's client pays for that extra cost. Unfortunately, the client, being a layman, does not always know that, but must depend upon his engineers for proper economy. Even the engineer sometimes makes the mistake of thinking that his case may be a "special one", that the mills will be glad to get his order and so will absorb the extra cost. There is no need to point out that this is false economy, but it happens constantly.

Very short deliveries are now being specified. The standard structural steels lend themselves to these short deliveries far better than the special steels, because they are standard product.

The economical problems placed before the designer and the producer by the introduction of these many new high-strength steels are not simple. Their solution will take time and study even if the demand for stronger and better steel is ever pressing.

G. K. Herzog,[120] Esq. (by letter).—The tremendous increase in the use of the stainless, high-alloy steels for structural purposes should make the paper by Mr. Morris of particular interest to civil engineers who are, perhaps, as yet not quite as familiar with their advantages as are mechanical and chemical engineers. For the latter they are the solution of many a heretofore insoluble problem. As Mr. Morris states, the method of manufacture and the production of these steels are in a state of flux and rapid progress is being made. The same is true of the modifications of the compositions of the various types of steel to increase their strength or corrosion resistance, or to improve their fabricating properties. He has briefly touched upon some of these modifications in composition, but has perhaps not emphasized their importance quite enough.

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[120] With Electro Metallurgical Co., New York, N. Y.

The addition of 2 to 4% molybdenum to the 18–8 (18% chromium and 8% nickel) type of steel not only increases its resistance to corrosion, but also has a marked favorable effect on its creep strength at elevated temperatures. The increased resistance to corrosion is such that in a number of applications for which ordinary or plain 18–8 steels cannot be used, the grade containing molybdenum is entirely satisfactory; that is, the addition of molybdenum so enhances the corrosion-resisting properties of this type of steel that it opens up entirely new fields of application.

The same is true of the use of columbium. The ordinary grade of Alloy 18–8 cannot be used satisfactorily in certain temperature ranges because of the structural changes which this steel undergoes when exposed for any length of time to temperatures within this range. This may result in premature failure of parts operating at these temperatures. The addition of ten times as much columbium as the carbon content stabilizes the steel and entirely prevents these structural changes. This improvement is brought about without in any way impairing any of the desirable physical or chemical properties of the steel.

In discussing the plain chromium, or ferritic type of stainless steel, Mr. Morris mentions briefly the addition of small quantities of nickel, molybdenum, and silicon to improve the physical properties and the corrosion resistance of this type. A very interesting and important development in these steels is the addition of nitrogen to improve their physical properties and workability. In the form of castings or ingots these steels, without nitrogen, possess a coarse grain structure which makes them relatively brittle and difficult to forge, roll, or otherwise hot work. The addition of a quantity of nitrogen equivalent to about 1 part per 120 parts of chromium greatly refines the grain, improves ductility and strength, and makes it much easier to hot work the steels.

Another important new development in the plain chromium steels is the addition of columbium to prevent so-called "air-hardening." When some of these steels are heated above a critical temperature (as, for example, in welding), and are then allowed to cool normally in air, they become brittle. This disadvantage is entirely eliminated by the use of the proper quantity of columbium. Furthermore, these improved properties are obtained without any sacrifice of resistance to corrosion at ordinary or elevated temperatures.

These improvements in physical and chemical properties are perhaps of greatest interest to the chemical and mechanical engineer who must design equipment to operate at higher temperatures and under more corrosive conditions than are in general encountered by civil engineers. However, the writer believes the latter will be interested in these developments.

John H. Meursinge,[121] Assoc. M. Am. Soc. C. E. (by letter).—This Symposium happens to be of unusual interest for the Civil Engineering Profession, because it calls attention to the fact that structural engineering has entered into the third phase of its existence, the period called "neotechnics" by the late Patrick Geddes.

As many civil engineers will not be familiar with the symptoms which accompany this event, it will be necessary to dwell for a moment on the philosophy of that great Scotchman. His ideas have recently been worked over and broadened by Lewis Mumford[122] whose work has furnished most of the information which will be used for this discussion.

Mumford has shown[122] how everything on earth grows or develops according to the same pattern. This phenomenon holds true for Man and beast, for machines, architecture, political science, etc.—and also for structural engineering. The pattern distinguishes three periods that fade out in each other. For human beings one may refer to babyhood, childhood, and manhood or womanhood. For machinery, utilities, etc., Mumford calls these phases: Eotechnics, paleotechnics and neotechnics. In the eotechnic stage, the object is still close to Nature; it is poorly developed, but nevertheless it is at most times still attractive to the eye. In the paleotechnic stage, the object increases its speed (if it has any) and it grows quickly, quite often out of proportion; it becomes ugly. In the neotechnic stage, it becomes more efficient, it improves its appearance, and it adapts itself to its surroundings.

Each phase utilizes its own power and materials. According to Mumford: "The eotechnic phase is a water and wood complex, the paleotechnic phase is a coal and iron complex, and the neotechnic phase is an electricity and alloy complex."

If the structural engineer wishes to use the alloys to their fullest advantage he must understand the characteristics of the "neotechnic" period. As a matter of fact he has already obtained results that suit the demands of the neotechnique. Where in the paleotechnic area the cost of a refined design was often greater than that of the materials that could be saved, in the "neotechnic" area, due to the more expensive alloys, the refined design has become a profitable item. This change has been emphasized by most of the authors of the Symposium. Mr. Karpov states: "More refined designs will involve additional engineering work." In the "Synopsis" of his paper, Mr. Brahtz states: "The more expensive alloys make it imperative to the designer, that he avail himself of every possible means of refined stress analysis"; and Mr. Winston states: "* * * careful attention to small details has been an important factor in the success of these applications." Mumford also has stressed the importance of the same fact:

"The lightness of aluminum is a challenge to the more careful and more accurate designer in such machines [and utilities] as still use iron and steel.

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[121] Reinforced Concrete Designer, Gen. Petroleum Co., Vernon, Calif.

[122] "Technics and Civilization", by Lewis Mumford.

The gross over sizing of standard dimensions with an excessive factor of safety based upon a judicious allowance for ignorance, is intolerable in the finer designs of airplanes; and the calculations of the airplane engineer must in the end react back upon the design of bridges, cranes, steel buildings; in fact, such a reaction is already in evidence. Instead of bigness and heaviness being a happy distinction, these qualities are now recognized as handicaps; lightness and compactness are the emergent qualities of the 'neotechnic' area."

The technical philosopher and the structural engineer have reached the same conclusion. The importance of this fact should not be under-estimated, for it proves that they can work together. The structural engineer can profit by this in regard to his future.

What can he expect? To sum up the characteristics of the neotechnic area: It does not call for bigger structures, but for more efficient, better looking, edifices which adapt themselves to their surroundings. Whereas the paleotechnic complex (which, for many structures, is still in existence), called for a centralization of the population, the neotechnic complex tends toward decentralization. Subsequently, modern demands will be for structures the sizes of which are within the present scope. Therefore, it is interesting to note that Messrs. Moisseiff and Hartmann call attention to the limitations of the alloys. Mr. Winston calls for shorter members, at the same time mentioning the high fatigue endurance of this alloy.

This last characteristic means higher efficiency, a typical requirement of the neotechnic phase. This higher efficiency goes also with all the non-corrosive alloys. To quote Mr. Aston: "* * * the tangible depreciation of the cost of abandoning structures is exceeded in a monetary sense by the more intangible effects of designing structures heavier than the requirements of working stresses * * *." Stronger and lighter bridges, earthquake-proof buildings, and better homes will call for the application of the new alloys. The structural engineer and the architect should find the world waiting for the application of these new materials.

The demands for the architect's services should increase as shape (appearance) plays an important rôle in the neotechnic complex. In certain areas where high wind pressures are the rule, stream-lined buildings may have a future. Shape will not be less important for the earthquake-proof structures. A material equally distributed throughout the entire building will keep the earthquake stresses low, not to mention the advantages already obtained by the use of the lighter material.

The average structural engineer should have no difficulties in adapting himself to those first two requirements of the neotechnic phase. The third requirement, adaptation to its surroundings, may cause more difficulties, because this means a better place in present-day society or, better stated, the neotechnic society. Although many people cannot see it, the fact remains that pecuniary differences will disappear. To quote Mumford:

> "There is no qualitative difference between a poor man's electric bulb of a given candle power and a rich man's, to indicate their differing pecuniary status in society, although there was an enormous difference between the rush or stinking tallow of the peasant and the wax candles or sperm oil used by the upper classes before the coming of gas or electricity."

The structural engineer should understand that the improvements of the neotechnic phase tend toward a classless society. Those who disagree should not waste their efforts on the design and the application of the new materials, for only a mind freed from all the traditions, prejudices, and superstitions of the present era can expect to be successful in this kind of work. Hence, fiction writers, whose imagination is not kept within certain limits by tradition, cost estimates, or conservative employers, have often been able to encroach into the structural engineer's territory.

In the Nineteenth Century the French writer, Jules Verne, visualized a modern submarine long before any engineer had given thought to it. To-day, history is ready to repeat itself. Writers such as H. G. Wells and others indicate that the structural engineer is lagging behind in imagination and application. Is he going to take up the challenge?

If he wants to, he should prepare himself for a struggle. The structural engineer will have to adapt himself to the new circumstances he has brought forth by his own efforts. Many of them soon will find themselves in "no man's land." On one side neotechnics will call for an elaborate design at a high engineering cost, and on the other side will be the employer who is still firmly submerged in the paleotechnic ideas of the profit system. The last one, no doubt, will insist on keeping the engineering cost down to the lowest possible level. The structural engineer, by the use of a great diplomacy, will have to make the best of it, so that he may continue his services to mankind. To succeed he will have to study his technical heritage.

P. G. LANG, JR.,[123] M. AM. SOC. C. E. (by letter).—A scholarly and illuminating treatment of this subject, Mr. Moisseiff's paper merits careful study by all whose duties concern the design, construction, or maintenance of steel structures. The statements made therein concerning mass and automotive transportation, long-span bridges, and highway bridges for motor traffic are most appropriate, and are complete in a degree which leaves no margin for elaboration or criticism. The paper epitomizes in concise and logical form the present status and prevailing trends of the structural steel industry, especially in its relation to bridges.

In forecasting tendencies in railroad bridge design, a factor of major importance must necessarily be the anticipated developments in railroad equipment, and the present indications are, and have been for some time, that the average weight of such equipment and the consequent severity of its effect on railroad structures is likely to undergo little if any increase in the near future. The demand for increased speed on railroads, however, is continuous and unabated, and it is impossible to surmise any limit to developments in this field. Increasing speeds are inevitably accompanied by increased vibration in the bridges carrying such traffic, and this, in turn, calls attention to questions of impact and fatigue. Apropos of this phase of the matter, it is worthy of note that Mr. Moisseiff establishes a clear distinction between bridge and building work, on the basis of the primary difference that impact and stress reversal, characteristic of the former, are absent in the latter.

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[123] Engr. of Bridges, B. & O. R. R., Baltimore, Md.

Specific research objectives, designed to benefit producer and consumer, alike, may well include enhancement of the corrosive-resistant properties of structural metal. This is of obvious importance from a maintenance standpoint. In an effort to improve the corrosive-resistant quality of steel, some specifications were amended to provide for a copper content of 0.2 per cent.

Considering the fact that present-day, ordinary, carbon steel has been in use for forty years or more, and that its reliability has been well tested in practice, when alloy steels are used some consideration should be given to temperature effects, particularly the effects produced by comparatively low temperatures. Structures are frequently built in locations where temperatures range well below zero Fahrenheit, 40° to 50° below being not uncommon in the United States.

Since steel is a mass production commodity, economy of manufacture, in a major degree, is predicated upon uniformity of requirement on the part of the consumers. Any deviation from established practice is expensive and disruptive, and it is probable that in the steel industry the conflict between scientific and economic justification of new processes is most sharply defined. Each producing unit represents a vast investment, and changes in procedure involve alterations in equipment and accessories which are admitted only when absolutely unavoidable. Thus, the development of appliances and processes tends to stagnate. From the standpoint of the consumer, it is necessary that the material be purchasable at a moderate price, and, if it possesses the qualities that fit it for structural use, there is little incentive to assume the additional cost necessary to procure a steel of improved quality, unless this improvement is so great as to leave no room for doubt of economic justification. It is only natural that any specification which provides for material differing from the standard product should meet with some opposition.

Steel of the quality in common use for structural purposes at this time was specified and manufactured as early as 1889, nearly fifty years ago. During this long period, although there have been occasional demands for quantities of high-strength steel for large and important bridges, the demand which composes the major volume of the steel business, and determines its policies and practices, has been for material to be used in ordinary bridges, of moderate size. The actual need as to material for structural work, at this time, is for an improved quality of steel to be used in structures of this latter character, and the inability to produce material of more desirable characteristics seems more apparent than real.

Until 1926, structural steel for use on the Baltimore and Ohio Railroad was purchased to a minimum specification requirement of 30 kips per sq in. A review of test reports covering the actual properties of about 14 000 tons of steel used in bridges on that railroad, and bought to the specification named, indicated that more than 99% of the tonnage represented possessed an elastic limit of 35 kips per sq in., or more, and the specification was changed accordingly to establish that value as the minimum. This change was followed by prompt and vigorous protest, which subsided as soon as it was demonstrated that for a long time steel had actually been furnished which met or exceeded the new specification requirement, and that, obviously, no increase of cost was involved.

Within the recent past, welding has achieved recognition—and, for work of certain classes, or work situated in certain localities it has received preference—as an articulative process for metal structures. The stability of structures with such connections is necessarily conditioned upon the strength and homogeneity of the jointure welds, and, in devising procedure and materials to insure this condition, the character of steel work becomes a factor of much greater importance than in the case of a structure with riveted or bolted connections.

For many years no attempt has been made to impose a specification requirement limiting the carbon content of structural steel, and, at present, only a small proportion of the structural steel produced and used is subjected to a formal carbon limitation. With the introduction and extensive adoption of structural welding, this phase of the problem has become accentuated, and research indicates that for steel structures thus connected it is highly desirable that carbon be limited to 0.25% on check analysis. The difficulties of production largely vanish when the pertinent facts are ascertained and considered. Although little attempt has been made to impose a definite limitation on carbon for many years, tests of ordinary steel have included ladle and check analyses of carbon content, and a review of test reports covering a very large tonnage of steel work actually furnished for structures of various kinds, principally railroad bridges, clearly shows that only approximately 1% would have been rejected on the basis of carbon content had the 0.25% maximum been stipulated. Hence, no economic grounds exist for objection to such a requirement.

To-day's apparent tendency in the development of specifications for steel construction is toward precision and exactitude, taking cognizance in the design of every ascertainable element, and reducing to the minimum the factor of safety or, as it is sometimes denominated, the "factor of ignorance." Continued progress in this direction is dependent upon the accumulation of exact knowledge concerning the characteristics of structural steel. With respect to the economic phase in its relation to design, as stated by Mr. Moisseiff, it is necessary to balance carefully, the advantages of reduced cost arising from the adoption of stronger steel and, consequently, lighter sections against possible objectionable flexibility in the structure.

In the case of certain alloy steels the cost increment is unquestionably a factor of considerable importance, requiring economic justification. The prices of nickel and silicon steel exceed that of ordinary carbon steel by about 3 cents and 1 cent per lb, respectively. Furthermore, the use in any structure of a steel differing materially from that readily procurable on the market, or the use of steels of different characteristics in the same structure, should be admitted with caution, in view of complications which are likely to arise incident to repairs or partial replacements in the future.

Apart from those portions of Mr. Moisseiff's paper which relate to abstract physical characteristics of the steel, the discussion concerning the necessity for considering deflections and buckling of plates develops a topic which it is impossible to emphasize too strongly.

With the growing importance of welding in connection with steel construction, the distribution of stress at joints acquires new importance. Questions of this character have arisen on numerous occasions in connection with riveted work, and their recurrence in connection with welded joints does not constitute a new problem; it merely represents a novel phase of one which has existed since the beginning of steel construction.

In conclusion, it may be remarked, as a matter of actual experience, that it is possible without increased cost to manufacture steel in conformity with Specifications A-7-34 of the American Society for Testing Materials, which form the basis of the Specifications for Steel Railway Bridges of the American Railway Engineering Association. At best, laboratory tests are only approximate indications of quality and behavior. As a final thought, it seems fitting to state that, all other factors being equal, the best structure is that having the greatest mass. This final observation applies to the usual structure, and does not, of course, apply to movable bridges or to bridges of extremely long spans.

W. L. WARNER,[124] ESQ. (by letter).—Because of the fundamental metallurgical welding principles which are stated therein, the paper by Messrs. Bain and Llewellyn is of interest not only to the structural engineer and designer, but also to the welding engineer and metallurgist. From the welding standpoint it is believed that no criticism can be made. However, there are several statements in the paper to which attention should be directed.

In discussing the effect of carbon it is stated (see heading, "Characteristics of Individual Alloying Elements: Carbon") that "long experience indicates that at about 0.20% to 0.25% (with comparatively low content of other elements), a broadly applicable optimum is reached for structural carbon steels." This value of 0.25% carbon is the limit which has been set for alloy steel plate material used at the Watertown Arsenal, at Watertown, Mass., in building welded structures for gun carriages as determined experimentally from welding tests.

In the same discussion, the following statements should be emphasized as of particular interest to those concerned with the fabrication of welded structures:

> "Furthermore, even with a minor content of other elements, carbon exerts a definite influence toward hardenability; only in its presence, and in proportion to its concentration, are other elements able to exert substantial influences toward hardenability. * * * However, where welding is to be utilized, the usual structural steel derives about all the strength that carbon alone can provide without excessively restricting other valuable properties."

The latter statement means simply that higher strength should be obtained by adding alloy elements rather than carbon when a weldable higher tensile steel is desired.

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[124] Welding Engr., Watertown Arsenal, Watertown, Mass.

Another point in connection with the composition of weldable structural alloy steels is brought out by the following statement in the paper by Messrs. Bain and Llewellyn (see heading, "Complex Steels: Simultaneous Addition of Several Elements"): "A fortunate circumstance is that the concurrent use of moderate quantities of several elements appears to produce a more favorable combination of properties than would result from a single element in an amount sufficient to produce an equivalent strength increase." At Watertown Arsenal, it has been found, for example, that in welding structural nickel steel plate containing approximately $3\frac{1}{2}\%$ of nickel, better physical properties are obtained by using a combination of nickel and molybdenum in the electrode than by using even larger percentages of either of these two elements alone.

In the discussion of carbon content of the steels shown in Table 3, the following statement is made: "At the same time wherever welding is not used, or in cases amenable to stress-relief annealing following welding, the higher carbon steels are applicable, and they may possess still higher strength." In this connection, it is desired to inject a word of caution that, even when stress-relief annealing is used, this treatment cannot prevent trouble from cracks due to hardness adjacent to the weld during the period between the time when the welding is done and the moment when the heat is turned on in the annealing furnace. Sometimes, this interval may be as much as ten days or two weeks. Therefore, even if the heat treatment is practiced, a carbon content of more than 0.25% in an alloy steel makes the freedom from cracks an uncertain proposition unless some preheating is done before welding.

Elmer K. Timby,[125] Assoc. M. Am. Soc. C. E. (by letter).—The types of tests which may confront the designer of structures, the test methods and materials which he may use, and the technique necessary for the correct design of the model and interpretation of experimental data are outlined clearly in Mr. Templin's excellent paper. It is the purpose of this discussion to show to what extent models are actually used by the profession in the advancement of structural design methods.

Every useful structure develops a definite stress system under a given load condition. The determination of this stress system is the concern of the designing engineer. A simple structure may be solved by the well known equations of equilibrium; hyperstatic structures require a greater number of equations than can be written from the condition of equilibrium, and the sources usually drawn from are the principle of least work and the theory of consistent elastic distortions. The formation of these equations frequently involves assumptions of important magnitude—assumptions that may be so far in error, if improperly made, as to vitiate completely the results of the solution. The model, on the other hand, automatically solves the problem, avoiding the use of equations and the assumptions on which they are based.

The summaries and illustrations that follow are an attempt to review briefly the results of past work in this field in the hope that those who have not had an intimate connection with the development of this line of attack may have a better conception of the true amount of work done and of the effectiveness of the method. The results presented have been assembled partly by circularizing engineers known to be interested in this work. The response to these inquiries was most gratifying, and it is hoped that the summary represents a typical cross-section of the opinions of the profession. It is pointed out that these examples by no means represent all the work which has been done. Many very important studies have been omitted because of lack of space.

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[125] Asst. Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

One classification includes those models which have formed the basis of research projects sponsored by technical societies, universities, or industrial organizations for the purpose of promoting the progress of engineering. One of the most outstanding examples of this type of work was the extensive use of models by the Committee on Arch Dam Investigation.

The Stevenson Creek Test Dam, studied by this Committee, might be called a full-sized model. It was constructed at a particularly favorable location along Stevenson Creek, in California, where a deep reservoir having a very small capacity was possible. A large source of water, the flow of which could be controlled, was available up stream. The dam was 60 ft high, 2 ft thick at the top, and thickened along a curve on the down-stream side for the lower half of its height.

This dam was loaded and unloaded repeatedly and many observations were taken in an effort to determine its stress distribution. The results obtained have been published in several volumes by the Engineering Foundation[136]. Many organizations and individuals co-operated in making the tests and the results were of sufficient importance to warrant the Committee to make certain general recommendations helpful in designing, constructing, and testing future arch dams, and in testing existing dams to establish their safety, or to determine the feasibility of increasing the height or making other alterations.

The cost of conducting a test on such a large scale was necessarily very great and for this reason the Committee decided to investigate the same structure by use of a celluloid model. This was done at Princeton University under the direction of George E. Beggs, M. Am. Soc. C. E. The model was made of amber celluloid, placed in a concrete valley and loaded with mercury. The results obtained agreed so well with the tests on the full-sized model that future studies to determine the effect of increasing the height of the dam were completed by use of additional celluloid models at a cost much less than would have been required to raise the concrete dam.

It is interesting to note one point brought out by the model studies. Under certain conditions of loading, a part of the dam deflected up stream against a uniformly varying water load. After the model had demonstrated that this was true, a theory was promptly furnished which would lead to this result. Theory had previously erred in assumptions made; experiment corrected the error, and, consequently, improved the theory.

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[136] Rept of the Committee on Arch Dam Investigation, Engineering Foundation, Vol. 1, 2, and 3.

The particular usefulness of the tests on the model of the Calderwood Dam[127], together with the tests of the structure itself, was that they emphasized the inadequacy of considering an arch dam as a simple series of arches and indicated definitely that some more complete method of analysis, such as the "trial-load", method is necessary. They further emphasized that in any analysis the character and deformation of the foundation must be considered.

Supporting these observations relative to model studies of dams are the following conclusions which are quoted from a paper by J. L. Savage, M. Am. Soc. C. E., presented at the 1934 Annual Meeting of the Society for the Promotion of Engineering Education[128]:

"The careful testing of accurately constructed model dams furnishes a satisfactory and reliable basis for checking the action and safety of concrete arch, arch-gravity, and gravity type dams as well as a satisfactory means of verifying analytical methods used in designing such structures."

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"Plaster-celite models of maximum sections of arch, arched-gravity, and gravity dams are useful in determining load deflections, general stress conditions, special stress conditions at corners, stresses around galleries, and so forth; but may give misleading stress curves in the foundation material due to the necessity of using rigid boundary conditions."

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"The experimental analysis of dams offers an unusually fertile field for further research and for the instruction of students."

Models also have been extremely useful in the construction of huge dams, as well as of assistance in design. The use of models in connection with the construction of the Bonneville Dam (Oregon) will serve as a timely illustration. Of this project, J. C. Stevens, M. Am. Soc. C. E., has stated[129]:

"Hydraulic models of the Bonneville Dam have saved many times their cost and have substantially accelerated the construction schedule of the project. The experiments have been carried on in an outdoor hydraulic laboratory built especially for the purpose at Government Moorings, in Portland."

In describing a model type of baffle adopted to prevent scour and consisting essentially of blocks staggered in two rows, Mr. Stevens stated[130]:

"It was a surprise to find that the downstream sloping face of the baffle block was very effective in preventing scour, but once the fact was established a theory to account for it was discovered in the phenomena of turbulence and stray currents."

The coffer-dam design and installation were studied thoroughly by models.

The Special Committee of the Society on Concrete and Reinforced Concrete Arches made many model studies in connection with concrete arch bridges, on rigid abutments, on elastic piers, with and without high and low superstructure, and with and without expansion joints. A number of both concrete and celluloid models were used in these tests. The experimental work on the concrete arches was conducted at the University of Illinois under the supervision of W. M. Wilson, M. Am. Soc. C. E.[16] Celluloid model studies of some of the same arches were made at Ohio State University under the direction of Clyde T. Morris, M. Am. Soc. C. E., and at Princeton University, by Professor Beggs[131]. As a result of these and other tests conducted by it, the Committee has made definite recommendations for design practice.

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[127] "Model of Calderwood Arch Dam", by A. V. Karpov and R. L. Templin, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E. Vol. 100 (1935), p. 185.

[128] A condensed version of this paper was published in *Engineering News-Record*, December 6, 1934, under the title, "Dam Stresses Studied by Slice Models."

[129] *Civil Engineering*, October, 1936, p. 674.

[130] *Loc. cit.*, p. 675.

Cases of the use of models to improve theory have been cited by Mr. Templin. It might be well to consider a case in which a model was used to verify a new theoretical approach. The geometric properties of the Williot diagram were used to express moments in terms of angular displacement and thus furnish a sufficient number of equations for the stress analysis of the tower of the Golden Gate Suspension Bridge. This was a large and unusual cellular structure combining set-backs with portal construction above the roadway. Below the roadway cross-bracing connected the two legs of the tower. It was considered desirable to verify the theoretical design by means of an independent study. Professor Beggs constructed a structural model and made strain and deflection measurements at many points. Then he computed the model, using the theory which was being applied in the design in the full-sized tower.

This model was fabricated from stainless steel by shot-welding to a scale of 1 to 56. Tensometers were used to measure strains; deflections were measured with dials; and rotations were measured by the use of mirrors and reflected light. Structurally, the model was a duplicate of the tower as regards area of members, distances to extreme fibers, and moments of inertia. However, the entire comparative theoretical work was based on a calculation of the model. The results of the experiment were in excellent agreement with the results of the theoretical calculations. The model was used in this case to verify a method. There could be no question of lack of similarity since the identical structure was used in each of the two methods. The theoretical method can now be applied with confidence to structures of a similar nature.

In addition to the verification of a particular method this model represented a new type of model construction. Stainless steel is a material of exceptional quality and when fabricated properly, produces a structure in which there is no question as to the rigidity of joints. Some engineers may question the use of welding to represent structures which in practice are assembled by riveting. It is the general practice, however, for designers to consider the gross area and moment of inertia of either riveted or reinforced concrete members during the calculations of resultant forces, and to make allowance for net sections, or cracked sections, only when the point at which the unit stresses are to be evaluated has been reached. For this reason it is thought that the use of welding is entirely justified. Allowance for net sections may be made when interpreting the observed strains. Theoretical deflection calculations are also generally based upon gross sections and, therefore, need no correction when welding is used instead of riveting.

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[131] Final Report of the Special Committee on Concrete and Reinforced Concrete Arches, *Transactions*, Am. Soc. C. E. Vol. 100 (1935), p. 1427.

It seems entirely reasonable that the use of gross sections should give results close to the actual even when a structure is assembled with rivets. The rivet holes form such a small part of the structure that their total effect in any case would be slight. Furthermore, the irregularity of stress around a rivet hole argues against too great refinement when considering the over-all action of a structure using theories postulating uniform stress variation.

Another example of the use of models is in connection with suspension bridges. In the final report[132] on the Delaware River Bridge, Leon S. Moisseiff, M. Am. Soc. C. E., presented a treatise on suspension bridge theory. D. B. Steinman, M. Am. Soc. C. E., applied this theory to the Mt. Hope Suspension Bridge, in Rhode Island, and furnished rather complete theoretical calculations for this structure[133]. Since such complete data were available, Professor Beggs selected this structure as the prototype for a model study to determine whether a model of a suspension bridge would yield satisfactory design information.

The results of this study were in excellent agreement with the available theoretical data, and it was evident that a vast amount of information could be obtained from such a model in a relatively short time. Slight but consistent differences between experimental and theoretical values were explained by the fact that the theory assumed the application of uniform suspender loads, as a matter of convenience, whereas the model showed definitely (as was expected) a non-uniform distribution of load[134].

Although the model of the Mt. Hope Suspension Bridge was made and tested purely as a research problem, it did prove so practical that Professor Beggs later undertook preliminary studies of the San Francisco-Oakland Bay Bridge, using models of the type developed in the Mt. Hope studies. The models, three in number, followed preliminary designs of the full-sized structure and were made from the following data furnished by these designs: (1) Principal dimensions of the structure; (2) cross-sectional areas and elastic moduli of cables and suspenders; (3) moments of inertia of the stiffening trusses and the elastic modulus of their material; (4) elastic constants of the towers; (5) magnitudes of the various loads; (6) general proportions of certain parts, such as the tower saddles and cable anchorages; and (7) range of temperature conditions.

Information obtained from the tests included tower deflections, cable deflections (both lateral and vertical), truss deflections (both lateral and vertical), cable stresses, grade changes on the roadway, and lateral and vertical truss moments. Instrument readings were converted directly into values for the prototype by means of properly constructed charts based upon elastic constants, scale factors, and calibration factors. The results[135] were most useful, both from a research and from a design point of view. One of the models conformed very closely to the design finally adopted and was partly dismantled, during the construction of the prototype, and used to plan the sequence for erecting stiffening-truss sections without producing excessive tower deflections, and to determine the order of erecting the floor steel, and the order of placing concrete paving.

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[132] Final Report of the Board of Engineers on the Bridge Over the Delaware River Connecting Philadelphia, Pa., and Camden, N. J., Appendix D, p. 96.

[133] "Suspension Bridges", by D. B. Steinman, Second Edition.

[134] "Suspension Bridge Stresses Determined by Model", by George E. Beggs, M. Am. Soc. C. E., Elmer K. Timby, Assoc. M. Am. Soc. C. E., and Blair Birdsall, Jun. Am. Soc. C. E., *Engineering News-Record,* June 9, 1932.

[135] "Tests on Structural Models of Proposed San-Francisco-Oakland Suspension Bridge," by Messrs. Beggs, Davis, and Davis, Univ. of California Press, 1933.

The model furnished a means of providing easier, quicker, cheaper, and probably more reliable information than could have been obtained analytically. Early measurements of tower deflection varied 10% or more from those measured on the prototype. The variation, however, was uniform and after the law of variation was established, proper allowance was made and subsequent behavior predicted with satisfactory accuracy. Construction operations were expedited to an extent that made the mechanical analysis more than self-supporting. No accurate costs of this test are available, but it is estimated that it was less than 0.1% of the contract. The tests were made jointly by the engineer and the contractor and were accepted by the engineer as a basis for the approval of the erection program.

Another interesting experiment which led from the Mt. Hope model was the model which was used during the erection of the George Washington Bridge across the Hudson River, at New York City. During the erection of this bridge some of the junior engineers in the field had witnessed a demonstration of the Mt. Hope model. Upon returning to the job, one of them, Mr. A. O. Bergholm, decided that it would be comparatively easy and extremely interesting to construct a model of the partly completed bridge and have a private check for his own use of the erection data furnished by the design office. Accordingly, at his own expense, he began the construction of a model in the basement of the building being occupied as the Field Office. His project was discovered, however, by the Resident Engineer who was so favorably impressed by the possibilities that he authorized the necessary expenditures for constructing the model.

In this model the cable and suspenders were made from piano wire, the dead weights from cloth bags and shot, and the towers and foundation from wood. Dials were utilized to measure saddle motions and a vertical scale was used to determine the position of each panel point. In addition to being used many times, the model saved its cost by solving one particular problem.

Steel for the roadway was delivered to the structure by barge. On one day the steel arriving on the barge was different from that called for on the erection schedule. The question arose as to whether the erection schedule could be altered safely and the erection of the steel which was on the barge permitted. The engineers went to the model, applied an amount of lead shot which corresponded to the steel in question and observed the deflections. The results indicated that the steel could be erected safely at that time and the erection proceeded without delay.

The rapid determination of an erection schedule is always extremely vital to the bridge fabricator. Years may have been spent by the promoters of a project before arriving at a decision to construct, in smoothing out legal difficulties, and in actually undertaking the contract; but, when the fabricator comes "into the picture", time is at such a premium that the fabricator may be liable to a late penalty of as much as $1 000 per day on a large project. The fabricator, therefore, is anxious to use a reliable means of determining his erection schedule quickly because upon this hinges the sequence of his mill and equipment orders, and the profitable completion of his contract.

The use of models of suspension bridges by bridge companies for the determination of erection data is one of the notable examples of the practical application of models in the field of bridge construction. One company has constructed models of four bridges, in each case for the primary purpose of determining definite erection data for the trusses and roadway. The first such model used by this company was utilized in determining the erection schedule for the reconstruction of the Ambassador Bridge, at Detroit, Mich. This bridge has an 1 850-ft main span and unloaded side spans.

The model was ingeniously designed, of inexpensive material, in such a manner as to be quickly fabricated. Nominal $\frac{1}{8}$-in. bead chains, similar to pull-chains for electric lamps, were used for the cable. This was flexible, permitted easy location of the suspenders, and had a weight which gave a practical scale reduction factor. The saddles of the prototype were mounted on rollers during erection and, to simulate this freedom of motion in the model, the tower was replaced with a hinged tension member that could be made vertical by adjustment and comparison with a plumb-bob. Being vertical it exerted no horizontal component, and thus simulated the rollers. For that part of the span with suspenders only hanging from the cables, narrow strips of paper simulated the weight of the suspenders. Pieces of soft iron wire were used to represent the weight of the suspender plus the truss and floor system. One man spent two weeks in designing and erecting the model.

Information obtained from this model consisted of the shape of the cable under unsymmetrical or partial erection loads with particular reference to kinks at cable bands and at the edges of the saddles, vertical deflections of the cable, horizontal movements of the saddles, and the allowable or necessary sequence of closing the joints in the trusses. This information was determined for successive conditions of erection and for various numbers of travelers and the total time required to obtain all this information was one man's time for one week. As a result of the studies, the eight floor-beams at mid-span were suspended from the cable before the travelers left the towers. This pre-loading eliminated excessive cable kinks.

The results obtained from this simple model were entirely satisfactory for field use and were subsequently checked by available and well established, but nevertheless time-consuming, analytic methods. The agreement between experiment and theory was also entirely satisfactory.

The fact that the aforementioned model was practical and economical as well as rapid is best established by noting that the same company has since used three similar models in the determination of erection data.

A model was made for the Triborough Bridge in New York City. The engineer for this bridge had fixed definite limits for the tower deflections. Therefore, it was essential to have material arrive on the job in an order which would insure freedom from delays caused by revision of the erection schedule, which might be necessary should the adopted plan of erection produce excessive tower deflections. Model studies furnished a theoretically ideal erection schedule and allowable variations therefrom. This schedule was used during the actual erection of the bridge and proved to be entirely satisfactory.

The Triborough Bridge was chosen as a test bridge for the comparison of erection data obtained from a model, from observation of the bridge during erection, and by calculation. Comparisons have been entirely satisfactory for erection purposes and the model has shown itself to be superior to computation even when the erection schedule had been definitely predetermined and, therefore, all trial calculations eliminated.

Since 1928 another bridge company has used two models to determine the dampening effect of various types of storm systems. Any one familiar with suspension bridge erection will know of the complicated nature of these systems on long-span bridges and will be able to appreciate the effectiveness of a model in studying their action. Other models have been made so that spinning equipment could be operated in miniature. The rapid strides made in the art of spinning parallel wire cables is ample proof that the models have been useful. The equipment used and the spinning speed attained during the spinning of the cables for the Golden Gate Bridge mark another milestone in suspension-bridge construction. By the use of triple spinning wheels and double tramways, twenty-four wires were placed with each trip of the wheels as compared with only four wires on the George Washington Bridge, the cables of which were erected only about eight years before. Models have played no small part in this rapid progress. All the models used by the company have either reduced the cost of the construction or the time required for design to such an extent as to be more than self-supporting. As a matter of interest, where records are available, these various model studies cost from 0.05% to a maximum of 2.5% of the cable contract.

Other models have been used by the company to study erection equipment for foot-bridges, cables during the erection of the roadway, and suspended cable-car transportation systems. All materials used have been inexpensive, and the models have been fabricated in an exceedingly practical manner. The erection procedures for the cables of the George Washington Bridge, the "Sky-ride" at the Chicago World's Fair (1933), and the Golden Gate Bridge have all been determined by model studies. These models have proved to be such a great aid in facilitating design and solving erection procedure problems that the company considers them indispensable on any major project.

Models have other uses, however, than those cited by Mr. Templin and in this discussion. Many structures have been designed either as a whole or in part as a result of stress analyses made with models. Probably the most extensively used system of model analysis is the deformeter method[136], which is particularly popular with State Highway, County and City Engineering Departments. The engineers in these departments are frequently charged with the design of relatively large and complicated, continuous, reinforced concrete structures.

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[136] "Indeterminate Structures Mechanically Analyzed", by George Erle Beggs, M. Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 18, 1922.

The deformeter was used to advantage by Arthur G. Hayden, M. Am. Soc. C. E., in developing methods of design for the rigid frame bridge which was perfected by Mr. Hayden for use in grade separations in an extensive parkway system[137]. Six single-span, concrete, rigid-frame bridges of about 60-ft span, and two double-span, concrete, rigid-frame bridges were analyzed by this method. Each span of the double-span bridges was about 42 ft. After the construction program was well under way for this system of bridges, quick analytical methods were developed, which replaced the model studies. The model studies pointed the way to the efficient analytical method, and the accuracy of the method was established by comparison with the mechanical analyses.

The Missouri State Highway Department has made mechanical analyses as a part of its regular design procedure. The deformeter was applied in the Department's design of a self-anchored suspension bridge tower of the Vierendeel type. The tower was about 60 ft high and had all diagonal bracing omitted. This stress analysis was considered by the designing engineer of the State Highway Department to be more reliable than the tedious mathematical analysis. The results were very satisfactory, the cost being 0.1% of the cost of the structure. The latter was reduced by reason of the mechanical analysis to an extent which made the model study more than self-supporting.

The Missouri State Highway Department also used the deformeter in the analysis of a structure which consists of five reinforced concrete open-spandrel arches of 195-ft span with the deck structure continuous over each span. The two end spans were unsymmetrical. The purpose of the mechanical analysis was to show the effect of the deck structure. This deck structure was not considered in the mathematical analysis because the arches were on high piers and it would have been necessary to solve 285 simultaneous equations. The model was constructed of celluloid to represent the full length of the bridge. The scale chosen made the model about 14 ft long. Again, the results were satisfactory, the cost in this case being 0.2% of the cost of the structure, and the model was more than self-supporting.

Citation of other and varied examples of the usefulness of this device could be continued indefinitely.

Because it could be easily fabricated for the purpose, bronze was selected as the material for a model of the Bayonne Arch (Kill van Kull), the longest two-hinged spandrel-braced steel arch bridge in the world[138]. The model was built in 1929 under the direction of the consulting engineer and was used partly to check the results derived from theoretical investigations and partly to determine more specifically the importance of sway-bracing for distributing unsymmetrical loads between the two arch ribs, and the behavior of the portals in transmitting wind stresses. This model falls in a class mentioned by Mr. Templin as being different in some details from the prototype, but nevertheless very useful, and emphasizes the point which he made that models do not need to be exact scale reproductions, nor do they need to satisfy every theoretical requirement for exact similitude. Built-up sections of the full-sized structure were represented by solid members in the model. It was not possible, therefore, to reduce the area in the required proportion and, at the same time, assure equivalent stiffness against buckling. Areas of all members were reduced in the same ratio. The proportional behavior of the two structures was not affected by the departures from strict similitude. Static loads were applied to the model and the resulting strains and deflections were read with commercial instruments. The results obtained from this study were entirely satisfactory.

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[137] "The Rigid Frame Bridge," by Arthur G. Hayden, John Wiley and Sons, 1931.

[138] "Design, Materials and Erection of the Kill van Kull (Bayonne Arch)," by Leon S. Moisseiff, M. Am. Soc. C. E., *Journal,* Franklin Inst., May, 1932.

Welding, a method of fabrication which is increasing in popularity, has had a distinct effect in revealing and improving poor designs in some fields of construction. The newer designs have often presented difficult stress problems due to the shapes made possible by flame-cutting and the greater thicknesses which can be joined by welding. The safety of these new structures has been determined mainly by one of the following experimental methods: Testing of full-sized structures to destruction; non-destructive tests of full-sized structures in which the overstressed points are detected by the cracking of rosin or the flaking of a lime wash applied to the surface before testing; non-destructive tests in which strain-gage measurements are taken over the surface; and testing miniature models of like or of different material.

Many engineers feel that the use of miniature models in this field is not particularly fruitful. The departure of stress variation from the usually assumed straight-line distribution, the variable ability of ductility to adjust overstress in plates of different thicknesses, the relatively small size of the completed units, the residual stresses caused by the welding, and the new shop technique necessary to make a scalar reproduction, all tend to convince the producer of welded structures that he obtains more reliable information from a full-sized test.

In 1930, H. V. Spurr, M. Am. Soc. C. E., published[139] a method of design for wind bents in which he utilized a direct and rational method of design in contradistinction to the approximate-investigate-re-distribute-re-analyze methods common prior to that time.

As a check upon the Spurr method, a steel model was constructed and tested at the Experiment Station of Ohio State University[140]. The results of this investigation have been published as an authoritative study on the reliability of the Spurr method of design. The conclusions and recommendations are too numerous to mention herein, but the investigation serves as an excellent illustration of the usefulness of model studies in establishing the validity, or in making desirable revisions of practical, and somewhat approximate, design methods, so that they can be used with confidence by the profession. Without doubt this model, and the information obtained therefrom, will have a marked effect on the design of many of the buildings of the future.

The knowledge of wind pressures and velocities at surfaces of exposed structures is necessary to the successful application of any theory for the calculation of wind stresses in members within the structure. In general, more thought must be given to necessary violation of the laws of similitude when dealing with dynamic or moving loads than when considering a static system. For this reason tests on the airship hangar at Lakehurst, N. J., conducted under the joint sponsorship of the Bureau of Yards and Docks of the U. S. Navy Department, The National Advisory Committee for Aeronautics, and Rensselaer Polytechnic Institute, should be of particular value.

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[139] "Wind Bracing," by Henry V. Spurr, McGraw-Hill Book Co., 1930.

[140] "Tests and Design of Steel Wind Bents for Tall Buildings," by G. E. Large, Assoc. M. Am. Soc. C. E., Samuel T. Carpenter, Jun. Am. Soc. C. E., and Clyde T. Morris, M. Am. Soc. C. E., Eng. Experiment Station. *Bulletin No. 93*, Ohio State University.

Wind-tunnel tests were made on 1 to 800, 1 to 400, 1 to 200, and 1 to 40-scale models and actual pressure measurements are now (1936) being made on the full-sized hangar itself. The measurements on the full-sized hangar plus those made on the models will serve to provide a more reliable basis than has been available in the past for the evaluation of wind-tunnel tests on small models. The science of fluid dynamics, applied as it is at present, to problems involving all kinds of motion in any medium, owes much of its progress to the results of model studies.

Mention will be made of one type of test which is in the field of the Mechanical Engineer, but which also has a real interest for the Civil Engineer. Most of the monumental dams recently built and now under construction would have been impractical had it not been that equipment was being developed to utilize the hydraulic power which they made available. Elaborate equipment has been perfected for testing new designs of turbine rotors[141]. Models are made to include new features and are tested to determine their ability to resist fatigue stresses. The effective operation of the rotor requires irregularities of shape which make analytical calculations extremely difficult. On the other hand, the unit as a whole must meet continuous operating requirements. Failure of a rotor, revolving at present high speeds, cannot be contemplated—the design must be good. Inexpensive models may be allowed to fail without danger or inconvenience and the designer, in this way, is at liberty to try new and perhaps novel ideas. The publications of men interested in this field have repeatedly pointed out the usefulness of models of this type used in an extremely practical manner to determine fatigue limits of new materials or new designs.

Still another method of model investigation which should not go unmentioned is the method of testing underground structures which has been developed by Philip B. Bucky, Professor of Mining at Columbia University, in New York City. In this method the model is made an exact duplicate of the structure and strained by placing it in a centrifuge which, with the revolutions per minute properly regulated, produces centrifugal forces equivalent to the gravitational forces encountered in a sub-surface structure. The cost of an experimental analysis is small. Safe spans for flat and arched roofs, and the stress distribution within pillars, have been determined successfully in this manner by the combined use of stroboscopic and photo-elastic equipment. The practical application of this method requires a rather complete geological knowledge of the strata in which the proposed structure is to be built, together with a comprehensive and thorough understanding of theory and experimental technique.

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[141] "Fatigue Tests of Model Turbo-Generator Rotors", by R. E. Peterson, *Mechanical Engineering*, March, 1931.

Considerable experimental research work has been done by the faculty and graduate students of the universities. The Beggs deformeter, photo-elasticity, and loaded models have all been used extensively and from reports which have been gathered, it would appear that much effort has been wasted because many men have studied the same problem, only to have the results filed away and lost to the profession. These efforts, however, are not entirely wasted. A student will gain an understanding of structural action from working with models which might take years of study to acquire. Models, without doubt, serve a useful purpose when used to supplement class-room instruction. It is regrettable, nevertheless, that more co-ordinated and continued studies cannot be made. That this is possible is demonstrated by the tests which have been conducted on a miniature building frame by various students at Iowa State College under the direction of R. A. Caughey, M. Am. Soc. C. E. This miniature model has been revised and used repeatedly to solve various problems.

Illustrations of successful model studies could continue at great length, but the effort seems unnecessary. Mr. Templin has indicated the possibilities of making experimental studies; this discussion has attempted to show by means of actual illustration that experimental studies are more than possible. They are a much used tool of many present-day designers.

Models, however, are not just built. There are strict requirements of similitude. A real danger exists in that some designers may be led, by reports of startling results obtained with models, to believe that the model is a substitute for analytical ability. Such is definitely not the case. The successful model investigations have been made by engineers skilled in theoretical analysis, capable of designing a model, recognizing its limitations, and interpreting the experimental data. An engineer who has mastered the technique of using models has the ability to construct calculating machines which will assist him to design structures of unprecedented magnitude and importance. Mr. Templin has made a valuable contribution to engineering literature.

In an exceedingly vivid description of the photo-elastic method of determining stress, Mr. Brahtz raises an old point, which attains new importance in the design of the larger present-day structures, when he states that it is not always a simple matter to keep the probable error as low as 10 per cent. The writer is of the opinion that the results are very acceptable in the majority of cases if no greater discrepancies exist.

Engineers have always used a factor of safety. With loading conditions as unknown and variable as they are, this seems essential. However, the factor of safety has cared for three uncertainties: (1) Unknown loading conditions; (2) variability of the material which carries the loads; and (3) approximations in the design method. Human nature, and Nature herself, being what they are, there seems to be little hope of decreasing the factor of safety by reducing therein the requirements of Item (1). Much has been done to eliminate uncertainties in structural materials, and a consequent increase in working stresses has resulted, thus reducing the requirements for Item (2). The "factor of ignorance" (Item (3)), seems to be the most vulnerable point of attack.

Many structures, costing millions of dollars each, have been built recently, and the designers, almost without exception, have been using to advantage the newly perfected methods offered by experimental analyses. In some cases, errors of appreciable magnitude have been discovered in accepted theoretical methods—in particular, those dealing with hyperstatic structures. In almost all cases the experimental results have furnished a confidence which fostered a more efficient design.

The writer doubts whether results are in general within 10% of the actual, even if they may be within a small percentage of that required by the assumed loads. Mr. Brahtz is to be commended for calling attention to the difficulties of technique involved in a reliable mechanical analysis. The photo-elastic method, as a practical tool, is both new and fascinating. As such, it is subject to both incorrect and too frequent use. The results obtained from the polariscope and other auxiliary devices certainly must be judged in the light of all factors, and not merely by the inherent accuracy of the method.

WERNER LEHMAN,[142] ESQ. (by letter).—Many phases of applications of aluminum alloys in the field of excavating machinery have been investigated by the writer, in which light weight structures are desirable in long booms, buckets, and other parts that must be counterbalanced and revolved at long radii. A reduction in weight of such parts is easily found to be economical through an increase in output or an increase in reach or working radius of the machines. The selling of such a benefit is often more difficult than the proof of economy.

Engineers are mostly interested in the capacity of a finished product to perform the intended purpose and for that reason the writer agrees with the importance of the questions presented by Messrs. Jeffries, Nagel, and Wood (see heading, "Effect of Cold Working"): (1) Will the new structure possess adequate strength? and (2) will it retain that strength under the anticipated service conditions?

The first question can be well defined by the engineer using the proper proportions for the characteristic of the building material. Structural Aluminum 17ST can be trusted as close to the values given in the data books as steel structures, provided the material is handled correctly in the process of manufacture so that no parts are changed in their characteristic properties by excess heat or local over-stress. With the proper instructions one can rely on the manufacturing department so that no deductions are made for improper handling. The aluminum alloys are not more difficult to handle than high-strength heat-treated steels. To avoid errors, the writer does not advocate any heating of aluminum in manufacturing processes.

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[142] Chf. Engr., Bucyrus-Erie Co., South Milwaukee, Wis.

Closely spaced rivets are driven at random to prevent local over-heating. Riveting with hot steel rivets has shown little ill effect on the heat-treated alloy, 17ST, used in long dragline booms. Most holes for bolts and rivets are drilled.

The second question is not so easily answered, because experience is not available over long periods as with steel. On an average, aluminum alloys can be trusted as well as steel. Concerning the structures of excavating machinery with their constant shock, vibrations, rapidly changing loads, and often occurring unforeseen strains, there is not enough knowledge of the long-time fatigue factor. It seems that occasional rest periods have the same healthy effect of aging and normalizing as in steel structures.

The writer agrees with Mr. Hartmann concerning the need for careful study of slenderness in compression members as a composite strut as well as the detail of the sections. Considerable test data are available. His statements concerning savings on long dragline booms agree with the writer's experience. The resistance factor of aluminum to corrosion does not enter into the designs and uses for excavation machinery. Abrasion resistance is more important in the design of buckets for excavators where it may be a deciding factor in the selection of material. The coefficient of expansion and the modulus of elasticity are both unsuitable to combine aluminum with steel in the same section.

Probably the strongest competitor to light weight alloys is the progress in the art of welding of steel and the recently developed weldable steel alloys of great strength and resistance to abrasion. The peculiar shapes of dippers and drag-buckets does not lend itself to extensive economical use of aluminum mainly because the resistance to abrasion and shock is all important and the use of wearing plates defeats the benefit derived from light weight alloys.

When saving of weight is important, any designer of machinery should keep the excellent properties of structural heat-treated aluminum alloys in mind and should weigh their possibilities not only in the detail, but as to their effect on the complete structure.

OTIS E. HOVEY,[143] M. AM. SOC. C. E. (by letter).—In the Specifications for Steel Railway Bridges of the American Railway Engineering Association, dated August, 1935 (in Part III, "Alloy Steels, Foreword", p. 49, Article 9), it is stated that "no steel should be considered satisfactory for bridge construction if its yield point exceeds 70 per cent of the ultimate strength." In some cases, this rule has been quoted as an authoritative requirement for all structural alloy steels and particularly with reference to the low-alloy steel the chemical and physical properties of which are given by Messrs. Bain and Llewellyn (see heading, "Complex Steels: Simultaneous Addition of Several Elements"). It will be interesting to trace the origin and application of the limiting yield point ratio of 70 per cent.

During an extensive research into the properties of heat-treated carbon-steel eye-bars, in 1914, the late C. G. Emil Larsson, M. Am. Soc. C. E., and the writer studied the results of many tests with respect to physical properties developed and with particular reference to ductility. The suggestion to limit the yield point ratios to 70% of the ultimate tensile strength was made for the purpose of insuring the desired ductility in the finished product. It should be remembered that carbon steel, and not alloy steel, was under investigation at that time. At a later date the same limiting yield point ratio was inserted in the Specifications of the American Railway Engineering Association for alloy steels.

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[143] Cons. Engr., New York, N. Y.

The paper by Messrs. Bain and Llewellyn shows that there are several combinations of low-carbon steel with various alloying elements that produce low-alloy steels which have satisfactory ductile properties, even if their yield points in some cases are greater than 70% of their ultimate tensile strengths.

The writer believes that arbitrary rules limiting the yield point ratio should not be made, but that attention should be given to the ductile properties, the endurance limits, and to the impact properties, in order to be sure that a particular steel will be suitable for bridges and other structures.

Mr. Moisseiff has presented a paper of much interest and great value at a time when many engineers are keenly interested in the development of high-strength steels appropriate for use in heavy and long-span bridges and in other structures in which a stronger steel is needed.

The presentation of this paper brought to mind a matter of some historic interest, since it deals with the evolution of what may be termed low-alloy high-strength steels. As a matter of fact the steel first used in the principal truss members of a long-span bridge was an alloy. This refers to the trussed arches of the first Mississippi River Bridge, at St. Louis, Mo., commonly known as the Eads Bridge. The main members of the arches are hollow cylinders about 18 in. in diameter, in panel lengths, and each section is made up of six steel staves bound together by outside cylindrically formed plates. The material in the staves is a chrome alloy steel.

The contract for the superstructure, erected in place, was signed on February 26, 1870. The celebration of the completion of the bridge occurred on July 4, 1874. During 1870 and 1871 an investigation was made of various steels, which resulted in the choice of the chrome alloy steel. A study of the specifications reveals that the modulus of elasticity was considered to be one of the most important physical properties of the steel. A few quotations are given herewith:

> "The modulus of elasticity of the steel in the staves shall not be less than 26 000 000 nor more than 30 000 000 pounds. This variation should be avoided if possible, in which case the lower amount will be preferable. Each bar will be tested by the contractor, and the modulus stamped on it by the inspector. * * *."
>
> "Each tube is to be composed of six staves having, as near as possible, the same modulus of elasticity."
>
> "One specimen bolt from each twenty staves will be made by which to test their limits of elasticity and ultimate strengths in tension."
>
> "The specimen bolts must be able to sustain a tensile strain of 40 000 pounds per square inch, without permanent set, and must have an ultimate strength of 90 000 pounds per square inch of section."

Elsewhere, it is stated that "should some of the sections of a tube when under compression yield more than others, it is obvious that those others would be forced to bear more than their share of the load." Furthermore, "small specimens of the steel were, however, continually tested to destruction to ascertain both the modulus of elasticity and the ultimate strength. The modulus was kept at about 27 000 000 and the ultimate strength at about 120 000 pounds per square inch." The stress-strain line of a full-sized test of a stave was nearly straight up to a compressive unit stress of 50 321 lb per sq in. "In compression almost any degree can be obtained by the addition of chrome. To avoid difficulty, however, in finishing the steel in the lathes, it is only made sufficiently hard to meet the requirements of the specifications." A supplementary contract provided that "the staves composing the tubes shall not be required to stand a compressive strain exceeding 50 000 pounds, or a tensile strain exceeding 40 000 pounds, per square inch of section, without permanent set."

At a time when there is so much interest in the development of alloy steels, particularly adapted to special applications, the writer deems it proper to call attention to the attitude of Captain Eads and his staff, 66 yr ago, toward the use of alloy steel in the first long-span bridge, which now carries live loads far in excess of those for which it was designed. Much additional interesting data may be found in "A History of the St. Louis Bridge", by Professor Calvin Milton Woodward, published in 1881.

R. G. Sturm,[144] Assoc. M. Am. Soc. C. E. (by letter).—In the various papers presented in this Symposium many problems confronting the users of new materials in structures have been presented. Special emphasis should be given to a consideration of the stability problem, several phases of which have been treated in the Symposium. The writer wishes to connote a conception of stability in the more general sense; that is, stability against buckling, stability against vibration, stability against shock, and stability against excessive deformation with time. In connection with this general definition of stability, it is proposed to consider the actual volume of the metal (without relation to weight) that goes to make up a given member of any structure, and the effect of that volume on each of the aforementioned types of stability.

Stability against buckling is influenced not only by the modulus of elasticity of the material, but also by the shape and proportions of the cross-section of the member. The member must be strong enough not only to carry its load as a whole, but each component part must also be able to carry its share of the load without local buckling. Since the stress at which local buckling occurs varies as the square of the thickness, it is possible that the thickness of the material may influence the stability more than the modulus of elasticity. Structural members often fail by twisting, in which case the torsional rigidity of the member plays an important part in its stability.

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[144] Research Engr. Physicist, Aluminum Co. of America, New Kensington, Pa.

The torsional rigidity of thin open members varies as the cube of the thickness of the material. It may be concluded from these considerations that in some cases the actual volume of the member might have a greater effect than the modulus of elasticity.

Stability against vibration is achieved by two means: First, the natural frequency of vibration may be made so high that resonance or near resonance will not occur in practice; and, second, if resonance is approached, the damping effect of the system may be made great enough to prevent excessive amplitudes of vibration. The frequency of vibration of any member is greatly increased by the rigidity of its end connections, and the rigidity of a connection is influenced greatly by the torsional rigidity of the members framing into the joint. Therefore, in this case also, the actual thickness or the volume of the material is an important factor in stability against vibration.

In general, the natural frequency of vibration of a given structure may be fairly well approximated by the equation:

$$\text{Frequency} = \frac{3.13}{\sqrt{\text{Static deflection}}} \quad \ldots\ldots\ldots\ldots\ldots(26)$$

One finds that the weight and modulus included with the design control the static deflection of the structure under its own weight and, hence, its natural frequency. Resonance between applied impulses and the natural frequency of a structure is one of the greatest factors causing dynamic effects of high-speed traffic on large bridges.

Stability against shock is a difficult quantity to measure. Elastic resistance to shock is greatly increased by lowering the natural frequency of vibration, which is just the opposite of the condition sought for to provide stability against vibration. Therefore, a balance between these two conditions must be sought. In general, it is desirable to have members that will withstand emergency shocks without complete fracture even if they do take a permanent set. Very thin sections generally tend to tear or buckle, whereas heavier sections can deform plastically and take considerable permanent set without a complete fracture Therefore, the stability against shock will be influenced greatly by the volume of the material in the member as well as by other factors. Another factor very significant in resisting shock is the ductility of the material considered. Ductility is intended herein to connote the capacity of the material to become elongated over relatively long lengths rather than to "neck down" at a local point.

The stability against excessive deformation with time requires a balance between the beneficial effects and the deleterious effects of creep. High concentrated stresses are unavoidable at some localities in structures. If, by virtue of a slight amount of creep, the metal can adjust itself, these concentrated stresses are distributed, and the danger of failure at that point is greatly reduced. If the material at the point of concentrated stresses is not thick enough to permit such a redistribution of stress, minute buckling or tearing through local instability may occur before creep has an opportunity to take place. In such event, the actual volume of the material at the point of highly stressed concentration is of a great significance. In general, over-all deformation of structural members under stress can be controlled by a proper choice of design stresses for the material considered, whereas the local deformations around joints cannot be so controlled.

Many of the foregoing factors have been considered in the Symposium but is seems well to emphasize the fact that the general stability of a structure may be influenced greatly by the thickness of the material in the members of the structure and that with materials of low specific gravity it is possible to obtain a lighter dead weight and still maintain adequate thickness to provide stability.

E. ROBERT DELUCCIA,[145] ESQ. (by letter).—In the popular mind structural aluminum alloy is known only as duralumin, the material from which is fabricated the framework of large rigid airships, such as the *Akron* and the *Macon*; and even in the average engineering mind the thought of using structural aluminum is associated with such special structures as airships and airplanes. In his excellent paper, Mr. Hartmann indicates other uses for this material. He states that but little difference exists in the methods of shop fabrication between aluminum and other structural metals. However, he does state that the workmanship should be of a high quality. It is doubtful whether the average practice for structural steel, say, would be at all suitable for aluminum alloys. The difference does not lie so much in the use of the proper tools but in the care that the shop personnel is willing to exercise in the fabrication of the structure. This does not belong so much in the category of workmanship as it does in the proper appreciation of a new and useful material and in the will to assist the designer to the fullest in his attempt to produce a fine, closely designed structure. Certain lapses, which occur even in the best shops, such as occasional rough handling and nicking of material, over-driving and over-heating of rivets, accidental bending of plates and shapes, and consequent straightening, although possibly tolerated without danger to structural steel, would probably damage aluminum material and, therefore, should not be allowed. The rejection of even a small quantity of aluminum material rapidly mounts into a considerable sum of money and, therefore, it is important to the fabricator that he understand clearly the requirements necessary for successful fabrication in this metal. It would appear to be better to stress the proper fabrication of aluminum as such than to compare it to some other material, such as steel. Of course, steel has been used so long as the standard structural metal that it is a temptation to point out the similarity of other metals to its advantages. Nevertheless, it seems important to think in aluminum while working with it, if satisfactory results are to be obtained.

Although the thought of treating aluminum as a sovereign metal is important to the fabricator, it is of the utmost importance to the designer. Mr. Hartmann indicates this somewhat, but it should be emphasized that, due to properties which are peculiarly its own, aluminum should be considered as a quite different material, the various properties of which are subject to successful exploitation in design only when they are thoroughly understood and appreciated. As an illustration, there is the relatively low modulus of elasticity of aluminum which produces deflections that generally would be considered excessive in structural steel. The first thought of the designer, newly introduced to aluminum, is to deepen or thicken the member under consideration in an attempt to reduce the deflection. Such a procedure, of course, subtracts from the property which probably had most to do with the selection of the aluminum, namely, its relatively light weight. The designer, therefore, should understand, and expect fully, that relatively excessive deflection will occur and he should be willing to face this fact squarely, accepting it as proper and usual, and designing his structure accordingly.

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[145] Chf. Designing Engr., U. S. Engr. Office, Huntington, W. Va.

A short description of the dam across the Ohio River 9 miles below Gallipolis, Ohio, is given to demonstrate the type of problems that can arise in connection with the design of an up-stream emergency bulkhead for the roller-gates, and their satisfactory solution by the use of a light-weight alloy. The dam consists principally of nine concrete piers (16 ft wide and about 135 ft high) eight gate-hoists, eight steel roller-gates (each with a clear opening of 125.5 ft and a damming height of 29.5 ft) and a concrete sill upon which the roller-gates are seated. The pool above the dam is maintained during increasing flows in the river by raising the gates. In times of flood, the gates are raised completely above the water; as the flow decreases, they are lowered. In the event that a gate is prevented from being lowered due to some accident, the pool is saved by lowering a bulkhead in specially constructed recesses in the piers up stream from the gates. The bulkhead is also used for unwatering the gates for routine inspection, painting, and repairing.

The studies for the design of the dam indicated that the aforementioned size of roller-gates, although unprecedented, would be the most feasible. It was realized, however, that the relatively long span and high head would require a special solution for the up-stream emergency bulkhead. The problem was further complicated by the fact that the bulkhead would have to be capable of being placed across an opening with the gate raised and passing water through the full 125.5-ft width, under a head of more than 20 ft.

A further consideration in the design of the bulkhead was the method of handling it. It was desired to place it by the use of a whirler derrick-boat. In order that this could be done without endangering the boat, it was necessary that its boom be of sufficient length to enable the boat to lay against an undamaged gate while placing the bulkhead across the opening of the damaged gate. This would result in a boom more than 125 ft long, with the load being lifted at a radius of about 117 ft. It will be readily seen, therefore, that the weight of the bulkhead was a prime consideration.

Various designs for the bulkhead fabricated from structural steel, alloy steel, or structural aluminum alloy were considered. It was found that the structure fabricated from an aluminum alloy would be by far the lightest in weight and, therefore, this material was selected. To facilitate handling, the bulkhead was separated into seven units of equal size and weight. The weights of the bulkhead, fabricated from the various materials considered, are as follows:

Material	Weight, in Tons	
	Each unit	Total bulkhead
Structural steel	78	546
Alloy steel	44	308
Structural aluminum alloy	28	196

As constructed the bulkhead has a total damming height of 30.33 ft, each unit having a damming height of 4.33 ft. Fig. 51 (submitted through the courtesy of the Huntington District, U. S. Engineer Office, Huntington, W. Va.) shows the seven units as stored temporarily. These units are built from two, simple trusses of the Pratt type, which are held apart by channel and angle struts between the chord members. On the inside of the lower chord and extending between the two trusses is a $\frac{3}{8}$-in. skin-plate which serves to make the unit water-tight. Each truss is 127.5 ft long and 12.5 ft deep (measured between working centers). On the outsides of the lower chord of each truss are 4 by 4-in. oak timbers, running the full length of the unit, which effect a seal between the units, and between the lowest unit and the sill, when placed in the dam. Timbers are also provided at the panel points of the top chords to serve as buffer-blocks between the units. On each end of each unit is a housing containing a roller nest which, in turn, contains two rollers $18\frac{1}{4}$ in. in diameter which bear on a steel plate embedded in the down-stream face of the recesses in the piers. The rollers are placed on the down-stream side of the unit and the roller nest is arranged to swivel about trunnion pins so that as the trusses deflect the rollers remain in full contact with the plate in the recess.

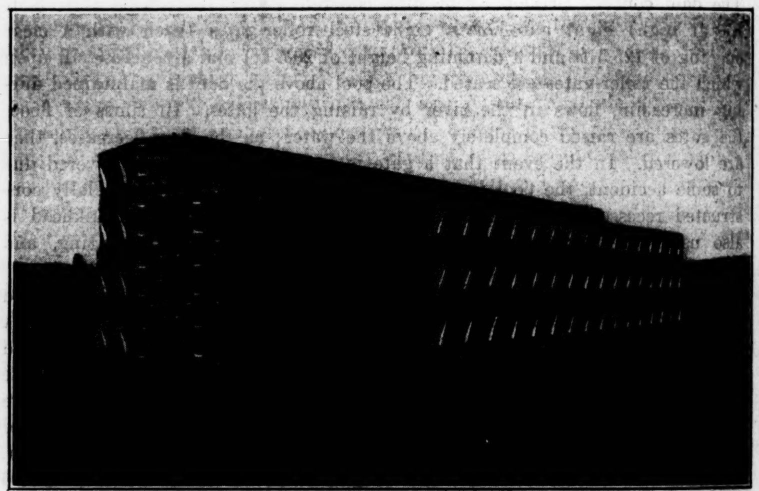

FIG. 51.—SEVEN UNITS, AS TEMPORARILY STORED, OF GALLIPOLIS DAM, OHIO RIVER.

Guide rollers are provided on the ends and up-stream side of the housing to prevent binding in the recess in the event that the unit tips in raising or lowering. A specially designed pick-up beam, from which hooks are suspended to engage pins at approximately the third points of each unit, is used to pick up the units.

The truss is fabricated from Aluminum Alloy 27 S-T (see Table 7, Item No. 10) with the exception of the bent hip plates and the rivets which are of structural grade steel. The aluminum alloy has the following chemical composition: Aluminum (minimum) 92%; copper, 3.9% to 4.9%; manganese, 0.5% to 1.1%; silicon, 0.5% to 1.1%; tin, 0.03% to 0.07%; iron (maximum), 0.9%; magnesium (maximum) 0.03%; and, other elements (maximum), 0.03 per cent. It is used structurally in the heat-treated condition and has the following physical characteristics: Ultimate tensile strength, 58 kips per sq in.; yield strength (permanent set of 0.2% of the initial gage length), 45 kips per sq in.; and, elongation in 2 in. (minimum), 8 per cent.

The following are the maximum stresses, in pounds per square inch, allowed in the design of the Gallipolis bulkhead:

(1) Tension, on net section = 22 000; (2) compression, axial loads, on gross section for $\frac{a\,L}{k}$ less than 67, $= 22\,000 - 219\,\frac{a\,L}{k}$; $\frac{a\,L}{k}$ more than 67, $= 33\,000\,000 \left(\frac{k}{a\,L}\right)^2$; and, for truss members, the riveted connections, $a = 0.75$; (3) compression in beam and girder flanges $= 20\,000 - 230\,\frac{L}{b'}$; (4) maximum shear on gross area = 10 000; and, on net area = 13 000, limited always, however, by the formula:

$$s = 12\,000\,000\ t^2 \left(\frac{1}{d^2} + \frac{1}{l^2}\right) \qquad (27)$$

in which d = clear depth of web, in inches; and l = clear stiffener spacing, in inches; and (5) bearing = 26 000. In Items (1) to (3) L = unsupported length of flange, in inches; $b' = b\left(c + \frac{t}{d}\right)$; b = width of compression flange, in inches; t = web thickness, in inches; d = over-all depth of beam or girder, in inches; $c = 1.0$ for symmetrical members, with web and flange rolled or extruded integrally; $c = 0.7$ for symmetrical members with cover-plate, flanges and web not integral; $c = 0.5$ for symmetrical members with no cover-plate, flanges and web not integral; and, the value of $\frac{L}{b'}$ was kept less than 40.

The top chord of each truss consists of four 6 by 4 by $\frac{9}{16}$-in. angles, one 16 by $\frac{3}{4}$-in. web-plate, and one $8\frac{3}{4}$ by $\frac{5}{8}$-in. cover-plate arranged in the form of an I-section. The cover-plate extends over the four center panels only. The web-plate is cut at the gussets for the diagonals and milled for bearing. The angles are continuous over the gussets. The bottom chord is similar to the top chord, except that the web-plates are not milled at the gussets. Splice-plates are used on the web-plates to transmit the tension in the web across the gussets and are carried a sufficient distance beyond the gusset to take care of the negative moment resulting from the water load on the skin-plate. The diagonals are all rolled or extruded shapes, and range in size from two 8-in. channels weighing 5.78 lb per lin ft to two $3\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{5}{16}$-in. angles. The verticals range from two 10-in. car channels, weighing 9.59 lb per lin ft, to two $3\frac{1}{2}$ by $2\frac{1}{2}$ by $\frac{5}{16}$-in. angles. In three cases, the verticals are built-up sections of four 4 by 3 by $\frac{1}{2}$-in. angles, with spreader plates, $6\frac{1}{2}$ by $\frac{3}{4}$ by 8 in. long; four 3 by 3 by $\frac{3}{8}$-in. angles, with spreader plates, $9\frac{1}{2}$ by $\frac{3}{4}$ by 10 in. long; and four 3 by 3 by $\frac{3}{8}$-in. angles, with spreader plates $6\frac{1}{2}$ by $\frac{3}{4}$ by 7 in. long. Drain holes are provided in the webs of both top and bottom chords so that they will drain as the units are lifted clear of the water. The bent hip-plate between the inclined end post and the top chord is of structural steel. This substitution of steel for the aluminum was made in order to avoid bending the latter. A camber of 4 in. which is approximately one-half the total calculated deflection, was provided in each truss. The remaining deflection is cared for by allowing the ends of the truss to rotate freely about trunnion pins on the ends of the nest castings containing the bearing rollers. The trunnion pins are received by bronze-bushed, cast, nickel-steel bearings bolted to the end gusset-plates of the truss with turned steel bolts.

The roller-nest casting is of special steel having an ultimate tensile strength of 100 kips per sq in., a minimum yield point of 60 kips per sq in., and an elongation of 20% in 2 in. After the castings were made they were thoroughly annealed to remove any strains incident to casting. Due to the limited space available for this nest, it was necessary to use relatively high-strength material and, therefore, cast steel was substituted for aluminum. The trunnion pins about which the trusses rotate were cast integral with the roller nest and chromium plated as a precaution against corrosion and to minimize friction.

The bearing rollers $18\frac{1}{4}$ in. in diameter are of cast nickel steel and are bushed in manganese bronze for the nickel-steel pins about which they rotate. Nickel steel was used in lieu of aluminum not only because a stronger material was necessary, but also due to the fact that the relatively small diameter of the rollers and the heavy bearing load made the use of aluminum unsuitable in this instance. The guide-rollers are of cold rolled steel. Their weight is so small relative to the total weight of the unit that it was felt that in this case the more expensive aluminum would not be justified.

Nickel-steel plates are used at the pick-up points due to the higher bearing value of this material. The pick-up pins are also of nickel steel and are adjusted in the units by means of nickel-steel adjusting screws. The seals are of white oak. Wood was chosen for this use due to its easy replacement, shock protection, and capacity to swell under water and so assist in making the seal.

All rivets are of steel and are, in general, $\frac{7}{8}$ in. in diameter. Steel rivets were used because it was felt that aluminum rivets would be impracticable, due to the special heating equipment and close temperature control necessary for their successful use. Where possible all rivets were cold-driven. From examination of both hot and cold-driven steel rivets in aluminum, it appears that less local deformation of the riveted material accrues with the use of the latter method. It is necessary, however, that some provision be made to avoid the use of excessive pressures while cold squeezing. All rivet holes were subpunched $\frac{1}{16}$ in. smaller and reamed $\frac{1}{16}$ in. larger than the nominal size of the rivet.

After the units were fabricated, the aluminum was thoroughly cleaned with a chemical cleaner and rinsed off with clean water. A red iron-oxide primer was then applied. After the primer had dried thoroughly, two coats of aluminum paint were applied. The steel was painted with red lead and two coats of aluminum.

The approximate weights, in kips, of the various materials in one bulkhead unit are as follows:

Structural Aluminum Alloy No. 27 S-T	37.9
Structural steel, including rivets	7.3
Structural nickel steel	0.5
Cast nickel steel	3.8
Cast steel (special)	1.9
Miscellaneous bolts, pins, bushings, etc	2.1
Timber	2.5
Total	56.0

The make-up of the sections of the top and bottom chords and of the diagonals is considerably different from the forms that would have been adopted had steel been used. Large aluminum alloy shapes are not available, ordinarily, due to the lack of rolling equipment. They can be obtained by using steel passes but only at a sacrifice in strength of about 10 per cent. The largest shape that can be extruded is one that can be contained in a circle 12 in. in diameter, due to the limitations of the size of the ingot. Shapes made by the extrusion process or by rolls made specially for rolling aluminum suffer no reductions in strength. Therefore, only shapes that could be contained in a circle 12 in. or less in diameter were used. The relatively high base cost of this material made it economical to allow considerably more fabrication per pound than is ordinarily the case for steel. For this reason, even small sections were built up instead of using a single shape of somewhat greater weight. If fabricated from structural steel, the chords would probably be made up of two channels, a fill-plate, and a cover-plate. The design adopted, however, while increasing fabrication utilized the minimum possible weight of material.

The deflection of the unit is calculated to be approximately 8 in., which is about three times the deflection which would be expected for steel. However, by cambering the trusses 4 in. and by using a swiveled roller nest in the housings at the supports, it was possible to accept the apparently high deflection. As might be expected, secondary stresses of considerable magnitude occur in the trusses when they are under load, and consideration was given them in the design.

O. J. HORGER,[146] Esq. (by letter).—Those responsible for the non-failure of materials in their application to structural and machine parts find greater need to-day for an intelligent use of stress theories. For this reason Mr. Karpov's paper, reviewing the knowledge available, is of considerable interest.

It is not clear from observation of Fig. 3(*a*) for the un-notched tension specimen why the stress is not uniformly distributed for values within the yield point[147]. The same question applies to Fig. 9 for the case of a rectangular bar subjected to bending where the bending stress within the yield point should be linear.

The fatigue diagrams presented in Fig. 10 are apparently based on plain specimens; that is, without stress concentration. It would be interesting to see comparable diagrams on the basis of notched specimens. Such diagrams would be the ones actually required in most design problems since failure usually occurs in members due to local stress concentration, such as fillets, holes, etc.

Fig. 10(*b*) indicates that the lighter metal has a strength about equal to nickel steel on a weight-strength basis. When the lighter alloy is used the section dimensions must be increased over the steel so that a size effect is also involved in the calculation of comparable fatigue strength. This factor of size effect may be small, considering plain specimens, but with stress concentration it is important and must be considered. The writer is familiar with size-effect factors using steels as given in published literature, and would like to ask whether the author can give similar data on the light alloys?

Considering the effect of the aforementioned factors of stress concentration and size effect does the author believe that these considerations would alter the relative comparison shown in the diagrams of Fig. 10(*a*) and Fig. 10(*b*)?

A. W. DEMMLER,[148] Esq. (by letter).—Attention has been called by Mr. Beard to U. S. Navy Specification No. 48 S 5e covering manganese-vanadium steel in connection with steels suitable for fusion welding. The 0.18% carbon, 1.45% manganese, and 0.25% silicon, listed in this specification, are maximum values. The presence of vanadium in this combination results in distinct grain refinement with uniformity and also serves to insure toughness and freedom from sensitive hardenability.

Mr. Bain has also mentioned these features and, in discussing orally the cumulative effect of various alloy additions, made the true and interesting point that simple arithmetic does not give the full picture in that "two plus two plus two may equal seven" in the behavior of the final product. In the absence of vanadium, 1.45% manganese would be frowned upon for fusion welding; yet with 0.10% to 0.12% vanadium a piece of 0.5-in. plate water-quenched from 2 400° F, and machined free of any decarburization, will have a Brinell hardness number of only 400, while air-hardening from this temperature will give a Brinell hardness number of about 250. Therefore, any danger of encountering a glass-hard condition from a welding operation is out of the question.

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[146] Research Engr., The Timken Roller Bearing Co., Canton, Ohio.

[147] "Photo-Elasticity", by Coker and Filon, 1931, Cambridge Univ. Press, England; also, "Theory of Elasticity", by S. Timoshenko, 1934, McGraw-Hill Book Co., New York, N. Y.

[148] Metallurgical Engr., Vanadium Corp. of America, Bridgeville, Pa.

THEODORE BELZNER,[149] AFFILIATE, AM. SOC. C. E. (by letter).—So carefully has Mr. Templin covered the methods of operation, and the precautions, required in the manipulation of the various types of strain-gages for precise work on engineering structures and their models, that little can be added. Possibly a few references could be made to the measurements required after the preliminary work.

The skill required to obtain reliable results in strain-gage measurements is not in the possession of all; and the writer agrees with Mr. Templin, that,

> "In the testing of structures, the errors arising from the personal equations involved in the manipulation of the instruments used, must be considered, in addition to those already mentioned. The magnitude of these errors becomes less, at a diminishing rate, as the experience of the observer increases."

During 1913 and 1914, a comprehensive series of extensometer investigations was made in connection with the strengthening of the end spans of the Williamsburg Bridge, by the Department of Bridges (later the Department of Plant and Structures), City of New York, in co-operation with the National Bureau of Standards, under the auspices of the late James E. Howard, Engineer Physicist, on the important truss members at the main towers and legs of the Brooklyn intermediate towers.

During these investigations, extending over a long period of measurements, a Howard extensometer of 20-in capacity was used throughout the entire test; and the precision attained with the strain-gage proved to be an invaluable aid, and its merits were illustrated in certain vital operations, especially with the various stages of wedging, and in the transferring of stress from the old to the new diagonal members of the end trusses.

The results obtained firmly convinced the writer that the most important objective to be considered in taking stress-strain measurements (other factors being equal) is to obtain consistent results; and in order to secure such results (which are absolutely essential), persistent patience and painstaking thoroughness, combined with good judgment on the part of the observer, are required, until he experiences little or no difficulty, and it is upon the observer that the value of such measurements will depend.

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[149] Insp. of Steel, and Bridge Insp.-in-Charge, Brooklyn Bridge, Dept. of Plant and Structures, City of New York, Brooklyn, N. Y.

J. P. GROWDON,[150] M. AM. SOC. C. E. (by letter).—The engineer who is concerned with the design of structures, such as bridges, rather than the design of structural parts of machines, such as dragline booms, will be interested in the use of strong aluminum alloys when these alloys will: (1) Permit him to design and build a structure that would not otherwise be possible; and (2), when it will permit him to design and build a more efficient structure than can be obtained by the exclusive use of other material.

The first situation occurs in certain fields, such as that of aircraft design, but will be rare in the structural field because all the structures which are now required can be successfully constructed of other materials. The second situation will be more frequently encountered by the structural engineer, who can, in most cases, measure the efficiency of his design in dollars, either in the form of reduced maintenance cost or of less capital investment.

Reduced maintenance cost can often be achieved by utilizing the high resistance to corrosion possessed by aluminum alloys of the structural group as compared to alloys of other materials. Less capital investment can be achieved in many specific cases by utilizing the bulk-weight ratio as well as the high strength-weight ratio of these alloys.

The extent to which aluminum alloys can be used economically in any given structure will depend upon their cost, relative to the cost of other materials which can be used for the same purpose, the type and size of the structure, the physical characteristics of the alloy, the skill with which the designer makes use of it, and the price at which it can be purchased.

To-day, the cost of aluminum per pound used to replace low alloy steels will be five or six times the cost of such alloy steels. This cost differential effectively limits the use of structural aluminum to those places where 1 lb of the newer material will save several pounds of steel.

This use of structural aluminum has been well illustrated in bridges, both new and reconstructed. Referring to new bridges, of the suspension, cantilever, or truss type, this limitation will mean that aluminum can be used only in those parts of the structure where the saving in dead weight will be most effective in reducing the weight and cost of other parts. In new short-span bridges such a saving is small, whereas in new long-span bridges the possible saving in dead weight (and, hence, in cost) becomes very large; so that one may say that in short-span bridges no aluminum can be used economically for structural parts, but as the length of span increases, the possibility of saving money by its use will also increase. Furthermore, the longer the span, the greater will be the number of places in which it can be used to advantage.

Various types of bridges will require aluminum in different parts of the structure in order to secure the most economical result. In a suspension bridge, it will be used first in the deck and with longer spans in the stringers, floor-beams, and stiffening trusses. In a cantilever bridge aluminum would find its most profitable use in the suspended span, where a saving in weight would reduce progressively the weight of the cantilever arms, the anchor arms, the towers, and the size of the foundations. Thus far, aluminum has found little utility in truss bridges except for railing and decorative parts.

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[150] Asst. Chf. Hydr. Engr., Aluminum Co. of America, Pittsburgh, Pa.

In a bascule bridge a pound of weight in the outer end of the leaf requires several pounds of material in other parts of the structure to support it. A lift span, constructed entirely, or in part, of aluminum, would require a lighter counter-weight, lighter towers, smaller motors, and less power to operate it. In all bridges of the bascule or lift type aluminum deserves careful consideration from the standpoint of economy and efficiency.

In the rehabilitation and reconstruction of old bridges the high strength weight-ratio of aluminum has found a most effective use. Many old bridges, inadequate for modern traffic, can thus be reconstructed so as to reduce the dead load, increase the traffic capacity, or improve the character of the roadway. In some cases all three results may be accomplished.

From the many aluminum alloys available Mr. Hartmann has selected a single alloy of the duralumin type, (17S-T), and has largely limited his discussion to that single alloy. In thus selecting Alloy 17S-T as the typical alloy he has been wise since this alloy was the first of its type to be developed and has long been established in the structural field. As pointed out by Mr. Hartmann, there are many other alloys available to the structural engineer, and it is believed that certain of the newer alloys now available have physical characteristics which particularly adapt them for use in such structures as bridges. Among these characteristics may be mentioned: Alloys 53S-T and 27S-T, included in the paper by Messrs. Jeffries, Nagel, and Wood (see Table 7, Items Nos. 10 and 13). They differ from Alloy 17S-T, because of the lower tensile and yield strengths of the latter, (which are, respectively, 33 and 20 kips per sq in.), but more especially because of its extraordinary resistance to corrosion.

Item No. 10, Table 7, differs from Item No. 6 in that it has a greater yield strength (50 kips per sq in.) and a somewhat smaller percentage of elongation (9 to 13). These characteristics make Item No. 13 particularly suitable for applications where maximum resistance to corrosion is desirable, but where maximum strength is not required. Bridge railing is an example of such an application. The characteristics of Item No. 10 make it particularly suitable for those applications in which the maximum strength-weight ratio is desirable.

To secure satisfactory results, it is not sufficient merely to substitute an aluminum member for a similar steel member. The designer must take advantage of the particular properties of the material so as to utilize these properties to the best advantage and to minimize the effect of disadvantages which are inherent in certain of them. In addition to the ultimate strength and the yield strength, the designer must keep constantly in mind the modulus of elasticity, the temperature coefficient, the weight, and the cost. The relatively high price of aluminum makes it necessary for the designer to have accurate knowledge of the properties of the material, the loads to which the structure will be subjected, and the stress which these loads will produce in every part of the structure. He must exercise his skill, ingenuity, and best judgment, in addition to his knowledge, not only to see that every part of the structure is adequate for the work which it must do, but that no material is wasted.

A properly designed structure utilizing aluminum in a greater or less degree will be different from a steel structure serving the same purpose, because the properties of the material are different. As an illustration of how the properties affect the design, it is interesting to note the effect of modulus of elasticity, which, of course, in an aluminum structure having a modulus of elasticity of 10 000 000 lb per sq in. will produce deflections much greater than in an identical steel structure having a modulus of elasticity of 29 000 000 lb per sq in. In many cases, this excessive deflection can be reduced within satisfactory limits by utilizing continuity wherever possible in the design. Other factors in which the design of an aluminum structure will differ from that of a steel structure will occur to the engineer who thinks seriously on the subject.

The Smithfield Street Bridge, across the Monongahela River, at Pittsburgh, Pa., was reconstructed in 1933 by replacing the entire wood and steel floor system with an aluminum floor system of Alloy 27S-T.[151] It has functioned satisfactorily since reconstruction and is an excellent example of the successful application of one of the newer light alloys in the bridge field.

KARL ARNSTEIN,[152] ESQ. (by letter).—An interesting presentation of the most important phases of models and model testing is contained in the paper by Mr. Templin. The modern trend toward light-weight designs of larger structures, requires that more consideration be given to the subject of general buckling. Mr. Templin refers to this problem in a general way only. It should be emphasized that, in order to obtain reliable general information on buckling in scaled down or scaled up models, very rigid specifications are required in designing the model and model members. Not only can general buckling be sensitive to the axial, bending, torsional, and shear characteristics of its individual members, but it is essential that the unit linear deformation (and, hence, also the relative angular deformation), under any system of forces or moments, be the same as the corresponding unit linear, and relative angular, deformation in the corresponding prototype member, under the properly scaled corresponding forces and moments.

For most types of engineering structures, the shear stiffness of its individual members is sufficiently large to have only a small effect on the general stability (unless a built-up section is represented by a single member in the model) and can be neglected. To meet the remaining requirements, it is necessary, for reasons mentioned by Mr. Templin, that the model member be designed so that the axial, bending, and torsional stiffness can be varied independently. Such a model girder has been developed and used by the writer's company and has been explained, together with other interesting comments on this subject, by Mr. L. H. Donnell[153]. The application of this type of model representation may remove some of the objections raised by Mr. Templin in his valuable paper.

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[151] *Civil Engineering*, March, 1934.

[152] Chf. Engr., Aeronautical Dept., The Goodyear Tire & Rubber Co., Akron, Ohio.

A. CHRISTIANSON,[154] ESQ. (by letter).—The weight savings accomplished through the use of structural aluminum in the framing of car bodies have proved to be a substantial addition to the weight saved through the other major application of aluminum in this field, namely, inside finish and fittings. The extra cost of providing these weight savings is offset by the savings in power and improvements in performance. The trend away from the conventional steam locomotive, of course, has accelerated the demand for light-weight construction.

Aluminum alloys are well adapted to light-weight car construction because they are available in forms that fit the needs of the car builder. Large-sized sheets with a high degree of flatness are readily obtained, and the shapes, particularly the special extrusions, meet all the requirements of form, thickness, and length.

The low unit weight of the aluminum alloys permits light-weight construction without drastic reductions of thickness of sheet and shapes. This has been found to simplify both design and construction. The car designer, having available a greater volume of metal, is able to distribute it effectively in the members and still not depart radically from conventional sizes which have been established through years of experience in car-building practice. The use of the greater volume of light-weight metal and the resulting thicker sections greatly reduces the tendency for buckling failures and other complications due to instability. The pleasing and substantial external appearance which always accompanies freedom from wrinkles is readily obtained in aluminum construction with no extra effort in the shop.

Mr. Hartmann makes some interesting comments on workmanship, indicating that aluminum construction should receive careful attention in the shop in order to help approach the degree of perfection for which the designer is striving. The importance of this cannot be too highly emphasized. It has long been the writer's contention that no construction can function as the designer intended, unless the shop work is perfect. Excellence in fabrication is of importance to aluminum construction for the same reasons that it is important to all other construction, and is accomplished in exactly the same manner. Aluminum alloy construction presents almost no new problems to the shop experienced in producing a high quality of workmanship in other metals.

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[153] "Model Measurements and Airship Stress Analysis", pub. in "Report on Airships Forum", July 25-26, 1935.

[154] Chf. Engr., Eng. Dept., Pullman-Standard Car Mfg. Co., Chicago, Ill.

Robert E. Glover,[155] **Esq.** (by letter).—In his discourse on "The Extent of Present Theoretical Knowledge" Mr. Karpov comments on the lack of information regarding the electric and magnetic forces of the electrons, and expresses the belief that the fundamental avenue of approach to the problems of stress is now closed because of a lack of essential basic information. He then refers to the "theory of stress" and concludes that, because no general solution of the differential equations is known, the designer is forced to fall back upon a number of more or less reliable assumptions among which the most important are that the material follows Hooke's law and that the stress follows a straight line distribution. The writer can not agree that the designer need be reduced to such straits. It is well known among students of the subject that the general solution of a partial differential equation may be of little use in cases where a particular problem must be solved. Since the differential equations of elasticity are of this type the lack of such a solution need not be cause for great concern. The question of the validity of Hooke's law does not admit of a general answer applying to all materials, but with regard to a given material, may be answered at the nearest testing machine.

To the writer's knowledge, there are at least four good, sound, carefully written texts on the theory of elasticity which contain a large number of solutions of the elastic equations, many of them illustrated with solved examples for particular cases. With these at hand, supplemented, perhaps, by some other excellent available texts on special subjects, such as photoelasticity, plasticity, slab theory, elastic stability, and the contributions which are constantly appearing in technical literature, the designer should not find his problem of stress determination hopeless.

The uniqueness theorem[156] has a bearing on some of the statements made by Mr. Karpov. This theorem was published in 1859 by the distinguished investigator, G. Kirchhoff, whose name is so well known in the electrical world. He proved that under certain stipulated conditions (which, indeed, apply to the great majority of engineering structures), the differential equations of elasticity can have no more than one solution. If the existence theorem were also available, the case would be complete, and it could be inferred that the elastic equations, as they exist to-day, solved in conjunction with the supplementary conditions imposed by any particular problem are completely adequate under the postulated conditions. There seems to be small reason to doubt that this is actually the case, but if the required solution is known or can be found the existence theorem may be dispensed with, since the possession of a solution is ample proof of existence, and the designer can proceed with the assurance that, in his case, the problem is properly set. Furthermore, since no other solution can exist, he also has the assurance that the results he arrives at can not be altered by whatever information some later investigator may succeed in prying out of the electrons.

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[155] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[156] "Mathematical Theory of Elasticity", by A. E. H. Love, Fourth Edition, Paragraph 118.

It is true that the theory of elasticity is mathematically difficult, but it is also true that so many of these difficilties have been overcome by able investigators that the charge of practical uselessness can no longer be sustained. The writer has had occasion to apply the methods of the theory of elasticity to a number of problems and has obtained very satisfactory results. The Kirchhoff theorem has been of great assistance in connection with the trial-load method for the design of arch dams, by making it possible to determine what constitutes an adequate analysis.[157] These methods were also applied, under the writer's direction, to the problem of developing formulas for designing the supporting rings for the penstocks at the Boulder Dam, which is located about 30 miles southeast of Las Vegas, Nev. These formulas were obtained and used with entire satisfaction, and the results have been confirmed by extensive tests both in the laboratory and on the penstocks themselves in the field. These and similar formulas have been used more recently in the design of the Malheur siphon on the Owyhee project in Eastern Oregon. This pipe is 80 in, in diameter and is constructed with 60-ft spans, without intermediate stiffener rings.

Arthur C. Ruge,[158] Assoc. M. Am. Soc. C. E. (by letter).—A remarkably complete and lucid discussion of the types and purposes of tests of engineering structures and their models has been presented by Mr. Templin. Although he does not specifically make a critical comparison of the testing of full-sized structures with that of their models, one feels that he is doubtless aware that defects in the model method are sometimes over-emphasized in relation to those defects which are inherent in the testing of many full-sized structures. The principal difference in this respect lies in the relative ease with which the model test conditions can be controlled and varied.

As a case in point, one might consider the dynamic tests of structures, which are becoming important as a means of interpreting the effects of earthquake motions. Consider a tall masonry chimney in which it is desired to study the vibration periods for the purpose of checking theoretical calculations. If the model method is selected, difficulties arise only in securing the proper material to represent the masonry and the foundation material. These difficulties may be very serious; but they are not insuperable.

If, on the other hand, one chose to test the actual chimney in the field, one might be misled at first by the apparent simplicity of the problem, but sooner or later one would discover defects more serious than in the model technique. The first great difficulty would be in generating the modes of vibration, and, assuming that has been done, one cannot know if the periods measured will be maintained for the large amplitudes which are really the most important, but which the investigator dares not, or cannot, produce in the test. For example, a chimney laid up with completely bare bricks would exhibit a set of natural periods not greatly different from those of the same chimney laid up in good mortar; but the dynamic behavior under large amplitudes would be vastly different.

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[157] "Fundamentals of the Trial Load Method for the Design of Arch Dams", by R. E. Glover, presented as a thesis to the Univ. of Nebraska in partial fulfillment of the requirements for the degree of Civil Engineer.

[158] Research Associate in Seismology, Dept., Civ. and San. Eng., Mass. Inst. Tech., Cambridge, Mass.

Next, it is impossible from field test data to separate the effect of the foundation from that of the masonry in producing the observed periods, unless broad assumptions are adopted as to the correctness of the very theory to be checked. In such a problem the model method is not nearly at such a disadvantage as it might at first seem. In the model, the conditions of amplitude, stress, and foundations can be controlled and varied in such a manner as to give a positive check on the theory involved. To a less extent the same arguments will be found to hold for certain purely statical tests of structures.

It is not to be ignored, of course, that a tremendous amount of work remains to be done in the development of model materials and artifices for overcoming difficulties in similarity; but it needs to be emphasized that there may be (and often are) pitfalls in the testing of actual structures which are likely to be overlooked entirely. In this connection, Mr. Templin very rightly points out the importance, at times, of considering the model itself as a structure, without assuming a simple relation to its prototype.

The statement (under the heading, "Test Methods: Models Made of the Same Material as the Prototype") that a "photographic" model of a gusseted truss would not be truly similar, needs to be clarified somewhat. Although it is quite true as stated that a geometrically scaled model will not result in the same scale ratio for areas as for moments of inertia, this fact alone cannot preclude the realization of perfect similarity conditions. In fact, unless these ratios are different, true similarity cannot exist in the usual sense of the word.

If, for simplicity, the assumption is made that the connections are welded, or that the riveted connections do not slip, it is difficult to understand why Mr. Templin makes the statement that "the degree of end restraint will not be the same as in the prototype, and the behavior of the various members of the model, under combined axial loads and bending, will be different from that of the corresponding members of the prototype." The elastic action of a gusset-plate cannot be said to be of a different nature from that of a beam or strut, although the stress distribution may not be equally simple; and, fortunately, models do not have the means to avoid complex stress distributions as is commonly done in analytical work.

Mr. Templin's point is very good in regard to the limitations of models; but he has chosen an unfortunate example for illustration. To present a better picture of the difficulties of scale effects in similarity problems, one needs only to consider structures in which more than one type of physical forces act simultaneously. In the gusseted truss only elastic forces are involved. In the case of a structure like an elevated water tank (considered dynamically) both gravity and elastic forces are found interacting in such a way that a "photographic" model actually fails to represent the prototype to a marked degree.

The classical dimensional analysis in the water-tank problem promptly leads to the result, $\lambda = 1$; or, the model must be made full size to give true similarity. By proper distortion of scale, however, one easily preserves the similarity in spite of the first result, it being only necessary to find the means of keeping the same force scale for both gravity and elastic forces. To be sure, in practical work of this kind, one may have to be content with securing true similarity only in those features of the behavior he desires to study, but there can scarcely be objection to that.

Alexander Klemin,[159] Esq. (by letter).—In the construction of aircraft one encounters both complicated structures and complicated distributions of loads. The loads applied are air pressures and not only is the distribution peculiar but it varies with each condition of flight. The complete structure is very frequently tested to destruction.

Models are used infrequently. In the structural testing of aircraft, the technique is not too well defined and methods of determining stresses and strains are likely to be improvised for a particular construction. Those engaged in aircraft engineering have learned much from the Civil Engineering Profession and Mr. Templin's paper will add to such lessons. Every aeronautical engineer who makes structural tests should follow the principles enunciated.

Paul E. Gisiger,[160] Assoc. M. Am. Soc. C. E. (by letter).—In connection with the very informative paper on stainless high alloy structural steels by Mr. Morris, it may be of interest to cite a recent application of non-corrosive steel in the field of hydraulic engineering.

In the hydro-electric power plant of the Pennsylvania Water and Power Company at Holtwood, Pa., the trash screens which protect the turbine intakes against floating logs, trash, etc., and which were installed between 1910 and 1913, are now (1937) approaching the end of their useful life. Screening equipment consists of cast-iron guides bolted to the piers between individual intake openings (of which there are four for each of the ten main turbines and one for each of the two exciter turbines) and of individual screen units (five per opening or twenty per turbine), which slide in the guides and can be removed, changed, or replaced easily with overhead cranes.

The original screen units are made of structural steel channel frames, 11 ft high and 9 ft 6 in. wide. The screen bars are $4\frac{1}{2}$ by $\frac{3}{8}$-in. flat bars spaced $4\frac{5}{16}$ in. and held to the frames with bent and slotted attachment plates and bolts. Each such unit weighs about 4 500 lb so that the total of 210 units represents a considerable tonnage.

The established maintenance practice is to sand-blast and repaint the screens once every four years. During recent years it became increasingly necessary, however, to add miscellaneous repairs to this maintenance schedule and, by 1935, it was evident that replacement of at least a part of the screens could no longer be postponed.

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[159] Prof., Daniel Guggenheim School of Aeronautics, Coll. of Eng., New York Univ., New York, N. Y.

[160] Structural Engr., Pennsylvania Water & Power Co., and Safe Harbor Water Power Corporation, Baltimore, Md.

An investigation was then made into the use of materials which would require less maintenance and possibly would last longer than the structural steel. In addition to several kinds of corrosion-resisting alloy steels this investigation included bronze, copper,* and aluminum. In order to be economically justified a material other than low carbon structural steel had to give reasonable assurance that the increased first cost would be offset by less maintenance and a longer life.

Comparative estimates in which such factors are of large influence are naturally somewhat uncertain and will have to be checked by experience. They were conclusive enough in this instance to decide in favor of new screens with a frame of structural steel as before, but with screen bars made of high-strength chromium alloy of approximately similar composition as Item No. 2, Table 4. Wearing strips of the same material were also provided along the contact line between screens and guides. This resulted in somewhat more than one-half the entire surface area of the screens being non-corrosive and the parts which still have to be painted are those which suffer the least wear are easiest to paint.

Each of the new screens which have the same outside dimensions as the old ones includes 1 050 lb of corrosion-resistant alloy steel and 2 350 lb of standard structural steel. The total weight of the new screens, therefore, is only 75% of the old ones which fact in itself is very desirable because it makes for easier handling. The saving results mostly from the higher strength of the alloy material, but also from the all-welded construction.

Another advantage resulting from the use of high-strength material as well as from welding is the very considerable reduction in the area that obstructs the water passage. This is quite evident when pictures of the old and new screens are compared, and the improved hydraulic effect is expected to be further emphasized by decreased friction between water and screen bars which do not become roughened by corrosion.

The welding of the high-strength chromium steel and especially its connection by welding to a different material were made the subject of close investigation. A number of types of welding rod were tried and welds examined and tested. The results indicated that generally a somewhat better penetration is obtained if the welding rod is of a composition similar to the base metal and that, therefore, the joints between ordinary steel and alloy steel for which a coated alloy steel electrode was used, had to be welded with more than the usual care. During all these investigations and tests the manufacturers of non-corrosive steel were most helpful in co-operating to arrive at the most suitable materials and methods. Of the old screens in the Holtwood Plant, 25% have now (1937) been replaced and it is intended to replace the remainder progressively during the next few years.

There is no doubt that hydro-electric structures offer a wide field for the application of non-corrosive high strength steels and that their use for such structures will increase rapidly as knowledge concerning these metals advances and as production on a larger scale will make it possible to lower their cost.

Russell C. Brinker,[161] **Jun. Am. Soc. C. E.** (by letter).—Engineers are given a clear-cut analysis of the possibilities and limitations of light-weight alloys in the papers of this Symposium. Several statements from the various papers might conveniently be linked for the purpose of greater emphasis.

In their "Summary," Messrs. Bain and Llewellyn state that, "* * * corrosive attack does not reduce the effective metal section in proportion to the thickness, but irrespective of section." Simple arithmetic thus shows that with the thinner sections of material with higher unit strength, the relative effect of corrosion is greater since a $\frac{1}{16}$-in. reduction in thickness due to rusting of a $\frac{3}{8}$-in. plate means a loss of 16.7 per cent. The same loss from a similar $\frac{1}{2}$-in. ordinary steel plate is equal to a section loss of only 12.5 per cent. With total strengths in direct proportion to the thicknesses, the high-strength steel would thus suffer more. If, however, the thickness was governed by other factors, such as column action, stiffness, etc., the effect might be more, or, in exceptional cases, less serious than indicated by these values.

Mr. Aston illustrates substantially the same condition (see heading, "Low-Alloy Structural Steel") and suggests a design solution in his analysis by means of a "unit corrosion-merit" value, his statement being, "in effect, therefore, in the selection of structural materials, the benefit to be derived, as far as it relates to corrosion, will be only to the extent that the corrosion-merit ratio exceeds the strength ratio for the metals under consideration."

Incidentally, Mr. Karpov's statement (see "Application to Actual Designs: Safety Factors and Working Stress") that, "in all engineering structures the factor of safety decreases as time goes on", will apply to the case of structures weakened by corrosion. Part of the spread between the design stress and the elastic limit is assumed to take care of this feature along with secondary stress, increased loading, etc. Hence, in choosing a design stress, the corrosive resistance of a new alloy steel should be known. In the past only too often paint maintenance has been assumed by the designer, but failed to come up to expectations. Rusted stringer flanges and connection angles, corroded to a point that makes support of only the dead load appear miraculous, are not uncommon sights. A classic example is that of a steel plate girder bridge that carried traffic practically to the time of its replacement when the deteriorated web could be punched through by a single blow of an ordinary sledge hammer. Truly, corrosive resistance will have to be specified along with ultimate strengths as alloys of greater and greater strength are put on the market.

The esthetic possibilities suggested by the alloy steels are not to be overlooked. Many American structures appear less graceful than similar European designs, at least in past constructions. The attitude in the United States has inclined toward the liberal use of materials and the limited use of the designers' time, whereas European economy has dictated the need of ample design time to analyze and produce a light and materials-saving structure. Perhaps with the double incentives of lighter but costlier material, the clumsy appearance of many "railroad-type" bridges will gradually disappear from future highways. A deterring factor, however, is that, in general, the longer structures which have received the necessary close design scrutiny have attained greater esthetic beauty than the intermediate or short spans; but it is in the former and not in the latter classes that the high-strength steels will find their greatest usefulness. There has been no excuse for bulky structures in the long-span field where an extra pound of dead load weight at the bridge center would require as much as 5 lb of added metal to carry it. Certainly, if the use of alloy steels should extend to the shorter spans, many of the wasteful "standard designs" would be eliminated by the more costly material.

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[161] Exchange Teacher in Civ. Eng., Worcester Polytechnic Inst., Worcester, Mass.

Arshag G. Solakian,[162] Esq. (by letter).—Dealing with several phases of photo-elasticity (such as history, application, limitations, methods, apparatus, model materials, etc) the valuable paper of Mr. Brahtz is most timely. It reflects the wide interest shown in the last decade, in America as well as abroad, in this interesting optical method of measuring stress intensities in transparent models. The writer's discussion is intended simply to supplement certain parts of Mr. Brahtz's paper, and he has preserved the order of representation used by the author.

It is true that Sir David Brewster has been universally recognized as the founder of photo-elasticity, because of his discovery in 1816 of the stress-optical phenomenon known as double refraction of polarized light in transparent isotropic materials under stress. It was Erasmus Bartholinus, however, who, in 1669, first observed the phenomenon of polarization by the double refraction of ordinary light in its passage through a uni-axial crystal, such as Iceland spar, a material used extensively for polarizing prisms in polariscopes. It is evident that without polarized light, Brewster would have not found it possible to observe the double-refraction effect in a stressed plate.

In America, the pioneers in the art of photo-elasticity have been Carus A. C. Willson[163] and the late Charles Lee Crandall, M. Am. Soc. C. E., and Anson Marston,[164] Past-President, Am. Soc. C. E., but it has been due to the numerous valuable researches and publications of Mesnager, in France, and Filon and Coker, in England, that photo-elasticity has become a practical and useful method for the analysis of stress distributions in machines and structures of a two-dimensional nature.

Mr. Brahtz refers to a paper[165] by the late A. H. Beyer, M. Am. Soc. C. E., and the writer, in connection with the possibility of stress analysis in reinforced concrete through composite models, made of bakelite and cast with metal rods of circular cross-section embedded. As a necessary improvement on this type of test, the circular-section reinforcement may be replaced advantageously by flat strips, having a width equal to the thickness of the model. This modification will distribute the metal uniformly across the entire thickness and also will eliminate any three-dimensional stress effect on particles in the immediate vicinity of such rods as were formerly proposed. It may be of interest to add, in this connection, that a photo-elastic material of preferably liquid form (such as marblette,[166] for example) that can be handled easily under laboratory conditions as a matrix for the models, and also affords good adhesion with the metal inserts and develops the least amount of initial strain effects after being cast, will be of great help to investigators in work of this nature.

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[162] Lecturer in Civ. Eng., Columbia Univ., New York, N. Y.

[163] "The Influence of Surface Loading on the Flexure of Beams", *Philosophical Magazine*, December, 1891.

[164] "Friction Rollers", *Transactions*. Am. Soc. C. E., Vol. XXXII (1894), p. 99.

[165] "Photo-Elastic Analysis of Stresses in Composite Materials", *Transactions*. Am. Soc. C. E., Vol. 99 (1934), p. 1196.

Mr. Brahtz emphasizes the necessity for developing a cement that can be used to combine models of the same or different elastic properties, thus opening a wide field of research in a great many problems of practical importance to engineers, such as shrinkage stresses in dams, welded connections, built-up structural sections, space frames with rigid joints, etc. It should be noted that such a cement, made of liquid marblette and hydrochloric acid and setting under room-temperature conditions, has been used by the writer in the qualitative analysis of shrinkage stresses in dams, as reported in a discussion[167] of a paper by Howard G. Smits, Jun. Am. Soc. C. E.

A special type of polariscope (with aluminum-coated mirror reflectors instead of lenses) has been advocated for instruments producing a large field of parallel rays, with the minimum space and lowest cost as governing factors. The writer doubts that such instruments with mirror reflectors will command much popularity, especially in view of the recent development of

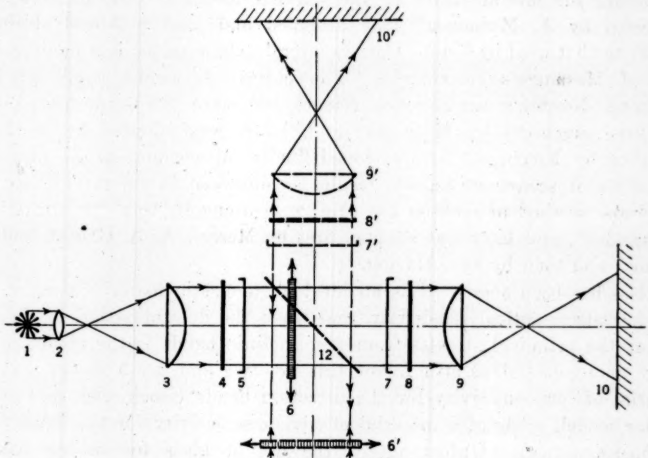

FIG. 52.—UNIVERSAL POLARISCOPE.

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[166] "A New Photo-Elastic Material," by A. G. Solakian, *Mechanical Engineering*, December, 1935.

[167] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 937.

large-sized and moderately priced artificial polarizing media in the United States. A polariscope having a field of 10-in. parallel rays, for example, has been designed recently by the writer, using polarizing glass plates both for the polarizer and analyzer, with a total length (not including the screen) of about 6 ft. By the addition of a glass plate (at 45° with the axis of the instrument) and a reflection mirror, with the second half of the instrument turned 90° (as shown by the dotted-line arrangement in Fig. 52), this "transmission" type of polariscope is easily transformed into the "reflection" type, which latter is of extreme importance in a great many specific cases that could not be handled by the transmission type of polariscope.

Referring to Fig. 52, when the apparatus is set up as a transmission type of polariscope, the light source is at Point 1; Point 2 indicates the condenser lens; Points 3 and 9 the lenses for parallel light; Point 4 is the polarizer; Points 5 and 7 are quarter-wave plates; Point 6 is the model; Point 8 is the analyzer; and Point 10, the screen. In the converted set-up (the reflection type), Points 1 to 5, inclusive, are the same as before; Point 12 is a glass plate; Point 6′ is the model; Point 11, the mirror, and Points 7′ to 10,′ inclusive, are the same as in the transmission type, except that they are turned through 90 degrees.

The idea of evaluating the sum of the principal stresses at a point, by measuring the lateral strain of the stressed model at that point was first suggested by A. Mesnager[168] who designed and used a lateral strain-gage similar to that used by Vose. Coker's lateral extensometer is a more practical type of Mesnager's instrument. The optical flat-model combination for obtaining Newton's interference fringes, to serve the same purpose, was also first suggested by Mesnager, and, later, was adopted by V. Tesar[169] and then by Frocht, to be abandoned finally on account of the mechanical difficulties of securing accurate results encountered in its application. The membrane method of making the same measurements was first suggested by L. Prandtl[170], and later was adopted first by Messrs. A. A. Griffith and G. I. Taylor[171] and then by Den Hartog.

Glass has been suggested as an ideal photo-elastic material because of its very low stress-optical sensitivity, and when the determination of the direction of the principal stresses from the isoclinic bands is the object in view. Many photo-elasticians still follow this practice and use a model made of a material of low sensitivity for the direction bands (isoclinics), and another, similar model, made of a material of higher sensitivity for the stress fringes (monochromatics). Unfortunately, the use of glass for models has been found to be impractical, primarily because of the difficulty of machining it into intricate shapes and also because it is extremely fragile under stresses of moderate intensity. The writer has recently found that a commercial synthetic plastic, having a stress-optical sensitivity only 1.25 times greater than that of glass, with a transparency rivaling that of glass, and possessing none of the disadvantages of the latter, can very advantageously replace the use of glass in models.

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[168] "Sur une Methode pour déterminer a l'avance les tensions qui se produisent dans une Construction", *Comptes Rendus* November, 1912.

[169] "Photo-Elasticite", *Revue d'Optique theorique et instrumentale,* Tome II, 1932.

[170] *Physicalischer Zeitschrift,* Vol. 4, 1903.

[171] Technical Rept., Advisory Committee on Aeronautics, Vol. 3, 1917–1918.

Considering the problem of determining stresses in models of machines and structures of the three-dimensional type, it has been suggested the previously silvered surface of a strip of photo-elastic material be cemented against the metallic surface of the prototype considered. By using polarized light reflected through and back from this cemented strip, the surface stresses (which, in general, are the largest of all across a given cross-section) can be determined from the isoclinics and monochromatics, in the usual manner. This idea was first suggested by Mesnager.[172] In 1936 the writer cited[167] the adaptability of liquid marblette as a coating on polished metallic surfaces. Successful results along this line have been thus far obtained only with very thin coatings, not of sufficient thickness to produce a fringe pattern under stresses of low intensity. Although such stresses can be measured by one of the compensation methods known to photo-elasticians (such as color matching, quartz-wedge compensators, Coker's tension bar, etc), these methods do not possess the ease and simplicity of the fringe-pattern method now universally in use. By additional coatings on the original one, the thickness of the skin layer can be increased to any desired amount, but the non-uniformity of the layer thus deposited may be objectionable. With the marblette liquid cement the writer has been able to produce a satisfactory bond between the surface of a metallic strip and a silvered surface of a piece of $\frac{1}{8}$-in. marblette. As this cement sets under room temperature conditions, initial strain effects in the cemented photo-elastic material are eliminated. Fig. 53 is a fringe pattern of stress in a model thus prepared, under a uniform bending moment applied to the metallic strip.

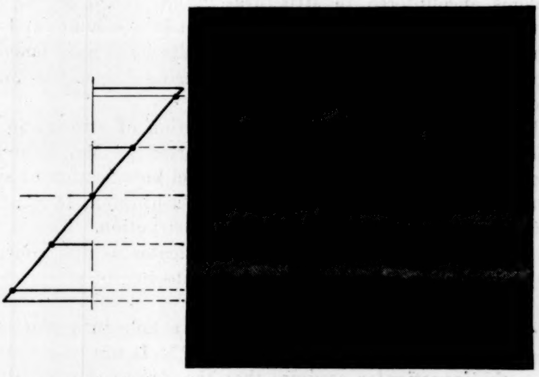

FIG. 53.—SURFACE STRESSES IN METALLIC STRUCTURES AND MACHINES FROM A STRIP OF PHOTO-ELASTIC MATERIAL CEMENTED TO THE PROTOTYPE.

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[172] "Sur la détermination optique des tensions intérieures dans les solides à trois dimensions", *Comptes Rendus*, 1930, p. 190.

Finally, it is interesting to note that the interferometer (or rather the interferometer-polariscope) as designed and built by the engineers of the U. S. Bureau of Reclamation is similar in principle as well as in construction to that first designed by H. Favre,[173] in Switzerland. A brief translation in English of Favre's paper concerning the theory and use of this instrument was made by the writer[174] in 1932. An interferometer-polariscope of the Favre type was developed by the writer for the Photo-Elastic Laboratory of Columbia University. Similar instruments are reported to have been constructed for McGill University and the University of Buenos Aires.

H. D. Hussey,[175] **M. Am. Soc. C. E.** (by letter).—Structural engineering, as it involves the modern alloy steels, has developed to a highly technical state. The papers in this Symposium emphasize this fact. The use of these steels requires a better understanding of the phenomena of fatigue, stress concentrations at holes and fillets, buckling of plates, etc., as well as a better knowledge of the materials themselves. The writer wishes to emphasize the statements by the various authors concerning some of these factors.

Mr. Karpov gives a general outline of certain stress phenomena that are receiving much study, including an exposition of the theory of fatigue. He states that stability problems are less understood than problems of stress and that the tendency is to use higher factors of safety with respect to stability as compared to stress. It should be emphasized, however, that many stability problems encountered in structural design (such as the buckling of web-plates in a girder, plates under compression in a column, and outstanding flanges) have been solved,[176] and that the results have been embodied in the Specifications of the American Railway Engineering Association for Steel Railway Bridges (1935).

A problem of much importance is the solution of stresses in rigid-frame knee-sections. The over-sized model test reported by Mr. Templin is interesting as corroborating tests on a structural steel knee-section of similar shape made at the National Bureau of Standards, Washington, D. C., in collaboration with the American Institute of Steel Construction. The stress diagram on Line *A-A'*, Fig. 14, across the corner, indicates a high concentration of stress on the inside, or curved, face and the absence of stress at the outside corner.

The problem of stress concentration about a hole in a plate has received much study, both theoretical and experimental.[177] It must be observed that in such studies the investigator assumes that the stresses are within the elastic range at all times. The phenomenon of stress concentration about a hole in a plate, when the theoretical stress exceeds the yield point, has not been explored so widely. Theory indicates that the stress at the edge of a hole in a wide plate may be three times the average stress on the net section. If the average stress is one-half the yield point the stress condition adjacent to the hole is shown in Fig. 54. (By "theoretical stress" is meant the stress that would result if proportionality of stress and strain continued to any limit.) Since the maximum stress cannot be much greater than the yield point there is some re-adjustment of stress due to yielding of the material adjacent to the hole. Due to this yielding there results a small permanent set on release of the load. Repetition of this stress condition produces no further yielding, but a sufficiently large number of repetitions will lead to failure by fatigue.

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[173] "Sur une Nouvelle Methode optique", *Revue d'optique theorique et instrumentale*, No. 8, 1929.

[174] "New Developments in Photo-Elasticity", *Journal*, Optical Soc. of America, May, 1932.

[175] Designing Engr., Am. Bridge Co., New York, N. Y.

[176] "Elastic Stability of Plates," by Otis E. Hovey, M. Am. Soc. C. E., *Proceedings*, 36th Annual Convention, Am. Ry. Eng. Assoc., Vol. 36, 1935, p. 715.

[177] "Theory of Elasticity", by S. Timoshenko, 1935, p. 75, and "Stress Concentration Produced by Holes and Notches", by A. M. Wahl and R. Beeuwkes, Jr., *Transactions*, Am. Soc. Mech. Engrs., Vol. 56, 1934, Paper APM-56-11.

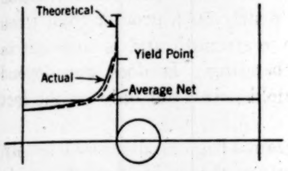

FIG. 54.—STRESS CONDITIONS ADJACENT TO A HOLE IN A PLATE.

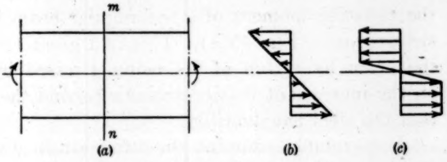

FIG. 55.

The problem of riveted joints and stress concentration at rivet holes has received the consideration of several of the authors. A better understanding of this problem depends, among other factors, upon a more thorough knowledge of the theory of plasticity.

A problem of much importance is the question of proper, or permissible, ratio of yield point to tensile strength. Mr. Moisseiff calls attention to the fact that several of the new alloy steels have a high ratio. In the field of bridges and buildings, it has been considered that this ratio should not exceed 70 per cent. It seems appropriate to inquire whether this, or any, limitation should be applied to ductile materials such as those under discussion.

The reason for a low ratio of yield point to tensile strength is to provide a reserve of strength against high concentrations of stress at joints, holes, fillets, etc. It is apparent at once that this reserve strength cannot provide against an increase of direct stress, either tension or compression, beyond the yield point, because, being the point of transition from an elastic to a plastic state, this is the useful limit of any member under those stresses. This reserve strength, therefore, can operate only under conditions of bending stresses beyond the yield point.

A study of stresses beyond the yield point shows that the bending resistance of a beam continues to increase after the stress in the outer fiber has reached the yield point, depending upon the make-up of the beam. For a rectangular section this increase may be as high as 50 per cent.[178] Fig. 55(*b*) shows the stress diagram on Section *m*-*n* of the beam in Fig. 55(*a*) when stressed to the yield point. The stress diagram of a beam stressed beyond the yield point can be represented with sufficient accuracy by Fig. 55(*c*). The deformation increases with the increase in bending moment on the beam, but the stress does not increase much beyond the yield point. When the total deformation is in the order of about ten times the yield-point deformation the resisting moment of a rectangular beam is nearly 50% greater than when stressed as in Fig. 55(*b*). This is a great reserve strength and is inherent in the beam by virtue of its being stressed by bending. It does not depend on the increase of tensile strength beyond the yield point, but only on the fact that the steel has ductility.

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[178] "Strength of Materials", by S. Timoshenko, Pt. I 1930, p. 237.

For example, examine the stress-strain diagram (Fig. 2) discussed by Mr. Karpov. When strains beyond the yield point exist, the amount of the strain is influenced very little by the shape of the stress-strain curve. It is apparent that the relief of stress concentrations is caused by the yielding of the material at the over-stressed parts and not by the numerical value of the tensile strength. Indeed, were it not for plastic elongation of the material there would be no relief. In such cases the important characteristic of the stress-strain curve is the amount of elongation rather than the increase in stress beyond the yield point. This leads to the conclusion that the physical properties of greatest importance are yield point and ductility (as measured by elongation and reduction of area), and that the tensile strength is secondary to these. Thus, if it is possible to produce a satisfactory ductility, the ratio of yield point to tensile strength is of minor importance.

The writer welcomes the papers by metallurgists in Section II of the Symposium. The paper by Messrs. Bain and Llewellyn is of interest to the structural engineer because of the explanation of the metallurgy of alloying elements. Their discussion of "Interpretation of Specimen Tensile Tests" is worthy of note. Such discussion, coming from those with a different point of view, should be of value to the engineer in appraising, correctly, the materials he uses.

RALPH FREEMAN,[179] M. AM. SOC. C. E. (by letter).—The evolution of high strength steels in structural engineering in Great Britain has led to the adoption of qualities of steel differing materially from those used in the United States, as recorded in the paper by Mr. Moisseiff. Nickel steel has not been used to any significant extent for bridges or for other structures designed in Great Britain, no doubt because no bridge has been constructed having a span long enough to require the use of a superior quality of steel, obtainable only at a material increase of cost.

As stated by Mr. Moisseiff (see heading, "History of Silicon Steel"), "silicon" steel was first produced in 1907 for the construction of part of the upper structure of the *Mauretania* and data regarding this steel and the silicon steel in the *Lusitania* have been reported by Sir Robert Hadfield.[180]

The plates rolled were of large dimensions (to 38 ft in length, 8 ft in width, and 1¼ in. in thickness), requiring ingots as heavy as 16 tons.

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[179] Managing Partner (Sir Douglas Fox & Partners) Westminster, London, S. W. 1, England.

[180] "Metallurgy and Its Influence on Modern Progress", by Sir Robert Hadfield.

The plates were also subjected to the most severe tests in order to prove their resistance to sudden shock and violent stress beyond the yield point. In one series of these tests plates, 6 ft by 3 ft by 1 in. thick, were placed on supports 4 ft apart and tested by dropping a 3-ton steel ball on them from heights increased successively to 31 ft. Presumably, this was the maximum practicable value since the plates did not crack although they buckled.

In spite of the apparent successful application of this steel for shipbuilding, it does not appear to have been used again for this purpose and was not adopted for bridge building until the construction of the Sydney Harbor Bridge during the period, 1923–1928.

When preparing the designs for the Sydney Harbor Bridge, the writer considered the advisability of using various alternative qualities of steel, from mild steel (with a yield stress of about 15 tons per sq in.) to alloys having a yield point of 50 tons per sq in. The conclusion was reached that no high-class alloy steel effected a saving in weight which justified the extra expense but that "silicon" steel, having a yield stress of 20 tons per sq in., or one-third higher than that of mild steel, would effect a substantial reduction of cost. This steel complied with the following analysis:

	Percentage
Carbon	0.32 to 0.42
Manganese	0.60 to 1.00
Silicon	0.15 to 0.25

The rolled material included plates as wide as 8.25 ft and 2.25 in. thick, weighing 10 tons, and angles of 12 in. by 12 in. by 1.25 in. The permitted working stresses were 30% higher in tension and 25% higher in compression. The cost of the raw material was about 25% in excess of that of mild steel. No special difficulties of fabrication were experienced and the cost of fabrication was little more than that of mild steel. Nearly 40 000 tons of this steel were used for the Sydney Harbor Bridge, but as far as the writer is aware, "silicon" steel has not been used by British engineers for any other structural work.

This circumstance, no doubt, arose from the fact that at the time of the construction of Sydney Harbor Bridge, attention was being given in Great Britain to the production of high tensile steel superior to "silicon" steel, especially for shipbuilding, and led to the production of various alloys containing small percentages of manganese and chromium which could be produced at a cost very little more than that of silicon steel and would have distinctly superior physical properties.

These high-strength tensile steels are now extensively used for shipbuilding, bridge construction, and other structural steelwork. Steel used for the latter purpose is defined by a British Standard Specification which declares the physical properties of the steel, but does not state the chemical analysis. The yield point must be higher than 23 tons per sq in. for sections or plates not exceeding 1.25 in. in thickness, reduced to 22 tons for thicknesses up to 1.75 in. Actual manufactured plates and sections of the kind in most general use (that is, of thicknesses as great as 1 in.), have a yield stress of 24 tons per sq in. The breaking strength is specified to be between 37 and 43 tons per sq in. The quantity of steel of this character produced in recent years in Great Britain exceeds 300 000 tons.

The steel of this quality which has been produced most extensively is a chromium-copper-manganese alloy complying with the following composition:

	Percentage
Carbon (maximum)	0.30
Chromium	0.70 to 1.1
Copper	0.25 to 0.50
Manganese	0.70 to 1.0
Silicon	0.20

Another alloy contains 1.0% to 1.5% of manganese, as much as 0.5% of copper, and no chromium. Other types contain smaller proportions of chromium and copper.

High tensile steel plates have a tensile strength transverse to the direction of rolling equal to that in the direction of rolling and sustain the usual cold bend tests (bending through 180° over a diameter equal to three times the thickness of the test piece without cracking).

The yield stress of high tensile steel of the foregoing type is approximately 50% higher than that of British mild steel. The permissible stresses allowed for this quality of steel are usually taken as 50% in excess of those allowed for mild steel.

The raw material cost of British "high tensile" steel can be taken roughly as 25% in excess of that for mild steel, and fabrication costs are not significantly higher, except as the consequence of the reduced tonnage resulting from the use of a stronger material.

Rivet steel commonly used for structural steelwork has a tensile stress of 26 to 30 tons per sq in., and it has been felt that with rivets of this class full advantage could not be taken of the superior strength of high tensile steel. Rivet steels of improved quality are produced, having tensile strengths (before driving) as high as 30 tons per sq in., and a strength in shear in high tensile plates exceeding that of mild steel rivets in mild steel plates by about 50 per cent. These rivets have not yet been used extensively.

Modern "evolution" of high strength steels when all is said appears to be anything but evolution when one achievement is recalled which occurred in an age now apparently forgotten in its own country. The steel used for the arch ribs of the Eads Bridge over the Mississippi River, at St. Louis, Mo. (completed in 1874), is not only one of the most astonishing features of this wonderful bridge, but an example of quality in a structural material that has probably never been surpassed.

In 1881, Mr. C. N. Woodward recorded[181] the fact that the steel staves that formed the ribs of the arches, were of chromium alloy which rolled very well. This was in 1872. The steel actually used is said to have had a tensile strength of 120 000 lb per sq in. Tests made in 1869 for the late James B. Eads, M. Am. Soc. C. E., apparently of the steel proposed for the bridge, indicated a superior quality. One series of these tests, made at Messrs. Kirkaldy's Testing House, in London, in 1869, of chrome steel bars, showed that the limit of elasticity in tension had an average value of 82 000 lb per sq in., the corresponding average ultimate strength being 155 000 lb per sq in.!

The writer subscribes entirely to Mr. Moisseiff's views regarding the advantages and limitations of high strength steel. He has found that high tensile steel is a source of great economy for long spans and has the effect of greatly increasing the economic span. It is also economical for small spans where freight costs are high. For a light road bridge recently built in Rhodesia, crossing a river, 1 000 ft wide, with difficult foundation conditions, it was found to be advantageous to use a single arch span of high tensile steel with no river piers. The use of such steel made this type of bridge practicable. Had mild steel been used the bridge would have been of a different type (girder spans on piers) and would have been far more costly.

Another bridge carrying a precisely similar roadway and having the same span is now (1937) about to be built, but in a locality where the freight cost will be about £19 per ton. In this case, the suspension type of bridge with high-tensile steel stiffening trusses is the least costly the economy arising to a large extent in consequence of its smaller total weight and the reduction of freight costs.

The saving of freight costs to distant parts of the British Empire, from Great Britain, by using high tensile steel, is also in many cases the means of affecting considerable savings. For sites where the total freight may be as much as £19 per ton, this will be readily appreciated.

In using high tensile steel for bridge structures the writer has paid particular attention to reducing to a minimum the number of different sections to be used, and finds that when this aspect of design is carefully studied the small number of sections which is really essential is somewhat surprising. With a class of steel which is not in general use this is important; otherwise, costs may be greatly increased.

The possible combination of normal and high tensile steels is frequently advantageous and is made readily practicable by the fortunate, but somewhat remarkable, fact that for all steels used for structural purposes the modulus of elasticity is the same. As far as the writer has been able to ascertain, there is no more variation between values for the modulus of elasticity for different kinds of steel than there is for different specimens of the same kind. This uniformity of the modulus permits the interconnection of various classes of steel without risk of indeterminate stresses which would be caused by different moduli of elasticity.

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[181] "A History of the St. Louis Bridge", by C. N. Woodward, pub. by G. I. Jones & Co., 1881.

A. V. KARPOV,[182] M. AM. SOC. C. E. (by letter).—The purpose of the Symposium is exceptionally well defined in the seven terse questions given in the "Foreword" by Mr. Jones. The papers and the discussions give at least partial answers to each of these questions, although they show also that, at the present stage of engineering development, and in all probability for many years to come, no final answer can be given on many of these problems. Engineering development forges ahead, and it seems that no better goal can be accepted by the Structural Division of the Society than to keep up to date the valuable data collected in this Symposium.

The most fundamental conclusion that can be drawn from the Symposium is the impossibility of limiting the field of advanced structural engineering to stationary structures of fundamental character. The broader viewpoint is necessary, expanding into border fields of structural design which are commonly assigned to mechanical engineering.

The determination of actual stresses in structural elements is leading into fields which are different from conventional design practice. New methods of attack are developing both in the theoretical and experimental handling of structural problems. Fatigue, impact, and creep test are supplementing the work of the ordinary metal testing laboratories; small and large scale models are being used extensively; indirect methods of strain and stress measurements have been added to the direct strain measuring methods that were used in the past; and the vibrator test is being used in the laboratory as well as in the field.

This very extensive theoretical and experimental work, which at present is being conducted in a large number of laboratories in the United States, seems to suffer from lack of co-ordination and insufficient exchange of information concerning the work proposed or being done. The different testing programs in the field of structural research that were mentioned in the Symposium, suggest the necessity for steps to be taken to remedy these conditions. The Society, which by its very nature is interested in promotion of engineering knowledge without paying undue attention to the commercial aspects, is the most logical organization which could undertake successfully such a co-ordination and co-operation and could bring closer together the different educational and industrial organizations working in the field of structural research. Such a co-operation would result in a properly planned and coordinated research program, eliminating unnecessary overlapping and wasteful repetition of work.

Part II of the Symposium brought out forcefully the fact that structural engineering is no longer a single-metal field. As in all fields of engineering, materials must be chosen, depending upon the suitability for each particular design. The requirements are becoming so exact that it does not seem probable that in the near future a universal alloy will be introduced that can take the place that open-hearth mild steel took in the past.

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[182] Chairman, Committee of the Structural Division, on Fundamentals Controlling Structural Design; Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

In line with the papers and discussions of the Symposium there seem to be at present three outstanding problems that should be solved:

First.—A more suitable definition of terms that have a clear meaning in their application to ordinary structural steel, but which lose this clearness if applied to high-grade steel alloys or to light-weight alloys. More appropriate definitions of ultimate strength, yield point strength, and related physical properties of materials should be developed. The particular and most pressing objective would be the proper definition of yield point strength and its relation to other fundamental properties.

Second.—The problem, made clear in the Symposium, concerns the necessity of introducing additional characteristics of structural materials. The time-honored coefficients such as ultimate strength, yield point, and elongation are still the basic coefficients that are necessary to form engineering judgment as to the suitability of a material; but they are obviously insufficient: Fatigue, creep, impact, corrosion, and welding properties must also be considered. To give a practical definition of these properties and to outline the extent to which they should be used in structural designs is an important field that should be properly covered.

Third.—Welding is rapidly growing in importance in structural designs. The problem of weldability of alloys is encountered more in the field of metallurgy; but the manner in which welds are made is becoming a problem in structural engineering. The researches that were and are being made have disclosed the importance of fatigue considerations in welded joints. The remarkable increase in fatigue strength that may be obtained by changes in the design of welded joints, the comparison with riveted joints, and the determination of conditions under which different kinds of joints should be considered, are fields which undoubtedly will occupy the attention of the structural designer.

Part III must be considered in light of the theoretical, Part I, and the metallurgical, Part II. "The picture" that can be gained from Part III, gradually emerging from a single material stage and developing into multiple material designs, can be taken as a guide to an understanding of the development trend in structural engineering. The self-sufficiency of the structural designer of the past is gone, probably forever. The best handbooks available cannot keep a structural designer abreast of the latest practice. A correct design may result only from the correlated work of the theoretical designer, structural laboratory engineer, and metallurgist. If the structural designer wishes to maintain a leading part in this work, which naturally is his, he must possess the insight and ability to understand the problems of all the professions involved, as was evidenced to such marked degree by the author of one of the papers in Part III.

Part IV is the logical conclusion of the entire Symposium. Light-weight designs are encountered in the most recent and most modern field. The importance of this field is not often realized. Many such designs cannot be rationalized due to lack of engineering knowledge. The stability problems, which were more or less neglected in the past, are becoming of the same importance as the stress problems. Corrosion, expected life span of a structure, fatigue and creep properties, and stress concentrations, are all problems whose importance is considerably accentuated in these designs. It is interesting to note that these problems are of particular importance in two structural fields which are as far apart as can be imagined—in aircraft and in long span bridges. The modifications of standard design formulas that were proposed during this session of the Symposium may be accepted as a clear indication of the necessity to consider the difference in properties of new alloys, as compared with structural steel. Different treatment is necessary to obtain the entire advantage of the light-weight design.

A useful field of endeavor would be to watch the trend of light-weight designs and to bring to the attention of the structural designer the most important and practical developments.

LEON S. MOISSEIFF,[139] M. AM. SOC. C. E. (by letter).—The valuable papers presented in the Symposium and the many discussions it has brought forth testify to the timeliness of the undertaking. The Society, through its Structural Division, has succeeded, at an opportune time in gathering and presenting to the Engineering Profession the knowledge, the information, and the experience of engineers active in the making of structural metals and in fabricating them, in the planning of structures, and in their design, erection, and utilization. The Society, moreover, has been fortunate in securing a group of papers which has proved interesting enough to provoke a many-sided and illuminating discussion by men of information and reflection.

The authors of the Symposium wisely did not attempt to offer complicated mathematical analyses and subtle discussions of the various phases of structures built of metal. Instead, they have presented to the profession the thoughts which have matured in their minds during many years of experience, in making and forming of metal and in designing and building structures. The Symposium has been successful in producing a representative cross-section of current American engineering thought. A comprehensive study of the papers reveals the present trend of structural development.

The Symposium has brought forth a symphony of many instruments. The low tones of the heavy metals and the high ones of the light metals unite in one sustained theme. They announce that to create structures for the wants and comforts of the many, such structures must be efficient and economical. They must be built of materials which possess suitable qualities to fulfill the special requirements and functions, and they must be workable, dependable, uniform, and enduring in order to justify the trust placed in them. It is the constant business of producers and fabricators to search and test the products of the mills and shops, and to establish the capacities and limitations of these products to sustain and endure the attacking forces and agents in space and time.

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[139] Cons. Engr., New York, N. Y.

These capacities must be utilized further by the planners and designers to the best advantage so as to produce efficient and economical structures. It is the task of engineers, therefore, to strive to approach the fullest utilization of their materials in strength and endurance, in economy and comfort. Only then when the fullest knowledge of materials and structures will be gained will this aim have been attained. To acquire this knowledge it devolves upon engineers to study the true behavior of structures under the action of forces by analysis and strain measurements on models, by tests in laboratories, and by observations in the field.

The Symposium shows that engineers are realizing that the elementary approach in analyzing structures was well enough for the smaller dimensions where cheaper materials could be used and some of it could be wasted, but that for large structures the most suitable material is demanded and almost all of it should be utilized. Engineers realize that a deeper insight into the behavior of structures is required and that much of it can be attained by modern methods of analysis and search, and they feel that the application of such knowledge is imperative in the design of important structures. It must not be forgotten that the admission of higher unit stresses and larger strains demand more care in analysis and proportioning and that all advance in this direction is based on the mental probity of the engineers.

A. V. KARPOV,[184] M. AM. SOC. C. E. (by letter).—The purpose of the paper was to bring to the attention of the structural designer the fact that the theories on which structural designs are based by no means can be considered as perfect. These theories are based not only on approximations but in many instances on assumptions that are contrary to present knowledge. The result is that instead of developing theoretically correct methods, structural research is leaning toward a larger use of empirical and not correlated coefficients. The developments of fatigue data and stress concentration factors are probably good illustrations of this thought.

Professor Mirabelli's discussion gives an interesting method of a more satisfactory correlation of the experimentally determined stress-strain relation and the actual behavior of structural elements. Such investigations should be conducted extensively. Mr. Horger raises a number of most important questions in his short discussion. The boundary conditions and their influence on stress distributions is a problem that requires considerable additional study. If the influence of boundary conditions is accepted then there should be a different stress distribution within the thickness of the material and at the boundary. Figs. 3(*a*) and 9 are indicative of the most probable change of stress distribution at the boundaries for specimens subjected to tension-compression and bending, respectively. The additional data with reference to fatigue strength, size effect, and stress concentrations are not yet available and, therefore, the last question asked by Mr. Horger can not be answered at present.

The writer does not agree with Mr. Glover's conclusions that the investigators of the past gave all the necessary information that can be further developed. Theories based on incorrect assumptions cannot be satisfactorily developed beyond very narrow limits. His work on trial load arch dams and supporting rings for penstocks, mentioned in his discussion, would seem to be of very considerable interest.

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[184] Designing Engr., Hydr. Dept., Aluminum Co. of America, Pittsburgh, Pa.

R. L. Templin,[185] M. Am. Soc. C. E. (by letter).—Not only has Professor Timby amplified many points raised in the paper but he has also indicated some additional uses of models. In commenting on the merits that he has emphasized, which obtain when using stainless steel models fabricated by "shot" or spot welding, the writer wishes to state that he has used numerous spot-welded aluminum alloy models of structural units with very satisfactory results. Although it is scarcely probable that any one individual would be familiar with all the recent developments in model testing and analysis, it is apparent that Professor Timby has assembled information on a group of model studies which covers the apparent range of application quite thoroughly. The examples cited both in the paper and in the discussion, represent considerable evidence to show the increasing extent of the use of models in various engineering fields.

Many improvements have been made on strain-gages since the time the Howard gage (mentioned by Mr. Belzner) was introduced. Nevertheless, as was stated in the paper, there is still considerable of the personal equation involved in the manipulation of the modern strain-gage and this presents a factor which must be considered in obtaining and interpreting strain-gage results.

The model girder referred to by Dr. Arnstein would seem to offer some interesting advantages in certain model studies.

A problem very similar to that mentioned by Professor Ruge was encountered in the model of the Santeetlah Pipe Line.[14] The load was applied by gravity and although the model could be scaled down photographically, the result was that the stresses were also reduced photographically. Consequently, it was not sufficient simply to rely upon the model test directly, but to use it to substantiate the theoretical development for this type of loading. The example suggested by Professor Ruge appears to be likewise of this type and, therefore, an excellent one to illustrate the point raised. The gusset-plate problem that he has mentioned will not give exact similitude because the modulus of elasticity for the model would not be reduced from that of the prototype in the same proportion as the scale ratio.

Professor Klemin's reference to full-sized structural tests of aircraft is an excellent example of a case in which the load conditions on the actual structure are so indefinite as to make model testing relatively unreliable, except in the study of component parts of the structures.

J. H. A. Brahtz,[186] Esq. (by letter).—In closing this paper, the writer wishes to express his appreciation for the many interesting discussions that have appeared. These discussions have brought out many points of interest and valuable details which were not included, or were insufficiently elaborated, in the paper. In reviewing the discussions briefly, these will be taken up in the order in which they appeared.

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[185] Chf. Engr. of Tests, Aluminum Co. of America, New Kensington, Pa.

[186] Director, Photo-Elastic Laboratory, U. S. Bureau of Reclamation, Denver, Colo.

In his excellent discussion, Mr. Vose emphasizes the difficulties in producing a truly two-dimensional state of stress. It is a fact that it never really occurs. In order to have a plane state of stress the plane plate $(x\,y)$ should be infinitely thin, and in the case of plane strain the plate should be infinitely thick in order to comply completely with the definitions of those states, in which the stresses σ_x, σ_y, and τ_{xy}, are independent of the elastic constant of the material. In photo-elastic experimentation, of course, a small finite model thickness is necessary. Consequently, the average stresses over the thickness of the model are measured. It is true that these stresses will be effected to some extent by the lateral stresses near discontinuities on the boundaries. On the other hand, the same effect occurs in the prototype so that the deviations between experimental and actual stresses in this respect are practically only caused by the ratios of the elastic properties in the two materials. This is one of the many reasons why the maximum errors of experimentation in such regions cannot be expected to be less than 10% and often may be even greater.

Mr. Vose calls attention to the convenient portable reflecting polariscope designed at the Massachusetts Institute of Technology, which should be of great help in the type of observations mentioned in his discussion.

In a very lucid discussion Professor Rathbun describes a number of experiments with riveted connections. The very ingenious technique of multi-materials of varying optical sensitivity no doubt will prove extremely useful in the future. The aforementioned three-dimensional difficulties now become acute. It was this problem that the writer had in mind when stating in the paper that the elastic properties of the model materials should be such as to imitate as closely as possible the conditions in the prototype. Due to plastic flow, the writer has had difficulties in calibrating the highly sensitive materials, accurately. Professor Rathbun's discussion brings out clearly the truth of the statement that "photo-elasticity has proved its value, is here to stay, and new apparatus, model materials, and technique are being developed continuously."

In his very valuable discussion Professor Timby explains clearly the three contributing reasons for an adequate safety factor. The writer is in full agreement with these viewpoints. The live loads, including wind and temperature effects, are seldom accurately known and can only be assumed for many types of structures, as for example, bridges, buildings, ships, etc. In such structures as dams these effects are defined more clearly, but many other uncertainties enter, among which may be mentioned earthquake effects and uncertainties in the quasi-elastic properties of the materials in the dam proper relative to those of the foundation. The photo-elastic laboratory of the U. S. Bureau of Reclamation has proved extremely useful in connection with designs of hyperstatic structures (for example, large drum-gates) in obtaining the most economical sections.

The interesting discussion by Mr. Solakian deals to a large extent with the history of the fundamental physical phenomena, which enables present-day photo-elastic experimentation. He argues that the polariscope designed and constructed by the writer at the California Institute of Technology and having the large field lenses replaced by aluminum-coated parabolic or spherical mirrors, will be less popular since the introduction of new "polarizing media." These polarizing glass plates will successfully replace the far more expensive Nicol prisms. The writer fails to see the point since it is the high cost of high-grade lenses *versus* the comparatively low cost of high-grade reflectors that matters.

In conclusion, the writer can state that the new interferometer of the Favre type which was recently designed and constructed by the engineers of the U. S. Bureau of Reclamation, has proved highly satisfactory in its application. The polariscope is now being used only in: (1) Such experiments, in which merely the boundary stresses are needed; (2) for preliminary design studies in order to obtain an optimum section, which is then analyzed in detail by the interferometer; and (3) for checking the principal stress differences obtained in the interferometer.

The interferometer differs from the instrument described by Favre in several respects. For example, it gives both principal stresses and their direction in one operation, whereas Favre obtains the direction and magnitude of the principal stresses in separate instruments.

M. J. R. MORRIS,[187] ESQ. (by letter).—Referring to Mr. Herzog's discussion, the writer agrees that he has not gone into detail as far as possible, as it was the primary intention to cover as large a field in general as possible; his comments, therefore, are very well taken. There is no doubt that the addition of alloying elements, such as molybdenum and columbium, will open up new fields to the stainless alloys, and will help greatly in the actual fabrication of some of these alloys by making them more easily handled and possibly more nearly "fool-proof." As time goes on and improvements follow experimental work, these alloys will come into more and more prominence and broader use with the civil engineer just as they have already been a boon to chemical and industrial engineers in general.

Mr. Gisiger cites the application of stainless bars in connection with trash screws protecting the turbine intakes against floating logs, trash, etc., of the Holtwood Plant, of the Pennsylvania Water and Power Company, at Holtwood, Pa. This is an excellent example of the reduction in weight made possible through the use of corrosion-resistant stainless steel over structural carbon steel; also, the increased efficiency provided by decreasing the area covered by the bars, themselves, which obstruct the passage of the water.

In connection with welding these stainless alloy bars to structural carbon steel, the writer has found that, in general, a heavy, flux-coated alloy electrode will give the most satisfactory weld.

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[187] Chf. Metallurgical Engr., Central Alloy Dist., Republic Steel Corporation, Massillon, Ohio.

If mild steel electrodes are used, the weld metal (which will have a smaller cross-section than the structural steel) will be subject to any localized corrosion that may be set up as a result of the stainless alloy being in contact with mild steel. If the weld, itself, is stainless, then the corrosion will be transferred to the heavy, structural shape, which will be much more capable of distributing and minimizing its effect so as to be of very little consequence.

In laying a stainless weld on mild steel, it is best to deposit at least two beads on top of each other to eliminate the effect of pollution from the steel. The lower bead fused to the steel will be less corrosion resistant than the record on subsequent beads. Such applications well illustrate previous statements as to the broad use of stainless alloys in the hydraulic field.

Zay Jeffries,[188] **Esq., C. F. Nagel, Jr.,**[189] **Esq., and R. T. Wood,**[190] **Esq.** (by letter).—In commenting on the writers' paper, Mr. Hobrock has given emphasis to the desirability of consulting the manufacturers of materials before making final selection of a particular alloy. Quite correctly, he points out that this is not a feature peculiar to aluminum, or to any one metal, but applies to all. However, it probably applies less to those older metals for which many years of use in a particular service have built up a present satisfactory practice by the process of "cut and try," the soundness of which has been reasonably well proved or at least accepted. In passing, it might not be out of order to suggest that following "tradition" carries with it certain dangers—namely, what was quite adequate yesterday may be inadequate tomorrow—if not to-day. Engineers should not necessarily select alloys or materials to-day because they were satisfactory a few years ago.

Materials suppliers are constantly making progress, both in the greater uniformity of the properties of materials, and in the development of improved products possessing superior qualities, range of sizes and forms, and closer tolerances. Furthermore, the manufacturer is constantly enlarging his knowledge regarding the properties of those materials.

It is the latter point that Mr. Hobrock has emphasized. Some danger, from a commercial angle, may lurk in the possibility that a purchasing agent, becoming aware of the existence of characteristics other than those commonly mentioned in purchase specifications, may unnecessarily include specifications covering non-pertinent properties. Nevertheless, as Mr. Hobrock properly states, the satisfactory performance of a structure may not depend upon the more commonly published properties, but possibly upon other characteristics such as, for example, the strength at certain elevated temperatures. Although these other fields of properties have not been fully charted, a substantial beginning has been made, and the knowledge available should be put to use. This should not result in every writer of materials specifications attempting to state and define the limits of every conceivable characteristic (that would be utterly unwarranted and abusive of human intelligence), but it does mean that the engineer should ferret out to just what actions his particular structure will be subjected, and then ascertain from the manufacturer what information is available as to the properties of his product with respect to those conditions. One should not expect that the desired "road map" will be forthcoming in all details, but in many cases numerous useful "land marks" can be shown.

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[188] With Incandescent Lamp Dept., General Electric Co., Nela Park, Cleveland, Ohio.
[189] Chf. Metallurgist, Fabricating Div., Aluminum Co. of America, Pittsburgh, Pa.
[190] Chf. Metallurgist, American Magnesium Corporation, Cleveland, Ohio.

Mr. Hobrock's contribution to this subject of "consulting the supplier" deals mainly with the uncommonly published properties of alloys. This is important. The writers' main reason for presenting this thought was somewhat different, although not at all contradictory. Progress is continually being made in the perfection of range of dimensions, both as to maximum sizes and tolerances and, more important, in knowing just what can and cannot be accomplished from an economical viewpoint. This feature is of special importance in the field of castings, although it is not confined to that field.

Slight changes in the detail design of a casting, for example, or a more complete understanding on the part of the foundryman as to how the product is to be used, may permit of a more favorable method of gating, which, in turn, may lower the cost of the product, or may permit the production of a casting in which the metal at critical locations will possess more favorable characteristics.

An alloy is defined by its composition alone. The properties of Alloy *A*, for example, may well be different when in the form of a thin sheet, as compared with a rolled shape. The processes by which the original ingot is converted into various wrought shapes may differ widely and, hence, the final characteristics may differ. Furthermore, the characteristics of the ingot (caused by the method of manufacture and the dimensions) may vary depending upon the particular wrought product for which it is intended. The structures of those different ingots are not all alike, and the effect of this original ingot difference may affect the final characteristics and, hence, one should not casually refer to "properties of the alloy" when considering the design of a structure but, rather, should think in terms of the properties of the particular type of material concerned. Many published tables fail to reveal whether the properties stated apply to sheet alone, to forgings alone, or to rolled shapes, etc. When using any table for design purposes, therefore, one should know to what particular type of product those properties apply.

JAMES ASTON,[191] ESQ. (by letter).—The logic of Mr. Clapp's suggestions will be admitted by all who are actively interested in the corrosion problem. Engineers are concerned primarily with the serviceability of materials, and a systematic study which would evaluate their utility in particular types of service, or would enable one to apportion properly the effects of service factors, would be invaluable in enabling a more correct appraisal and use of metals to be made. Service data have been gathered in considerable amounts, largely by individual efforts of competing manufacturers, with release prompted largely by the biased discretion of the compiling interest.

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[191] Metallurgist, Pittsburgh, Pa.

Actually, the task is of great complexity and magnitude. One must have the several metals in indisputably parallel conditions of service; they must remain undisturbed throughout their service life; and, particularly, interest and records must be kept alive through the long life span of many metals in many services. In corrosion testing, the temptation is to accelerate the test, or to attempt to control conditions in a laboratory set-up. With such procedure, unbalancing of the several factors involved will probably seriously distort the results as compared with what might appear to be similar conditions in actual service. Furthermore, results from a controlled or specific type of service are applicable only to that condition, and cannot be interpreted more broadly. Finally, failure of material is the only true end point of a test, with the criterion of failure dependent upon the commodity involved and the service requirements.

By way of illustration, one may cite the sheet metal tests conducted by the Corrosion Committee of the American Society for Testing Materials. In this field, one finds a co-operating personnel of outstanding qualifications. The tests were carefully planned, were carried out on a comprehensive scale, and were painstakingly observed and recorded. The contribution to knowledge has been of great value; yet 1937 sees the passing of the twentieth anniversary of the tests, with a clearing up, at best, of only a few points related to atmospheric exposure of bare sheet steel. There is still almost as much controversy as in the beginning due to disputed points in the conduct of the tests; or a doubt regarding the practical value of tests of bare sheets when actual utilization requires paint or metallic protection; and particularly because in a twenty-year period many real or fancied advances have been made in the production of steel, with all the uncertainties and arguments formerly prevailing with respect to their relative merits.

E. C. HARTMANN,[192] ASSOC. M. AM. SOC. C. E. (by letter).—Many of those who have discussed the writer's paper have commented on various phases of shop practice. In his excellent description of the design and construction of the emergency bulkheads for the Gallipolis Dam, Mr. deLuccia emphasizes the need for great care in workmanship. He raises a doubt as to whether the average shop practice for structural steel is suited for aluminum alloys. Mr. Christianson also emphasizes the need for careful shop work, but states that aluminum alloys present no new problems to the shop experienced in producing a high quality of workmanship in other metals. Mr. Lehman states that with proper instructions one can rely on the shop to handle the material properly so that no parts are damaged. All these viewpoints may be summarized in the term,

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[192] Research Engr., Aluminum Co. of America, New Kensington, Pa.

"good shop practice". In the writer's experience with fabrication methods in various types of construction, he has yet to find the shop which, when properly instructed, cannot produce a first-class aluminum alloy structure.

The question of riveting aluminum has received some attention by the discussers. Mr. Lehman describes the use of hot-driven steel rivets in aluminum structures and Mr. deLuccia describes cold-driven steel rivets, which is a newer development. To complete this picture a number of examples of the use of aluminum alloy rivets could be mentioned as, for instance the truss-type, traveling cranes of 76-ft spans built in 1930 and fabricated throughout with hot-driven 0.75-in. aluminum alloy rivets.

Professor Hunsaker has questioned the advisability of using steel rivets in aluminum structures because of the possibility of corrosion at the junction of the two metals in cases where the paint protection is not adequate. The possibility of such corrosion cannot be denied, and because of this possibility steel-riveted aluminum-alloy structures must receive adequate paint protection. On the other hand, tests have shown that unpainted full-sized structural joints, involving hot-driven steel rivets, subjected to sea-coast conditions for two years retain their original strength. It should be remembered in this connection that visible surface corrosion can occur without appreciable reduction of strength, particularly in structural shapes and plates which have the advantage of greater bulk compared to the thinner materials used in aircraft.

Both Mr. deLuccia and Mr. Growdon have commented on the necessity for thinking in terms of aluminum when designing structures of aluminum, especially with reference to the question of deflection. The writer was particularly pleased with this phase of Mr. deLuccia's description of the bulkheads of the Gallipolis Dam. It is quite evident that in these structures arbitrary limitations based on conventional practice were not permitted to interfere with the logical development of the design of a light-weight structure to serve a specific purpose. Aluminum structures, of course, can be designed without this approach to the problem, but maximum economy and minimum weight are rarely attained without it.

Mr. Sturm's discussion presents a refreshing and thought-provoking concept of the relation of bulk of metal to stability. Stability problems are almost certain to be encountered more frequently in light-weight construction than in conventional construction, whether the weight savings are accomplished by the use of light-weight, low-modulus materials, or by the use of thin sections of the heavier materials. The question of bulk of metal is important in both instances. In the structures made up of thin sections of the heavier metals the reduction in bulk compared to conventional construction is largely responsible for the stability problems encountered; and in light-weight materials the relatively greater bulk is advantageous in overcoming the effect of lower modulus.

Professor Plummer has suggested a more complete treatment of the economic factors which influence the selection of materials for light-weight construction. In this connection this writer was pleased to note the comments of Mr. Lehman, Mr. Growdon, and Mr. Christianson, indicating that in three fields as widely separated as construction equipment, bridges, and railway cars, problems in economics arising in connection with specific applications of aluminum alloys have been solved satisfactorily. Mr. Lehman's interesting statement that, "the selling of such a benefit is often more difficult than the proof of economy," no doubt will be echoed with feeling by all who have been associated with the commercial development of the newer materials.

Two of the discussers have described aluminum alloys not specifically mentioned in the paper. As pointed out in the paper, there are several such alloys available, and the selection of the best one for a specific purpose involves a consideration of many factors. The day has passed when each engineer could expect to be familiar enough with all available materials to make his selections entirely on the basis of his own experience. Any logical modern approach to the problem of selection of material should certainly include consultation with the producers of the materials.

MEMOIRS OF DECEASED MEMBERS

ONWARD BATES, Hon. M. and Past-President, Am. Soc. C. E.[1]

DIED APRIL 4, 1936

Onward Bates was born February 24, 1850, in St. Charles County, Missouri, the eldest son of Judge Barton and Caroline Matilda (Hatcher) Bates. The family home was known as Cheneaux Farm and here his early youth was spent. He attended the local schools, but considerable of his early education was directed by his parents, and also by his distinguished grandfather, the Hon. Edward Bates.

On March 1, 1865, Onward, at the age of 15, entered the Fulton Iron Works, at St. Louis, Mo., as an Apprentice to learn the machinist trade. Edward Bates recorded in his diary this event as follows:

> "Feby. 28 tuesday morning — Comes my grand-son, Onward Bates who arrived in town last night, with his aunt Sarah — Julian's wife. He comes, by previous arrangement of his father (Barton Bates), to enter himself Apprentice to Gerard B. Allen (Allen & Filley) to learn the trade of a Machinist. He is a very ingenuous youth, of good, solid parts: Not brilliant, but uncommonly heedful and apt to learn, and retentive of all he learns He is well advanced, for a boy of 15 — Has read more than most lads (perhaps without much method) — Is, as I hear, pretty well versed in Geography, Grammar, Arithmetic and Algebra — Knows a little Latin, and perhaps a smattering of German. In temper, he is docile and tractable, and in manners, respectful, orderly and moral — Yet of a firm and independent character of mind.
>
> "I have high hopes that he will avail himself of all his opportunities, and make himself a man of whom his near kindred may be proud
>
> "He will live with us — that is, spend his nights and spare time at our house. And so, we hope to be able to exercise a fair degree of care over his habits and associations."

Before the completion of the apprenticeship, the late Charles Shaler Smith, M. Am. Soc. C. E., who was building a bridge over the Missouri River, at St. Charles, Mo., engaged Mr. Bates as Draftsman and Inspector. This was the beginning of his engineering work. After about a year he accepted the position of General Assistant to Captain William S. Nelson, who had a contract for the caissons for the piers of the St. Louis Bridge.

In 1871, Mr. Bates entered Rensselaer Polytechnic Institute, at Troy, N. Y., as a Special Student. He attended the Institute for two years, until 1873, when he was again employed by Colonel Smith as Assistant Engineer and Inspector on the East Approach of the St. Louis Bridge and other work.

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[1] Memoir prepared by a Committee consisting of T. L. Condron, Edgar S. Nethercut, and W. A. Rogers, Members, Am. Soc. C. E.

The late Don Juan Whittemore, Past-President and Hon. M. Am. Soc. C. E., has commented[2] on this employment as follows:

> "Col. Smith was at that time President of the Baltimore Bridge Co., which has the contract for this east approach. While inspecting this work at the shops of the Detroit Bridge & Iron Works, he was appointed by Capt. Eads, the Chief Engineer of the St. Louis Bridge, as shop inspector for the bridge company. He thus served on the same work in the double capacity of inspector for the contractor and for his client as well. While this work was being manufactured, Mr. Willard S. Pope, President of the Detroit Bridge & Iron Works, thought there should be an adjustment of the schedule of prices paid his company by the Baltimore Bridge Co. for the metal work, and upon taking this question up with Col. Smith the latter instructed Mr. Bates to arrange with Mr. Pope a new schedule of prices which would be fair to both parties. The circumstances of the manufacture of this metal work are worth recording as showing, at that period, the confidence and respect existing between engineers, when one inspector could serve both parties and when the contract prices could be disregarded and readjusted to secure equity."

Upon the completion of this work later in 1874, Mr. Bates entered the employ of the Cincinnati Southern Railway Company as Draftsman under the late Louis Gustave Frederic Bouscaren, M. Am. Soc. C. E., Principal Assistant to the late Thomas Davis Lovett, M. Am. Soc. C. E., Chief Engineer. The late Charles Louis Strobel, Hon. M. Am. Soc. C. E., was Mr. Bates' immediate superior in the office and in charge of the design of the bridge over the Ohio River. Later, Mr. Bates was promoted by Mr. Bouscaren to be Inspector of Iron Bridges and Trestles, and he continued in this capacity until June, 1877.

He then became Inspector of Iron Bridges for the Chicago, Milwaukee and St. Paul Railway Company, under Mr. Whittemore, Chief Engineer, with headquarters at Milwaukee, Wis.

In March, 1878, Mr. Bates was sent to Australia by Colonel Smith and Mr. William Sellers, President of the Edge Moor Iron Company, of Wilmington, Del., as their representative. He remained in Australia for more than three years, returning to the United States in August, 1881. In Australia, he designed and built several iron bridges and viaducts.

In the United States, Mr. Bates was again employed with Colonel Smith and, later, became President of the Pittsburgh Bridge Company. From March, 1884, until April, 1886, he was again associated with Colonel Smith as Assistant in his practice of Consulting Engineer. For one year, from April, 1886, to April, 1887, he was engaged in mining in Mexico, following which he was with the Edge Moor Iron Works, at Wilmington, Del., until February, 1888.

Mr. Bates, at this time, returned to the Chicago, Milwaukee and St. Paul Railway Company as Engineer and Superintendent of Bridges and Buildings. For thirteen years, he remained with this Company, until March, 1901, when he organized the Bates and Rogers Construction Company. He withdrew from this Company in 1907, selling his interest to his associates, and sought to retire from active engineering. On important occasions he was chosen as

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[2] *Engineering News*, February 11, 1909.

Arbitrator on engineering and contracting matters for which service his training, judgment, and fairness conspicuously qualified him.

Although he did not complete his college work, he was on many occasions honored by degrees. In 1897, the University of Wisconsin, at Madison, Wis., conferred on him the Honorary Degree of Civil Engineer. In 1918, he received the Honorary Degree of Doctor of Engineering from Rensselaer Polytechnic Institute, and, in 1924, the degree of Doctor of Laws from the University of Missouri, at Columbia, Mo.

He represented the Society on the John Fritz Medal Board of Award for four years—from 1910 to 1913—and, in 1911, he was the President of that Board. In 1918, Mr. Bates was appointed Chairman of the Committee on Development of the American Society of Civil Engineers which Committee was discharged in 1919. In this position, he showed remarkable ability and fairness. He was a Member and Honorary Member of the Western Society of Engineers and served as its President in 1899.

Mr. Bates was a member of the Institution of Civil Engineers of Great Britain; the Franklin Institute; the Royal Society of Arts (England); the University Club of Chicago, Ill.; the Chicago Engineers Club; the Theta Xi Fraternity; and for many years, was a Trustee of the Chicago Bureau of Public Efficiency.

He contributed many papers and discussions to the publications of the Society and to those of the Western Society of Engineers. Among the latter should be mentioned a paper on "Fundamentals",[3] presented in 1911, for which he received the Octave Chanute Medal, and also a paper on "Arbitration"[4] presented in 1912. He wrote and presented addresses before many engineering colleges, including Purdue University, and the Universities of Illinois and Wisconsin. His subjects for these addresses were related to citizenship and character. While with the Chicago, Milwaukee and St. Paul Railway Company, he called frequent conferences of his Staff and his addresses on these occasions were intimate and inspiring.

During his retirement he wrote delightfully regarding his family, for private circulation, and these booklets included "Bates, *et al* of Virginia and Missouri", "A Good Name", and "Onward and Onward: A Romance of Four Continents." There was a charm in his writing which was most evident in his personal correspondence which he continued until a short time before his death.

While the engineering accomplishments of Onward Bates were conspicuous and important, he will be remembered because of his outstanding character. By his devotion to the principles of his religion, he became the friend and counselor of all who came in contact with him. He sought to do well by every one and gave of his ripe judgment to establish right and justice in his dealings, as well as in his engineering practice.

In 1892, Mr. Bates was married to Virginia Castleman Breckinridge, of St. Louis, Mo. Mrs. Bates survives him. He had no children to bear his name, but it is with pride that those, who either worked under his direction or were associated with him in his societies, claim to have received from him inspiration for better engineering and life. Mr. Bates always evidenced a great desire to be helpful, by counsel and inspiration, to the young men with whom he came into contact.

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[3] *Journal*, Western Soc. of Engrs., Vol. XVI, November, 1911, p. 769.

[4] *Loc. cit.*, Vol. XVII, June, 1912, p. 481.

He was a member of the Presbyterian Church which he joined in 1866, and which he served as an Elder for many years. His high ideals were inspired by his Christian experiences and his love of the good name which he inherited and which he maintained honorably through his entire life. He has left behind him a memory of inspiration, counsel, example, and love. The profession has profited by his life.

Mr. Bates was elected a Member of the American Society of Civil Engineers on January 4, 1882, and an Honorary Member on October 15, 1923. He served as Vice-President of the Society in 1906 and 1907 and as President in 1909.

MILO SMITH KETCHUM, Hon. M. Am. Soc. C. E.[1]

DIED DECEMBER 19, 1934

Milo Smith Ketchum was born on January 26, 1872, on a farm three miles north of Elmwood, Ill., the son of Smith Ketchum and Ann (Clement) Ketchum. He was the ninth in direct descent from Edward Ketchum who came from England to Ipswich, Mass., in 1634. The history of the Ketchum family is typical of many families which go to make up the history of America. The children and the children's children of Edward spread to Connecticut, Long Island, New York, and 1800 found them near Cayuga, N. Y. While many of them remained in the United States after the Revolution, some of them were loyalists and moved to Canada, settling in the Provinces of Ontario and New Brunswick.

The branch of the family to which Milo Smith Ketchum belonged went to Crawford County, Ohio, in 1822, and settled near Bucyrus, on the northern edge of Sandusky plains in Whetstone Township. Other New England families to whom the possession of land was a great ambition were settled in this vicinity. His father, Smith Ketchum, was born near Bucyrus, Ohio, in 1840, and went to Illinois in 1852. He was one of a family of six tall men, all of whom were more than 6 ft in height. The lack of educational opportunities was no measure of his education. He attended an academy near his home where he studied the common branches of knowledge, ancient history, and algebra, and thus gained the foundation for two striking abilities he had later in life, that of an excellent public speaker and conversationalist. For more than forty years Smith Ketchum was a Minister in the Primitive Baptist Church, preaching without compensation and ministering to the people of his community who made up that brotherhood. He was a powerful and formative influence in the life of his son, as well as on the lives of those about him. He had a fine and discriminating feeling for the dignity of personality. Self-determination and States' rights were a part of his family tradition which went back to a great-grandmother who was a descendant of Roger Williams. This feeling led to his interest and sympathy for the Confederacy during the Civil War.

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[1] Memoir prepared by W. C. Huntington, M. Am. Soc. C. E.

Milo Smith Ketchum grew up in the companionship of his father and in the tradition of freedom. From him during the long days on the farm, he learned to know intimately the wild life of the plains. This love of the open and the soil was never lost, for during the whole of his mature life Dean Ketchum owned and directed several large farms in Colorado and Kansas.

The country schools of Illinois provided the elementary education of Milo Smith Ketchum. These schools were ungraded, so that it was possible to advance in any subject as fast as the student was able to progress. His interest from the beginning was in mathematics and history. He finished the regular nine-term course in the Elmwood, Ill., High School, in five terms, and took a teacher's examination in Peoria, Ill. The following years were occupied in the task of teaching country schools. During this time he took the examination for the United States Military Academy, at West Point, N. Y., but the appointment went elsewhere and this was an end to his interest in military affairs.

His great interest in mathematics led him to enter the University of Illinois, in 1891, where he chose Civil Engineering because that course contained more of his favorite study than any of the other divisions. His college days were filled with concentrated study in the field of his choice.

During the summer of 1893, Mr. Ketchum taught surveying at the Michigan School of Mines, at Houghton, Mich., and it was here that he was first initiated into the problems of mine structures. From these, and from later experiences in the Coeur d'Alene, in Idaho, he drew the material for his book on "Mine Structures" (1912).

In June, 1895, he received the degree of Bachelor of Science in Civil Engineering, with the distinction of being the Valedictorian of his Class. The two following years were spent as Assistant in Civil Engineering and Mathematics at the University of Illinois. Since there was little or no opportunity for graduate work in Civil Engineering at that time, he turned his activities to the career of a practicing engineer. In 1900, he was awarded the professional degree of Civil Engineer by his Alma Mater.

During the period of nine years immediately following his graduation from the University of Illinois, he served as Instructor and Assistant Professor of that institution; as Structural Engineer with the Gillette-Herzog Manufacturing Company, in Montana and Idaho, during the period of labor troubles shortly after the Bunker Hill-Sullivan catastrophe; and as Contracting Manager at Kansas City, Mo., for the American Bridge Company. His period of service with the American Bridge Company, although short, meant much to him. In this organization he became acquainted with many men whose friendship he valued very highly throughout his life. He was glad to be known as an "American Bridge Company man".

In 1904, Professor Ketchum was appointed Head of the Department of Civil Engineering at the University of Colorado, at Boulder, Colo., and a year later was also made Dean of the College of Engineering. The next fifteen years were among the happiest of his life. Throwing himself into the numerous and varied duties of building an outstanding institution from a very small beginning, he made many life-long friends among his students and colleagues. In this position, he had a marked influence on the lives of many of the men who came in contact with him in their early manhood and who prized his friendship highly in later years. The success of many of his former students can be attributed quite largely to their association with him, and not a few of them chose careers in college teaching because of the inspiration received from him.

In 1919, Dean Ketchum resigned his position at the University of Colorado to become Director of Civil Engineering at the University of Pennsylvania, at Philadelphia, Pa., and, in 1922, he joined the Staff of the University of Illinois, at Urbana, Ill., as Dean of the College of Engineering and Director of the Engineering Experiment Station. Here he enjoyed his associations with many of his friends of earlier days and with other members of the Staff who worked with him in advancing the already high standards of these organizations. "'Alertness and a spirit of co-operation throughout the staff were outstanding accomplishments of Dean Ketchum's leadership". Because of ill-health he resigned as Dean and Director at the College of Engineering at the University of Illinois in 1933, and was appointed Research Professor of Civil Engineering. In this capacity he again took up the investigation of pressures in deep bins, the subject on which he had written his Bachelor's thesis and which had continued to interest him. In September, 1934, he retired from the active Staff, but he remained in Urbana, and his affiliation with the University as Dean Emeritus continued until his career was ended.

Dean Ketchum's first book, "The Surveying Manual," which appeared in 1900, was written jointly with W. D. Pence, M. Am. Soc. C. E., then a member of the Staff at the University of Illinois, and was the outgrowth of his early teaching experience in surveying. His next book, "Steel Mill Buildings," was developed at the request of the publishers from a paper presented by him in 1902 before the Illinois Society of Engineers and Surveyors, later known as the Illinois Society of Engineers. Dean Ketchum wrote the remaining four books of this remarkable series while he was at the University of Colorado, serving as Dean and teaching a full-time schedule in the Civil Engineering Department. "Walls, Bins, and Grain Elevators" appeared in 1907; "Highway Bridges" in 1908; "Mine Structures" in 1912; and the "Structural Engineers Handbook" in 1914. From that date much of his time was required in keeping his many books abreast of the times. The total sales of Dean Ketchum's books have been considerably more than 100 000 volumes. His books should continue to exert a strong influence on structural practice under the direction of his competent and well-trained son, Milo Smith Ketchum, Jr., Jun. Am. Soc. C. E.

Although primarily a teacher, college administrator, and author, Dean Ketchum devoted quite a portion of his time to his consulting practice. Much of this work was done along with other duties, but on two occasions he was on leave of absence from the University of Colorado. During the years, 1909 and 1910, he joined H. S. Crocker, Past-President, Am. Soc. C. E., to carry on an extensive engineering practice under the firm name of Crocker and Ketchum, with headquarters in Denver, Colo. During the World War, Dean Ketchum was assigned the almost impossible task of representing the War Department in the construction of the $70 000 000 smokeless powder plant at Nitro, W. Va. This was a rush job necessarily started with incomplete plans, with many organizations at work, with conflicting interests, and with no clear definition of authority and responsibility at the beginning. Twenty thousand men were at work placing nearly three hundred car loads of materials per day. He performed this task with the greatest credit to himself and to those whom he represented.

Dean Ketchum was always active in the affairs of technical societies and inspired younger men to become interested in such organizations. During his earlier years at the University of Colorado, nearly one-fourth of his income was devoted to attending engineering society meetings. Then, and particularly later, he felt fully repaid for this sacrifice. He was a member of the American Society for Testing Materials, the American Railway Engineering Association, the American Concrete Institute, the Western Society of Engineers, the Illinois Society of Engineers, the Society for the Promotion of Engineering Education, Sigma Xi, and Tau Beta Pi. He was a national honorary member of Chi Epsilon Civil Engineering Fraternity.

Honors came to Dean Ketchum from many sources. He was elected Secretary and President of the Society for the Promotion of Engineering Education; Director (1918–1920) and Vice-President (1925–1926), of the American Society of Civil Engineers, and shortly before his death he was elected an Honorary Member of that Society, an honor which comes to few. He was awarded the honorary degree of Doctor of Science in 1926 by the Colorado School of Mines, and, in 1927, by the University of Colorado.

From the humble beginning of a country school-master, Dean Ketchum rose to be one of the nation's foremost educators. To train men for life by means of engineering rather than to teach them engineering was his philosophy.

As an author he won distinction through his many outstanding books. Having combined teaching and practice in his own experience and showing rare judgment in the selection and preparation of material and skill in its presentation, he produced books which were pioneers in their respective fields and which were of service alike to student and practicing engineer.

As an engineer, his services were often sought as a consultant in structural design, as an arbitrator in controversies arising out of construction operations, and as an expert on patent litigation.

He was frank and definite in stating his position on matters that were referred to him. He valued highly the friends which he had in all parts of the United States and in all walks of life. In his last address given to a group of civil engineering students just four days before his death, he spoke feelingly of the meaning of friendship.

He was an intensely human man, large in stature and intellect, with strong and upright character, dynamic personality, and rare judgment who seemed destined to lead. He had the capacity for making quick decisions which would stand the test of deliberate analysis, and he never wavered in the face of difficulty or opposition but was inspired by them.

Dean Ketchum's life was rich in accomplishment and his influence will continue through his many books, through the organizations in the development of which he played an important part, and through the many men who are stronger because of their association with him.

He was married on September 17, 1903, to Mary Esther Beatty, of Newton, Iowa, and is survived by his widow and three children, Martha Esther (Mrs. N. C. Debevoise), Elizabeth Jane, and Milo Smith Ketchum, Jr.

Dean Ketchum was elected an Associate Member of the American Society of Civil Engineers on September 4, 1901; a Member on March 3, 1908; and an Honorary Member on October 2, 1934.

CHARLES LOUIS STROBEL, Hon. M. Am. Soc. C. E.

Died April 4, 1936

Charles Louis Strobel was born in Cincinnati, Ohio, on October 6, 1852, the son of Karl and Ida L. (Merker) Strobel. His early education was received in the public schools of Cincinnati, which he left in 1869 to enter the Royal Institute of Technology, in Stuttgart, Germany, from which he was graduated in 1873 with the degree of Civil Engineer.

It seems significant that there is no record of any scientific analysis of stresses in framed structures prior to the Sixteenth Century, nor any similar record from that time until about 1840. Construction was carried on by the "fit-and-try" method and successful attempts were recorded and duplicated while others were improved or discarded. Essentially all the fundamental development in the use of iron and steel in framed structures occurred during the 50-yr period succeeding 1840.

Following the early discovery of the scientific mathematical principles of design, and during this development period, there lived a group of engineers who, by their initiative, developed not only the scientific principles of construction, but also the means of carrying on the work. Charles Louis Strobel was one of this group.

From 1874 to 1878, he was Assistant to the late L. G. F. Bouscaren, M. Am. Soc. C. E., Chief Engineer, Cincinnati Southern Railway Company (now a part of the Southern Railway Company). Mr. Strobel, being in

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[1] Memoir prepared by the following Committee: Albert F. Reichmann, *Chairman*, Edward Haupt, and George T. Horton, Members, Am. Soc. C. E., for publication jointly by the American Society of Civil Engineers and the Western Society of Engineers.

charge of the design of bridges and viaducts, found it necessary to analyze competitive designs submitted by competing bridge companies. Two of the notable structures designed and built under his supervision were the Ohio River Bridge, in Cincinnati, with one span of 519 ft, and the Kentucky River High Bridge built on the cantilever principle. The construction on this railroad was in advance of the times, and Mr. Strobel introduced, probably for the first time in the United States, the method of calculating stresses from definite locomotive-wheel concentration followed by uniform train loads. This method has since become common practice. He further contributed to the development of the scientific design of structures by computing the bearing and shear values of rivets and the bending, as well as shear and bearing values, of pins.

Mr. Strobel was Engineer and Assistant to the President of the Keystone Bridge Company, in Pittsburgh, Pa., from 1878 to 1885, and, from 1885 to 1893, its Consulting Engineer and Agent in Chicago, Ill., as well as Consulting Engineer for Carnegie, Phipps and Company, Limited, the Chicago, Milwaukee and St. Paul Railway Company, Burnham and Root, Architects, and others. While acting as Consulting Engineer for Carnegie, Phipps and Company, Limited, Mr. Strobel standardized rolling-mill practice by designing standard sections for I-beams and channels.

He originated Z-bar columns which were used as towers by the Chicago, Milwaukee and St. Paul Railway Company for a viaduct in connection with the Randolph Bluffs Bridge, over the Missouri River, and many other structures.

The steel work for the Chicago Auditorium, Adler and Sullivan, Architects, was designed under his supervision, the late Edgar Marburg, M. Am. Soc. C. E., assisting in the design. Mr. Strobel supplied the steel to Mr. H. A. Streeter for the Tacoma Building, and also for the Woman's Temple, Home Insurance Building Annex, in Chicago, and many other buildings since replaced in many instances by more modern and larger structures.

Mr. Strobel's connection with the development of steel skeleton construction and numerous railroad and highway bridges over the Missouri, Mississippi, Ohio, and other rivers, is a matter of record.

One of his most notable achievements was the design and construction, in 1890, of the 525-ft span for the Ohio Connecting Railway, over the Ohio River below Pittsburgh. This structure was erected on falsework, 143 ft high, on coal barges and floated into place. The structure weighed 900 tons and the falsework an additional equal amount, making a total of 1 800 tons.

Probably the most important contribution of Mr. Strobel to Engineering Science was the development and publication of a handbook of information on the use of iron and steel in construction. This book was first edited by him in 1881 and was entitled "A Pocket Companion of Useful Information and Tables Appertaining to the Use of Wrought Iron for Engineers, Architects and Builders." Later editions giving the properties of steel sections were prepared and edited by Mr. Strobel in 1884, 1887, 1890, and

1892, and published by Carnegie, Phipps and Company. These early publications were the bases of later editions generally known as the "Carnegie Handbook." Supplemented by similar books of other manufacturers, these handbooks did much to promote the use of steel in construction by supplying a reliable source of information not otherwise available to prospective users of steel.

In 1894, Mr. Strobel entered into a contract with the City of Chicago for the construction of the first Rolling Lift Bascule Bridge, which was designed by the late William Scherzer, M. Am. Soc. C. E. This connection led to his activities in this type of construction and, under his direction and supervision, a number of bascule bridges of various types were designed or constructed. In collaboration with Mr. Theodore Rall, the "Rall" type was successfully developed.

In 1895, Mr. Strobel designed for Mr. James B. York, wide, flanged, I-beam sections, 8, 9, 10, 12, and 15 in. deep, as well as 18, 24, and 30-in. I-beams, all to be rolled in universal mills. The smaller sections were the forerunners of the present column sections and the 30-in. section of the present girder beams. Mr. York failed to interest American manufacturers in his process which later was developed abroad.

Mr. Strobel organized the Strobel Steel Construction Company in 1905 to take over his personal business, and he was active in its management until his retirement in 1926

In 1890, he was married to Henrietta Baxter, of Chicago, who died in 1905. There were two children by this marriage, Charles L. Strobel, Jr., and Marion (Strobel) Mitchell. The latter has attained prominence in the literary world as a poetess and authoress as well as a book reviewer. In 1910, he married Mary Wilkins who, with his two children, survives him.

Mr. Strobel was very much interested in literature, the theater, music, and world affairs and subscribed to many publications. Being possessed of a retentive memory he could discuss intelligently foreign as well as domestic affairs. He was independent in politics, but always interested in electing the best men to office.

He attended services at St. James Protestant Episcopal Church and also those of the Chicago Ethical Society conducted by Dr. Horace J. Bridges.

Mr. Strobel was a man of high integrity and was moderate in all things. His modesty and diffidence at times created the impression of aloofness which did not exist, and on close acquaintance he was most friendly and affable.

Those fortunate enough to be present recall with much pleasure the celebration of his eightieth birthday at the Chicago Engineers Club, when he and his lifelong friend, the late Onward Bates, Past-President and Hon. M. Am. Soc. C. E., were presented with Honorary Membership in the Club.

In the summer of 1934, while on vacation in Santa Barbara, Calif., he suffered an illness which rendered him temporarily blind and although he recovered in a small measure, he severely felt the handicap of not being able to read as before. When able, he continued his attendance at the

Chicago Orchestra Concerts and Sunday Church services. His inability to get around freely and maintain his former friendly contacts caused him much regret.

His passing on April 4, 1936, removed another of that group of truly great pioneer engineers who, by establishing the fundamentals of scientific design and construction, have built the bridges over which the present and future generations may pass.

He was a member of the Western Society of Engineers (Trustee from 1892 to 1895), and a member of the Institution of Civil Engineers of Great Britain, the New York Engineers' Club, the Chicago Club, as well as the Onwentsia, University, Commercial, Casino, and Arts Clubs, of Chicago.

Mr. Strobel was elected a Member of the American Society of Civil Engineers on December 3, 1879, and an Honorary Member on October 3, 1932. He served as a Director of the Society for the term ending 1886, and again in 1894, to fill a vacancy. He was Vice-President in 1911–1912, but later declined the nomination for President as he felt that the office should be filled only by an engineer engaged in purely professional work.

WILLIAM ANDERSON AYCRIGG, M. Am. Soc. C. E.[1]

DIED MAY 30, 1936

Born in Passaic, N. J., on April 26, 1859, William Anderson Aycrigg, the son of Benjamin Bogert Aycrigg and Catherine Elizabeth (Anderson) Aycrigg, received his preliminary education at the D. S. Evertson School, in New York, N. Y. He entered Rensselaer Polytechnic Institute, at Troy, N. Y., in 1880, from which he was graduated in June, 1884, with the degree of Civil Engineer.

During the first months after his graduation, Mr. Aycrigg was employed in the Engineering Department of the City of Omaha, Nebr., and his special work was in connection with the surveying and laying out of new streets in that young but rapidly growing city. The following year found him with the Union Pacific Railroad Company as Assistant Engineer, with headquarters at Omaha, which position he held for three years.

In 1888, Mr. Aycrigg was with the Birmingham (Ala.) Bridge and Bolt Company, from which he went to the Edge Moor Bridge Works, at Wilmington, Del., with which Company he was Assistant Engineer from 1889 to 1891. He then went to New York City where he opened an office as Consulting Engineer and where he remained until the end of 1898. At that time he accepted the position of Supervising Bridge Engineer with the Union Pacific

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[1] Memoir prepared by Edward W. Aycrigg, Esq., Stamford, Conn.

Railroad Company at Omaha. He was in charge of the replacement of all the wooden trestles with steel bridges from Omaha as far west as the State of Utah. Mr. Aycrigg remained with the Union Pacific Railroad Company until the fall of 1905. He then went to San Francisco, Calif., where he was engaged as Bridge Engineer for the Western Pacific Railroad Company which was building a new railroad line to the coast. Most of his work was the construction of bridges in the Feather River Canyon, through which the roadbed was being laid. He remained with the Western Pacific Railroad Company until the spring of 1908, when he returned to Omaha and opened an office for private practice as a Consulting Engineer.

Two years later he moved to Stamford, Conn., where he continued his practice as a Consulting Engineer for a number of years before finally retiring from business.

Mr. Aycrigg was a Thirty-second Degree Mason, a member of the Chi Phi Fraternity, the Stamford Yacht Club, and the Woodway Country Club. He was a communicant of the Protestant Episcopal Church and, at the time of his death, was a Vestryman at St. John's Episcopal Church, in Stamford. He was also very much interested in the Boy Scouts of America. He had been connected with that organization since 1912, and had received the Silver Beaver as a token of his work. In regard to Mr. Aycrigg's "service to boyhood", the following editorial appeared in the *Stamford Advocate,* on January 15, 1932:

> "Our fellow townsman, William A. Aycrigg, has every reason to be proud of the distinguished service medal awarded to him last Wednesday by the Executive Committee of the Boy Scouts of America, and Stamford shares in the honor due to one of her citizens. The service rendered by him to the boys of Stamford has been, not only distinguished, but has been long.
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> "Mr. Aycrigg early saw the possibilities in the Boy Scout movement and started in as the first Scout leader of Stamford as long ago as 1912. His interest was not a flash in the pan. Through all these years he has been an active supporter of the movement.
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> "The job of being a Scoutmaster is not an easy one. The master has to have tact and patience. Unless he is regular in his own attendance, he can not expect regularity in the attendance of the boys. It is not enough to have the motive of loyalty to the community. The leader in such an enterprise must have real liking for the boys and must have retained, in large measure, his own boy spirit.
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> "Mr. Aycrigg's distinction is that he has been a guide and counsellor of the individual boys and has also been a guide and counsellor of the Boy Scout activities in Stamford. He has been zealous in promoting the formation of other troops and in encouraging the expansion of the work. The distinguished service medal is a symbol of the high value that lies in disinterested work in behalf of the rising youth of any community."

On October 20, 1887, he was married to Jessie Kelsey Wilcox. Mrs. Aycrigg, a daughter, Mrs. Lee Roy Robbins, a son, Edward W. Aycrigg, and seven grandchildren survive him. A son, William Arthur Aycrigg, died in 1910.

Mr. Aycrigg was elected an Associate Member of the American Society of Civil Engineers on May 4, 1892, and a Member on May 4, 1898.

JOHN CAPRON BALCOMB, M. Am. Soc. C. E.[1]

Died November 25, 1936

John Capron Balcomb was born in San Diego, Calif., on March 2, 1878. He was the son of Robert G. and Frances M. (Spencer) Balcomb. Both parents originally came from the East, his father from Salem, Mass., and his mother from Ithaca, N. Y. Soon after John's birth the family moved to Denver, Colo., where he received his education in the public schools and in the Denver High School.

Mr. Balcomb started his career in the engineering field at the age of 20. Between 1898 and 1901 he was engaged in work for the State of New Mexico as Draftsman and Instrumentman on river gaging and canal surveys. In September, 1901, he joined the Engineering Staff of the Santa Fé Central Railway Company (later, the New Mexico Central Railway Company), first as Engineer in charge of thirty miles of grading, and, later, as Chief Draftsman in charge of mapping and designing; he located and surveyed a 75-mile pipe line between Deming, N. Mex., and El Paso, Tex., and did all the mapping and drafting for this work.

Between October, 1904, and March, 1906, Mr. Balcomb had field charge of main-line reconstruction and bridge work for the Denver and Rio Grande Railroad Company; from April to December, 1906, he was Resident Engineer for the St. Louis, Rocky Mountain and Pacific Railroad Company, at Raton, N. Mex., in charge of grading, bridge work, and track-laying on two residencies.

Mr. Balcomb was with the Chicago, Milwaukee, and St. Paul Railway Company, in Montana, from January, 1907, to May, 1911, as Resident Engineer on heavy main-line construction, and on the location and mapping of town sites, including the field work and mapping of a 12 000-acre irrigation project. From June to August, 1911, he was with the Idaho-Northern Railway Company, in Idaho, as Resident Engineer in charge of heavy mountain construction.

In September, 1911, he left for Brazil where he was employed by the Rio de Janeiro Tramway, Light and Power Company, Limited, in charge of all engineering, including surveys and design and the construction of the 5¼-mile Pirahy Tunnel; he became Chief Engineer of the Company and, for two years, was principally occupied in investigating, and in making reports on, hydro-electric projects.

Mr. Balcomb returned to the United States in July, 1916, and was engaged in contracting work as a member of the firm of George V. Slack and Company, until the end of 1917. In January, 1918, he went as Division Engineer for the Chile Exploration Company, to Chuquicamata, Chile, in charge of work involving heavy excavation, reinforced concrete and structural steel erection for smelters, machine shops, foundries, mill buildings, etc. From August,

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[1] Memoir prepared by Eugene E. Halmos, M. Am. Soc. C. E.

1919, to February, 1922, he was in business for himself as a Contractor on sewer construction in New York City. In March, 1922, he returned to Brazil as Engineer in charge on reclamation and dam work at Ceara, for Dwight P. Robinson and Company.

In March, 1924, Mr. Balcomb joined the Staff of Stevens and Wood, Engineers and Constructors, and remained with that organization for eight years. In this period he constructed three large steam power stations, one at Toronto, Ohio, another at Deep Water, N. J., and the third at Sioux City, Iowa. Between September, 1929, and March, 1931, he was in charge of the construction of the extension of the Spier Falls Hydro-Electric Station on the Hudson River, in New York State. It was during the execution of this difficult undertaking that the writer first had an opportunity to become intimately acquainted with Mr. Balcomb. Between September, 1932, and November, 1933, he was Superintendent in charge of highway construction and the erection of the beautiful three-hinged arch bridge over the Ausable Chasm, in Essex County, New York.

In the early part of 1934, Mr. Balcomb was appointed Surveyor for the Forest Service of the Department of Agriculture, with headquarters at Charleston, S. C. In September, 1934, he joined Parsons, Klapp, Brinckerhoff and Douglas, Consulting Engineers, as Supervising Construction Engineer of the Sutherland Project, on the writer's staff at North Platte, Nebr., and continued in this position until July, 1936, when he was transferred to the Engineering Department of the New York World's Fair of 1939. He became ill at his desk about six weeks prior to his death.

Mr. Balcomb always regretted that, due to lack of money, he was not able to acquire a college education. He spent much of his spare time in studying and educating himself in the theory of engineering and always kept himself well informed of the advances in the profession as recorded in textbooks and periodicals He had a splendid physique, was an indefatigable worker, and was most interested in the execution of large and difficult undertakings, which he was eminently fitted to study from both a practical and a theoretical point of view. He had a remarkable knowledge of construction machinery and particularly of all types of earth-moving equipment; his judgment of the choice of proper construction methods and machinery was infallible. He was very methodical and accurate in keeping costs of construction work and the data thus collected enabled him to make reliable estimates on projected work. He was a kind-hearted but exacting executive, and was respected and liked by both his superiors and by those under him.

In private life he was a splendid family man, a good companion, and a brilliant conversationalist. He was very fond of classical music and never missed an opportunity to hear a worth-while operatic performance or a concert.

Mr. Balcomb was a Mason, a member of Montezuma Lodge, of Santa Fé, N. Mex., a Knight Templar, and a member of the Mystic Shrine; he was also an active Rotarian and lectured at many Rotary Club meetings on technical subjects.

He was married, in 1916, to Helen Pope Robertson, of Charleston, S. C. She and one son, John Duncan Balcomb and two brothers, Kenneth C. and Spencer Balcomb, survive him.

Mr. Balcomb was elected a Member of the American Society of Civil Engineers on January 19, 1920.

ANGUS FRANCIS BARCLAY, M. Am. Soc. C. E.[1]

Died January 31, 1937

Angus Francis Barclay, the son of William and Elizabeth (Anderson) Barclay, was born in New Orleans, La., on January 20, 1879. He was educated in the public schools of New Orleans and the Soule Business College, at that time a semi-technical as well as a business school.

In 1897 and 1898 Mr. Barclay was employed as Rodman on the location and construction of the New Orleans and Northeastern Railroad (later, a part of the Southern Railway System).

He held a position in the fabricating shops of the Berlin Iron Bridge Company, at East Berlin, Conn., in 1899 and 1900. Leaving this position in 1900, he returned to railroad engineering and was employed, successively, by the Illinois Central Railroad Company on line and grade revision work in Tennessee; by the St. Louis and San Francisco Railroad Company on the construction of the Chalmette Terminal, in New Orleans; and by the New Orleans and Northeastern Railroad Company as Locating Engineer.

In 1906, Mr. Barclay accepted a position as Rodman with the New Orleans Public Belt Railroad and was promoted, successively, to Field Engineer, Chief Engineer, and General Manager, which positions he held during the entire time from 1906 until his death, with the exception of a period between 1912 and 1916, during which time he served as Construction Engineer for the Trans-Mississippi Terminal Company (later, a part of the Texas Pacific-Missouri Pacific System) on the construction of its freight and passenger terminals in New Orleans. Fully 90% of the New Orleans Public Belt Railroad was constructed under Mr. Barclay's direct supervision.

In 1926 and 1927, he served as a member of the Advisory Board of Engineers with Ralph Modjeski, the late Daniel E. Moran, and Howard C. Baird, Members, Am. Soc. C. E., which Board was charged with the design of the foundations and superstructure of the Mississippi River Bridge, at New Orleans, and although the construction of the bridge was under the direct supervision of Modjeski, Masters, and Case, Mr. Barclay kept closely in touch with all the details of the construction.

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[1] Memoir prepared by a Committee of the Louisiana Section, consisting of V. J. Bedell, J. A. McNiven, and A. F. Theard, Members, Am. Soc. C. E.

Always interested in the welfare of the Engineering Profession, he was an active member of the Louisiana Engineering Society.

Mr. Barclay enjoyed the respect and admiration of his fellow members of the Society and his associates and the leading citizens of New Orleans, and his outstanding character and integrity reflected high honor upon his profession.

On July 3, 1907, he was married to Kentucky Oliver, of Brookhaven, Miss., who died in 1929. He is survived by three children: Mrs. Allen B. Rogers, William O. Barclay, and Dorothy Barclay.

Mr. Barclay was elected a Member of the American Society of Civil Engineers on **January 16, 1928.**

ALBIN HERMANN BEYER, M. Am. Soc. C. E.[1]

Died April 19, 1936

Albin Hermann Beyer, the son of Hermann Albin and Theckla (Schmidt) Beyer, was born near Freiburg, in Saxony, Germany, on September 19, 1880.

Although the area near Freiburg had been a great mining center in earlier days, the first mining school in the world having been founded there in 1702, the principal industry in the small village where the Beyer family was settled was the manufacture of wooden toys. Hermann Albin Beyer apparently realized that machine methods would displace the older hand methods, even in this field, and was interested in developing this necessary machine equipment. In connection with the manufacture and distribution of these machines, he came to the United States in 1893, bringing his son, Albin, with him. Young Beyer, then thirteen years old, was placed in public school and, later, entered the Boys' High School, in Brooklyn, N. Y., from which he was graduated in 1899, entering the Civil Engineering Course at Columbia University in New York, N. Y., in the fall of the same year.

As a student at Columbia University, Mr. Beyer won high honors and was elected to Tau Beta Pi and Sigma Xi, honorary fraternities. Although he was of a studious type of mind and did not enter organized athletics, he was of strong physical build, took part in many class battles, and was an expert swimmer. He enjoyed in particular the outdoor life and experience at the Summer Surveying Camp, Camp Columbia, near Litchfield, Conn., which has been an important element in the Civil Engineering Course at Columbia for more than fifty years. In fact, upon graduation with the degree of Civil Engineer in June, 1903, he accepted an appointment for the summer as Assistant at the Camp, and also found it possible to return in the summers of 1904 and 1905.

Upon the close of the Camp in 1903, Mr. Beyer was appointed Instructor in Mechanics at Cornell University, at Ithaca, N. Y., holding this position until September, 1905, when he entered active practice in connection with a railroad location survey in Indiana.

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[1] Memoir prepared by J. K. Finch, M. Am. Soc. C. E.

His experience at Cornell under that outstanding teacher of mechanics, Professor Church, was always recalled by Mr. Beyer, not only with great pleasure, but as a most valuable element in his engineering training. The work at Cornell also included one season with the Cornell Summer Surveying Group.

When, upon completion of the railroad location survey in the fall of 1906, the project did not go forward to construction, Mr. Beyer returned to New York City to become Time-Keeper and, later, Superintendent of Construction on the Pelham Bay Bridge. This work, consisting of six concrete arch spans and a Scherzer lift, was built by the Godwin Construction Company. This job having been completed, Mr. Beyer, in October, 1907, became Assistant Superintendent of the power-house and yards of the Church E. Gates Lumber Company, in Brooklyn, and, later, he superintended a building operation in The Bronx. It was in October, 1908, however, that he entered upon an engagement as Designing Engineer and Principal Assistant to Alexander Potter, M. Am. Soc. C. E., a Consulting Engineer, in New York City, engaged on the designs of various water supply and municipal works. This position Mr. Beyer held for nine years.

It will be recalled that the technique of analysis and design in reinforced concrete was being rapidly developed during this period. Mr. Beyer, possessing to an unusual degree a keen analytical mind in mechanics, and also having an intensely practical viewpoint, originated for Mr. Potter a number of designs in this then comparatively new material, which established his reputation as a versatile and practical designing and construction engineer. These designs were part of plans for water supply, sewerage, and drainage systems for a number of cities in the United States, Canada, and Cuba, such as Coatesville, Pa., Muskogee, Okla., Corpus Christi and San Antonio, Tex., etc., but his work also involved as well appraisals and the design of miscellaneous other structures.

In October, 1917, Mr. Beyer received an appointment as Associate in the Department of Civil Engineering at Columbia University, and the remainder of his career was devoted to teaching. In 1920, he was appointed Associate Professor, and, in 1928, became Professor of Civil Engineering.

At the time Professor Beyer joined the Faculty at Columbia, it was anticipated that, although American engineers had been slow and conservative in following European trends in this field, continuous structures would become of increasing importance in American engineering practice. Professor Beyer's first problem, therefore, was to broaden the instruction in Strength of Materials and Structural Analysis and Design to meet these anticipated developments. The structural tradition in the Department had been established by the late William Hubert Burr, M. Am. Soc. C. E., and Professor Beyer, responsible for this broadened Structural Engineering program, carried on this tradition of a progressive and forward-looking viewpoint in a most effective and constructive manner. In spite of the fact that he was intensely individualistic in his manner and methods, Professor Beyer was a born teacher. In addition, he brought to his teaching that rare combination of practical experience and a highly developed ability in the field of mathematical and mechanical analysis which is possessed by so few men. He treated his students as he did his own sons, and the high regard in which they held him both as a friend and as a teacher is attested by the fact that they dedicated to him the Yearbook of the Engineering School which was published just about the time of his death.

His duties at Columbia also involved the direction of the work of the Civil Engineering Research Laboratories. The personnel of these Laboratories have not only conducted many of the approval tests for the Building Departments of the several Boroughs of New York City, and a large number of special tests and researches for various public organizations and engineering industries, but they have also undertaken a considerable amount of fundamental scientific and engineering research. Professor Beyer's direction of all this work, in addition to his teaching duties, even with the aid of a loyal and enthusiastic staff, was a very considerable burden. His duties also involved the maintenance and operation of the Columbia Fire-Testing Station at Greenpoint, Brooklyn, N. Y., and the development of a photo-elastic laboratory which has done pioneer work in this new and interesting field. Professor Beyer's enthusiasm and willingness to talk over with and assist his associates in the solution of their problems remain as a permanent inspiration and influence in this field. He always gave the same attention and care to his University duties that he devoted to his own personal affairs.

Professor Beyer maintained an extensive consulting practice during his almost twenty years of University service. He served on many committees having to do with building, construction, and machinery problems, such as the Committee on the Revision of the New York Building Code, the Committee of the American Society of Mechanical Engineers on Shafts and Bearings, and several committees of the American Society for Testing Materials. He was particularly interested in the development of fire-resistant construction in buildings.

He was a member of the American Society for Testing Materials, the American Concrete Institute, and the American Fire Protective Association.

In 1909, he was married to Emma Eiermann, who died in 1934. Two of his sons, Theodore and Harold, survive him.

Professor Beyer was elected a Member of the American Society of Civil Engineers on July 11, 1921.

JOHN BIDDLE, M. Am. Soc. C. E.[1]

DIED JANUARY 18, 1936

John Biddle was born in Detroit, Mich., on February 2, 1859, the son of William Shepherd and Susan Dayton (Ogden) Biddle. His forebears had been identified with the military service of their country since the middle of the Eighteenth Century. During the War of 1812, his grandfather, John Biddle, was sent on duty from Philadelphia, Pa., to Detroit. After the war, he was given a grant of land near the latter city and, instead of returning to Philadelphia which had been his home, he settled on his new property.

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[1] Memoir prepared by General Biddle's niece, Miss Susan Dayton Copland, Detroit, Mich.

John Biddle—his grandson—had his early schooling in and near Detroit and in Philadelphia. In his "Memoirs" which he wrote informally in 1931, he described at length his childhood days in the country near Detroit, where he and his brothers enjoyed their outdoor life, hitching a young steer to their toboggan, riding horseback, fishing, and nutting. Schooling here was in the typical little school house of the MacGuffey Reader days, and he and a brother were paid $1.50 per week for keeping the school clean and the stove fires going. When he was thirteen years old, he went to school in Geneva, Switzerland, for three years, and then to Heidelberg, Germany, for six months. At seventeen years of age, Mr. Biddle entered the University of Michigan, at Ann Arbor, Mich., where he took the "Latin Scientific" Course for a year. He found it possible, in addition to his required work, to read extensively in the University Library, making himself "especially well acquainted with the early English dramatists." (At that time, 1876–1877, there were no organized athletics and no gymnasium. A nondescript football game was played in which each side consisted of as many boys "as happened to turn up".) During the year at Ann Arbor, he became a member of the Delta Kappa Epsilon Fraternity.

His entrance to the United States Military Academy, at West Point, N. Y., hung on a thread of chance. One day early in August, 1877, his father saw a newspaper notice of a competitive examination to be held in Detroit for the cadetship at West Point. His son took the examination and received the appointment from Michigan. He was graduated from the Academy in 1881, second in his Class.

Following West Point, he was a student at the Engineer School of Application, at Willet's Point, N. Y., from September 30, 1881, to June 15, 1884; Engineer Officer, Department of Dakota, U. S. Army, in charge of all surveys and reconnaissances for military purposes in that Department to December 15, 1887; and Assistant Instructor of Practical Military Engineering, at West Point, N. Y., to April 1, 1891. During this tour of duty, he was sent on detached service to Johnstown, Pa., after the disastrous flood of 1889, and rendered valuable service in the relief work there. He was then stationed, until June, 1898, at Nashville, Tenn., as Assistant and in charge of river and harbor work. Until June, 1893, he was Assistant in the Engineer District, in local and responsible charge of a survey of part of the Tennessee River, and of open channel work on the Upper Tennessee and Cumberland Rivers, and their tributaries, including, snagging, excavation, dam-building, a dike at the mouth of the Cumberland River, and extensive shore protection at the mouth of the Tennessee. From June, 1893, to June, 1898, he was in charge of the improvement of the following rivers: The Tennessee, above Chattanooga, Tenn., and below Bee Tree Shoals; the Cumberland above and below Nashville, in Kentucky, and Tennessee; and the Hiawassee, Caney Fork, French Broad, Little Pigeon, Clinch, and Obion Rivers, in Tennessee. He was also in charge of the supervision and construction of various bridges across these rivers.

In February, 1898, he was designated a Delegate to the Eighth International Congress of Navigation which was held in Brussels, Belgium, in July of that year.

At the beginning of the Spanish-American War, he was commissioned Lieutenant-Colonel and Chief Engineer in the 6th Corps, 1st Division, U. S. Volunteers, the first troops to enter Puerto Rico. From August to December, 1898, he was stationed at New York City, Lexington, Ky., and Macon, Ga. Following this, he was Chief Engineer of various Departments in Cuba, until September 19, 1899. On January 1, 1899, the City and Fortress of Matanzas, Cuba, was delivered to Lieut-Col. Biddle, who was commanding the American forces there, and the American flag was raised on the Fortress and all public buildings.

Lieut.-Col. Biddle was discharged from Volunteer Service in May, 1899. In August, of that year, Gen. James H. Wilson, U. S. Army, then of the Headquarters Department of Matanzas and Santa Clara, Cuba, wrote to Brig.-Gen. John M. Wilson, Chief of Engineers, as follows:

> "Now that they are raising additional volunteer regiments and are going to prosecute the War in the Philippines with vigor, I am very anxious that John Biddle shall have an opportunity to participate in the campaign * * *. He is one of the very best and most active officers in your corps. He is particularly energetic and enterprising in reconnoitering and surveying country, and I am sure he would gain distinction for himself and shed lustre upon the engineers if he had a chance in the Philippines. I have therefore to ask if you can not arrange to send him there * * *. If you would do this you would confer not only a favor upon me and him, but I am sure you would contribute largely to the success of the war in that far away region."

From September, 1899, to March 29, 1900, Captain Biddle was en route to and in Manila, Philippine Islands, as Engineer Officer of the Department of the Pacific. He was then Chief Engineer Officer of the Philippines Division until April 28, 1901, on which date he was commissioned a Major in the Regular Army. He was en route to and in the United States, serving on a Military Board until August 1, 1901.

Following this, Major Biddle was assigned for six years as Engineering Commissioner for the District of Columbia, a longer period of service than any of his predecessors. The tributes paid him at a farewell reception given in May, 1907, by fellow commissioners, department heads, and employees, suggest his work and the quality of service which he gave during this period. Commissioner MacFarland addressed the assembly as follows:

> "In all the general work of the Board of Commissioners determining policies, deciding disputed questions, advocating the District's interest before Congress and administering the District's affairs, you have done

your full part. The recommendations you have made to the Commissioners as to the affairs especially assigned to you as Engineer Commissioner have always been well considered, judicious, and progressive. They gave the Board just the basis it needed for intelligent action and showed your thorough grasp of the duties of the Engineer Department. At the public hearings of citizens, so necessary and illuminating under our form of government, at meetings of citizens' associations, and in private conferences, you have shown that spirit of co-operation which is essential to the relations between the Commissioners and the citizens and have helped to maintain the good understanding which makes so strongly for effectiveness in the common effort for the advancement of the national capital. We wish you could have remained for the opening of the District Government building, the union station, the sewage disposal system, and the Connecticut Avenue bridge, all representing so much of what especially turned on your recommendations."

Speaking for the employees the Collector of Taxes, E. G. Davis, added, in reference to the public improvements with which Major Biddle had been associated:

"These things do not successfully progress by chance, they need not only technical ability but careful and continued attention. But what has most endeared you to the employees in this building is your uniform kindness and courtesy to your subordinates, which is, after all the test of a true gentleman * * *. We believe that you have appreciated our fidelity to you and to the public service and we want you to know that your kindness to us without ostentation has made it a privilege to serve under you. For my own part I desire to say that I have felt a keener intellectual edge after every interview with you."

While Major Biddle was serving in this capacity, he was detailed, in 1902, to Gen. James H. Wilson as Senior Aide-de-Camp to attend the coronation of King Edward the Seventh.

Commissioned a Lieutenant Colonel in 1907, Colonel Biddle was assigned as Chief Engineer Officer of the Department of California and Division Engineer of the Pacific Division. His service in this capacity lasted until 1911, and included river and harbor work at various points in California and at Honolulu, Hawaiian Islands, with headquarters at San Francisco. Commissioned Colonel in 1911, he was detailed, until 1914, as a member of the General Staff in charge of the War College Division. That he performed his duties with distinction is evident from a letter of June 30, 1914, written to him by Maj. Gen. W. W. Witherspoon, U. S. Army, then Chief of Staff, which expresses "deep regret at your separation from the General Staff during my term of office as chief of that body. The earnestness of purpose and the industry and acumen that you brought to bear upon all the work you had as head of the War College Division of the General Staff, in my opinion, places you at the head of the eligibles for most important classes of duty."

This opinion of his work was confirmed by Maj.-Gen. Leonard Wood, U. S. Army, the same month when he wrote to Colonel Biddle:

"I have just learned of your relief from duty on the General Staff, and, although no longer Chief of Staff, it may not be improper, inasmuch as most of your service on the General Staff came under my direct observation, to express to you my very high appreciation of the manner in which you conducted the affairs of the War College Section of the General Staff. The work done by you was particularly satisfactory and the results attained reflect great credit upon you, not only because of their value and merit, but because they were secured under conditions of considerable difficulty incident to the injection into the General Staff of a very large amount of new personnel. I regret exceedingly, that the General Staff is no longer to have the benefit of your experience, information, and sound judgment."

Colonel Biddle's next service was in Savannah, Ga., where he was in charge of rivers and harbors from July to September, 1914. The World War had broken out in Europe in August, 1914, and he was assigned as Military Observer with the Austro-Hungarian Army in Western Galicia and Poland from November, 1914, to June, 1915. He returned to the United States to be in charge of rivers and harbors in Baltimore, Md., and Wilmington, Del., for a few months. He also served as a member of the Board of Engineers on Rivers and Harbors.

On July 1, 1916, he was appointed Superintendent of the United States Military Academy, at West Point, by President Wilson. He served as such until May, 1917, when he was assigned to organize and command the 6th United States Engineers, a special regiment which was among the first of the troops sent abroad by this country in the World War. He had been given the rank of Brigadier-General, U. S. Army, on May 15, 1917, and was promoted to Major-General, National Army, on August 15, 1917. He commanded the United States Railway Regiments with the British Army in France from July through October, 1917, laying railroads for the use of the American and Allied troops.

He then was ordered to Washington, D. C., to become Assistant Chief of Staff "acting as Chief much of the time through General Bliss' absence in Europe on the Supreme War Council." In March, 1918, he was sent to England to command Base Section No. 3, including all American troops and activities in the United Kingdom. He served in this capacity until July, 1919.

In a letter to General Biddle dated June 13, 1919, the Hon. Newton D. Baker, Secretary of War, wrote,

"Now that we are nearing the end of our work abroad, I am glad to have an oportunity to tell you how valuable it seems to me your work in England has been. From the beginning of your command there, I have never had a complaint or an unfavorable comment. Indeed, all returning soldiers and travelers speak with uniform gratitude and approval of you and your staff. This has given me deep satisfaction, and I am sure it will be pleasant to you to know that your labors have been thus successful."

Later, Mr. Baker again wrote in commendation of his service there, as follows:

"General Biddle performed his task in England with the greatest possible success; the dispatch of many hundred thousand soldiers through England was accomplished under his direction smoothly; the relations between American soldiers and English soldiers were maintained in a cordial way, and the military business of the War Department with the War Office in London was dispatched promptly and intelligently."

Appreciation of his work during this period was expressed by General Pershing, also, in a letter written June 29, 1919:

> "It would be difficult to say all the things I could say of the way your big task has been handled. Your consummate tact and your understanding of the British have continued to leave us a record that we are all proud of.
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> "You are leaving with the good will of the people among whom you have lived and served, and with the appreciation of all your own countrymen who know of your fine record.'"

The British, likewise, were generous in their praise of General Biddle's services. In July, 1919, J. A. Corcoran, of the War Office in London, wrote the General, "The (Army) Council for their part would take this opportunity of recording the high sense which they entertain of the spirit of co-operation and comradeship with which their endeavors have at all times been met in the task which lay before the two Governments and Armies."

In recognition of his work abroad, General Biddle was made an Honorary Member of the Military Division, Knight Commander, Order of the Bath in Great Britain, and was given the award of the Royal Victorian Order, and the Distinguished Service Medal, "for exceptionally meritorious and distinguished services. In command of American troops in England, by his tact and diplomacy in handling intricate problems, he made possible the transshipment of thousands of men to France. To his executive ability the efficient handling, control and dispatch of casual troops through England is largely due." He was also elected Honorary Knight Vice-President of the Ancient Club of the Knyttes of Ye Round Table.

While in London, General Biddle, with several American officers, took as their residence the house of Sir John and Lady Harrington (the latter the daughter of the late Senator James McMillan of Michigan). Through their establishment they were able to fulfill the social obligations which fell to them as representatives of their country. The Prince of Wales attended their dances, and the association of these years probably accounted for General Biddle's traveling with him as a member of his staff on the Prince's subsequent visit to the United States. In this connection, it is interesting that during President Wilson's stay in England, General Biddle accompanied him on his visit to King George V.

On returning to the United States from England in August, 1919, General Biddle was in command at Camp Travis, Texas, and, later, of Camp Custer, Michigan, until he was retired as Brigadier-General at his own request on December 1, 1920, after forty-three years of service.

In 1924, General Biddle was notified of the following citation: "John Biddle, Brigadier General, United States Army, retired, then Lieutenant Colonel, Chief Engineer, 6th Army Corps, United States Volunteers.

For gallantry in action against Spanish forces at Coamo, Porto Rico, August 9, 1899." This entitled him to wear the Silver Star for gallantry in action.

On June 21, 1930, he was promoted to Major-General, U. S. Army, (Retired). He maintained no residence after his retirement, but traveled extensively in Europe and America, finding friends wherever he went. His health gradually failed and, after two years of invalidism, he died at Fort Sam Houston, Texas, on January 18, 1936, at the age of seventy-six.

General Biddle will be remembered because of his brilliant military career; but, more than this, because he was a genius in the art of human relationships. His friends were legion, and of all ages and stations in life. He will be remembered by them for his loyalty, his ability to project himself into their lives and see through their eyes, his light touch of humor, and his personal sacrifices for them. His devotion to the members of his large "family," whether or not they conformed to his code, was outstanding, and he opened opportunities for many of the younger generation at his own expense and personal inconvenience. Never marrying, he undertook to educate and launch a number of boys in whom he became interested from time to time.

The numerous quotations included herein suggest that his tact and ability to endear himself to others were invaluable assets in the performance of his military duties. Assignments to European duty of one type or another throughout his career point to a recognition of his eligibility for such service because of these qualities, his background, and his facility with languages.

As his life is reviewed, it is striking how General Biddle steadily advanced to the rich fulfillment of the promise of his youth. That he was destined for responsibility and achievement seemed evident from his boyhood. In 1874, when he was fourteen years old, his professor in Geneva, Switzerland, wrote of him:

> "John is a serious young man, conscientious and very gifted. He has worked well, he has achieved success without the least appearance of struggle or fatigue. He is a pupil as I like them and how I wish that mine were all like him. We are already very fond of him, and his behavior at home is not only beyond reproach but extremely amiable.
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> "Although he is serious minded his maturity is not the artificial kind that comes from lack of vitality. Indeed he knows very well how to laugh and enjoy himself with his comrades. He has a certain shyness, very gracious at his age because it does not come from lack of decision or energy. I have great hopes for John and anticipate a great future for him."

That he lived up to this promise and his later opportunities is evident from these pages. His achievement is epitomized in the terse final paragraph of the announcement of his death issued by Maj-Gen. E. M. Markham, Chief of Engineers, U. S. Army, M. Am. Soc. C. E.:

> "Admired by all who knew him, General Biddle's distinguished service in a long and varied career will always serve as an inspiration to the officers

of the Corps of Engineers. He excelled both as a Military leader and as an Engineer."

General Biddle was elected a Member of the American Society of Civil Engineers on July 4, 1894.

HARRY CHESTER BOYDEN, M. Am. Soc. C. E.[1]

Died October 25, 1936

Harry Chester Boyden, the son of Mason A. and Cornelia W. Boyden, was born in Readville, Mass., on April 23, 1873. Descended from a family prominent in New England life since 1700, his father and grandfather lived in Worcester, Mass., and it was here that young Harry received his Grammar and High School education. He then entered Worcester Polytechnic Institute, from which he was graduated in 1894 with the degree of Bachelor of Science in Civil Engineering. He returned to the Institute later to receive the degree of Civil Engineer in 1923.

The first year after his graduation from the Institute Mr. Boyden spent in Melrose and Waltham, Mass. This was followed by seven years in the Engineering Department of the Boston and Albany Railroad Company on construction, maintenance of way, and the design of grade separation structures.

In 1902, he started West, going to Corning, N. Y., where he was Assistant Resident Engineer on double-tracking of the New York Central and Hudson River Railroad. In December of the same year, he went with the Delaware, Lackawanna and Western Railroad Company as Assistant Engineer on Construction and built the Car Repair and Locomotive Shops, at Scranton, Pa. He was also engaged on the construction of third-track and a tunnel near Scranton.

Mr. Boyden then went to California with the Atchison, Topeka and Santa Fé Railway Company and was one of the first Resident Engineers of the Los Angeles County Highway Commission, under the late Frank Hall Joyner, M. Am. Soc. C. E., a former New Englander, who was its Chief Engineer. The California Highway Commission was being organized at this time, and its Chief Engineer, the late A. B. Fletcher, M. Am. Soc. C. E., sought Mr. Boyden who held several important residencies.

A few days after the United States entered the World War, Mr. Boyden was commissioned a Captain in the Engineer Corps. Although scheduled three times for overseas service he was kept on training work at Camp Humphreys, Virginia, being advanced to the ranks of Major and Lieutenant Colonel. Later, he was commissioned Colonel in the Engineer Reserve Corps.

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[1] Memoir prepared by George D. Whittle, M. Am. Soc. C. E.

After the war, Colonel Boyden spent some years with the Portland Cement Association on promotional work, and then followed a period as Dean of the College of Engineering, Ohio Northern University, at Ada, Ohio, and five years with the Celite Company, lecturing at universities throughout the United States.

During this period, his health began to fail, as a result of a weak heart which followed an attack of influenza during the war, and pleurisy and diabetes forced him to retire from active work in 1929. Retirement, however, did not bring back health, and a lowered vital resistance sent him in 1935 to the Veterans' Hospital, at Livermore, Calif., where he died on October 25, 1936. Funeral services were held at the Chapel of the Chimes, in Oakland, Calif.

On December 22, 1896, he was married to Emilie Louise Schreiner, of Springfield, Mass. He is survived by his widow, a daughter, Marion Alice Skiolvig, a son, Robert Eldridge Boyden, and two grandchildren.

Colonel Boyden was a member of Painted Post Lodge, A. F. and A. M., of Corning, N. Y., and of Berkeley (Calif.), Post No. 7 of the American Legion.

During the years prior to his retirement, he wrote many articles on concrete and related subjects. His judgment in engineering matters was respected by his associates. He was a man of high ideals, of strong convictions, and a firm believer in the American home and for what it stands.

Colonel Boyden was elected a Member of the American Society of Civil Engineers on June 3, 1908.

EDWARD GATLING BRADBURY, M. Am. Soc. C. E.[1]

Died March 11, 1936

Edward Gatling Bradbury was born in Brooklyn, N. Y., on June 14, 1870, the son of Edward Emerson and Sarah Jane (Sykes) Bradbury, and was a lineal descendant of Thomas Bradbury, of Wicken Bonant, England, who came to America in 1630 as an agent of Sir Fernando Gorges, Governor of Maine, and who, settling in Agamenticus (York) Me., and, later, in Salisbury, Mass., became quite an important figure in early Colonial affairs—his wife, Mary, having been condemned to death as a witch during the Salem insanity, but not executed.

In 1881, Mr. Bradbury's parents, after residing some years in Dedham, and Somerville, Mass., moved to Providence, R. I., where he received a public school education and was graduated from the High School in 1888.

Immediately after his graduation, Mr. Bradbury obtained employment on the Staff of the City Engineer of Providence, the late Samuel Merrill Gray,

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[1] Memoir prepared by G. Gale Dixon, Clyde T. Morris, and Frank A. Barbour, Members, Am. Soc. C. E.

M. Am. Soc. C. E., and served as "Assistant" under Mr. Gray and his successor, the late J. Herbert Shedd, M. Am. Soc. C. E., until 1890, when he resigned to take a position as Recorder and, later, as Inspector in the United States Army Engineer Office, at Newport, R. I., in charge of Major W. R. Livermore and, later, of the late General (then Captain) William Herbert Bixby, U. S. Army, M. Am. Soc. C. E., who subsequently became Chief of Engineers, U. S. Army. His work included wreck removal, dredging, and the building of breakwaters and jetties.

In the summer of 1892, Mr. Bradbury found employment under F. Herbert Snow, M. Am. Soc. C. E., then City Engineer of Brockton, Mass., as Chief of Party on surveys preliminary to the construction of the sewerage system of that city. He continued his service under Mr. Snow as Assistant Engineer on sewer design and grade crossing abolition until November, 1897, when he became the first employee of the newly formed engineering partnership of Snow and Barbour. In 1899, when this firm opened an office in Columbus, Ohio, he moved to that city as its Western Representative and continued as such until 1905. During this period he supervised the construction of sewers and sewage disposal systems in Mansfield and Lakewood, Ohio, and at numerous institutions, and made plans for sewerage systems at Springfield, Cuyahoga Falls, and Wadsworth, Ohio.

In 1905, Mr. Bradbury took over the Columbus Office and continued in private practice in his own name until 1922—except for a period of two years when Mr. George P. Shute was associated with him under the firm name of Bradbury and Shute. In this private practice, Mr. Bradbury designed and supervised the construction of twenty or more sewerage and water supply systems, including those at Galion, Sidney, Bremen, Medina, Columbiana, and Alliance, in Ohio, and Crystal Falls, Mich. In his private practice, he also investigated and reported on numerous water supply, sewerage and sewage disposal projects, including such municipal systems as those at Newark, Zenia, Norwalk, Niles, Delaware, Salem, and Van Wert, in Ohio, and Iron River, Mich.

From 1910 to 1917, Mr. Bradbury was associated with Frank A. Barbour, M. Am. Soc. C. E., of Boston, Mass., under the firm name of Barbour and Bradbury, in various professional undertakings, independently of the individual practice of the two members. Under this association, the improved water supply system of Akron, Ohio, was designed and constructed, Mr. Bradbury acting as Resident Engineer on the entire work. Other work done by the firm included investigations and reports on the water supply for the Goodyear Tire and Rubber Company, the Goodrich Rubber Company, Columbia Chemical Company, and on flood protection at Tiffin, Ohio.

In 1917, on a call from the Supervising Engineer at Camp Devens, Massachusetts, Mr. Bradbury promptly responded, and designed and supervised the construction of the sewerage system and, in its later stages, of the water supply system at that cantonment.

Returning to Ohio late in 1917, Mr. Bradbury was appointed County Sanitary Engineer of Franklin County—coincidentally with the creation of the department and office—and continued to hold this position until his death. In 1922, the expansion of the County work compelled the practical abandonment of his private practice.

As County Engineer, he designed and supervised the construction of approximately 200 miles of water mains and sewers, involving the expenditure of more than $3 000 000. He served as Secretary for many years and also as President of the organization known as the Ohio Conference of County Sanitary Engineers, and he has been credited with having been responsible, more than any other person, for the County Sanitary Law now operative in Ohio. In the development of the method of assessing betterments now largely in use in this County work, he was particularly interested. He was the author of the paper entitled "County Sewer District Work and Assessment of Cost According to Benefits."[2]

Mr. Bradbury was a member of many technical organizations and a Past-President of the Ohio Engineering Society, the Engineers Club of Columbus, and of the Central Ohio Section of the Society. His ethical standard was of the highest and his personal integrity above reproach. "Brad", as he was known to his intimates, was a many-sided personality—a steadfast friend, considerate, alert, humorous, and a distinct social addition to any group. His capacity for rapid, quantitative thinking was notable and presumably inherited. An uncle had been the author of "Bradbury's Arithmetic" and other textbooks well known to past generations of school boys. An unusual flair for the impromptu production of humorous verse was also a notable characteristic.

In his family life, Mr. Bradbury was fortunate. He was married in 1894 to Cora Bell Gay, daughter of Alford and Frances I. (Swett) Gay, of Brockton, Mass., and three children were born to the union. His widow and two sons, Alford Gay and Irving Lewis, survive him.

At its meeting on March 11, 1936, the Central Ohio Section of the Society passed the following resolution:

> "*Whereas,* Edward Gatling Bradbury was a Past-President and honored member of this Central Ohio Section of the American Society of Civil Engineers; and
>
> "*Whereas,* Through his untiring service and staunch loyalty this Section was organized and has enjoyed continued success; and
>
> "*Whereas,* His fellow members feel deeply the loss of Mr. Bradbury as a respected friend and valued associate because of his fine character, outstanding ability and congenial fellowship:
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> "*Be It Resolved* that we, the Central Ohio Section of the American Society of Civil Engineers, do and hereby does express to Mrs. Bradbury and all the other members of his family our heartfelt sympathy at their great loss.
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> "*Be It Further Resolved* that a copy of this resolution be sent to his family."

Mr. Bradbury was elected an Associate Member of the American Society of Civil Engineers on November 7, 1906, and a Member on August 31, 1909.

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[2] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 445.

ERNEST WILLIAM BRANCH, M. Am. Soc. C. E.[1]

DIED MARCH 8, 1936

Ernest William Branch was born in Smithfield, Me., on March 26, 1863, the son of William David and Ann Eliza (Newcomb) Branch. He was graduated from Williston Seminary, at Easthampton, Mass., in 1883; received the degree of Bachelor of Arts from Boston University, at Boston, Mass., in 1888; the degree of Bachelor of Laws from Suffolk Law School, at Boston, in 1916; and was admitted to the Massachusetts Bar the same year.

Mr. Branch began his professional career with the firm of Whitman, Breck and Company, Civil Engineers, of Boston, while still in college, and upon graduation he remained in the employ of that Company. In 1896, he was a member of the firm of Whitman and Branch.

On August 1, 1896, the Board of Sewerage Commissioners of the City of Quincy, Mass., appointed Mr. Branch as Engineer for the Board on the construction of a sewerage system for the city. On February 20, 1905, the system was substantially completed. This work comprised the building of forty-five miles of sewers, a pumping station, and a force main about three miles long, connecting with the Boston Outfall Sewer in Squantum, Mass.

Upon the completion of this sewerage system, Mr. Branch returned to private practice, with an office in the Adams Building, in Quincy. For a number of years, he also maintained an office in Boston. His general practice covered most of the Northeastern States. He was a Registered Engineer and Surveyor in New York State.

During the World War he served as Project Engineer for the United States Housing Corporation on its three tracts in Quincy, and, in addition, gave instruction in Surveying to men in training for the United States Army.

Mr. Branch was a member of the Boston Society of Civil Engineers, and Beta Theta Pi, and President of The Boston University Alumni Association. He served as Councilor-at-Large in the City Council of Quincy during 1909 and 1910.

He was prominently identified with Bethany Congregational Church, of Quincy, and for many years was Superintendent of the Sunday School.

Mr. Branch was a man of cheerful disposition, a great lover of children, of which he had none, and was greatly respected in his community. As an Engineer, he was a hard and indefatigable worker, and always had the ethics of the profession at heart.

He died on March 8, 1936, at Daytona Beach, Fla., where he maintained a winter residence. He was buried at Camden, Me.

On March 18, 1890, he was married to Fairvilla A. Gould, of Camden, Me., who survives him.

Mr. Branch was elected a Member of the American Society of Civil Engineers on September 6, 1905.

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[1] Memoir prepared by George H. Newcomb, Treas., Ernest W. Branch, Inc., Quincy, Mass.

JOHN BRUNNER, M. Am. Soc. C. E.[1]

DIED JUNE 15, 1936

John Brunner was born in Weddige, Sweden, on November 22, 1864, the son of John and Anna Brunner. He received his early education in the public schools of Sweden, and, later, attended special schools preparatory for college. He entered the Royal Institute of Technology, at Stockholm, Sweden, in September, 1883, and was graduated in May, 1887, with the degree of Civil Engineer.

Mr. Brunner served in the Engineering Corps of the Swedish Government during the summers of 1884, 1885, and 1886, and after his graduation, until December, 1887, he was Assistant Engineer on Construction, in connection with the survey and the construction of the Northern Trunk Line, Swedish State Railroad, to the iron ore deposits in Northern Sweden.

In March, 1888, he came to the United States and his first position in this country was as Draftsman with the Stonemetz Printing Machine Company, at Millbury, Mass., where he was employed from May to July, 1888, at which time he left to take the position of Assistant Engineer in the Bridge Department of the Boston and Maine Railroad Company. He held this position until June, 1890, when he was appointed Assistant Engineer with the Mount Vernon Bridge Company, at Mount Vernon, Ohio. In September of that year, Mr. Brunner was promoted to the position of Chief Engineer, which position he held until May, 1895. During this period the Mount Vernon Bridge Company built numerous bridges under his supervision for the Pennsylvania Railroad Lines, the Cleveland, Cincinnati, Chicago and St. Louis Railroad Company; the Cleveland, Akron, and Columbus Railroad Company; the Chicago, Peoria, and St. Louis Railroad Company; the West Side Metropolitan Elevated Company, in Chicago, and for the Cities of Chicago, Ill., and Cleveland, Ohio. The Bridge Company also fabricated the steel work for the Administration Building and the Art Galleries for the World's Columbian Exposition in Chicago.

In May, 1895, Mr. Brunner accepted the position of Assistant Engineer in the Engineering Department of the Carnegie Steel Company, and shortly thereafter was promoted to be Assistant Chief Engineer of the Structural Division. It was at this time that the Duquesne (Pa.) Works of the Carnegie Steel Company was rebuilt and new equipment and buildings were added to the Homestead (Pa.) Works and the Edgar Thomson Works. Mr. Brunner also had charge of the steel designs for the Call Building and the Ferry Building, in San Francisco, Calif., which buildings survived the earthquake without any damage as a result of the precautionary measures taken in the design of the steel work to withstand earthquake shocks.

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[1] Memoir prepared by a Committee of the Illinois Section consisting of Albert F. Reichmann, M. Am. Soc. C. E., Walter G. Zimmermann, Assoc. M. Am. Soc. C. E., and B. F. Affleck, Esq., of Chicago, Ill.

In July, 1896, Mr. Brunner became Bridge Engineer, and, later, Chief Engineer, of the City of Pittsburgh, Pa. Under his direction several new bridges were built and old bridges remodeled; and the construction of Bigelow Boulevard was begun and completed. Extensive relief sewers, carried through tunnels, were also constructed to take care of the excessive rainfall which occasionally occurred in the East Liberty Section of the city. Mr. Brunner left this position in 1902 to accept that of Assistant General Superintendent of the North Works of the Illinois Steel Company, in which capacity he served until 1912. During this period new mills and new ore-handling equipment were constructed at the South Works of the Company. The designs of the structural work of buildings and ore-handling equipment and the fabrication and erection were handled by the North Works. The erection of buildings at the Gary (Ind.) Works and the designs for some of the structural work were also handled by the North Works.

In April, 1912, Mr. Brunner was transferred to the Chicago Office of the Illinois Steel Company, in charge of Metallurgy and Inspection, and from July, 1923, to January, 1936, he was Manager of the Department of Metallurgy and Inspection. In January, 1936, he was appointed Consulting Engineer of the Company.

During Mr. Brunner's service with the Illinois Steel Company, extensive experiments and investigations of rails were made, beginning in 1912 with investigations of the strength of rails at low temperatures, and other experiments on repeated stresses to determine the cause of so-called "transverse fissures."

The metallurgy and inspection of steels obtained from the Illinois Steel Company by foreign countries during the first years of the World War (1914–1918) were in Mr. Brunner's charge. Later, after the United States entered the war, he was placed in charge of the metallurgy and inspection for the munition and ordnance steels furnished by the Illinois Steel Company to munition and gun-makers for the United States Government. This work was done in consultation with the Ordnance Divisions of the Army and Navy, in Washington, D. C.

After the war, he again took up the investigation of rails with thermal treatment which resulted in the adoption of comparatively simple and practicable methods of operation by means of which rails of longer durability and greater safety under traffic are rolled. Other carbon and alloy steels were also investigated and improved through proper thermal treatment.

Mr. Brunner was married on November 16, 1892, to Cora Mitchell, of Mount Vernon, Ohio, who survives him.

In 1919 he was knighted by the King of Sweden in the Royal Order of the North Star (Kungliga Nordstjerne Orden) for his achievements in engineering research. In February, 1936, he was awarded the John Ericsson Medal by the American Society of Swedish Engineers for development work in improving steel rails by heat treatment.

Mr. Brunner was a member of the American Society for Testing Materials, Western Society of Engineers, American Iron and Steel Institute, American

Railway Engineering Association, Army Ordnance Association, Association of American Steel Manufacturers (President, two years), Art Institute of Chicago (life member), and a Fellow of the American Geographical Society. His club activities included the Chicago Engineers and Alpine of Canada. He was a member of the Masonic Fraternity.

Mr. Brunner was elected a Member of the American Society of Civil Engineers on March 2, 1898.

ROBERT DUNN BUDD, M. Am. Soc. C. E.[1]

DIED JULY 30, 1935

Robert Dunn Budd was born at Petersburg, Va., on October 29, 1873, the son of Judge Joseph S. and Frances E. (Dunn) Budd. After receiving his preliminary education at the famous McCabe School, at Petersburg, he attended Hampden-Sidney College, at Hampden-Sidney, Va., for two years, and the Virginia Military Institute, at Lexington, Va., for three years.

From 1894 to 1898 Mr. Budd was associated with Captain R. T. Dunn, of Petersburg, in private engineering practice. He was then, for two years, with the Richmond, Petersburg and Carolina Railroad Company as Instrumentman and, for a time, as Resident Engineer.

After engaging in non-technical activities for about three years, Mr. Budd, in 1903, was employed on engineering work for the City of Petersburg, the Virginia Passenger and Power Company, and the W. Brill Lumber Company. From October, 1903, to August, 1904, he was with the Richmond, Fredericksburg and Potomac Railroad Company as Draftsman and Engineer.

In August, 1904, Mr. Budd was appointed as City Engineer of Petersburg, in which capacity he served until his retirement, because of ill health, in October, 1930. Upon retirement as City Engineer, he was appointed Consulting Engineer to the city, holding that position until the time of his death.

In his work as City Engineer, Mr. Budd exercised a profound influence upon the development and improvement of the community and made his greater reputation. He was recognized among his associates and professional acquaintances as an all-round well-versed engineering authority and was often consulted by them. Having an unusually retentive memory, his long tenure as City Engineer of Petersburg gave him an almost uncanny knowledge of the various engineering structures of that city, particularly of the underground structures.

Between the time he took office and his retirement, he played the major engineering part in enlarging inadequate and limited water and sewerage systems until only extremely isolated sections of the city were without adequate water and sanitation. The source of water supply was changed to provide adequate water for any demands, under almost any conditions. The reservoir, pumping plant, and filter plant were modernized and enlarged to provide both safe water for domestic use and an adequate supply for fire protection and for any industrial uses likely for years to come. He had charge of the transformation of the City's thoroughfares from a few streets, with antiquated pavements and sidewalks, to a city with most of its streets improved with modern sidewalks, curbs, gutters, and roadway pavements. He designed, installed, or improved storm drainage for a large part of the City's water-sheds. Former difficult approaches to the city over temporary bridge structures were replaced under his direction with permanent structures of adequate design.

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[1] Memoir prepared by William A. Smith, Esq., Petersburg and Richmond, Va.

For a number of years Mr. Budd was a member of the Virginia Board for the Examination and Certification of Professional Engineers, Architects, and Land Surveyors.

His devotion to his work and his unswerving loyalty to his community were equalled only by his unselfishness in being helpful to all those to whom he could be of service. He was especially distinguished for his interest in improving the knowledge and abilities of young men who, from time to time, worked under him. He was patient with the inexperienced and unstintingly gave them, both during and after working hours, the benefit of his broad experience in both the theory and practice of engineering. This unselfish interest developed better and most loyal subordinates, and equipped them to meet the engineering probems with which they were confronted when leaving his employ for larger fields of engineering endeavor under less benevolent superiors.

He is survived by his widow, who was Marie T. Steel, of Petersburg (to whom he was married on June 14, 1900), and by two children, Robert Dunn Budd, Jr., and Elizabeth D. Budd, and by a brother and two sisters.

Mr. Budd was elected a Member of the American Society of Civil Engineers on October 4, 1910.

MOSES BURPEE, M. Am. Soc. C. E.[1]

DIED AUGUST 18, 1936

Moses Burpee was born on February 25, 1847, at Sheffield, New Brunswick, Canada, the fourth of six children of George and Phebe Burpee. At the age of five, the boy began attending the local schools where he remained until he had completed all the courses offered, which included those required for college entrance. During this time, he helped his eldest brother with work on the farm and, for himself, developed a liking for mechanics and some facility in the use of tools. An aunt was able to give him lessons in free-hand drawing which helped him in his later work.

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[1] Memoir prepared by H. W. Oxnard, M. Am. Soc. C. E.

In the fall of 1865, he left home for Philadelphia, Pa., where an uncle, Coburn Burpee, gave him work in his machine shop. Here, Mr. Burpee gained practical experience in mechanics and in the operation of the steam power plant, and also, for three winters, he attended Evening Classes in Mechanical Drawing at the Franklin Institute and at the Polytechnic Institute. Drawings which he then made are still preserved and show an unusual degree of perfection.

During these years the promotion of railroads in New Brunswick had reached the stage of construction and, in 1868, Mr. Burpee returned to work on one of the first of these lines, the Fredericton Railway. On its completion early in 1869, he was transferred to the main line, chartered as the European and North American Railroad Company, for work on a section between the eastern terminus at Fairville and Tracy, N. B. When operation of the line began in November, 1869, Mr. Burpee was made Station Agent at Fredericton Junction. In the fall of 1870, he was put in charge of the location and construction of a 3-mile extension of the road, from Fairville to a deep-water terminal at Carleton, later West St. John, N. B.

From 1871 to 1878, Mr. Burpee was employed as Assistant Engineer by the New Brunswick Railway Company in which position he gained valuable experience in the field work of location and construction, as well as the making of maps and profiles and the design of Howe trusses in the office. This road was begun in 1871 at a point on the east bank of the St. John River, opposite Fredericton, N. B., and during Mr. Burpee's term of service had reached Grand Falls, N. B., with a short branch to Woodstock, N. B., and a longer one to Caribou, Me. There were three crossings of the St. John River.

Construction of new lines having ceased for a time in New Brunswick, Mr. Burpee decided to look elsewhere. In March, 1879, he went to Chicago, Ill., and from there to Milwaukee, Wis., where the Chief Engineer of the Chicago, Milwaukee, and St. Paul Railroad Company offered him work on a new line about to be built in Dakota. For the remainder of that year Mr. Burpee served as Draftsman in the Construction Office, first, at Canton, (So.) Dakota, and, later, as the work progressed, at Parker, (So.) Dakota. In 1880, he was in charge of location surveys from Ortonville, Minn., to Lake Traverse, and from Aberdeen, (So.) Dakota, running first 40 miles south and then 40 miles north; then on construction west from Mitchell, (So.) Dakota. The year following, Mr. Burpee was Division Engineer on construction in Iowa, with headquarters first at Marion and, later, at Tama. During 1882, he continued in charge of the construction of 70 miles of road between Mitchell and Chamberlain, (So.) Dakota, and of a branch line in Iowa, from Spenser to Okoboji and Spirit Lakes.

In the spring of 1883, Mr. Burpee became a Division Engineer for the Canadian Pacific Railway Company in charge of construction of 100 miles from Maple Creek, Saskatchewan, to Calgary, Alberta. Grading on this line was relatively light and rapid progress was made, a speed of 10 miles per day being attained on track-laying. In the fall, Mr. Burpee made a survey for a branch line in Manitoba and before the end of the year returned to New Brunswick.

In the summer of 1884, he began a survey for the Central Railway of New Brunswick, but this project was not successful and had to be abandoned.

During the winter of 1884–85, Mr. Burpee made a preliminary survey for the International Railway Company, a predecessor of the Canadian Pacific Railway Company, in Maine, between Lincoln and Moosehead Lake, across which a long trestle bridge was planned. The survey was finally abandoned in favor of one running around the south shore of the lake, but Mr. Burpee had gained some knowledge and experience of much value to him in later work in this same forest country.

In the spring of 1885, he was made Chief Engineer of the New Brunswick Railway, which now included all the lines in New Brunswick on which he had previously worked as well as some additions made during his absence. He was at once confronted with the difficult job of restoring the road after it had suffered severely from freshets in the St. John River. Two of the river bridges had been washed out and had to be replaced while water was still at flood stage. In 1887, this road became a part of the Canadian Pacific Railway with which Mr. Burpee remained as Chief Engineer of the Atlantic Division. With the addition of 160 miles more of the road his circuit was extended to Megantic, Province of Quebec.

F. W. Cram, General Manager of the New Brunswick Railway, declined to remain with the road after it was taken over by the Canadian Pacific and instead began the promotion of a new line from Bangor to the northern part of Maine. Early in 1891, he had accomplished the organization of the Bangor and Aroostook Railroad Company, of which he offered Mr. Burpee the post of Chief Engineer. Mr. Burpee accepted this offer and commenced a service which continued with satisfaction to all concerned as long as he lived. For thirty-seven years he was active as Chief Engineer and then, from January 1, 1928, continued in an advisory capacity as Consulting Engineer.

The Bangor and Aroostook Railway Company acquired the Bangor and Piscataquis Railroad which comprised a main line from Old Town to Greenville, Me., and a branch from Milo, Me., to Katahdin Iron Works by way of Brownville, Me., at which point the new construction was to begin. Mr. Burpee soon had three surveying parties in the field which, within a year, had completed the location from Brownville to Caribou, with branches to Fort Fairfield, Me., and to Ashland, Me. A general contract for construction was let early in 1893, and work was rushed, permitting operation as far as Houlton, Me., to begin on January 1, 1894, to Caribou and Fort Fairfield, on January 1, 1895, and to Ashland on January 6, 1896. Later additions of considerable length were made in 1899, 1902, 1905, 1907, and 1910, in each of which years it was necessary to organize new location and construction forces. Because of these successive organizations a comparatively large number of young engineers came under Mr. Burpee's influence, none of whom was too inexperienced to attract his interest and attention, and few of whom failed to profit from his example of kindliness and devotion to duty.

Mr. Burpee was truly a railroad pioneer, beginning his career at a time when railroads in any territory were in their infancy and in that in which he began his work were still unknown. From this early stage to the present, he was not only a very interested observer, but a contributor as well to the development of the art.

He was a Life Member of the Engineering Institute of Canada and the American Railway Bridge and Building Association, and, for many years, a member of the American Railway Engineering Association.

Mr. Burpee was deeply interested in religion, being a member of the First Congregational Church, of Houlton, of which for many years he was a Deacon. He also served as a Trustee of the Bangor Theological Seminary for twenty years. His recreations were sketching, water-color painting, and photography, in all of which he had much talent.

On March 4, 1880, he was married to Caroline Alexander, of Fredericton Junction, N. B., who survives him, as do his two children, Mary G. Burpee, a teacher of voice culture at Houlton, and George W. Burpee, M. Am. Soc. C. E., a member of the firm of Coverdale and Colpitts, Consulting Engineers, in New York City.

In recent years Mr. Burpee enjoyed many interesting and extended motor trips, including the ever enjoyable ones from his home at Houlton to Fredericton and Sheffield. It was on one of these trips that he caught the cold which developed into pneumonia and resulted in his passing on August 18, 1936. Thus ended a life full of usefulness, of unusual kindliness and thoughtfulness for others, and one which had won the respect and affection of all who were privileged to know him.

Mr. Burpee was elected a Member of the American Society of Civil Engineers on September 3, 1884.

GEORGE J. CALDER, M. Am. Soc. C. E.[1]

DIED MARCH 18, 1936

George J. Calder was born at Sonora, Calif., on August 5, 1884. His early education was received in the public schools of his native State. In 1905, after graduation from the Stockton, Calif., High School, he entered the University of California, College of Civil Engineering, at Berkeley, from which he was graduated with the degree of Bachelor of Science, in 1909, with honorable mention.

Mr. Calder devoted his professional career to civil engineering construction, in which he showed proficiency, both as an Engineer of Design and as a Contractor. He combined successfully the viewpoints of the Engineer who plans and the Contractor who builds. He had the faculty to understand men, to work with them, and to earn their loyalty and support.

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[1] Memoir prepared by Charles Derleth, Jr., M. Am. Soc. C. E.

His first engagements after leaving college in 1909 took him to San Francisco, Calif., and Stockton, mainly upon the design of reinforced concrete and steel framed buildings. He soon became a successful Superintendent of Construction for buildings, particularly in the region of Sacramento, Calif. Beginning in 1915, Mr. Calder served the City of Sacramento by preparing reports and designs for flood-control projects, such as the Sacramento By-Pass Weir; later, he was engaged as Superintendent of Construction on various municipal buildings, and sanitary and storm-sewer installations. He was prominent in private structural design of steel and concrete buildings, with Sacramento architects.

In January, 1920, Mr. Calder joined Charles Gilman Hyde, M. Am. Soc. C. E., in the planning, design, and construction of the Sacramento Filtration Plant, for which, later, he served as Resident Engineer. In October, 1923, the writer as Chief Engineer for the Carquinez Highway Bridge across Carquinez Strait, retained him as Resident Engineer for the American Toll Bridge Company. Throughout the design and construction of this notable bridge, Mr. Calder rendered valuable services not only as the responsible Engineer in the field, but also in his counsels with the Chief Engineer and with the officials of the Company. When the Carquinez Bridge neared completion in November, 1926, Mr. Calder was elected Vice-President of the American Toll Bridge Company. He deserves credit for developing the bridge approaches, building adequate toll houses, and devising successful methods for collecting tolls. When the Carquinez Bridge was opened to traffic in May, 1927, Mr. Calder was selected by the Bridge Company to succeed the writer as Chief Engineer, and, at the same time, he remained a member of the Board of Directors for the Bridge Company until 1935.

As Engineer and Contractor, Mr. Calder was an expert in heavy underwater construction. In January, 1933, he became President of the Duncanson-Harrelson Company, of San Francisco, Specialist Bridge, Dock, and Wharf Contractors. As the directing head of this Company, he built the Mare Island Causeway for the United States Navy Department, and much under-water work in and about San Francisco Bay. His firm did diamond drilling and exploration work for the San Francisco-Oakland Bay Bridge.

Throughout his life Mr. Calder was active in fraternal circles, notably in the Masonic Order. During his residence in Sacramento, he was a prominent Rotarian. He never forgot his Alma Mater, and was always ready to help students at the University of California. He remained a member of the Engineering Honor Societies. He endeared himself to succeeding classes of students at Berkeley, and when called was ever ready to attend their meetings. On the evening of March 17, 1936, he attended a meeting of the Honor Society, Tau Beta Pi, as its chief speaker. Just as he was finishing this speech of counsel and cheer to the boys, he was stricken with a cerebral hemorrhage to which he succumbed on the morning of March 18, 1936.

His friends will long remember Mr. Calder's loyalty to his Alma Mater and his great interest in and love of young men. It was in service to youth that he was stricken at the zenith of a successful career. His future had great promise. He died a young man at fifty-one, leaving a reputation for a kindly and generous nature.

He is survived by his widow, Mrs. Geneva Calder, a daughter, Georgene, and a son, George B. Calder.

Mr. Calder was elected a Member of the American Society of Civil Engineers on April 23, 1928.

JOHN HIRST CATON, 3d, M. Am. Soc. C. E.[1]

Died May 2, 1936

John Hirst Caton, 3d, the son of John H. and Mary (Cranston) Caton, Jr., was born in Philadelphia, Pa., on July 27, 1883, where he lived until he was twelve years old. His family moved to Rhode Island, finally settling in Edgewood, where John attended the Cranston Grammar School. He was graduated from the Cranston High School in 1900 with honors and was an outstanding football enthusiast.

During the following two years Mr. Caton worked as an Apprentice Civil Engineer with J. A. Latham and Company, of Providence, R. I., on surveying. The next year he was engaged as Engineer Surveyor for Mr. J. A. Nickerson, of the Shady Hill Nursery Company, Boston, Mass., on landscape construction work. The following year he entered the service of F. A. Gammino, a Contractor, of Providence, who had extensive sewer contracts in Dedham, Mass.

Mr. Caton then decided to apply his time toward acquiring a technical education, and, in 1904, he entered the Massachusetts Institute of Technology, at Boston, Mass., studying Civil Engineering until 1908. During his summer vacations he was occupied in engineering on municipal construction work in the vicinity of Boston.

In August, 1908, Mr. Caton passed the Federal Civil Service examinations and was assigned to the Bureau of Public Works, at Manila, Philippine Islands. Here, he served for nine years, successively, as Assistant, District, Structural, and Construction Engineer, in charge of Insular Government construction and maintenance in Manila.

From August, 1917, to August, 1919, he was in the United States Army. He entered as a Captain, but soon was made Major of Engineers, becoming Company and, finally, Regimental Commander of the 33d Engineers. He had charge of quarries and road construction, at Base Section 5, American Expeditionary Forces. His record includes consulting on port work at Brest, in

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[1] Memoir prepared by E. F. Kriegsman, M. Am. Soc. C. E.

France, the design, purchase, and installation of five rock-crusher plants, with harbor loading and unloading facilities, etc.

In September, 1919, Major Caton became an Assistant Engineer, United States Engineer Department, Office of Public Buildings and Grounds, at Washington, D. C. He had charge of the construction of the Lincoln Memorial Reflecting Pool.

In January, 1920, at the urgent request of his family, he accepted the position of Assistant General Manager of the Industrial Chemical Company, at Providence, R. I. Industrial management did not satisfy Major Caton's disposition to keep physically active, and, in November, 1920, he was designated as Director General, Bureau of Public Works, at Santo Domingo, Dominican Republic. In this position, he had charge of administration, involving the design and construction of all public works in the Republic under the American Naval Administration, the cost of which was more than $3 000 000. He remained in this capacity after the government was returned to the elected Dominican Government.

In October, 1925, Major Caton became associated with R. W. Hebard and Company, Incorporated, New York, N. Y., as Chief Engineer and Superintendent of Construction on contracts with the Republic of Salvador for the construction of several highways centering in the capital City of San Salvador. Extensive work was done also in the city for various municipal improvements.

Near the completion of this work Major Caton was sent to the Republic of Colombia for the same Company. There, he carried out the work of the construction of the "Carretero al Mar" (Road to the Sea), from Medellin across the Andes Mountains to the Pacific Ocean. The work was an historic piece of highway construction, being the first of its kind ever carried out, under very primitive conditions, over the rugged and picturesque Andes.

With these experiences, Major Caton considered the possibility of future public works construction in Latin America as a fertile field for development. In April, 1929, when the work in Colombia was discontinued, due to the exhaustion of public funds, he became associated with Winston Brothers, of Minneapolis, Minn., which Company had contracts for railroad, highway, and other improvements in Latin America. Major Caton was engaged by this Company on highway construction, taking advantage of the opportunities afforded him. Shortly after, he was made Latin-American Representative for the Company (with general power of attorney). In this position, he developed opportunities for several large contracts for general, heavy construction, and municipal improvement work, most of which was projected in Chile and the Argentine Republic.

In June, 1932, Major Caton returned to the United States, the depression in this country having had the effect of disrupting the finances of the Latin-American countries.

In September, 1933, he was appointed Deputy Engineer Examiner for Rhode Island under L. A. Hoffman, State Engineer, for the Federal Emergency Administration of Public Works, of Rhode Island and Connecticut. In this position, Major Caton developed the Emergency Public Works projects for the 1933 program for the State of Rhode Island, the cost of which amounted to $11 000 000. This program consisted largely of municipal and State buildings, schools, water-works, and sewage disposal projects.

Concurrent with this position, he was named one of the members and Vice-Chairman of the Rhode Island State Planning Board, by Governor Theodore Francis Green, in 1934, and was re-appointed in 1935.

In March, 1935, Major Caton was appointed Chief of the Division of Roads and Bridges, Department of Public Works, State of Rhode Island, and as such was engaged in carrying out the Federal Aid Highway Program for the State, including grade-crossing elimination installations and the maintenance and construction of the State Highway System. On April 24, 1936, President Roosevelt appointed Major Caton to be State Director, Public Works Administration, for Rhode Island.

On May 2, 1936, while at his camp at Wallum Lake, Rhode Island, where he had gone to spend the week-end, he was stricken, and was taken to a hospital in Providence where he died from a cerebral hemorrhage shortly afterward.

Major Caton always insisted that his most important work was done in the Republic of Santo Domingo and in the Latin-American countries, where he was well known. He spoke Spanish fluently and was popular with all who came in contact with him.

He was an engineer of exceptional ability and, in addition, he possessed those human qualities which endeared him to his associates and friends. He was a brilliant conversationalist and had that happy faculty of meeting people and enjoying social intercourse.

He was a member of Corregidor Lodge, A. F. and A. M., Manila, Philippine Islands, which affiliation he always considered a distinction. He was also a member of the following scientific and technical societies: American Society of Military Engineers; American Association of Engineers; Association of State Highway Officials; Association of State Highway Officials in North Atlantic States; Association of State Planning Officials, Rhode Island State Planning Board; Providence Engineering Society; and the National Geographic Society.

He also had membership in the following social clubs: Massachusetts Institute of Technology Club of Rhode Island; Santo Domingo Country Club; Club Union, Colombia; Club Union, Buenos Aires, Argentine Republic; and the Philippine Club, New York City.

He was married on July 18, 1908, to Sadie Wilson, of Newton Center, Mass., who survives him. He also left four sons, John H., 4th; Frederick W.; David C.; and Richard B. Caton; two sisters, Mrs. E. A. Dow, of Worcester, Mass., and Mrs. Thomas H. Roberts, of Edgewood, R. I.; and two brothers, Jesse H. and W. Stuart Caton, both of Edgewood, R. I.

Major Caton was elected an Associate Member of the American Society of Civil Engineers on June 11, 1917, and a Member on January 20, 1922.

WILLIAM BOWDOIN CAUSEY, M. Am. Soc. C. E.[1]

Died August 10, 1936

William Bowdoin Causey was born in Suffolk, Nansemond County, Va., on June 24, 1865, the eldest child of Charles Henry Causey, a Captain of Cavalry in the Confederate Army, and Martha Josephine (Prentis) Causey. On his father's side he traced his ancestry to the Causeys who settled quite early in Delaware and to James Colvin who came from Scotland to Philadelphia, Pa., in 1817, with his wife, Jane Campbell, and their two young daughters. His mother's people had lived in Tidewater, Virginia, since 1620, when Sir George Yeardley, afterward Deputy Governor of Virginia, came to Jamestown. Colonel Causey's great-grandfather, Judge Joseph Prentis, was a member of the Virginia Legislature during the Revolutionary period and soon after became Judge of that State's first Admiralty Court. Judge Prentis' father, William Prentis, had come to Williamsburg, Va., from Norwolk County, England, about 1725, and engaged in a general merchandising business; in the restoration of Williamsburg, the shop in which Prentis and Company conducted its business in the latter half of the Eighteenth Century has been restored to its original condition. Colonel Causey loved Tidewater, Virginia, and his many associations there and, after his death, in Chicago, Ill., his body was taken to Suffolk and laid to rest in the family lot in Cedar Hill Cemetery within almost a stone's throw of the house in which he was born.

He was the eldest of eight children and, as his childhood and youth fell in the difficult period just following the Civil War, he learned early the meaning of hard work. He attended a local military academy, but his formal schooling came to an end when he was seventeen and he went to work as a Chainman with the engineers who were building the Atlantic and Danville Railroad (later, a part of the Southern Railway System). He continued with the Atlantic and Danville Railroad Company until 1889, when he had become Assistant to the Chief Engineer. Because of the death of his father and grandfather in 1889 and 1890, his presence was required at home. After affairs at home were settled, he spent a year in charge of construction forces on track elevation near Boston, Mass., for the New York, New Haven, and Hartford Railroad Company. After this, he and his brother, James C. Causey, were in the logging business at Suffolk.

In 1898, Mr. Causey became Roadmaster and Assistant Superintendent of the Milwaukee Division of the Chicago and Northwestern Railroad. In 1901, he was Division Engineer of the Chicago Great Western Railroad Company, at Des Moines, Iowa, and, in 1902 and 1903, he was Chief Engineer of the Elgin, Joliet, and Eastern and of the Chicago, Lake Shore, and Eastern Railway Companies. In 1903, he returned to the Chicago Great

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[1] Memoir prepared by W. G. Atwood, M. Am. Soc. C. E.

Western Railroad Company as General Superintendent of Construction, and, from 1904 to 1908, he was Engineer of Maintenance of Way and Superintendent of the Illinois Lines of the Chicago and Alton Railroad Company. He then returned again to the Chicago and Great Western Railroad Company as Division Superintendent and remained with this Company until 1914, when he was made Vice-President of the Norwood White Coal Company. He had just left this position to become Fuel Agent for the Buick Motor Car Company when the United States entered the World War. Although past the draft age, he at once volunteered and received a commission as Captain of Engineers and was assigned to the Seventeenth Engineers (Railway).

Captain Causey sailed for France with the regiment on July 28, 1917, as Battalion Adjutant of the First Battalion. Regimental headquarters were established at St. Nazaire, France, and the Commanding Officer was in charge of all construction in Base Section No. 1, with an area not much less than that of the State of Massachusetts. Captain Causey, who was soon promoted to Major and, later, to Lieutenant Colonel, was stationed at Nantes, and represented the Section Engineer in charge of all construction in the eastern half of the Section.

Among the projects under his supervision were: The construction of the Aerial Gunnery School, at St. Jean de Monts, with a very large flying field and about 15 acres of buildings; the construction of small holding yards, and the reconstruction of the track systems, on many of the French docks in and near Nantes; the St. Luce storage yard, with 150 000 yd of grading, about 10 miles of track, and 5 acres of warehouses; a flying junction, with about 15 miles of yard tracks and an engine terminal, at Saumur; five hospitals, at Nantes and Angers, with a combined capacity of 15 000 beds; an artillery training camp, at Coetquidan, with 58 acres of buildings, many roads, and a firing range where three regiments of field artillery could fire at one time, as well as a large number of smaller projects.

Colonel Causey's tireless energy and driving force, together with his diplomatic skill and, above all, his capacity for making and holding friends, enabled him to administer this work to the great satisfaction of his superiors and great credit to himself.

After the Armistice, Colonel Causey was relieved from duty in Base Section No. 1 and assigned to duty with the American Relief Administration. He accompanied the first mission to Austria as a transportation expert. His first telegraphic report was a classic: "The Austrian railroads are worse than * * * Railroad," naming a road that was notorious for the low grade of its maintenance.

When the Relief Administration found it necessary to supervise the operation of the railroads transporting relief supplies, Colonel Causey was placed in charge of the operation of the roads from Trieste and Fiume, supplying Austria, Hungary, and Czechoslovakia. He increased the daily tonnage from 200 to 8 000 in a very short time. This did much, not only for the relief of distress, but for the pacification of Central Europe.

After the disbandment of the Relief Administration forces in October, 1919, Colonel Causey was employed as Technical Adviser by the Republic of Austria. In addition to strictly technical services he was called on to represent Austria in many international negotiations, and was one of the official representatives of the Republic at the Genoa and Rapallo Conferences.

He served Austria for about five years, and was so universally admired and loved by the people that when he left Vienna, the Imperial rooms in the station were opened, and he was sent away with all the honors that could be given him.

After his return to the United States, Colonel Causey was City Manager of Norfolk, Va., for several years, and afterward, to the time of his death, he was Vice-President of the M. E. White Construction Company, of Chicago, Ill. He had also served as United States Assistant Commissioner to the Century of Progress Exposition at Chicago in 1933.

Colonel Causey was an engineer of ability, an administrator of still greater ability, and a man who attracted and held the love and admiration of his associates to an extent that is rare. He had great opportunity for service to others in his European experience, took full advantage of it, and left a record that will be an inspiration to all who know it.

He received a citation from General Headquarters, American Expeditionary Forces, was made an Officer of the Legion of Honor by France, received the Order of St. Sava, Grade II, from the Jugoslav Government, and the Great Silver Cross of Honor from Austria.

Colonel Causey was elected a Member of the American Society of Civil Engineers, on October 11, 1920.

ROBERT CARR CHURCHILL, M. Am. Soc. C. E.[1]

Died September 9, 1936

Robert Carr Churchill was born at Roanoke, Va., on October 20, 1890, the son of the late Charles Samuel Churchill, M. Am. Soc. C. E., and Anna (Green) Churchill. His early education was received in the public schools of Roanoke. In 1909, after graduation from Mercersburg Academy, Mercersburg, Pa., with honors, he entered Yale University and, in June, 1912, was graduated from the Sheffield Scientific School, with the degree of Bachelor of Philosophy.

Upon leaving college, he was engaged in keeping cost data for Mr. David W. Flickwir, of Flickwir and Bush, which firm was then building the Nicholson (Pa.) Viaduct for the Delaware, Lackwanna, and Western Railroad Company. Mr. Churchill remained with Mr. Flickwir until 1915, when he entered the employ of the Norfolk and Western Railroad Company, as Resident Engineer on a valuation survey at Suffolk, Va. In July, 1916, he became Plant Engineer for the Westinghouse Machine Company, at East Pittsburgh, Pa., later being transferred to Westinghouse, Church, Kerr and Company, to supervise the layout of the Essington Plant.

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[1] Memoir prepared by William T. Gilbert, Esq., New York, N. Y.

During the World War Mr. Churchill served in France as First Lieutenant in the Twelfth Engineers, from September, 1917, to April, 1919, seeing active service at St Mihiel, the Argonne, and during the final drive around Toul. He was cited for exceptionally meritorious and conspicuous services in the Toul Sector, and in October, 1934, was awarded the decoration of the Purple Heart, by the Congress of the United States.

From April to August, 1919, he was in Philadelphia, Pa., as Plant Engineer for Westinghouse Electric and Manufacturing Company, at the Essington Plant, making a special study of oils. After August, 1919, Mr. Churchill returned to his native city, Roanoke, with J. F. Barbour and Sons, Building Contractors. In April, 1920, he went into business for himself operating a quarry. In 1922 Mr. Churchill took up contracting in conjunction with the quarry business, and was engaged in building streets for the Cities of Roanoke and Vinton and doing work for the Virginian and the Norfolk and Western Railway Companies.

In 1928, he built two large viaducts, the Jefferson Street and Walnut Avenue Viaducts, for the City of Roanoke. From then, until his death on September 9, 1936, Mr. Churchill was engaged in general contracting and the ready-mixed concrete business in and around Roanoke.

Throughout his life Mr. Churchill was known for his fairness and high sense of business responsibility and civic leadership He was a member of St John's Episcopal Church, the Rotary Club of Roanoke, Chi Phi Fraternity, Elks Club, Shenandoah Club, Country Club, and American Legion Post No. 3. From 1934 to 1935, he served on the School Board of the City of Roanoke.

Besides his widow, Mrs. Dorothy (Lacy) Churchill, and two children, Robert Carr Churchill, Jr., and Dorothy Fontaine Churchill, he is survived by his mother, Mrs. Charles S. Churchill, and by three sisters, Mrs. P. V. Littlejohn, Ethel Churchill, and Mrs. Cecil Bertie.

Mr. Churchill was elected an Associate Member of the American Society of Civil Engineers on November 27, 1917, and a Member on January 17, 1927.

EDWARD IVAN CLAWITER, M. Am. Soc. C. E.[1]

DIED AUGUST 12, 1936

Edward Ivan Clawiter was born at Mount Eden, Calif., on December 21, 1879, the son of Edward and Anna Clawiter. His early education was received in the schools of his native city, and, later, he was graduated from the High School, at Hayward, Calif. In 1896, Mr. Clawiter entered the University of California, at Berkeley, and was graduated in 1900, with the degree of Bachelor of Law; he was admitted to the Bar of California in the same year. He then decided on Civil Engineering as a life work, and was graduated from the Van der Naillen School of Civil Engineering in 1903.

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[1] Memoir prepared by Frederick R. Muhs, M. Am. Soc. C. E.

Immediately after his graduation in 1903, he went to Manila, Philippine Islands, where he was employed as Assistant City Engineer. In January, 1904, Mr. Clawiter became Chief Engineer of the Atlantic, Gulf, and Pacific Company of Manila in which position he continued until 1909. His experience in Manila was broad, including both design and construction of breakwaters, dredging, reclamation, harbor improvements, wharves, buildings, sewers, water systems, and tunnels.

On his return to the United States, for eighteen months, Mr. Clawiter was Assistant Engineer in the field for the Contractor, on the construction of the New York Barge Canal between Whitehall and Fort Edward, N. Y.

During 1910 and 1911 he was Engineer for J. G. White and Company, at Buenos Aires, Argentine Republic, in charge of extensions and reconstructions for Ferro-Carril Central de Buenos Aires, later representing the Trussed Concrete Steel Company in the Argentine and Brazil, where he remained until the end of 1914. He was then transferred to the Los Angeles, Calif., Office of the same Company.

During the latter part of 1915 and the first six months of 1916 he was Assistant Engineer for Miller and Lux, a large land-holding Company in California, for which he designed and constructed extensive drainage and irrigation works.

In July, 1916, Mr. Clawiter became Chief Engineer of the San Francisco Bridge Company, engaged in the building of dry docks, harbor, and bridge work. He resigned from that Company in December, 1929, to become Division Engineer for the Hydraulic Dredging Company, of Oakland, Calif., which position he held until the time of his retirement in 1935.

He was married, in 1907, to Letitia Foley. His widow survives him.

Mr. Clawiter was elected an Associate Member of the American Society of Civil Engineers, on June 30, 1911, and a Member on June 2, 1920.

FREDERICK HOSMER COOKE, M. Am. Soc. C. E.[1]

Died August 28, 1936

Frederick Hosmer Cooke was born in Cincinnati, Ohio, on March 11, 1879. He came of a family long distinguished in the annals of New England. The forebears of his father, Henry P. Cooke, settled in and around Boston, Mass., as early as 1636, and his mother's family in and around Salem, Mass., between 1630 and 1640. His grandfather, the Rev. Sylvester Cooke, was a member of the first graduating class of Amherst College, while his father, together with four brothers, served with distinction throughout the Civil

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[1] Memoir prepared by Commander E. C. Seibert (C. E. C.), U. S. N., M. Am. Soc. C. E.

War. Abner Hosmer, a member of the family of his mother, Mary R. (Barker) Cooke, was among those killed at the Battle of Concord Bridge.

Frederick Hosmer Cooke attended the public schools in Cincinnati, and was graduated in 1900 from the Massachusetts Institute of Technology, at Boston, with the degree of Bachelor of Science in Civil Engineering. However, before obtaining his engineering education, he had received a training in Latin, Greek, and the classics, that broadened his mind and gave him a charm of diction and a vocabulary seldom encountered in an engineer. His fluent knowledge of French later became of inestimable value to him during the periods of his work in France and Haiti.

After graduation, he held various engineering positions, in civil life, in Cleveland, Ohio, Boston, Mass., and at the Navy Yard, at Portsmouth, N. H. This was prior to his being commissioned, after competitive examinations in which he stood first (with a grade of 91.6), an Assistant Civil Engineer, United States Navy, with the rank of Lieutenant (Junior Grade), on January 1, 1904.

In November, 1906, he was given the grade of Civil Engineer, U. S. Navy, with the rank of Lieutenant; in July, 1914, he received a promotion to the rank of Lieutenant Commander; and in July, 1918, to temporary rank of Commander, and to Commander in October, 1921; and, finally, to Captain in June, 1924.

In the Naval Service, Captain Cooke had a professional career marked by interesting and important duties. From February, 1904, to February, 1906, at the Navy Yard, Mare Island, California, he was Assistant to the late Rear Admiral H. H. Rousseau (C.E.C), U. S. N., M. Am. Soc. C. E., in charge of the Drafting Room. The outstanding work then in progress was the commencement of the construction of dikes for regulating and increasing the depth of water in Mare Island Strait.

From March, 1906, to November, 1908, he was in charge of the Department of Yards and Docks at the Naval Station, Cavite, Philippine Islands, during which period a considerable addition was built to the Canacao Hospital, mostly by day labor of Filipinos and Chinese.

From November, 1908, to January, 1909, Lieutenant Cooke was en route from the Philippine Islands to the United States *via* India, Egypt, and Europe. Then followed, for a period of about three months, temporary duty at the Bureau of Yards and Docks, at Washington, D. C.; from which he went to the Naval Training Station, Great Lakes, Ill., and thence, again, for a period of one year and four months to the Bureau of Yards and Docks where he had charge of the Drafting Room.

Between October, 1910, and January, 1911, he made a voyage to Europe and return with the United States Atlantic Fleet, inspecting commercial dock yards and marine works in England, and naval establishments at Chatham, Sheerness, Portsmouth, and Devonport, in England; at Kiel and Wilhelmshafen, in Germany; and at Cherbourg and Brest, in France. His description of these dock yards and marine structures later was published, with commentary, by the Bureau of Yards and Docks.[2]

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[2] *Confidential Bulletin No. 6*, Bureau of Yards and Docks, Navy Dept., Washington, D. C., September 1, 1911.

In 1911, for a ten-month period, Lieutenant Cooke was Assistant to the late Rear Admiral Frank Taylor Chambers (C. E. C.), U. S. N., M. Am. Soc. C. E., at the Navy Yard, Norfolk, Va. Late in the same year he made an inspection of coaling plants and coal-handling appliances at several points on the Great Lakes, and along the Atlantic seaboard, at the request of the Isthmian Canal Commission.

From the early part of 1912 until the end of 1916, he served with the Panama Canal on the Isthmus, first, as Assistant Engineer, and then as Designing Engineer, under Admiral Rousseau, who was a member of the Isthmian Canal Commission and, later, Engineer of Terminal Construction. Lieutenant (and before he left Panama, Lieutenant Commander) Cooke was in charge of the design, and, to some extent, of the construction, of the Cristobal Coaling Plant, the Balboa Dry Dock, the two 250-ton floating cranes, *Ajax* and *Hercules*, and of the Naval Radio Stations at Darien, Colon, and Balboa.

Once again, from March to July, 1917, after a three-month leave, Lieutenant Commander Cooke served at the Bureau of Yards and Docks. This was the formative time for the real preparation for the participation of the United States in the World War. In November, 1917, he went to Norfolk, Va., to make appraisal of the just compensation due owners of property commandeered for the Naval Operating Base at Hampton Roads, Virginia.

In the latter part of 1917, he went to the Navy Yard, at Portsmouth, for duty, and then, again, in 1918, he returned to the Bureau for duty preparatory to going to France in charge of the construction of the Lafayette (high-power) Radio Station, at Croix d'Hins, near Bordeaux.

Upon returning to the United States, Commander Cooke was made a member of a board on vacation of properties commandeered for naval purposes during the war. This Board was charged with the appraisal of numerous items of real estate all over the country, and its praiseworthy reports were largely the result of Commander Cooke's careful and diligent efforts.

This work was followed by duty at the Training Station, Great Lakes (September, 1921, to June, 1924), as Public Works Officer of the Station, and of the 9th Naval District; and from July, 1924, to August, 1928, as Engineer-in-Chief to the Republic of Haiti. In this latter capacity he did much toward the rehabilitation of the Island Republic, employing several thousand men under the guidance of American engineers in the construction of roads, telegraph and telephone systems, wharves, drainage, sanitary works, water-works, irrigation systems, hospitals, schools, public buildings, and other works necessary in laying the foundation of an orderly government.

Following his tour in Haiti, Captain Cooke went to Philadelphia, Pa., as Yard and District Public Works Officer (October, 1928 to June, 1932); thence, in July, 1932, to Pearl Harbor, Hawaii, as Station and District Public Works Officer. As such, he had charge of important and very extensive work, including a large Repair Basin at the Navy Yard.

In July, 1935, he returned to the United States *via* Europe, and made inspections of dockyards in Japan, Singapore, Italy, Malta, and France. Upon the completion of this inspection, he went to his last duty, at the Navy Yard, Boston, Mass., as Yard and 1st Naval District Public Works Officer.

He was a member of the Military Order of the Loyal Legion of the United States in succession to his father, and was also a member of the Army and Navy Club of Washington, of the Army and Navy Club of Manila, of Sigma Alpha Epsilon Fraternity, and of the Alumni Association of the Massachusetts Institute of Technology.

The Government of the Republic of France conferred on Captain Cooke the grade of Chevalier of the Legion of Honor for his work in connection with the construction of the Lafayette Radio Station near Bordeaux, during the World War; the Republic of Haiti, for his work as Engineer-in-Chief of the Republic (1924–1928), conferred upon him the rank of Commandeur of the National Order of Honneur and Merite.

He was married to Olga Faure, on December 2, 1914, at Ancon, Canal Zone. Mrs. Cooke survives him, as do their sons, Frederick A. F. and Henry J. H. Cooke, and their daughter, Olga Cooke.

Captain Cooke will be sorely missed by his associates and by many others, of divers nationalities, who came in contact with him. It can truthfully be said that his friends were myriad. His unusual understanding in all his relations with others, combined with a fine generosity, endeared him to the masses. His wit and humor, combined with his personal dignity, endeared him to his intimates. His remarkable memory served him well in recounting his experiences, in the telling of which he interjected a wealth of anecdote to the delight of his friends. His industry and thorough-going methods in his work and his vigor of attack of professional problems gained the admiration of his co-workers. Truly, he was an outstandingly capable, vigorous, and lovable character.

He died at the Naval Hospital, at Chelsea, Mass., and was buried with full military honors of his rank, at the National Cemetery, at Arlington, Va.

Captain Cooke was elected an Associate Member of the American Society of Civil Engineers on July 9, 1906, and a Member on April 26, 1921.

JOHN JOSEPH DONOVAN, M. Am. Soc. C. E.[1]

DIED JANUARY 9, 1937

One of seven children of Patrick and Julia (O'Sullivan) Donovan, natives of Ireland, John Joseph Donovan was born at Rumney, N. H., on September 8, 1858. After completing his High School course he attended the State Normal School, at Plymouth, N. H., from which he was graduated in 1877.

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[1] Memoir prepared by Joseph Jacobs, M. Am. Soc. C. E.

For three years thereafter (1877–1880), he taught school in New Hampshire and Massachusetts and then entered Worcester Polytechnic Institute, at Worcester, Mass., to prepare himself for the Engineering Profession which he then desired to make his life work. As Valedictorian of his Class Mr. Donovan, was graduated from that institution in 1882 with the degree of Bachelor of Science, to be followed, a few years later, by the award of the degree of Civil Engineer. In 1932, on the occasion of the Fiftieth Anniversary of the graduation of his Class, he was the Commencement Speaker and at that time he was made an Honorary Doctor of Science of the Institute. In recognition of his marked ability as a student, Mr. Donovan, together with another brilliant student, J. Q. Barlow, M. Am. Soc. C. E., who also was destined to rise high in the Engineering Profession, was selected for service with the Northern Pacific Railroad Company in the construction of its western extension to the Pacific Coast.

Mr. Donovan's initial assignment was to Missoula, Mont., where he began work as a Rodman in August, 1882. From this modest start, he rose rapidly through the grades of Levelman, Transitman, Assistant Engineer, and then Location and Construction Engineer in charge of important new work. During this early period, Mr. Donovan participated, as a young engineer observer, in the ceremonies which marked the joining of the Eastern and Western Divisions of the Northern Pacific Railroad, at Gold Creek, Mont., in September, 1883—ceremonies that were presided over by the President of the Railroad Company, Mr. Henry Villard, and which had among others of its distinguished guests, Gen. Ulysses S. Grant, former President of the United States. In 1886 and 1887, Mr. Donovan was stationed in Washington, in charge of heavy mountain work on the Cascade Division West; he also had charge of location along the Snoqualmie River approaching Puget Sound. During the latter part of 1887, he was again assigned to duty in Montana and was made Engineer in Charge of a number of Northern Pacific Branch Line Subsidiaries, as follows: Drummond and Phillipsburg Railroad, Helena and Northern Railroad, Helena and Red Mountain Railroad, Helena, Boulder Valley and Platte Railroad, Helena, Gallatin and National Park Railroad, and Helena and Madison Railroad. He also had charge of surveys for other projected branches in that region. During a brief vacation, in 1887, he made a hurried trip to Alaska and also to his home in the East.

Mr. Donovan returned to New England for his marriage in 1888 and then, with his young bride, promptly returned to Montana to resume his engineering work. Shortly thereafter, the couple moved to Tacoma, Wash., where, however, they made but a brief stay. Associating himself with certain corporations then interested in developing Bellingham Bay, Mr. Donovan and his wife, in December, 1888, moved to Fairhaven (later a part of the City of Bellingham), Wash., where they made their permanent home and where Mr. Donovan established headquarters for his extensive future professional and business activities. His initial work there, as Chief Engineer of the Fairhaven Land Company, Skagit Coal and Transportation Company, and Fairhaven and Southern Railway Company, included the building of a railroad, opening coal mines on the Skagit River, platting the town site of Fairhaven, and constructing its first wharves. The Tacoma-Montana Syndicate, which financed these operations, began, in 1890, with Mr. Donovan again as Chief Engineer, the construction of a railroad intended to extend from Victoria, B. C., Canada, to Portland, Ore., on the south and to Spokane, Wash., on the east. With appropriate official ceremonies, in which Mr. Donovan took a prominent part, the Canadian and American Sections were joined at the International Boundary on February 14, 1891, and a short time later, when eighty miles of line had been completed, the property was acquired by the Great Northern Railway Company.

Upon the termination of this last engagement, Mr. Donovan, after his return from another hurried trip to the East Coast, served for a short time as Chief Engineer for the local Tideland Appraisers. Following this, his engineering engagements for a number of years included the following: Chief Engineer and Manager, Blue Canyon Coal Mining Company (1891–1902); and Chief Engineer and Manager, Bellingham Bay and Eastern Railroad Company (1898–1906), later a branch of the Chicago, Milwaukee, St. Paul and Pacific Railroad. An interesting sidelight on this branch line is that when it was originally built, which was prior to Mr. Donovan's connection with it, its principal owner was the late Ogden Mills and that its first locomotive, which had already seen service in California, had been brought around "the Horn" to San Francisco, Calif., by Mr. Mills who had a strong sentimental attachment for that particular locomotive. It will be noted, from what has preceded, that Mr. Donovan had served as Chief Engineer of the several lines that eventually became the terminal sections, for entry into Bellingham, of three great transcontinental rail systems, namely, the Great Northern Railway, the Northern Pacific Railroad, and the Chicago, Milwaukee, St. Paul and Pacific Railroad.

During the latter period of his more active railroad work, Mr. Donovan also became definitely identified with the logging and lumber industry—an industry which eventually claimed his major attention and became his major business concern. In 1898, in association with Messrs. Larson and Bloedel, he organized the Lake Whatcom Logging Company, Mr. Donovan serving as its Vice-President. Greatly expanding their activities, these same men subsequently organized the Larson Lumber Company and, in 1913, the Company was re-organized as the Bloedel-Donovan Lumber Mills under which name it has since then been continuously operated. Mr. Bloedel, as President, supervised the Manufacturing and Sales Departments, while Mr. Donovan, as Vice-President, handled all logging railroad and general logging operations. This Company is one of the large operators in this industry. It has extensive timber holdings, and operates large mills and logging camps in several of the northwest counties of Washington. It maintains Sales Offices in New York, N. Y., Chicago, Ill., and other large cities throughout the United States and, at times, it has employed as many as 3 000 men, with a monthly payroll in excess of $300 000. Mr. Donovan was also, in 1925, Vice-President and General Manager of the Columbia Valley Lumber Company.

His interests were by no means confined to his business and professional activities. He was an ideal citizen—charitable, public-spirited, and always helpful in civic affairs. He gave freely of his time and talents to the study of National, State, and local social and economic problems. In politics, he was a Republican and frequently attended the conventions of that party, but he never held partisan political office. In religion, he was a Roman Catholic and was ardently devoted to his church and to its affiliate organizations, such as the Knights of Columbus, its hospitals, and its other charitable institutions. The breadth of his interests and activities in matters not directly related to business is evidenced in the large number of social and civic organizations to which he belonged and the numerous public commissions upon which he had served at various times. Some of these are, as follows: Member, first two City Councils of Fairhaven; first Charter Commission of Bellingham; Trustee, Bellingham State Normal School; Member, first State Highway Commission; State Commission on Forest Legislation; Executive Committee, State Conservation Association; President, State Good Roads Association; State Chamber of Commerce; Director, Columbia Basin League; Member, State Board of Charities and Correction; National Child Labor Commission; National Municipal League; National Civic Federation; National Security League; Navy League; Adviser, St. Joseph's Hospital; President, Catholic State Federation; and Member, American-Irish Historical Society.

Among his trade association and technical society connections were the following: President, Bellingham Chamber of Commerce, Bellingham Rotary Club, Pacific Logging Congress, and Northwest Rivers and Harbors Congress; Vice-President, National Foreign Trade Council; Member, National Geographic Society, and the Montana Society of Engineers. Mr. Donovan was a frequent contributor to the press on economic questions and he had written considerably on railroading and logging. At the Summer Meeting of the Society at Seattle, in 1926, he presented a paper[2] entitled "The Engineer in the Lumber Industry." His principal social club affiliations were the Rainier Club of Seattle, and the Golf and Country Club and the Twentieth Century Club of Bellingham. His major recreations were horseback riding, tramping, climbing, and travel. Well read and exceptionally well informed on public affairs, a widely traveled man who had seen much of Europe and the Orient, he was much in demand as a speaker and as a presiding officer at public functions.

In 1888, Mr. Donovan was married to Clara Isobel Nichols, of Melrose, Mass., a lady of unusual charm and culture who trod with him, to the end, all of life's long path ahead, and she contributed her full share to his success as a man of affairs. Three children resulted from this union—Helen Donovan (Mrs. Leslie Craven); J. N. Donovan, who succeeded his father in the lumber business; and Philip Donovan, of Portland, Ore. Mrs. Donovan passed on in June, 1936, which sad event was followed only a few months later by the passing of Mr. Donovan. In consequence of his long years of heavy toil and service, and of the special demands put upon his time and energy during the early years of the depression, Mr. Donovan had a nervous breakdown in 1933 which compelled his retirement from active business and which, after three years of invalidism, caused his demise on January 9, 1937. He is survived by his three children and seven grandchildren. Attended by his family and a large part of the citizenry of his home city, and mourned by thousands throughout the Pacific Northwest, he was laid to rest in Bay View Cemetery, Bellingham, on January 12, 1937.

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[2] *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 460.

Mr. Donovan was an able engineer and a man of talent, aggressiveness, and fine business acumen. When he settled in Bellingham a half century ago that region was largely a forested wilderness waiting to be subdued and developed by just such hardy pioneers as progressively pushed the Western Frontier across the continent to the Pacific Seaboard and helped to make the United States a great nation. Mr. Donovan was of that type—"their tribe were God Almighty's men"—and he played an important part in the development of the Pacific Northwest. He will be missed by his fellows in their future counsels. The following is quoted from an editorial in the *Bellingham Herald*:

> "Mr. Donovan was a man of faith and a man of action whose life of service has enriched this community and this State. His name is deeply graven in the records of the Northwest as one of our greatest builders and most influential leaders. Courage and ambition flamed in the heart of this son of Irish Emigrants and led him on to seek an education and then to throw the power of his young body and brain into the work of building the West. His intelligence, integrity, energy and initiative made him a leader in that work. He achieved wealth, position, and honor. Here is inspiration for the youth of this land, yet young and in need of leaders and doers."

Mr. Donovan was elected a Junior of the American Society of Civil Engineers on April 7, 1886, and a Member on April 4, 1888.

OTTO RAE ELWELL, M. Am. Soc. C. E.[1]

DIED NOVEMBER 13, 1935

Otto Rae Elwell was born on August 15, 1890, in Aberdeen, Scotland, the son of Lewis M. and Martha (Kirkwood) Elwell. His parents were both American born, the father's family having been in this country since 1632. The mother was of Scotch descent.

Lewis M. Elwell was in Scotland for about ten years, in charge of a plant producing monumental granite for export to the American firm which he represented. Three sons and one daughter were born while the family was resident in Scotland. About 1898 the Elwells returned to the United States and lived for six years in Quincy, Mass., at the end of which time they moved to Zanesville, Ohio, where Otto Rae Elwell was graduated from the High School.

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[1] Memoir prepared by M. S. Woodin, Assoc. M. Am. Soc. C. E.

In 1908, he went to the State of Washington where he began his career as an engineer by joining a surveying party working in the Yakima Valley for the Union Pacific Railway Company, locating the system known as the Yakima Valley Transportation Company. In this connection, Mr. Elwell was, successively, Chainman, Rodman, and Instrumentman. In 1911, he entered the employ of the Canadian Northern Railway Company, with which for nearly three years, he was an Instrumentman and in charge of field parties on location and construction. After the completion of this work, Mr. Elwell remained in Canada as Assistant Engineer in charge of the preliminary surveys, location, and the preparation of plans and estimates for the High River and Hudson Bay Railway and for the Alberta and Great Waterways Railway.

In 1915, he returned to Ohio, and was employed as an Assistant Engineer with the State Highway Department. This service was interrupted in 1917 when, at the entrance of the United States into the World War, he enlisted in the 23d United States Engineers, at Columbus, Ohio, and was sent to Camp Meade, Maryland, for training. In November, 1917, the regiment reached France where, as a Sergeant, Mr. Elwell remained in the service until July, 1919, participating in the regiment's arduous work of constructing roads, bridges, and docks.

After the conclusion of the war, he returned to Ohio and resumed his work with the Highway Department, remaining until March, 1921. He then took a position with the Idaho State Highway Department as Assistant Engineer and was in charge of field parties on the construction of roads and highway bridges. When this work was finished, Mr. Elwell returned to Washington and for a short time was engaged as Construction Engineer for Thurston County. He then entered the service of the Washington State Department of Highways, at Olympia, in January, 1922. In this service, he was engaged for several years as Bridge Designer in connection with many important projects. Some of the structures which Mr. Elwell designed during this period were the concrete approaches to the Puyallup River Bridge, at Tacoma; the concrete bridges between Snoqualmie Pass and Easton, on State Road No. 2; and the Nooksack River Bridge, near Ferndale.

In 1927, he was placed in charge of the Bridge Division of the Department of Highways, with the title of Assistant Bridge Engineer, and later, was made Bridge Engineer, which position he held until his death. While in charge of the Bridge Division Mr. Elwell was responsible to the Director of Highways for the design of several million dollars worth of work distributed over all parts of the State.

In July, 1935, Mr. Elwell was advised to enter a sanitorium at Milwaukee, Ore., for rest and treatment. It was thought that a few months would restore him to good health and, for a time, he did apparently improve. His friends were encouraged and not alarmed when it became known that he had returned to Olympia on November 11 but this was quickly followed by the shock of learning that he had passed away at his home, two days later.

Otto Elwell, or "Bud" Elwell, as he was more generally known, had a real talent for making friends and was held in high esteem by a host of people with whom he came in contact in his many activities. He was an ardent sportsman, interested in nearly all branches of athletics, was an active basket-ball player during his younger days, and a capable tennis player. He was an enthusiastic fisherman and hunter and became widely known in the State as an expert trap and skeet shooter. He was interested in the Boy Scouts and for a considerable time was in charge of one of the local troops.

In appreciation for his many years of service in the Washington Department of Highways, the new bridge across the Nisqually River, east of Olympia, has been designated as the O. R. Elwell Memorial Bridge.

Mr. Elwell is survived by his widow, Bess Williams Elwell, to whom he was married in Columbus, Ohio, on October 27, 1917; by his parents; by three brothers, E. W. Elwell, Lewis M. Elwell, Jr., and Roy A. Elwell; and by a sister, Olga Elwell.

He was a member of the American Legion and belonged to the Masonic Fraternity, having been affiliated with the Blue Lodge and Royal Arch Lodge of the York Rite in Zanesville, Ohio, and with the Scottish Rite Lodge, in Olympia.

Mr. Elwell was elected a Member of the American Society of Civil Engineers on October 14, 1930.

JOSEPH FIRTH, M. Am. Soc. C. E.[1]

DIED MAY 10, 1936

Joseph Firth, the son of Frederick William and Ann (Ardron) Firth, was born in Philadelphia, Pa., on October 16, 1878. Both parents died in Mr. Firth's early childhood and he was reared by his paternal grandfather. His early education began with his entrance into Girard College, where he pursued his studies until he was graduated in 1895. After two years with the Pennsylvania Railroad Company he entered Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1901 with the degree of Civil Engineer.

Following his graduation Mr. Firth remained for one year at his Alma Mater as an Instructor in Mathematics and Surveying. In 1902 and 1903, he was with the Foundation Company, of New York, N. Y., as Assistant Engineer on pneumatic foundations in New York City and bridge piers at Omaha, Nebr. Continuing with that Company, he installed a water supply at Ardmore, Okla., in 1904, serving as Resident Engineer in full responsible charge. From 1904 to 1907, with the exception of one year as Instructor in Civil Engineering at Lehigh University, at Bethlehem, Pa., he was engaged by the late T. Chalkley Hatton, M. Am. Soc. C. E., on the design and construction of a number of municipal improvements—water supplies for Ridgley, Md., and Benton, Pa.; sewerage systems for Hyattsville, Md., and Mt. Carmel, Pa.; and as Mr. Hatton's Principal Assistant in charge of the design and construction of the sewerage systems at Charleston, W. Va., and Pensacola, Fla.

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[1] Memoir prepared by a Committee of the Philadelphia Section consisting of Dudley T. Corning, *Chairman*, C. S. Shaughnessy, C. E. Myers, and W. E. A. Doherty, Members, Am. Soc. C. E.

In 1907, the City of Charlotte, N. C., engaged Mr. Firth as its City Engineer. During his incumbency (until 1914) he re-organized the Engineering Department and practically rehabilitated the city's water supply, its streets, and its sewage disposal plant. In 1915, he was appointed Commissioner of Public Works of Winston-Salem, N. C., and there he continued his meritorious municipal work. In 1919, he took up the duties of City Manager of West Palm Beach, Fla., but after one year, he returned to Charlotte, N. C., as City Engineer to rebuild its sewage disposal plant.

Always at heart a teacher, Mr. Firth went to Oklahoma Agricultural and Mechanical College, at Stillwater, Okla., as Associate Professor of Civil Engineering, in 1927. From there, in 1929, he returned to the place of his birth—Philadelphia, Pa.,—where he was engaged in the Bureau of Highways as Engineer of Tests and Materials, until his death.

Mr. Firth's career that culminated so suddenly, was one of great personal accomplishment. He was a thorough student and had the kindly faculty of imparting his knowledge to his subordinates and associates in a way that never savored of officiousness. Many knotty problems were made easy by a suggestion from his resourceful brain. It is a tribute worthy of aspiration that he left the world a little better than it would have been without him.

At Rensselaer Polytechnic Institute, he was honored by election to Sigma Xi; and at Oklahoma Agricultural and Mechanical College by election to Phi Kappa Phi. Mr. Firth was a Thirty-second Degree Mason; and a member of the Shrine, Oasis Temple, of Charlotte, N. C. He was also a member of the American Road Builders Association.

He was married to Maud Hunt, of Germantown, Pa., on October 22, 1907, and is survived by his widow and two children, James Hunt and Marjorie Delvalere Firth.

Mr. Firth was elected an Associate Member of the American Society of Civil Engineers on June 5, 1907, and a Member on July 6, 1925.

ROBERT FLETCHER, M. Am. Soc. C. E.[1]

Died January 7, 1936

Robert Fletcher was born in New York, N. Y., on August 23, 1847, the son of Edward H. and Mary (Hill) Fletcher. His early education was obtained in the public schools of New York City, followed by three years' attendance at the New York Free Academy, later the College of the City of New York, where he chose the Classical Course, including Greek, Latin, and rhetoric. In August, 1864, he won by competitive examination a coveted appointment to the United States Military Academy, at West Point, N. Y., and that month he passed his entrance examination and was enrolled as a Cadet in the Class of 1868. At the New York Free Academy, and at the U. S. Military Academy, he received the liberal arts education and the technical training which were to serve as his background for directing and teaching a Civil Engineering curriculum. The technical subjects studied were physics, chemistry, mathematics, descriptive geometry, free-hand drawing, surveying, and spherical astronomy.

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[1] Memoir prepared by Frank W. Garran, M. Am. Soc. C. E., and William P. Kimball, Assoc. M. Am. Soc. C. E.

Upon graduation in June, 1868, he was commissioned a Second Lieutenant in the First United States Artillery, serving for one year on garrison duty at Brownsville, Tex., on the Mexican frontier. In the spring of 1869, he was transferred with his Battery to Fort Trumbull, at New London, Conn. In August of that year his active military career ended, and a remarkable life as an educator of young men was undertaken in an appointment as Instructor of Mathematics under Professor A. E. Church, at the U. S. Military Academy, at West Point.

Lieutenant Fletcher's outstanding character and ability in this position led to a meeting with Gen. Sylvanus Thayer, U. S. Army, revered "father" of the U. S. Military Academy and veteran of the War of 1812 and of the Civil War. General Thayer, a man of broad vision, keen perspicacity, and remarkable singleness of purpose, viewed with concern the tendency toward narrow technical training of engineers, and was inspired with the vision of an engineering school for students prepared for a full life by college training in humanities and social studies. A graduate of Dartmouth College, at Hanover, N. H., General Thayer naturally chose this Liberal Arts College as the place for the engineering school which he wished to endow. Quite apparently the selection of a man to organize and direct the school along the lines outlined by its founder was a delicate task. General Thayer was a man of rigid ideals, both professional and moral, and many applicants for the position were rejected for a wide variety of reasons. Faced with the collapse of his plan for the lack of a qualified leader, General Thayer was finally advised by Professor Church of a brilliant young Lieutenant serving as Instructor of Mathematics in the Military Academy. Thus, it was that Lieutenant Fletcher found the ideal that was to constitute a major portion of his life work. An excerpt from a history of the Thayer School of Civil Engineering, founded in 1871, gives this insight into the character of the 24-yr old lad who was chosen to guide the destiny of this unique experiment in engineering education:

> "On the eighth of July [1870] Fletcher arrived in Hanover. Smith's [A. D. Smith then President of Dartmouth] reaction was that the young lieutenant's good manners and evident 'moral tone' were pleasing additions to his quiet personality. He was impressed with Fletcher's classical background gained from three years of liberal college training at C. C. N. Y. In a letter to Thayer he urged Fletcher's immediate appointment.

"The following day * * * Fletcher had a seven-hour interview with Thayer. * * * Thayer approved of Fletcher's scholastic accomplishments, his earnestness and energy and was impressed with the recommendation given by Professor Church of the West Point Mathematics Department."

In December, 1870, Lieutenant Fletcher resigned his commission in the U. S. Army and moved to Hanover, N. H., where a truly amazing task awaited him. If any present-day analogy could be drawn, it would be of a third-year engineering student faced with the task of setting up a complete curriculum of civil engineering in a liberal arts college, and, having set up the course, teaching every subject. To be sure the field of engineering has broadened since the day Robert Fletcher arrived by sleigh in the snow-bound village of Hanover; but so has the available literature increased. It is no longer necessary for engineering teachers to write their own texts, compose their own problems, and outline the very essentials of the subject-matter they teach.

It is impossible to write of Robert Fletcher without seeming to write a history of the school which was his life work, just as it is impossible to write of Thayer School during that period, without emphasizing the part played by Robert Fletcher during its forty-seven years under his direction.

Director Fletcher's first accomplishment was to prepare examinations, with the aid of Professor E. T. Quimby, of the Dartmouth College Faculty, to test the eligibility of candidates for the course specified in broad terms by General Thayer. These examinations covered subjects of mensuration, trigonometry, surveying, descriptive geometry, analytical geometry, calculus, mechanics, physics, astronomy, chemistry, geology, physical geography, and meteorology.

In June, 1871, Robert Fletcher was awarded the Honorary Degree of Master of Arts by Dartmouth College, and three months later he began the instruction of the first class of the Thayer School of Civil Engineering. During that year he taught the subjects of surveying, railroad engineering, industrial drawing, topographical drawing, fortifications, stone-cutting, statics of mechanics, dynamics of mechanics, lettering, geodesy, mining, hydrography, retaining walls, arches, strength of materials, architectural, structural, and machine drawing, materials of construction, hydrostatics, hydrodynamics, and the theory of machines described in his own words as dealing with "transmission of work by machines; modulus of machines moving with uniform or periodical motion and with variable motion; velocity of machines under variable motion; coefficients of the modulus; motion about a cylindrical axis; wheel and axle; pulley; sheaves; modulus of compound machine; capstan; Chinese capstan; whim gin; friction of cords; friction brake bands; theory of the teeth of wheels; involute teeth and how to describe them; epi-cycloidal and hypo-cycloidal teeth and methods of describing them; a train of wheels; strength of teeth; teeth of rack and pinion; lantern or trundle; conical or bevel wheels and the method of describing them", and, until 1876, he continued instruction in these courses and added to this curriculum the subjects of bridge construction, analytical chemistry, construction of cranes and hoists, drainage of lands, towns and buildings, astronomy, and foundations. During his forty-seven years of active direction and teaching in the school which he organized and led to a position of eminence in the field of Civil Engineering education, Professor Fletcher taught courses in twelve distinct branches of engineering. This breadth of scope was perhaps his greatest attribute, and distinguished him throughout his life time from his contemporaries who were outstanding in specialized fields in a world of increasingly specialized and restricted endeavor.

The influence of his personality on the students was profound. His classes were always small, so that it was possible for him to know intimately every student who attended his school. He made the most of this opportunity, and because of his keen interest in the characters as well as in the minds of his boys, his teaching was always alive. Fully appreciative of the value of technical learning as he was, he nevertheless felt that the greatest value he could give was in the development of high ideals, mental discipline, and an appreciation of human nature. He recognized that the laws of Nature are immutable, and he taught that the greatest folly of Man is disregard for or denial of these laws; but he recognized also that the laws of human behavior are variable, and he taught that eminence in engineering as in all things could be attained only by those who are able to shape and use the laws of human behavior to the accomplishment of ends not inconsistent with the laws of Nature.

Professor Fletcher was not a dogmatist; nor did he try to force on others his own convictions. His approach was indirect. He lived openly and consistently by his own lights, teaching the truth of his convictions by example rather than by the ever less effectual method of insistence. He was a sincerely religious man and rigidly temperate, never permitting himself the relaxing luxury of tobacco, alcohol, or profanity. Yet, rigorously observant of his own ideals, he recognized in these things too the variability of human behavior.

The tributes which his former students and associates have paid to Professor Fletcher are numerous and coming as they do from some of the leading engineers of the United States, they constitute the best testimonial to his true worth. The following is chosen as being as graphic as any words ever written of Robert Fletcher:

> "Professor Fletcher made a deep impression on me as a young man in his classes. His sincerity, his fine and very lovable character, together with his deep knowledge of many things, made it a privilege to be in his classes. We are all, perhaps, prone to visualize a composite of groups. As the figure of Uncle Sam or John Bull is symbolical, or as the word Southerner creates an image in our mind's eye, so the old-fashioned and much abused term, gentleman, in the finest sense of the word is associated in my mind with my remembrance of Dr. Fletcher. Some years ago a man who was then President of the American Society of Civil Engineers remarked in conversation to me, 'I met a friend of yours at the Annual Meeting. I have forgotten his name, but he looked like a character from Dickens.' I knew at once that it was Dr. Fletcher. He was rare; his kind is not so easy to find these days; and it will always be a source of pride to me that I knew him and was honored with his friendship."

These thoughts are typical of the opinions expressed time and time again by those with whom Robert Fletcher had contacts, yet the love and esteem of his fellow men were not the result of a soft or pliable nature. His early training at the U. S. Military Academy and his rigorous years in sole charge of the Engineering School at Dartmouth taught him the value of unwavering concentration, hard work, and self-discipline. That he could be severe when necessary is well remembered. He taught his students to spell the word rest, "r-u-s-t". His only criticism of contemporary Civil Engineering education was that the students are not required to work hard enough.

In 1881, at the age of 34, he was honored by Dartmouth with an Honorary Degree of Doctor of Philosophy, and thirty-seven years later, on the occasion of his retirement as Director of the Thayer School, he was awarded the Honorary Degree of Doctor of Science by Dartmouth. At this time he was appointed Director Emeritus of the Thayer School, a position which he held until his death. During this period he was also active as Clerk of the Board of Overseers of the Thayer School.

Small and slight of stature, he was physically active and energetic. He had great power of concentration and physical stamina, and he was a prodigious worker. Only a man of great determination and capability could have carried on the school duties, demanding twelve to sixteen hours' work per day over a period of many pears. Few men could add to that the pursuit of intellectual hobbies, active participation in church and public affairs, and a consulting engineering practice.

Both because of the demands of other activities and because of the somewhat remote location of Hanover, Robert Fletcher's professional accomplishments, outside the educational field, were not great as measured in terms of dollars or tons. He designed and supervised the construction of steel bridges across the Connecticut and White Rivers, and of water-works for the Towns of Hanover and Enfield, N. H., and was Consulting Engineer for water-works and reservoirs in Lebanon, N. H., and in Hartford and Woodstock, Vt. For more than forty years he was President and Engineer of the Hanover Water Company, as well as a member of the New Hampshire State Board of Health, of which he was President until 1934. In the latter capacity he performed great public services, most notably in the development of the tight, free-flowing septic tank. Owing to his work in developing and sponsoring this type of tank for homes and summer resorts, the old-fashioned, often obnoxious, cesspool has practically disappeared from the State, and the pollution of recreational streams and lakes has been greatly reduced. He was in charge of one-half the very extensive survey to determine the New Hampshire-Vermont boundary line, a notorious boundary dispute finally settled by a decision of the United States Supreme Court in 1933.

Throughout his life Director Fletcher contributed freely to technical periodicals and bulletins, and, occasionally, when the spirit moved him and the ignorance of his fellow men seemed too abyssmal, he wrote words of homely wisdom for the public press. Although these contributions covered a wide field of engineering and lay knowledge, including bridge design, water supply engineering, sewage treatment and disposal, geology, surveying, and mechanics, his outstanding work was "A History of the Development of Wooden Bridges", prepared in conjunction with the late Jonathan Parker Snow, M. Am. Soc.

C. E., and published with the voluminous discussion thereon by the Society.[2] All the work on this major contribution was carried on after Robert Fletcher had passed the age of 83, and this fact is only one of many indications that his mind was keen, his interest unfailing, and his sense of obligation to his profession as deep as ever during the last years of his life. This feeling of obligation to his fellows was expressed by Professor Fletcher himself who frequently quoted John Smeaton: "The abilities of the individual are a debt due to the common stock of public well being."

His hobbies and recreations were of a type consistent with his nature; most men would consider them intelluctual pursuits and would perhaps prefer lighter, more trivial, relaxation. One of these hobbies was an intense study of volcanology, both in delving into theoretical and technical causes and effects and in following the sometimes too apparent manifestations of these phenomena. A related hobby was archæology, and still another was scientific research in the history of the Bible lands, and the development of civilization in all parts of the world.

He enjoyed one relaxation, however, which really overshadowed all the other pleasures of his life; that is, following the fortunes and accomplishments of his former students. He found this easy to do, for the affection and esteem in which his students held him, prompted them to write often of their trials and tribulations and triumphs. In 1916, after forty-five years as Director of the Thayer School, Professor Fletcher traveled from New England to New Orleans, La., to the Pacific Coast, and back East by a northerly route, visiting at frequent intervals engineers who had formerly been his students. These visits took him to some of the greatest engineering works of the time. It was a truly rich reward to him to see these men, whom he had taught as boys, in responsible charge of such projects. This trip, with the lasting pleasure which it brought, was perhaps Robert Fletcher's major self-indulgence.

He was an active member of the Society for the Promotion of Engineering Education from the time of its organization in 1892 and served as its President in 1901–02.

He was married in June, 1872, to Ellen M. Huntington who, with their daughter, Mary A., and his two sisters, Harriet and Helen, survives him. His only son, Robert H. Fletcher, Professor at Grinnell College, Grinnell, Iowa, died in 1919.

On January 9, 1936, Robert Fletcher was buried in the Old Cemetery in Hanover, N. H., within a few blocks of the house in which he had spent sixty years of his long and noble life, and whether we knew him as Professor, or Doctor, or "Bobby", his passing leaves a gap in our lives which can never be filled.

At the time of his death, he had been associated with the Society longer than any other man—sixty-one years. This distinction, however, was not one which he would have chosen to contemplate, for he was always a little ashamed of growing old and disliked to recognize in himself symptoms of the passing years.

Professor Fletcher was elected an Affiliate of the American Society of Civil Engineers on November 4, 1874, and a Member on August 31, 1909.

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[2] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 814. (Paper No. 1864.)

SAMUEL GOURDIN GAILLARD, M. Am. Soc. C. E.[1]

DIED NOVEMBER 17, 1936

Samuel Gourdin Gaillard was born on July 17, 1853, at Hayden Hill Plantation, Upper St. Johns, Berkeley County, South Carolina, the son of Eugene M. and Camilla (Richebourg) Gaillard, both of that section of South Carolina. He was of French Huguenot descent on both sides. Following the revocation of the Edict of Nantes, his ancestors, together with many other Huguenot refugees, left France and settled on the Santee River, at Jamestown, S. C., early in the 1680's. These French Huguenot families played an important part in the history of South Carolina, and Mr. Gaillard's ancestors were no exception—one, Captain Peter Gaillard, was an Officer with the Colonial Forces during the Revolutionary War; another, the Hon. John Gaillard, was for many years, United States Senator from South Carolina. Colonel David Dubose Gaillard, the engineer in charge of the Culebra Cut of the Panama Canal and for whom that Cut was named after his death, was a cousin and close friend. During the Civil War, all the men of his family able to bear arms were in active service on the side of the Confederacy. Mr. Gaillard, himself, although only a boy of ten or eleven years, was old enough to remember some of the incidents of the latter years of the war—especially the visit, in the dead of night, of Northern troops to his family's plantation, and the complete despoiling of that place. Although still very young, he took an active part in the difficult Reconstruction Period, and was one of Hampton's "Red Shirts" Later, he obtained a scholarship at the Porter Military Academy, at Charleston, S. C., and upon graduation obtained another scholarship at Union College, at Schenectady, N. Y. There, he made many life-long friends, and was graduated as a Civil Engineer in 1881. While an undergraduate at Union College he was elected a member of Kappa Alpha Society.

Upon graduation from Union College, Mr. Gaillard obtained a position as Assistant Engineer with the Shenandoah Valley Railroad Company in Virginia. This position came to him on the recommendation of the Professor of Civil Engineering at Union College to whom the late Foster B. Morss, Civil Engineer, a graduate of the Class of 1864, had written, asking him to name a recent graduate qualified to be his Assistant on the construction of the Shenandoah Valley Railroad (now a part of the Norfolk and Western Railroad System). After about two years in this position, Mr. Gaillard accepted the offer of Mr. Morss to go with him as his Assistant with the railroad company being organized in Connecticut to purchase the New England Railroad and build connections to New York, N. Y., and Boston, Mass., and, in the capacity of Assistant Engineer, he surveyed much of the proposed line of the Hartford and Harlem Railroad. This road was not built because the plans for financing failed. In 1884, he went to the Norfolk and Western Railroad Company as Assistant Engineer with the Construction Department. From this position he was promoted to be Resident Engineer, Engineer in Charge of Construction, and, finally, Assistant to the President, with headquarters in Philadelphia, Pa.

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[1] Memoir prepared by Edward M. Gaillard, Esq., New Haven, Conn., assisted by Foster G. Morss, M. Am. Soc. C. E.

When the Norfolk and Western Railroad Company went into Receivership, the position of Assistant to the President was abolished and Mr. Gaillard was returned to the Engineering Department as Division Engineer. In 1896, at the solicitation of Arthur W. Sewall, Secretary of the Mack Manufacturing Company, of Philadelphia and New Cumberland, W. Va., Mr. Gaillard accepted the position of Superintendent of Mines and Factories of that Company at New Cumberland. This Company was engaged in the manufacture of vitrified paving brick and vitrified clay sewer pipe and although Mr. Gaillard had had no previous experience in that line, he secured every publication available on the subject and became an authority on the manufacture of clay products He held, successively, the positions of General Manager, Vice-President, and President of the Company, and continued in this last capacity until 1919 when control of the property was sold to new interests. In 1920, Mr. Sewall, then President of the Barber Asphalt Company (General Asphalt Company), now Chairman of the Board, appointed Mr. Gaillard Chief Consulting Engineer of the Uintah Railway Company, a subsidiary operating in Western Colorado and Eastern Utah, and under his supervision surveys were made for an extension of the railway into the Gilsonite fields in Utah and for a grade revision survey over the Book Cliff Mountains where a line was run with 3% grades and 16° curves to replace the existing 5% and 7½% grades and 66° curves. For a few years before retiring. he held a position with the General Asphalt Company of Philadelphia.

In the course of his engineering work, he was the author of numerous reports on ores, and, while he was with the Mack Manufacturing Company, he made minor inventions in brick machinery and improvements in the design and construction of the plants

Mr. Gaillard was fond of all kinds of sports, especially of shooting, and most of his vacations were spent on hunting expeditions with friends in South Carolina.

In June, 1891, he was married to Esther Lynch McCrady, a daughter of Professor and Mrs. John McCrady, of Charleston, S. C. After retiring from active business about five years ago (1931), Mr. Gaillard moved to New Haven, Conn., in order to be near his sons, Samuel G. Gaillard, Jr., and Edward M. Gaillard, who, with his widow, survive him.

In addition to his business activities, Mr. Gaillard was for six years President and Commissioner of Hancock County, West Virginia. He was a life long member of the Protestant Episcopal Church. During the World War, he was a member of the Navy League. He was also a member of the National Geographic Society, Society for Scientific Research, and a Life Member of the Huguenot Society of South Carolina.

Mr. Gaillard was elected a Member of the American Society of Civil Engineers on October 1, 1912.

CHRISTOPHER LAWRENCE GATES, M. Am. Soc. C. E.[1]

Died August 30, 1920

Christopher Lawrence Gates was born in Nüremberg, Germany, on June 13, 1848, the son of Christopher Lawrence and Anna Sibylla Goetz. His early education was gained in the common schools of that city, where, after competitive examinations, he was admitted to the Polytechnic School at the age of 12.

In 1865, Mr. Gates came to the United States, and settled in Fort Wayne, Ind., where he was engaged as a Machinist. From Fort Wayne, he moved to Milwaukee, Wis., where he was employed by the Milwaukee Bridge and Iron Works and the Chicago, Milwaukee, and St. Paul Railroad Company as a Designer of railroad bridges until 1886.

He spent a year with the Clinton (Iowa) Bridge Company, and then went to Toledo, Ohio, as Railroad Bridge Engineer for the Smith Bridge Company, continuing in that capacity after this Company sold its interests to the Toledo Bridge Company. His naturalization papers were taken out about the time he came to Toledo.

When the Toledo Bridge Company was absorbed by The American Bridge Company in 1900, Mr. Gates severed his connection with it, and for the next twenty years did occasional consultation work. He represented the Toledo Bridge and Crane Company in the erection of the United States Naval Coaling Station in Guantanamo Bay, Cuba, in 1908, and in the erection of wireless towers in Darien, Panama, in 1914. He was also Consultant on bascule bridges for the City of Saginaw, Mich., and for the Ann Arbor and Pere Marquette Railroad Companies, the Saginaw Valley Traction Company, and the City of Toledo.

He was a Director in the Toledo Bridge Company, The Toledo-Massillon Bridge Company, The Toledo Bridge and Crane Company, and The Toledo and Indiana Interurban Company.

Mr. Gates became a Mason in 1879, as a member of Independence Lodge, in Milwaukee. In Toledo, he was connected with Toledo Commandery No. 7, Knights Templars, Fort Meigs Chapter, Sanford L. Collins Lodge, Scottish Rite, Zenobia Shrine, and Fraters Club.

For many years, he was a Trustee of the Central Congregational Church, of Toledo, and represented that body on its merger with the First Congregational Church.

Mr. Gates was an active member of the Toledo Engineering Society, serving in various official capacities, and had a wide acquaintance both in and out of engineering circles. He was a man of pronounced and outspoken convictions for which he was admired by his many friends. Mr. Gates passed away in Milwaukee, his old home, where he had gone on a business trip, after a few days of very sudden illness.

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[1] Memoir prepared by A. H. Smith, Assoc. M. Am. Soc. C. E.; L. M. Gram, M. Am. Soc. C. E., and W. G. Clark, Jun. Am. Soc. C. E.

He was married on May 30, 1872, to Jennie Young Walker, who passed away in 1909. Of eight children, but three survive their father: Arthur O. Gates, of Salt Lake City, Utah; and the Misses Florence A. and Louise W. Gates, of Toledo. Christopher Lawrence Gates, Jr., his youngest son, who accompanied him to the Annual Convention of the Society and the trip to the Panama Canal, in 1911, was drowned the following year at Lafayette, Ind., where he was a student at Purdue University.

Mr. Gates was elected a Junior of the American Society of Civil Engineers on December 4, 1878, and a Member on September 5, 1883.

MARTIN GAY, M. Am. Soc. C. E.[1]

DIED NOVEMBER 23, 1935

Martin Gay, the son of Sydney Howard and Elizabeth Johns (Neall) Gay, was born on Staten Island, New York, on May 15, 1854. He came of a long line of Quaker and New England ancestry. His father was a journalist of note, whose principal work was done in Civil War times as Editor of the *Anti-Slavery Standard,* in 1844, and of the *New York Tribune.* Subsequently, he became Editor of the *Chicago Tribune* and, later, of the *New York Evening Post.*

His paternal progenitor—John Gay—came from England in 1630 and, with his wife, Joanna, located at Watertown, Mass. John Gay was one of the founders of Dedham in that State. His grandson, the Rev. Ebenezer Gay, was noted among early clergymen for his vitality and unflagging interest in his religious work at Hingham, Mass. Ebenezer Gay died in his ninety-first year on returning to his home after preaching in the Old Ship's Church. A model of this church is in the Colonial Exhibition in the Metropolitan Museum of Art in New York City, and the original is still standing and in good repair, at Hingham. From Ebenezer the line of descent is through his son, Martin, who set up in Boston, the first copper foundry in the Colonies, and the latter's son, Ebenezer, who was a lawyer, the father of Sydney Howard, who was the grandfather of Martin Gay. The ancestral home of the Gays, which was built by the Rev. Ebenezer Gay, still stands in Hingham. For the last thirty years of his life, Martin Gay with his family spent the summers in this old home, which was occupied by the Gays for more than 200 yr. He loved the old home, its furniture, and environment, and liked to work there maintaining it.

From the maternal side of his family, all of whom were Quakers, Martin Gay inherited a patience and tolerance seldom found in the swift tempo of life to-day. A straight line of descent may be drawn from the Mifflins, Nealls, and Justices, all of whom were early settlers in and about Philadelphia, Pa. Warner Mifflin, son of John Mifflin, "before God and my con-science, feeling it wrong to enslave my fellow man * * *", was the first man to liberate his slaves, and this action took place nearly one hundred years before the Civil War and many years before such action was taken in England. His original deed of manumission remains a cherished heirloom of the family. The site of the old Mifflin homestead is marked with a tablet and comprised, what is now a substantial part of Fairmont Park, in Philadelphia.

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[1] Memoir prepared by J. A. Knighton and Arthur W. Tidd, Members, Am. Soc. C. E., and Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E.

Martin Gay was educated first in private schools in Chicago, Ill., and on Staten Island. In 1877, he was graduated from the Massachusetts Institute of Technology, at Boston, Mass. During vacation periods he had his first professional work in Nebraska in railroad surveying. It was pioneering work, sleeping in sod houses, posting sentries against Indians, and long distances to go for a water supply. He was in nowise appalled by these hardships and liked to tell of them retrospectively. Later, he did some work in connection with the construction of the Second Avenue Elevated Railroad in New York City.

On September 17, 1877, he was appointed an Axeman with the Department of Public Works of the old City of New York, serving on the work in the Croton Water-Shed under the Chief Engineer, George W. Birdsall, and was promoted through the various grades until he reached the rank of Assistant Engineer on November 20, 1883. He was in charge of the construction of a section of the Bronx River 48-in. pipe line and of the Williamsbridge Reservoir. Then, about 1894, he was in charge of the construction of Macomb's Dam Bridge over the Harlem River for the Department of Parks, which was designed by the late Alfred P. Boller, M. Am. Soc. C. E. Mr. Gay remained on this work until its completion. During this period he was also Engineer in charge of the completion of the Third Avenue Bridge, over the same waterway, for the Department of Public Works, under the late George W. Birdsall, Chief Engineer. This bridge was designed by the late Thomas Curtis Clarke, Past-President, Am. Soc. C. E.

With the consolidation of the City of New York into its present form, on January 1, 1898, Mr Gay was transferred to the Department of Bridges, later, called the Department of Plant and Structures, with which he remained for twenty-three years until he retired on January 1, 1921. His outstanding work was with that Department. At first, he was assigned to the preparation of a plan designed to make the City's newly created Department of Bridges function smoothly and to co-ordinate its various activities. The Brooklyn Bridge, formerly under the supervision of a separate commission, was naturally the focal point of his plan, that structure being the largest bridge in the jurisdiction of the Department, and the only one in operation at that time over the East River.

Among other bridges over the Harlem River the construction of which he supervised were those at Willis and Madison Avenues. While thus employed, Mr. Gay was also in charge of the maintenance of the Harlem River bridges. It has been stated, and is a fact worth noting, that all his bridge work was so carefully planned and the construction co-ordinated with such vigilance and care that no man was ever killed in the operations. He took much pride and satisfaction in this record. His creditable and faithful service with the Department of Bridges and the Department of Plant and Structures was identified principally with these Harlem River bridges with which construction and maintenance he had so much to do, and the details of which he knew so well. One who went to Mr. Gay's office was sure of a courteous and kindly welcome and received the information he came for if it was possible to give it to him. He was a cultivated gentleman, a scholarly Engineer, and a loyal friend, greatly beloved by all who had the privilege of knowing him, and was withal a real man.

On his retirement from active service on January 1, 1921, the Department of Plant and Structures by formal resolution, sent to his widow on his death, testified that he had endeared himself to his associates by his devotion to the City's welfare and by his justice, kindness, cheerfulness, and humanity. Following his retirement, Mr. Gay's interest in the work continued, and he kept up his visits to the office as long as his health permitted. He often gave lectures illustrated with lantern slides, intended for engineers, but in which there was a keen popular interest. In the leisure of his retirement he made an exhaustive study and prepared a history of the bridges of New York City—not including the East River bridges. He had the literary and historic sense as well as the technical talent to do justice to his topic, and spent many hours of investigation of old records, newspapers, and reports. This history is said to be practically the only account that is available of the "birth" of the earlier bridges and is a valuable document. He was a man of infinite patience and loved to do any such work that required research. He upheld the ideals of his profession and was a credit to the Engineering Service of the City he served so long, efficiently, and unostentatiously.

Mr. Gay was domestic in his tastes, and was not an active club man. He was fond of reading and was never at a loss for occupation. He liked carpentry. He had watches that had been in his family for six generations, and he made a box to fit them. He knew his grandfather clocks and could mend them and make them keep correct time when they were the despair of the modern clockmaker. Out of such things and indeed out of life in general, he extracted much of humor and sunshine for he had the genial gayety that goes with a mind at rest because he was always just and considerate. His was a beautiful character. He lived up to the favorite maxim of his grandfather, "Be good, do good, and be ever mindful of the rights of others." He was a close reader of current events, a student of economics and history, and an earnest advocate of the single tax theory, being a personal friend of its chief exponent, Henry George.

Mr. Gay died in New York City on November 23, 1935—a kindly compassionate man of unusual altruism who gave freely and quietly to private charities and whose personal probity was above reproach His body was taken to his ancestral home in Hingham, and, with a simple and impressive service, was buried in the cemetery there.

On September 5, 1895, he was married to Julia DeWitt Stone, of Staten Island, New York. She survives him, as do two daughters, Mrs. Martha

(Gay) Whiting, of Hingham, Mass., and Mrs. Elizabeth Neall (Gay) Pierce, wife of William Curtis Pierce, of New York City.

Mr. Gay was elected a Junior of the American Society of Civil Engineers on June 4, 1884, and a Member on June 5, 1889. He was also a member of the Municipal Engineers of the City of New York.

GEORGE ARCHIBALD GRAHAM, M. Am. Soc. C. E.[1]

DIED FEBRUARY 11, 1937

George Archibald Graham was born on October 2, 1878, at Fort Ricasolia, Valetta, Malta, in the Mediterranean Sea, the son of the late Captain W. J. Graham, of the British Army, and Huldah (Barnes) Graham. He received his early education at Oxford, England, later attending Oxford University, where he took a Civil Engineering Course.

Soon after leaving college, Mr. Graham became Town Engineer for Whitney, Oxfordshire, England, where he remained for eight years. He then emigrated to Canada, and became connected with the Trans-Continental Railway, having been engaged in exploration, surveying, and construction for a number of years. Attracted by the lure of the South, he went to Jacksonville, Fla., in September, 1911, where for six and one-half years, he was Assistant Engineer in the City Engineering Department. During this time, he assisted in the preparation for a complete survey of the city, besides aiding in the remodeling of the water-works and underground electrical construction.

At the outbreak of the World War, Mr. Graham volunteered for active service, and was commissioned a First Lieutenant in the United States Corps of Engineers. He served at Camp Humphreys, in Virginia, and, for nearly two years, at Camp Dix, in New Jersey. After his discharge from the Army he went to Daytona Beach, Fla., in November, 1922, and immediately began to practice his profession. Among the subdivisions in the Halifax country which Mr. Graham surveyed may be mentioned the following: El Pina Parque, Kahnway Heights Seabreeze Park, and about one hundred others. He also prepared more than three hundred maps of the Halifax country.

Throughout his life Mr. Graham was noted for his fairness and high sense of business responsibility. He was a member of the American Association of Engineers and the American Water Works Association. He was also associated with the Institution of Civil Engineers of Great Britain. Locally, he was a member of the Chamber of Commerce and of the Masonic Fraternity.

For the last three years of his life, Mr. Graham was employed by the Government as Chief of Party No. 13, of the United States Coast and Geodetic Survey of Clay County, in Florida.

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[1] Memoir prepared from information on file at the Headquarters of the Society and from data from "A History of Volusia County, Florida".

Besides his widow, Mrs. Alice Isobel Graham, and two children, Edgar J. and June Graham, he is survived by his sister, Rose Graham, of Oxford, England, and a brother, Professor Richard Graham, of McGill University, at Montreal, Que., Canada.

Mr. Graham was elected an Associate Member of the American Society of Civil Engineers on August 9, 1920, and a Member on April 12, 1926.

PAUL EVANS GREEN, M. Am. Soc. C. E.[1]

Died March 12, 1937

Paul Evans Green was born at Richmond, Ind., on April 23, 1879, the son of George Donaldson and Carrie Belle (Evans) Green.

He attended public and private schools in Wheeling, W. Va., and received his technical education at the State University of West Virginia, at Morgantown, W. Va., from which he was graduated in 1899 with the degree of Bachelor of Science in Civil Engineering. He was a member of the Kappa Alpha (South) Fraternity.

Mr. Green's first engineering work was with the Illinois Central Railroad Company as Rodman and Inspector. From 1900 to 1906, he was employed as Instrumentman, and, later, as Assistant Engineer in charge of new construction, and masonry and steel bridges, in Minnesota, the Dakotas, and Wisconsin. During 1906, he was Assistant Engineer on the New York State Barge Canal as Designer of reinforced concrete bridges, spillways, locks, dams, and other hydraulic works.

After 1906, Mr. Green made his home in Chicago, Ill. He was Division Engineer for the City, in charge of street improvements on the North Side. For the last thirty years of his life, he had been in private practice as a Consulting Engineer. The firm of Marr, Green, and Company was formed in 1908, but twenty years later, the name was changed to Marr, Green, and Opper. With these firms Mr. Green served more than one hundred cities and villages in the Middle West, as Engineer on water supply, sewerage, paving, and other municipal works.

As a citizen, Mr. Green was actively interested in the betterment of civic affairs. His abilities, as an independent consultant, were recognized. He made an investigation and report for the State's Attorney of Cook County on the operations of the Sanitary District of Chicago. He also served as a member of the Advisory Board of Engineers for the West Chicago Parks, on the proposed super-highway, and was Consultant on the design of the double-decked Wacker Drive improvement. The Finance Committee of Chicago engaged Mr. Green as its adviser on cost examinations.

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[1] Memoir prepared by L. K. Sherman, M. Am. Soc. C. E.

During the World War, he was in Washington, D. C., with the United States Bureau of War Housing and Transportation, under the Chief Engineer, John W. Alvord, Hon. M. Am. Soc. C. E. Mr. Green had charge of about fourteen projects connected with munition plants.

In 1934, he served as an Associate Consultant for the National Resources Board on the inventory of water resources of the Great Lakes District. Mr. Green also served as Assistant Chief Engineer with the Civil Works Administration for the State of Illinois in 1933, and as Chief Engineer in 1934.

Much of Paul Green's work as an Engineer had to do with men, rather than materials. His genial personality won him many friends. When he was a subordinate, he was loyal, dependable, and free from envy. As a leader, he was respected for his calm judgment, integrity, prompt decision, and kindly interest in the welfare of his associates.

He was an enthusiast at golf, a member of the Evanston, Ill., Golf Club, and did considerable work in the layout, drainage, and sprinkling systems of golf courses.

Mr. Green was the author of a number of technical papers. One of his latest was a study of the cause of failure of a water supply storage dam near Herrin, Ill. He was a member and Past-Director of the Western Society of Engineers. He was also a member of the American Water Works Association, American Society for Testing Materials, and the Illinois Society of Engineers, and served as President of the latter for the year 1936.

In 1911, he was married at Fort Wayne, Ind., to Jessie Godfrey Winslow. Mrs. Green, his mother, a sister, and a brother survive him.

Mr. Green was elected an Associate Member of the American Society of Civil Engineers on July 10, 1907, and a Member on October 1, 1912.

JOHN HERBERT GREGORY, M. Am. Soc. C. E.[1]

Died January 18, 1937

In the passing of John Herbert Gregory in the midst of an active and eminent career, the profession has lost one of its pioneers in modern sanitary engineering, that branch of engineering in which the great advances in the art of water purification and sewage treatment have taken place during the last four or five decades.

John Herbert Gregory was born in Cambridge, Mass., on August 7, 1874, the son of John P. and Mary C. (Stone) Gregory. His early education was obtained in the public schools, and his technical education at the Massachusetts Institute of Technology, from which he was graduated in Civil Engineering in 1895.

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[1] Memoir prepared by the late Harrison P. Eddy, Past-President, Am. Soc. C. E., and J. Trueman Thompson, M. Am. Soc. C. E.

In college, he was recognized, by teachers and students alike, as a leader in scholarship and an exceptionally able, orderly, and neat worker. Mr. Gregory always remained deeply interested in the Institute and took just pride in his Class, many members of which have attained distinction.

His early work was chiefly in Massachusetts, where he was employed by the Metropolitan Sewage Commission during college vacations and, later, by the Metropolitan Water Board upon problems of design relating largely to the Wachusett Dam and the Wachusett Aqueduct under the Chief Engineer, the late Frederic P. Stearns, Past-President, Am. Soc. C. E.

Mr. Gregory was intimately associated with early water purification practice in the United States. At Albany, N. Y., while in the employ of the late Allen Hazen, M. Am. Soc. C. E., he was Assistant Engineer and, later, Resident Engineer on the design and construction of the slow sand-filter plant, the first large covered plant of its kind in this country. Thus, in less than five years after graduation, Mr. Gregory held a position of responsibility in design and construction.

From 1900 to 1902, he was Assistant Engineer in Charge of Designs upon a project for the improvement, extension, and filtration of the water supply of Philadelphia, Pa., including the City's first four water filter plants, of which the Torresdale Plant, with a capacity of 100 000 000 gal per day, was then the largest in the world.

In 1902, Mr. Gregory was Division Engineer in charge of building the 22-mile conduit from Boonton, N. J., to Jersey City, N. J., for the new water-works of that city. Following this, he was Principal Assistant Engineer in the Filtration Department of the Commission on Additional Water Supply of the City of New York. In 1903, for the late Rudolph Hering and George W. Fuller, Members, Am. Soc. C. E., he personally made the designs for the rapid sand-filter plant built by the Hackensack Water Company, at New Milford, N. J., with a capacity of 24 000 000 gal per day. This was the second large rapid sand-filter plant built in the United States.

In some ways, Mr. Gregory's most noteworthy professional work was for the Water Department of the City of Columbus, Ohio, which he served intermittently for a period of 33 yr. Beginning in 1904, he was, successively, Engineer of Design, Principal Assistant Engineer, and Engineer in Charge. His first undertaking was the design and construction of the Julian Griggs Dam and Reservoir. He then designed and put in operation the first large sewage testing station built by an American city. In addition to the testing of various methods of sewage treatment, extensive research was carried on in the field of water softening. As Engineer in Charge of Improved Water and Sewage Works, Mr. Gregory was in charge of both the design and construction of the water purification and softening works (30 000 000 gal per day) and of the sewage treatment works (20 000 000 gal per day). The water treatment plant was the third large rapid sand-filter plant in the United States, and the largest water-softening plant then built. The trickling filters at the Sewage Treatment Works were the first large trickling filters designed and placed under construction in the United States. In 1910, Mr. Gregory was awarded the Thomas Fitch Rowland Prize by the Society for his paper entitled "The Improved Water and Sewage Works of Columbus, Ohio."[2]

From 1920 to 1925, Mr. Gregory served as Consulting Engineer to the City of Columbus, in connection with the design and construction of additional water supply works which had been recommended by him in a report made to the City in 1919. These additions included mainly the enlargement of the water-purification and water-softening works from a capacity of 30 000 000 to 54 000 000 gal daily, and the construction of the O'Shaughnessy Dam and Reservoir on the Scioto River. For a paper entitled "The O'Shaughnessy Dam and Reservoir"[3] written by Mr. Gregory in collaboration with C. B. Hoover and C. B. Cornell, Members, Am. Soc. C. E., the James Laurie Prize was awarded them by the Society.

From 1926 to the time of his death Mr. Gregory served the City of Columbus as Consulting Engineer, in connection with the design and construction of many miles of relief and separate sewers and storm drains, and the new intercepting sewers and sewage treatment plant. The storm stand-by tanks which were built in connection with the sewage treatment works were the first to be built in the United States. The Rudolph Hering Medal of the Sanitary Engineering Division of the Society was awarded in 1935 for the paper entitled "Intercepting Sewers and Storm Stand-By Tanks at Columbus, Ohio"[4], written by Mr. Gregory in collaboration with Robert A. Allton, Orris Bonney and the late R. H. Simpson, Members, Am. Soc. C. E. In recognition of Mr. Gregory's services well and faithfully performed over a long period of years, first, in the regular employ of the City and, subsequently, in the capacity of Consulting Engineer, the City Council of Columbus passed a resolution of appreciation on January 25, 1937.

On the Passaic Valley Sewage Project in 1909 and 1910, he was Resident Engineer in charge of surveys and designs for the 20-mile Passaic Valley trunk sewer. He then entered the employ of the Metropolitan Sewerage Commission of New York, and was engaged primarily on investigations and studies relating to the collection, treatment, and disposal of sewage of the City of New York. One of the sites selected by him for sewage treatment works was the northeasterly end of Ward's Island, where the City now has nearly completed an activated-sludge treatment plant.

From 1911 to 1917, Mr. Gregory was a member of the firm of Hering and Gregory. Consulting Engineers and Sanitary Experts. After Mr. Hering's retirement from active practice in 1917, Mr. Gregory continued the practice until 1919, except for a period in 1918 when he was a Captain in the Sanitary Corps, United States Army. During the period devoted exclusively to private practice, he was Consulting Engineer to a large number of cities and towns, being engaged particularly upon problems relating to water, sewage, and municipal refuse.

Mr. Gregory became a member of the Faculty of the School of Engineering of The Johns Hopkins University, at Baltimore, Md., in 1919. In 1920, he was appointed Professor of Civil and Sanitary Engineering, occupying at the same time a similar Chair in the School of Hygiene and Public Health. Both these offices he held until his death. His wide experience in the management of affairs not connected with University life brought to the Advisory Boards of both these Schools many practical suggestions of great value. His services were also much in demand by other committees of the University.

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[2] *Transactions*, Am. Soc. C. E., Vol. LXVII (1910), p. 206.

[3] *Loc. cit.*, Vol. 93 (1929), p. 1428.

[4] *Loc. cit.*, Vol. 99 (1934), p. 1295.

Aside from his work-a-day relations with his students, his principal interest of an extra-curricular nature was that which he showed for many years in the Johns Hopkins Student Chapter of the Society. He was responsible for its establishment in 1921, and served as its Adviser for a number of years. A day or two before his death, he generously assumed the financial responsibility of a trip by the Senior members of the Chapter to the Annual Meeting of the Society in New York City.

During his connection with the University, he continued to act in a consulting and advisory capacity in so far as his teaching obligations would permit, and a large proportion of his most important work was done in this period. A few of his notable engagements, in addition to his work at Columbus, may be selected for special mention.

For about ten years Professor Gregory served as Consulting Engineer to the Sewer Department of the City of Baltimore in connection with the design and construction of large sewers and the enlargement of the sewage treatment works. For many years he served, and was serving at the time of his death, as Consulting Engineer to the Public Improvement Commission of the City of Baltimore on water supply matters. Among the works built and planned were the Prettyboy Dam and the Prettyboy Reservoir which was created by the construction of the dam. He served from 1932 to 1935 as one of three advisory engineers named to make an investigation and report on the need of an additional water supply for Baltimore. Professor Gregory was serving as one of the same group in an advisory capacity in connection with carrying out one of the steps recommended, namely, a duplicate water-works tunnel, 12 ft in diameter and 6 miles long, from the existing source of supply to the city. A resolution appreciative of his services to the City of Baltimore was adopted by the Public Improvement Commission on January 25, 1937.

In 1925, he served as one of the twenty-eight members of the Engineering Board of Review of the Sanitary District of Chicago, Ill., in connection with the controversy over diversion of water from Lake Michigan and the lowering of the levels of the Great Lakes which was alleged to have resulted therefrom.

In the same year, he served as one of three consulting engineers appointed by the Special Committee on Sewage Disposal of the City of Detroit, Mich., to advise as to the collection and disposal of the sewage of that city. The Board recommended the building of intercepting sewers and sewage treatment works. In 1936, Professor Gregory and his former associates were retained to advise upon the design and construction of these works.

He was chosen in 1933 as one of the three consulting engineers to make an investigation of the sewage disposal problem of the National Capital. Subsequent to their report, they were again retained in connection with the design and construction of remedial works.

At the time of his death, Professor Gregory was serving as Consulting Engineer to Samuel A. Greeley and Paul Hansen, Members, Am. Soc. C. E., on the design of the sewage disposal works at Buffalo, N. Y.

His services as expert witness were sought in important Court cases. Among those in which he participated were the States of Wisconsin, Minnesota, Ohio, Pennsylvania, Michigan, and New York *vs.* the State of Illinois and the Sanitary District of Chicago, in which an injunction was sought to prevent the diversion of water from Lake Michigan through the Main Drainage and Calumet-Sag Channels, Des Plaines River, and Illinois River to the Mississippi River; the State of Connecticut *vs.* the Commonwealth of Massachusetts, in which an injunction was sought to prevent the diversion of water from the Ware and Swift Rivers, tributaries of the Connecticut River, for use in the Metropolitan District of Boston; and the condemnation proceedings brought by the Passaic Valley Water Commission of New Jersey to acquire the water supply works of the Montclair Water Company, then supplying water to Paterson, Passaic, and other cities and towns in Northern New Jersey.

In 1932, Professor Gregory was appointed and served for a year as one of the five members of the Engineers' Advisory Board of the Reconstruction Finance Corporation. The function of the Engineers' Advisory Board was to advise the Board of Directors of the Corporation as to the merits of the applications made for funds to build so-called self-liquidating public works, for the financing of which Congress had appropriated $1 500 000 000. Professor Gregory's appointment was due to his wide experience in the design and construction of water supply and sewerage works. Of the loans made, 58% were for such projects.

The technical papers and discussions written by Professor Gregory are numerous and have appeared in various publications. The most important are the three papers presented to the Society, for which Society prizes were awarded as previously noted. Another literary field in which he participated was the reviewing of technical books, especially those dealing with Hydraulics and Sanitary Engineering.

Thus, for more than forty years, Professor Gregory had devoted his time, energy, and skill to the practice of Sanitary Engineering. His work as designer, supervisor of construction, teacher, consulting engineer, author, and collaborator in professional society activities was at all times marked by accuracy, refinement of detail, and fidelity to the highest standards and ideals of the Engineering Profession.

In the course of his professional career, Professor Gregory accumulated a comprehensive reference library of more than 5 000 volumes, pertaining to sanitary and municipal engineering and allied subjects. In accordance with his wishes, this library has been presented to the City of Columbus, Ohio, as an expression of appreciation of the confidence placed in him during his many years of service to that city. The library is now established in the City Hall Building, where it is available to any one who desires to make use of it.

Professor Gregory's participation in professional society activities, his voluminous correspondence in quest of information, and his contacts with teachers and students gave him a wide acquaintance and a large circle of close friends who recognized his sterling qualities.

He was married, in Boston, on July 16, 1900, to Sarah Ann James. Mrs. Gregory and one son, Richard Sears Gregory, survive him.

He held membership in the Boston Society of Civil Engineers, American Society for Testing Materials, American Public Works Association, American Public Health Association, American Water Works Association, New England Water Works Association, California Sewage Works Association, Maryland-Delaware Water and Sewerage Association, Engineers' Club of Baltimore, and the Society of Sigma Xi. He was an Honorary Member of Tau Beta Pi.

Professor Gregory was elected a Junior of the American Society ot Civil Engineers on January 3, 1899; an Associate Member on April 3, 1901; and a Member on December 4, 1906. He was a Director for the 3-yr term, 1932-34, serving on the Publication Committee, of which he was Chairman during his last year on the Board of Direction.

GEORGE TILLINGHAST HAMMOND, M. Am. Soc. C. E.[1]

DIED AUGUST 9, 1936

George Tillinghast Hammond was born in Newport, R. I., on August 14, 1863, the son of George Tillinghast and Mary Elizabeth (Shipman) Hammond, His father was the publisher of the *Newport Daily News* and also served as Deputy Collector of Customs, New York, N. Y., during the administration of President Grant. His grandfather, William G. Hammond, had been Collector of Customs of Newport, R. I. The Hammonds were among the earliest of the settlers in Rhode Island and were among the most conspicuous of the earliest families. His Grandfather Shipman was the Chief Engineer of the Long Island Railroad during the early development of that road between 1836 and 1848.

Mr. Hammond prepared in his youth to enter the field of medicine and studied at the College of Physicians and Surgeons, in New York City, and was Assistant to his cousin, Dr. George Tillinghast Bull, who was a noted surgeon of a generation ago.

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[1] Memoir prepared by J. C. Riedel, M. Am. Soc. C. E.

Leaving the field of medicine in 1888 to enter the profession of Civil Engineering, he was engaged by the Aerated Fuel Company, a subsidiary of the Standard Oil Company, in connection with the design and installation of some of the earliest oil burners for steam generation. He became Chief Engineer of the Aerated Fuel Company and remained in this field until 1891, when he became an Assistant Engineer in the Department of City Works of the former City of Brooklyn, N. Y., in which position he was engaged upon the design and construction of water-works, aqueducts, and pumping equipment. He then became an Assistant Engineer in the Department of Sewers in the newly created Greater City of New York. During this period, he also designed the sewerage system of Ceballos, Cuba.

From 1898 until 1906, Mr. Hammond was attached to the Department of Sewers and then to the Bureau of Sewers in the Borough of Brooklyn, as Assistant Engineer in charge of the building of some the largest sewers which had been constructed to that time.

In 1906, he obtained a leave of absence from the City of New York and was appointed Division Engineer in charge of the design of the storm-water sewers of the City of Baltimore, Md. Upon completing the design of this system, he returned to Brooklyn, and was appointed Engineer of Design, in the Bureau of Sewers, which position he held until his retirement from active service in 1933. As such, he designed the Classon Avenue Relief Sewer System, as well as the sewerage systems of other parts of the Borough. The value of the work for which designs were made under the supervision of Mr. Hammond was more than $100 000 000.

During his connection with the Bureau of Sewers of Brooklyn, Mr. Hammond was in charge of the experiments performed in 1913 to 1918 at the 26th Ward Sewage Disposal Works, upon all known methods and processes of sewage treatment and disposal. His work resulted in the development of processes now accepted as standard. Mr. Hammond was in charge of the design of the pumping station erected at the head of Gowanus Canal, which cleared the waters of that Canal twice daily, the capacity of the plant being 30 000 cu ft per min.

Mr. Hammond was one of the foremost experts on sewage disposal in the United States. In 1912, he visited Europe with the late Rudolph Hering, M. Am. Soc. C. E., who was then the Dean of American engineers specializing in Sanitary Engineering, and inspected all the important sewage treatment works there.

During the World War, Mr. Hammond entered the United States Public Health Service, was appointed a Surgeon, and, as such, designed and supervised the construction of sewage treatment works in the camps of the Army in the United States. He was in charge of the health and sanitation of thirty-eight shipyards between Boston, Mass., and Philadelphia, Pa., which employed about 100 000 men. He also served in the Housing and Transportation Division of the Shipping Board.

Mr. Hammond also acted as Consultant upon sewerage projects and water supply systems for Batavia, Java, Dutch East Indies, other cities in the

Orient, and for a number of cities in the United States. He was frequently consulted by agencies of the Japanese and Chinese Governments on these related matters.

He was a member of the American Society for Testing Materials and was Chairman of one of its committees for many years.

In his early career he wrote frequently on scientific subjects for the *New York Tribune*, the *New York Sun*, and the *New York World*, and was a member of the Press Club in the City of New York.

Mr. Hammond studied law from 1900 to 1903, after office hours, at the Law School of New York University, and was admitted to the Bar in 1904. He served as Arbitrator and Counselor in many engineering matters.

He was a member of the Delta Chi Fraternity, the Long Island Historical Society, the Kings County Historical Society, Altaire Lodge 601, F. and A. M., and the Washington Club, and was a Fellow of the American Association for the Advancement of Science.

Mr. Hammond had an encyclopedic mind, was deeply interested in scientific developments, was an ardent research worker, and kept in contact with the most recent developments in the fields of physics, chemistry, and engineering. He possessed high artistic ability and in his youth painted many fine portrayals of landscapes of the country about his early homestead.

He possessed a charming and interesting personality and was always highly regarded and highly respected by his countless friends and associates.

He was never married, and is survived by four brothers: John F. Hammond, M. Am. Soc. C. E., Henry B. Hammond, a member of the New York Bar; William G. Hammond, the famous organist and composer, all of New York City; and Charles F. Hammond, a lawyer, of Watch Hill, R. I.; and a sister, Mrs. Mary K. Smith, of New York City.

Mr. Hammond was elected a Member of the American Society of Civil Engineers on February 7, 1906.

THEODORE HENRY HINCHMAN, M. Am. Soc. C. E.[1]

DIED JULY 16, 1936

Theodore Henry Hinchman, the son of John Marshall Hinchman and Ella (Cropsey) Hinchman, was born in Detroit, Mich., on June 24, 1869. His forebears were important people in Detroit back in the days when it began to develop from a town almost exclusively French in character, into an American city. His great-grandfather, Dr. Marshall Chapin, was one of Detroit's early physicians, having left his home in New York State in 1809 to settle there. In 1819, his grandfather, Theodore Henry Hinchman, established a wholesale drug business, which, after the founder's death, was carried on by his sons under the name of T. H. Hinchman's Sons, and which ultimately became The Michigan Drug Company of the present time. With such a background, the subject of this memoir may well be called a "Detroiter", since he and his relatives have taken an active part in the growth and development of the city for more than one hundred and twenty-five years.

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[1] Memoir prepared by William R. Kales, M. Am. Soc. C. E.

As a boy, Mr. Hinchman attended the Detroit Public Schools, and was graduated from the Old Detroit High School in 1887. He then entered the University of Michigan, at Ann Arbor, Mich., from which he was graduated from the Liberal Arts Department, in 1891, with the degree of Bachelor of Arts, and from the Engineering Department, in 1893, with the degree of Bachelor of Science. While acquiring his training as an engineer, he had the good fortune to study under Mortimer E. Cooley, Hon. M. Am. Soc. C. E., beginning a friendship which continued for the remainder of his life.

During college vacations he worked on the Engineering Staff of the Michigan Central Railroad Company, and under Mr. Henry M. Leland, with the Leland, Faulkner, Norton Company. After his graduation, in 1893 and for a part of 1894, he served under Professor Cooley and under Mr. Jesse M. Smith, on consulting engineering work.

In the latter part of 1894, the partnership of Field and Hinchman, Consulting Mechanical and Electrical Engineers, was formed. Times were hard in those years immediately after the panic of 1893; and it took real courage to start a new business, venturing all one's small financial resources. Fifteen hundred dollars was the paid in capital with which the new firm started. Of this amount, Theodore Hinchman contributed $1 000 which had come to him through a legacy from his maternal grandfather, Mr. John G. Cropsey, of Chittenango, N. Y. Mr. George H. Field of the firm had also been recently graduated from the University of Michigan.

As can be seen by the dates, this was long before the development of the automobile industry in Michigan. Detroit was a slow, easy-going city of about 250 000 people—approximately one-sixth of its present size. It was a pleasant, clean place in which to live, being largely a jobbing center with comparatively little manufacturing. The first steel-frame office building was constructed about this time. There was very little work for the consulting engineer in such a community, and it was very difficult to secure enough of it "to keep the wolf from the door."

A year later, on October 24, 1895, Theodore Hinchman was married to Emma MacAllan Ballentine, at her home in Port Huron, Mich.

When war was declared against Spain in 1898, Mr. Hinchman temporarily left his work in Detroit to enlist in the Navy as Chief Machinist on the U. S. S. *Yosemite*, serving under his old friend and professor, Mortimer E. Cooley, who was Chief Engineer of that ship. The *Yosemite* served in the blockades of Santiago, Cuba, and of San Juan, Puerto Rico. At one time, while alone on the latter station, she was attacked simultaneously by several gunboats and from Morro Castle. Dean Cooley gives the following account of this naval action:

> "While on the blockade off San Juan, the *Antonio Lopez*, flagship of the Lopez Line, crept by us in the dark the night of June 27–28, 1898, missed the port and sailed westward. Discovering her mistake in the early dawn, she was returning when we saw her about four-thirty A. M.

"We immediately cleared ship for action, and intercepted her. To escape she turned toward the shore, and we ran in between her and the port. This brought us within range of Morro Castle, which began to fire at us with her nine-inch guns. Simultaneously, several torpedo boats came out from the harbor, one of them armed with a gun from the fort. Also the *Antonio Lopez* was armed and took part in the engagement. We drove the gunboats back into the harbor; and as the shots from the fort were dropping closer and closer, we went back out of range. This part of the engagement lasted from about four-thirty A. M. to about eight A. M.

"After breakfast we went in to destroy the *Lopez*, which was on the shore. The fort and the torpedo boats attacked us again. We drove the latter back into the harbor a second time, and went back and played safe. The fort did some clever firing. The flashes from the heliograph in the hills helped them find the range. One shot came so close as to splash water in on the gun deck. Another barely grazed the top of our smoke pipe. Most of the shells fell astern, as we kept up the speed of six or eight knots an hour.

"I timed the flight of a shell from the flash until it struck—31 seconds, which would mean the range was about six miles. One torpedo boat, armed with a Hontura from the fort, fired one shell that went lengthwise of the bridge, and so low that everybody dropped flat, including the Captain. Had one of the plunging shells from the fort struck us on the deck, it would have gone straight on through us. The nearest land was about a mile and a half below. The water there is very deep.

"In the Spanish War, we were the only ship in the Navy to fight against superior odds. As a result of it we received double prize money, a considerable sum by the way. The law awarding prize money was later done away with. We were the last ship to receive it."

After the war, Mr. Hinchman resumed his work as Consulting Engineer; and by hard, faithful, and effective service, built up his reputation.

The advent of the automobile business, and the increase in manufacturing in Detroit after 1900, brought more demand for the services of a man who had proved his ability and reliability during the preceding years. There was a great increase in the use of structural steel, and reinforced concrete construction was introduced. Building work required the services of the engineer as well as those of the architect. On March 30, 1903, an Architectural Department was added to the firm's activities; and the business was incorporated under the name of Field, Hinchman, and Smith.

In an old city like Detroit, many of the down-town office and commercial buildings occupied the best sites, and were hopelessly inadequate for the requirements. Obviously, they would soon have to come down, and be replaced by modern structures. New factories, embodying up-to-date manufacturing methods of production, were constantly going up. In planning the construction and equipment of such buildings, the harmonious and co-operative work of engineers and architects was necessary.

At the end of 1906, Mr. Field left the organization; and the present name—Smith, Hinchman and Grylls—was adopted early in 1907.

The following are some of the important pieces of work planned and carried to completion under Mr. Hinchman's supervision; Buhl Building (26 stories); Union Guardian Building (34 stories); Penobscot Building (44 stories); J. L. Hudson Co. Department Store; Crowley Milner Co. Department Store; Second National Bank of Saginaw, Mich.; the University Club (Detroit); the Detroit Country Club; Dodge Brothers Automobile Plant;

Hiram Walker & Company's great distillery recently completed in Peoria, Ill.; the Mistersky Power Plant and Transmission System for the Public Lighting Commission of Detroit; the High-Pressure Fire Protection System of Detroit; the Fairview Sewage Pumping Plant for Detroit; and the Power Plant, the Chemistry Building, Sub-Station, Power House, East Engineering Building, Yost Field House, Intramural Building, and Graduate School, of the University of Michigan.

In 1905, Mayor George P. Codd recognized the ability of Theodore Hinchman, and the need the City of Detroit had for his services, by appointing him a member of the Public Lighting Commission. He was also a member of the Michigan State Planning Commission, receiving his appointment from Governor Fitzgerald, in January, 1935. At the time of his death, he was President of the Village of Grosse Pointe Farms, where he had his home.

Soon after starting in business in 1894, Mr. Hinchman assisted in organizing the Detroit Engineering Society. For several years he served as Secretary of this Society; and held this office until it had become firmly established as a factor in the Detroit community. Later, he became, successively, Vice-President and President of the Engineering Society.

He joined the American Society of Mechanical Engineers as a Junior Member shortly after his graduation from college; and, when his age and experience justified his doing so, became a Member. His affiliations also included membership in the Detroit Board of Commerce, of which he was at one time a Director, the Detroit Country Club, the University Club, and the Detroit Club, of which a few years ago, he was President.

Coming from an old Presbyterian family, Mr. Hinchman was always a member and consistent worker in that church, and, at the time of his death, he was President of the Board of Trustees of the Grosse Pointe Memorial Presbyterian Church.

Theodore Hinchman is survived by Mrs. Hinchman and their three sons, Theodore Henry Hinchman, Jr., David Ballentine Hinchman, and John Marshall Hinchman, II. The second son, David, is married and has two children, one of whom bears the name of Theodore Henry Hinchman. He was a loving and much loved husband and father.

In writing this memoir, the attempt has been made very briefly to sketch the career of one who represented the highest type of American citizen. He was fortunate in having back of him an ancestry that stood for the best things in life—integrity, reliability, and honor He was endowed with an exceptionally bright and logical mind, and the capacity for unflagging industry, so that year by year he became more useful and more efficient. His education only began in his school and college days. It ended only with his death, which occurred at his home in Grosse Pointe Farms, a suburb of Detroit, on July 16, 1936.

Mr. Hinchman had a cheerful friendly disposition that inspired in all with whom he came in contact, feelings of confidence, trust, and loyalty. He began his professional career at the "bottom of the ladder", relying for success on his own ability and industry. By consistent hard work, he progressed until he built up one of the finest engineering and architectural organizations in the United States.

When his services were needed by his friends, his church, his city, his State, or his country, they were freely given; he never shirked his responsibilities. It is an honor to have been his friend.

Mr. Hinchman was elected a Member of the American Society of Civil Engineers on February 25, 1924.

JOHN HISLOP, M. Am. Soc. C. E.[1]

DIED FEBRUARY 22, 1901

John Hislop was born at Strasburg, Waterloo County, Ont., Canada, on February 1, 1856. He entered McGill University, at Montreal, Que., Canada, in September, 1881, and was graduated therefrom in April, 1884, with the degree of Bachelor of Applied Science.

During his vacation of 1882, he was engaged in making surveys on the Great Eastern and Pontiac Pacific Railways, in the Province of Quebec, Canada. In 1884, he served as Rodman on the Canadian Pacific Railway; in 1885, on surveys on the Burlington and Missouri River Railway; and afterward on construction work for the Burlington and Northern Railway Company.

In 1886, Mr. Hislop was employed as Transitman on the Burlington and Northern Railway; and, in 1887, as Transitman and Track Engineer, and, in 1888, as Assistant Division Engineer, Maintenance of Way, on the Chicago, Santa Fé and California Railway.

During February and March, 1889, he served as Assistant Engineer on Construction, on the Carbon Cut-Off, for the Union Pacific Railway Company. Later, he was appointed First Assistant Engineer, on the exploration and surveys of the Colorado River of the West, for the Denver, Colorado Canyon, and Pacific Railway Company. This work proved his courage and efficiency beyond all question.

Mr. Hislop was engaged in general engineering practice, in Denver, Colo., in 1890, and in the winter and spring of 1891, he was Resident Engineer in charge of construction on the Denver, Lakewood, and Golden Electric Railway. Later, he was appointed Chief Engineer of the Colorado Grand Canyon Mining and Improvement Company.

From January to April, 1892, he was Acting County Surveyor of Weld County, and City Engineer, of Greeley, Colo., and while holding this position he made surveys and estimates for the proposed Highline Reservoir, in South Park. He then became Assistant Engineer of the Payette Valley Irrigation and Water Power Canal, in Idaho, in charge of location and construction of twelve miles of canal, including head and waste-gates, flumes, bridges, siphons, etc.

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[1] Memoir compiled from information supplied by the late C. LeR. Annan, M. Am. Soc. C. E., and from data on file at the Headquarters of the Society.

In January, 1894, Mr. Hislop became Assistant Engineer of the Castaica and San Feliciana, and Hocumac Mining Companies, with headquarters at Los Angeles, Calif. Later, he went to Alaska, where he had charge of the construction of the White Pass and Yukon Railroad. This road was between Skagway and the interior, 110 miles in length, and required the most expert work under very difficult conditions. At this time, he also served as Mayor of the City of Skagway.

Mr. Hislop had great athletic skill, and was able to keep his poise under all circumstances. Although very reserved, he was considered a good fellow among his associates, and was ever ready to assist in any arduous undertaking.

He was accidentally killed in Chicago, Ill., on February 22, 1901, by falling beneath the wheels of a moving train which he was trying to board.

Mr. Hislop was elected a Member of the American Society of Civil Engineers on May 1, 1895.

RICHARD CARMICHAEL HOLLYDAY, M. Am. Soc. C. E.[1]

Died November 17, 1936

Richard Carmichael Hollyday, Civil Engineer and Naval Officer, the son of Richard Carmichael and Marietta Fauntleroy (Powell) Hollyday, was born at Ratcliffe Manor, in Talbot County, Maryland, the ancestral home of the fine old Southern family from which he sprung, on November 13, 1859.

Educated in private schools, the Shenandoah Valley Academy, and Washington and Lee University, at Lexington, Va., Mr. Hollyday subsequently completed the necessary course of study and was admitted to the practice of law; but, because of an inclination for an outdoor life, he sought and obtained a position on a railroad survey and, thereafter, followed the profession of civil engineering.

Beginning his professional career on preliminary and location surveys of the Philadelphia Branch of the Baltimore and Ohio Railroad, he was employed, successively, on bridge construction—foundations and superstructure erection—for crossing of the Susquehanna River, at Havre de Grace, Md., the Schuylkill River, at Gray's Ferry, Philadelphia, Pa., and the Christiana River for the City of Wilmington, Del. After a short period at the Baltimore Headquarters of the Baltimore and Ohio Railroad Company, he was transferred to the Ohio Division of the System as Engineer of Maintenance of Way, with headquarters at Newark, Ohio. Upon the termination of this connection, he resigned from the railroad service and was variously employed on a gas-holder project, in Philadelphia, Pa.; blast furnaces, at Bristol, Tenn., and Radford, Va.; a pipe foundry, at Bristol; an inclined railway, at Lookout Mountain, Chattanooga, Tenn.; and on the construction of an ore plant, at Shelby, Ala.

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[1] Memoir prepared by Leonard M. Cox, Captain (C. E. C,). U. S. N. (*Retired*), M. Am. Soc. C. E.

On March 15, 1894, Mr. Hollyday was appointed by President Cleveland as a Civil Engineer, United States Navy, with the relative rank of Lieutenant, Junior Grade, and served on active duty continuously thereafter in the Corps of Civil Engineers, until he reached the statutory age for retirement.

In April, 1894, Civil Engineer Hollyday began his naval career as Assistant to the late Civil Engineer, U. S. G. White, U. S. N., then attached to the Puget Sound Navy Yard. In December of the same year, through the detachment of his superior, Civil Engineer Hollyday became head of his Department in charge of Public Works. Thence, until May, 1897, he played a most important part in the building of the then new station, a part that included superintendence of construction of the timber dry dock, general layout for the Yard, design and construction of water system, shops, and officers' quarters.

From May, 1897, to August, 1901, Civil Engineer Hollyday was Yard Civil Engineer—corresponding to the present position of Public Works Officer —at the Mare Island Navy Yard, in California. During this duty the disastrous earthquake of March 30, 1898, afforded opportunity for the display of his exceptional abilities. He virtually rebuilt the Yard—and in what was considered record-breaking time. Many of the Yard shops and storehouses were either demolished, or badly damaged, all officers' quarters had to be condemned for use, and Yard operations were virtually suspended. Civil Engineer Hollyday wired estimates and a request for funds, and secured a special appropriation. Work was started immediately and, by January, 1900, the Yard was to all intents and purposes rebuilt. Undoubtedly, the record then made for handling such an emergency was the deciding factor in his later selection as Chief of Bureau of Yards and Docks. He was the officer who inaugurated the methodical study of the hydraulics of Mare Island Strait, which culminated during his administration as Chief of Bureau of Yards and Docks, in the appointment of the so-called Biddle Board and, by that Board's recommendations, in the adoption of the dike layout which has proved so successful in maintaining the desired depths of channel.

From August, 1901, to January, 1903, he was assigned to duty at the Boston (Mass.) Navy Yard, in charge of all new construction and of Yard maintenance; and from the last-mentioned date until March, 1907, he held the corresponding position at the then ranking naval shore station, the New York Yard, in Brooklyn, N. Y.

In March, 1907, Commander Hollyday—he held that rank at the time— was commissioned Chief of Bureau of Yards and Docks of the Navy, with the relative rank of Rear-Admiral during incumbency. Within the term of his administration many important—some serious—problems were solved satisfactorily, and many improvements effected in Bureau procedure. One of the most important advances in Bureau policy was the elimination of irresponsible bidders and the issuance of "invitations to bid" to selected contractors of known ability and financial standing in the case of Dry Dock No. 3, at the New York Yard.

He administered the highest office in his Corps during the trying period when the ambition to re-organize the Navy Department, among newly appointed Department officials, assumed something like epidemic proportions. Boards and commissions were charged with the planning of procedure changes affecting nearly every phase of the Department's activity from filing systems to works management and supervision. Carried away by the unquestioned need for improvement in the way of reduction of plant and supervisory duplication, attempts were made to consolidate the Naval Staff Corps—a project which, had it been put into effect, would have resulted in placing authority over one class of technical work in the hands of men trained for entirely different work. Admiral Hollyday saved the situation—for his Corps and for the best interests of the Navy—not by the militant methods advocated by many able officers, but by a calm demeanor throughout the period, by a willingness to concede the merit of the plan within limits of practicability, and by the unanswerable logic of his arguments.

In January, 1912, Admiral Hollyday, his term of office having expired, was detached from duty as Chief of Bureau of Yards and Docks, and automatically reverted to his permanent rank of Captain. As Dean of his Corps he served from this date until retirement as Public Works Officer of the Washington, D. C., Norfolk, Va., and New York Navy Yards, and successfully administered the expenditures of large amounts of public funds in naval shore construction.

In November, 1923, while stationed at the New York Yard—with additional duty as Public Works Officer of the Third Naval District—Captain Hollyday reached the retirement age and was relieved from active duty.

He was recalled subsequently to active duty for temporary service on selection boards for the promotion of civil engineer officers, but his active professional and naval career ended with retirement. Until his death, he and Mrs. Hollyday resided in New York City, or at his family home in Easton, Md.

Captain Hollyday was married, on April 28, 1894, to Mary Holton King, of Newark, Ohio, and enjoyed an exceptionally happy married life. Of their two sons one, Richard Carmichael, educated for his father's profession, but preferring a business career, is a member of the firm of Culver, Hollyday and Company, Realtors, of New York City; the other, King, like his Colonial ancestors, is an agriculturist, residing at St. Michaels, near Easton, Md. He is also survived by his sister, Mrs. William P. Semple, of Louisville, Ky.

Captain Hollyday entered the Naval Service with the rank of Lieutenant, Junior Grade. On June 2, 1897, he received his first promotion—to the rank of Lieutenant. He was commissioned with the rank of Lieutenant Commander on April 6, 1902; Commander, on November 26, 1906; Chief of Bureau of Yards and Docks, with the rank, during incumbency, of Rear-Admiral, on January 18, 1908; and Captain (regular list), on March 13, 1911.

His social clubs were the Army and Navy, Chevy Chase, and University, of Washington, D. C.

Captain Hollyday was an able engineer, a naval officer of the highest type, and a gentleman of the old school—true to the traditions he inherited from a long line of Virginia and Maryland ancestors who played their part in

American history from early Colonial days. In times of stress, he was cool and deliberate; ever kind and considerate to his subordinates; to those who failed to measure up to his own standards of professional or naval ethics, he was slow to forgive—but never vindictive. Those who were fortunate enough to serve under or with him in close association, loved him, and will not soon forget their "Old Chief."

Captain Hollyday was elected a Member of the American Society of Civil Engineers on September 3, 1903.

LAURENCE COOPER HOUGH, M. Am. Soc. C. E.[1]

DIED JANUARY 13, 1937

Laurence Cooper Hough, the eldest child of Elida C. and Pamelia (Rice) Hough, was born on December 5, 1891, at East Falls Church, Fairfax County, Va. He was of straight English Colonial stock on both sides of his house. His direct ancestor, William Hough, of Westchester, England, came over to the Plymouth Colony in 1640. William Hough soon moved to Connecticut, and family tradition has it that he built the first house at what is now the City of Meriden, in 1642, and settled there.

Mr. Hough's great-grandfather, Dr. H. G. Hough, moved over into Northern New York State and practiced medicine as a pioneer in the Black River country. Dr. Hough's son, Dr. Franklin B. Hough, was elected to Phi Beta Kappa Fraternity while a student at Union College, at Schenectady, N. Y., about 1842, and, later, studied science at Case Institute in Ohio, walking from Albany, N. Y., to Cleveland, Ohio, for the purpose. He practiced medicine at Lowville, N. Y., served as Surgeon and Major with the 90th New York Volunteers during the Civil War, being present at the battles of Fredericksburg, and Antietam, and in other campaigns. In 1875, he was appointed United States Commissioner of Forestry, and, as such, established the office which is now the United States Forest Service.

Mr. E. C. Hough, the father of Laurence Cooper Hough, was born at Lowville, N. Y., was graduated from Cornell University, at Ithaca, N. Y., in 1885, with the degree of Bachelor of Arts, and spent almost his entire working life as Examiner for the United States Patent Office, in Washington, D. C.

Laurence Cooper Hough attended the public school at Falls Church, Va., and was also a student at Force School, Washington, D. C. He spent one year at the Western High School and three years at the McKinley Manual Training School, in the District of Columbia, and was graduated from the latter in 1910.

In 1914, he was graduated from Cornell University with the degree of Civil Engineer. His summers during his college course were spent in engineering work. In 1911, he was a Draftsman, in Washington, on Patent Office drafting. During the summer of 1912, he acted as Transitman at the Cornell University Engineers' Camp on a topographical survey. The following summer he was employed as Chainman for the United States Land Office, on a surveying project in the Swan River country of Western Montana.

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[1] Memoir prepared by Edgar K. Wilson and Egbert D. Case, Members, Am. Soc. C. E.

The summer after his graduation, Mr. Hough worked for The Whitney Company, in New York City, as Estimator on Construction. Later, he was employed for a short time by the late James H. Fuertes, M. Am. Soc. C. E., after which he served as Chief of Party on subway construction in New York City for the Public Service Commission, until June, 1917. During this latter engagement, part of his duties was to check the elevations of buildings adjacent to the construction work to determine whether they were sinking.

In June, 1917, he became connected with The Pitometer Company, Engineers, and this proved to be the engagement to which the remainder of his life was devoted. With the exception of a period from August, 1917, to November, 1918, when he served in the World War, he was engaged in making water-waste surveys and other investigations for water-works distribution systems, first as Assistant and then as Engineer in Charge in surveys of Buffalo, N. Y., Boston, Mass., and Providence, R. I., as well as of many smaller communities.

In 1924, due to his thorough knowledge of this work, his pleasing personality, and his growing acquaintance with water-works officials, Mr. Hough was appointed a District Manager of the Company, first with headquarters at Boston, Mass., covering the New England States; but, later, with an office at Albany, with New York State, part of New Jersey, Georgia, and Florida, added to his territory.

During the years following his appointment as District Manager Mr. Hough almost literally lived in his automobile, driving long distances to call on his clientele, or to attend meetings of officials, and he was known to hundreds of water-works men, including many with whom he had no direct business contact, as a man of character, genial and kindly in all his relations with his fellows.

Mr. Hough always had a taste for music and as a boy rather seriously studied the violin. In later years, he derived much pleasure from a piano.

He was always greatly interested in military matters, and during his fourth year in preparatory school served as Captain and Regimental Adjutant of the Cadet Corps. When the United States entered the World War, Mr. Hough promptly applied for admission to the First Officers' Training Camp and was bitterly disappointed when he failed to receive an appointment. However, he was appointed to the Second Camp at Plattsburgh, N. Y., and received a commission as Second Lieutenant of Coast Artillery. After a brief assignment at Fortress Monroe, Virginia, he was sent to France, and assigned to I Battery, 51st Coast Artillery. This outfit was using 14-in. naval guns on railway mounts. Unfortunately, or perhaps fortunately, Mr. Hough saw little action, although he underwent frequent aerial bombardments. On one occasion a severe attack of the mumps, on another a quarantine against measles suffered by a tent mate, and on a third a school assignment removed him from his outfit when it was in combat. His education and talent as a Civil Engineer were made use of in laying out the curved trackage for new positions, by which means the guns were trained to left or to right, and, at other times, he was responsible for the supply of ammunition to the guns. One of his war stories is to the effect that while moving their guns it was found that one gun with its car would not enter a tunnel by a scant 2 in. This was remedied after considerable delay by changing to smaller car wheels. When the gun train finally passed through, a Rear Admiral was in the locomotive cab, in his jubilation holding down the whistle cord. After the war, Mr. Hough became a member of the Coast Artillery Reserve and served until 1934 when he retired with the rank of Captain.

Mr. Hough's pioneering ancestry showed itself in his insatiable interest in people, historical places, and things around him. His accurate knowledge of the happenings of former days in the cities and countryside within his reach was amazing. Not only did he have a store of interesting knowledge about such places, but he was always eager to tell about them and show them to his friends, even at considerable inconvenience to himself.

However, inconvenience was not in his vocabulary, when there was a question of doing a favor for a friend or an associate. By all who knew him Mr. Hough was recognized as a man of the highest character, upright and fair in all his dealings, an able engineer, a true friend, and a valued associate.

He was married on October 7, 1922, to Jennie Marion Ponti, who survives him. Mr. and Mrs. Hough were members of the Old South Church, of Boston.

He was also a member of the Cornell Society of Civil Engineers, American Water Works Association, New England Water Works Association, Maine Water Utilities Association, and of the Joseph Warren Lodge, A. F. and A. M., of Boston.

Mr. Hough was elected an Associate Member of the American Society of Civil Engineers on July 11, 1921, and a Member on October 7, 1929.

JOSEPH MILTON HOWE, M. Am. Soc. C. E.[1]

DIED MARCH 22, 1937

Joseph Milton Howe was born on July 30, 1874, in Houston, Tex., the son of the late Milton Grosvenor Howe, M. Am. Soc. C. E., and Jessie (Briscoe) Howe. His grandparents were pioneers of Texas. His grandfather, Captain Andrew Briscoe, fought at the Battle of San Jacinto; he was one of the signers of the Texas Declaration of Independence and was the first Judge of Harris County. His father built the Houston and Texas Central Railway, and, later, became its Chief Engineer. Mr. Howe was a descendant of John R. Harris, founder of Harrisburg, which is now a suburb of Houston, and for whom Harris County was named.

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[1] Memoir prepared by A. J. Wise, M. Am. Soc. C. E.

He was educated in private schools, at Phillips Academy, at Andover, Mass., and received his degree from the Massachusetts Institute of Technology, in Boston, Mass., in 1896. Later, he attended the Law School of the University of Texas, at Austin, for a year.

Mr. Howe ended his academic career to enter the Engineering Department of the Santa Fé Railway Company in 1897, joined the Engineering Staff of the Houston and Texas Central Railway Company in 1899, and became Acting Engineer, Maintenance of Way, and Assistant Engineer, Maintenance of Way, of the Company before he resigned to become Principal Assistant Engineer, Maintenance of Way, of the Southern Pacific Railroad Company (Atlantic System). He resigned this position in January, 1904, to enter private practice in Houston, and was engaged in the location and construction of short-line railroads and drainage. Among his achievements were the Montana Panuco y Monclova Railroad, in Mexico; the design and construction of Drainage District No. 1, Liberty County, Texas; the location of storage tanks and pipe lines for various oil companies; and the design of a sewage disposal plant in Estacion Monclova, Coahuila, Mexico.

In July, 1909, he formed a partnership with A. J. Wise, M. Am. Soc. C. E., and organized the firm of Howe and Wise which association continued until his death. During the period of this partnership Mr. Howe devoted most of his time to road construction, paving, and other municipal engineering work, and to hydraulic and railroad engineering. He personally supervised design and construction for the Gulf Coast Irrigation Company, the Texas Exploration Company, as well as the Dayton-Goose Creek Railroad Company, as Cousulting Engineer of the Brays Bayou Drainage District, one of the most important projects in Houston and Harris County.

The firm of Howe and Wise served as County Engineers of Harris County from 1911 to 1915 and again in 1917 until 1929. The bridge and road improvements that brought national attention to this area were designed under its supervision. In 1935 and 1936, Mr. Howe again acted as Consulting Engineer for Harris County. From 1911 to 1936, about $20 000 000 was expended on road and bridge construction in that County under his supervision.

He was prominent in professional and business movements throughout the United States. He was Vice-President of the Houston National Bank, a Director of the Trust Company of Texas, President of the Houston Realty Syndicate, Oakland Realty Company, and the Brookline Development Company. He was a member of the State Board of Health and a Trustee of Hermann Hospital Estate. He belonged to the Museum of Fine Arts, University Club, Rotary Club, and Sigma Chi Fraternity.

The high regard in which Mr. Howe was held in the City of Houston and the State of Texas was expressed editorially by the *Houston Post,* on March 24, 1937, as follows:

"America's modern industrial civilization is largely the creation of engineers.

"Business men, promoters and politicians get much of the credit, but the engineers furnish the brains.

"Because of their technical skill railroads have been pushed across rugged mountain ranges, automobiles dash around curves in highways at sixty miles an hour in perfect safety, and huge bridges span the Mississippi River and San Francisco bay, bodies of water once considered unconquerable.

"Look behind any modern triumph of construction or invention and you will find an engineer bent over a desk, pipe in mouth, scanning blueprints, filling pages with figures.

"His name does not appear on the bronze plates or marble cornerstones, but without him and his knowledge it would be impossible to build a three-story building that would not topple down on the heads of its occupants.

"J. Milton Howe, who was born in Houston and who died here Monday, was an engineer who had won recognition throughout America. He was known for two outstanding characteristics that dominated his entire professional career—technical skill and unswerving honesty.

"In a long and brilliant engineering career, he spent his lifetime above all usefully. He leaves behind him railroads in Mexico, highways in the United States, and scores of other projects as monuments to his constructive genius. Many of the fine highways which bear traffic of Harris County and Houston were built under his supervision and his name has been associated with notable construction projects throughout the country.

"He was honored repeatedly by the American Society of Civil Engineers and by the Texas chapter which he helped organize. Nearly every honor that can come to an engineer as a reward for proficiency in his profession came to Mr. Howe. Throughout his career, he was as interested in the advancement of engineering as a science as in his own affairs.

"As a man he was kindly, sincere, honorable and wholly lovable. He made friends who clung to him because of his genuine worth, his lack of pretense and his unconquerable spirit.

"His friends and surviving members of his family will miss him sorely. The engineering profession has lost a man who followed in his work and his every day life its highest idealism."

And, again, by the *Houston Chronicle,* of March 25, 1937, as follows:

"J. Milton Howe leaves an enduring record of achievement in the field of civil engineering. Internationally known both as railroad and highway builder, he applied to his work not only the time-tested sound principles of engineering but also an originality that produced new methods and gained him wide recognition.

"Mr. Howe followed in the footsteps of his ancestors, who were builders and pioneers. His great-grandfather was John R. Harris, founder of Harrisburg and the man for whom Harris County was named. More remote forebears of his founded Harrisburg, Pa. Other of his ancestors figured largely in Texas History. His grandfather was Capt. Andrew Briscoe, who fought at San Jacinto and was the first judge of Harris County. His father, Capt. Milton G. Howe, was one of the engineers who built the Houston & Texas Central Railroad.

"With such a background it is not surprising that he became a well-known engineer and that he figured prominently in the civic life of his county and state.

"His long service as Harris County engineer, in connection with his partner, A. J. Wise, was preceded by a number of years in the railroad building field in this country and in Mexico.

"The work he and Mr. Wise did along irrigation and drainage, road and bridge building lines gained national recognition. They pioneered the present method of settling highway embankments within a few weeks time, soaking the roadbed throughout its depth by means of long pipes, supplemented by ponding on the surface. This method produced in a short time a more compact foundation, without the usual soft spots, than the old method of traffic-packing obtained in months or even years. The plan is now standard in Texas and other States with similar soil conditions.

"Mr. Howe's death is a blow to a number of civic organizations as well as to the engineering profession. His passing will be widely mourned over the nation as well as in his home community."

A fitting tribute and appreciation of his likable personality was expressed in the *Houston Press*, of March 22, 1937:

"There was something in the appearance of J. Milton Howe that gave you the impression of rugged honesty. That was his outstanding characteristic. He was fair and square in all his dealings and thousands in Houston held him in high esteem. His death Monday was not a shock to his friends, for his serious illness was known to them, but it caused much sadness."

And, again, on March 23, 1937:

"J. Milton Howe's friends will miss his sparkling smile, his quiet humor and the stories of his adventures when he was a young railroad engineer in Mexico.

"At 62 he was still young in spirit and body and he had more zest for living and greater interest in the people and things about him than most men of 25. It's hard to believe that yesterday he died.

"Behind him he left many evidences of his engineering skill. As county engineer from 1911 to 1914 and from 1917 until 1929, he supervised the building of many of the county's important roads and bridges. As a consultant, his advice was sought widely.

"He was a native of Houston. His forebears were among the pioneers of Texas. Their spirit was strong in him, and it never grew dim."

From his partner, A. J. Wise, M. Am. Soc. C. E.:

"Mr. Howe was more than a business associate, he was a companion. His charming personality and many delightful characteristics contributed much to ease us over the rough spots which were encountered during our twenty-eight years of partnership. He was always honest and reasonable with his employees as well as myself. He was sociable, he loved parties and his pipes. In going through his desk we found six full-fledged smoke pipes. His son, Knox, informed me that he had about that many more at home."

Mr. Howe was married to Rowena Thomson, of Cleburne, Tex., in 1901, and is survived by one son, Knox Briscoe Howe.

He took a leading part in the formation of the Texas Section of the Society and never missed a meeting of that organization. He served a term in all its offices.

Mr. Howe was elected an Associate Member of the American Society of Civil Engineers on February 4, 1903, and a Member on January 7, 1908. He served as a Director of the Society from 1924 to 1926, and as a Vice-President—the first Texan ever to hold that office in the organization—in 1930 and 1931.

LAURENCE BRACKETT HOYT, M. Am. Soc. C. E.[1]

DIED JUNE 1, 1936

Laurence Brackett Hoyt, the son of Frank Murray Hoyt and Mary Ellen (Brackett) Hoyt, was born on September 10, 1891, at Greenland, N. H., the home of his mother's people. He was a descendant of John and Priscilla Alden, his great-grandmother being an Adams, a descendant of Joseph Adams, an uncle of President John Adams.

The boy was educated in the public schools of Melrose, Mass., and was graduated from the High School in 1909. He then entered the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated in 1913 with the degree of Bachelor of Science in Civil Engineering. In 1913 and 1914, he did graduate work in Civil Engineering at the Massachusetts Institute of Technology. In 1922, he returned to the Institute for graduate work in Engineering Administration and Highway Engineering, including business management, statistics, mathematical laboratory, hydrology, highway materials testing, and chemistry of road materials.

During his summer vacations as a Student at Massachusetts Institute of Technology, Mr. Hoyt served as Rodman for the Boston Elevated Street Railway Company; he was also engaged as Rodman for the Power Construction Company on the Hoosac Tunnel, and as Instrumentman for the Directors of the Port of Boston, Mass.

He served as Instructor at the Technology Surveying Camp, at Gardiner's Lake, Maine, during the summer session in 1913, following which he was appointed Assistant in the Civil Engineering Department at the Institute and served during the school year of 1913 and 1914. This included private work for Charles M. Spofford, M. Am. Soc. C. E., on bridge and masonry designs.

For the next ten years (1914–1924), Mr. Hoyt was with the Massachusetts Department of Public Works on the various highway engineering projects being carried out by that Department.

He served as Director of Public Works, of Manchester, N. H., in 1925 and 1926. In this position he was in executive charge of highways, bridges, sewers, refuse disposal, street cleaning, traffic regulation, and of the various Engineering Departments of the City.

Returning to Boston, Mr. Hoyt entered the employ of the firm of Metcalf and Eddy, Consulting Engineers. During 1927 and 1928, he made investigations and reports on work carried on by that firm for Braintree, Mass. He was also engaged on studies on the feasibility of the extension of Riverside Drive from New York City to Tarrytown, N. Y., for the Westchester County (New York) Park Commission.

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[1] Memoir prepared from information supplied by the family and on file at the Headquarters of the Society.

In October and November, 1928, Mr. Hoyt was engaged by Mr. John D. Rockefeller, Jr., as Consulting Engineer on highways for the Town of Seal Harbor, Me. In February, 1929, he was appointed as Assistant Professor of Civil Engineering at the University of Maine, at Orono, Me., to fill a vacancy of a teacher on leave. He remained in this position until June, 1929, and was offered a continuance of the work, but declined it to accept another offer in Indiana. During the summer of 1929, he served as Special Engineer with the State Highway Department of Maine, with headquarters at Solon, Me.

In the fall of 1929, Mr. Hoyt went to Evansville, Ind., to accept the position of Director of Engineering and Professor of Civil Engineering, at Evansville College. He remained in this position until the fall of 1931, at the time the College was re-organized as a part of Purdue University.

After leaving Evansville College in September, 1931, he was engaged until March, 1932, on an industrial investigation at Columbia, S. C., for Meade Johnson Company, of Evansville, Ind. For the remainder of 1932 and during 1933, he was again engaged with the State Highway Department of Maine, with headquarters at Bangor.

In 1934, Professor Hoyt entered the employ of the State Highway Department of New Hampshire, as Resident Engineer on jetty construction, including shore dikes and hydraulic fill, for Hampton, N. H., Harbor. He continued in this position until his sudden death from a heart attack on June 1, 1936.

Professor Hoyt was a member of the Boston Society of Civil Engineers, the Canopy Club of Massachusetts, the Boston Chess Club, and the Society of American Magicians. He was a Mason, a member of Fidelity Lodge, A. F. and A. M., and the Waverly Royal Arch Chapter, of Melrose Mass.

He had taken and passed with high marks all the Civil Service examinations given to Civil Engineers in Massachusetts.

In writing of Professor Hoyt and his work, the President of Evansville College stated:

> "In his relation to the work of teaching I have discovered that he has a keenly analytical mind, and that either in writing or in speech he expresses himself with crystal clarity. He always has the courage of his convictions about matters professional or academic, but being a reasoning man, he is always reasonable in debate and discussion. He is therefore a most satisfactory person to deal with.
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> "In the few contacts he has had during his period here, * * * he has won the undeniable respect of professional men in the city. In the course of a very thoroughgoing survey of our college and its work, particularly in the field of engineering, made by five of the leading officials of Purdue University, the impression settled upon these men that 'Professor Hoyt is a man who knows his business thoroughly,' to speak in the terms used by one of the men."

He is survived by his widow and seven children, and by his father, Frank M. Hoyt, of Melrose, Mass.

Professor Hoyt was elected an Associate Member of the American Society of Civil Engineers on May 8, 1922, and a Member on July 7, 1925.

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EDWARD SHERMAN JACKSON, M. Am. Soc. C. E.[1]

Died April 4, 1935

Edward Sherman Jackson was born in Walden, Vt., on August 3, 1865, the son of Marshall Dexter Jackson, a farmer, and Jane Elizabeth (Johnson) Jackson. He was educated in the public schools of Walden, later attending the University of Vermont, at Burlington, from which he was graduated as an Honor Student, with the degree of Civil Engineer, in the Class of 1890.

Immediately upon graduation, Mr. Jackson accepted employment with the Chicago, Burlington, and Quincy Railroad Company and was assigned as Assistant Engineer on Maintenance of Way at McCook, Nebr., and Denver, Colo. In March, 1894, he was appointed Assistant Division Engineer on Construction in Wyoming and Montana, which marked the beginning of his steady advancement through the grades of Resident Engineer, Location Engineer, Superintendent of Construction, and Train Master, to Engineer, Maintenance of Way, for the Wyoming District, in 1906. During this period, his engineering responsibilities included the construction of the Big Horn River Bridge, near Fort Custer, Montana; the location of lines in the North Platte Valley, in Nebraska; and heavy railroad construction between Alliance and Bridgeport, Nebr., and between Northport, Nebr., and Guernsey, Wyo.

In September, 1906, Mr. Jackson became Chief Engineer of the Intermountain Railroad, at Boise, Idaho, where he supervised the preliminary surveys, location, plans, and specifications for one hundred miles of railroad lines north and east of that city. He also had charge of extensive repairs on the Barber Dam across the Boise River. These activities led to his appointment as Superintendent of Construction and Operation of the Idaho Southern Railroad, and Engineer of Construction and Superintendent and Chief Engineer, of the Milner and North Side Railroad. He remained with these Companies until February, 1916.

Following a year on construction at the Bremerton Navy Yard, at Seattle, Wash., he became associated with the Great Northern Railway Company as Maintenance Engineer, on the Great Falls (Mont.) Division, later, being identified with the relocation and double-tracking of the main line through the main range of the Rockies in the vicinity of Glacier National Park. Mr. Jackson was assigned as Engineer, in charge of location and construction, on the Eastern Section of the Cascade Tunnel, on the Great Northern Railway, at Berne, Wash., including the Mill Creek shaft and headings. At the conclusion of this work, in July, 1927, he was transferred to resident charge of the Chumstick cut-off and approach to the Cascade Tunnel, which is considered the most important single refinement of the Great Northern Railway's Transcontinental Line attempted since 1912. He continued with the Great Northern Railway Company in various capacities until November, 1933, including location work on the proposed

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[1] Memoir prepared by R. H. Willcomb, Assoc. M. Am. Soc. C. E.

New Rockford cut-off, in the vicinity of Sand Springs, Mont., the Klamath Falls Line, between Bend and Klamath Falls, Ore.; and the location of rail connections with the Grand Coulee Dam, in Washington.

Mr. Jackson relinquished his more arduous responsibilities incident to railroad engineering in 1933, but did not retire entirely from his chosen work. He took charge of the location of a section of the Great Falls-Fort Peck Dam high-power transmission line for the United States Government. Completing this work, he became associated with the Montana State Highway Commission as Project Engineer and Bridge Inspector, which positions he held at the time of his sudden death.

He was married on October 8, 1902, to Ruth Ethelwyn Bogue, at Alliance, Nebr. Mrs. Jackson died on April 14, 1916. He is survived by two daughters, Mrs. Harry B. Ward, of Garden City, Long Island, N. Y., and Mrs. Alfred P. Shepherd, of Winchester, Mass.; a son, Edward S. Jackson, Jr., of Jackson, Mich.; a brother, Elmer Jackson, of St. Johnsbury, Vt.; and one grandchild.

Among Masonic circles he was a member of the York Rites, receiving his degrees in McCook, Nebr. He later affiliated with the Masonic bodies at Alliance, Nebr., and, at the time of his death, was a member of Lincoln Lodge No. 59, A. F. and A. M. and King Solomon Chapter, R. A. M., both of Gooding, Idaho. He served as Excellent High Priest of the latter body in 1914, and was appointed and installed as Grand Master of the Third Veil of the Grand Chapter of Royal Arch Masons of Idaho in May, 1916. He demitted from Bunah Commandery No. 26, Knights Templars, of Alliance, Nebr., in 1913.

Mr. Jackson was singularly proficient and exacting in any work which he undertook. Quiet and unassuming to the point of reticence, he went ahead with each responsibility confidently and without hesitation. The many engineering projects which were completed under his guidance and supervision attest to his accuracy, thorough dependability, and unfailing integrity as an Engineer. He was a gentleman, unusually considerate and helpful, particularly toward his younger professional brothers, and a trustworthy friend to all his associates. He was a member of the Association of Railway Superintendents from 1908 to 1916.

Mr. Jackson was elected a Member of the American Society of Civil Engineers on March 4, 1915.

JOHN QUENTIN JAMIESON, M. Am. Soc. C. E.[1]

Died September 28, 1932

John Quentin Jamieson, the son of John and Mary (Duncan) Jamieson, was born in Pittsburgh, Pa., on April 19, 1855. Both his parents were of Scotch ancestry. His grandfather, Quentin Jamieson, was a Captain in the

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[1] Memoir prepared by J. P. Newell, M. Am. Soc. C. E.

Yeomanry, in the time of George III. His father, John Jamieson, came as a boy, from Maybole, Ayrshire, Scotland, to Pittsburgh, and was engaged in the manufacture of iron.

Thomas and Helen Duncan, great-grandparents of Mary Duncan Jamieson, left Peebles, Scotland, and settled in York, Pa., before the Revolution. One of their sons, James, was a minister of the United Presbyterian Church, noted for his learning. Late in life he was put out of the Church for heresy, preferring to break off the association of a lifetime rather than sacrifice his convictions. Thomas, a son of James, and the father of Mary, was a Quartermaster in the American Army in the War of 1812.

John Quentin Jamieson received his technical education at the Western University of Pennsylvania, at Pittsburgh. He served his apprenticeship under that master of the art of railroad location, the late William Harlin Kennedy, M. Am. Soc. C. E. From 1879 to 1883, Mr. Jamieson was Topographer and Assistant Engineer for the Oregon Railroad and Navigation Company, on the line then being built along the Columbia River and over the mountains of Eastern Oregon, from Portland to the Idaho line. From 1884 to 1887, he was Assistant Engineer on the Northern Pacific Railroad, in charge of the construction of the two-mile Stampede Pass Tunnel at the summit of the Cascade Range and of the six miles of very heavy work on each approach to the tunnel.

Through 1888, 1889, and 1890, he was Division Engineer in charge of location and construction of the Oregon Railroad and Navigation Company's line from Tekoa, Wash., over the mountains to Coeur d'Alene Lake, and thence along the river of the same name to Wallace, Idaho, In 1891, Mr. Jamieson served as Division Engineer on the Portland and Puget Sound Railroad for the Union Pacific Railway Company. In 1892, he had charge of surveys for the San Francisco and Great Salt Lake Railroad from Oroville, Calif., through the Sierra Nevada Mountains to Beckwith Pass, at the Nevada line, along the route later followed by the Western Pacific Railroad.

Following the lull in railroad construction in 1892, Mr. Jamieson turned to another field and for eighteen months was in charge of the construction of the head-works and part of the pipe line of the Portland, Ore., Water-Works, in a feat then unprecedented, the drawing of an urban water supply from the heart of a snow-capped mountain range.

Back to railroad work again, he was for four years Engineer in Charge of Construction of the Astoria and Columbia River Railroad from Goble to Astoria, Ore. An incident illustrates the ingenuity of the pioneer: A large part of this line consists of tunnels and heavy cuts through rocky promontories and embankments across deep bays filled to a little above high tide with fine sediment brought down by the Columbia River and deposited in the tidal stretch from Bonneville, Ore., to the sea. Material borrowed from the mountain sides was used for these fills, and the subsidence was excessive. In order to secure reliable measurements of pay quantities, Mr. Jamieson devised a plan of placing heavy wooden platforms cross-wise at suitable intervals in the base of the fill, to which upright pieces were attached. Careful watch was kept, and as the fill sank into the soft bottom the uprights were extended and the heights noted. When equilibrium was finally reached, the settlement was known within reasonable limits. The estimates thus obtained were acceptable to the contractors.

For the succeeding five years Mr. Jamieson was again with the Oregon-Washington Railroad and Navigation Company, on location and heavy construction work in the Snake River Valley, or on miscellaneous examinations and reports.

In 1905, he went to Alaska to report on the development of extensive copper deposits, but finding that the interests of his employer were in conflict with those of a former client for whom he had made a reconnoissance in the same territory a short time before, he declined to continue the engagement.

In the same year (1905) the construction of the Western Pacific Railroad was begun, and Mr. Jamieson was among the first in the field. His Division comprised the entire crossing of the Sierra Nevada Mountains from the Sacramento Valley to the Nevada line, and was more than four years in building. In those tremendous canyons and among those mighty peaks there was ample opportunity for the exercise of the skill and knowledge he had acquired in thirty years of struggle with mountain, desert, storm, and flood. In all this there was nothing of the romance which a later generation may see. Among the last survivors of that mighty race of engineers who led the "iron horse" from the great plains through pathless wilderness to the Western Sea, to him it was all part of the day's work.

With the construction of this line, Mr. Jamieson's professional work was ended. Engineers to-day will look with something of envy on his record of thirty-one years of work of a kind necessarily discontinuous yet with a total of only sixteen months between engagements.

On November 22, 1910, he was married to Kathleen Ward, who survives him, together with two sisters, Maria and Agnes D. Jamieson, all of Portland, Ore. Following his marriage, he retired to a small farm near Oswego, Ore., a few miles from Portland, saying, that with only twenty-five acres he could loaf afternoons. In later years he was wont to remark, with a twinkle in his eye, that one acre would have been more nearly his size. The career of the citizen Jamieson was marked by the same faithful performance of duty as that of the engineer. From spending millions in building railroads, he came down, without loss of dignity, to the position of Treasurer of a local Farmers' Telephone Company. He served on the District School Board. Finding the neighborhood lacking in good water for domestic use, he persuaded his neighbors to form an Assessment District and obtain water through an extension of the Portland mains. In every community matter, he bore his part.

The affection and esteem in which he was held are evidenced by many expressions, of which a few are quoted.

W. D. Clarke, M. Am. Soc. C. E., states:

> "My association with him, beginning as a mere boy on the Bull Run Pipe Line and continuing through our service together on the Western Pacific, gave me an appreciation of him as a gentleman, an engineer, and a friend, that brought me to a respect and love for him second only to that for my own father."

From Richard Sachse, an Engineer with the Western Pacific Railroad Company, comes the following:

"At first, he appeared to me as a very severe, almost hard, superior, who did not welcome any personal or friendly relations with employees under his direction. Later on, I learned that this was the outward shell only, and that underneath that shell there was an understanding and sympathetic note and a staunch and loyal friend who could be depended upon under all conditions."

Mr. Robert E. Duniway, a friend and neighbor, writes:

"I was well acquainted with Mr. Jamieson from 1920 until his death, and often visited at his home. On these occasions he was a charming and genial host, though more inclined to listen to the conversation of his guests than to contribute from his own experiences.

"In 1920, he became interested in the problem of securing City water for the rural district where his home was located. A large part of the remainder of his life was devoted to this project; first making the plans and tediously raising the necessary funds, later in superintending the installation, and finally reading the meters and collecting rentals to keep the system on a business basis. For all of this work he received no compensation whatever, save the esteem and appreciation of those neighbors who knew him best. * * * His loss is deeply mourned by all who knew him."

Mrs. Charlotte Boykin Parker, a friend of many years, states:

"In the death of John Quentin Jamieson there passed from this world one of the best men and citizens, one of the rarest spirits. He was a man of unusual dignity, which came from his utter sincerity and modesty. This quiet, simple manner never concealed, from those who knew him, the big heart and soul, the high idealism.

"There was another quality almost hidden beneath these gentle ways—kept in the background through modesty, but always there as the thing on which life was founded—a strength of character like iron. Fidelity, conscientiousness and integrity were not merely laws with him, but parts of his human fabric."

True to his Covenanter ancestry, Mr. Jamieson was a deeply religious man. Soon after coming West, he joined the First Presbyterian Church of Portland and was a faithful member for fifty-two years. In a friendship of more than forty years, the writer has been aware of nothing inconsistent with his profession of faith. His outstanding characteristic was his absolute integrity. He had a keen sense of the responsibility of the engineer as an impartial judge between employer and contractor, and was as scrupulous in maintaining the rights of the one as of the other. The late Mr. Kennedy once said: "There are times when I would lose faith in the honesty of human beings if it were not for John Jamieson."

Reserved, a man of few words, yet he had a keen sense of humor, and was a warm and constant friend. A great engineer, a faithful citizen, a good neighbor, nothing affords deeper insight into the heart of the man than that little children ran to meet him, climbed into his lap, chattered of their affairs, and minded not his silence nor his brief replies, content to be with him.

Mr. Jamieson was elected a Member of the American Society of Civil Engineers on November 6, 1889.

ALBERT FREDERICK JOHNTZ, M. Am. Soc. C. E.[1]

Died October 17, 1936

Albert Frederick Johntz was born in Abilene, Kans., on July 3, 1887, the son of Frederick and Louisa P. (Muskopf) Johntz. On both sides of the family he was descended from a line of Swiss stock which came to the United States in 1830 and settled in Ohio. He was one of two children. His brother, Harry, later became a distinguished engineer, dying in service as Engineer of Maintenance and Way for the Missouri, Kansas, and Texas Railroad Company.

Albert Frederick Johntz received his preliminary education in the public schools of Abilene. He entered the University of Kansas, at Lawrence, Kans., in 1906, from which he was graduated in 1910 with the degree of Bachelor of Science in Civil Engineering.

After his graduation from college, he was employed by the City Engineer of Kansas City, Mo., and, later, by the firm of Worley and Black, on designs for municipal improvements in Middle Western cities. From the service of the Winnipeg, Salina, and Gulf Railway Company, he entered the employ of the Kansas City Terminal Company and, during the planning and construction of these terminal facilities, was advanced to the position of Assistant Resident Engineer, and, later, as Assistant Engineer assumed the supervision of the construction of the viaducts, subways, retaining walls, sewers, and track work of this project.

In 1915, Mr. Johntz returned to the University of Kansas as a Graduate Student and candidate for a higher engineering degree, but left in August of the same year to accept the position of Assistant Engineer in the office of the Chief Engineer of the Cuba Railroad Company, at Camagüey, Cuba.

His duties during the three years he was in the employ of the Cuba Railroad Company were varied and were performed with distinction. They included the survey and construction of two lines to the coast, the replacement of main-line bridges under traffic, the $1 000 000 improvements to the terminal, and the construction of the new shops and gravity yards in Camagüey. In addition to the engineering problems faced with the assistance and direction of the Chief Engineer, Mr. A. C. Reed, the construction problems were met in the field by Mr. Johntz with great credit to his ability. The success that attended his efforts in this connection, requiring the apt handling of all types and nationalities of labor, proved a measure of the man.

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[1] Memoir prepared by Mason Garber, M. Am. Soc. C. E.

Due to the World War, he returned to the United States in 1918 and, later, was commissioned a Captain in the Reserve Corps of the Construction Division of the Army. During his tour of duty—1918 to 1925—he served, successively, as Civil Engineer, Supervising Engineer of Construction, later acting as Principal Assistant to the Constructing Quartermaster in Maryland. His principal services during this time included the supervision of the construction of buildings, gun emplacements, roads, sewers, and railroads, at the Aberdeen Proving Grounds, in Maryland. Later, at Fort Benning, Georgia, he assisted in the design and construction of many of the buildings and facilities at that post.

In 1925, Mr. Johntz resigned from the Government service to become associated as Engineer with the Southern District Office of the North-Eastern Construction Company. For several years his work was that of planning, designing, and supervising, in the field, the construction of some of the more important projects which that Company had under way. These assignments included the terminal facilities and Union Passenger Station at Winston-Salem, N. C., the Winston-Salem City Hall, the Winston-Salem Young Men's Christian Association building, and other similar projects.

In 1928, he was placed in charge of the construction of the grade-elimination program of the Southern Railway Company and the City of Greensboro, N. C., for the North-Eastern Construction Company. This contract approximated $1 500 000 and contemplated the construction of twelve underpasses and over-passes. In spite of unexpected soil conditions, this program was carried through to successful completion, with great credit to Mr. Johntz personally and to his Company. Most of the work was constructed under traffic and required the underpinning and care of adjacent buildings. On the completion of his work in Greensboro, he returned to his position as Chief Engineer of the Company's Southern District Office, in Winston-Salem. His work, in this capacity, required direct field supervision of the Company's more important projects in North Carolina, South Carolina, and Virginia. Among the more recent projects completed under Mr. Johntz's direct supervision was the Engineering Group for the University of Virginia, at Charlottesville, Va.

His engineering ability, together with a forcefulness as a leader of men, is best attested by his completed works. They stand as fitting monuments to his memory.

In February, 1916, Mr. Johntz was married, at St. Augustine, Fla., to Keene Fones, of Lyons, Kans. He is survived by his widow and three sons, Frederick Fones, Robert Fones, and William Fones.

He was a member of the Twin City Country Club, the Masonic Fraternity, and the Officers Reserve Corps, U. S. Army.

Mr. Johntz was elected an Associate Member of the American Society of Civil Engineers on October 9, 1917, and a Member on January 13, 1933.

WILLIAM DATUS KELLEY, M. Am. Soc. C. E.[1]

Died January 21, 1936

William Datus Kelley was of New England ancestry. The first generation of which there is a definite record was located in Norwich, Conn., in 1690, and the third generation immigrated to Lowville, Oneida County, N. Y., about one hundred years later. Here, the family occupied an outstanding position in the business, educational, and religious life of the community.

Early in the Nineteenth Century, in that period of international disturbance that resulted in the War of 1812, began the migration to the "Far West", as the Western Reserve of Connecticut was then considered. As a part of this movement in 1811 went Datus Kelley, grandfather of William, to Cleveland, Ohio. Fifteen years before an uncle, Joshua Stow, had accompanied the surveying party of Moses Cleaveland who had been one of the original members of the Connecticut Land Company, which purchased the Western Reserve from the State of Connecticut. Others of the family, including two of his brothers, Alfred and Irad Kelley, settled in Cleveland, or the vicinity, about the same time.

Datus Kelley settled just west of Rocky River on a large tract of land extending from Lake Erie to the North Ridge where he resided for thirty-five years. In 1833 he, with his brother Irad, purchased what is now Kelley's Island (then Cunningham's Island) of about 3 000 acres, one of the group of islands in Western Lake Erie. It was in the bay on the south side of this island that Commodore Perry assembled his fleet for the historic and successful naval battle with the Lake Erie Naval Forces of Great Britain in 1813. In 1836, Datus Kelley moved with his family to the island where he developed the valuable limestone deposits as well as the forest and agricultural resources, shipping in 1834 the first cargo of limestone from the island. Since that time a large tonnage—in the year 1912 alone more than 500 000 tons—of limestone has been quarried and shipped from the island for use in the steel industry as well as for construction purposes. The stone for the first locks at Sault Ste. Marie came from Kelley's Island. Datus Kelley was a patriarch in the community and with his descendants exercised a lasting influence on its members. Among his other activities, he served as a Land Surveyor both on the island and on the mainland.

The activities of others of this generation of this family seem of sufficient interest to engineers to record here, particularly those of Alfred Kelley, due to his active participation in transportation facilities, more notably canal and railroad building. In 1815, Alfred Kelley was elected the first President of the recently incorporated Village of Cleveland, and, in 1816, with his two brothers, Irad and Datus, he incorporated the Cleveland Pier Company which was formed to build the first pier at Cleveland for the accommodation of vessels navigating Lake Erie.

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[1] Memoir prepared by Charles H. Spilstone and Wendell P. Brown, Members, Am. Soc. C. E.

Known as the "Father of Ohio Canals", Alfred Kelley as the first State Canal Commissioner was the dominating force in the building of the Lake Erie and Ohio Canal, completed in 1832. In 1847, he became President of the Cleveland, Columbus, and Cincinnati Railroad (the "Big Four"), and is credited with turning the first shovel of earth and with laying the last rail. Records of 1850 note him as President of the Cleveland, Painesville, and Ashtabula Railroad (later, the Lake Shore and Michigan Southern Railway), now with the "Big Four", a part of the New York Central System.

The brother, Irad Kelley, a prominent merchant in Cleveland, was interested in lake transportation and operated a sailing vessel between Cleveland and Detroit, Mich. He is said to have acted as a pilot for Commodore Perry's fleet. Horace Kelley, a nephew of Alfred and Datus, was another member of this capable and aggressive family, who contributed a considerable portion of his fortune, more than $500 000 for the founding of an art gallery and school. This, later, with contributions from other sources, provided for the establishment of The Cleveland Art School and the building of the Cleveland Museum of Art.

William Datus Kelley was born on Kelley's Island, on May 11, 1859, son of William Dean and Marcella (Dean) Kelley. He was a grandson of Datus Kelley and a grand-nephew of Alfred and Irad Kelley. In this inspiring and interesting environment he passed his boyhood years, receiving there his first impressions and early education in public and private schools. While waiting for his age to permit him to enter Cornell University, at Ithaca, N. Y., he went to Wooster University, later, Wooster College, at Wooster, Ohio. He later attended and was graduated from Cornell University, in 1880, with the degree of Bachelor of Science. In 1881, he acquired a degree in Civil Engineering.

In 1880, Mr. Kelley served as a Rodman on the New York and New England Railroad, later a part of the New Haven System, and as a Transitman and Leveler on the Catskill Mountain Railroad. In 1881, he was Assistant Engineer on the surveys for the South Penn Railroad from Harrisburg to Brownsville, Pa., and also Assistant Engineer on the Somerset Division until the construction work was stopped by the sale of the road to a competing line.

The year, November, 1885, to November, 1886, was spent in France and Germany in travel and study. From 1886 to 1890, he was Special Assistant Engineer on the New Croton Aqueduct for New York, N. Y., with headquarters at Tarrytown, N. Y.

In 1891 and 1892, Mr. Kelley was Engineer in Chief of the Third Corps of the International Railway Commission in Ecuador and Peru, under the sponsorship of the United States War Department, where he ably displayed the characteristics and ability of his pioneering family. The outstanding feature of this engagement was the running of a continuous back-sight line with stadia and topography from the equator near Quito, Ecuador, to Lake Titicaca, on the Bolivian boundary, perhaps the longest transit line—3 000 miles—ever produced with one instrument and by one party, crossing the main range of the Andes Mountains at nine different water divides and also the head-waters of the Amazon River 500 miles above navigation limits. This feat required living in encampments under varying conditions from the extreme tropical temperature of the Amazon Valley to the frigid regions of ice and snow 2 000 ft above the perpetual snow line of the Andes.

In addition to the survey of the railroad line, the authority involved reporting on the mineral and agricultural resources as well as the animal and vegetable life. The complete report of this project required the preparation of two large volumes of maps, estimates, and data which were published by the United States War Department.

In 1892 and 1894, Mr. Kelley was engaged in general engineering practice, designing and constructing a sewerage system for North Tarrytown, N. Y.

From January, 1894, to 1927, he was engaged in contracting on sewers and public utilities in New York City and Cleveland, as President of the firm of James R. F. Kelly and Company, later Kelly and Kelley, Incorporated, of New York City, and also of the Kelley Demarest Company, of Cleveland. During this period he developed and patented a hinged arch center providing for the renewal of sewer arch supports without demolishing the supporting centers, with a consequential major saving of labor and material.

He owned and maintained a summer home on Kelley's Island where he was interested in fruit growing, and after his retirement from active business, in writing a genealogy of the Dean family, who with the Kelley family were prominent in the history of New England and the Western Reserve.

Mr. Kelley will be remembered as a loyal, conscientious, and dependable engineer and a gentleman possessing an interesting and agreeable personality with a deep sense of duty to his friends and associates. He died in Cleveland, on January 21, 1936, and his remains were interred on the Island in Lake Erie named for and developed by this family of exceptional ability and character of which he was a worthy representative.

He was married to Isabelle Silver, of Tarrytown, N. Y., who survives him, as does a son, William D. Kelley, Jr.

Mr. Kelley was elected a Member of the American Society of Civil Engineers on November 2, 1887, and was a Past-President of the Cornell Society of Civil Engineers.

GEORGE BERTRAM de BETHAM KERSHAW, M. Am. Soc. C. E.[1]

DIED JULY 30, 1935

George Bertram de Betham Kershaw, the third son of the late Rev. G. S. W. Kershaw and Isabella Marie (Fitz-Hardinge) Kershaw, was born at Boughton, Nottinghamshire, England, on August 22, 1874. His father was Rector of the Parish of Fledborough, England. He was educated at Malvern, England, and obtained his early experience of sewerage, sewage disposal, and water supply as Assistant to Messrs. John Taylor and Sons and Santo Crimp, of Westminster, London, England.

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[1] **Memoir prepared by H. P. Kaufman, Esq., Westminster, London, England.**

At the inception of the Royal Commission on Sewage Disposal, appointed in England in 1898 to inquire into the methods of disposing of sewage and trade effluents, Mr. Kershaw was appointed Engineer to the Commission, in which capacity he did excellent and most noteworthy work for sixteen years. The work which he accomplished during these years can be appreciated to some extent by those who have studied the voluminous reports presented by the Commission.

When the Commission was dissolved in 1914, Mr. Kershaw engaged in private practice as a Consulting Engineer, and the experience he had gained while acting as Engineer to the Royal Commission proved of inestimable value to him in the many sewerage, sewage disposal, trade waste, and water supply projects he designed and carried out for a large number of important local authorities and manufactories in England. Mr. Kershaw also visited Egypt, India, and Bermuda to advise the Towns of Khartoum, Bombay, and Hamilton, on their sewerage and sewage disposal problems.

In 1911, he published his well-known book, "Modern Methods of Sewage Purification", and, in 1914, a further work entitled "Sewage Purification and Disposal", of which a Second Edition appeared in 1925. He was also the author of several other publications and a number of articles written by him on the subject of the treatment of trade wastes were published in technical journals from time to time.

He was a member of the Institution of Civil Engineers, the Association of Consulting Engineers, a Fellow of the Royal Sanitary Institute, a Fellow and a Past-President of the Institution of Sanitary Engineers, a member of the Institution of Water Engineers, and of other learned bodies connected with the Engineering Profession.

Mr. Kershaw was a man for whom all who knew him had the deepest regard, a man of gentle actions, and loved by his friends. He was held with the greatest esteem in his profession.

He was married, in 1900, to Edith, daughter of the late John Charles Pickin, of Fledborough Manor, Nottinghamshire, and is survived by his widow.

Mr. Kershaw was elected a Member of the American Society of Civil Engineers on April 2, 1913.

BURDETT KIPP, M. Am. Soc. C. E.[1]

DIED JANUARY 23, 1937

Burdett Kipp was born in New York, N. Y., on August 4, 1874. He was the son of Lieut.-Col. William H. Kipp, associated with the 7th Regiment, New York National Guard, since the Civil War, and Emily

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[1] Memoir prepared by R. Prosper Gustin, Assoc. M. Am. Soc. C. E.

A. Kipp, both of whom were descended from old New York families. After his preparatory education in New York schools, he entered the School of Mines, Columbia University, and was graduated from its Civil Engineering Course in June, 1898.

In September, 1898, Mr. Kipp became an employee of the City of New York as Topographical Draftsman and Transitman, and did drafting, computing, and surveying in the five Boroughs. In September, 1900, he was transferred to the Rapid Transit Commission as Assistant Engineer. From September, 1903, until September, 1906, he was Engineer in responsible charge of construction, for the Rapid Transit Commission, of the Interborough Rapid Transit Railroad Subway under Broadway from south of Battery Place to Vesey Street (approximate value, $2 000 000).

From September, 1906, to September, 1919, he was with the Degnon Contracting Company as Principal Assistant Engineer and, later, as Engineer in Charge of Construction of the following work in New York City: Hudson-Manhattan Railroad Subway from 12th Street to 33d Street (value $4 000 000); construction of the foundations for the Gimbel Building and the McAlpin Hotel (value, $500 000); New York Municipal Railroad, under Broadway, from Park Place to Walker Street (value, $2 500 000); Interborough Rapid Transit Railroad Subway, under Greenwich Street and West Broadway, from Vesey and Franklin Street, and under Varick Street to Beach Street (value $3 100 000); and under 14th Street from Irving Place to Avenue B (value, $2 000 000).

From 1922 to 1928, Mr. Kipp was employed by Niewenhous Company as Chief Engineer and Superintendent of Construction on the following work: The Bronx Wholesale Terminal Market (largest of its kind; value, $3 000 000); Concourse Plaza Hotel (value, $2 225 000); Mount Saint Michael's Institute (value, $660 000); Rockland State Hospital, Orangeburg, N. Y. (value $8 000 000); Biological Plant, Brooklyn, N. Y. (value $560 000); Hall of Records and Power House, Newark, N. J. (value, $2 200 000); also, banks, schools, and industrial buildings.

From February, 1928, to September, 1936, he was Chief Engineer with Atwell-Gustin-Morris, Incorporated, on the following work: Independent Subway System in Nott Avenue and Jackson Avenue, from Van Alst Avenue to Harris Avenue, Long Island City (value, $5 000 000); and Independent Subway System under Broadway from Northern Boulevard to Baxter Avenue, Elmhurst (value, $8 000 000).

From September, 1936, to time of his death, Mr. Kipp was employed by the G. A. M. Construction Corporation (successors to Atwell-Gustin-Morris, Inc.), as Chief Engineer on the Queens Construction Shaft of the Midtown Tunnel.

On June 2, 1913, he was married to Georgina Sullivan. Mrs. Kipp, with his daughter, Emily, survives him.

He will be remembered for his pleasing personality and for his dependability both in business matters and socially. He was a man of fine character, always cheerful, and always kindly. By those who knew him well, he was beloved.

Mr. Kipp was elected a Member of the American Society of Civil Engineers on August 9, 1920.

AUGUST GUSTAVE KLEINBECK, M. Am. Soc. C. E.[1]

DIED FEBRUARY 8, 1936

August Gustave Kleinbeck, the son of Peter and Fredericka (Best) Kleinbeck, was born on February 17, 1852, in Württemberg, Germany. He received his education in the Royal Technical School, of Stuttgart, Germany, from which he was graduated in 1869, with the degree of Building Technic.

From 1869 to 1872, he was in the service of the German Government on the construction of a railroad through the Black Forest in the Valley of the Nagoldie River. Following this, until 1875, Mr. Kleinbeck was in the military service of Germany. From 1875 to 1878, he was employed on the construction of the Hohenzollern Railway running from Tubingen, Württemberg, to Sigmarigen, Hohenzollern, Germany. In 1879, he passed the State Examination in Stuttgart, and received the title of Building Master.

In 1880, prospects in the United States attracted Mr. Kleinbeck and he came to Chicago, Ill., and was immediately employed by Mr. Jennings, an Architect. From 1881 to 1882, he was engaged on the construction of a narrow-gauge railroad running from Toledo to Cincinnati, Ohio, and to St. Louis, Mo. While with this Company he designed wooden trestles for a bridge across the Okaw River. The following two years he served as Draftsman for the Columbus and Cincinnati Midland Railroad Company, and the next four years found him employed as Chief Engineer for a railroad running from Peoria, Ill., to St. Louis. It was at this time that he superintended the sinking of a shaft and the construction of a coal mine north of Mt. Olive, Ill. From 1885 to 1892, he served as Civil Engineer on the Tuscaloosa, Montgomery, and Bainbridge Railroad, in Georgia.

In 1893 Mr. Kleinbeck deserted railroading and was engaged in other lines of engineering work until 1897. During this time, he supervised the laying of gas mains in New York, N. Y., for the East River Gas Company, and, later, managed the Litchfield Mining and Power Company, of Litchfield, Ill.

Returning to railroad work, in 1898, he was appointed Civil Engineer on the construction of a line from Columbia, Mo., to the Missouri River, and from 1900 to 1904, he was Chief Engineer in charge of all bridge work for the Frisco Railroad Company. The years 1905 and 1906 found him acting as Chief Engineer for the St. Louis, Brownsville, and Mexico Railroad Company. He made extensive soundings for bridges across the Brazos River and the Rio Grande, in Texas.

One of Mr. Kleinbeck's most important accomplishments was the design and construction of a bridge across the Brazos River, consisting of one draw-span of 235 ft, two girder spans of 100 ft, the pivot and end pier of the draw-span resting on caisson foundations. The other piers rested on piles. He also designed and built a draw-span and two end spans, each 60 ft long, for the St. Louis, Brownsville and Mexico Railroad Company.

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[1] Memoir prepared by A. G. Kleinbeck, Jr., Esq., Litchfield, Ill.

In 1907, because of ill health, he gave up his professional work and lived a retired life in Litchfield. Mr. Kleinbeck died of necrosis of the jaw bone, at St. Francis Hospital, in Litchfield, on February 8, 1936.

On July 14, 1887, at Litchfield, he was married to Matilda Lange who, with two sons, J. C. Kleinbeck, of Chicago, and A. G. Kleinbeck, Jr., of Litchfield, and a daughter, Mrs. R. J. Blank, of West Palm Beach, Fla., survives him.

Mr. Kleinbeck was elected a Member of the American Society of Civil Engineers on February 3, 1897.

CARL GUSTAF EMIL LARSSON, M. Am. Soc. C. E.[1]

Died April 1, 1936

Carl Gustaf Emil Larsson was born in Geddeholm, Sweden, on February 29, 1864, the son of Lars Larsson and Anna Lovisa Larsson. He was a student in the higher Elementary School, in Westeräs, and in the Real School, in Stockholm. After having been graduated from the latter institution he entered the Royal Institute of Technology, in Stockholm, from which, after three years of study, he received the diploma of Mechanical Engineer and, after an additional year, was given the diploma of Civil Engineer in 1887. He came to United States in the autumn of 1887.

From January, 1888, to July, 1889, he was a Draftsman with the Wrought Iron Bridge Company, at Canton, Ohio, the Edge Moor Bridge Works, at Edge Moor, Del., and the Hilton Bridge Company, at Albany, N. Y. In July, 1889, Mr. Larsson returned to the Edge Moor Bridge Works as a Designer, holding that position until the organization of the American Bridge Company, in May, 1900.

On July 17, 1900, he was transferred to the Pencoyd Office of the American Bridge Company as a Designing Engineer, and, in March, 1901, was appointed Division Engineer in charge of the designing offices and drawing rooms of the Company in the eastern part of the United States. In January, 1904, Mr. Larsson was made Assistant Chief Engineer of the Company, with office first at Ambridge, Pa., and, later, in New York City. For a short time in 1927 he was Acting Chief Engineer of the Company, and, in September, 1927, he was appointed Chief Consulting Engineer which position he held until he retired on June 30, 1933.

In 1918, he visited France on an important mission for the Ordnance Department of the United States Army.

His scholastic training in Sweden was thorough and differed from contemporaneous methods in the United States. One illustration of this is that the teaching of theoretical and applied mechanics began in the preparatory school before entering the Royal Institute of Technology and was continued in some form for a period of seven years. Mr. Larsson acquired an unusually thorough fundamental knowledge of theory which remained outstanding throughout his long service in engineering work.

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[1] Memoir prepared by Otis E. Hovey, and William M. White, Members, Am. Soc. C. E.

During an association with him of more than thirty years, he and the senior writer had discussed many difficult problems in analysis. His methods of approach were so different from those commonly in use in this country that his verification of the results of analysis were most valuable.

His early work as a designer demonstrated that he was logical and original in his methods, that he had a keen sense of the economics of engineering, and was quick to take advantage of local conditions favorable to simplicity in design and construction. He was ingenious in devising simple and efficient methods of erecting bridges under unusual circumstances and in developing economical designs to meet difficult conditions.

Mr. Larsson came in close contact with many younger engineers, was much interested in their progress, and was glad to advise them, to give them the benefit of his experience, and to assist them to acquire direct and effective methods in analysis and design.

It frequently happens that an engineer employed by a corporation does not become so well or so favorably known as he would have been, had he been in practice under his own name. Mr. Larsson was an exception in this respect. His work brought him in contact with the building of many of the important bridges in America and his skill and resourcefulness contributed much to the design and construction of these structures.

Some of the most difficult problems in bridge design and construction are encountered where an old bridge must be rebuilt without interrupting traffic. Mr. Larsson had unusual skill in devising safe and economical means to carry out such work. Sometimes, a new bridge is designed so that it is difficult or nearly impossible to build it as originally planned. Some of his most brilliant work was done in overcoming such conditions. A long list could be given of bridges to the construction of which Mr. Larsson made important contributions. A few of them are the Hell Gate (arch) Bridge, of the New York Connecting Railroad; the renewal of the Ohio Connecting Railroad Bridge, at Brunot's Island, Pittsburgh, Pa.; the Bessemer and Lake Erie Railroad Bridge across the Allegheny River; the Cincinnati Southern Bridge over the Ohio River at Cincinnati, Ohio; and others of comparable magnitude. On the Bessemer and Lake Erie Bridge he originated the modern method of jacking the continuous spans of fixed bridges to the desired reactions, a method that had long been in use for fixing the reactions and designing the end lifts of continuous swing spans.

In addition to the structural problems of bridge construction, Mr. Larsson contributed to the improvement of the materials of the art. He was an early advocate of the use of silicon steel as a high-strength material for bridges. Also, he was largely responsible for the development of heat-treated eye-bars which have been used for the tension members of many large bridges. Heat-treated eye-bars of high strength also were used in the suspension chains of the Florianopolis Bridge, in Brazil.

On April 26, 1893, Mr. Larsson was married to Eliza Gano, who survives him. Mrs. Larsson is a daughter of Charles K. Gano and Anna E. Bartram, of Wilmington, Del. Their only son, Gano Larsson, who was a student at Massachusetts Institute of Technology, died in Sweden in 1919.

He was a member of the American Iron and Steel Institute, the American Society for Testing Materials, the American Railway Engineering Association, and the Army Ordnance Association.

Mr. Larsson was elected a Member of the American Society of Civil Engineers on April 1, 1903.

EDWARD HERVEY LEE, M. Am. Soc. C. E.[1]

DIED JANUARY 11, 1937

Edward Hervey Lee was born in Dayton, Ohio, on January 29, 1863, the son of the Rev. Dr. John Newton and Julia (Sheldon) Lee, members of a noted Ohio family of ministers and educators. Mr. Lee, the eldest of a family of four, was educated at Ohio University, and the University of Wooster, at Wooster, Ohio.

In 1880, he entered the railroad service, being employed, successively, as a Rodman on the Scioto Valley Railway from April, 1880, to January, 1881; Rodman and Instrumentman on the New York, Chicago, and St. Louis Railway from January, 1881, to January, 1882; Instrumentman on the Ohio Central Railway from January to August, 1882; Instrumentman on the New York, Chicago, and St. Louis Railway from August, 1882, to January, 1883; Resident Engineer, in charge of construction, on the Ohio River Railway from January, 1883, to January, 1884; Instrumentman on surveys in West Virginia from January to July, 1884; Resident Engineer on the construction of the Wisconsin Central Railway, from August, 1885, to January, 1886; Division Engineer, in charge of construction, on the Omaha and Republican Valley Branch of the Union Pacific Railway from January, 1886, to March, 1887; Resident Engineer on the Wisconsin Central Railway from March to September, 1887, in charge of the construction of ore yards, engine-houses, tracks on dock, etc.; and Draftsman, Office Engineer, and Chief Engineer of the Elgin, Joliet and Eastern Railway Company from September, 1887, to June, 1893.

In January, 1894, Mr. Lee left the railroad service to take charge of a field party on the construction of the Sanitary District of Chicago and continued in that work until March, 1898. He also acted as Contractors' Engineer, superintending steam-shovel work, and concrete masonry and sewer construction from June, 1896, to June, 1897. He returned to railroad work as Principal Assistant Engineer on joint grade separation for a number of railroads at 16th and Clark Streets, in Chicago, Ill., from March to December, 1898.

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[1] Memoir prepared by F. E. Morrow. M. Am. Soc. C. E., and W. F. Krahl, Assoc. M. Am. Soc. C. E., in collaboration with Committees of the Am. Ry. Eng. Assoc., and the Western Soc. of Engrs.

In December, 1898, Mr. Lee entered the service of the Chicago and Western Indiana Railroad Company and The Belt Railway Company of Chicago as Engineer and General Roadmaster. He advanced, successively, to the positions of Chief Engineer, and Vice-President and Chief Engineer, acting as President during the period of Federal Control. On January 1, 1927, he was made President of these roads, remaining in that position until April 1, 1932, when he retired with the title of "President, Retired."

Almost all of Mr. Lee's active life was devoted to the construction, maintenance, and operation of steam railroads. During the time of his employment by the Chicago and Western Indiana Railroad Company and The Belt Railway Company of Chicago, he had charge of a large amount of grade-crossing elimination between the railroad lines and highways, and between railroads. He was a pioneer in the development of methods and means of eliminating grade crossings in Chicago and was considered an authority on such work. He was also in charge of considerable work in connection with the development and enlargement of these properties. He devoted much time and study to the development and improvement of steam railroad terminal facilities and terminal operations, and his opinions and judgments on many phases of these problems were highly valued by railroad engineers and executives.

Mr. Lee always had an active interest in the well-being and advancement of the Engineering Profession. He became a Charter Member of the American Railway Engineering Association upon its organization in 1899 and was quite prominent in the affairs of that Association, serving as a member of the Committee on Uniform General Contract Forms from 1908 to 1921, inclusive, being Chairman from 1917 to 1920, inclusive. He served as a Director, of the Association in 1918, 1919, and 1920, as Vice-President in 1921 and 1922, and as President in 1924. He became a member of the Western Society of Engineers on November 7, 1899, and contributed much to the success of that Society serving as President in 1914. For many years, Mr. Lee was an active member of the Chicago Engineers Club, of which he was President in 1919. He was also an active member of the Traffic Club of Chicago.

Mr. Lee was married, in 1916, to Ruth Sheldon Brooke, eldest daughter of the late Bishop of Oklahoma, the Rt. Rev. Francis Key Brooke. He died on January 11, 1937, at his home in Chicago at the age of 73. He is survived by his widow, a sister, Mrs. Mary Lee Stewart, and one brother, John H. S. Lee. His youngest brother, Dr. W. George Lee, died in 1927.

From his very early youth, Mr. Lee was an accomplished pianist and a lover of music. During his active life his principal recreations were handball and golf. In his younger days, he was an expert handball player and rated as one of the best players in Chicago. Until his retirement, he had been for many years an active member of the Chicago Athletic Association, the Edgewater Golf Club, and the Olympia Fields Country Club.

He was possessed of a fine quality of humor and was widely known for his ability as a story-teller. He was admired and respected by all his associates and friends on acount of his sterling character, his ability as an engineer, his good judgment, and for his human qualities that made him liked by all with whom he came in contact, and loved by those who knew him best. In spirit and memory, Edward Hervey Lee will continue to live in the hearts of a host of warm and loyal friends, for he, himself, was the very embodiment of the spirit of fine and genuine friendship wherein association with him was always delightful.

Mr. Lee was elected a Member of the American Society of Civil Engineers on June 6, 1900.

GEORGE CASPER DOERING LENTH, M. Am. Soc. C. E.[1]

Died May 11, 1936

George Casper Doering Lenth was born in Garrett, Ind., on February 1, 1882, the son of Charles L. and Anna (Doering) Lenth. Both his parents were born in Germany.

George Lenth received his early education in the public, Grammar, and High Schools of Chicago, Ill. He was graduated from the Massachusetts Institute of Technology at Boston, Mass., in 1903, with a degree of Doctor of Science in Civil Engineering.

Following his graduation Mr. Lenth returned to Chicago, and was engaged, for a short time, with the Chicago and North Western Railroad Company and in the Assessor's Office of Cook County. Later, he entered the service of the City of Chicago, in the Bridge Division of the Department of Public Works. In 1905, he was in the employ of the Board of Local Improvements as a Rodman. The following year, he became an Assistant Engineer in the Sewer Division and, later, Assistant Chief Sewer Engineer. Sewerage work and special assessments became his specialty for the remainder of his life. In the absence of the late C. D. Hill, M. Am. Soc. C. E., during 1910 and 1911, Mr. Lenth was appointed Engineer of the Board of Local Improvements, serving in that capacity until Mr. Hill's return. During the World War, he was engaged in construction work at Camp Knox, in Kentucky.

In 1921, Mr. Lenth was granted leave of absence by the Board of Local Improvements, to take the position of Secretary of the Clay Products Association. He was also engaged in private practice as Consulting Engineer, which included service in that capacity for the City of Chicago on many important projects. He was a member of the Subway Commission appointed by the Mayor of Chicago to make plans for a subway in State Street.

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[1] Memoir prepared by Loran D. Gayton, M. Am. Soc. C. E.

Mr. Lenth was a member of the Illinois Society of Engineers, American Ceramic Society, Chicago Engineers' Club, Municipal Employees Society of Chicago, Chicago Municipal Employees Credit Union, Lake Shore Athletic Club, and the Royal League Club.

In 1905, he was married to Lillian Julien Holton, and is survived by his widow and two sons and two daughters.

Mr. Lenth was elected a Member of the American Society of Civil Engineers on July 9, 1912.

CLIFFORD SHERRON MacCALLA, M. Am. Soc. C. E.[1]

DIED FEBRUARY 4, 1936

Clifford Sherron MacCalla was born in Wallingford, Pa., on March 31, 1876, the son of Clifford P. MacCalla, who was an attorney, writer, and prominent Mason, in Philadelphia, Pa., and Helen (Arrison) MacCalla. His parents were both of American ancestry dating back for several generations. His great-great-great-grandfather, John MacCalla, born in 1712, came from the Island of Islay, one of the Hebrides off the southwestern coast of Scotland, to settle in America. This ancestor became a "Prominent Rebel" during the Revolutionary War, and on account of his work in encouraging the cause of the Revolution was confined for a time in the "Jersey prison ship" by the British. He was buried in the churchyard at Roadstown, N. J.

Many of Clifford Sherron MacCalla's most admirable traits were inherited from his father. In a resolution offered June 8, 1892, by the Bethlehem, Pa., Lodge No. 283, F. & A. M., the Right Worshipful Past Grand Master, Clifford P. MacCalla, was eulogized as "a man of noble and sincere soul", of whom it can truly be said that, "as a man, a scholar and a Mason, he lived respected and died regretted, having performed his labours with honor and reputation."

Clifford Sherron MacCalla was educated at the Friends' Central High School, in Philadelphia, and prepared for college at Ulrich's School, in Bethlehem, Pa. He was graduated from Lehigh University, in Bethlehem, in 1896 with the degree of Electrical Engineer. While maintaining a good scholastic standard, he took an active interest in class affairs and social events. He was a Charter Member of the revived Alphi Rho Chapter of the Sigma Chi Fraternity and was instrumental in the acquisition of the present Chapter House, at Bethlehem. He was also a member of the Sophomore Cotillion Club. In athletics, his favorite sport was lacrosse. His eastern rearing was reflected throughout his life by his clipped speech and precise manner which, however, was tempered by considered and genuine friendliness for others.

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[1] **Memoir prepared by W. A. MacCalla, Esq., Mt. Lebanon, Pa.**

After his graduation from Lehigh Mr. MacCalla's initial work was in the Installation Department of the Philadelphia Bell Telephone Company. After a few months, he quit a $7 per week job with that Company to take a $5 per week job as "trouble shooter" with the Edison Electric Light Company of Philadelphia, as he thought there was a greater future in the electric industry. Having made this choice, his rise within the industry was rapid at every place he worked. In 1897, he went with the Edison Electric Illuminating Company of Brooklyn, N. Y., as Draftsman. He was then moved to the Construction Department installing machinery and, in three years, was advanced to be Construction Boss and General Foreman of Construction.

One of his colleagues in this early work, now (1936) an engineer for the United States Government with the Works Progress Administration, writes concerning Mr. MacCalla:

> "Mac and I began our real work together in the Brooklyn Edison Company. I departed from the industry and became a sort of 'beloved vagabond.' Mac stayed with it and became a pillar of industry. I wish that men would say of me when I am gone the sort of tributes paid to Mac."

In 1901, Mr. MacCalla was engaged in experimental and structural work for the General Electric Company, at Schenectady, N. Y. Upon hearing that the Company proposed to construct a large electrical installation for the New South Wales (Australia) Government, he asked to be transferred to the Foreign Department and was sent as Principal Assistant Engineer in charge of construction. This installation consisted of three 1 500-kw steam-driven generators, together with sub-stations and distribution system. This was the largest three-phase transmission plant south of the equator. In addition, he put in a complete installation of five rotary converter sub-stations and railway equipment for the Sydney tram lines of the New South Wales Government.

In the spring of 1903, after the completion of this job, Mr. MacCalla returned to the Schenectady Factory to conduct steam turbine and special tests. That summer he rejoined the Edison Electric Illuminating Company of Brooklyn as Assistant Electrical Engineer in charge of the Operating Department, including two main steam-generating plants and fourteen distributing sub-stations.

In the fall of 1903, he went to Spokane, Wash., as Assistant to the General Manager of the Washington Water Power Company. In this position he set a mark for others to follow, for in the fifteen years that he was associated with this Company he was advanced to Assistant General Manager, Chief Engineer, General Manager, and finally, Vice-President and General Manager, which position he held during his last five years with that Company.

In his first capacity, he was in charge of engineering and construction, practically all of which the Company did with its own organization. From 1904 to 1906, Mr. MacCalla designed and had charge of the construction of the Post Falls Hydraulic Power Plant, the approximate cost of which was $1 000 000, including electrical, hydraulic, and construction features. From 1907 to 1908, he directed the electrical design, erection of buildings, and installation of all machinery for the $900 000 steam turbine plant in Spokane.

Following this, he directed the electrical, mechanical, and hydraulic design and construction of the $2 000 000 Little Falls Plant. From 1913 to 1915, he designed and built the Long Lake Hydro-Electric Plant on the Spokane River. This plant had the highest spillway dam in the United States, being 227 ft from bed-rock to forebay level, and designed for 19 ft of water on the crest of the spillway. It had the four largest hydraulic turbines built at that time, each developing 22 500 hp.

As Vice-President and General Manager, Mr. MacCalla had full charge of all operations, including engineering, sales, rates, etc. The Company, with a load of 55 000 kw in 1918, served Spokane and thirty-three smaller towns within a radius of 100 miles, and also distributed power to the Coeur D'Alene mines in Idaho. It had about 33 000 accounts in a population of 150 000, and employed 1 400 persons, including the construction force. It owned the city street and interurban railways. Mr. MacCalla had charge of equipping the city railway system with one-man cars, Spokane being the first city in the United States with a population of 100 000 or more to have complete one-man operation. This change was accomplished with the cordial co-operation of the street railway employees.

During the time that he lived in Spokane, he was very active in civic affairs and was particularly interested in the activities of the University Club.

In 1918, Mr. MacCalla returned to the General Electric Company in Schenectady. While on the staff of the Vice-President in charge of manufacturing he was Chairman of the Furnace Committee of the Schenectady Works and devoted his attention also to the development of a number of miscellaneous lines of manufacturing, including high-voltage insulators, overhead line materials, and X-ray tubes. Later, he had charge of organizing a new factory in Rochester, N. Y., designed for an output of 8 000 small motors per week and employing from 1 000 to 1 400 hands. He was then made the first Manager of the Rochester Works of the General Electric Company.

In 1920, he joined the Virginia Power Company, at Charleston, W. Va., as Vice-President and General Manager in responsible charge of all operations. This Company had an installed generating capacity of 60 000 kw in steam turbines. At the time he took over the management of this Company the post-war period of high prices was having an adverse effect on its earning power. Within the next two years Mr. MacCalla secured two rate increases for his Company and, later, had dismissed an attempt to reduce rates before the Public Service Commission of West Virginia.

In addition to his office with the Power Company, Mr. MacCalla was Vice-President of the Coalburg Colliery Company, at Ronda, W. Va., from 1922 to 1923. While in Charleston, he was appointed Chairman of the State Board for Registration of Engineers by the Governor of West Virginia. He was also appointed by the Governor as a Delegate to represent West Virginia at the Water Power Conference at New York, N. Y., in February, 1922.

Mr. MacCalla's first contact in Youngstown, Ohio, where he was located during the remainder of his career, was in 1923 when he became Vice-President and General Manager of the Pennsylvania-Ohio Power and Light Company, under R. P. Stevens and the late John T. Harrington, the former of the Republic Railway and Light Company. In addition, he became President and General Manager of the Youngstown Municipal Railway Company and the Pennsylvania-Ohio Coach Lines Company; President and General Manager of the Pennsylvania Power Company; Vice-President of the Penn-Ohio Edison Company; and President and General Manager of ten subsidiaries of the Penn-Ohio System. Later, the ownership of the Penn-Ohio System passed to the Allied Power and Light Company, with Mr. MacCalla still occupying the same positions. In 1926 and 1928, under his management, the Penn-Ohio property received the Charles A. Coffin Gold Medal for distinguished contribution to the development of electric light and power for the convenience of the public and the benefit of industry. This is the only instance in which this award was made twice to the same organization. The Pennsylvania-Ohio Power and Light Company received the award in the Forbes Competition in 1927 for outstanding performance in the line of public relations. In 1928, the Penn-Ohio System won the Anthony N. Brady Gold Medal awarded by the American Museum of Safety, in conjunction with the American Electric Railway Association. The inscription on the gold medal reads: "Awarded to the Penn-Ohio System for achievement in safety and sanitation by electric railway organizations in the United States—1927." Always active in national affairs in the industry, Mr. MacCalla was for a time President of the East-Central Division of the National Electric Light Association and presided at the convention in Louisville, Ky., May 7 to 10, 1929.

On July 1, 1930, the Ohio Edison Company was formed by a merger of the Pennsylvania-Ohio Power and Light Company with electric properties in Akron and Springfield, Ohio, and Mr. MacCalla then became Vice-President and Division Manager and a Director of the Ohio Edison Company. He was also named a Director of the Pennsylvania Power Company, at New Castle, Pa., a subsidiary company. He continued in this capacity until his death in Youngstown on February 4, 1936. He was buried in Mt. Moriah Cemetery, Philadelphia, Pa.

Besides his widow, the former Mrs. Anna Lanter Clark, of Olathe, Kans., to whom he was married in 1922, he is survived by two children by a previous marriage in 1906 to Agnes Wilson Purves, of Sydney, N. S. W., Australia. The children are Sylvia (MacCalla) Ryder, of Cleveland, Ohio, and Willard A. MacCalla, of the West Penn Power Company, Pittsburgh, Pa.

A statement of facts such as the foregoing may seem tedious and uninteresting, but Clifford MacCalla's life and career were anything but that. He had a capacity for leadership and a joy for work that were outstanding. Few executives were better liked by employees. His influence and character touched the most remote employee, making him his friend. Humble employees were "Bob, Jim, and Dick", just as were the many important men he numbered among his friends. He often "went to bat" against higher officers of the Company on behalf of his employees, and frequently stated that the hardest job he was given was to lay off men during the depression. He had learned every step of the utility business from the ground up, and when his employees encountered problems which they were unable to solve, Mr. MacCalla would show them exactly how the job should be done. Then he would add, "You know I've done it and I know."

The feeling of his employees toward their Chief was expressed editorially in the February number of the *Ohio Edison Magazine*, in these words:

> "The loss of a capable executive and cooperative business associate is a misfortune to any organization, but when that person was also endowed with the rare talent of making and keeping friends, his passing is mourned as a real tragedy. C. S. MacCalla was such a man. He possessed a genuine liking for his fellow men. Their occupation or station in life made no difference in the cordial word he always gave them. In return he was held in highest esteem and respect by his close friends and employees alike. He inspired a friendship that lives—and to live in the hearts of friends left behind is not to die.
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> "His success in his chosen field was the result of progressive endeavor combined with a generous personality. He never progressed so far that he outgrew his friends.
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> "Those who were present at the Safety Dinner in Youngstown will always feel honored that theirs was the privilege of being with Mr. MacCalla at his last group gathering; they will remember him at his best—a cordial host, a brilliant speaker and master of ceremonies, and best of all a genuine friend."

During the week following his death, a memorial window in the Youngstown Electric Shop, of the Ohio Edison Company, was appropriately arranged with a photograph of Mr. MacCalla, beneath which were inscribed these words: "To those who knew him best he leaves the memory of countless loving deeds, the richest legacy that man can leave to man."

In addition to his duties as Vice-President of the Ohio Edison Company, Mr. MacCalla had a wide variety of interests and because of a magnetic personality, he possessed countless friends wherever he went. As Director of the Youngstown Chamber of Commerce and Chairman of the Allied Industries Committee of the Mahoning Valley, he was intimately known and liked by many business men. Hs was an active Rotarian, having served as a member of the Board of Directors and on various committees. He was also a member of the local Torch Club and took a leading part in its activities. He organized and was elected President of the Youngstown Alumni Chapter of Lehigh University and kept an active interest in Sigma Chi, his college fraternity. He was a member of the Youngstown Club, the Youngstown Country Club, the Bankers Club, of New York City, and the Academy of Political Science. He was also a Fellow of the American Institute of Electrical Engineers.

Describing his personality, Esther Hamilton, Columnist of the *Youngstown Telegram*, writes:

> "Mr. MacCalla 'got around.' He would be seen dancing in costume at the Youngstown Players ball one evening and presiding at a civic group the next. He knew everybody and everybody literally knew him.
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> "Quite frequently in later years Mr. MacCalla walked to work, swinging down Fifth Avenue in the early morning. Of a Sunday he liked to carry a cane which he did handsomely.

"He was a member of the First Presbyterian Church and was active in the men's group of that church. In politics, he had always been a Republican.

Mr. MacCalla played hard and he worked hard. He liked to play golf and he liked to swim. He was attached to the theater and no first night was complete in Youngstown without Mr. MacCalla and his wife.

"He was especially interested in the Youngstown Players and on one occasion took time out of his busy life to play a rôle in one of the shows, never missing a rehearsal. He was seldom ill and his constant flow of energy amazed his closest associates. 'He never goes to bed', one of his friends said of him recently."

Upon learning of his death, civic leaders and associates of Clifford MacCalla united in praising him as a gentleman and "one of the outstanding electrical engineers of the United States."

J. E. E. Royer, Vice-President and General Manager of The Washington Water Power Company wrote: "His work in helping to build the physical properties of The Washington Water Power Company, especially the hydroelectric plants, is outstanding and will stand through the years as a memorial to his engineering genius and ability."

A. C. Blinn, Executive Vice-President of the Ohio Edison Company, said:

"Ohio Edison Company and I, personally, feel deeply the loss of Mr. Clifford S. MacCalla who for more than twelve years had been in active charge of our Youngstown properties.

"Mr. MacCalla was not only active in the affairs of our own company, but was generally known and respected in the public utilities field, contributing materially to its development. He was respected and admired by his associates not only for his business capacity and leadership, but also for his genuine personal work. His family and friends have suffered an irreparable loss."

John Rowland, President of the Mahoning National Bank, stated: "He was one of the greatest authorities on electric power in the country. In his death, Youngstown loses a master of business and a civic-minded gentleman. I lose a close friend."

Attorney Charles F. Smith said::

"I have known Mr. MacCalla since he became manager of the Pennsylvania-Ohio Power & Light Company in 1923. He was an affable gentleman and one of the most skilled men I have ever known in the operation of an electric power and light company. He knew the business in all phases from the ground up, as well as being an expert in management and public relations. He always was interested in giving patrons the greatest service possible.

"Ohio Edison has lost one of its most valued employees, this community one of its best citizens, and I, a fine and loyal friend."

R. N. Graham, of the Youngstown Municipal Railway, wrote: "I worked directly with Mr. MacCalla until the railway was segregated. He was a wonderful man, intensely loyal to men who worked for him in big and small matters. I have never known a man so thoughtful and interested in the welfare of the men who worked for him."

Charles F. Owsley, Architect, stated: "I am very much shocked at his death. In Mr. MacCalla's death, I lose a close personal friend for whom I have the highest esteem. He was a civic-minded man of science and business with high intellectual attainments."

According to the *Youngstown Telegram*:

"The sudden death of Clifford S. MacCalla came as a shock to his community because of the part he has played in public and private life ever since he came to the city.

"He was an outstanding figure as an Ohio Edison executive. He was almost equally prominent as an enthusiastic participant in other community affairs, such as activities of The Youngstown Players and of the Rotary Club.

"A succession of promotions and advancements in all phases of the Engineering Profession preceded his appointment here. His connection in an executive and advisory capacity with many social and industrial groups testified to his ability and the confidence placed in him by other leaders.

"He was an aggressive business man and at the same time a charming gentleman whose graciousness in serving those in his own organization and his community endeared him to many.

"Clifford MacCalla had that happy faculty of differing with acquaintances and friends in and out of business relationships without permitting those differences in opinion to become personal differences.

"Youngstown has suffered a great loss in the death of a leader among its corporation executives, and in social activity; in the death of a personal friend."

According to the *Youngstown Vindicator*:

"Youngstown has to thank Clifford S. MacCalla for its dependable light and power service. When he came here a dozen years ago these indispensable utilities were still in the experimental stage. While he exaggerated a little when he said that 'the lights went out whenever it thundered', it is true that both light and power were interrupted with every high wind or heavy rain. Mr. MacCalla gave Youngstown the great boon of electric service so dependable that we no more expect it to be interrupted than we are in doubt as to whether we shall get water or not when we go to a faucet.

"Improvement of the irregular service of the early 1920's was Mr. MacCalla's purpose in coming to Youngstown. He brought modern ideas and a new standard of ability to the management of the local plant. Thoroughly grounded in his profession, and with his technical training reinforced by wide experience in its various phases, both in this country and abroad, he at once gave the city the impression that he knew what he was about.

"The years of the company's greatest development followed. One improvement succeeded another, until his plans reached their culmination in the building of the great power plant at Toronto and the construction of substations and service lines to replace the old ramshackle system and assure service as nearly continuous as man has been able to make it.

"It is not too much to say that any one who can do for Youngstown what Mr. MacCalla did is a public benefactor. He did not look upon himself in that light, and what he did came from pride in his profession and delight in achievement. Personally, he was keen and wide awake, well informed on the rapidly changing developments in the electrical industry and eager to adopt whatever he found helpful. His mind was intent on his work, and his habit of concentration was revealed both in his manner and his crisp mode of expression.

"In the little time Mr. MacCalla found for recreation he made many friends. He brought to his social life the energy and ability which made him a leader in his profession, and he added life and interest to any gathering which he joined. As a man as well as an engineer he was an asset to Youngstown."

Mr. MacCalla was elected a Member of the American Society of Civil Engineers on March 1, 1910.

ROBERT WENTWORTH MACINTYRE, M. Am. Soc. C. E.[1]

DIED MAY 30, 1936

Robert Wentworth Macintyre was born in London, England, on July 14, 1867, the son of John Stevenson and Emily Sarah (Wiggins) Macintyre.

After completing his school education in 1884, some years of which were spent at Redland Hill House, Clifton, Bristol, England, and some in Germany, Mr. Macintyre began the study of engineering as a pupil under E. Wilson and Company, Consulting Engineers, of Westminster, London, England. His experience with this Company was of the greatest value in his future career, giving him an insight into harbor, bridge, and railway construction and maintenance, and also roads and sewers.

In 1888, he went to Canada, and after spending one year on a farm, he entered the service of the Lethbridge and Great Falls Railway Company. At this time in the history of Western Canada, there was a marked movement toward the development of large areas under irrigation systems, and from 1892 to 1904, Mr. Macintyre, except for an autumn's work with the Red Deer Valley Railway Company, was engaged under the Dominion Government in subdivision and contour surveys, hydrographic work, preliminary location of the Red Deer and Rosebud Canals, and the distribution system of the proposed Bow River Canal, later covering also the investigation of water rights' applications and the inspection of works.

From 1904 to 1905, Mr. Macintyre was Assistant Chief Engineer of the North West Territories, with headquarters at Regina, continuing, on the formation of the Provinces of Saskatchewan and Alberta, in the same position for the latter Province until 1911. This position necessitated constant traveling, as it included charge of outside public works, such as bridges, ferries, and roads—both maintenance and construction—and the holding of arbitration proceedings for the expropriation of necessary lands.

Moving still farther west to the Pacific Coast, he spent a couple of years (1911–1913) as Assistant Engineer in the Sewer Department of the City of Victoria, B. C., Canada. During the following two years Mr. Macintyre was engaged in private practice, before becoming, in 1915, Assistant to the late Francis Clarke Gamble, M. Am. Soc. C. E., Chief Engineer of Railways for the Government of British Columbia; on Mr. Gamble's retirement in 1918, Mr. Macintyre served in the same position under Mr. A. F. Proctor until the latter's retirement in 1922, when he became Engineer of Railways for the Province.

In 1921, torrential rains in the mountains of Northern Vancouver caused unprecedented floods that did much damage to roads and bridges; Mr. Macintyre then introduced the use of wire netting in the construction of training groynes of rock, and was very successful in the protection works built in this manner.

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[1] Memoir prepared by E. G. Marriott, Esq., Victoria, B. C., Canada.

From 1924 to 1928, Mr. Macintyre was again engaged in private practice, and then spent a couple of years as Municipal Engineer for the Municipality of North Cowichan, on Vancouver Island. It was at this time that he first became aware of that insidious enemy, diabetes, which brought his engineering activities to a close.

While not engaged at any time on works of great magnitude, in the light of developments of to-day, Mr. Macintyre's wide experience, coupled with a capacity for hard work, and an unfailing perseverance that refused to be beaten, enabled him to render extremely useful service to his fellow men.

He was elected Secretary of the Victoria Branch of the Canadian Society of Civil Engineers (now the Engineering Institute of Canada) at its formation in 1911, and by his personal interest, and the giving of much time, he not only infused life into the conduct of local affairs, but took a leading part in obtaining greater recognition for engineers in Western Canada; later, he filled the position of Chairman of the Victoria Branch.

When, in 1920, there was obtained from the British Columbia Legislature an Act to incorporate the Association of Professional Engineers of the Province of British Columbia, by which the practice of professional engineering was to be regulated, Mr. Macintyre was duly registered as a "Civil Engineer", and continued as a member of the Association until his retirement in 1931.

Apart from his garden, his chief relaxations were books and music, and many of his friends took advantage of his generous invitation to make use of his library; organ music was particularly his delight, and G. Jennings Burnett, Organist of St. John's Anglican Church, which Mr. Macintyre attended, often played special compositions at his request.

When the men from the front began to return to Victoria during the World War, he took a vigorous part in obtaining for them the recognition he felt they deserved, and the following extract from the local press by a leading member gives him the credit through his friendly encouragement and cheerful persistence of having a leading part in the founding of the Great War Veterans' Association:

> "Mr. Macintyre was the first person in Canada to offer assistance to the organizers of the veteran movement.
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> " At the second meeting of the eleven members of the Returned Soldiers' Association held May 16, 1916, * * * Mr. Macintyre appeared as an unknown and uninvited guest, * * *. The bare walls, broken plaster, and candles stuck in bottle necks, roused his ire, * * * before the next meeting he arranged with the late Mr. C. T. Cross for their spacious, well-furnished rooms in Belmont House to be at the Association's disposal. * * *
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> "A social gathering, held in the Belmont House board room was organized by Mr. and Mrs. Macintyre, and in a few weeks the little band of veterans was the centre of a host of friends, and Canada's first organization of veterans of the Great War was firmly established. * * *".

Speaking before the Canadian Club in the Empress Hotel, at Victoria, B. C., on July 7, 1919, Colonel Purney, Dominion President of the Great War Veterans' Association, said: "This powerful organization, of which I am proud to be the head, had its birth in your city."

Mr. Macintyre was married, at Fort Saskatchewan, Canada, on November 24, 1907, to Marion Ada Sharpe, who, with their daughter, Ruby M. R. Macintyre, survives him. Another daughter, by a previous marriage, Mrs. W. E. Huntington, resides in Vancouver, B. C., Canada.

Mr. Macintyre was elected a Member of the American Society of Civil Engineers on October 10, 1916.

CHARLES PATTERSON McCAUSLAND, M. Am. Soc. C. E.[1]

Died November 4, 1936

Charles Patterson McCausland, the son of William H. and Laura B. (Hoop) McCausland, was born at Lonaconing, Md., on August 27, 1881. His early days were spent at Phillipsburg, Pa., where he was educated in the public and high schools.

He began his career as a Civil and Mining Engineer in Clearfield County, Pennsylvania, and, in 1898, was engaged in surveying and railroad construction work as a Rodman on the preliminary surveys for the West Branch Valley Railroad from Clearfield to Williamsport, Pa. In 1900, Mr. McCausland was employed as Axeman on the New York Central and Hudson River Railroad on construction work at Clearfield.

In 1901, he turned his activities to mining, and was employed by the Sharon Coal and Coke Company on construction work, sinking two mine shafts and building the Town of Ronco, Pa.

He then became associated with A. V. Hoyt, Civil Engineer, at Phillipsburg, as Transitman on mining and all classes of general engineering. In 1903, Mr. McCausland became a partner of the Lorain Engineering Company, of Phillipsburg, and was associated with this firm from September, 1903, to August, 1905, and from September, 1907, to May, 1910.

Returning to railroad work, he was employed from August to December, 1905, as Resident Engineer, and from March, 1906, to September, 1907, as Locating Engineer by the Pittsburgh, Binghamton and Eastern Railroad Company. For a short period, from December, 1905, to March, 1906, he was Assistant Engineer in charge of preliminary surveys for the Kentucky Railroad, along the Big Sandy River.

In April, 1910, Mr. McCausland entered the service of the Western Maryland Railway Company as Field Bridge Engineer. With the exception of a short period from 1914 to 1916 when he was engaged in private practice as a Civil and Mining Engineer in Canton, Pa., his service was continuous with the Western Maryland Railway. His ability was well recognized by the Company, and he served as Locating Engineer from 1911 until 1919 when he became Engineer of Surveys. He took a keen interest in the development of coal properties along the line. Throughout this long period of service with the

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[1] Memoir prepared by E. M. Killough, Assoc. M. Am. Soc. C. E.

Western Maryland Railway Company, Mr. McCausland made many friends who will long remember his genial personality.

On February 17, 1913, Mr. McCausland was married, in Canton, Pa., to Aileen M. Hickey, and is survived by his widow and a son, Charles Patterson McCausland, Jr.

During the last year or more of his life, he hid from his closest associates a heart affliction which ultimately was the cause of his death, on November 4, 1936.

He was a member of the American Railway Engineering Association, and served on its Yards and Terminals Committee. He was also a member of the Masonic Order.

Mr. McCausland was elected an Associate Member of the American Society of Civil Engineers on December 6, 1910, and a Member on July 2, 1913.

CHARLES ALBERT McKENNEY, M. Am. Soc. C. E.[1]

DIED MARCH 23, 1935

Charles Albert McKenney was born in Washington, D. C., on October 14, 1870, the son of James Hall and Virginia Dorcas (Walker) McKenney. He was descended from those McKenneys who came from the north of Ireland and settled in Maine in 1668. His Great-Grandfather McKenney moved to Maryland in 1791, since which time the family has remained in Maryland and the District of Columbia. His father was Clerk of the Supreme Court of the United States from 1880 until his death in 1913.

His preliminary schooling was received at Emerson Institute, in Baltimore, Md., and at the Columbian Preparatory School of Columbian College (later, George Washington University), at Washington, D. C.; his college preparatory work was done at the Lawrenceville School, in New Jersey, and his technical education was obtained at Princeton University, at Princeton, N. J., from which he was graduated in 1892 with the degree of Civil Engineer.

A few months after his graduation Mr. McKenney entered the service of the District of Columbia and, thereafter, for the next twenty years, he was intimately associated with the major engineering problems of the Nation's Capital and many of its monumental projects bear evidence of his energy and skill. Beginning as an Axeman in the Sewer Department, he advanced through the grades of Chainman, Rodman, Draftsman, Instrumentman, and Assistant Engineer in that Department. Entering upon this work with enthusiasm and continuing to apply himself to his books and endeavoring to absorb the experience of his associates, he made rapid progress and, in 1896, was selected to take charge of the construction of a sewerage system for Fortress Monroe, Virginia. For this purpose he was furloughed from his regular employment and loaned to the United States War Department. On the successful completion of this work Mr. McKenney returned to the District of Columbia and, in 1897, was made a Special Assistant to the Engineer Commissioner. He was placed in charge of the surveys for the bridge across Anacostia River, and of the preparation of plans for the elimination of railway grade crossings in the District. He afterward served as Engineer of Roads and Bridges and, later, as Engineer and Superintendent of Construction of the Water Department. In this latter capacity he had charge of the construction of the 30 000 000-gal Brightwood, or Sixteenth Street, Reservoir, which at the time it was built was a new departure in reservoir types. He also built the Bryan Street pumping plant, including the installation of pumps and other equipment. When the new Municipal Building was constructed (1904–1908), he served as Engineer for the Municipal Building Commission in charge of the construction of this monumental structure. Shortly after the completion of this assignment, Mr. McKenney resigned his position with the District Government and established an Office as Consulting Engineer, in Washington.

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[1] Memoir prepared by L. M. Gray, M. Am. Soc. C. E.

His professional service was interrupted by the World War, but shortly after his discharge from the Army, he joined with the late Major-General William M. Black, U. S. A. (*Retired*), M. Am. Soc. C. E., formerly Chief of Engineers, and John Stewart, M. Am. Soc. C. E., in the organization, at Washington, of the firm of Black, McKenney, and Stewart, Engineers, and thus began an association which lasted until General Black retired in 1930, and the firm was dissolved. During this period, Colonel McKenney was engaged on work for which he was peculiarly adapted and which he thoroughly enjoyed. Gifted with a personality of rare charm he made friends easily and readily established the basic contacts essential for his firm's success. He was a convincing talker and, possessed of a keen and ready yet kindly wit, he was ideally fitted for carrying on the negotiations preliminary to the making of contracts. Combined with a rare facility of expression and a vocabulary enriched by extensive reading, he had the ability to arrange his thoughts clearly and logically and to express them forcibly. The reports he prepared during the ten years of the firm's existence are examples of his logical reasoning and clarity of expression.

The firm of Black, McKenney, and Stewart handled many important assignments; among the first works undertaken was an investigation and report on some harbor work at Tampa, Fla.; then followed a report on the harbor at Baltimore. In 1921, the firm was engaged to make a report on the canalization of the Magdalena River, in Colombia, across the bar at the Bocas de Ceniza. The field work on this project, including extensive surveys and an investigation of all available data, was undertaken by Colonel Stewart, but the report was prepared by Colonel McKenney in consultation with General Black. The work was done for, and the report was submitted to, the Compañia Colombiana de las Bocas de Ceniza, but the project recommended, which included the construction of two rubble mound jetties, was accepted subsequently by the Colombian Government. When, in 1925, that Government was ready to proceed with the work, the firm of Black, McKenney, and Stewart was engaged as Consulting Engineers to supervise and direct the construction, and continued as such until work was discontinued, in 1929, with the jetties only partly completed. Enough work had been done, however, to demonstrate the soundness of the project. Subsequently, Sir Alexander Gibb. M. Am. Soc. C. E., of London, England, was engaged by the Colombian Government to advise it with regard to matters relating to rivers and harbors, and in his report on the Bocas de Ceniza project, he insisted that the work be completed exactly in accord with the plans of Black, McKenney and Stewart. Although Colonel McKenney had never visited this project during its construction, he was always in touch with it in an advisory capacity and conducted all the negotiations with the Colombian Legation at Washington.

In 1924, the firm was engaged by the City of Guayaquil, Ecuador, to prepare plans for a light and power plant for that city, and Colonel McKenney went to Ecuador to make the necessary field investigations and to conduct the surveys. While there, he visited Quito and concluded an arrangement with the President of the Republic whereby the firm would act as Technical Adviser to the Government. However, tthe contract that was then signed, was never approved by the Congress of Ecuador.

During 1924 and 1925, Colonel McKenney handled all the negotiations connected with the firm's participation in the project for the construction of a gigantic rail and water terminal at Bayonne, N. J. Under his direction, and in consultation with General Black, preliminary study plans and estimates were prepared for this $150 000 000 project. At this time, too, the firm prepared a report on a vast rail and river terminal at St. Louis, Mo.; a project for the improvement of the Lower Brazos River, in Texas, for presentation to the U. S. Engineer Department; and designs for breakwaters at Milwaukee, Wis.; it managed many other projects in which Colonel McKenney had a leading part.

In 1927, the firm was engaged by the Brotherhood of Locomotive Engineers to prepare plans for an extensive development at Venice, Fla., and Colonel McKenney directed all phases of the firm's work on this project and personally wrote the reports.

The last important work he did for the firm of Black, McKenney, and Stewart, was the preparation of a report, after exhaustive investigations, on the improvement of the Chattahoochie River below Columbus, Ga. After the submission of this report and upon the dissolution of the firm, he was retained by the City of Columbus as Consulting Engineer to continue the work he had been doing on the project.

When the firm of Black, McKenney and Stewart was dissolved in 1930, Colonel McKenney again established an office in Washington as Consulting Engineer and spent the remaining years of his life in the leisurely practise of the profession to which he had devoted all his active years.

Colonel McKenney had always been an interested student of military matters. Through his long and intimate association, while in the service of the District of Columbia, with the officers of the Corps of Engineers, he had, in a way, absorbed the traditions of that Corps and had seemed actually to belong to the Army. With this background it was natural that, in 1916, when it appeared to be certain that the United States would ultimately become involved in the World War, he would want to prepare himself to take part in the nation's military effort. Accordingly, he was one of that group of patriotic men who attended the 1916 Plattsburg Camp, and afterward applied for a commission in the Officers Reserve Corps. His application was favorably considered, and he accepted appointment as Major, Engineer Section, Officers Reserve Corps, on May 23, 1917, and was assigned to active duty in the office of the Chief of Engineers at Washington, on July 26, 1917, in charge of engineer supplies and as Engineer Liaison Officer in matters affecting the Engineer Corps and other branches of the War Department, as well as the civil agencies having to do with war materials. He was commissioned Lieutenant Colonel, Engineers, National Army, on March 18, 1918. He was assigned to duty in the Purchase, Storage, and Traffic Division, General Staff, on May 29, 1918, and detailed as War Department Representative with the Priorities Division, War Industries Board, charged with fixing the priority of production, distribution, and transportation of supplies for the Army. He was commissioned Colonel, United States Army, on November 4, 1918. After the signing of the Armistice, Colonel McKenney was assigned as Assistant to the Director of Purchase, U. S. Army, with duties involving the settlement of purchase contracts, and, in January, 1919, he was appointed a member of the War Department Claims Board. On October 25, 1919, he was honorably discharged from the Service.

After his discharge from active duty, Colonel McKenney was appointed Colonel, Engineer Officers Reserve Corps, on February 3, 1920, which appointment he accepted on April 1, 1920; he was re-appointed in the same grade on February 3, 1925, and was transferred as a Colonel to the Auxiliary Reserve on February 3, 1930, and re-appointed in the same grade and status on February 2, 1935.

His engineering ability is attested by the character of the works he planned and constructed. The reports he prepared, with their lucid statement of facts and sound and logical conclusions, show the brilliant quality of his mind. He was a gentleman in the finest sense of the word; of a distinguished family the influence of a long line of cultured forebears showed in all his actions. A gifted conversationalist and a delightful companion, he made friends wherever he went and retained them always. He was a considerate employer and a staunch and loyal friend. One who was closely associated with him for the last fifteen years of his life wrote:

> "Colonel McKenney was an engineer of exceptional ability. He had a keen and penetrating intelligence, possessed sound judgment and the ability clearly and forcibly to express his thoughts. Full of sympathy and understanding he appreciated all the problems of his subordinates and was always helpful in their solution. He was one of the finest gentlemen I have ever known."

Colonel McKenney was a member of the Metropolitan Club and the Chevy Chase Country Club, and took an active part in the social life of Washington.

He was married in 1901 to Frances Marion Miller, of Washington, who survives him. He is also survived by three children, James Hall McKenney 3d, Francesca McKenney, and Charles A. McKenney, Jr.

Colonel McKenney was elected a Junior of the American Society of Civil Engineers on December 4, 1894; an Associate Member, on December 1, 1897; and a Member on March 2, 1909.

FRANK PAPE McKIBBEN, M. Am. Soc. C. E.[1]

DIED NOVEMBER 27, 1936

Frank Pape McKibben was born on November 13, 1871, at Fort Smith, Ark., the son of Frank Read McKibben and Minnie Elizabeth (Pape) McKibben. He was a student at the University of Arkansas, at Fayetteville, Ark., from 1887 to 1890 and at Massachusetts Institute of Technology, at Boston, Mass., from which he graduated in 1894 with the degree of Bachelor of Science in Civil Engineering.

From 1894 to 1903, Mr. McKibben was engaged as Instructor in Civil Engineering at the Massachusetts Institute of Technology; from 1903 to 1906, he was Assistant Professor of Civil Engineering, and from 1906 to 1907, Associate Professor at the same institution. He was then appointed Professor of Civil Engineering at Lehigh University, at Bethlehem, Pa., where he remained until 1919, when he became Professor of Civil Engineering at Union College, at Schenectady, N. Y.

In 1897, he designed the New Bedford, Mass., Bridge; he also served as Engineer for the bridge at Oil City, Pa. From 1898 to 1901, he was employed as Assistant Engineer, with the Boston Elevated Railway Company, and from 1901 to 1907, he served as Assistant Engineer, with the Massachusetts Railroad Commission. In 1918, Professor McKibben was Supervisor of Technical Training for the Emergency Fleet Corporation, in which position he did wonderful work. He was held in the highest esteem by every one in Schenectady, N. Y., in which, as City Engineer, he developed many outstanding improvements. From 1913 to 1926, he was variously engaged as Consulting Engineer for Venango, Lehigh, Northampton, Berks, and Luzerne Counties, in Pennsylvania, the Lehigh Coke Company, the Bethlehem Steel Company, the American Locomotive Company, and, in later years, the General Electric Company.

During this time, Professor McKibben wrote the chapter on "Arches," in the book entitled "Concrete, Plain and Reinforced," by the late Frederick W. Taylor and Sanford E. Thompson, M. Am. Soc. C. E.; also, the chapter on "Steel," for the American Civil Engineers Pocket Book.

Professor McKibben had more than thirty-two years of teaching experience in Massachusetts Institute of Technology, Lehigh University, and Union College. He was a Bridge Engineer of national reputation, having been Designer, Engineer, or Constructor, of many outstanding bridges of masonry, steel, and concrete. He traveled extensively abroad and during these travels inspected many famous old and new bridges. He also was an authority on electric welding, having been for several years Consultant for the General Electric Company relative to structural steel welding in which capacity as a Lecturer before engineering societies in all parts of the United States, he contributed valuable information on this subject. In recognition of his knowledge in this field, he was elected President of the American Welding Society in April, 1932. Professor McKibben was in charge of the design, construction, and equipment of the John Fritz Engineering Laboratory, at Lehigh University, in 1910 and 1911. He was Consulting Engineer to the Attorney General of Pennsylvania in the investigation of the failure of the Austin Dam in 1911, as well as Consulting Engineer for the Pennsylvania Water Supply Commission in 1914 and 1915. In 1915, he helped to organize the People's Trust Company, of Bethlehem, Pa., and was its Vice-President from 1915 to 1919. It is amazing how he did all these things, and in such a quiet, modest manner. The statement of one of his co-workers that, "it is remarkable how one man could have accomplished all this in a single life time," is particularly significant.

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[1] Memoir prepared by T. B. Wood, Esq., Chambersburg, Pa.

Professor McKibben was a man of the highest type, as an engineer, citizen, and Christian. His knowledge of the animal and plant life of the forest and of its birds was amazing, and no less was his knowledge of stamps and oriental rugs, with which he loved to work in his spare time, when most busy men would be taking their rest or ease. He also was extremely fond of Astronomy, giving more time to it than any of his regular problems required; it might be said that this was his real hobby. In addition to all these works, he also was greatly interested in the growing of apples and in farming.

A most kindly man, a wonderful companion, at every turn on a trip around the country side something presented itself to cause a remark by him, filled with information, stated in an interesting and instructive manner, that added so much to the pleasure of the trip. His was an analytical mind; that is what made him a good engineer. He imparted his knowledge analytically, yes, but tempered it with kindness, common sense, and knowledge of the human equation; that is what made him a good teacher. His ideas of society were so broad that he gave to each person his own right to do as he saw fit. He was an Elder in the Falling Spring Presbyterian Church, of Chambersburg, Pa. Above all else, he was a high type Christian man, and the world is better for the place he held in it.

Professor McKibben was a member of many organizations and societies for the advancement of science and served in many positions of importance in education, scientific investigation, engineering, and construction. He held many positions, in important engineering projects, as the author of various engineering books, and as a teacher. His engineering ability and general good judgment relative to civic and other problems were so well known and thought of, that he was chosen Consulting Engineer for a number of cities, counties, and of large business interests.

He was married to Arabella Almy, of Tiverton, R. I., in January, 1899, to which union was born a son, Elliot S. McKibben. Mrs. McKibben died in May, 1921. On March 24, 1923, he was married to Ariana Kennedy Elder, of Chambersburg.

He died on November 27, 1936, and interment was in Falling Spring Church Cemetery, at Chambersburg. He is survived by his widow; his son, Elliot; a sister, Mrs. J. Raymond Williams, and a brother, Dr. William McKibben, both of Miami, Fla.

Professor McKibben was elected a Junior of the American Society of Civil Engineers on January 3, 1895; an Associate Member on March 6, 1901; and a Member on October 3, 1905.

FREDERIC OZANAM XAVIER McLOUGHLIN, M. Am. Soc. C. E.[1]

Died June 28, 1936

Frederic McLoughlin was beloved by his associates for his human qualities and admired for his ability as an engineer. Almost his entire professional life was devoted to the organization and development of the School of Technology, College of the City of New York. His memory will live on in the ideals of that school and in the spirit of its staff. Beloved by all for his friendly and generous spirit, he was, nevertheless, a man of fearless convictions and outspoken opinions. He never sacrificed principle to good nature; yet so frank and amiable was his attitude that he could win a colleague's affection even while opposing him on a matter of policy.

Frederic Ozanam Xavier McLoughlin was born in New York, N. Y., on March 12, 1888, and attended the public schools of that city. He earned his Bachelor of Science degree at The City College, New York City, in 1909, being honored by election to Gamma Chapter of Phi Beta Kappa and receiving the Ralph Weinberg Memorial Prize. He also became a member of Phi Gamma Delta Fraternity. In 1913, he took his Civil Engineering degree at Columbia University, New York City, and, in the following year, was awarded the degree of Master of Arts in Highway Engineering by the same institution. At Columbia, he was the recipient of the J. Pierpont Morgan Scholarship and was elected to the Columbia Chapter of Phi Beta Kappa. During the summer vacations from 1911 to 1913, he was employed as an Inspector of the New York Board of Water Supply on the Catskill Aqueduct.

Professor McLoughlin was a teacher at The College of the City of New York from 1910 until his death. He rose to the rank of Instructor in 1916, was made Assistant Professor in 1918, Associate Professor in 1925, and Professor in 1934. He was identified with courses in Civil Engineering from the beginning of his career and became one of the original members of the Faculty of The School of Technology at the time the latter was made a separate unit of the College in 1919. From 1917 to 1919, he served as Civilian Instructor attached to the 30th Service Company, United States Signal Corps, in charge of instruction in Military Surveying.

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[1] Memoir prepared by R. E. Goodwin, M. Am. Soc. C. E.

At The City College he was Secretary of the Faculty of The School of Technology from the time of its founding until his death, and served as a member of the City College Co-Operative Store Committee. He established and organized the Summer Surveying Camp, founded and developed the Evening Courses in Building Construction, organized post-graduate courses in Highway Engineering, and was Chairman of the Curriculum Committee. He was the author of a Field Manual in Elementary Surveying, which is used in the Elementary Surveying Course at the College.

Professor McLoughlin was a leader in furthering student activities and student welfare. For a long time he was Faculty Adviser to the City College Student Chapter of the American Society of Civil Engineers, which he organized in 1921. He was Faculty Adviser to the student publications, *Tech News* and *Tech Journal*, and founded the Tech Council, a committee for co-ordinating the activities of the Student Chapters of the several National Engineering Societies.

He was a member of the Engineering Council for Professional Development Delegatory Committee for Region No. 2, a committee charged with the responsibility for examining and accrediting Technical Schools in this district; a member of the Society for the Promotion of Engineering Education; and for six years he was Secretary of Gamma Chapter of Phi Beta Kappa. His services as a Consulting Engineer were frequently in demand. He was an active proponent of the plan for standardizing engineering degrees.

On August 8, 1917, Professor McLoughlin was married to E. Margaret Brown, of Cornwall-on-Hudson, N. Y., who survives him.

He died at his summer home in Big Indian, N. Y., on June 28, 1936. Memorial services were held for him at The City College on July 1, 1936, at which the Great Hall of the College was filled with his friends and associates who came to pay their respects and to do honor to his memory. In his remarks at these services, Dr. Frederick B. Robinson, President of The City College, said in part:

> "Frederic McLoughlin had a clear and vigorous mind. This made him effective as a teacher and valuable as a counselor in all matters of academic policy. He was a moving force in the development of The School of Technology. He was a man of generous heart, given to warm friendships with students and colleagues. He was a big brother to many a boy who needed help. Many a graduate of the college owes his success to the timely help of the McLoughlins and also to the entrée into engineering practice arranged by the Professor.
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> "Professor McLoughlin was not given to preachment, but the purest and loftiest of human ideals were expressed in his deeds. He was honorable, and dependable in all the relations of life—personal, official and civic. He wished the noblest destiny for his country, his college, his students and friends, and those bound to him by the most intimate ties of affection. He was unsparing of himself in practical efforts to carry those wishes into effect. Professor McLoughlin was a patriot in his unselfish loyalty to his country, a scholar in his labors for more knowledge and more adequate expression of the truth,

a doer of good deeds for his fellow creatures, and a devoted husband. We grieve over his loss to us in the flesh, but we thank the source of all good for the companionship we had with Frederic McLoughlin, and we know that his spirit will live in this college as an institution and in the lives of those who love him."

Professor McLoughlin was elected a Junior of the American Society of Civil Engineers on April 1, 1914; an Associate Member on April 19, 1920; and a Member on October 24, 1932. He was also a member of the Metropolitan Section of the Society and served on its Committee on Vocational Guidance.

ELWOOD MEAD, M. Am. Soc. C. E.[1]

Died January 26, 1936

Elwood Mead was born near Patriot, Ind., on January 16, 1858, the son of a pioneer Switzerland County farmer, Daniel, and his wife, Lucinda (Davis) Mead. From these humble beginnings, an inquisitive and clear-thinking mind and hard labor carried him to the very top of his chosen specialty—irrigation and reclamation engineering.

The boy who experimented with a home-made transit on his father's farm, and worked his way through school as a rodman on a survey crew, later was to guide the United States Bureau of Reclamation in mapping, planning, and launching the greatest reclamation construction program in history.

The man who was to head the Bureau of Reclamation during its entire period of construction of the mighty Boulder Dam, began his career (after having been graduated from Purdue University, at La Fayette, Ind., with a degree of Bachelor of Science in 1882 and a degree of Master of Science in 1884) as an Assistant Engineer in the United States War Department on a survey of the Wabash River in his native Indiana.

As a teacher and administrator in the field of irrigation, Dr. Mead was renowned throughout the world. In his specialty, he was a pioneer, and the influence of his research, innovations, and ideas is reflected in laws governing irrigation practice in the arid States of Western United States and in many other parts of the world.

In bestowing upon Dr. Mead an honorary degree of Doctor of Laws, in 1925, the University of Michigan, issued this statement:

> "Dr. Elwood Mead, Commissioner of the Bureau of Reclamation, engineer, law-giver and administrator, whose labors will endure through ages yet to come. By framing and putting into operation the irrigation laws of the State of Wyoming, he established a precedent followed not only by the newer States of the West, but also by Canada, Australia, South Africa, and New Zealand. He brought order out of confusion and opened a way where none had been."

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[1] Memoir prepared by R. F. Walter and William H. Code, Members, Am. Soc. C. E., with valuable assistance from Professor Frank Adams, of the Univ. of California, Berkeley, Calif.

In addition to the degree of Doctor of Laws, Dr. Mead also held a Civil Engineering Degree bestowed by the Iowa State College of Agriculture in 1883, and an honorary degree of Doctor of Engineering received in 1904 from Purdue University (the first honorary degree given by that University). In June, 1934, Dr. Mead was also awarded the degree of Doctor of Laws by the University of Wyoming.

The endeavors of Dr. Mead for the rights of humanity to the orderly use of water for agriculture by irrigation can be divided roughly into five parts, as follows: (1) Territorial Engineer of Wyoming, 1888–1889, and State Engineer, 1889–1899; (2) Chief of the Division of Irrigation and Drainage Investigations, United States Department of Agriculture, 1899–1907; (3) Chairman of State Rivers and Water Supply Commission, Victoria, Australia, 1907–1914; (4) Professor of Rural Institutions, University of California, Berkeley, Calif., 1915–1923; Chairman of the State Land Settlement Board, of California, 1917–1923; and (5) Commissioner of Reclamation, United States Department of the Interior, 1924 until his death.

After two years as Instructor in Mathematics, he was appointed, in 1886, Professor of Irrigation Engineering at the Colorado State College of Agriculture, at Fort Collins, Colo., the first such chair held in an American college. In addition, he served Colorado as Assistant to the State Engineer. He went to Wyoming as Territorial Engineer, in 1888, and served that State as its first State Engineer from 1889 to 1899. It was during this period that he first gained prominence.

When the new State was being organized, Dr. Mead, as State Engineer, proposed an entirely new Water Law for inclusion in the State Constitution, one in which the people turned their backs upon the common law principle of riparian rights which had thrown into confusion the legal status of water in so many communities of the arid West. Under the Mead plan, the State claimed title to all water, surface and underground. Diversion from streams and all appropriations of water in Wyoming since that time have been under the control of State Officials.

The success of the fight for this reform made the young engineer known throughout the entire West. His biennial reports discussed the determination of, and limitations to, the rights to water, their adjudication, and approval of permits to appropriate water, and his later reports emphasized the importance of having the land and water under one control; and, just as cities owned water-works, so he believed that irrigation communities dependent on a single large canal should own that canal. With the spread of his reputation as an authority on irrigation and water laws, the young engineer was in demand as a speaker at civic and technical associations, and delivered a very comprehensive address on "The Arid Public Lands—Their Reclamation, Management, and Disposal", before the then recently organized American Society of Irrigation Engineers. His reputation reached the East, and he wrote for the Departments of Agriculture and the Interior a series of *Bulletins* on water rights.

Elwood Mead served as Chief of the Irrigation and Drainage Division of the Department of Agriculture for eight years ending in 1907, and served the University of California concurrently as Professor of Institutions and Practice of Irrigation. During this period, he visited Italy and studied the institutional and legal phases of irrigation in that country. He was detailed as an expert in the celebrated Kansas-Colorado case to aid in the broad policy to be laid down by the Courts, and wrote several papers on the influence of State boundaries on water-right controversies and water rights within the States. Early in 1901, he lost his right arm in a street car accident, in Washington.

In 1907, Dr. Mead went to Australia as Chairman of the State Rivers and Water Supply Commission of Victoria. He inaugurated a comprehensive plan for water conservation and reclamation in the State of Victoria during his eight years of service, that is considered one of the models of the British Empire to-day. On reaching Australia, he found conditions somewhat like those in Wyoming at the earlier date, except in this case it was the "land hog" instead of the "water hog" who was standing in the way of development. The large land-owners were using the water as a kind of insurance for stock water in dry seasons, paying a very small sum for the large expense incurred in bringing the water long distances to the land and preventing land settlement. Here, again, he encountered the same old fight of vested interests, but by appealing to decency, justice, and common sense, Dr. Mead won his victory as he had done in Wyoming years before.

Large estates were cut up, and as a result of the controversy the Closer Settlement Act was enacted and a policy of State aid in placing settlers on the land was inaugurated. Dr. Mead assisted in settling controversies over the waters of the River Murray between the States of Victoria and New South Wales, suggesting a form of compact between the two States and advocating that irrigation and not navigation should have first place. Several of his addresses were published in American journals, including articles entitled "What Australia Can Teach America", and "Irrigation in Victoria."[3]

In 1913, Dr. Mead, then one of the leading authorities on land settlement, resigned his position in Victoria, Australia, but remained there until 1915. He then returned to the United States to accept the position of Professor of Rural Institutions at the University of California, and, also in 1917, of Chairman of the State Land Settlement Board. Shortly after his return to the United States, in 1915, he was engaged as Chairman of the Central Board of Review to aid in deciding some of the settlement problems on Federal irrigation projects. In 1917, he acted as Consulting Engineer on a board in connection with the plans for the All-American Canal to Imperial Valley in California.

Later, he was called upon by Secretary Lane, of the Department of the Interior, for expert advice regarding settlement problems in connection with placing World War veterans on farms. A pamphlet by him on a "Summary of Soldier Settlements in English Speaking Countries", was widely distributed.

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[3] *Engineering Record*, August 14, 1909.

In 1923, Dr. Mead was appointed by the Secretary of the Department of the Interior as a Special Adviser on Reclamation, to serve with four others on the Fact-Finding Commission set up to investigate Federal reclamation projects. The Commission's report was the basis for reforms put into effect by Congress by the omnibus Reclamation Act of 1924.

On April 3, 1924, his intimate knowledge of irrigation and land settlement was again recognized by his appointment by President Coolidge as Commissioner of Reclamation. Under his leadership Federal reclamation has been placed on a still firmer foundation of usefulness to the nation. He was appointed by the President, a Special Commissioner on the International Water Commission, on December 27, 1924.

Dr. Mead was loaned to foreign governments in need of expert advice on water and irrigation problems. He went to Palestine to advise the Zionists upon the problems involved in reclaiming the arid lands of the Holy Land; he served on commissions in Cuba and Haiti, and acted as Adviser to the Governments of New South Wales, Australia, Canada, Hawaii, Java, and Mexico.

Perhaps no man knew the terrain of Western United States better than Dr. Mead. With 'Buffalo Bill" Cody, he explored some of the more difficult rivers of Wyoming in 1888. Repeated trips in the years following gave him first-hand knowledge of virtually every creek and range of hills in the arid region. In recent years, as Commissioner of Reclamation, he had made a tour of the West each year, inspecting Federal irrigation projects, examining areas where new projects have been proposed, and renewing acquaintances which were to be found in every hamlet from Denver, Colo., to San Francisco, Calif., and from Spokane, Wash., to El Paso, Tex.

Dr. Mead's administration of the Bureau of Reclamation was business-like and humane. He introduced several reforms one of the most important of which was the establishment of a policy of selecting settlers for new Federal reclamation projects on the basis of their qualifications for irrigation farming. Irrigation farming, Dr. Mead believed, required a high degree of skill for success. He took great pride in the growth and development of the communities and cities on Federal projects in the West.

He was an Honorary Member of the American Society of Agricultural Engineers, and a member of the Institution of Civil Engineers of Great Britain. He was also a member of the Water Resources Committee of the National Resources Board, and served on many other governmental committees and commissions. He was the author of two books, that are used widely as texts, entitled, "Helping Men Own Farms" and "Irrigation Institutions."

Dr. Mead is survived by his widow, Mrs. Mary Lewis Mead, by three sons, Tom C. Mead, Arthur Mead, and Lt. John Mead, U. S. Army; by two daughters, Mrs. Lucy Marston, wife of Maj. Morrill Marston, U. S. Army, and Mrs. Sue Kaiser; and by seven grandchildren.

In a letter of condolence to Mrs. Mead, Franklin Delano Roosevelt, President of the United States, wrote "Dr. Mead was one of the country's outstanding engineers. He was a builder with vision whose loss will be keenly felt."

Secretary of the Interior Harold L. Ickes, stated,

"The death of Dr. Mead is a profound shock to me personally. His place in the official family of the Interior Department will be almost impossible to fill. He was an outstanding man, famed in his field throughout the world. All who knew him loved and respected him.

"As Commissioner of the Bureau of Reclamation, Dr. Mead has left for the Nation many monuments in work well done. Perhaps no man contributed more to the planning of Boulder Dam and certainly no one had a more important part in the actual construction of it than Dr. Mead. If his place in the memory of his Nation were to rest on that accomplishment alone, it would be secure. But there were many other accomplishments in his long and exceptional career as an engineer, law-giver, teacher, and administrator in the field of irrigation.

"He showed the way when he formulated the water provisions of the constitution of the State of Wyoming, which he served as its first State Engineer, where some of the knottiest problems in the arid regions of the world existed. Wyoming, at Dr. Mead's urging, discarded the riparian theory of water ownership and adopted a system under which the State retained all water rights. Development of this primary resource was retained under the jurisdiction of the State. Revolutionary in the field of water law at the time, the plan since has been adopted by other Western States and by many other nations.

"The death of Dr. Mead is a distinct loss to the country, which by his life he has so enriched through original thought, and devotion to duty and ideals."

Miss Mae A. Schnurr was closely associated with Commissioner Mead during the twelve years of his official connection with the Bureau of Reclamation, first as his Secretary, and, later, as his Assistant. In speaking for the employees of the Bureau, she stated:

"It is an honor to speak for those privileged to have been associates of Dr. Mead. In his death we have suffered the loss, not only of a very able Commissioner of Reclamation, but of a kindly friend and counselor who made of each associate a competent and loyal assistant. His ability inspired respect, his kindly thoughtfulness made us admiring friends. His memory will stay fresh and revered in the hearts of all of us."

His successor, John C. Page, M. Am. Soc. C. E., then Chief of the Engineering Division of the Commissioner's Office, writes:

"The death of Dr. Mead means not only a tremendous loss to the Bureau of Reclamation, but it also creates a vacancy among the most eminent engineers in the world.

"Through a long and useful life, Dr. Mead exemplified the highest professional ethics and was recognized as an engineer with wide conception of the human as well as the technical side of the profession.

"Engineers throughout the world looked toward Dr. Mead for leadership in the field of reclamation. He had won his place in their esteem through sound developments and outstanding construction."

E. K. Burlew, Administrative Assistant to the Secretary of the Interior, when notified of Dr. Mead's death, made the following statement:

"Through 12 years of close association with Dr. Mead, I have come to esteem him as a friend and official. His professional attainments gave him an eminence that few men attain in their chosen field and his human attributes endeared him to all who had occasion to seek his kindly wisdom and straightforward honesty in the solution of their problems. He spent his life in creating homes for many thousands of people and in developing the natural resources necessary to maintain them. His passing will be mourned by a great multitude in the West who have him to thank more than any other man for their hearth and their economic independence."

The Federal Board of Geographic Names paid Dr. Mead a signal and appropriate honor in bestowing his name upon the new lake formed by Boulder Dam, the largest artificial lake in the world, an important new geographical feature of the nation, and a vital influence on the lives of millions of people in the Southwest.

The official designation by the Board of Geographic Names is as follows:

> "*Lake Mead.*—An artificial lake in Mojave County, Ariz., and Clark County, Nev., formed by damming the Colorado River in Upper Black Canyon. Named in honor of Dr. Elwood Mead (born Jan. 16, 1858, died Jan. 26, 1936), Commissioner, Bureau of Reclamation, 1924–1936, under whose supervision the dam that impounds the lake was built."

Lake Mead began forming February 1, 1936, when the gates at the diversion tunnels of Boulder Dam were closed.

Noteworthy tributes were paid to this gentle and modest man by fine editorials and articles in scores of leading newspapers and magazines throughout the nation. Space will permit only brief reference to them. The following comment is from *Engineering News-Record.*

> "A well merited tribute was paid to the memory of Elwood Mead and to the value of his long labors in western development by the action of the Board of Geographic Names in designating the inland sea formed by Boulder Dam as Lake Mead."

The Washington, D. C., *Daily News* published an especially fine editorial from which is quoted the following:

> "It is to his credit and to that of Presidents Coolidge, Hoover, and Roosevelt that no one knew or cared what his politics were. * * * Such administrators as Dr. Mead are all too rare in state and national affairs."

Quoting from the *Arizona Republic:*

> "His task was not an easy one. Reclamation projects are similar in one respect, their purpose. Each particular project has its multitude of problems, peculiar to it only. The ability of Dr. Mead to cope with the individual reclamation problems of each section was remarkable."

Throughout the West memorial services were held for Dr. Mead. The week of February 16 was set aside in his memory by proclamation of Governor B. B. Moeur, of Arizona, who said: "The entire citizenship of Arizona mourns the loss of Dr. Elwood Mead. We have lost one of our most constructive friends." Governor Clarence D. Martin, of Washington, also by proclamation, declared that week, "Elwood Mead Memorial Week" in his State. Fine eulogies were delivered by statesmen and others who knew the man and his works intimately, and many resolutions were passed by important State bodies in California and elsewhere.

Dr. Mead was a man of many friends. Throughout the seventeen Western States comprising the arid and semi-arid region of the United States, there is scarcely a community where the results of his labors during the past fifty years have not left their mark of progress. Under his leadership the construction of the great Boulder Dam was initiated and completed. Under him the Bureau of Reclamation constructed monumental works which now stand in nearly every Western State as evidence of the breadth of his vision.

Although one of the kindest and most lovable characters, Dr. Mead was also a doughty and determined fighter when his plans for what he believed to be in the public interest, were attacked, either at home or in the foreign country where he was located for many years.

No tribute to Dr. Mead would be complete unless mention were made of his fondness for the game of "Five Hundred", in which he was never too tired to engage after a strenuous day in the field or office. Dr. Mead knew this game with all its ramifications and, given an equal chance, no one could best him in the final score.

Dr. Mead's early ambition was to attend the United States Military Academy, at West Point, N. Y., for which he qualified by taking the necessary examinations. Subsequent ill health compelled him to give up this plan. He was greatly pleased to have a son graduated from that fine school, and at the time of his death was looking forward to the possibility that his grandson also might enter the Academy. And thus Dr. Mead's early frustrated ambitions are being realized by his descendants.

Although his life work might be considered as the antithesis of one dealing with war and its strategies, the facts are that in the pioneer days of irrigation, shovels, shot-guns, and rifles were common implements on display in settling disputes over water rights. His subsequent wise water laws pointed the way of peace by arbitration.

He always had the courage of his convictions. In recent years, when even some of his best friends began to think that perhaps he was too optimistic in planning for additional irrigation projects in the West, the Middle West suffered a period of devastation occasioned by drought followed by ruinous winds and dust storms. This calamity demonstrated conclusively the wisdom of the reclamation policies of Dr. Mead and his predecessors in office to whom he would be one of the last to deny full credit.

Friends of Dr. Mead traveling in Europe have heard warm tributes paid him in many unexpected quarters, namely: From a high official of the Italian Emigration Service, in Rome, who stated that Dr. Mead had been a great friend of Italy in advising on its land and water investigations in Australia and elsewhere; from the highest salaried agronomist of the British Empire, encountered in the Blue Nile country, who was in full accord with Dr. Mead's land and water policies; and from an Australian met casually at the British Exposition, in Wembley, in 1925. He had formerly been a tenant farmer in England, but was induced to go to Australia and take up land upon Dr. Mead's recommendations. He paid Dr. Mead the warmest of tributes.

An excerpt quoted from a fine eulogy given in Spokane, Wash., by a close friend reveals that the eight years spent in Australia by Dr. Mead, were not all a bed of roses:

"He advised the Government that its primary need was not one of more stored water and canals to carry it to the land, but a supplanting of large wheat and sheep ranches with group settlements and the creation of a type of agriculture best suited to irrigation, thus laying the foundation for a program of planned rural development and aided and directed settlement that might well serve as a pattern for the rest of the world. It is interesting to note that upon the completion of this work, the town whose leading newspaper had denounced the 'foreign interloper' requested and obtained the privilege of giving the official banquet to the departing engineer.

"Upon returning to the United States in 1915, Doctor Mead resumed his teaching at the University of California, becoming Professor of Rural Institutions. He also served as Chairman of the California Land Settlement Board, created by the State for the purpose of giving deserving and qualified persons assistance in acquiring small and improved farms and to demonstrate the value of adequate capital and organized direction in preparing agricultural land for settlement."

The years succeeding the launching of these two State colonization projects were most unpropitious for land settlement of either State or private character, both classes failing to meet the expectation of their promoters. The blame for the disappointing aftermath of the State enterprises was laid entirely upon Dr. Mead, and was courageously assumed by him, although he was only one on a board of several prominent and "hard-headed" citizens of the State the other members of which also believed in his land-settlement policies.

In a conversation with a friend during 1935, Dr. Mead expressed the belief that, in regard to the unfortunate land-settlement projects in California, the future would demonstrate that it was a case of the "stone rejected by the masons."

Dr. Mead had firm friends among the Jewish philanthropists back of the Zionist movement in New York, N. Y., London, England, and Paris, France.

He was most unselfish, lived a life of service in the interests of his fellow men, and literally "died in harness." He was a man of the highest integrity, and although the Commission and the Bureau of which he was in charge for so many years at home and abroad were engaged in vast projects, involving the expenditure of scores of millions of dollars, during his incumbency of office, his own high sense of honor and that of his many loyal assistants have been such as to leave a spotless record.

All engineers are proud when outstanding men in their profession leave such records of integrity. The personnel of the Reclamation Service especially can take great pride in this record as from the inception of the Bureau it has been in charge of men of similar high character. Dr. Mead and his predecessors in office have left a wonderful heritage which will be an inspiration to future successors and to all employees of the U. S. Bureau of Reclamation.

Dr. Mead was elected a Member of the American Society of Civil Engineers on June 7, 1893. He served as a Director of the Society from 1903 to 1905.

FRANK HOWARD NEFF, M. Am. Soc. C. E.[1]

DIED OCTOBER 11, 1936

Frank Howard Neff was born in Cleveland, Ohio, on July 30, 1865. His parents were William A. and Eliza (Mong) Neff, well known and highly respected citizens of Cleveland, whose homestead at the corner of 105th Street and Carnegie Avenue, was long a landmark.

His early education was acquired at the Cleveland Public Schools; he then entered Case School of Applied Science in 1883, and was graduated therefrom with the degree of Bachelor of Science in 1887. The degree of Civil Engineer was conferred upon him in 1892. The Class of 1887 was the third to be graduated at Case and among Mr. Neff's ten classmates were Dr. A. W. Smith, for many years Professor of Chemistry at Case, and Dr. Herbert H. Dow, founder of the Dow Chemical Company. The Civil Engineering Department at Case, at that time was in charge of the late Cady Staley, M. Am. Soc. C. E., of beloved memory, and a close friend of Professor Neff in later years.

Following his graduation, Mr. Neff served for a year as Instructor in Mathematics and Civil Engineering at Case. He then went abroad and studied at the Ecole des Ponts et Chaussées and at the Sorbonne, in Paris, France. Returning to Case School in 1890, he was appointed Assistant Professor of Civil Engineering, under Dr. Staley, who served both as President of the Faculty and as Professor of Civil Engineering at that time. In 1897, he was made full professor, in which capacity he served Case School of Applied Science until his retirement in 1931, with the title of Professor Emeritus of Civil Engineering. At the 1931 Alumni Reunion, Professor Neff was presented with an impressive testimonial of appreciation from the Alumni, which reads as follows:

> "Your former students and fellow members of the Alumni Association of Case School of Applied Science wish to present this resolution of appreciation to you, Prof. Frank H. Neff, '87. Your forty-four years of splendid service to this college is an achievement worthy of a lifetime of endeavor.
>
> "The spirit of your teaching has gone into great bridges and towering skyscrapers through the deeds of your students whom you taught to be engineers and men.
>
> "In some measure we hope you can feel that what we do today and tomorrow is made possible by your teaching yesterday.
>
> "We want you to know that we sincerely appreciate the vision and the spirit of unselfishness you possessed in keeping diligently at this task and doing it well."

Desirous of supplementing his teaching with active engineering practice, Professor Neff spent his summers for some years in the field, in charge of a survey party engaged in location work for the Cincinnati Southern Railway

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[1] Memoir prepared by Wilbur J. Watson, M. Am. Soc. C. E., largely from data supplied by the Alumni Office of Case School of Applied Science, Cleveland, Ohio.

Company, then as a Structural Designer for the King Bridge Company, of Cleveland, at that time one of the leading bridge companies, and one year as Assistant on sewer construction.

His colleagues long considered two features of his work as outstanding—his unexcelled supervision of design in the drafting-room and his meticulous devotion to detail. These substantial qualities were said to have developed many of the best known engineers in the United States.

Professor Neff was of a quiet and somewhat retiring disposition, the perfect gentleman and scholar, seldom showing anger, always endeavoring to help his students, in his quiet patient way. In addition to teaching, he had served for more than thirty years as Treasurer of the Athletic Association and for many years as Treasurer of *Case Tech*, the student paper.

He was an ardent, although apparently not always successful, hunter and fisherman. He liked to spend at least a part of each summer at the Case Faculty Camp on Tomahawk Island, in Georgian Bay, about twelve miles above Penetanguishene, Ont., Canada ("place of the shining sand"). Perhaps even now in the cold winds that are whistling across Penetanguishene the spirit of this great and faithful teacher, true friend of all Case men, hovers peacefully, awaiting a far-off dawn. Throughout his life, Professor Neff was a faithful member of the Euclid Avenue Congregational Church, in Cleveland.

Professor Neff was a member of the American Railway Engineering Association, the American Society for the Advancement of Science, the Society for the Promotion of Engineering Education, the Cleveland Engineering Society, Cleveland Chamber of Commerce, the University Club, and the Rowfant Club. He was one of the organizers of the Case Chapter of Zeta Psi.

In later years, he became President of the Electric Railway Improvement Company, of Cleveland, manufacturers of electric railway equipment, which office he held at the time of his death.

His wife, Ida (Brown) Neff, died in 1914. Two sons, Frank H., Jr., and Edward B., survive him. Also surviving him are a brother and sister, Mr. H. A. Neff, and Mrs. B. K. King.

Professor Neff was elected an Associate Member of the American Society of Civil Engineers on May 6, 1903, and a Member on March 16, 1925.

EDWARD SHERMAN NETTLETON, M. Am. Soc. C. E.[1]

DIED JULY 23, 1936

Edward Sherman Nettleton was born in New Haven, Conn., on October 24, 1868, the son of John F. Nettleton and Sarah (Peck) Nettleton. He was a direct descendant in the eighth generation of Samuel Nettleton who settled in Totoket, later Branford, Conn., in 1644. He was educated in Dwight Public School, of New Haven, the Hillhouse High School, Class of 1890, and was graduated from Sheffield Scientific School of Yale University in 1892, with the degree of Bachelor of Philosophy.

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[1] Memoir prepared by a Committee of the Connecticut Section, consisting of Frederick L. Ford and W. Vincent Barry, Members, Am. Soc. C. E.

At Yale he took the course in Civil Engineering under the late Augustus Jay DuBois, M. Am. Soc. C. E., and in Military Science and Tactics under Lieut. Charles Totten, U. S. A.

After his graduation from Yale, Mr. Nettleton traveled in Europe for three months and on his return was first employed at Fort Wadsworth, Staten Island, New York, on Government fortification work. His next job was in Waterbury, Conn., where, for a short time, he was connected with the construction of a pipe line for the new city water supply. From April, 1895, to April, 1896, he was engaged as a Draftsman for Purdy and Henderson, Structural Engineers, in New York, N. Y.

In 1896, Mr. Nettleton returned to New Haven, where he entered the employ of the Department of Public Works as a Transitman in the Bureau of Engineering. After many years of experience in municipal engineering, and upon the resignation of City Engineer Frederick L. Ford, M. Am. Soc. C. E., Mr. Nettleton became Acting City Engineer of New Haven in May, 1920. On October 27, 1921, he was appointed head of the Bureau of Engineering by the late John J. Lane, then Director of Public Works of New Haven.

During the fourteen years that Mr. Nettleton served as City Engineer of New Haven, a total of $6 500 000 in bond issues for public improvements, was expended under his supervision, both in the design and in the execution of the various projects involved. To relieve the congestion in traffic at the intersection of Broadway and York Street, his plan for the widening of Grove Street and its extension to Broadway was completed during the administration of Mayor John B. Tower, and the name, Tower Parkway, was given to the new highway beginning at York Street. Mr. Nettleton also planned the relocation of the street railway tracks on Broadway and the present layout of Broadway Parks, resulting in a greatly improved traffic flow through that artery. A present-day comparison of the status of the vicinity of Broadway and York Square, previous to the present layout of Tower Parkway and Broadway, speaks well for the knowledge and foresight with which he conceived and completed this great improvement to the appearance and traffic facilities of the central section of the city.

With the exception of the expenditures for permanent pavements the two major projects during Mr. Nettleton's term of office as City Engineer were the Tomlinson Bridge contracts and the construction of the East Street Sewage Treatment Works. Tomlinson Bridge, opened to traffic in April, 1925, is situated on the Boston Post Road, U. S. Route No. 1, crossing New Haven Harbor, and is of the Strauss trunnion bascule type, with approach spans and causeway. The cost to the City in bond issues was $1 150 000. The East Street Treatment Works is a gravity-flow plant.

consisting of mechanical screens, grit chamber, three sedimentation tanks, and sludge disposal by barging to deep water in Long Island Sound. Operation of this plant which has a maximum capacity of 30 000 000 mgd and cost about $725 000, was begun in January, 1931. The execution of these contracts and the economical expenditure of the funds appropriated were under Mr. Nettleton's supervision as head of the Bureau of Engineering.

By an Act of the Connecticut Legislature, in 1921, the Mayor of New Haven was authorized to appoint a commission to write a zoning ordinance for the city and, in the same year, such a commission was appointed. City Engineer Nettleton was made a member of the Commission and acted as Secretary. Through an entire summer, public meetings were held for the benefit of citizens in every ward of the City and the details of the zoning district boundaries were developed on section maps of the city furnished by the Bureau of Engineering. It was not, however, until December, 1926, that the present ordinance was finally adopted on petition to the Board of Aldermen, by the City Plan Commission.

On June 4, 1901, he was married, in West Haven, Conn., to Elizabeth Hyndman, the daughter of William Hyndman, of Ketchum, Idaho. Their children are John Edward, of Milford, Conn., and Elizabeth (Mrs. P. Starr Cressy), of Malden, Mass. Besides the members of his immediate family, a brother, George Nettleton, and three grandchildren survive him.

After his marriage, Mr. Nettleton established his home on Townsend Avenue, in the 32d Ward of New Haven, and soon became identified with the community life of that neighborhood. An organization, called the Fairmont Association, was formed by the taxpayers in the Ward to raise funds for police and fire protection, and Mr. Nettleton was elected the first Clerk and Treasurer of the organization and served until 1922. As a member of the Church of the Epiphany (Protestant Episcopal), on Forbes Avenue, he served as Vestryman, Treasurer of the Church, and teacher in the Sunday School. A local newspaper well summarized his character in these words;

> "Besides a (career man) civil engineer he was a Churchman. Long a Sunday School teacher and Vestryman, he demonstrated solid spiritual, as well as practical engineering, qualities. Rather reserved by nature, he was nevertheless gracious in his co-operation with all who sought it and he will be missed at City Hall where he worked so long and in the community with whose physical development he was so long associated."

At the time of his death Mr. Nettleton was Secretary of the City Plan Commission of New Haven, a member of the Chamber of Commerce, and a Director of the Elm City Building and Loan Association. He was a member of the Connecticut Society of Civil Engineers, the American Road Builders Association, the American City Planning Institute, and the Graduate Club of New Haven.

Mr. Nettleton was elected a Member of the American Society of Civil Engineers on April 12, 1926.

GEORGE ISRAEL OAKLEY, M. Am. Soc. C. E.[1]

Died November 6, 1936

With an abiding loyalty to his country, his family, his many friends, his profession, George Israel Oakley died as he had lived, beloved by all for his sterling character. His death came as a shock to every one; and, in his passing, the profession lost a skillful technician, one with a quiet genius for organizing the many details of large engineering projects, one with the true spirit of craftsmanship, in a word, an engineer of the highest type. That he had chosen Civil Engineering as his profession is clear evidence of the disposition of his mind and heart; but a great deal of his work had to do with the installation of intricate mechanical details as part of the project as a whole. He would have been equally successful in any engineering field because of his scientific approach combined with the practical viewpoint which, due to his wide experience, he was able to bring to bear on each task assigned to him. That he performed those tasks with a high degree of ability and conscientious application is testified to by his associates on each job.

When he died, Mr. Oakley was in his 59th year, having been born on August 14, 1878, in Roslyn, Long Island, N. Y. He was the son of Charles Oakley and Lydia Ann (Ludlam) Oakley. He attended the elementary school at East Williston, Long Island, and entered the Jamaica High School, at Jamaica, Long Island, from which he was graduated in 1898. He then entered Union College, at Schenectady, N. Y., graduating with the degree of Bachelor of Engineering in 1902. While at college, he received a stage appointment, was a member of the Union Engineers' Club, the track team, and played class football and baseball.

After graduating from Union College, from 1902 to 1903, Mr. Oakley was, successively, a Transitman on the Baltimore and Ohio Railroad on Maintenance of Way and New Construction; an Assistant Engineer, with the New York and Long Island Traction Company, in charge of a field party on location surveys of 20 miles, and 5 miles of construction of interurban electric railway; a Transitman on a topographic survey on the Sacandaga River, above Northville, N. Y.; and he made an assessor's map for Nassau County, Long Island, of farm and village property remaining in the Town of North Hempstead, after the changes in boundary line, due to the formation of the new County of Nassau.

From 1903 to 1904, Mr. Oakley worked as Rodman for the Rapid Transit Subway Construction Company of New York, N. Y., and was engaged in checking bids for extra work and making formal reports on them. He was also engaged in checking monthly estimates, and in charge of construction work in the Lenox Avenue car storage yard and repair shops, in New York City.

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[1] **Memoir prepared by Thomas K. A. Hendrick, Assoc. M. Am. Soc. C. E.**

From 1904 to 1905, he was employed by the Borough of Queens, New York City, in charge of a field party on triangulation surveys in that Borough and the relocation of street-line monuments.

From August, 1905, to 1915, Mr. Oakley was with the Department of New York State Engineer and Surveyor, for 7 yr as Assistant Engineer, and for 2½ yr as Resident Engineer, on the New York State Barge Canal System. As Assistant Engineer, he was in responsible charge of parties on base line, center line, topographical surveys, land appropriation surveys, rock borings, land and river drive-rod soundings, and general studies for the canal prism and structural location of 30 miles of the canal through the Mohawk River Valley. He also established a line of gages on the river for the measurement of stream flow and especially for ascertaining the heights and the duration of normal and extreme high waters in the river. Pipe wells were driven and water levels obtained along various sections to obtain information for studies on the ground-water movements, especially as affected by floods in the river. Later, he was in responsible charge of construction work on Contracts Nos. 10, 18 and 30 (value $2 500 000) which consisted of canal locks, guard-gates, fixed and movable dams, six fixed bridges and seven lift bridges, culverts, retaining walls, earth and rock excavation, dredging, etc.

As Resident Engineer, Mr. Oakley had full and responsible charge of Residency No. 4; 25 miles of canal, at Little Falls, N. Y.; contracts to the value of $5 000 000; general supervision of all construction work, reports, and recommendations on new work; estimates for monthly and final payments to contractors; and the preparation of plans for the canal prism and incidental structures. The force of men under him varied from 25 to 75. He made formal reports on all claims for land appropriation and damages from canal construction. On one particular claim for water damages in the Village of Herkimer (10 000 inhabitants), alleged to have been caused by the changes in the river channel, Mr. Oakley took an important part in the hydraulic and hydrological studies necessary for the preparation of the defense and in giving expert testimony before the State Court of Claims. The case was subsequently decided in favor of the State in that it was not liable for the alleged damages.

From January to April, 1915, he was Engineer for the Acme Engineering and Construction Company, at Herkimer, N. Y., in charge of special investigations; and from May, 1915, to October, 1917, he was City Engineer for Little Falls, N. Y. He was Executive Officer of the Board of Public Works and Chief Engineer of the Little Falls Water-Works, a $500 000 plant, consisting of four reservoirs, slow sand filter, 20 miles of transmission and distribution mains, chlorine purification plant, distribution system, and meter system. He was in responsible charge of all design, construction, and maintenance of streets, pavements, bridges, sewers, parks, cemeteries, and the water-works system. During his term, he completed $300 000 worth of work. His city force consisted of from 50 to 75 men of different grades. Aside from his duties with the Board of Public Works, the City Engineer was the Consulting and Advisory Engineer for the Mayor and the Common Council.

As City Engineer, Mr. Oakley recommended to the Board of Public Works the following improvements which were made under his direction and supervision: Purchase of additional land for sanitary protection of the water-shed; a general yearly plan for the reforestation of the cleared land on the water-shed; the rebuilding of a $\frac{3}{4}$-acre sand filter; the installation of a liquid chlorine plant for sterilizing all the water used in the system; and a general plan for the conservation of the water supply by the installation of Venturi water meters; water waste surveys; and the general installation of domestic and industrial water meters throughout the city. He made topographical surveys, and had measurements taken of the quantity of water from various springs and streams; he had rainfall records tabulated and made into usable data in that two United States Government Weather Bureau stations were located on City property and read daily by men in the City Engineer's Office. Other general information was collected and studies were made for possible future extensions.

From November, 1917, to 1919, Mr. Oakley was with Henry A. Van Alstyne, M. Am. Soc. C. E., in responsible charge of the design and supervision of the reconstruction of a 20-in. hydraulic dredge and other general engineering work connected with dredging contracts in Boston, Mass., and New York, N. Y., harbors. He also worked for the late Patrick McGovern, Affiliate, Am. Soc. C. E., of New York, on the construction of a compressed air and open rock tunnel under the East River at 60th Street, in New York City. He then became Assistant Superintendent on the construction of the open caisson foundation piers for the Boston Army Supply Base, which building was 120 by 1 500 ft.

From February, 1919, to January, 1922, Mr. Oakley was with the Guarantee Construction Company, in responsible charge of various contracts in Virginia, New York State, and Massachusetts. From September, 1922, to September, 1930, he was with the J. G. White Engineering Corporation as a Designing Engineer in the Hydraulic Department. From September, 1931, to December, 1932, he served the Board of Water Supply, City of New York, as Assistant Engineer, Grade 4. From December, 1933, to February, 1935, he was with the Public Service Commission of the State of New York in valuation and appraisal work. From February to July, 1935, he was with the Corps of Engineers, United States Army, 1st District, New York City, as Assistant Engineer on special design work for the improvement of the Winooski River, in Vermont. From July, 1935, to the time of his death, Mr. Oakley was an Assistant Engineer with the Department of Sanitation, City of New York, and had charge of important design work in connection with the vast new sewage disposal system of that city.

During the last few years of his life, Mr. Oakley acquired the site for his new home at 49 Crescent Road, Beacon Hill, Port Washington, Long Island. He himself made the plans and supervised the actual building of this new house, a model home in every respect. It was like one of his hobbies to him; and of these he had many, such as a woodwork bench, machine shop work, about a dozen experimental radios, collections of rocks and minerals and miscellaneous firearms, antique and modern, along with his zeal in spending many hours on auto-mechanics. He took a great interest in describing various intricate points on how and why "the wheels went round"; and the care with which he kept his extensive engineering library was known only to his intimates. In the recreation room of his new home, he took great delight in entertaining informally his valued friends.

Mr. Oakley is survived by his widow, the former Vera Forrest Chapman, daughter of the late B. E. Chapman, of Little Falls, N. Y., to whom he was married on February 10, 1910, their son, Alan C., and one sister, Mrs. U. G. Williams, of Newport, N. Y. He was buried on November 9, 1936, in the family plot at West Hills, near Huntington, Long Island, where his parents and grandparents are also buried.

He was a Licensed Professional Engineer and Land Surveyor of the State of New York. He was a Mason, a member of Morton Lodge, No. 63, F. and A. M., of Hempstead, Long Island, having been raised September, 23, 1901; Astorogan Chapter, Royal Arch Masons, and the Little Falls, N. Y. Commandery, Knights Templars, of which he was also a Past Commander in 1915–1916. In addition, he was a member of the Central City Syracuse Scottish Rite Bodies, having been created a Thirty-second Degree Mason, and a member of Ziyara Temple, A.A.O.N.M.S. Funeral services were conducted by the Knights Templars, Noble George Willcox of Little Falls, N. Y., officiating.

Mr. Oakley was elected a Junior of the American Society of Civil Engineers on October 6, 1903; an Associate Member on November 6, 1907; and a Member on June 20, 1922.

VINCENT PHILLIP ODONI, M. Am. Soc. C. E.[1]

Died August 9, 1936

Vincent Phillip Odoni was born of Swiss parentage in Lucerne, Switzerland, on May 28, 1879. He received his schooling in Lucerne, where he attended the primary and High Schools preparatory to his enrollment as a student in the Swiss Polytechnic University, at Zurich, from which he was graduated with the degree of Civil Engineer in 1904.

The first years of Mr. Odoni's engineering practice were spent in Switzerland, where he was associated first with Colonel F. von Schumacher, of Lucerne, in the design and estimates of cost for water supply systems, hydro-electric plants, and cable railroads, and, subsequently, as Construction Engineer with the Chemin de Fer Electric de la Gruyere, Bulle, Canton de Fribourg. His last engagement in Switzerland was as Construction Engineer with the Government Railroads at the depot in Basle.

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[1] Memoir prepared by A. L. Sonderegger, M. Am. Soc. C. E.

In 1907, Mr. Odoni came to the United States and found employment in Los Angeles, Calif., with the writer, in the design and construction of works for water supplies and irrigation. He was also in charge of dredging and jetty construction in Long Beach, Calif., Harbor.

In 1909, Mr. Odoni moved to Denver, Colo., to take employment with the Central Colorado Power Company in the design and estimates of cost for structures for the Boulder and Shoshone Power Plants. He was in charge of construction of the Fall River Power Plant, near Idaho Springs, Colo. The year 1910 and part of 1911 found him associated with the Field, Fellows, and Hinderlider Engineering Company, of Denver, his work including the investigation, design, and estimates for irrigation systems, dams, hydro-electric power development, etc., and, in the latter part of 1911, the construction of the outlet system of the Lyman Dam, in Arizona.

A new chapter in Mr. Odoni's life opened with his removal to Tuscon, Ariz., where he remained as Chief Engineer of Tucson Farms Company from the inception of this project until its completion in 1917. The water development connected with this enterprise included extensive studies of the movement and yield of ground-water supplies and of the economic production thereof on a very large scale. The full development of the project saw more than eighty well-pumping plants in operation and a successful irrigation enterprise "in full swing" in a semi-arid region.

The gigantic project of the Miami Conservancy District lured Mr. Odoni to Dayton, Ohio, where he remained until 1918 when he joined the United States Navy. During the two years following, he served as a Lieutenant in the Civil Engineering Corps and Project Manager of the Marine Corps Base, at San Diego, Calif. With the completion of this work, he returned to Tucson as Chief Engineer and Vice-President of the Tucson Farms Company, in charge of its irrigation system.

A new episode in Mr. Odoni's experience began in 1923 when he accepted the position of Chief Civil Engineer with the Haytian-American Sugar Company, at Port au Prince, Haiti. This position allowed full sway for his engineering ability. The work included the drilling and equipping of irrigation wells, the development of water and of irrigation systems for new cane plantations, the operation of steam and electric irrigation pumping plants, the construction of bridges, residences, pipe lines, and wharves for molasses delivery to tankers. He was also charged with soil studies and drainage systems for a number of plantations which suffered from high alkali and from high-water plane.

Engineers who have found themselves in remote areas, thrown upon their own resources, responsible for the construction, maintenance, and operation of a multiplicity of plants and structures, and called upon to meet emergencies of widely differing character with unskilled labor, will appreciate the ability required to fill such a position.

During his residence in Tucson, Mr. Odoni met and married Olive Haugen, who accompanied him to Haiti, making it possible for him to enjoy the comforts of a home in an otherwise lonely existence. He left the employ of the Sugar Company in 1931 because of ill health, and returned to California.

After his return to California, his health continued to fail and for five years his life was one of sickness and suffering. The supreme patience with which he bore his long illness was exceeded only by that of his loving wife who devoted her energies and efforts to the comfort and aid of her husband. Being exceedingly fond of music, Mr. Odoni found solace in the excellent programs which the radio offered. His interest in engineering science and his studies in art remained active during the years of his illness and served to alleviate his sufferings.

He was an engineer of ability, having the capacity for original research and yet being able to adapt himself to circumstances and conditions, and to master the innumerable problems that present themselves to an engineer in general practice.

Mr. Odoni was elected an Associate Member of the American Society of Civil Engineers on April 4, 1911, and a Member on January 18, 1916.

MILNOR PECK PARET, M. Am. Soc. C. E.[1]

DIED AUGUST 16, 1936

Milnor Peck Paret was born in Pierpont Manor, Jefferson County, N. Y., on June 17, 1857. He was the son of William Paret and Maria G. (Peck) Paret. His father, who was of French parentage originating in Servillac, France, was born in New York, N. Y., on September 23, 1826. He was educated in the graded schools of New York City, and in Hobart College, Geneva, N. Y. In 1884, he was elected and ordained a Bishop of the Protestant Episcopal Church in the Diocese of Maryland and lived at Baltimore until his death, on January 18, 1911. His mother was of English parents, who resided in Flushing, Long Island, N. Y.

Mr. Paret was educated in the public schools of Elmira, N. Y., and the High School, of Williamsport, Pa. He was graduated from Lehigh University, in Bethlehem, Pa., in 1878 with the degree of Civil Engineer. He was a member of the Sigma Phi Fraternity. While at Lehigh he took an active part in college athletics being a member of both the baseball and the track teams.

Following his graduation in 1878, Mr. Paret spent a year traveling in Europe inspecting engineering works, and particularly the docks and terminals at Liverpool, England, and the railroad, tunnel, and highway construction in Switzerland and Italy.

On his return to the United States, he was employed (1879–1882) as Assistant Engineer under the late General W. P. Craighill, Chief of Engineers, U. S. Army, Hon. M. Am. Soc. C. E., on James River channel and harbor improvements and fortifications and also on the Chesapeake and Delaware Ship Canal. For some time he was detailed to make reports and hydrographic surveys relating to projected jetty and harbor work at principal points on the Georgia and Florida coasts.

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[1] Memoir prepared by R. A. Thompson, M. Am. Soc. C. E.

Mr. Paret engaged in private practice under the firm name of Paret and Farquhar, in Baltimore, in 1882–1883, and built a number of water-works and sewerage systems. He also designed and erected, in Baltimore, the first bicycle track built in the United States along scientific lines for racing purposes.

From 1883 to 1887, he served as Assistant Engineer in charge of the location and the heavy construction of the Pennsylvania Lines both East and West of Pittsburgh, Pa. He designed and built several important bridges for this Company, among others, a number of large spiral stone arches.

In 1888, Mr. Paret joined the Engineering Forces of the Kansas City Southern Railway Company, which was then being promoted and built by Mr. Arthur E. Stillwell, as the "Kansas City, Pittsburgh and Gulf Railway", from Kansas City, Mo., to Port Arthur, Tex., on the Gulf of Mexico. As Resident Engineer and Assistant Chief Engineer under the late Robert Gillham, M. Am. Soc. C. E., he located and built several sections of this railway in Indian Territory and Arkansas, and, later, the lower section in Louisiana and the terminals at Lake Charles. He made a special investigation for Chief Engineer Gillham of the feasibility of constructing a ship canal from Sabine River to Port Arthur, and it was upon his favorable report presented to the Secretary of War by Mr. Gillham that this outstanding improvement was authorized and built.

In 1901, Mr. Paret became Chief Engineer of the Kansas City, Mexico, and Orient Railway, which was then being promoted by Mr. Stillwell and his associates, as the short route from Kansas City to tidewater at Topolobampo Bay, on the Pacific Coast, *via* Wichita, Kans., San Angelo, Tex., and Chihuahua, Mexico. He built several hundred miles of this railroad from Wichita to Alpine, Tex.; east and west out of Chihuahua; and from Topolobampo Bay east to the foot of the Sierra Madre.

He resigned in 1909 to form a partnership with Mr. E. J. Beard as Consulting Engineers (Paret and Beard) in Kansas City, specializing in railroad, and hydro-electric and general engineering practice in the West. During this time, Mr. Paret served as Chief Engineer of the Utah Railway Company, in Salt Lake City, Utah, and the Utah and Grand Valley Railway—an electric line.

In 1914, he was appointed District Engineer by the Interstate Commerce Commission on railroad valuation work in the Pacific District, with headquarters at San Francisco, Calif. The engineers of this District made an inventory and reported on the value of about 56 000 miles of main-line railroad west of the Mississippi River. In 1917, he returned to Lake Charles, La., to manage the estate of his deceased father-in-law, Captain George Lock, a wealthy lumberman and land-holder. In 1920, when this estate was liquidated, Mr. Paret again entered private practice at Lake Charles as a Consulting Engineer, specializing in railroad, irrigation, and land practice. He remained at Lake Charles until his death.

In 1898, Mr. Paret was married to Letitia Lock, daughter of Captain Lock, of Lake Charles. His widow and sons, Milnor Peck, Jr., and George Lock, and his daughters, Irene and Helen, survive him.

Mr. Paret was a member of the Protestant Episcopal Church and served as a Vestryman of his Parish for many years. At the time of his death he was a Director in the First National Bank, Edgewood Land and Logging Company, and Swift Coal and Timber Company, all of Lake Charles. During the World War, he was Chairman of the Council of Defense for Calcasieu Parish, Louisiana.

During his life, he wrote extensively on engineering subjects for many technical journals. He was the author of a number of articles on railroad location, track, tunnel construction, and similar subjects, among which was a series of highly technical papers written by him for *Engineering News*, from 1883 to 1887, while he was Engineer for the Pennsylvania Lines, describing the design and construction of a number of oblique stone arches which he built for this Company. These papers have since been accepted as the authority on this subject.

Through his long years of active service in his profession, Mr. Paret was regarded as an outstanding member of the Society, a man of high technical attainments, and an engineer of broad and successful experience. He was an able executive and a sound counselor. These qualifications in 1913 under the Civil Service tests given by the Interstate Commerce Commission for Senior Railroad Valuation Engineer for the United States at large, gave to him the highest rating among approximately 7 000 engineer competitors who also took the examination.

He was beloved and respected in the highest degree by his numerous friends and by his associates in professional as well as in public life. He was a kind husband and father, a patriotic citizen, and always gave freely of his time and energy to the welfare and civic organizations of the communities in which he lived.

Mr. Paret was elected a Member of the American Society of Civil Engineers on September 2, 1885. He was also member of the American Railway Engineering Association.

JAMES EDWIN PARKER, M. Am. Soc. C. E.[1]

DIED FEBRUARY 24, 1934

James Edwin Parker was born in Birmingham, Ala., on December 22, 1884. He was the son of Thomas F. and Nannie (Barker) Parker, pioneer residents of Birmingham. He received his education in the public schools of his home town before entering Howard College from which he was graduated in June, 1903, with the degree of Bachelor of Civil Engineering.

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[1] Memoir prepared by O. G. Thurlow, M. Am. Soc. C. E.

Mr. Parker began his practical experience in 1901 as Rodman with the Tennessee Coal, Iron, and Railroad Company. He worked with this Company as Draftsman and Instrumentman until January, 1904, when he became connected with the Southern Railway Company. From 1904 until 1907, he held positions with that Company and with the Seaboard Air Line Railway Company as Resident Engineer in charge of the construction of various spurs and other railroad work.

He spent the years, 1911 and 1912, with the late James Nisbet Hazlehurst, M. Am. Soc. C. E., as Resident Engineer in charge of the design and construction of a water-works and sewerage system for Kirkwood, Ga.—a $100 000 job—and at Key West, Fla., in charge of street paving, where 250 000 sq yd of vitrified brick and asphalt block pavement was built.

From 1912 to 1919, Mr. Parker was Principal Assistant Engineer of the River and Canal Commission, of Augusta, Ga., under the late Nisbet Wingfield, M. Am. Soc. C. E., Chief Engineer. As such, he designed a large part, and was in full charge of the entire construction, of a $2 000 000 flood-protection project at Augusta. The work consisted chiefly of: 11 miles of earth levee: 25 000 cu yd of reinforced concrete in flood-control bulkheads, gates, and retaining walls; raising two highway bridges over the river and building a new 110-ft span, combined street railway and highway; moving and rebuilding warehouses, factories, residences, railway yards, and terminals; the complete re-arrangement and construction of the entire sewerage system of the city, with numerous street changes, etc.; the construction of a modern steamboat terminal and warehouse; bank protection, paving, etc. During and after its construction this work was inspected by prominent consulting engineers, including the United States Board of Engineers for Rivers and Harbors, and was pronounced by all to be the equal of any similar work with which they were familiar.

This project was designed and built to protect the City of Augusta, and the adjacent territory from floods of the Savannah River. Since its completion it has successfully withstood several severe floods and has proved beyond question its value, having saved the community from damage considerably more than its cost.

Mr. Parker became Assistant Engineer, United States Civil Service, at Birmingham, in 1919, and worked with the writer who was Consulting Engineer for the Government, on the design of the Wilson Dam and Power House, at Muscle Shoals, Ala. After the completion of this project he was retained by several different municipalities in the design of waterways and sewerage systems and in flood-protection projects.

In September, 1920, Mr. Parker became associated with the Alabama Power Company as Assistant Engineer to the Chief Engineer, at Birmingham, and from that time until his death, in 1934, he was actively engaged in the work carried on by that Company. He designed and supervised the construction of a coal-handling and washing plant, at Gorgas, Ala., with a capacity of 120 tons per hr, and spent many years on the design of dams and hydro-electric power-plant developments.

Mr. Parker was a man of unusual ability and character. He stood high in his profession, taking a keen interest in every task and giving meticulous care in preparing and carrying out any work in which he was engaged. He was well liked by all who knew him and left a host of friends who sincerely regretted his death.

He was a member of the Alabama Technical Association, Sigma Nu Fraternity, and of the Masonic Fraternity.

He is survived by his widow, Mrs. Mary (Jones) Parker, and several brothers and sisters.

Mr. Parker was elected an Associate Member of the American Society of Civil Engineers on October 1, 1913, and a Member on August 28, 1922.

WALTER CAMP PARMLEY, M. Am. Soc. C. E.[1]

DIED FEBRUARY 19, 1934

Walter Camp Parmley was born about twenty-five miles south of Madison, in Rock County, Wisconsin, on December 8, 1862. He was the son of Russel and Lucy E. Parmley, of New England Colonial ancestry. After his preparatory education, he entered the University of Wisconsin in 1883 and was graduated from its Civil Engineering Course in 1887.

Immediately after his graduation from college, Mr. Parmley went to California and opened an office in San Bernardino to practice as a Civil Engineer in a partnership, under the firm name of Parmley and Finkle. The firm was immediately successful. The work included surveys of important projects and the construction of the Jurupa Canal and the Vivienda Pipe Line, together with other irrigation systems. In 1889, Mr. Parmley accepted the position of Assistant Engineer on the Bear River Canal Project, in Utah, and the construction of a part of the Ogden City Water-Works, involving an expenditure of about $300 000. In February, 1891, he became City Engineer of Ogden. It was here that he first became directly interested in water supply and sewer construction.

In 1893, it became necessary for him to relinquish this work and return to his home in Wisconsin. While there, he designed and laid out a system of sewers for the Village of Elkhorn. Later, he went to Peoria, Ill., and, as an Assistant in the Engineering Department of that city, designed and directed the construction of sewers costing more than $800 000. He opened an office for private practice in 1895 and, in that connection, designed a sewerage system and chemical precipitation disposal plant for Kewanee, Ill.

In May, 1896, Mr. Parmley became Assistant City Engineer in the Department of Special Sanitation, in Cleveland, Ohio, and had an important part in devising the sewer plans covering a large part of that city. He also assisted in the design of the Walworth Run Sewer with its intercepting sewers, and had charge of its construction.

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[1] Memoir prepared by C. T. Purdy, M. Am. Soc. C. E.

The work in Cleveland, with its many responsibilities, continued until 1903. In connection with this work, he made an extensive study of the flow of water over weirs. The results of this study were embodied in a discussion[2] of the paper presented to the Society, in 1900, by the late George W. Rafter, M. Am. Soc. C. E., relating to the flow of water over dams. He also discussed other papers presented to the Society on similar subjects.

During this period, Mr. Parmley also began a study of the reinforcement of concrete used in sewers, sewer pipes, and tunnel construction. It involved an examination of the different kinds of soils and other loads to which they are subjected, the amount and character of the pressure produced, the stresses that are caused thereby, and exactly where and how such underground structures should be reinforced to secure the best results and, at the same time, an economical construction. He probably made the most thorough analysis of this subject that had been undertaken up to that time. The investigation also involved the derivation of various formulas for the determination of the stresses and the reinforcement required.

In this connection Mr. Parmley designed concrete blocks for use in the construction of large sewers, instead of brick or massive concrete material, the blocks forming segments of a circle, with special provisions for their reinforcement.

He also served in a consulting and designing capacity for the Water Supply Tunnel which was constructed in Cleveland during this period. It was 10 ft in inside diameter, about 3 miles long, and made of reinforced concrete block construction. Before leaving Cleveland he designed sewers in Columbus, and in Sandusky, Ohio, in which his improvements in reinforcement were used.

In 1904, Mr. Parmley opened an office in New York, N. Y., as a Consulting Engineer. This marked the end of a distinct period in his life and the beginning of another for which the first one had particularly fitted him.

Before he came to New York he was an employed engineer, working most of the time for some city or other interest. He was governed by their activities, he went where they required him to go, and he did what they called upon him to do. In a measure, they absorbed his life. For sixteen years it was substantially a continuous drive with an increasing responsibility with each new engagement. After he opened his office, these conditions gradually changed. It was the end of his moving about from one place to another. He established his home in Montclair, N. J., and it remained there. He was able to arrange his activities better.

His knowledge of sanitation, his ability to analyze unusual structural problems, his mathematical proficiency, and the breadth of his experience, eminently fitted him to serve as a counselor and designer for difficult and important projects.

For a considerable time Mr. Parmley's work called him to all parts of the Eastern, Middle, and Southern States, and concerned many undertakings in sewer construction. During these years, he continued his research work and the study of new and better forms of construction. He devised new methods of computations and new formulas relating to them.

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[2] *Transactions*, Am. Soc. C. E., Vol. XLIV (1900), p. 346.

He was greatly interested in the construction of the vehicular tunnel under the Hudson River, between New York City and New Jersey[3]. Of his own initiative he executed a design for the tunnel to be made entirely of a concrete segmental block construction instead of a cast-iron lining, which received much commendation.

Mr. Parmley's most valuable research work was done during the last years of his life. It pertained to the making of large concrete pipe conduits capable of carrying water satisfactorily under great pressure. Concrete pipes had been used mostly for drainage and sewer construction. For such uses they are not subject to outward pressure. When they have been used to carry water, the interior pressure has been relatively low. It was Mr. Parmley's purpose to design a concrete pipe that could be used for any requirement and thus give such construction a much greater range of usefulness.

The shrinkage of ordinary reinforced concrete pipe in the making causes a compression in the reinforcement and tension in the concrete, so that hair-cracks are likely to occur. When the pipe is put in place in the ground, the load of earth over it may add to the tension in the concrete, and when the pipe is filled with water under pressure the tension is materially augmented, hair-cracks are increased and enlarged, the pipe leaks, and the water is wasted. Ordinary reinforcement does not prevent it.

Mr. Parmley surmounted these difficulties by coating the reinforcement with a preparation that prevents the concrete from adhering to it. This alone overcomes the initial difficulty. The shrinkage occasions no stress in either material and the hair-cracks due to shrinkage are avoided. He provided nuts for the ends of the reinforcement rods, which were set in iron boxes buried in the concrete. The pipe is pre-stressed by tightening these nuts, putting tension into the rods and compression into the concrete. When the pipe is filled with water the outward pressure reduces the compression in the concrete as it increases the tension in the reinforcement rods.

To make this a practical development, Mr. Parmley studied every detail of the design, every possible reaction that could occur, and the machinery and methods that might be used in its manufacture. These factors were studied critically from the standpoint of cost, in order to keep the construction as economical as possible. To make sure that he had succeeded in his purpose, three sizes of large pipe, taken direct from the factory, were turned over to Purdue University, at Lafayette, Ind., to be thoroughly tested. The results were published by Purdue University[4] shortly after Mr. Parmley's death.

In all his investigations Mr. Parmley's developments followed mathematical analyses. In many cases his calculations went far afield, and resulted in new working formulas. Testimony from Purdue University, regarding the pre-stressed pipe, is to the effect that it is evident that Mr. Parmley's mathematical treatment of it was sound and that it betrayed his analytical ability and the correctness of his fundamental ideas. Pre-stressed pipe provides a new use for concrete, and a new method of designing long water conduits for the supply of cities and for penstock construction. Sometimes, it may be used to great advantage instead of iron or heavy masonry.

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[3] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 459.

[4] *Research Bulletin No. 46,* Purdue Univ., May, 1934.

Mr. Parmley's acquaintance among engineers was extensive and his interest in the Society was maintained throughout his professional career. Many years ago, he was made a member of a commission in Essex County, New Jersey, his home county, for the extermination of mosquitos. He was particularly fitted for this undertaking, and his contribution to the work was noteworthy. In later years, he served as President of this Commission.

In 1889, two years after he was graduated from the University of Wisconsin, Mr. Parmley was married to Rose A. Webster. Mrs. Parmley and two daughters, Marjorie (Mrs. William Lentz, of Baltimore, Md.), and Florence (Mrs. Roy W. Chesnut, of New York City), survive him.

Mr. Parmley was a member of the Congregational Church and was prominent in its counsels and its activities. He was interested in most matters pertaining to his community. He was an amateur photographer of wide experience and artistic ability. He was very successful in taking color pictures.

He will be remembered longest for his pleasing personality and for his dependability, both in business matters and socially. He was a man of fine character, unassuming, always cheerful, and always kindly. By those who knew him well, he was beloved.

Mr. Parmley was elected an Associate Member of the American Society of Civil Engineers on April 1, 1896, and a Member on June 1, 1898.

HUGH PATTISON, M. Am. Soc. C. E.[1]

DIED AUGUST 20, 1936

Hugh Pattison was born on August 3, 1872, near Cambridge, Md., the son of John Richard Pattison and Emily (DeValin) Pattison and a descendant from three of the earliest Colonial Maryland families—all early settlers in Dorchester County, namely, the Pattisons, the LeCompts, and the Skinners. Of these, Thomas Pattison first settled in Calvert County, but moved to Dorchester County in 1671. He was Clerk of the Court of Dorchester County and, in 1688, was appointed his Lordship's Attorney for that County. The home farm, about four miles from Cambridge, where Hugh Pattison and his brothers and sisters were born, has been left as part of his estate.

Mr. Pattison attended the McDonogh School for boys near Baltimore, Md., from June, 1883, to December, 1889, with a record as one of the outstanding students in ability and character, very much interested in machine shop work, and excelling as a draftsman. In January, 1890, he entered Johns Hopkins University, at Baltimore for the course in Electrical Engineering which he completed in June, 1892, receiving a Certificate of Proficiency in Applied Electricity (P. A. E.) such as all graduates in the Electrical Engineering Course received at Johns Hopkins in those days. In 1927 he was awarded a diploma as Bachelor of Science *Extraordinem*, by the same institution.

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[1] Memoir prepared by Alex McIver, Esq., New York, N. Y.

During his college vacations, Mr. Pattison worked for the Baxter Motor Company, of Baltimore, as Armature Winder and as Draftsman. After his graduation, he obtained employment in the Equipment Department, at the United States Navy Yard, at Norfolk, Va., and remained there several months in charge of the installation of wiring on three cruisers—some of the earliest, if not the earliest, electrical installation work on American war vessels. Then, with Sprague, Duncan, and Hutchinson, Limited, Consulting Engineers, he was Assistant Engineer in the Baltimore Office of the firm, engaged on the work of preparing plans and specifications and supervising electrical installations in office buildings and on the vessels of the Merchants and Miners Transportation Company. In 1894, he was placed in charge of the construction of the electric plant and all wiring work in the Congressional Library, at Washington, D. C., and after completing this work, stayed on in charge of operation of the plant until 1898.

Mr. Pattison then accepted a position with the Sprague Electric Company as Secretary to the Vice-President and Technical Director, in which capacity, for some time, he followed office work in connection with the first multiple-unit electric-train equipment on the South Side Elevated Railway, in Chicago, Ill., the Brooklyn (N. Y.) Elevated Railway, the elevator equipments on the Central London (underground) Railway, and, later, in charge of installing and testing multiple-unit railway equipments in Brooklyn and in Boston, Mass.

In 1901, he left the Sprague Electric Company to accept a position as Resident Engineer (representing the Consulting Engineer) in charge of the design and erection of a new power plant for the Pennsylvania Steel Company (later, Bethlehem Steel Company), at Steelton, Pa., including sub-stations, high-tension transmission lines, and the lighting and motor equipment in the new shops.

After the completion of the work at Steelton in 1903, Mr. Pattison went with Westinghouse, Church, Kerr and Company, New York, N. Y., as Engineer, and, with that Company, was engaged on the foundation work of the Long Island City Power House and later on the design and installation of electric car equipment and the erection of car inspection shops for the Long Island Railroad Company. After the completion of this work, he was made Assistant Engineer of Electric Traction with the Pennsylvania Tunnel and Terminal Railroad Company and the Long Island Railroad Company, and was placed in charge of the electrification of the West Jersey and Seashore Railroad, from Camden to Atlantic City, N. J. Later, he had charge of the construction and operation of an experimental single-phase, alternating-current, electric railway on Long Island for the purpose of obtaining information about alternating-current equipment to help in deciding the question as to whether direct current or alternating current should be used for the Pennsylvania Railroad tunnel electrification. During 1907, he served as Field Engineer in charge of a series of tests on the West Jersey and Seashore Railroad made by the Pennsylvania Railroad Company to determine whether there were any features in the known designs of electric locomotives which might introduce strains on the track in excess of those exerted by steam locomotives, and to obtain information which eventually would lead to a mechanically satisfactory design of electric locomotive. Side pressures on rails were measured by the impressions made on steel plates by hardened steel balls, with ties of special construction having roller-bearings under the rail supports. High-speed tests were made with different types of steam and electric locomotives.

Mr. Pattison was next appointed Superintendent of Construction (under the Chief Engineer of Electric Traction) in charge of the construction work in connection with the electrification of the Pennsylvania Railroad in the New York Area, including the foundations and the erection of steel transmission-line towers across the New Jersey meadows from Manhattan Transfer through the New York Terminal to Sunnyside Yard, making and erecting reinforced concrete telegraph and telephone lines, all equipment in sub-stations, various electric circuits in the tunnels, station, etc.

After the completion of the Pennsylvania Railroad Terminal in 1911, Mr. Pattison was appointed Electrical Engineer for the Chicago Association of Commerce Committee of Investigation on Smoke Abatement and Electrification of Railway Terminals, in Chicago. He remained in this position in exhaustive studies until the completion of the work in 1915. Then, from 1915 to 1919, he was Electrical Engineer, Superintendent of Inspection (during the World War), and Assistant to the General Manager of the Eddystone Rifle Plant, at Eddystone, Pa.

For the next two years Mr. Pattison was engaged in making studies and analyses of existing steam railroad electrification, including the Chicago, Milwaukee and St. Paul Railway, the Norfolk and Western, the New York, New Haven and Hartford, and the Pennsylvania Railroads and making reports on comparative operating and maintenance costs for the Westinghouse Electric and Manufacturing Company.

During 1921 and 1922, Mr. Pattison served as Electrical Engineer of the Electrification Commission for the Illinois Central Railroad, Chicago, to decide upon plans for and the system of electrification to be used. He was a delegate to the International Railway Congress which met in Rome, Italy, in April, 1922. After the completion of the work of the Illinois Central Railroad Commission, Mr. Pattison spent several months preparing a proposal for the electrification of the Virginian Railway, and, then, from 1923 to 1933, he was Engineer of Electric Traction of the Virginian Railway Company, having general engineering supervision over the electrification work between Mullens, W. Va. and Roanoke, Va., covering power plant, transmission and over-head lines, sub-stations, and locomotives during the construction period, and, subsequently, over the electrical operation.

In 1933, Mr. Pattison accepted the position as Engineer of Car Equipment with the Third Avenue Railway Company, New York City, at the time when this Company was embarking upon a new, extensive program of rehabilitation of cars and equipment. He had charge of the Drafting-Room work and was responsible for all revisions in car and equipment designs and for new designs. Due to his wide experience, Mr. Pattison was also consulted on the technical side of almost all engineering projects involved in the Mechanical and Building Departments of the Third Avenue Railway Company. His last important work was the supervision of the designs of entirely new light-weight intermediate side-door cars of aluminum alloy and steel alloy, respectively, which may prove to be the ultimate standards to be adopted. He was busily engaged in this work until he went to the New York Hospital, less than a week before his death, for an operation from which he did not recover.

As a man and as an Engineer, Mr. Pattison always commanded the respect and admiration of his associates and of men under him. He was eminently fair and just in all his dealings, always thoughtful and considerate, and always ready with encouragement or help. He became deeply interested in any task or engineering problem which his duties imposed, went about his work in a quiet and unassuming manner, doing what he believed was right, and his example was always a source of inspiration to his co-workers He had a remarkable memory for incidents and details, the gift of expressing his ideas clearly, a keen sense of humor, and, as a conversationalist, was always entertaining.

One of Mr. Pattison's associates in the Third Avenue Railway Company has written about him as follows:

"Hugh Pattison accepted the position of Engineer of Equipment in the Mechanical Department of the Third Avenue Railway Company in June, 1933, just when this particular department greatly needed attention. He brought to the position technical ability of the highest order and a rich background of experience, but, above all, he brought a fine influence in the spirit in which he did his work and in the steadily splendid example which he set. His loss is one that you and I and the world at large can ill afford. There are entirely too few men like Hugh Pattison."

Dr. Cary T. Hutchinson, of the early firm of Sprague, Duncan, and Hutchinson, Limited, with whom, later, Mr. Pattison was associated at different times, has written as follows:

"Long association with Hugh Pattison beginning in 1893 and continuing well through his life saw the development of a man of the highest integrity both personal and intellectual. He never deviated from what he thought was right, no matter how great the pressure upon him might be. His loyalty to friends was unfailing and his modesty was extreme. His engineering knowledge and ability were of the highest order. It will be hard to find his equal. His friends will long remember him."

Mr. Pattison was never married. He is survived by two brothers, Judge John R. Pattison, of Cambridge, Md., for many years a Judge of the Court of Appeals of Maryland, and Mr. James B. Pattison, of Philadelphia, Pa., and by several nieces. Former Governor Robert E. Pattison of Pennsylvania was a first cousin.

He was a member of the American Institute of Electrical Engineers and of the American Society of Mechanical Engineers.

Mr. Pattison was elected a Member of the American Society of Civil Engineers on March 4, 1913.

DWIGHT PORTER, M. Am. Soc. C. E.[1]

Died February 26, 1935

Dwight Porter was born in Hartford, Conn., on August 28, 1855, the son of James Timothy and Elizabeth Ann (King) Porter. His early education was received in the public schools of Hartford, but before entering college, he was employed in a bank for several years. Entering the Sheffield Scientific School of Yale University, at New Haven, Conn., he was graduated with the degree of Bachelor of Philosophy, the form of degree at that time granted to graduates in Engineering. While a student at Yale, Mr. Porter received a number of prizes, both for excellence in special subjects and for general excellence. He was President of his Class in his Junior year as well as a member of the Class Cup Committee. He served as Vice-President of the Young Men's Christian Association of the Sheffield School. He was also a member of the Colony and the Berzelius Societies.

Following his graduation in 1880, he became a Special Expert Agent of the Tenth United States Census. His work consisted of investigations and reports on water powers of the United States, as follows: (1) The water power of the region tributary to Long Island Sound; (2) the water power of the Hudson River and of Lake George Outlet; (3) the water powers tributary to Lake Ontario and the New York State Canal; and (4) the water powers of the Eastern Gulf Slope.

The reports were very thoroughly prepared, contained much valuable information, and served a very useful purpose for many years. They were, in fact, for at least twenty years after their publication, the outstanding source of information on the water powers of these districts and even to-day are very often consulted for information of value. Mr. Porter was engaged in this work until 1883.

He then became Instructor in Mathematics and Drawing at the Massachusetts Institute of Technology, at Boston, Mass., under Gen. Francis A. Walker, who had recently been appointed President of the Institute after having served as Superintendent of the Tenth Census of the United States. Thereafter Mr. Porter became, successively, Assistant Professor and Associate Professor of Civil Engineering, until 1896, when his title was changed to Professor of Hydraulic Engineering and, in 1914, to Professor of Hydraulic and Sanitary Engineering. He was Head of the course in Sanitary Engineering, one of the established courses in engineering, known as Course XI, from 1909 until his retirement in 1921, when he was given the title of Professor Emeritus. The Faculty elected him Chairman of the Faculty for two successive terms as was then the custom.

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[1] Memoir prepared by C. Frank Allen, M. Am. Soc. C. E.

To aid in his teaching, he wrote two textbooks for the use of his students, one on "Hydraulic Measurements," and the other on "Stereotomy," a subject which he taught to the Civil Engineering students for several years.

Professor Porter, however, was more than a teacher in the classroom sense. Students and others closely associated with him thought of him quite as much as a counselor and friend.

His profession was that of Engineering Teaching. This was his main interest. In it, he was painstaking, thorough, and precise. To this work he gave unstintingly of his time and energy and this tended to restrict, in some degree, opportunity for the exercise of engineering practice. Nevertheless, as a Consulting Engineer, he made examinations and studies, followed by reports, depositions, and testimony, for various clients over a considerable range of subjects, including storm damage, tide damage, back-water damage, stream diversion, power diversion, fire protection, Venturi and other meter tests, as well as certain patent cases.

Somewhat more specifically there were reports in connection with the Charles River Dam, at Boston, and particularly as to the effect upon the sewers entering the river from Beacon Street houses bordering on the river and the proposed basin. His report on certain tenement districts in Boston was rated as more complete than any similar report for any other city which had been made before this time.

Previous to the construction of the Cape Cod Canal, Professor Porter was asked to make a study of the probable conditions to be met in cutting the proposed channel. When the Canal was finally built, engineers expressed surprise at the remarkable degree to which his predeterminations proved true.

Furthermore, in 1896 and 1897, he made a series of reports on runoff, rainfall, and available water power. These reports included the various rivers of the State of Maine.[2] He also made similar reports between 1898 and 1905, concerning the Merrimac River, Blackstone River, Taunton River, and other streams in their vicinity, which, however, have never been published by the U. S. Geological Survey. Nevertheless, they have been of material use to the District Engineers of the Survey in their work.

He was for a considerable time, a member of the Examining Board of the Massachusetts Civil Service Commission for the examination of State and City Civil Engineers.

For many years Professor Porter had his residence in Malden, Mass. He was President of the University Club of Malden, a member of the Malden Historical Society, a Trustee of the Public Library, and had served the usual term as Deacon of the First (Congregational) Church, of Malden, where his counsel and advice were most welcome and appreciated. The pastor of this Church at the funeral services referred to Professor Porter as the most courteous man he had ever known, and this appealed to his friends as a very apt characterization. It may be further said of him that he was a competent engineer, a successful teacher, a useful citizen, a good neighbor, a man of fine character throughout.

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[2] These repts. were published in the Annual Repts. of the U. S. Geological Survey.

Professor Porter was a member of the Boston Society of Civil Engineers, the New England Water Works Association, the Yale Club of Boston, the Technology Club, the Faculty Club of Technology, and was one of the Founder Members of the Society for the Promotion of Engineering Education.

He was married on September 21, 1881, in Hartford, to Alice Case Marsh, who died on April 11, 1908. He is survived by two daughters, Katherine Elizabeth (Mrs. Charles B. Waterman), and Anne Alice; and a son, James Marsh.

Professor Porter was elected a Member of the American Society of Civil Engineers on October 4, 1893.

ANDREW JACKSON POST, 2D, M. Am. Soc. C. E.[1]

Died February 25, 1936

Andrew Jackson Post, the second of that name, was born in Jersey City, N. J., on November 17, 1871, the son of Andrew Jackson Post, 1st, and Margaret (Combe) Post.

Mr. Post was born and reared in an engineering atmosphere. His grandfather, Simeon S. Post, was one of the founders of the American Society of Civil Engineers, his membership dating from November 5, 1852. Simeon S. Post had been Chief Engineer of the New York and Erie Railroad and of the Bergen Tunnel and the Long Dock Company and, at the time of his death in 1872, he was Principal Assistant to Edwin F. Johnson, Chief Engineer, of the Northern Pacific Railroad Company. Simeon S. Post gave special attention to the application of iron to bridge construction and, in 1863, obtained letters patent for his improvements in iron truss bridges which resulted in the well-known "Post truss." He is also credited with the invention of the parabolic head-light reflector used upon locomotives, and with originating the system of railroad baggage checks and the first diagram for making railroad time-tables.

His father, Andrew Jackson Post, 1st, was also a member of the American Society of Civil Engineers, having studied Civil Engineering under his father and having specialized in bridge building. In 1877, he turned his attention to the construction of the iron work of buildings and with the late William H. McCord formed the firm of Post and McCord. It was about this time that the evolution of what was to become the structural frame of the modern skyscraper began, and the history of the firm is coincident with the development of the steel industry as applied to building construction in New York, N. Y. The train-shed of the old Grand Central Station and the roof of the amphitheatre of the original Madison Square Garden were among the important works of the firm in those early days. Two of the early apartment houses, the Dakota, at 72d Street and Central Park West, and the Chelsea, on West 23d Street, were also constructed by the firm.

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[1] Memoir prepared by Aubrey Weymouth, M. Am. Soc. C. E.

Under the inspiration of this family background it was natural that an engineering career should appeal to Andrew Jackson Post, 2d, as a profession. He was educated at Stevens Institute of Technology, at Hoboken, N. J., from which he was graduated in 1892 with the degree of Mechanical Engineer. While in college Mr. Post was a member of the lacrosse team and also a member of the Chi Phi Fraternity, of which he was, successively, National Treasurer and National President after graduation.

In 1892 he entered the employ of Post and McCord and from that time until his death he was continuously engaged in the structural steel industry. One of his earliest responsibilities with the firm placed him in charge of the construction of the steel frame work of the Empire Building, at 71 Broadway, New York City, the present home of the United States Steel Corporation.

Mr. Post's father died in 1896 and, in 1900, Mr. McCord disposed of his fabricating plant in the Greenpoint Section of Brooklyn, N. Y., to the newly formed American Bridge Company, and it was operated by that Company for a number of years as its Brooklyn Plant. From 1900 to 1903, Mr. Post was Engineer in Charge of this plant.

In 1903 with Mr. W. H. McCord as President, the firm of Post and McCord was re-established as a Corporation to engage in the work of furnishing and erecting steel construction. Mr. Post was appointed Chief Engineer and afterward, successively, Secretary, and Vice-President. Upon Mr. McCord's death in 1912, he became President of the Company.

Notable among the long list of steel frames of New York's monumental structures built under Mr. Post's direction are the following: The Metropolitan Tower, Hotel Pennsylvania, Standard Oil Building, New York Telephone Company's Barclay-Vesey Building, American Telephone and Telegraph Building, the large power houses of the Consolidated Edison Company and the New York Steam Corporation, the Chrysler Building and the famous Empire State Building, the tallest building in the world. When Post and McCord obtained the contract in 1931 for the steelwork of the group of buildings for Rockefeller Center amounting to 125 000 tons, it was the largest single order for structural steel in building history.

In 1918, during the World War, the firm of Post and McCord, under Mr. Post's direction, executed the general contract as well as the steel work for all the buildings of the Federal Shipbuilding Company, at Kearny, N. J., for the U. S. Steel Corporation.

Mr. Post's reputation for clear judgment and wise counsel caused him to be drafted for many activities outside his immediate business. He was one of the founders of the New York Building Congress and was always much interested in its work. He was formerly a Vice-President of the Congress and a member of the Executive Committee from its organization. As a member of the Committee of the Merchants Association, he assisted in the writing of the proposed new Building Code of the City of New York. He was Treasurer of the Architects' Building, at 101 Park Avenue, and Treasurer of the 115 East 61st Street Building Corporation. He was also Treasurer of the American Institute of Steel Construction and a member of its Board of Directors and of the Board of Directors of the National Erectors Association. For many years he was President of the Iron League of New York, the local association of steel contractors.

Mr. Post was an enthusiastic golfer and was one of the organizers and President of the Woodway Country Club, at Stamford, Conn. He was also a member of the Union League Club of New York and the Architectural League of New York.

His home was at Old Greenwich, Conn. He was married in 1894 to Mary Abbett, a daughter of *ex*-Governor Leon Abbett, of New Jersey. He is survived by Mrs. Post and two sons, Andrew J. Post, Jr., and Leon Abbett Post; a brother, Robert C. Post; and a sister, Mrs. Edgar L. Bradley, of Keene, Va.

Mr. Post's relations with his clients, his business associates, and his employees were characterized by the finest quality of consideration and fairness. The vexatious problems of a busy contractor's life were met with admirable serenity and able resourcefulness. No better evidence could be submitted of the deep impression which his clean personality and fine character had made on the members of the Building Industry than is found in the following action of the New York Building Congress:

> "When a member of a group is an acknowledged leader, when he is known for his probity, when he is respected by his business associates and employees, then his passing is a great loss to the community.
>
> "Such an one was Andrew Jackson Post, for twenty-three years President of Post and McCord, Inc., Engineers and Erectors of the frames for many of the country's finest buildings, a Founder and a Member of the Executive Committee of the New York Building Congress; President of the Iron League of New York; Treasurer and Director of the American Institute of Steel Construction.
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> "It was with deepest sorrow that the Executive Committee of the New York Building Congress learned of the passing of a valued friend and adviser. In session this Eleventh Day of March, Nineteen Hundred Thirty Six, the members of the Executive Committee express their sincere sympathy to the family and business associates of our former member now departed this life."

Mr. Post was elected a Member of the American Society of Civil Engineers on April 30, 1912.

WILLIAM FULLERTON REEVES, M. Am. Soc. C. E.[1]

DIED SEPTEMBER 18, 1936

William Fullerton Reeves was born on Van Dam Street, in New York, N. Y., on December 13, 1859. He died on September 18, 1936, of a heart attack after a few days' confinement at the apartment which he owned and which for several years had been his home, 1 West 67th Street, New York City. He was buried in the family plot at Woodlawn Cemetery. He was of a robust physique, and although his friends had noticed a slowing up of his movements for a year, none realized his serious condition as he cheerfully carried on the fight to the end.

He was the son of Abraham M. and Julia (Jenkins) Reeves. His mother's sister married a brother of Judge William Fullerton, for whom he was named. His father was a Contractor and was connected with the early construction of the New York Elevated Railroads. The boy was taken out of school and put to work, but with characteristic determination, he arranged to be coached in his studies after working hours and received a diploma of graduation from high school, on July 3, 1877. He continued his studies while still at work, and, from 1900 to 1908, took private courses in civil, hydraulic, mechanical, and electrical engineering, and under Professor Collins P. Bliss, of New York University, made tests of boilers and the strength of materials, in the University laboratories.

From February, 1879, to 1880, Mr. Reeves was employed as Checker, Rodman, and Levelman with the New York Land and Improvement Company and for contractors on the construction of the Elevated Railroads. In August, 1880, he entered the employ of the Manhattan Railway Company and remained in continuous service with that Company and its lessor, the Interborough Rapid Transit Company, to the time of his death, a service of fifty-six years. His positions were, successively, Rodman, Levelman, Transitman, Draftsman, and Assistant Engineer, in which latter capacity he was the Engineering Representative of the Law Department.

It is difficult to do justice to Mr. Reeves in a memoir because his always calm and friendly personality and desire to extend fellowship added so greatly to his services to society and the Engineering Profession. His very social nature inspired sociability among engineers with whom he was in contact, thus leading to an understanding and appreciation of one another and often facilitating business dealings.

Having been born and having always resided in New York City, he experienced its many transformations and enjoyed its many and distinctive successive phases of living. He was the pitcher of the first baseball club formed in the City. As a member of the Salmagundi Club, he associated with artists, and was himself an artist of modest pretentions in painting, with much skill in wood carving, producing several ambitious creations as an amateur. His hobby was cabinet work in which he excelled.

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[1] Memoir prepared by a Committee comprising George H. Pegram, Past-President and Hon. M. Am. Soc. C. E., *Chairman*, Robert Ridgway, Past-President and Hon. M. Am. Soc. C. E., and Arthur S. Tuttle, Past-President, Am. Soc. C. E.

He took much interest in the development of the City and throughout his life collected photographs and records which might have led to his writing an interesting history of New York. In 1915, Mr. Reeves contributed a paper on "Transit Problems in American Cities" to the International Engineering Congress, held in San Francisco, Calif. In this paper he presented a general review of the development of transit facilities in a number of large cities.

He was induced to write the history of the Elevated Railroads which was published later by the New York Historical Society in 1935. It is interspersed with photographs which, incidentally, depict the character of street conditions at various periods.

Beginning work with the Elevated Railroads with their inception, Mr. Reeves became the Engineering Expert for the lawyers who had the difficult task of overcoming the obstructions and restrictions imposed by such radical changes in the use of the streets traversed by these railroads. This naturally drew him somewhat away from the usual duties of strictly engineering work, as his capability in this line was so great that he was induced to continue as the Engineering Representative in the Law Department throughout his fifty-six years of service. The Transportation System which he served included the elevated roads, the subways, and, for twelve years, the street-car lines.

Mr. Osgood Nichols, of the Law Department of the Interborough Rapid Transit Company, thus describes his services:

> "It is difficult to respond to a request for a brief statement of the connection of Mr. William F. Reeves with our legal work, so extensive and diverse has it been over a period of half a century. Throughout the course of the litigation and settlements through which the Manhattan and Interborough Companies have acquired the rights to street easements of abutting property owners, Mr. Reeves, beginning in 1890, testified practically in every trial. Of this litigation the Court of Appeals said: 'It is unprecedented in the records of this Court.' For many years so great was its volume, that a term of the Supreme Court was devoted solely to the trial of these cases, in practically all of which Mr. Reeves testified as to the elevated railroad structure and the operation of the system in its relation to the abutting properties. So accurate and satisfactory was his testimony that he was called in a great proportion of the 1 975 cases tried in Court as a witness for the claimants, receiving often the commendation of the Court, and always the respect and confidence of the litigants.
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> "Following the third-tracking and extension of the elevated lines under the Certificates of 1913, twenty-three commissions were appointed to try these cases. More than two thousand parcels were thrown into these proceedings in which, during their continuance, several hearings were often held daily. A policy of settlement resulted in disposing of nearly all claims of this character during the past twenty-five years, and in the thousands of settlements involved, Mr. Reeves afforded the Legal Department the necessary engineering details.
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> "In large measure, his engaging personality, candor, and fairness were instrumental in securing amicable adjustments in avoidance of litigation costs. His memory as to engineering details, particularly of the Manhattan System,

was extraordinary, reinforced as it was, by research into the history of the Manhattan Railway and a unique collection of photographs and data relating thereto.

"Mr. Reeves' work in connection with cinder and smoke conditions in relation to the owners of property adjoining our power stations was of great importance. It was largely due to his efforts and contact with owners that complaints have been minimized and litigation arising from these conditions avoided.

"In the preparation of maps involving a great variety of conditions and in collecting data relative to the valuation of Interborough and Manhattan properties, Mr. Reeves' work was most valuable and extensive. In proceedings before the Transit and Public Service Commissions, Mr. Reeves' work has been important and extensive. He was also called upon to assist the Legal Department in negligence cases.

"It has been a great privilege and pleasure throughout more than forty years to have been associated with Mr. Reeves in a service and friendship which can never be replaced."

His friends were many and they will long miss his companionship and their association with him. He took a keen interest in the affairs of the Society and served helpfully on many of its Committees and on those of the Metropolitan Section, of which he was a Vice-President, from 1930 to 1932.

In 1882, Mr. Reeves was married to Mary L. Crocker, a lady of prominent family and much social charm, who died in 1928. They had no children, and he is survived only by a married sister.

He was a member of the Engineers Club, the Salmagundi Club, the Pleiades Club, and the New York Historical Society.

Mr. Reeves was elected a Member of the American Society of Civil Engineers on October 1, 1912.

DOUGLAS WILLIAM ROSS, M. Am. Soc. C. E.[1]

Died June 22, 1935

Douglas William Ross was best known as a pioneer in the field of Irrigation Engineering. Although his professional record of forty-two years includes several years in mining and railroad work, his greatest accomplishments were in his chosen field. Due to his brilliance of mind and the high quality of his leadership, all his engagements after the first few years carried a high degree of responsibility. Geographically, he left his record of engineering accomplishments in Canada, Colorado, Kansas, Wyoming, Utah, Nevada, Idaho, Oregon, California, Arkansas, Tennessee, the Gulf States, and in Mexico. His last assignment was in Washington, D. C., as Chief Examining Engineer on Reclamation, Flood Control, and Power Projects, for the Federal Emergency Administration of Public Works.

Douglas William Ross was born on May 7, 1865, in Dunwich Township, Elgin County, Ontario, Canada, the son of John and Mary (Griffin) Ross.

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[1] Memoir prepared by J. L. Savage, M. Am. Soc. C. E.

He was graduated from the Collegiate Institute, at St. Thomas, Ont., Canada, at the age of 17 years, and, later, attended the University of Toronto, at Toronto, Ont., Canada.

Mr. Ross' first work was as an Instrumentman during 1883 and 1884 on public land surveys in the Northwest Territories of Canada. He left Canada, and, for three years beginning in 1886, he was engaged on location and construction surveys for the Atchison, Topeka and Santa Fé Railway Company, in Kansas; for the Union Pacific Railway Company, in Colorado and Nevada; and, finally, for the Sioux City and Pacific Railway Company. This completed his railroad experience. In 1890, he obtained the position of Topographer and Assistant to the late Arthur De Wint Foote, M. Am. Soc. C. E., on surveys and investigations for the reclamation of desert lands in Idaho. This work was being done under the direction of the United States Geological Survey. It was on this assignment that Mr. Ross became intensely interested in irrigation work. During this same year of 1890 he was employed by the Idaho Mining and Irrigation Company, and, under the direction of C. H. Tompkins, M. Am. Soc. C. E., Chief Engineer, and the late Mr. Foote, Consulting Engineer, was placed in charge of the location of the New York Canal in the Boise Valley, Idaho. This canal, with a base width of 40 ft, a length of nearly 100 miles, and a designed capacity sufficient to irrigate more than 200 000 acres of desert land, was the initial development of the Boise Irrigation Project later to be taken over and completed by the United States Reclamation Service.

In 1891, Mr. Ross was appointed Superintendent of the Phyllis Canal of the Idaho Mining and Irrigation Company, which was 55 miles long and provided for the irrigation of 25 000 acres of fertile valley land. In this engagement he gained experience in the operation of irrigation works that was of inestimable value to him later in the location, construction, and planning of many major projects. In 1892, he left Idaho to accept a position, under the late William Hammond Hall, M. Am. Soc. C. E., Chief Engineer in charge of the construction of the Sunnyside Canal, in Yakima County, Washington. Returning to Idaho a year later Mr. Ross organized a company and built the Riverside Canal for the irrigation of about 15 000 acres in Canyon County, Idaho. From 1893 to 1896, he was Superintendent of an irrigation system in Payette Valley, Idaho, and from 1896 to 1899, he was Manager for the Pioneerville Gravel Gold Company, Limited, in Boise County, Idaho. During both these latter connections, he also acted in a consulting capacity for various irrigation enterprises in the State.

In 1899, Mr. Ross was made State Engineer of Idaho, which position he held for four years. During the latter part of his term of office he drafted an Irrigation Code which was enacted into law by the Idaho State Legislature in 1903. Following this service with the State, he was appointed Supervising Engineer for the U. S. Reclamation Service in charge of its work in Idaho and part of the time in Oregon and Utah. He retained this important executive position until 1908. It was during this period that the main features of the great Minidoka and Boise Irrigation Projects were planned and partly constructed, including four important dams, two power plants, and the irrigation systems for nearly 300 000 acres of land. In the summer of 1908, Mr. Ross resigned his position with the Reclamation Service to accept an engagement as Consulting Engineer for the irrigation and land interests of the American Water Works and Guaranty Company, of Pittsburgh, Pa. The development work of this Company was located in Idaho and California. He was Manager and Chief Engineer of its land and irrigation project in the Sacramento Valley, California, upon which more than $8 000 000 was expended. Governor Hiram Johnson appointed him a member of the Irrigation Board of California for a period of four years.

During the World War, Mr. Ross was in immediate charge of the examination of cut-over and other waste lands in the Gulf States, and in Arkansas and Tennessee, in connection with the plans of the Secretary of the Interior, the Hon. Franklin K. Lane, for the rehabilitation of returned soldiers. For nearly two years after the war, he was located in Washington, D. C., in the interests of national legislation for the reclamation of lands, representing reclamation and land interests chiefly of Louisiana, working through the New Orleans Association of Commerce and with the co-operation of an association representing the Public Land States, the National Reclamation Association, the United States Chamber of Commerce, the Legislative Committee of the American Legion, and other civic organizations. Afterward, for nearly four years, from 1922 to 1926, he was engaged in reporting on land colonization and irrigation projects in Mexico.

From 1927 until 1932, Mr. Ross was first Consulting Engineer and then Chief Engineer for the Brown County Water Improvement District No. 1, with offices in Brownwood, Tex. In the latter capacity he had direct charge of the construction of the Brownwood Dam, an earth-fill structure, impounding 125 000 acre-ft of water for flood control, irrigation, and municipal supply. This dam was completed in May, 1933. In July of that year, Mr. Ross was appointed a member of a Board of Engineers to examine and pass on projects proposed for building with Public Works Administration Funds. He held this responsible position with the PWA in Washington, D. C., until his death about two years later.

A few excerpts from letters received from many of Mr. Ross' associates and other friends, are quoted, as follows:

"Mr. Ross had the spirit of a true promoter and an insight into the future possibilities of this western country. As State Engineer in 1903 Mr. Ross wrote the first comprehensive reclamation law that was ever passed in the State of Idaho. As a member of the Legislature I was associated with him in this work and can testify to his patient and persevering study of irrigation problems that were finally embodied in a bill passed by the Legislature, and which, as I have said, was the foundation of the future irrigation developments of the State.

"Upon the passage of the National Reclamation Act Mr. Ross immediately visioned the development which later transformed the Boise Valley through the building of Government works. All through the preliminary stages of work Mr. Ross was the guiding mind. I was in Washington with him in company with Governor Steunenberg and Judge Richards when Secretary Hitchcock gave his first approval of the Payette-Boise Project and allocated the first money for the commencement of work. I consider that Mr. Ross had high attainments as an engineer, but his ability reached beyond the technical part of his profession, and he had the ability to see the economic and social results which followed the irrigation development of this Western Country." (J. H. Lowell, Secretary, The College of Idaho, Caldwell, Idaho.)

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"I first met Mr. Ross when he came to Idaho in the early history of the Reclamation Service. He came there in connection with the Boise-Payette Project. His life was lived in the service of reclamation and conservation. He was a true friend of the West. He knew its problems and shared with others in a desire to correct the present evils and make it a better place for future generations. He was devoted to his work. He was honest, painstaking, and considerate of the feelings and aspirations of others. In his passing, reclamation lost one of its greatest advocates." (The Hon. Theodore A. Walters, First Assistant Secretary of the Interior, Washington, D. C.)

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"I consider the most important phase of Mr. Ross' engineering career began with his appointment as District Engineer of the U. S. Geological Survey. In this position he had supervision of the investigations and construction of projects covering the reclamation of arid lands in the States of Idaho, Oregon, and Utah. This work was undertaken by the Federal Government under the recently enacted Reclamation Act.

"As soon as the Reclamation law became effective, there was a demand for immediate construction. To select the first projects to be undertaken and to prepare plans and specifications for their construction required organization of forces on a large scale and in a short space of time. In this work, Mr. Ross showed his ability both as an engineer and as an executive.

"Mr. Ross was thoroughly human; he seemed to see the best there was in a person and was always ready to defend those he knew against attacks reflecting upon their integrity or ability." (J. B. Bond, M. Am. Soc. C. E.)

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"About 1915 I was assigned the task of compiling the history of the Boise Reclamation Project from its inception to the end of 1911. With complete access to all files and records the writer had ample opportunity to determine the persons responsible for the planning and setting in motion the forces necessary to secure the approval and construction of the first unit of the project. It is certainly true that the will power and drive of D. W. Ross was the beginning of all. Seldom is energy and fire coupled with education and technical training as in this man. Without the prompt and effective action of Mr. Ross, Idaho's two very successful irrigation projects, the Boise and Minidoka, might never have been built." (Gilbert Hogue, Engineer, U. S. Bureau of Reclamation, Friant, Calif.)

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"I knew Mr. Ross in the capacity of Supervising Engineer for the U. S. Reclamation Service, in charge of irrigation investigations and construction in the State of Idaho during the period of 1903 to 1908. It was my privilege to work under him during this period, on the surveys and construction of, first, the Minidoka Project and, later, the Boise Project. Mr. Ross was an able engineer and a man of unusual executive ability for his young age; also he had a splendid and aggressive personality." (C. C. Fisher, M. Am. Soc. C. E.)

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"Mr. Ross was an able engineer of scholarly and well trained mind. He had an unusually broad experience with the building of works of this class [referring to the construction of the Brownwood Dam] in Utah, Wyoming, and California * * *." (R. A. Thompson, M. Am. Soc. C. E.)

"It was not my privilege to have met Mr. Ross until late in his long and notable career although I had previously known considerable of him and his work for many years and had acquired a high regard for his ability and leadership as an engineer and executive especially in the reclamation development of the West.

"When I did have close association with him, his qualities as a man and engineer that undoubtedly had made him such a leader and builder were clearly manifested He worked with great energy, enthusiasm and thoroughness. He analyzed the problems at hand by clear thinking, sound judgment, and honesty of purpose and seemed to possess remarkable ability in discerning the economic and human factors involved. He commanded and was freely given the respect and loyalty of his associates and he extended to all a fine friendliness and consideration. I wish I might have known him earlier and longer." (W. H. Nalder, M. Am. Soc. C. E.)

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"I first met Mr. Ross when I was a young assistant engineer, in charge of a comparatively important engineering investigation, on which I needed advice from some one with experience and mature judgment. I appealed to Mr. Ross who was considerably older and held the important position of Supervising Engineer in the Reclamation Service at the time. I will always remember the patience and careful consideration he gave to my problems although not in his assigned line of duty.

"I later came in contact with him from time to time and on one occasion, a short time before his death, when he visited the Denver Office of the Bureau of Reclamation seeking data and advice in connection with a difficult investigation he was making under his important assignment in the Public Works organization at Washington, D. C.

"I have always been greatly impressed by Mr. Ross' sterling character and his innate sense of fairness and responsibility. He was sensitive to a fault but his high qualities as a gentleman were recognized by all who knew him. He was a great inspiration to the younger engineers and his splendid influence is still discernible in many engineers throughout the Western States who started their professional careers under him in 1903 on the Minidoka and Boise projects of the U. S. Reclamation Service.

"The death of Mr. Ross removes from the Engineering Fraternity one of its outstanding and most distinguished members." (R. F. Walter, M. Am. Soc. C. E.)

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"It was my good fortune and pleasure to have Mr. Ross closely associated with me from July, 1933 to May, 1935, when illness necessitated his return to his home in California. During this period Mr. Ross acted as my Examining Engineer on Reclamation, Flood Control, and Power Projects.

"The selection of projects to be included in the reclamation program for which the Public Works Administration allotted funds in excess of $100 000 000 was based largely on the studies and recommendations made by Mr. Ross. His extensive knowledge of reclamation, gained through intimate contact with this work in many sections of the West over a long period of years, his keen appraisal of the engineering and economic factors involved, and his sound judgment and honesty of purpose, made his services invaluable.

"He was a man of the highest character, considerate, sincere, and respected by every one who knew him." (Fred E. Schnepfe, Assoc. M. Am. Soc. C. E.)

On May 10, 1892, Mr. Ross was married to Edith E. Spofford, at Boise, Idaho, to which union two daughters, Katherine and Mary Kenneth, were born. Mrs. Edith E. Ross died in 1904, and, later, Mr. Ross was married to Inez V. Spofford, by which marriage three sons were born, Douglas

William, Jr., Donald Spofford, and David Goodall Ross. Mr. Ross was stricken with his fatal illness while engaged with the PWA, in Washington, but returned to his home in Berkeley, Calif., where he passed away on June 22, 1935. His widow and all five children survive him.

Mr. Ross was elected a Member of the American Society of Civil Engineers on March 1, 1899.

DONALD BENJAMIN RUSH, M. Am. Soc. C. E.[1]

Died February 15, 1937

Donald Benjamin Rush was born on May 8, 1886, at Dana, Ind., the son of Fred and Annie (Hinkle) Rush. After completing his Grammar and High School work, he entered Rose Polytechnic Institute, at Terre Haute, Ind., in 1906, and was graduated with the degree of Bachelor of Science in Civil Engineering in 1910. In 1921, he received the degree of Master of Science from the Institute.

Early in life, Mr. Rush evidenced a natural bent for engineering work. His father, Fred Rush, was a Mining and Municipal Engineer of the firm of Rush and Emerson, Terre Haute, Ind. During the vacations of his college course, Donald Benjamin Rush worked as an Assistant on mine surveys, railroad construction, and municipal engineering, so that at the time of his graduation he had already acquired a considerable amount of practical experience.

From 1911 to 1912, he was in the Engineering Department of the Chicago and Alton Railroad Company, as Assistant Bridge Engineer. From 1913 to 1914 he was connected with the Chicago Electrification of Railroads Commission, and assisted materially in the development of the modern types of catenary construction.

From 1914 to 1918, his work was in connection with analysis of railroad construction, operation, and economics, as Assistant to the late John Findley Wallace, Past-President, Am. Soc. C. E., and to Bion J. Arnold, M. Am. Soc. C. E.

In 1918 Mr. Rush joined the staff of Robert W. Hunt Company, in Chicago, Ill., remaining with that Company until 1935. From 1923 to 1935, he was Manager of the Cement and Concrete Department of the Company. In 1936, he organized and became President of the Rush-Roberts Engineering Company, specializing in consultation, inspection, and testing of building materials and construction. Mr. Rush was particularly interested in the subject of concrete mixtures and strengths, foundations, and soil pressures, and had done some very able work along these lines. His work on the testing of soils to determine safe allowable pressures, contributed materially to the proper engineering methods of handling this difficult problem, and he developed some very efficient data on this subject.

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[1] Memoir prepared by I. F. Stern, M. Am. Soc. C. E.

He was also deeply interested in mining, and at various times was called upon to make analyses and reports on various mining projects, in which his early training was adequately put to use.

Mr. Rush was always deeply considerate of the rights of others. He had a keen sense of humor and was noted among his associates for his dry wit. His home was in Glen Ellyn, Ill.

In 1912, he was married to Rose E. Duenweg, of Terre Haute, Ind., and is survived by his widow, his two sons, Richard, and James A., his mother, and one brother.

Mr. Rush was elected a Member of the American Society of Civil Engineers on May 12, 1930.

FREDERICK EDWARD SCHALL, M. Am. Soc. C. E.[1]

Died August 6, 1936

Frederick Edward Schall was born in Kuchen, Geislingen, Württemberg, Germany, on December 19, 1857, the son of John and Mary Schall. He received his elementary education in the public schools, attended the Royal Building Academy Trade School, at Stuttgart, and worked as an Apprentice to learn stone-cutting. In 1876, he entered the Government Railway Service, and was engaged in railroad construction for the Provinces of Hesse, Nassau, and Baden, Germany; and, from 1877 to 1878, he was Assistant Engineer to a contractor on railroad construction. From the late fall of 1878 to the spring of 1880, he was in the German Army, from which he was honorably discharged on November 7, 1880, and relieved of any further military duties.

From 1880 to 1881, he assisted his eldest brother, a Registered Land Surveyor of Württemberg. He sailed for the United States in July, 1881. Upon his arrival in this country, Mr. Schall worked at stone and marble cutting until he could in some measure master the English language. In order to accomplish this as quickly as possible, he attended night school at the Business College of Easton, Pa.

On July 15, 1883, he entered the service of the Lehigh Valley Railroad Company as a Draftsman in the Engineering Department, which position he held until 1887 when he transferred to the Bridge Department as Draftsman and Bridge Designer. From 1888 to 1897, he served as Chief Draftsman in the Chief Engineer's Office, at Bethlehem, Pa. During this time Mr. Schall also designed and built all the bridges on the new line of the Lehigh Valley from Geneva to Buffalo, N. Y., co-operating with the late Paul S. King, M. Am. Soc. C. E., Chief Engineer of Construction. In 1897, he was made Assistant Engineer in charge of the Chief Engineer's Office, and, in 1899, was promoted to the position of Bridge Engineer, which he held until April 1, 1929, when he was appointed Consulting Bridge Engineer; this latter position he held until his death.

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[1] Memoir prepared by George T. Hand, M. Am. Soc. C. E., and H. T. Rights, Assoc. M. Am. Soc. C. E.

During Mr. Schall's fifty-three years with the Lehigh Valley Railroad Company, he saw practically all the bridges on the line between New York, N. Y., and Buffalo rebuilt twice on account of the fast changing conditions and the increase in the weight of locomotives. He also saw the construction of the Vosburg and Rockport Tunnels, and prepared plans and specifications for the construction of the new Musconetcong Tunnel in 1927.

Mr. Schall was always a very active supporter of engineering societies and activities. He had been a member of the American Railway Bridge and Building Association since 1899, and had served as Vice-President for five years. He was elected President of the Association for the year from 1911 to 1912. He had also been a member of the American Railway Engineering Association since 1905 and had served on its Masonry Committee since 1908. He was elected President of the Lehigh Valley Section of the Society for the year 1931.

In addition to the societies previously noted, Mr. Schall was a member of the American Society for Testing Materials, The Engineers' Club of Philadelphia, Pa., The New York Railroad Club, and the Engineers' Club of the Lehigh Valley.

He was a man of sterling character, a staunch friend, a strict disciplinarian of himself and his employees and had the highest ideals of duty to his employer. He never spared himself when there was work to be done. He was a man of strong conviction and believed in stating his opinions so that there could be no doubt as to his position on the question under consideration.

He was married on January 18, 1883, to Mary K. Kurtz, of Belvidere, N. J., who died on February 15, 1921. He is survived by his two daughters, Emma and Fanny Schall, and a son, Walter Schall.

Mr. Schall was elected a Member of the American Society of Civil Engineers on October 1, 1902.

LEIDY RUDY SHELLENBERGER, M. Am. Soc. C. E.[1]

Died July 25, 1936

Leidy Rudy Shellenberger was born on a farm near Perkasie, Bucks County, Pa., on April 5, 1865. His parents were of German descent, and he received his early education in the rural school of the district in which they lived. Later, he taught in a country school for three years and while thus engaged he studied at the Millersville (Pa.) State Normal School from which he was graduated in 1886.

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[1] Memoir prepared by S. J. Harwi, M. Am. Soc. C. E.

He entered the College of Engineering, Lehigh University at Bethlehem, Pa., in 1887, where he studied under the late Mansfield Merriman, M. Am. Soc. C. E., and was graduated with the degree of Civil Engineer in 1891. Mr. Shellenberger gained his early professional experience, from 1891 to 1895, in the drafting-rooms of several small structural iron works.

From 1895 to 1903, he was Assistant Engineer with the Pencoyd Iron Works, at Pencoyd, Pa., and had charge of the design and detailing of the Richmond Viaduct, at Richmond, Va.; the Rondout Viaduct, in New York State, for the New York Central Railroad Company; several highway bridges; and a number of mill and other buildings.

From September, 1903, to November, 1904, Mr. Shellenberger was associated with the late Charles C. Schneider, Past-President, Am. Soc. C. E., and made preliminary surveys and designs for a new Union Station at Cincinnati, Ohio, the estimated cost of which was $10 000 000. From 1904 to 1908, he made many preliminary and final studies and designs for the Architects of the Grand Central Station, in New York City. He also supervised the erection of the steel for this work.

He was with the Public Service Commission of New York City from 1908 to 1912, and was engaged principally in making preliminary and final designs for subways.

In 1912, he organized the Bayonne (N. J.) Launch Company, of which he was Secretary and Treasurer. In 1913, Mr. Shellenberger returned to the Public Service Commission of New York City, as Designer, and had charge of the design of subway work valued at approximately $10 000 000. He resigned in 1916, and went with the Chile Exploration Company as Designer and Squad Engineer for a period of two years, after which he was with the Dwight P. Robinson Company in a similar capacity until 1921, when he became connected, as Designer, with the Engineering Force for the Holland Tunnel, with which he remained until 1924.

He then became Chief Designer of Bridges and Structures for the New Jersey State Highway Department, at Jersey City, N. J. As such, he had charge of the design of the Route 25 Connecting Link, commonly known as the Pulaski Skyway, which is one of the largest viaducts of its kind. Upon the completion of this work, in 1933, he retired from active engineering. He died after a short illness on July 25, 1936, and was buried at Perkasie, Pa.

Mr. Shellenberger was a tireless, energetic, and thorough worker, and possessed sound engineering judgment. He was a man of sturdy character, simple habits, and had a genial personality. He was a member of the First Reformed Church, at Bayonne, N. J., and, at the time of his death, was an Elder of that Church. He was a Thirty-second Degree Mason and also a member of the Shrine, Salaam Temple, of Newark, N. J.

On March 29, 1902, he was married to Mary Lavinia Stump, of Stouchsburg, Pa., and is survived by his widow and two sons, John and William Howard Shellenberger.

Mr. Shellenberger was elected an Associate Member of the American Society of Civil Engineers on May 2, 1900, and a Member on December 15, 1924.

ZENAS HARRISON SIKES, M. Am. Soc. C. E.[1]

DIED AUGUST 1, 1936

Zenas Harrison Sikes, the son of Zenas L. and Emma (Harrison) Sikes, was born at Suffield, Conn., on March 5, 1874. His preliminary education was obtained at the Suffield School and at the Williston Academy, at Easthampton, Mass. He was graduated with the degree of Bachelor of Philosophy from the Sheffield Scientific School of Yale University, at New Haven, Conn., in 1898.

After his graduation in 1898, Mr. Sikes became connected with the Pennsylvania Lines West of Pittsburgh, first as Assistant Engineer in the Maintenance-of-Way Department and then as Designer and Draftsman in the office of the Bridge Engineer.

In 1901, he took a position with the Riter-Conley Manufacturing Company as Detailer on various types of steel structures and, later, as Designer in the Bridge Department. In 1906, Mr. Sikes started his career with the New York Central Railroad Company with which he was connected until his death. He spent three years as Designer and Estimator on various railroad structures. In 1910, he was promoted to the position of Assistant Engineer of Structures, in which capacity he was engaged in the preparation of designs and general detail plans for various types of railroad and highway bridges. In this position he had an important part in several undertakings of considerable magnitude, such as tthe bridge carrying the Hudson River Connecting Railroad over the Hudson River, at Castleton, N. Y.; several Strauss trunnion bascule railroad bridges; the bridges of the track elevation and the elimination of grade crossings through the City of Syracuse, N. Y., and the bridges for the extensive West Side Improvements, in New York, N. Y.

In 1900, Mr. Sikes was married to Jennie Fuller, of Suffield, Conn. Mrs. Sikes, together with a son, Dr. Ralph F. Sikes, and a daughter, Mrs. Ernest Warner, survives him.

Mr. Sikes was a member of the American Railway Engineering Association, the American Association of Engineers, the New York State Society of Professional Engineers, and the Yale Engineering Association. He was a Mason and a member of the Yale Club of New York City. He was active in religious work, having been Trustee of the Young Men's Christian Association, at Yonkers, N. Y., the Railroad Young Men's Christian Association, in New York City, and the Baptist Church of the Redeemer, of Yonkers.

He will be remembered by all those with whom he came in contact for his exceptional Christian character which was exemplified particularly through a marked degree of unselfishness and devotion. Mr. Sikes' able and conscientious efforts made him recognized as a worthy member of the Engineering

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[1] Memoir prepared by B. S. Voorhees, M. Am. Soc. C. E.

Profession, and his generous contributions toward welfare and religious work won him admiration in the community in which he lived.

Mr. Sikes was elected an Associate Member of the American Society of Civil Engineers on June 30, 1910, and a Member on March 14, 1916.

JAMES CUMMIN STEVENSON, M. Am. Soc. C. E.[1]

Died December 29, 1936

James Cummin Stevenson, the son of James Oliver and Emilie Blanche (Rattiseau) Stevenson, was born in Galveston, Tex., on December 16, 1885. He attended the public schools of Galveston, and, in 1904, he entered the University of Texas, at Austin, Tex., from which he was graduated with the degree of Civil Engineer in 1908. The excellency of Mr. Stevenson's work as a student was recognized by the Faculty of the University by his appointment as a Student Assistant in the Engineering Department. During his last year in college his classmates paid a tribute to his popularity by electing him President of the Senior Class.

Mr. Stevenson's first engineering experience was acquired during the vacation periods of his college years, when he worked for the Gulf, Colorado and Santa Fé Railway Company as Chainman, Rodman, and Instrumentman. From June, 1908, to December, 1910, he served, successively, as Inspecting Engineer, Draftsman, Assistant Engineer, and Assistant City Engineer of the City of Galveston, in connection with the raising of the grade of the city as a flood preventive. From December, 1910, to May, 1911, he was Assistant Engineer for the United States Boundary Commission in connection with the delineation of the boundary between the United States and Mexico along the Rio Grande. From 1911 until 1932, except for a period of four months during 1918, when he was Executive Assistant to the Construction Quartermaster at the Army Base, at Brooklyn, N. Y., Mr. Stevenson was connected with H. L. Stevens and Company of Chicago, Ill., on the design and construction of fireproof buildings, successively, as Designing Engineer, Assistant Superintendent of Construction, General Superintendent, Assistant Chief Engineer, Chief Engineer, and Vice-President and Managing Partner.

Mr. Stevenson's building experience, embracing a period of twenty-five years, covered a large number of structures. The last twenty-three years of his life were spent exclusively in hotel construction and operation. Among the buildings which he supervised from their inception to their completion, including the securing and execution of the contracts with the owners, the design and construction, and, frequently, the financing and operation, are the following: Kirkwood Hotel, Des Moines, Iowa; Both-well Hotel, Sedalia, Mo.; Bankhead Hotel, Birmingham, Ala.; St. Nicholas Hotel, Springfield, Ill.; Hotel Chieftain, Council Bluffs, Iowa; Hotel Tall Corn, Marshalltown, Iowa; Hotel President, Waterloo, Iowa; Hotel Wenonah, Bay City, Mich.; Hotel Northland, Marquette, Mich.; Hotel Vicksburg, Vicksburg, Miss.; Capitol Hotel, Lincoln, Nebr.; Onesto Hotel, Canton, Ohio; and the Metropole Hotel, Cincinnati, Ohio.

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[1] Memoir prepared by W. F. Krahl, Assoc. M. Am. Soc. C. E.

In 1932, after the depression had put an end to new hotel construction, Mr. Stevenson opened his own office, in Chicago, as a Consulting Engineer and specialized in the re-habilitation and financial re-organization of hotel properties. In this work, too, he was successful. His buildings were economically designed, admirably planned, and soundly constructed. His intimate knowledge of construction costs influenced advantageously the planning and construction of his projects, and his experience in hotel management helped his clients to avoid errors involving unnecessary operating expenses.

He was a reader and a profound student of economics. He had the ideal engineering mind—keen, analytical, and well balanced. He was experienced in at least three branches of engineering, successful as a salesman, a financier, and a business executive, a builder of note, exceedingly versatile, modest, and unassuming, but the master of any situation which presented itself. Mr. Stevenson's memory will live not only in the stone and mortar of the many fine permanent structures which he visualized and built, but in the affections of those whom he served so well, and in the hearts of the many who called him "friend."

In his home life he was very fortunate and supremely happy. On June 18, 1913, he was married to Mary Pearl Melton, of San Antonio, Tex. His widow, two daughters, Mary Jane and Catherine Melton, and one son, James Cummin, III, survive him. For them it may be said, "Love hath a tie which Time nor Death can sever."

Mr. Stevenson was elected an Associate Member of the American Society of Civil Engineers on March 4, 1913, and a Member on September 9, 1919.

EARL STIMSON, M. Am. Soc. C. E.[1]

DIED MAY 27, 1936

Earl Stimson was born at Cincinnati, Ohio, on September 2, 1873, the son of Earl W. and Florence Pritchard (Cummins) Stimson. His ancestry of Scotch-Irish origin, settled in New England, his father and mother moving later to the Middle West. His early education included the usual course in the public schools of his native city, with one year (1892), at the University of Cincinnati, and two years (1893–1894), at Cornell University, at Ithaca, N. Y.

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[1] Memoir prepared by the following Committee: J. E. Greiner, Hon. M. Am. Soc. C. E., and H. A. Lane, and P. G. Lang, Jr., Members Am. Soc. C. E.

The entire period of Mr. Stimson's long and useful professional career was spent in the service of The Baltimore and Ohio Railroad Company and its affiliated lines, which he entered in June, 1895, in the capacity of Rodman in the Maintenance-of-Way Department, at Cincinnati. In 1896, he was advanced to the position of Assistant Engineer, at Cincinnati, and, in 1898, he was transferred to Chillicothe, Ohio. Between February, 1899, and May, 1901, he occupied the position of Resident Engineer at Osgood, Ind., in charge of revision of grades and alignment between Milan and North Vernon, Ind. In May, 1901, he was appointed Assistant Division Engineer at Chillicothe, and in April, 1902, became Division Engineer of the Springfield Division, with headquarters at Flora, Ill., being subsequently transferred to the Illinois Division at Washington, Ind.

In May, 1905, Mr. Stimson was made Engineer, Maintenance of Way, of The Baltimore and Ohio Southwestern System, at Cincinnati. In September, 1907, he was made Chief Engineer, Maintenance of Way, of the same System, continuing in that position until April, 1910. During that month, and shortly after the operations of the Southwestern were merged with those of The Baltimore and Ohio Railroad, he was promoted to the position of Chief Engineer, Maintenance of Way, of the latter System, with offices at Baltimore, Md. Mr. Stimson remained in that position until August, 1918, when, due to the re-organization of the operations of the System incident to Federal Control, his jurisdiction was extended over the Western Maryland Railroad and other neighboring lines which at that time were allied with The Baltimore and Ohio Railroad Company in the Allegheny Region under the direction of Mr. C. W. Galloway as Federal Manager, Mr. Stimson's title being General Superintendent, Maintenance of Way and Structures. At the termination of Federal Control, on March 1, 1920, he became Chief Engineer, Maintenance, of The Baltimore and Ohio System, in which position he spent the remainder of his life. Mr. Stimson's rapid rise in the short period of fifteen years from the position of Rodman to the head of a large and important Department extending over the entire System, is an eloquent tribute to his ability as an Engineer and Administrator.

Between Mr. Stimson's entrance into railroad service, in 1895, and his death in 1936, there occurred a period of profound change and re-adjustment in all important phases of the railroad and construction industries. The major technical problems of his life were concerned, primarily, with the co-ordination of railroad maintenance procedure with the advancing tide of knowledge and improvement. For many years he was actively associated with important national organizations devoted to the furtherance of engineering and maintenance interests. Although his direct professional activities for a period of more than forty-one years were spent on The Baltimore and Ohio System, his influence upon the development of maintenance technique, exerted through his participation in the work of such organizations, was subject to no limitations of extent or importance. To the problems which he was called upon to meet and solve, he applied a combination of an analytical mind and resourcefulness and ability for laborious and thorough research with systematic classification.

For many years Mr. Stimson was active in the affairs of the American Railway Engineering Association, serving as Director in 1914; Vice-President, in 1917 and 1918; and as its President for the term, 1919–1920. He held memberships in the American Railway Engineering Association's Standing Committees on Standardization, Economics of Railway Labor, and Wood Preservation, and between 1927 and his death, he was Chairman of its Committee on Rail. He was also a member of the Special Committee on Stresses in Railroad Track.

As Chairman of the Rail Committee of the American Railway Engineering Association, his constructive activities were especially marked. These activities comprised the conclusion of an arrangement with Dr. Elmer A. Sperry which resulted in the perfection of the transverse fissure detector car, the development of standard design for 112-lb and 131-lb rail and fastenings, revision of rail specifications, and the conclusion of arrangements with rail manufacturers and the University of Illinois for an exhaustive study of transverse fissures by Professor H. F. Moore.

The list of professional societies with the work of which Mr. Stimson was associated, also includes the American Wood Preservers' Association, in which he served as a member of the Committee on Fireproofing. He also was appointed a representative of America to the International Railway Congress Association and prepared and presented a report on "Maintenance Organization and Economic Methods," read at the Ninth Congress, held in Rome, Italy, in April, 1922.

His association and work in the National Societies did not make him lose sight of the value of such activity in his own community, and for fifteen years, Mr. Stimson was a member of the Engineers' Club of Baltimore, serving as a Director for a number of years and being elected its President for the term, 1923–1924. A further evidence of the progressive character of his professional work is furnished by his numerous contributions to the columns of technical periodicals, and his addresses at association meetings and before college students.

Mr. Stimson will always occupy a high place among those who have faithfully and efficiently discharged their duties in the walk of life to which they were called, and the respect and esteem which his achievements have won for him in the field of professional activities, are surpassed only by the tribute and regard voluntarily accorded him by those whose good fortune it was to enjoy the privilege of a close association with him.

Beneath his close adherence to strict discipline, he displayed a real humanitarianism toward the members of the organization under his direction. His social instincts found expression through membership in the Merchants Club of Baltimore, the Maryland Country Club, and the Ohio Society in Maryland.

On November 30, 1899, Mr. Stimson was married to Katherine S. Hopper, of Cincinnati. Mrs. Stimson and their children, Earl, Jr., Elizabeth, Alfred, Katherine, and Emma, survive him.

It is a tribute to Earl Stimson's character and devotion to duty that his death at Massillon, Ohio, on May 27, 1936, occurred during an inspection trip. His long tenure of office, the wide variety of his railroad and engineering activities, his outside associations, had all brought an increasing circle of friends, to whom his passing leaves a sense of deep personal loss.

To his survivors, Mr. Stimson has bequeathed an example of a life of untiring activity and notable achievements, a genial and natural personality, and if his memory proves an inspiration to those of his own generation and those of the rising generation who have come under his leadership and have been associated with him, he will not have lived in vain.

Mr. Stimson was elected a Member of the American Society of Civil Engineers on January 19, 1920.

CHARLES EUGENE SUDLER, M. Am. Soc. C. E.[1]

Died May 3, 1936

Charles Eugene Sudler was born in Baltimore, Md., on October 3, 1876, the son of Colonel and Mrs. John Emory Sudler. His father was a practicing civil engineer, and, for two years after graduating from High School, young Charles received practical education in engineering from him.

In 1896, he secured employment as a Mechanical Draftsman in the machine shop of the Murrill and Keizer Company, of Baltimore. In the latter part of 1897, Mr. Sudler and several other young engineers formed a partnership which operated a small yacht-building yard in Baltimore; as Manager and Engineer of the firm he designed and built a number of boats ranging from small tenders and gasoline launches to a schooner yacht of 65-ft length. Due to lack of capital and inexperience in business, the undertaking was not profitable, and was abandoned in 1900. However, this experience left a lasting impression on Mr. Sudler and even in later years he was happiest when he was engaged on maritime work and on subjects related to shipping.

From 1901 to 1903, he was employed as a Draftsman in the office of A. M. Kinsman, Engineer of Construction and, later, Chief Engineer, of the Baltimore and Ohio Railroad during the rebuilding and double-tracking of this line west of the Ohio River. In addition to much diversified work on plans, estimates, calculations, etc., Mr. Sudler, during this period, had been given considerable opportunity to observe actual construction work; he often said that he was indebted to Mr. Kinsman for the real foundation of his career.

In 1903, he became Superintendent and Engineer for the Hoover and Kinnear Company, Contractors, of Columbus, Ohio, which firm specialized in railroad construction, including tunnels, earthwork, and bridges. Mr. Sudler had charge of the design of coffer-dams, arch centering, and the construction plant for the Company's various contracts, for which he prepared estimates and organized the field and office forces. Contracts for the reconstruction and track elevation of the Baltimore and Ohio Railroad, at Wheeling, W. Va., and at Bellaire, Ohio, and other contracts of similar character for the Pennsylvania Railroad Company, were under his supervision. He also had charge of the construction of the Big Walnut Bridge for the Baltimore and Ohio and the Pennsylvania Railroad Companies near Columbus, Ohio. Other contracts under his charge included bridges, culverts, and tunnels on the Buffalo and Susquehanna and Wabash Railroads.

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[1] Memoir prepared by Eugene E. Halmos, M. Am. Soc. C. E.

In 1906, he became Chief Engineer of the Hamilton Manufacturing Company, of Columbus, Ohio, in the development of the first successful mine-loading machine under the Hamilton patents. A new design of coal-storage and reclaiming machinery was developed and built for the Illinois Steel Company; this machine stocked and reclaimed coke at a rate of 3 tons per min; it was accepted and paid for on the strength of its first day's trial run.

In 1908, Mr. Sudler returned to Baltimore, having been appointed Principal Assistant Engineer under the late Oscar Francis Lackey, Assoc. M. Am. Soc. C. E., Harbor Engineer. At this time, following the disastrous fire of 1904, the City of Baltimore had purchased a large part of its water-front and was carrying forward an ambitious program of municipal maritime construction. Mr. Sudler had active charge of the design and construction of wharves, piers, bridges, buildings, floating equipment, dredging, etc. The steel and concrete arch bridge over Jones Falls, at Pratt Street, Baltimore, included in the foregoing, received considerable notice in engineering literature.

In 1911, Mr. Sudler became Engineer and Manager of the Furst Concrete Scow Construction Company. This was a new plant, consisting of a 1 000-ton railway dry dock, machine and wood-working shops, concrete barge plant, and office building, all of which were constructed under his plans and direction. In addition to the first large concrete barge built in the East, his Company produced composite and wooden scows, and made general repairs on vessels and machinery.

Between 1913 and 1915, Mr. Sudler acted as Superintendent of Construction for the million-dollar Perry Memorial, at Put-In-Bay, Ohio. This very beautiful but little advertised structure is located on a small island in Lake Erie. From 1916 to 1918, he was engaged in the development of mineral and timber property in Eastern Tennessee, as part owner.

In 1919, Mr. Sudler became associated with Parsons, Klapp, Brinckerhoff & Douglas, Consulting Engineers, New York, N. Y., and for two years worked in collaboration with the writer on the design of graving and floating dry docks and ship-repair plants. For the succeeding two years he was in charge of the Hydrology Division of the New York Water Power Investigation, conducted by the firm, under the direction of John P. Hogan, M. Am. Soc. C. E. This work embraced the analysis of the flow and the power possibilities of all New York State streams, and included the study of rainfall and run-off relations, flood flows, etc. Some of the results of the study were presented to the Society under the title, "Storage Required for the Regulation of Stream Flow."[2] For this paper, Mr. Sudler was awarded the Norman Medal in 1928.

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[2] *Transactions*, Am. Soc. C. E., Vol. 91 (1927), p. 622.

In 1924, he took a position with the New York Times Company on research work and the construction of a new rotogravure plant. In 1927, Mr. Sudler became connected with the Port of New York Authority in the preparation of studies for the Suburban Transit Engineering Board on the suburban passenger problem for the entire Metropolitan District, and on an analysis of construction cost estimates. From the time of the consolidation of the Port of New York Authority and the New York-New Jersey Tunnel Commission, in 1930, to his death, Mr. Sudler was assigned to the Chief Consulting Engineer, Ole Singstad, M. Am. Soc. C. E., to assist in investigations, preliminary studies, lay-outs, and other work contemplated in connection with improvement of traffic in and around New York City. Mr. Sudler did particularly fine work in connection with the studies and preliminary planning of the Manhattan approach for the Midtown Hudson Tunnel. He also assisted Mr. Singstad in the preparation of a detailed history of the design, construction, and operation, of the Holland Tunnel. His last work consisted in the preliminary planning and estimation of the cost of the proposed Queens-Midtown Tunnel under the East River.

Mr. Sudler was greatly admired by his associates for his remarkably able, versatile, and analytical mind; he was as much at home in the field of Mechanical Engineering as in matters of Civil Engineering. He had a most likable personality, and all who came in contact with him remember him with the kindest thoughts.

Mrs. Sudler, the former Helen Hamilton, a son, Hamilton G., two daughters, Mrs. Sarah S. Westerman and Mrs. Josephine C. Buell, two brothers, Ralph Sudler, and Emory Sudler, M. Am. Soc. C. E., and two sisters, Annette and Elsa Sudler, and his mother, Mrs. John E. Sudler, of Washington, D. C., survive him.

Mr. Sudler was elected a Member of the American Society of Civil Engineers on October 1, 1913.

WILLIAM STANTON TWINING, M. Am. Soc. C. E.[1]

DIED FEBRUARY 8, 1937

William Stanton Twining was born near Titusville, Pa., on February 20, 1865, the son of Charles and Mary (Stanton) Twining. His parents removed to Union City, Pa., in 1870, where he entered the public schools, from which he was graduated in 1881. He then entered the employ of the Industrial Iron Works, at Union City, as a Machinist's Apprentice and remained there three years. In September, 1884, he entered Cornell University, at Ithaca, N. Y. A year later, he entered Allegheny College, at Meadville, Pa., from which he was graduated in 1887 as a Civil Engineer with the degree of Bachelor of Science. He remained at Allegheny College as an Instructor in Civil Engineering and Chemistry until 1890, having been granted the degree of Bachelor of Arts in 1889.

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[1] Memoir prepared by Charles H. Stevens, M. Am. Soc. C. E., and Joseph W. Silliman, Assoc. M. Am. Soc. C. E.

Mr. Twining's long connection with the development of the electric street-railway industry began when he entered the Electric Railway Department of the Thompson-Houston Electric Company, in Boston, Mass., as Assistant Engineer, and was associated with the design and erection of steam-driven electric power plants at Indianapolis, Ind., Allentown, Pa., and Toledo, Ohio. In 1891, he entered the employ of the Harlem Bridge, Fordham and Morrisania Railway Company (later, a part of the Union Railway Company of New York City) as Principal Assistant Engineer in charge of the electrification of the street railway system, and, in the following year, he was engaged on similar work with the Atlantic Avenue Railway Company (later, a part of the Brooklyn-Manhattan Rapid Transit System).

In 1893, Mr. Twining entered the employ of the Peoples Traction Company, of Philadelphia, Pa., as Assistant to the Chief Engineer, and was made Chief Engineer in 1895. In that year, the principal street railway lines of Philadelphia were consolidated as the Union Traction Company and Mr. Twining was made Chief Engineer of the Consolidated System.

During this period, the street car lines passed from the horse-drawn type to electric operation, and the successful reconstruction of the roadbed, with its overhead trolley system, new cars, large generating plants, with their far-flung distribution system, is an enduring monument to the genius, resourcefulness, and skill of Mr. Twining as an Engineer.

In 1902, the Philadelphia Rapid Transit Company was formed, and this Company leased the electrified street railway system, controlled and operated by the Union Traction Company. This Company also secured public franchises authorizing it to construct and operate a number of high-speed subway and elevated passenger railway lines.

With the transfer of the Union Traction System to the Philadelphia Rapid Transit Company, Mr. Twining continued as Chief Engineer, in which position he directed and supervised the construction and equipment of the Market Street Subway and Elevated Passenger Railway; and a two-track subway in Market Street, extending from the Delaware River to the Schuylkill River, at which point it became a two-track elevated railway structure of steel and concrete, to the western boundary of the City near Sixty-ninth Street. This system is six miles in length, and its construction embraced some outstanding improvements over the existing structures of this period.

In 1910, he became associated with the firm of Ford, Bacon and Davis, of New York, N. Y., as an Engineering Executive and, in this capacity, on behalf of the firm, studied the operating structure of many of the prominent public utility systems of the United States, submitting reports and recommendations covering operation, valuation, and improvement of the properties.

In 1912, Mr. Twining returned to Philadelphia as a representative of Ford, Bacon, and Davis, which firm had been retained to advise the City of Philadelphia in the formulation of a comprehensive plan for the improvement of the passenger transportation system and to develop and recommend a program for the construction of a system of high-speed transportation lines by the City of Philadelphia. These studies continued until 1916, at which time, Mr. Twining severed his connections with the firm of Ford, Bacon, and Davis, to become the Director of the Department of City Transit of the City of Philadelphia, a position that he filled for a period of eight years.

During this time the Frankford Elevated Railway, a two-track elevated structure of steel and concrete, six miles in length, was constructed and equipped for operation. This line connected with the eastern terminus of the Market Street Subway, and extended to the northeast section of the City, locally known as Frankford. Also, a beginning was made on the construction of the Broad Street Subway on that part of the subway under City Hall.

In 1924, Mr. Twining retired from the public service and devoted his time to consulting practice, and to the study and development of matters of a lifetime interest.

He possessed, to a rare degree, the ability to conceive, envision, and plan comprehensive engineering systems and structures; he was a master worker in his chosen field of endeavor; his warm personal charm drew his associates into a close relationship and encouraged them to give to their work the best that was in them.

Throughout his professional career, Mr. Twining was steadfast in his convictions and adhered to his conceptions as to what constituted the right and proper course of procedure. He was loved and esteemed by those who were privileged to be associated with him as co-workers and also by those who were admitted to the closer and more intimate tie of friendship.

Mr. Twining was a member of the Union League, the Franklin Institute, and the Engineers Club of Philadelphia. His membership in technical societies included the American Society of Mechanical Engineers, the American Institute of Electric Engineers, the American Academy of Political and Social Science, and the Phi Kappa Psi Fraternity.

In 1916, he was married to Mrs. Harriett P. Rundell, of Toledo, Ohio, who survives him.

Mr. Twining was elected a Member of the American Society of Civil Engineers on September 3, 1913.

RAY BENEDICT WEST, M. Am. Soc. C. E.[1]

Died June 3, 1936

Ray Benedict West was born at Ogden, Utah, on October 21, 1882. His grandparents were pioneers of the West. He was the son of Joseph Alva and

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[1] Memoir prepared by a Committee of the Utah Section, consisting of O. W. Israelsen, *Chairman*, R. B. Ketchum, Richard R. Lyman, and R. K. Brown, Members, Am. Soc. C. E., and George D. Clyde, Assoc. M. Am. Soc. C. E.

Josephine (Richards) West. His father, with whom he worked as a youth, was a very successful civil engineer.

Ray Benedict West was graduated from the Utah State Agricultural College, at Logan, Utah, with the degree of Bachelor of Science in Civil Engineering, in 1904. He went to Cornell University, at Ithaca, N. Y., from which he received the degree of Civil Engineer, in 1906. In 1928 and 1929, he did graduate work at the University of California, at Berkeley, Calif.

Following his graduation from Cornell University, Mr. West practised his profession in the field of railroad location and construction, working at various times with the Eureka Railroad, the Sumter Valley Railroad, and the Oregon Short Line Railroad Companies. Later, he and his brother, Mr Joseph W. West, engaged in the private practice of engineering at Portland, Ore.

In 1913, he was made Professor of Agricultural Engineering at the Utah State Agricultural College, at Logan, and, in 1916, Dean of the Schools of Agricultural Engineering and Mechanic Arts. In 1927, he was made Dean of the newly created School of Engineering, which position he held at the time of his death.

While at the Utah State Agricultural College, Dean West had charge of the operation and maintenance of the State Power Plant, which produced and distributed electric power for some of the public institutions of the State; he designed and supervised the construction of the Athletic Stadium and also the Amphitheatre, two remarkably beautiful and well-built structures. He also designed and supervised the construction of a new water system for the College. He was active and influential in all phases of the College building program, including the making of plans and selection of locations for major campus buildings. He had general supervision of the design and construction of the Livestock Building, which was completed in 1917; the Engineering Building, and the Plant Industry Building, in 1919; the Library, in 1930; and, finally, the Home Economics-Commons Building completed in 1935.

Dean West consecrated his time, his energies, his vision, and his judgment to the educational advancement, the development, and the building of the Utah State Agricultural College continuously for twenty-one years. During the World War he had charge of the soldier training work at the College and after the war, he directed the training of disabled soldiers. In the administrative councils of the College, his sound judgment and knowledge of practical affairs were in frequent demand. For many years, he was Chairman of the College Athletic Council.

He was a Fellow of the American Society for the Advancement of Science and a member of the following societies: The American Society of Agricultural Engineers; American Association of Engineers; Society for the Promotion of Engineering Education; and Phi Kappa Phi Scholastic Fraternity.

During 1934 and 1935 Dean West was a member of the State Planning Board of Utah, and, in 1936, he was appointed by the Governor of the State as the Director of this important Board. It was while thus engaged, and on leave of absence from the College, that he was stricken with pneumonia and passed away within a week.

The School of Engineering at the Utah State Agricultural College grew at a rapid rate under his direction. The course of study was strengthened, and the Faculty enlarged. The graduates of the School, who were trained under his direction, have made remarkably fine records, particularly in the fields of irrigation and highway construction. Dean West was not only an excellent teacher and a fine school administrator, but a generous, public-spirited citizen. He was active in the Kiwanis Club of Logan, was Chairman of important committees of the Chamber of Commerce, and, for four years, acted as Bishop in one of the leading Logda wards of the Latter Day Saints Church.

Many of the leaders of the State in religious, educational, and civic matters, including the Governor, paid high tribute to his great worth. He was probably best known, however, for his kindly, pleasing, and genuine personal qualities, which endeared him to his fellow workers and to the many students who came within his influence and to whom he was always a great inspiration.

A fitting tribute and appreciation of his character and accomplishments was prepared by the Faculty and the Board of Trustees of the Utah State Agricultural College. Quoting from this splendid tribute:

> "He created, with the help of a faculty whose ideals coincided lovingly with his, an Engineering School * * * which today has no superior in the West. From it have gone, as graduates bearing the stamp of their association with Dean West, approximately one hundred and fifty engineers who, as leaders in this western commonwealth, are a significant monument to his name not only as a competent administrator, but also as a great teacher.
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> "Nor was his influence as a builder of men limited to members of the Engineering School. As chairman of the Athletic Council for a number of years, he ushered in a new era for competitive sports at the College. As a worker on the Attendance and Scholarship Committee, on the Social Affairs Committee, as a willing and wise adviser to all who sought his help, he became a veritable shaper and saver of youth.
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> "Through his multitudinous activities, Dean West came into closer contact with all departments of the College than did any of his colleagues. He was a useful man who won the esteem of all. All shared the fruits of his productive career. For this we offer our gratitude. In his passing we shall experience an irreparable loss.
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> "The Nation was quick to see his worth during the World War, during which he bore a large part of the responsibility of managing the soldiers assigned to the College for training, a duty meritoriously performed. * * *
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> "His wisdom, his willingness, his practical grasp and understanding of the problems of the hour made him a public servant of such value to the State and Nation that the vacancy caused by his death will be difficult to fill."

He was married on September 20, 1905, to Mary S. Morrell, of Logan, Utah, who with five sons, Ray B., Jr., Allen M., Philip L., Willard J., and Robert E., and one daughter, Mary A., survives him.

Dean West was elected a Member of the American Society of Civil Engineers on August 27, 1928.

ARTHUR CHAMBERS WHEELER, M. Am. Soc. C. E.[1]

DIED JUNE 3, 1935

Arthur Chambers Wheeler was born at Detroit, Mich., on November 17, 1880, the son of the late Levi Lockwood Wheeler, M. Am. Soc. C. E., and Isabel (Chambers) Wheeler. He was graduated from the University of Michigan, at Ann Arbor, Mich., in 1903, with the degree of Bachelor of Science in Civil Engineering.

Most of his life was spent in an engineering atmosphere as his father, "Captain" Wheeler, until his death in 1927, was employed on river and harbor improvement work by the United States Corps of Engineers, on the Great Lakes, the Mississippi River, and the Hennepin Canal in Illinois.

After his graduation from the University of Michigan, Mr. Wheeler was employed on Hennepin Canal and Illinois River surveys and improvement work for three years, when he accepted a position with the Philippine Commission and completed various engineering projects in the Philippine Islands, including the sub-division of the City of Baguio, until 1907 when he returned to the Territory of Hawaii.

In Hawaii, Mr. Wheeler served as Assistant Engineer, United States War Department; Assistant Superintendent of the Territorial Department of Public Works; County Engineer for the County of Hawaii; and the Hawaiian Contracting Company. For several years prior to his death he practiced privately as a Consulting Engineer and Surveyor.

He was married on June 23, 1914, at Honolulu, to Harriet E. Grant. Mrs. Wheeler died in 1922. Surviving him is one son, Lee Grant, Jr.; a brother, Frank D. Wheeler; and two sisters, Mrs. William W. Clingan and Mabel Wheeler.

Mr. Wheeler was a member of the Masonic Fraternity, of the Benevolent and Protective Order of Elks, and of the Hawaiian Engineering Association.

He was a man of quiet and pleasing personality, slow to anger, and with a tendency to make few but loyal and lasting friendships. His engineering work was always marked with a fine degree of accurate detail and completeness.

Mr. Wheeler was elected an Associate Member of the American Society of Civil Engineers on October 7, 1908, and a Member on April 12, 1926.

FRANK ORMOND WHITNEY, M. Am. Soc. C. E.[2]

DIED MAY 13, 1936

Frank Ormond Whitney, the only child of Jonas and Elizabeth Corey (Rice) Whitney, was born at Fitchburg, Mass., on July 21, 1851. He was a direct descendant of John and Eleanor Whitney who settled in Watertown, Mass., in 1635, and seven of his direct ancestors served with the Colonial forces in the Revolutionary War. He was educated in the public schools of Fitchburg, being graduated from the High School in 1868, following which he entered the Worcester Polytechnic Institute, at Worcester, Mass., which was established in that year, and was then known as the Worcester Technical School. He was graduated from the Institute in 1871, in the first class of seventeen members, with the degree of Bachelor of Science.

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[1] Memoir prepared by G. K. Larrison, M. Am. Soc. C. E.

[2] Memoir prepared by Frank E. Winsor, M. Am. Soc. C. E.

Mr. Whitney was employed in the City Engineer's Office in Fitchburg during the summers of 1869 and 1870, and was with the Fitchburg Water Department in 1871. He entered the employ of the City of Boston, Mass., in the Surveying and Engineering Departments on December 4, 1871, and remained in that service until February 1, 1924, when he retired. He was Chief Engineer of the Street Laying Out Department for twenty-five years, during which period he had charge of many important projects by which the street system of old Boston was transformed from that of the "horse and buggy" age to meet the demands of present-day traffic. City improvements aggregating more than $50 000 000 were constructed under his direction. After his retirement at the age of seventy-three, Mr. Whitney continued to be active, established a consulting practice, and devoted his time to a great variety of other interests.

He became a member of the Boston Society of Civil Engineers in 1879, was a valued contributor to its publications, and served as a Director in 1894 and 1895 and as Treasurer from March, 1915 to March 1931, when he refused a renomination.

For thirty-five years, he was a Director of the Merchants Cooperative Bank, of Boston, served as Second Vice-President in 1919 and 1920, as First Vice-President from 1921 to 1933, and as President from 1933 to the time of his death. He was also a Trustee of the Eliot Savings Bank, of Boston. He was a member of the Walnut Avenue Congregational Church, of Roxbury, Mass., and served as Superintendent of the Sunday School, as a Deacon, and as Chairman of its Prudential Committee. Among other affiliations, Mr. Whitney was a member of Aberdour Lodge, A. F. and A. M., and of the Sons of the American Revolution. He was a devoted alumnus of his Alma Mater, rarely missing an Alumni Meeting in Boston or Worcester, and left a substantial bequest to the Institute. A week before his death, he was informed that the Board of Trustees had voted to confer upon him the honorary degree of Doctor of Engineering.

On October 26, 1881, he was married to Anna Myrick Snow, of Boston, who died on September 6, 1917. His only child, Franklin Snow Whitney, was born on June 4, 1883, was graduated from Harvard University in 1906, and died on February 23, 1924.

Although Mr. Whitney outlived most of his contemporaries, he retained his youthful vision, keeping in contact with new developments and with younger associates, and carrying on his varied interests with characteristic energy even to the end. He leaves many warm friends who will miss his genial presence and wise counsel.

Mr. Whitney was elected a Junior of the American Society of Civil Engineers on May 3, 1876, and a Member on January 5, 1887.

CHARLES VICTOR WITT, M. Am. Soc. C. E.[1]

DIED DECEMBER 6, 1936

Charles Victor Witt was born on September 25, 1877, in Gleiwitz, Germany, where he attended the public schools and, later, the High School. In 1897, he entered the Royal Technical Institute, in Dresden, from which he was graduated in 1900.

From 1900 to 1901, he was employed by Rietschel and Henneberg, specialists on large central heating systems. In 1901, Mr. Witt came to the United States and found immediate employment with the Carnegie Steel Company, at Pittsburgh, Pa., as Special Engineer for the development of machinery and processes. In 1902, he entered the employ of the Westinghouse Electric and Manufacturing Company, at East Pittsburgh, Pa., under the Engineer of Works who had charge of the arrangement of machinery, the development and maintenance of the power plant, as well as the buildings and grounds. Mr. Witt became Assistant Engineer of Works and, eventually, had full charge of this Office until 1906.

In 1906, he took the position of Manager with James McNeill and Brother Company, steel plate construction fabricators, at Pittsburgh, and shortly was made General Manager, Vice-President, and Director. Mr. Witt had full charge of the office, works, and Construction Department; became an expert in steel-plate construction; and directed the production of large steel pipe lines for municipal water supplies for many of the largest cities in the United States. He was with this firm for a period of twenty years, until 1926, and during this time had become acquainted with—and was friend of—a large circle of water-works engineers and city officials throughout the United States.

In 1926, Mr. Witt founded the Witt Steel Company, later the Witt-Humphrey Steel Company, of Greensburg, Pa. He was President of this Company and continued in the same line of steel-plate work, including a large water supply line for the City of Philadelphia, Pa. He resigned, however, from this Company in 1931, when a position as General Manager and Chief Engineer in the Steel Watermains Association, also of Greensburg, Pa., was offered to him. With a number of the largest steel pipe manufacturers of the East as members of this Association, it was organized with a view to collecting all available information and data on municipal water supply lines for ready reference, not only to the individual members of the Association but also to the water-works engineers, city officials, and the general public. Because of his expert knowledge in this line and his large circle of acquaintances among water-works engineers, Mr. Witt was chosen to head this Association. He undertook this work with his usual energy, organized the office, and made necessary and desirable contacts. In 1935, however, the new concern fell a victim to the effects of the depression and was closed. Mr. Witt's health having become seriously impaired, he finally decided in 1935 to take a trip abroad. He stayed at one of the German health resorts and returned considerably improved, but planned a complete rest. However, a heart trouble from which he had suffered, unfortunately returned, and caused his death at the comparatively early age of 59 years.

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[1] Memoir prepared by C. F. W. Rys, Esq., Pittsburgh, Pa.

Mr. Witt was a member of the Engineers Society of Western Pennsylvania and of the German-American Technological Society.

For those who knew him, it is easy to understand why he made friends wherever he went, because in addition to a clear constructive brain, he possessed a good humor and a warm heart ready to help those who needed help.

Mr. Witt was elected a Member of the American Society of Civil Engineers on April 18, 1927.

HANS HERMANN WOLFF, M. Am. Soc. C. E.[1]

DIED OCTOBER 1, 1935

Hans Hermann Wolff, the son of Norbert and Clementine Wolff, was born in New York, N. Y., on August 12, 1877. His early education was secured in private schools, after which he entered the Engineering School of Columbia University, from which he was graduated in 1898 with the degree of Civil Engineer.

Mr. Wolff's first employment after graduation was as a Rodman for the Third Avenue Railroad Company, of New York City. He then went to Ecuador where he spent two years as Leveler, Transitman, and Topographer on the Guayaquil and Quito Railroad. On his return to the United States, he spent a short period in the Engineering Office of the Kansas City Southern Railroad Company and then went to Cape Girardeau, Mo., where he was in charge of construction on the St. Louis and San Francisco Railroad, which was then being built from St. Louis, Mo., to Memphis, Tenn. He was afterward employed for several years in railroad, bridge, and masonry construction in Pennsylvania and New York State for the New York Central, Buffalo and Susquehanna, and the New York, Pennsylvania, and Southern Railroad Companies.

As a member of the contracting firms of Allen and Wolff and Allen, Connolly, and Wolff, Mr. Wolff had charge of sewer construction in Newton, N. Y., and the building of five miles of the Delaware and Eastern Railroad, near East Branch, N. Y.

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[1] Memoir prepared by Gustavus Sessinghaus, Esq., Denver, Colo.

He served as District Engineer in the State of Washington during the construction of the Chicago, Milwaukee, and Puget Sound Railroad. After the completion of this work, he established an office in Seattle, Wash., and was a member of the firm of Cross and Wolff. He was also Northwestern Representative of Hildreth and Company.

After the United States entered the World War, Mr. Wolff joined the Engineer Officers Training School and was promoted to the rank of Captain in the Engineer Reserve. He was assigned as Commanding Officer of the 556th Engineers Service Battalion, at Camp Humphreys, Virginia. This battalion was soon disbanded, and Captain Wolff was attached to the 353d Engineers Service Battalion. He was discharged from service on December 12, 1918. In 1932, he was given a certificate of capacity for promotion to the rank of Major, Engineer Reserves. He had also served as Regimental Adjutant of the 328th Engineers.

Following the war, he spent some time in New York City, giving attention to the business which his father had established. He then took up his residence at Denver, Colo., became interested in oil and mining, and spent a year at the Colorado School of Mines, at Golden, taking courses in Geology. For several years Mr. Wolff was engaged in handling oil leases, but returned to New York City in February, 1932, expecting to remain there but a short time; business matters, however, prolonged his stay until 1935.

In the spring of 1935 he had to undergo an operation for appendicitis from which he never fully recovered. His death occurred on October 1, 1935, from angina pectoris. He was buried on October 5, 1935, with military honors, in Arlington National Cemetery, at Fort Myer, Virginia. He is survived by his widow, the former Frances A. Babcock, of Kansas City, Mo., to whom he was married on October 25, 1915.

He was a member of the Colorado Scientific Society, of Denver; the Army and Navy Club, of Washington, D. C.; Arcade Lodge, F. & A. M., of Arcade, N. Y.; Denver Chapter No. 2, R. A. M., Denver; Colorado Commandery No. 1, Knights Templars; Columbia University Alumni Association; and the Reserve Officers Association of the United States.

Mr. Wolff was a capable engineer and a fine type of citizen. As a youth he had instruction in the violin, became an accomplished performer, and displayed a taste for artistic and cultural things. As a student of history, he understood well the causes of the rise and fall of the different peoples. He argued zealously for the preservation of the rights, liberties, and prerogatives of the American citizen. He protested against any manifestation of intolerance. He kept up his military training and preserved his interest in military matters. With a wide circle of friends and acquaintances, his association with them was of a most wholesome nature. Modest and unassuming, life meant much to him; he enjoyed living; his passing was untimely, and he will be missed by those who knew him.

Mr. Wolff was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908, and a Member on September 5, 1911.

BRUCE CLINTON YATES, M. Am. Soc. C. E.[1]

DIED AUGUST 10, 1936

Bruce Clinton Yates was born on a farm near Grafton, Taylor County, W. Va., on September 10, 1869. His parents came to Grafton (then in Virginia) soon after the first settlers arrived there in 1852, and engaged in farming near the Tygart River. When the boy was twelve years old, his parents moved with their family of four boys and one girl to Lancaster County, Nebraska, settling on a farm near Emerald, seven miles from Lincoln, at that time the new Capital of the State.

He attended a country school, and studied two years in preparation for entrance to the University of Nebraska, at Lincoln, successfully passing the entrance examinations in 1888. Pursuing the course in Civil Engineering, he was graduated in 1892, with the degree of Bachelor of Civil Engineering.

The Engineering Department of the Burlington and Missouri River Railroad Company, in Nebraska (later, the "lines west of the Missouri River," of the Chicago, Burlington and Quincy Railroad Company) was located in Lincoln, and was busy with the construction of the line to Billings, Mont. Like many another engineering student of the State University at Lincoln, Mr. Yates found service during three summer vacations on this railroad construction in Wyoming and Montana. From June, 1892, he continued in such work until October, 1894, when the line was completed to Billings, and the forces were reduced, due to the general depression in business. As Instrumentman and Computer on grading construction, Assistant on track-laying, Inspector on pile-driving, and Division Engineer on construction, his experience was varied and extensive.

From October, 1894, to February, 1895, Mr. Yates (with others released by the Burlington and Missouri River Railroad Company) was with the Butte, Anaconda and Pacific Railroad Company on location in the Rocky Mountain region in Montana. From April to November, 1895, he was again on construction work with the Burlington and Missouri River Railroad Company, this time on the Spearfish Line in the Northern Black Hills, in South Dakota, where he came in contact with conditions surrounding the Homestake Mine and with mining in the gold fields of the Black Hills.

From November 1895, to October, 1897, Mr. Yates was in general engineering practice at Deadwood, S. Dak., in partnership with Mr. Frank S. Peck, largely on Government survey contracts and mine surveys.

On October 1, 1897, he began his life work with the Homestake Mine, as a member of its Engineering Staff. Thereafter, he was promoted through the positions of Assistant Engineer, Chief Engineer, Assistant Manager, and General Manager. As Chief Engineer, he devised the system of mining by shrinkage stopes which displaced the square sets of earlier days; as General Manager, during the period of Homestake's great growth and modernization, he directed and co-ordinated these plans. Increasingly, as the years passed, his merits were appreciated throughout the State, and during these recent years of depression, he was frequently called to the State's service.

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[1] Memoir prepared by F. T. Darrow, M. Am. Soc. C. E.

In addition to his duties as General Manager of the Homestake Mining Company, Mr. Yates also had charge of the Wyodak Coal Mine, at Gillette, Wyo.—a strip mine operating a 90-ft vein of lignite coal.

The Homestake Mine is an old mine. As a producer of gold from low-grade ores, it has been worked profitably for sixty years. It is the leading industry of the Black Hills in mines and lumbering, and is one of the largest gold mines in the world. It owns most of the Town of Lead, S. Dak. Three generations of many families have grown up about it and found work with it.

Since January 1, 1918, Mr. Yates, as General Manager with headquarters at Lead, had been its operating head. As such he had carried on the Homestake policy of friendly co-operation with all interests of the community and a helpful and liberal policy with the employees.

A quality of humanness endeared Mr. Yates to every one with whom he came in contact. At heart, he had an abiding interest in the welfare, not only of Company employees, but of all townspeople, and the industrial peace of Lead as well as the staunch loyalty of Homestake employees to Mr. Yates were substantial evidences of his greatness.

He never regarded another man's difficulties as too trivial to concern him. Employees had implicit faith in his sense of fairness. Those who came in contact with him along lines of charitable enterprises, always were surprised at his knowledge of unfortunate townsfolk.

It is recalled that during the influenza scourge of 1918, which turned the Homestake Recreational Building into a community hospital, Mr. Yates made a personal inspection every day. That incident was characteristic of his kindly interest in the townspeople. Countless incidents of small personal charities and kindlinesses not generally known to the public could be cited.

Mr. Yates was a member of many societies, among which were the American Institute of Mining and Metallurgical Engineers, the American Mining Congress, and the National Safety Council, of which he had served as Chairman of the Mining Section. He introduced safety measures in mine work as well as methods which have been fully proved and adopted generally in such work. He also held the Honorary Degree of Engineer of Mines from the South Dakota School of Mines, at Rapid City.

He was married on November 21, 1896, to Ada Myers, of Beatrice, Nebr., who died in Lead, on May 26, 1928. He was married, in 1929, to Carmen Martin, of Iron Mountain, Mich. He is survived by his widow, Mrs. Carmen Yates; a sister, Gertrude Yates; and five children, Arthur, George C., Mrs. Mary Cadwallader; and Mrs. A. D. Bell. Another son, Bruce C., Jr., died in Berkeley, Calif., on November 17, 1936.

His death was due to an attack of coronary thrombosis. Over-exertion and loss of rest on preceding days, when a forest fire was threatening the lumber camp at Moskee, Wyo., may have contributed to his death.

Mr. Allan J. Clark, Chief Metallurgist of the Homestake Mining Company, writes[2] of Mr. Yates as follows:

> "But it was not alone his friendliness that made Yates so readily accessible to all, of whatever station in life. It was his humanness, his ready sympathy with the need of others. Some months ago, he was lightly told he might expect an early visit from one of the chronically aggrieved, and the suggestion was made that he should spare himself from many such visits. His reply, to the effect that even if he gave no help, the visitor felt better for having told his troubles, was so typical of his consideration that it merits setting down here."

Mr. Yates was elected an Associate Member of the American Society of Civil Engineers on June 7, 1905, and a Member on October 15, 1923.

AARON STANTON ZINN, M. Am. Soc. C. E.[1]

DIED MAY 7, 1936

On Saturday afternoon, May 9, 1936, in a little cemetery adjacent to a country churchyard, a few miles south of Deer Creek, Ind., there was laid to rest an Engineer who had participated in an important capacity on one of the nation's greatest engineering projects—the Panama Canal.

As the writer stood under the shade of a big cedar tree and looked over the fence at a farmer furrowing, there came to his mind the first verse of Gray's "Elegy Written in a Country Churchyard":

"The curfew tolls the knell of parting day,
The lowing herd winds slowly o'er the lea,
The ploughman homeward plods his weary way,
And leaves the world to darkness and to me."

And that other verse, which Wolfe quoted the night before he took Quebec,

"The boast of heraldry, the pomp of pow'r,
And all that beauty, all that wealth e'er gave,
Await alike the inevitable hour.
The paths of glory lead but to the grave."

There was a striking contrast to the quiet of the eternal home of Aaron Stanton Zinn with the surroundings of his home for many years at Empire, Canal Zone, just on the edge of the Culebra or Gaillard Cut, where one could look down on myriads of steam shovels, steaming construction trains, and hear the thunder of the dynamite roaring the removal of that rock cut.

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[2] *Mining and Metallurgy*, October, 1936.

[1] Memoir prepared by Alonzo J. Hammond, Past-President, Am. Soc. C. E.

Aaron Stanton Zinn, the son of George and Eliza (Nutt) Zinn, was born near Deer Creek, Ind., on August 26, 1862. His great-grandparent on his father's side came from Germany in 1749 and settled in Pennsylvania; and on the mother's side from Switzerland, also settling in Pennsylvania. Each side had a son, Jacob, in the Revolutionary War.

Jacob Zinn, the grandfather, moved to Carroll County, Indiana, in 1851, locating on a 480-acre farm near Deer Creek, later giving the land needed for the cemetery and the United Brethren Church. In this little cemetery, there lie thirty-two Zinn relatives, and now Aaron Stanton Zinn is with them.

George Zinn, the father of Stanton, in 1856, settled on a 160-acre farm near that of Jacob, the grandfather, and in 1865 the family moved to Logansport, Ind. Stanton Zinn was a twin in a family of eight children, four boys and four girls.

Mr. Zinn was educated in the public schools of Logansport, was graduated from the High School in 1884, and, later, took a 3-yr course in Civil Engineering at Rose Polytechnic Institute, at Terre Haute, Ind.

Leaving college in June, 1887, he entered the employ, as Draftsman, of the Atchison, Topeka and Santa Fé Railway Company, following which he was connected, successively, with the Illinois Central Railway Company; the Harney Peak Tin Mining Company; the Pennsylvania Railroad Company; and the Elgin, Joliet and Eastern Railway Company (Chicago Outer Belt Line), designing shops at Joliet, Ill., for the last-named line.

Mr. Zinn then entered politics and was elected County Surveyor of Will County, at Joliet, and from 1892 to 1895 he also carried on a general engineering business in partnership with Mr. O. R. Rauchfuss, a former classmate.

From 1895 to 1900, he was with the Vandalia Railway Company as Assistant Engineer, Maintenance of Way, at Terre Haute, in charge of construction surveys, spiraling curves, and the location and construction of more than 100 miles of track to coal mines, etc.

An important event occurred in June, 1897, when he was married to Mabel Gray Cooper, of Marquette, Mich., a charming, vivacious girl whose restless energy was actuating in the life partnership which resulted. From this union there was one child, a son, Kenyon Cooper Zinn.

From 1900 to 1904, Mr. Zinn was Assistant Engineer on track elevation for the Chicago and Western Indiana Railway Company, at Chicago, Ill. During 1904 and 1905 he was with the Rock Island Railway Company, first as Division Engineer of the Oklahoma Division, and, later, as Principal Assistant Engineer, at Oklahoma City, Okla., in charge of the maintenance and construction of about 1 800 miles of road.

After another engagement with the Chicago and Western Indiana Railway Company, for which he designed a 52-stall elliptical engine-house, he took charge, as Construction Engineer, for the Michigan Central Railway Company, at St. Thomas, Ont., Canada, of the design and construction of 95 miles of second main track. While on this work in October, 1906, he was called to the Panama Canal by John F. Stevens, Past-President and Hon. M. Am. Soc. C. E., Chief Engineer, and appointed Resident Engineer on the

Culebra, or Central, Division (32 miles) where he remained for seven and one-half years.

As is well known, the work on the Culebra Cut was a transportation problem, so that the railway experience of Mr. Zinn, which had attracted the attention of Mr. Stevens when he also was with the Rock Island Railway Company as Vice-President, was of vital importance in the handling of the railway problems involved, and Mr. Zinn's services, during the latter part of Mr. Steven's term, no doubt assisted materially in the placing into operation of that effective railroad system which produced the economies of operation for the Culebra Cut.

When the late Col. (afterward Maj. Gen.) George Washington Goethals, U. S. Army, M. Am. Soc. C. E., became Chief Engineer and Col. David St. Pierre Gaillard, U. S. Army, Division Engineer of the Central Division, Mr. Zinn acted as Division Engineer during the illness of Colonel Gaillard. When he decided in 1914 to leave the Canal, he was recommended by Colonel Goethals to the President of Panama for the position of Consulting Engineer and took charge of the building of the railway from Pedregal to Boquette, or from the coast to the coffee ranches. He spent two and one-half years on engineering work for the Republic of Panama, the opening of the railroad being attended by much pomp and ceremony, with President Belisaro Porras of Panama attending.

During 1917 and 1918, Mr. Zinn was engaged in war work, first as Assistant General Superintendent of Construction for Camp Sherman (Chillicothe, Ohio, Cantonment) and, later, with the New York Nitrates Corporation, in New York, N. Y. From 1919 to 1923 he was with the Missouri Pacific Railroad Company at St. Louis, Mo., most of the time, in the Valuation Department.

In April, 1923, Mr. Zinn removed to Los Angeles, Calif., and took a position as Chief Draftsman on the Harbor Plans for Los Angeles, for the Chamber of Commerce, continuing in this work until 1925.

From 1925 until his death, with the exception of a leave of absence on account of ill health, he was connected with Los Angeles County, engaged principally in the designing of sewers. His illness became serious when on a visit to his sister, at South Bend, Ind.; perhaps the All-Wise Providence arranged it so he could be laid in his last resting place, as provided by his forefathers. He is survived by his widow and his son.

Mr. Zinn held memberships in the Western Society of Engineers and the American Railway Engineering Association, and had contributed articles to the technical press, as well as to the publications of the Western Society of Engineers, covering the construction of the Central Division, Panama Canal, and the Chiriqui Railway for the Republic of Panama, the latter a most interesting presentation of engineering work in a tropical country with native labor.

He was a man of sterling character; his friendship was genuine and complete; his praise for those he admired was without stint, and he had the confidence and regard of all his associates, as instanced decidedly by his contacts on the Panama Canal.

Mr. Zinn was elected a Member of the American Society of Civil Engineers on October 5, 1909.

ALBERT READ BAKER, Assoc. M. Am. Soc. C. E.[1]

DIED SEPTEMBER 7, 1936

Albert Read Baker was born at Norwalk, Ohio, on May 28, 1882, the son of Daniel Albert and Arabella (Benson) Baker. His family moved to California in 1894, and he received his early education in the public and High Schools of San Diego, and his technical education at the University of California, at Berkeley, Calif.

After leaving college in 1905, Mr. Baker's first work was in connection with the construction of the Stanislaus River hydro-electric power development on which he spent four years. This engagement was followed by two years as an Assistant Engineer on the reconstruction of the sewerage system and the high-pressure fire-protection system of San Francisco, Calif., which was undertaken as an aftermath of the 1906 earthquake and fire. Following this work, he spent a few months with the Snow Mountain Water and Power Company as Assistant Engineer on penstock construction, and nearly a year in British Columbia as Office Engineer in connection with the Sooke Water Supply for the City of Victoria.

In November, 1912, Mr. Baker returned to California to become Chief Engineer of the Marin Municipal District organized to provide a water supply for about ten or twelve towns and adjacent territory in Marin County, across the Golden Gate from San Francisco—a task which was to absorb his best efforts for more than six years, and the successful consummation of which blazed the way for a number of similar developments throughout the State. The principal physical features of the project were the Alpine Dam, a gravity concrete structure, 100 ft in height; the Pine Mountain Tunnel, 8 ft in diameter and 8 700 ft long; and seven miles of pipe line, with the necessary roads and appurtenant structures. The construction of these works, however, was preceded by a task of even greater magnitude, namely, the development, in the face of strenuous opposition, of the finally approved plan for the project, the overcoming of legal and political obstacles, and the final campaign resulting in the approval by the electorate of a $3 000 000 bond issue for construction purposes. All this work was carried on under the inspiring leadership of the late M. M. O'Shaughnessy, M. Am. Soc. C. E., Consulting Engineer for the project, and a member of its Board of Directors, but its success may be ascribed, in no small measure, to Mr. Baker's firm persistency of purpose and undeviating courage in the face of seemingly insurmountable obstacles, as well as to Mr. O'Shaughnessy's engineering vision, unfailing optimism, and fighting spirit.

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[1] Memoir prepared by Charles A. Bissell, M. Am. Soc. C. E.

On the completion of this work late in 1919, Mr. Baker was for a year with the Southern California Edison Company, in charge of the diamond drill exploration of the foundations of the Shaver Lake Dam, followed by work with the Pacific Gas and Electric Company on the construction of the Spring Gap Power House on the Stanislaus River. During 1922 and 1923, he was in charge of all construction work at Early Intake, on the Hetch Hetchy water supply project for the City of San Francisco, including the intake section of the Hetch Hetchy Tunnel and the Early Intake diversion dam.

After a year spent in valuation work for the East Bay Municipal Utilities District, Mr. Baker, in June, 1925, became connected with the construction of the Oakland-Alameda Estuary tube, a vehicular under-pass connecting the East Bay cities of Oakland and Alameda, Calif. His first work in this connection was as Engineer for Robinson, Roberts, and Rohl, the Sub-Contractor for the Approach and Tunnel Section at the Oakland end. Later, he served as Engineer and Assistant Superintendent for the General Contractor, the California Bridge and Tunnel Company, on the construction of the twelve pre-cast reinforced concrete sections, each 203 ft in length, and having an interior diameter of 32 ft, which were constructed in the dry dock at Hunters Point, in San Francisco, and one after another floated across the Bay and sunk to line and grade to form the subaqueous section of the tube. An interesting and comprehensive paper by Mr. Baker on the construction of this tube was published in the technical press.[3]

In September, 1929, Mr. Baker joined the Engineering Staff of The Metropolitan Water District of Southern California, an organization of municipalities formed for the purpose of bringing Colorado River water across the State of California, over 250 miles of desert and mountain, to provide a supplemental domestic water supply for the population of the Southern California coastal plain. At this time preliminary surveys had largely been completed, the selection of the route for the 1 500 sec-ft aqueduct had been practically determined, and one of the immediate tasks was the development of the details of plans on a self-liquidating basis for financing the project and the repayment of its cost, which was estimated to be in excess of $200 000 000. Into these financial studies for which his earlier experience with the Marin Municipal District had given him valuable training, Mr. Baker plunged with characteristic energy and enthusiasm. During the two-year period preceding the bond election of September 29, 1931, at which the issue was carried by a 5 to 1 majority, and subsequent thereto, he was responsible for the production of a large volume of financial studies of great importance in establishing the feasibility and sound financial basis of the enterprise.

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[3] *Engineering* (London, England), September 26 and October 10, 1930.

As construction on this project got under way (the work was begun almost simultaneously at numerous points in the effort to relieve the prevalent unemployment crisis), Mr. Baker was given the responsibility as Materials Co-Ordinator for planning and expediting the inspection and delivery of construction materials for the entire aqueduct and distribution system work, both contract and force account. It was in the midst of this work, in the prime of his life, that after a brief illness he passed away on September 7, 1936.

Mr. Baker possessed a pleasing personality and high character. He was thoroughly reliable and possessed a fine balanced judgment. He was unusually co-operative in working with others. Mr. Baker lived simply with his family to whom he was exceptionally devoted.

On March 12, 1913, he was married to Ivy Hayward, who, with one son, John Raymond, and two daughters, Marjorie Hayward and Katharine Read, survives him.

Mr. Baker was elected an Associate Member of the American Society of Civil Engineers on October 1, 1913.

LeROY WRIGHT BARBOUR, Assoc. M. Am. Soc. C. E.[1]

Died September 14, 1936

LeRoy Wright Barbour was born in Greenwich, N. Y., on October 22, 1882, the son of Sidney Matthew and Elizabeth Remington (Wright) Barbour, whose ancestors came to America in the Seventeenth Century.

In 1888, Mr. Barbour's parents moved to California, taking up their residence at La Manda Park, near Los Angeles. Shortly thereafter the family settled at Corona in Riverside County, California. Here, Mr. Barbour attended the grade school, and finished his college preparatory work at the Palo Alto High School. In the fall of 1903, he enrolled at Stanford University, taking up the course in Mining and Metallurgy. This line of study did not exactly appeal to him and he finally decided upon a Civil Engineering course, receiving his degree in 1909.

Immediately after his graduation, Mr. Barbour obtained employment with the Banco Del Oro Mining Company, at Magdalena, Sonora, Mexico. This work was in charge of Horace Pomeroy, an experienced Mining Engineer and a former graduate of Stanford University. Here, Mr. Barbour acted as Assistant Engineer and Refiner and obtained much valuable experience.

The period from October, 1910, until July, 1913, he spent in making surveys on a water project for the City of Los Angeles, near Blyth, Calif., and acting as Resident Engineer for the Nevada, California Power Company, at Bishop, Calif.

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[1] Memoir prepared by E. D. Cole, Esq., Culver City, Calif.

During the following three years Mr. Barbour was employed as Construction Superintendent for the firm of Mahoney Bros., Allinson, and Cole, of San Francisco, Calif. This firm was engaged in building large earthen reservoirs for the storage of heavy crude oil. The inside area of these reservoirs was completely lined with a 3-in. slab of concrete. As it was essential that these slabs should be oil-proof and free from checks and cracks, it was imperative that great care should be exercised in their construction. The fact that these reservoirs are still in use to-day and have proved satisfactory in every way is history, and a large share of the credit is due to the painstaking care and the thoroughness with which Mr. Barbour directed this work.

Early in 1917, Mr. Barbour entered the employ of the Empire Gas and Fuel Company, at Bartlesville, Okla., where he acted as Resident Engineer of the Company's Gainesville Refinery. He remained with this Company until 1920, under Mr. J. J. Allinson, then Chief Engineer. During this time, he acted as Resident Engineer at the refinery at Cushing, Okla., and, later, at the Ponca City, Okla., plant.

About this time, Mr. Allinson resigned as Chief Engineer to take a similar position with the Pierce Oil Corporation, and he had become so favorably impressed with Mr. Barbour's work and ability that he decided to take him with him. Thus, it was that early in 1923, Mr. Barbour was established as Resident Engineer at the refinery at Tampico, Tamps, Mexico. From that time until 1927, he was, successively, Resident Engineer, Assistant Superintendent, and Superintendent of the plant.

In 1930, Mr. Barbour was made Manager of Production and Refineries for the Pierce Corporation, with headquarters at Tulsa, Okla. This position he held until the Pierce interests were acquired by the Sinclair Refining Company in 1931. He was then transferred to Fort Worth, Tex., as Superintendent of the Fort Worth and Gladewater Refineries. This position he held at the time of his death.

Mr. Barbour possessed a keen analytical mind and boundless energy. These qualities, in addition to an entirely likable nature, made him outstanding on any work that he undertook. "Roy," as he was familiarly known, was a natural leader of men and was highly respected by all and loved by those who knew him well. His zeal and intense interest in his work were unfortunately the contributing factors that led to his death. It was on one of his frequent tours of inspection around the refinery that he noticed that the workmen making an excavation were confused over some detail of the work. Mr. Barbour leaped down into the excavation and accidentally bruised his knee against an excavating tool. This bruise, although not considered serious at the time, eventually developed into sarcoma which, later, caused his death.

Mr. Barbour was married to Blanche Mankins, the daughter of Henry and Josephine Mankins, of San Luis Obispo, Calif., in 1915, and to this union four daughters, Josephine, Elizabeth, Mabel, and Billie, were born. Mrs. Barbour and his four daughters survive him. He is also survived by his father and mother and by one sister.

He was a Mason, a member of Delta Lodge No. 425 of Tulsa, Okla., and of the Cushing, Okla., Chapter No. 81, Royal Arch Masons.

He will perhaps be remembered best because of his infectious laugh that endeared him to many. He was essentially a man's man and a tried and true friend, and to those of us who knew him best he leaves a priceless heritage in that his life was an exemplification of the meaning of friendship in its truest sense.

Mr. Barbour was elected an Associate Member of the American Society of Civil Engineers on April 19, 1920.

ARTHUR TAYLOR BRAGONIER, Assoc. M. Am. Soc. C. E.[1]

Died June 29, 1936

Arthur Taylor Bragonier was born at Shepherdstown, W. Va., on August 20, 1890, the son of the late Joseph S. and Rosalie B. Bragonier. He was a descendant of an old Southern family, his grandfather having served as a Captain in Stonewall Jackson's Brigade during the Civil War.

He was graduated from Shepherd College and then studied at Washington and Lee University, at Lexington, Va., and at Lehigh University, at Bethlehem, Pa. He received the degree of Bachelor of Science in Civil Engineering from West Virginia University in 1916.

Mr. Bragonier began his professional career as Chainman for the Norfolk and Western Railway Company in 1912, and for nearly five years, from June, 1914, until December, 1919, he enjoyed a varied engineering career, as follows: Assistant to the County Road Engineer on road surveys in McDowell and Logan Counties, in West Virginia; this was followed by an engagement as Chainman, with some office work, with the Western Maryland Railway Company; Draftsman in the Valuation Department of the Norfolk and Western Railway Company; and in charge of a field party for the Baltimore and Ohio Railway Company. More than a year was spent as Transitman with The Koppers Company, of Pittsburgh, Pa., on the layout and checking of toluol recovery plants in Cleveland, Ohio, Washington, D. C., and New Orleans, La., and in charge of the construction of an 1 800-ton reinforced concrete coal bin at Norristown, Pa. In July and August, 1919, Mr. Bragonier served as County Road Engineer of Jackson County, West Virginia, on the preparation of Federal Aid maps and on the layout and construction of concrete roads; and from August to December, 1919, he was Assistant to the County Road Engineer of Doddridge County, West Virginia, on the layout and supervision of construction of a concrete road.

In January, 1920, he was appointed Instructor in Surveying in the College of Engineering at West Virginia University, which position he retained until

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[1] Memoir prepared by Professor R. P. Hron, Huntington, W. Va., assisted by Howard A. Levering, M. Am. Soc. C. E.

July, 1922, his summer vacations having been spent with the State Road Commission. In July and August, 1922, he was employed as Designer in the Division Engineer's Office of the State Road Commission, with headquarters at Morgantown, W. Va.

In recognition of Mr. Bragonier's achievements in the field of Highway Engineering, he was given the Detroit-Edison Fellowship in Highway Engineering, in August, 1922. This Fellowship permitted him to study at the University of Michigan, Ann Arbor, Mich., from which he received the degree of Master in Civil Engineering in 1923.

From June, 1923, to October 22, 1925, he served as Assistant Engineer with the West Virginia State Road Commission, Division No. 2, Huntington, W. Va., in charge of the construction of concrete and bituminous macadam road surface and on road design.

In September, 1925, Mr. Bragonier was appointed first as Instructor and then as Assistant Professor of Applied Mathematics at Marshall College, at Huntington, W. Va.; and, later, on account of his excellent teaching ability, he was made Associate Professor, in which position he continued until his death. At the time he was taken ill, Professor Bragonier was engaged in a thorough study of traffic accidents with a view to their prevention. He was greatly instrumental in the development of the two-year course in Engineering at Marshall College and a number of young engineers, now eminent in the profession, gained their basic engineering knowledge from him. In 1935, he was again honored when West Virginia University conferred on him the degree of Civil Engineer.

Professor Bragonier was a member of the American Society for the Promotion of Engineering Education, and the West Virginia Society of Professional Engineers. He was also a member of the Masonic Fraternity (Mt. Nebo Lodge No. 91, A. F. and A. M.; Macklenberg Chapter No. 31, Royal Arch Masons; and Potomac Commandery No. 5, Knights Templars). He was also a member of Trinity Protestant Episcopal Church, at Huntington.

He died at his home, in Huntington, after a short illness of bacterial endocarditis, on June 29, 1936, and was buried in the family plot at Elmwood Cemetery, at Shepherdstown, where four generations of his family are interred.

He is survived by his widow, Mrs. Dorothy (Berry) Bragonier, whose native home was Martinsburg, W. Va., and to whom he was married on June 10, 1924, and a brother Dr. Richard Keene Bragonier, of Keystone, W. Va. Mrs. Bragonier is also a teacher in the Teacher's Training School of Marshall College, at Huntington.

Professor Bragonier was an excellent teacher, and was intensely interested in his work. He always stood for excellence in scholarship, and had little patience with students who did not put forth their best efforts. He was a scholar by nature, and will be greatly missed by both the Faculty and the students of Marshall College. He taught so many engineering subjects that it will be difficult to find a man who can successfully take his place.

Professor Bragonier was elected an Associate Member of the American Society of Civil Engineers on February 23, 1932.

CHARLES WILLIS CHASSAING, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 1, 1934

Charles Willis Chassaing, the son of Joseph Henry and Marie Louise (Gratiot) Chassaing, was born in Gratiot, Wis., on December 1, 1872. He received his technical education at Washington University, in St. Louis, Mo., from which he was graduated in 1896 with the degree of Bachelor of Science.

From July, 1896, to March, 1898, Mr. Chassaing was Assistant Engineer in the Chief Engineer's Office of the Union Pacific Railway Company, on computations and plans for railway bridges; he also acted as Field Inspector during the erection of several railway bridges.

He was with the St. Louis Water Company, in the Extension Office, from June, 1898, to April, 1899, and assisted in plate-girder design, railway bridge design, and structural steel work for buildings.

In April, 1899, Mr. Chassaing entered the employ of The Union Iron and Foundry Company, of St. Louis, serving as Draftsman until about June, 1902, when he became Chief Draftsman with supervision of all shop drawings and of listing and ordering material. He also designed the steel work for several buildings, consisting of roof trusses, plate girders, columns, etc.

He served as Engineer with Eames and Young, Architects, of St. Louis, from April, 1907, to August, 1909. In this position he designed the steel work for the Crunden Branch Library and The David Rankin, Jr., School of Mechanical Trades, at St. Louis, and for hospital buildings for the Federal Penitentiaries, at Atlanta, Ga., and Leavenworth, Kans., and other buildings.

From August, 1909, to September 1910, Mr. Chassaing was engaged with E. J. Eckel and Company, Architects, at St. Joseph, Mo., as Engineer and Superintendent of Construction. As such, he designed steel-frame and reinforced concrete structures, and also supervised construction work in the field.

From September, 1910, until his death, he was in the employ of Selden-Breck Construction Company, General Contractors, of St. Louis, in the capacity of Structural Engineer and Estimator. His activities with this Company were many, including designing, quantity surveys, purchasing, and the administration of construction work. During the World War, he had charge of the surveying and laying out of the work in connection with the construction of both Camp Doniphan, at Fort Sill, Oklahoma, and Fort Benning, at Columbus, Ga.

Modest, a man of few words, conscientious, and of integrity beyond reproach, Mr. Chassaing gained the respect and admiration of all with whom he came in contact, and gave willingly to any one needing assistance He took great delight in solving technical problems in engineering.

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[1] Memoir prepared by Selden-Breck Construction Co., St. Louis, Mo., from information on file at the Headquarters of the Society.

Although he never married, he was fond of home life, where he spent the greater part of his leisure time in reading. His main recreation was hunting and trap-shooting.

Mr. Chassaing was elected an Associate Member of the American Society of Civil Engineers on September 6, 1910.

WILLIAM DRISCOLL, Assoc. M. Am. Soc. C. E.[1]

Died December 6, 1935

William Driscoll was born on May 23, 1870, at New Britain, Conn. He was the son of John and Mary (Halloran) Driscoll. His father and mother, natives of Ireland, came to the United States in early youth and were married in this country.

His early education started in the public schools of Greenfield, Mass., and was supplemented while a cadet-student by special courses in Civil Engineering at Norwich University, at Northfield, Vt. Although he was not graduated from the University, he often spoke with deep affection of his college days and of friends made at Norwich.

Mr. Driscoll's unusually broad and varied experience as a Locating and Construction Engineer carried him through many foreign countries. He blazed many trails through remote wilderness zones of Mexico, the West Indies, Central America, and Brazil, on railroad location and construction. He was also called to take responsible charge of hydro-electric power developments in Spain prior to the World War. On several assignments, after the projects were built, he was also retained as a Field Engineer on maintenance and operation.

Mr. Driscoll won an enviable reputation as an Engineer who could be trusted to undertake and carry on responsible assignments in the most difficult wilderness zones where the engineer must master all field problems, ranging from reconnaissance into location, and carry on through to the finish of construction. He could be depended upon to lay out his camps and to organize and direct the work of both skilled craftsmen and the primitive workers of the jungles and the mountains. He handled his field forces ably, with tact and discretion, and obtained results with minimum friction.

The best evidence of Mr. Driscoll's trustworthiness, constancy, and intelligent devotion to duty is shown by the fact that he was repeatedly recalled by his superiors. Often when completing one assignment in a foreign country, he received cabled offers simultaneously from previous associates inviting him to go to other countries where his former chiefs were inaugurating new projects. A list of the railroad, power, and general development projects on which Mr. Driscoll was employed, reads much as if it were a compilation of the more difficult feats of railroad and hydro-electric power engineering through the mountainous wilderness zones of Mexico, Central America, and South America, not to mention many interesting and difficult tasks completed in various parts of the West Indies.

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[1] Memoir prepared by J. A. Sargent, M. Am. Soc. C. E.

Soon after finishing an assignment for the Cuba Railroad Company, prior to the World War, he was called to Spain as Assistant Engineer, resident on the Lerida Weir Dam that was being built on the Segre River up stream from the ancient Town of Lerida. This was the location where, about 49 B. C., Julius Cæsar shortened and widened the river by cutting several parallel channels across the "short end of the horseshoe", following one of the notorious destructive floods of the Rio Segre. The whole story of this earlier engineering job is told in Caesar's Commentaries, covering the "Siege of Ilerda." During the construction of the present Lerida Dam on the identical site of Caesar's operations, William Driscoll obtained and read the leading works describing this job which had been started nearly twenty centuries before and which Mr. Driscoll helped to finish. It was a deep satisfaction to him that he was permitted to see the finish of such a project devoted to the beneficient use of water instead of strategic use in war.

A well-known General Manager of railroad construction and operation puts "in a nutshell" what many other friends and associates have stated in effect:

> "His work was always satisfactory. He was very methodical and constant. The training that Bill Driscoll received at the Military Academy in his earlier days stayed with him through life. This was shown by his exceptional orderliness."

Another friend and admirer remarked casually that it was always a pleasure, after a hard ride, to reach one of "Bill" Driscoll's camps. No matter in which jungle, forest, or mountain wilderness the trails crossed, his camps would be well located, as nearly like "Spotless Town" as painstaking care could make them. They were sanitary and supplied with as many of the "comforts of home" as ingenuity and foresight could contrive.

Like so many engineers who have blazed wilderness trails, William Driscoll for years looked forward with deep longing to the possession of a farm and a home. His habit of carefully planning ahead showed again in planning his retirement. About the time he had completed his last field assignment in 1927, in connection with the International Railways of Central America, he bought a farm near St. Inigoes, Md., and lived there. He died at St. Inigoes on December 6, 1935. His friends and former co-workers in many countries remember him with affection. He always gave his best to the task in hand. He was a good friend with whom to live and work.

He had three sisters and one brother, and is survived by one sister, Mrs. Catherine Sullivan, of Middletown, Conn., and by his brother, Mr. J. E. Driscoll, of Savannah, Ga.

Mr. Driscoll was elected an Associate Member of the American Society of Civil Engineers on September 12, 1916.

HARRY HENRY FROST, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 26, 1922

Harry Henry Frost was born in Bay City, Mich., on November 27, 1886. He was the son of Herman Charles Frost and Dorothea (Small) Frost. His father came to Michigan from New York, N. Y., about 1870; his mother's parents were pioneers in Michigan, in the vicinity of Detroit. He was a graduate of the Bay City High School in 1904, and of the Civil Engineering Department of the University of Michigan, Ann Arbor, Mich., in 1908. In recognition of his scholastic attainments he was elected, in his Senior year, to Sigma Xi Honorary Fraternity.

Mr. Frost began the practice of his profession, immediately upon his graduation, as a Surveyor in connection with coal mines in the Saginaw Valley, in Michigan. On August 26, 1908, he was employed as an Assistant Civil Engineer with the Department of Water Supply, at Detroit, Mich., but on May 7, 1910, he resigned to become Chief Civil Engineer to the Department of Parks and Boulevards, of Detroit. In January, 1911, he accepted the position of Superintendent of the City Water-Works Company at Akron, Ohio; and, after the acquisition of the property of the Water Company by the City of Akron on April 1, 1912, he retained his position as Superintendent, through five administrations, until his death. He also acted as Superintendent of the Akron Garbage Disposal Plant during 1915-16, while that plant was under construction and until it was in successful operation.

Mr. Frost possessed an unusually pleasing personality. Endowed with a jovial spirit and a natural kindliness, he "had fine capacity for friendship and was generally beloved." He was popularly known as "Brakie" and "Jack." He was President of his High School Class and of his Senior Class at the University of Michigan. A former employer writes: "I was very fond of Harry Frost;" and a former associate relates of "Jack" that: "On one occasion, late on a cold night, he and the trouble foreman crawled through water, curb deep, in a low spot on a down-town street, fumbling for the valve-box, to shut off a break. That was his style and the men loved it." In a spirit of "service" Mr. Frost took a personal interest "in each of his fellow workers" which strengthened their attachment to him.

He was an excellent administrator in public office. He had the "knack" of handling his employees, the public, and the politician. From 1910 to 1920, the population of Akron increased from 69 000 to 208 000, and Mr. Frost was successful in his efforts to meet the demands for increased water-works facilities required by this remarkable growth. The following is quoted from the Akron Rotary publication of February 7, 1922:

"Through the change from private to municipal ownership of the City Water Plant, and through all political and partisan changes that have taken place in the decade, there was never any question as to who should head this important branch of our municipal government. And his retention was based solely on merit and proved achievement."

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[1] Memoir prepared by Louis E. Ayres, M. Am. Soc. C. E.

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"During a period when the tremendous growth of the City required public officials to do miracles in engineering accomplishment, Harry Frost measured up fully to the necessities of the day."

For three years prior to his death, Mr. Frost suffered from diabetes. He knew the implications in a young man, but he never let the shadow affect him, other than to put his affairs into the best order possible. He had an optimistic philosophy which he expressed in the words: "You may be disappointed, but never discouraged." To the end he replied to many solicitous inquiries regarding his health that he "was getting along just fine." Death came suddenly from a throat infection which could not be corrected because of the diabetic condition.

Mr. Frost was active in the civic and business affairs of Akron. He was interested in banking, real estate development, and automobile manufacturing. He was a member of the Akron City Club, University Club, Rotary Club, and the Masonic Order.

On June 24, 1915, he was married to Mareta Lyon, daughter of Dr. O. A. Lyon, of Akron. He is survived by his widow and a daughter, Betsy Jane Frost.

Mr. Frost was elected an Associate Member of the American Society of Civil Engineers on June 4, 1913.

CONWAY ROBINSON HOWARD, Assoc. M. Am. Soc. C. E.[1]

Died August 26, 1936

Conway Robinson Howard was born in Richmond, Va., on March 27, 1881, the son of Major Conway Robinson Howard, a distinguished Civil Engineer, and Jane (Colston) Howard.

Mr. Howard came of an old Virginia family, prominent in the State since Colonial days. His early youth was spent in the city of his birth, where he attended a school conducted by his Aunt, Mrs. A. B. Camm, and another conducted by a Mr. Nolley. During the session of 1895–1896, he was a student at the Baylor School, in Chattanooga, Tenn. In the fall of 1896 he matriculated at the Virginia Military Institute, at Lexington, Va., and was graduated therefrom in the Class of 1900 in Civil Engineering.

After his graduation, Mr. Howard entered the service of The Chesapeake and Ohio Railway Company in the Engineering Department. He served that Company and the Lexington and Eastern Railway Company as Assistant and Resident Engineer in charge of heavy construction work.

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[1] Memoir prepared by a Committee of the Virginia Section, consisting of C. W. Johns and Fontaine Jones, Members, Am. Soc. C. E.

In 1908 he became associated with the City of Richmond, on cadastral topographical surveys of territory which had become annexed to that City in December, 1906. Mr. Howard was engaged in this work until 1910, when he was tendered a position with the Great Southern Lumber Company and the New Orleans Great Northern Railroad Company, at Bogalusa, La. In 1911, he was appointed Chief Engineer of the Great Southern Lumber Company, in which capacity he served until 1917.

Mr. Howard joined the 17th Engineers (Railway) in June, 1917, with the rank of First Lieutenant. The regiment sailed for France on July 28, 1917, landing first in England, and was included in the first contingent of volunteers sent overseas. From England the regiment went to France and was stationed at St. Nazaire for the duration of the World War.

Among other works, Lieutenant Howard had charge of the construction of the junction yard and the flying connection of the main line of the Paris-Orleans Railroad and the line from Bordeaux to Samur. There was about 15 miles of track in the yard with engine-terminal facilities, etc., as well as an overhead crossing of the main line. He reported directly to the late Lieut.-Col. William Bowdoin Causey, M. Am. Soc. C. E., who had charge of the work in the eastern part of Base Section No. 1. Lieutenant Howard was promoted to the rank of Captain in 1918 and, after the Armistice, was detached from his regiment and assigned to the transportation rehabilitation of Central Europe under Colonel Causey, with station in Vienna, Austria. This work was under the direction of Col. William G. Atwood, M. Am. Soc. C. E., American Transportation Representative on the Supreme Economic Council and the American Relief Administration, under Herbert Hoover, Hon. M. Am. Soc. C. E.

Captain Howard was discharged from the Army in Europe and remained with Colonel Causey for several years while the latter was Technical Adviser to the Austrian Republic.

Before going to Vienna, Captain Howard was first sent to Serbia, with headquarters at Zagreb. He remained there for about nine months, most of which time he was the only American in the town. The efficiency of his work in connection with railway rehabilitation and the administration of relief was attested by two decorations, the Order of St. Save (Fourth Class) from the Serbian Government and the Cross of the Serbian Red Cross.

Mr. Howard returned to the United States in 1921 on account of the illness of his mother. In 1922, he went back to Europe with the American Relief Administration in Russia, being stationed chiefly near the Siberian border, first at Ufa during the typhus epidemic, and, later, in Ekaterinburg, the town where the Czar and his family were murdered.

Returning to America in 1923, Mr. Howard again entered the service of The Chesapeake and Ohio Railway Company and from 1923 to 1926 was assigned to special duties in the Chief Engineer's Office, in Richmond.

In 1926, Mr. Howard became associated with a concern in Louisiana making mattresses out of a moss grown in that section. This venture, however, did not prove a success, and he returned to Richmond late in 1927. He returned to The Chesapeake and Ohio Railway Company in 1927, serving in various engineering capacities and reporting to the Chief Engineer at Richmond. From 1932 to 1934, Mr. Howard was variously employed. For a short time he was connected with the Department of Public Utilities, City of Richmond, preparing maps of the gas and water distribution system. In the latter part of 1934, he was engaged as Resident Engineer-Inspector with the Public Works Administration, in charge of building construction at the Virginia Polytechnic Institute, at Blacksburg, Va.

He was a member of the Commonwealth Club of Richmond; a Knight Templar, York Rite, and member of the Ancient Arabic Order of Nobles of the Mystic Shrine, Jerusalem Temple, of New Orleans.

During 1935, Mr. Howard suffered a nervous breakdown and, after making a trip to South America in the hope of regaining his health, found it necessary to retire from active practice. He died at Ambler Heights Sanatorium, Asheville, N. C., on August 26, 1936, and was buried in Hollywood Cemetery, in Richmond.

He is survived by three sisters, Mrs. Vance C. McCormick, of Harrisburg, Pa., Mrs. Frank Shoup, of Dallas, Tex., and Jane Howard, of Richmond, Va.

Mr. Howard was elected an Associate Member of the American Society of Civil Engineers on December 5, 1906.

GILES MATTHEW JOWERS, Assoc. M. Am. Soc. C. E.[1]

DIED MARCH 24, 1936

Giles Matthew Jowers was born at Thomasville, Ala., on September 19, 1869. When four years of age he lost his father, Wilson Jowers, a Professor of Mathematics and a Civil Engineer whose analytical mind was a natural heritage of the son. Giles Matthew Jowers received his education in the public schools in Thomasville and Mobile, Ala., and was graduated from the Springhill Academy, at Mobile, in 1885. In 1904, he completed a course of study in Railroad Engineering at the Armour School of Technology, at Chicago, Ill.

In 1888, Mr. Jowers moved to Texas where, in 1906, he surveyed and subdivided the old Circle S Ranch, laying out the now prosperous Winter Garden City of Crystal City, Tex. Employed as Location Engineer and, later, as Division Engineer by the San Antonio, Uvalde and Gulf Railroad Company from 1908 until 1913, he located and supervised the construction of 300 miles of railroad which opened a vast section of Texas to development. In 1914, he was in charge of the construction of the irrigation system of the Winter Garden Irrigation Company in Zavala and Dimmit Counties, Texas. He returned to this work in 1925 and 1926 to extend and complete the system.

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[1] Memoir prepared by H. P. Stockton, Jr., Assoc. M. Am. Soc. C. E.

In private practice, with headquarters in San Antonio, Tex., in 1915 and 1916, Mr. Jowers was engaged in surveying, irrigation, and municipal work. In 1917, he was employed by the United States Government in laying out and constructing railway facilities, streets, and sewers in the several training camps and flying fields then being expanded and developed in the San Antonio Area. In 1918 and 1919, he was employed as Special Engineer by the International Boundary Commission in locating and marking the border and determining the division of irrigational waters between the United States and Mexico from Del Rio to the Gulf of Mexico. With Val Verde County, Texas, from 1920 to 1925, he located and supervised the construction of 150 miles of State and Federal highways.

From 1926 until his death, Mr. Jowers was employed by the Texas State Highway Department as Resident Engineer on the location, design, and construction of highways. In this capacity he located several hundred miles and designed and supervised the construction of more than 100 miles of highways of the Texas System in Zavala, Kerr, Kendall, Bexar, Bandera, Bell, Coryell, and Hamilton Counties.

Possessed of a keen and inquiring mind and of remarkable strength and energy, Mr. Jowers gave to his work that character which makes it an outstanding monument to his ability. He had a large part in the development of the Winter Garden District and of the highway system of Texas.

The respect of all with whom he came in contact and the firm friends he left are evidences of his pleasing personality, earnest endeavor, and fidelity. The admiration of his colleagues gauges his professional standing.

He is survived by his widow, Margaret M. Jowers, and a son, Thomas Giles Jowers, of San Antonio; his daughter, Mrs. Lucy (Jowers) Storck, of Bronxville, N. Y.; a sister, Mrs. Stella Ford, of Thomasville, Ala.; and two brothers, V. E. and Albert W. Jowers, of Buffalo, Tex.

Mr. Jowers was elected an Associate Member of the American Society of Civil Engineers on March 9, 1920.

RALPH LONG KELL, Assoc. M. Am. Soc. C. E.[1]

DIED MAY 24, 1936

Ralph Long Kell was born on September 8, 1881, in Loysville, Perry County, Pa., the son of Peter G. and Ella (Long) Kell. Following the completion of his preparatory schooling at the New Bloomfield Academy, Mr. Kell entered Pennsylvania State College, at State College, Pa., from which he was graduated with the degree of Bachelor of Science in Civil Engineering in 1905.

His first engineering work was with the Maintenance-of-Way Department of the Pennsylvania Railroad Company, and he served with this Company from 1905 to 1917 as Rodman, Transitman, and Assistant Supervisor of Track. His work included plans, surveys, estimates, examinations, and reports in connection with railroad maintenance and construction. During this period, Mr. Kell was located at various times at Altoona, Harrisburg, Freeport, and Chester, Pa

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[1] Memoir prepared by John H. Wickersham, M. Am. Soc. C. E.

In 1917, Mr. Kell accepted a position with Frank H. Shaw, M. Am. Soc. C. E., Consulting Engineer, of Lancaster, Pa., as Principal Engineer in charge of the Lancaster Office and served in this capacity until 1918. The work of this office embraced designs, plans, specifications, and the supervision of construction on highway bridges and water-works.

From 1918 to 1919, he was with the Water Supply Commission of Pennsylvania, as Engineer on surveys, examinations, and reports on flood control, stream encroachments, and stability and construction of dams. In 1919, Mr. Kell returned to Lancaster, and was engaged as Engineer with John H. Wickersham, M. Am. Soc. C. E., Engineer and Contractor, until 1924. His services with this firm were extremely valuable and included the design, plans and specifications, and the management of general building construction for numerous projects throughout Eastern and Central Pennsylvania.

In 1924, his health required him to engage in less confining work, and he accepted a position as Construction Supervisor for Franklin and Marshall College, Lancaster, on an extensive building program. His work with the College covered a period of two years. Following its completion, Mr. Kell served, during 1927, as Construction Supervisor for Charles H. Bear and Company, York, Pa., on alterations and addition to its Department Store.

Mr. Kell was also engaged as Construction Supervisor for the School District of Lancaster, during the period, 1927 and 1928, on the erection of two Junior High Schools and two Grade Schools. He served in the same capacity for the York Hospital, at York, on the erection of a new building, a Nurses Home, and a power plant, during 1929 and 1930.

In January, 1934, he was appointed City Engineer of Lancaster and served with distinction in this office until the time of his death. He was stricken while marching with the Knights Templars to attend the annual Ascension Day services, and died shortly afterward.

Although modest and unassuming, Ralph Kell was a man of high ideals and integrity. A conscientious and capable engineer, he left behind him a record of many accomplishments which will be remembered by his associates, who held him in great esteem.

He was married on April 30, 1910, to Rowena Millar, of Harrisburg, Pa., who survives him. He is also survived by a sister, Mrs. Mary Kell Fry, of York, Pa., and a brother, Harry, of Huntingdon, Pa.

He was a member of the Torch Club and the Penn State Alumni Club, of Lancaster. His fraternal affiliations in the Masonic Order, included: Lamberton Lodge, No. 476, F. and A. M.; Royal Arch Chapter, No. 43, of which he was Past High Priest and District Deputy Grand High Priest; Goodwin Council, No. 19, of which he was Past Thrice Illustrious Master; Lancaster Commandery, No. 13, Knights Templars; and Lancaster Forest, No. 27, Tall Cedars of Lebanon.

Mr. Kell was elected an Associate Member of the American Society of Civil Engineers on March 14, 1916.

JOHN BIGGER LEEPER, Assoc. M. Am. Soc. C. E.[1]

Died December 3, 1936

John Bigger Leeper was born at Hookstown, Pa., on February 1, 1868, the son of Robert and Elizabeth (Dallas) Leeper. In 1892, he was graduated from Lafayette College at Easton, Pa., with the degree of Bachelor of Science.

Between July, 1892, and February, 1893, he was Draftsman for the Boston Bridge Works; from February, to July, 1893, he served as Draftsman with the McMyler Manufacturing Company, at Cleveland, Ohio; from July, 1893, to February, 1895, he was Rodman, Leveler, Draftsman, and Masonry Inspector with the Lake Shore and Michigan Southern Railway Company; and from February, 1895, to September, 1897, he was in the employ of Lewis and Daily, at Cleveland, as Designer, Estimator, Inspector, and Superintendent of Structural Work.

On September 1, 1897, Mr. Leeper became affiliated with the Keystone Bridge Works, at Pittsburgh, Pa., as Estimator and Draftsman. The Keystone Bridge Works was merged into the American Bridge Company in 1900, and he was transferred to the Engineering Department, Pittsburgh Division, as Estimator, and, after 1904, was in charge of the estimating forces of the Pittsburgh Division.

Mr. Leeper did not confine his thoughts to his work at hand. In 1904, he instigated an experimentation in the manufacture of spelter pans with no riveted joints (pressed from a single plate), which finally led to an extensive development of pressed spelter pans, such as are now in extensive use.

In 1908, he began work in connection with high-tension transmission towers, and soon gave up his post as Chief of the Estimating Department. He then became Manager and Engineer of a Tower Department that was organized by the American Bridge Company, his duties including design, detail, and sales. His influence was soon felt throughout the power industry. He developed several patents on power-transmission equipment, and his unremitting efforts developed a distinctive advancement in the design and detail of transmission towers and transformer stations. He retired from active service on May 31, 1933.

He was the author of a Hand Book on Transmission Towers that was published in 1925 and which became an authoritative reference book for engineers and engineering schools.

Mr. Leeper made an outstanding reputation as a Designer and Salesman. His methods were original and in conformity with a strict code of honorable dealing which was exemplified in his social contacts with all men. He was a most lovable character. Friendliness and serenity were the keynotes of his personality, but he had an active mentality that was awake to every opportunity. However, he carefully analyzed every opportunity to assure himself that it corresponded with his code in every respect. He enjoyed a wide circle of friends, whose confidence was never violated and that confidence extended to the people with whom he had business dealings, as well as among those with whom he labored, and among his neighbors and acquaintances from all stations of life. Pomp and circumstance had no place in his plan of life. The finger of scorn and the darts of criticism were never aimed at him during the course of a life that was well-nigh blameless, and he will be fondly remembered as a distinctive character, a capable Engineer, and an honorable gentleman.

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[1] Memoir prepared by Marshall Williams, M. Am. Soc. C. E.

Mr. Leeper was a devotee of outdoor life. His home in the hills back of Glenfield, Pa., was an open house to his friends, who shared his enjoyment of a home that was removed from the busy lines of traffic and where he gave full swing to his love for communion with Nature. He also maintained a lodge on an island in Hollow Lake, Ontario, Canada, where he spent his summers in a fisherman's Paradise, and enjoyed the labor of improvement in the living quarters and equipment of the place.

On November 29, 1899, Mr. Leeper was married to Margaret Campbell, of Pittsburgh, who, with a daughter, Mildred, survives him to mourn the loss of a devoted husband and father.

He was an Associate Member of the Engineers' Society of Western Pennsylvania, and from 1914 to 1934, a Member of the American Institute of Electrical Engineers.

Mr. Leeper was elected an Associate Member of the American Society of Civil Engineers on January 3, 1900.

JAMES WILLIAM NORTON, Assoc. M. Am. Soc. C. E.[1]

Died May 27, 1936

James William Norton was born in Nicholas County, Kentucky, on May 16, 1889, the only child of Jesse H. and Mary (Huffstetter) Norton. His early education was received in the public schools of Carlisle, Ky., followed by preparatory training in the Millersburg Military Institute, at Millersburg, Ky., and, finally, entry into the University of Kentucky, at Lexington, where he enrolled as a student in Mining Engineering.

About 1905, Mr. Norton left the University before completing his course and went to work for the Tennessee Coal, Iron, and Railroad Company, at Birmingham, Ala., and, later, at Ensley, Ala. At the latter place an accident happened that had much to do with shaping his future. While attempting to catch a moving yard engine, his foot slipped, and he was caught by a drive-wheel and carried around by it, striking the hard road-bed many times before the locomotive could be stopped. The resulting nervous shock and the loss of one foot caused him much suffering throughout the remainder of his life; and often made it necessary for him to have either medical attention, or care in a hospital, for periods of several weeks at a time.

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[1] Memoir prepared by J. T. Madison, M. Am. Soc. C. E.

While living in the South Mr. Norton served as Engineer for one or more municipalities; and, for a time, was engaged in the business of contracting. After returning to Kentucky, he was Road Engineer for Nicholas County, from 1915 to 1920; he then engaged in private practice for about a year at Carlisle.

In 1921, Mr. Norton was appointed Assistant District Engineer for the Kentucky Highway Department at its Covington Office, where he served until 1922. Then, acting on the recommendation for a change of climate made by his medical advisers, he moved to Lakeland, Fla., in 1923, where he was employed as Designer of Sewers and Streets in the City Engineering Department until the close of 1924.

In January, 1925, he, with several associates, entered private practice in Lakeland to specialize in municipal and highway work. For about four months of the same year, Mr. Norton served as City Engineer of Lakeland. He continued in private practice at Lakeland until 1933, when he returned to Kentucky.

For several years following his return, Mr. Norton was associated with highway contractors, as Superintendent of Construction on projects in Central Kentucky. Subsequently, he had been engaged on Federal projects in a number of counties in Kentucky. The last assignment was at Noble, Ky.

Mr. Norton had many friends and acquaintances who were attracted by his pleasing personality and good sense of humor. His manner was mild, but his arguments forceful. One of his outstanding qualities was an ability to impress the members of governing bodies and convert them to his views. The great handicap resulting from the physical injuries received at Ensley often bore heavily upon him; this, however, was admitted only to his most intimate friends who understood how he suffered.

He entered the Masonic Lodge in 1910, and at the time of his death was affiliated with Daugherty Lodge F. and A. M., at Carlisle.

Mr. Norton was elected an Associate Member of the American Society of Civil Engineers on March 9, 1920.

GEORGE PAUL O'CONNELL, Assoc. M. Am. Soc. C. E.[1]

DIED JANUARY 3, 1936

George Paul O'Connell, one of six sons of Daniel and Johanna (Brassil) O'Connell, was born in Holyoke, Mass., on January 4, 1881. His father was a prominent contractor in Western Massachusetts and an influence in Holyoke from the period following shortly after the Civil War. After the father's death, the contracting business was continued by his sons, Daniel, Charles, and John, under the name of Daniel O'Connell's Sons, Inc.

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[1] Memoir prepared by James L. Tighe, M. Am. Soc. C. E.

George Paul O'Connell received his elementary education in Holyoke, and was graduated from the Holyoke High School, in the Class of 1898. He then entered the Massachusetts Institute of Technology, at Boston, Mass., from which he was graduated in 1902, receiving the degree of Bachelor of Science in Civil Engineering. His early engineering experience included work as Assistant Engineer with the firm of Ellsworth and Kirkpatrick, of Holyoke, the Burr-Hering-Freeman Commission, of New York, N. Y., the Passaic Water Company, of Paterson, N. J., and the Aqueduct Commission of New York City.

From 1906 to 1913, Mr. O'Connell was an Assistant Engineer with the New York City Board of Water Supply, and was engaged on the construction of the Catskill Supply System, including surveys for the Ashokan Reservoir, surveys and studies for the proposed Prattsville Reservoir, and the construction of the Ashokan Reservoir and the East Dike and Waste Weir, at Brown Station, N. Y.

Toward the end of 1913, he returned to Holyoke to become Engineer of Daniel O'Connell's Sons, Inc., General Contractors, of which Company he was also Vice-President at the time of his death. While associated with this Corporation, he directed the construction of many engineering projects, including the building of dams, bridges, water supply pipe lines, sewerage systems, State and county highways, city streets, churches, mills, and other industrial establishments.

For several years before, and until the time of, his death, Mr. O'Connell was a member of the Planning Board of the City of Holyoke. During the World War, he served as a Captain in the Engineer Corps, being stationed for the most part at Camp Humphreys, Virginia. He was a member of the Holyoke Post, American Legion, and the Holyoke Lodge of the Benevolent and Protective Order of Elks.

He was not married, and is survived by two brothers, Daniel and John, and by several nephews and nieces.

Mr. O'Connell was elected an Associate Member of the American Society of Civil Engineers on September 6, 1910.

RODERIC PEARSON, Assoc. M. Am. Soc. C. E.[1]

DIED FEBRUARY 4, 1937

Roderic Pearson, the son of the late L. K. Pearson and Minnie G. Pearson, was born at Portland, Ore., on October 1, 1893. He was educated in the local schools and at Oregon State Agricultural College, at Corvallis, where he was a member of the local fraternity, Gamma Tau Beta, and from which he was graduated with the degree of Civil Engineer in 1916.

After his graduation from college, Mr. Pearson spent a season with the United States Geological Survey, and several months with the firm of Meese and Godfried, on the design of conveying machinery. In January, 1918, he was employed as Draftsman for the United States Bureau of Public Roads, at Portland. He left the service of the Bureau in October of the same year to enlist in the Army, being assigned to the 17th Recruit Company, at Fort McDowell, California. He was honorably discharged the month after the Armistice, and returned to the employ of the Bureau of Roads, at Portland. He remained with this organization until his death, serving 2 yr, at Portland; 2 yr, at Helena, Mont.; $2\frac{1}{2}$ yr, at Ogden, Utah; and the last $12\frac{1}{2}$ yr, at San Francisco, Calif. During all this time, Mr. Pearson was engaged on bridge work, advancing through the grades to the position of Highway Bridge Engineer. For a period he was also Materials Engineer in the San Francisco District, and during the heavy program of grade separation, in 1936, he had charge of this work in Nevada.

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[1] Memoir prepared by George D. Whittle, M. Am. Soc. C. E.

Mr. Pearson was at his best when engaged on some difficult problem of structural design, mathematical analysis, or the cost of construction materials.

For several years he suffered from anemia and despite continual medical care, his condition gradually became worse. He was confined to his bed for three months preceding his death, at Letterman General Hospital, in San Francisco. He was buried in the National Cemetery at the Presidio on a protected slope overlooking the Golden Gate.

He was not married, and is survived by his mother and one brother, L. K. Pearson, Jr.

Mr. Pearson's chief hobby was photography and he did some very creditable work therein as a member of the San Francisco Camera Club. He also made a hobby of his engineering library and equipment, as evidenced by the fact that at the time of his death he possessed eleven different slide-rules.

He was made a Mason in Portland, in 1918, and was a member of Oregon Lodge No. 101 and Al Kader Temple of the Mystic Shrine.

Mr. Pearson was elected an Associate Member of the American Society of Civil Engineers on August 28, 1922.

JOHN MELVIN REARDON, Assoc. M. Am. Soc. C. E.[1]

Died September 29, 1936

John Melvin Reardon was born on March 10, 1900, in St. Paul, Minn., the son of Michael Joseph and Alice (McGorry) Reardon. He received his early education in the public schools of St. Paul. He afterward attended, and was graduated from, St. Thomas College, at St. Paul, in 1915. He then enrolled in the Civil Engineering College of the University of Minnesota, at Minneapolis, Minn., from which he was graduated in 1919 with a degree of Bachelor of Science in Civil Engineering. The United States entered the

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[1] Memoir prepared by William N. Carey, M. Am. Soc. C. E.

World War before Mr. Reardon was graduated from the University, but at the outbreak of the war he enlisted in the United States Army and served in the University Training Corps until November 26, 1918, when he was honorably discharged.

After his graduation from the University, and until December, 1923, Mr. Reardon was employed as Surveyor and as Construction Engineer, principally on highway paving construction in Minnesota.

In December, 1923, he joined the forces of the Department of Public Works, in St. Paul, where he served until his death. As Assistant Superintendent of the Bureau of Construction and Repair, Department of Public Works, City of St. Paul and, later, as Superintendent of the Bureau and Assistant City Engineer, Mr. Reardon was responsible for supervising much of the paving and sewer construction carried on by the City. This work extended over a 12-yr period and involved a construction expenditure of more than $11 000 000. Although principally concerned with the construction of sewers, pavement, and similar improvements, Mr. Reardon was keenly interested in the specifications and the design of Public Works Department improvements in his city. The plans and specifications originating in that Department showed the influence of his sound and practical construction judgment.

In the field, on the supervision of contract construction, Mr. Reardon displayed the unusually happy faculty of combining firmness of purpose with a natural warm-hearted geniality. He was able thereby to enforce contract compliance with the absolute minimum of disagreement as between Engineer and Contractor. His immediate associates as well as the contractors and job superintendents with whom he came in contact in his work, held him in the highest respect both as a man and as an Engineer.

He was married on October 6, 1925, to Helen Albeck, who, with a son and a daughter, survives him.

Mr. Reardon was elected an Associate Member of the American Society of Civil Engineers on March 5, 1928.

CLARENCE HORACE SCHWARTZ, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 28, 1936

Clarence Horace Schwartz, known to his many friends as "Charley", was born near Carbondale, Ill., on December 22, 1893, to William and Etta (Thomas) Schwartz. His ancestry was a mixed lineage with English, Irish, and German predominating. Two of his ancestors most worthy of mention were General James Wolfe, conqueror of Quebec, and Thomas Parr, who was noted for his longevity, and who is buried in Westminster Abbey, in London, England.

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[1] Memoir prepared by Lieut. Col. E. Reybold, Corps of Engrs., U. S. Army; Dist. Engr., Memphis, Tenn.

Mr. Schwartz's early education was received in the public schools at Wickliffe, Ky., where he was graduated from the High School in 1910. He entered the University of Kentucky, at Lexington, Ky., in the same year, and was graduated in 1914, Cum Laude, receiving the degree of Bachelor of Science in Civil Engineering. He was a member of Tau Beta Pi Fraternity.

Mr. Schwartz obtained his elementary engineering experience after graduation, with the Delaware, Lackawanna and Western Railroad Company, working on physical evaluation appraisals; with the Kentucky State Highway Department as an Inspector; and with the Morgan Engineering Company on plans for flood-control improvements for the Miami Conservancy District, at Dayton, Ohio. With the Morgan Engineering Company, Mr. Schwartz received his first flood-control and hydraulic engineering experience, which he was later to use in his most important works.

A heart ailment of long standing, which was eventually the cause of his death, almost prevented Mr. Schwartz from joining the Army when the United States entered the World War, but his persistence and refusal to accept the first rejection, finally won for him entrance to the Second Officers' Training School. He was commissioned Second Lieutenant in the Engineering Corps on October 4, 1917, and promoted to the rank of First Lieutenant on August 24, 1918. He was discharged from the Army on February 4, 1919.

The Miami Conservancy District employed Mr. Schwartz after the war and put him in charge of the flood-control works at West Carrollton, Ohio. On the completion of this work, F. G. Mueller, Architect, placed him in charge of construction of the hydro-electric power plant at Hamilton, Ohio, for the Ford Motor Company. After the hydro-electric plant was completed, the Miami Conservancy District re-employed Mr. Schwartz, giving him full responsibility for its Division Office, which position he held until these flood-control works were completed. He then joined the forces of the Lloyd Construction Company, of Willoughby, Ohio, in the building of filtration and sewage disposal plants. In 1927, the U. S. Engineer Office at New Orleans, La., engaged him to make hydraulic studies on flood-control works on the Mississippi River. The Portland Cement Association at New York, N. Y., employed him next as Contact Engineer to meet engineers, architects, and contractors. After two years, friends who knew of his ability on flood-control studies induced him to join the forces in the District Office of the United States Engineers, at Memphis, Tenn., where he remained until his death. The last five years of his service were in cost accounting, which work he conducted in a highly efficient manner, and for which he received official commendation.

Having made a success of engineering, Mr. Schwartz took up the study of law and was admitted to the Bar in Tennessee in 1932. He was graduated from the University of Memphis Law School, in 1933. Immediately upon his graduation, Mr. Schwartz formed a partnership law firm with Mr. Milton Picard, but was prevented from taking an active part in the firm's affairs because of ill health.

He spent five months in the hospital from January to June, 1934, and although the doctors gave him no encouragement, he recovered sufficiently to resume his work, which he carried on for two years until his passing on June 28, 1936.

He was a Mason, and a member of the Memphis University Club, and the Memphis Engineers' Club. He served as Secretary of the Mid-South Section of the Society for the year 1933 and 1934.

Mr. Schwartz was a constant student and a reader of diversified subjects. He was a pleasing conversationalist because of his breadth of knowledge, which, coupled with his interest in and his desire to help his fellow companions and his unfailingly cheerful disposition, earned for him the friendship of all with whom he came in contact. His brilliant mind would have carried him much farther in his chosen career; his passing was a distinct loss to his friends and to his profession.

He was never married, and is survived by his parents, two sisters, Mrs. Ruth Spalding and Mrs. Grace G. Carter, and one brother, John, all of Chicago, Ill., and vicinity.

Mr. Schwartz was elected an Associate Member of the American Society of Civil Engineers on November 9, 1920.

EDWARD RALPH TAYLOR, Assoc. M. Am. Soc. C. E.[1]

DIED JUNE 4, 1935

Edward Ralph Taylor was born in Trenton, N. J., on June 3, 1898, the son of Edward Watson and Ida Frances (Skillman) Taylor. He received his Grammar School and High School education at the local schools.

After leaving High School, Mr. Taylor was employed in the Distribution Department of the Public Service Electric Company, at Trenton, as a Draftsman. In June, 1916, he became Chief Draftsman, which position he held until July, 1918. During this time, he attended Drexel Institute, in Philadelphia, Pa., and was graduated from its Evening Course in Electrical Engineering. While with the Public Service Electric Company, he had not only experience in special transmission-line design, but he had considerable field work as Instrumentman and Chief of Party on the survey of the transmission line which had to be run from Mt. Holly, N. J., to the Army Cantonment at Camp Dix, New Jersey.

Directly after this, and until October, 1918, Mr. Taylor served as Electrical Draftsman and Designer of Industrial Wiring and Sub-Stations for the Plant Engineer at the Harriman, Pa., Ship Yards. At this time, he joined the Student Army Training Corps at Lafayette College, at Easton, Pa., pursuing further his study of Electrical Engineering.

The need for the Student Corps being over by the end of 1918, Mr. Taylor returned to Trenton and became a member of the Tramway and Bridge Engineering Department of the John A. Roebling's Sons Company, in January, 1919. He was engaged on the study and design of suspension bridges, cableways, tramways, and various wire-rope applications. Later, he was placed in charge of construction in the field. He also made engineering studies and surveys of the use of wire rope for mines, oil fields, logging, and coal and ore-handling equipment.

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[1] Memoir prepared by A. J. Morgan, Esq., Trenton, N. J.

In furthering his study of bridges, Mr. Taylor was employed on the field work during the erection of the cables for the Bear Mountain Bridge over the Hudson River, and for the Philadelphia-Camden Bridge. About that time, he was instrumental in introducing the new parallel-strand type of bridge cable on the Grand 'Mere Bridge, in Canada.

In January, 1928, he became Assistant Resident Engineer for the John A. Roebling's Sons Company, on the New Jersey side of the George Washington Bridge over the Hudson River at New York City, and continued in this position until about the end of that year, when he was made Assistant Chief Engineer of the Wire Rope Department of the Company. He then continued his engineering study of wire rope and its proper application to all types of installations in the field.

In September, 1934, he was appointed Mill Representative for the Sales Department, which position he held at the time of the unfortunate accident which caused his death. He died on June 4, 1935, at Honolulu, Hawaii, due to a broken vertebra, the result of a so-called shallow dive the day before. His body was brought to Trenton for burial.

Mr. Taylor was well known throughout the wire rope industry and the trade, and his personality and engineering ability made many friends among bridge builders and industries using wire rope.

He was a Licensed Professional Engineer and Land Surveyor of the State of New Jersey. He was a member of Column Lodge, F. and A. M. of Trenton, a Thirty-second Degree Scottish Rite Mason, and a member of Crescent Temple.

He was married at Trenton, on June 23, 1920, to Mabel Randall Towers of that city, who survives him.

Mr. Taylor was elected an Associate Member of the American Society of Civil Engineers on December 3, 1928.

WILLIAM JOHN UBBINK, Assoc. M. Am. Soc. C. E.[1]

Died February 8, 1937

William John Ubbink was born in Port Washington, Wis., on August 10, 1890, the son of Joseph and Susan (Weyker) Ubbink, both of whom were natives of Ozaukee County, Wisconsin. His paternal grandparents emigrated from Holland to the United States in 1840, and at about the same time his maternal grandparents came to this country from Germany, both families settling in Ozaukee County.

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[1] Memoir prepared by Joe J. Ubbink, Esq., Port Washington, Wis.

His grandfather, Bernard Ubbink, established a masonry contracting business, in Port Washington, in 1848, and, in 1876, was joined in that enterprise by his son, Joseph, with whom he carried it forward until his death in 1890. Joseph Ubbink thereafter conducted the business alone until 1908, when he was joined by his son, William. Five years later, in 1913, Joseph and Arnold Ubbink, the two other sons of the family, likewise associated themselves with the firm. The Company was continued until the death of the father, which occurred in 1929. The family still has the continuous set of books, covering the details of the business over a period of more than eighty years. Among the many important structures erected by the Ubbinks may be mentioned three of the leading hotels in Port Washington; the plants of the Wisconsin Chair Company and the Gilson Manufacturing Company, both of Port Washington; the Port Washington malt-house; oil-burning lime kilns, at Dubuque, Iowa; the Lower Wisconsin Avenue Viaduct, in Milwaukee, Wis.; the Wisconsin Street School Building, in Port Washington; St. Mary's Roman Catholic Church and School; and many bridges, large and small, throughout the State.

William John Ubbink received his early education in the parochial schools of Port Washington and in the local High School. He attended the Rheude Architectural School, in Milwaukee, until that institution was discontinued on the death of Mr. Rheude, and afterward entered the Chicago Technical College, at Chicago, Ill., where he completed a course in Civil Engineering in 1909.

On his return to Port Washington, Mr. Ubbink became associated, as stated, with his father in the contracting business, to which he devoted his attention until 1917, when he enlisted for service in the World War. He was on active duty with the 42d United States Engineers—a road and bridge building regiment—in the Lorraine Sector, France, and was honorably discharged on July 1, 1919.

On his return to Port Washington, he rejoined his father, with whom he was associated in business until January 1, 1921, when he was made Highway Commissioner of Ozaukee County, which position he held until his death.

During his term of office the County floated a bond issue of $1 750 000 for the construction of roads and bridges, all of which has been completed, together with the new Highway Office Building into which the Highway Department had just moved. Mr. Ubbink's administration of the office held by him was marked by conscientious regard for the interests of the taxpayers of the County and he earned high praise for the marked improvement in the condition of the streets and highways.

At Lake Church, Wis., on June 23, 1922, Mr. Ubbink was married to May Johann, the daughter of Carl and Mary (Frantz) Johann, natives of Germany. Afflicted by a chronic heart ailment, Mr. Ubbink died on February 8, 1937, and is survived by his widow and a daughter, Mary Ruth, and by two brothers and three sisters.

He was a member of Port Washington Post No. 82, of the American Legion; the Knights of Columbus, Fourth Degree; the Rotary Club, of

Port Washington; the County Highway Commissioners Association of Wisconsin; the Engineering Society of Wisconsin; and the Port Washington Chamber of Commerce, of which he was President. He was also Vice-President of the First National Bank; Vice-President of The Ubbink Fuel and Dock Company; and President of the Port Development Company. He was very highly regarded as a civil engineer; in private life, he was a loyal citizen, and a dependable friend and neighbor.

Mr. Ubbink was elected an Associate Member of the American Society of Civil Engineers on August 12, 1935.

REENEN JACOB VAN REENEN, Assoc. M. Am. Soc. C. E.[1]

DIED OCTOBER 19, 1935

Reenen Jacob van Reenen was born at Calvinia, Union of South Africa, on April 15, 1884. He was the son of Albert Johannes van Reenen, a prominent attorney at Calvinia, and Susanna (de Villiers) van Reenen. His education was received principally at the South African College, at Cape Town, Union of South Africa, from which he took his Bachelor's Degree and won the Victoria Scholarship.

In 1903, he came to the United States and entered the Civil Engineering Department at Lehigh University, at South Bethlehem, Pa., from which he was graduated in 1906. During this period, he became a member of Kappa Sigma and Tau Beta Pi, and played on the University football team.

On the completion of his studies in 1906, Mr. van Reenen became Resident Engineer to the Tri-State Land and Irrigation Company, in Nebraska, and it is interesting to note that the writer, during a visit to the United States in 1934, met several members of the Society who remembered Mr. van Reenen on this particular work.

Toward the end of 1907 Mr. van Reenen returned to his native land, where he became associated with the construction of the Reservoir and Irrigation Distribution Works of the Smartt Syndicate, at Britstown, in the Cape Province, Union of South Africa. On the completion of this work, in the middle of 1909, he entered the Government Civil Service and was occupied in the design and construction of several irrigation projects on the Gamtoos River—one of which was named for him.

In May, 1912, he was appointed Superintendent of Roads and Local Works in the Orange Free State Province, Union of South Africa, which position he retained until 1926. The excellent highway system which that Province enjoys, bears eloquent testimony to his ability. During this period, Mr. van Reenen was appointed a member of the Drought Investigation Commission (1920–24), and, later, Chairman of the Drought Investigation Commission of the mandated territory of South West Africa (1923–24). In

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[1] Memoir prepared by Philip R. R. Bisschop, Assoc. M. Am. Soc. C. E.

October, 1924, he was appointed a member of the Irrigation Finance Commission and, on the completion of that work, he was appointed Chairman of the Special Irrigation Commission. During this latter period, he also served as a member of the Angola Boundary Commission, which was detailed to determine the boundary between the Portuguese territory of Angola and that of South West Africa. In August, 1926, he was appointed Chairman of the Permanent Irrigation Commission which position he held with great distinction until his death. Apart from these duties, Mr. van Reenen was called upon to serve first as a member and, later, as Chairman of the National and Historical Monuments Commission. In 1933, he served as Chairman of the Low Grade Ore Commission, appointed by the Government to report on the life of the low-grade gold mines of the Witwatersrand. In 1934, he was appointed Chairman of the Industrial Legislation Commission the report of which was issued shortly before his untimely death.

In addition to these many Government appointments, Mr. van Reenen was a recognized authority on ancient Bushman culture and paintings, educational matters, and Afrikaans literature. Besides giving numerous lectures on these subjects before scientific and academic bodies, he published a number of volumes in Afrikaans and had just completed writing a book entitled, "Resisting Drought." His book on "Die Agterste Voortrekker", can be found on the shelves of the Library of his American Alma Mater, Lehigh University. Mr. van Reenen was a member of the Afrikaner Kring, Die Suid Afrikaanse Akademie vir Lettere, the South African Society for the Advancement of Science, the South African Society of Civil Engineers, and the Ramblers Club, at Bloemfontein, Union of South Africa.

In 1908, Mr. van Reenen was married to Elizabeth Lilian Roos, a member of a well-known Cape Town family, and the sister of the late South African statesman, the Hon. Tielman Roos. Of this union there are two daughters and a son, who, with Mrs. van Reenen, survive him.

Known among his intimates as "Voorsitter" (that is, "Chairman"), he was a hard worker, a brilliant man, generous and kind, diplomatic, loyal, and straight. Full of humor, appreciative of, and a narrator of, a good story, in short, a lovable fellow. Chief among his many hobbies were sketching, cabinet-making, and photography. All that he produced along these lines was of exceptional merit. When visiting in New York City in 1932, he obtained a number of camera studies, and was particularly proud of the photographs that he took at night from the top of the Empire State Building.

As a public servant, Mr. van Reenen served his country well. There are no brass tablets to record his deeds, but his findings on the many Government bodies upon which he served, will benefit his country, farmers, and townsmen. South Africa, the Government Service, and his friends have lost one of the best, and are the poorer for his departure.

"Pass, Friend, 'Totsiens, Voorsitter'."

Mr. van Reenen was elected an Associate Member of the American Society of Civil Engineers on October 3, 1911.

JOHN JUNIOR WILSON, Assoc. M. Am. Soc. C. E.[1]

DIED NOVEMBER 21, 1936

John Junior Wilson was born at Georgetown, Colo., on September 2, 1878. He was the son of the Rev. John Wilson, a noted pioneer minister in Colorado and New Mexico, and Elizabeth (Boyce) Wilson. John Junior Wilson was educated in the public schools of Longmont, Colo., and the State Preparatory School, at Boulder, Colo., and took his engineering course in the University of Colorado, at Boulder. He was interested in athletics and was a member of the University football team. He served as a Sergeant of the Signal Corps of the Colorado National Guard during the strike at Cripple Creek.

After leaving the University, Mr. Wilson began his career with the Hill's Engineering Company, of Cripple Creek, Colo. Although subsequently he was engaged in other lines of engineering work, especially in the construction of dams, Mining Engineering was his specialty and his last active work was in that branch of service.

In 1907, he accepted a position with the Teziutlan Copper Company, of LaAurora, Pueblo, Mexico, and was with that Company for a considerable length of time. Returning to the United States he became Engineer for the construction of a large earthwork dam for irrigation purposes, at Two Buttes, near Lamar, Colo.

When construction was begun on the Barker Meadows Dam, near Nederland, Colo., Mr. Wilson was assigned to check the work as Deputy State Inspector and remained in that position until the dam was completed. He then went to Minnesota and became a Bridge Engineer for the Great Northern Railroad Company. He was afterward elected a member of the Faculty of the Agricultural Engineering Department of the University of Minnesota, Minneapolis, Minn.

In 1915, his chosen work as Mining Engineer lured him back and he took a position with the Ray Consolidated Copper Company, at Ray, Ariz. He later returned to Colorado and took charge of the construction work at the Silver Lake and Albion Dams, for the water supply of the City of Boulder. He then became Engineer in charge at the tungsten mines of the Primos Mining Company, at Lakewood, Colo.

While superintending some underground work in those mines, Mr. Wilson met with a serious accident, and for the last twenty years of his life he was incapacitated from active service. He made his home at Boulder, where he died on November 21, 1936. He was never married, and is survived by four sisters, Mrs. Jessie W. Dwyer, and Mrs. Agnes W. Morris, of Boulder; Mrs. Jennie B. Brown, of Longmont, Colo.; Mrs. Irvin E. Keeler, of Long Beach, Calif., and one brother, the Rev. Harry Noble Wilson, D.D., of St. Paul, Minn.

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[1] Memoir prepared by the Rev. Harry Noble Wilson, D.D., St. Paul, Minn.

John Junior Wilson was a man of fine intellectual gifts, a stalwart character, and possessed of a pleasing personality. During the years when he was laid aside from his regular calling, he never lost his courage or cheerfulness, and he continued to pursue his studies and kept in touch with modern research. He was always greatly interested in the Society and always wore his badge of membership. Although he was a member of other organizations, the pin of the Society, at his request, was the only insignia that was buried with him. His calling away is a source of real sorrow to his many friends.

Mr. Wilson was elected an Associate Member of the American Society of Civil Engineers on February 4, 1913.

LEONARD JOHN BUTLER, Jun. Am. Soc. C. E.[1]

Died April 7, 1936

Leonard John Butler was born in Flagstaff, Ariz., on May 21, 1912, the son of the Rev. and Mrs. John Butler, who, for many years, have served as missionaries to the Navajo tribe of Indians of Northern Arizona. After completing his studies in the graded school on the Indian Reservation, he attended High School, at Tucson, Ariz. On the completion of his four years' High School Course, he entered the University of Arizona at Tucson, selecting the Civil Engineering Course, and was graduated in May, 1935, with the degree of Bachelor of Science in Civil Engineering.

Soon after his graduation, Mr. Butler entered the employ of the Government, as a Civil Engineer in the Soil Conservation Service of the United States Department of Agriculture. He continued in this work until his tragic death on April 7, 1936, at Santa Barbara, Calif., caused by an accidental 150-ft fall from the top of the large Gibraltar Dam, on the Santa Ynez River, to the rocks below. He had been assigned to that location temporarily to make silt tests.

During the brief period of his professional work, which began at Elephant Butte Dam, in New Mexico, Mr. Butler served on various similar projects, in soil and erosion work in that State as well as in Arizona and California. Although yet quite young (being only twenty-three at the time of his death), his potential possibilities in his particular profession seemed rapidly coming to the fore, assuring a possible brilliant future. His engineering associates who contacted him most in his professional work and social life paid lavish tribute to the high qualities of his character, his exceptional ability and keenness as a young engineer, and his conscientious disposition to do his work with an interest, skill, and thoroughness, which, in due time, would undoubtedly have commanded attention and given to him the advancement in his chosen profession which his fitness deserved.

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[1] Memoir prepared by the Rev. John Butler, Tuba City, Ariz.

Mr. Butler was of a strong and robust constitution, with a splendid physique and high ideals of life. Although generous in a quiet and unpretentious way, he was also prudent in all his affairs, as the adjustment of his business after his death so satisfactorily disclosed. His exceptionally fine spirit, his quiet, earnest, yet modest and unassuming ways, with his other fine qualities, quickly won for him the warm friendships which bore valid witness to his intrinsic worth. None but an All-Wise Providence can satisfactorily account for the sudden and tragic taking of this young and promising life, so early in the morning of what seemed a worthy career, for which he had been well trained, and which seemed fraught with such happy and splendid possibilities.

His mother, Mrs. Sue Woodward Butler, died when he was only six years of age. He is survived by his father, his sisters, Florence M. Butler, and Mrs. Grace W. Wells, the wife of the Rev. Lloyd D. Wells, of Yuma, Ariz., and by his older brother, Warren W. Butler.

Mr. Butler was elected a Junior of the American Society of Civil Engineers on January 13, 1936.

MARSHALL HUDSON REESE, Jun. Am. Soc. C. E.[1]

DIED SEPTEMBER 26, 1936

Marshall Hudson Reese, the son of Hilliard H. and Claudia (Coyle) Reese, was born at Arcadia, La., on October 5, 1904. He acquired his early education at the Grade and High Schools, at Baskin, La. In 1928, he received the degree of Bachelor of Science in Civil Engineering, at the College of Engineering, Louisiana State University, at Baton Rouge, La., entering the service of the United States Coast and Geodetic Survey on September 1, 1928. In 1935, he received the Professional Degree of Civil Engineer from Louisiana State University, for a graduate thesis entitled "The Development of Aerial Surveying in the U. S. Coast and Geodetic Survey."

Lieutenant Reese served as a junior officer on the U. S. Coast and Geodetic Survey Ships, *Lydonia* and *Natoma*, and, at the time of his death, was attached to the *Surveyor*. He was one of the first assigned to the duty of flight checking the sectional airway maps, and also spent time in the Washington Office of the U. S. Coast and Geodetic Survey in compiling aerial photographs.

Lieutenant Reese was lost on September 26, 1936, when a dory, in which he and Quartermaster McLees were leaving camp near Wislow Island, Aleutian Islands, Alaska, capsized in the outer line of breakers. A strong offshore gale carried the men and the overturned boat out into Bering Sea. All efforts to reach them by their comrades on shore were unsuccessful, and the men were last seen clinging to the overturned boat, drifting seaward in a strong offshore wind.

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[1] Memoir prepared by Paul Albert Smith, Assoc. M. Am. Soc. C. E.

The party had been encamped on an isolated section of the north coast of Unalaska, and news of the accident was carried to Dutch Harbor by two of his men, who arrived at the Coast Guard Station thirty hours after the accident. The Coast Guard Cutter, *Alert,* and the U. S. Coast and Geodetic Survey Ship, *Surveyor,* searched a wide area and the coasts in the vicinity of Wislow Island without finding any trace of the men or the boat.

Lieutenant Reese was efficient and unusually well liked, and his many friends and associates feel the loss of a real friend. He was outstanding in his profession, and, in his death, the U. S. Coast and Geodetic Survey has lost a most promising young officer.

He was a member of Delta Sigma Phi; Sigma Tau Sigma; Scabbard and Blade; and the Society of American Military Engineers.

On December 14, 1935, he was married to Doris Anderson, of Pensacola, Fla., who survives him.

Lieutenant Reese was elected a Junior of the American Society of Civil Engineers on January 14, 1929.

ALBERT FARWELL BEMIS, Affiliate, Am. Soc. C. E.[1]

DIED APRIL 11, 1936

Albert Farwell Bemis was born in Boston, Mass., on November 11, 1870, the son of Judson Moss and Alice (Cogswell) Bemis. He received his primary education at schools in Newton, Mass., until 1882 when he went to Colorado. He was a student at Cutler Academy, Colorado Springs, and at Colorado College, until 1889 when he returned to Boston. He then entered for the course in Civil Engineering at the Massachusetts Institute of Technology from which he was graduated in 1893 with the degree of Bachelor of Science.

After his graduation from college, Mr. Bemis became associated with the Bemis Bro. Bag Company. This Company operates bag factories, cotton mills, a bleachery, paper mill, machine shop, and ink factory, and Mr. Bemis had supervision of the design and plans of all new construction for the Company from 1894 to 1922. In 1897 he became Secretary, and in 1909 President, of the Company, continuing in the latter position until 1925 when he was made Chairman of the Board of Directors. He held the latter office until his retirement in 1934.

Mr. Bemis was also a Director of the Boott Mills, of Lowell, Mass., from 1905 to 1934, and in 1912 he organized the Angus Jute Company, Limited, of Calcutta, India, which operates a jute mill and jute machinery works in India. Mr. Bemis also served as a Director of this Company until 1934. Included in his business interests were his Directorships of the Second National Bank of Boston from 1916 to 1926 and of the Federal Reserve Bank of Boston from 1928 to 1932.

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[1] Memoir prepared from information supplied by Milton Brooks, Esq., of Boston, Mass., and on file at the Headquarters of the Society.

He was deeply interested in research and economics. This interest led him to become associated with the work of the National Industrial Conference Board of which he was a member from its organization in 1916. In 1928 he was appointed a member of its Executive Committee and served as such until 1932. In 1934 he accepted a re-appointment to the Executive Committee and at the time of his death he was, in addition, Chairman of the Advisory Council on Research. He had been for many years a member of the International Chamber of Commerce and of the Royal Economic Society of London, England.

As an active and constant supporter of housing research, Mr. Bemis became a nationally known authority on the theory of building and the improvement of construction methods. He was largely responsible for the newer dormitory construction at the Massachusetts Institute of Technology to which he was a large contributor and he assisted in making possible the restoration of Lincoln Cathedral in England, both by large financial contributions and also co-operation with the authorities in charge of the restoration. In 1918 he organized the Housing Company in Boston, which Company was responsible for many group housing developments in New England and elsewhere. He also founded the course in building at Hampton Institute in Virginia. He was an active supporter of the production of better housing for the lower service groups and in connection with this interest he published a three-volume work under the title of "The Evolving House." The first two volumes of this work were "A History of the Home", and "The Economics of Shelter" and the third volume, published just prior to his death, was entitled "Rational Design."

Mr. Bemis was actively interested in his Alma Mater, the Massachusetts Institute of Technology. He served as President of its Alumni Association in 1910, and had been a Life Member of the Corporation since 1914. He also served at various times as a member or Chairman of Visiting Committees of the Departments of Civil Engineering, Architecture, and Naval Architecture, as well as the Division of Industrial Co-Operation and Research. The younger graduates of the Institute were always of great interest to Mr. Bemis and many of them are indebted to him for placing them in positions from which they rose to prominence.

He was a frequent traveler and had visited many countries of the world. The promotion of Anglo-American relations was one of his major interests. He held a Life Governorship in the English Speaking Union, of London, and was also a Fellow of the Royal Society of Arts. He was a member of the Club International, of Geneva, Switzerland.

As a member of the Commission on Foreign Inquiry of the National Civic Federation, Mr. Bemis contributed the sections of its report published in 1919 on "Social and Industrial Relations" and "Housing and Agricultural Reconstruction" in Great Britain and France. In 1927 he visited Russia and afterward prepared a paper on his observations which was published[2] by the American Academy of Political and Social Science. He also visited the Orient and in connection with the operations of his Company, the Angus

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[2] *Annals*, Am. Academy of Political and Social Science, July, 1928.

Jute Company, Limited, at Calcutta, India, he framed policies which did much to improve living conditions for its employees and for the public residing in its vicinity, from the point of view of health and sanitation. The medical work conducted by the Company was highly commended by the Royal Commission on Labor in India in a report to the British Parliament.

Mr. Bemis was a member and Past-President of the National Association of Cotton Manufacturers and a member of the American Cotton Manufacturers Association and, in 1911 prepared a report on "Co-Operation Between Departments of the Federal Government and Textile Manufacturers" for the former as well as a report, in 1910, on "Cotton Goods Sale Note."

He served as Alderman of the City of Newton, Mass., from 1911 to 1914; was a Director of the Boston Chamber of Commerce from 1914 to 1916; a Trustee of the Newton Young Men's Christian Association from 1923 to 1926; a Trustee of the Cambridge School of Architecture from 1927 to his death; and Chairman of the Trustees for Real Estate of the Boston Young Men's Christian Association in 1935.

Mr. Bemis was also a member of the Bostonian Society, the Ipswich Historical Society the New England Historic Genealogical Society, and the Boston Athenæum. He had been a member of the Old South Church, in Boston, since 1908.

His social clubs included the City, Union, Commercial, Technology, Exchange, University, and Engineers Clubs, of Boston; the Longwood Cricket Club, the Brookline Country Club, Cohasset Golf Club, and the Manchester Yacht Club. He was also a member of the Engineers and Technology Clubs, of New York, N. Y.

As stated by a friend of long standing the attributes for which Mr. Bemis will be remembered were "his kindliness and humor, his humanity and sympathy, and his self-sacrificing effort to render the greatest possible service to his fellow man."

On December 30, 1899, he was married to Faith Gregg, of Colorado Springs, Colo., and is survived by his widow, four sons, and three daughters.

Mr. Bemis was elected an Affiliate of the American Society of Civil Engineers on September 12, 1916.

THANE ROSS BROWN, Affiliate, Am. Soc. C. E.[1]

DIED JUNE 30, 1934

Thane Ross Brown (or "T.R.", as he was called by those who were intimate with him) was born at Oxford, Ohio, on June 21, 1870, the son of Edwin W. and Harriet (Ross) Brown. Soon after his birth the family moved to Kansas and, after a short stay, returned to Oxford and then moved to Indianapolis, Ind., where Mrs. Brown died in 1878. This broke up the family, and an aunt at Oxford assumed the care of the children—three boys, Thane, his twin brother, and a younger brother.

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[1] Memoir prepared by E. D. Coddington, Esq., New York, N. Y.

In 1882, the father succeeded in getting his boys together again on a farm near Topeka, Kans., and here Thane Ross Brown grew from boyhood to manhood, getting his first college work at Washburn College, at Topeka. From Washburn, he went to the University of Wisconsin, at Madison, Wis., from which he was graduated, from the College of Engineering, in 1895.

Mr. Brown immediately began work relating to the profession for which he had been training and, from June to August, 1895, he was engaged in surveying in University Heights, Madison, and from August 1 to November 25 of the same year, he held the position of Levelman with the Field Engineering Force supervising the construction and erection of a bridge for the Pacific Short Line Railroad Company, at Sioux City, Iowa, Lee Treadwell, M. Am. Soc. C. E., being the Engineer in Charge.

Mr. Brown's experience during the construction of this bridge undoubtedly had a great influence in leading him to decide on specializing in bridge construction, and after completing his work at Sioux City in the fall of 1895, he entered, in November, the service of the Wisconsin Bridge and Iron Company, at Milwaukee, Wis.

In this Company, which Mr. Brown served so faithfully for many years, when designs are made, contracts taken, and working drawings begun, it is upon the engineers and draftsmen of the Shop Drafting Room that the task of putting the designer's ideas into practical shape for actual construction falls, and, in this work, Mr. Brown was a main stand-by for many years.

Few engineers have handled a greater variety of work than did Mr. Brown during his connection with the Wisconsin Bridge and Iron Company. From small highway spans to heavy cantilever bridges across such rivers as the Ohio and the Mississippi, from the simplest types of lift and draw-bridges to heavy double-track railroad lift spans with their machinery; from small factory buildings to the largest types of industrial plants; from small storage bins to immense ore docks—Mr. Brown was engaged in all this work doing as much of it as it was possible for one engineer. If difficult counterweight calculations or deflections diagrams for long spans were required, where willingness to work, and where much patience and mathematical ability were necessary, Mr. Brown was the man. He was the very personification of steadiness and reliability and was held in highest respect by all who worked with him.

Mr. Brown's father served his country during the Civil War and showed his patriotism as a soldier in doing his part in some of the most difficult campaigns and battles of that struggle. His patriotism and loyalty to duty found expression again in his son, Thane, who identified himself in the community in which he lived by always answering unselfishly any call from either Church or public, and whatever he undertook he could be depended upon to finish.

Mr. Brown's services with the Wisconsin Bridge and Iron Company covered a period of more than thirty-seven years and when failing physical powers, finally ending in death on June 30, 1934, compelled him to lay down his tools, his loss was felt by all who knew him and most keenly by those in closest touch with him.

On July 6, 1904, he was married to Cora Treadwell, of Kilbourn, Wis., who died in January, 1914. Their union was blessed by two children, Helen and Thane B. Brown, who survive them.

Although he had very definite beliefs and set for himself the strictest rules of conduct toward his fellow men, he was always very tolerant toward the beliefs and actions of others. Although his life was not of the spectacular kind, his was one of the finest examples of American manhood, and truly it can be said of him that he served his day and generation faithfully and well.

Mr. Brown was elected a Junior of the American Society of Civil Engineers on March 31, 1896, and an Affiliate, on October 2, 1900.

ROBERT WHITMAN LESLEY, Affiliate, Am. Soc. C. E.[1]

Died November 10, 1935

Robert Whitman Lesley was born in Philadelphia, Pa., on July 3, 1853, the son of James Lesley, Jr., and Elizabeth (Thomson) Lesley. His ancestors on both sides were Scotch, his grandfather, Judge Thomson, of Chambersburg, Pa., who had served as a member of Congress, having married a Graham, or in the old spelling, Graeme. His father was a literary man, a student of poetry, and a linguist. His mother was a sister of the eminent oculist, the late Dr. William Thomson, and of the late Frank Thomson, President of the Pennsylvania Railroad Company. A few years after his birth, his father was appointed American Consul to Lyons, and, later, to Nice, France, where he died in 1865.

Robert Whitman Lesley received his preparatory education in France and at the Langdon School, in Philadelphia. In 1867, he entered the University of Pennsylvania, but left college before he was graduated to enter the office of the Philadelphia *Public Ledger*, then owned by George W. Childs and Anthony J. Drexel. Mr. Lesley served in the Business Department of the newspaper and, later, became Assistant Editor. He afterward studied law in the office of Benjamin Harris Brewster, United States Attorney General, and was admitted to the Philadelphia Bar in 1879. In 1908 he was granted the degree of Master of Arts by the University of Pennsylvania as of the Class of 1871.

In 1874, Mr. Lesley organized the firm of Lesley and Trinkle in connection with the importation and distribution of cement. In 1876, he became associated with the late David O. Saylor, of Allentown, Pa., in the introduction of Saylor's Portland cement. From 1880 to 1883, Mr. Lesley was interested in the manufacture of natural cement in Maryland, and with Professor E. J. De Smedt, United States Chemist, of Washington, D. C., he carried on a series of experiments and tests looking to the economical and commercial manufacture and distribution of Portland cement. This work resulted in a series of patents granted to them for improvement in the art, especially in the preparation of the "slurry" for the kilns by the use of gas tar pitch at that time a waste product.

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[1] Prepared from information on file at the Headquarters of the Society.

Under these inventions Mr. Lesley and others, in 1883–84, organized the American Improvement Cement Company, of which he served as Treasurer and a Director. The Company had three plants at Egypt, Pa., and one at Jordan, N. Y., and was at that time the largest manufacturer of Portland cement in the United States. Subsequently, the Company was re-organized as the American Cement Company, and Mr. Lesley was its first President, also serving as Treasurer and Director. It is now operated as the Giant Portland Cement Company, of which Mr. Lesley was a Director at the time of his death. In 1924, jointly with Mr. George S. Bartlett, he wrote and published a history of the Portland Cement Industry, and, in 1925, he laid the cornerstone of the Portland Cement Association Building, in Chicago, Ill.

In addition to his cement interests, Mr. Lesley was also a Director of the Surface Combustion Corporation, The Philadelphia and Camden Ferry Company, the Bearings Company of America, Bonzano Rail Joint Company, Jackson Mills Emery Company, Incorporated, The Tidewater Land Company and the Concrete Cement Age Publishing Company. He also served as Consulting Editor of *Concrete*, published in Chicago.

Mr. Lesley was always interested in science and was connected with the Clarke Thomson Research Fund. The Company was organized just before the World War to develop new inventions in the field of aviation, and was associated with the National Advisory Committee for Aeronautics. During Mr. Thomson's absence in the Army, Mr. Lesley took over the management of the Company. He also presented a Cement Laboratory to the University of Pennsylvania.

During the World War, Mr Lesley assisted in the organization of the Philadelphia War Chest and served as Captain of teams appointed to secure money for the various Liberty Loans, War Savings Stamps, and the Red Cross.

Mr. Lesley was an Honorary Member and Past Vice-President of the American Society for Testing Materials; Honorary Member and Past-President of the Portland Cement Association; a member of the American Concrete Institute; The American Chemical Society; British Chemical Society; the Franklin Institute; and the Philadelphia Chamber of Commerce. He had also served as a member of the Special Committee of the American Society of Civil Engineers on Concrete and Reinforced Concrete; and as Chairman of the Sub-Committee on Ways and Means of the Joint Committee on Concrete and Reinforced Concrete; on Committee C–1 of the Committee of the American Society for Testing Materials on Cement Specifications; Presiding Officer of the Cement Section of the International Congress of the Association for Testing Materials, in New York, N. Y., in 1924; and as a member of the Government Board for Testing Fuels and Materials.

In addition to his commercial and financial interests, Mr. Lesley found time to devote to other interests, of which golf was one of his particular hobbies. He served as a Director of the Philadelphia Golf Association, and as President of the United States Senior Golf Association, in 1924. From 1923 until his death, he also served as President of the Merion Cricket Cub. He was the donor of the Lesley Cup, for competition between golf teams from New York State, Pennsylvania, Massachusetts, and Eastern Canada. It was his endeavor always to establish all forms of sport on the highest plane.

His social clubs included the Rittenhouse, Midday, Pine Valley, Radnor Hunt, Bryn Mawr Polo, and the Engineers Clubs of New York and Philadelphia, as well as the Railroad Club of New York.

As a resident on the "Main Line", Mr. Lesley was interested in civic and charitable matters. He was also a Director of the Main Line Citizens Association for many years, and it was during his administration that it was re-organized as the Community Health and Civic Association with its various fields of nursing, zoning, roads, and other local matters. In addition, he was in charge of the Welfare Federation Drive for the Main Line territory, and was interested as a Director of Regional Planning in beautifying the Creek Valleys on the Main Line.

Although brought up a Unitarian in religion, he joined the Roman Catholic Church some years before his death, and found great happiness in the practice of that faith.

Mr. Lesley was a man of wide experience and his charming personality, sympathy, generosity, sense of humor, and joy of life endeared him to his many friends. He was interested in and acquainted with all current events and was well versed in history, art, and literature, and read and spoke both French and German. Although he was eighty-three years of age when he died, Mr. Lesley was active both mentally and physically until shortly before the end. He had established a health program many years before to which he adhered rigidly and to which he attributed his splendid physical condition.

He was married on October 23, 1879, to Eulalia Willcox, of Glen Mills, Pa., and Philadelphia, whose ancestors at the old Ivy Mills, and, later, at the Glen Mills, made the Colonial and Continental Currency paper; they also made the paper for the currency used in the North during the Civil War. Mr. Lesley is survived by his widow and by an only daughter, Mrs. Richard Berridge, of London, England, and Screebe Lodge, County Galway, Ireland. He is also survived by several grand-children and great-grandchildren.

Possessed of a buoyant personality, a sparkling humor, a keen sense of responsibility, and a priceless reservoir of enthusiasm and good cheer, Robert Whitman Lesley will be missed as a man, a loyal friend, and a co-worker in the Cement Industry. He will also be remembered widely, for his presence made the world a somewhat better place in which to live.

Mr. Lesley was elected an Affiliate of the American Society of Civil Engineers on January 31, 1893.

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